CENGAGE

Cengage EXAM CRACK Series®

PHYSICS for JEE MAIN

3e

Detailed Theory with 5000+ Topicwise Questions

Theory-based Solved Illustrations

Concept Application Exercises

Solved Examples of Higher Difficulty Level

Chapterwise JEE Main 2002-18 Questions in the Book

FREE TESTS
Available on Android and Windows

Cengage App available on

See inside cover for details

Free Booklet containing Chapterwise Solved Questions of JEE Main 2019 (All Sets – January and April 2019) with this book

B. M. Sharma

Physics

for

JEE Main

Third Edition

B. M. Sharma

CENGAGE

Australia • Brazil • India • Mexico • Singapore • United Kingdom . United States

CENGAGE

Physics
for JEEMain. ***3e***

B. **M. Sharma**

ISBN-13: 978-93-5350-132 7
ISBN-10:93-5350-132-6

Cengage Learning India Pvt. Ltd.
T-303,3rd Floor
DAV Complex, Plot 3
Mayur Vihar Phase 1
Delhi 110091

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Australia, Brazil, India, Mexico, Singapore, United Kingdom, and United States. Locate your local office at:
www.cengage.com/global

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

For product information, visit www.cengage.co.in

Preface

Over the years, students appearing for the Joint Entrance Examination (JEE) have banked on conventional methods and shortcuts provided by books to crack the examination. However, the paper-setting patterns during the recent years have made it clear that only conventional techniques arc not enough as most of the questions are based on concepts rather than on just formulae.

It is necessary for students appearing for JEE to gain a thorough knowledge and understanding of the concepts, and this book helps students in acquiring the same. This book uses a right mix of fundamental principles and concepts, illustrations which highlight the application of these concepts, and exercises for practice. It is designed to help students to get an all-round grasp of the subject so as to be able to make its useful application in all its contexts.

The objective of the book is to help students develop their problem-solving skills/accuracy, the ability to reach the crux of the matter, and the speed to get answers in limited time.

It is imperative to note that this book would be as profound and useful as you want it to be. Therefore, in order to get maximum benefit from this book, we recommend the following study plan for each chapter:

Step 1: Go through the entire opening discussion about the fundamentals and concepts.

Step 2: After learning the theory/concept, follow the illustrative examples to get an understanding of the theory/concept.

Step 3: Attempt in-text concept application exercises. This will ensure your understanding and implementation of concepts and formulae.

Step 4: Move to the section that lists solved problems. This will help better your understanding of the chapter theory.

Step 5: Finally, there are many exercises/problems to try your hands on. These problems warrant the recall of all the formulae and concepts involved. Archives is a collection of solved problems from previous years' examinations, which will help students understand the trend(s) behind JEE.

The purpose of this book is to help you actually understand what you are doing, rather than blindly applying memorized techniques. It is recommended that you solve the problems given in the book by yourself, without the help of the solution at the end of the section. The idea is that if you discover something on your own, you will remember it better than if you had simply seen the a nswer. So, refer to the solution only if you fail to solve a problem after several attempts or when you have already found a solution a nd want to check your approach.

Although this book h as been written on the basis of the syllabus prescribed for JEE, it will prove useful also to the student s preparing for other entrance examinations since the content of *physics* remains almost the same for all examinations.

We hope this book helps you immensely in acquiring the kind of acumen that is vital for students appearing for JEE. Since we believe that there is always scope for improvement, any suggestions are welcome.

You have taken the first step. A bright future beckons you. We wish you success in this endeavour and in life.

B.M. Shanna

Contents

Chapter 1

Units, Dimensions and Measurement

PHYSICAL QUANTITY

A physical quantity is represented completely by its magnitude and unit. For example, 10 m means a length which is 10 times the unit of length. Here 10 represents the numerical value of the given quantity and meter represents the unit of quantity under consideration. Thus, in expressing a physical quantity, we choose a unit and then find that how many times that unit is contained in the given physical quantity, i.e.,

Physical quantity (Q) = Magnitude × Unit = $n \times u$

where n represents the numerical value and u represents the unit. Thus, while expressing a definite amount of physical quantity, it is clear that as the unit u changes, the magnitude (n) will also change but product nu will remain same.

i.e. $\quad n\,u = \text{constant}$

or $\quad n_1u_1 = n_2u_2 = \text{constant} \Rightarrow n \propto \dfrac{1}{u}$

i.e., the magnitude of a physical quantity and units are inversely proportional to each other. Larger the unit, smaller will be the magnitude.

FUNDAMENTAL AND DERIVED QUANTITIES

Fundamental quantities Out of a large number of physical quantities which exist in nature, there are only few quantities which are independent of all other quantities and do not require the help of any other physical quantity for their definition. Therefore, these are called absolute quantities. These quantities are also called fundamental or base quantities, as all other quantities are based upon and can be expressed in terms of these quantities.

Derived quantities All other physical quantities can be derived by suitable multiplication or division of different powers of fundamental quantities. These are, therefore, called derived quantities.

For example, if length is defined as a fundamental quantity, then area and volume are derived from length and are expressed in term of length with power 2 and 3 over the term of length.

Table 1.1: Fundamental quantities in SI system and their units

S. No.	Physical quantity	Name of unit	Symbol of unit
1.	Mass	Kilogram	kg
2.	Length	Meter	m
3.	Time	Second	s
4.	Temperature	Kelvin	K
5.	Luminous Intensity	Candela	Cd
6.	Electric Current	Ampere	A
7.	Amount of Substance	Mole	mol

Table 1.2: Supplementary quantities in SI system and their units

S. No.	Physical quantity	Name of unit	Symbol of unit
1.	Plane angle	Radian	rad
2.	Solid angle	Steradian	sr

FUNDAMENTAL AND DERIVED UNITS

Normally each physical quantity requires a unit or standard for its specification so it appears that there must be as many units as there are physical quantities. However, it is not so. It has been found that if in mechanics we arbitrarily choose units of any three physical quantities we can express the units of all other physical quantities in mechanics in terms of these. Arbitrarily, the physical quantities *mass, length,* and *time* are choosen for this purpose. So any unit of mass, length, and time in mechanics is called a *fundamental, absolute, or base unit*. Other units which can be expressed in terms of fundamental units are called derived units. For example, light year or km is a fundamental units as it is a unit of length while s^{-1}, m^2, or $kg\,m^{-1}$ are derived units as these are derived from units of time, mass, and length, respectively.

System of units A complete set of units, both fundamental and derived, for all kinds of physical quantities is called system of units. The common systems are given in Table 1.3.

Table 1.3: Fundamental quantities in SI system and their units

	MKS System	CGS System	FPS System	SI units
Length	m (meter)	cm (centimeter)	ft (foot)	It is an extended form of MKS system. It includes four more fundamental units (in addition to three basic units) which represent fundamental quantities in electricity, magnetism, heat, and light.
Mass	kg (kilogram)	g (gram)	lb (pound)	
Time	s (second)	s (second)	s (second)	

DIMENSIONS OF A PHYSICAL QUANTITY

When a derived quantity is expressed in terms of fundamental quantities, it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

To make it more clear, consider the physical quantity force:

Force = Mass × Acceleration

$$= \frac{\text{Mass} \times \text{Velocity}}{\text{Time}} = \frac{\text{Mass} \times \text{Length/Time}}{\text{Time}}$$

$$= \text{Mass} \times \text{Length} \times (\text{Time})^{-2} \quad \text{(i)}$$

Thus, the dimensions of force are 1 in mass, 1 in length and –2 in time. Here the physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to indicate that the equation is among dimensions and not among magnitudes.Thus, (i) can be written as [force] = $[MLT^{-2}]$. Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional equation. If we consider only the RHS of the equation, the expression is termed as dimensional formula. Thus, the dimensional formula for force is $[MLT^{-2}]$.

APPLICATIONS OF DIMENSIONAL ANALYSIS

To find the unit of a physical quantity in a given system of units:

By writing the definition or formula for the physical quantity, we find its dimensions. Now in the dimensional formula replacing M, L, and T by the fundamental units of the required system, we get the unit of physical quantity. However, sometimes to this unit, we further assign a specific name, e.g.,

Work = Force × Displacement

So $[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

So, its units in CGS system will be g cm^2 s^{-2} which is called erg while in MKS system will be kg m^2 s^{-2} which is called joule.

To convert a physical quantity from one system to the other:

The measure of a physical quantity is nu = constant.

If a physical quantity X has dimensional formula $[M^aL^bT^c]$ and if the derived units of that physical quantity in two systems are $[M_1^aL_1^bT_1^c]$ and $[M_2^aL_2^bT_2^c]$ and n_1 and n_2 be the numerical values in the two systems, respectively, then

$$n_1[u_1] = n_2[u_2]$$

$$\Rightarrow \quad n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$$

$$\Rightarrow \quad n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

where M_1, L_1, and T_1 are the fundamental units of mass, length, and time in the first (known) system and M_2, L_2, and T_2 are the fundamental units of mass, length, and time in the second (unknown) system, respectively. Thus, knowing the values of fundamental units in two systems and the numerical value in one system, the numerical value in the other system may be evaluated.

ILLUSTRATION 1.1 Convert 1 joule to ergs.

Solution. Joule: SI system, erg: CGS system

Work = Force × Distance = Mass × Acceleration × Length

$$= \text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Length}$$

Dimensions of work = $[W] = [M^1L^2T^{-2}]$

$\therefore \quad a = 1, b = 2, c = -2.$

Now

SI system	CGS system
$M_1 = 1$ kg	$M_2 = 1$ g
$L_1 = 1$ m	$L_2 = 1$ cm
$T_1 = 1$ s	$T_2 = 1$ s

Here $N_1 = 1, N_2 = ?$

$$\therefore \quad \text{Using } N_2 = N_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 1\left[\frac{1\text{kg}}{1\text{ g}}\right]^1\left[\frac{1\text{ m}}{1\text{ cm}}\right]^2\left[\frac{1\text{ s}}{1\text{ s}}\right]^{-2}$$

$$= 1\left[\frac{1000\text{ g}}{1\text{ g}}\right]^1\left[\frac{100\text{ cm}}{1\text{cm}}\right]^2 = 10^7$$

$\therefore \quad N_2 = 10^7$

So $\quad 1\text{ J} = 10^7$ erg

ILLUSTRATION 1.2 In CGS system, the magnitude of the force is 100 dynes. In another system where the fundamental physical quantities are kilogram, meter, and minute, find the magnitude of the force.

Solution. We have $n_1 = 100$, $M_1 = 1$ g, $L_1 = 1$ cm, $T_1 = 1$ s and $M_2 = 1$ kg, $L_2 = 1$ m, $T_2 = 1$ min, the dimensional formula of forces is $[M^1L^1T^{-2}]$, where $a = 1, b = 1, c = -2$.

By substituting these values in the following conversion formula, we have

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$= 100\left[\frac{1\text{ g}}{1\text{ kg}}\right]^1\left[\frac{1\text{ cm}}{1\text{ m}}\right]^1\left[\frac{1\text{ s}}{1\text{ min}}\right]^{-2}$$

$$= 100\left[\frac{1\text{ g}}{10^3\text{g}}\right]^1\left[\frac{1\text{ cm}}{10^2\text{cm}}\right]^1\left[\frac{1\text{ s}}{60\text{ s}}\right]^{-2} = 3.6$$

To check the dimensional correctness of a given physical relation: This is based on the *principle of homogeneity*. According to this principle, the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$,

then according to the principle of homogeneity, we have

$$[X] = [A] = [(BC)^2] = [\sqrt{DEF}].$$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

ILLUSTRATION 1.3 Find out the unit and dimensions of the constants a and b in the van der Waals equation $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P is pressure, V is volume, R is gas constant, and T is temperature.

Solution. We can add and subtract only like quantities.

$\Rightarrow$ Dimensions of P = Dimensions of $\frac{a}{V^2}$ (i)

and Dimensions of V = Dimensions of b (ii)

From (i),

Dimensions of a = Dimensions of $P \times$ Dimensions of V^2

$$[a] = [M^1L^{-1}T^{-2}] \times [L^3]^2 = [M^1L^5T^{-2}]$$

Unit of a = Unit of $P \times$ Unit of $V^2 = \frac{\text{N}}{\text{m}^2} \times \text{m}^6 = \text{N m}^4$

From (ii), $[b] = [V] = [M^0L^3T^0]$

So unit of b = Unit of $V = \text{m}^3$

ILLUSTRATION 1.4 A famous relation in physics relates the moving mass m to the rest mass m_0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of the special theory of relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes $m = \frac{m_0}{(1 - v^2)^{1/2}}$. Guess where to put the missing c.

Solution. According to the principle of homogeneity of dimensions, powers of M, L, T on either side of the formula must be equal. For this, on RHS, the denominator $(1 - v^2)^{1/2}$ should be dimensionless. Therefore, instead of $(1 - v^2)^{1/2}$, we should write $(1 - v^2/c^2)^{1/2}$. Hence, the correct formula would be

$$m = \frac{m_0}{(1 - v^2/c^2)^{1/2}}$$

As a research tool to derive new relations: If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, a relation between the quantities can be derived. Let us understand this point through following examples:

Time period of a simple pendulum: Let the time period of a simple pendulum be a function of the mass of the bob (m), effective length (l), and acceleration due to gravity (g), then assuming the function to be the product of power function of m, l, and g, i.e., $T = Km^x l^y g^z$, where K = dimensionless constant.

If the above relation is dimensionally correct, then by substituting the dimensions of quantities,

$$[T] = [M]^x [L]^y [LT^{-2}]^z$$

or $$[M^0L^0T^1] = [M^xL^{y+z}T^{-2z}]$$

Equating the exponents of similar quantities, we get $x = 0$, $y = 1/2$, and $z = -1/2$.

So, the required physical relation becomes $T = K\sqrt{\frac{l}{g}}$.

The value of dimensionless constant is found (2π) through experiments, so $T = 2\pi\sqrt{\frac{l}{g}}$.

Stoke's law: When a small sphere moves at low speed through a fluid, the viscous force F, opposing the motion, is found experimentally to depend on the radius r, the velocity of the sphere v, and the viscosity η of the fluid.

So $F = f(\eta, r, v)$

If the function is the product of power functions of η, r, and v, $F = K\eta^x r^y v^z$, where K is the dimensionless constant.

If the above relation is dimensionally correct,

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^x [L]^y [LT^{-1}]^z$$

or $$[MLT^{-2}] = [M^xL^{-x+y+z}T^{-x-z}]$$

Equating the exponents of similar quantities $x = 1$; $-x + y + z = 1$; and $-x - z = -2$.

Solving these for x, y, and z, we get $x = y = z = 1$.

So it becomes $F = K\eta rv$

On experimental grounds, $K = 6\pi$; so $F = 6\pi\eta rv$

This is the famous Stoke's law.

ILLUSTRATION 1.5 If velocity (V), force (F), and time (T) are chosen as fundamental quantities, express (a) mass and (b) energy in terms of V, F, and T.

Solution. Let M = (Some number)$(V)^a(F)^b(T)^c$

Equating dimensions of both the sides, we get

$$M^1L^0T^0 = (1)\,[L^1T^{-1}]^a\,[M^1L^1T^{-2}]^b\,[T^1]^c$$

$$= M^b L^{a+b} T^{-a-2b+c}$$

Get $a = -1, b = 1, c = 1$.

M = (Some number) $(V^{-1} F^1 T^1)$ or $[M] = [V^{-1} F^1 T^1]$

Similarly, we can also express energy in terms of V, F, and T.

Let $[E]$ = [Some number] $[V]^a [F]^b [T]^c$

$\Rightarrow$ $[ML^2T^{-2}] = [M^0L^0T^0] [LT^{-1}]^a [MLT^{-2}]^b [T]^c$

$\Rightarrow$ $[M^1L^2T^{-2}] = [M^b L^{a+b+c} T^{-a-2b+c}]$

$\Rightarrow$ $1 = b; 2 = a + b + c; -2 = -a - 2b + c$

Get $a = 1; b = 1; c = 1$.

$\Rightarrow$ E = (Some number) $V^1F^1T^1$ or $[E] = [V^1][F^1][T^1]$.

LIMITATIONS OF DIMENSIONAL ANALYSIS

Although dimensional analysis is very useful, it cannot lead us too far due to the following reasons:

1. If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example, if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$, it may be work or energy or torque.
2. Numerical constant having no dimensions $[K]$ such as (1/2), 1, 2π, etc., cannot be deduced by the methods of dimensions.
3. The method of dimensions cannot be used to derive relations other than the product of power functions. For example, $s = ut + (1/2)\,at^2$ or $y = a \sin \omega t$ cannot be derived by using this theory (try if you can). However, the dimensional correctness of these can be checked.
4. The method of dimensions cannot be applied to derive formula if in mechanics a physical quantity depends on more than three physical quantities as then there will be less number (= 3) of equations than the unknowns (>3). However, still, we can check the correctness of the given equation dimensionally. For example, $T = 2\pi\sqrt{I/mgl}$ cannot be derived by theory of dimensions but its dimensional correctness can be checked.
5. Even if a physical quantity depends on three physical quantities, out of which two have same dimensions, the formula cannot be derived by the theory of dimensions, e.g., formula for the frequency of a tuning fork $f = (d/L^2)\,v$ cannot be derived by the theory of dimensions but can be checked.

CONCEPT APPLICATION EXERCISE 1.1

1. The number of particles is given by $n = -D\dfrac{n_2 - n_1}{x_2 - x_1}$ crossing a unit area perpendicular to X-axis in unit time, where n_1 and n_2 are the number of particles per unit volume for the value of x meant to x_2 and x_1. Find the dimensions of D called diffusion constant.
2. The potential energy of a particle varies with distance x from a fixed origin as $U = \dfrac{A\sqrt{x}}{x^2 + B}$, where A and B are dimensional constants, then find the dimensional formula for AB.
3. Convert 1 MW power on a new system having basic units of mass, length, and time as 10 kg, 1 dm, and 1 min, respectively.
4. If the present units of length, time, and mass (m, s, kg) are changed to 100 m, 100 s, and 1/10 kg, then how will the new unit of force change?
5. Suppose we employ a system in which the unit of mass equals 100 kg, the unit of length equals 1 km and the unit of time 100 s and call the unit of energy *eluoj* (*joule* written in reverse order), then what is the relation between *eluoj* and *joule*?
6. If velocity (V), force (F), and energy (E) are taken as fundamental units, then find the dimensional formula for mass.

SIGNIFICANT FIGURES

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity:

All non-zero digits are significant.

Example: 42.3 has three significant figures.
243.4 has four significant figures.
24.123 has five significant figures.

A zero becomes a significant figure if it appears between two non-zero digits.

Example: 5.03 has three significant figures.
5.604 has four significant figures.
4.004 has four significant figures.

Leading zeros or the zeros placed to the left of the number are never significant.

Example: 0.543 has three significant figures.
0.045 has two significant figures.
0.006 has one significant figure.

Trailing zeros or the zeros placed to the right of the number are significant.

Example: 4.330 has four significant figures.
433.00 has five significant figures.
343.000 has six significant figures.

In exponential notation, the numerical portion gives the number of significant figures.

Example: 1.32×10^{-2} has three significant figures.
1.32×10^4 has three significant figures.

Rounding Off

While rounding off measurements, we use the following rules by convention:

If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example: $x = 7.82$ is rounded off to 7.8, again $x = 3.94$ is rounded off to 3.9.

If the digit to be dropped is more than 5, then the preceding digit is raised by 1.

Example: $x = 6.87$ is rounded off to 6.9, again $x = 12.78$ is rounded off to 12.8.

If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by 1.

Example: $x = 16.351$ is rounded off to 16.4, again $x = 6.758$ is rounded off to 6.8.

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is left unchanged, if it is even.

Example: $x = 3.250$ becomes 3.2 on rounding off, again $x = 12.650$ becomes 12.6 on rounding off.

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by 1, if it is odd.

Example: $x = 3.750$ is rounded off to 3.8, again $x = 16.150$ is rounded off to 16.2.

Significant Figures in Calculation

In most of the experiments, the observations of various measurements are to be combined mathematically, i.e., added, subtracted, multiplied, or divided as to achieve the final result. Since all the observations in measurements do not have the same precision, it is natural that the final result cannot be more precise than the least precise measurement. The following two rules should be followed to obtain the proper number of significant figures in any calculation.

1. The result of an addition or subtraction in the number having different precisions should be rounded off the same number of decimal places as are present in the number having the least number of decimal places. The rule is illustrated by the following examples:

(a) 33.3 ← (has only one decimal place)
3.11
+ 0.313
36.723 ← (answer should be rounded off one decimal place)

Answer = 36.7

(b) 3.1421
0.241
+ 0.09 ← (has 2 decimal places)
3.4731 ← (answer should be rounded off 2 decimal places)

Answer = 3.47

(c) 62.831 ← (has 3 decimal places)
−24.5492
38.2818 ← (answer should be rounded off 3 decimal places)

Answer = 38.282

2. The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation. The rule is illustrated by the following examples:

(a) 142.06
× 0.23 ← (two significant figures)
32.6738 ← (answer should have two significant figures)

Answer = 33

(b) 51.028
× 1.31 ← (three significant figures)
66.84668

Answer = 66.8

(c) $\frac{0.90}{4.26} = 0.2112676$

Answer = 0.21

CONCEPT APPLICATION EXERCISE 1.2

1. The length, breadth, and thickness of a block are measured as 125.5 cm, 5.0 cm, and 0.32 cm, respectively. Which one of the measurement is most accurate?
2. The length of a rectangular sheet is 1.5 cm and the breadth is 1.203 cm. Find the area of the face of a rectangular sheet to the correct number of significant figures
3. Each side of a cube is measured to be 5.402 cm. Find the total surface area and the volume of the cube in appropriate significant figures.
4. Taking into account the significant figures, what is the value of 9.99 m + 0.0099 m?
5. Find the value of the multiplication 3.124×4.576 correct to three significant figures.
6. If the value of resistance is 10.845 Ω and the value of current is 3.23 A, the potential difference is 35.02935 V. Find its value in significant number.

ERRORS OF MEASUREMENT

The measuring process is essentially a process of comparison. Inspite of our best efforts, the measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

Absolute error Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times. Let the measured value be $a_1, a_2, a_3, \ldots, a_n$. The arithmetic mean of these values is $a_m = \frac{a_1 + a_2 + \ldots + a_n}{n}$.

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\cdots$$
$$\Delta a_n = a_m - a_n$$

The absolute errors may be positive in certain cases and negative in certain other cases.

Mean absolute error It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus,

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \cdots + |\Delta a_n|}{n}$$

Hence, the final result of measurement may be written as

$$a = a_m \pm \overline{\Delta a}$$

This implies that any measurement of the quantity is likely to lie between $(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$.

Relative error or fractional error The relative error or fractional error of measurement is defined as the ratio of mean absolute error to the mean value of the quantity measured. Thus, relative error or fractional error

$$= \frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{\overline{\Delta a}}{a_m}$$

Percentage error When the relative/fractional error is expressed in percentage, we call it percentage error.

Thus, percentage error $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

Propagation of Errors

Error in sum of the quantities Suppose $x = a + b$

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x, i.e., sum of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$.

Percentage error in the value of $x = \dfrac{(\Delta a + \Delta b)}{a + b} \times 100\%$

Error in difference of the quantities Suppose $x = a - b$.

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x, i.e., difference of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \dfrac{(\Delta a + \Delta b)}{a - b} \times 100\%$

Error in product of quantities Suppose $x = a \times b$.

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x, i.e., product of a and b.

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

Error in division of quantities Suppose $x = \dfrac{a}{b}$.

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x, i.e., division of a and b.

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

Error in quantity raised to some power Suppose $x = \dfrac{a^n}{b^m}$.

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(n\dfrac{\Delta a}{a} + m\dfrac{\Delta b}{b}\right)$

Percentage error in the value of $x = n$(Percentage error in value of a) + m(Percentage error in value of b)

- The quantity which have maximum power must be measured carefully because it's contribution to error is maximum.

ILLUSTRATION 1.6 The length and breadth of a rectangle are (5.7 ± 0.1) cm and (3.4 ± 0.2) cm, respectively Calculate the area of rectangle with error limits.

Solution. Here, $l = (5.7 \pm 0.1)$ cm, $b = (3.4 \pm 0.2)$ cm.

Area, $A = l \times b = 5.7 \times 3.4 = 19.38\ \text{cm}^2 = 19.0\ \text{cm}^2$ (rounding off to two significant figures)

$$\therefore \quad \frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right)$$
$$= \pm\left(\frac{0.1}{5.7} + \frac{0.2}{3.4}\right) = \pm\left(\frac{0.34 + 1.14}{5.7 \times 3.4}\right) = \pm\frac{1.48}{19.38}$$
$$\Rightarrow \quad \Delta A = \pm\frac{1.48}{19.38} \times A = \pm\frac{1.48}{19.38} \times 19.38 = \pm 1.48$$

$\Delta A = \pm 1.5$ (rounding off to two significant figures);

Area $= (19.0 \pm 1.5)\ \text{cm}^2$.

ILLUSTRATION 1.7 A physical quantity x is calculated from the relation $x = \dfrac{a^2 b^3}{c\sqrt{d}}$. If the percentage error in a, b, c, and d are 2%, 1%, 3%, and 4%, respectively, what is the percentage error in x?

Solution. As $x = \dfrac{a^2 b^3}{c\sqrt{d}}$,

$$\therefore \quad \frac{\Delta x}{x} = \pm\left[2\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{1}{2}\frac{\Delta d}{d}\right] \times 100$$

$$= \pm\left[2 \times 2\% + 3 \times 1\% + 3\% + \frac{1}{2} \times 4\%\right]$$

$$= \pm[4\% + 3\% + 3\% + 2\%] = \pm 12\%$$

ILLUSTRATION 1.8 The length and breadth of a field are measured as: $l = (120 \pm 2)$ m and $b = (100 \pm 5)$ m, respectively. What is the area of the field?

Solution. Now $\dfrac{\Delta A}{A} = \dfrac{\Delta l}{l} + \dfrac{\Delta b}{b} = \left(\dfrac{2}{120} + \dfrac{5}{100}\right) = 0.0667$;

$$\Delta A = 0.0667 \times A$$

Now $A = l \cdot b = 120 \times 100 = 12000\ \text{m}^2$

$\Rightarrow \quad \Delta A = 0.0667 \times 12000 = 800.4\ \text{m}^2$

Area of the field $= A \pm \Delta A$

$$= 12000 \pm 800.4 = (1.2 \pm 0.08) \times 10^4\ \text{m}^2$$

Concept Application Exercise 1.3

1. A research worker takes 100 observations in an experiment. If he repeats the same experiment by taking 500 observations, how is the probable error affected?
2. A body travels uniformly a distance of (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. Find the velocity of the body within error limits and the percentage error.
3. The error in the measurement of the radius of a sphere is 1%. Find the error in the measurement of volume.
4. Given $R_1 = 5.0 \pm 0.2\ \Omega$, and $R_2 = 10.0 \pm 0.1\ \Omega$. What is the total resistance in parallel with possible % error?
5. The value of resistance is 10.845 Ω and the current is 3.23 A. On multiplying them, we get the potential difference = 35.02935 V. What is the value of potential difference in terms of significant figures?
6. The length of one rod is 2.53 cm and that of the other is 1.27 cm. The least count of the measuring instrument is 0.01 cm. If the two rods are put together end to end, find the combined length.
7. The pressure on a square plate is measured by measuring the force on the plate and the length of the sides of the plate. If the maximum error in the measurement of force and length are, respectively, 4% and 2%, Find the maximum errors in the measurement of pressure.
8. A physical quantity P is given by $P = \dfrac{A^3 B^{1/2}}{C^{-4} D^{3/2}}$. Which quantity among A, B, C, and D brings in the maximum percentage error in P?

Solved Examples

1. From the equation $\tan\theta = \dfrac{rg}{v^2}$, one can obtain the angle of banking θ for a cyclist taking a curve (the symbols have their usual meanings). Then, say, it is
   (a) Both dimensionally and numerically correct
   (b) Neither numerically nor dimensionally correct
   (c) Dimensionally correct only
   (d) Numerically correct only

Sol. (c) Given equation is dimensionally correct because both sides are dimensionless but numerically wrong because the correct equation is $\tan\theta = v^2/rg$.

2. From the dimensional consideration, which of the following equation is correct?
   (a) $T = 2\pi\sqrt{\dfrac{R^3}{GM}}$ (b) $T = 2\pi\sqrt{\dfrac{GM}{R^3}}$
   (c) $T = 2\pi\sqrt{\dfrac{GM}{R^2}}$ (d) $T = 2\pi\sqrt{\dfrac{R^2}{GM}}$

Sol. (a) By substituting the dimensions in $T = 2\pi\sqrt{\dfrac{R^3}{GM}}$, we get $\sqrt{\dfrac{L^3}{M^{-1}L^3T^{-2} \times M}} = T$

3. The equation of the state of some gases can be expressed as $\left(P + \dfrac{a}{V^2}\right) = \dfrac{R\theta}{V}$, where P is the pressure, V the volume, θ the absolute temperature, and a and b are constants. The dimensional formula of a is
   (a) $[ML^5T^{-2}]$ (b) $[M^{-1}L^5T^{-2}]$
   (c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-5}T^{-2}]$

Sol. (a) By the principle of dimensional homogenity,

$$[P] = \left[\frac{a}{V^2}\right]$$

$$\Rightarrow [a] = [P] \times [V^2] = [ML^{-1}T^{-2}][L^6] = [ML^5T^{-2}]$$

4. The dimension of $\dfrac{a}{b}$ in the equation $P = \dfrac{a - t^2}{bx}$, where P is pressure, x is distance and t is time, is
   (a) MT^{-2} (b) M^2LT^{-3}
   (c) ML^3T^{-1} (d) LT^{-3}

Sol. (a) $[a] = [T^2]$ and $[b] = \dfrac{[a - t^2]}{[P][x]} = \dfrac{T^2}{[ML^{-1}T^{-2}][L]}$

$\Rightarrow \quad [b] = [M^{-1}T^4]$

So $\quad \left[\dfrac{a}{b}\right] = \dfrac{[T^2]}{[M^{-1}T^4]} = [MT^{-2}]$

5. The dimension of $e^2/4\pi\varepsilon_0 hc$, where e, ε_0, h and c are electronic charge, electric permittivity, Planck's constant and velocity of light in vacuum, respectively, is
   (a) $[M^0L^0T^0]$ (b) $[M^1L^0T^0]$
   (c) $[M^0L^1T^0]$ (d) $[M^0L^0T^1]$

Sol. (a) $[e] = [AT]$, $\varepsilon_0 = [M^{-1}L^{-3}T^4A^2]$ $[h] = [ML^2T^{-1}]$

and $[c] = [LT^{-1}]$

$$\left[\frac{e^2}{4\pi\varepsilon_0 hc}\right] = \left[\frac{A^2T^2}{M^{-1}L^{-3}T^4A^2 \times ML^2T^{-1} \times LT^{-1}}\right] = [M^0L^0T^0]$$

6. If the acceleration due to gravity is 10 ms^{-2} and the units of length and time are changed in kilometer and hour respectively, the numerical value of the acceleration is
   (a) 360000 (b) 72000
   (c) 36000 (d) 129600

Sol. (d) $n_2 = n_1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2} = 10\left[\frac{\text{meter}}{\text{km}}\right]^1\left[\frac{\text{sec}}{\text{hr}}\right]^{-2}$

$$n_2 = 10\left[\frac{\text{m}}{10^3\text{m}}\right]^1\left[\frac{\text{sec}}{3600\,\text{sec}}\right]^{-2} = 129600$$

7. With the usual notations, the following equation $S_t = u + \frac{1}{2}a(2t-1)$ is
   (a) Only numerically correct
   (b) Only dimensionally correct
   (c) Both numerically and dimensionally correct
   (d) Neither numerically nor dimensionally correct

Sol. (c) We can derive this equation from equations of motion. So it is numerically correct.

$$S_t = \text{Distance travelled in } t^{\text{th}} \text{ second} = \frac{\text{Distance}}{\text{Time}} = [LT^{-1}]$$

$$u = \text{Velocity} = [LT^{-1}] \text{ and } \frac{1}{2}a(2t-1) = [LT^{-1}]$$

As dimensions of each term in the given equation are same, hence equation is dimensionally correct also.

8. The speed of light (c), gravitational constant (G) and Planck's constant (h) are taken as the fundamental units in a system. The dimension of time in this new system should be
   (a) $G^{1/2}h^{1/2}c^{-5/2}$ (b) $G^{-1/2}h^{1/2}c^{1/2}$
   (c) $G^{1/2}h^{1/2}c^{-3/2}$ (d) $G^{1/2}h^{1/2}c^{1/2}$

Sol. (a) Time $\propto c^xG^yh^z \Rightarrow T = kc^xG^yh^z$

Putting the dimensions in the above relation

$\Rightarrow$ $[M^0L^0T^1] = [LT^{-1}]^x[M^{-1}L^3T^{-2}]^y[ML^2T^{-1}]^z$

$\Rightarrow$ $[M^0L^0T^1] = [M^{-y+z}L^{x+3y+2z}T^{-x-2y-z}]$

Comparing the powers of M, L and T

$-y + z = 0$ ...(i)

$x + 3y + 2z = 0$ ...(ii)

$-x - 2y - z = 1$ ...(iii)

On solving equations (i) and (ii) and (iii)

$$x = \frac{-5}{2}, y = z = \frac{1}{2}$$

Hence dimension of time are $[G^{1/2}h^{1/2}c^{-5/2}]$

9. In the relation $P = \frac{\alpha}{\beta}e^{-\frac{\alpha Z}{k\theta}}$ P is pressure, Z is the distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be
   (a) $[M^0L^2T^0]$ (b) $[M^1L^2T^1]$
   (c) $[M^1L^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

Sol. (a) In given equation, $\frac{\alpha z}{k\theta}$ should be dimensionless

$$\therefore \quad \alpha = \frac{k\theta}{z} \Rightarrow [\alpha] = \frac{[ML^2T^{-2}K^{-1} \times K]}{[L]} = [MLT^{-2}]$$

and $P = \frac{\alpha}{\beta} \Rightarrow [\beta] = \left[\frac{\alpha}{p}\right] = \frac{[MLT^{-2}]}{[ML^{-1}T^{-2}]} = [M^0L^2T^0]$.

10. The frequency of vibration of string is given by $\nu = \frac{p}{2l}\left[\frac{F}{m}\right]^{1/2}$. Here p is number of segments in the string, l is the length and F is the tension in the string. The dimensional formula for m will be
   (a) $[M^0LT^{-1}]$ (b) $[ML^0T^{-1}]$
   (c) $[ML^{-1}T^0]$ (d) $[M^0L^0T^0]$

Sol. (c) $\nu = \frac{P}{2l}\left[\frac{F}{m}\right]^{1/2} \Rightarrow \nu^2 = \frac{P^2}{4l^2}\left[\frac{F}{m}\right] \therefore m \propto \frac{F}{l^2\nu^2}$

$$\Rightarrow \quad [m] = \left[\frac{MLT^{-2}}{L^2T^{-2}}\right] = [ML^{-1}T^0]$$

11. A wire has a mass 0.3 ± 0.003 g, radius 0.5 ± 0.005 mm and length 6 ± 0.006 cm. The maximum percentage error in the measurement of its density is
   (a) 1 (b) 2
   (c) 3 (d) 4

Sol. (d) $\because$ Density, $\rho = \frac{M}{V} = \frac{M}{\pi r^2 L}$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}$$

$$= \frac{0.003}{0.3} + 2 \times \frac{0.005}{0.5} + \frac{0.06}{6}$$

$$= 0.01 + 0.02 + 0.01 = 0.04$$

$$\therefore \quad \text{Percentage error} = \frac{\Delta\rho}{\rho} \times 100 = 0.04 \times 100 = 4\%$$

12. A physical parameter a can be determined by measuring the parameters b, c, d and e using the relation $a = b^\alpha c^\beta / d^\gamma e^\delta$. If the maximum errors in the measurement of b, c, d and e are $b_1\%$, $c_1\%$, $d_1\%$ and $e^1\%$, then the maximum error in the value of a determined by the experiment is

(a) $(b_1 + c_1 + d_1 + e_1)\%$
(b) $(b_1 + c_1 - d_1 - e_1)\%$
(c) $(\alpha b_1 + \beta c_1 - \gamma d_1 - \delta e_1)\%$
(d) $(\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$

Sol. (d) $a = b^\alpha c^\beta / d^\gamma e^\delta$

So, maximum error in a is given by

$$\left(\frac{\Delta a}{a}\times 100\right)_{\max} = \alpha\cdot\frac{\Delta b}{b}\times 100 + \beta\cdot\frac{\Delta c}{c}\times 100 + \gamma\cdot\frac{\Delta d}{d}\times 100 + \delta\cdot\frac{\Delta e}{e}\times 100$$

$$= (\alpha b_1 + \beta c_1 + \gamma d_1 + \delta e_1)\%$$

13. The relative density of material of a body is found by weighing it first in air and then in water. If the weight in air is (5.00 ± 0.05) Newton and weight in water is (4.00 ± 0.05) Newton. Then the relative density along with the maximum permissible percentage error is

(a) $5.0 \pm 11\%$ (b) $5.0 \pm 1\%$
(c) $5.0 \pm 6\%$ (d) $1.25 \pm 5\%$

Sol. (a) Weight in air $= (5.00 \pm 0.05)$N

Weight in water $= (4.00 \pm 0.05)$N

Loss of weight in water $= (1.00 \pm 0.1)$N

Now relative density $= \dfrac{\text{weight in air}}{\text{weight loss in water}}$

$$R.D = \frac{5.00 \pm 0.05}{1.00 \pm 0.1}$$

Now relative density with max permissible error

$$= \frac{5.00}{1.00} \pm \left(\frac{0.05}{5.00} + \frac{0.1}{1.00}\right)\times 100 = 5.0 \pm 11\%$$

14. The length of a cylinder is measured with a meter rod having least count 0.1 cm. Its diameter is measured with venier calipers having least count 0.01 cm. Given that length is 5.0 cm. and radius is 2.0 cm. The percentage error in the calculated value of the volume will be

(a) 1% (b) 2%
(c) 3% (d) 4%

Sol. (c) Volume of cylinder $V = \pi r^2 l$

Percentage error in volume

$$\frac{\Delta V}{V}\times 100 = \frac{2\Delta r}{r}\times 100 + \frac{\Delta l}{l}\times 100$$

$$= \left(2\times\frac{0.01}{2.0}\times 100 + \frac{0.1}{5.0}\times 100\right) = (1+2)\% = 3\%$$

15. In an experiment, the following observation's were recorded: $L = 2.820$ m, $M = 3.00$ kg, $l = 0.087$ cm, Diameter $D = 0.041$ cm. Taking $g = 9.81$ m/s^2, using the formula, $Y = \dfrac{4Mg}{\pi D^2 l}$, the maximum permissible error in Y is

(a) 7.96% (b) 4.56%
(c) 6.50% (d) 8.42%

Sol. (c) $Y = \dfrac{4MgL}{\pi D^2 l}$ so maximum permissible error in

$$Y = \frac{\Delta Y}{Y}\times 100 = \left(\frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\Delta D}{D} + \frac{\Delta l}{l}\right)\times 100$$

$$= \left(\frac{1}{300} + \frac{1}{9.81} + \frac{1}{9820} + 2\times\frac{1}{41} + \frac{1}{87}\right)\times 100$$

$$= 0.065 \times 100 = 6.5\%$$

EXERCISES

Basic Concept of Units

1. If the unit of length and force be increased four times, then the unit of energy is

(a) Increased 4 times (b) Increased 8 times
(c) Increased 16 times (d) Decreased 16 times

2. If u_1 and u_2 are the units selected in two systems of measurement and n_1 and n_2 their numerical values, then

(a) $n_1u_1 = n_2u_2$ (b) $n_1u_1 + n_2u_2 = 0$
(c) $n_1n_2 = u_1u_2$ (d) $(n_1 + u_1) = (n_2 + u_2)$

3. To determine the Young's modulus of a wire, the formula is $Y = \dfrac{F}{A}\times\dfrac{L}{\Delta L}$; where L = length, A = area of cross-section of the wire, ΔL = change in length of the wire when stretched with a force F. The conversion factor to change it from CGS to MKS system is

(a) 1 (b) 10 (c) 0.1 (d) 0.01

4. In C.G.S. system the magnitude of the force is 100 dynes. In another system where the fundamental physical quantities are kilogram, metre and minute, the magnitude of the force is

(a) 0.036 (b) 0.36 (c) 3.6 (d) 36

5. A physical quantity is measured and its value is found to be nu where n = numerical value and u = unit. Then which of the following relations is true

(a) $n \propto u^2$ (b) $n \propto u$ (c) $n \propto \sqrt{u}$ (d) $n \propto \dfrac{1}{u}$

6. If $x = at + bt^2$, where x is the distance travelled by the body in kilometres while t is the time in seconds, then the unit of b is

(a) km/s (b) km-s (c) km/s^2 (d) km-s^2

7. If $S = a + bt + ct^2$. S is measured in metres and t in *seconds*. The unit of c is

(a) None (b) m (c) ms^{-1} (d) ms^{-2}

8. A watt is

(a) kg m/s^2 (b) kg m^2/s^3 (c) kg m/s (d) kg m^2/s^2

9. Experiment shows that two perfectly neutral parallel metal plates separated by a small distance d attract each other

via a very weak force, known as the Casimir force. The force per unit area of the plates, F, depends only on the Planck constant h, on the speed of light c, and on d. Which of the following has the best chance of being correct for F?

(a) $F = \dfrac{hc}{d^2}$ (b) $F = \dfrac{hc}{d^4}$

(c) $F = \dfrac{hd^2}{c}$ (d) $F = \dfrac{d^4}{hc}$

Dimensional Analysis

10. Suppose refractive index μ is given as

$$\mu = A + \frac{B}{\lambda^2}$$

where A and B are constants and λ is wavelength, then dimensions of B are same as that of

(a) Wavelength (b) Volume
(c) Pressure (d) Area

11. A physical quantity x depends on quantities y and z as follows: $x = Ay + B\tan(Cz)$, where A, B, and C are constants. Which of the followings do not have the same dimensions?

(a) x and B (b) C and z^{-1}
(c) y and B/A (d) x and A

12. If L and R denote inductance and resistance, respectively, then the dimensions of L/R are

(a) $M^1L^0T^0Q^{-1}$ (b) $M^0L^0TQ^0$
(c) $M^0L^1T^{-1}Q^0$ (d) $M^{-1}LT^0Q^{-1}$

13. Force F is given in terms of time t and distance x by $F = A\sin Ct + B\cos Dx$. Then the dimensions of A/B and C/D are

(a) $[M^0L^0T^0]$, $[M^0L^0T^{-1}]$
(b) $[MLT^{-2}]$, $[M^0L^{-1}T^0]$
(c) $[M^0L^0T^0]$, $[M^0LT^{-1}]$
(d) $[M^0L^1T^{-1}]$, $[M^0L^0T^0]$

14. The frequency (n) of vibration of a string is given as $n = \dfrac{1}{2l}\sqrt{\dfrac{T}{m}}$, where T is tension and l is the length of vibrating string, then the dimensional formula is

(a) $[M^0L^1T^1]$ (b) $[M^0L^0T^0]$
(c) $[M^1L^{-1}T^0]$ (d) $[M^1L^0T^0]$

15. In the relation $y = r\sin(\omega t - kx)$, the dimensions of ω/k are

(a) $[M^0L^0T^0]$ (b) $[M^0L^1T^{-1}]$
(c) $[M^0L^0T^1]$ (d) $[M^0L^1T^0]$

16. The dimensions of $\varepsilon_0\mu_0$ are

(a) $[LT^{-1}]$ (b) $[LT^{-2}]$ (c) $[L^2T^{-2}]$ (d) $[L^{-2}T^2]$

17. If frequency F, velocity V, and density D are considered fundamental units, the dimensional formula for momentum will be

(a) DVF^2 (b) DV^2F^{-1}
(c) $D^2V^2F^2$ (d) DV^4F^{-3}

18. If force F, acceleration a, and time T are taken as the fundamental physical quantities, the dimensions of length on this system of units are

(a) FAT^2 (b) FAT (c) FT (d) AT^2

19. The frequency f of vibrations of a mass m suspended from a spring of spring constant k is given by $f = Cm^xk^y$, where C is a dimensionless constant. The values of x and y are, respectively.

(a) $\frac{1}{2}, \frac{1}{2}$ (b) $-\frac{1}{2}, -\frac{1}{2}$

(c) $\frac{1}{2}, -\frac{1}{2}$ (d) $-\frac{1}{2}, \frac{1}{2}$

20. If C (the velocity of light) g, (the acceleration due to gravity), and P (the atmospheric pressure) are the fundamental quantities in MKS system, then the dimensions of length will be same as that of

(a) C/g (b) C/P (c) PCg (d) C^2/g

21. The quantities A and B are related by the relation $A/B = m$, where m is the linear mass density and A is the force, the dimensions of B will be

(a) Same as that of pressure
(b) Same as that of work
(c) That of momentum
(d) Same as that of latent heat

22. Write the dimensions of $a \times b$ in the relation $E = \dfrac{b - x^2}{at}$, where E is the energy, x is the displacement, and t is the time.

(a) ML^2T (b) $M^{-1}L^2T^1$ (c) ML^2T^{-2} (d) MLT^{-2}

23. If the velocity of light C, the universal gravitational constant G, and Planck's constant h are chosen as fundamental units, the dimensions of mass in this system are

(a) $h^{1/2}C^{1/2}G^{-1/2}$ (b) $h^{-1}C^{-1}G$
(c) hCG^{-1} (d) hCG

24. In the relation $V = \dfrac{\pi}{8}\dfrac{Pr^4}{\eta l}$, where the letters have their usual meanings, the dimensions of V are

(a) $M^0L^3T^0$ (b) $M^0L^3T^{-1}$
(c) $M^0L^{-3}T^{-1}$ (d) $M^1L^3T^0$

25. If the velocity (V), acceleration (A), and force (F) are taken as fundamental quantities instead of mass (M), length

(L), and time (T), the dimensions of Young's modulus (Y) would be

(a) FA^2V^{-4} (b) FA^2V^{-5} (c) FA^2V^{-3} (d) FA^2V^{-2}

26. Which of the following product of e, h, μ, G (where μ is the permeability) be taken so that the dimensions of the product are same as that of the speed of light?

(a) $he^{-2}\mu^{-1}G^0$ (b) $h^2eG^0\mu$

(c) $h^0e^2G^{-1}\mu$ (d) $hGe^{-2}\mu^0$

27. The mass of the liquid flowing per second per unit area of cross section of the tube is proportional to P^x and v^y, where P is the pressure difference and v is the velocity, then the relation between x and y is

(a) $x = y$ (b) $x = -y$ (c) $y^2 = x$ (d) $y = -x^2$

28. The potential energy of a particle varies with distance x as $U = \dfrac{Ax^{1/2}}{x^2 + B}$, where A and B are constants. The dimensional formula for $A \times B$ is

(a) $M^1L^{7/2}T^{-2}$ (b) $M^1L^{11/2}T^{-2}$

(c) $M^1L^{5/2}T^{-2}$ (d) $M^1L^{9/2}T^{-2}$

29. Using mass (M), length (L), time (T), and electric current (A) as fundamental quantities, the dimensions of permittivity will be

(a) $[MLT^{-1}A^{-1}]$ (b) $[MLT^{-2}A^{-2}]$

(c) $[M^{-1}L^{-3}T^4A^2]$ (d) $[M^2L^{-2}T^{-2}A]$

30. Assuming that the mass m of the largest stone that can be moved by a flowing river depends upon the velocity v of the water, its density ρ, and the acceleration due to gravity g. Then m is directly proportional to

(a) v^3 (b) v^4 (c) v^5 (d) v^6

Errors in Measurement

31. The best method to reduce random error is

(a) To change the instrument used for measurement
(b) To take help of experienced observer
(c) To repeat the experiment many times and to take the average results
(d) None of the above

32. A length is measured as 7.60 m. This is the same as

(a) 7600 mm (b) 0.0076 mm
(c) 760 cm (d) 0.76 dm

33. If the percentage errors of A, B, and C are a, b, and c, respectively, then the total percentage error in the product ABC is

(a) abc (b) $a + b + c$

(c) $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}$ (d) $ab + bc + ca$

34. Which of the following numbers has least number of significant figures?

(a) 0.80760 (b) 0.80200
(c) 0.08076 (d) 80.267

35. If $X = a + b$, the maximum percentage error in the measurement of X will be

(a) $\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right) \times 100\%$

(b) $\left(\dfrac{\Delta a}{a+b} - \dfrac{\Delta b}{a+b}\right) \times 100\%$

(c) $\left(\dfrac{\Delta a}{a+b} + \dfrac{\Delta b}{a+b}\right) \times 100\%$

(d) $\left(\dfrac{\Delta a}{a} \times \dfrac{\Delta b}{b}\right) \times 100\%$

36. A physical quantity X is represented by $X = (M^xL^{-y}T^{-z})$. The maximum percentage errors in the measurement of M, L, and T, respectively, are $a\%$, $b\%$ and $c\%$. The maximum percentage error in the measurement of X will be

(a) $(ax + by - cz)\%$ (b) $(ax - by - cz)\%$
(c) $(ax + by + cz)\%$ (d) $(ax - by + cz)\%$

37. The length l, breadth b, and thickness t of a block of wood were measured with the help of a measuring scale. The results with permissible errors (in cm) are

$l = 15.12 \pm 0.01$, $b = 10.15 \pm 0.01$, and $t = 5.28 \pm 0.01$.

The percentage error in volume up to proper significant figures is

(a) 0.28% (b) 0.35% (c) 0.48% (d) 0.64%

38. The relative density of a material of a body is found by weighing it first in air and then in water. If the weight of the body in air is $W_1 = 8.00 \pm 0.05$ N and the weight in water is $W_2 = 6.00 \pm 0.05$ N, then the relative density $\rho_r = W_1/(W_1 - W_2)$ with the maximum permissible error is

(a) $4.00 \pm 0.62\%$ (b) $4.00 \pm 0.82\%$
(c) $4.00 \pm 3.2\%$ (d) $4.00 \pm 5.62\%$

39. The percentage errors in the measurement of mass and speed are 2% and 3%, respectively. How much will be the maximum error in the estimation of KE obtained by measuring mass and speed?

(a) 5% (b) 1% (c) 8% (d) 11%

40. An experiment measures quantities a, b, and c, and then X is calculated from $X = \dfrac{a^{1/2}b^2}{c^3}$. If the percentage errors in a, b, and c are $\pm1\%$, $\pm3\%$, and $\pm2\%$, respectively, then the percentage error in X can be

(a) $\pm12.5\%$ (b) $\pm7\%$ (c) $\pm1\%$ (d) $\pm4\%$

41. The resistance of a metal is given by $R = V/I$, where V is potential difference and I is the current. In a circuit, the potential difference across resistance is $V = (8 \pm 0.5)$ V and current in resistance, $I = (4 \pm 0.2)$ A. What is the value of resistance with its percentage error?

(a) $(2 \pm 5.6\%)\,\Omega$ (b) $(2 \pm 0.7\%)\,\Omega$
(c) $(2 \pm 35\%)\,\Omega$ (d) $(2 \pm 11.25\%)\,\Omega$

42. The specific resistance ρ of a circular wire of radius r, resistance R, and length l is given by $\rho = \pi r^2 R/l$. Given: $r = 0.24 \pm 0.02$ cm, $R = 30 \pm 1\ \Omega$, and $l = 4.80 \pm 0.01$ cm. The percentage error in ρ is nearly
(a) 7% (b) 9% (c) 13% (d) 20%

43. The mass of a body is 20.000 g and its volume is 10.00 cm^3. If the measured values are expressed to the correct significant figures, the maximum error in the value of density is
(a) 0.001 g cm^{-3} (b) 0.010 g cm^{-3}
(c) 0.100 g cm^{-3} (d) None of these

44. While measuring the acceleration due to gravity by a simple pendulum, a student makes a positive error of 1% in the length of the pendulum and a negative error of 3% in the value of time period. His percentage error in the measurement of g by the relation $g = 4\pi^2(l/T^2)$ will be
(a) 2% (b) 4% (c) 7% (d) 10%

45. While measuring acceleration due to gravity by a simple pendulum, a student makes a positive error of 2% in the length of the pendulum and a positive error of 1% in the value of time period. His actual percentage error in the measurement of the value of g will be
(a) 3% (b) 0% (c) 4% (d) 5%

46. The relative density of a material is found by weighing the body first in air and then in water. If the weight in air is (10.0 ± 0.1) gf and the weight in water is (5.0 ± 0.1) gf, then the maximum permissible percentage error in relative density is
(a) 1 (b) 2 (c) 3 (d) 5

47. The dimensional formula for a physical quantity x is $[M^{-1}L^3T^{-2}]$. The errors in measuring the quantities M, L, and T, respectively, are 2%, 3%, and 4%. The maximum percentage of error that occurs in measuring the quantity x is
(a) 9 (b) 10 (c) 14 (d) 19

48. The heat generated in a circuit is given by $Q = I^2Rt$, where I is current, R is resistance, and t is time. If the percentage errors in measuring I, R, and t are 2%, 1%, and 1%, respectively, then the maximum error in measuring heat will be
(a) 2% (b) 3% (c) 4% (d) 6%

Problems Based on Mixed Concepts

49. The wavelength associated with a moving particle depends upon p^{th} power of its mass m, q^{th} power of its velocity v and r^{th} power of Planck's constant h. Then the correct set of values of p, q and r is
(a) $p = 1, q = -1, r = 1$
(b) $p = 1, q = 1, r = 1$
(c) $p = -1, q = -1, r = -1$
(d) $p = -1, q = -1, r = 1$

50. A gas bubble from an expression under water, oscillates with a period T proportional to $p^a d^b E^c$, where p is the static pressure, d is the density of water and E is the total energy of explosion. Find the value of a.
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $-\frac{1}{4}$ (d) $-\frac{5}{6}$

51. In the relation $P = \frac{\alpha}{\beta} e^{-\alpha Z/k\theta}$, P is the pressure, Z is the distance, k is the Boltzmann constant and θ is the temperature. The dimensional formula of β will be
(a) $[M^0L^2T^0]$ (b) $[M^1L^2T^1]$
(c) $[M^1L^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

52. In a direct impact, loss in kinetic energy is given by
$$\Delta K = \frac{M_1M_2}{2(M_1+M_2)}\cdot(V_1-V_2)^2(1-k^2)$$
with usual notations (except k). The quantity k will have dimensional formula
(a) $[M^0L^2T^{-2}]$ (b) $[MLT^{-1}]$
(c) $[M^0L^0T^0]$ (d) $[M^0LT^{-1}]$

53. If area (A) velocity (v) and density (ρ) are base units, then the dimensional formula of force can be represented as
(a) $Av\rho$ (b) $Av^2\rho$ (c) $Av\rho^2$ (d) $A^2v\rho$

54. A bus travels distance x_1 when accelerates from rest at constant rate a_2 for some time and after that travels a distance x_2 when decelerates at a constant rate a_2 to come to rest. A student established a relation $x_1 + x_2 = \frac{a_1a_2t^2}{2(a_1+a_2)}$.
Choose the correct option(s).
(a) The relation is dimensionally correct
(b) The relation is dimensionally incorrect
(c) The relation may be dimensionally correct
(d) None of the above

55. If energy E, length L and time T are taken as fundamental quantities. The dimensional formula of gravitational constant is
(a) $[FL^6E^{-2}]$ (b) $[FL^5T^{-1}]$
(c) $[E^2FL^6T^3]$ (d) $[E^3F^{-1}L^6T^5]$

56. A wire has a mass 0.4 ± 0.004 g and length 8 ± 0.08 (cm). The maximum percentage error in the measurement of its density is 4%. The radius of the wire is $r \pm \Delta r$, find Δr.
(a) $0.02r$ (b) $0.01r$ (c) $0.03r$ (d) $0.1r$

57. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in MKSQ system?

(a) $[M^{-2}L^{-2}T^{3}Q^{4}]$ (b) $[M^{-1}L^{-2}T^{4}Q^{4}]$
(c) $[M^{-2}L^{-2}T^{4}Q^{4}]$ (d) $[M^{-1}L^{-1}T^{3}Q^{4}]$

58. The error in measurement of unknown resistance of X is minimum in a meter bridge when $l = 70$ cm, where l is the distance of null point from one end. If $X = \left[\frac{l}{(A-l)}\right]R$, find the value of A, where R is known resistance.
(a) 35 cm (b) 105 cm (c) 140 cm (d) 210 cm

59. A body travels uniformly a distance of $(S + \Delta S)$ in a time $(t \pm \Delta t)$. What may be the condition so that body within the error limits move with a velocity $\left(\frac{S}{t} \pm \frac{\Delta S}{\Delta t}\right)$?

(a) $\frac{\Delta t}{t} + \frac{S(\Delta t)^2}{(\Delta S)t^2} = \pm 1$ (b) $\frac{\Delta t}{t} + \frac{S\,\Delta t}{\Delta S\, t} = \pm 1$

(c) $\frac{\Delta t}{t} + \frac{(\Delta S)t}{S(\Delta t)} = \pm 1$ (d) $\frac{\Delta t}{t} + \frac{S^2\Delta t}{(\Delta S)^2 t} = \pm 1$

60. If an unknown quantity, $\phi = \frac{ma}{\alpha}\log\left(1 + \frac{\alpha l}{ma}\right)$, where m = mass, a = acceleration and l = length.

(a) $[MLT^{-2}]$ (b) $[MT^{-2}]$
(c) $[M^0LT^0]$ (d) $[ML^{-3}]$

61. If $F = \frac{v}{C\ln(xb)}$, then
(a) F and v denote force and velocity, the dimensions of C are $[MT]$
(b) x denotes distance, the dimensions of b are $[L^{-1}]$
(c) the dimension of $\frac{v}{C}$ never be same as F
(d) the dimensions of x must be same as $\frac{v}{cb}$

62. If m, e, ε_0, h and c denote mass electron, charge of electron, Planck's constant and speed of light, respectively. The dimensions of $\frac{me^4}{\varepsilon_0^2 h^3 c}$ are

(a) $[M^0L^0T^{-1}]$ (b) $[M^0L^{-1}T^{-1}]$
(c) $[M^2LT^{-3}]$ (d) $[M^2L^{-1}T^0]$

63. $\int \frac{dt}{\sqrt{2at - t^2}} = a^x \sin^{-1}\left[\frac{t}{a} - 1\right]$. The value of x is
(a) 1 (b) −1 (c) 0 (d) 2

64. A physical quantity P is given by $P = \frac{A^3B^{1/2}}{C^{-4}D^{3/2}}$. The quantity which brings in the maximum percentage error in P is
(a) A (b) B (c) C (d) D

65. Number of particles is given by $n = -D\frac{n_2 - n_1}{x_2 - x_1}$ crossing a unit area perpendicular to X-axis in unit time, where n_1 and n_2 are number of particles per unit volume for the value of x meant to x_2 and x_1. Find dimensions of D called as diffusion constant
(a) M^0LT^2 (b) $M^0L^2T^{-4}$ (c) M^0LT^{-3} (d) $M^0L^2T^{-1}$

66. The position of a particle at time t is given by the relation $x(t) = \left(\frac{v_0}{\alpha}\right)(1 - e^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 and α are respectively
(a) $M^0L^1T^{-1}$ and T^{-1} (b) $M^0L^1T^0$ and T^{-1}
(c) $M^0L^1T^{-1}$ and LT^{-2} (d) $M^0L^1T^{-1}$ and T

ARCHIVES

1. Identify the pair whose dimensions are equal.
(a) torque and work (b) stress and energy
(c) force and stress (d) force and work
(AIEEE 2002)

2. The dimensions of pressure is equal to
(a) Force per unit volume
(b) Energy per unit volume
(c) Force
(d) Energy **(AIEEE 2002)**

3. Which two have same dimensions?
(a) Force and strain
(b) Force and stress
(c) Angular velocity and frequency
(d) Energy and strain **(AIEEE 2002)**

4. The physical quantities not having same dimensions are
(a) speed and $(\mu_0\varepsilon_0)^{-1/2}$
(b) torque and work
(c) momentum and Planck's constant
(d) stress and Young's modulus **(AIEEE 2003)**

5. The dimensions of $1/\mu_0\varepsilon_0$, where symbols have their usual meaning, are
(a) $[LT^{-1}]$ (b) $[L^{-1}T]$ (c) $[L^{-2}T^2]$ (d) $[L^2T^{-2}]$
(AIEEE 2003)

6. Which one of the following represents the correct dimensions of the coefficient of viscosity?
(a) $[ML^{-1}T^{-2}]$ (b) $[MLT^{-1}]$
(c) $[ML^{-1}T^{-1}]$ (d) $[ML^{-2}T^{-2}]$
(AIEEE 2004)

7. Out of the following pairs, which one does not have identical dimensions?
   (a) work and torque
   (b) moment of inertia and moment of a force
   (c) impulse and momentum
   (d) angular momentum and Planck's constant.
   (AIEEE 2005)

8. Which of the following units denotes the dimensions $[ML^2Q^{-2}]$, where Q denotes the electric charge?
   (a) H/m^2 (b) weber (Wb)
   (c) Wb/m^2 (d) henry (H) **(AIEEE 2006)**

9. Rad is the correct unit used to report the measurement of
   (a) the biological effect of radiation.
   (b) the rate of decay of a radioactive source.
   (c) the ability of a beam of gamma ray photons to produce ions in a target.
   (d) the energy delivered by radiation to a target.
   (AIEEE 2006)

10. The dimensions of magnetic field in M, L, T, and C (coulomb) is given as
    (a) $[MLT^{-1}C^{-1}]$ (b) $[MT^2C^{-2}]$
    (c) $[MT^{-1}C^{-1}]$ (d) $[MT^{-2}C^{-1}]$
    (AIEEE 2008)

11. The respective number of significant figures for the numbers 23.023, 0.0003, and 2.1×10^{-3} are
    (a) 5, 1, 2 (b) 5, 1, 5 (c) 5, 5, 2 (d) 4, 4, 2
    (AIEEE 2009)

12. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurements of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is:
    (a) 6% (b) zero (c) 1% (d) 3%
    (AIEEE 2012)

13. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then:
    (a) $[\varepsilon_0]=[M^{-1}L^{-3}T^4A^2]$ (b) $[\varepsilon_0]=[M^{-1}L^2T^1A^{-2}]$
    (c) $[\varepsilon_0]=[M^{-1}L^2T^1A]$ (d) $[\varepsilon_0]=[M^{-1}L^{-3}T^2A]$
    (JEE Main 2013)

14. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:
    (a) 92 ± 2 s (b) 92 ± 5.0 s
    (c) 92 ± 1.8 s (d) 92 ± 3 s
    (JEE Main 2016)

15. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides the main scale line?
    (a) 0.75 mm (b) 0.80 mm
    (c) 0.70 mm (d) 0.50 mm
    (JEE Main 2016)

16. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5 % and 1 %, the maximum error in determining the density is
    (a) 6% (b) 2.5% (c) 3.5% (d) 4.5%
    (JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	2. (a)	3. (c)	4. (c)	5. (d)	6. (c)	7. (d)	8. (b)	9. (b)	10. (d)
11. (d)	12. (b)	13. (c)	14. (c)	15. (b)	16. (d)	17. (d)	18. (d)	19. (d)	20. (d)
21. (d)	22. (b)	23. (a)	24. (b)	25. (a)	26. (a)	27. (b)	28. (b)	29. (c)	30. (d)
31. (c)	32. (c)	33. (b)	34. (c)	35. (c)	36. (c)	37. (b)	38. (d)	39. (c)	40. (a)
41. (d)	42. (d)	43. (d)	44. (c)	45. (b)	46. (d)	47. (d)	48. (d)	49. (d)	50. (d)
51. (a)	52. (c)	53. (b)	54. (a)	55. (a)	56. (b)	57. (b)	58. (c)	59. (a)	60. (c)
61. (b)	62. (d)	63. (c)	64. (c)	65. (d)	66. (a)				

Archives

1. (a)	2. (b)	3. (c)	4. (c)	5. (c)	6. (c)	7. (b)	8. (d)	9. (d)	10. (c)
11. (d)	12. (c)	13. (a)	14. (a)	15. (b)	16. (d)				

Chapter 2

Vectors

VECTORS AND SCALARS

Certain physical quantities are completely described by a numerical value alone (with units specified) and are added according to the ordinary rules of algebra. Such quantities are called *scalars*.

If a physical quantity is a vector, it has a direction, but the converse may or may not be true, i.e., if a physical quantity has a direction, it may or may not be a vector, e.g., pressure, surface tension, current, etc., have directions but are not vectors because they do not obey parallelogram law of addition.

The magnitude of a vector ($\vec{A}$) is the absolute value of a vector and is indicated by $|\vec{A}|$.

Example of vector quantity: displacement, velocity, acceleration, force, etc.

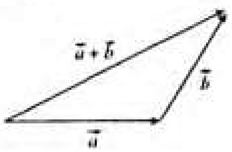
REPRESENTATION OF VECTOR

A vector is represented by a straight arrow. The tail is the staring point, the head is the ending point and the line measures the direction. The length of the line gives the value or magnitude of the vector. To express them mathematically, we need to draw the origin and the coordinate systems.

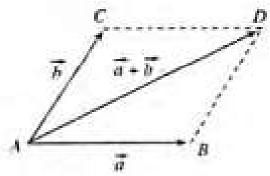
Mathematically, vector is represented by $\vec{A}$. Sometimes, it is represented by bold letter *A*.

ADDITION OF VECTORS

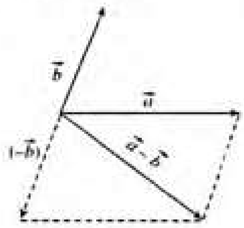
Triangle law of addition of vectors: If $\vec{a}$ and $\vec{b}$ are two vectors to be added, a diagram is drawn in which the tail of $\vec{b}$ coincides with the head of $\vec{a}$ (see figure). The vector joining the tail of $\vec{a}$ with the head of $\vec{b}$ is the vector sum of $\vec{a}$ and $\vec{b}$. Figure shows the construction.



Parallelogram law of addition of vectors: The same rule may be stated in a slightly different way. We draw the vectors and $\vec{b}$ with both the tails coinciding (see figure). Taking these two as the adjacent sides we complete the parallelogram. The diagonal through the common tails gives the sum of the two vectors. Thus, in the figure, $\overline{AB}+\overline{CD}=\overline{AD}$

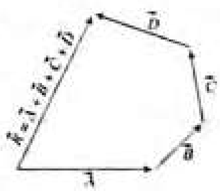


Subtraction of vectors: Let $\vec{a}$ and $\vec{b}$ be two vectors. We define $\vec{a}-\vec{b}$ as the sum of the vector $\vec{a}$ and the vector $(-\vec{b})$. To subtract $\vec{b}$ from $\vec{a}$, invert the direction of $\vec{b}$ and add to $\vec{a}$. The figure shows the process.

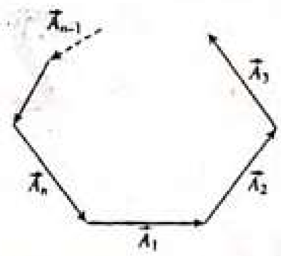


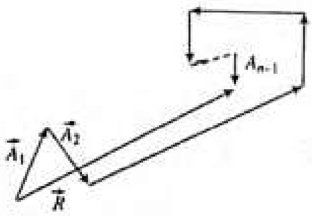
Polygon Law of Addition of Vectors

A geometric construction can also be used to add more than two vectors as shown in figure for the case of four vectors. The resultant vector $\vec{R}=\vec{A}+\vec{B}+\vec{C}+\vec{D}$ is the vector that completes the polygon. In other words, $\vec{R}$ is the vector drawn from the tail of the first vector to the tip of the last vector. This technique for adding vectors is often called the "head to tail method."



If *n* vectors are represented by the sides of a polygon of *n* sides in cyclic manner, the resultant of these vectors is a zero vector. Triangle and rectangle are the polygons of sides three and four, respectively. The non-coplanar vectors can be represented by a polygon of sides which cannot lie in a single plane. Sometimes, for coplanar vectors represented by a polygon, the resultant (closing side of the polygon) may cross the component vectors (other sides of the polygon) as shown in the figure.



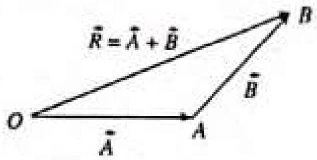


- Resultant of two unequal vectors cannot be zero.
- Resultant of three co-planar vectors may or may not be zero.
- Resultant of three non-co-planar vectors cannot be zero.

Vector Addition by Analytical Method

Triangle Law of Vector Addition of Two Vectors

If two non-zero vectors are represented by two sides of a triangle taken in the same order, then the resultant is given by the closing side of triangle in opposite order, i.e., $\vec{R} = \vec{A} + \vec{B}$



$\because \quad \overline{OB} = \overline{OA} + \overline{AB}$

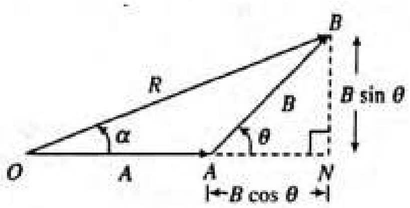
Magnitude of resultant vector: In ΔABN

$$\cos\theta = \frac{AN}{B}$$

$\therefore \quad AN = B\cos\theta$

$$\sin\theta = \frac{BN}{B}$$

$\therefore \quad BN = B\sin\theta$



In ΔOBN, we have $OB^2 = ON^2 + BN^2$

$$\Rightarrow \quad R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$= A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

$$= A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$= A^2 + B^2 + 2AB\cos\theta$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of resultant vectors: If θ is the angle between $\vec{A}$ and $\vec{B}$, then

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN} = \frac{B\sin\theta}{A + B\cos\theta}$$

Special Cases

Case 1: When the given vectors act in the same direction $(\theta = 0°)$.

So $\quad R = \sqrt{A^2 + B^2 + 2AB\cos 0°}$

$$= \sqrt{A^2 + B^2 + 2AB}$$

$$= \sqrt{(A+B)^2} = A + B \qquad [\therefore \cos 0° = 1]$$

or $\quad |\vec{R}| = |\vec{A}| + |\vec{B}|$ which represents the magnitude of the resultant.

Case 2: When the given vectors act in opposite directions $(\theta = 180°)$.

So $\quad R = \sqrt{A^2 + B^2 + 2AB\cos 180°}$

$$= \sqrt{A^2 + B^2 - 2AB}$$

$$= \sqrt{(A-B)^2} \qquad [\because \cos 180° = -1]$$

$$= \pm(A - B) = A - B \text{ or } B - A$$

or $\quad |\vec{R}| = \left||\vec{A}| - |\vec{B}|\right|$ which represents the magnitude of the resultant.

Case 3: When the given vectors $\vec{A}$ and $\vec{B}$ act at right angle to each other $(\theta = 90°)$.

$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB\cos 90°}$

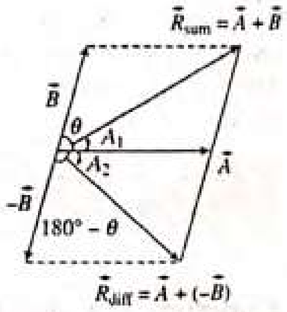
$$= \sqrt{A^2 + B^2} \qquad [\because \cos 90° = 0]$$

or $\quad |\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2}$

and $\quad \tan\beta = \dfrac{B\sin 90°}{A + B\cos 90°}$ or $\tan\beta = \dfrac{B}{A}$ $[\because \sin = 90° = 1]$

Subtraction of Vectors

Two vectors $\vec{A}$ and $\vec{B}$ are inclined at as angle θ (see figure). The summation of both vectors is represented by the bigger diagonal of parallelogram and difference of vectors is represented by the smaller diagonal of parallelogram.



Since, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

and $\quad |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$\Rightarrow \quad |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180° - \theta)}$$

Since, $\cos(180° - \theta) = -\cos\theta$

$$\Rightarrow \quad |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\tan\alpha_1 = \frac{B\sin\theta}{A + B\cos\theta}$$

and $\quad \tan\alpha_2 = \dfrac{B\sin(180° - \theta)}{A + B\cos(180° - \theta)}$

But $\sin(180° - \theta) = \sin\theta$

and $\cos(180° - \theta) = -\cos\theta$

$$\Rightarrow \quad \tan\alpha_2 = \frac{B\sin\theta}{A - B\cos\theta}$$

Illustration 2.1 The greatest and least resultant of two forces acting at a point is 10 N and 6 N, respectively. If each force is increased by 3 N, find the resultant of new forces when acting at a point at an angle of 90° with each other.

Solution. Let A and B be the two forces.

Greatest resultant $= A + B = 10$ ... (i)

Least resultant $= A - B = 6$ ... (ii)

Solving (i) and (ii), we get, $A = 8$ N and $B = 2$ N.

When each force is increased by 3 N then

$A' = A + 3 = 8 + 3 = 11$ N, then

$B' = B + 3 = 2 + 3 = 5$ N

As the new forces are acting at an angle of 90° (i.e., $\theta = 90°$)

So $R' = \sqrt{A'^2 + B'^2} = \sqrt{(11)^2 + (5)^2} = \sqrt{146}$ N

Illustration 2.2 Two equal forces have their resultant equal to either. At what angle are they inclined?

Solution. Here $A = F$; $B = F$; $R = F$; $\theta = ?$

Now $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$$F = \sqrt{F^2 + F^2 + 2F.F.\cos\theta}$$

$$= F\sqrt{2(1+\cos\theta)}$$

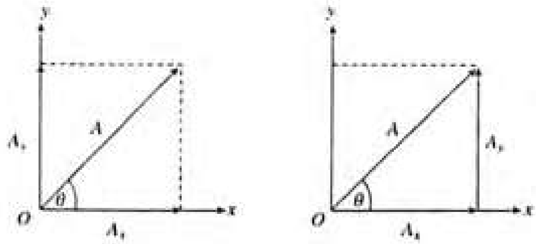
$\Rightarrow \quad 1 = 2(1 + \cos\theta)$

$$\cos\theta = -\frac{1}{2} = \cos 120° \Rightarrow \theta = 120°$$

COMPONENTS OF A VECTOR

The graphical method of adding vectors is not recommended whenever high accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the projection of vectors along coordinate axes. These projections are called components of the vector. Any vector can be completely described by its components.

Consider a vector A lying in the xy plane and making an arbitrary angle θ with the positive x-axis, as shown in Fig. (a). This vector can be expressed as the sum of two other vectors A_x and A_y. From Fig. (b), we see that the three vectors form a right triangle and that $A = A_x + A_y$. We shall often refer to the "components of a vector A," written A_x and A_y (without the boldface notation). Component A_x represents the projection of A along the x-axis and component A_y represents the projection of A along the y-axis. These components can be positive or negative. The component A_x is positive if A_x points in the positive x direction and is negative if A_x points in the negative x direction. The same is true for the component A_y.



(a) (b)

From Fig. (b) and the definition of sine and cosine, we see that $\cos\theta = A_x/A$ and that $\sin\theta = A_y/A$. Hence, the components of A are

$A_x = A\cos\theta$ ...(i)

$A_y = A\sin\theta$ ...(ii)

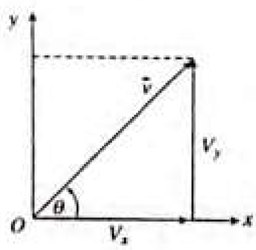
These components form two sides of a right triangle with a hypotenuse of length. Thus, it follows that the magnitude and direction of A are related to its components through the expressions

$$A = \sqrt{A_x^2 + A_y^2} \qquad \text{...(iii)}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \qquad \text{...(iv)}$$

Illustration 2.3 A particle is moving with velocity $v = 100$ m s^{-1}. If one of the rectangular components of a velocity is 50 m s^{-1}. Find the other component of velocity and its angle with the given component of velocity.

Solution. Here we are given net velocity $v = 100$ m s^{-1}.



Let $v_x = 50$ m s^{-1} $\Rightarrow v_x = v\cos\theta$

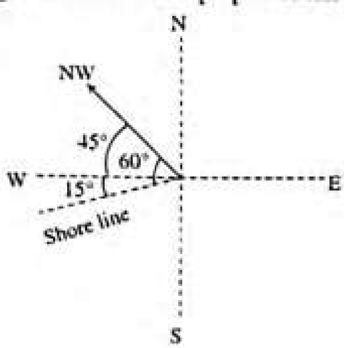
$\Rightarrow \quad 50 = 100\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$ or $\theta = 60°$

$\therefore \quad v_y = v\sin\theta = 100\sin 60°$

$$= 100 \times \frac{\sqrt{3}}{2} = 50\sqrt{3} \text{ m s}^{-1}$$

Illustration 2.4 A man rows a boat with a speed of 18 km h^{-1} in the north-west direction (see figure). The shoreline makes an angle of 15° south of west. Obtain the components of the velocity of the boat along the shoreline and perpendicular to the shoreline.



Solution. The north-west direction of the boat makes an angle of 60° with the shoreline.

Component of the velocity of boat along the shoreline

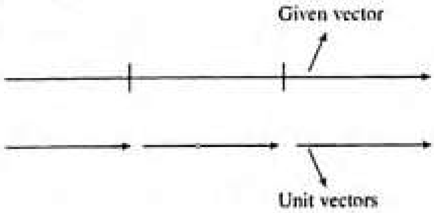
$$= 18\cos 60^\circ \text{ km h}^{-1} = 9 \text{ km h}^{-1}$$

Component of the boat velocity along a line normal to the shoreline

$$= 18\sin 60^\circ \text{km h}^{-1} = 18 \times \frac{\sqrt{3}}{2} \text{ km h}^{-1} = 9\sqrt{3} \text{ km h}^{-1}$$

UNIT VECTORS

When we divide an arrow which represents a vector of magnitude 3 units into three equal parts, each arrow represents a magnitude of one unit (see figure).



In general, when we divide any vector $\vec{A}$ by its magnitude $|\vec{A}|$, it gives us a unit vector of that vector denoted by the symbol $\hat{a} = \vec{A}/|\vec{A}|$. Hence, any vector $\vec{A}$ can be given as its magnitude times its unit vector, $\vec{A} = |\vec{A}|\hat{a}$. Hence, $\vec{A} \parallel \hat{a}$. The unit vectors have no dimensions and no units. A unit vector shows the orientation of the corresponding vector in space.

Suppose a force of magnitude 10 N acts in direction of $\hat{A}$ vector, this force $\vec{F}$ can be expressed as $\vec{F} = (10\hat{A})$ N.

Illustration 2.5 Find the unit vector of $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$.

Solution. $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

We know that $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

where $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$= \sqrt{2^2 + 3^2 + 2^2} = \sqrt{4+9+4} = \sqrt{17}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{i} + 3\hat{j} + 2\hat{k}}{\sqrt{17}}$$

Illustration 2.6 Determine that vector which when added to the resultant of $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives unit vector along y-direction.

Solution. Here $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$

Resultant $\vec{R} = \vec{A} + \vec{B}$

$$= (3\hat{i} - 5\hat{j} + 7\hat{k}) + (2\hat{i} + 4\hat{j} - 3\hat{k}) = 5\hat{i} - \hat{j} + 4\hat{k}$$

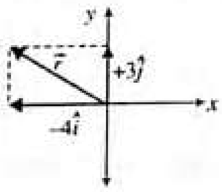
Let the vector to be added is $\vec{X}$. As the unit vector along y direction is $\hat{j}$

$$\therefore \quad \hat{j} = 5\hat{i} - \hat{j} + 4\hat{k} + \vec{X}$$

So, required vector, $\vec{X} = \hat{j} - (5\hat{i} - \hat{j} + 4\hat{k}) = -5\hat{i} + 2\hat{j} - 4\hat{k}$

Illustration 2.7 A car is moving in direction $\vec{r} = -4\hat{i} + 3\hat{j}$ with a speed of 10 m s^{-1}. Write the velocity vector of car in unit vector notation.

Solution. The direction of the motion of car is along the unit vector $\hat{r}$.



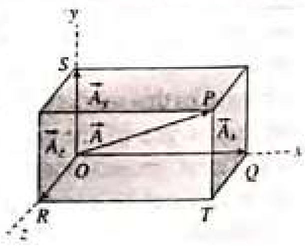
$$\therefore \quad \hat{r} = \frac{-4\hat{i} + 3\hat{j}}{\sqrt{4^2 + 3^2}} = -\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$$

Thus, velocity vector of car equal to

$$\vec{v} = |\vec{v}|\hat{r} = (10 \text{ m s}^{-1})\left(-\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right) = (-8\hat{i} + 6\hat{j}) \text{ m s}^{-1}$$

RECTANGULAR COMPONENTS OF A VECTOR IN THREE DIMENSIONS

When a vector is split into mutually perpendicular directions in 3-D space, the component vectors obtained are called rectangular components of the given vector in 3-D space.



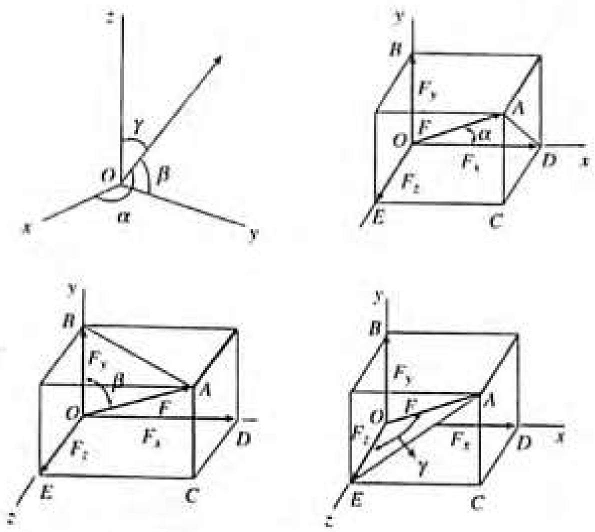
The figure shows vector $\vec{A}$ represented by $\overline{OP}$.

Here $\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

The magnitude of $\vec{A}$ is given by $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Direction Cosines

Let A be a point in space whose coordinates are (x, y, z). Then its position vector w.r.t. the origin of coordinate system is given by: $\vec{r} = \overline{OA} = x\hat{i} + y\hat{j} + z\hat{k}$.



and $r = OA = \sqrt{x^2 + y^2 + z^2}$

Angles with x-, y-, and z-axes, respectively, are given by:

$$\cos\alpha = \frac{x}{r} = l, \cos\beta = \frac{y}{r} = m, \cos\gamma = \frac{z}{r} = n$$

The direction cosines l, m, and n of a vector are the cosines of the angles α, β, and γ, which a given vector makes with x-axis, y-axis, and z-axis, respectively.

Now squaring and adding l, m, and n,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2 + y^2 + z^2}{r^2}$$

or $\quad l^2 + m^2 + n^2 = \frac{r^2}{r^2} = 1$

It means the sum of the squares of the direction cosines of a vector is always unity.

Illustration 2.8 A bird moves with velocity 20 ms^{-1} in a direction making an angle of 60° with the eastern line and 60° with vertical upward. Represent the velocity vector in rectangular form.

Solution. Let the eastern line be taken as x-axis, northern as y-axis, and vertical upward as z-axis. Let the velocity v makes angles α, β, and γ with x-, y-, and z-axes, respectively. Then $\alpha = 60°$ and $\gamma = 60°$.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

We have $\cos^2 60° + \cos^2\beta + \cos^2 60° = 1$

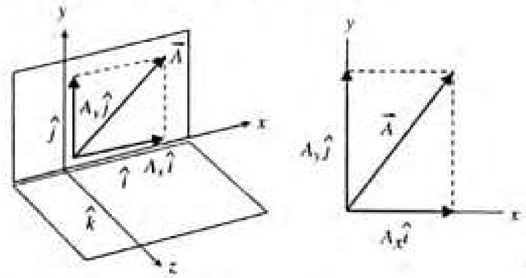
or $\quad \cos\beta = \frac{1}{\sqrt{2}}$

$$\vec{v} = v\cos\alpha\,\hat{i} + v\cos\beta\,\hat{j} + v\cos\gamma\,\hat{k}$$

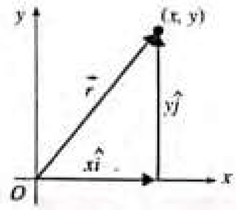
$$= 20\left[\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}\right] = 10\hat{i} + 10\sqrt{2}\,\hat{j} + 10\hat{k}$$

POSITION VECTOR

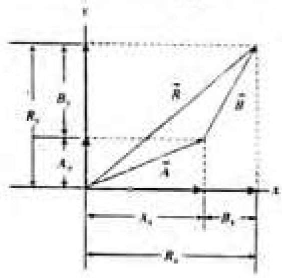
Consider a vector $\vec{A}$ lying in the xy plane as shown in the figure. The product of the component A_x and the unit vector $\hat{i}$ is the component vector $\vec{A}_x = A_x\hat{i}$, which lies on the x-axis and has magnitude $|A_x|$. Likewise, $\vec{A}_y = A_y\hat{j}$ is the component vector of magnitude $|A_y|$ lying on the y-axis. Therefore, the unit vector notation for the vector $\vec{A}$ is $\vec{A} = A_x\hat{i} + A_y\hat{j}$.



For example, consider a point lying in the xy plane and having Cartesian coordinates (x, y) as in the figure. The point can be specified by the position vector $\vec{r}$, which in unit vector form is given by $\vec{r} = x\hat{i} + y\hat{j}$. This notation tells us that the components of $\vec{r}$ are the coordinates x and y.



Now let us see how to use components to add vectors when the graphical method is not sufficiently accurate. Suppose we wish to add vector $\vec{B}$ to vector $\vec{A}$ as shown in the figure, where vector $\vec{B}$ has components B_x and B_y. Because of the book-keeping convenience of the unit vectors, all we do is add the x and y components separately. The resultant vector $\vec{R} = \vec{A} + \vec{B}$ is



$$\vec{R} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j})$$

$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Because $\vec{R} = R_x\hat{i} + R_y\hat{j}$, we see that the components of the resultant vector are $R_x = A_x + B_x$ and $R_y = A_y + B_y$.

Therefore, we see that in the component method of adding vectors, we add all x components together to find the x component of the resultant vector and use the same process for y components. We can check this addition by components with a geometric construction as shown in the figure.

The magnitude of $\vec{R}$ and the angle it makes with the x-axis are obtained from its components using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

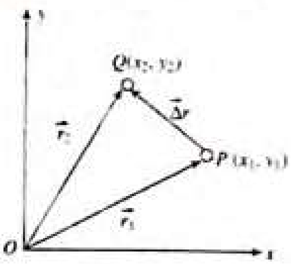
$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x}$$

DISPLACEMENT VECTOR

Displacement vector is a vector which gives the position of a point with reference to a point other than the origin of the co-ordinate system.

It gives the position with reference to a point other than the origin.

Applying the triangle law of vectors, we get



$$\vec{r}_1 + \overrightarrow{\Delta r} = \vec{r}_2 \quad \text{or} \quad \overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1$$

Thus, displacement vector is merely the difference of two position vectors.

If (x_1, y_1) and (x_2, y_2) are the co-ordinates of P and Q, respectively, then $\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$ and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$.

$$\therefore \quad \overrightarrow{\Delta r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

$$|\overrightarrow{\Delta r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Generalizing this result for three dimensions, we get

$$|\overrightarrow{\Delta r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Illustration 2.9 A particle is initially at point $A(2, 4, 6)$ m moves finally to the point $B(3, 2, -3)$ m. Write the initial position vector, final position vector, and displacement vector of the particle.

Solution. Initial position vector: $\vec{r}_1 = 2\hat{i} + 4\hat{j} + 6\hat{k}$

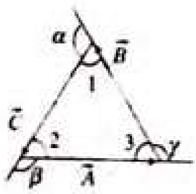
Final position vector: $\vec{r}_2 = 3\hat{i} + 2\hat{j} - 3\hat{k}$

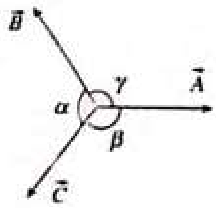
Displacement: $\vec{d} = \vec{r}_2 - \vec{r}_1 = (3-2)\hat{i} + (2-4)\hat{j} + (-3-6)\hat{k}$

$$= \hat{i} - 2\hat{j} - 9\hat{k}$$

Lami's Theorem

It states that if the resultant of three vectors is zero, then the magnitude of a vector is directly proportional to the sine of angle between other two vectors (see figure). Or it can be stated as if the resultant of three vectors is zero, then the ratio of magnitude of a vector to the sine of angle between other two vectors is constant, i.e.,

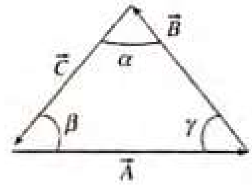




$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Illustration 2.10 Given that $\vec{A} + \vec{B} + \vec{C} = \vec{0}$. Out of three vectors, two are equal in magnitude and the magnitude of the third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Find the angles between the vectors.

Solution. Given



$$A = B,\ C = \sqrt{2}A = \sqrt{2}B.$$

From the figure, $\alpha = \beta$ and $\alpha + \beta + \gamma = 180°$

$\Rightarrow \quad \gamma = 180° - 2\alpha$

Apply Lami's theorem: $\dfrac{A}{\sin\alpha} = \dfrac{C}{\sin\gamma}$

$$\Rightarrow \quad \frac{A}{\sin\alpha} = \frac{\sqrt{2}A}{\sin(180 - 2\alpha)}$$

$$\Rightarrow \quad \frac{1}{\sin\alpha} = \frac{\sqrt{2}}{\sin 2\alpha}$$

$$\frac{1}{\sin\alpha} = \frac{\sqrt{2}}{2\sin\alpha\cos\alpha} \Rightarrow \cos\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45°$$

$\Rightarrow \quad \beta = 45° \quad$ and $\quad \gamma = 180° - 2\alpha = 90°$

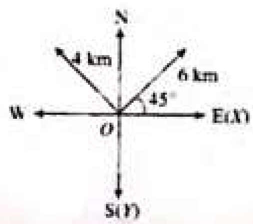
Angle between $\vec{A}$ and $\vec{B}$ = $180° - \gamma = 90°$.

Angle between $\vec{B}$ and $\vec{C}$ = $180 - \alpha = 135°$.

Angle between $\vec{C}$ and $\vec{A}$ = $180° - \beta = 135°$.

CONCEPT APPLICATION EXERCISE 2.1

1. A car travels 6 km towards north at an angle of 45° to the east and then travels distance of 4 km towards north at an angle of 135° to the east (see figure). How far is the point from the starting point? What angle does the straight line joining its initial and final position makes with the east?



2. There are two force vectors, one of 5 N and other of 12 N. At what angle should the two vectors be added

to get the resultant vector of 17 N, 7 N, and 13 N respectively?

3. If $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 6\hat{i} + 8\hat{j}$, then find the magnitude and direction of $\vec{A} + \vec{B}$.
4. A truck travelling due north at 20 m s^{-1} turns west and travels at the same speed. Find the change in its velocity.
5. If the sum of two unit vectors is a unit vector, then find the magnitude of difference.
6. The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitudes of forces?
7. Two forces $F_1 = 1$ N and $F_2 = 2$ N act along the lines $x = 0$ and $y = 0$, respectively. Then find the resultant of forces.
8. Let $\vec{A} = 2\hat{i} + \hat{j}$, $B = 3\hat{j} - \hat{k}$ and $\vec{C} = 6\hat{i} - 2\hat{k}$. Find the value of $\vec{A} - 2\vec{B} + 3\vec{C}$.

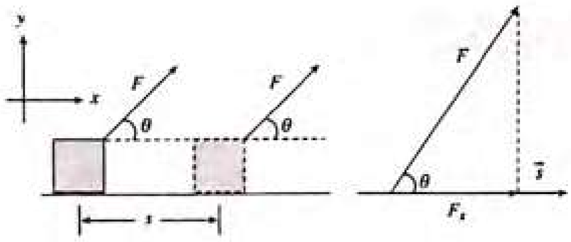
PRODUCT OF TWO VECTORS

Scalar Product of Two Vectors

If we multiply a vector by a scalar produces another vector. The scalar product is different from this multiplication. The scalar product is a way to multiply two vectors to yield a scalar result.

In physics, we defined some scalar quantities such as work, power, flux, etc., as the product of two vectors. For example, work done W by a force $\vec{F}$ is defined as the product of the displacement of a point at which the force is acting and the component of the force along the displacement (direction of motion of the point).

Symbolically, $W = |\vec{F}_x||\vec{s}|$, where $|\vec{F}|$ $|\vec{F}|\cos$.



This gives $W = |\vec{F}||\vec{s}|\cos\theta$. Since the scalar quantity W is defined as a product of two vectors $\vec{F}$ and $\vec{s}$, we can call this product "scalar product."

In short, substituting cos θ by a dot (.), we write $W = \vec{F}\cdot\vec{s}$. Hence the scalar product can also be termed as a "dot product."

There are many instants in physics where we need to express some scalar quantities such as flux, pressure, power as products of two vectors. For this, we need to develop the idea of scalar (dot) product of two vectors.

Note: If, in general, a scalar quantity C is defined as the scalar (or dot) product of any two vectors and $\vec{B}$, C is given as $C = |\vec{A}||\vec{B}|\cos\theta = \vec{A}\cdot\vec{B}$, where θ is the smaller angle between the vectors $\vec{A}$ and $\vec{B}$.

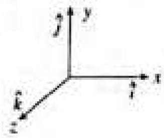
Special Cases

1. If $\theta = 0°$, $\vec{A}\cdot\vec{B} = AB$ (maximum value) [$\because \cos 0° = 1$]
2. If $\theta = 180°$, $\vec{A}\cdot\vec{B} = -AB$ (negative maximum value) [$\because \cos 180° = -1$]
3. If $\theta = 90°$, $\vec{A}\cdot\vec{B} = 0$ (minimum value) [$\because \cos 90° = 0$] So if two vectors are perpendicular, then their dot product is zero.
4. If θ is acute then, $\vec{A}\cdot\vec{B}$ is +ve. [$\because$ cos θ is +ve when θ is acute.]
5. If θ is obtuse, then $\vec{A}\cdot\vec{B}$ is –ve. [$\because$ cos θ is –ve when θ is obtuse.]

NOTE: The dot product of two vectors is always a scalar quantity.

Dot Product of Unit Vectors

The dot product of a unit vector with itself is unity and with other perpendicular unit vector is zero.



$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1 \text{ and}$$

$$\hat{i}.\hat{j} = \hat{j}.\hat{i} = \hat{j}.\hat{k} = \hat{k}.\hat{j} = \hat{k}.\hat{i} = \hat{i}.\hat{k} = 0$$

In component form, the product is expressed as:

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}, \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Then $\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$$= A_x\hat{i}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}).$$

So $\vec{A}\cdot\vec{B} = A_xB_x(\hat{i}.\hat{i}) + A_xB_y(\hat{i}.\hat{j}) + A_xB_z(\hat{i}.\hat{k})$

$$+ A_yB_x(\hat{j}.\hat{i}) + A_yB_y(\hat{j}.\hat{j}) + A_yB_z(\hat{j}.\hat{k})$$

$$+ A_zB_x(\hat{k}.\hat{i}) + A_zB_y(\hat{k}.\hat{j}) + A_yB_z(\hat{k}.\hat{k})$$

$$= A_xB_x + A_yB_y + A_zB_z$$

Properties of Dot Product

Dot product is commutative: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Applying the commutative property to unit vectors, we get

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i}, \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j}, \hat{k}\cdot\hat{i} = \hat{i}\cdot\hat{k}$$

Dot product is distributive: $\vec{A}(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

This may be extended to any number of vectors.

$$\vec{A}\cdot(\vec{B}+\vec{C}+\vec{D}+\cdots)=\vec{A}\cdot\vec{B}+\vec{A}.\vec{C}+\vec{A}\cdot\vec{D}+\cdots$$

Dot product of perpendicular vectors is zero

Proof. If $\vec{A}$ is perpendicular to $\vec{B}$, then $\theta = 90°$.

$$\vec{A}\cdot\vec{B} = AB\cos 90° = 0$$

This leads us to the following condition of perpendicularity of two vectors.

"Two given non-zero vectors will be perpendicular to each other if and only if their dot product is zero."

Applying the result to unit vectors, we get

$$\hat{i}\cdot\hat{j}=0,\ \hat{j}\cdot\hat{k}=0, \text{ and } \hat{k}\cdot\hat{i}=0.$$

Dot product of a vector with itself: A vector is parallel to itself. So, angle of a vector with itself is zero.

$\therefore$ $\quad \vec{A}\cdot\vec{A} = A\,A\cos 0° = A^2 \qquad [\because \cos 0° = 1]$

Hence, the dot product of a vector with itself is the square of its magnitude.

We can also write: $\vec{A}\cdot\vec{A}=|\vec{A}|^2 \Rightarrow |\vec{A}|=\sqrt{\vec{A}\cdot\vec{A}}$

For example: $|\vec{P}+\vec{Q}|=\sqrt{(\vec{P}+\vec{Q})\cdot(\vec{P}+\vec{Q})}$

(taking $\vec{A}=\vec{P}+\vec{Q}$)

$$\Rightarrow\quad |\vec{P}+\vec{Q}|^2=(\vec{P}+\vec{Q})\cdot(\vec{P}+\vec{Q})$$

$$= P^2+Q^2+2\vec{P}\cdot\vec{Q} = P^2+Q^2+2PQ\cos\theta$$

Similarly, $|\vec{P}-\vec{Q}|^2 = P^2+Q^2-2\vec{P}\cdot\vec{Q}$

$$= P^2+Q^2-2PQ\cos\theta$$

Important Results

1. Angle between two vectors can be calculated from:

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{AB}=\frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$$

2. If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ is 90°

Proof: Given $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$

Squaring both sides, we get $|\vec{A}+\vec{B}|^2=|\vec{A}-\vec{B}|^2$

$$\Rightarrow\quad A^2+B^2+2\vec{A}\cdot\vec{B}=A^2+B^2-2\vec{A}\cdot\vec{B}\Rightarrow 4\vec{A}\cdot\vec{B}=0$$

$$\Rightarrow\quad \vec{A}\cdot\vec{B}=0$$

Hence, $\vec{A}$ is perpendicular to $\vec{B}$.

3. If $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=0$, then $\vec{A}$ and $\vec{B}$ are equal in magnitude, i.e., $A = B$.

Proof: Given $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B})=0$

$$\Rightarrow\quad \vec{A}\cdot\vec{A}-\vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{A}-\vec{B}\cdot\vec{B}=0$$

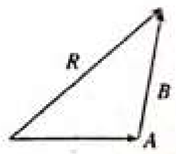
$$\Rightarrow\quad A^2-B^2=0$$

$$\Rightarrow\quad A^2=B^2$$

$\Rightarrow\quad A=B \qquad$ (Hence proved)

4. We can find the addition of two vectors using dot product.

In the figure, $\vec{R}=\vec{A}+\vec{B}$



$$\Rightarrow\quad |\vec{R}|=|\vec{A}+\vec{B}|$$

or $\quad |\vec{R}|^2=|\vec{A}+\vec{B}|^2$

or $\quad R^2=(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B}) = A^2+B^2+2\vec{A}\cdot\vec{B}$

or $\quad R=\sqrt{A^2+B^2+2\vec{A}\cdot\vec{B}}=\sqrt{A^2+B^2+2AB\cos\theta}$

Similarly, we can find subtraction of two vectors also.

Illustration 2.11 Find the value of m so that the vector $3\hat{i}-2\hat{j}+\hat{k}$ may be perpendicular to the vector $2\hat{i}+6\hat{j}+m\hat{k}$.

Solution. The given vectors will be perpendicular if their dot product is zero.

$$(3\hat{i}-2\hat{j}+\hat{k})\cdot(2\hat{i}+6\hat{j}+m\hat{k})=0$$

$$6(\hat{i}\cdot\hat{i})-12(\hat{j}\cdot\hat{j})+m(\hat{k}\cdot\hat{k})=0$$

or $\quad 6-12+m=0$

or $\quad m-6=0$ or $m=6$

Illustration 2.12 What is the angle between the following pair of vectors ?

$$\vec{A}=\hat{i}+\hat{j}+\hat{k} \text{ and } \vec{B}=-2\hat{i}-2\hat{j}-2\hat{k}.$$

Solution. $\vec{A}\cdot\vec{B}=AB\cos\theta$ or $\cos\theta=\dfrac{\vec{A}.\vec{B}}{AB}$ ...(i)

But, $\quad \vec{A}\cdot\vec{B}=(\hat{i}+\hat{j}+\hat{k})\cdot(-2\hat{i}-2\hat{j}-2\hat{k})$

$$\vec{A}\cdot\vec{B}=-2-2-2=-6$$

Again $A=|\vec{A}|=\sqrt{(1)^2+(1)^2+(1)^2}=\sqrt{3}$;

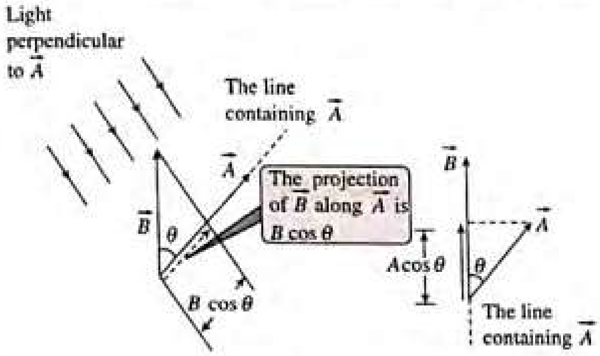
$$B=|\vec{B}|=\sqrt{(-2)^2+(-2)^2+(-2)^2}=\sqrt{12}=2\sqrt{3}$$

Now, $\cos\theta=\dfrac{-6}{\sqrt{3}\times 2\sqrt{3}}=-1$

$$\Rightarrow\quad \theta=180°$$

Geometric Interpretation of Dot Product

In order to geometrically interpret the scalar product, we draw $\vec{A}$ and $\vec{B}$ drawn with their tails together, we drop a perpendicular from the tip of $\vec{B}$ to line containing $\vec{A}$ (see figure). The quantity $B\cos\theta$ is called projection of $\vec{B}$ or component of $\vec{B}$ on the line containing $\vec{A}$. Imagine light shinning perpendicular to $\vec{A}$ then the shadow of vector $\vec{B}$ on the line containing $\vec{A}$, has length equal to the projection of $\vec{B}$ or component of $\vec{B}$ on line of $\vec{A}$.



We can also take projection the other way around.

$$\vec{A}\cdot\vec{B} = A(B\cos\theta) = (A\cos\theta)B$$

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

Illustration 2.13 $\hat{i}$ and $\hat{j}$ are unit vectors along x- and y-axes respectively. What is the magnitude and direction of the vectors $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$? What are the components of a vector $\vec{A}=2\hat{i}+3\hat{j}$ along the direction $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

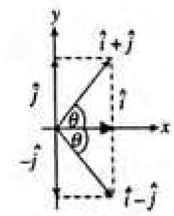
Solution.

(a) $|\hat{i}+\hat{j}| = \sqrt{1^2+1^2+2\times1\times1\times\cos 90^\circ}$

$= \sqrt{2} = 1.414$ units

$\tan\theta = \frac{1}{1} = 1$

$\therefore\ \theta = 45^\circ$



So, the vector $\hat{i}+\hat{j}$ makes an angle of -45° with x-axis.

(b) $|\hat{i}-\hat{j}| = |\hat{i}+(-\hat{j})|$

$= \sqrt{1^2+(-1)^2+2(1)(-1)\cos 90^\circ} = \sqrt{2}$

$\tan\theta = \frac{1}{-1} = -1 \quad \therefore\ \theta = -45^\circ$

The vector $(\hat{i}-\hat{j})$ makes an angle of -45° with x-axis.

Let us now determine the component of $\vec{A} = 2\hat{i}+3\hat{j}$ in the direction of $\hat{i}+\hat{j}$.

Then, $\vec{B} = \hat{i}+\hat{j}$

$\vec{A}.\vec{B} = AB\cos\theta = (A\cos\theta)B$

So component of $\vec{A}$ in the direction of $\vec{B}$.

$$= \frac{\vec{A}.\vec{B}}{B} = \frac{(2\hat{i}+3\hat{j}).(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}} = \frac{2.1+3.1}{\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ units}$$

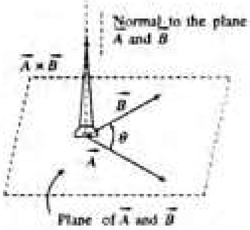
Component of $\vec{A}$ in the direction of $\hat{i}-\hat{j}$

$$= \frac{(2\hat{i}+3\hat{j}).(\hat{i}-\hat{j})}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \text{ units}$$

Here, PM is the component of $\vec{A}$ in the direction of $\vec{B}$. So, dot product of two vectors $\vec{A}$ and $\vec{B}$ may be defined as the product of the magnitude of $\vec{B}$ and the component of $\vec{A}$ in the direction of $\vec{B}$.

Vector Product of Two Vectors

The cross product of two vectors $\vec{A}$ and $\vec{B}$ is equal to the product of the magnitude of $\vec{A}$ and $\vec{B}$ and sine of the shortest angle between them, i.e., $\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$, where $\hat{n}$ is the unit vector which represents the direction of $\vec{A}\times\vec{B}$ and it is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$. It is given by right-handed screw rule depicted in the figure. Note that $\hat{n}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.



Magnitude of $\vec{A}\times\vec{B}$: $|\vec{A}\times\vec{B}| = AB\sin\theta$

From here, we can write: $\vec{A}\times\vec{B} = |\vec{A}\times\vec{B}|\hat{n}$

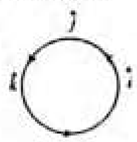
$$\Rightarrow \quad \hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

Unit Vectors and their Cross Product

$\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors along x, y, and z axis, respectively. The magnitude of each vector is 1 and the angle between any of two unit vectors is 90°.

So $\hat{i}\times\hat{j} = (1)(1)\sin 90^\circ\ \hat{n} = \hat{n}$, where $\hat{n}$ is a unit vector perpendicular to the plane containing vector, $\hat{i}$ and $\hat{j}$.

To find out the resultant of any unit vector in cross product, use the following rule (see figure).



1. Multiplication of any two unit vectors in anticlockwise direction gives the third unit vector with positive sign.
2. Multiplication of any two unit vectors in clockwise direction gives the third unit vector with negative sign.

From these rules, we obtain the following results:

From Rule 1

1. $\hat{i} \times \hat{j} = \hat{k}$ 2. $\hat{j} \times \hat{k} = \hat{i}$
3. $\hat{k} \times \hat{i} = \hat{j}$

From Rule 2

1. $\hat{j} \times \hat{i} = -\hat{k}$ 2. $\hat{k} \times \hat{j} = -\hat{i}$
3. $\hat{i} \times \hat{k} = -\hat{j}$

Cross product

$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_xB_x(\hat{i}\times\hat{i}) + A_yB_x(\hat{j}\times\hat{i}) + A_zB_x(\hat{k}\times\hat{i})$$

$$+A_xB_y(\hat{i}\times\hat{j}) + A_yB_y(\hat{j}\times\hat{j}) + A_zB_y(\hat{k}\times\hat{j})$$

$$+A_xB_z(\hat{i}\times\hat{k}) + A_yB_z(\hat{j}\times\hat{k}) + A_zB_z(\hat{k}\times\hat{k})$$

[As $\hat{i}\times\hat{i} = 0,\ \hat{j}\times\hat{j} = 0,\ \hat{k}\times\hat{k} = 0$ and $\hat{i}\times\hat{j} = \hat{k}$,

$\hat{j}\times\hat{i} = -\hat{k}, \hat{k}\times\hat{i} = \hat{j}, \hat{i}\times\hat{k} = -\hat{j}, \hat{j}\times\hat{k} = \hat{i}\ \hat{k}\times\hat{j} = -\hat{i}$]

So, we have $\vec{A} \times \vec{B}$

$$= A_yB_x(-\hat{k}) + A_zB_x\hat{j} + A_xB_y\hat{k}$$

$$+ A_zB_y(-\hat{i}) + A_xB_z(-\hat{j}) + A_yB_z(\hat{i})$$

$$= (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

Properties of Vector Cross Product

Anti commutative property: The vector product of two vectors is anti commutative

$$\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n} \text{ and } \vec{B} \times \vec{A} = BA \sin\theta(-\hat{n})$$

$$= -AB \sin\theta\, \hat{n} = -(\vec{A} \times \vec{B})$$

So $\vec{B} \times \vec{A} = -(\vec{A} \times \vec{B})$

It means $\vec{B} \times \vec{A} \neq \vec{A} \times \vec{B}$

Distributive property: Vector product is distributive, i.e.,

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

Associative Property:

$$(\vec{A} + \vec{B}) \times (\vec{C} + \vec{D}) = \vec{A} \times \vec{C} + \vec{A} \times \vec{D} + \vec{B} \times \vec{C} + \vec{B} \times \vec{D}$$

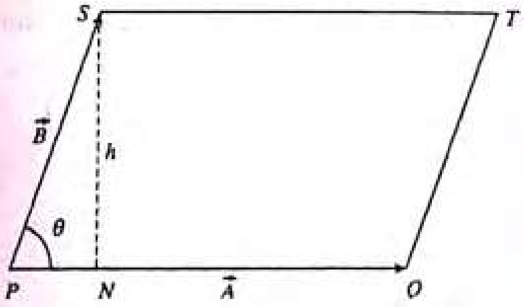
Cross product of two parallel vectors: Cross product of the parallel vectors is zero.

As $\theta = 0°$ (for parallel vectors)

So $(\vec{A} \times \vec{B}) = AB \sin 0°\, \hat{n} = 0$

Important Results

If two vectors represent the two adjacent sides of a parallelogram, then the magnitude of the cross product will give the area of the parallelogram. Mathematically:



Two vectors $\vec{A}$ and $\vec{B}$ are represented by the two adjacent sides PQ and PS, respectively, of the parallelogram as shown in the figure.
Now, from magnitude of cross product:

$$|\vec{A} \times \vec{B}| = AB\sin\theta = Ah$$

Illustration 2.14

(a) Prove that the vectors $\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = \hat{i} - 3\hat{j} + 5\hat{k}$, and $\vec{C} = 2\hat{i} + \hat{j} - 4\hat{k}$ form a right-angled triangle.

(b) Determine the unit vector parallel to the cross product of the vectors $\vec{A} = 3\hat{i} - 5\hat{j} + 10\hat{k}$ & $\vec{B} = 6\hat{i} + 5\hat{j} + 2\hat{k}$.

Solution.

(a) The given vectors will constitute a triangle only if one of the given vectors is equal to vector sum of the remaining two vectors. In the given problem, $\vec{B} + \vec{C} = \vec{A}$
So, the given vectors do form a triangle. This triangle will be right-angled only if the dot product of two vectors (out of the given three) is zero.

$$\vec{A}\cdot\vec{B} = (3\hat{i} - 2\hat{j} + \hat{k})\cdot(\hat{i} - 3\hat{j} + 5\hat{k})$$

$$= 3(\hat{i}\cdot\hat{i}) + 6(\hat{j}\cdot\hat{j}) + 5(\hat{k}\cdot\hat{k}) = 3 + 6 + 5 = 14$$

$$\vec{B}\cdot\vec{C} = (\hat{i} - 3\hat{j} + 5\hat{k})\cdot(2\hat{i} + \hat{j} - 4\hat{k})$$

$$= 2(\hat{i}\cdot\hat{i}) - 3(\hat{j}\cdot\hat{j}) - 20(\hat{k}\cdot\hat{k}) = 2 - 3 - 20 = -21$$

$$\vec{C}\cdot\vec{A} = (2\hat{i} + j - 4\hat{k})\cdot(3\hat{i} - 2\hat{j} + \hat{k})$$

$$= 6(\hat{i}\cdot\hat{i}) - 2(\hat{j}\cdot\hat{j}) - 4(\hat{k}\cdot\hat{k}) = 6 - 2 - 4 = 0$$

Since the dot product of $\vec{C}$ and $\vec{A}$ is zero, therefore, it implies that $\vec{C}$ is perpendicular to $\vec{A}$.

(b) The unit vector parallel to $(\vec{A} \times \vec{B})$ is given by

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

So, let us first determine $\vec{A}\times\vec{B}$.

Now $\vec{A}\times\vec{B} = (3\hat{i}-5\hat{j}+10\hat{k})\times(6\hat{i}+5\hat{j}+2\hat{k})$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -5 & 10\\ 6 & 5 & 2\end{vmatrix}$$

$$=\hat{i}(-10-50)+\hat{j}(60-6)+\hat{k}(15+30)$$

$$=-60\hat{i}+54\hat{j}+45\hat{k}$$

Magnitude: $|\vec{A}\times\vec{B}|=\sqrt{(-60)^2+(54)^2+(45)^2}=\sqrt{8541}$

So, required unit vector: $\hat{n}=\dfrac{-60\hat{i}+54\hat{j}+45\hat{k}}{\sqrt{8541}}$

Concept Application Exercise 2.2

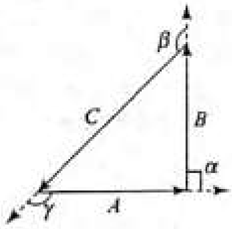
1. $\vec{A}=2\hat{i}+4\hat{j}+4\hat{k}$ and $\vec{B}=4\hat{i}+2\hat{j}-4\hat{k}$ are two vectors. Find the angle between them.
2. If two vectors $2\hat{i}+3\hat{j}-\hat{k}$ and $-4\hat{i}-6\hat{j}-\lambda\hat{k}$ are parallel to each other, then find the value of λ.
3. In Q. 2, if vectors are perpendicular to each other then find the value of λ.
4. If $\vec{A}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{B}=-\hat{i}+3\hat{j}+4\hat{k}$, then find the projection of $\vec{A}$ on $\vec{B}$.
5. A body, acted upon by a force of 50 N, is displaced through a distance 10 m in a direction making an angle of 60° with the force. Find the work done by the force.
6. A particle moves from position $3\hat{i}+2\hat{j}-6\hat{k}$ to $14\hat{i}+13\hat{j}+9\hat{k}$ due to a uniform force of $4\hat{i}+\hat{j}+3\hat{k}$ N. If the displacement is in meters, then find the work done.
7. If for two vectors $\vec{A}$ and $\vec{B}$, sum $(\vec{A}+\vec{B})$ is perpendicular to the difference $(\vec{A}-\vec{B})$. Find the ratio of their magnitude.

Solved Examples

1. Given that $\vec{A}+\vec{B}+\vec{C}=0$ out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then the angles between vectors are given by

(a) 30°, 60°, 90° (b) 45°, 45°, 90°
(c) 45°, 60°, 90° (d) 90°, 135°, 135°

Sol. (d) From polygon law, three vectors having summation zero should form a closed polygon (Triangle). Since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. That is, the triangle should be right angled triangle



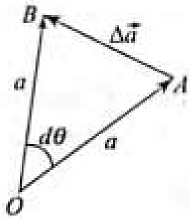
Angle between A and B, $\alpha = 90°$
Angle between B and C, $\beta = 135°$
Angle between A and C, $\gamma = 135°$

2. A vector $\vec{a}$ is turned without a change in its length through a small angle $d\theta$. The value of $|\Delta\vec{a}|$ and Δa are respectively

(a) $0, a\,d\theta$ (b) $a\,d\theta, 0$
(c) 0, 0 (d) None of these

Sol. (b) From the figure $|\vec{OA}|=a$ and $|\vec{OB}|=a$

Also from triangle rule $\vec{OB}-\vec{OA}=\vec{AB}=\Delta\vec{a}$



$\Rightarrow \quad |\Delta\vec{a}|=AB$

Using angle $=\dfrac{\text{arc}}{\text{radius}}$

$\Rightarrow \quad AB=a\cdot d\theta$

So $\quad |\Delta\vec{a}|=a\,d\theta$

Δa means change in magnitude of vector, i.e., $|\vec{OB}|-|\vec{OA}|$

$\Rightarrow \quad a-a=0$

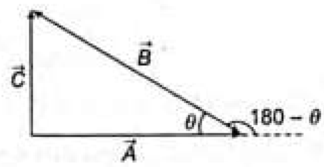
So $\quad \Delta a=0$

3. Given that $\vec{A}+\vec{B}=\vec{C}$ and that $\vec{C}$ is perpendicular to $\vec{A}$. Further if $|\vec{A}|=|\vec{C}|$, then what is the angle between $\vec{A}$ and $\vec{B}$?

(a) $\dfrac{\pi}{4}$ radian (b) $\dfrac{\pi}{2}$ radian
(c) $\dfrac{3\pi}{4}$ radian (d) π radian

Sol. (c) $A=C$

$\tan\theta=\dfrac{C}{A}=1$



$\Rightarrow \quad \theta=45°=\dfrac{\pi}{4}$

Angle between $\vec{A}$ and $\vec{B}$ is

$$180-\theta=180°-45°=135°=\frac{3\pi}{4}\text{rad}$$

4. Which pair of the following forces will never give resultant force of 2 N?

(a) 2 N and 2 N (b) 1 N and 1 N
(c) 1 N and 3 N (d) 1 N and 4 N

Sol. (d) If two vectors A and B are given then range of their resultant can be written as $(A-B)\le R\le(A+B)$.

i.e. $\quad R_{max}=A+B$ and $R_{min}=A-B$

If $B = 1$ and $A = 4$ then their resultant will lies in between 3 N and 5 N. It can never be 2 N.

If these three vectors are represented by three sides of triangle then they form equilateral triangle.

5. If the sum of two unit vectors is a unit vector, then magnitude of difference is

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) $1/\sqrt{2}$ (d) $\sqrt{5}$

Sol. (b) Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is

$$\hat{n}_s = \hat{n}_1 + \hat{n}_2 \text{ or } n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta$$
$$= 1 + 1 + 2\cos\theta$$

Since it is given that n_s is also a unit vector, therefore

$$1 = 1 + 1 + 2\cos\theta \Rightarrow \cos\theta = -\frac{1}{2}$$

$\therefore \quad \theta = 120°$

Now the difference vector is

$$\hat{n}_d = \hat{n}_1 - \hat{n}_2$$

or $\quad n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1 + 1 - 2\cos(120°)$

$\therefore \quad n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$

6. The sum of the magnitudes of two forces acting at point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitudes of forces?

(a) 12, 5 (b) 14, 4

(c) 5, 13 (d) 10, 8

Sol. (c) Let P be the smaller force and Q be the greater force, then according to the problem

$$P + Q = 18 \quad \text{...(i)}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12 \quad \text{...(ii)}$$

$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90 = \infty$$

$\therefore \quad P + Q\cos\theta = 0 \quad$ ...(iii)

By solving (i), (ii) and (iii) we will get $P = 5$, and $Q = 13$

7. A person moves 30 m north and then 20 m towards east and finally $30\sqrt{2}$ m in south-west direction. The displacement of the person from the origin will be

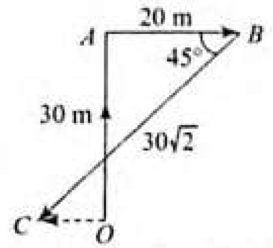
(a) 10 m along north

(b) 10 m long south

(c) 10 m along west

(d) Zero

Sol. (c) From figure, $\overline{OA} = 0\hat{i} + 30\hat{j}, \overline{AB} = 20\hat{i} + 0\hat{j}$



$$\overline{BC} = -30\sqrt{2}\cos 45°\hat{i} - 30\sqrt{2}\sin 45°\hat{j} = -30\hat{i} - 30\hat{j}$$

$\therefore \quad$ Net displacement, $\overline{OC} = \overline{OA} + \overline{OB} + \overline{BC} = -10\hat{j} + 0\hat{j}$

$|\overline{OC}| = 10$ m

8. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$. Then the value of α is

(a) -1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) 1

Sol. (c) Given vectors can be rewritten as $\vec{A} = 2\hat{i} + 3\hat{j} + 8\hat{k}$ and $\vec{B} = -4\hat{i} + 4\hat{j} + \alpha\hat{k}$

Dot product of these vectors should be equal to zero because they are perpendicular.

$\therefore \quad \vec{A}\cdot\vec{B} = -8 + 12 + 8\alpha = 0 \Rightarrow 8\alpha = -4 \Rightarrow \alpha = -1/2$

9. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

(a) are equal to each other in magnitude

(b) are not equal to each other in magnitude

(c) cannot be predicted

(d) are equal to each other

Sol. (a) If two vectors are perpendicular then their dot product must be equal to zero. According to problem

$$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0 \Rightarrow \vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$$

$\Rightarrow \quad A^2 - B^2 = 0 \Rightarrow A^2 = B^2$

$\therefore \quad A = B$ i.e., two vectors are equal to each other in magnitude.

10. If for two vector $\vec{A}$ and $\vec{B}$, sum $(\vec{A} + \vec{B})$ is perpendicular to the difference $(\vec{A} - \vec{B})$. The ratio of their magnitude is

(a) 1 (b) 2

(c) 3 (d) None of these

Sol. (a) $(\vec{A} + \vec{B})$ is perpendicular to $(\vec{A} - \vec{B})$. Thus

$$(\vec{A} + \vec{B})\cdot(\vec{A} - \vec{B}) = 0$$

or $\quad A^2 + \vec{B}\cdot\vec{A} - \vec{A}\cdot\vec{B} - B^2 = 0$

Because of commutative property of dot product $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$\therefore \quad A^2 - B^2 = 0$ or $A = B$

Thus the ratio of magnitudes $A/B = 1$

11. If $\vec{A} \times \vec{B} = \vec{C}$, then which of the following statements is wrong

(a) $\vec{C} \perp \vec{A}$ (b) $\vec{C} \perp \vec{B}$

(c) $\vec{C} \perp (\vec{A} + \vec{B})$ (d) $\vec{C} \perp (\vec{A} \times \vec{B})$

Sol. (d) From the property of vector product, we notice that $\vec{C}$ must be perpendicular to the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$ and $(\vec{A} + \vec{B})$ vector also must lie in the plane formed by vector $\vec{A}$ and $\vec{B}$. Thus $\vec{C}$ must be perpendicular to $(\vec{A} + \vec{B})$ but the cross product $(\vec{A} \times \vec{B})$ gives a vector $\vec{C}$ which cannot be perpendicular to itself. Thus the last statement is wrong.

12. A vector $\vec{A}$ points vertically upward and $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is

(a) Zero (b) Along west

(c) Along east (d) Vertically downward

Sol. (b) Direction of vector A is along z-axis

$\therefore \quad \vec{A} = a\hat{k}$

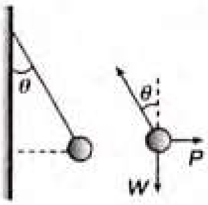
Direction of vector B is towards north

$\therefore \quad \vec{B} = b\hat{j}$

Now $\vec{A} \times \vec{B} = a\hat{k} \times b\hat{j} = ab(-\hat{i})$

$\therefore$ The direction is $\vec{A} \times \vec{B}$ is along west.

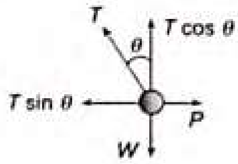
13. A metal sphere is hung by a string fixed to a wall. The sphere is pushed away from the wall by a stick. The forces acting on the sphere are shown in the second diagram. Which of the following statements is wrong?



(a) $P = W \tan \theta$ (b) $\vec{T} + \vec{P} + \vec{W} = 0$

(c) $T^2 = P^2 + W^2$ (d) $T = P + W$

Sol. (c)



As the metal sphere is in equilibrium under the effect of three forces therefore $\vec{T} + \vec{P} + \vec{W} = 0$

From the figure

$$T \cos \theta = W \qquad \text{...(i)}$$

$$T \sin \theta = P \qquad \text{...(ii)}$$

From equations (i) and (ii), we get $P = W \tan \theta$ and $T^2 = P^2 + W^2$

EXERCISES

Vector Addition

1. The magnitudes of vectors $\vec{A}, \vec{B}$ and are 3, 4 and 5 units respectively. If $\vec{A} + \vec{B} = \vec{C}$, the angle between $\vec{A}$ and $\vec{B}$ is

(a) $\frac{\pi}{2}$ (b) $\cos^{-1}(0.6)$

(c) $\tan^{-1}\left(\frac{7}{5}\right)$ (d) $\frac{\pi}{4}$

2. Five equal forces of 10 N each are applied at one point and all are lying in one plane. If the angles between them are equal, the resultant force will be

(a) Zero (b) 10 N

(c) 20 N (d) $10\sqrt{2}$ N

3. A car travels 6 km towards north at an angle of 45° to the east and then travels distance of 4 km towards north at an angle of 135° to the east. How far is the point from the starting point? What angle does the straight line joining its initial and final position makes with the east?

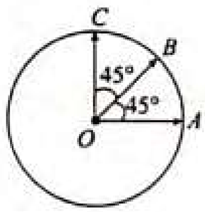
(a) $\sqrt{50}$ km and $\tan^{-1}(5)$

(b) 10 km and $\tan^{-1}(\sqrt{5})$

(c) $\sqrt{52}$ km and $\tan^{-1}(5)$

(d) $\sqrt{52}$ km and $\tan^{-1}(\sqrt{5})$

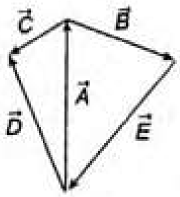
4. Find the resultant of three vectors $\overline{OA}, \overline{OB}$ and $\overline{OC}$ shown in the following figure. Radius of the circle is R.



(a) $2R$ (b) $R(1 + \sqrt{2})$

(c) $R\sqrt{2}$ (d) $R(\sqrt{2} - 1)$

5. In figure, $\vec{E}$ equals



(a) $\vec{A}$ (b) $\vec{B}$
(c) $\vec{A}+\vec{B}$ (d) $-(\vec{A}+\vec{B})$

6. If $\vec{P}=\vec{Q}$ then which of the following is NOT correct
(a) $\hat{P}=\hat{Q}$ (b) $|\vec{P}|=|\vec{Q}|$
(c) $P\hat{Q}=Q\hat{P}$ (d) $\vec{P}+\vec{Q}=\hat{P}+\hat{Q}$

7. Three concurrent forces of the same magnitude are in equilibrium. What is the angle between the forces? Also name the triangle formed by the forces as sides
(a) 60° equilateral triangle
(b) 120° equilateral triangle
(c) 120°, 30°, 30° an isosceles triangle
(d) 120° an obtuse angled triangle

8. A force is inclined at 60° to the horizontal. If its rectangular component in the horizontal direction is 50 N, then magnitude of the vertical component of force is approximately
(a) 25 N (b) 84 N
(c) 87 N (d) 90 N

9. In the following options you are given the magnitudes of three forces in Newton acting simultaneously on a body. Find the set for which the resultant force on the body can be zero.
(a) 10, 8, 2 (b) 15, 30, 14
(c) 40, 19, 17 (d) 10, 20, 35

10. Given that $\vec{A}+\vec{B}=\vec{C}$ and that $\vec{C}$ is perpendicular to $\vec{A}$. Further if $|\vec{A}|=|\vec{C}|$, then what is the angle between $\vec{A}$ and $\vec{B}$?
(a) $\frac{\pi}{4}$ radian (b) $\frac{\pi}{2}$ radian
(c) $\frac{3\pi}{4}$ radian (d) π radian

Resultant of Vectors

11. What is the angle between $\vec{P}$ and the resultant of $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$
(a) Zero (b) $\tan^{-1}(P/Q)$
(c) $\tan^{-1}(Q/P)$ (d) $\tan^{-1}(P-Q)/(P+Q)$

12. A force of 6 kg and another of 8 kg can be applied together to produce the effect of a single force of
(a) 1 kg (b) 9 kg
(c) 15 kg (d) 22 kg

13. There are two force vectors, one of 5 N and other of 12 N at what angle the two vectors be added to get resultant vector of 17 N, 7 N and 13 N respectively
(a) 0°, 180° and 90° (b) 0°, 90° and 180°
(c) 0°, 90° and 90° (d) 180°, 0° and 90°

14. Let $\vec{C}=\vec{A}+\vec{B}$ then
(a) $|\vec{C}|$ is always greater then $|\vec{A}|$
(b) It is possible to have $|\vec{C}|<|\vec{A}|$ and $|\vec{C}|<|\vec{B}|$
(c) C is always equal to $A+B$
(d) C is never equal to $A+B$

15. When two vectors of magnitudes P and Q are inclined at an angle θ the magnitude of their resultant $2P$. When the inclination is changed to $180-\theta$ the magnitude of the resultant is halved. Find the ratio of P to Q.
(a) $\sqrt{2}:\sqrt{3}$ (b) $1:\sqrt{3}$
(c) $1:\sqrt{2}$ (d) $\sqrt{3}:\sqrt{2}$

16. Two forces of magnitudes P and Q are inclined at an angle (θ) the magnitude of their resultant is $3Q$. When the inclination is changed to $(180-\theta)$ the magnitude of the resultant force becomes Q. The ratio of the forces $\left(\frac{P}{Q}\right)$ is
(a) $\frac{4}{1}$ (b) $\frac{2}{1}$
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

17. Two forces, each of magnitude F have a resultant of the same magnitude F. The angle between the two forces is
(a) 45° (b) 120°
(c) 150° (d) 60°

18. The resultant of $\vec{A}+\vec{B}$ is $\vec{R}_1$ On reversing the vector $\vec{B}$ the resultant becomes $\vec{R}_2$. What is the value of $R_1^2+R_2^2$?
(a) A^2+B^2 (b) A^2-B^2
(c) $2(A^2+B^2)$ (d) $2(A^2-B^2)$

19. The sum of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpendicular to minimum force then the forces are
(a) 6 N and 10 N (b) 8 N and 8 N
(c) 4 N and 12 N (d) 2 N and 14 N

20. If vectors P, Q and R have magnitude 5, 12 and 13 units and $\vec{P}+\vec{Q}=\vec{R}$, the angle between Q and R is
(a) $\cos^{-1}\frac{5}{12}$ (b) $\cos^{-1}\frac{5}{13}$
(c) $\cos^{-1}\frac{12}{13}$ (d) $\cos^{-1}\frac{7}{13}$

Expressing Vectors in Unit Vector Notation

21. The expression $\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\right)$ is a
(a) Unit vector
(b) Null vector
(c) Vector of magnitude $\sqrt{2}$
(d) Scalar

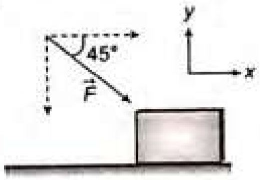
22. The unit vector along $\hat{i}+\hat{j}$ is
(a) $\hat{k}$ (b) $\hat{i}+\hat{j}$
(c) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{j}}{2}$

23. A vector is represented by $3\hat{i}+\hat{j}+2\hat{k}$. Its length in XY plane is
(a) 2 (b) $\sqrt{14}$
(c) $\sqrt{10}$ (d) $\sqrt{5}$

24. The angle made by the vector $A=\hat{i}+\hat{j}$ with x- axis is
(a) 90° (b) 45°
(c) 22.5° (d) 30°

25. If a unit vector is represented by $0.5\hat{i}+0.8\hat{j}+c\hat{k}$, then the value of '$c$' is
(a) 1 (b) $\sqrt{0.11}$
(c) $\sqrt{0.01}$ (d) $\sqrt{0.39}$

26. A person pushes a box kept on a horizontal surface with force of 100 N. In unit vector notation force can be expressed as



(a) $100(\hat{i}+\hat{j})$ (b) $100(\hat{i}-\hat{j})$
(c) $50\sqrt{2}(\hat{i}-\hat{j})$ (d) $50\sqrt{2}(\hat{i}+\hat{j})$

27. Determine a vector which when added to the resultant of $\vec{A}=2\hat{i}+5\hat{j}-\hat{k}$ and $\vec{B}=3\hat{i}-4\hat{j}-\hat{k}$ gives unit vector along negative y direction.
(a) $-5\hat{i}-2\hat{j}+2\hat{k}$ (b) $-5\hat{i}-\hat{j}+\hat{k}$
(c) $-5\hat{i}-\hat{j}+2\hat{k}$ (d) $5\hat{i}-\hat{j}+2\hat{k}$

28. If a particle moves from point P (2,3,5) to point Q (3,4,5). Its displacement vector be
(a) $\hat{i}+\hat{j}+10\hat{k}$ (b) $\hat{i}+\hat{j}+5\hat{k}$
(c) $\hat{i}+\hat{j}$ (d) $2\hat{i}+4\hat{j}+6\hat{k}$

29. Following forces start acting on a particle at rest at the origin of the co-ordinate system simultaneously
$\vec{F}_1=-4\hat{i}-5\hat{j}+5\hat{k}$, $\vec{F}_2=5\hat{i}+8\hat{j}+6\hat{k}$, $\vec{F}_3=-3\hat{i}+4\hat{j}-7\hat{k}$ and $\vec{F}_4=2\hat{i}-3\hat{j}-2\hat{k}$ then the particle will move
(a) In x–y plane (b) In y–z plane
(c) In x–z plane (d) Along x-axis

30. The unit vector parallel to the resultant of the vectors $\vec{A}=4\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{B}=-\hat{i}+3\hat{j}-8\hat{k}$ is
(a) $\frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k})$ (b) $\frac{1}{7}(3\hat{i}+6\hat{j}+2\hat{k})$
(c) $\frac{1}{49}(3\hat{i}+6\hat{j}-2\hat{k})$ (d) $\frac{1}{49}(3\hat{i}-6\hat{j}+2\hat{k})$

Dot Product

31. Angle between the vectors $(\hat{i}+\hat{j})$ and $(\hat{j}-\hat{k})$ is
(a) 90° (b) 0°
(c) 180° (d) 60°

32. Two forces $\vec{F}_1=5\hat{i}+10\hat{j}-20\hat{k}$ and $\vec{F}_2=10\hat{i}-5\hat{j}-15\hat{k}$ act on a single point. The angle between $\vec{F}_1$ and $\vec{F}_2$ is nearly
(a) 30° (b) 45°
(c) 60° (d) 90°

33. Let $\vec{A}=\hat{i}A\cos\theta+\hat{j}A\sin\theta$ be any vector. Another vector $\vec{B}$ which is normal to A is
(a) $\hat{i}B\cos\theta+\hat{j}B\sin\theta$ (b) $\hat{i}B\sin\theta+\hat{j}B\cos\theta$
(c) $\hat{i}B\sin\theta-\hat{j}B\cos\theta$ (d) $\hat{i}B\cos\theta-\hat{j}B\sin\theta$

34. The angles which a vector $\hat{i}+\hat{j}+\sqrt{2}\,\hat{k}$ makes with X, Y and Z axes respectively are
(a) 60°, 60°, 60° (b) 45°, 45°, 45°
(c) 60°, 60°, 45° (d) 45°, 45°, 60°

35. If a vector $\vec{P}$ making angles α, β, and γ respectively with the X, Y and Z axes respectively.
Then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$
(a) 0 (b) 1
(c) 2 (d) 3

36. Consider a vector $\vec{F}=4\hat{i}-3\hat{j}$. Another vector that is perpendicular to $\vec{F}$ is
(a) $4\hat{i}+3\hat{j}$ (b) $6\hat{i}$
(c) $7\hat{k}$ (d) $3\hat{i}-4\hat{j}$

37. If $|\vec{V}_1+\vec{V}_2|=|\vec{V}_1-\vec{V}_2|$ and V_2 is finite, then
(a) V_1 is parallel to V_2
(b) $\vec{V}_1=\vec{V}_2$
(c) V_1 and V_2 are mutually perpendicular
(d) $|\vec{V}_1|=|\vec{V}_2|$

38. If $\vec{P}\cdot\vec{Q} = PQ$, then angle between $\vec{P}$ and $\vec{Q}$ is
(a) $0°$ (b) $30°$
(c) $45°$ (d) $60°$

39. When $\vec{A}\cdot\vec{B} = -|A|\,|B|$ then
(a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other
(b) $\vec{A}$ and $\vec{B}$ act in the same direction
(c) $\vec{A}$ and $\vec{B}$ act in the opposite direction
(d) $\vec{A}$ and $\vec{B}$ can act in any direction

40. The component of vector $A = 2\hat{i} + 3\hat{j}$ along the vector $\hat{i}+\hat{j}$ is
(a) $\dfrac{5}{\sqrt{2}}$ (b) $10\sqrt{2}$
(c) $5\sqrt{2}$ (d) 5

Cross Product

41. Vector $\vec{A}$ makes equal angles with x, y and z axis. Value of its components (in terms of magnitude of $\vec{A}$) will be
(a) $\dfrac{A}{\sqrt{3}}$ (b) $\dfrac{A}{\sqrt{2}}$
(c) $\sqrt{3}A$ (d) $\dfrac{\sqrt{3}}{A}$

42. If for two vectors $\vec{A}$ and $\vec{B}$, $\vec{A}\times\vec{B} = 0$, the vectors
(a) are perpendicular to each other
(b) are parallel to each other
(c) act at an angle of $60°$
(d) act at an angle of $30°$

43. What is the angle between $(\vec{P}+\vec{Q})$ and $(\vec{P}\times\vec{Q})$?
(a) 0 (b) $\dfrac{\pi}{2}$
(c) $\dfrac{\pi}{4}$ (d) π

44. What is the unit vector perpendicular to the following vectors $2\hat{i}+2\hat{j}-\hat{k}$ and $6\hat{i}-3\hat{j}+2\hat{k}$?
(a) $\dfrac{\hat{i}+10\hat{j}-18\hat{k}}{5\sqrt{17}}$ (b) $\dfrac{\hat{i}-10\hat{j}+18\hat{k}}{5\sqrt{17}}$
(c) $\dfrac{\hat{i}-10\hat{j}-18\hat{k}}{5\sqrt{17}}$ (d) $\dfrac{\hat{i}+10\hat{j}+18\hat{k}}{5\sqrt{17}}$

45. If A = 5 units, B = 6 units and $|\vec{A}\times\vec{B}| = 15$ units, then what is the angle between $\vec{A}$ and $\vec{B}$?
(a) $30°$ (b) $60°$
(c) $90°$ (d) $120°$

46. Three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B} = 0$ and $\vec{A}.\vec{C} = 0$. The vector $\vec{A}$ is parallel to
(a) $\vec{B}$ (b) $\vec{C}$
(c) $\vec{B}\times\vec{C}$ (d) $\vec{B}+\vec{C}$

47. If a vector $\vec{A}$ is parallel to another vector $\vec{B}$, then the resultant of the vector $\vec{A}\times\vec{B}$ will be equal to
(a) A (b) $\vec{A}$
(c) Zero vector (d) Zero

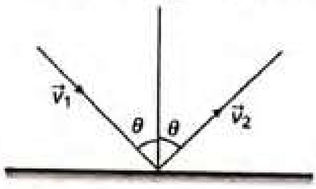
48. Which of the following is the unit vector perpendicular to $\vec{A}$ and $\vec{B}$?
(a) $\dfrac{\hat{A}\times\hat{B}}{AB\sin\theta}$ (b) $\dfrac{\hat{A}\times\hat{B}}{AB\cos\theta}$
(c) $\dfrac{\vec{A}\times\vec{B}}{AB\sin\theta}$ (d) $\dfrac{\vec{A}\times\vec{B}}{AB\cos\theta}$

49. Two vector $\vec{A}$ and $\vec{B}$ have equal magnitudes. Then the vector $\vec{A}+\vec{B}$ is perpendicular to
(a) $\vec{A}\times\vec{B}$ (b) $\vec{A}-\vec{B}$
(c) $3\vec{A}-3\vec{B}$ (d) All of these

50. The value of $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$ is
(a) 0 (b) $A^2 - B^2$
(c) $\vec{B}\times\vec{A}$ (d) $2(\vec{B}\times\vec{A})$

Problems Based on Mixed Concepts

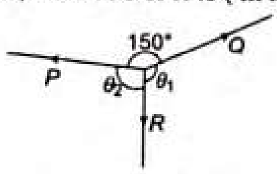
51. An object of m kg with speed of v m/s strikes a wall at an angle θ and rebounds at the same speed and same angle. The magnitude of the change in momentum of the object will be



(a) $2\,mv\cos\theta$ (b) $2\,mv\sin\theta$
(c) 0 (d) $2\,mv$

52. Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy the relation $\vec{a}\cdot\vec{b} = 0$ and $\vec{a}.\vec{c} = 0$. The vector $\vec{a}$ is parallel to
(a) $\vec{b}$ (b) $\vec{c}$
(c) $\vec{b}.\vec{c}$ (d) $\vec{b}\times\vec{c}$

53. P, Q and R are three coplanar forces acting at a point and are in equilibrium. Given P = 1.9318 kg wt, $\sin\theta_1 = 0.9659$, the value of R is (in kg wt.)



(a) 0.9659 (b) 2
(c) 1 (d) $\dfrac{1}{2}$

54. If the resultant of two forces of magnitude p and $2p$ is perpendicular to p, then the angle between the forces is
   (a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$
   (c) $\frac{4\pi}{5}$ (d) $\frac{5\pi}{6}$

55. A car is moving on a straight road due north with a uniform speed of 50 km h^{-1} when it turns left through 90°. If the speed remains unchanged after turning, the change in the velocity of the car in the turning process is
   (a) zero
   (b) $50\sqrt{2}$ km h^{-1} S-W direction
   (c) $50\sqrt{2}$ km h^{-1} N-W direction
   (d) 50 km h^{-1} due west

56. What can be the angle between $(\vec{P}+\vec{Q})$ and $(\vec{P}-\vec{Q})$?
   (a) 90°
   (b) 0° only
   (c) any angle between 0° and 180°
   (d) 180° only

57. A sail boat sails 2 km due East, 5 km 37° South of East and finally an unknown displacement. If the final displacement of the boat from the starting point is 6 km due East, determine the third displacement.
   (a) 3 km, North (b) 4 km, South
   (c) 5 km, East (a) 3 km, West

58. A particle travels with speed 50 m/s from the point (3 m, −7 m) in a direction $7\hat{i}-24\hat{j}$. Find its position vector after 3 seconds.
   (a) $(45\hat{i}-125\hat{j})$ m (b) $(45\hat{i}-151\hat{j})$ m
   (c) $(45\hat{i}+125\hat{j})$ m (b) $(35\hat{i}-115\hat{j})$ m

59. A particle moves so that its position vector varies with time as $\vec{r}=A\cos\omega t\,\hat{i}+A\sin\omega t\,\hat{j}$. The initial velocity of the particle is
   (a) $A\omega\hat{i}$ (b) $A\omega\hat{j}$
   (c) $A\omega(\hat{i}+\hat{j})$ (d) $A\omega(\hat{i}-\hat{j})$

60. A vector of magnitude 10 N acting in XY-plane has components 8 N and 6 N along positive X-axis and positive Y-axis, respectively. The coordinate system is rotated about Z-axis through angle 90° in anti-clockwise direction. Find x-component and y-component in new coordinate system.
   (a) $F_x = 8\,\text{N}, F_y = 6\,\text{N}$
   (b) $F_x = 6\,\text{N}, F_y = 8\,\text{N}$
   (c) $F_x = 6\,\text{N}, F_y = -8\,\text{N}$
   (d) $F_x = 6\,\text{N}, F_y = -8\,\text{N}$

ARCHIVES

1. A particle moves towards east with velocity 5 m/s. After 10 seconds its direction changes towards north with same velocity. The average acceleration of the particle is
   (a) Zero (b) $\frac{1}{\sqrt{2}}$ m/s^2 N – W
   (c) $\frac{1}{\sqrt{2}}$ m/s^2 N – E (d) $\frac{1}{\sqrt{2}}$ m/s^2 S – W
   (IIT-JEE 1982)

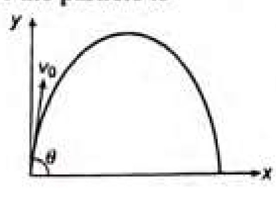
2. A force $\vec{F}=-K(y\hat{i}+x\hat{j})$ (where K is a positive constant) acts on a particle moving in the x-y plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and then parallel to the y-axis to the point (a, a). The total work done by the forces $\vec{F}$ on the particle is
   (a) $-2Ka^2$ (b) $2Ka^2$
   (c) $-Ka^2$ (d) Ka^2 **(IIT-JEE 1998)**

3. The sum of the magnitudes of two forces acting at point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the, magnitudes of forces
   (a) 12, 5 (b) 14, 4
   (c) 5, 13 (d) 10, 8
   (AIEEE 2002)

4. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$ then the angle between A and B is
   (a) $\pi/2$ (b) $\pi/3$
   (c) π (d) $\pi/4$ **(AIEEE 2004)**

5. A particle has an initial velocity $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s
   (a) 10 units (b) $7\sqrt{2}$ units
   (c) 7 units (d) 8.5 units
   (AIEEE 2009)

6. A small particle of mass m is projected at an angle q with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t<\frac{v_0\sin\theta}{g}$, the angular momentum of the particle is

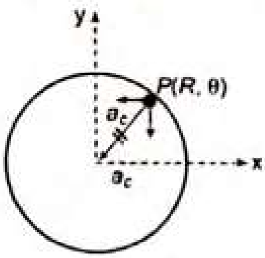


(a) $-mgv_0t^2\cos\theta\,\hat{j}$ (b) $mgv_0t\cos\theta\,\hat{k}$

(c) $-\frac{1}{2}mgv_0t^2\cos\theta\,\hat{k}$ (d) $\frac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$

(AIEEE 2010)

7. For a particle in uniform circular motion the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (here θ is measured from the x-axis)



(a) $-\frac{v^2}{R}\cos\theta\,\hat{i}+\frac{v^2}{R}\sin\theta\,\hat{j}$

(b) $-\frac{v^2}{R}\sin\theta\,\hat{i}+\frac{v^2}{R}\cos\theta\,\hat{j}$

(c) $-\frac{v^2}{R}\cos\theta\,\hat{i}-\frac{v^2}{R}\sin\theta\,\hat{j}$

(d) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$ **(AIEEE 2010)**

8. A particle is moving with velocity $\vec{v}=K(y\hat{i}+x\hat{j})$, where K is a constant. The general equation for its path is

(a) $y=x^2+\text{constant}$ (b) $y^2=x+\text{constant}$

(c) $xy=\text{constant}$ (d) $y^2=x^2+\text{constant}$

(AIEEE 2010)

9. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10$ m/s^2, the equation of its trajectory is

(a) $y=x-5x^2$ (b) $y=2x-5x^2$

(c) $4y=2x-5x^2$ (d) $4y=2x-25x^2$

(JEE Main 2013)

ANSWER KEY

Exercises

1. (a)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (c)
11. (a)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (c)
21. (a)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (c)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (a)
31. (d)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (c)	**36.** (c)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (a)
41. (a)	**42.** (b)	**43.** (b)	**44.** (c)	**45.** (a)	**46.** (c)	**47.** (c)	**48.** (c)	**49.** (a)	**50.** (d)
51. (a)	**52.** (d)	**53.** (c)	**54.** (a)	**55.** (b)	**56.** (c)	**57.** (a)	**58.** (b)	**59.** (b)	**60.** (c)

Archives

1. (b)	**2.** (c)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (d)	**9.** (a)

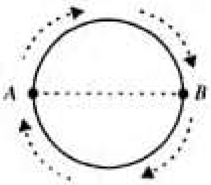
Chapter 3

Motion in One Dimension

DISPLACEMENT AND DISTANCE

When we speak of distance, we should not mistake it as displacement. These two are entirely different quantities. Displacement is a vector quantity. Its magnitude is equal to the "shortest distance" between the initial position and final position of the particle.

ILLUSTRATION 3.1 A particle is moving in a circle of radius R as shown in figure.



(a) What is its displacement when it covers (i) half the circle, (ii) full circle?

(b) What is its distance when it covers (i) half the circle and (ii) full circle?

Solution.

(a) Displacement is the vector drawn from the initial position to the final position, and its magnitude is equal to the shortest distance between the initial and final positions.

(i) Hence, displacement is equal to the diameter of the circle = $2R$.

(ii) As initial and final positions are same, i.e., displacement will be zero.

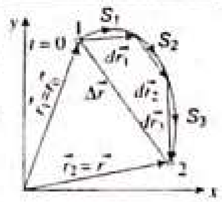
(b) Distance is the length of the path travelled by the particle. Hence, distance travelled in case (i) will be equal to πR. Hence, distance travelled in case (ii) will be equal to $2\pi R$.

Important Results

- Distance is a scalar quantity and displacement is a vector quantity.
- Distance between a given set of initial and final positions can have infinite values but displacement is unique.
- Displacement can be negative, zero, or positive, but distance is never negative. When a body returns to its initial position of starting, its displacement is zero but distance or path length is not zero.
- The magnitude of displacement can never be greater than distance.
- In uniform motion, displacement is equal to distance.
- Displacement is zero if particle returns to initial position.
- Distance does not decrease with time and never be a zero for a moving body.
- Displacement can decrease with time, can be zero or even be negative if the body is returning to its initial position, got initial position, and moves back to the initial position. So the magnitude of displacement is not equal to distance; however, it can be so if the motion is along a straight line without change in direction.
- In general, distance ≥ displacement.

AVERAGE VELOCITY AND AVERAGE SPEED

Average velocity: If at any time t_1 position vector of the particle is and at time t_2 position is $\vec{r}_2$.
Hence, on an average, the position vector changes with time at a rate $\Delta\vec{r}/\Delta t$.



$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

To find the average velocity, we need to know only the total displacement from initial to final position and need not consider the nature of motion between initial and final positions.

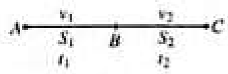
Physical meaning of average velocity: It is that uniform velocity with which if the object is made to move, it will cover the same displacement in a given time as it does with its actual velocity in the same time.

Average speed: If the particle is going a distance D during the time $\Delta t(= t_2 - t_1)$, the average rate of distance covered by it is $D/\Delta t$ for this time interval. We can call this ratio, "average speed" denoted by v_{av}. It is the ratio of total distance d travelled by the particle to the total time taken t in which this distance is travelled.

$$v_{av} = \frac{\text{Total distance}}{\text{Total time taken}} = \frac{s_1 + s_2 + s_3}{\Delta t}$$

NOTE: If motion takes place in same direction, then average speed and average velocity are same.

In the figure, a particle goes from A to C. Distances, velocities and time taken are shown.



$$S_1 = v_1 t_1 \Rightarrow t_1 = \frac{S_1}{v_1};$$

$$S_2 = v_2 t_2 \Rightarrow t_2 = \frac{S_2}{v_2}$$

$$v_{av} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} = \frac{S_1 + S_2}{\frac{S_1}{v_1} + \frac{S_2}{v_2}}$$

NOTE:

- If $t_1 = t_2 = t$, then $v_{av} = \frac{v_1 + v_2}{2}$; average speed is equal to the arithmetical mean of individual speeds.
- If $S_1 = S_2 = S$, then $v_{av} = \frac{2v_1 v_2}{v_1 + v_2}$; average speed is equal to the harmonic mean of individual speeds.

ILLUSTRATION 3.2 A train travels from city A to city B with a constant speed of 10 m s^{-1} and returns back to city A with a constant speed of 20 m s^{-1}. Find its average speed during its entire journey.

Solution. Let the distance between the two cities A and B be x m.

Time taken by the train to travel from A to B $= \frac{x}{10} = t_1$ (say)

Time taken to come back from B to A $= \frac{x}{20} = t_2$ (say)

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{x + x}{t_1 + t_2} = \frac{2x}{\frac{x}{10} + \frac{x}{20}} = \frac{40}{3}\ \text{m s}^{-1}$$

INSTANTANEOUS VELOCITY

When you move to some position and come back to same initial position, you undergo a zero displacement. Hence, your average velocity is zero. It does not mean that you were not moving. The average velocity cannot always define the motion of a particle. Then we require to define the instantaneous velocity. We need to know the instantaneous value of the velocity and speed of a particle to understand its motion.

The time rate of change of position (x) or displacement (s) at any instant of time (t) is known as *instantaneous velocity* or simply *velocity* at that instant of time. It is denoted by v.

Mathematically: $\vec{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{x}}{\Delta t}\right) = \frac{d\vec{x}}{dt}$

or $\quad \vec{v} = \frac{d\vec{x}}{dt} \quad \left[\text{Also } \vec{v} = \left(\frac{d\vec{s}}{dt}\right)\right]$

In general, if the particle moves in space, then $\vec{r}$ will change and the time rate of change of position vector is known as velocity. Thus,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[x\hat{i} + y\hat{j} + z\hat{k}\right]$$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\Rightarrow \quad \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

where $v_x = \frac{dx}{dt}$, $v_y = \frac{dy}{dt}$, and $v_z = \frac{dz}{dt}$.

ILLUSTRATION 3.3 A particle moves so that its position vector varies with time as $\vec{r} = A\cos\omega t\,\hat{i} + A\sin\omega t\,\hat{j}$. Find the

(a) initial velocity of the particle.

(b) angle between the position vector and velocity of the particle at any time, and

(c) speed at any instant.

Solution.

(a) Position at time t

$\vec{r} = A\cos\omega t\,\hat{i} + A\sin\omega t\,\hat{j}$

Instantaneous velocity, $\vec{v} = \frac{d\vec{r}}{dt}$

We have $\vec{v} = A\frac{d}{dt}(\cos\omega t)\hat{i} + A\frac{d}{dt}(\sin\omega t)\hat{j}$

$= -A\omega\sin\omega t\,\hat{i} + A\omega\cos\omega t\,\hat{j}$

At $t = 0, v = -A\omega\sin 0\,\hat{i} + A\omega\cos 0\,\hat{j} = A\omega\hat{j}$

(b) For calculating the angle between two vectors, we use the concept of dot product of the vectors. The angle θ between $\vec{r}$ and $\vec{v}$ can be given as

$$\theta = \cos^{-1}\frac{\vec{r}.\vec{v}}{|\vec{r}||\vec{v}|}$$

where

$\vec{r}.\vec{v} = (A\cos\omega t\,\hat{i} + A\sin\omega t\,\hat{j}).(-A\omega\sin\omega t\,\hat{i} + A\omega\cos\omega t\,\hat{j})$

$= \omega A^2\,(-\cos\omega t\sin\omega t + \sin\omega t\cos\omega t) = 0$

Hence, $\theta = \cos^{-1} 0 = \pi/2$

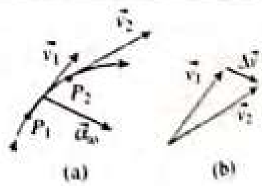
That means $\vec{v} \perp \vec{r}$.

(c) Speed at any time is the magnitude of instantaneous velocity, i.e.,

$v = |\vec{v}| = \sqrt{(-A\omega\sin\omega t)^2 + (A\omega\cos\omega t)^2} = A\omega$

AVERAGE ACCELERATION

If the velocity of a particle at instant t is $\vec{v}_1$ and at instant t_2 is $\vec{v}_2$, then the average acceleration is mathematically given by:



$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

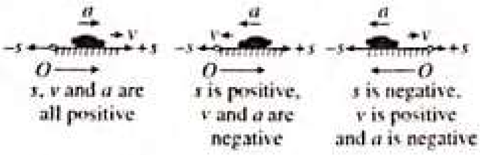
(**Remember** Change in velocity = Final velocity – Initial velocity)

The time rate of change of velocity at any instant of time is known as *instantaneous acceleration* or simply *acceleration* $\left(\vec{a} = \frac{d\vec{v}}{dt}\right)$. Actually, acceleration at an instant is defined as the limit of the average acceleration as the time interval Δt around that instant becomes infinitesimally small.

$$\vec{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{v}}{\Delta t}\right) = \frac{d\vec{v}}{dt}$$

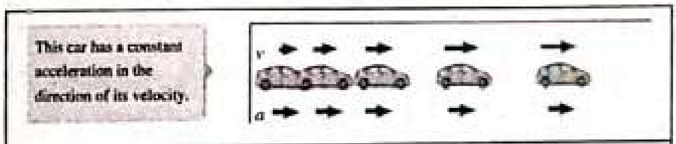
MOTION IN A STRAIGHT LINE

While moving in a straight line, we have two possible directions for acceleration, velocity, and displacement of the particle. Hence, for the sake of simplicity, we need not mention them with their usual vector symbols. Rather we prefer to write them like scalars a, v, s and r and use plus (+) and minus (–) to treat them as vectors keeping their senses (directions) in our minds. Hence, we call the given quantities (s, v, and a) as "algebraic scalars." Conventionally, when $\vec{a}, \vec{v}$, and $\vec{s}$, are directed along positive s-axis we write them as $+a$, $+v$, and $+s$, respectively. If they are directed along negative s-axis, we can write them as negative quantities such as $-a$, $-v$, and $-s$, respectively, as shown in the figure.



Formulae and Concepts for Uniformly Accelerated Motion in a Straight Line

NOTE: Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant. There will be one-dimensional motion if the initial velocity and acceleration are parallel or anti-parallel to each other.



Scalar form	Vector form
$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$v^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

ILLUSTRATION 3.4 Consider a particle initially moving with a velocity of 5 m s^{-1} starts decelerating at a constant rate of 2 m s^{-2}.

(a) Determine the time at which the particle becomes stationary.

(b) Find the distance travelled in the second second.

(c) Find the distance travelled in the third second.

Solution.

(a) Here $u = 5$ m s^{-1}, $a = -2$ m s^{-2}, $v = 0$, $t = ?$

Using $v = u + at$, where

$0 = 5 - 2t$

$\Rightarrow \quad t = 2.5$ s

(b) Here $u = 5$ m s^{-1}, $a = -2$ m s^{-2}, $n = 2$.

Using $x_n = u + \frac{a}{2}(2n-1) = 5 - \frac{2}{2}[2(2)-1)] = 2$ m

(c) Here, if we use the above formula, we will get $x_n = 0$, but in reality it is not zero. This formula is not applicable for the third second because velocity becomes zero in the third second, i.e., at $t = 2.5$ s.

The particle has a turning point at $t = 2.5$ s. We have to indirectly calculate the distance travelled in this particular second. That is, we have to determine the distance travelled between 2 s $\le t \le$ 2.5 s and 2.5 s $\le t \le$ 3 s, and then add the two.

Displacement of the particle at $t = 2.5$ s is

$$x_{2.5} = \frac{u^2}{2a} = \frac{(5)^2}{2(2)} = 6.25 \text{ m}$$

Due to symmetry, the displacement of the particle at $t = 2$ s and $t = 3$ s are same, i.e.,

$$x_3 = x_2 = 5(2) + \frac{1}{2}(-2)(2)^2 = 6 \text{ m}$$

Thus, the distance travelled in the third second is

$$x = (x_{2.5} - x_2) + (x_{2.5} - x_3)$$
$$= (6.25 - 6) + (6.25 - 6) = 0.5 \text{ m}$$

Important Results

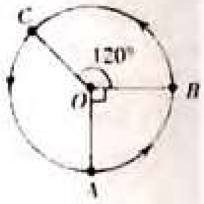
- If a body starts from rest and moves with uniform acceleration, then distance covered by the body in t seconds is proportional to t^2 (i.e. $s \propto t^2$). So we can say that the ratio of distance covered in 1 s, 2 s, and 3 s is or 1 : 4 : 9.
- If a body starts from rest and moves with uniform acceleration, then distance covered by the body in nth second is proportional to $(2n - 1)$ [i.e., $s_n \propto (2n - 1)$].
- So we can say that the ratio of distance covered in I second, II second, and III second is 1 : 3 : 5.
- A body moving with a velocity u is stopped by the application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it, then it will come to rest after covering a distance of n^2s. As $v^2 = u^2 - 2as$

  $\Rightarrow 0 = u^2 - 2as \Rightarrow s = u^2/2a \quad \Rightarrow s \propto u^2$ [since a is constant] So we can say that if u becomes n times, then s becomes n^2 times that of previous value.
- A particle moving with uniform acceleration from A to B along a straight line has velocities v_1 and v_2 at A and B, respectively. If C is the mid-point between A and B, then velocity of the particle at C is equal to

  $$v = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

Concept Application Exercise 3.1

1. A body covers 10 m in the seconds second and 25 m in fifth second of its motion. If the motion is uniformly accelerated, how far will it go in the seventh second?
2. A body moving with uniform acceleration in a straight line describes 25 m in the fifth second and 33 m in the seventh second. Find its initial velocity and acceleration.
3. Figure shows a particle starting from point A, traveling up to B with a speed s, then up to point C with a speed $2s$, and finally upto A with a speed of $3s$. Determine is average speed.



4. A particle moving in a straight line covers half the distance with a speed of 3 ms^{-1}. The other half of the distance is covered in two equal time intervals with speeds of 4.5 ms^{-1} and 7.5 ms^{-1}, respectively. Find the average speed of the particle during this motion.
5. Find the ratio of the distance moved by a free-falling body from rest in fourth and fifth seconds of its journey.
6. Two balls of different masses (one lighter and other heavier) are thrown vertically upwards with the same speed. Which one will pass through the point of projection in the downward direction with greater speed?
7. A car runs at a constant speed on a circular track of radius 200 m, taking 62.8 s on each lap. Find the average velocity and average speed on each lap.
8. A train accelerates from rest for time t_1 at a constant rate a and then it retards at the constant rate b for time t_2 and comes to rest. Find the ratio t_1/t_2.

ONE-DIMENSIONAL MOTION IN A VERTICAL LINE (MOTION UNDER GRAVITY)

When the motion takes place under the effect of *gravitational attractive force* only, the motion is known as free fall. Here free fall does not mean that the particle is falling down only. Even if the particle is rising up or may be momentarily at rest at highest point, but if only gravitational force is acting on it, then also motion will be called free fall.

Sign convention: Any vector quantity directed upward will be taken as positive and directed downward will be taken as negative. According to this sign convention,

1. Displacement will be taken as positive if the final position lies above initial position and negative if the final position lies below initial position.

2. Velocity (initial or final) will be taken as positive if it is upward and negative if it is downward.
3. Acceleration a is always taken to be $-g$.

In equations of motions, when we replace a by $-g$ (minus sign, because acceleration is always directed downward), we get

$$v = u - gt$$

$$s = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gs$$

Important Results

- The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity, i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.
- In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance.

  Time of descent (t_1) = Time of ascent $(t_2) = u/g$

  $\therefore$ Total time of flight, $T = t_1 + t_2 = \dfrac{2u}{g}$
- In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

  As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.
- A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent $(t_2 > t_1)$.

  Let u be the initial velocity of body, then time of ascent

  $$t_1 = \frac{u}{g+a} \quad \text{and} \quad h = \frac{u^2}{2(g+a)}$$

  where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will act vertically downward.

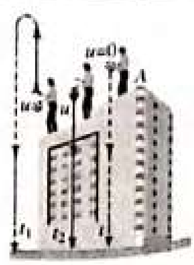
  For downward motion, a and g will act in opposite direction because a always acts in direction opposite to motion and g always acts vertically downward.

  So $h = \frac{1}{2}(g-a)t_2^2$

  $$\Rightarrow \frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)t_2^2 \Rightarrow t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$$

  Comparing t_1 and t_2, we can say that $t_2 > t_1$ since $(g+a) > (g-a)$

ILLUSTRATION 3.5 A body is thrown vertically upwards from A, the top of a tower (figure). It reaches the ground in time t_1. If it is thrown vertically downwards from A with the same speed it reaches the ground in time t_2. If it is allowed to fall freely from A, then find the time it takes to reach the ground.



Solution. Suppose the body be projected vertically upwards from A with a speed u_0.

Using equation $\quad s = ut + \left(\frac{1}{2}\right)at^2$

For first case, $\quad -h = u_0t_1 - \left(\frac{1}{2}\right)gt_1^2 \qquad$ (i)

For second case, $\quad -h = -u_0t_2 - \left(\frac{1}{2}\right)gt_2^2 \qquad$ (ii)

From (i) − (ii) $\Rightarrow \; 0 = u_0(t_2 + t_1) + \left(\frac{1}{2}\right)g(t_2^2 - t_1^2)$

or $\quad u_0 = \left(\frac{1}{2}\right)g(t_1 - t_2) \qquad$ (iii)

Put the value of u_0 in (ii), we get

$$-h = -\left(\frac{1}{2}\right)g(t_1 - t_2)t_2 - \left(\frac{1}{2}\right)gt_2^2$$

$\Rightarrow \quad h = \frac{1}{2}gt_1t_2 \qquad$ (iv)

For third case, $u = 0$, $t = ?$

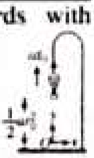
$$-h = 0 \times t - \left(\frac{1}{2}\right)gt^2$$

or $\quad h = \left(\frac{1}{2}\right)gt^2 \qquad$ (v)

Combining (iv) and (v), we get

$$\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \quad \text{or} \quad t = \sqrt{t_1t_2}$$

ILLUSTRATION 3.6 A balloon starts rising upwards with constant acceleration a and after time t_0 second, a packet is dropped from it which reaches the ground after t seconds of dropping (figure). Determine the value of t.



Solution. *Analysis of situation:* $t = 0$ is the time when the balloon started rising up. At $t = t_0$, when the packet is dropped, the balloon is moving up with velocity $v = 0 + at_0 = at_0$. Hence, initial velocity of the packet will be $v_0 = at_0$ (upward). As the balloon has started rising upwards with constant acceleration a, so after t_0 seconds, its height from the ground is $y_0 = \frac{1}{2}at_0^2$.

For packet: $s = ut - -gt$

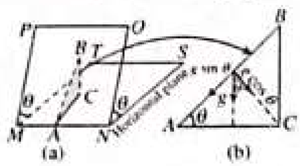
$$\Rightarrow \quad -\frac{1}{2}at_0^2 = at_0t - \frac{1}{2}gt^2 \quad \Rightarrow \quad gt^2 - 2at_0t - at_0^2 = 0$$

Solving the quadratic equation, we get $t = \frac{at_0}{g}\left[1 + \sqrt{1 + \frac{g}{a}}\right]$

MOTION UPON AN INCLINED PLANE

Any plane inclined to the horizontal at a definite angle is said to be an inclined plane, and the corresponding angle is known as the angle of inclination (or simly inclination).

Figure shows a horizontal plane $MNST$ being cut by an inclined plane $MNOP$. Along the line MN, consider any point B upon the inclined. From B, drop perpendiculars BA and BC to the line MN and horizontal plane $MNST$, respectively. Then $\angle BAC$ is said to be the *angle of inclination of the inclined plane.*

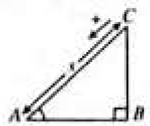


Line AB is said to be the line of greatest slope. If a body is allowed to move freely along the inclined plane, it always chooses the line of greatest slope to move. However, it can be made to move along other line by allowing it to slide along a groove in the desired direction. Figure (b) shows an inclined plane AB with angle of inclination θ. θ is such that $\sin\theta = l/n$. Usually l is chosen as 1.

The inclination is often referred to as "1 in n." It implies that a "1 in n" inclination is such that, there is a vertical rise of 1 unit for every n unit distance travelled along the plane up.

ILLUSTRATION 3.7 (a) Show that the velocity acquired by a particle in sliding down an inclined plane is the same as that acquired by a particle falling freely from rest through a distance equal to the height of the inclined plane. (b) Find the time taken in sliding a particle down the whole length of the incline.

Solution.



(a) Let a particle sliding from C to A along the inclined plane CA acquire a final velocity v_1, covering a distance s (figure). If θ is the angle of inclination, then

$$\sin\theta = \frac{h}{s}$$

Now for the sliding particle, $s = s$, $u = 0$, $a = g\sin\theta$, $v = v_1$.
[Taking the direction C to A as positive]
Using, $v^2 = u^2 + 2as$

$$\Rightarrow \quad v_1^2 = 2(g\sin\theta)s = 2g\left[\frac{h}{s}\right]s = 2gh$$

$\therefore \quad v_1 = \sqrt{2gh} \qquad$ ...(i)

Again, let a particle fall from rest along a distance h and acquire a velocity v_2, then from $v^2 = u^2 + 2as$,

$v_2^2 = 0 + 2gh \Rightarrow v_2 = \sqrt{2gh} \qquad$ ...(ii)

From (i) and (ii), we get the same result.

(b) Time taken in sliding a particle down the entire length of the incline:
For the motion of particle from C to A (figure), $u = 0$, $s = s$, $a = g\sin\theta$, $t = ?$.
Using $s = ut + \frac{1}{2}at^2$, we get $s = \frac{1}{2}(g\sin\theta)t^2$

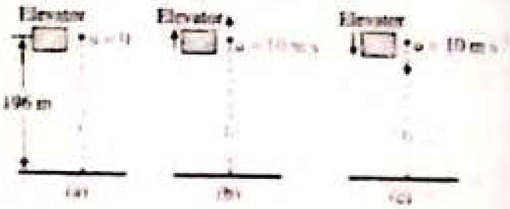
$$\Rightarrow \quad t^2 = \frac{2s}{g\sin\theta} = \frac{2h}{g\sin^2\theta} \quad \left[\because \sin\theta = \frac{h}{s}\right]$$

$$\therefore \quad t = \sqrt{\frac{2h}{g}}\operatorname{cosec}\theta$$

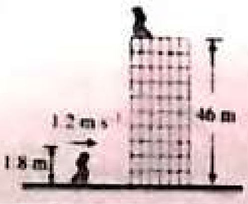
Thus, the time taken varies directly as the cosecant of the angle of inclination. Now, since cosecant is a decreasing function in the first quadrant with an increase in θ, time t would decrease.

CONCEPT APPLICATION EXERCISE 3.2

1. A ball is dropped from an elevator at an altitude of 200 m (figure). How much time will the ball take to reach the ground if the elevator is



   (a) Stationary?
   (b) Ascending with velocity 10 m s^{-1}?
   (c) Descending with velocity 10 m s^{-1}?
2. A particle is projected vertically upwards. Prove that it will be at three-fourth of its greatest height at times which are in the ratio 1 : 3.
3. A balloon rises from rest on the ground with constant acceleration $g/8$. A stone is dropped from the balloon when the balloon has risen to a height of H. Find the time taken by the stone to reach the ground.
4. A parachutist, after bailing out, falls 50 m without friction. When the parachute opens, it decelerates at 2 m s^{-2}. He reaches the ground with a speed of 3 m s^{-1}. At what height did he bail out?
5. A ball is dropped from the top of a tower of height h. It covers a distance of $h/2$ in the last second of its motion. How long does the ball remain in air?
6. When a ball is thrown up, it reaches a maximum height h travelling 5 m in the last second. Find the velocity with which the ball should be thrown up.
7. You are on the roof of the physics building, 46.0 m above the ground (figure). Your physics professor, who is 1.80 m tall, is walking alongside the building at a constant speed of 1.20 m s^{-1}. If you wish to drop a flower on your professor's head, where should the professor be when you release the flower? Assume that the flower is in free fall.



RELATIVE MOTION IN ONE DIMENSION

Consider the motion of a car moving towards right and two observers O_1 and O_2 are coming from opposite directions as shown in the figure.

Observer O_1 finds that the car is moving slower while observer O_2 finds that the car is moving faster in comparison to when observer is at rest. The motion of same object looks different for two different observers. To understand such observations, there is need of introduction of the concept of relative velocity.

In figure, two particles A and B are moving with velocities v_A and v_B and accelerations a_A and a_B, respectively.

If x_A and x_B are their respective displacements with respect to the fixed origin, then

- The relative displacement of B w.r.t A is defined as

$$\vec{x}_{BA} = \vec{x}_B - \vec{x}_A \quad \text{...(i)}$$

Differentiating (i) w.r.t time.

- The relative velocity of B with respect to A is defined as

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A \quad \text{...(ii)}$$

Differentiating (ii) w.r.t time.

- The relative acceleration of B with respect to A is defined as

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

The equations of motion in one dimension are modified as:

$$\vec{v}_{BA} = \vec{u}_{BA} + \vec{a}_{BA}t \quad \Rightarrow \vec{v}_{rel} = \vec{v}_{rel} + \vec{v}_{rel}t$$

$$\vec{x}_{BA} = \vec{u}_{BA}t + \frac{1}{2}\vec{a}_{BA}t^2 \quad \Rightarrow \quad \vec{x}_{rel} = \vec{u}_{rel}t + \frac{1}{2}\vec{a}_{rel}t^2$$

$$v_{rel}^2 - u_{rel}^2 = 2a_{rel}\, x_{rel}$$

The relative velocity of a particle A with respect to B is defined as the velocity with which A appears to move if B is considered to be at rest. In other words, it is the velocity with which A appears to move as seen by B considering itself to be at rest.

NOTE: All velocities are relative and have no significance unless observer is specified. However, when we say "velocity of A," what we mean is that the velocity of A w.r.t. ground which is assumed to be at rest.

$$\vec{v}_{\text{Object, Observer}} = \vec{v}_{\text{Object}} - \vec{v}_{\text{Observer}}$$

or $\quad \vec{v}_{rel} = \vec{v}_{\text{Object}} - \vec{v}_{\text{Observer}}$

ILLUSTRATION 3.8 A car A moves with velocity 20 m s^{-1} and car B with velocity 15 m s^{-1} as shown in figure. Find the relative velocity of B w.r.t. A and A w.r.t. B.

Solution. Let us assume that the right direction is positive.

We are given: $v_A = 20 \text{ m s}^{-1}$, $v_B = 15 \text{ m s}^{-1}$.

Relative velocity of B w.r.t. A:

$$v_{B/A} = v_B - v_A = 15 - 20 = -5 \text{ m s}^{-1}$$

(Negative sign indicates that this relative velocity is in the left direction.)

Relative velocity of A w.r.t. B:

$$v_{A/B} = v_A - v_B = 20 - 15 = 5 \text{ m s}^{-1}$$

(Positive sign indicates that this relative velocity is in the right direction.)

Here the separation between A and B is decreasing with time, hence the velocity of approach of A w.r.t. B is 5 m s^{-1}

ILLUSTRATION 3.9 A car A moves with velocity 15 m s^{-1} and B with velocity 20 m s^{-1} are moving in opposite directions as shown in figure. Find the relative velocity of B w.r.t. A and A w.r.t. B.

Solution. Here also let us assume right direction is positive, then left direction will be negative.

Given: $v_A = -15 \text{ m s}^{-1}$, $v_B = 20 \text{ m s}^{-1}$.

Relative velocity of B w.r.t. A:

$$v_{B/A} = v_B - v_A = 20 - (-15) = 35 \text{ m s}^{-1}$$

(Positive sign indicates that this relative velocity is in right direction.)

Relative velocity of A w.r.t. B:

$$v_{A/B} = v_A - v_B = -15 - 20 = -35 \text{ m s}^{-1}$$

(Negative sign indicates that this relative velocity is in backward direction.)

Here the separation between A and B is increasing with time, hence the velocity of separation is 35 m s^{-1}.

NOTE: $\vec{v}_{B/A}$ and $\vec{v}_{A/B}$ are of equal magnitude but have opposite directions.

ILLUSTRATION 3.10 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

Solution. Let speed of each bus $= v \text{ km h}^{-1}$

The distance between the nearest buses plying on either car $= vT$ km ...(i)

For buses going from town A to B:

Relative speed of bus in the direction of motion of man $= (v - 20)$

Buses plying in this direction go past the cyclist after every 18 min. Therefore, separation between the buses $= (v-20)\times\dfrac{18}{60}$

From (i), $(v-20)\times\dfrac{18}{60} = vT$ ...(ii)

For buses coming from B to A:

The relative velocity of bus with respect to man $= (v + 20)$

Buses coming from town B past the cyclist after every 6 min,

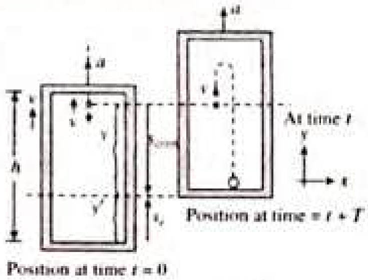
$$\therefore \quad (v+20)\times\frac{6}{60} = vT \qquad \text{... (iii)}$$

Solving (ii) and (iii), we get $v = 40 \text{ km h}^{-1}$ and $T = \dfrac{2}{30}$ h

ILLUSTRATION 3.11 An elevator is moving with an upward acceleration a. A coin is dropped from rest from the roof of the elevator, relative to you. After what time the coin will strike the base of the elevator?

***Solution.* Method 1:** Observation from ground frame

Let us assume that the elevator has a velocity v_0 when the coin loses contact with the elevator at time t. Let the coin strike the base of the elevator after time $t = T$, just below the point of loosing contact with roof of the coin.



In time T, the elevator moves up the displacement y' (upward) and the coin has net displacement $(h - y') = y$ (downward)

For motion of coin

Displacement of coin $= -y = -(h - y)$

Using equation $S = ut + \dfrac{1}{2}at^2$,

$$-(h - y') = v_0 T - \frac{1}{2}gT^2 \qquad \text{(i)}$$

For motion of elevator:

Displacement of elevator during time $T = +y'$

$$+y' = v_0 T + \frac{1}{2}aT^2 \qquad \text{(ii)}$$

Adding (i) and (ii), we have $h = \dfrac{1}{2}(g+a)T^2$

This yields $T = \sqrt{\dfrac{2h}{g+a}}$

Method 2: Analyze the motion of coin with respect to the observer standing in the elevator. As the coin releases from rest inside elevator, its velocity with respect to ground is equal to the velocity of elevator.

Initial relative velocity of coin w.r.t. observer in the elevator,

$$[\vec{u}_{coin}]_{elevator} = [\vec{u}_{coin}]_{ground} - [\vec{u}_{elevator}]_{ground} = u - u = 0$$

and acceleration of coin with respect to the observer in the elevator,

$$[\vec{a}_{coin}]_{elevator} = [\vec{a}_{coin}]_{ground} - [\vec{a}_{elevator}]_{ground}$$

$$= -g - (+a) = -(g + a)$$

Now using second equation for relative motion

$$[\vec{s}_{coin}]_{elevator} = [\vec{u}_{coin}]_{elevator}[\vec{u}_{coin}]_{elevator}t + \frac{1}{2}[\vec{a}_{coin}]_{elevator}t^2$$

$$-h = 0 - \frac{1}{2}(g + a)t^2$$

This yields $t = \sqrt{\dfrac{2h}{g+a}}$

River–Man Problem in One Dimension

The velocity of river water current is u and that of man in still water is v, i.e., man can swim in water with velocity v.

Case 1: Man swimming downstream (along the direction of river flow)

In this case, velocity of river $v_R = +u$

Velocity of man w.r.t. river $v_{mR} = +v$

(a)

Now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = u + v$

Case 2: Man swimming upstream (opposite to the direction of river flow)

In this case, velocity of river $\vec{v}_R = -u$

(b)

Velocity of man w.r.t. river $\vec{v}_{mR} = +v$

Now $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R = (v - u)$

ILLUSTRATION 3.12 A swimmer capable of swimming with velocity v relative to water jumps in a flowing river having velocity u. The man swims a distance d down stream and returns back to the original position. Find out the time taken in complete motion.

Solution. Total time = Time of swimming downstream + Time of swimming upstream

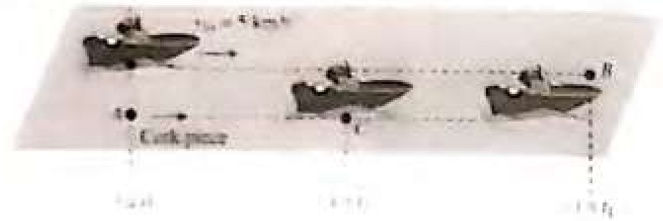
Velocity of the man during swimming downstream = $v + u$

Velocity of the man during swimming upstream = $v - u$

$$t = t_{down} + t_{up} = t_{down} + t_{up} = \frac{d}{v+u} + \frac{d}{v-u} = \frac{2dv}{v^2 - u^2}$$

ILLUSTRATION 3.13 Let us consider a boat which moves with a velocity $v_{bw} = 5\ \text{km h}^{-1}$ relative to water. At time $t = 0$, the boat passes through a piece of cork floating in water while moving downstream. If it turns back at time $t = t_1$, when does the boat meet the cork again? Assume $t_1 = 30$ min.

Solution. Time of travelling of boat from A to B (t_1) and then B to C (t_1') = time of moving the cork from A to C.



Velocity of boat from A to B

$$\vec{v}_{b,w} + \vec{v}_w = (5 + u)\ \text{km h}^{-1}$$

And velocity of boat from B to C

$$\vec{v}_{b,w} - \vec{v}_w = (5 - u)\ \text{km h}^{-1}$$

Distance moved by boat in time t_1

$$AB = (5 + 4)t_1$$

And distance moved by boat in time $t_1' = BC = (5 - u)t_1'$

Distance moved by cork during this time

$$AC = u(t_1 + t_1')$$

But $AB = AC + BC$

$$(5 + u)t_1 = u(t_1 + t_1') + (5 - u)t_1'$$

$$5t_1 = 5t_1' \Rightarrow t_1' = t_1 = 30 \text{ min}$$

Hence, the cork meets the boat again after 1 h.

CONCEPT APPLICATION EXERCISE 3.3

1. A train 200 m long is moving with a velocity of 72 km h^{-1}. Find the time taken by the train to cross the bridge 1 km long.
2. Two cars A and B are moving on the straight parallel paths with speeds 36 km h^{-1} and 72 km h^{-1}, respectively, starting from the same point in the same direction. After 20 min, how much behind is car A from car B?
3. Two trains 110 m and 90 m long, respectively, are running in opposite directions with velocities 36 km h^{-1} and 54 km h^{-1}. Find the time taken by the two trains to completely cross each other.
4. A moving sidewalk in an airport terminal building moves at a speed of 1.0 m s^{-1} and is 35.0 m long. If a woman steps on at one end and walks at 1.5 m s^{-1} relative to the moving sidewalk, then find the time that she requires to reach the opposite end (a) when she walks in the same direction the sidewalk is moving and (b) when she walks in the opposite direction.

5. A passenger and a good train are headed in the same direction on parallel tracks. The passenger train is 240 m long and has a constant velocity 72 km h^{-1}. Beginning from the time the engine of the passenger train approaches the last wagon of the goods train, it takes 25 s to be in level with the engine of the goods train. It took 30 s more to completely overtake the goods train. Determine the length and speed of the goods train.

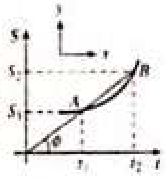
GRAPHS IN MOTION IN ONE DIMENSION

Graphical Representation of Motion

For graphical representation, we require two coordinate (reference) axes, one variable being taken along one axis. The usual practice is to take the independent variable along x-axis and the dependent on along y-axis. In general cases, involving time as one of the variables, time being independent, is usually taken along x-axis.

Graphs play very important role in analyzing a motion. Some times it becomes difficult to solve the problems analytically. But with the help of graphs, we can solve the problems easily and without much calculation.

Position–Time Relation



Let us take a general motion of a particle and plot its position–time graph. From the graph the position of the particle in time interval from t_1 to t_2 changes from s_1 to s_2.

Hence, average velocity, $v_{av} = \dfrac{\Delta s}{\Delta t} = \tan\phi$.

$v_{av} = \dfrac{\Delta s}{\Delta t}$ = Slope of the line joining the points in s–t graph.

NOTE: The slope of a straight line connecting any two points in s–t graph gives the velocity averaged over the time interval $(t_2 - t_1)$.

Different Cases in Position–Time Graph

Particle is stationary: Let a particle be at some point P at time $t = 0$ which is at a distance x_0 from origin. Since the particle is stationary. So at any further time the particle will remain at point P. Hence, position–time graph for a stationary particle is parallel to time axis (figure).

Particle is moving with constant velocity towards right: Equation to be used: $x = x_0 + vt$. Graph will be a straight line. Let the particle be at some point P initially at time $t = 0$ which is at a distance of x_0 from origin. Since the particle is moving towards right so its distance from origin goes on increasing. Hence, the position–time graph for a particle moving with constant velocity towards right will be a straight line inclined to time axis making an acute angle α (figure).

Recall that $\tan\alpha$ is the slope of position–time graph which is equal to the velocity of particle.

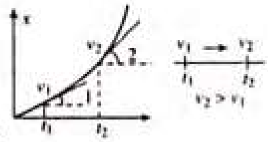
Particle is moving with constant velocity towards left: Equation to be used: $x = x_0 - vt$. Graph will be a straight line.

Let the particle be at some point P at time $t = 0$ which is at a distance of x_0 from origin. Since the particle is moving toward left so first its distance from origin goes on decreasing and then its distance from the origin goes on increasing in negative direction. Hence, position–time graph for a particle moving with constant velocity towards left will be a straight line inclined to time axis making an obtuse angle α (figure). Here $\tan\alpha$, the slope, will be negative, which indicates negative velocity.

Particle is moving with constant acceleration directed rightward (positive acceleration): Since acceleration is towards right so velocity increases in right direction. Hence, slope of position–time graph goes on increasing.

Equation to be used:

$$x = ut + \frac{1}{2}at^2$$



Graph will be a curved line (parabolic).

$$v_1 = \tan\alpha_1,\ v_2 = \tan\alpha_2$$

It is clear from the figure that as time passes, slope goes on increasing.

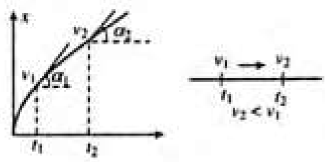
Now $\alpha_2 > \alpha_1 \Rightarrow \tan\alpha_1 > \tan\alpha_2 \Rightarrow v_2 > v_1$

So, velocity goes on increasing.

Particle is moving with constant acceleration directed leftward (negative acceleration): Since acceleration is towards left so velocity decreases in right direction and increases in left direction.

Equation to be used:

$$x = ut + \frac{1}{2}at^2$$



Graph will be a curved line (parabola).

$$v_1 = \tan\alpha_1,\ v_2 = \tan\alpha_2$$

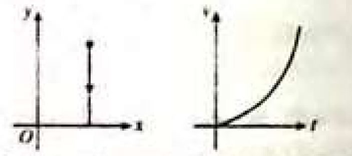
It is clear from the figure that as time passes, slope goes on decreasing.

Now $\alpha_2 < \alpha_1 \Rightarrow \tan\alpha_2 < \tan\alpha_1 \Rightarrow v_2 < v_1$

So, velocity goes on decreasing.

Important Result

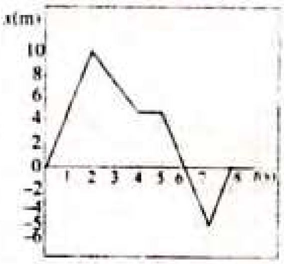
When the displacement of a particle along y-axis is expressed as the function of displacement of the particle along x-axis, we call it "locus equation" or "equation of trajectory." But when the displacement of a particle along y-axis is expressed as the function of time, it is displacement–time graph. For example, when we drop a stone from rest, it moves down in a straight line. Hence, the locus is a straight line.



Trajectory of a particle Position–time graph

If you plot the displacement versus time you will get a parabola. We call it displacement (or position)–time (r–t) graph which shows the variation of position with time.

ILLUSTRATION 3.14 The position versus time graph for a certain particle moving along the x-axis is shown in figure. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 2 s to 4 s, and (c) 4 s to 7 s.



Solution. We are given the values of time, but the values of the position are to obtained from the graph corresponding to the given time interval under consideration. We know that the slope of the displacement–time graph represents velocity.

Formula to be used: $v_{av} = \dfrac{x_2 - x_1}{t_2 - t_1}$, where x_1 and x_2 are the initial and final positions, respectively, and t_1 and t_2 are the initial and final values of time, respectively.

(a) Here $x_1 = 0$ m, $x_2 = 10$ m, $t_1 = 0$ s, $t_2 = 2$ s.
Therefore, the required average velocity is given by

$$v_{av} = \frac{10-0}{2-0} = 5\,\mathrm{m\,s^{-1}}$$

(b) Here $x_1 = 10$ m, $x_2 = 5$ m, $t_1 = 2$ s, $t_2 = 4$ s.
So the required value of average velocity is given by

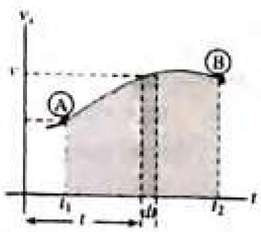
$$v_{av} = \frac{5-10}{4-2} = -2.5\ \mathrm{m\,s^{-1}}$$

(c) Here $x_1 = 5$ m, $x_2 = -5$ m, $t_1 = 4$ s, $t_2 = 7$ s.
So the required value of average velocity is given by

$$v_{av} = \frac{-5-5}{7-4} = \frac{-10}{3} = -3.3\ \mathrm{m\,s^{-1}}$$

Velocity–Time Graph

From v–t graph, we can find (a) displacement, (b) distance, (c) average acceleration, and (d) instantaneous acceleration.

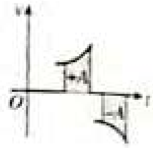


Displacement: When the velocity of a particle is given as the function of time, i.e., $v = f(t)$, we find the total displacement $\vec{s}$ by summing up (integrating) the elementary displacements $d\vec{s}$.

The elementary displacement $d\vec{s} = \vec{v}\,dt$ can be given as the area of the shaded elementary strip. Summing up all the elementary areas between $t = t_1$ and $t = t_2$, we have the total area under v–t graph enclosed within the time interval which gives the total displacement of the particle during the time interval $t(= t_2 - t_1)$.

$$\vec{s} = \int \vec{v}\,dt = A(= \text{Area under } v\text{–}t \text{ graph})$$

When the area lies above the time axis, it is said to be negative. A positive area reveals a positive displacement (displacement directed along positive directions of coordinate system) and negative area signifies a negative displacement.



Distance: Since the distance covered D is given as $D = \int |\vec{v}|\,dt$, it can be given graphically as the sum of the magnitude of positive and negative areas under v–t graph.

$$D = \int |\vec{v}|\,dt = |A_+| + |A_-|$$

Average acceleration: As derived earlier, the average acceleration, $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Then, $\vec{a}_{av}$ = slope of the line joining A and B in v–t graph as shown in figure.

The slope of the straight line AB that joins two points of time t_i and t_f under consideration gives the average acceleration over the time interval, $\Delta t = t_2 - t_1$.

Instantaneous acceleration: The instantaneous acceleration is given as $\vec{a} = \dfrac{d\vec{v}}{dt}$.

Then, $\vec{a}$ = slope of v–t graph.

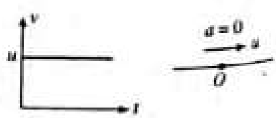
As we reduce the time interval Δt, the slope of the straight line AB tends to be equal to the slope of the tangent drawn at B. The slope of a line on v–t graph, that is (dv/dt) gives the acceleration.

If the slope of v–t graph is positive at any point of time on the graph, the acceleration of the particle is positive at that time. If the slope of v–t graph is zero, it signifies that the acceleration is zero. When the slope of v–t graph is negative, it means negative acceleration (or retardation).

Velocity–Time Graph of Various Types of Motions of a Particle

Particle is moving with a constant velocity Since velocity is constant, so $a = 0$.

Let at any time velocity of the particle is u. Since velocity remains constant so at any time velocity remains same. Hence, velocity–time graph for a particle moving with constant velocity is a straight line parallel to time axis.



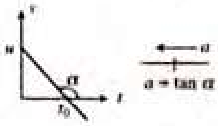
Particle is moving with a constant positive acceleration: Equation to be used: $v = u + at$.

As the time passes, velocity goes on increasing. Hence, velocity-time graph for a particle moving with constant positive acceleration is a straight line inclined to time axis making an acute angle α. Here, $\tan\alpha$ is the slope of velocity-time graph (figure).

Note that the slope of velocity-time graph is equal to acceleration.

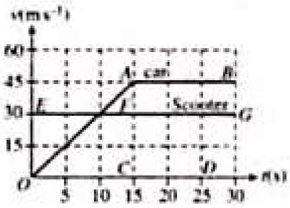
Particle is moving with a constant negative acceleration (Retardation): Equation to be used: $v = u + at$.

As the time passes, the velocity goes on decreasing; at time $t = t_0$, velocity becomes zero and after that it increases in negative direction. Hence, velocity-time graph for a particle moving with constant retardation is a straight line inclined to time axis making an obtuse angle α (figure).



Since α is obtuse angle, hence slope $\tan\alpha$ becomes negative. Hence, acceleration is negative.

ILLUSTRATION 3.15 As soon as a car just starts from rest in a certain direction, a scooter moving with a uniform speed overtakes the car. Their velocity-time graph is shown in the figure. Calculate



(a) The difference between the distances travelled by the car and the scooter in 15 s.

(b) The distance of car and scooter from the starting point at that instant.

Solution.

(a) The distance travelled by car in 15 s

= Area of ΔOAC

$= \frac{1}{2}\times 15\times 45 = 337.5$ m

Distance travelled by scooter in 15 s

= Area of rectangle $OCEF$

$= 15 \times 30 = 450$ m

Thus, difference between distance travelled by them

$= 450$ m $- 337.5$ m $= 112.5$ m

(b) Let after time t from start car will catch up the scooter. In time t, the distance travelled by them are equal.

Distance travelled by car $= -\times 15\times 45 + 45(\ -15)$

Distance travelled by scooter $= 30t$

$$\frac{1}{2}\times 15\times 45 + 45(t-15) = 30t$$

which gives $t = 22.5$ s

Distance travelled by car or scooter in 22.5 s $= 30 \times 22.5$

$= 675$ m

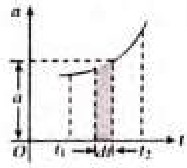
So the car catches the scooter when both are at 67.5 m from the starting point.

Acceleration–Time Graph

Let the acceleration be given as the function of time, i.e., $a = f(t)$. Then the elementary change in velocity during a time dt is: $d\vec{v} = \vec{a}\,dt$.

We can observe that in a–t graph, $\vec{a}\,dt$ is equal to the area of the shaded elementary strip (rectangle of sides a and dt; (see figure). Summing up all the elementary areas dA, we have the total area

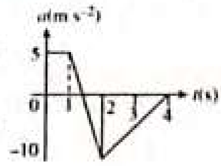
$$A = \int dA = \int \vec{a}\,dt$$



Since $\int_{t_1}^{t_2} \vec{a}\,dt = \int_{v_1}^{v_2} d\vec{v} = \Delta\vec{v}$

we have $\Delta\vec{V} = \int \vec{a}\,dt = A$ = Area under $a - t$ graph

ILLUSTRATION 3.16 A particle moves along x-axis with an initial speed $v_0 = 5$ m s^{-1}. If its acceleration varies with time as shown in a–t graph in figure.



(a) Find the velocity of the particle at $t = 4$ s.

(b) Find the time when the particle starts moving along $-x$ direction.

Solution. The velocity of the particle at $t = 4$ s can be given as

$$\vec{v}_4 = \vec{v}_0 + \Delta\vec{v} \qquad \text{...(i)}$$

where $\Delta\vec{v} \equiv A$

(= area under a–t graph during first four seconds)

Referring to a–t graph (figure), we have

$$A = A_1 + A_2 - A_3 - A_4 \qquad \text{...(ii)}$$

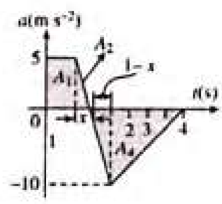
where $A_1 = 5\times 1 = 5$, $A_2 = \frac{1}{2}\times x\times 5$,

$$A_3 = \frac{1}{2}\times(1-x)\times 10, \text{ and } A_4 = \frac{1}{2}\times 2\times 10 = 10$$

We can find x as following:

Using properties of similar triangles, we have $\frac{x}{5} = \frac{1-x}{10}$

This yields $x = \frac{1}{3}$.



Substituting $x = \frac{1}{3}$ in $A_2 = \frac{1}{2}\times x\times 5$

and $A_3 = \frac{1}{2}(1-x)\times 10$, we have

$$A_2 = \frac{5}{6} \text{ and } A_3 = \frac{10}{3}.$$

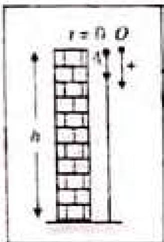
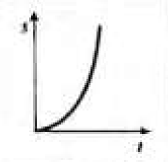
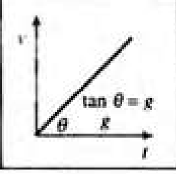
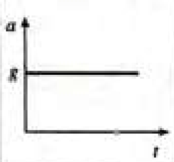
Then substituting A_1, A_2, A_3, and A_4 in (ii), we have $A = -7.5$. Negative area tells us that change in velocity is along $-x$ direction

$$\Delta\vec{v} = -7.5 \text{ m s}^{-1}$$

Hence, substituting in (i), $\vec{v}_0 = 5 \text{ m s}^{-1}$ and $\Delta\vec{v} = -7.5 \text{ m s}^{-1}$, we have $\vec{v}_4 = \vec{v}_0 + \Delta\vec{v} = 5 - 7.5 = -2.5 \text{ m s}^{-1}$.

Graphical Representation of Motion of A Particle Moving Under Gravity

If a body dropped from some height (initial velocity zero): Taking initial position as origin and direction of motion (i.e., downward direction) as a positive.

	Position–time graph	Velocity–time graph	Acceleration–time graph
			

If a body is projected vertically upward: Taking initial position as origin and direction of motion (i.e., vertically up) as positive.

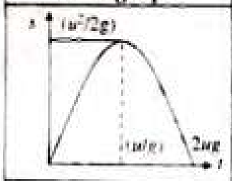
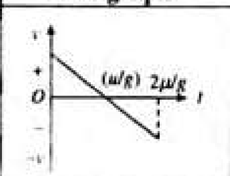
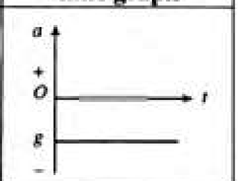
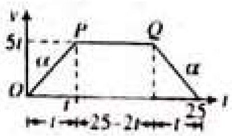
Position–time graph	Velocity–time graph	Acceleration–time graph
		

ILLUSTRATION 3.17 A car starts moving rectilinearly, first with acceleration $\alpha = 5 \text{ m s}^{-2}$ (the initial velocity is equal to zero), then uniformly, and finally, decelerating at the same rate α comes to a stop. The total time of motion equals $t = 25$ s. The average velocity during this time is equal to $\langle v \rangle = 72 \text{ km h}^{-1}$. How long does the car move uniformly?

Solution. Let t be the time up to which the car accelerates or decelerates. The maximum velocity attained in this duration is $5t$. The time upto which car moves uniformly $= 25 - 2t$. The velocity–time graph of the motion of car is drawn as shown in the figure.

Given the average velocity is whole time of motion

$$v_{av} = \frac{72 \times 5}{18} = 20 \text{ m s}^{-1}$$



The average velocity from the graph can be obtained as

$$v_{av} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{\text{Area of } \vec{v} - t \text{ graph}}{\text{Total time}}$$

$$20 = \frac{\frac{1}{2} \times [25 + (25 - 2t)] \times 5t}{25} = \frac{\frac{1}{2} \times [50 - 2t)] \times 5t}{25}$$

$$200 = 50t - 2t^2$$

$\Rightarrow$ $t^2 - 25t + 100 = 0$

$\Rightarrow$ $(t - 20)(t - 5) = 0 \Rightarrow t = 5 \text{ s or } 20 \text{ s}$

But $t = 20$ s is not possible.

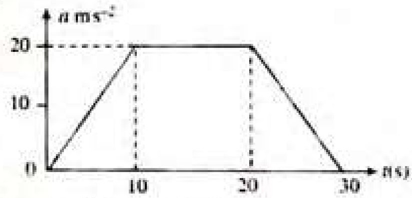
Hence, $t = 5$ s

The time up to which car moves uniformly

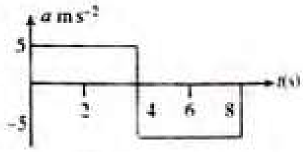
$= 25 - 2t = 25 - 2 \times 5 = 15$ s

CONCEPT APPLICATION EXERCISE 3.4

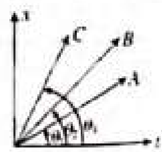
1. See figure. Find the average acceleration in first 20 s. (**Hint:** Area under $a - t$ graph is equal to the change in velocity)



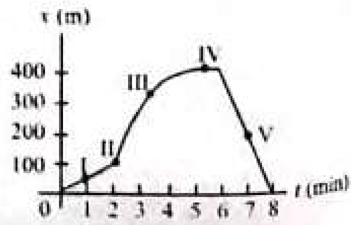
2. At $t = 0$, a particle starts from rest and moves along a straight line, whose acceleration–time graph is shown in figure. Convert this graph into velocity–time graph. From the velocity–time graph, find the maximum velocity attained by the particle. Also find from v–t graph, the displacement and distance travelled by the particle from 2 to 6 s.



3. You are given the position–time graph of three different bodies A, B, and C. Find which will have greater velocity and which will have least velocity.

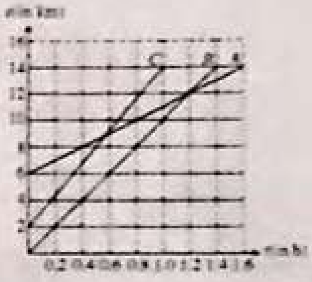


4. A physics professor leaves her house and walks along the sidewalk towards campus. After 5 min, it starts to rain and she returns home. Her distance from her house as a function of time is shown in figure. At which of the labeled points is her velocity?



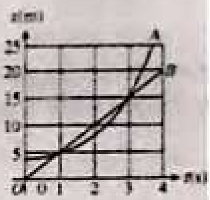
(a) Zero
(b) Constant and positive
(c) Constant and negative
(d) Increasing in magnitude

5. Figure shows the position–time graphs of three cars A, B, and C. On the basis of the graphs, answer the following questions:



(a) Which car has the highest speed and which the lowest?
(b) Are the three cars ever at the same point on the road?
(c) When C passes A, where is B?
(d) What is the time interval during car A travel between the time it passed cars B and C?
(e) What is the relative velocity of car C with respect to car A?
(f) What is the relative velocity of car B with respect to car C?

6. Two cars, A and B, move along the x-axis. Car A starts from rest with constant acceleration while car B moves with constant velocity.



(a) At what time(s), t(s), if any, do A and B have the same position?
(b) At what time(s), if any, do A and B have the same velocity? What is the velocity of car B at this time.
(c) Graph velocity versus time for both A and B.
(d) At what time(s), if any, does car A pass car B?
(e) At what time(s), if any, does car B pass car A?

SOLVED EXAMPLES

1. A thief is running away on a straight road in jeep moving with a speed of 9 ms^{-1} A police man chases him on a motor cycle moving at a speed of 10 ms^{-1}. If the instantaneous separation of the jeep from the motorcycle is 100 m, how long will it take for the police to catch the thief

(a) 1 s (b) 19 s
(c) 90 s (d) 100 s

Sol. (d) The relative velocity of policeman w.r.t. thief
$= 10 - 9 = 1$ m/s.

$\therefore$ Time taken by police to catch the thief $= \dfrac{100}{1} = 100$ s

2. Two trains one of length 100 m and another of length 125 m, are moving in mutually opposite directions along parallel lines, meet each other, each with speed 10 m/s. If their acceleration are 0.3 m/s^2 and 0.2 m/s^2, respectively, then the time they take to pass each other will be

(a) 5 s (b) 10 s
(c) 15 s (d) 20 s

Sol. (b) Relative velocity of one train w.r.t. other
$= 10 + 10 = 20$ m/s.

Relative acceleration $= 0.3 + 0.2 = 0.5\ m/s^2$

If trains cross each other, then from $s = ut + \dfrac{1}{2}at^2$

As, $s = s_1 + s_2 = 100 + 125 = 225$

$\Rightarrow \quad 225 = 20t + \dfrac{1}{2} \times 0.5 \times t^2 \quad \Rightarrow 0.5t^2 + 40t - 450 = 0$

$\Rightarrow \quad t = -\dfrac{40 \pm \sqrt{1600 + 4.(005) \times 450}}{1} = -40 \pm 50$

$\therefore \quad t = 10$ sec (Taking +ve value).

3. A body starts from rest with uniform acceleration. If its velocity after n second is v, then its displacement in the last two seconds is

(a) $\dfrac{2v(n+1)}{n}$ (b) $\dfrac{v(n+1)}{n}$

(c) $\dfrac{v(n-1)}{n}$ (d) $\dfrac{2v(n-1)}{n}$

Sol. (d) $\because v = 0 + na \Rightarrow a = v/n$

Now, distance travelled in n sec $\alpha \Rightarrow S_n = \dfrac{1}{2}an^2$ and distance travelled in $(n-2)$ sec $\Rightarrow S_{n-2} = \dfrac{1}{2}a(n-2)^2$

$\therefore$ Distance travelled in last two seconds,

$= S_n - S_{n-2} = \dfrac{1}{2}an^2 - \dfrac{1}{2}a(n-2)^2$

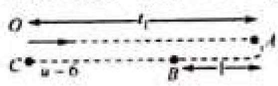
$= \dfrac{a}{2}[n^2 - (n-2)^2] = \dfrac{a}{2}[n + (n-2)][n - (n-2)]$

$= a(2n-2) = \dfrac{v}{n}(2n-2) = \dfrac{2v(n-1)}{n}$

4. A particle is moving in a straight line and passes through a point O with a velocity of 6 ms^{-1}. The particle moves with a constant retardation of 2 ms^{-2} for 4 s and there after moves with constant velocity. How long after leaving O does the particle return to O?

(a) 3 s (b) 8 s
(c) Never (d) 4 s

Sol. (b) Let the particle moves toward right with velocity 6 m/s. Due to retardation, after time t_1, its velocity becomes zero.



From $v = u - at \Rightarrow 0 = 6 - 2 \times t_1$

$\Rightarrow \quad t_1 = 3$ sec

But retardation works on it for 4 sec. It means after reaching point A, direction of motion gets reversed and acceleration works on the particle for next one second.

$$S_{OA}=ut_1-\frac{1}{2}at_1^2=6\times 3-\frac{1}{2}(2)(3)^2=18-9=9\text{m}$$

$$S_{AB}=\frac{1}{2}\times 2\times(1)^2=1\text{ m}$$

$\therefore$ $S_{BC}=S_{OA}-S_{AB}=9-1=8$ m

Now velocity of the particle at point B in return journey

$v=0+2\times 1=2$ m/s

In return journey from B to C, particle moves with constant velocity 2 m/s to cover the distance 8 m.

$$\text{Time taken}=\frac{\text{Distance}}{\text{Velocity}}=\frac{8}{2}=4\text{ s}$$

Total time taken by particle to return at point O is

$T=t_{OA}+t_{AB}+t_{BC}=3+1+4=8$ s.

5. A particle is projected with velocity v_0 along x-axis. The deceleration on the particle is proportional to the square of the distance from the origin i.e., $a=\alpha x^2$. The distance at which the particle stops is

(a) $\sqrt{\frac{3v_0}{2\alpha}}$ (b) $\left(\frac{3v_0}{2\alpha}\right)^{\frac{1}{3}}$

(c) $\sqrt{\frac{3v_0^2}{2\alpha}}$ (d) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Sol. (d) $a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=v\frac{dv}{dx}=-\alpha x^2$ (Given)

$$\Rightarrow \int_{v_0}^{0} v\,dv=-\alpha\int_0^S x^2dx \Rightarrow \left[\frac{v^2}{2}\right]_{v_0}^{0}=-\alpha\left[\frac{x^3}{3}\right]_0^S$$

$$\Rightarrow \frac{v_0^2}{2}=\frac{\alpha S^3}{3} \Rightarrow S=\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$$

6. A body is projected vertically up with a velocity v and after some time it returns to the point from which it was projected. The average velocity and average speed of the body for the total time of flight are

(a) $\vec{v}/2$ and $v/2$ (b) 0 and $v/2$

(c) 0 and 0 (d) $\vec{v}/2$ and 0

Sol. (b) Average velocity = 0 because net displacement of the body is zero.

$$\text{Average speed}=\frac{\text{Total distance covered}}{\text{Time of flight}}=\frac{2H_{max}}{2u/g}$$

$$\Rightarrow v_{av}=\frac{2u^2/2g}{2u/g} \Rightarrow v_{av}=u/2$$

Velocity of projection = v (given)

$\therefore$ $v_{av}=v/2$

7. A balloon rises from rest with a constant acceleration $g/8$. A stone is released from it when it has risen to height h. The time taken by the stone to reach the ground is

(a) $4\sqrt{\frac{h}{g}}$ (b) $2\sqrt{\frac{h}{g}}$

(c) $\sqrt{\frac{2h}{g}}$ (d) $\sqrt{\frac{g}{h}}$

Sol. (b) The velocity of balloon at height h, $v=\sqrt{2\left(\frac{g}{8}\right)h}$

When the stone is released from this balloon, it will go upward with velocity, $v=\frac{\sqrt{gh}}{2}$ (same as that of balloon). In this condition, time taken by stone to reach the ground is

$$t=\frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$$

$$=\frac{\sqrt{gh}/2}{g}\left[1+\sqrt{1+\frac{2gh}{gh/4}}\right]=\frac{2\sqrt{gh}}{g}=2\sqrt{\frac{h}{g}}$$

8. A body falls freely from the top of a tower. It covers 36% of the total height in the last second before striking the ground level. The height of the tower is

(a) 50 m (b) 75 m

(c) 100 m (d) 125 m

Sol. (d) Let height of tower is h and body takes t time to reach to ground when it fall freely.

$$\therefore h=\frac{1}{2}gt^2 \quad ...(i)$$

In last second, i.e., t^{th} sec body travels = $0.36h$

It means in rest of the time i.e. in $(t-1)$ sec it travels

$=h-0.36h=0.64h$

Now applying equation of motion for $(t-1)$ sec, we get

$$0.64h=\frac{1}{2}g(t-1)^2 \quad ...(ii)$$

From (i) and (ii), we get $t=5$ sec and $h=125$ m.

9. A projectile is fired vertically upwards with an initial velocity u. After an interval of T seconds, a second projectile is fired vertically upwards, also with initial velocity u.

(a) They meet at time $t=\frac{u}{g}$ and at a height $\frac{u^2}{2g}+\frac{gT^2}{8}$

(b) They meet at time $t=\frac{u}{g}+\frac{T}{2}$ and at a height $\frac{u^2}{2g}+\frac{gT^2}{8}$

(c) They meet at time $t=\frac{u}{g}+\frac{T}{2}$ and at a height $\frac{u^2}{2g}-\frac{gT^2}{8}$

(d) They never meet

Sol. (c) For first projectile, $h_1 = ut - \frac{1}{2}gt^2$

For second projectile, $h_2 = u(t-T) - \frac{1}{2}g(t-T)^2$

When both meet i.e. $h_1 = h_2$:

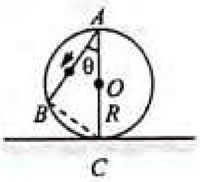
$$ut - \frac{1}{2}gt^2 = u(t-T) - \frac{1}{2}g(t-T)^2$$

$$\Rightarrow \quad uT + \frac{1}{2}gT^2 = gtT$$

$$\Rightarrow \quad t = \frac{u}{g} + \frac{T}{2}$$

and $h_1 = u\left(\frac{u}{g} + \frac{T}{2}\right) - \frac{1}{2}g\left(\frac{u}{2} + \frac{T}{2}\right)^2 = \frac{u^2}{2g} - \frac{gT^2}{8}$

10. A frictionless wire AB is fixed on a sphere of radius R. A very small spherical ball slips on this wire. The time taken by this ball to slip from A to B is



(a) $\frac{2\sqrt{gR}}{g\cos\theta}$

(b) $2\sqrt{gR}\cdot\frac{\cos\theta}{2}$

(c) $2\sqrt{\frac{R}{g}}$

(d) $\frac{gR}{\sqrt{g\cos\theta}}$

Sol. (c) Acceleration of body along AB is

Distance travelled in time t sec = $AB = \frac{1}{2}(g\cos\theta)t^2$

From ΔABC, $AB = 2R\cos\theta$; $2R\cos\theta = \frac{1}{2}g\cos\theta t^2$

$$\Rightarrow \quad t^2 = \frac{4R}{g} \text{ or } t = 2\sqrt{\frac{R}{g}}$$

11. A body is slipping from an inclined plane of height h and length l. If the angle of inclination is θ, the time taken by the body to come from the top to the bottom of this inclined plane is

(a) $\sqrt{\frac{2h}{g}}$ (b) $\sqrt{\frac{2l}{g}}$

(c) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ (d) $\sin\theta\sqrt{\frac{2h}{g}}$

Sol. (c) Force down the plane = $mg\sin\theta$

Acceleration down the plane = $g\sin\theta$

Since $l = 0 + \frac{1}{2}g\sin\theta t^2$

$$t^2 = \frac{2}{g\sin\theta} = \frac{2h}{g\sin^2\theta} \Rightarrow t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$$

12. A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is

(a) 4.0 m/s (b) 5.0 m/s

(c) 5.5 m/s (d) 4.8 m/s

Sol. (a) If t_2 and $2t_2$ are the time taken by particle to cover first and second half distance, respectively, then

$$t_1 = \frac{x/2}{3} = \frac{x}{6} \quad \text{...(i)}$$

$x_1 = 4.5\,t_2$ and $x_2 = 7.5\,t_2$

So, $x_1 = x_2 = \frac{x}{2} \Rightarrow 4.5t_2 + 7.5t_2 = \frac{x}{2}$

$$t_2 = -\frac{x}{24} \quad \text{...(ii)}$$

Total time $t_1 + x_2 + 2t_2 = \frac{x}{6} + \frac{x}{12} = \frac{x}{4}$

So, average speed 4 m/sec.

13. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β and comes to rest. If the total time elapsed is t, then the maximum velocity acquired by the car is

(a) $\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)t$ (b) $\left(\frac{\alpha^2-\beta^2}{\alpha\beta}\right)t$

(c) $\frac{(\alpha-\beta)t}{\alpha\beta}$ (d) $\frac{\alpha\beta t}{\alpha+\beta}$

Sol. (d) Let the car accelerate at rate α for time t_1. Then maximum velocity attained, $v = 0 + \alpha t_1 + \alpha t_1$

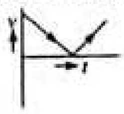
Now, the car decelerates at a rate β for time $(t - t_1)$ and finally comes to rest. Then,

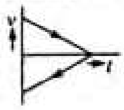
$$0 = v\,\beta(t - t_1) \Rightarrow 0 = \alpha t_1 - \beta t + \beta t_1$$

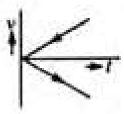
$$\Rightarrow \quad t_1 = \frac{\beta}{\alpha+\beta}t$$

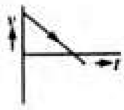
$$\therefore \quad v = \frac{\alpha\beta}{\alpha+\beta}t$$

14. A ball is thrown vertically upwards. Which of the following graph/graphs represent velocity-time graph of the ball during its flight (air resistance is neglected)

(a) 

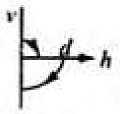
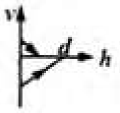
(b) 

(c) 

(d) 

Sol. (d) In the positive region, the velocity decreases linearly (during rise) and in the negative region, velocity increases linearly (during fall) and the direction is opposite to each other during rise and fall. Hence, fall is shown in the negative region.

15. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground is correctly shown in

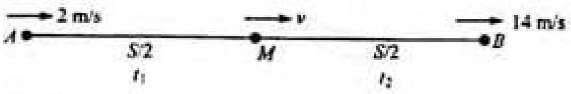
(a) 
(b) 
(c) 
(d) 

Sol. (a) For the given condition, initial height $h = d$ and velocity of the ball is zero. When the ball moves downward its velocity increases and it will be maximum when the ball hits the ground; just after the collision, it becomes half and in opposite direction. As the ball moves upward, its velocity again decreases and becomes zero at height $d/2$. This explanation match with graph (A).

16. A particle moves with uniform acceleration along a straight line AB. Its velocities at A and B are 2 m/s and 14 m/s, respectively. M is the mid-point of AB. The particle takes t_1 seconds to go from A to M and t_2 seconds to go from M to B. Then t_2/t_1 is

(a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) 3 : 1

Sol. (c) $v^2 = 2^2 + \frac{2as}{2}$



$\Rightarrow \quad v^2 - 4 = as$ (i)

$$14^2 - v^2 = \frac{2as}{2}$$

$\Rightarrow \quad 196 - v^2 = as$ (ii)

From (i) and (ii),

$$v^2 - 4 = 196 - v^2 \Rightarrow v = 10 \text{ m/s}$$

Now $t_1 = \frac{v-2}{a} = \frac{10-2}{a} = \frac{8}{a}$

$$t_2 = \frac{14-v}{a} = \frac{14-10}{a} = \frac{4}{a} \Rightarrow \frac{t_2}{t_1} = \frac{4/a}{8/a} = \frac{1}{2}$$

17. A ball is thrown straight upward with a velocity v_0 from a height h above the ground. The time taken for the ball to strike the ground is

(a) $\frac{v_0}{g}\left(1+\frac{2gh}{v_0^2}\right)$ (b) $\frac{v_0}{g}\left(\sqrt{1+\frac{2gh}{v_0^2}}\right)$

(c) $\frac{v_0}{g}\left(1-\sqrt{1+\frac{2gh}{v_0^2}}\right)$ (d) $\frac{v_0}{g}\left(1+\sqrt{1+\frac{2gh}{v_0^2}}\right)$

Sol. (d) Apply $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad -h = v_0 t - \frac{1}{2}gt^2 \Rightarrow gt^2 - 2v_0 t - 2h = 0$$

$$\Rightarrow \quad t = \frac{2v_0 \pm \sqrt{4v_0^2 + 8gh}}{2g}$$

As t is not negative, so ignoring negative sign, we get

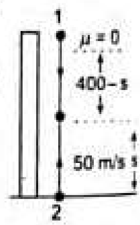
$$t = \frac{2v_0 + 2\sqrt{v_0^2 + 2gh}}{2g} = \frac{v_0}{g}\left[1 + \sqrt{1+\frac{2gh}{v_0^2}}\right]$$

18. From the top of the tower of height 400 m, a ball is dropped by a man, simultaneously from the base of the tower, another ball is thrown up with a velocity 50 m/s; at what distance will they meet from the base of the tower?

(a) 100 m (b) 320 m
(c) 80 m (d) 240 m

Sol. (c) Let the first ball meet at a height s from ground

$$400 - s = \frac{1}{2}gt^2 \quad \text{...(i)}$$



and for second ball: $s = 50t - \frac{1}{2}gt^2$ ...(ii)

Adding $50\,t = 400$, we get

$t = 8$ sec.

Now substituting the value of 't' in (ii),

we get $s = 50 \times 8 - \frac{1}{2} \times 10 \times 64 = 80$ m

19. A student is standing at a distance of 50 metres from a bus. As soon as the bus begins its motion (starts moving away from student) with an acceleration of 1 m s^{-2}, the student starts running towards the bus with a uniform velocity u. Assuming the motion to be along a straight road, the minimum value of u, so that the student is able to catch the bus is:

(a) 5 m s^{-1} (b) 8 m s^{-1}
(c) 10 m s^{-1} (d) 12 m s^{-1}

Sol. (c) $50 + x = ut$ ...(1)

and $x = \frac{t^2}{2}$ ...(2)

From (1) and (2),

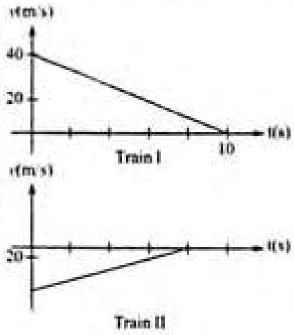
$$50+\frac{t^2}{2}=ut \Rightarrow t^2-2ut+100=0$$

For real roots, discriminant = 0

For minimum velocity, discriminant = 0

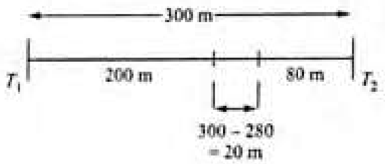
$\Rightarrow \quad 4u^2-400=0 \quad \Rightarrow u=10$ m/s

20. Two trains, which are moving along different tracks in opposite directions, are put on the same track due to a mistake. Their drivers, on noticing the mistake, start slowing down the trains when the trains are 300 m apart. Graphs given below show their velocities as function of time as the trains slow down. The separation between the trains when both have stopped is



(a) 120 m (b) 280 m
(c) 60 m (d) 20 m

Sol. (d) Initial distance between trains is 300 m. Displacements of trains can be calculated by area under V-t graph.



$$\text{Displacement of train 1} = \frac{1}{2}\times 10\times 40 = 200 \text{ m}$$

$$\text{Displacement of train 2} = \frac{1}{2}\times 8\times(-20) = -80 \text{ m}$$

which means it moves towards left.

$\therefore$ Distance between the two is 300 – 200 – 80 = 20 m.

EXERCISES

General Kinematics and Motion with Constant Acceleration

1. The magnitude of displacement is equal to the distance covered in a given interval of time if the particle
(a) Moves with constant acceleration along any path
(b) Moves with constant speed
(c) Moves in same direction with constant velocity or with variable velocity
(d) Moves with constant velocity

2. The distance travelled by a particle in a straight line motion is directly proportional to $t^{1/2}$, where t is the time elapsed. What is the nature of motion?
(a) Increasing acceleration
(b) Decreasing acceleration
(c) Increasing retardation
(d) Decreasing retardation

3. The position x of a particle varies with time (t) as $x=at^2-bt^3$. The acceleration at time t of the particle will be equal to zero, where t is equal to
(a) $\frac{2a}{3b}$ (b) $\frac{a}{b}$ (c) $\frac{a}{3b}$ (d) zero

4. Between two stations, a train accelerates from rest uniformly at first, then moves with constant velocity, and finally retards uniformly to come to rest. If the ratio of the time taken is 1 : 8 : 1 and the maximum speed attained be 60 kmh^{-1}, then what is the average speed over the whole journey?
(a) 48 kmh^{-1} (b) 52 kmh^{-1}
(c) 54 kmh^{-1} (d) 56 kmh^{-1}

5. The velocity acquired by a body moving with uniform acceleration is 30 ms^{-1} in 2 s and 60 ms^{-1} in 4 s. The initial velocity is
(a) zero (b) 2 ms^{-1} (c) 3 ms^{-1} (d) 10 ms^{-1}

6. A particle starts from the origin with a velocity of 10 ms^{-1} and moves with a constant acceleration till the velocity increases to 50 ms^{-1}. At that instant, the acceleration is suddenly reversed. What will be the velocity of the particle, when it returns to the starting point?
(a) Zero (b) 10 ms^{-1} (c) 50 ms^{-1} (d) 70 ms^{-1}

7. When the speed of a car is u, the minimum distance over which it can be stopped is s. If the speed becomes nu, what will be the minimum distance over which it can be stopped during the same time?
(a) s/n (b) ns (c) s/n^2 (d) n^2s

8. The relation between time t and distance x is $t=\alpha x^2+\beta x$ where α and β are constants. The retardation is
(a) $2\alpha v^3$ (b) $2\beta v^3$ (c) $2\alpha\beta v^3$ (d) $2\beta^2 v^3$

9. The displacement x of a particle moving in one dimension under the action of a constant force is related to time t by the equation $t=\sqrt{x}+3$, where x is in meters and t is in seconds. Find the displacement of the particle when its velocity is zero.
(a) Zero (b) 12 m (c) 6 m (d) 18 m

10. A drunkard is walking along a straight road. He takes five steps forward and three steps backward and so on. Each step is 1 m long and takes 1 s. There is a pit on the road 11 m away from the starting point. The drunkard will fall into the pit after
(a) 29 s (b) 21 s (c) 37 s (d) 31 s

11. A body travels 200 cm in the first 2 s and 220 cm in the next 4 s with deceleration. The velocity of the body at the end of the seventh second is
(a) $5\ cm s^{-1}$ (b) $10\ cm s^{-1}$
(c) $15\ cm s^{-1}$ (d) $20\ cm s^{-1}$

12. A body starts from rest and travels a distance S with uniform acceleration, then moves uniformly a distance $2S$ uniformly, and finally comes to rest after moving further $5S$ under uniform retardation. The ratio of the average velocity to maximum velocity is
(a) 2/5 (b) 3/5 (c) 4/7 (d) 5/7

13. A police party is chasing a dacoit in a jeep which is moving at a constant speed v. The dacoit is on a motorcycle. When he is at a distance x from the jeep, he accelerates from rest at a constant rate. Which of the following relations is true if the police is able to catch the dacoit?
(a) $v^2 \le \alpha x$ (b) $v^2 \le 2\alpha x$
(c) $v^2 \ge 2\alpha x$ (d) $v^2 \ge \alpha x$

14. A moving car possesses average velocities of $5\ m s^{-1}$, $10\ m s^{-1}$, and $15\ m s^{-1}$ in the first, second, and third seconds, respectively. What is the total distance covered by the car in these 3 s?
(a) 15 m (b) 30 m
(c) 55 m (d) None of these

15. The average velocity of a body moving with uniform acceleration after travelling a distance of 3.06 m is $0.34\ m s^{-1}$. If the change in velocity of the body is 0.18 $m s^{-1}$ during this time, its uniform acceleration is
(a) $0.01\ m s^{-2}$ (b) $0.02\ m s^{-2}$
(c) $0.03\ m s^{-2}$ (d) $0.04\ m s^{-2}$

16. A point moves with uniform acceleration and v_1, v_2, and v_3 denote the average velocities in the three successive intervals of time t_1, t_2, and t_3. Which of the following relations is correct?
(a) $(v_1 - v_2):(v_2 - v_3) = (t_1 - t_2):(t_2 + t_3)$
(b) $(v_1 - v_2):(v_2 - v_3) = (t_1 + t_2):(t_2 + t_3)$
(c) $(v_1 - v_2):(v_2 - v_3) = (t_1 - t_2):(t_1 - t_3)$
(d) $(v_1 - v_2):(v_2 - v_3) = (t_1 - t_2):(t_2 - t_3)$

17. A 2-m wide truck is moving with a uniform speed $v_0 = 8\ m s^{-1}$ along a straight horizontal road. A pedestrian starts to cross the road with a uniform speed v when the truck is 4 m away from him. The minimum value of v so that he can cross the road safely is
(a) $2.62\ m s^{-1}$ (b) $4.6\ m s^{-1}$
(c) $3.57\ m s^{-1}$ (d) $1.414\ m s^{-1}$

Problems Based on Motion Under Gravity

18. A ball is released from the top of a tower of height h. It takes time T to reach the ground. What is the position of the ball (from ground) after time $T/3$?
(a) $h/9$ m (b) $7h/9$ m (c) $8h/9$ m (d) $17h/18$ m

19. A body is released from the top of a tower of height H m. After 2 s, it is stopped and then instantaneously released. What will be its height after next 2 s?
(a) $(H - 5)$ m (b) $(H - 10)$ m
(c) $(H - 20)$ m (d) $(H - 40)$ m

20. A stone is dropped from the top of a tower of height h. After 1 s another stone is dropped from the balcony 20 m below the top. Both reach the bottom simultaneously. What is the value of h? Take $g = 10\ m s^{-2}$.
(a) 3125 m (b) 312.5 m (c) 31.25 m (d) 25.31 m

21. A person is throwing two balls in the air one after the other. He throws the second ball when the first ball is at the highest point. If he is throwing the balls every second, how high do they rise?
(a) 5 m (b) 3.75 m (c) 2.50 m (d) 1.25 m

22. A stone thrown upwards with speed u attains maximum height h. Another stone thrown upwards from the same point with speed $2u$ attains maximum height H. What is the relation between h and H?
(a) $2h = H$ (b) $3h = H$ (c) $4h = H$ (d) $5h = H$

23. A body dropped from the top of a tower covers a distance $7x$ in the last second of its journey, where x is the distance covered in the first second. How much time does it take to reach the ground?
(a) 3 s (b) 4 s (c) 5 s (d) 6 s

24. The distances moved by a freely falling body (starting from rest) during 1st, 2nd, 3rd, ..., nth second of its motion are proportional to
(a) Even numbers (b) Odd numbers
(c) All integral numbers (d) Squares of integral numbers

25. A stone is dropped from a certain height which can reach the ground in 5 s. It is stopped after 3 s of its fall and then it is again released. The total time taken by the stone to reach the ground will be
(a) 6 s (b) 6.5 s (c) 7 s (d) 7.5 s

26. A body sliding on a smooth inclined plane requires 4 s to reach the bottom, starting from rest at the top. How much time does it take to cover one-fourth the distance starting from rest at the top?
(a) 1 s (b) 2 s (c) 4 s (d) 16 s

27. B_1, B_2, and B_3 are three balloons ascending with velocities v, $2v$, and $3v$, respectively. If a bomb is dropped from each when they are at the same height, then
(a) Bomb from B_1 reaches ground first
(b) Bomb from B_2 reaches ground first
(c) Bomb from B_3 reaches ground first
(d) They reach the ground simultaneously

28. A ball is thrown from the top of a tower in vertically upward direction. The velocity at a point h meter below the point of projection is twice of the velocity at a point h meter above the point of projection. Find the maximum height reached by the ball above the top of tower.
(a) $2h$ (b) $3h$ (c) $(5/3)h$ (d) $(4/3)h$

29. A juggler keeps on moving four balls in air throwing the balls after regular intervals. When one ball leaves his hand (speed = 20 m s^{-1}), the position of other balls (height in meter) will be (take $g = 10$ m s^{-2})
(a) 10, 20, 10 (b) 15, 20, 15
(c) 5, 15, 20 (d) 5, 10, 20

30. A particle slides from rest from the topmost point of a vertical circle of radius r along a smooth chord making an angle θ with the vertical. The time of descent is
(a) Least for $\theta = 0$ (b) Maximum for $\theta = 0$
(c) Least for $\theta = 45°$ (d) Independent of θ

31. A body is thrown vertically upwards from A, the top of a tower. It reaches the ground in time t_1. If it is thrown vertically downwards from A with the same speed, it reaches the ground in time t_2. If it is allowed to fall freely from A, then the time it takes to reach the ground is given by
(a) $t = \dfrac{t_1 + t_2}{2}$ (b) $t = \dfrac{t_1 - t_2}{2}$
(c) $t = \sqrt{t_1 t_2}$ (d) $t = \sqrt{\dfrac{t_1}{t_2}}$

32. A stone is dropped from the 25th storey of a multistoried building and it reaches the ground in 5 s. In the first second, it passes through how many storeys of the building? ($g = 10$ m s^{-2})
(a) 1 (b) 2
(c) 3 (d) none of these

33. A body is projected upwards with a velocity u. It passes through a certain point above the ground after t_1. The time after which the body passes through the same point during the return journey is
(a) $\left(\dfrac{u}{g} - t_1^2\right)$ (b) $2\left(\dfrac{u}{g} - t_1\right)$
(c) $3\left(\dfrac{u^2}{g} - t_1\right)$ (d) $3\left(\dfrac{u^2}{g^2} - t_1\right)$

34. A parachutist drops first freely from an aeroplane for 10 s and then his parachute opens out. Now he descends with a net retardation of 2.5 m s^{-2}. If he bails out of the plane at a height of 2495 m and $g = 10$ m s^{-2}, his velocity on reaching the ground will be
(a) 5 m s^{-1} (b) 10 m s^{-1}
(c) 15 m s^{-1} (d) 20 m s^{-1}

35. Water drops fall from a tap on the floor 5 m below at regular intervals of time, the first drop striking the floor when the fifth drop begins to fall. The height at which the third drop will be from ground (at the instant when the first drop strikes the ground) will be ($g = 10$ m s^{-2})
(a) 1.25 m (b) 2.15 m (c) 2.75 m (d) 3.75 m

Relative Velocity in One Dimension

36. A thief is running away on a straight road in a jeep moving with a speed of 9 m s^{-1}. A policeman chases him on a motor cycle moving at a speed of 10 m s^{-1}. If the instantaneous separation of the jeep from the motor cycle is 100 m, how long will it take for the policeman to catch the thief?
(a) 1 s (b) 19 s (c) 90 s (d) 100 s

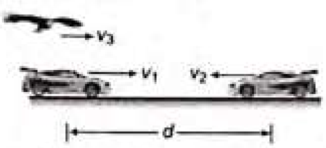
37. A train 100 m long travelling at 40 m s^{-1} starts overtaking another train 200 m long travelling at 30 m s^{-1}. The time taken by the first train to pass the second train completely is
(a) 30 s (b) 40 s (c) 50 s (d) 60 s

38. A train is moving at a constant speed V when its driver observes another train in front of him on the same track and moving in the same direction with constant speed v. If the distance between the trains is x, then what should be the minimum retardation of the train so as to avoid collision?
(a) $\dfrac{(V+v)^2}{x}$ (b) $\dfrac{(V-v)^2}{x}$
(c) $\dfrac{(V+v)^2}{2x}$ (d) $\dfrac{(V-v)^2}{2x}$

39. A person A is sitting in one train while another person B is in the second train. The trains are moving with velocities 60 m/s and 40 m/s, respectively, in the same direction. Then the velocity of B relative to A will be
(a) 100 m/s in the direction of trains
(b) 20 m/s in the direction of trains
(c) 100 m/s in the direction opposite to that of trains
(d) 20 m/s in the direction opposite to that of the trains

40. Imagine yourself standing in an elevator which is moving with an upward acceleration $a = 2$ m/s^2. A coin is dropped from rest from the roof of the elevator, relative to you. The roof to floor height of the elevator is 1.5 m. (Take $g = 10$ m/s^2). Find the velocity of the coin relative to you when it strikes the base of the elevator.
(a) 3 m/s (b) 6 m/s (c) 2 m/s (d) 1.5 m/s

41. A bird flies to and fro between two cars which move with velocities $v_1 = 20$ m/s and $v_2 = 30$ m/s. If the speed of the bird is $v_3 = 10$ m/s and the initial distance of separation between them is $d = 2$ km, find the total distance covered by the bird till the cars meet.



(a) 2000 m (b) 1000 m (c) 400 m (d) 200 m

42. A man swimming downstream overcome a float at a point M. After travelling distance D he turned back and passed the float at a distance of $D/2$ from the point M, then the ratio of speed of swimmer with respect to still water to the speed of the river will be
(a) 2 (b) 3 (c) 4 (d) 2.5

43. It takes one minute for a passenger standing on an escalator to reach the top. If the escalator does not move it takes him 3 minute to walk up. How long will it take for the passenger to arrive at the top if he walks up the moving escalator?
(a) 30 sec (b) 45 sec (c) 40 sec (d) 35 sec

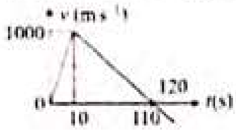
44. A bus is moving with a velocity 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what velocity should the scooterist chase the bus?
(a) 50 ms^{-1} (b) 40 ms^{-1} (c) 30 ms^{-1} (d) 20 ms^{-1}

Understanding Motion Through Graphs

45. The variation of velocity of a particle moving along a straight line is shown in figure. The distance travelled by the particle in 12 s is

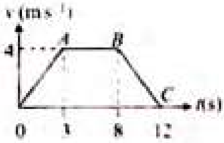
(a) 37.5 m
(b) 32.5 m
(c) 35.0 m
(d) None of these

46. The following graph (figure) shows the variation of velocity of a rocket with time. Then the maximum height attained by the rocket is



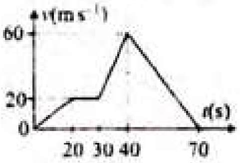
(a) 1.1 km
(b) 5 km
(c) 55 km
(d) None of these

47. From the velocity–time graph, given in figure of a particle moving in a straight line, one can conclude that



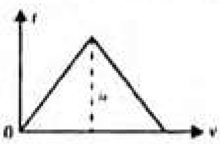
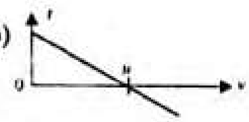
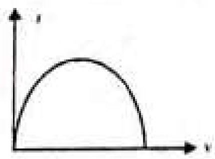
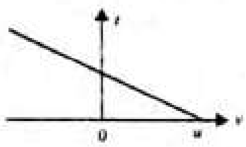
(a) Its average velocity during the 12 s interval is 24/7 $m\,s^{-1}$.
(b) Its velocity for the first 3 s is uniform and is equal to 4 $m\,s^{-1}$.
(c) The body has a constant acceleration between $t = 3$ s and $t = 8$ s.
(d) The body has a uniform retardation from $t = 8$ s to $t = 12$ s.

48. The velocity–time graph of a body is given in figure. The maximum acceleration in $m\,s^{-2}$ is

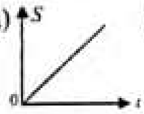
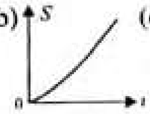
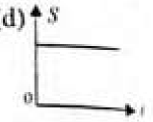


(a) 4 (b) 3
(c) 2 (d) 1

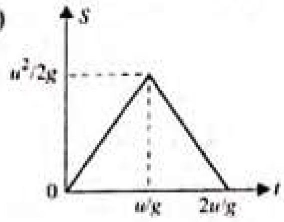
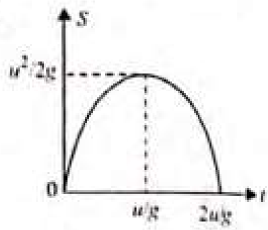
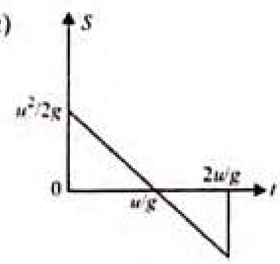
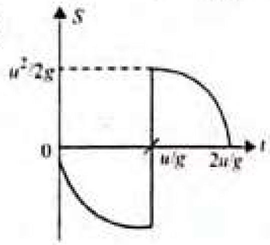
49. An object is thrown up vertically. The velocity–time graph for the motion of the particle is

(a) 
(b) 
(c) 
(d) 

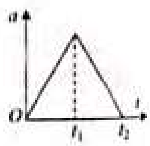
50. From a high tower, at time $t = 0$, one stone is dropped from rest and simultaneously another stone is projected vertically up with an initial velocity. The graph of distance S between the two stones plotted against time t will be

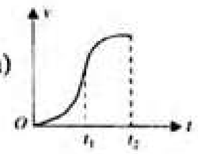
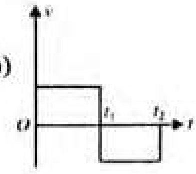
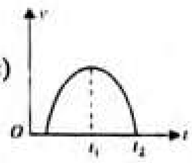
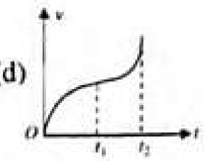
(a) 
(b) 
(c) 
(d) 

51. An object is vertically thrown upwards. Then the displacement–time graph for the motion is as shown in

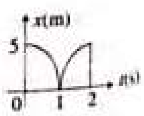
(a) 
(b) 
(c) 
(d) 

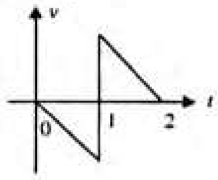
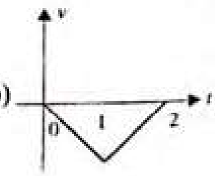
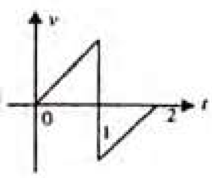
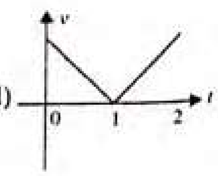
52. The acceleration versus time graph of a particle is shown in figure. The respective v–t graph of the particle is



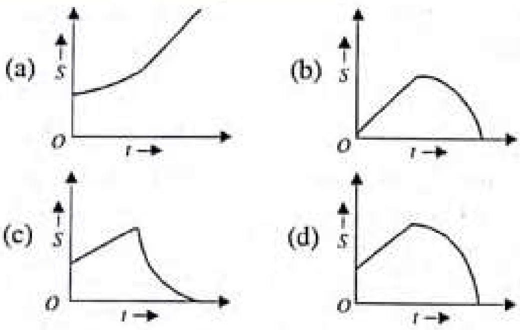
(a) 
(b) 
(c) 
(d) 

53. The displacement–time graph of a moving particle with constant acceleration is shown in figure. The velocity–time graph is given by

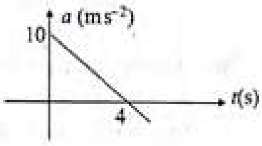


(a) 
(b) 
(c) 
(d) 

54. Two balls are dropped from the top of a high tower with a time interval of t_0 second, where t_0 is smaller than the time taken by the first ball to reach the floor, which is perfectly inelastic. The distance S between the two balls, plotted against the time lapse t from the instant of dropping the second ball, is best represented by

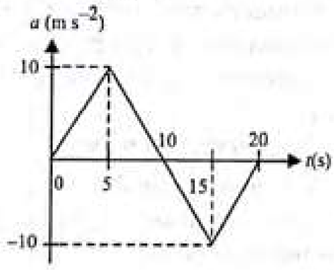


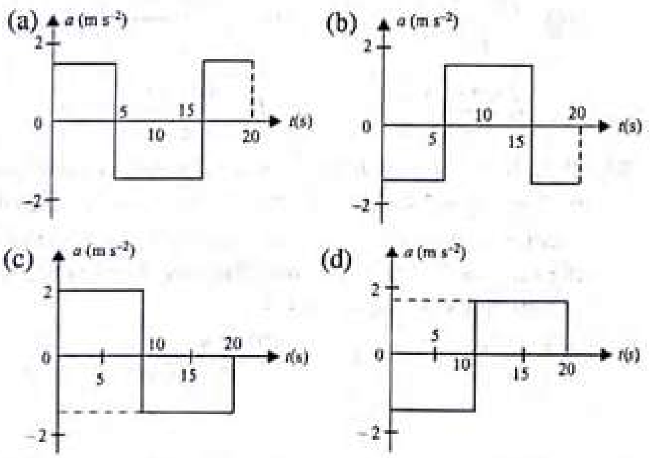
55. The acceleration–time graph of a particle moving along a straight line is as shown in figure. At what time the particle acquires its initial velocity?



(a) 12 s (b) 5 s (c) 8 s (d) 16 s

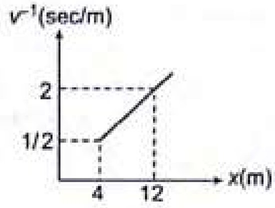
56. Plot the acceleration–time graph of the velocity–time graph given in the figure.





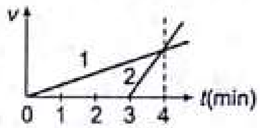
Problems Based on Mixed Concepts

57. Graph of $(1/v)$ vs. x for a particle under motion is as shown, where v is velocity and x is position. The time taken by particle to move from $x = 4$ m to $x = 12$ m is



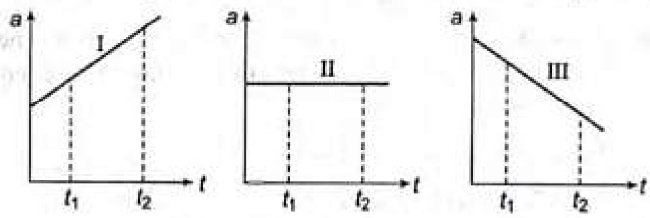
(a) 16/3 sec (b) 10 sec
(c) 8 sec (d) 12 sec

58. The drawing shows velocity (v) versus time (t) graphs for two cyclists moving along the same straight segment of a highway from the same point. The second cyclist starts moving at $t = 3$ min. At what time do the two cyclists meet?



(a) 4 min (b) 6 min (c) 8 min (d) 12 min

59. Each of the three graphs represents acceleration versus time for an object that already has a positive velocity at time t_1. Which graphs show an object whose speed is increasing for the entire time interval between t_1 and t_2?



(a) graph I, only (b) graphs I and II, only
(c) graphs I and III, only (d) graphs I, II and III

60. A lift performs the first part of ascent with uniform acceleration a and the remainder with uniform retardation $2a$. The lift starts from rest and finally comes to rest. If t is the time of ascent. Find the height ascended by lift.

(a) $\frac{at^2}{4}$ (b) $\frac{at^2}{3}$ (c) $\frac{at^2}{2}$ (d) $\frac{at^2}{8}$

61. A train normally travels at a uniform speed of 72 km/h on a long stretch of straight level track. On a particular day, the train was forced to make a 2.0 minute stop at a station along this track. If the train decelerates at a uniform rate of 1.0 m/s^2 and accelerates at a rate of 0.50 m/s^2, how much time is lost in stopping at the station?

(a) 2 min (b) 2 min 30 s
(c) 30 s (d) 2 min 20 s

62. A particle is thrown up inside a stationary lift of sufficient height. The time of flight is T. Now it is thrown again with same initial speed v_0 with respect to lift. At the time of second throw, lift is moving up with speed v_0 and uniform acceleration g upward (the acceleration due to gravity). The new time of flight is

(a) $\sqrt{2}\,T$ (b) $\frac{T}{2}$

(c) T (d) $2T$

63. A particle is projected upwards from the top of a tower. Treat point of projection as origin and upwards as positive. Consider three point on its path such that at (s represents displacement from origin and v represents velocity)
(i) $s > 0, v > 0$ (ii) $s > 0, v < 0$
(iii) $s < 0, v < 0$
Which of the following correctly indicates the distance travelled by particle since its projection to respective points in appropriate order
(a) (i) > (ii) > (iii)
(b) (i) = (ii) < (iii)
(c) (i) < (ii) < (iii)
(d) ordering depends on exact location of points

64. Two objects moving along the same straight line are leaving point A with an acceleration a, $2a$ and velocity $2u$, u respectively at time $t = 0$. The distance moved by the object with respect to point A when one object overtakes the other is
(a) $\frac{3u^2}{a}$ (b) $\frac{2u^2}{a}$ (c) $\frac{4u^2}{a}$ (d) $\frac{6u^2}{a}$

65. Each of the four particles move along an x-axis. Their coordinates (in metres) as function of time (in seconds) are given by
Particle 1: $x(t) = 3.5 - 2.7t^3$
Particle 2: $x(t) = 3.5 + 2.7t^3$
Particle 3: $x(t) = 3.5 + 2.7t^2$
Particle 4: $x(t) = 3.5 - 3.4t - 2.7t^2$
which of these particles is speeding up for $t > 0$?
(a) All four (b) only 1
(c) only 1, 2 and 3 (d) only 2, 3 and 4

66. When two bodies move uniformly towards each other, the distance between them diminishes by 16 m every 10 s. If bodies move with velocities of the same magnitude and in the same direction as before the distance between then will decrease 3 m every 5 s. The velocity of each body is.
(a) 1.5 m/s, 0.8 m/s (b) 1.1 m/s, 0.5 m/s
(c) 2.4 m/s, 1.5 m/s (d) 3.0 m/s, 1.5 m/s

67. Two particles P and Q start from rest and move for equal time on a straight line. Particle P has an acceleration of X m/s^2 for the first half of the total time and $2X$ m/s^2 for the second half. The particle Q has an acceleration of $2X$ m/s^2 for the first half of the total time and Xm/s^2 for the second half. Which particle has covered larger distance?
(a) both have covered the same distance
(b) P has covered the larger distance
(c) Q has covered the larger distance
(d) none of these

68. A particle is thrown upwards from ground. It experiences a constant air resistance force which can produce a retardation of 2 m/s^2. The ratio of time of ascent to the time of descent is
(a) 1 : 1 (b) $\sqrt{\frac{2}{3}}$ (c) $\frac{2}{3}$ (d) $\sqrt{\frac{3}{2}}$

69. An insect moving along a straight line, travels in every second distance equal to the magnitude of time elapsed. Assuming acceleration to be constant, and the insect starts at $t = 0$. Find the magnitude of initial velocity of insect
(a) $\frac{3}{2}$ unit (b) $\frac{5}{4}$ unit (c) 2 unit (d) $\frac{1}{2}$ unit

70. A particle moving along a straight line with a constant acceleration of -4 m/s^2 passes through a point A on the line with a velocity of +8 m/s at some moment. Find the distance travelled by the particle in 5 seconds after that moment.
(a) 26 m (b) 18 m (c) 9 m (d) 32 m

71. A stone is dropped from the top of a tower. When it has fallen by 5 m from the top, another stone is dropped from a point 25 m below the top. If both stones reach the ground at the same moment, then height of the tower is (take $g = 10$ m/s^2)
(a) 45 m (b) 50 m (c) 60 m (d) 65 m

72. A particle starts from rest with uniform acceleration and its velocity after n seconds is v. The displacement of the body in last two seconds is
(a) $\frac{v(n+1)}{n}$ (b) $\frac{2v(n+1)}{n}$
(c) $\frac{2v(n-1)}{n}$ (d) $\frac{v(n-1)}{n}$

73. Two bikes A and B start from a point. A moves with uniform speed 40 m/s and B starts from rest with uniform acceleration 2 m/s^2. If B starts at $t = 10$ and A starts from the same point at $t = 10$ s, then the time during the journey in which A was ahead of B is
(a) 20 s (b) 8 s
(c) 10 s (d) A is never ahead of B

ARCHIVES

1. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest?
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm **(AIEEE 2002)**

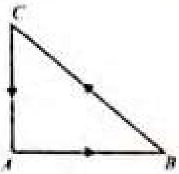
2. Two balls A and B of same masses are thrown from the top of a building. A, thrown upward with velocity V and B, thrown downward with velocity V, then
(a) Velocity of A is more than B at the ground
(b) Velocity of B is more than A at the ground

(c) Both A and B strike the ground with same velocity
(d) None of these **(AIEEE 2002)**

3. Speed of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances in which the two cars are stopped from that instant is
(a) 1 : 1 (b) 1 : 4
(c) 1 : 8 (d) 1 : 16 **(AIEEE 2002)**

4. A car moving with a speed of 50 km/h can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/h, the minimum stopping distance is
(a) 12 m (b) 18 m
(c) 24 m (d) 6 m
(AIEEE 2003)

5. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity



(a) $\vec{v}$ remaining unchanged.
(b) less than $\vec{v}$.
(c) greater than $\vec{v}$.
(d) $|\vec{v}|$ in the direction of the largest force BC.
(AIEEE 2003)

6. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball at $T/3$ second?
(a) $(8h)/9$ metre from the ground
(b) $(7h)/9$ metre from the ground
(c) $h/9$ metre from the ground
(d) $(17h)/18$ metre from the ground
(AIEEE 2004)

7. A car travelling with a speed of 60 km/h can stop within a distance of 20 m. If the car is going twice as fast, i.e., 120 km/h, the stopping distance will be
(a) 60 m (b) 40 m
(c) 20 m (d) 80 m **(AIEEE 2004)**

8. A car starting from rest accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed is $15S$, then
(a) $S = \frac{1}{4}ft^2$ (b) $S = \frac{1}{2}ft^2$
(c) $S = \frac{1}{6}ft^2$ (d) $S = ft$ **(AIEEE 2005)**

9. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is
(a) $2av^2$ (b) $-2av^3$
(c) $2bv^3$ (d) $-2abv^2$ **(AIEEE 2005)**

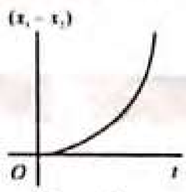
10. A parachutist after bailing out falls 50 m without friction. When his parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height did he bail out?
(a) 111 m (b) 293 m
(c) 182 m (d) 91 m **(AIEEE 2005)**

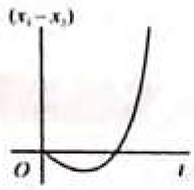
11. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest assuming that it faces constant resistance to motion?
(a) 1.0 cm (b) 1.5 cm
(c) 2.0 cm (d) 3.0 cm **(AIEEE 2005)**

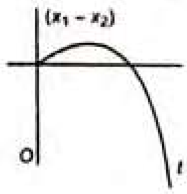
12. A particle located at $x = 0$ at time $t = 0$ starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as
(a) $t^{1/2}$ (b) t^3
(c) t^2 (d) t **(AIEEE 2006)**

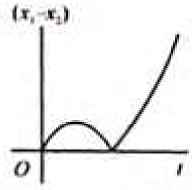
13. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is
(a) $v_0 + (g/2) + (f/3)$ (b) $v_0 + g + f$
(c) $v_0 + (g/2) + f$ (d) $v_0 + 2g + 3f$
(AIEEE 2007)

14. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?

(a)


(b)


(c)


(d)


(AIEEE 2008)

15. A particle has an initial velocity $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is
(a) 10 units (b) $7\sqrt{2}$ units
(c) 7 units (d) 8.5 units **(AIEEE 2009)**

16. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is
(a) $y = x^2 +$ constant (b) $y^2 = x +$ constant
(c) $xy =$ constant (d) $y^2 = x^2 +$ constant
(AIEEE 2010)

17. An object moving with a speed of 6.25 m/s, is decelerated at a rate given by:
$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is instantaneous speed. The time taken by the object, to come to rest, would be:

(a) 1 s (b) 2 s

(c) 4 s (d) 8 s **(AIEEE 2011)**

18. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be

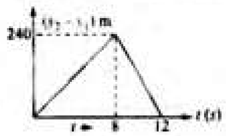
(a) $20\sqrt{2}$ m (b) 10 m

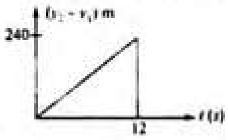
(c) $10\sqrt{2}$ m (d) 20 m **(AIEEE 2012)**

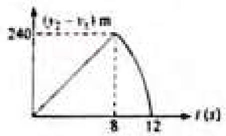
19. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graphs best represents the time variation of relative position of the second stone with respect to the first ?

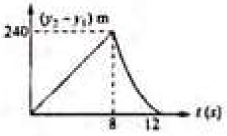
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10$ m/s^2)

(The figures are schematic and not drawn to scale.)

(a) 

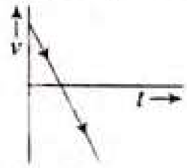
(b) 

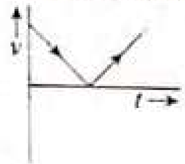
(c) 

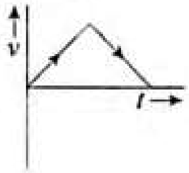
(d) 

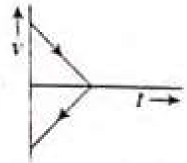
(JEE Main 2015)

20. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs. time?

(a) 

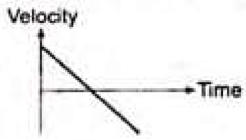
(b) 

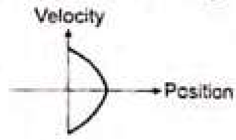
(c) 

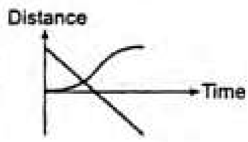
(d) 

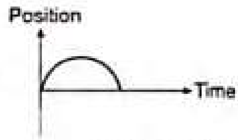
(JEE Main 2017)

21. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(a) 

(b) 

(c) 

(d) 

(JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (a)	**10.** (a)
11. (b)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (d)	**20.** (c)
21. (a)	**12.** (c)	**23.** (b)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (d)
31. (c)	**32.** (a)	**33.** (b)	**34.** (a)	**35.** (d)	**36.** (d)	**37.** (a)	**38.** (d)	**39.** (d)	**40.** (b)
41. (c)	**42.** (b)	**43.** (b)	**44.** (d)	**45.** (a)	**46.** (c)	**47.** (d)	**48.** (a)	**49.** (d)	**50.** (a)
51. (b)	**52.** (a)	**53.** (a)	**54.** (d)	**55.** (c)	**56.** (a)	**57.** (b)	**58.** (b)	**59.** (d)	**60** (b)
61. (b)	**62.** (b)	**63.** (c)	**64.** (d)	**65.** (a)	**66.** (b)	**67.** (c)	**68.** (b)	**69.** (d)	**70.** (a)
71. (a)	**72.** (c)	**73.** (d)							

Archives

1. (a)	**2.** (c)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (d)	**8.** (No option)		**9.** (b)
10. (b)	**11.** (a)	**12.** (c)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (c)
20. (a)	**21.** (a)								

Chapter 4

Motion in Two Dimensions

PROJECTILE MOTION

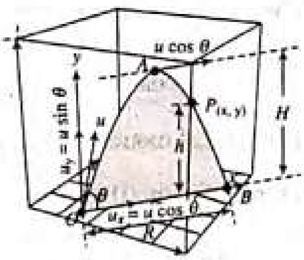
Generally "projectile motion" refers to the motion of a point object thrown in earth's gravitational field. However, the above definition of a projectile is still valid when you throw a point object in gravitational field of any other planet which may not be practically possible for everyone. This tells us that the motion of rockets, crackers, etc., driven by some other external forces other than gravity, cannot be termed as projectile motion in general sense.

The assumptions of projectile motion are as follows:

- There is no resistance due to air.
- The effect due to the curvature of earth is negligible.
- The effect due to the rotation of earth is negligible.
- For all points of the trajectory, the acceleration due to gravity g is constant in magnitude and direction.

Calculation of Various Parameters in Projectile Motion

Let a particle is projected with velocity u at angle θ with the horizontal from point O as shown in the figure. We can write the x and y components of the initial velocity as $u_x = u\cos\theta$, $u_y = u\sin\theta$.



The acceleration of the particle in x- and y-direction is $a_x = 0$ and $a_y = -g$.

Time of Flight

The total time taken by the projectile to go up (O to A) and come down (A to B) to the same level from which it was projected is called **time of flight.**

For vertical upward motion, using $v_y = u_y + a_y t$, we get $u_y = u\sin\theta$ and $a_y = -g$

Hence, $0 = u_y + a_y t = u\sin\theta - gt$

$$\Rightarrow \quad t = \frac{u_y}{g} = \frac{u\sin\theta}{g}$$

Now as the time taken to go up is equal to the time taken to come down, so time of flight, L

$$T = 2t = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$$

Some important points regarding time of flight

- For complementary angles of projection θ and $90° - \theta$,

  Ratio of time of flight $= \dfrac{T_1}{T_2} = \dfrac{2u\sin\theta / g}{2u\sin(90-\theta)/g}$

  which gives $\dfrac{T_1}{T_2} = \tan\theta$

- If t_1 is the time taken by the projectile from point O to P and t_2 is the time taken in moving from point P to B at ground level, then $t_1 + t_2 = \dfrac{2u\sin\theta}{g} =$ time of flight.

  $u\sin\theta = \dfrac{g(t_1+t_2)}{2}$ and height of the point P is given by

  $$h = u\sin\theta\, t_1 - \frac{1}{2}gt_1^2 = g\frac{(t_1+t_2)}{2}t_1 - \frac{1}{2}gt_1^2$$

  By solving, we get $h = \dfrac{g t_1 t_2}{2}$.

Maximum Height

It is the maximum height from the point of projection, a projectile can reach.

Using $v^2_y = u^2_y + 2a_y s \Rightarrow 0 = (u\sin\theta)^2 - 2gH$, we get

$$H = \frac{u^2\sin^2\theta}{2g}$$

Some important points regarding maximum height

- Maximum height can also be expressed as:

  $$H = \frac{u_y^2}{2g}$$

  where u_y is the vertical component of initial velocity.

- $H_{max} = \dfrac{u^2}{2g}$ (when $\sin^2\theta = \max = 1$ i.e., $\theta = 90°$)

  i.e., for maximum height, the body should be projected vertically upward. So it falls back to the point of projection after reaching the maximum height.

- For complementary angles of projection θ and $90° - \theta$
- Ratio of maximum height

$$= \frac{H_1}{H_2} = \frac{u^2 \sin^2\theta/2g}{u^2 \sin^2(90° - \theta)2g} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

ILLUSTRATION 4.1 Which of the path (I) or (II) of a projectile has more time of flight? Use necessary assumptions.

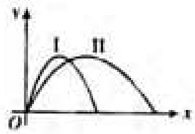
Solution. Given $H_1 = H_2$.

$$\frac{(u_y)_1^2}{2g} = \frac{(u_y)_2^2}{2g} \Rightarrow \frac{u_1^2\sin^2\theta_1}{2g} = \frac{u_2^2\sin^2\theta_2}{2g}$$

$\Rightarrow \quad u_1 \sin\theta_1 = u_2 \sin\theta_2$ Or $(u_y)_1 = (u_y)_2$

We know $T_1 = \frac{2(u_y)_1}{2} = \frac{2(u_y)_2}{2} = T_2$

Hence, both paths take same time.



Horizontal Range

It is the horizontal distance travelled by a body during the time of flight.

In horizontal direction, the acceleration of the particle is zero, i.e., horizontal component of the velocity is constant. Hence, displacement in the horizontal direction can be written as $x = R = u_x \times T$.

$$\Rightarrow \quad R = u\cos\theta\, T = u\cos\theta \times \left(\frac{2u\sin\theta}{g}\right)$$

$$= \frac{u^2(2\sin\theta\cdot\cos\theta)}{g} = \frac{u^2\sin 2\theta}{g}$$

ILLUSTRATION 4.2 A batsman hits a ball at an angle of 30° with an initial speed of 30 m s^{-1}. Assuming that the ball travels in a vertical plane, calculate

(a) The time at which the ball reaches the highest point
(b) The maximum height reached
(c) The horizontal range of the ball
(d) The time for which the ball is in the air

Solution. Here $\theta = 30°$, $u = 30$ m s^{-1}.

(a) The time taken by the ball to reach the highest point is half the total time of flight. As the time of ascending and descending is same for a projectile without air resistance, the time to reach the highest point

$$t_H = \frac{T}{2} = \frac{u\sin\theta}{g} = \frac{30}{10} \times \sin 30° = 1.5\text{ s}$$

(b) The maximum height reached is

$$\frac{u^2\sin^2\theta}{2g} = \frac{(30)^2 \times (\sin 30°)^2}{2g} = \frac{900}{2 \times 10 \times 4} = 11.25\text{ m}$$

(c) Horizontal range $= \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(30)^2 \sin 2(30°)}{10}$$

$$= \frac{900\sqrt{3}}{20} = 45\sqrt{3}\text{ m}$$

(d) The time for which the ball is in air is same as its time of flight, i.e.,

$$\frac{2u\sin\theta}{g} = \frac{2 \times 30 \times \sin 30°}{10} = 3\text{ s}$$

ILLUSTRATION 4.3 A bullet with muzzle velocity 100 m s^{-1} is to be shot at a target 30 m away in the same horizontal line. How high above the target must the rifle be aimed so that the bullet will hit the target?

Solution. Horizontal range of bullet is 30 m.

Using range formula, $R = \frac{u^2\sin 2\theta}{g} = 30$

or $\quad \sin 2\theta = \frac{30 \times 10}{(100)^2}$ or $\sin 2\theta = 0.03$

For small θ, $\sin\theta \approx \theta = \tan\theta$, i.e., $2\theta = 0.03$

Therefore, $\theta = 0.015$ rad

In ΔOAP, $\tan\theta = \frac{AP}{OP} \Rightarrow AP = OP\tan\theta$

The rifle must be aimed at an angle $\theta = 0.015$ above horizontal.

Height to be aimed = 30 tan $\theta \approx 30(\theta) = 30 \times 0.015 = 45$ cm

Equation of Trajectory

A projectile thrown with velocity u at an angle θ with the horizontal. The velocity u can be resolved into two rectangular components: $u\cos\theta$ component along X-axis and $u\sin\theta$ component along Y-axis.

For horizontal motion, $x = u\cos\theta \times t$

$$\Rightarrow \quad t = \frac{x}{u\cos\theta} \quad \text{...(i)}$$

For vertical motion, $y = (u\sin\theta)t - \frac{1}{2}gt^2 \quad$ ...(ii)

From equation (i) and (ii),

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

$$= x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} \quad \text{...(iii)}$$

This equation shows that the trajectory of projectile is parabolic because it is similar to the equation of parabola.

$y = ax - bx^2$

It is known as the equation of trajectory. It is an equation of parabola. Hence, path of a projectile is parabolic.

Equation (iii) can also be written as:

$$y = x\tan\theta - \frac{x^2}{\frac{2u^2\cos^2\theta}{g}}$$

$$= x\tan\theta - \frac{x^2}{\left(\frac{2u^2\cos\theta\cdot\sin\theta}{g}\right)}\frac{\sin\theta}{\cos\theta}$$

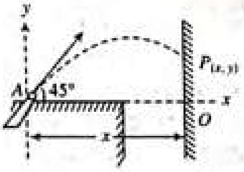
$$= x\tan\theta - \frac{x^2\tan\theta}{R} \quad y = x\tan\theta\left(1-\frac{x}{R}\right)$$

NOTE: The equation of oblique projectile also can be written as $y = x\tan\theta\left[1-\frac{x}{R}\right]$

(where R = horizontal range $= \frac{u^2\sin 2\theta}{g}$)

The shape of the curve above x-axis is symmetrical about a vertical line passing through the highest point.

ILLUSTRATION 4.4 A jet of water is projected at an angle $\theta = 45°$ with horizontal from point A which is situated at a distance $x = OA$ = (a) 1/2 m, (b) 2 m from a vertical wall. If the speed of projection is $v_0 = \sqrt{10}\ \text{m s}^{-1}$, find point P of striking of the water jet with the vertical wall.



Solution. Point P lies at the trajectory of jet of water. Hence, the coordinate of point P (x, y) should satisfy the trajectory equation.

$$y = x\tan\theta\left(1-\frac{x}{R}\right) \quad \text{...(i)}$$

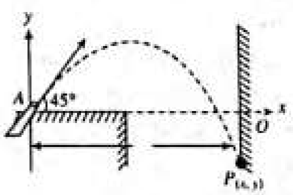
$$R = \frac{u^2\sin 2\theta}{g} = \frac{(\sqrt{10})^2\cdot\sin 2\times 45°}{10} = 1\ \text{m}$$

(a) If $x = 1/2$ m, from (i), $y = \frac{1}{2}\tan 45°\left(1-\frac{1/2}{1}\right) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$ m

Hence, the coordinate of $P = \left(\frac{1}{2}\text{m}, \frac{1}{4}\text{m}\right)$

(b) If $x = 2$ m, from (i),

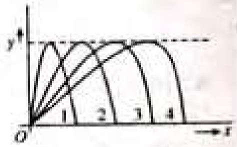
$$y = 2\cdot\tan 45°\left(1-\frac{2}{1}\right) = -1\ \text{m}$$



Hence, water jet will strike below the horizontal dotted line (x-axis) at coordinate (2 m, –1m).

CONCEPT APPLICATION EXERCISE 4.1

1. A particle is thrown with velocity u at an angle α from the horizontal. Another particle is thrown with the same velocity at an angle α from the vertical. What will be the ratio of times of flight of two particles?
2. The friction of the air causes vertical retardation equal to one-tenth of the acceleration due to gravity (take $g = 10\ \text{m s}^{-2}$). Find the decrease in the time of flight.
3. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m s^{-1} at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ($g = 10\ \text{m s}^{-2}$, $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$.
4. Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to the initial horizontal velocity component, highest first.



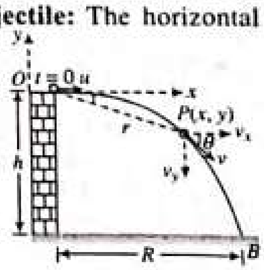
5. A projectile is thrown into space so as to have maximum horizontal range R. Taking the point of projection as origin, find out the co-ordinates of the point where the speed of the particle is minimum.
6. A large number of bullets are fired in all directions with same speed u. What is the maximum area on the ground on which these bullets will spread?
7. A projectile thrown with an initial speed u and the angle of projection 15° to the horizontal has a range R. If the same projectile is thrown at an angle of 45° to the horizontal with speed $2u$, what will be its range?
8. The range R of a projectile is same when its maximum heights are h_1 and h_2. What is the relation between R and h_1 and h_2?

HORIZONTAL PROJECTILE

A body is projected horizontally from a certain height y vertically above the ground with initial velocity u. If friction is considered to be absent, then there will be no other horizontal forces which can affect the horizontal motion. The horizontal velocity, therefore, remains constant and so the object covers equal distance in horizontal direction in equal intervals of time.

- **Trajectory of horizontal projectile:** The horizontal displacement x is governed by the equation

$$x = ut \Rightarrow t = \frac{x}{u} \quad \text{...(i)}$$



The vertical displacement y is governed by

$$y = -\frac{1}{2}gt^2 \quad \text{...(ii)}$$

(Since the initial vertical velocity is zero)

By substituting the value of t in (ii), $y = -\frac{1}{2}\frac{gx^2}{u^2}$

- **Displacement of projectile ($\vec{r}$):** After time t, horizontal displacement $x = ut$ and vertical displacement $y = \frac{1}{2}gt^2$.

  So, position vector, $\vec{r} = ut\,\hat{i} - \frac{1}{2}gt^2\,\hat{j}$

  $$\therefore \quad r = ut\sqrt{1+\left(\frac{gt}{2u}\right)^2}$$

  and $\alpha = \tan^{-1}\left(\frac{gt}{2u}\right) = \tan^{-1}\left(\sqrt{\frac{gy}{2}}\Big/u\right)\left(\text{as } t = \sqrt{\frac{2y}{g}}\right)$

- **Instantaneous velocity:** Throughout the motion, the horizontal component of the velocity is $v_x = u$.
  The vertical component of velocity increases with time and is given by

  $$v_y = 0 - g\,t = -g\,t \qquad (\text{from } v = u + gt)$$

  So, $\vec{v} = v_x\hat{i} - v_y\hat{j} = \vec{v} = u\hat{i} - gt\,\hat{j}$

  i.e., $v = \sqrt{u^2 + (gt)^2} = u\sqrt{1+\left(\frac{gt}{u}\right)^2}$

  Again $\vec{v} = u\hat{i} - \sqrt{2gy}\,\hat{j}$

  i.e., $v = \sqrt{u^2 + 2gy}$

- **Direction of instantaneous velocity:** $\tan\phi = \frac{v_y}{v_x}$

  $$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{\sqrt{2gy}}{u}\right) = \tan^{-1}\left(\frac{gt}{u}\right)$$

  where ϕ is the angle of instantaneous velocity from the horizontal.

- **Time of flight:** If a body is projected horizontally from a height h with velocity u and the time taken by the body to reach the ground is T, then for vertical motion,

  $$-h = 0 - \frac{1}{2}gT^2 \quad \Rightarrow \quad T = \sqrt{\frac{2h}{g}}$$

- **Horizontal range:** Let R be the horizontal distance travelled by the body. The acceleration in x-direction is zero. Hence,

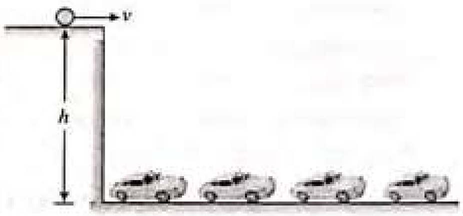
  $$R = uT + \frac{1}{2}\,0\,T^2 \quad \Rightarrow \quad R = u\sqrt{\frac{2h}{g}}$$

Important Points

1. If projectiles A and B are projected horizontally with different initial velocity from same height and a third particle C is dropped from same point, then

   (a) All three particles will take equal time to reach the ground.
   (b) Their net velocity would be different but all three particles possess same vertical component of velocity.
   (c) The trajectory of projectiles A and B will be straight line w.r.t. particle C.
2. If a particle is released from an airplane moving horizontally at a certain height, the analysis of the motion of the particle will be same as the particle thrown from the certain height as discussed above.

ILLUSTRATION 4.5 A rubber ball escapes from the horizontal roof with a velocity $v = 5\ \text{m s}^{-1}$. The roof is situated at a height, h = 20 m. If the length of each car is equal to x_0 = 4 m, with which car will the ball hit?



Solution. The initial velocity of the ball is in horizontal direction. The initial velocity in vertical direction is zero.

For vertical motion, $u_x = 5\ \text{m s}^{-1}$, $u_y = 0$.

Displacement in vertical direction, $y = -h = -20$ m

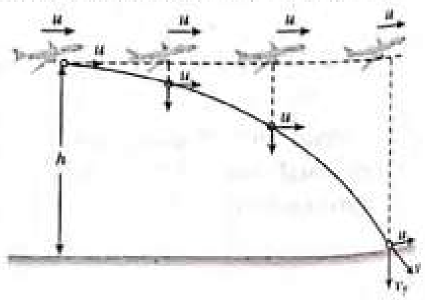
$$-h = \frac{1}{2} - gt^2 \Rightarrow \frac{2\times 20}{10} = t^2 \Rightarrow t = 2\ \text{s}$$

Horizontal displacement: Distance covered in 2 s,

$$x = d = u_x \times t = 5\times 2 = 10\ \text{m}$$

Since the length of one car is 4 m, it will hit the third car for which 8 m < d < 12 m.

ILLUSTRATION 4.6 A relief food package is dropped from a airplane which is moving horizontal with a velocity of $30\ \text{m s}^{-1}$ at a height $h = 50$ m. Find the (a) time of flight of the package, (b) location of the point of striking of the food package, (c) velocity of the package at the time of striking the ground, and (d) displacement of the food package.



Solution. $u_x = 30 \text{ m s}^{-1}$, $u_y = 0 \text{ m s}^{-1}$, and $h = 50$ m.

$$h = u_y t + \frac{1}{2} g t^2$$

$$\Rightarrow \quad \frac{50 \times 2}{10} = t^2 \Rightarrow t = \sqrt{10} \text{ s}$$

Distance (horizontally) covered in $t = \sqrt{10}$ s

$$x = u_x t = 30\sqrt{10} \text{ m}$$

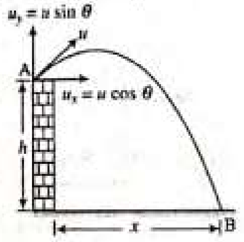
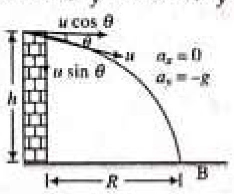
$$v_x = u_x = 30 \text{ m s}^{-1}$$

$$v_y = u_y + gt = 10\sqrt{10} \text{ m s}^{-1}$$

$$v = \sqrt{900 + 1000} = 10\sqrt{19} \text{ m s}^{-1}$$

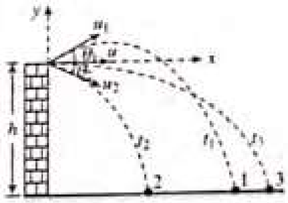
$$\text{Displacement} = \sqrt{h^2 + x^2} = \sqrt{2500 + 9000} = 10\sqrt{115} \text{ m}$$

Projectile from Height at Certain Angle with Horizontal

Case I	**Case II**
Projection at an angle θ above horizontal $u_x = u\cos\theta$; $a_y = -g$ 	Projection at an angle θ below horizontal $u_x = u\cos\theta$; $u_y = -u\sin\theta$; $a_y = -g$ 

Equation of motion between A and B (in Y direction) $S_y = -h$, $u_y = u\sin\theta$, $a_y = -g$, $t = T$ $S_y = u_y t + \frac{1}{2} a_y t^2$ $\Rightarrow -h = (u\sin\theta)T - \frac{1}{2} gT^2$ Solving the above equation, we will get the time of flight, T. Range, $R = u_x T = u\cos\theta\, T$ Also, $v_y^2 = u_y^2 + 2a_y S_y$ $= u^2\sin^2\theta + 2gh$ $v_x = u\cos\theta$ $v_B = \sqrt{v_y^2 + v_x^2}$ $v_B = \sqrt{u^2 + 2gh}$	$S_y = u_y t + \frac{1}{2} a_y t^2$ $S_y = -h$, $u_y = -u\sin\theta$ $t = T$, $a_y = -g$ $\Rightarrow -h = -(u\sin\theta)T - \frac{1}{2} gT^2$ $\Rightarrow \quad h = (u\sin\theta)T + \frac{1}{2} gT^2$. Solving the above equation, we will get the time of flight, T. Range, $R = u_x T = u\cos\theta\, T$ $v_x = u\cos\theta$ $v_y^2 = u_y^2 + 2a_y S_y$ $= u^2\sin^2\theta + 2(-g)(-h)$ $= u^2\sin^2\theta + 2gh$ $v_B = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

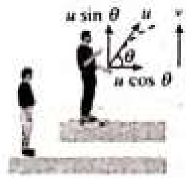
Important observation: If three objects are thrown from the same height in different directions as shown in the figure, then



- if $\quad u_1 = u_2 = u$
  All the three objects will strike the ground with the same final speed.
  $u_1 \neq u_2 \neq u$, then all the three objects will strike the ground with the different final speeds.
- If $|u_1 \sin\theta_1| = |u_2 \sin\theta_2|$, then the time of flights of the objects are related as
  $t_3 = \sqrt{t_1 t_2}$

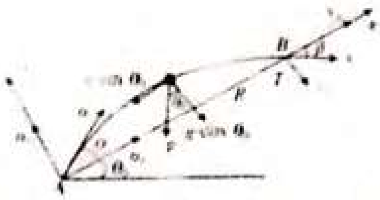
Projectile from a Moving Platform

In given four cases an observer 'A' is moving with constant velocity and he throws a ball. An another observer 'B' is standing on ground and observing the ball.

Case I	**Case II**
The observer B standing on road will see the ball moving in a parabolic path. The horizontal speed of the ball is equal to the speed of the skateboard.	The horizontal and vertical components of ball's velocity w.r.t. observer B standing on the ground, respectively, is $u_x = u\cos\theta + v$ $u_y = u\sin\theta$.
Case III	**Case IV**
The horizontal and vertical components of ball's velocity w.r.t. observer B standing on the ground, respectively, is $u_x = u\cos\theta - v$ $u_y = u\sin\theta$	The horizontal and vertical components of ball's velocity w.r.t. observer B sitting on the ground, respectively, is $u_x = u\cos\theta$ $u_y = u\sin\theta + v$

Projectile Motion on an Inclined Plane

Consider a plane is inclined at an angle θ_0 with horizontal. Let a projectile is projected from the foot of the inclined pane A, with velocity u and at an angle α with the inclined plane as shown in the figure. The particle strikes the plane at point B. We want to find the range of projectile on the inclined plane which is $R = AB$.



Take x-axis parallel to the inclined plane and y-axis perpendicular to the inclined plane as shown in the figure. We can write the following:

$u_x = u\cos\alpha$, $u_y = u\sin\alpha$, $a_x = -g\sin\theta_0$, $a_y = -g\cos\theta_0$

From A to B: $s_x = R$, $s_y = 0$ and let time taken from A to B is T

From A to B, apply $s_y = u_yt + \frac{1}{2}a_yt^2$, we get

$$0 = (u\sin\alpha)T - \frac{1}{2}g\cos\theta_0 T^2 \Rightarrow T^2 = \frac{2u\sin\alpha}{g\cos\theta_0} \quad \text{...(i)}$$

Now apply $s_x = u_xt + \frac{1}{2}a_xt^2$, we get

$$R = (u\cos\alpha)T - \frac{1}{2}g\sin\theta_0 T^2 \quad \text{...(ii)}$$

Put the value of T from (i) in (ii), we get

$$R = u\cos\theta\frac{2u\sin\alpha}{g\cos\theta_0} - \frac{1}{2}g\sin\theta_0\left(\frac{2u\sin\alpha}{g\cos\theta_0}\right)^2$$

$$= \frac{2u^2\sin\alpha\cos\alpha}{g\cos\theta_0} - \frac{2u^2\sin^2\alpha\sin\theta_0}{g\cos^2\theta_0}$$

$$= \frac{u^2}{g\cos^2\theta_0}[\sin 2\alpha\cos\theta_0 - \sin\theta_0 2\sin^2\alpha]$$

$$= \frac{u^2}{g\cos^2\theta_0}[\sin 2\alpha\cos\theta_0 - \sin\theta_0(1-\cos 2\alpha)]$$

$$= \frac{u^2}{g\cos^2\theta_0}[\sin(2\alpha+\theta_0) - \sin\theta_0] \quad \text{...(iii)}$$

The above equation (iii) gives the range of a projectile on an inclined plane.

Maximum range: We want to find angle α for which the range of the projectile is maximum on an inclined plane.

Range will be maximum if $\sin(2\alpha + \theta_0) = 1$

$$\Rightarrow \quad 2\alpha + \theta_0 = \pi/2 \Rightarrow \alpha = \frac{\pi}{4} - \frac{\theta_0}{2} \quad \text{...(iv)}$$

The above equation (iv) gives the angle α for which the range is maximum. Put the vale of α in (iii), we get

$$R_{max} = \frac{u^2}{g\cos^2\theta_0}\left[\sin\left[2\left(\frac{\pi}{4}-\frac{\theta_0}{2}\right)+\theta_0\right] - \sin\theta_0\right]$$

$$= \frac{u^2}{g\cos^2\theta_0}[1 - \sin\theta_0]$$

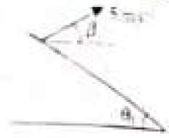
$$= \frac{u^2(1-\sin\theta_0)}{g(1-\sin^2\theta_0)} = \frac{u^2}{g(1+\sin\theta_0)} \quad \text{...(v)}$$

The above equation (v) gives the maximum range.

Important Points

As the projectile accelerates down perpendicular to the inclined plane with an acceleration $g\cos\beta$, its speed perpendicular to the inclined plane decreases from $v_0\sin\phi$ at O to zero at P and gradually increases to $v_0\sin\phi$ at Q just before striking.

ILLUSTRATION 4.7 An inclined plane makes an angle $\theta_0 = 30°$ with the horizontal. A particle is projected from this plane with a speed of 5 m s^{-1} at an angle of elevation $\beta = 30°$ with the horizontal as shown in the figure.



(a) Find the range of the particle on the plane when it strikes the plane.

(b) Find the range of the particle for $\beta = 120°$.

Solution.

(a) $\alpha' = \theta_0 + \beta = 60°$

$$R = \frac{u^2}{g\cos^2\theta_0}[\sin(2\alpha' - \theta_0) + \sin\theta_0]$$

$$= \frac{5^2}{10\cos^2 30°}[\sin(2\times 60° - 30°) + \sin 30°] = 5\text{ m}$$

(b) $\alpha + \theta_0 + 120° = 180° \Rightarrow \alpha = 30°$

$$R = \frac{u^2}{g\cos^2\theta_0}[\sin(2\alpha + \theta_0) - \sin\theta_0]$$

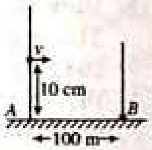
$$= \frac{5^2}{10\cos^2 30°}[\sin(2\times 30° + 30°) - \sin 30°] = 5/3\text{ m}$$

CONCEPT APPLICATION EXERCISE 4.2

1. A staircase contains three steps each 10 cm high and 20 cm wide. What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the lowest plane?

2. For a given velocity of projection from a point on the inclined plane, the maximum range down the plane is three times the maximum range up the incline. Then find the angle of inclination of the inclined plane.
3. A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30°$ and the angle of the barrel to the horizontal $\beta = 60°$. The initial velocity v of the shell is 21 m s^{-1}. Then find the distance of point from the gun at which the shell will fall. (use $g = 9.8$ m/s^2)

4. The maximum range of a rifle bullet on the horizontal ground is 6 km. Find its maximum range on an inclined of 30°.
5. A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude $9.0\ \mathrm{m\,s^{-1}}$. Find the motorcycle's position, distance from the edge of the cliff and velocity after 0.5 s.
6. A fighter plane moving with a speed of $50\sqrt{2}\ \mathrm{m\,s^{-1}}$ upward at an angle of 45° with the vertical releases a bomb when it was at a height 1000 m from ground. Find
   (a) the time of flight
   (b) the maximum height of the bomb above ground
7. A bullet is fired from the bottom of the inclined plane at angle $\theta = 37°$ with the inclined plane. The angle of incline is 30° with the horizontal. Find the (a) position of the maximum height of the bullet from the inclined plane; (b) time of light; (c) range along the incline; (d) the value of θ at which the range will be maximum; (e) maximum range.
8. Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect the resistance of paper and air.

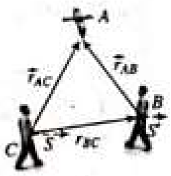


RELATIVE MOTION

Analysis of Relative Motion in General

Let us assume that observers B and C are fixed with the reference frames S' and S, respectively, and observe the motion of object A.

Let the observers B and C measure the position vectors of A as $\vec{r}_{AB}$ and $\vec{r}_{AC}$, respectively. If the position vector of B relative to C is $\vec{r}_{BC}$, following the triangle law of vectors, we have $\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC}$



Differentiating the above expression with time, we have

$$\frac{d\vec{r}_{AC}}{dt} = \frac{d\vec{r}_{AB}}{dt} + \frac{d\vec{r}_{BC}}{dt}$$

Substituting $\dfrac{d\vec{r}_{AB}}{dt} = \vec{v}_{AB}, \dfrac{d\vec{r}_{AC}}{dt} = \vec{v}_{AC}$, and $\dfrac{d\vec{r}_{BC}}{dt} = \vec{v}_{BC}$, we have

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

The above expression tells us that simultaneously different observers (B and C) will record different velocities ($\vec{v}_{AB}$ and $\vec{v}_{BC}$) of the object (A).

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$

If observer B is located on ground $\vec{v}_{AC} = \vec{v}_{A,\text{Earth}} + \vec{v}_{\text{Earth},C}$

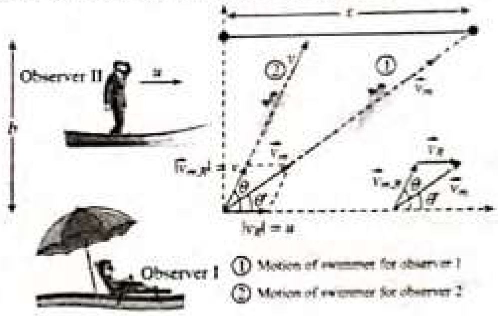
or $\quad \vec{v}_{A,C} = \vec{v}_{A,\text{Earth}} - \vec{v}_{C,\text{Earth}} = \vec{v}_A - \vec{v}_C$

or $\quad \vec{v}_{\text{Object, Observer}} = \vec{v}_{\text{Object}} - \vec{v}_{\text{Observer}}$

or $\quad \vec{v}_{\text{rel}} = \vec{v}_{\text{Object}} - \vec{v}_{\text{Observer}}$

Motion of Man Swimming in a River

Consider a man swimming in a river with a velocity of $\vec{v}_{m,R}$ relative to river water at an angle of θ with the river flow(figure).



The velocity of river water is $\vec{V}_R$.

Let there be two observers I and II, observer I is on ground and observer II is on a raft floating along with the river and hence moving with the same velocity as the river. Hence motion w.r.t. observer II is same as motion w.r.t. river, i.e., the man will appear to swim at an angle θ with the river flow for observer II.

For observer I, the velocity of man will be

$$\vec{v}_m = \vec{v}_{m,R} + \vec{v}_R$$

Hence, the swimmer will appear to move at an angle θ' with the river flow.

River Problem in Two Dimensions (Crossing River)

Consider a man swimming in a river with a velocity of $\vec{v}_{m,R}$ relative to river at an angle of θ with the river flow.

The velocity of river is $\vec{v}_R$ and the width of the river is d.

$$\vec{v}_m = \vec{v}_{m,R} + \vec{v}_R$$
$$= (v\cos\theta\,\hat{i} + v\sin\theta\,\hat{j}) + u\,\hat{i}$$
$$= (v\cos\theta + u)\hat{i} + v\sin\theta\,\hat{j}$$

Here $v \sin\theta$ is the component of the velocity of man in the direction perpendicular to the river flow.

This component of velocity is responsible for the man crossing the river. Hence, if the time to cross the river is t, then

$$t = \frac{d}{v_y} = \frac{d}{v\sin\theta}$$

It is defined as the displacement of man in the direction of river flow.

It is simply the displacement along x-axis during the period the man crosses the river. ($v\cos\theta + u$) is the component of the velocity of man in the direction of river flow, and this component of velocity is responsible for drift along the river flow. If the drift is x, then

Drift $= v_x \times t$

$$x = (v\cos\theta + u) \times \frac{d}{v\sin\theta}$$

Crossing the River in Shortest Time

As we know that $t = d/v \sin\theta$. Clearly t will be minimum when $\theta = 90°$, i.e., time to cross the river will be minimum if the man swims perpendicular to the river flow. Which is equal to d/v.

Crossing the River in Shortest Path, Minimum Drift

The minimum possible drift is zero. In this case, the man swims in the direction perpendicular to the river flow as seen from the ground. This path is known as *shortest path*.

Here $x_{min} = 0 \Rightarrow (v\cos\theta + u) = 0$

or $\quad \cos\theta = -\dfrac{u}{v}$

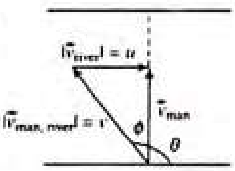
Since $\cos\theta$ is –ve, therefore, $\theta > 90°$, i.e., for minimum drift the man must swim at some angle ϕ with the perpendicular in upstream direction, where

$$\sin\phi = \frac{|\vec{v}_R|}{|\vec{v}_{m,R}|} = \frac{u}{v}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{-v_R}{v_{m,R}}\right) \Rightarrow \frac{u}{v} \le 1$$

i.e., $u \le v$

i.e., minimum drift is zero if and only if the velocity of man in still water is greater than or equal to the velocity of river.



Time to cross the river along the shortest path.

$$t = \frac{d}{v\sin\theta} = \frac{d}{\sqrt{v^2 - u^2}}$$

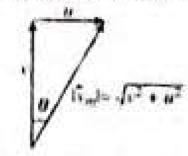
NOTE:

If $|\vec{v}_R| > |\vec{v}_{m,R}|$, then it is not possible to have zero drift. In this case, the minimum drift (corresponding to shortest possible path is non-zero and the condition for minimum drift can be proved to be $\cos\theta = -\dfrac{v}{u}$ or $\sin\phi = \dfrac{u}{v}$ or $\theta = \dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{u}{v}\right)$ for minimum but non-zero drift.

ILLUSTRATION 4.8 A man wishes to cross a river in a boat. If he crosses the river in minimum time he takes 10 min with a drift of 120 m. If he crosses the river taking shortest route, he takes 12.5 min. Find the velocity of the boat with respect to water.

Solution. **Case 1:** If the man crosses the river in minimum time he should move perpendicular to bank or normal to the direction of water flow.

Let $|\vec{v}_{m,w}| = v$ and $|\vec{v}_w| = u$

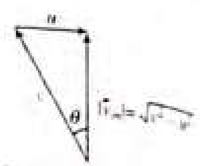


(a) Man crossing river taking minimum time

Time to cross river, $t_1 = 10 = \dfrac{d}{v} \Rightarrow d = 10\,v$.

$x = ut_1 \Rightarrow 120u = u \times 10 \Rightarrow u = 12$ m min^{-1}

Case 2: If the man crosses river taking the shortest route, the drift should be zero. Time to cross river,



(b) Man crossing river taking shortest

$$t_2 = 12.5 = \frac{d}{\sqrt{v^2 - u^2}}$$

From case 1, $d = 10v$ and $v = 12$ m s^{-1}

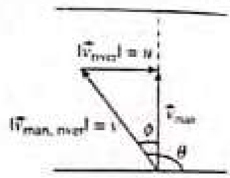
$$12.5 = \frac{10v}{\sqrt{v^2 - 12^2}} \Rightarrow v = 20\,\text{m s}^{-1}$$

ILLUSTRATION 4.9 A man can swim at the rate of 5 km h^{-1} in still water. A 1-km wide river flows at the rate of 3 km h^{-1}. The man wishes to swim across the river directly opposite to the starting point.

(a) Along what direction must the man swim?
(b) What should be his resultant velocity?
(c) How much time will he take to cross the river?

Solution.

(a) Velocity of man with respect to river water, $v = 5$ km h^{-1}. This is greater than the river flow velocity. Therefore, he can cross the river directly (along the shortest path or no drift condition from flow velocity). The angle of swim,



$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{u}{v}\right)$$

$$= 90° + \sin^{-1}\left(\frac{u}{v}\right)$$

$$= 90° + \sin^{-1}\left(\frac{3}{5}\right) = 90° + 37° = 127° \text{ w.r.t. the river}$$

flow or 37° w.r.t. perpendicular in upstream direction

(b) Resultant velocity or velocity of mass will be

$$v_m = \sqrt{v^2 - u^2} = \sqrt{5^2 - 3^2} = 4\,\text{km h}^{-1}$$

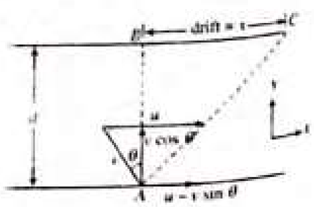
In the direction perpendicular to the river flow.

(c) time taken to cross the river

$$t = \frac{d}{\sqrt{v^2 - u^2}} = \frac{1\,\text{km}}{4\,\text{km h}^{-1}} = \frac{1}{4}\,\text{h} = 15\,\text{min}$$

ILLUSTRATION 4.10 A boat moves relative to water with a velocity v which is n times less than the river flow velocity u. At what angle to the stream direction must the boat move to minimize drifting?

Solution. In this case, the velocity of boat is less than the river flow velocity. Hence, boat cannot reach the point directly opposite to its starting point, i.e., drift can never be zero. Suppose the boat starts at an angle θ from the normal direction up stream as shown in the figure.



Component of the velocity of boat along the river,

$v_x = u - v\sin\theta$

and velocity perpendicular to the river, $v_y = v\cos\theta$.

Time taken to cross the river is $t = \dfrac{d}{v_y} = \dfrac{d}{v\cos\theta}$

Drift $x = (v_x)t = (u - v\sin\theta)\dfrac{d}{v\cos\theta} = \dfrac{ud}{v}\sec\theta - d\tan\theta$

The drift x is minimum when $\dfrac{dx}{d\theta} = 0$

or $\left(\dfrac{ud}{v}\right)(\sec\theta\cdot\tan\theta) - d\sec^2\theta = 0$

or $\dfrac{u}{v}\sin\theta = 1 \Rightarrow \qquad \sin\theta = \dfrac{v}{u}$

i.e., for minimum drift, the boat must move at an angle $\theta = \sin^{-1}\theta$

$= \sin^{-1}\left(\dfrac{v}{u}\right) = \sin^{-1}\dfrac{1}{n}$ from the normal direction.

Rain–Man Problems

If rain is falling vertically with a velocity $\vec{v}_r$ and an observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to the observer will be

$$\vec{v}_{r,m} = \vec{v}_r - \vec{v}_m \text{ or } v_{r,m} = \sqrt{v_r^2 + v_m^2}$$

and direction $\theta = \tan^{-1}\left(\dfrac{v_m}{v_r}\right)$ with the vertical as shown in the figures.

Different situations in rain-man problem

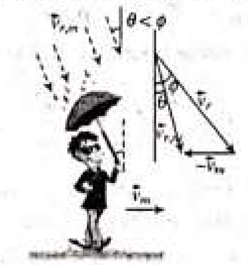
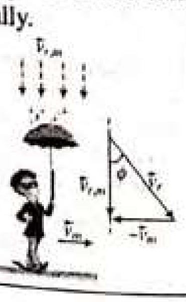
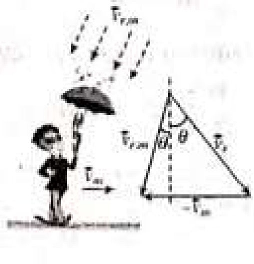
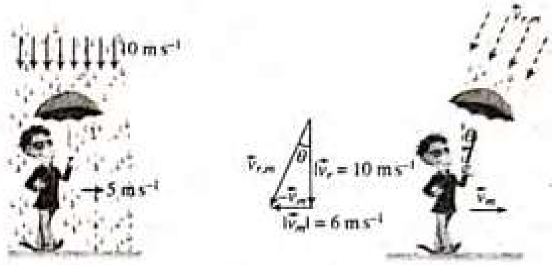
The man is stationary and the rain is falling at his back at an angle ϕ with the vertical. 	The man starts moving forward. The relative velocity of rain w.r.t man shifts towards vertical direction. 
As the man further increases his speed, then at a particular value, the rain appears to be falling vertically. 	If the man increases his speed further more, then the rain appears to be falling from the forward direction. 

ILLUSTRATION 4.11 Rain is falling vertically and a man is moving with velocity 6 m s^{-1}. Find the angle at which the man should hold his umbrella to avoid getting wet.

Solution. $\vec{v}_{\text{rain}} = -10\hat{j} \Rightarrow \vec{v}_{\text{man}} = 6\hat{j}$

$\vec{v}_{\text{rain w.r.t man}} = -10\hat{j} - 6\hat{i}$

$\tan\theta = \dfrac{6}{10} \Rightarrow \theta = \tan^{-1}\left(\dfrac{3}{5}\right)$

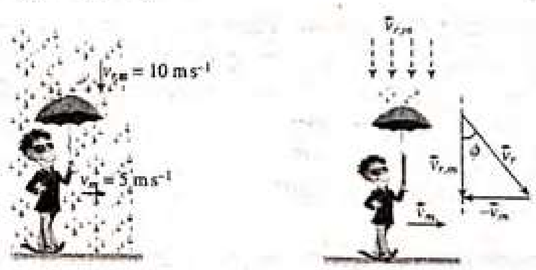


where θ is the angle with vertical.

ILLUSTRATION 4.12 A man moving with 5 m s^{-1} observes rain falling vertically at the rate of 10 m s^{-1}. Find the speed and direction of the rain with respect to ground.

Solution. $\vec{v}_{r,m} = -10\hat{j}\,\text{m s}^{-1}, \vec{v}_m = 5\hat{i}\,\text{m s}^{-1}$

$\vec{v}_{r,m} = \vec{v}_r - \vec{v}_m$



$\vec{v}_r = \vec{v}_{r,m} + \vec{v}_m = (-10\hat{j}) + (5\hat{i})\,\text{m s}^{-1}$

$\Rightarrow \quad |\vec{v}_r| = 5\sqrt{5}\ \text{m s}^{-1}$

Inclination of the rain with vertical, $\tan\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = \tan^{-1}\dfrac{1}{2}$

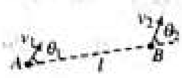
Velocity of Approach

If two particles are moving with velocities $\vec{v}_1$ and $\vec{v}_2$, as shown in the figure, at any time, the separation between the particles is l. Then the velocity of approach of the particles can be written as:

$$V = v_1\cos\theta_1 + v_2\cos\theta_2 = -\frac{dl}{dt}$$

If particles are moving in same direction as in the figure,

$$v_1\cos\theta_1 - v_2\cos\theta_2 = -\frac{dl}{dt}$$



(provided $v_1\cos\theta_1 > v_2\cos\theta_2$)

ILLUSTRATION 4.13 Two particles A and B are moving with constant velocities v_1 and v_2. At $t = 0$, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B. Find their velocity of approach.

Solution. Velocity of approach is the relative velocity along line AB

$$v_{APP} = v_1 \cos \theta_1 + v_2 \cos \theta_2$$

Condition for Uniformly Moving Particles to Collide

If two particles are moving with uniform velocities and the relative velocity of one particle w.r.t. other particle is directed towards each other, then they will collide.

ILLUSTRATION 4.14 Two particles A and B are moving with constant velocities v_1 and v_2. At $t = 0$, v_1 makes an angle θ_1 with the line joining A and B and v_2 makes an angle θ_2 with the line joining A and B.

(a) Find the condition for A and B to collide.

(b) Find the time after which A and B will collide if separation between them is d at $t = 0$

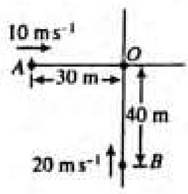
Solution.

(a) For A and B to collide, their relative velocity must be directed along the line joining them.
Therefore their relative velocity along the perpendicular to this line must be zero.
Thus $v_1 \sin\theta_1 = v_2 \sin\theta_2$.

(b) $v_{App} = v_1 \cos\theta_1 + v_2 \cos\theta_2$

$$t = \frac{d}{v_{app}} = \frac{d}{v_1 \cos\theta_1 + v_2 \cos\theta_2}$$

ILLUSTRATION 4.15 Two particles A and B are moving with uniform velocity as shown in figure given below at $t = 0$.

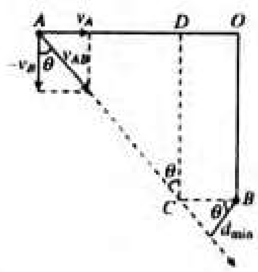


(a) Will the two particles collide?

(b) Find out the shortest distance between two particles.

Solution. Observing A from the frame of B

Let us draw $\vec{v}_{AB}$,



$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = \vec{v}_A + (-\vec{v}_B)$$

$$\tan\theta = \frac{10}{20} = \frac{1}{2}$$

Again $\quad \tan\theta = \frac{AD}{CD} = \frac{AD}{40} = \frac{1}{2}$

$\Rightarrow \quad AD = 20 \Rightarrow DO = 10$

$\Rightarrow \quad BC = 10$

Hence, the shortest distance between the particles

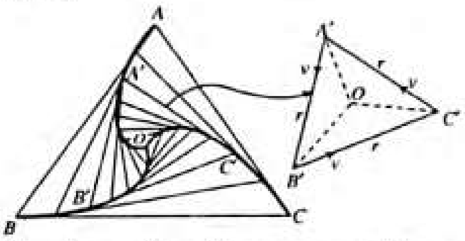
$$d_{short} = BC' \cos\theta = 10\cos\theta = \frac{10 \times 2}{\sqrt{5}} = 4\sqrt{5}\text{ m}$$

Since the closest distance is non-zero, therefore, the particles will not collide.

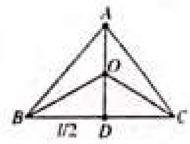
ILLUSTRATION 4.16 Three particles A, B and C are situated at the vertices of an equilateral triangle of side r at $t = 0$. The particle A heads towards B, B towards C, C towards A with constant speeds v. Find the time of their meeting.

Solution. The motion of the particles is roughly sketched in figure. By symmetry they will meet at the centroid O of the triangle. As the speed of all the particles is equal they will cover equal distance in any given interval of time.

If we join the instantaneous position of the particle at any time, the particles will form an equilateral triangle of same centroid as initial triangle.



Let us consider the motion of any one particle say A. At any instant, its velocity makes an angle 30° with line AO.



The component of the velocity along AO is $v \cos 30°$. This component will be equal to the rate of change of distance between A and O.

At $t = 0$, distance between A and O,

$$AO = \frac{2}{3}AD = \frac{2}{3}\sqrt{l^2 - \left(\frac{l}{2}\right)^2} = \frac{l}{\sqrt{3}}$$

At time $t = T$, the separation between A and O is zero.

Hence, time taken for AO to become zero,

$$T = \frac{(l/\sqrt{3})}{v\cos 30°} = \frac{(l/\sqrt{3})}{v(\sqrt{3}/2)} = \frac{2d}{3v}$$

After time t, let the distance of separation between the insect be r. The velocity of approach (component of relative velocity v_{rel} between the insects along the line of their separation) is $= v + v \cos 60° = 3v/3$

Since $(v_{rel})_{\parallel} dt = -dr$, substituting $(v_{rel})_{\parallel} = \frac{3v}{2}$, we obtain

$$\frac{3v}{2}dt = -dr$$

As at time of meeting, integrating both sides, we have

$$\frac{3v}{2}\int_0^t dt = -\int_l^0 dr$$

This gives $t = \frac{2l}{3v}$

Concept Application Exercise 4.3

1. A 400-m wide river is flowing at a rate of 2.0 m s^{-1}. A boat is sailing with a velocity of 10 m s^{-1} with respect to the water, in a direction perpendicular to the river.
   (a) Find the time taken by the boat to reach the opposite bank.
   (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank?
   (c) In what direction does the boat actually move?
2. A man wishing to cross a river flowing with velocity u jumps at an angle θ with the river flow.
   (a) Find the net velocity of the man with respect to ground if he can swim with speed v in still water.
   (b) In what direction does the boat actually move?
   (c) Find how far from the point directly opposite to the starting point does the boat reach the opposite bank, if the width of the river is d.
3. Find the time an aeroplane having velocity v takes to fly around a square with side a if the wind is blowing at a velocity u along one side of the square.
4. A boatman finds that he can save 6 s in crossing a river by the quickest path than by the shortest path. If the velocity of the boat and the river be, respectively, 17 m s^{-1} and 8 m s^{-1}, find the river width.
5. A man directly crosses a river in time t_1 and swims down the current a distance equal to the width of the river in time t_2. If u and v be the speed of the current and the man respectively, show that $t_1 : t_2 :: \sqrt{v+u} : \sqrt{v-u}$.

KINEMATICS OF CIRCULAR MOTION

To express the linear quantities, i.e., position vector, displacement, velocity, let us choose the center of the circular as the origin of the coordinate system.

Angular Position

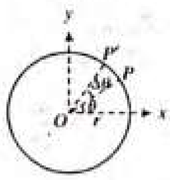
To decide the angular position of a point in space, we need to specify (1) origin and (2) reference line.

The angle made by the position vector w.r.t. origin, with the reference line is called *angular position.*

Clearly angular position depends on the choice of the origin as well as the reference line.

Angular Displacement

Angle through which the position vector of the moving particle rotates in a given time interval is called its *angular displacement*. Angular displacement depends on origin, but it does not depend on the reference line. As the particle moves on above circle its angular position θ changes. Suppose the point rotates through an angle $\Delta\theta$ in time, Δt, then $\Delta\theta$ is angular displacement.



☞ Important Points

- Angular displacement is a dimensionless quantity. Its SI unit is radian; some other units are degree and revolution.
  2π rad = 360° = 1 rev
- Infinitesimally small angular displacement is a vector quantity, but finite angular displacement is a scaler, because while the addition of the infinitesimally small angular displacements is commutative, the addition of finite angular displacement is not.
  $d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$ but $\theta_1 + \theta_2 \neq \theta_2 + \theta_1$
- The direction of small angular displacement is decided by right hand thumb rule. When the fingers are directed along the motion of the point, then the thumb will represent the direction of angular displacement.

Average Angular Velocity

It is defined as the total angular displacement divided by the total time taken.

or $$\omega_{av} = \frac{\text{Angular displacement}}{\text{Total time taken}}$$

$$= \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2, respectively. Since angular displacement is a scalar, average angular velocity is also a scalar quantity.

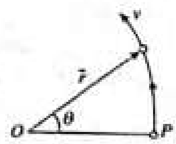
Angular Speed and Angular Velocity

It is the limit of average angular velocity as Δt approaches zero, i.e.,

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} = \frac{d\vec{\theta}}{dt}$$

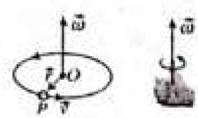
When point P moves in a circular path, the angle θ made by radius vector $\vec{r}$ changes with time.

The unit of ω is radian per second. If ω is more, the particle turns more quickly about O and vice-versa.



Since infinitesimally small angular displacement $\overline{d\theta}$ is a vector quantity, instantaneous angular velocity $\vec{\omega}$ is given by "right hand thumb rule." The magnitude of the angular velocity is given as angular speed.

If we wrap the fingers of the right hand in the sense of revolution (turning) of the point P, the extended thumb will give us the direction of the angular velocity.



Angular Acceleration

The angular acceleration of P relative to O can be given as, $\alpha = d\omega/dt$.

where $\omega = \dfrac{d\theta}{dt}$

Then, $\alpha = \dfrac{d^2\theta}{dt^2}$

When ω increases, $\vec{\alpha}$ is directed along $\vec{\omega}$; when ω decreases, $\vec{\alpha}$ is directed opposite to $\vec{\omega}$.

Important Points

- Angular velocity has dimension of $[T^{-1}]$ and SI unit rad/s.
- If a body makes n rotations in t seconds, then the average angular velocity in radian per second will be
  $\omega_{av} = \dfrac{2\pi n}{t}$
- If T is the period and f the frequency of uniform circular motion.
  $\omega_{av} = \dfrac{2\pi}{T} = 2\pi f$
- Even though $\vec{v}$ is tangential for all curvilinear motion, it is not always essentially perpendicular to the position vector $\vec{r}$. However, in circular motion, the velocity of the particle always perpendicular to its position vector drawn relative to the center of the circle.
- Both average and instantaneous angular acceleration are axial vectors with dimension $[T^{-2}]$ and unit rad s^{-2}.
- If $\alpha = 0$, circular motion is said to be uniform.
- Angular displacement in time t, $\theta = \omega t$.

Centripetal Acceleration

It is responsible for change in the direction of velocity. In circular motion, there is always a centripetal acceleration.

Centripetal acceleration is always considered variable because it changes in direction. It is also called radial acceleration or normal acceleration.

It is given by $a_r = \dfrac{v^2}{R}$; Here v is the speed of the particle and R is the radius of the circle.

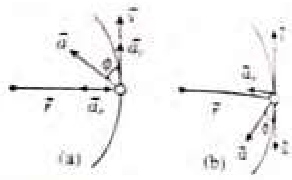
Tangential Acceleration

When a particle moves in a curve, the magnitude of its velocity, that is, the speed of the particle may change. Since the velocity changes its magnitude along the tangent to the path, the rate at which the magnitude of velocity changes along the tangent is called *tangential acceleration*.

It is defined as, $a_t = \dfrac{dv}{dt} = \dfrac{d|\vec{v}|}{dt}$ = Rate of change of speed.
$a_t = \alpha t$

In vector form, it is given as $\vec{a}_t = \dfrac{d|\vec{v}|}{dt}\hat{\tau}$, where $\hat{\tau} = \dfrac{\vec{v}}{|\vec{v}|}$

If the speed $|\vec{v}|$ increases, $d|\vec{v}|/dt$ is positive. Then $\vec{a}_t$ points in the direction of motion $(\vec{v})$. We can call it "tangential acceleration." If the speed $|\vec{v}|$ decreases, $d|\vec{v}|/dt$ is negative. Hence, $\vec{a}_t$ is directed opposite to the direction of motion $(\vec{v})$. We can call it "retardation" or "deceleration."

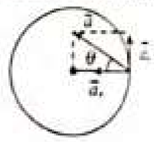


Important Points

- In vector form, $\vec{a}_t = \vec{\alpha} \times \vec{r}$
- If tangential acceleration is directed in the direction of velocity, then the speed of the particle increases.
- If tangential acceleration is directed opposite to the velocity, then the speed of the particle decreases.

Total Acceleration

Total acceleration is the vector sum of centripetal acceleration and tangential acceleration.



$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{a}_r + \vec{a}_t$$

$$a = \sqrt{a_t^2 + a_r^2}\ ;\ \tan\theta = \frac{a_r}{a_t}$$

Important Points

- Differentiation of speed gives tangential acceleration.
- Differentiation of velocity $(\vec{v})$ gives total acceleration.
- $\left|\dfrac{d\vec{v}}{dt}\right|$ and $\dfrac{d|\vec{v}|}{dt}$ are not same physical quantity. $\left|\dfrac{d\vec{v}}{dt}\right|$ is the magnitude of rate of the change of velocity, i.e., magnitude of the total acceleration and $\dfrac{d|\vec{v}|}{dt}$ is the rate of change of speed, i.e., tangential acceleration.

ILLUSTRATION 4.17 A particle moves in a circle of radius 2 cm at a speed given by $v = 4t$, where v is in cm s^{-1} and t is in seconds.

(a) Find the tangential acceleration at $t = 1$s.

(b) Find total acceleration at $t = 1$s.

Solution.

(a) Tangential acceleration,

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(4t) = 4\,\text{cm s}^{-2}$$

(b) $a_c = \dfrac{v^2}{R} = \dfrac{(4)^2}{2} = 8\ \text{cm s}^{-2}$

$\therefore \quad a = \sqrt{a_t^2 + a_C^2} = \sqrt{(4)^2 + (8)^2} = 4\sqrt{5}\ \text{cm s}^{-2}$

ILLUSTRATION 4.18 A particle moves on a circle of radius r with centripetal acceleration as function of time as $a_c = k^2rt^2$, where k is a positive constant. Find the following quantities as function of time at an instant:

(a) The speed of the particle
(b) The tangential acceleration of the particle
(c) The resultant acceleration, and
(d) Angle made by the resultant with tangential direction.

Solution.

(a) In the problem, it is given that $a_c = k^2rt^2$

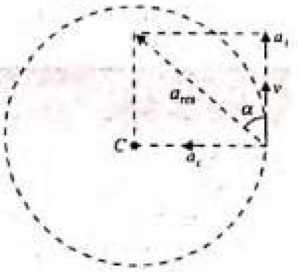
Since $a_c = \dfrac{v^2}{r} \Rightarrow \dfrac{v^2}{r} = k^2rt^2$

$\Rightarrow \quad v^2 = k^2r^2t^2$

Taking square root on both sides, we get $v = krt$

(b) Tangential acceleration,

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(krt) = kr$$



(c) Resultant acceleration,

$$a_{res} = \sqrt{a_c^2 + a_t^2}$$

$$= \sqrt{(k^2rt^2)^2 + (kr)^2}$$

$$= kr\sqrt{k^2t^4 + 1}$$

(d) From figure, $\tan\alpha = \dfrac{a_c}{a_t} = \dfrac{k^2rt^2}{kr} = kt^2$

$\Rightarrow \quad \alpha = \tan^{-1}(kt^2)$

ILLUSTRATION 4.19 A particle moves in a circular path such that its speed v varies with distance s as $v = \alpha\sqrt{s}$ where α is a positive constant. Find the acceleration of the particle after traversing a distance s.

Solution. Total acceleration, $a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(\dfrac{dv}{dt}\right)^2 + \left(\dfrac{v^2}{R}\right)^2}$

where $v = \alpha\sqrt{s}$

Differentiating $v = \alpha\sqrt{s}$ with respect to time, we have

$$\frac{dv}{dt} = \alpha\frac{s^{-1/2}}{2}\frac{ds}{dt}$$

Substituting $\dfrac{ds}{dt} = \alpha\sqrt{s}$, we have $\dfrac{dv}{dt} = \dfrac{\alpha^2}{2}$

Now substituting dv/dt and v in the expression of a, we have

$$a = \sqrt{\left(\frac{\alpha^2}{2}\right)^2 + \left[\frac{\alpha\sqrt{s}}{R}\right]^2}$$

This gives $a = \alpha^2\sqrt{\dfrac{1}{4} + \dfrac{s^2}{R^2}}$

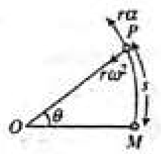
Motion with Constant Angular Acceleration

Circular motion with constant angular acceleration is analogous to one-dimensional translational motion with constant acceleration. Hence, even here, the equations of motion have same form.

$\omega = \omega_0^2 + \alpha t$	$\omega_0 \Rightarrow$ Initial angular velocity
$\theta = \omega_0 t + \dfrac{1}{2}\alpha t^2$	$\omega \Rightarrow$ Final angular velocity
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$\alpha \Rightarrow$ Constant angular acceleration
$\theta = \left(\dfrac{\omega + \omega_0}{2}\right)t$	$\theta \Rightarrow$ Angular displacement
$\theta_{n\text{th}} = \omega_0 + \dfrac{\alpha}{2}(\theta_n - \theta_{n-1})$	

Relation between Linear and Angular Quantities

Let point P describe an arc MP whose length s can be given as $s = r\,\theta$.



Differentiating both sides with time, we have $\dfrac{ds}{dt} = r\dfrac{d\theta}{dt}$, where $ds/dt = v$ and $d\theta/dt = \omega$. Then, $v = r\omega$

Vector relation between speed and angular velocity: $\vec{v} = \vec{\omega} \times \vec{r}$. Here, $\vec{v}$ is the velocity of the particle, $\vec{\omega}$ is the angular velocity about the center of circular motion and $\vec{r}$ is the position of particle w.r.t. the center of circular motion.

Again differentiating both sides with time, we have

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

where $dv/dt = a_t$ and $d\omega/dt = \alpha$. Then, $a_t = r\omega$

Since the radial acceleration $a_r = v^2/r$, putting $v = r\omega$, we have $a_r = r\omega^2$.

ILLUSTRATION 4.20 A fan is rotating with angular velocity $100\ \text{rev s}^{-1}$. Then it is switched off. It takes 5 min to stop.

(a) Find the total number of revolution made before the fan stops. (assume uniform angular retardation).
(b) Find the value of angular retardation.
(c) Find the average angular velocity during this interval.

Solution.

(a) $\theta = \left(\dfrac{\omega + \omega_0}{2}\right)t = \left(\dfrac{100+0}{2}\right) \times 5 \times 60 = 15000$ rev.

(b) $\omega = \omega_0 + \alpha t$

$\Rightarrow \quad 0 = 100 - \alpha(5 \times 60) \Rightarrow \alpha = \dfrac{1}{3}$ rev s^{-2}

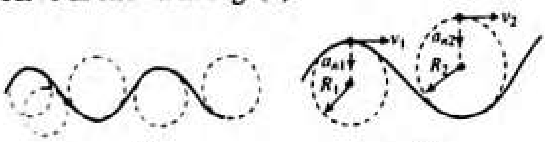
(c) $\omega_{av} = \dfrac{\text{Total angle of rotation}}{\text{Total time taken}}$

$= \dfrac{15000}{50 \times 60} = 50$ rev s^{-1}

Radius of Curvature

When a particle moves in a curve, it has a tangential acceleration $\vec{a}_t$ due to change in its speed and it must have a radial (normal) acceleration $\vec{a}_r$ because of the change in the direction of its velocity. Then, the total acceleration, $\vec{a} = \vec{a}_t + \vec{a}_n$, where $a_t = d|\vec{v}|/dt$ and $a_n = v^2/R$.

Any curved path can be assumed to be made of infinite circular arcs. The radius of curvature at a point is defined as the radius of that circular arc which fits at that particular point on the curve as shown in Fig. (a).



(a) (b)

If we know the magnitude and direction of the total acceleration $\vec{a}$ of the particle, resolving $\vec{a}$ perpendicular to the velocity of the particle, we have $\vec{a}_r = \vec{a}_n = a\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{v}$. Substituting $a_n = v$, we have

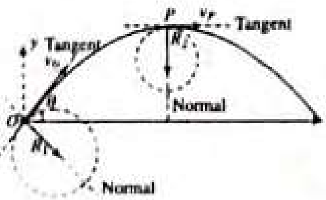
$$R = \frac{v^2}{a_n}$$

where R is the radius of the effective circle at any instant when the particle has a velocity $\vec{v}$ and acceleration $\vec{a}$. This is known as *radius of curvature* of the path followed by the particle at the given position (or time).

ILLUSTRATION 4.21 A particle is projected at angle θ with horizontal with velocity v_0 at $t = 0$. Find

(a) tangential and normal acceleration of the particle at $t = 0$ and at highest point of its trajectory.

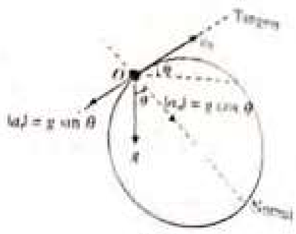
(b) the radius of curvature at $t = 0$ and highest point.



Solution.

(a) The direction of tangential acceleration is in the line of velocity and the direction perpendicular to velocity direction. The tangential and normal directions at O and P are shown in Figs. (a) and (b), respectively.

The net acceleration of the particle during motion is acceleration due to gravity, i.e., g is acting vertically downward.



(a)

At O (at $t = 0$):

Tangential acceleration,

$a_t = -g \sin\theta$

Normal acceleration,

$a_n = g\cos\theta$

At P (at highest point):

Tangential acceleration,

$a_t = 0$ and $a_n = g$

(b)

(b) Let the radius of curvature at O be R_0. The normal acceleration at O be $g\cos\theta$.

$$a_n = \frac{v^2}{R} \Rightarrow R_0 = \frac{v_0^2}{a_n} = \frac{v_0^2}{g\cos\theta}$$

Radius of curvature at P:

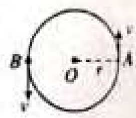
Normal acceleration at P is g. Hence,

$$R_P = \frac{v^2}{a_n} = \frac{(v_0\cos\theta)^2}{g} = \frac{v_0^2\cos^2\theta}{g}$$

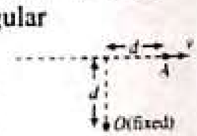
CONCEPT APPLICATION EXERCISE 4.4

1. A particle is moving with constant speed in a circle as shown in figure. Find the angular velocity of the particle A with respect to fixed point B and C if angular velocity with respect to O is ω.

2. Particles A and B move with constant and equal speeds in a circle as shown in figure. Find the angular velocity of the particle A with respect to B, if the angular velocity of particle A w.r.t. O is ω.



3. Find the angular velocity of A with respect to O at the instant shown in figure.



4. A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changes from 5.0 m s^{-1} to 6.0 m s^{-1} in 2.0 s, find the angular acceleration.
5. Find the magnitude of the acceleration of a particle moving in a circle of radius 10 cm with uniform speed completing the circle in 4 s.

SOLVED EXAMPLES

1. A stone projected with a velocity u at an angle θ with the horizontal reaches maximum height H_1. When it is projected with velocity u at an angle $\left(\dfrac{\pi}{2} - \theta\right)$ with the horizontal, it reaches maximum height H_2. The relation between the horizontal range R of the projectile, H_1 and H_2 is

(a) $R = 4\sqrt{H_1 H_2}$ (b) $R = 4(H_1 - H_2)$

(c) $R = 4(H_1 + H_2)$ (d) $R = \frac{H_1^2}{H_2^2}$

Sol. (a) $H_1 = \frac{u^2\sin^2\theta}{2g}$ and $H_2 = \frac{u^2\sin^2(90-\theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$

$$H_1 H_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g} = \frac{(u^2\sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$$

$\therefore$ $R = 4\sqrt{H_1 H_2}$

2. An object is projected with a velocity of 20 m/s making an angle of 45° with horizontal. The equation for the trajectory is $h = Ax - Bx^2$ where h is height, x is horizontal distance. A and B are constants. The ratio $A : B$ is ($g = 10\ \text{ms}^{-2}$)

(a) 1 : 5 (b) 5 : 1

(c) 1 : 40 (d) 40 : 1

Sol. (d) Standard equation of projectile motion

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Comparing with given equation

$$A = \tan\theta \text{ and } B = \frac{g}{2u^2\cos^2\theta}$$

So $\frac{A}{B} = \frac{\tan\theta \times 2u^2\cos^2\theta}{g} = 40$

(As $\theta = 45°$, $u = 20$ m/s, $g = 10\ \text{m/s}^2$)

3. For a given velocity, a projectile has the same range R for two angles of projection if t_1 and t_2 are the times of flight in the two cases then

(a) $t_1 t_2 \propto R^2$ (b) $t_1 t_2 \propto R$

(c) $t_1 t_2 \propto \frac{1}{R}$ (d) $t_1 t_2 \propto \frac{1}{R^2}$

Sol. (b) For same range angle of projection should be θ and $90 - \theta$

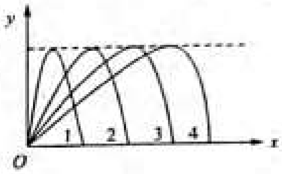
So, time of flights $t_1 = \frac{2u\sin\theta}{g}$ and

$$t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$$

By multiplying $t_1 t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$

$$t_1 t_2 = \frac{2}{g}\frac{(u^2\sin 2\theta)}{g} = \frac{2R}{g} \Rightarrow t_1 t_2 \propto R$$

4. Figure shows four paths for a kicked football. Ignoring the effects of air on the flight, rank the paths according to initial horizontal velocity component, highest first



(a) 1, 2, 3, 4

(b) 2, 3, 4, 1

(c) 3, 4, 1, 2

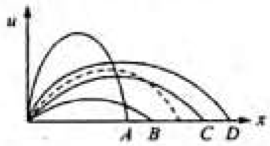
(d) 4, 3, 2, 1

Sol. (d) $R = \frac{u^2\sin 2\theta}{g} = \frac{2u_x \upsilon_y}{g}$

$\therefore$ Range υ horizontal initial velocity (u_x)

In path 4 range is maximum so football possess maximum horizontal velocity in this path.

5. The path of a projectile in the absence of air drag is shown in the figure by dotted line. If the air resistance is not ignored then which one of the path shown in the figure is appropriate for the projectile



(a) B

(b) A

(c) D

(d) C

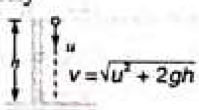
Sol. (a) If air resistance is taken into consideration then range and maximum height, both will decrease.

6. A man standing on the roof of a house of height h throws one particle vertically downwards and another particle horizontally with the same velocity u. The ratio of their velocities when they reach the earth's surface will be

(a) $\sqrt{2gh+u^2} : u$ (b) 1 : 2

(c) 1 : 1 (d) $\sqrt{2gh+u^2} : \sqrt{2gh}$

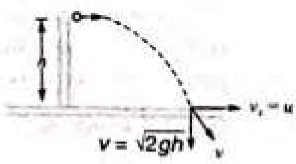
Sol. (c) When particle is thrown in vertical downward direction with velocity u, then the final velocity at the ground level is



$$v^2 = u^2 + 2gh$$

$\therefore$ $v = \sqrt{u^2 + 2gh}$

Another particle is thrown horizontally with same velocity then at the surface of earth.



Horizontal component of velocity $v_x = u$

Therefore, resultant velocity,

$$v = \sqrt{u^2 + 2gh}$$

For both the particles, final velocities when they reach the earth's surface are equal.

7. A cannon on a level plane is aimed at an angle θ above the horizontal and a shell is fired with a muzzle velocity v_0 towards a vertical cliff a distance D away. Then the height from the bottom at which the shell strikes the side walls of the cliff is

(a) $D\sin\theta - \dfrac{gD^2}{2v_0^2\sin^2\theta}$ (b) $D\cos\theta - \dfrac{gD^2}{2v_0^2\cos^2\theta}$

(c) $D\tan\theta - \dfrac{gD^2}{2v_0^2\cos^2\theta}$ (d) $D\tan\theta - \dfrac{gD^2}{2v_0^2\sin^2\theta}$

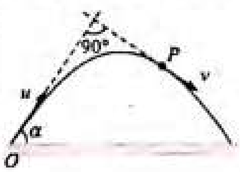
Sol. (c) Equation of trajectory for oblique projectile motion

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Substituting $x = D$ and $u = v_0$

$$h = D\tan\theta - \frac{gD^2}{2u_0^2\cos^2\theta}.$$

8. A particle is projected from point O with velocity u in a direction making an angle α with the horizontal. At any instant its position is at point P at right angles to the initial direction of projection. Its velocity at point P is

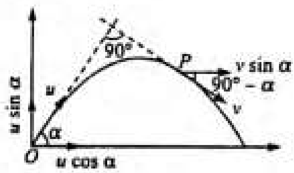


(a) $u\tan\alpha$
(b) $u\cot\alpha$
(c) $u\operatorname{cosec}\alpha$
(d) $u\sec\alpha$

Sol. (b) Horizontal velocity at point $'O' = u\cos\alpha$

Horizontal velocity at point $'P'$ $= v\sin\alpha$

In projectile motion horizontal component of velocity remains constant throughout the motion



$\therefore$ $v\sin\alpha = u\cos\alpha$

$\Rightarrow$ $v = u\cot\alpha$

9. Two seconds after projection a projectile is travelling in a direction inclined at 30° to the horizontal after one more second, it is travelling horizontally, the magnitude and direction of its velocity are

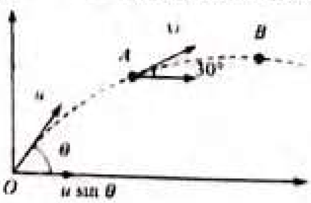
(a) $2\sqrt{20}$ m/s, 60° (b) $20\sqrt{3}$ m/s, 60°

(c) $6\sqrt{40}$ m/s, 30° (d) $40\sqrt{6}$ m/s, 30°

Sol. (b) Let in 2 seconds, the body reaches up to point A and after one more second up to point B.

Total time of ascent for a body is given 3 sec, i.e., $t = \dfrac{u\sin\theta}{g} = 3$

$\therefore u\sin\theta = 10\times 3 = 30$ ...(i)



Horizontal component of velocity remains always constant

$u\cos\theta = v\cos 30°$ ...(ii)

For vertical upward motion between point O and A,

$v\sin 30° = u\sin\theta - g\times 2$ [Using $v = u - gt$]

$v\sin 30° = 30 - 20$ [As $u\sin\theta = 30$]

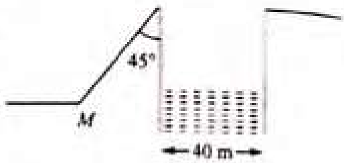
$\therefore v = 20$ m/s.

Substituting this value in equation (ii),

$u\cos\theta = 20\cos 30° = 10\sqrt{3}$ ....(iii)

From equations (i) and (iii), $u = 20\sqrt{3}$ and $\theta = 60°$

10. A body is projected up a smooth inclined plane (length = $20\sqrt{2}$ m) with velocity u from the point M as shown in the figure. The angle of inclination is 45° and the top is connected to a well of diameter 40 m. If the body just manages to cross the well, what is the value of v?



(a) 40 m s^{-1} (b) $40\sqrt{2}$ m s^{-1}

(c) 20 m s^{-1} (d) $20\sqrt{2}$ m s^{-1}

Sol. (d) At point N, the angle of projection of the body will be 45°. Let, the velocity of projection at this point is v.

If the body just manages to cross the well, then

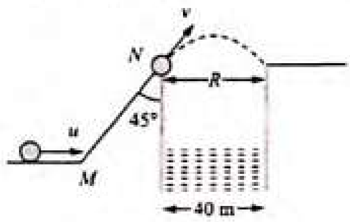
Range = Diameter of well

$$\frac{v^2\sin 2\theta}{g} = 40$$

[As $\theta = 45°$]

$v^2 = 400$

$\Rightarrow v = 20$ m s^{-1}



But we have to calculate the velocity (u) of the body at point M.

For motion along the inclined plane (from M to N)

Final velocity (v) = 20 m s,

acceleration (a) = $-g\sin\alpha = -g\sin 45°$, distance of inclined plane (s) = $20\sqrt{2}$ m

$$(20) = u^2 - 2\frac{g}{\sqrt{2}}\cdot 20\sqrt{2} \qquad \text{[Using } v^2 = u^2 + 2as\text{]}$$

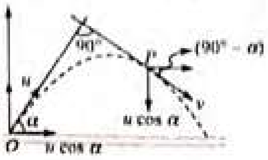
$u^2 = 20^2 + 400 \Rightarrow 20\sqrt{2}$ m/s.

11. A particle is projected from a point O with a velocity u in a direction making an angle α upward with the horizontal. After some time at point P, it is moving at right angle with its initial direction of projection. The time of flight from O to P is

(a) $\dfrac{u\sin\alpha}{g}$ (b) $\dfrac{u\operatorname{cosec}\alpha}{g}$

(c) $\dfrac{u\tan\alpha}{g}$ (d) $\dfrac{u\sec\alpha}{g}$

Sol. (b) When the body projected with initial velocity $\vec{u}$ by making angle α with the horizontal, then after time t, (at point P) its direction is perpendicular to $\vec{u}$. Magnitude of velocity at point P is given by $v = u \cot \alpha$.



For vertical motion: Initial velocity (at point O) = $u \sin \alpha$

Final velocity (at point P) = $-v \cos \alpha = -u \cot \alpha \cos \alpha$

Time of flight (from point O to P) = t

Applying first equation of motion $v = u - gt$

$$-u \cot \alpha \cos \alpha = u \sin \alpha - g\,t$$

$$\therefore \quad t = \frac{u \sin\alpha + \cot\alpha\cos\alpha}{g}$$

$$= \frac{u}{g \sin\alpha}[\sin^2\alpha + \cos^2\alpha] = \frac{u\,\mathrm{cosec}\alpha}{g}$$

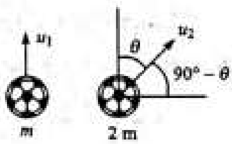
12. Pankaj and Sudhir are playing with two different balls of masses m and $2m$, respectively. If Pankaj throws his ball vertically up and Sudhir at an angle θ, both of them stay in our view for the same period. The height attained by the two balls are in the ratio

(a) 2 : 1 (b) 1 : 1
(c) 1 : cosθ (d) 1 : secθ

Sol. (b) Time of flight for the ball thrown by Pankaj, $T_1 = \frac{2u_1}{g}$

Time of flight for the ball thrown by Sudhir

$$T_2 = \frac{2u^2 \sin(90° - \theta)}{g} = \frac{2u^2 \cos\theta}{g}$$



According to problem $T_1 = T_2$

$$\Rightarrow \frac{2u_1}{g} = \frac{2u^2 \cos\theta}{g}$$

$$\Rightarrow u_1 = u_2 \cos\theta$$

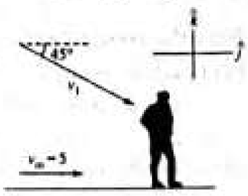
Height of the ball thrown by Pankaj $H_1 = \frac{u_1^2}{2g}$

Height of the ball thrown by Sudhir

$$H_2 = \frac{u_2^2 \sin^2(90° - \theta)}{2g} = \frac{u_2^2 \cos^2\theta}{2g}$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{u_1^2/2g}{u_2^2 \cos^2\theta/2g} = 1 \qquad [\text{As } u_1 = u_2 \cos\theta]$$

13. A man walks horizontally in rain with a velocity of 5 m/s. The raindrops strike on his back at angle of 45° with the horizontal. Then for the actual rain's velocity



(a) horizontal component = vertical component
(b) horizontal component > vertical component
(c) horizontal component < vertical component
(d) none of these

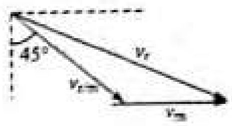
Sol. (b) Velocity of man: $\vec{v}_m = 5\hat{i}$

Velocity of rain w.r.t man:

$$\vec{v}_{r/m} = v_1 \cos 45° \hat{i} - v_1 \sin 45° \hat{k} = \frac{v_1}{\sqrt{2}}\hat{i} - \frac{v_1}{\sqrt{2}}\hat{k}$$

Hence, velocity of rain:

$$\vec{v}_r = \vec{v}_{r/m} + \vec{v}_m$$



$$\Rightarrow \vec{v}_r = \left(\frac{v_1}{\sqrt{2}} + 5\right)\hat{i} - \frac{v_1}{\sqrt{2}}\hat{k}$$

Horizontal component of rain's velocity $= \frac{v_1}{\sqrt{2}} + 5$

Vertical component of rain's velocity $= \frac{v_1}{\sqrt{2}}$

So horizontal component > vertical component

14. A ship A sailing due east with a velocity of 10 km/h happens to appear sailing due north with a velocity of 5 km/h, to a person, sitting in a moving ship B. Determine the velocity (absolute) of ship B.

(a) $5\sqrt{5}$ km/h, $\tan^{-1}(1/2)$ S of E

(b) $5\sqrt{5}$ km/h, $\tan^{-1}(1/2)$ E of S

(c) $4\sqrt{5}$ km/h, $\tan^{-1}(1/2)$ S of E

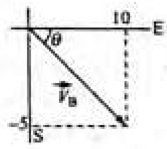
(d) $4\sqrt{5}$ km/h, $\tan^{-1}(1/2)$ E of S

Sol. (a) Here we are given velocity of 'A', $\vec{v}_A = 10\hat{i}$

Velocity of 'A' w.r.t 'B', $\vec{v}_{A/B} = 5\hat{j}$

Now $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

$$5\hat{j} = 10\hat{i} - \vec{v}_B \Rightarrow \vec{v}_B = 10\hat{i} - 5\hat{j}$$



Hence velocity of B,

$$v_B = \sqrt{10^2 + 5^2} = 5\sqrt{5} \text{ km/h}$$

$$\tan\theta = \frac{5}{10} = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) \text{ S of E}$$

15. A helicopter is flying horizontally at 8 m/s at an altitude 180 m when a package of emergency medical supplies is ejected horizontally backward with a speed of 12 m/s relative to the helicopter. Ignoring air resistance, what is the horizontal distance between the package and the helicopter when the package hits the ground?

(a) 120 m (b) 24 m
(c) 36 m (d) 72 m

Sol. (d) Time taken by packet to reach the ground:

$$T=\sqrt{\frac{2\times 180}{10}}=6\,\text{s}$$

Horizontal distance travelled by helicopter in this time
$= 8 \times 6 = 48$ m.

Velocity of package w.r.t. ground
$= 12 - 8 = 4$ m/s in backward direction.

Horizontal distance travelled by package in time,
$T = 4 \times 6 = 24$ m.

So, horizontal distance between them $= 48 + 24 = 72$ m

16. In horizontal level, ground to ground projectile if at any instant velocity becomes perpendicular to initial velocity, then what can you say about projection angle with horizontal?

(a) $\theta = 45°$
(b) $\theta \le 45°$
(c) $\theta \le 45°$
(d) for any value of θ it is possible

Sol. (b) Velocity at any time, $\vec{v}=\vec{u}+\vec{g}\,t$

$$\Rightarrow \quad \vec{v}=u\cos\theta\, i+(u\sin\theta-gt)\, j$$

Let at any time, this velocity becomes perpendicular to initial velocity. Then $\vec{v}\cdot\vec{u}=0$

Solve to get $t=\dfrac{u}{g\sin\theta}$

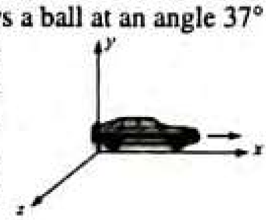
Now t should be less than/equal to time of flight.

So $\quad t \le T$

$$\frac{u}{g\sin\theta}\le\frac{2u\sin\theta}{g}\ \Rightarrow\ \sin^2\theta\ge\frac{1}{2}$$

$$\Rightarrow \quad \sin\theta\ge\frac{1}{\sqrt{2}}\ \Rightarrow\ \theta\ge 45°$$

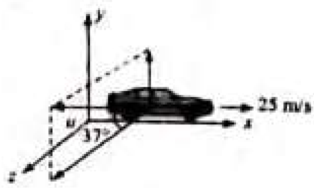
17. A car is traveling on a highway at a speed of 25 m/s along x-axis. A passenger in a car throws a ball at an angle 37° with horizontal in a plane perpendicular the motion of the car. The ball is projected with a speed of 10 m/s relative to car. What may be the initial velocity of ball in unit vector notation?



(a) $25\hat{i}+8\hat{j}+6\hat{k}$ (b) $10\hat{i}+8\hat{j}+6\hat{k}$

(c) $10\hat{i}+25\hat{j}+6\hat{k}$ (d) $25\hat{i}+6\hat{j}+8\hat{k}$

Sol. (d) Velocity of the car, $\vec{v}_c=25\hat{i}$



Velocity of ball with respect to car

$$\vec{v}_{b/c}=u\sin 37°\hat{j}+u\cos 37°\hat{k}$$

$$=10\times\frac{3}{5}\hat{j}+10\times\frac{4}{5}\hat{k}=6\hat{j}+8\hat{k}$$

Now: using $\vec{v}_{b/c}=\vec{v}_b-\vec{v}_c$

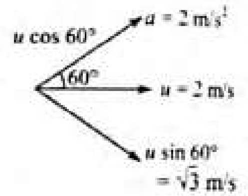
$$\Rightarrow \quad \vec{v}_b=\vec{v}_b+\vec{v}_{b/c}=25\hat{i}+6\hat{j}+8\hat{k}$$

18. Velocity of a particle at time $t = 0$ is 2 m/s. A constant acceleration of 2 m/s^2 act on the particle for 2 s at an angle 60° with the initial velocity. The magnitude of velocity of particle at the end of 2 s will be

(a) $2\sqrt{5}$ m/s (b) $2\sqrt{7}$ m/s

(c) $2\sqrt{6}$ m/s (d) $2\sqrt{3}$ m/s

Sol. (b) Let us make the components of initial velocity in the direction and perpendicular to the direction of acceleration as shown in the figure.



In the direction of acceleration:

Using $v = u + at$, we get

$$v_1=u\cos 60°+at$$

$$=2\times\frac{1}{2}+2\times 2=5\,\text{m/s}$$

Velocity perpendicular to direction of acceleration remains same, which is

$$u\sin 60°=\sqrt{3}\,\text{m/s}$$

So net velocity at the end of 2 s:

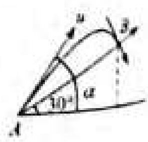
$$v=\sqrt{v_1^2+(\sqrt{3})^2}=\sqrt{5^2+3}=\sqrt{28}=2\sqrt{7}\,\text{m/s}$$

19. A particle is projected with a certain velocity at an angle α above the horizontal from the foot of an inclined plane of inclination 30°. If the particle strikes the plane normally, then α is equal to

(a) $30°+\tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ (b) 45°

(c) 60° (d) $30°+\tan^{-1}(2\sqrt{3})$

Sol. (a) t_{AB} = time of flight of projectile



$$=\frac{2u\sin(\alpha-30°)}{g\cos 30°}$$

Now the component of velocity along the plane becomes zero at point B.

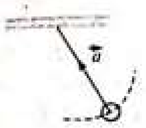
$$0=u\cos(\alpha-30°)-g\sin 30°\times t_{AB}$$

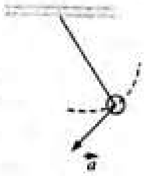
or $\quad u\cos(\alpha-30°)=g\sin 30°\times\dfrac{2u\sin(\alpha-30°)}{g\cos 30°}$

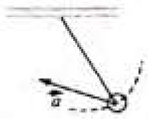
or $\quad \tan(\alpha-30°)=\dfrac{\cot 30°}{2}=\dfrac{\sqrt{3}}{2}$

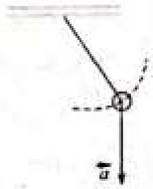
$$\alpha = 30^\circ + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

20. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in

(a) 

(b) 

(c) 

(d) 

Sol. (c) a_c = centripetal acceleration
a_t = tangential acceleration
a_N = net acceleration = Resultant of a_c and a_t

EXERCISES

Problems Based on Basic Concept of Projectile Motion

1. A particle is projected with a velocity v so that its range on a horizontal plane is twice the greatest height attained. If g is acceleration due to gravity, then its range is

(a) $\frac{4v^2}{5g}$ (b) $\frac{4g}{5v^2}$ (c) $\frac{4v^3}{5g^2}$ (d) $\frac{4v}{5g^2}$

2. During a projectile motion, if the maximum height equals the horizontal range, then the angle of projection with the horizontal is

(a) $\tan^{-1}(1)$ (b) $\tan^{-1}(2)$ (c) $\tan^{-1}(3)$ (d) $\tan^{-1}(4)$

3. A particle is projected from ground at some angle with the horizontal. Let P be the point at maximum height H. At what height above the point P should the particle be aimed to have range equal to maximum height?

(a) H (b) $2H$ (c) $H/2$ (d) $3H$

4. A shot is fired from a point at a distance of 200 m from the foot of a tower 100 m high so that it just passes over it horizontally. The direction of shot with horizontal is

(a) 30° (b) 45° (c) 60° (d) 70°

5. A projectile has a time of flight T and range R. If the time of flight is doubled, keeping the angle of projection same, what happens to the range?

(a) $R/4$ (b) $R/2$ (c) $2R$ (d) $4R$

6. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?

(a) Yes, 60° (b) Yes, 30° (c) No (d) Yes, 45°

7. Two paper screens A and B are separated by 150 m. A bullet pierces A and B. The hole in B is 15 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is ($g = 10\ \text{m s}^{-2}$)

(a) $100\sqrt{3}\ \text{m s}^{-1}$ (b) $200\sqrt{3}\ \text{m s}^{-1}$

(c) $300\sqrt{3}\ \text{m s}^{-1}$ (d) $500\sqrt{3}\ \text{m s}^{-1}$

8. A projectile can have same range R for two angles of projection. It t_1 and t_2 are the times of flight in the two cases, then what is the product of two times of flight?

(a) $t_1t_2 \propto R^2$ (b) $t_1t_2 \propto R$

(c) $t_1t_2 \propto \frac{1}{R}$ (d) $t_1t_2 \propto \frac{1}{R^2}$

9. A ball is thrown at different angles with the same speed u and from the same point and it has the same range in both the cases. If y_1 and y_2 are the heights attained in the two cases, then $y_1 + y_2$ is equal to

(a) $\frac{u^2}{g}$ (b) $\frac{2u^2}{g}$ (c) $\frac{u^2}{2g}$ (d) $\frac{u^2}{4g}$

10. The equation of motion of a projectile is $y = 12x - \frac{3}{4}x^2$. The horizontal component of velocity is $3\ \text{m s}^{-1}$. What is the range of the projectile?

(a) 18 m (b) 16 m (c) 12 m (d) 21.6 m

11. The range R of projectile is same when its maximum heights are h_1 and h_2. What is the relation between R, h_1, and h_2?

(a) $R = \sqrt{h_1h_2}$ (b) $R = \sqrt{2h_1h_2}$

(c) $R = 2\sqrt{h_1h_2}$ (d) $R = 4\sqrt{h_1h_2}$

12. At what angle with the horizontal should a ball be thrown so that the range R is related to the time of flight as $R = 5T^2$? (Take $g = 10\ \text{m s}^{-2}$)

(a) 30° (b) 45° (c) 60° (d) 90°

13. A ball thrown by one player reaches the other in 2 s. The maximum height attained by the ball above the point of projection will be about

(a) 2.5 m (b) 5 m (c) 7.5 m (d) 10 m

14. At a height 0.4 m from the ground, the velocity of a projectile in vector form is $\vec{v} = (6\hat{i} + 2\hat{j})\ \text{m s}^{-1}$. The angle of projection is

(a) 45° (b) 60° (c) 30° (d) $\tan^{-1}(3/4)$

15. A projectile is thrown in the upward direction making an angle of 60° with the horizontal direction with a velocity of $150\ \text{m s}^{-1}$. Then the time after which its inclination with the horizontal is 45° is

(a) $15(\sqrt{3}-1)$ s (b) $15(\sqrt{3}+1)$ s

(c) $7.5(\sqrt{3}-1)$ s (d) $7.5(\sqrt{3}+1)$ s

16. A number of bullets are fired in all possible directions with the same initial velocity u. The maximum area of ground covered by bullets is

(a) $\pi\left(\frac{u^2}{g}\right)^2$ (b) $\pi\left(\frac{u^2}{2g}\right)^2$

(c) $\pi\left(\frac{u}{g}\right)^2$ (d) $\pi\left(\frac{u}{2g}\right)^2$

17. Two stones are projected with the same speed but making different angles with the horizontal. Their ranges are equal. If the angle of projection of one is $\pi/3$ and its maximum height is h_1, then the maximum height of the other will be

(a) $3h_1$ (b) $2h_1$ (c) $h_1/2$ (d) $h_1/3$

18. A ball is projected from the ground at angle θ with the horizontal. After 1 s, it is moving at angle 45° with the horizontal and after 2 s it is moving horizontally. What is the velocity of projection of the ball?

(a) $10\sqrt{3}$ ms (b) $20\sqrt{3}\ \text{m s}^{-1}$

(c) $10\sqrt{5}\ \text{m s}^{-1}$ (d) $20\sqrt{2}\ \text{m s}^{-1}$

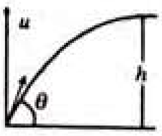
19. A hose lying on the ground shoots a stream of water upward at an angle of 60° to the horizontal with the velocity of $16\ \text{m s}^{-1}$. The height at which the water strikes the wall 8 m away is

(a) 8.9 m (b) 10.9 m (c) 12.9 m (d) 6.9 m

20. A golfer standing on level ground hits a ball with a velocity of $u = 52\ \text{m s}^{-1}$ at an angle α above the horizontal. If $\tan\alpha = 5/12$, then the time for which the ball is at least 15 m above the ground will be (take $g = 10\ \text{m s}^{-2}$)

(a) 1 s (b) 2 s (c) 3 s (d) 4 s

21. If a stone is to hit at a point which is at a distance d away and at a height h (see figure) above the point from where the stone starts, then what is the value of initial speed u if the stone is launched at an angle θ?



(a) $\frac{g}{\cos\theta}\sqrt{\frac{d}{2(d\tan\theta - h)}}$ (b) $\frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$

(c) $\sqrt{\frac{gd^2}{h\cos^2\theta}}$ (d) $\sqrt{\frac{gd^2}{(d-h)}}$

22. The speed of a projectile at its maximum height is $\sqrt{3}/2$ times its initial speed. If the range of the projectile is P times the maximum height attained by it, P is equal to

(a) 4/3 (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) 3/4

23. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a and b are constants and x and y are, respectively, horizontal and vertical distances of the projectile from the point of projection. The maximum height attained by the particle and the angle of projection from the horizontal are

(a) $\frac{b^2}{2a}, \tan^{-1}(b)$ (b) $\frac{a^2}{b}, \tan^{-1}(2b)$

(c) $\frac{a^2}{4b}, \tan^{-1}(a)$ (d) $\frac{2a^2}{b}, \tan^{-1}(a)$

24. A projectile is given an initial velocity of $\hat{i} + 2\hat{j}$. The cartesian equation of its path is ($g = 10\ \text{m s}^{-2}$).

(a) $y = 2x - 5x^2$ (b) $y = x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $y = 2x - 25x^2$

25. A particle is projected from the ground with an initial speed of v at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

(a) $\frac{v}{2}\sqrt{1+2\cos^2\theta}$ (b) $\frac{v}{2}\sqrt{1+2\cos^2\theta}$

(c) $\frac{v}{2}\sqrt{1+3\cos^2\theta}$ (d) $v\cos\theta$

26. Two balls A and B are thrown with speeds u and $u/2$, respectively. Both the balls cover the same horizontal distance before returning to the plane of projection. If the angle of projection of ball B is 15° with the horizontal, then the angle of projection of A is

(a) $\sin^{-1}\left(\frac{1}{8}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$

(c) $\frac{1}{3}\sin^{-1}\left(\frac{1}{8}\right)$ (d) $\frac{1}{4}\sin^{-1}\left(\frac{1}{8}\right)$

27. The horizontal range and maximum height attained by a projectile are R and H, respectively. If a constant horizontal acceleration $a = g/4$ is imparted to the projectile due to wind, then its horizontal range and maximum height will be

(a) $(R+H), \frac{H}{2}$ (b) $\left(R+\frac{H}{2}\right), 2H$

(c) $(R+2H), H$ (d) $(R+H), H$

28. A particle is projected with a certain velocity at an angle α above the horizontal from the foot of an inclined plane of

inclination 30°. If the particle strikes the plane normally, then α is equal to

(a) $30° + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (b) 45°

(c) 60° (d) $30° + \tan^{-1}(2\sqrt{3})$

Projectile from Moving Frame and from Inclined Plane

29. Two bullets are fired horizontally with different velocities from the same height. Which will reach the ground first?
(a) Slower one
(b) Faster one
(c) Both will reach simultaneously
(d) Cannot be predicted

30. A ball rolls off the top of a staircase with a horizontal velocity u $\mathrm{m\,s^{-1}}$. If the steps are h metre high and b metre wide, the ball will hit the edge of the nth step, if

(a) $n = \frac{2hu}{gb^2}$ (b) $n = \frac{2hu^2}{gb}$

(c) $n = \frac{2hu^2}{gb^2}$ (d) $n = \frac{hu^2}{gb^2}$

31. A body is projected horizontally from the top of a tower with initial velocity 18 $\mathrm{m\,s^{-1}}$. It hits the ground at angle 45°. What is the vertical component of velocity when strikes the ground?

(a) 9 $\mathrm{ms^{-1}}$ (b) $9\sqrt{2}$ $\mathrm{ms^{-1}}$

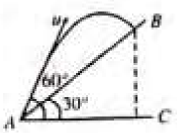
(c) 18 $\mathrm{m\,s^{-1}}$ (d) $18\sqrt{2}$ $\mathrm{ms^{-1}}$

32. A plane flying horizontally at 100 $\mathrm{m\,s^{-1}}$ releases an object which reaches the ground in 10 s. At what angle with horizontal it hits the ground?
(a) 55° (b) 45° (c) 60° (d) 75°

33. A body of mass m is projected horizontally with a velocity v from the top of a tower of height h and it reaches the ground at a distance x from the foot of the tower. If a second body of mass 2m is projected horizontally from the top of a tower of height 2 h, it reaches the ground at a distance $2x$ from the foot of the tower. The horizontal velocity of the second body is
(a) v (b) $2v$ (c) $\sqrt{2}v$ (d) $v/2$

34. A car is moving horizontally along a straight line with a uniform velocity of 25 $\mathrm{m\,s^{-1}}$. A projectile is to be fired from this car in such a way that it will return to it after it has moved 100 m. The speed of the projection must be
(a) 10 $\mathrm{ms^{-1}}$ (b) 20 $\mathrm{ms^{-1}}$ (c) 15 $\mathrm{ms^{-1}}$ (d) 25 $\mathrm{ms^{-1}}$

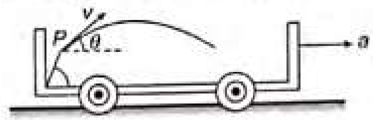
35. In the figure, the time taken by the projectile to reach from A to B is t. Then the distance AB is equal to



(a) $\frac{ut}{\sqrt{3}}$ (b) $\frac{\sqrt{3}ut}{2}$

(c) $\sqrt{3}\,ut$ (d) $2ut$

36. A particle is projected from a trolley car with a velocity $\bar{v}$. If the trolley car moves with an acceleration $\vec{a}$ towards right, which of the following remain unchanged relative to both ground and trolley car?



(a) Range (b) Maximum range
(c) Time of flight (d) Horizontal velocity

37. A boy projects a stone vertically perpendicular to the trolley car with a speed v. If the trolley car moves with a constant velocity u, the time of flight of the stone is:

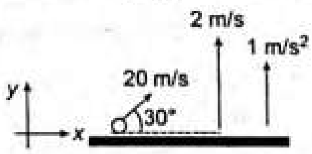
(a) $\frac{u+v}{g}$ (b) $\frac{2v}{g}$

(c) $\frac{2u}{g}$ (d) none of these

38. An aeroplane is flying with speed 60 m/s at angle 30° with the horizontal. A shell is fired horizontally from the aeroplane. Find the direction and magnitude of velocity of the shell so that it falls vertically along a straight line with respect to the observer on the ground.

(a) horizontally opposite to motion of plane with velocity $30\sqrt{3}$ m/s
(b) horizontally opposite to motion of plane with velocity 60 m/s
(c) at angle 30° with the horizontal opposite to motion of plane with velocity 60 m/s
(d) at angle 30° with the horizontal opposite to motion of plane with velocity $30\sqrt{3}$ m/s.

39. A very broad elevator plateform is going up vertically with a constant acceleration 1 $\mathrm{ms^{-2}}$. At the instant when the velocity of the lift is 2 m/s, a stone is projected from the plateform with a speed of 20 m/s relative to the floor at an elevation 30°. The time taken by the stone to return to the floor will be



(a) $\frac{30}{11}$ sec (b) $\frac{70}{11}$ sec

(c) $\frac{20}{11}$ sec (d) $\frac{90}{11}$ sec

40. A particle is projected up with a velocity of $v_0 = 10$ m/s at an angle of $\theta_0 = 60°$ with horizontal onto an inclined plane. The angle of inclination of the plane is 30°. The time of flight of the particle till it strikes the plane is (take $g = 10$ m/s^2)

(a) 1 s (b) 1/2 s

(c) $\frac{2}{\sqrt{3}}$ s (d) $\frac{1}{2\sqrt{3}}$ s

41. Referring to question-40, the time after which the particle attains maximum height is

(a) $\frac{1}{2\sqrt{3}}$ s (b) $\frac{\sqrt{3}}{2}$ s

(c) $\frac{2}{\sqrt{3}}$ s (d) 1 s

42. Referring to question-40, the ratio of the range of the particle and its maximum range in the inclined plane is

(a) 1 : 2 (b) $1:\sqrt{2}$

(c) $1:\sqrt{3}$ (d) 1 : 1

43. Referring to question-40, if the particle is projected down onto the inclined plane at same speed and angle with the inclined plane the ratio of the time of flight and time of flight for upward projection is

(a) $1:\sqrt{2}$ (b) $\sqrt{2}:1$

(c) 1 : 1 (d) None of these

44. Referring to question-40, the ratio of ranges for upward and downward projection is

(a) 1 : 2 (b) 1 : 3

(c) 1 : 4 (d) 1 : 1

45. Referring to question-40, the ratio of component of velocity striking perpendicular to the inclined plane for upward and downward projection is

(a) 1 : 1 (b) 1 : 2

(c) 1 : 3 (d) $1:\sqrt{3}$

46. Referring to question-40, the ratio of speeds of striking for upward and downward projection is

(a) 1 : 1 (b) $1:\sqrt{3}$

(c) $1:\sqrt{5}$ (d) $1:\sqrt{7}$

Relative Velocity in Two Dimensions

47. A bird is flying towards north with a velocity 40 km h^{-1} and a train is moving with velocity 40 km h^{-1} towards east. What is the velocity of the bird noted by a man in the train?

(a) $40\sqrt{2}$ km h^{-1} N-E (b) $40\sqrt{2}$ km h^{-1} S-E

(c) $40\sqrt{2}$ km h^{-1} N-W (d) $40\sqrt{2}$ km h^{-1} S-W

48. A river is flowing from west to east at a speed of 5 m min^{-1}. A man on the south bank of the river, capable of swimming at 10 m min^{-1} in still water, wants to swim across the river in the shortest time. Finally, he will move in a direction

(a) $\tan^{-1}(2)$ E of N (b) $\tan^{-1}(2)$ N of E

(c) 30° E of N (d) 60° E of N

49. A boat is moving with a velocity $3\hat{i}+4\hat{j}$ with respect to ground. The water in the river is moving with a velocity $-3\hat{i}-4\hat{j}$ with respect to ground. The relative velocity of the boat with respect to water is

(a) $8\hat{j}$ (b) $-6\hat{i}-8\hat{j}$ (c) $6\hat{i}+8\hat{j}$ (d) $5\sqrt{2}$

50. A car is moving towards east with a speed of 25 km h^{-1}. To the driver of the car, a bus appears to move towards north with a speed of $25\sqrt{3}$ km h^{-1}. What is the actual velocity of the bus?

(a) 50 km h^{-1}, 30° E of N

(b) 50 km h^{-1}, 30° N of E

(c) 25 km h^{-1}, 30° E of N

(d) 25 km h^{-1}, 30° N of E

51. A swimmer wishes to cross a 500-m river flowing at 5 km h^{-1}. His speed with respect to water is 3 km h^{-1}. The shortest possible time to cross the river is

(a) 10 min (b) 20 min (c) 6 min (d) 7.5 min

52. A train of 150 m length is going toward north direction at a speed of 10 m s^{-1}. A parrot flies at a speed of 5 m s^{-1} toward south direction parallel to the railway track. The time taken by the parrot to cross the train is equal to

(a) 12 s (b) 8 s (c) 15 s (d) 10 s

53. A man can swim in still water with a speed of 2 m s^{-1}. If he wants to cross a river of water current speed $\sqrt{3}$ m s^{-1} along the shortest possible path, then in which direction should he swim?

(a) At an angle 120° to the water current

(b) At an angle 150° to the water current

(c) At an angle 90° to the water current

(d) None of these

54. A river flows with a speed more than the maximum speed with which a person can swim in still water. He intends to cross the river by the shortest possible path (i.e., he wants to reach the point on the opposite bank which directly opposite to the starting point). Which of the following is correct?

(a) He should start normal to the river bank.

(b) He should start in such a way that he moves normal to the bank, relative to the bank.

(c) He should start in a particular (calculated) direction making an obtuse angle with the direction of water current.

(d) The man cannot cross the river in that way.

55. Rain, driven by the wind, falls on a railway compartment with a velocity of 20 m s^{-1}, at an angle of 30° to the vertical. The train moves, along the direction of wind flow, at a speed of 108 km h^{-1}. Determine the apparent velocity of rain for a person sitting in the train.

(a) $20\sqrt{7}$ m s^{-1} (b) $10\sqrt{7}$ m s^{-1}

(c) $15\sqrt{7}$ m s^{-1} (d) $10\sqrt{7}$ km h^{-1}

56. An aeroplane is flying vertically upwards. When it is at a height of 1000 m above the ground and moving at a speed of 367 m/s, a shot is fired at it with a speed of 567 m/s from a point directly below it. What should be the acceleration of aeroplane so that it may escape from being hit?
(a) $>5\ m/s^2$ (b) $>10\ m/s^2$
(c) $<10\ m/s^2$ (d) Not possible

57. A boat travels from south bank to north bank of a river with a maximum speed of 8 km/h. To arrive at a point opposite to the point of start, the boat should start at an angle
(a) $\tan^{-1}(1/2)$ W of N (b) $\tan^{-1}(1/2)$ N of W
(c) 30° W of N (d) 30° N of W

58. A swimmer crosses a flowing stream of width w to and fro in time t_1. The time taken to cover the same distance up and down the stream is t_2. If t_3 is the time the swimmer would take to swim a distance $2w$ in still water, then
(a) $t_1^2 = t_2 t_3$ (b) $t_2^2 = t_1 t_3$
(c) $t_3^2 = t_1 t_2$ (d) $t_3 = t_1 + t_2$

59. A man who can swim at the rate of 2 km/hr (in still river) crosses a river to a point exactly opposite on the other bank by swimming in a direction of 120° to the flow of the water in the river. The velocity of the water current in km/hr is
(a) 1 (b) 2
(c) 1/2 (d) 3/2

60. A man wishes to cross a river in a boat. If he crosses the river in minimum time he takes 10 minutes with a drift of 120 m. If he crosses the river taking shortest route, he takes 12.5 minutes, find velocity of the boat with respect to water.
(a) 20 m/min (b) 12 m/min
(c) 10 m/min (d) 8 m/min

61. Wind is blowing in the north direction at speed of 2 m/s which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him?
(a) 2 m/s south (b) 2 m/s north
(c) 4 m/s west (d) 4 m/s south

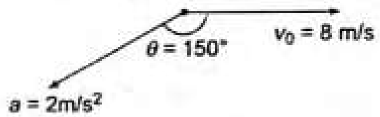
62. A person walks at the rate of 3 km/hr. Rain appears to him in vertical direction at the rate of $3\sqrt{3}$ km/hr. Find the magnitude and direction of true velocity of rain.
(a) 6 km/hr, inclined at an angle of 45° to the vertical towards the person's motion.
(b) 3 km/hr, inclined at an angle of 30° to the vertical towards the person's motion.
(c) 6 km/hr, inclined at an angle of 30° to the vertical towards the person's motion.
(d) 6 km/hr, inclined at an angle of 60° to the vertical towards the person's motion.

63. A swimmer crosses the river along the line making an angle of 45° with the direction of flow. Velocity of the river water is 5 m/s. Swimmer takes 12 seconds to cross the river of width 60 m. The velocity of the swimmer with respect to water will be
(a) 10 m/s (b) 5 m/s
(c) $5\sqrt{5}$ m/s (d) $5\sqrt{2}$ m/s

64. A jet airplane travelling from east to west at a speed of 500 km h^{-1} ejected out gases of combustion at a speed of 1500 km h^{-1} with respect to the jet plane. What is the velocity of the gases with respect to an observer on the ground?
(a) 1000 km h^{-1} in the direction west to east
(b) 1000 km h^{-1} in the direction east to west
(c) 2000 km h^{-1} in the direction west to east
(d) 2000 km h^{-1} in the direction east to west

Kinematics of Circular Motion

65. The figure shows the velocity and acceleration of a point like body at the initial moment of its motion. The acceleration vector of the body remains constant. The minimum radius of curvature of trajectory of the body is



(a) 2 m (b) 3 m
(c) 8 m (d) 16 m

66. A body is moving in a circle with a speed of 1 m s^{-1}. This speed increases at a constant rate of 2 m s^{-1} every second. Assume that the radius of the circle described is 25 m. The total acceleration of the body after 2 s is
(a) 2 ms^{-2} (b) 25 ms^{-2}
(c) $\sqrt{5}$ ms^{-2} (d) $\sqrt{7}$ ms^{-2}

67. A body is moving in a circular path with a constant speed. It has
(a) A constant velocity
(b) A constant acceleration
(c) An acceleration of constant magnitude
(d) An acceleration which varies with time in magnitude

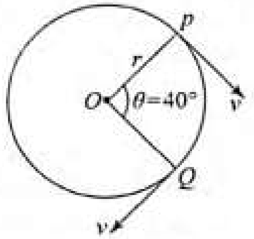
68. A particle is moving along a circular path with uniform speed. Through what angle does its angular velocity change when it completes half of the circular path?
(a) 0° (b) 45° (c) 180° (d) 360°

69. A particle is moving along a circular path. The angular velocity, linear velocity, angular acceleration, and centripetal acceleration of the particle at any instant, respectively, are $\vec{\omega}, \vec{v}, \vec{\alpha}$, and $\vec{a}_c$. Which of the following relations is not correct?
(a) $\vec{\omega} \perp \vec{v}$ (b) $\vec{\omega} \perp \vec{\alpha}$ (c) $\vec{\omega} \perp \vec{a}_c$ (d) $\vec{v} \perp \vec{a}_c$

70. The angular velocity of a particle moving in a circle of radius 50 cm is increased in 5 min from 100 revolutions per minute to 400 revolutions per minute. Find the tangential acceleration of the particle.

(a) 60 m s^{-2} (b) $\pi/30$ m s^{-2}
(c) $\pi/15$ m s^{-2} (d) $\pi/60$ m s^{-2}

71. A particle is moving on a circular path of radius r with uniform velocity v. The change in velocity when the particle moves from P to Q is ($\angle POQ = 40°$)



(a) $2v\cos 40°$ (b) $2v\sin 40°$
(c) $2v\sin 20°$ (d) $2v\cos 20°$

72. A particle P is moving in a circle of radius 'a' with a uniform speed v. C is the centre of the circle and AB is a diameter. When passing through B the angular velocity of P about A and C are in the ratio

(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1

73. When a ceiling fan is switched off its angular velocity reduces to 50% while it makes 36 rotations. How many more rotation will it make before coming to rest (Assume uniform angular retardation)?

(a) 18 (b) 12
(c) 36 (d) 48

74. A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'a' meter/sec^2, then the resultant acceleration will be

(a) $\sqrt{\left\{\frac{v^2}{r^2}-a^2\right\}}$ (b) $\sqrt{\left\{\frac{v^4}{r^2}+a^2\right\}}$

(c) $\sqrt{\left\{\frac{v^4}{r^2}-a^2\right\}}$ (d) $\sqrt{\left\{\frac{v^2}{r^2}+a^2\right\}}$

75. Position vector of a particle moving in xy plane at time t is $\vec{r} = a(1-\cos\omega t)\hat{i} + a\sin\omega t\,\hat{j}$. The path of the particle is

(a) a circle of radius a and center at $(a, 0)$
(b) a circle of radius a and center at $(0, 0)$
(c) an ellipse
(d) neither a circle nor an ellipse

76. A particle moves in xy plane. The position vector at any time t is $\vec{r} = \{(2t)\hat{i} + (2t^2)\hat{j}\}$ m. The rate of change of θ at time t = 2 second (where θ is the angle which its velocity vector makes with positive x-axis) is

(a) $\frac{2}{17}$ rad/s (b) $\frac{1}{14}$ rad/s
(c) $\frac{4}{7}$ rad/s (d) $\frac{6}{5}$ rad/s

77. A train 1 moves from east to west and another train 2 moves from west to east on the equator with equal speeds relative to ground. The ratio of their centripetal accelerations $\frac{a_1}{a_2}$ relative to centre of earth is:

(a) > 1 (b) = 1
(c) < 1 (d) ≥1

78. In uniform circular motion which of the following remains constant?

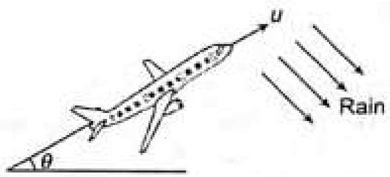
(a) $\vec{r}$ (b) $\vec{a}$
(c) $\vec{v}$ (d) $|\vec{a}|$

79. The diagram shows a CD rotating clockwise (as seen from above) in the CD-player. After turning it off, the CD slows down. Assuming it has not come to a stop yet, the direction of the acceleration of point P at this instance is

(a) ↘ (b) ↓
(c) ↗ (d) →

Problems Based on Mixed Concepts

80. Rain is falling with speed $12\sqrt{2}$ m/s at an angle of 45° with vertical line. A man in a glider going at a speed of u at angle of 37° with respect to ground. Find the speed of glider so that rain appears to him falling vertically. Consider motion of glider and rain drops in same vertical plane.



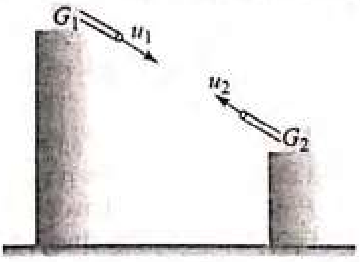
(a) 15 m/s (b) 30 m/s
(c) 10 m/s (d) 25 m/s

81. A stone is projected from a horizontal plane. It attains maximum height 'H' and strikes a stationary smooth wall and falls on the ground vertically below the maximum height. Assuming the collision to be elastic the height of the point on the wall where ball will strike is

(a) $\frac{H}{2}$ (b) $\frac{H}{4}$

(c) $\frac{3H}{4}$ (d) None of these

82. Two guns are mounted (fixed) on two vertical cliffs that are very high from the ground as shown in figure. The muzzle velocity of the shell from G_1 is u_1 and that from G_2 is u_2. The guns aim exactly towards each other The ratio $u_1 : u_2$ such that the shells collide with each other in air is (Assume that there is no resistance of air)



(a) 1 : 2
(b) 1 : 4
(c) will not collide for any ratio
(d) will collide for any ratio

83. Two particles are projected from the same point with the same speed at different angles θ_1 and θ_2 to the horizontal. They have the same range. Their times of flight are t_1 and t_2 respectively.

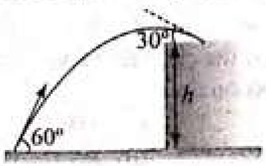
(a) $\frac{t_1}{t_2} = \tan^2\theta_1$ (b) $\frac{t_1}{\sin\theta_1} = \frac{t_2}{\cos\theta_2}$

(c) $\frac{t_1}{t_2} = \tan\theta_1$ (d) $\frac{t_1}{t_2} = \tan^2\theta_2$

84. Three stones A, B and C are simultaneously projected from same point with same speed. A is thrown upwards, B is thrown horizontally and C is thrown downwards from a building. When the distance between stone A and C becomes 10 m, then distance between A and B will be

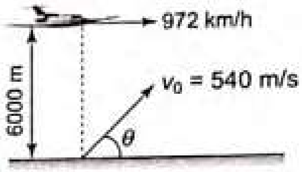
(a) 10 m (b) 5 m
(c) $5\sqrt{2}$ m (d) $10\sqrt{2}$ m

85. A stone projected at an angle of 60° from the ground level strikes at an angle of 30° on the roof of a building of height 'h'. Then the speed of projection of the stone is:



(a) $\sqrt{2gh}$ (b) $\sqrt{6gh}$
(c) $\sqrt{3gh}$ (d) $\sqrt{gh}$

86. An aircraft moving with a speed of 1000 km/h is at a height of 6000 m, just overhead of an anti-aircraft gun. If the muzzle velocity of the gun is 540 m/s, the firing angle θ for the bullet to hit the aircraft should be



(a) 73° (b) 30°
(c) 60° (d) 45°

87. An aeroplane is to go along straight line from A to B, and back again. The relative speed with respect to wind is V. The wind blows perpendicular to line AB with speed v. The distance between A and B is l. The total time for the round trip is:

(a) $\frac{2\ell}{\sqrt{V^2 - v^2}}$ (b) $\frac{2v\ell}{V^2 - v^2}$

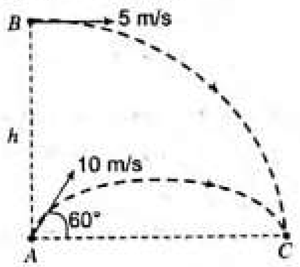
(c) $\frac{2V\ell}{V^2 - v^2}$ (d) $\frac{2\ell}{\sqrt{V^2 + v^2}}$

88. A cat runs along a straight line with constant velocity of magnitude v. A dog chases the cat such that the velocity of dog is always directed towards the cat. The speed of dog is 'u' and always constant. At the instant both are separated by distance x and their velocities are mutually perpendicular, the magnitude of acceleration of dog is.

(a) $\frac{uv}{x}$ (b) $\frac{u^2}{x}$

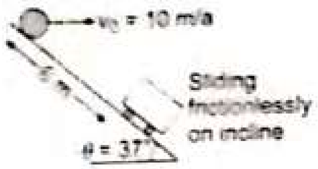
(c) $\frac{v^2}{x}$ (d) $\frac{u^2+v^2}{x}$

89. A particle A is projected from the ground with an initial velocity of 10 m/s at an angle of 60° with horizontal. From what height h should an another particle B be projected horizontally with velocity 5 m/s so that both the particles collide in ground at point C if both are projected simultaneously (g = 10 m/s^2)



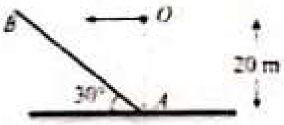
(a) 10 m (b) 30 m
(c) 15 m (d) 25 m

90. A ball is projected horizontally from an incline so as to strike a cart sliding on the incline. Neglect height of cart and point of projection of ball above incline. At the instance the ball is thrown, the speed of cart is v (in m/s). Find v so that the ball strikes the cart.



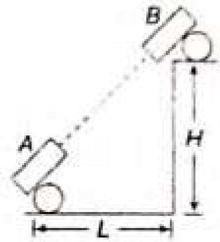
(a) 4 (b) 6
(c) 2 (d) 8

91. AB is an inclined plane of inclination 30° with horizontal. Point O is 20 m above point A. A particle is projected horizontally from O leftwards, and it collides with the plane AB perpendicularly. Speed of the particle at the time of projection should be ($g = 10$ m/s^2)



(a) 13 m/s (b) $8\sqrt{3}$ m/s
(c) $4\sqrt{5}$ m/s (d) $2\sqrt{3}$ m/s

92. Canon A is located on a plain a distance L from a wall of height H. On top of this wall is an identical cannon (cannon B). Ignore air resistance throughout this problem. Also ignore the size of the cannons relative to L and H. The two groups of gunners aim the cannons directly at each other. They fire at each other simultaneously, with equal muzzle speed v_0. What is the value of v_0 for which the two cannon balls collide just as they hit the ground?



(a) $\frac{1}{2}\sqrt{\frac{g(L^2+H^2)}{H}}$ (b) $\sqrt{\frac{g(L^2+H^2)}{H}}$

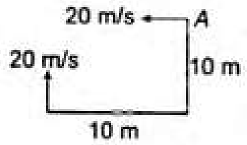
(c) $\sqrt{\frac{2g(L^2+H^2)}{H}}$ (d) $\sqrt{\frac{g(L^2+H^2)}{2H}}$

93. A particle is projected vertically upwards from O with velocity v and a second particle is projected at the same instant from P (at a height h above O) with velocity v at an angle of projection θ. The time when the distance between them is minimum is

(a) $\frac{h}{2v\sin\theta}$ (b) $\frac{h}{2v\cos\theta}$

(c) h/v (d) $h/2v$

94. Two balls are thrown from A and B simultaneously. Find the minimum distance between them during the motion.



(a) 1 m (b) zero
(c) 4 m (d) 2 m

ARCHIVES

1. A ball is projected with kinetic energy E at an angle of 45° to the horizontal. At the highest point during its flight, its kinetic energy will be

(a) Zero (b) $\frac{E}{2}$
(c) $\frac{E}{\sqrt{2}}$ (d) E **(AIEEE 2002)**

2. In a projectile motion, velocity at maximum height is

(a) $\frac{u\cos\theta}{2}$ (b) $u\cos\theta$
(c) $\frac{u\sin\theta}{2}$ (d) None of these

(AIEEE 2002)

3. The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by

(a) $\sqrt{\alpha^2+\beta^2}$ (b) $3t\sqrt{\alpha^2+\beta^2}$
(c) $3t^2\sqrt{\alpha^2+\beta^2}$ (d) $t^2\sqrt{\alpha^2+\beta^2}$ **(AIEEE 2003)**

4. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground? $g = 10$ m/s^2

(a) 8.66 m (b) 5.20 m
(c) 4.33 m (d) 2.60 m **(AIEEE 2003)**

5. A projectile can have the same range R for two angles of projection. If T_1 and T_2 are the times of flight in the two cases, then the product of the two times of flight is directly proportional to

(a) R (b) $1/R$
(c) $1/R^2$ (d) R^2 **(AIEEE 2004)**

6. Which of the following statements is false for a particle moving in a circle with a constant angular speed?

(a) The acceleration vector points to the centre of the circle.
(b) The acceleration vector is tangent to the circle.

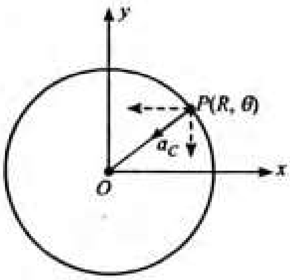
(c) The velocity vector is tangent to the circle.
(d) The velocity and acceleration vectors are perpendicular to each other. **(AIEEE 2004)**

7. A ball is thrown from a point with a speed v_0 at an elevation of angle θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ?
(a) No (b) Yes, 30°
(c) Yes, 60° (d) Yes, 45° **(AIEEE 2004)**

8. For a given velocity, a projectile has the same range R for two angles of projection. If t_1 and t_2 are the times of flight in the two cases then
(a) $t_1t_2 \propto R^2$ (b) $t_1t_2 \propto R$
(c) $t_1t_2 \propto \frac{1}{R}$ (d) $t_1t_2 \propto \frac{1}{R^2}$ **(AIEEE 2004)**

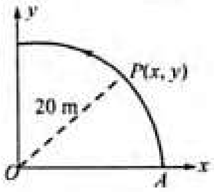
9. A particle is moving eastwards with a velocity of 5 m/s. In 10 s, the velocity changes to 5 m/s northwards. The average acceleration in this time is
(a) $1/\sqrt{2}$ m/s² towards north-west
(b) zero
(c) 1/2 m/s² towards north
(d) $1/\sqrt{2}$ m/s² towards north-east **(AIEEE 2005)**

10. For a particle in uniform circular motion the acceleration $\vec{a}$ at a point $P(R, \theta)$ on the circle of radius R is (here θ is measured from the x-axis).



(a) $-\frac{v^2}{R}\cos\theta\,\hat{i}+\frac{v^2}{R}\sin\theta\,\hat{j}$
(b) $-\frac{v^2}{R}\sin\theta\,\hat{i}+\frac{v^2}{R}\cos\theta\,\hat{j}$
(c) $-\frac{v^2}{R}\cos\theta\,\hat{i}-\frac{v^2}{R}\sin\theta\,\hat{j}$
(d) $\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$ **(AIEEE 2010)**

11. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 2$ s is nearly



(a) 13 m/s² (b) 12 m/s²
(c) 7.2 m/s² (d) 14 m/s² **(AIEEE 2010)**

12. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is
(a) $\pi\frac{v^2}{g}$ (b) $\pi\frac{v^4}{g^2}$
(c) $\frac{\pi}{2}\frac{v^4}{g^2}$ (d) $\pi\frac{v^2}{g^2}$ **(AIEEE 2011)**

13. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is
(a) $m_1 r_1 : m_2 r_2$ (b) $m_1 : m_2$
(c) $r_1 : r_2$ (d) 1 : 1 **(AIEEE 2012)**

14. A projectile is given an initial velocity of $(\hat{i}+2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along thevertical. If $g = 10$ m/s², the equation of its trajectory is
(a) $y = 2x - 5x^2$ (b) $y = x - 5x^2$
(c) $4y = 2x - 5x^2$ (d) $y = 2x + 5x^2$
(JEE Main 2013)

15. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is:
(a) $2gH = nu^2(n-2)$ (b) $gH = (n-2)u^2$
(c) $2gH = n^2u^2$ (d) $gH = (n-2)2u^2$
(JEE Main 2014)

16. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is
(a) $2gH = nu^2(n-2)$ (b) $gH = (n-2)u^2$
(c) $2gH = n^2u^2$ (d) $gH = (n-2)2u^2$
(JEE Main 2014)

ANSWER KEY

Exercises

1. (a)	2. (d)	3. (a)	4. (b)	5. (d)	6. (a)	7. (d)	8. (b)	9. (c)	10. (b)
11. (d)	12. (b)	13. (b)	14. (c)	15. (c)	16. (a)	17. (d)	18. (c)	19. (a)	20. (b)
21. (b)	22. (c)	23. (c)	24. (a)	25. (c)	26. (b)	27. (d)	28. (a)	29. (c)	30. (c)
31. (c)	32. (b)	33. (c)	34. (b)	35. (a)	36. (c)	37. (b)	38. (a)	39. (c)	40. (c)
41. (b)	42. (d)	43. (c)	44. (a)	45. (a)	46. (d)	47. (c)	48. (b)	49. (c)	50. (a)
51. (a)	52. (d)	53. (b)	54. (d)	55. (b)	56. (b)	57. (c)	58. (a)	59. (a)	60. (a)
61. (b)	62. (c)	63. (b)	64. (b)	65. (c)	66. (c)	67. (c)	68. (a)	69. (b)	70. (d)
71. (c)	72. (b)	73. (b)	74. (b)	75. (a)	76. (a)	77. (c)	78. (d)	79. (a)	80. (a)
81. (c)	82. (d)	83. (c)	84. (c)	85. (c)	86. (c)	87. (a)	88. (a)	89. (c)	90. (a)
91. (c)	92. (a)	93. (d)	94. (b)						

Archives

1. (b)	2. (b)	3. (c)	4. (a)	5. (a)	6. (b)	7. (c)	8 (b)	9. (a)	10. (c)
11. (d)	12. (b)	13. (c)	14. (a)	15. (a)	16. (a)				

Chapter 5

Newton's Laws of Motion (Without Friction)

NEWTON'S LAWS OF MOTION

The entire classical mechanics is based upon Newton's laws of motion. These, in fact, are simply known as laws of motion. These laws provide the basis for understanding the effect that forces have on an object.

Newton's First Law of Motion

According to this law, a body continues to be in its state of rest or uniform motion along a straight line unless it is acted upon by some external force to change the state.

According to Newton's first law of motion, a body continues to be in the state of rest or of uniform motion along a straight line unless it is acted upon by an external force to change its state. This means a body, on its own, cannot change its state of rest or state of uniform motion along a straight line. This inability of a body to change by itself its state of rest or state of uniform motion along a straight line is called inertia of the body. Hence, Newton's first law defines inertia and is rightly called the *law of inertia*.

Inertia of rest It is the inability of a body to change by itself, its state of rest. This means a body at rest remains at rest and cannot start moving by its own.

Inertia of motion It is the inability of a body to change itself its state of uniform motion, i.e., a body in uniform motion can neither accelerate nor retard by its own.

Linear momentum, p Linear momentum is a vector quantity. It is the quantity of motion in a body. It is given by the product of mass and velocity. Thus,

$$p = mv$$

In vector form, $\vec{p} = m\vec{v}$. The direction of $\vec{p}$ is same as that of $\vec{v}$.

Newton's Second Law of Motion

According to this law, the rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the force applied.

When an unbalanced force is applied on a body, the momentum of the body changes. The rate of change of momentum with respect to time is defined as the net external force acting on the body, i.e., $\vec{F}_{ext} = d\vec{p}/dt$, where linear momentum $\vec{p} = m\vec{v}$, m is the mass of the body, and $\vec{v}$ is the instantaneous velocity.

$$\vec{F}_{ext} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

If m is constant, i.e., $\dfrac{dm}{dt} = 0 \Rightarrow \vec{F}_{ext} = m\vec{a}$

If the mass of the body is constant, the acceleration of the body is inversely proportional to its mass and directly proportional to the resultant force acting on it, i.e., $\Sigma\vec{F} = m\vec{a}$. This vector equation is equivalent to three algebraic equations:

$$\Sigma F_x = ma_x, \Sigma F_y = ma_y, \text{ and } \Sigma F_z = ma_z$$

Important Results

- $\vec{F} = 0$, second law gives $\vec{a} = 0$; therefore, it is consistent with the first law.
- The second law of motion is a vector law. It means whatever be the direction of the instantaneous velocity of a particle, if any net external force is acting on it, this force will change only that component of the velocity which is in the direction of force.
- Strictly speaking, this law applies to particles, i.e., point masses only. However, with the introduction of the concept of the center of mass, this law can now be applied in case of extended bodies or a system of point masses also. You will study this in the chapter on systems of particles and rotational motion.
- The second law is a local law. This means that it applies to a particle at a particular instant without taking into consideration any history of the particle or its motion.
- As $\vec{F} = \dfrac{d\vec{p}}{dt}$, where $\vec{p}$ denotes momentum, the slope of momentum versus time graph gives force. In the figure, $\tan\alpha$ gives the force at $t = t_1$ and $\tan\beta$ gives the force at $t = t_2$. The force $\vec{F}$ in the law stands for the net external force. Any internal forces in the system are not included in $\vec{F}$.

Newton's Third Law of Motion

According to this law, to every action, there is always an equal and opposite reaction, i.e., the forces of action and reaction are always equal and opposite.

Whenever a body exerts a force on another body, the second body also exerts a force on the first. This force is equal in magnitude, is in the opposite direction, and has the same line of action, acting simultaneously.

Any of the two forces making action–reaction pair can be called action, and the other reaction.

Forces always occur in pairs. If body A exerts a force F on body B, then B will exert equal and opposite force on body A. We can write it as $\vec{F}_{BA} = -\vec{F}_{AB}$.

To every action, there is always an opposed equal reaction. It does not matter which force we call action and which we call reaction. The important thing is that they are co-pairs of a single interaction, and neither force exists without the other.

- When you walk, you interact with the floor. You push against the floor and the floor pushes against you. The pair of forces occurs at the same time (they are simultaneous).
- Likewise, the tires of a car push against the road while the road pushes back on the tires—the tires and road simultaneously push against each other.
- In swimming, you interact with water, pushing water backwards, while water simultaneously pushes you forward—you and water push against each other.

IMPULSE

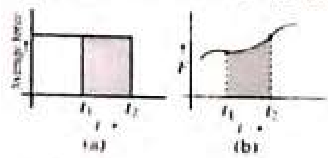
According to Newton's second law, the momentum of a particle changes if a net force acts on the particle. Knowing the change in momentum caused by a force is useful in solving some problems. To build a better understanding of this important concept, let us assume that a net force $\Sigma\vec{F}$ acts on a particle and that this force may vary with time. According to Newton's second law,

$$\Sigma\vec{F} = d\vec{P}/dt \quad \text{or} \quad d\vec{P} = \Sigma\vec{F}\,dt \qquad \text{...(i)}$$

We can integrate this expression to find the change in the momentum of a particle when the force acts over some time interval. If the momentum of the particle changes from $\vec{P}_i$, at time t_i, to $\vec{P}_f$, at time t_f, integrating Eq. (i) gives

$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = \int_{t_i}^{t_f} \sum \vec{F}\,dt \qquad \text{...(ii)}$$

Equation (ii) is an important statement known as the impulse–momentum theorem: if we plot a graph between average force and time [Fig. (a)], the area under the curve will give the impulse imparted during the time interval under consideration.



$$J = \int_{t_1}^{t_2} F\,dt = \int_{t_1}^{t_2} dp = \text{Total change in linear momentum} = \text{Impulse}$$

ILLUSTRATION 5.1 An iron ball of mass $m = 50$ g falls from a height of $h_1 = 5$ m and rises upto $h_2 = 3.2$ m after colliding with the horizontal surface. If the time of contact of the glass half is $\Delta t = 0.02$ s, find the average contact force exerted on the ball by the horizontal surface.

Solution. The change in linear momentum during collision is

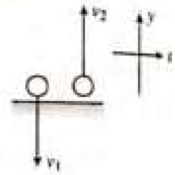
$$\Delta\vec{p} = m\vec{v}_2 - m\vec{v}_1$$
$$= m(\vec{v}_2 - \vec{v}_1)$$
$$= m[v_2\hat{j} - (-v_1\hat{j})] = m[(v_1 + v_2)]\hat{j}$$

The average force during the collision is

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{m(v_1 + v_2)\hat{j}}{\Delta t}$$

We can calculate v_1 and v_2 by using kinematics.

Using $v^2 = u^2 + 2as$



Putting $a = g$ and $s = h$, we get

$$v_1 = \sqrt{2gh_1}\,,\ v_2 = \sqrt{2gh_2}$$

or $$\vec{F} = \frac{m(\sqrt{2gh_1} + \sqrt{2gh_2})\hat{j}}{\Delta t}$$

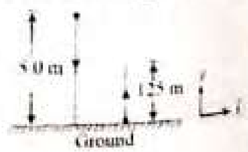
$$= \left(\frac{50}{1000}\right)\frac{(\sqrt{2\times10\times5} + \sqrt{2\times10\times3.2})}{0.02}\hat{j} = 45\hat{j}\text{ N}$$

During collision, the horizontal surface pushes the ball up with an average force of 45 N.

CONCEPT APPLICATION EXERCISE 5.1

1. Figure shows the position–time graph of a particle of mass 0.04 kg. Suggest a suitable physical context for this motion. What is the time between two consecutive impulses received by the particle? What is the magnitude of each impulse?

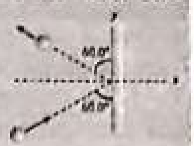
2. A rubber ball of mass 50 g falls from a height of 5 m and rebounds to a height of 1.25 m. Find the impulse and the average force between the ball and the ground if the time for which they are in contact was 0.1 s.



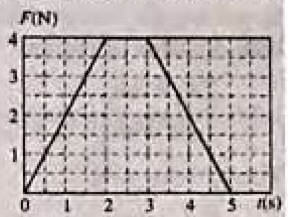
3. Water falls without splashing at a rate of 0.250 L s^{-1} from a height of 1.8 m into a 0.750 kg bucket placed on a scale. If the bucket is originally empty, what does the scale read 3 s after water starts to accumulate in it?

4. A liquid of density ρ is flowing with a speed v through a pipe of cross sectional area A. The pipe is bent in the shape of a right angle as shown. What force should be exerted on the pipe at the corner to keep it fixed?

5. A 3-kg steel ball strikes a wall with a speed of 10.0 ms^{-1} at an angle of 60.0° with the surface of the wall. The ball bounces off with the same speed and same angle (see figure). If the ball was in contact with the wall for 0.2 s, find the average force exerted by the wall on the ball.



6. The magnitude of the net force exerted in the x direction on a 2.50-kg particle varies with time as shown in (see figure). Find (a) the impulse of the force, (b) the final velocity the particle attains if it is originally at rest, (c) its final velocity if its original velocity is $-2.0\ ms^{-1}$, and (d) the average force exerted on the particle for the time interval between 0 and 5 s.



EQUILIBRIUM OF A PARTICLE

The equilibrium of a particle in mechanics refers to the situation when the net external force acting on the particle is zero.

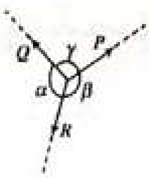
The above condition is correct and complete as far as the equilibrium of a particle (i.e., a point mass) is concerned. In case of rigid bodies (i.e., extended bodies), there are two conditions to be satisfied for such bodies to have equilibrium. First, the net external force acting on the body should be zero. Second, the net external torque acting on the body should be zero.

Concurrent Forces

If two or more forces act on the same particle, we call them concurrent forces.

Lami's Theorem

If three concurrent forces P, Q, and R acting on a particle keep the particle in equilibrium, then Lami's theorem states



$$\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}$$

Here α is the angle opposite to $\vec{P}$, β is the angle opposite to $\vec{Q}$, and γ is the angle opposite to $\vec{R}$.

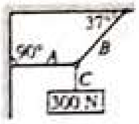
NOTE: If concurrent forces are coplanar but more than three, then it is generally convenient to resolve all of them along two mutually perpendicular directions and then the resultant of each set of these resolved components will be zero.

$$\Sigma F_x=0;\ \Sigma F_y=0$$

If the forces are not coplanar, then they can be resolved along any three mutually perpendicular directions and then the following conditions apply:

$$\Sigma F_x=0;\ \Sigma F_y=0;\ \Sigma F_z=0$$

ILLUSTRATION 5.2 A block of mass 30 kg is suspended by three strings as shown in figure. Find the tension in each string.



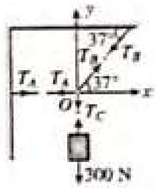
Solution. **Method I:** Considering equilibrium of each part of system The whole system is in equilibrium; therefore, for each part $\Sigma\vec{F}=0$. From the free-body diagram of block C, $T_C=300\ N$.

Now consider the equilibrium of point O,

$$\Sigma F_x=0 \text{ or } T_B\cos 37°-T_A=0$$

$$\therefore\quad T_A=T_B\cos 37°=T_B\cdot\frac{4}{5} \qquad ...(i)$$

and $\quad \Sigma F_y=0 \text{ or } T_B\sin 37°-T_C=0 \qquad ...(ii)$



$$\therefore\quad T_B=\frac{T_C}{\sin 37°}=\frac{300}{3/5}=500\,\text{N}$$

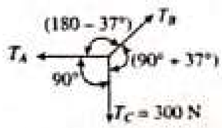
From Eq. (i), we get

$$T_A=\frac{4}{5}T_B=\frac{4}{5}\times 500=400\text{ N}$$

Method II: Using Lami's theorem, we have

$$\frac{T_A}{\sin(90+37°)}=\frac{T_B}{\sin 90°}=\frac{T_C}{\sin(180-37°)}$$

But $\quad TC=300$ N

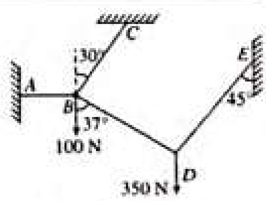


and $\quad T_B=\dfrac{T_C}{\sin 37°}=\dfrac{300}{3/5}=500\,\text{N}$

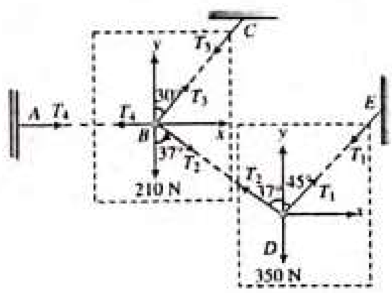
$$T_A=T_C\left(\frac{\sin(90+37°)}{\sin(180-37°)}\right)=300\left(\frac{\cos 37°}{\sin 37°}\right)=400\text{ N}$$

ILLUSTRATION 5.3 Two particles of masses 10 kg and 35 kg are connected with four strings at points B and D as shown in figure.

Determine the tensions in various segments of the string.



Solution. The free-body diagram of the whole system is shown in figure.



Analyzing the equilibrium of point D

$$\Sigma F_x = 0 \text{ or } T_1 \sin 45^\circ - T_2 \sin 37^\circ = 0 \quad \text{...(i)}$$

and $$\Sigma F_y = 0 \text{ or } T_1 \cos 45^\circ + T_2 \cos 37^\circ = 350 \quad \text{...(ii)}$$

From (i), we have $T_2 = \dfrac{T_1 \sin 45^\circ}{\sin 37^\circ}$

Now from (ii), $T_1 \cos 45^\circ + \dfrac{T_1 \sin 45^\circ}{\sin 37^\circ} \times \cos 37^\circ = 350$

or $$\frac{T_1}{\sqrt{2}} + \frac{T_1}{\sqrt{2}} \times \frac{4}{3} = 350$$

or $$\frac{T_1}{\sqrt{2}}\left[1 + \frac{4}{3}\right] = 350 \Rightarrow T_1 = 150\sqrt{2}\text{ N}$$

and $$T_2 = \frac{T_1 \sin 45^\circ}{\sin 37^\circ} = \frac{150\sqrt{2} \times \left(\frac{1}{\sqrt{2}}\right)}{3/5} = 250\text{ N}$$

Analyzing the equilibrium of point B

$$\Sigma F_x = 0 \text{ or } T_2 \sin 37^\circ + T_3 \sin 30^\circ - T_4 = 0 \quad \text{...(i)}$$

and $$\Sigma F_y = 0 \text{ or } T_3 \cos 30^\circ - T_2 \cos 37^\circ - 100 = 0 \quad \text{...(ii)}$$

From (ii), $T_3 \cos 30^\circ - 250 \times \dfrac{4}{5} - 100 = 0$

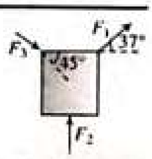
$\Rightarrow \quad T_3 = 200\sqrt{3}$ N

From (i), $T_4 = 150 + 100\sqrt{3} = 50(3 + 2\sqrt{3})$ N

APPLICATIONS OF NEWTON'S LAWS OF MOTION IN DIFFERENT SITUATIONS

We can better understand the application of Newton's laws of motion through studying different situations.

ILLUSTRATION 5.4 Three boys, each of mass 45 kg, pull simultaneously a block on a smooth surface. The mass of block is 20.0 kg.

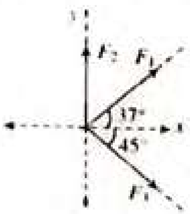


(a) Find the acceleration of the block.

(b) Find the acceleration of the boy exerting force F_1. Assume no friction between the boy and the surface.

[Given $F_1 = 90$ N, $F_2 = 114$ N, and $F_3 = 128\sqrt{2}$ N]

Solution. First we will resolve all the forces acting on the block into x and y components.



$$\Sigma F_x = F_1 \cos 30^\circ + F_3 \cos 45^\circ$$

$$\Sigma F_y = F_1 \sin 30^\circ + F_2 - F_3 \sin 45^\circ$$

Now, $a_x = \dfrac{\Sigma F_x}{m}$ and $a_y = \dfrac{\Sigma F_y}{m}$

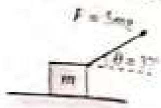
$$a_x = \frac{(90.0)\cos 37^\circ + 128\sqrt{2}\cos 45.0^\circ}{20.0} = 10\text{ ms}^{-2}$$

$$a_y = \frac{(90.0)(\sin 37^\circ) + 114 - (128\sqrt{2}\sin 45.0^\circ)}{20.0} = 2\text{ ms}^{-2}$$

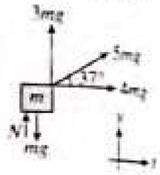
According to Newton's third law, the force exerted on the block by the boy must be equal to the force exerted on the boy by the block. Therefore,

$$a_1 = \frac{F_1'}{m} = \frac{90.0}{45} = 2\text{ ms}^{-2}$$

ILLUSTRATION 5.5 A block of mass m is placed on a horizontal surface. If the block is pulled by applying a force of magnitude $F = 5mg$ at an angle $\theta = 37^\circ$ with horizontal as shown in figure, find the acceleration of the block at the given instant.



Solution. Three forces act on the block (a) $mg \downarrow$ (b) $F = 5mg \nearrow$ (c) $N \uparrow$



Resolving the forces along x- and y-axis, we have equation of motion

$$\Sigma F_x = ma_x$$

$$4\,mg = ma_x$$

This gives $a_x = 4g$ ...(i)

$$\Sigma F_y = ma_y$$

$$3mg - mg + N = ma_y$$

$$2mg + N = ma_y \quad \text{...(ii)}$$

Let $a_y = 0$. Then $N = -2mg$

The negative result signifies that N will be directed down (opposite to the assumed direction), but the ground cannot pull the block down. Hence, the block will lose contact with the ground.

or $N = 0$

Hence, from (ii), $a_y = 2g \uparrow$

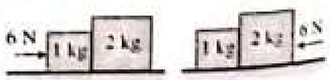
Hence, the net acceleration, $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$

where $a_x = 4g$ from Eq. (i) and $a_y = 2g$ from Eq. (ii).

This gives $a = \sqrt{a_x^2 + a_y^2} = \sqrt{(4g)^2 + (20)^2} = 2\sqrt{5}\,g$

Analysis of Newton's laws of motion in connected bodies: Problems based on normal reaction

ILLUSTRATION 5.6 Two blocks of masses 1 kg and 2 kg are placed in contact on a smooth horizontal surface as shown in Figs. (a) and (b). A horizontal force of 6 N is applied (a) on a 1-kg block (b) a 2-kg block. Find the force of interaction of the blocks.



(a) (b)

Solution. Since both the blocks are in contact, therefore, they will move together with an acceleration

$$a = \frac{F_{net}}{M_{total}} = \frac{6}{2+1} = 2\text{ ms}^{-2}$$

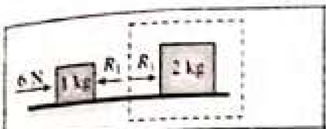
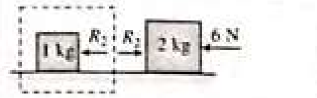
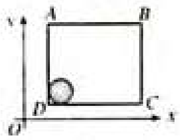
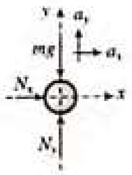
	
Let the force of interaction between them is R_1. By Newton's second law for 2-kg block. Normal reaction on 2-kg block. $R_1 = 2a = 2 \times 2 = 4$ N	Let the force of interaction between them is R_2. By Newton's second law for 1-kg block, normal reaction on 2-kg block. $R_2 = 1a = 1 \times 2 = 2$ N

ILLUSTRATION 5.7 A solid sphere of mass 2 kg is resting inside a cube as shown in figure. The cube is moving with a velocity $\vec{v} = (5t\hat{i} + 2t\hat{j})\,\text{m s}^{-1}$. Here t is time in seconds. All surfaces are smooth. The sphere is at rest with respect to the cube. What is the total force exerted by the sphere on the cube?



Solution. The velocity of the sphere is same as that of the cube, which is given as $\vec{v} = 5t\hat{i} + 2t\hat{j}$.



Hence, acceleration of the sphere: $\vec{a} = \dfrac{d\vec{v}}{dt}$

or $\quad \vec{a} = (5\hat{i} + 2\hat{j})\,\text{m s}^{-2}$

Hence, $a_x = 5\ \text{m s}^{-2}$ and $a_y = 2\ \text{m s}^{-2}$

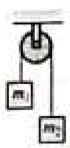
From FBD of sphere,

$$N_x = ma_x = 2 \times 5 = 10\ \text{N}$$

$$N_y - mg = ma_y \Rightarrow N_y = 2 \times 10 + 2 \times 2 = 24\ \text{N}$$

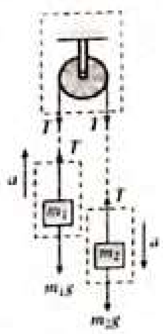
Total force $= \sqrt{N_x^2 + N_y^2} = \sqrt{(10)^2 + (24)^2} = 26\,\text{N}$

ILLUSTRATION 5.8 Two masses m_1 and m_2 are attached to a flexible inextensible massless rope, which passes over a frictionless and massless pulley. Find the accelerations of the masses and tension in the rope.



Solution. Fix an inertial reference frame to the ground to observe the motion of the masses.

Let the tension in the rope be T (in fact, tension is the property of a point of the rope). In this case with ideal pulley (massless and frictionless) and ideal rope (inextensible, massless, and flexible), the tension will remain constant throughout the rope.



Let the acceleration of m_2 be a, vertically downwards, and acceleration of m_1 will also be a, vertically upwards. This is because the rope is inextensible; during motion, the length of the rope must not change, and the rope must not slacken either.

From the above statement, you must not conclude that the accelerations of the masses connected by a rope are always equal. The relationship between the accelerations of the masses depends on the configuration of the pulley–rope system, which can be obtained from the fact that the length of an ideal rope must not change and the rope must not slacken.

Using equation $\sum \vec{F} = m\vec{a}$ for the force diagrams of m_1 and m_2,

$$T - m_1 g = m_1 a \qquad \text{(i)}$$

$$m_2 g - T = m_2 a \qquad \text{(ii)}$$

Adding (i) and (ii), $m_2 g - m_1 g = m_1 a + m_2 a$

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g \qquad \text{(iii)}$$

Substituting this value of a in (i), we get

$$T = m_1 g + m_1 \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g = \frac{2m_1 m_2}{(m_1 + m_2)} g \qquad \text{(iv)}$$

☞ Important Results

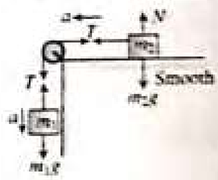
These type of problems in which all connected bodies have same acceleration magnitude can be solved by the following method:

For calculating a use $a = \dfrac{F_{\text{net}}}{M_{\text{total}}} = \dfrac{\text{unbalanced load}}{\text{Total mass}}$

For unbalanced load:

1. If mass moves vertically, take mg.
2. If mass moves horizontally, take zero.
3. If mass moves on inclined plane of inclination, take $mg \sin \theta$.

- Bodies accelerated on a horizontal surface by a falling body:



$$T = m_2 a \qquad \text{(i)}$$

$$m_1 g - T = m_1 a \qquad \text{(ii)}$$

Acceleration, $a = \left(\dfrac{m_1}{m_1 + m_2}\right) g$

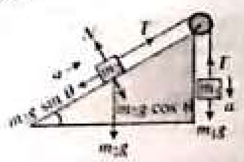
Tension, $T = \left(\dfrac{m_1 m_2}{m_1 + m_2}\right) g$

Solving this problem using shortcut method:

Here unbalanced forced $= m_1 g$.

$\Rightarrow$ Acceleration, $a = \dfrac{F_{\text{net}}}{M_{\text{total}}} = \dfrac{\text{Unbalanced load}}{\text{Total mass}} = \dfrac{m_1 g}{m_1 + m_2}$

- Motion on a smooth inclined plane:



$$m_1 g - T = m_1 a \qquad \text{...(i)}$$

$$T - m_2 g \sin \theta = m_2 a \qquad \text{...(ii)}$$

$$\therefore\ a = \left(\frac{m_1 - m_2 \sin \theta}{m_1 + m_2}\right) g$$

and $T = \dfrac{m_1 m_2 (1 + \sin \theta) g}{(m_1 + m_2)}$

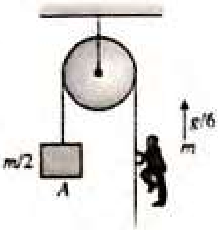
Solving this problem using shortcut method:

Here unbalanced forced = $m_1 g - m_2 g \sin\theta$.

$$\Rightarrow \text{Acceleration, } a = \frac{F_{net}}{M_{total}} = \frac{\text{Unbalanced load}}{\text{Total mass}}$$

$$= \frac{m_1 g - m_2 g \sin\theta}{m_1 + m_2}$$

ILLUSTRATION 5.9 Block A of mass $m/2$ is connected to one end of light rope which passes over a pulley as shown in figure. A man of mass m climbs the other end of rope with a relative acceleration of $g/6$ with respect to rope. Find the acceleration of block A and tension in the rope.



Solution. Let the acceleration of block be (a_0)up. The acceleration of man with respect to ground is a.

$$\vec{a}_{\text{man, rope}} = \vec{a}_{\text{man}} - \vec{a}_{\text{rope}}$$

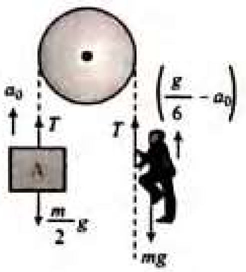
$$\vec{a}_{\text{man}} = \vec{a}_{\text{man, rope}} + \vec{a}_{\text{rope}}$$

$$= \frac{g}{6} + (-a_0) \qquad \Rightarrow \quad a = \frac{g}{6} - a_0$$

From the force diagram of man,

$$T - mg = ma = m\left(\frac{g}{6} - a_0\right) \quad \text{...(i)}$$

From force diagram of block A,



$$T - \frac{m}{2}g = \frac{m}{2}a_0 \qquad \text{...(ii)}$$

Subtracting (i) from (ii),

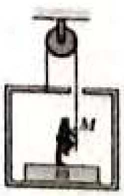
$$\frac{m}{2}g = \frac{m}{2}a_0 - \frac{mg}{6} + ma_0$$

$$\text{or} \quad \frac{3}{2}ma_0 = \frac{mg}{2} + \frac{mg}{6} \Rightarrow a_0 = \frac{4}{9}g$$

From (ii), $T = \frac{m}{2}a_0 + \frac{m}{2}g$

$$= \frac{m}{2} \times \frac{4}{9}g + \frac{m}{2}g \Rightarrow T = \frac{13mg}{18}$$

ILLUSTRATION 5.10 A man of mass M is standing on a plank kept in a box. The plank and box as a whole has mass m. A light string passing over a fixed smooth pulley connects the man and box. If the box remains stationary, find the tension in the string and force exerted by the man on the plank. (Given $M > m$)



Solution. The fixed pulley is taken as frame of reference. The forces on man and box with plank are shown in figure.

The forces are as follows:

(a) Weight of the man = Mg

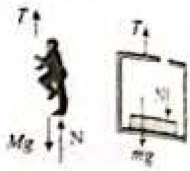
(b) Tension in the string = T

(c) Normal contact force between the man and the plank = N

(d) The weight of the plank and box = mg

The box remains at rest, the man will have to be at rest

Referring to free-body diagram of man, the equation of motion of the man is given as



$$T + N - Mg = 0 \qquad \text{...(i)}$$

Referring to the free-body diagram of lift,

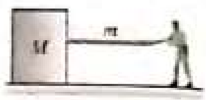
$$T - N - mg = 0 \qquad \text{...(ii)}$$

Solving (i) and (ii), we obtain

$$T = \frac{(M+m)g}{2} \text{ and } N = \frac{(M-m)g}{2}$$

Problems of string with mass

ILLUSTRATION 5.11 A block of mass M is being pulled with the help of a string of mass m and length L. The horizontal force applied by the man on the string is F.



Determine

(a) Find the force exerted by the string on the block and acceleration of system.

(b) Find the tension at the mid point of the string.

(c) Find the tension at a distance x from the end at which force is applied.

Assume that the block is kept on a frictionless horizontal surface and the mass is uniformly distributed in the string.

Solution.

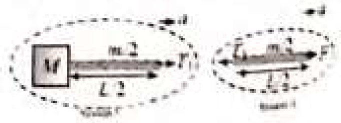
(a) Let the force applied by string to the block be T. For part (a), consider one system is block and other string. Let the acceleration of the system (block + string) be a. Now we apply Newton's law to each of them.

For system II: $F - T = ma$ ...(i)

For system I: $T = Ma$ ...(ii)

After solving (i) and (ii), $a = \frac{F}{M+m}$, $T = \frac{MF}{M+m}$

(b) Now we have to redefine our system. Choose system 1 as block and half string and system 2 as the other half string. On applying Newton's second law to system 1 and system 2, we have



System II: $F - T_1 = \frac{m}{2} \times a$ ...(iii)

Mass per unit length of string = m/L

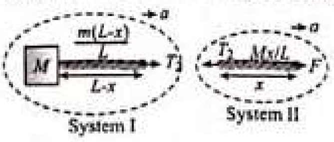
Hence, mass of $L/2$ length of string $= \frac{m}{L} \times \frac{L}{2} = \frac{m}{2}$

System I: $T_1 = \left(M + \frac{m}{2}\right)a$ ...(iv)

After solving, (iii) and (iv), we get

$$a = \frac{F}{M+m},\ T_1 = \frac{(M+m/2)F}{(M+m)}$$

(c) Now we can redefine our system in the block and string of length $L-x$ (system I) and string of length x (system II)



System II: $F - T_2 = \left(\frac{m}{L} \times x\right)a$ ...(v)

System I: $T_2 = \left\{\frac{m}{L}(L-x) + M\right\}a$ ...(vi)

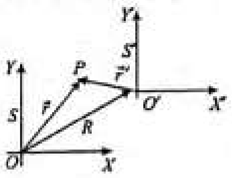
Solving, (v) and (vi), we get

$$a = \frac{F}{M+m} \quad \text{and} \quad T_2 = \frac{\{m(L-x)/L + M\}F}{M+m}$$

Here we see that acceleration in each part is same, but tension changes along the string.

NON-INERTIAL FRAME OF REFERENCE AND PSEUDO (FICTITIOUS) FORCE

A coordinate system whose origin either accelerates or rotates is called a non-inertial frame of reference. The motion of a particle (P) is studied from two frames of references, S and S'. S is an inertial frame of reference, and S' is a non-inertial frame of reference. At any time, the position vectors of the particle with respect to those two frames are $\vec{r}$ and $\vec{r}'$, respectively. At the same moment, the position vector of the origin of S' is $\vec{R}$ with respect to S as shown in figure.



From the vector triangle $OO'P$, we get $\vec{r}' = \vec{r} - \vec{R}$

Differentiating this equation twice with respect to time, we get

$$\frac{d^2\vec{r}'}{dt^2} = \frac{d^2(\vec{r})}{dt^2} - \frac{d^2}{dt^2}(\vec{R}) \Rightarrow \vec{a}' = \vec{a} - \vec{A}$$

Here $\vec{a}'$ is the acceleration of the particle P relative to S', is the acceleration of the particle relative to S, and $\vec{A}$ is the acceleration of S' relative to S.

Multiplying the above equation by m (mass of the particle), we get $m\vec{a}' = m\vec{a} - m\vec{A}$

$$\Rightarrow \quad \vec{F}' = \vec{F}_{(\text{real})} - m\vec{A} \Rightarrow \vec{F}' = \vec{F}_{(\text{real})} + (-m\vec{A})$$

We can observe an additional force $(-m\vec{A})$ apart from real face, that is acting on the particle as seen from an observer observing from reference frame S'.

This force has no existence in reality but has been included only to suit the calculations involved, by Newton's second law, while working out a problem w.r.t. a non-inertial reference frame. This imaginary force is known as "pseudo force" or "fictitious force" or "inertial force." (The term "pseudo" means something which is not real.)

ILLUSTRATION 5.12 A man of mass M stands on a weighing machine in an elevator accelerating upwards with an acceleration a. Draw the free-body diagram of the man as observed by the observer A (stationary on the ground) and observer B (stationary on the elevator). Also, calculate the reading of the weighing machine.

Solution. **Method 1:** Observation from observer. A on ground

From FBD of man w.r.t. A, using Newton's second law,

$$N - Mg = Ma \Rightarrow N = M(g+a)$$

FBD of the man w.r.t. observer A

N is the reading of weighing machine.

Method 2: Observation from observer B on elevator. The observer 'B' is moving in accelerated frame of reference. If he observes the man on weighing machine he will observe the man at rest w.r.t. elevator. If he is asked to draw a free-body diagram of M he will apply pseudo force on M apart from real forces acting on him.

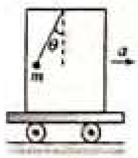
FBD of the man w.r.t. observer 'B'

From free-body diagram:

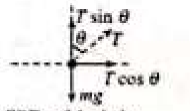
$$N = Mg + ma \Rightarrow N = M(g+a)$$

The value of N calculated from both the methods is same.

ILLUSTRATION 5.13 A bob of mass $m = 50$ g is suspended from the ceiling of a trolley by a light inextensible string. If the trolley accelerates horizontally, the string makes an angle $\theta = 37°$ with the vertical. Find the acceleration of the trolley.



Solution. **Method 1:** Problem solving from ground frame. The observer on the ground will observe the bob moving with acceleration a same as trolley. In FBD, he will observe only two real forces, weight and tension force.



FBD of the bob w.r.t observer standing on ground

The bob is at equilibrium in vertical direction, hence writing equation of motion in vertical direction

$$T\cos\theta = mg \quad ...(i)$$

Along horizontal direction, the bob is accelerating with acceleration a.

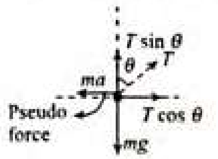
$$T\sin\theta = ma \quad ...(ii)$$

Using (i) and (ii), $\tan\theta = a/g$

$\therefore \quad a = g\tan\theta = 10\tan 37° = 7.5\ \text{m s}^{-2}$

Therefore, acceleration of the trolley is $7.5\ \text{m s}^{-2}$

Method 2: Problem solving from non-inertial frame. If an observer sitting in the trolley and observes the bob, he will observe the bob at rest. If he asks to draw free-body diagram of bob he will apply pseudo force apart from real forces acting on the bob.



FBD of the bob w.r.t. observer in trolley

The bob is at equilibrium in vertical direction, hence writing equation of motion in vertical direction

$$T\cos\theta = mg \qquad \text{...(iii)}$$

Along horizontal direction the bob is at rest.

$$T\sin\theta = ma \qquad \text{...(iv)}$$

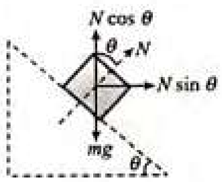
Using (iii) and (iv), $\tan\theta = a/g$

$\therefore \quad a = g\tan\theta = 10\tan 37° = 7.5\ \text{m s}^{-2}$

ILLUSTRATION 5.14 A block of mass m is placed on an inclined plane. With what acceleration A towards right should the system move on a horizontal surface so that m does not slide on the surface of inclined plane? Also calculate the force supplied by wedge on the block. Assume all surfaces are smooth.

Solution. **Method 1:** Analysis of forces on m relative to ground

If the motion of m is analyzed from ground, its acceleration is A and the forces acting on it are its weight mg and normal reaction N.



As m is at rest, moving with same acceleration as wedge in horizontal direction but in vertical direction, the block is at rest.

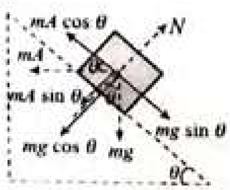
$$\Sigma \vec{F}_y = 0 \Rightarrow N\cos\theta = mg \qquad \text{...(i)}$$

$$\Sigma \vec{F}_x = \Sigma m_i \vec{a}_i \Rightarrow N\sin\theta = mA \qquad \text{...(ii)}$$

On solving (i) and (ii), we get $A = g\tan\theta$ and $N = \dfrac{mg}{\cos\theta}$

Method 2: Analysis of forces on m relative to the inclined plane

If the motion of m is analyzed from the view point of an observer standing on the inclined plane (i.e., relative to the plane), its acceleration is 0 and the forces acting on it are: its weight, the normal reaction, and a pseudo force of magnitude mA towards left.



$$mA\sin\theta = mA\cos\theta$$

$\Rightarrow \quad A = g\tan\theta$

Also: $N = mg\cos\theta + mA\sin\theta$

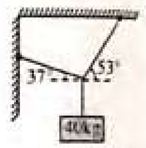
$$= mg\cos\theta + mg\tan\theta\sin\theta = \frac{mg}{\cos\theta}$$

NOTE: If we resolve the forces along the sloping side, we have net force along sloping direction $F = mA\cos\theta - mg\sin\theta$

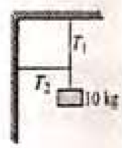
- If $mA\cos\theta > mg\sin\theta$, F is directed up along the slant. Then, the block will accelerate relative to the wedge with acceleration $a = A\cos\theta - g\sin\theta$
- If $mA\cos\theta = mg\sin\theta$, $F = 0$. Hence, the block remains stationary relative to the wedge (or more with constant velocity relative to the wedge).
- If $mA\cos\theta < mg\sin\theta$, F is directed down along the slant. Hence, the block accelerates down the slant with acceleration $a = g\sin\theta - A\cos\theta$

CONCEPT APPLICATION EXERCISE 5.2

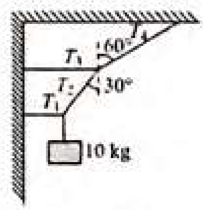
1. The object in the figure weighs 40 kg and hangs at rest. Find the tensions in the three cords that hold it.



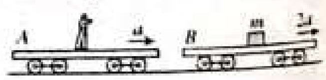
2. A block of mass m = 10 kg is suspended with the help of three strings as shown in the figure. Find the tensions T_1 and T_2.



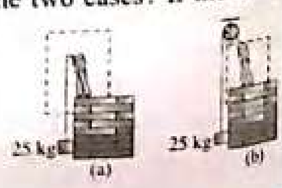
3. Determine tension T_4 in the figure.



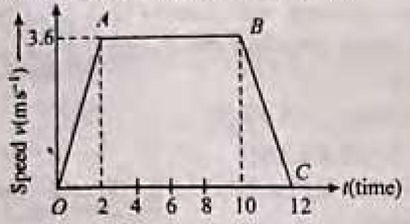
4. Two trolleys A and B are moving with accelerations a and $2a$, respectively, in the same direction. To an observer in trolley A, find the magnitude of the pseudo force acting on a block of mass m on trolley B.



5. A block of mass 25 kg is raised by a 50-kg man in two different ways as shown in the figure. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding?



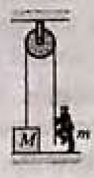
6. A homogeneous rod of length L is acted upon by two forces F_1 and F_2 applied to its ends and directed opposite to each other. With what force F will the rod be stretched at the cross section at a distance l from the end where F_1 is applied?
7. A lift is going up. The total mass of the lift and the passengers is 1500 kg. The variation in the speed of the lift is given by the graph (see figure).



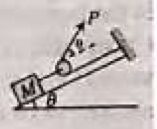
(a) What will be the tension in the rope pulling the lift at time t equal to
i. 1 s ii. 6 s iii. 11 s?
(b) What will be the average velocity and the average acceleration during the course of the entire motion?

8. A body hangs from a spring balance supported from the roof of an elevator.
(a) If the elevator has an upward acceleration of 2.45 ms^{-2} and the balance reads 50 N, what is the true weight of the body?
(b) Under what circumstances will the balance read 30 N?
(c) What will be the balance reading if the elevator cable breaks?

9. In the figure, the block of mass M is at rest on the floor. At what acceleration with which should a boy of mass m climb along the rope of negligible mass so as to lift the block from the floor?

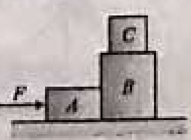


10. What should be the minimum force P to be applied to the string so that block of mass M just begins to move up the frictionless plane?



11. In the figure, the man and the platform together weigh 950 N. The pulley can be treated as frictionless. Determine how hard the man has to pull on the rope to lift himself upward above the ground with constant velocity. If the weight of man is 550 N, what is the normal reaction between them?

12. The masses of blocks A, B, and C are 1 kg, 2 kg, and 0.5 kg, respectively. All surfaces are smooth. If force $F = 50$ N acts as shown in figure at the instant shown, find the force which A exerts on B and the acceleration of C.



CONSTRAINT RELATIONS

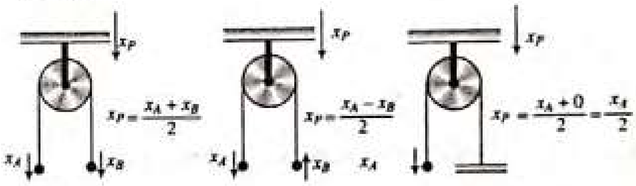
Constraints mean that two bodies (in this case the bodies which are attached to the pulley) are not free to move the way they want. The accelerations between them are dependent on each other. We need to find out the relationship to solve the problems of Newton's laws of motion.

The equations showing the relation of the motions of a system of bodies, in which one motion is constrained by the other motions, are called *constrained relations*.

Pulley Constraint

Pulley constraints are applicable when the bodies concerned are connected through pulleys and the rope connecting them is inextensible.

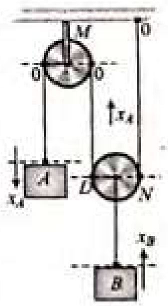
In the cases, where pulley moves along with the blocks connected on both sides, we can say that the displacement of the pulley is the average of the displacement on both sides of the pulley.



If one end of the string is connected with the fixed end, the displacement of that end can be considered zero.

Analysis of case I using shortcut method

- As pulley M is fixed, the displacement should be zero. If the displacement of block A is x_A (down), then the displacement of other end should be x_A (up).



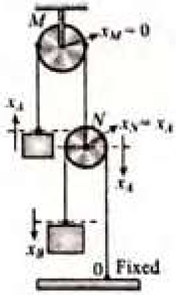
$$x_M = 0 = \frac{x_A + x_D}{2} \Rightarrow x_D = -x_A$$

- Displacement of block B = Displacement of pulley 2

$$x_B = \frac{x_A + 0}{2} \Rightarrow x_A = 2x_B$$

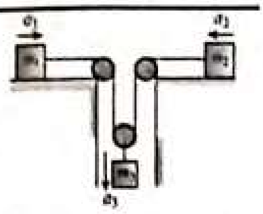
Analysis of case II using shortcut method

- As pulley M is fixed, $|x_{P,2}| = |x_A|$.
- If block A moves up by x_A, pulley N should move x_A in downward direction as shown in figure.
- For pulley 2,



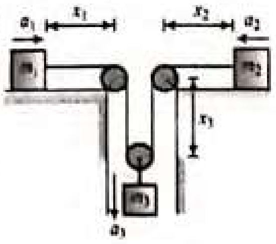
$$x_{P,2} = x_A = \frac{x_B + 0}{2} \Rightarrow x_B = 2x_A$$

ILLUSTRATION 5.15 In the arrangement of three blocks as shown in the figure, the string is inextensible. If the directions of accelerations are as shown in the figure, then determine the constraint relation.



Solution. Let us assume the respective distance of each block as shown in the figure. Since the length of the string is constant, $x_1 + x_2 + 2x_3 =$ constant.

On differentiating twice w.r.t. time, we get



$$\frac{d^2x_1}{dt^2}+\frac{d^2x_2}{dt^2}+2\frac{d^2x_3}{dt^2}=0$$

Since x_1 and x_2 are assumed to be decreasing with time,

$$\frac{d^2x_1}{dt^2}=-a_1 \text{ and } \frac{d^2x_2}{dt^2}=-a_2$$

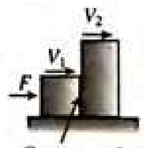
and x_3 is assumed to be increasing with time. Therefore,

$$\frac{d^2x_3}{dt^2}=+a_3$$

Thus, $-a_1-a_2+2a_3=0$ or $a_1+a_2=2a_3$

Wedge Constraint: Normal Constraint

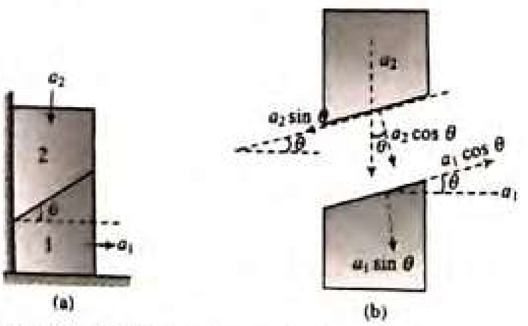
Consider two blocks moving on a surface and always remaining in contact. In order to maintain the contact component of velocity the vector perpendicular to the contact surface must be same, i.e., $\vec{V}_1=\vec{V}_2$.



Contact surface

Similarly, $\vec{a}_1=\vec{a}_2$

In the figure, find the relation between the accelerations of wedges 1 and 2.



(a) (b)

If wedges (1) and (2) are to remain in contact, the component of acceleration perpendicular to the contact surface must be same.

$$a_1 \sin\theta = a_2\cos\theta$$

$$\frac{a_1}{a_2}=\cot\theta$$

In the figure, plane 1 and wedge 2 are free to move. Obtain relation, between their acceleration $a_1 = a_2 \sin\theta$.

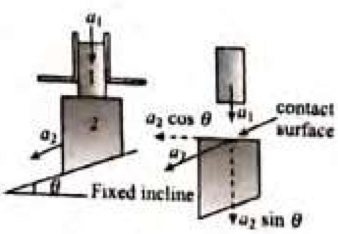
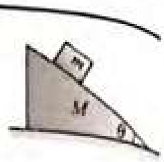
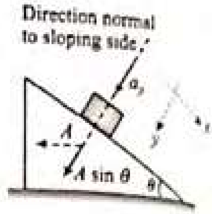


ILLUSTRATION 5.16 A block of mass m is placed on the inclined surface of a wedge as shown in the figure. Calculate the acceleration of the wedge and the block when the block is released. Assume all surfaces are frictionless.



Solution. We consider the motion of the block parallel to incline and perpendicular to inclined surface. Let the components of acceleration of block with respect to ground along these direction are a_x and a_y, respectively.



Then we can write $a_y = A\sin\theta$.

For wedge: $N\sin\theta = MA$ (i)

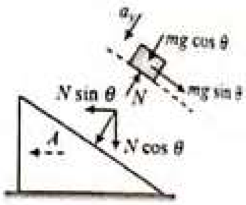
For block: considering the block in the direction perpendicular to sloping surface.

$$mg\cos\theta - N = ma_y$$

But $a_y = A\sin\theta$

Hence,

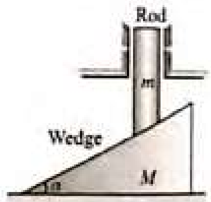
$$mg\cos\theta - N = mA\sin\theta \quad \text{(ii)}$$



From (i) and (ii), we get

$$A=\frac{mg\sin\theta\cos\theta}{M+m\sin^2\theta}$$

ILLUSTRATION 5.17 A rod of mass m is supported on a wedge of mass M shown in the figure. Find the accelerations of rod a and wedge A in the arrangement. The friction between all contact surfaces is negligible.

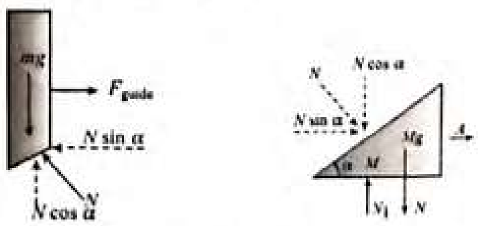


Solution. The rod is constrained to move in the vertical direction (with the help of the guides) and the wedge will move along the surface in the horizontal direction. Initially, the system is held at rest.

Constraint relation: The fact that the rod (or a particle on the wedge) and the wedge must not lose contact is usually called *wedge constraints*. For this, the component of the acceleration of the rod perpendicular to the wedge plane = component of acceleration of the wedge perpendicular to wedge plane.

$$a\cos\alpha = A\sin\alpha \Rightarrow a = A\tan\alpha$$

The forces acting on the rod are:



- The weight mg, vertically downwards
- The normal force N, normal to the bottom surface of the rod
- The force (F_{guide}) exerted by the guide to nullify the horizontal component of N as for the rod $a_{\text{Horizontal}} = 0$.

Motion in vertical direction

$$mg - N\cos\alpha = ma \quad \text{...(ii)}$$

and the forces acting on the wedge are

- The weight, Mg
- N, reaction of N acting on the rod
- N_1, normal force by the surface

The force equations are

$$N_1 - Mg - N\cos\alpha = 0 \quad \text{(iii)}$$

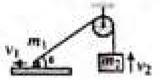
and $N\sin\alpha = MA$ (iv)

Solving the above equations for a and A, we get

$$a = \frac{mg\tan\alpha}{m\tan\alpha + M\cot\alpha} \text{ and } A = \frac{mg}{m\tan\alpha + M\cot\alpha}$$

General Constraints

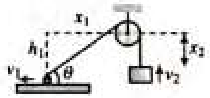
ILLUSTRATION 5.18 In the figure, a ball of mass m_1 and a block of mass m_2 are joined together with an inextensible string. The ball can slide on a smooth horizontal surface. If v_1 and v_2 are the respective speeds of the ball and the block, then determine the constraint relation between the two.



Solution. **Method 1:** Distances are assumed from the center of the pulley as shown in the figure.

Constraint: Length of the string remains constant.

$$\sqrt{x_1^2 + h_1^2} + x_2 = \text{constant}$$



Differentiating both the sides w.r.t. time, we get

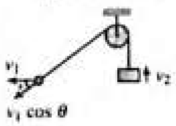
$$\frac{2x_1}{2\sqrt{x_1^2 + h_1^2}}\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

Since the ball moves so as to increase x_1 with time and block moves so as to decrease x_2 with time,

$$\frac{dx_1}{dt} = +v_1 \quad \text{and} \quad \frac{dx_2}{dt} = -v_2$$

Also $\frac{x_1}{\sqrt{x_1^2 + h_1^2}} = \cos\theta$ or $v_2 = v_1\cos\theta$

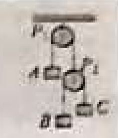
Method 2: The problem can be solved very easily if we look at the problem from a different viewpoint and identify a different constraint, i.e., the velocity of any two points along the string is same. Obviously, from the figure, we have $v_1\cos\theta = v_2$.



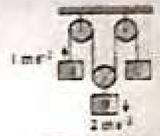
CONCEPT APPLICATION EXERCISE 5.3

1. In the figure, find the acceleration of B, if the acceleration of A is $2\ \text{m s}^{-2}$.

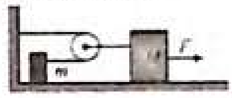
2. The three blocks shown in the figure move with constant velocities. Find the velocity of blocks A and B. Given $v_{P_2} = 10\ \text{ms}^{-1}\downarrow$, $v_C = 2\ \text{ms}^{-1}\uparrow$.



3. For the system as shown in the figure, find the acceleration of C. The acc0elerations of A and B with respect to ground are marked.

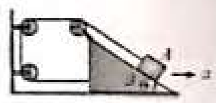


4. Find the acceleration of blocks in the figure. The pulley and the strings are massless.

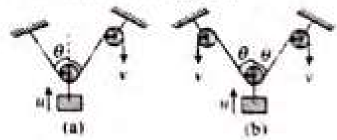


5. A ring A which can slide on a smooth wire is connected to one end of a string as shown in the figure. Other end of the string is connected to a hanging mass B. Find the speed of the ring when the string makes an angle θ with the wire and mass B is going down with a velocity v.

6. Figure shows a block A constrained to slide along the inclined plane of the wedge B shown. Block A is attached with a string which passes through three ideal pulleys and connected to the wedge B. If wedge is pulled toward right with an acceleration a, find
   (a) the acceleration of the block with respect to wedge.
   (b) the acceleration of the block with respect to ground.



7. If the string is inextensible, determine the velocity u of each block in terms of v and θ.



8. Calculate the accelerations of blocks A and B in cases (a), (b), and (c).

SPRING FORCE AND COMBINATIONS OF SPRINGS

In mechanics, we come across the system of bodies connected with springs. For this purpose, we need to develop the ideas how exactly a spring responds to an external force applied on it.

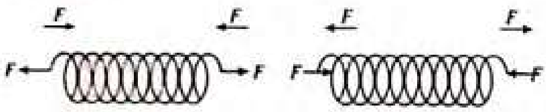
- The force offered by the spring, that is, "spring force" F_s points (acts) opposite to the displacement of the force end of the spring.
- The amount of springs force increase linearly with the deformation (compressi]on or elongation) of the spring; when we plot the variation of F_s versus x, we obtain a straight line up to certain (limited) value of x, which is known as elastic limit.

Robert Hooke experimented in seventh century who put forth all the above facts in a single vector formula in scalar form.

Force ∝ Stretch (or compression), i.e., $F = kx$ (i)

i.e., restoring force is linear. This force in a spring is not constant and depends on stretch (or compression) x. Greater the stretch (or compression), greater will be the force and vice-versa.

k is called the force constant of the spring and is equal to the slope of force versus–stretch curve. It has dimensions $[F/L] = [MT^{-2}]$ and units $\mathrm{N\,m^{-1}}$. Greater the force constant of a spring, lesser will be the stretch (or compression) for a given force and more stiffer is said to be the spring. The force constant k of a spring depends on wire (its length, radius r, and material) used to make the spring, radius of spring R, and length l of spring.



Stretching a spring   Compressing a spring

To produce extension or compression in a spring, two equal and opposite forces are to be applied and in equilibrium restoring force developed due to the elasticity of spring is equal to either force, i.e.,

$F = F'$ and always opposite to applied force.

For small stretch or compression, springs obey Hooke's law.

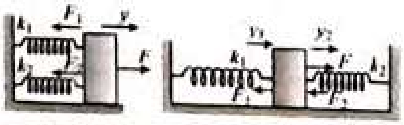
Force Constant of Composite Springs

If a number of springs are connected to a body and we want to produce it to a single spring, following three cases of common interest are possible.

Springs in Parallel

This situation is shown in the figure If the force F pulls the mass m by y, the stretch in each spring will be y.

i.e., $y_1 = y_2 = y$ (i)



Now as for a spring $F = ky$ and as force constants are not equal, so $F_1 \neq F_2$, but for equilibrium,

$$F = F_1 + F_2, \text{ i.e., } k_y = k_1y_1 + k_2y_2 \text{ [as } F = ky]$$

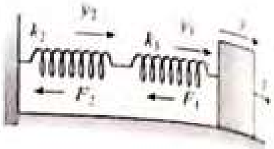
which in the light of (i) reduces to

$$k = k_1 + k_2 + \cdots$$

This is like capacitors in parallel or resistance in series.

Springs in Series

This situation is shown in the figure.



As springs are massless, force in these must be same, i.e.,

$$F = F_2 = F \quad \text{(i)}$$

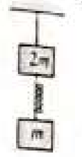
Now as $F = ky$ and force constants are not equal, stretches will not be equal, i.e., $y \neq y_2$.

But, $y = y_1 + y_2$ or $\dfrac{F}{k} = \dfrac{F_1}{k_1} + \dfrac{F_2}{k_2}$

[as for $F = ky$, $y = (F/k)$]

which in the light of Eq. (i) reduces to $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \ldots$

ILLUSTRATION 5.19 Two blocks are connected by a spring. The combination is suspended, at rest, from a string attached to the ceiling, as shown in the figure. The string breaks suddenly.

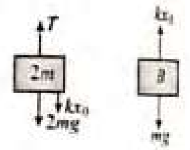


Immediately after the string breaks, what is the initial downward acceleration of the upper block of mass $2m$?

Solution. **Step I:** *Before cutting the string.*

From the force diagram of lower block,

$$kx_0 = mg$$



From the force diagram of upper block,

$$T = 2mg + kx_0$$

Step II: *After cutting the string.*

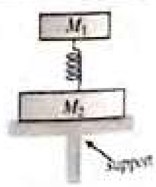
$$2mg + kx_0 = 2ma$$

or $\quad 2mg + mg = 2ma$

or $\quad 3mg = 2ma$

$\therefore \quad a = \dfrac{3}{2}g$

ILLUSTRATION 5.20 The system of two weights with masses M_1 and M_2 are connected with weightless spring as shown in the figure. The system is resting on the support S. Find the acceleration of each of the weights just after the support S is quickly removed.



Solution. The force of spring does not change instantaneously so find spring force at initial instant

Initially, $M_1g = kx$

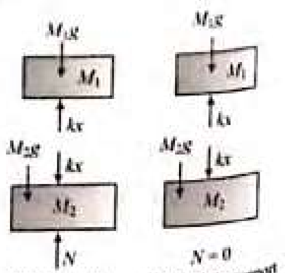
When support is removed, spring force does not change.

For M_1: $M_1g - kx = M_1a_1$

or $\quad a_1 = 0$

For M_2: $M_2g + kx = M_2a_2$

or $\quad a_2 = \dfrac{(M_1 + M_2)g}{M_2}$

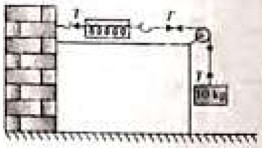


FBD initially   FBD when support is removed

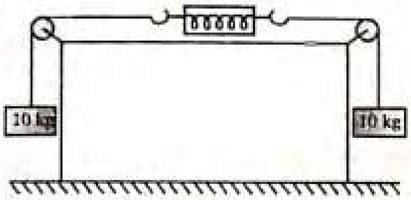
Concept Application Exercise 5.4

1. (a) A 10-kg block is supported by a cord that runs to a spring scale, which is supported by another cord from the ceiling as shown in the figure. What is the reading on the scale?

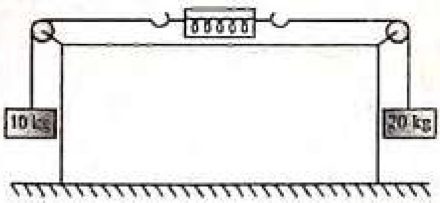
(b) In the figure, the block is supported by a cord that runs around a pulley and to a scale. The opposite end of the scale is attached by a cord to a wall. What is the reading of the scale?



(c) In the figure, the wall has been replaced with a second 10-kg block. What is the reading on the scale now?

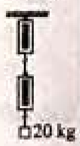


2. What is the reading of the spring balance in the following device?



3. Two blocks A and B of same mass m attached with a light spring are suspended by a string as shown in the figure. Find the acceleration of block A and B just after the string is cut.

4. A block of mass 20 kg is suspended through two light spring balances as shown in the figure. Calculate the:
   (a) reading of spring balance (1).
   (b) reading of spring balance (2).

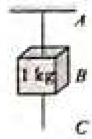


5. Two blocks A and B of same mass m attached with a light string are suspended by a spring as shown in the figure. Find the acceleration of block A and B just after the string is cut..

6. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 m s^{-2} as shown in the figure

Solved Examples

1. A mass of 1 kg is suspended by a string A. Another string C is connected to its lower end (see figure). If a sudden jerk is given to C, then
   (a) the portion AB of the string will break
   (b) the portion BC of the string will break
   (c) none of the strings will break
   (d) the mass will start rotating



Sol. (b) When a sudden jerk is given to C, an impulsive tension exceeding the breaking tension develops in C first, which breaks before this impulse can reach A as a wave through block.

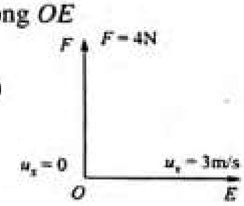
2. In the above question, if the string C is stretched slowly, then
   (a) the portion AB of the string will break
   (b) the portion BC of the string will break
   (c) none of the strings will break
   (d) none of the above

Sol. (a) When the spring C is stretched slowly, the tension in A is greater than that of C. This is due to the weight mg and the former reaches breaking point earlier.

3. A body of mass 2 kg has an initial velocity of 3 meters per second along OE and it is subjected to a force of 4 N in a direction perpendicular to OE. The magnitude of displacement of the body from O after 4 seconds will be
   (a) 12 m (b) 20 m
   (c) 8 m (d) 48 m

Sol. (b) Displacement of body in 4 s along OE

$s_x = v_x t = 3 \times 4 = 12$ m



Force along OF (perpendicular to OE) $= 4$ N

$\therefore \quad a_y = \dfrac{F}{m} = \dfrac{4}{2} = 2$ m/s^2

Displacement of body in 4 s along OF

$\Rightarrow \quad s_y = u_y t + \dfrac{1}{2} a_y t^2 = \dfrac{1}{2} \times 2 \times (4)^2 = 16$ m [As $u_y = 0$]

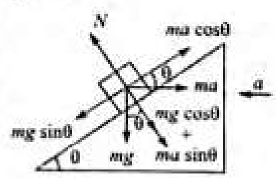
$\therefore \quad$ Net displacement $s = \sqrt{s_x^2 + s_y^2} = \sqrt{(12)^2 + (16)^2} = 20$ m

4. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block (g is acceleration due to gravity) will be
   (a) $mg\cos\theta$ (b) $mg\sin\theta$
   (c) mg (d) $mg/\cos\theta$

Sol. (d) When the whole system is accelerated towards left, then pseudo force (ma) works on a block towards right.

For the condition of equilibrium:

$mg\sin\theta = ma\cos\theta$



$\Rightarrow \quad a = \dfrac{g\sin\theta}{\cos\theta}$

Therefore, force exerted by the wedge on the block

$$N = mg\cos\theta + ma\sin\theta$$

$$= mg\cos\theta + m\left(\frac{g\sin\theta}{\cos\theta}\right)\sin\theta$$

$$= \frac{mg(\cos^2\theta+\sin^2\theta)}{\cos\theta} = \frac{mg}{\cos\theta}$$

5. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?

(a) One (b) Four
(c) Two (d) Three

Sol. (d) u = Velocity of bullet

$\dfrac{dm}{dt}$ = Mass fired per second by the gun

Mass of bullet (m_B) × Bullets fired per sec (N)

Maximum force that man can exert, $F = u\left(\dfrac{dm}{dt}\right)$

$\therefore \quad F = u \times mB \times N$

$$\Rightarrow \quad N = \frac{F}{m_B \times u} = \frac{144}{40\times10^{-3}\times1200} = 3$$

6. N bullets each of mass m kg are fired with a velocity v ms^{-1} at the rate of n bullets per second upon a wall. The reaction offered by the wall to the bullets is given by

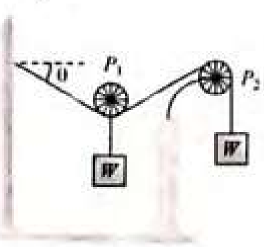
(a) nmv (b) $\dfrac{Nmv}{n}$
(c) $n\dfrac{Nm}{v}$ (d) $n\dfrac{Nv}{m}$

Sol. (a) Total mass of bullets = Nm, time $t = \dfrac{N}{n}$

Momentum of the bullets striking the wall = Nmv

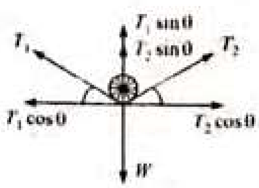
Rate of change of momentum (Force) = $\dfrac{Nmv}{t} = nmv$.

7. In the following diagram, pulley P_1 is movable and pulley P_2 is fixed. The value of angle θ will be



(a) 60°
(b) 30°
(c) 45°
(d) 15°

Sol. (b) From free body diagram of pulley P_1



At the pulley is in equilibrium, considering horizontal direction

For horizontal equilibrium,

$$T_1\cos\theta = T_2\cos\theta$$

$\Rightarrow \quad T_1 = T_2$ and $T_1 = T_2 = W$ ...(i)

Now considering for vertical equilibrium

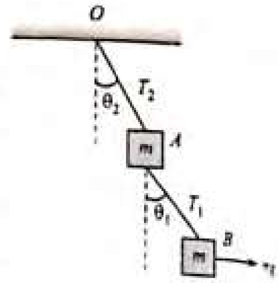
$$T_1\sin\theta + T_2\sin\theta = W \quad ...(ii)$$

From (i) & (ii), we get

$$W\sin\theta + W\sin\theta = W$$

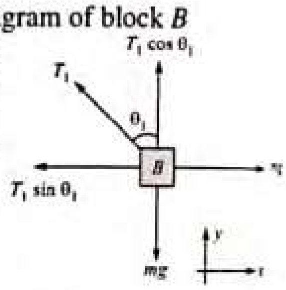
$\therefore \quad \sin\theta = \dfrac{1}{2}$ or $\theta = 30°$

8. In the following figure the masses of the blocks A and B are same and each equal to m. The tensions in the strings OA and AB are T_2 and T_1, respectively. The system is in equilibrium with a constant horizontal force mg on B. Then T_1 is



(a) mg (b) $\sqrt{2}\,mg$
(c) $\sqrt{3}\,mg$ (d) $\sqrt{5}\,mg$

Sol. (b) Considering free body diagram of block B



$\Sigma F_x = 0;\ T_1\cos\theta_1 = mg$ (i)

$\Sigma F_y = 0;\ T_1\cos\theta_1 = -mg$ (ii)

By squaring and adding equations (i) & (ii), we get

$$T_1^2(\sin^2\theta_1 + \cos^2\theta_1) = 2(mg)^2$$

$\therefore \ T_1 = \sqrt{2}mg$

9. In the previous problem, the angle θ_1 is

(a) 30° (b) 45°
(c) 60° (d) $\tan^{-1}\left(\dfrac{1}{2}\right)$

Sol. (b) From the solution of previous problem, if we divide equation(ii) by equation (i), we get

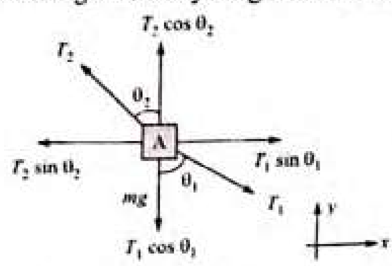
$$\frac{T_1\sin\theta_1}{T_1\cos\theta_1} = \frac{mg}{mg}$$

$\therefore \quad \tan\theta_1 = 1$ or $\theta_1 = 45°$

10. In the above question, tension T_2 will be

(a) mg (b) $\sqrt{2}\,mg$
(c) $\sqrt{3}\,mg$ (d) $\sqrt{5}\,mg$

Sol. (d) Considering free body diagram of block A



$\Sigma F_y = 0$; in vertical direction,

$$T_2\cos\theta_2 = mg + T_1\cos\theta_1$$

$$T_2\cos\theta_2 = mg + \sqrt{2}mg\cos45° \quad ....(i)$$

$$T_2\cos\theta_2 = 2mg$$

In horizontal direction $\Sigma F_x = 0$

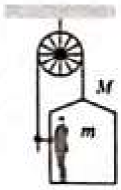
$$T_2 \sin\theta_2 = T_1 \sin\theta_1 = \sqrt{2}mg\sin 45^\circ$$

$$T_2 \sin\theta_2 = mg \quad \text{.....(ii)}$$

by squaring and adding (i) and (ii) equilibrium

$$T_2^2 = 5(mg)^2 \text{ or } T_2 = \sqrt{5}mg$$

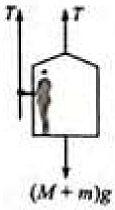
11. A man of mass m stands on a crate of mass M. He pulls on a light rope passing over a smooth light pulley. The other end of the rope is attached to the crate. For the system to be in equilibrium, the force exerted by the men on the rope will be



(a) $(M+m)g$ (b) $\frac{1}{2}(M+m)g$

(c) Mg (d) mg

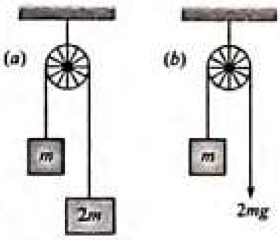
Sol. (b) Considering free body diagram of man and crate system in vertical direction.



$$2T = (M+m)g$$

$$\therefore \quad T = \frac{(M+m)g}{2}$$

12. The two pulley arrangements shown in the figure are identical. The mass of the rope is negligible. In (a) the mass m is lifted up by attaching a mass $2m$ to the other end of the rope. In (b), m is lifted up by pulling the other end of the rope with a constant downward force of $2mg$. The ratio of accelerations in two cases will be

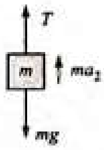


(a) 1 : 1 (b) 1 : 2

(c) 1 : 3 (d) 1 : 4

Sol. (c) For first case,

$$a_1 = \frac{m_2 - m_1}{m_1 + m_2} g = \frac{2m - m}{m + 2m} = \frac{g}{3} \quad \text{.....(i)}$$



For second case,

From free body diagram of m,

$$ma_2 = T - mg$$

$$ma_2 = 2mg - mg \quad [\text{As } T = 2mg]$$

$$a_2 = g \quad \text{........(ii)}$$

From (i) and (ii), $\frac{a_1}{a_2} = \frac{g/3}{g} = 1/3$

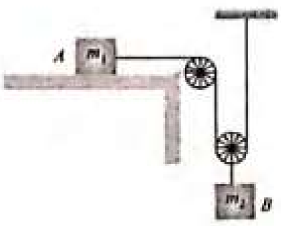
13. The acceleration of block B in the figure will be

(a) $\frac{m_2 g}{(4m_1 + m_2)}$

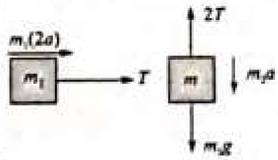
(b) $\frac{2m_2 g}{(4m_1 + m_2)}$

(c) $\frac{2m_1 g}{(m_1 + 4m_2)}$

(d) $\frac{2m_1 g}{(m_1 + m_2)}$



Sol. (a) When block m_2 moves downward with acceleration a, the acceleration of mass m_1 will be $2a$ because it covers double distance in the same time in comparison to m_2.



Let T be the tension in the string.

By drawing the free body diagram of A and B,

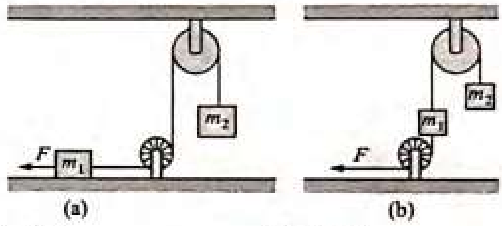
$$T = m_1\, 2a \quad \text{...(i)}$$

$$m_2 g - 2T = m_2 a \quad \text{...(ii)}$$

By solving (i) and (ii),

$$a = \frac{M_2 g}{(4m_1 + m_2)}$$

14. The ratio of tensions in the string connected to the block of mass m_2 in Figure (a) and Figure (b) respectively, is (friction is absent everywhere): [$m_1 = 50$ kg, $m_2 = 80$ kg and $F = 1000$ N].



(a) 7 : 2 (b) 2 : 7

(c) 3 : 4 (d) 4 : 3

Sol. (c) In Figure (a): Let tension in the string is T_1

$$F - T_1 = m_1 a$$

$$\Rightarrow \quad 1000 - T_1 = 50a \quad \text{(i)}$$

$$T_1 - m_2 g = m_2 a$$

$$\Rightarrow \quad T_1 - 80g = 80a \quad \text{(ii)}$$

From (i) and (ii) $T_1 = \frac{12000}{13}$ N

In Figure (b): Let tension in the string is T_2.

$$F + m_1 g - T_2 = m_1 a$$

$$\Rightarrow \quad 1000 + 50g - T_2 = 50a \quad \text{(iii)}$$

$$T_2 - 80g = 80a$$

$$T_2 - 80g = 80a \quad \text{(iv)}$$

From (iii) and (iv),

$$T_2 = \frac{16000}{13}\text{N} \Rightarrow \frac{T_1}{T_2} = \frac{12000}{16000} = \frac{3}{4}$$

15. Five forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$ and $\vec{F}_5$ are acting on a particle of mass 2.0 kg so that it is moving with 4 m/s^2 in east direction. If $\vec{F}_1$ force is removed, then the acceleration becomes 7 m/s^2 in north, then the acceleration of the block if only $\vec{F}_1$ is acting will be

(a) 16 m/s^2 (b) $\sqrt{65}$ m/s^2

(c) $\sqrt{260}$ m/s^2 (d) $\sqrt{33}$ m/s^2

Sol. (b) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 = 2(4\hat{i})$ (i)

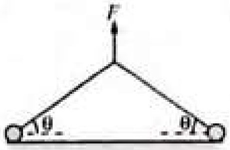
and $\vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 = 2(7\hat{j})$ (ii)

From (i) and (ii), $\vec{F}_1 = 8\hat{i} - 14\hat{j}$

$$\vec{a}_1 = \frac{\vec{F}_1}{m} = 4\hat{i} - 7\hat{j}$$

$\Rightarrow$ $a_1 = \sqrt{16+49} = \sqrt{65}$ m/s^2

16. Two small spheres each of mass m connected by a string of length $2l$ are kept on a smooth horizontal surface. A vertical force F is applied at the middle of the string. What is maximum value of F for which the spheres do not lose contact with the surface?



(a) $2mg$ (b) mg

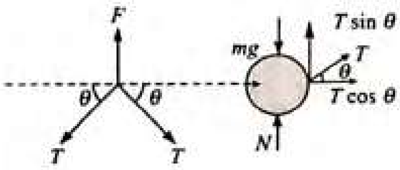
(c) $\frac{3mg}{2}$ (d) $4mg$

Sol. (a) $F = 2T\sin\theta$ or $T\sin\theta = \frac{F}{2}$ ...(i)

$T\sin\theta + N = mg$

$\Rightarrow$ $N = mg - T\sin\theta$ ...(ii)

For no loss of contact:



$N > 0$

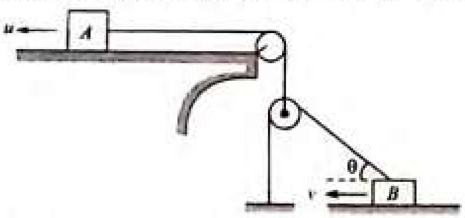
$\Rightarrow$ $mg - T\sin\theta > 0$

$\Rightarrow$ $mg - \frac{F}{2} > 0$

$\Rightarrow$ $F < 2mg$

So $F_{max} = 2mg$

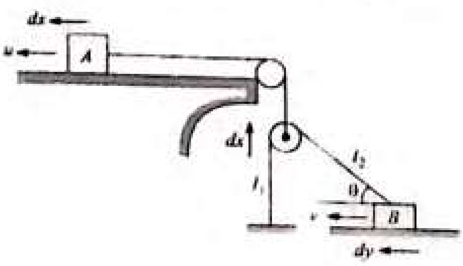
17. In figure, lower pulley is free to move in vertical direction only. Block A is given a uniform velocity u as shown, what is velocity of block B as a function of angle θ.



(a) $u\cos\theta$ (b) $\frac{u}{\cos\theta}$

(c) $\frac{u[1+\sin\theta]}{\cos\theta}$ (d) $\frac{u[1+\cos\theta]}{\sin\theta}$

Sol. (c)



Let in a small time dt, displacement of A is dx and displacement of B is dy. Increase in length $l_1 = dx$.

Increase in length $l_2 = dx\sin\theta - dy\cos\theta$

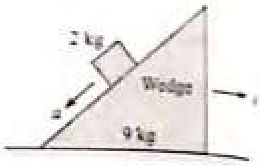
But net increase in length should be zero, so

$dx + dx\sin\theta - dy\cos\theta = 0$

$$\Rightarrow \quad dy = \frac{(1+\sin\theta)dx}{\cos\theta}$$

$$\Rightarrow \quad \frac{dy}{dt} = \left(\frac{1+\sin\theta}{\cos\theta}\right)\frac{dx}{dt} \Rightarrow v = \left(\frac{1+\sin\theta}{\cos\theta}\right)u$$

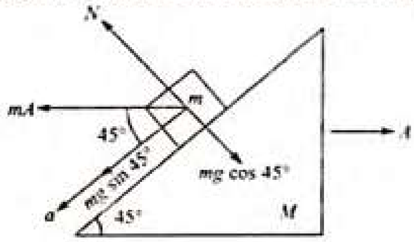
18. A block of mass 2 kg slides down the face of a smooth 45° wedge of mass 9 kg as shown in the figure. The wedge is placed on a frictionless horizontal surface. Determine the acceleration of the wedge.



(a) 2 m/s^2 (b) $\frac{11}{\sqrt{2}}$ m/s^2

(c) 1 m/s^2 (d) none of these

Sol. (c) Suppose a is the acceleration of block w.r.t. wedge.



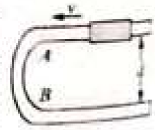
For block:

$N + mA\sin 45° = mg\cos 45°$ (i)

For wedge: $N\sin 45° = mA$ (ii)

From (i) and (ii) $A = 1$ m/s^2

19. A U-shaped wire has a rough semicircular bending between A and B as shown in the figure. A bead of mass m moving with uniform speed v through the wire enters the semicircular bend at A and leaves at B with velocity $v/2$ after time T. The average force, exerted by the bead on the part AB of the wire is



(a) $\frac{mv}{2T}$ (b) $\frac{3mv}{2T}$

(c) $\frac{3mv}{T}$ (d) None of these

Sol. (b) The momentum of the bead at A is $\vec{p}_i = -mv\hat{i}$

The momentum of the bead at B $\vec{p}_f = (mv/2)\hat{i}$

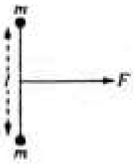
Therefore, the magnitude of the change in momentum between A and B is

$$\Delta p = |\vec{p}_f - \vec{p}_i| = (3/2)mv$$

Average force exerted by the bead on the wire is

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{3mv}{2T}$$

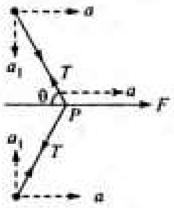
20. Two identical small masses each of mass m are connected by a light inextensible string on a smooth horizontal floor. A constant force F is applied at the mid point of the string as shown in the figure. The acceleration of each mass towards each other is,



(a) $\dfrac{F}{2\sqrt{3}m}$ (b) $\dfrac{\sqrt{3}F}{2m}$

(c) $\dfrac{2}{\sqrt{3}}\dfrac{F}{m}$ (d) None of these

Sol. (a) Let the tension in the string be T at any angular position θ, the acceleration of each ball along x and y axes be a and a_1 respectively. Writing the equation of motion of m, we obtain



$$\Sigma F_x = ma$$

$$\Rightarrow \quad T\cos\theta = ma \qquad \text{...(1)}$$

$$\Sigma F_y = ma_1$$

$$\Rightarrow \quad T\sin\theta = ma_1 \qquad \text{...(2)}$$

At point P, as it is accelerating with an acceleration a, therefore $F - 2T\cos\theta = m_p a$ where m_p = mass of the string at the point $P \cong 0$

$$\Rightarrow \quad F = 2T\,\text{sos}\theta \qquad \text{...(3)}$$

$$(2) \div (1) \Rightarrow \quad \tan\theta = \frac{a_1}{a}$$

$$\Rightarrow \quad a_1 = a\tan\theta$$

where $a = \dfrac{T\,cos\theta}{m}$ from (1).

Putting $T = \dfrac{F}{2\,cos\theta}$, from (3), we obtain

$$a_1 = \frac{F}{2m}\tan\theta$$

Putting $\theta = 30° \Rightarrow a_1 = \dfrac{F}{2\sqrt{3}m}$

EXERCISES

Problems Based on Basic Concept of Projectile Motion

1. Two skaters have weight in the ratio 4 : 5 and are 9 m apart, on a smooth frictionless surface. They pull on a rope stretched between them. The ratio of the distance covered by them when they meet each other will be

(a) 5 : 4 (b) 4 : 5 (c) 25 : 16 (d) 16 : 25

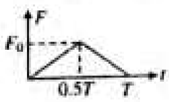
2. Three forces are acting on a particle of mass m initially in equilibrium. If the first two forces (R_1 and R_2) are perpendicular to each other and suddenly the third force (R_3) is removed, then the acceleration of the particle is

(a) $\dfrac{R_3}{m}$ (b) $\dfrac{R_1+R_2}{m}$ (c) $\dfrac{R_1-R_2}{m}$ (d) $\dfrac{R_1}{m}$

3. n balls each of mass m impinge elastically each second on a surface with velocity u. The average force experienced by the surface will be

(a) mnu (b) 2 mnu (c) 4 mnu (d) $mnu/2$

4. A ball of mass m moving with a velocity u rebounds from a wall. The collision is assumed to be elastic and the force of interaction between the ball and wall varies as shown in the figure. Then the value of F_0 is



(a) mu/T (b) $2\,mu/T$ (c) $4\,mu/T$ (d) $mu/2\,T$

5. A rope is stretched between two boats at rest. A sailor in the first boat pulls the rope with a constant force of 100 N. First boat with the sailor has a mass of 250 kg whereas the mass of second boat is double of this mass. If the initial distance between the boats was 120 m, the time taken for two boats to meet each other is (neglect water resistance between boats and water)

(a) 15 s (b) 20 s (c) 5 s (d) 30 s

6. A particle of mass 2 kg moves with an initial velocity of $\vec{v} = 4\hat{i} + 4\hat{j}$ m s^{-1}. A constant force of $\vec{F} = 20\,\hat{j}$ N is applied on the particle. Initially, the particle was at (0, 0). The x-coordinate of the particle when its y-coordinate again becomes zero is given by

(a) 1.2 m (b) 4.8 m (c) 6.0 m (d) 3.2 m

7. A block of metal weighing 2 kg is resting on a frictionless plane. It is struck by a jet releasing water at a rate of 1 kg s^{-1} and at a speed of 5 m s^{-1}. The initial acceleration of the block is

(a) $\dfrac{5}{3}$ m s^{-2} (b) $\dfrac{25}{4}$ m s^{-2}

(c) $\dfrac{26}{6}$ m s^{-2} (d) $\dfrac{5}{2}$ m s^{-2}

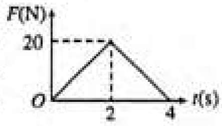
8. A wooden box is placed on a table. The normal force on the box from the table is N_1. Now another identical box is kept on first box and the normal force on lower block due to upper block is N_2 and normal force on lower block by the table is N_3. For this situation, mark out the correct statement(s).

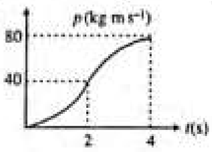
(a) $N_1 = N_2 = N_3$ (b) $N_1 < N_2 = N_3$

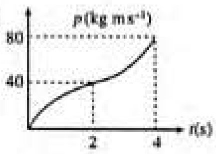
(c) $N_1 = N_2 < N_3$ (d) $N_1 = N_2 > N_3$

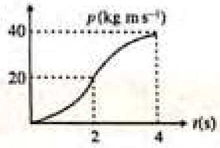
9. A particle of mass 2 kg moves with an initial velocity of $(4\hat{i}+2\hat{j})\ \text{m s}^{-1}$ on the x-y plane. A force $\vec{F}=(2\hat{i}-8\hat{j})$ N acts on the particle. The initial position of the particle is (2 m, 3 m). Then for $y = 3$ m,
   (a) Possible value of x is only $x = 2$ m
   (b) Possible value of x is not only $x = 2$ m, but there exists some other value of x also
   (c) Time taken is 2 s
   (d) All of the above

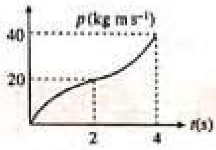
10. Figure shows the variation of force acting on a body with time. Assuming the body to start from rest, the variation of its momentum with time is best represented by which plot?



(a)


(b)


(c)


(d)


11. A particle is moving in the x-y plane. At certain instant of time, the components of its velocity and acceleration are as follows: $v_x = 3\ \text{m s}^{-1}$, $v_y = 4\ \text{m s}^{-1}$, $a_x = 2\ \text{m s}^{-2}$ and $a_y = 1\ \text{m s}^{-2}$. The rate of change of speed at this moment is
   (a) $\sqrt{10}\ \text{m s}^{-2}$ (b) $4\ \text{m s}^{-2}$
   (c) $\sqrt{5}\ \text{m s}^{-2}$ (d) $2\ \text{m s}^{-2}$

12. A machine gun is mounted on a 2000 kg car on a horizontal frictionless surface. At some instant the gun fires bullets of mass 10 gm with a velocity of 500 m/sec with respect to the car. The number of bullets fired per second is ten. The average thrust on the system is
   (a) 550 N (b) 50 N
   (c) 250 N (d) 250 dyne

13. A body of mass 2 kg has an initial velocity of 3 meters per second along OE and it is subjected to a force of 4 N in a direction perpendicular to OE. The distance of the body from O after 4 seconds will be
   (a) 12 m (b) 20 m
   (c) 8 m (d) 48 m

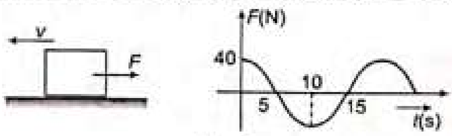
14. A particle moves in the xy-plane under the action of a force F such that the components of its linear momentum p at any time t are $p_x = 2\cos t$, $p_y = 2\sin t$. The angle between F and p at time t is
   (a) 90° (b) 0°
   (c) 180° (d) 30°

15. A body A of mass m_1 exerts a force on another body B of mass m_2. If the acceleration of B be a_2, then the acceleration (in magnitude) of A is
   (a) $\dfrac{m_2}{m_1}a_2$ (b) $\dfrac{m_1}{(m_1+m_2)}$
   (c) $\dfrac{m_1}{m_2}a_2$ (d) $\dfrac{(m_1+m_2)}{m_1}$

16. Five forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$ and $\vec{F}_5$ are acting on a particle of mass 2.0 kg so that it is moving with 4 m/s^2 in east direction. If $\vec{F}_1$ force is removed, then the acceleration becomes 7 m/s^2 in north, then the acceleration of the block if only $\vec{F}_1$ is acting will be
   (a) 16 m/s^2 (b) $\sqrt{65}$ m/s
   (c) $\sqrt{260}$ m/s^2 (d) $\sqrt{33}$ m/s^2

17. Five persons A, B, C, D and E are pulling a cart of mass 100 kg on a smooth surface and cart is moving with acceleration 3 m/s^2 in east direction. When person A stops pulling, it moves with acceleration 1 m/s^2 in the west direction. When person B stops pulling, it moves with acceleration 24 m/s^2 in the north direction. The magnitude of acceleration of the cart when only A and B pull the cart keeping their directions same as the old directions, is
   (a) 26 m/s^2 (b) $3\sqrt{71}$ m/s^2
   (c) 25 m/s^2 (d) 30 m/s^2

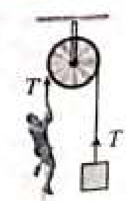
18. A 15 kg block is initially moving along a smooth horizontal surface with a speed of $v = 4$ m/s to the left. It is acted by a force F, which varies in the manner shown. Determine the velocity of the block at $t = 15$ seconds.



Given that, $F = 40\cos\left(\dfrac{\pi}{10}\right)t$
   (a) 12.5 m/s (b) 8.5 m/s
   (c) 20 m/s (d) 9.5 m/s

Application of Newton's Laws of Motion in Base Situations

19. In order to raise a mass of 100 kg, a man of mass 60 kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with acceleration $5g/4$ relative to the rope. The tension in the rope is (take $g = 10\ \text{m s}^{-2}$)



   (a) $\dfrac{4875}{8}$ N (b) $\dfrac{4875}{2}$ N
   (c) $\dfrac{4875}{4}$ N (d) $\dfrac{4875}{6}$ N

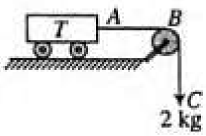
20. A plumb bob is hung from the ceiling of a train compartment. The train moves on an inclined track of inclination 30° with horizontal. The acceleration of train up the plane is $a = g/2$. The angle which the string supporting the bob makes with normal to the ceiling in equilibrium is

(a) 30° (b) $\tan^{-1}(2/\sqrt{3})$
(c) $\tan^{-1}(\sqrt{3}/2)$ (d) $\tan^{-1}(2)$

21. A balloon of mass M is descending at a constant acceleration a. When a mass m is released from the balloon, it starts rising with the same acceleration a. Assuming that its volume does not change, what is the value of m?

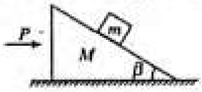
(a) $\dfrac{a}{a+g}M$ (b) $\dfrac{2a}{a+g}M$
(c) $\dfrac{a+g}{a}M$ (d) $\dfrac{a+g}{2a}M$

22. A trolley T of mass 5 kg on a horizontal smooth surface is pulled by a load of 2 kg through a uniform rope ABC of length 2 m and mass 1 kg. As the load falls from $BC = 0$ to $BC = 2$ m, its acceleration (in $m s^{-2}$) changes from



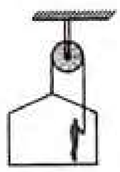
(a) $\dfrac{20}{6}$ to $\dfrac{30}{6}$ (b) $\dfrac{20}{8}$ to $\dfrac{30}{8}$
(c) $\dfrac{20}{5}$ to $\dfrac{30}{6}$ (d) None of these

23. Two wooden blocks are moving on a smooth horizontal surface such that the mass m remains stationary with respect to the block of mass M as shown in the figure. The magnitude of force P is



(a) $(M+m)g \tan\beta$ (b) $g \tan\beta$
(c) $mg \cos\beta$ (d) $(M+m)g \operatorname{cosec}\beta$

24. A man is raising himself and the crate on which he stands with an acceleration of $5\ m s^{-2}$ by a massless rope-and-pulley arrangement. Mass of the man is 100 kg and that of the crate is 50 kg. If $g = 10\ m s^{-2}$, then the tension in the rope is

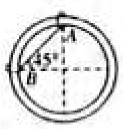


(a) 2250 N (b) 1125 N (c) 750 N (d) 375 N

25. In the above problem, the contact force between the man and the crate is

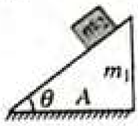
(a) 2250 N (b) 1125 N (c) 750 N (d) 375 N

26. Two objects A and B, each of mass m, are connected by a light inextensible string. They are restricted to move on a frictionless ring of radius R in a vertical plane (as shown in the figure). The objects are released from rest at the position shown. Then the tension in the cord just after release is



(a) Zero (b) mg (c) $\sqrt{2}\,mg$ (d) $mg/\sqrt{2}$

27. In the figure, the mass m_2 starts with velocity v_0 and moves with constant velocity on the surface. During motion, the normal reaction between the horizontal surface and fixed triangle block m_1 is N. Then during motion

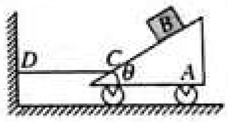


(a) $N = (m_1 + m_2)g$ (b) $N = m_1 g$
(c) $N < (m_1 + m_2)g$ (d) $N > (m_1 + m_2)g$

28. A lift is moving down with an acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift, and a man standing stationary on the ground are, respectively,

(a) a, g (b) $(g-a), g$ (c) a, a (d) g, g

29. Block B has mass m and is released from rest when it is on top of wedge A, which has a mass $3m$. Determine the tension in cord CD needed to hold the wedge from moving while B is sliding down A. Neglect friction.



(a) $2\,mg\cos\theta$ (b) $\dfrac{mg}{2}\cos\theta$
(c) $\dfrac{mg}{2}\sin 2\theta$ (d) $mg\sin 2\theta$

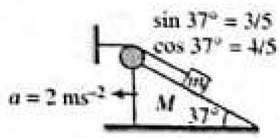
30. Three blocks A, B, and C are suspended as shown in the figure. Mass of each of blocks A and B is m. If the system is in equilibrium, and mass of C is M, then

(a) $M > 2m$ (b) $M = 2m$
(c) $M < 2m$ (d) None of these

31. An object is suspended from a spring balance in a lift. The reading is 240 N when the lift is at rest. If the spring balance reading now changes to 220 N, then the lift is moving

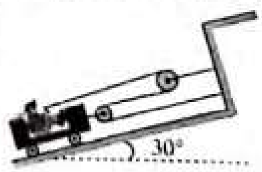
(a) Downward with constant speed
(b) Downward with decreasing speed
(c) Downward with increasing speed
(d) Upward with increasing speed

32. As shown in the figure, if acceleration of M with respect to ground is $2\ m s^{-2}$, then



(a) Acceleration of m with respect to M is $5\ m s^{-2}$.
(b) Acceleration of m with respect to ground is $5\ m s^{-2}$
(c) Acceleration of m with respect M is $2\ m s^{-2}$
(d) Acceleration of m with respect to ground is $10\ m s^{-2}$.

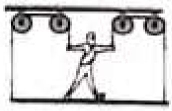
33. A man pulls himself up the 30° incline by the method shown in the figure. If the combined mass of the man and cart is 100 kg, determine the acceleration of



the cart if the man exerts a pull of 250 N on the rope. Neglect all friction and the mass of the rope, pulleys, and wheels.

(a) $4.5\ ms^{-2}$ (b) $2.5\ ms^{-2}$ (c) $3.5\ ms^{-2}$ (d) $1.5\ ms^{-2}$

34. A painter of mass M stands on a platform of mass m and pulls himself up by two ropes which hang over pulley as shown in the figure. He pulls each rope with force F and moves upward with a uniform acceleration a. Find a, neglecting the fact that no one could do this for long time.



(a) $\dfrac{4F+(2M+m)g}{M+2m}$ (b) $\dfrac{4F+(M+m)g}{M+2m}$

(c) $\dfrac{4F-(M+m)g}{M+m}$ (d) $\dfrac{4F-(M+m)g}{2M+m}$

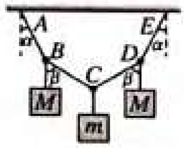
35. A block is lying on the horizontal frictionless surface. One end of a uniform rope is fixed to the block which is pulled in the horizontal direction by applying a force F at the other end. If the mass of the rope is half the mass of the block, the tension in the middle of the rope will be

(a) F (b) $2\ F/3$ (c) $3\ F/5$ (d) $5\ F/6$

36. A 60-kg man stands on a spring scale in a lift. At some instant, he finds that the scale reading has changed from 60 kg to 50 kg for a while and then comes back to original mark. What should be concluded?

(a) The lift was in constant motion upward.
(b) The lift was in constant motion downward.
(c) The lift while in motion downward suddenly stopped.
(d) The lift while in motion upward suddenly stopped.

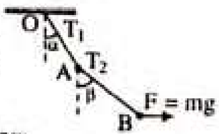
37. Figure represents a light inextensible string $ABCDE$ in which $AB = BC = CD = DE$ and to which are attached masses M, m, and M at the points B, C, and D, respectively.



The system hangs freely in equilibrium with ends A and E of the string fixed in the same horizontal line. It is given that $\tan\alpha = 3/4$ and $\tan\beta = 12/5$. Then the tension in the string BC is

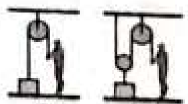
(a) $2\ mg$ (b) $(13/10)\ mg$
(c) $(3/10)\ mg$ (d) $(20/11)\ mg$

38. Two particles A and B, each of mass m, are kept stationary by applying a horizontal force $F = mg$ on particle B as shown in the figure. Then



(a) $2\tan\beta = \tan\alpha$ (b) $2T_1 = 5T_2$
(c) $T_1\sqrt{2} = T_2\sqrt{5}$ (d) None of these

39. In the figure, a person wants to rise a block lying on the ground to a height h. In both the cases, if the time required is same, then in which case he has to exert more force? Assume pulleys and strings light.

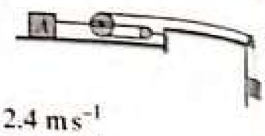


(i) (ii)

(a) (i) (b) (ii)
(c) Same in both (d) Cannot be determined

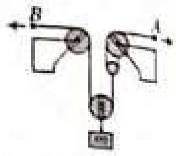
Problems Based on Constraint Relation

40. A block A has a velocity of $0.6\ ms^{-1}$ to the right. Determine the velocity of cylinder B.



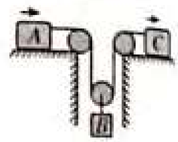
(a) $1.2\ ms^{-1}$ (b) $2.4\ ms^{-1}$
(c) $1.8\ ms^{-1}$ (d) $3.6\ ms^{-1}$

41. For the pulley system shown in the figure, each of the cables at A and B is given a velocity of $2\ ms^{-1}$ in the direction of the arrow. Determine the upward velocity v of the load m.



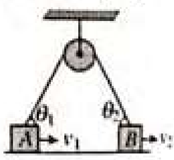
(a) $1.5\ ms^{-1}$ (b) $3\ ms^{-1}$ (c) $6\ ms^{-1}$ (d) $4.5\ ms^{-1}$

42. Blocks A and C start from rest and move to the right with acceleration $a_A = 12t\ ms^{-2}$ and $a_C = 3\ ms^{-2}$. Here t is in seconds. The time when block B again comes to rest is



(a) 2 s (b) 1 s (c) 3/2 s (d) 1/2 s

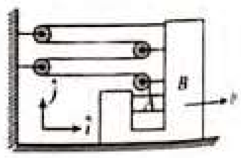
43. In the figure, blocks A and B move with velocities v_1 and v_2 along horizontal direction. Find the ratio of v_1/v_2.



(a) $\dfrac{\sin\theta_1}{\sin\theta_2}$ (b) $\dfrac{\sin\theta_2}{\sin\theta_1}$

(c) $\dfrac{\cos\theta_2}{\cos\theta_1}$ (d) $\dfrac{\cos\theta_1}{\cos\theta_2}$

44. If block B moves towards right with acceleration b, find the net acceleration of block A.



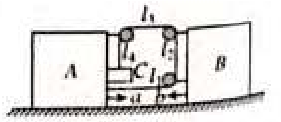
(a) $b\hat{i} + 4b\hat{j}$

(b) $b\hat{i} + b\hat{j}$

(c) $b\hat{i} + 2b\hat{j}$

(d) None of these

45. If the blocks A and B are moving towards each other with accelerations a and b as shown in the figure, find the net acceleration of block C.



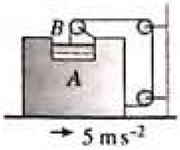
(a) $a\hat{i} - 2(a+b)\hat{j}$ (b) $-(a+b)\hat{j}$

(c) $a\hat{i} - (a+b)\hat{j}$ (d) None of these

46. In the following arrangement, the system is initially at rest. The 5-kg block is now released. Assuming the pulleys and string to be massless and smooth, the acceleration of block C will be

(a) Zero (b) 2.5 ms^{-2}
(c) $10/7 \text{ ms}^{-2}$ (d) $5/7 \text{ ms}^{-2}$

47. If block A is moving with an acceleration of 5 ms^{-2}, the acceleration of B w.r.t ground is

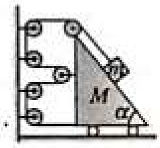


(a) 5 ms^{-2}
(b) $5\sqrt{2} \text{ ms}^{-2}$
(c) $5\sqrt{5} \text{ ms}^{-2}$
(d) 10 ms^{-2}

48. In the arrangement shown in the figure, if the acceleration of B is $\vec{a}$, then find the acceleration of A.

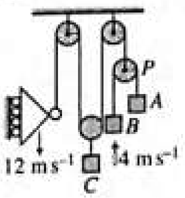
(a) $a \sin \alpha$
(b) $a \cot \theta$
(c) $a \tan \theta$
(d) $a(\sin \alpha \cot \theta + \cos \alpha)$

49. If the acceleration of wedge in the shown arrangement is $a \text{ ms}^{-2}$ towards left, then at this instant, acceleration of the block (magnitude only) would be



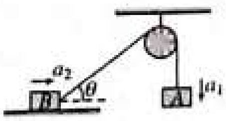
(a) $4a \text{ ms}^{-2}$
(b) $a\sqrt{17 - 8\cos\alpha} \text{ ms}^{-2}$
(c) $(\sqrt{17})a \text{ ms}^{-2}$
(d) $\sqrt{17} \cos\frac{\alpha}{2} \times a \text{ ms}^{-2}$

50. In the arrangement shown in the figure at a particular instant, the roller is coming down with a speed of 12 ms^{-1} and C is moving up with 4 ms^{-1}. At the same instant, it is also known that w.r.t. pulley P, block A is moving down with speed 3 ms^{-1}. Determine the motion of block B (velocity) w.r.t. ground.



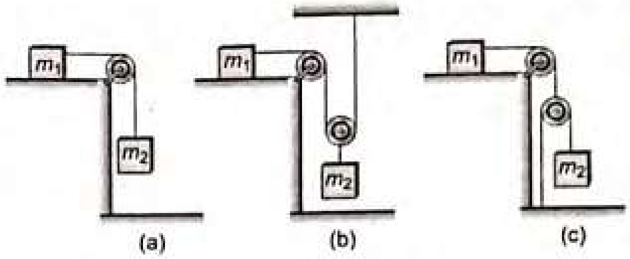
(a) 4 ms^{-1} in downward direction
(b) 3 ms^{-1} in upward direction
(c) 7 ms^{-1} in downward direction
(d) 7 ms^{-1} in upward direction

51. Figure shows two blocks, each of mass m. The system is released from rest. If accelerations of blocks A and B at any instant (not initially) are a_1 and a_2, respectively, then



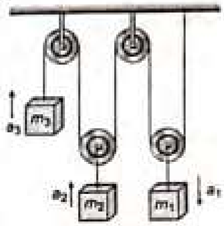
(a) $a_1 = a_2 \cos\theta$ (b) $a_2 = a_1 \cos\theta$
(c) $a_1 = a_2$ (d) None of these

52. In each of the three arrangements, the block of mass m_1 is being pulled left with constant velocity. There is no friction anywhere. The strings are light and inextensible and pulleys are massless. The ratio of the speed of the block of mass m_2 in the three cases respectively is:



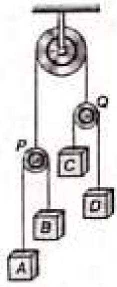
(a) $2:1:4$ (b) $2:4:1$
(c) $4:2:1$ (d) Cannot be calculated

53. If the blocks are moving as shown in the figure the relation between a_1, a_2 and a_3 will be



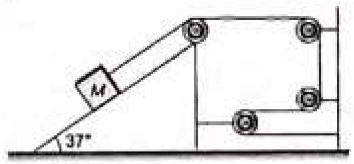
(a) $2a_1 + 2a_2 + a_3 = 0$
(b) $2a_1 - 2a_2 + a_3 = 0$
(c) $2a_1 - 2a_2 - a_3 = 0$
(d) $2a_2 - 2a_1 + a_3 = 0$

54. In the figure acceleration of A is 1 m/s^2 upwards, acceleration of B is 7 m/s^2 upwards and acceleration of C is 2 m/s^2 upwards. The acceleration of D will be



(a) 7 m/s^2 downwards
(b) 2 m/s^2 downwards
(c) 10 m/s^2 downwards
(d) 8 m/s^2 downwards

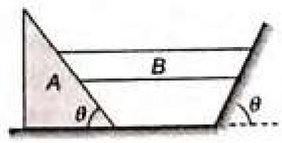
55. A block of mass m and wedge M is arranged as shown in the figure.



Initially the system is kept stationary, certain time system is released from rest. If acceleration of M is found to be $A = \sqrt{5} \text{ m/s}^2$ towards right, then find net acceleration of m with respect to ground

(a) $2\sqrt{5} \text{ m/s}^2$ (b) 3 m/s^2
(c) $3\sqrt{5} \text{ m/s}^2$ (d) none of these

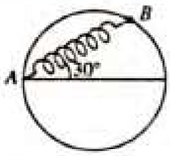
56. In the situation as shown in figure if acceleration of B is a then find the acceleration of A (B always remains horizontal)



(a) $a \sin\theta$ (b) $a \cot\theta$
(c) $2a \tan\theta$ (d) $2a \cos\theta$

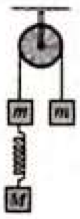
Kinematics of Circular Motion

57. A bead of mass m is attached to one end of a spring of natural length R and spring constant $k = \frac{(\sqrt{3}+1)mg}{R}$. The other end of the spring is fixed at a point A on a smooth vertical ring of radius R as shown in the figure. The normal reaction at B just after it is released to move is



(a) $mg/2$ (b) $\sqrt{3}\,mg$ (c) $3\sqrt{3}\,mg$ (d) $\frac{3\sqrt{3}\,mg}{2}$

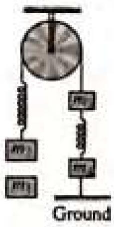
58. The system shown in the figure is released from rest. The spring gets elongated



(a) If $M > m$ (b) If $M > 2m$
(c) If $M > m/2$ (d) For any value of M

(Neglect the friction and masses of pulley, string, and spring.)

59. For the system shown in the figure, $m_1 > m_2 > m_3 > m_4$. Initially, the system is at rest in equilibrium condition. If the string joining m_4 and ground is cut, then just after the string is cut.



Statement I: m_1, m_2, m_3 remain stationary.
Statement II: The value of acceleration of all the four blocks can be determined.
Statement III: Only m_4 remains stationary.
Statement IV: Only m_4 accelerates.
Now, choose the correct options.
(a) All the statements are correct.
(b) Only I, II, and IV are correct.
(c) Only I and II are correct.
(d) Only II and IV are correct.

60. The tension in the spring is

(a) Zero (b) 2.5 N (c) 5 N (d) 10 N

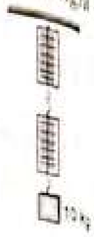
61. The masses of 10 kg and 20 kg respectively are connected by a massless spring as shown in figure. A force of 200 N acts on the 20 kg mass. At the instant shown, the 10 kg mass has acceleration 12 m/s^2. What is the acceleration of 20 kg mass?

(a) 12 m/s^2 (b) 4 m/s^2 (c) 10 m/s^2 (d) Zero

62. Figure shows a 5 kg ladder hanging from a string that is connected with a ceiling and is having a spring balance connected in between. A boy of mass 25 kg is climbing up the ladder at acceleration 1 m/s^2. Assuming the spring balance and the string to be massless and the spring to show a constant reading, the reading of the spring balance is (Take g = 10 m/s^2)

(a) 30 kg (b) 32.5 kg (c) 35 kg (d) 37.5 kg

63. A block of mass 10 kg is suspended through two light spring balances as shown in figure.

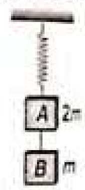


(a) Both the scales will read 10 kg
(b) Both the scales will read 5 kg
(c) The upper scale will read 10 kg and the lower zero
(d) The readings may be anything but their sum will be 10 kg

64. Initially the spring is undeformed. Now the force F is applied to B as shown in the figure. When the displacement of B w.r.t. A is x towards right in some time then the relative acceleration of B w.r.t. A at that moment is

(a) $\frac{F}{2m}$ (b) $\frac{F-kx}{m}$
(c) $\frac{F-2kx}{m}$ (d) none of these

65. Two blocks A and B of masses $2m$ and m, respectively, are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in the figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively



(a) $g, \frac{g}{2}$ (b) $\frac{g}{2}, g$
(c) g, g (d) $\frac{g}{2}, \frac{g}{2}$

66. Two blocks of mass 2 kg are connected by a massless ideal spring of spring constant k = 10 N/m. The upper block is suspended from roof by a light string A. The system shown is in equilibrium. The string A is now cut, the acceleration of upper block just after the string A is cut will be (g = 10 m/s^2)

(a) 0 m/s^2 (b) 10 m/s^2
(c) 15 m/s^2 (d) 20 m/s^2

Problems Based on Mixed Concepts

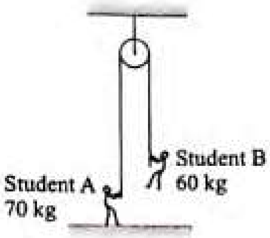
67. A rope of negligible mass passes over a pulley of negligible mass attached to the ceiling, as shown in figure. One end of the rope is held by Student A of mass 70 kg, who is at rest on the floor. The opposite end of the rope is held by Student B of mass 60 kg, who is suspended at rest above the floor. The minimum acceleration a_0 with which the Student B should climb up the rope to lift the Student A upward off the floor.

(a) $\frac{1}{3}$m/s^2

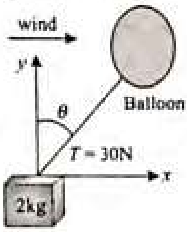
(b) $\frac{2}{3}$m/s^2

(c) $\frac{4}{3}$m/s^2

(d) $\frac{5}{3}$m/s^2



68. A balloon is tied to a block. The mass of the block is 2 kg. The tension of the string between the balloon and the block is 30 N. Due to the wind, the string has an angle θ relative to the vertical direction, $\cos\theta = 4/5$ and $\sin\theta = 3/5$. Assume the acceleration of gravity is $g = 10$ m/s^2. Also assume the block is small so the force on the block from the wind can be ignored. Then the x-component and the y-component of the acceleration a of the block.



(a) 9 m/s^2, 2 m/s^2 (b) 9 m/s^2, 12 m/s^2

(c) 18 m/s^2, 2 m/s^2 (d) 18 m/s^2, 12 m/s^2

69. A lift of total mass M is raised by cables from rest through a height h. The greatest tension which the cables can safely bear is nMg. The maximum speed of lift during its journey if the ascent is to made in shortest time is

(a) $\sqrt{2gh\left(\frac{n+1}{n}\right)}$ (b) $\sqrt{2gh}$

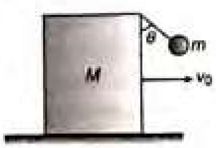
(c) $\sqrt{2gh\left(\frac{n}{n+1}\right)}$ (d) $\sqrt{2gh\left(\frac{n-1}{n}\right)}$

70. A bullet of mass m moving with velocity v_0 hits a wooden plank A of mass M placed on a smooth horizontal surface. The length of the plank is ℓ. The bullet experiences a constant resistive force F inside the block. The minimum value of v_0 such that it is able to come out of the plank is

(a) $\sqrt{\frac{F\ell/m}{M^2}}$ (b) $\sqrt{\frac{2F\ell(M+m)}{Mm}}$

(c) $\sqrt{\frac{2F\ell m}{M^2}}$ (d) $\sqrt{\frac{F\ell(M+m)}{Mm}}$

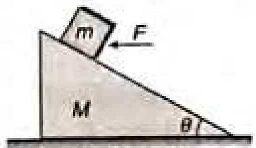
71. A wedge of mass m is pushed with a speed v_0 on a rough horizontal plane. The angle of friction between the wedge and horizontal plane is ϕ. The angle of inclination θ of the pendulum is



(a) ϕ (b) $90° - \phi$

(c) $\tan^{-1}\left(\frac{m}{M}\tan\phi\right)$ (d) None of these

72. A block of mass m lies on wedge of mass M, which lies on fixed horizontal surface. The wedge is free to move on the horizontal surface. A horizontal force of magnitude F is applied on block as shown, neglecting friction at all surfaces, the value of force F such that block has no relative motion w.r.t. wedge will be: (where g is acceleration due to gravity)



(a) $(M+m)\,g\tan\theta$ (b) $(M+m)\,g\cot\theta$

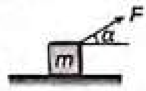
(c) $\frac{m}{M}(M+m)\,g\tan\theta$ (d) $\frac{m}{M}(M+m)\,g\cot\theta$

73. A particle of mass m is placed on the smooth face of an inclined plane of mass M and slope θ which is free to slide on a smooth horizontal plane in a direction perpendicular to the edge. If the particle slides with an accelerating a, the acceleration of the inclined plane towards right A will be

(a) $\frac{ma\cos\theta}{(M+m)}$ (b) $\frac{Mm\cos\theta}{(M+m)}$

(c) $\frac{Ma\cos\theta}{(M+m)}$ (d) $\frac{M\cos\theta}{(M+m)}$

74. A force $F = bt$ (where b is a constant) is applied at an angle to a mass m kept on a smooth horizontal plane. The velocity of mass m at the moment of its breaking off the plane is



(a) $\frac{mg^2\cos\alpha}{2b\sin^2\alpha}$ (b) $\frac{g^2\cos\alpha}{2mb\sin^2\alpha}$

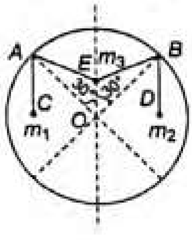
(c) $\frac{mg^2\sin^2\alpha}{2b\cos\alpha}$ (d) $\frac{bg^2\cos\alpha}{2m\sin\alpha}$

75. A flat plate moves normally with a speed v_1 towards a horizontal jet of water of uniform area of cross-section. The jet discharges water at the rate of volume V per second at a speed of v_2. The density of water is ρ. Assume that water splashes along the surface of the plate at right angles to the original motion. The magnitude of the force acting on the plate due to the jet of water is

(a) $\rho V v_1$ (b) $\rho V(v_1+v_2)$

(c) $\frac{\rho V}{v_1+v_2}v_1^2$ (d) $\rho\left[\frac{V}{v_2}\right](v_1+v_2)^2$

76. Two massless rings slide on a smooth circular loop of the wire whose axis lies in a horizontal plane. A smooth massless inextensible string passes through the rings, which carries masses m_1 and m_2 at the two ends and mass m_3 between the rings. If there is equilibrium when the line connecting

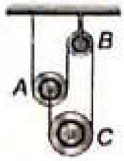


each ring with centre subtends an angle 30° with vertical as shown in figure. Then the ratio of masses are

(a) $m_1 = 2m_2 = m_3$ (b) $2m_1 = m_2 = 2m_3$

(c) $m_1 = m_2 = m_3$ (d) None of these

77. In the arrangement shown in the figure pulleys A and B are massless and the thread is inextensible. Mass of pulley C is equal to m. If friction in all the pulleys is negligible, then



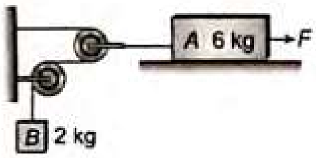
(a) tension in thread is equal to (1/2) mg

(b) acceleration of pulley C is equal to $g/2$ (downward)

(c) acceleration of pulley A is equal to g (upward)

(d) acceleration of pulley A is equal to $2g$ (upward)

78. The system starts from rest and A attains a velocity of 5 m/s after it has moved 5 m towards right. Assuming the arrangement to be frictionless everywhere and pulley and strings to be light, the value of the constant force F applied on A is



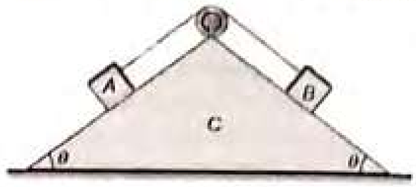
(a) 50 N (b) 75 N (c) 100 N (d) 96 N

79. In the following arrangement the system is initially at rest. The 5 kg block is now released. Assuming the pulleys and string to be massless and smooth, the acceleration of block C will be

(a) Zero (b) 2.5 m/s^2

(c) $\frac{10}{7}$ m/s^2 (d) $\frac{5}{7}$ m/s^2

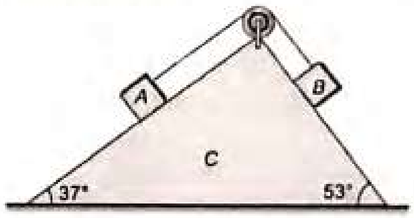
80. In the figure shown C is a fixed wedge on horizontal surface. Blocks A and B are of masses m and $2m$ respectively are kept as shown in figure. They can slide along the inclined plane smoothly. The pulley and string are massless. Take $\theta = 30°$ and $g = 10$ m/s^2. The inclined planes are very long. A and B are released from rest. 2 seconds after the release, B is caught for a moment and released again. Find out the speed of A just before the instant when the string becomes tight again.



(a) Zero (b) $\frac{9}{5}$ m/s

(c) $\frac{12}{5}$ m/s (d) $\frac{12}{7}$ m/s

81. In the figure shown blocks A and B are kept on a wedge C. A, B and C each have mass m. All surfaces are smooth. Find the acceleration of C.



(a) Zero (b) 2.0 m/s^2

(c) $\frac{12}{5}$ m/s^2 (d) 3.5 m/s^2

ARCHIVES

1. One end of a massless rope, which passes over a massless and frictionless pulley P, is tied to a hook C, while the other end is free. The maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in m/s^2) can a man of 60 kg climb on the rope?

(a) 16 (b) 6

(c) 4 (d) 8 **(AIEEE 2002)**

2. When forces F_1, F_2, and F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed, then the acceleration of the particle is

(a) F_1/m (b) F_2F_3/mF_1

(c) $(F_2 - F_3)/m$ (d) F_2/m **(AIEEE 2002)**

3. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are, respectively,

(a) g, g (b) $g - a$, $g - a$

(c) $g - a$, g (d) a, g **(AIEEE 2002)**

4. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is

(a) 8 : 1 (b) 9 : 7

(c) 4 : 3 (d) 5 : 3 **(AIEEE 2002)**

5. Two forces are such that the sum of their magnitudes is 18 N and their resultant, of magnitude 12 N, is perpendicular to the smaller force. Then the magnitudes of the forces are

(a) 12 N, 6 N (b) 13 N, 5 N
(c) 10 N, 8 N (d) 16 N, 2 N **(AIEEE 2002)**

6. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kilogram hangs from the former one. Which of the following statements about the scale reading is true?
(a) Both the scales read $M/2$ kilogram each.
(b) Both the scales read M kilogram each.
(c) The scale of the lower one reads M kilogram and of the upper one zero.
(d) The reading of the two scales can be anything but the sum of the reading will be M kilogram.
(AIEEE 2003)

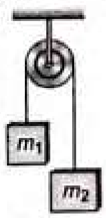
7. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is
(a) $\frac{PM}{M+m}$ (b) $\frac{Pm}{M+m}$
(c) $\frac{PM}{M-m}$ (d) P **(AIEEE 2003)**

8. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, the reading of the spring balance will be
(a) 49 N (b) 24 N
(c) 74 N (d) 15 N **(AIEEE 2003)**

9. A rocket with a lift-off mass 3.5×10^4 kg is blasted upward with an initial acceleration of 10 m/s^2. Then the initial thrust of the blast is
(a) 1.75×10^5 N (b) 3.5×10^5 N
(c) 7.0×10^5 N (d) 1.40×10^5 N
(AIEEE 2003)

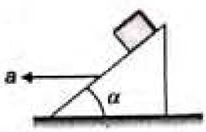
10. A machine gun fires a bullet of mass 40 g with a velocity of 1200 m/s. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
(a) two (b) four
(c) one (d) three **(AIEEE 2004)**

11. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when the system is left free to move? ($g = 9.8$ m/s^2)



(a) 5 m/s^2 (b) 9.8 m/s^2
(c) 0.2 m/s^2 (d) 4.8 m/s^2
(AIEEE 2004)

12. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then a is equal to



(a) $g \tan \alpha$ (b) g
(c) $g \operatorname{cosec} \alpha$ (d) $g/\tan \alpha$ **(AIEEE 2005)**

13. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin?
(a) 5 m/s^2 (b) 10 m/s^2
(c) 3 m/s^2 (d) 15 m/s^2 **(AIEEE 2005)**

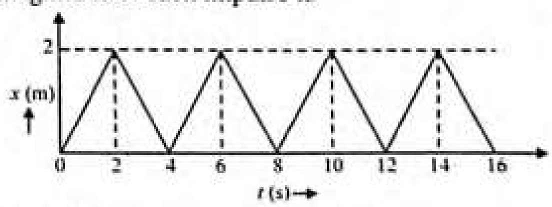
14. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is
(a) 30 N (b) 300 N
(c) 150 N (d) 3 N **(AIEEE 2006)**

15. A ball of mass 0.2 kg is thrown vertically upward by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes up to 2 m height further, find the magnitude of the force. Consider g = 10 m/s^2.
(a) 20 N (b) 22 N
(c) 4 N (d) 16 N **(AIEEE 2006)**

16. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m.
(a) $\frac{(M+m)F}{m}$ (b) $\frac{mF}{M+m}$
(c) $\frac{MF}{M+m}$ (d) $\frac{mF}{M}$ **(AIEEE 2007)**

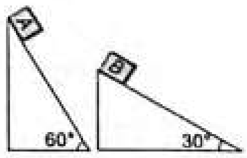
17. A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of 5.00 m/s. The magnitude of its momentum is recorded as
(a) 17.6 kg-m/s (b) 17.565 kg-m/s
(c) 17.56 kg-m/s (d) 17.57 kg-m/s **(AIEEE 2008)**

18. The figure shows the position–time (x–t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



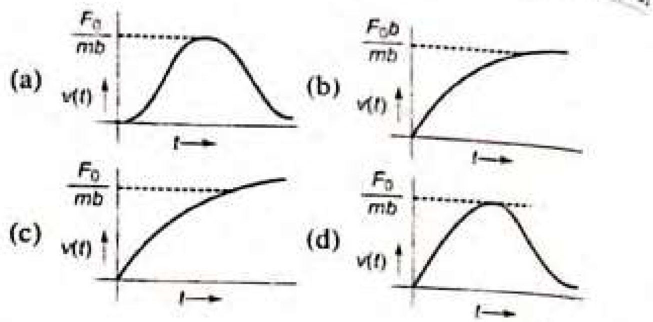
(a) 0.4 Ns (b) 0.8 Ns
(c) 1.6 Ns (d) 0.2 Ns **(AIEEE 2010)**

19. Two fixed frictionless inclined planes making angles 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



(a) 4.9 m/s^2 in horizontal direction
(b) 9.8 m/s^2 in vertical direction
(c) zero
(d) 4.9 m/s^2 in vertical direction **(AIEEE 2010)**

20. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves?



(AIEEE 2012)

ANSWER KEY

Exercises

1. (a)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (c)
11. (d)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (b)
21. (b)	**22.** (a)	**23.** (a)	**24.** (b)	**25.** (d)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (c)
31. (c)	**32.** (c)	**33.** (b)	**34.** (c)	**35.** (d)	**36.** (d)	**37.** (b)	**38.** (c)	**39.** (a)	**40.** (c)
41. (a)	**42.** (d)	**43.** (c)	**44.** (a)	**45.** (a)	**46.** (d)	**47.** (c)	**48.** (d)	**49.** (b)	**50.** (d)
51. (d)	**52.** (a)	**53.** (d)	**54.** (c)	**55.** (d)	**56.** (d)	**57.** (d)	**58.** (d)	**59.** (b)	**60.** (c)
61. (b)	**62.** (b)	**63.** (a)	**64.** (c)	**65.** (b)	**66.** (d)	**67.** (d)	**68.** (a)	**69.** (d)	**70.** (b)
71. (a)	**72.** (c)	**73.** (a)	**74.** (a)	**75.** (d)	**76.** (c)	**77.** (d)	**78.** (b)	**79.** (d)	**80.** (a)
81. (a)									

Archives

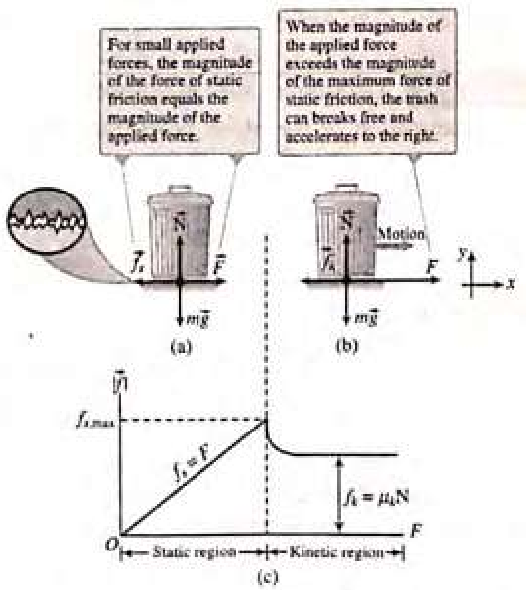
1. (c)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (b)	**7.** (a)	**8.** (b)	**9.** (c)	**10.** (d)
11. (c)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (d)	**20** (c)

Chapter 6

Newton's Laws of Motion (With Friction) and Dynamics of Circular Motion

STATIC FRICTION AND KINETIC FRICTION

Imagine you are trying to slide a heavy box on horizontal concrete surface by applying a force $\vec{F}$. You try to drag the box across the surface of your concrete floor. The concrete surface is a real, not an ideal, frictionless surface in a simplification model. If we apply an external horizontal force $\vec{F}$ to the box acting to the right, the box remains stationary if $\vec{F}$ is small. The force that counteracts $\vec{F}$ and keeps the box from moving acts to the left and is called the force of *static friction*, $\vec{f}_s$. As long as the box is not moving, it is modelled as a particle in equilibrium and $f_s = F$. Therefore, if $\vec{F}$ decreases, $\vec{f}_s$ also decreases. Experiments show that the friction force arises from the nature of two surfaces; because of their roughness, contact is made only at a few points, as shown in the magnified surface view in Fig. (a).



If we increase the magnitude of $\vec{F}$, the box eventually slips. When the box is on the verge of slipping, f_s is maximum. If F is excess $f_{s,\max}$, the box moves and accelerates to the right. While the box is in motion, the friction force is less than f_s, max. We call the friction force for an object in motion the force of *kinetic friction*. The net force $(F - f_k)$ in the x direction produces an acceleration to the right, according to Newton's second law. If we reduce the magnitude of $\vec{F}$ so that $F = f_k$, the acceleration becomes zero and the box moves to the right with constant speed. If the applied force is removed, the friction force acting to the left provides an acceleration of the box in the $-x$ direction and eventually brings it to rest.

- The magnitude of the force of static friction between any two surfaces in contact can have the values
  $$f_s \le \mu_s N \tag{i}$$
  where the dimensionless constant μ_s is called the coefficient of static friction and N is the magnitude of the normal force. The equality in Eq. (i) holds when the surfaces are on the verge of slipping, that is, when $f = f_{s,\max} \equiv \mu_s N$. This situation is called *impending motion*. The inequality holds when the component of the applied force parallel to the surfaces is less than this value.
- We define the static friction as tangential contact force which prevents relative sliding between the points of contact.
- Static friction can have any value between zero and its maximum (limiting) value $\mu_s N$. We can write $0 \le f_s \le \mu_s N$
- *Static friction is of self-adjusting nature and can change its magnitude and direction as per requirement.*
- The magnitude of the force of kinetic friction acting between two surfaces is $f_k = \mu_k N$
  μ_k is the coefficient of kinetic friction. In our simplification model, this coefficient is independent of the relative speed of the surfaces.
- The values of μ_k and μ_s depend on the nature of the surfaces, but μ_k is generally less than μ_s.
- The direction of the friction force on an object is opposite to the actual motion (kinetic friction) or the impending motion (static friction) of the object relative to the surface with which it is in contact.

LAWS OF LIMITING FRICTION

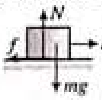
The force of friction always acts along such a direction so as to oppose the motion of the body relative to the other. The force of friction depends upon the nature and roughness of the two surfaces in contact. The rougher the surface, more will be the frictional forces.

- Limiting frictional force is independent of the apparent area of contact till the value of the normal reaction remains same.
- The direction of limiting frictional force is opposite to the direction in which one body is on the verge of starting its motion.
- The limiting frictional force depends upon the nature of surfaces in contact.

ILLUSTRATION 6.1 A block of mass $m = 1$ kg is at rest on a rough horizontal surface having coefficient of static friction $\mu_s = 0.2$ and kinetic force $\mu_k = 0.15$. Discuss the frictional forces if a horizontal force F is applied on the block.

S.No.	Value of F	
1.	1 N	
2.	2 N	
3.	2.5 N	

Solution. Assume no friction, the motion of the block will be in the direction of applied force. The friction opposes relative motion between two surfaces. The direction of friction will be opposite to relative motion, i.e., in leftward direction.



Now let us find the maximum friction force.

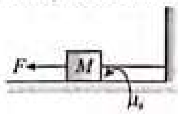
$$f_{max} = f_{lim} = \mu_s N = 0.2 \times 1 \times 10 = 2\text{ N}$$

(a) If $F = 1.0$ N
If the block is at rest friction acting on the block, $f = F = 1.0$ N.
As $f < f_{lim}$, hence, friction will be of static nature and acceleration of the block will be zero.

(b) If $F = 2.0$ N
Here the applied force equals limiting friction. The block is on the verge of slipping.
or $f = f_{lim} = 2.0$ N

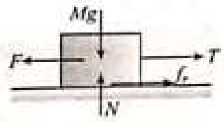
(c) If $F = 2.5$ N
Here the applied force is greater than the magnitude of the maximum nature of static friction. The nature of friction between the block and ground is of kinetic nature.
or $f = f_k = \mu_k N = 0.15 \times 1 \times 10 = 1.5$ N

ILLUSTRATION 6.2 In the figure, force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_S.



Solution. As the external force F is gradually increased from zero, it is compensated by the friction and the string bears no tension. When the limiting friction is achieved by increasing force F to a value till $\mu_S mg$, the further increase in F is transferred to the string.

$$\text{When} \begin{cases} F < \mu_s mg; \text{ the friction is static and } T = 0 \\ F \geq \mu_s mg; T = F - \mu_s mg \end{cases}$$



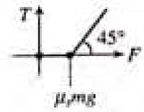
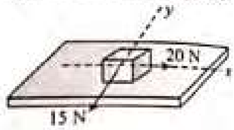


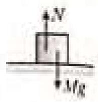
ILLUSTRATION 6.3 In the figure, an object of mass $M = 10$ kg is kept on a rough table as seen from above. Forces are applied on it as shown.

Find the direction of static friction if the object does not move. (Take μ = 0.4)



Solution. Here limiting value of friction force

$$f_{lim} = \mu_s N = 0.4 \times 10 \times 10 = 40\text{ N}$$

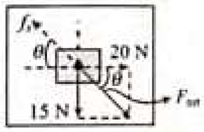


The resultant of external forces acting on the block (see figure).

$$F_{net} = \sqrt{(15)^2 + (20)^2} = 25\text{ N}$$

If the block is at rest, $f = F_{net} = 25$ N

As $f < f_{lim}$.



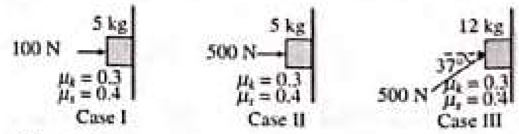
Here actual friction force acting on the block is less than f_{lim}. The friction in this case is of static nature.

For the direction of friction force, we draw the free body diagram of find the resultant force.

The direction of static friction is opposite to the direction of the resultant force FR. Its magnitude is equal to 25 N at angle,

$$\tan\theta = \frac{15}{20} = \frac{3}{4} \text{ or } \theta = 37° \text{ as shown in the figure.}$$

ILLUSTRATION 6.4 Determine the magnitude of frictional force and acceleration of the block in each of the following cases:

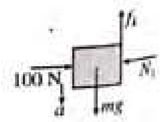


Solution. **Case I**

$$N_1 = 100\text{ N}, mg = 50\text{ N}$$

$$f_l = \mu_s N_1 = 0.4 \times 100 = 40\text{ N},$$

$$f_k = \mu_k N_1 = 0.3 \times 100 = 30\text{ N}$$



Here mg (driving force) is greater than maximum friction $f_l = 40$ N. Hence, the block will not be able to stay at rest. It will accelerate downwards. But when it starts slipping, then kinetic friction will come into play. Now

$$a = \frac{mg - f_k}{m} = \frac{50 - 30}{5} = 4\text{ m s}^{-2}$$

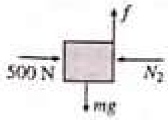
So, in this case, $f = f_k = 30$ N and $a = 4$ m s^{-2} (downwards)

Case II

$N_2 = 500$ N, $mg = 50$ N

$f_l = \mu_s N_2 = 200$ N,

$f_k = \mu_k N_2 = 150$ N



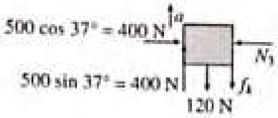
Here f_l is greater than mg (driving force). Hence, block will not move. So, in this case, $a = 0$, $f = mg = 50$ N.

Case III

$N_3 = 400$ N,

$f_l = \mu_s N_3 = 160$ N,

$f_k = \mu_k N_3 = 120$ N



Here driving force = 300 − 120 = 180 N in upward direction; hence, friction will act downwards. Driving force is more than f_l. So the block will accelerate upwards.

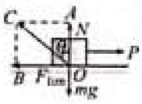
$$a = \frac{180 - f_k}{m} = \frac{180 - 120}{12} = 5 \text{ m s}^{-2} \text{(upwards)}$$

So, in this case, $f = f_k = 120$ N and $a = 5$ m s^{-2} (upwards)

ANGLE OF FRICTION

The angle of friction between any two surfaces in contact is defined as the angle which the resultant of the force of limiting friction F and normal reaction R makes with the direction of normal reaction R. It is represented by θ.

In the figure, OA represents the normal reaction R which balances the weight mg of the body. OB represents F, the limiting force of sliding friction, when the body tends to move to the right. Complete the parallelogram $OACB$. Join OC. This represents the resultant of R and F. By definition, $\angle AOC = \theta$ is the angle of friction between the two bodies in contact.



The value of angle of friction depends on the material and nature of the surfaces in contact.

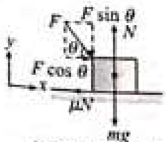
Relation between μ and θ

$$\text{In } \Delta AOC, \ \tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \frac{f_{\text{lim}}}{N} = \mu$$

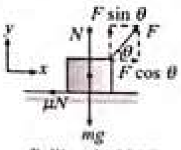
Hence, $\mu = \tan\theta$, i.e., coefficient of limiting friction between any two surfaces in contact is equal to tangent of the angle of friction between them.

Pull is Easier than Push

Push: Consider a block of mass m placed on a rough horizontal surface. The coefficient of static friction between the block and surface is μ. Let a push force F is applied at an angle θ with the horizontal.



Pushing the block



Pulling the block

As the block is in equilibrium along y-axis, thus, we have

$\Sigma f_y = 0$ or $N = mg + F \sin\theta$

To just move the block along x-axis, we have

$F \cos\theta = \mu N = \mu(mg + F \sin\theta)$

$$\text{or} \quad F = \frac{\mu mg}{\cos\theta - \mu \sin\theta} \qquad \text{(i)}$$

Pull: Along y-axis, we have

$\Sigma f_y = 0$ or $N = mg - F\sin\theta$

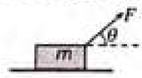
To just move the block along x-axis, we have

$F\cos\theta = \mu N = \mu(mg - F \sin\theta)$

$$\text{or} \quad F = \left(\frac{\mu mg}{\cos\theta + \mu\sin\theta}\right) \qquad \text{(ii)}$$

It is clear from above discussion that pull force is smaller than push force.

ILLUSTRATION 6.5 A block of mass m lying on a horizontal surface (coefficient of static friction = μ_s) is to be brought into motion by a pulling force F. At what angle θ with the horizontal should the force F be applied so that its magnitude is minimum? Also find this minimum magnitude.

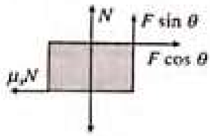


Solution. Let us first calculate the force F required to bring m into motion in terms of angle θ. Equation using laws of motion are as follows:

$N = mg - F \sin\theta$

and $F\cos\theta = \mu_s N$

$F\cos\theta = \mu_s(mg - F\sin\theta)$



$$\Rightarrow \quad F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

We have to find the angle θ for which this force F is minimum. Substituting $\mu_s = \tan\phi$ (for simplification), we get

$$F = \frac{mg\tan\phi}{\cos\theta + \tan\phi\sin\theta} = \frac{mg\sin\phi}{\cos(\theta - \phi)}$$

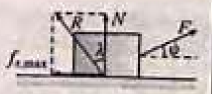
F is minimum if $\cos(\theta - \phi)$ is maximum. Hence, F is minimum for $\theta = \phi = \tan^{-1}\mu_S$ and $F_{\min} = mg \sin\phi$

To bring m into motion with least effort, force should be applied at an angle $\tan^{-1}\mu_S$ and should have a magnitude equal to

$$F_{\min} = mg\sin\phi = \frac{\mu_s mg}{\sqrt{1+\mu_s^2}}$$

Important Results

- The block can be moved with least effort on a rough surface (μ) if the force is applied at the angle of friction (λ). That is, $\phi = \tan^{-1}\mu_s = \lambda$



- The minimum magnitude of force is given by $F_{\min} = mg \sin\lambda$

ILLUSTRATION 6.6 A block of mass $m = 10$ kg is to be pulled on a horizontal rough surface with the minimum force.

(a) At what angle θ with horizontal the block should be pulled?

(b) What is the magnitude of the force F?

Solution.

(a) The block can be moved with least effort on a rough surface (μ) if the force is applied at the angle of friction (λ). That is, $\lambda = \theta = \tan^{-1}\mu_s$

The block should be pulled at the angle of friction, i.e.,

$$\theta = \tan^{-1}(0.75) = 37°$$

(b) The magnitude of external force is

$$F_{min} = mg \sin\theta$$

$$= (10)(10)\sin 37° = (100)\left(\frac{3}{5}\right) = 60 \text{ N}$$

ANGLE OF REPOSE OR ANGLE OF SLIDING

The angle of repose or angle of sliding is defined as the minimum angle of inclination of a plane with the horizontal such that a body placed on the plane just begins to slide down. Its value depends on the material and nature of the surfaces in contact.

In the figure, AB is an inclined plane such that a body placed on it just begins to slide down. $\angle BAC = \alpha =$ angle of repose. The various forces involved are:

1. Weight, mg, of the body acting vertically downwards
2. normal reaction, N, acting perpendicular to AB
3. Force of friction, f, acting upon the plane AB

Now, mg can be resolved into two rectangular components: $mg\cos\alpha$ opposite to R and $mg\sin\alpha$ opposite to F. In equilibrium,

$$f = mg\sin\alpha \quad \text{(i)}$$

$$N = mg\cos\alpha \quad \text{(ii)}$$

Dividing (i) by (ii), we get $\dfrac{f}{N} = \dfrac{mg\sin\alpha}{mg\cos\alpha} = \mu_s$

i.e., $\mu = \tan\alpha$ (iii)

Hence, the coefficient of limiting friction between any two surfaces in contact is equal to the tangent of the angle of repose between them.

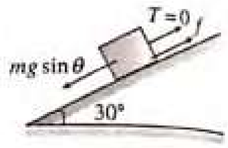
NOTE: Minimum angle for which a block starts sliding down an inclined plane is known as angle of repose. Angle of repose is independent of mass of the object.

ILLUSTRATION 6.7 A block of mass 1 kg is place on a rough inclined plane at angle $\theta = 30°$ will horizontal. The block is connected with a straight as shown in the figure. If $\mu_s = 3/4$, find the tension in string.

Solution. Let us first find the angle of repose, i.e., the angle of inclination of inclined plane with horizontal such that when a body placed on it just begins to slide down.

As we know the angle of repose,

$$\alpha = \tan^{-1}(\mu_s) = \tan^{-1}\left(\frac{3}{4}\right) = 37°$$

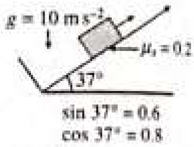


Here the angle of inclination of plane is less than the angle of repose. Here the block does not have tendency to slide. The friction force between the block and incline is static nature.

$$f = mg\sin\theta = 1 \times 10 \times \sin 30° = 5 \text{ N}$$

Hence, tension in string will be zero.

ILLUSTRATION 6.8 An object of mass 10 kg is to be kept at rest on an inclined plane making an angle of 37° to the horizontal by applying a force F along the plane upwards as shown in the figure. The coefficient of static fiction between the object and the plane is 0.2. Find the magnitude of force F. [Take $g = 10 \text{ m s}^{-2}$.]



Solution. mg sin 37° = 10 × 10 × 3/5 = 60 N

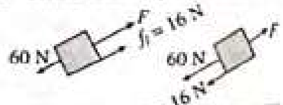
$$f_l = \mu N = 0.2\, mg\cos 37°$$

$$= 0.2 \times 10 \times 10 \times 4/5$$

$$= 16 \text{ N}$$

Friction force alone is unable to balance $mg\sin 37°$. Hence, F is required for balancing. Here two cases arise.

Case I: if the block has tendency to slide down, friction on block will act upwards.



Case (i) Case (ii)

$$F + 16 = 60$$

$$\Rightarrow \quad F = 44 \text{ N}$$

This is the minimum force required for balancing.

Case II: If the block has tendency to slide up, friction on block will act downwards.

$$F = 60 + 16 \quad \Rightarrow \quad F = 76 \text{ N}$$

This is the maximum force required for balancing.

Therefore, for balancing $44 \text{ N} \le F \le 76 \text{ N}$

ILLUSTRATION 6.9 A 5-kg block slides down a plane inclined at 30° to the horizontal. Find

(a) The acceleration of the block if the plane is frictionless.

(b) The acceleration if the coefficient of kinetic friction is $1/2\sqrt{3}$.

Solution.

(a) $mg\sin 30° = ma$

$a = g\sin 30° = 5 \text{ m s}^{-2}$, down the plane if plane is smooth.

(a) (b)

(b) $mg \sin 30° - \mu_k N = ma$

$a = g \sin 30° - \mu_k g \cos 30° = 5/2 \text{ m s}^{-2}$

Concept Application Exercise 6.1

1. What is the value of friction f for the following value of applied force F?

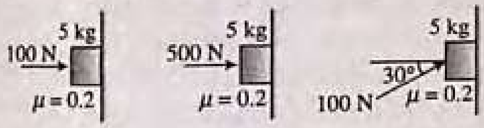
(a) 1 N (b) 2 N (c) 3 N
(d) 4 N (e) 20 N

Assume the coefficient of friction to be $\mu_s = 0.3$; $\mu_k = 0.25$. Mass of the body is $m = 1$ kg. (Assume $g = 10 \text{ m s}^{-2}$)

2. A block of mass 5 kg rests on a rough horizontal surface. It is found that a force of 10 N is required to make the block just move. However, once the motion begins, a force of only 8 N is enough to maintain the motion. Find the coefficients of kinetic and static friction between the block and the horizontal surface.

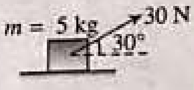
3. A body of mass m is kept on a rough horizontal surface of friction coefficient m. A force P is applied horizontally, but the body is not moving. Find the net force F exerted by the surface on the body.

4. Determine the magnitude of frictional force f in each of the following cases:



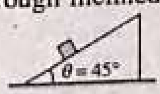
(a) (b) (c)

5. A 5-kg box is moving straight across the floor at a constant speed by a force of 30 N, as shown in the figure.



(a) How large a friction force impedes the motion of the box?
(b) Find μ_k between the box and the floor.

6. A block of mass $m = 3$ kg slides on a rough inclined plane of coefficient of friction 0.2. Find the resultant force offered by the plane on the block.

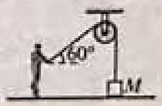


7. A block slides down an inclined plane (angle of inclination 60°) with an acceleration $g/2$. Find the coefficient of kinetic (dynamic) friction.

8. An insect crawls up a hemispherical surface very slowly. The coefficient of friction between the insect and the surface is 1/3. If the line joining the center of the hemispherical surface to the insect makes an angle α with the vertical, find the maximum possible value of α.

9. A box of bananas weighing 40.0 N rests on a horizontal surface. The coefficient of static friction between the box and the surface is 0.40 and the coefficient of kinetic friction is 0.20.
   (a) If no horizontal force is applied to the box and the box is at rest, how large is the friction force exerted on the box?
   (b) What is the magnitude of the friction force if a monkey applies a horizontal force of 6.0 N to the box and the box is initially at rest?
   (c) What minimum horizontal force must the monkey apply to start the box in motion?
   (d) What minimum horizontal force must the monkey apply to keep the box moving at constant velocity once it has been started?
   (e) If the monkey applies a horizontal force of 18.0 N, what is the magnitude of the friction force and what is the box's acceleration?

10. A man of mass 60 kg is pulling a mass M by an inextensible light rope passing through a smooth and massless pulley as shown in figure. The coefficient of friction between the man and the ground is $\mu = 1/2$. Find the maximum value of M that can be pulled by the man without slipping on the ground.



ANALYSIS OF FRICTION FORCE BETWEEN TWO OBJECTS IN CONTACT: CONDITION OF EXISTENCE OF STATIC FRICTION

In many situations, we need to analyze whether friction is present between the surface, and, if present what the nature of friction, kinetic or static is? Following are some important points to analyze this situation.

When two surfaces tend to slide (but do not slide) relative to each other, static friction comes into play.

Then, how do we confirm the tendency of relative sliding between any two surfaces in contact? Let us find a process for it.

First of all, mentally eliminate the friction between the contact points of the surfaces where we want to detect the presence of friction. Then, considering all other forces, we find the accelerations a_1 and a_2 of these points. If the relative acceleration $a_{rel}(=|\vec{a}_1 - \vec{a}_2|)$ between the surfaces is not zero, then surfaces tend to slide relative to each other.

In general, when two bodies are in contact, if the net forces acting on them (ignoring the frictional forces between the contacting surface under consideration) are $\vec{F}_1$ and $\vec{F}_2$, respectively, their accelerations are given as

$$\vec{a}_1 = \frac{\vec{F}_1}{m_1} \text{ and } \vec{a}_2 = \frac{\vec{F}_2}{m_2}$$

If $|\vec{a}_1 - \vec{a}_2| \neq 0$, static friction exists between the surface. If $\vec{a}_1 = \vec{a}_2$, we can say that there is no tendency of relative sliding. Hence, no friction exists; ($f_s = 0$) when $\vec{a}_1 = \vec{a}_2$.

ILLUSTRATION 6.10 Two blocks of masses M and m are arranged as shown in the figure. There is no friction between ground and block M.

The coefficients of static and kinetic friction between M and m are μ_s and μ_k, respectively.

(a) Calculate the maximum possible value of F so that both the bodies move together.

(b) Find the accelerations of the blocks if F is greater than that found in part (a).

Solution.

(a) If both blocks move together, i.e., no sliding between M and m, the friction between m and M will be static nature. Static friction force is a self-adjusting force $0 \le f \le \mu_s N$

Acceleration of system in this case, $a = \dfrac{F}{(M+m)}$ (i)

FBD of m and M:

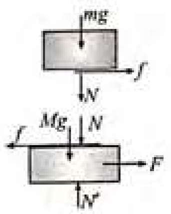
Equation of motion of m:

$$f = ma = m\frac{F}{(M+m)} \qquad \text{(ii)}$$

If there is no sliding between M and m, then $f \le f_{s\text{-max}}$

$$\Rightarrow \quad \frac{mF}{(M+m)} \le \mu_s(mg)$$

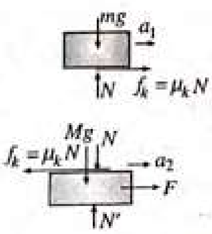
$$\Rightarrow \quad F \le \mu_s (M+m)g$$



(a)

(b) If $F > \mu_s (M+m)g$, then there will be relative sliding between M and m. When relative sliding between M and m starts.

The friction force reaches limiting value the friction force becomes $f_k = \mu_k N$



(b)

FBD of m and M:

Equation of motion of m: $\mu_k(mg) = ma_1$

Acceleration of m, $a_1 = \mu_k g$

Equation of motion of M: $F - \mu_k mg = Ma_2$

Acceleration of M, $a_2 = \dfrac{F - \mu_k mg}{M}$

ILLUSTRATION 6.11 Let us consider the next case in the previous illustration when there is no friction between ground and M. The coefficients of static and kinetic friction are μ_s and μ_k, respectively, and force F acts on upper block as shown in the figure.

(a) What is the maximum possible value of F so that the system moves together?

(b) If there is relative sliding between M and m, then calculate acceleration of M and m.

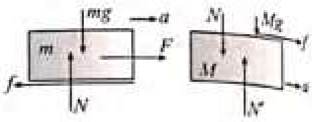
Solution. Let the systems move together, then $a = \dfrac{F}{(M+m)}$.

FBD of m and M

From FBD of M: $f = ma = M\left(\dfrac{F}{M+m}\right)$ (i)

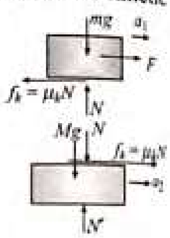
As there is no sliding between M and m,

$$M\left(\frac{F}{M+m}\right) \le \mu_s mg$$



$$\Rightarrow \quad F \le \mu_s \frac{m}{M}(M+m)g$$

If $F > \mu_s \dfrac{m}{M}(M+m)g$, the relative sliding between the blocks starts. The friction force between the blocks will be of kinetic nature and both blocks will move with different accelerations.

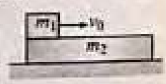


Acceleration of m, $a_1 = \dfrac{F - \mu_k mg}{m}$

Acceleration of M, $a_2 = \dfrac{\mu_k mg}{M}$

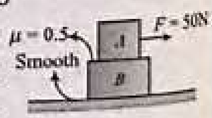
CONCEPT APPLICATION EXERCISE 6.2

1. A block of mass 1 kg is horizontally thrown with a velocity of 10 m s^{-1} on a stationary long plank of mass 2 kg whose surface has a $\mu = 0.5$. Plank rests on frictionless surface. Find the time when m_1 comes to rest w.r.t. plank.

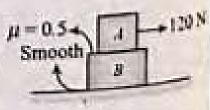


2. In the figure, initially the system is at rest. Find out minimum value of F for which sliding starts between the two blocks. Given $m_A = 10$ kg and $m_B = 20$ kg.

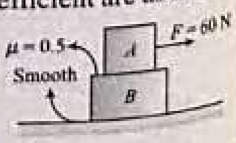
3. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure. Given $m_A = m_B = 10$ kg.



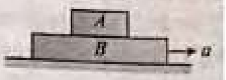
4. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure. Given $m_A = m_B = 10$ kg.



5. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure. Also find maximum F for which two blocks will move together. Given $m_A = 10$ kg and $m_B = 20$ kg.

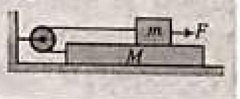


6. The block A is kept over a plank B. The maximum horizontal acceleration of the system in order to prevent slipping of A over B is $a = 2\,\text{m s}^{-2}$. Find the coefficient of friction between A and B.



7. A small block of mass m kept at the left end of a larger block of mass M and length l. The system can slide on a horizontal road. The system is started towards right with an initial velocity v. The friction coefficient between the road and the bigger block is μ and that between the blocks is $\mu/2$. Find the time elapsed before the smaller block separates from the bigger block.

8. A small block of mass m is placed on a plank of mass M. The block is connected to plank with the help of a light string passing over a light smooth pulley, shown in the figure. The co-efficient of static friction between the block and plank is μ. The co-efficient of friction between the plank and the horizontal surface is zero. What maximum horizontal force F applied on the block of mass m can make the block and plank not to slide relatively?



DYNAMICS OF CIRCULAR MOTION

Centripetal Force

Newton's second law tells us that when a particle accelerates, there must be a net force acting on the particle in the direction of its acceleration. Following this law, as the particle in uniform circular motion accelerates radially inwards, the particle must experience a net force directed radially inwards (centripetal). Hence, we call this *centripetal force*.

Centripetal force is not a special force. It can be any field force such as gravitational, electrostatics, magnetic, and others. This can be also be any constant force (reaction force), i.e., friction and normal reaction, etc. The string force and spring force can take the credit of pulling or pushing a particle in a circular track as described in this topic. Hence, any force or component of any force along the radial direction or sum (resultant) of two or many forces acting on a particle directed towards the center of a circle may be called centripetal force.

As $F = ma$, centripetal force = mass × centripetal acceleration, i.e.,

$$F = \frac{mv^2}{r} = m\omega^2 r$$

Centrifugal Force

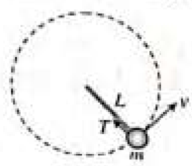
When you are sitting by the side of a window of a car while it negotiates a curve, you feel as if you are pushed away from the center of the curve. This is what we call centrifugal forces as it gives a physical sensation of being centrifuged (pushed radially away). Since you are sitting in an accelerating frame you think that you are not accelerating towards the center of the circle, rather you are centrifuged. But the actual situation can be viewed from ground frame. The actual thing is that "you are moving in a circular path for some time" by the inward reaction force (pressing) offered by the inner wall of the car. You may feel as if you are pressed against the walls of the vehicle. According to Newton's third law, the wall is pushing your muscles in contact, towards the center of the circular path. In fact, the reaction force given by the wall of the car is the centripetal force and in the rotating frames "$\vec{F}_{\text{pseudo}} = -m\vec{a}_{\text{CP}}$" is the centrifugal force which never exists in inertial frame.

Important Results

- As centrifugal force exists in rotating (accelerating) frames, it is a pseudo force. Hence, centripetal force has no action-reaction pair.
- The centrifugal force arises from the relative rotational motion of the observer (reference frame). In other words, F_{CF} arises from the kinematics of the rotating frame (but not from any interactions).
- Centripetal forces are real forces (arising from interaction) whereas centrifugal forces are imaginary or pseudo force (arising from the rotation of the reference frames). The centrifugal force acting on a particle depends only on "mass m," angular velocity $\vec{\omega}$ of the rotating frame (but not the angular velocity of the particle), and the perpendicular distance of the particle from the axis of rotation of the reference frame. In this way, the particle need not move in a circle to experience a centrifugal force.
- When a real force pushes a particle radially outward, we cannot call it centrifugal force of the first kind.
- The centrifugal force is directed "radially" outwards from the axis of rotation of the reference frame (or observer) along the line drawn from the particle perpendicular to the axis of rotation.
- The centrifugal force is a pseudo force, which is equal to $-m\vec{a}_{\text{CP}}$ and centripetal force is a real force. The centrifugal force is adopted to solve the problems in the rotating frame.

ILLUSTRATION 6.12 A block of mass 1 kg is tied to a string of length 1 m, the other end of which is fixed. The block is moved on a smooth horizontal table with constant speed $10\,\text{m s}^{-1}$. Find the tension in the string.

Solution. The ball in the figure does not accelerate vertically. Therefore, we model it as a particle in equilibrium in the vertical direction. It experiences a centripetal acceleration in the horizontal direction. In this case, centripetal force is provided by tension.



$$\Sigma F_y = N - mg = 0$$
$$N = mg \qquad \text{(i)}$$
$$\Sigma Fx = T = ma_c = \frac{mv^2}{r} \qquad \text{(ii)}$$
$$T = \frac{mv^2}{r} = \frac{1\times 10^2}{1} = 100\,\text{N}$$

ILLUSTRATION 6.13 A ball of mass m moves with speed v against a smooth, fixed vertical circular groove of radius R kept on smooth horizontal surface. Find the

(a) normal reaction of the floor on the ball.

(b) normal reaction of the vertical wall on the ball.

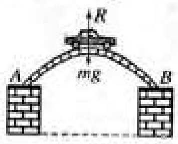
Solution. The ball in the figure does not accelerate vertically. Here centripetal force is provided by normal reaction of vertical wall. It experiences a centripetal acceleration in the horizontal direction (radial). In this case, centripetal force is provided by tension.

$$\Sigma F_y = N_{\text{Floor}} - mg = 0$$
$$N_{\text{Floor}} = mg \qquad \text{(i)}$$
$$\Sigma F_x = N_{\text{wall}} = ma_c = \frac{mv^2}{R} \qquad \text{(ii)}$$

ILLUSTRATION 6.14 A car of mass m moving over a convex bridge of radius r. Find the normal reaction acting on car when it is at the highest point of the bridge.

Solution. The motion of the motor car over a convex bridge AB is the motion along the segment of a circle AB (see figure).

The centripetal force is provided by the difference of weight mg of the car and the normal reaction R of the bridge.



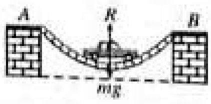
$$\therefore \quad mg - R = \frac{mv^2}{r} \quad \text{or} \quad R = mg - \frac{mv^2}{r}$$

Clearly $R < mg$, i.e., the weight of the moving car is less than the weight of the stationary car.

ILLUSTRATION 6.15 A car of mass m moving over a concave bridge of radius r. Find the normal reaction acting on car when it is at the lowest point of the bridge.

Solution. The motion of the motor car over a concave bridge AB is the motion along the segment of a circle AB (see figure). The centripetal force is provided by the difference of normal reaction R of the bridge and weight mg of the car.

$$\therefore \quad R - mg = \frac{mv^2}{r}$$



$$\text{or} \quad R = mg + \frac{mv^2}{r}$$

Clearly $R > mg$, i.e., the weight of the moving car is greater than the weight of the stationary car.

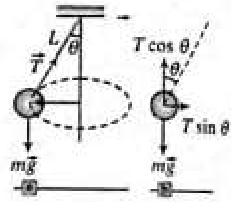
CONICAL PENDULUM

A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown in the figure. Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum.*

The string sweeps out a cone and that the ball moves in a horizontal circle. Let us find an expression for v.

The ball in the figure does not accelerate vertically. Therefore, we model it as a particle in equilibrium in the vertical direction. It experiences acentripetal acceleration in the horizontal direction, so it is modeled as a particle in uniform circular motion in this direction.

Let θ represent the angle between the string and the vertical. In the diagram of forces acting on the ball in the figure, the force $\vec{T}$ exerted by the string on the ball is resolved into a vertical component T cos θ and a horizontal component T sin θ acting toward the center of the circular path. Apply the particle in equilibrium model in the vertical direction:



$$\Sigma F_y = T\cos\theta - mg = 0$$
$$\text{or} \quad T\cos\theta = mg \qquad \text{(i)}$$
$$\Sigma F_x = T\sin\theta = ma_c = \frac{mv^2}{r} \qquad \text{(ii)}$$

Divide equation (ii) by (i) and use $\tan\theta = \dfrac{v^2}{rg}$

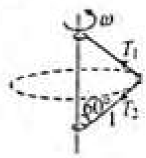
Solve for v: $v = \sqrt{rg\tan\theta}$

Incorporate $r = L \sin u$ from the geometry in figure:

$$v = \sqrt{Lg\sin\theta\tan\theta}$$

Notice that the speed is independent of the mass of the ball.

ILLUSTRATION 6.16 A small block is connected to one end of two identical massless strings of length $16\dfrac{2}{3}$ cm each with their other ends fixed to a vertical rod. If the ratio of tensions T_1/T_2 is 4 : 1, then what will be the angular velocity of the block? Take $g = 9.8\ \text{m s}^{-2}$.

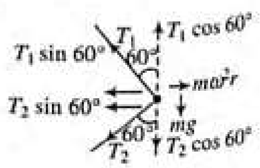


Solution. For horizontal,

$$T_1 \sin 60° + T_2 \sin 60° = m\omega^2 r = m\omega^2 l \sin 60°$$
$$\Rightarrow \quad T_1 + T_2 = m\omega^2 l \qquad \text{(i)}$$

For vertical equilibrium of the block,

$$T_1 \cos 60° = T_2 \cos 60° + mg$$
$$\Rightarrow \quad T_1 - T_2 = \frac{mg}{\cos 60°} = 2\,mg \qquad \text{(ii)}$$



Dividing (i) by (ii), $\dfrac{T_1 + T_2}{T_1 - T_2} = \dfrac{\omega^2 l}{2g}$

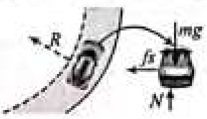
$$\Rightarrow \quad \frac{\frac{T_1}{T_2}+1}{\frac{T_1}{T_2}-1}=\frac{\omega^2 l}{2g} \quad \Rightarrow \quad \frac{4+1}{4-1}=\frac{\omega^2 l}{2g}$$

or $\quad \omega^2=\dfrac{10g}{3l}=\dfrac{10\times 9.8\times 3}{3\times 50\times 10^{-2}}=196$

or $\quad \omega = 14 \text{ rad s}^{-1}$

TURNING OF A VEHICLE ON HORIZONTAL CIRCULAR ROAD

A car moving on a flat, horizontal road negotiates a curve as shown in the figure. If the radius of the curve is R and the coefficient of static friction between the tires and dry pavement is μ_s, let us consider maximum speed the car can have and still it makes the turn successfully.



The force that enables the car to remain in its circular path is the force of static friction. (It is *static* because no slipping occurs at the point of contact between road and tires. The maximum speed, v_{max}, the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,max}=\mu_s N$. Apply equation in the radial direction for the maximum speed condition:

$$f_{s,max}=\mu_s N=\frac{mv_{max}^2}{r} \qquad \text{(i)}$$

The car is in equilibrium the vertical direction:

$$\sum F_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg \qquad \text{(ii)}$$

From equations (i) and (ii), we get

$$v_{max}=\sqrt{\frac{\mu_s NR}{m}}=\sqrt{\frac{\mu_s (mg)R}{m}}=\sqrt{\mu_s gR}$$

Notice that the maximum speed does not depend on the mass of the car, which is why curved highways do not need multiple speed limits to cover the various masses of vehicles using the road.

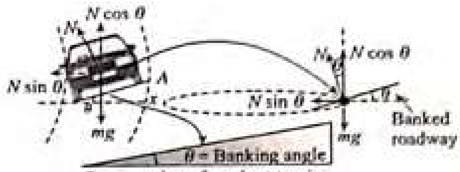
Banking of Roads

The curved roadways are designed in such a way that a vehicle will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. For achieving this, such roads are usually *banked,* which means that the roadway is tilted toward the inside of the curve as seen in the figure.

Banking of curved road provides following purposes:

- To contribute in providing necessary centripetal force
- To reduce frictional wear and tear of tyres
- To avoid skidding
- To avoid overturning of vehicles

Suppose the designated speed for the curved road ramp is to be v and the radius of the curve is R. Let us find at what angle should the curve be banked.



Cross section of roads at turning

The car is modeled as a particle in equilibrium in the vertical direction and a particle in uniform circular motion in the horizontal direction.

Write Newton's second law for the car in the radial direction, which is the x direction:

$$\sum F_r = N\sin\theta = \frac{mv^2}{R} \qquad \text{(i)}$$

Apply the particle in equilibrium model to the car in the vertical direction:

$$\sum F_y = N\cos\theta - mg = 0$$

$$N\cos\theta = mg \qquad \text{(ii)}$$

Divide Eq. (i) by Eq. (ii): $\tan\theta = \dfrac{v^2}{rg}$ (iii)

ILLUSTRATION 6.17 A turn of radius 20 m is banked for the vehicles going at a speed of 36 km h^{-1}. If the coefficient of static friction between the road and the tyre is 0.4, what are the possible speeds of a vehicle so that it neither slips down nor skids up?

Solution. Angle of banking for designed speed,

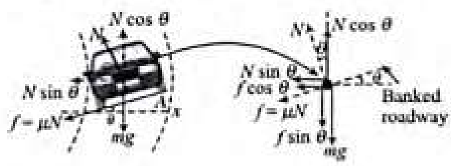
$$\tan\theta = \frac{v_0^2}{Rg} \qquad \text{(i)}$$

$$v_0 = 36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}$$

$$\Rightarrow \quad \tan\theta = \frac{v_0^2}{Rg}=\frac{10^2}{20\times 10}=\frac{1}{2}$$

The vehicle may have the tendency to slide up or down depending on the speed of the vehicle. If speed of the vehicle is more it has tendency to side up and visa-versa.

For speed greater than designed speed: The vehicle has the tendency to slide. Friction will act downward.



In vertical direction $\sum F_y = 0$, $N\cos\theta - \mu N\sin\theta = mg$

In horizontal direction $\sum F_r = \dfrac{mv_{max}^2}{R}$

$$N\sin\theta + \mu N\cos\theta = \frac{mv_{max}^2}{R} \qquad \text{(ii)}$$

$$N\cos\theta - \mu N\sin\theta = mg \qquad \text{(iii)}$$

From (ii) and (iii)

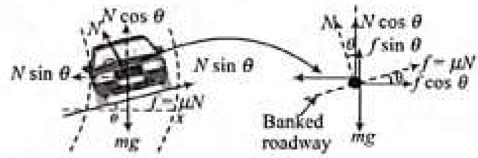
$$\frac{N(\sin\theta+\mu\cos\theta)}{N(\cos\theta-\mu\sin\theta)}=\frac{\frac{mv_{max}^2}{R}}{mg}\Rightarrow\frac{(\sin\theta+\mu\cos\theta)}{\cos\theta-\mu\sin\theta}=\frac{v_{max}^2}{Rg}$$

$$\Rightarrow\quad\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)=\frac{v_{max}^2}{Rg}\qquad\text{(iv)}$$

$$\Rightarrow\quad\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)=\left(\frac{0.5+0.4}{1-0.4\times0.5}\right)=\frac{v_{max}^2}{20\times10}$$

$$\Rightarrow\quad v_{max}=15\ \mathrm{m\,s^{-1}}$$

For speed less than designed speed: The vehicle has tendency to slide down friction will act upward.



In vertical direction $\sum F_y=0$, $N'\cos\theta+\mu N'\sin\theta=mg$ (v)

In horizontal direction $\sum F_r=\dfrac{mv_{max}^2}{R}$

$$N'\sin\theta-\mu N'\cos\theta=\frac{mv'^2}{R}\qquad\text{(vi)}$$

From (v) and (vi), we get $\dfrac{(\sin\theta-\mu\cos\theta)}{\cos\theta+\mu\sin\theta}=\dfrac{v'^2}{Rg}$

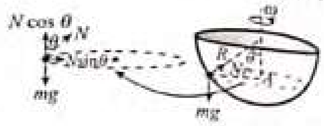
$$\left(\frac{\tan\theta-\mu}{1+\mu\tan\theta}\right)=\frac{v'^2}{Rg}\quad\Rightarrow\quad v_{min}=10\sqrt{\frac{1}{6}}\ \mathrm{m\,s^{-1}}$$

OTHER APPLICATIONS OF NEWTON'S LAWS OF MOTION IN CIRCULAR MOTION

- Remember $\dfrac{mv^2}{r}$ is not a force itself. It is just the value of the net force acting along the inside normal which is responsible for circular motion.
- This force may be friction, normal, tension, spring force, gravitational force or a combination of them. So to solve any problem in uniform circular motion, we identify all the forces acting along the normal (toward's center). Calculate their resultant and equate it to mv^2/r.
- If the circular motion is non-uniform, then in addition to above step we also identify all the forces acting along the tangent to the circular path, calculate their resultant and equate it to mdv/dt or $md|\vec{v}|/dt$.

ILLUSTRATION 6.18 A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is θ, find the angular speed at which the bowl is rotating.

Solution. Let ω be the angular speed of the rotation of the bowl. Two forces are (*a*) normal reaction, N and (*b*) weight, mg.



The ball is rotating in a circle of radius r (= $R\sin\theta$) with center at A at an angular speed ω. Thus,

$$N\sin\theta=mr\omega^2=mR\omega^2\sin\theta$$

$$N=mR\omega^2\qquad\text{(i)}$$

and $\quad N\cos\theta=mg\qquad$ (ii)

Dividing Eqs. (i) by (ii), we get

$$\frac{1}{\cos\theta}=\frac{\omega^2R}{g}$$

$$\omega=\sqrt{\frac{g}{R\cos\theta}}$$

CONCEPT APPLICATION EXERCISE 6.3

1. An amusement park ride consist of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drop away. The coefficient of static friction between person and wall is μ_s, and the radius of the cylinder is R.

   (a) Show that the maximum period of revolution necessary to keep the person from falling is $T=(4\pi^2R\mu_s/g)^{1/2}$.
   (b) Obtain a numerical value for T, taking $R=4.00$ m and $\mu_s=0.400$. How many revolutions per minute does the cylinder make?
   (c) If the rate of revolution of the cylinder is made to be somewhat larger, what happens to the magnitude of each one of the forces acting on the person? What happens to the motion of the person?
   (d) If instead the cylinder's rate of revolution is made to be somewhat smaller, what happens to the magnitude of each one of the forces acting on the person. What happens to the motion of the person?

2. A roller-coaster car has a mass of 500 kg when fully loaded with passengers.

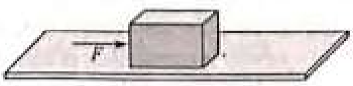
   (a) If the vehicle has a speed of $20.0\ \mathrm{m\,s^{-1}}$ at point A, what is the force exerted by the track on the car at this point?
   (b) What is the maximum speed the vehicle can have at point B and still remain on the track?

3. An air puck of mass m_1 is tied to a string and allowed to revolve in a circle of radius R on a frictionless horizontal table. The other end of the string passes through a small hole in the center of the table, and a load of mass m_2 is tied to the string? The suspended load remains in equilibrium while the puck on the tabletop revolves.

   (a) Find the tension in the string.
   (b) Find the radial force acting on the puck.
   (c) Find the speed of the puck.
   (d) Qualitatively describe what will happen in the motion of the puck if the value of m_2 is somewhat increased by placing an additional load on it.
   (e) Qualitatively describe what will happen in the motion of the puck if the value of m_2 is instead decreased by removing a part from the hanging load.
4. A simple pendulum is oscillating with angular displacement 90°. For what angle with vertical the acceleration of bob directed horizontally?
5. A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1500 at full speed. Consider a particle of mass 1 g sticking at the outer end of a blade.
   (a) How much force does it experience when the fan runs at full speed?
   (b) Who exerts this force on the particle?
   (c) How much force does the particle exert on the blade along its surface?
6. A simple pendulum is constructed by attaching a bob of mass m to a string of length L fixed at its upper end. The bob oscillates in a vertical circle. It is found that the speed of the bob is v when the string makes an angle α with the vertical. Find the tension in the string and the magnitude of net force on the bob at the instant.
7. A car is moving with uniform speed over a circular bridge of radius R which subtends an angle of 90° at its center. Find the minimum possible speed so that the car can cross the bridge without losing the contact any where.

SOLVED EXAMPLES

1. A block of mass 2 kg is kept on the floor. The coefficient of static friction is 0.4. If a force F of 2.5 N is applied on the block as shown in the figure, the frictional force between the block and the floor will be



   (a) 2.5 N (b) 5 N
   (c) 7.84 N (d) 10 N

Sol. (a) Applied force = 2.5 N

Limiting friction $\mu mg = 0.4 \times 2 \times 9.8 = 7.84$ N

For the given condition applied force is very smaller than limiting friction.

$\therefore$ Static friction on a body = Applied force = 2.5 N

2. A body of mass 10 kg slides along a rough horizontal surface. The coefficient of friction is $1/\sqrt{3}$. Taking $g = 10\,\text{m/s}^2$, the least force which acts at an angle of 30° to the horizontal is
   (a) 25 N (b) 100 N
   (c) 50 N (d) $\dfrac{50}{\sqrt{3}}$ N

Sol. (c) Let P force is acting at an angle 30° with the horizontal.

For the condition of motion $F = \mu R$

$$P\cos 30° = \mu(mg - P\sin 30°)$$

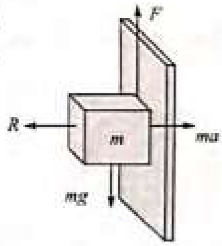
$$\Rightarrow \quad P\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(100 - P\frac{1}{2}\right) \Rightarrow \frac{3P}{2} = \left(100 - \frac{P}{2}\right)$$

$$\Rightarrow \quad 2P = 100 \Rightarrow P = 50 \text{ N}$$

3. A rough vertical board has an acceleration a so that a 2-kg block pressing against it does not fall. The coefficient of friction between the block and the board should be

   (a) $>g/a$ (b) $<g/a$
   (c) $=g/a$ (d) $>a/g$

Sol. (a) For the limiting condition, upward friction force between board and block will balance the weight of the block.



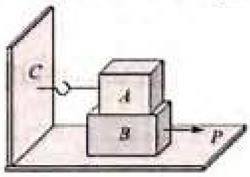
i.e. $F > mg$

$\Rightarrow \quad \mu(R) > mg$

$\Rightarrow \quad \mu(ma) > mg$

$\Rightarrow \quad \mu > \dfrac{g}{a}$

4. Block A weighing 100 kg rests on a block B and is tied with a horizontal string to the wall at C. Block B weighs 200 kg. The coefficient of friction between A and B is 0.25 and between B and the surface is 1/3. The horizontal force P necessary to move the block B should be ($g = 10\text{ m/s}^2$)



   (a) 1150 N (b) 1250 N
   (c) 1300 N (d) 1420 N

Sol. (b) Friction between block A and block B and between block B and surface will oppose the P

$$\therefore \quad P = F_{AB} + F_{BS}$$
$$= \mu_{AB} m_A g + \mu_{BS}(m_A + m_B)g$$

$$= 0.25 \times 100 \times 10 + \frac{1}{3}(100+200)\times 10 = 1250 \text{ N}$$

5. A body B lies on a smooth horizontal table and another body A is placed on B. The coefficient of friction between A and B is μ. What acceleration given to B will cause slipping to occur between A and B?

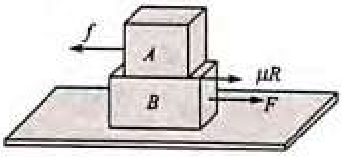
(a) μg (b) g/μ

(c) μ/g (d) $\sqrt{\mu g}$

Sol. (a) There is no friction between the body B and surface of the table. If the body B is pulled with force F, then

$F = (m_A + m_B)a$

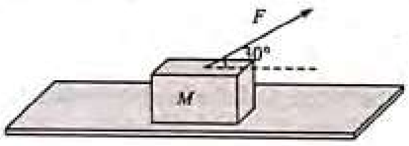
Due to this force, upper body A will feel the pseudo force in a backward direction.



$f = m_A \times a$

But due to friction between A and B, the body will not move. Body A will start moving when pseudo force is more than friction force, i.e., for slipping, $m_A a = \mu m_A g \Rightarrow a = \mu g$

6. A block of mass $M = 5$ kg is resting on a rough horizontal surface for which the coefficient of friction is 0.2. When a force $F = 40$ N is applied, the acceleration of the block will be ($g = 10$ m/s^2)

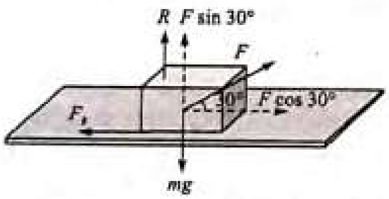


(a) 5.73 m/s^2 (b) 8.0 m/s^2

(c) 3.17 m/s^2 (d) 10.0 m/s^2

Sol. (a) Kinetic friction $= \mu_k R = 0.2(mg - F\sin 30°)$

$$= 0.2\left(5\times 10 \times 40 \times \frac{1}{2}\right) = 0.2(50-20) = 6 \text{ N}$$



$$\text{Acceleration of the block} = \frac{F\cos 30° - \text{Kinetic friction}}{\text{Mass}}$$

$$= \frac{40 \times \frac{\sqrt{3}}{2} - 6}{5} = 5.73 \text{ m/s}^2$$

7. A body of mass M is kept on a rough horizontal surface (friction coefficient μ). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on the body is F, where

(a) $F = Mg$ (b) $F = \mu Mgf$

(c) $Mg \le F \le Mg\sqrt{1+\mu^2}$ (d) $Mg \ge F \ge Mg\sqrt{1+\mu^2}$

Sol. (c) Maximum force by surface when friction works

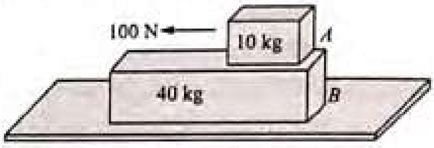
$$\sqrt{f^2 + R^2} = \sqrt{(\mu R)^2 + R^2} = R\sqrt{\mu^2+1}$$

Minimum force $= R$ when there is no friction

Hence, ranging from R to $R\sqrt{\mu^2+1}$ we get,

$$Mg \le F \le Mg\sqrt{\mu^2+1}$$

8. A 40 kg slab rests on a frictionless floor as shown in the figure. A 10 kg block rests on the top of the slab. The static coefficient of friction between the block and slab is 0.60 while the kinetic friction is 0.40. The 10 kg block is acted upon by a horizontal force 100 N. If $g = 9.8$ m/s^2, the resulting acceleration of the slab will be



(a) 1 m/s^2 (b) 1.5 m/s^2

(c) 2 m/s^2 (d) 6 m/s^2

Sol. (a) Limiting friction between block and slab

$= \mu_s m_A g = 0.6 \times 10 \times 10 = 60$ N

But applied force on block A is 100 N. So, the block will slip over a slab.

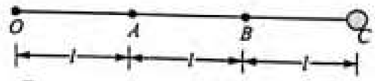
Now kinetic friction works between block and slab

$F_k = \mu_k m_A g = 0.4 \times 10 \times 10 = 40$ N

This kinetic friction helps to move the slab

$\therefore$ Acceleration of slab $\dfrac{40}{m_g} = \dfrac{40}{40} = 1$ m/s^2

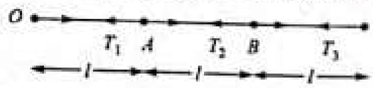
9. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is



(a) 3 : 5 : 7 (b) 3 : 4 : 5

(c) 7 : 11 : 6 (d) 3 : 5 : 6

Sol. (d) Let ω is the angular speed of revolution



$T_3 = m\omega^2 3l$

$T_2 - T_3 = m\omega^2 2l \Rightarrow T_2 = m\omega^2 5l$

$T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = m\omega^2 6l$

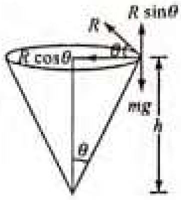
$T_3 : T_2 : T_1 = 3 : 5 : 6$

10. A particle describes a horizontal circle in a conical funnel whose inner surface is smooth with speed of 0.5 m/s. What is the height of the plane of circle from vertex of the funnel?

(a) 0.25 cm (b) 2 cm

(c) 4 cm (d) 2.5 cm

Sol. (d) The particle is moving in circular path



From the figure, $mg = R\sin\theta$ ...(i)

$$\frac{mv^2}{r} = R\cos\theta \quad \text{...(ii)}$$

From equation (i) and (ii) we get

$$\tan\theta = \frac{rg}{v^2} \text{ but } \tan\theta = \frac{r}{h}$$

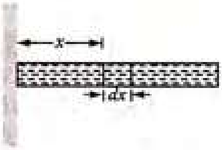
$$\therefore \quad h = \frac{v^2}{g} = \frac{(0.5)^2}{10} = 0.025\text{ m} = 2.5\text{cm}$$

11. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\dfrac{ML\omega^2}{2}$ (b) $ML\omega^2$

(c) $\dfrac{ML\omega^2}{4}$ (d) $\dfrac{ML^2\omega^2}{2}$

Sol. (a) $dM = \left(\dfrac{M}{L}\right)dx$



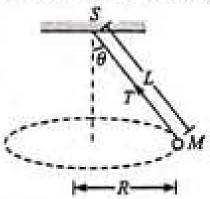
Force on dM mass is

$$dF = (dM)\,\omega^2 x$$

By integration, we can get the force exerted by whole liquid

$$\Rightarrow \quad F = \int_0^L \frac{M}{L}\omega^2 x\,dx = \frac{1}{2}M\omega^2 L$$

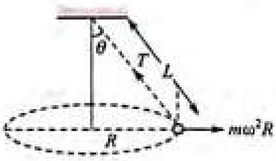
12. A string of length L is fixed at one end and carries a mass M at the other end. The string makes $2/\pi$ revolutions per second around the vertical axis through the fixed end as shown in the figure. Then tension in the string is



(a) ML (b) $2ML$

(c) $4ML$ (d) $16ML$

Sol. (d)



$$T\sin\theta = M\omega^2 R \quad \text{...(i)}$$

$$T\sin\theta = M\omega^2 L\sin\theta \quad \text{...(ii)}$$

From (i) and (ii)

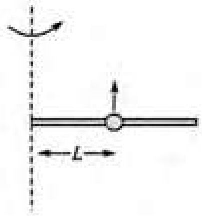
$$T = M\omega^2 L = M4\pi^2 n^2 L = M\,4\pi^2\left(\frac{2}{\pi}\right)^2 L = 16ML$$

13. A long horizontal rod has a bead which can slide along its length, and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(a) $\sqrt{\dfrac{\mu}{\alpha}}$ (b) $\dfrac{\mu}{\sqrt{\alpha}}$

(c) $\dfrac{1}{\sqrt{\mu\alpha}}$ (d) Infinitesimal

Sol. (a) Let the bead starts slipping after time t.



For critical condition

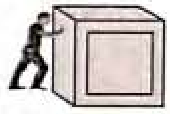
Frictional force provides the centripetal force

$$m\omega^2 L = \mu R = \mu m \times a_t = \mu L m\alpha$$

$$\Rightarrow \quad m\omega^2 L = \mu R = \mu m \times a_t = \mu L m\alpha$$

$$\Rightarrow \quad m(\alpha t)^2 L = \mu m L\alpha \quad t = \sqrt{\frac{\mu}{\alpha}} \qquad (\text{As } \omega = \alpha t)$$

14. A man of mass 75 kg is pushing a heavy box on a flat floor. The coefficient of kinetic and static friction between the floor and the box is 0.20, and the coefficient of static friction between the man's shoes and the floor is 0.80. If the man pushes horizontally (see figure), what is the maximum mass (in kg) of the box he can move?



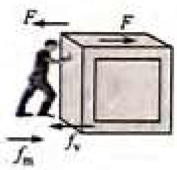
(a) 300 kg (b) 600 kg

(c) 900 kg (d) None of these

Sol. (a) From FBD of box, the box will move if

$$F \le f_b$$

$$\Rightarrow \quad F \ge 0.2mg \quad \text{(i)}$$



where m is the mass of box.

If man is moving the box, then there is no slipping of shoes of man on floor. So friction on man f_m will be static.

From FBD of man $\quad f_m = F$

but $\quad f_m \le \mu_s 75g$

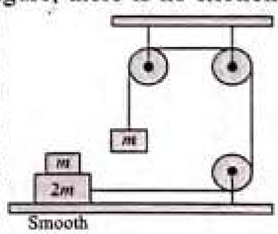
$$\Rightarrow \quad F \le \mu_s 75g \quad \text{(ii)}$$

From (i) and (ii)

$$0.2mg \le F \le \mu_s 75g \quad \Rightarrow \quad 0.2mg \le \mu_s 75g$$

$$\Rightarrow \quad m \le \frac{0.8}{0.2}75 \quad \Rightarrow \quad m \le 300\text{ kg}$$

15. In the arrangement shown in figure, there is no friction between the block of mass $2m$ and ground but there is friction between the blocks of masses m and $2m$. The block of mass m is stationary with respect to block of mass $2m$. The least value of coefficient of friction between m and $2m$ is



(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

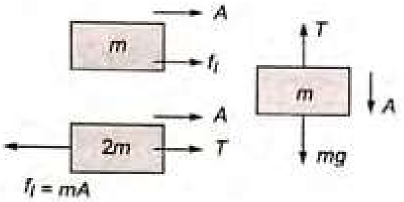
Sol. (c) For least value of coefficient of friction, limiting friction will exist between m and $2m$.

$mg - T = mA$

$T - f_l = 2\,mA$

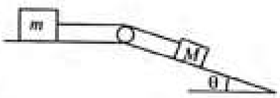
$f_l = mA$

Solving. $f_l = \dfrac{mg}{4}$



$$\mu mg = \frac{mg}{4} \;\Rightarrow\; \mu = \frac{1}{4}$$

16. Find the maximum value of (M/m) in the situation shown in figure so that the system remains at rest. Friction coefficient of both the contacts is μ, string is massless and pulley is friction less.



(a) $\dfrac{\cos\theta}{\sin\theta - \mu\cos\theta}$ (b) $\dfrac{\sin\theta}{\sin\theta - \mu\cos\theta}$

(c) $\dfrac{\mu\cos\theta}{\sin\theta - \mu\cos\theta}$ (d) $\dfrac{\mu}{\sin\theta - \mu\cos\theta}$

Sol. (d) The system is at rest ($F_{net} = 0$)

For maximum M/m ; Limiting friction will be acting on both blocks (at contact surfaces).

$\mu g + \mu Mg\cos\theta$ = Net pulling force on the whole system

$\mu mg + \mu Mg\cos\theta = Mg\sin\theta$

$Mg(\sin\theta - \mu\cos\theta) = \mu mg.$

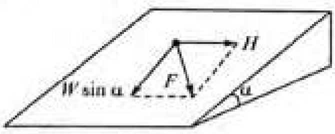
$$\frac{M}{m} = \frac{\mu}{(\sin\theta - \mu\cos\theta)}$$

17. A particle of weight W rests on a rough inclined plane which makes an angle α with the horizontal. If the coefficient of static friction $\mu = 2\tan\alpha$, find the horizontal force H acting transverse to the slope of the plane when the particle is about to slip.

(a) $2W\sin\alpha$ (b) $W\sin\alpha$

(c) $\dfrac{\sqrt{3}}{2}W\sin\alpha$ (d) $W\sqrt{3}\sin\alpha$

Sol. (d) Net force: $F = \sqrt{(W\sin\alpha)^2 + H^2}$



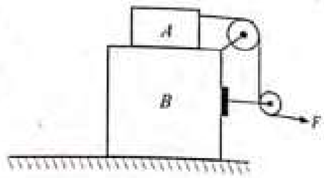
When the particle is about to slip:

$F = \mu N$

$\Rightarrow \sqrt{(W\sin\alpha)^2 + H^2} = (2\tan\alpha)\,W\cos\alpha$

$\Rightarrow H = \sqrt{3}\,W\sin\alpha$

18. In the arrangement shown in figure, $m_A = m_B = 2$ kg. String is massless and pulley is frictionless. Block B is resting on a smooth horizontal surface, while friction coefficient between blocks A and B is $\mu = 0.5$. The maximum horizontal force F that can be applied so that block A does not slip over the block B is



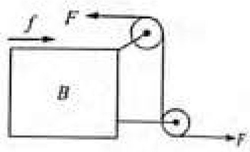
(a) 25 N (b) 40 N

(c) 30 N (d) 20 N

Sol. (d) Maximum value of friction on A,

$$f_{max} = \mu mg = 0.5 \times 2 \times 10 = 10\text{ N}$$

Hence, the block B is subjected to this frictional force in the forward direction. Since block A does not slip on B, both of them move with a common acceleration. The acceleration of B is the same as the acceleration of A.

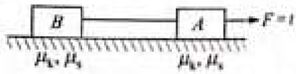


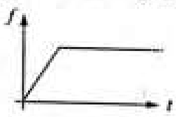
To determine acceleration of a, we find that $m_B a_{max} = f_{max}$

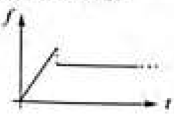
$$a_{max} = \frac{10}{2} = 5\text{m/s}^2$$

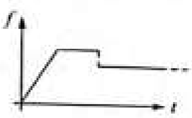
$$F_{max} = (m_A + m_B)a_{max} = 4 \times 5 = 20\text{ N}$$

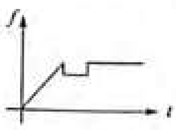
19. A force $F = t$ is applied to a block A as shown in figure, where t is time in seconds. The force is applied at $t = 0$ seconds when the system was at rest. Which of the following graph correctly gives the frictional force between A and horizontal surface as a function of time t. [Assume that at $t = 0$, tension in the string connecting the two blocks is zero].



(a) 

(b) 

(c) 

(d) 

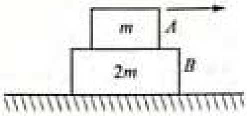
Sol. (c) Let m_A and m_B be the mass of blocks A and B, respectively.

As the force F increases from 0 to $\mu_s m_A g$, the frictional force on block A is such that $f = F$. When $F = \mu_s m_A g$, the frictional force f attains maximum value $f = \mu_s m_A g$.

As F is further increased to $\mu_s (m_A + m_B)g$, the block A does not move. In this duration frictional force on block A remains constant at $\mu_s m_A g$. After this blocks start moving and friction decrease.

Hence, (c) is correct choice.

20. In the figure block A of mass m is placed on block B of mass $2m$. B rests on floor. The coefficient of friction between A and B is μ and that between B and floor is $\mu/2$. Both blocks are given same initial velocity to the right. What is the acceleration of A with respect to B?



(a) $\frac{\mu g}{2}$ towards right

(b) μg towards left

(c) $\frac{\mu g}{4}$ towards left

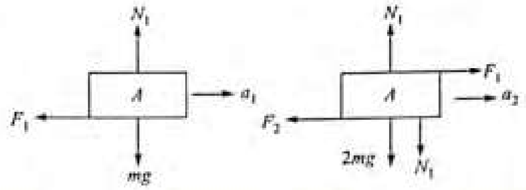
(d) $\frac{3\mu g}{4}$ towards left

Sol. (d) $F_1 = +\mu N_1$

$ma_1 = -\mu N_1$

$a_1 = -\mu g$

$F_1 - F_2 = (2m)\, a_2$



$$\mu mg - \frac{\mu}{2}(3mg) = 2ma_2$$

$$a_2 = -\frac{\mu g}{4}$$

Acceleration of A with respect to B

$$-\mu g - \left(-\frac{\mu g}{4}\right) = -\frac{3\mu g}{4}$$

$\Rightarrow$ $\frac{3\mu g}{4}$ towards left.

EXERCISES

Basic Concept of Static and Kinetic Friction

1. A box of mass 8 kg is placed on a rough inclined plane of inclination θ. Its downward motion can be prevented by applying an upward pull F and it can be made to slide upwards by applying a force $2F$. The coefficient of friction between the box and the inclined plane is

(a) $(\tan\theta)/3$ (b) $3\tan\theta$
(c) $(\tan\theta)/2$ (d) $2\tan\theta$

2. A horizontal force just sufficient to move a body of mass 4 kg lying on a rough horizontal surface is applied on it. The coefficient of static and kinetic friction between the body and the surface are 0.8 and 0.6, respectively. If the force continues to act even after the block has started moving, the acceleration of the block in $m\,s^{-2}$ is ($g = 10\ m\,s^{-2}$)

(a) 1/4 (b) 1/2 (c) 2 (d) 4

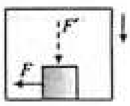
3. A passenger is travelling a train moving at 40 $m\,s^{-1}$. His suitcase is kept on the berth. The driver of train applies breaks such that the speed of the train decreases at a constant rate to 20 $m\,s^{-1}$ in 5 s. What should be the minimum coefficient of friction between the suitcase and the berth if the suitcase is not to slide during retardation of the train?

(a) 0.3 (b) 0.5 (c) 0.1 (d) 0.2

4. A rectangular wooden box 10 cm × 20 cm × 40 cm in size is kept on a horizontal surface with its face of largest area on the surface. A minimum force of 10 N applied parallel to the surface sets the box in sliding motion along the surface. If the box is now kept with its face of smaller area in contact with the surface, the minimum force applied parallel to the surface, to set the box in motion, is

(a) less than 10 N
(b) may be greater or less than 10 N
(c) greater than 10 N
(d) equal to 10 N

5. A block of mass m is kept on the floor of a freely falling lift. During the free fall of the lift, the block is pulled horizontally with a force of $F = 5$ N. $\mu_s = 0.1$. The frictional force on the block will be



(a) 5 N (b) 2 N (c) zero (d) 10 N

6. If in previous problem, an additional force $F' = 100$ N is applied in vertical direction as shown in figure. The friction force acting on the block is

(a) zero (b) 10 N (c) 20 N (d) 5 N

7. Figure shows a wooden block at rest in equilibrium on a rough horizontal plane being acted upon by forces $F_1 = 10$ N, $F_2 = 2$ N as shown.

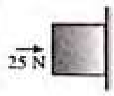
If F_1 is removed, the resultant force acting on the block will be

(a) 2 N towards left (b) 2 N towards right
(c) zero (d) cannot be determined

8. A lift is moving upwards with a uniform velocity v in which a block of mass m is lying. The frictional force offered by the block, when coefficient of friction is $\mu = 0.5$, will be

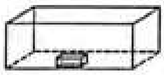
(a) zero (b) $mg/2$ (c) mg (d) $2mg$

9. A horizontal force of 25 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.4. The weight of the block is



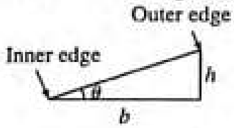
(a) 2.5 N (b) 20 N (c) 10 N (d) 5 N

10. A solid block of mass 2 kg is resting inside a cube as shown in the figure. The cube is moving with a velocity $\vec{v} = 5\hat{i} + 2\hat{j}$ m s^{-1}. If the coefficient of friction between the surface of cube and block is 0.2, then the force of friction between the block and cube is



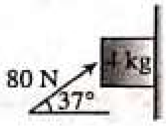
(a) 10 N (b) 4 N (c) 14 N (d) Zero

11. A vehicle is moving with a velocity v on a curved road of width b and radius of curvature R. For counteracting the centrifugal force on the vehicle, the difference in elevation required in between the outer and inner edges of the rod is



(a) v^2b/Rg (b) vb/Rg (c) vb^2/Rg (d) vb/R^2g

12. A block of mass 4 kg is pressed against the wall by a force of 80 N as shown in the figure. Determine the value of friction force and block's acceleration (take $\mu_s = 0.2$, $\mu_k = 0.15$).



(a) 8 N, 0 m s^{-2} (b) 32 N, 6 m s^{-2}
(c) 8 N, 6 m s^{-2} (d) 32 N, 2 m s^{-2}

13. A block of mass m, lying on a horizontal plane, is acted upon by a horizontal force P and another force Q, inclined at an angle θ to the vertical. The block will remain in equilibrium if the coefficient of friction between it and the surface is (assume $P > Q$)

(a) $(P \sin\theta - Q)/(mg - \cos\theta)$
(b) $(P - Q\sin\theta)/(mg + Q\cos\theta)$
(c) $(P\cos\theta + Q)/(mg - Q\cos\theta)$
(d) $(P + Q\sin\theta)/(mg + Q\cos\theta)$

14. A given object takes n times more time to slide down 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is

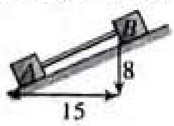
(a) $\sqrt{\dfrac{1}{1-n^2}}$ (b) $\sqrt{1-\dfrac{1}{n^2}}$

(c) $1-\dfrac{1}{n^2}$ (d) $\dfrac{1}{2-n^2}$

15. The upper half of an inclined plane with inclination ϕ is perfectly smooth while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom if the coefficient of friction for the lower half is given by

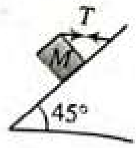
(a) $2\tan\phi$ (b) $\tan\phi$ (c) $2\sin\phi$ (d) $2\cos\phi$

16. Blocks A and B in the figure are connected by a bar of negligible weight. Mass of each block is 170 kg and $\mu_A = 0.2$ and $\mu_B = 0.4$, where μ_A and μ_B are the coefficients of limiting friction between blocks and plane, calculate the force developed in the bar ($g = 10\ \text{m s}^{-2}$)



(a) 150 N (b) 75 N (c) 200 N (d) 250 N

17. A block of mass 15 kg is resting on a rough inclined plane as shown in the figure. The block is tied by a horizontal string which has a tension of 50 N. The coefficient of friction between the surfaces of contact is



(a) 1/2 (b) 2/3 (c) 3/4 (d) 1/4

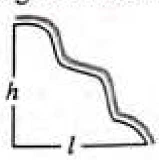
18. In the figure, the tension in the rope (rope is light) is

(a) $(M + m)g\sin\theta$
(b) $(M + m)g\sin\theta - \mu mg\cos\theta$
(c) Zero
(d) $(M + m)g\cos\theta$

19. A house is built on the top of a hill with 45° slope. Due to the sliding of material and sand from top to the bottom of hill, the slope angle has been reduced. If the coefficient of static friction between sand particles is 0.75, what is the final angle attained by hill? ($\tan^{-1} 0.75 \simeq 37°$)

(a) 8° (b) 45° (c) 37° (d) 30°

20. A uniform chain is placed at rest on a rough surface of base length l and height h on an irregular surface as shown in the figure. Then, the minimum coefficient of friction between the chain and the surface must be equal to

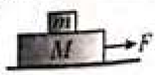


(a) $\mu = \dfrac{h}{2l}$ (b) $\mu = \dfrac{h}{l}$

(c) $\mu = \dfrac{3h}{2l}$ (d) $\mu = \dfrac{2h}{3l}$

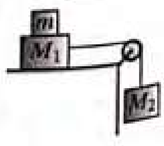
Application of Newton's Laws of Motion with Friction

21. A block of mass m is placed on another block of mass M, which itself is lying on a horizontal surface. The coefficient of friction between two blocks is μ_1 and that between the block of mass M and horizontal surface is μ_2. What maximum horizontal force can be applied to the lower block so that the two blocks move without separation?



(a) $(M + m)(\mu_2 - \mu_1)g$ (b) $(M - m)(\mu_2 - \mu_1)g$
(c) $(M - m)(\mu_2 + \mu_1)g$ (d) $(M + m)(\mu_2 + \mu_1)g$

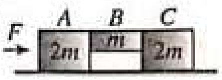
22. Two blocks of masses M_1 and M_2 are connected with a string passing over a pulley as shown in the figure. The block M_1 lies on a horizontal surface. The coefficient of friction between the block M_1 and the horizontal surface is μ. The system accelerates. What additional mass m should be placed on the block M_1 so that the system does not accelerate?



(a) $\dfrac{M_2 - M_1}{\mu}$ (b) $\dfrac{M_2}{\mu} - M_1$

(c) $M_2 - \dfrac{M_1}{\mu}$ (d) $(M_2 - M_1)\mu$

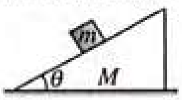
23. A system is pushed by a force F as shown in the figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from down ward slipping is



(a) $\left(\dfrac{3}{2\mu}\right)mg$ (b) $\left(\dfrac{5}{2\mu}\right)mg$

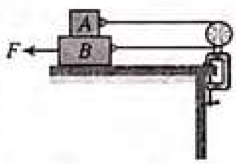
(c) $\left(\dfrac{5}{2}\right)\mu mg$ (d) $\left(\dfrac{3}{2}\right)\mu mg$

24. In the figure shown, the block of mass m is at rest relative to the wedge of mass M and the wedge is at rest with respect to ground. This implies that



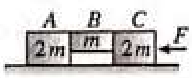
(a) Net force applied by m on M is mg.
(b) Normal force applied by m on M is mg.
(c) Force of friction applied by m on M is mg.
(d) None of the above.

25. Block A, as shown in the figure weighs 2.0 N and block B weighs 6.0 N. The coefficient of kinetic friction between all surfaces is 0.25. Find the magnitude of the horizontal force necessary to drag block B to the left at constant speed if A and B are connected by a light, flexible cord passing around a fixed, frictionless pulley.



(a) 2 N (b) 3 N (c) 5 N (d) 6 N

26. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ. Minimum value of F to prevent block B from downward slipping is



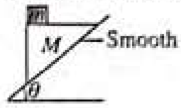
(a) $\left(\dfrac{5}{2\mu}\right)mg$ (b) $\left(\dfrac{5}{3\mu}\right)mg$

(c) $\left(\dfrac{5}{2}\right)\mu mg$ (d) $\left(\dfrac{3}{2}\right)\mu mg$

27. A block of mass M is being pulled along rough horizontal surface. The coefficient of friction between the block and the surface is μ. If another block of mass $M/2$ is placed on the block and it is again pulled on the surface, the coefficient of friction between the block and the surface will be

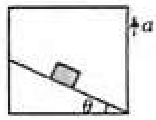
(a) μ (b) $\dfrac{3\mu}{2}$ (c) 2μ (d) $\dfrac{5\mu}{2}$

28. A triangular prism of mass M with a block of mass m placed on it is released from rest on a smooth inclined plane of inclination θ. The block does not slip on the prism. Then



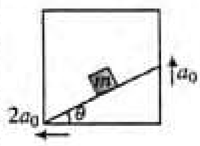
(a) The acceleration of the prism is $g \cos \theta$.
(b) The acceleration of the prism is $g \tan \theta$.
(c) The minimum coefficient of friction between the block and the prism is $\mu_{\min} = \cot \theta$.
(d) The minimum coefficient of friction between the block and the prism is $\mu_{\min} = \tan \theta$.

29. A block of mass m is at rest with respect to a rough incline kept in elevator moving up with acceleration a. Which of following statements is correct?



(a) The contact force between block and incline is parallel to the incline.
(b) The contact force between block and incline is of the magnitude $m(g + a)$.
(c) The contact force between block and incline is perpendicular to the incline.
(d) The contact force is of the magnitude $mg \cos \theta$

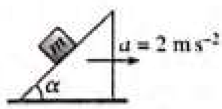
30. For the situation shown in the figure, the block is stationary w.r.t. incline fixed in an elevator. The elevator is having an acceleration of $\sqrt{5}\,a_0$ whose components are shown in the figure. The surface is rough and coefficient of static friction between the incline and block is μ_s. Determine the magnitude of force exerted by incline on the block. (Take $a_0 = g/2$ and $\theta = 37°$, $\mu_s = 0.2$.)



(a) $\dfrac{mg}{10}$ (b) $\dfrac{9mg}{25}$

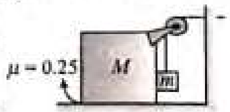
(c) $\dfrac{3mg}{25} \times \sqrt{41}$ (d) $\dfrac{\sqrt{13}\,mg}{2}$

31. A block of mass m is lying on a wedge having inclination angle $\alpha = \tan^{-1}\left(\dfrac{1}{5}\right)$. Wedge is moving with a constant acceleration $a = 2\text{ m s}^{-2}$. The minimum value of coefficient of friction μ so that m remains stationary w.r.t.wedge is



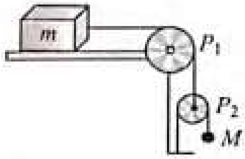
(a) 2/9 (b) 5/12 (c) 1/5 (d) 2/5

32. Two blocks (m and M) are arranged as in the figure. There is friction between ground and M only and other surfaces are frictionless. The coefficient of friction between ground and M is μ = 0.25. The maximum ratio of m and M (m/M) so that the system remains is at rest



(a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) 3 (d) $\frac{5}{2}$

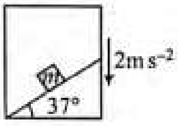
33. In the pulley arrangement shown in the figure, the pulley P_2 is movable. Assuming the coefficient of friction between m and surface to be μ, the minimum value of M for which m is at rest is



(a) $M = \frac{\mu m}{2}$ (b) $m = \frac{\mu M}{2}$

(c) $M = \frac{m}{2\mu}$ (d) $m = \frac{M}{2\mu}$

34. A block of mass m is kept on an inclined plane of a lift moving down with acceleration of $2\ \text{m s}^{-2}$. What should be the minimum coefficient of friction to let the block move down with constant velocity?



(a) $\mu = \frac{1}{\sqrt{3}}$ (b) $\mu = 0.4$

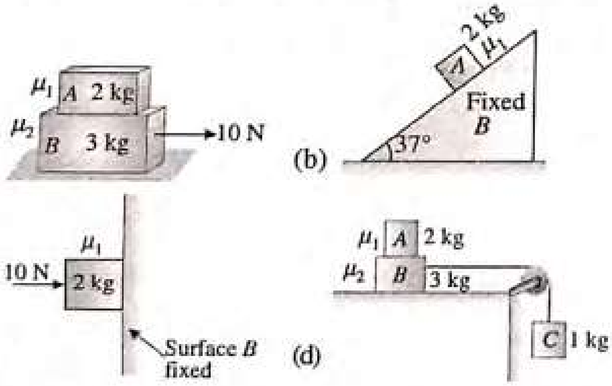
(c) $\mu = 0.8$ (d) $\mu =$ not defined

35. A body of mass M is resting on a rough horizontal plane surface, the coefficient of friction being equal to μ. At $t = 0$, a horizontal force $F = F_0 t$ starts acting on it, where F_0 is a constant. Find the time T at which the motion starts?

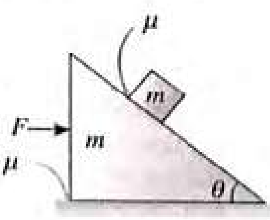
(a) $\mu Mg/F_0$ (b) $Mg/\mu F_0$

(c) $\mu F_0/Mg$ (d) None of these

36. In which of the following cases the friction force between 'A' and 'B' is maximum. In all cases $\mu_1 = 0.5$, $\mu_2 = 0$.



37. In the situation shown in figure a wedge of mass m is placed on a rough surface, on which a block of equal mass is placed on the inclined plane of wedge. Friction coefficient between plane and the block and the ground and the wedge (μ). An external force F is applied horizontally on the wedge. Given that m does not slide on incline due to its weight.



The value of F at which wedge will start slipping is:

(a) $= \mu\, mg$ (b) $= (3/2)\, \mu\, mg$

(c) $> 2\mu\, mg$ (d) $< \mu\, mg$

38. If in question no. 37, the wedge and block moves together. Then the value of F at which no friction will act on block on inclined plane, is

(a) $2\mu\, mg$ (b) $2\mu\, mg + 2\, mg \tan\theta$

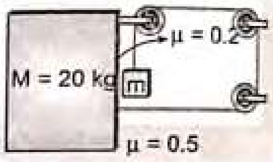
(c) $2\mu\, mg + mg \tan\theta$ (d) $2\mu\, mg + mg \sin\theta$

39. The minimum value of acceleration of wedge for which the block starts sliding on the wedge, is:

(a) $g\left(\frac{\cos\theta + \mu \sin\theta}{\sin\theta - \mu\cos\theta}\right)$ (b) $g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$

(c) $g\left(\frac{\sin\theta - \mu\cos\theta}{\sin\theta + \mu\cos\theta}\right)$ (d) $g\left(\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta}\right)$

40. In the arrangement shown in the figure, A block of mass $M = 20$ kg is placed on rough horizontal surface with $\mu = 0.5$. A block of mass m is arranged as shown. The maximum value of 'm' in order that system remains an equilibrium:



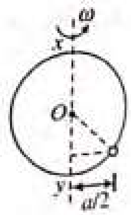
(a) $\frac{10}{3}$ kg (b) 10 kg (c) $\frac{40}{3}$ kg (d) $\frac{20}{3}$ kg

Dynamics of Circular Motion

41. A coin is placed at the edge of a horizontal disc rotating about a vertical axis through its axis with a uniform angular speed 2 rad s^{-1}. The radius of the disc is 50 cm. Find the minimum coefficient of friction between disc and coin so that the coin does not slip ($g = 10\ \text{m s}^{-2}$).

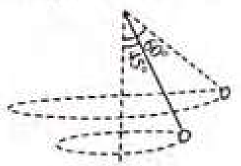
(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

42. A small ring P is threaded on a smooth wire bent in the form of a circle of radius a and center O. The wire is rotating with constant angular speed ω about a vertical diameter XY, while the ring remains at rest relative to the wire at a distance $a/2$ from XY. Then ω^2 is equal to



(a) $\frac{2g}{a}$ (b) $\frac{g}{2a}$ (c) $\frac{2g}{a\sqrt{3}}$ (d) $\frac{g\sqrt{3}}{2a}$

43. When the string of a conical pendulum makes an angle of 45° with the vertical, its time period is T_1. When the string makes an angle of 60° with the vertical, its time period is T_2. Then T_1^2/T_2^2 is



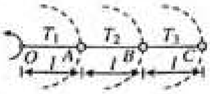
(a) 2

(b) $\sqrt{2}$

(c) 0.5

(d) none of these

44. Three identical particles are joined together by a thread as shown in the figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string (T_1: T_2: T_3 = ?) is

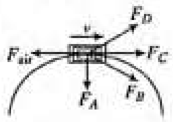


(a) 6 : 5 : 3 (b) 3 : 5 : 6
(c) 3 : 4 : 5 (d) none of these

45. A particle is projected with a speed $v_0 = \sqrt{gR}$. The coefficient of friction between the particle and the hemi-spherical plane is $\mu = 0.5$. Then, the acceleration of the particle is

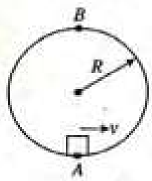
(a) $\sqrt{2}\,g \searrow$ (b) $\frac{\sqrt{5}\,g}{2} \nwarrow$
(c) $\sqrt{2}\,g \nwarrow$ (d) none of these

46. A car travels with constant speed on a circular road on level ground. In the figure, F_{air} is the force of air resistance on the car. Air resistance acts opposite to the motion of car. Which of the other forces shown best represents the horizontal force of the road on the car's tires?



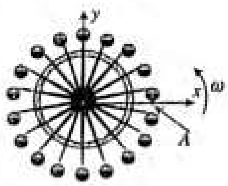
(a) F_C (b) F_D (c) F_A (d) F_B

47. A block of mass m is projected on a smooth horizontal circular track with velocity v. What is the average normal force exerted by the circular walls on the block during its motion from A to B?



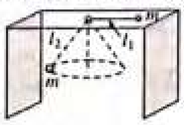
(a) $\frac{mv^2}{R}$ (b) $\frac{mv^2}{\pi R}$ (c) $\frac{2mv^2}{R}$ (d) $\frac{2mv^2}{\pi R}$

48. Consider the setup of a Ferris wheel in an amusement park. The wheel is turning in a counterclockwise manner. Contrary to the illustration, not all seats are aligned horizontally, i.e., parallel to the x-axis. Determine the orientation of the normal to seat as it passes point A.



(a) parallel to the x-axis
(b) in the first/third quadrants
(c) parallel to the y-axis
(d) in the second/fourth quadrants

49. Two identical particles are attached at the ends of a light string which passes through a hole at the center of a table. One of the particles is made to move in a circle on the table with angular velocity ω_1 and the other is made to move in a horizontal circle as a contact pendulum with angular velocity ω_2. If l_1 and l_2 are the length of the string over and under the table, then in order that particle under the table neither moves down nor moves up, the ratio l_1/l_2 is

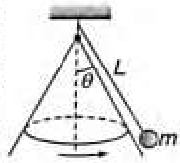


(a) $\frac{\omega_1}{\omega_2}$ (b) $\frac{\omega_2}{\omega_1}$ (c) $\frac{\omega_1^2}{\omega_2^2}$ (d) $\frac{\omega_2^2}{\omega_1^2}$

50. A circular road of radius 1000 m has banking angle 45°. The maximum safe speed (in m s^{-1}) of a car having a mass 2000 kg will be (if the coefficient of friction between tyre and road is 0.5)
(a) 172 (b) 124 (c) 99 (d) 86

51. A circular table of radius 0.5 m has a smooth diametrical groove. A ball of mass 90 g is placed inside the groove along with a spring of spring constant 10^2 N cm^{-1}. One end of the spring is tied to the edge of the table and the other end to the ball. The ball is at a distance of 0.1 m from the center when the table is at rest. On rotating the table with a constant angular frequency of 10^2 rad s^{-1}, the ball moves away from the center by a distance nearly equal to
(a) 10^{-1} m (b) 10^{-2} m (c) 10^{-3} m (d) 2×10^{-1} m

52. A small sized mass m is attached by a massless string (of length L) to the top of a fixed frictionless solid cone whose axis is vertical. The half angle at the vertex of the cone is θ. If the mass m moves around in a horizontal circle at speed v, what is the maximum value of v for which mass stays in contact with the cone? (g is acceleration due to gravity)



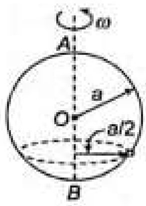
(a) $\sqrt{gL\cos\theta}$ (b) $\sqrt{gL\sin\theta}$
(c) $\sqrt{gL\sin\theta\tan\theta}$ (d) $\sqrt{gL\tan\theta}$

53. A circular road of radius R is banked for a speed v = 40 km/hr. A car of mass m attempts to go on the circular road, the friction co-efficient between the tyre and road is negligible:
(a) the car cannot make a turn without skidding
(b) if the car runs at a speed less than 40 km/hr, it will slip up the slope
(c) if the car runs at the correct speed of 40 km/hr, the force by the road on the car is equal to mv^2/r
(d) if the car runs at the correct speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than mv^2/r

54. An automobile enters a turn of radius R. If the road is banked at an angle of 45° and the coefficient of friction is 1, the minimum and maximum speed with which the automobile can negotiate the turn without skidding is
(a) $\sqrt{\frac{rg}{2}}$ and $\sqrt{rg}$ (b) $\frac{\sqrt{rg}}{2}$ and $\sqrt{rg}$
(c) $\frac{\sqrt{rg}}{2}$ and $2\sqrt{rg}$ (d) 0 and infinite

Problems Based on Mixed Concepts

55. A smooth wire is bent into a vertical circle of radius a. A bead P can slide smoothly on the wire. The circle is rotated about vertical diameter AB as axis with a speed ω as shown in figure. The bead P is at rest w.r.t. the circular ring in the position shown. Then ω^2 is equal to:



(a) $\frac{2g}{a}$ (b) $\frac{2g}{a\sqrt{3}}$ (c) $\frac{g\sqrt{3}}{a}$ (d) $\frac{2a}{g\sqrt{3}}$

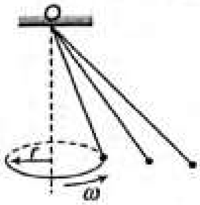
56. A particle is attached to an end of a rigid rod. The other end of the rod is hinged and the rod rotates always remaining horizontal. Its angular speed is increasing at constant rate. The mass of the particle is 'm'. The force exerted by the rod on the particle is $\vec{F}$, then:

(a) $F \geq mg$ (b) F is constant

(c) The angle between $\vec{F}$ and horizontal plane decreases

(d) The angle between $\vec{F}$ and the rod decreases

57. Three masses of small size are attached by light inextensible strings of various lengths to a point O on the ceiling. All of the masses swing round in horizontal circles of various radii with the same angular frequency ω (one such circle is drawn in the shown figure). Then pick up the correct statement.



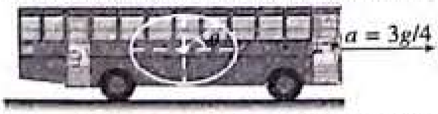
(a) The vertical depth of each mass below point of suspension from ceiling is different.

(b) The radius of horizontal circular path of each mass is same.

(c) All masses revolve in the same horizontal plane.

(d) All the particles must have same mass.

58. A bus is moving with a constant acceleration $a = 3g/4$ towards right. In the bus, a ball is tied with a rope and is rotated in vertical circle as shown. The tension in the rope will be minimum, when the rope makes an angle θ equal to.



(a) 53° (b) 37° (c) 180 − 53° (d) 180 + 37°

59. The force required just to move a body up an inclined plane is double the force required just to prevent the body sliding down. If the coefficient of friction is 0.25, the angle of inclination of the plane is

(a) 37° (b) 45° (c) 30° (d) 53°

60. Two blocks of masses m_1 and m_2 connected by a string are placed gently over a fixed inclined plane, such that the tension in the connecting string is initially zero. The coefficient of friction between m_1 and inclined plane is μ_1; between μ_2 and the inclined plane is μ_2. The tension in the string shall continue to remain zero if

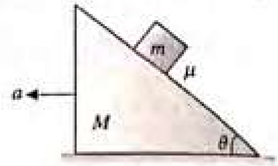
(a) $\mu_1 > \tan\alpha$ and $\mu_2 < \tan\beta$

(b) $\mu_1 < \tan\alpha$ and $\mu_2 > \tan\beta$

(c) $\mu_1 > \tan\alpha$ and $\mu_2 > \tan\beta$

(d) $\mu_1 > \tan\alpha$ and $\mu_2 > \tan\beta$

61. A block of mass m is at rest relative to the stationary wedge of mass M. The coefficient of friction between block and wedge is μ. The wedge is now pulled horizontally with acceleration 'a' as shown in figure. Then the minimum magnitude of 'a' for the friction between block and wedge to be zero is



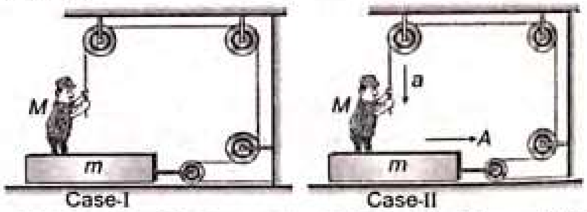
(a) $g\tan\theta$

(b) $\mu g\tan\theta$

(c) $g\cot\theta$

(d) $\mu g\cot\theta$

62. A board of mass $m = 11$ kg is placed on the floor and a man of mass $M = 70$ kg is standing on the board as shown (Case-I). The coefficient of friction between the board and the floor is $\mu = 0.25$. The maximum force that the man can exert on the rope so that the board does not slip on the floor is



(a) 125 N (b) 90 N (c) 162 N (d) None of these

63. If in case-II all surfaces are frictionless and the man hang on the rope without touching the board. The ratio of a/A is

(a) 2 (b) 1/2 (c) 1 (d) None of these

64. If in Case-II (all surfaces are frictionless) the acceleration of man will be

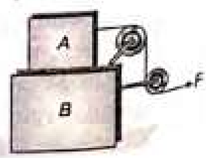
(a) $\frac{2800}{291}$ m/s^2 (b) $\frac{2218}{291}$ m/s^2

(c) 65 m/s^2 (d) None of these

65. A flat car is given an acceleration $a_0 = 2$ m/s^2 starting from rest. A cable is connected to a crate of weight 50 kg as shown whose other end is attached to a fixed support on ground. Neglect friction between the floor and the car wheels and also the mass of the pulley. Calculate corresponding tension in the cable if $\mu = 0.30$ between the crate and the floor of the car.

(a) 350 N (b) 250 N (c) 300 N (d) None of these

66. In the arrangement shown in figure, $m_A = m_B = 2$ kg. String is massless and pulley is frictionless. Block B is resting on a smooth



horizontal surface, while friction coefficient between blocks A and B is $\mu = 0.5$. The maximum horizontal force F that can be applied so that block A does not slip over the block B is

(a) 25 N (b) 40 N (c) 30 N (d) 20 N

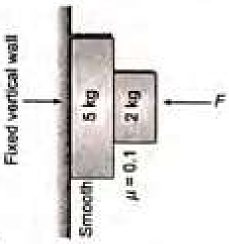
67. A block is placed at the bottom of an inclined plane and projected upwards with some initial speed. It slides up the plane and stops after time t_1. It begins to slide back down to the bottom in a further time t_2. The angle of inclination of plane is θ and the coefficient of friction between body and the surface is μ. Then

(a) $t_1 = t_2$ (b) $t_1 > t_2$ (c) $t_2 > t_1$ (d) $t_1 = 2t_2$

68. A block of mass 1 kg starts moving at $t = 0$ with velocity $\vec{v} = 3\hat{i}$ on a rough horizontal platform (with coefficient of friction $\mu = 0.3$) moving with velocity $4\hat{j}$. At $t = t_0$ block comes to rest with respect to platform. Then displacement of block with respect to ground in this time interval. (x-y plane is a horizontal plane)

(a) $\frac{7}{6}$ m (b) 2 m (c) 1 m (d) $\frac{3}{25}$ m

69. In the arrangement shown in figure the wall is smooth and friction coefficient between the blocks is $\mu = 0.1$. A horizontal force $F = 1000$ N is applied on the 2 kg block. Study following statements regarding this situation:



(i) The normal interaction force between the blocks is 1000 N
(ii) The friction force between the blocks is zero
(iii) Both the blocks accelerate downward with acceleration g m/s^2
(iv) Both the blocks remain at rest

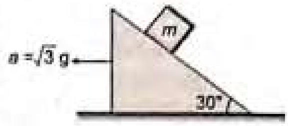
The correct statements are:

(a) only (ii) and (iv) (b) only (i), (ii) and (iii)
(c) only (ii) and (iii) (d) only (i) and (iv)

70. A plank of mass $3m$ is placed on a rough inclined plane and a man of mass m walks down the board. If the coefficient of friction between the board and inclined plane is $\mu = 0.5$, the minimum acceleration of does not slide is:

(a) 8 m/s^2 (b) 4 m/s^2 (c) 6 m/s^2 (d) 3 m/s^2

71. If the acceleration is towards right the frictional force exerted by wedge on the block will be (coefficient of friction between wedge and block is $\sqrt{3}/2$)

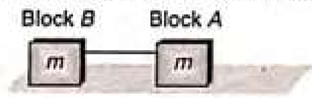


(a) mg (b) $\frac{3mg}{2}$ (c) $2mg$ (d) $\frac{mg}{2}$

72. A particle of mass m attached to a string of length l is describing circular motion on a smooth plane inclined at an angle α with the horizontal. For the particle to reach the highest point its velocity at the lowest point should exceed.

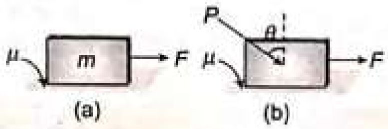
(a) $\sqrt{gl}$ (b) $\sqrt{5gl(\cos\alpha \tan+1)}$
(c) $\sqrt{5gl \tan\alpha}$ (d) $\sqrt{5gl \sin\alpha}$

73. Two blocks each of mass m are placed on a rough horizontal surface and connected by a massless inelastic string as shown. The coefficient of friction between each block and horizontal surface is μ. The string connecting both the blocks has zero tension. The minimum force to be applied on block A to just move the two block system horizontally (without the string getting slack) is:



(a) $2\mu mg$ (b) $\frac{2\mu mg}{\sqrt{\mu^2+1}}$ (c) $\frac{2\mu mg}{\sqrt{4\mu^2+1}}$ (d) $\frac{\mu mg}{\sqrt{\mu^2+1}}$

74. The coefficient of friction between the block and the horizontal surface is μ. The block moves towards right under action of horizontal force F [Fig. (a)]. Sometime later another force P is applied to the block making an angle θ (such that $\tan\theta = \mu$) with vertical as shown in [Fig. (b)]. After application of force P, the acceleration of block shall



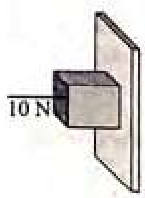
(a) increase (b) decrease
(c) remains same
(d) information insufficient for drawing inference

ARCHIVES

1. The minimum velocity (in m/s) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is

(a) 60 (b) 30
(c) 15 (d) 25 **(AIEEE 2002)**

2. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is



(a) 2 N (b) 20 N
(c) 50 N (d) 100 N **(AIEEE 2003)**

3. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. Then the coefficient of friction is
(a) 0.01 (b) 0.02
(c) 0.03 (d) 0.06 **(AIEEE 2003)**

4. A car is moving in a circular path of radius 500 m with a speed of 30 m/s. If the speed is increased at the rate of 2 m/s^2, the resultant acceleration will be
(a) 2 m/s^2 (b) 2.5 m/s^2
(c) 2.7 m/s^2 (d) 4 m/s^2 **(AIEEE 2003)**

5. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is (take $g = 10$ m/s^2)
(a) 1.6 (b) 4.0
(c) 2.0 (d) 2.5 **(AIEEE 2004)**

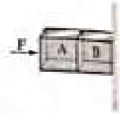
6. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on a rough incline than on a smooth incline. The coefficient of friction is
(a) $\mu_s = \sqrt{1-\frac{1}{n^2}}$ (b) $\mu_s = 1-\frac{1}{n^2}$
(c) $\mu_k = \sqrt{1-\frac{1}{n^2}}$ (d) $\mu_k = 1-\frac{1}{n^2}$ **(AIEEE 2005)**

7. Consider a car moving on a straight road with a speed of 100 m/s. The distance at which the car can be stopped is $[\mu_k = 0.5]$
(a) 400 m (b) 100 m (c) 1000 m (d) 800 m
(Note: It should be minimum distance in which the car can be stopped.)
(AIEEE 2005)

8. The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A body starting from the rest at top comes back to rest at the bottom if the coefficient of friction for the lower half is given by
(a) $\mu = \sin\theta$ (b) $\mu = \cot\theta$
(c) $\mu = 2\cos\theta$ (d) $\mu = 2\tan\theta$ **(AIEEE 2005)**

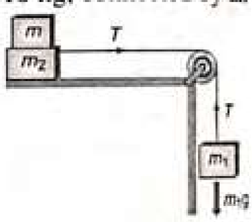
9. A block of mass m is placed on a surface with a vertical cross section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is
(a) $\frac{1}{3}$m (b) $\frac{1}{2}$m (c) $\frac{1}{6}$m (d) $\frac{2}{3}$m
(JEE Main 2014)

10. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is



(a) 100 N (b) 80 N
(c) 120 N (d) 150 N **(JEE Main 2015)**

11. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(a) 10.3 kg (b) 18.3 kg
(c) 27.3 kg (d) 43.3 kg **(JEE Main 2018)**

12. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then
(a) $T \propto R^{n/2}$ (b) $T \propto R^{3/2}$ for any n
(c) $T \propto R^{\frac{n}{2}+1}$ (d) $T \propto R^{(n+1)/2}$
(JEE Main 2018)

ANSWER KEY

Exercises

1. (a)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (d)
11. (a)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (a)	**16.** (a)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (b)
21. (d)	**22.** (b)	**23.** (b)	**24.** (a)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (d)
31. (b)	**32.** (a)	**33.** (a)	**34.** (a)	**35.** (a)	**36.** (b)	**37.** (c)	**38.** (b)	**39.** (b)	**40.** (d)
41. (b)	**42.** (c)	**43.** (c)	**44.** (a)	**45.** (b)	**46.** (d)	**47.** (d)	**48.** (d)	**49.** (d)	**50.** (c)
51. (b)	**52.** (c)	**53.** (d)	**54.** (d)	**55.** (b)	**56.** (c)	**57.** (c)	**58.** (a)	**59.** (a)	**60.** (c)
61. (c)	**62.** (b)	**63.** (a)	**64.** (a)	**65.** (a)	**66.** (d)	**67.** (c)	**68.** (a)	**69.** (b)	**70.** (a)
71. (a)	**72.** (d)	**73.** (b)	**74.** (c)						

Archives

1. (b)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (c)
11. (d)	**12.** (d)								

Chapter 7

Work, Energy and Power

WORK DONE BY A FORCE

When a constant force $\vec{F}$ acts on a particle while the particle moves through a displacement $\vec{s}$ (for a rigid body, $\vec{s}$ is the displacement of point of application of force on body w.r.t. the frame, in which we have to find the work), the force is said to do work W on the particle given by

$$W = \vec{F}.\vec{s}$$

$\vec{F}.\vec{s}$, the scalar (dot) product of $\vec{F}$ and $\vec{s}$ can be evaluated as:

$$W = \vec{F}.\vec{s} = Fs\cos\theta \quad \text{(i)}$$

$W = Fs\cos\theta = F(s\cos\theta) = fs_{\parallel}$

= Magnitude of the force × Component of the displacement in the direction of the force

$W = (F\cos\theta)s = F_{\parallel}s$

= Component of force in the direction of displacement × Magnitude of the displacement

Notice also that the displacement in Eq. (i) is that of *the point of application of the force.* If the force is applied to a particle or a rigid object that can be modeled as a particle, this displacement is same as that of the particle. For a deformable system, however, these displacements are not the same. For example, imagine pressing in on the sides of a balloon with both hands.

NOTE: If a number of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots, \vec{F}_n$ are acting on a body and they shift from position vector $\vec{r}_1$ to position vector $\vec{r}_2$, then

$$W = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \times \vec{F}_n).(\vec{r}_2 - \vec{r}_1) = (\Sigma\vec{F})\cdot(\Delta\vec{r})$$

In terms of rectangular components, the force and displacement vectors can be written as: $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$. Therefore,

Work done, $W = \vec{F}\cdot\vec{s}$

$$= (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= F_x x + F_y y + F_z z$$

Important Points

- We can calculate the work done by a force on an object, but that force is *not* necessarily the cause of the object's displacement. For example, if you lift an object (negative), work is done on the object by the gravitational force, although gravity is not the cause of the object moving upward.
- Work is defined for an interval or displacement. There is no term such as instantaneous work similar to instantaneous velocity.
- For a particular displacement, the work done by a force is independent of the type of motion, i.e.m whether it moves with constant velocity, constant acceleration or retardation, etc.
- For a particular displacement as work is independent of time. Work will be same for same displacement whether the time taken is small or large.
- When several forces act, the work done by a force for a particular displacement is independent of other forces.

ILLUSTRATION 7.1 A constant force $\vec{F} = (3\hat{i} + 2\hat{j} + 2\hat{k})$ N acts on a particle displacing it from a position $\vec{r}_1 = (-\hat{i} + \hat{j} - 2\hat{k})$ m to a new position $\vec{r}_2 = (\hat{i} - \hat{j} + 3\hat{k})$ m. Find the work done by the force.

Solution. The displacement vector, $\vec{s} = \vec{r}_2 - \vec{r}_1$

$$\vec{s} = (1+1)\hat{i} + (-1-1)\hat{j} + (3+2)\hat{k} = 2\hat{i} - 2\hat{j} + 5\hat{k}$$

From $W = \vec{F}\cdot\vec{s}$, we have

$$W = (3\hat{i} + 2\hat{j} + 2\hat{k})\cdot(2\hat{i} - 2\hat{j} + 5\hat{k}) = 6 - 4 + 10 = 12 \text{ J}$$

ILLUSTRATION 7.2 Three constant forces $\vec{F}_1 = 2\hat{i} - 3\hat{j} + 2\hat{k}$, $\vec{F}_2 = \hat{i} + \hat{j} - \hat{k}$, and $\vec{F}_3 = 3\hat{i} + \hat{j} - 2\hat{k}$ in newtons displace a particle from (1, –1, 2) to (–1, –1, 3) and then to (2, 2, 0) (displacement being measured in metres). Find the total work done by the forces.

Solution.

Net (resultant) force, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 6\hat{i} - \hat{j} - \hat{k}$ dyn

and net displacement, $\vec{s} = (2-1)\hat{i} + (2+1)\hat{j} + (0-2)\hat{k}$

$$= \hat{i} + 3\hat{j} - 2\hat{k} \text{ m}$$

Therefore, work done,

$$W = \vec{F}\cdot\vec{s} = (6\hat{i} - \hat{j} - \hat{k})\cdot(\hat{i} + 3\hat{j} - 2\hat{k}) \text{ Nm}$$

$$= (6 - 3 + 2) \times 10^{-5} \text{ J} = 5 \times 10^{-5} \text{ J}$$

Work Done By a Variable Force

Force is a vector quantity. When either its magnitude of direction or both magnitude and direction change, we say that the force is varying. For work done by a variable force, the work done for an infinitesimal displacement $\overrightarrow{ds}$ is given by

$$dW = \vec{F}.\overrightarrow{ds}$$

The total work done in going from A to B as shown in the figure is

$$W_{AB} = \int_A^B \vec{F}.\overrightarrow{ds} = \int_A^B F\,ds\cos\theta$$

In terms of rectangular components,

$$\overrightarrow{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

Work done by a variable force between points A and B,

$$W = \int \vec{F}.\overrightarrow{ds}$$

$$W_{AB} = \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_{x_A}^{x_B} F_x\,dx + \int_{y_A}^{y_B} F_y\,dy + \int_{z_A}^{z_B} F_z\,dz$$

ILLUSTRATION 7.3 A force $\vec{F} = 6x\hat{i} + 2y\hat{j}$ displaces a body from $\vec{r}_1 = 3\hat{i} + 8\hat{j}$ *to* $\vec{r}_2 = 5\hat{i} - 4\hat{j}$. Find the work done by the force.

Solution. Given force $\vec{F} = 6x\hat{i} + 2y\hat{j}$

Initial position $\vec{r}_1 = 3\hat{i} + 8\hat{j}$ and final position $\vec{r}_2 = 5\hat{i} - 4\hat{j}$

We know work done, $W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s} = \int_{s_1}^{s_2} \vec{F}\cdot(dx\hat{i} + dy\hat{j})$

$$W = \int_{s_1}^{s_2} (6x\hat{i} + 2y\hat{j})\cdot(dx\hat{i} + dy\hat{j}) = \int_3^5 6x\,dx + \int_8^{-4} 2y\,dy$$

$$= \left[3x^2\right]_3^5 + \left[y^2\right]_8^{-4} = [75 - 27] + [16 - 64] = 0 \text{ J}$$

ILLUSTRATION 7.4 A chain of length L and mass M is held on a frictionless table with $(1/n)$th of its length hanging over the edge (see figure). Calculate the work done in pulling the chain slowly on the table against gravity.

Solution. Let $\lambda = M/L$ = mass per unit length of the chain and y is the length of the chain hanging over the edge. So the mass of the chain of length y will be λy and the force acting on it due to gravity will be mgy.

The work done in pulling the dy length of the chain on the table.

$$dW = F(-dy) \qquad [dy \text{ is negative as } y \text{ is decreasing}]$$

As the chain is pulled slowly,

$$F = \text{Weight of the hanging chain} = \lambda yg$$

i.e., $\quad dW = (\lambda yg)(-dy)$

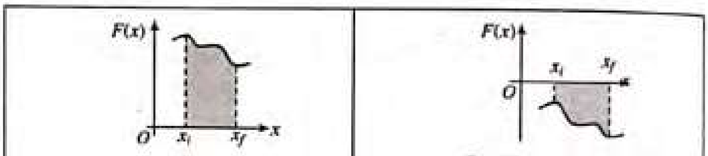
So the work done in pulling the hanging portion on the table,

$$W = -\int_{L/n}^{0} \lambda gy\,dy = -\lambda g\left[\frac{y^2}{2}\right]_{L/n}^{0} = \frac{\lambda g L^2}{2n^2} = \frac{MgL}{2n^2}$$

$$[\text{as } \lambda = M/L]$$

Graphical Interpretation of Work Done

Generally, the work done by a variable force $F(x)$ from an initial position x_i to final position x_f is interpreted as the area under the force-displacement curve.



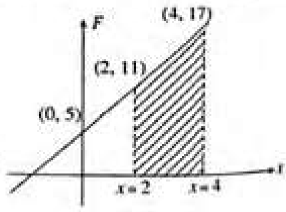
(a) Positive work (b) Negative work

ILLUSTRATION 7.5 Consider a variable force $F = (3x + 5)$ N acting on a body and if it is displaced from $x = 2$ m to $x = 4$ m, calculate the work done by this force.

Solution. If we plot the force in the function of displacement, the work done can be given by the shaded area shown in the figure. Thus, work done by this force is

W = Area of shaded trapezium

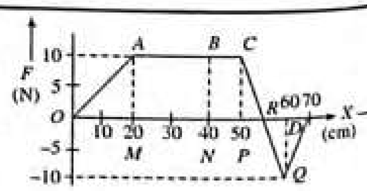
$$= \frac{1}{2} \times 2 \times (11 + 17) = 28$$



If we find the same using integration, we have

$$W = \int_2^4 (3x + 5)dx = \left(\frac{3x^2}{2} + 5x\right)_2^4 = 28 \text{ J}$$

ILLUSTRATION 7.6 From the figure, find the work done at the end of displacements: (a) 20 cm, (b) 40 cm, and (c) 60 cm.



Solution.

(a) Work done at the end of displacement 20 cm,

$$= \text{Area of triangle } OAM$$

$$= \frac{1}{2} \times (20 \times 10^{-2}) \times 10 = 1 \text{ J}$$

(b) Work done at the end of displacement 40 cm,

= Area of $OABN$

= Area of OAM + Area of rectangle $ABMN$

$= 1 + 20 \times 10^{-2} \times 10 = 3$ J

(c) Work done at the end of displacement 60 cm,

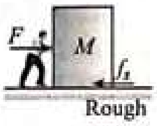
= Area of triangle OAM + Area of rectangle $ACMP$

$= 1 + 30 \times 10^{-2} \times 10 = 4.0$ J

Net area from 50 cm to 60 cm will be zero.

Work Done by Static Friction

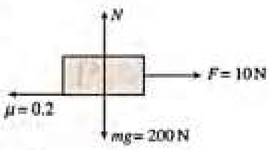
We have learnt about two types of frictional forces, i.e. static friction and kinetic friction. Let us discuss the work done by static friction.



If you push a box by applying a force F, say, let us assume that the box does not move (relative to fixed surface on which it is placed) as shown in the figure. Then the static friction f_s does not perform any work as the displacement of the box (displacement of point of application of friction force) is zero ($W_{f_s} = 0$).

ILLUSTRATION 7.7 A force of 10 N is acting on a block of 20 kg on a horizontal surface with coefficient of friction $\mu = 0.2$. Calculate the work done by the force.

Solution. $f_{max} = \mu N = 40$ N

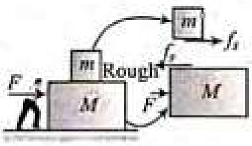


Driving force, $F < f_{max}$, therefore, $s = 0$.

Hence, $W = 0$.

The block does not displace ($s = 0$) because the applied force is less than the limiting friction.

Now take another example. A block of mass m is placed on the block of mass M as shown in the figure. The horizontal force $\vec{F}$ acts on M. The horizontal surface is smooth, assuming no relative sliding between the blocks. If the lower block moves through a distance x, the upper blocks m will also move through the same distance x. The direction of static friction on the upper block is in forward direction while the direction of static friction on lower block will be in backward direction. Work done by the friction can be given as:



On lower block: $(W_f)_M = -f_s x$

On upper block: $(W_f)_m = f_s x$

The total work done by static friction at the box–surface system (interface) is

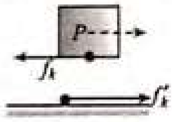
$$W_f = (W_f)_M + (W_f)_m = -f_s x + f_s x$$

Hence, the total "static frictional work," $W = 0$.

From the above discussion, we conclude the following points: the work done by static friction can be positive, negative, or zero. When we consider the net work done by the static friction at the contacting surfaces, since there is no relative displacement between the surfaces, the total work done by the static friction is zero.

Work Done by Kinetic Friction

Object is sliding over a fixed surface: As you know, kinetic friction acts on a particle when it slides on a surface. If the surface is fixed, the displacement of the particle is non-zero. That means, kinetic friction f_k will perform certain work. Since $\vec{f}_k$ points opposite to the displacement $\vec{s}$, the work done by kinetic friction is negative.



or $\quad W_{f_k} = \vec{f}_k \cdot \vec{s} = f_k s \cos 180° \Rightarrow W_{f_k} = -f_k s$

As the surface is assumed to be fixed ($s = 0$), the work done by the kinetic friction f_k' on the surface is zero. In this way, the total work done by kinetic friction on both the surfaces is negative.

ILLUSTRATION 7.8 A block of mass 2.0 kg is pushed down an inclined plane of inclination 37° with a force of $F = 20$ N acting parallel to the incline. It is found that the block moves down the incline with an acceleration of 10 m s^{-2}. If the block started from rest, find the work done

(a) by the applied force in the first second

(b) by the weight of the block in the first second

(c) by the frictional force acting on the block in the first second

Solution. Displacement of block in 1 s

$$s = \frac{1}{2} \times 10 \times 1^2 = 5 \text{ m}$$

The work done by the applied force in the first second,

$$W_F = \vec{F} \cdot \vec{s} = F \cdot s \cos\theta = 20 \times 5 = 100 \text{ J}$$

The work done by the weight of the block in the first second = Component of weight in the direction of displacement × Displacement

$$W_{\text{weight}} = (mg \sin\theta) \times d = \left(2 \times 10 \times \frac{3}{5}\right) \times 5 = 60 \text{ J}$$

Now we need to calculate friction force acting on block. For this, we need to write

Equation of motion of block, $mg \sin\theta + F - f = ma$

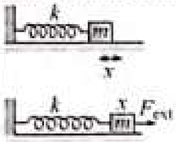
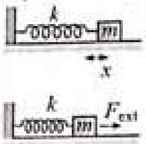
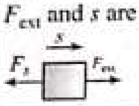
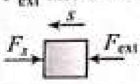
$$2 \times 10 \times \frac{3}{5} + 20 - f = 2 \times 10 \Rightarrow f = 12 \text{ N}$$

The work done by the frictional force acting on the block in the first second,

$$W_{\text{friction}} = \vec{f} \cdot \vec{d} = f \cdot d \cos 180° = -f \times d = -60 \text{ J}$$

Work Done by Spring Force

Whenever a spring is stretched or compressed, the spring force always tends to restore it to the relaxed position.

A spring stretched from its equilibrium position.	A spring is compressed from its equilibrium position.
	
F_s and s are antiparallel.	F_s and s are antiparallel.
F_{ext} and s are parallel.	F_{ext} and s are parallel.
	

If x be the displacement of the free end of the spring from its equilibrium position, then the magnitude of the spring force is

$$F_s = -kx$$

The negative sign indicates that the force is restoring.

The work done by the spring force for a displacement from x_i to x_f is given by

$$W_s = \int \vec{F}_s d\vec{x} = -\int_{x_i}^{x_f} kx\,dx = -\frac{1}{2}k(x_f^2 - x_i^2)$$

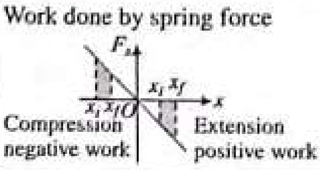
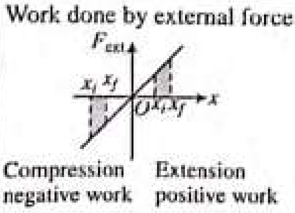
where x_i and x_f are the initial and final deformations of the spring. Hence, when a spring is deformed from $x = x_i$ to $x = x_f$,

- If $x_f > x_i$, W_{sp} is negative.
- If $x_f = x_i$, $W_{sp} = 0$. If $x_f < x_i$, W_{sp} is positive. That means a spring can perform positive, negative and zero work depending upon the initial and final deformations.
- When the spring is undeformed, we calculate the displacement of the free end P of the spring from the relaxed position of the spring. Substituting $x = 0$ and $x = x$, we have

$$W_{sp} = -\frac{1}{2}kx^2$$

The above expression tells us that a spring always performs negative work when deformed (compressed or elongated) from its relaxed (undeformed) position.

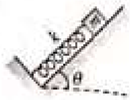
- The graph plotted between a spring force and the displacement from the equilibrium position is a straight line with negative slope.

Work done by spring force	The work done by spring force is negative both in compression and extension.
	
Work done by external force	The work done by external force is positive both in compressing or stretching the spring.
	

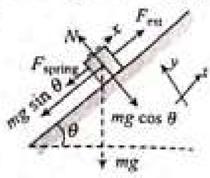
NOTE:

- Like gravity, the work done by spring force only depends on the initial and final positions.
- Also, the net work done by the spring force is zero for any path that returns to the initial position.

ILLUSTRATION 7.9 A block of mass m welded with a light spring of stiffness k is in equilibrium on an smooth inclined plane with angle of inclination θ. If a variable external force is applied slowly till the spring comes to its relaxed position, find the work done by spring force.



Solution. Referring to the FBD as shown in the figure, we have four forces (N, mg, F_{spring} and F_{ext}) acting on the particle.

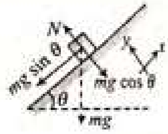


Initially, the block is at equilibrium.

$$kx_i = mg\sin\theta \Rightarrow x_i = \frac{mg\sin\theta}{k}$$

When the block is pulled up by an external force to bring the spring to relaxed length $x_f = 0$

Hence, work done by spring force,



$$W_{sp} = -\frac{1}{2}k(x_f^2 - x_i^2)$$

$$= -\frac{1}{2}\left[0 - \left(\frac{mg\sin\theta}{k}\right)^2\right]$$

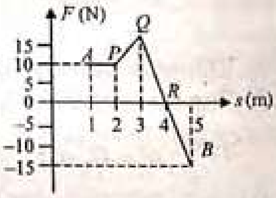
$$= \frac{1}{2}\frac{(mg\sin\theta)^2}{2k}$$

CONCEPT APPLICATION EXERCISE 7.1

1. A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposite to the motion.
   (a) How much work does the road do on the cycle?
   (b) How much work does the cycle do on the road?
2. A gardener moves a lawn roller through a distance of 100 m with a force of 50 N. Calculate his wages if he is to be paid 10 paise for doing 25 J of work. It is given that the applied force is inclined at 60° to the direction of motion.
3. A body constrained to move along the z-axis of a coordinate system is subjected to a constant force given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N, where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x-, y-, and z-axes of the system, respectively. What is the work done by this force in moving the body through a distance of 4 m along the z-axis?
4. An object is displaced from point A (2 m, 3 m, 4 m) to a point B (1 m, 2 m, 3 m) under a constant force $\vec{F} = (2\hat{i} + 3\hat{j} + 4\hat{k})$. Find the work done by this force in this process.

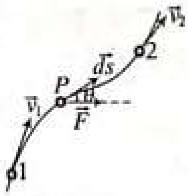
5. An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})$ m to $\vec{r}_2 = (4\hat{j} + 6\hat{k})$ m under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})$ N. Find the work done by the force.

6. A body moves from point A to B under the action of a force, varying in magnitude as shown in the figure. Obtain the work done. Force is expressed in newton and displacement in meter.



WORK–ENERGY THEOREM

One force on particle: For the sake of simplicity, let us assume a single force F acting on a particle P of mass m. If the particle moves in any arbitrary path, the work done by the force in an elementary displacement $d\vec{s}$ of the particle is



$$dW = \vec{F}.d\vec{s}$$

Substituting $\vec{F} = m\vec{a}$, we have

$$dW = m\vec{a}.d\vec{s}$$

Then the total work done is

$$W = \int dW = m\int \vec{a}.d\vec{s}$$

Substituting $\vec{a}\cdot d\vec{s} = v\,dv$, we have

$$W = m\int_{v_1}^{v_2} v\,dv$$

This gives $W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

where $\frac{1}{2}mv_1^2 = \text{KE}_1$ and $\frac{1}{2}mv_2^2 = \text{KE}_2$

Then, we have $W = \text{KE}_2 - \text{KE}_1 = \Delta\text{KE}$

When only a single force acts on a particle, *work done by the force is equal to the change in the kinetic energy of the particle.*

Many forces on one particle: If more than one forces act on a particle, we know that the sum of work done by all forces is equal to the work done by the resultant force acting on the particle. Since the resultant force $\vec{F}_{net}$ (say) decides the acceleration $\vec{a}$ of the particle, the total work done can be given as:

$$W_{total} = \sum W_{F_i} = W_F, \text{ where } W_F = \vec{F}_{net}.d\vec{s}$$

Then, $W_{total} = \int \vec{F}_{net}.d\vec{s}$

Substituting $\vec{F}_{net} = m\vec{a}$ and $\vec{a}.d\vec{s} = v\,dv$, we have

$$W_{total} = m\int_{v_1}^{v_2} v\,dv$$

which gives the same expression.

$$W_{total} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta\text{KE}$$

$$\sum W_{F_i} = \Delta K$$

The sum of work done by all forces acting on each particle of a system of particles is equal to the sum of change in the KE of each particle of the system. This is what we call work–energy theorem which signifies the "work" as "energy transfer."

NOTE:
For a system of particles (or system of rigid bodies),

$$\sum W_{ext} + \sum W_{int} = \Delta K$$

where $\sum W_{ext}$ is the work done by the external forces on the system. and $\sum W_{int}$ is the work done by the internal forces on the system.

Newton's third law of motion gives you that $\sum \vec{F}_{int} = 0$. But the work done by the internal force may or may not be equal to zero. Now it is more useful to write the work-energy theorem in non-inertial frame as

$$W_{external} + W_{internal} + W_{pseudo} + W_{other} = \Delta\text{KE}$$

- Positive work increases the kinetic energy and negative work decreases the kinetic energy.
- Work can be converted into kinetic energy and kinetic energy can be converted into work.

ILLUSTRATION 7.10 A particle slides along a track with elevated ends and a flat central part as shown in the figure. The flat part has a length $l = 3$ m. The curved portions of the track are frictionless. For the flat part, the coefficient of kinetic friction is $\mu_K = 0.2$. The particle is released at point A which is at height $h = 1.5$ m above the flat part of the track. Where does the particle finally come to rest?

Solution. The particle will finally come to rest on the flat part. Hence, displacement of the particle along vertical is h. If W_g be the work done on the particle by the gravity, then

$$W_g = mgh \quad \text{(i)}$$

where m is the mass of the particle.

If distance travelled by the particle on the flat part is x, the work done on the particle by the friction is

$$W_f = -\mu mgx \quad \text{(ii)}$$

Since, initially, particle was at rest and finally it comes to rest again. Hence, change in its KE is zero.

From work–energy theorem, $W_g + W_f = \Delta\text{KE}$

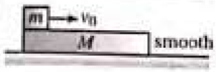
$$\Rightarrow \quad mgh - \mu mgx = 0$$

$$\Rightarrow \quad x = \frac{h}{\mu} = \frac{1.5}{0.2}\text{m} \Rightarrow x = 7.5 \text{ m}$$

Since $x > l$, the particle will reach C and then will rise up till the remaining KE at C is converted into potential energy. It will then again descend to C and will have the same kinetic energy as it had when ascending but now will move from C to B. At B, same thing will be repeated (because $7.5 > 2l$), and finally, the particle will stop at E such that

$$BC + CB + BE = 7.5$$
$$BE = 7.5 - 6 = 1.5 \text{ m}$$

ILLUSTRATION 7.11 A plank of mass M and length L is placed at rest on a smooth horizontal surface. A small block of mass m is projected with a velocity v_0 from the left end of it as shown in the figure.



The coefficient of friction between the block and the plank is m, and its value is such that the block becomes stationary with respect to the plank before it reaches the other end.

(a) Find the time and common velocity when relative sliding between the block and the plank stops.

(b) Find the work done by the friction force on the block during the period it slides on the plank. Is the work positive or negative?

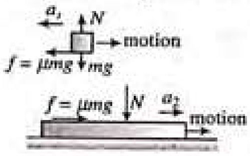
(c) Calculate the work done on the plank during the same period. Is the work positive or negative?

(d) Also, determine the net work done by friction. Is it positive or negative?

Solution.

(a) Initially, the block will slide on plank. The friction will be of kinetic nature. The friction will decrease the speed of the block and starts motion of the plank.

The free body diagrams of the block and the plank are shown in the figure.



Equation of motion

For block: $a_1 = \dfrac{f}{m} = \mu g$

Instantaneous velocity,

$v_1 = v_0 - \mu g t$

For plank: $a_2 = \dfrac{f}{M} = \dfrac{\mu mg}{M}$

Instantaneous velocity, $v_2 = \dfrac{\mu mgt}{M}$

Finally, both the block and the plank start moving together, i.e., $v_1 = v_2$, then

$$v_0 - \mu g t = \frac{\mu mgt}{M} \text{ or } t = \frac{Mv_0}{(M+m)\mu g}$$

and the final common velocity is $V = \dfrac{mv_0}{M+m}$.

If we take the block as system, friction is the only force which do work.

(b) The work done by friction on the block is equal to its change in kinetic energy, i.e.,

$$W_1 = K_f - K_i = \frac{1}{2}mV^2 - \frac{1}{2}mv_0^2$$

$$W_1 = \frac{1}{2}m\left(\frac{mv_0}{m+M}\right)^2 - \frac{1}{2}mv_0^2$$

$$= -\frac{1}{2}\frac{mM(M+2m)v_0^2}{(M+m)^2}$$

The work done by friction on the block is negative.

If we take plank as system only friction is only force which do work on plank.

(c) The work done by friction on the plank is given by

$$W_2 = K_f - K_i = \frac{1}{2}MV^2 - 0$$

$$W_2 = \frac{1}{2}M\left(\frac{mv_0}{m+M}\right)^2 = \frac{1}{2}\frac{m^2M}{(m+M)^2}v_0^2$$

The work done by friction on the block is positive.

(d) The net work done by friction is

$$W = W_1 + W_2 = -\frac{1}{2}\frac{mM}{M+m}v_0^2$$

The net work done by friction is negative.

The work done by tension $W_T = 0$

At the position of maximum deflection, the velocity of the bob is zero, $\Delta K = 0$.

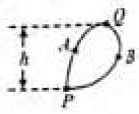
CONSERVATIVE AND NON-CONSERVATIVE FORCES

Consider an object moving downward near the surface of the earth. The work done by the gravitational force on the object does not depend on whether it falls vertically or slides down a sloping incline. All that matters is the change in the object's elevation. In other words, the path makes no difference when we consider the work done by the gravitational force, but it does make a difference when we consider energy transformation due to friction forces. We can use this varying dependence on path to classify forces as either conservative or non-conservative.

Conservative Force

Conservative forces have the following two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one for which the beginning point and the endpoint are same)



A particle is taken from point P to point Q via the path PAQ and then placed back to point P via the path QBP in the vertical plane. Let us calculate the work done by gravity on the body over this closed path. Here, displacement of the particle is $\overline{PQ}$; gravity is acting vertically downward. The vertical component of $\overline{PQ}$ is h upward. Hence,

$$W_{(PAQ)} = -mgh \qquad (1)$$

For the path QBP, component of the displacement along vertical is h (downward).

In this case, $W_{QBP} = mgh$

Therefore, total work done $= W_{PAQ} + W_{QBP} = 0$

For the case of the object–spring system, the work W_s done by the spring force is given by $W_s = \dfrac{1}{2}kx_i^2 - \dfrac{1}{2}kx_f^2$. We see

that the spring force is conservative because W_s depends only on the initial and final x coordinates of the object and is zero for any closed path.

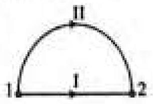
Non-Conservative Force

The work done by a non-conservative force not only depends on the initial and final positions but also on the path followed.

The common examples of such forces are frictional force and drag force in fluids.

The work done by a non-conservative force along a closed path is not equal to zero.

Let us choose two points 1 and 2 on a rough horizontal floor and drag an object slowly and horizontally between these points along paths I and II can observe that, in shifting the object along two paths. We feel more exhausted when we drag a body through more distance. In other words, work done by friction is more for a longer path. The work done due to friction for the paths I and II can be given as $W_1 = -fl_1$ and $W_2 = -fl_2$, respectively.

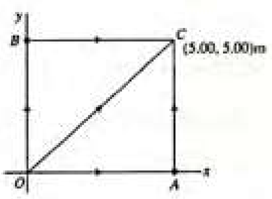


Since the work done by kinetic friction is path-dependent, for a round trip (when we drag the object from position 1 to 2 along the path I and we bring it back slowly to its initial position 1 from the path II), the total work done by friction is

$$W = W_1 + W_2 = -fl_1 + (-fl_2) = -f(l_1 + l_2) = -fl$$

where l is the length of the closed path.

ILLUSTRATION 7.12 A 4.00-kg particle moves from the origin to position C, having coordinate x = 5.00 m and y = 5 m. One force on the particle is the gravitational force acting in the negative y direction. Using equation $W = F\Delta r \cos\theta = \vec{F}\cdot\Delta\vec{r}$, calculate the work done by the gravitational force on the particle as it goes from O to C along (a) OAC, (b) OBC, and (c) OC. Your results should all be identical. Why?



Solution. $F_g = mg = (4.00\text{ kg})(10.0\text{ m s}^{-2}) = 40.0\text{ N}$

(a) Work done along OAC = Work along OA + Work along AC

$$= F_g(OA)\cos 90.0^\circ + F_g(AC)\cos 180^\circ$$
$$= (40.0\text{ N})(5.00\text{ m})\cos 90^\circ + (40.0\text{ N})(5.00\text{ m})\cos 180^\circ$$
$$= -200\text{ J}$$

(b) Work done along OBC = W along OB + W along BC

$$= (40.0)(5.00\text{ m})\cos 180^\circ + (40.0\text{ N})(5.00\text{ m})\cos 90.0^\circ$$
$$= -200\text{ J}$$

(c) Work done along $OC = F_g(OC)\cos 135^\circ$

$$= (40.0\text{ N})\ (5.00\times\sqrt{2}\text{ m})\left(-\frac{1}{\sqrt{2}}\right) = -200\text{ J}$$

Work done is same in all cases because gravitational force is a conservative force.

POTENTIAL ENERGY

Potential energy is defined as the ability of doing work by a conservative force. It arises from the configuration of the system or position of the particles in the system (conservative force fields like spring, gravity, etc.)

Work-potential energy theorem: The conservative force always does positive work at the expense of its potential energy sored in its field. For instance, when the compression or elongation of the spring decreases, the spring force decreases. Consequently the spring becomes more inactive. It means, the stability of work done by the spring decreases. In this way we can logically interpret that:

Work done by the spring is equal to the loss of its potential energy, $W_{sp} = -\Delta U_{sp}$; negative sign is used for the loss of potential energy.

Same logic is also valid for gravitational field. When the body moves down, its kinetic energy increases and the gravity does a positive work. It means, the gravitational potential energy decreases by an amount which is equal to the gravitational work done on the falling body, $W_{gr} = -\Delta U_{gr}$.

In general, we can state that: any conservative force field performs a work at the expense of its potential energy.

$$W_{\text{conc.}} = -\Delta U$$

The loss in potential energy is equal to the total work done by the conservative forces.

Elastic Potential Energy Stored in a Spring

As the spring deformation changes from x_1 to x_2, the work done by spring is

$$W_{sp} = -\frac{k}{2}(x_2^2 - x_1^2)$$

In this process, the spring potential energy changes from U_1 to U_2. Substituting the above value of W_{spring} in

$$W_{sp} = -\Delta U = -(U_2 - U_1)$$

we have $U_2 - U_1 = \dfrac{kx^2}{2} - \dfrac{1}{2}kx_1^2$

If $x_1 = 0$ and $x_2 = x$, we have $U_1 = 0$ and $U_2 = U = kx^2/2$. Then the potential energy U as the function of deformation x can be given as $U = kx^2/2$.

Change in Potential Energy

Change in potential energy between any two points is defined in the terms of the work done by the associated conservative force in displacing the particle between these two points without any change in kinetic energy.

$$U_2 - U_1 = -\int_{r_1}^{r_2}\vec{F}.d\vec{r} = -W \qquad \text{(i)}$$

We can define a unique value of potential energy only by assigning some arbitrary value to a fixed point called the reference point. Whenever and wherever possible, we take the reference point at infinity and assume potential energy to be zero there, i.e., if we take $r_1 = \infty$ and $r_2 = r$, then from eqn. (i),

$$U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W$$

In case of conservative force (field) potential energy is equal to negative of work done by conservative force in shifting the body from reference position to given position.

ILLUSTRATION 7.13 A conservative force held function is given by $F = k/r^2$, where k is a constant.

(a) Determine the potential energy function $U(r)$ assuming zero potential energy at $r = r_0$.

(b) Also, determine the potential energy at $r = \infty$.

Solution.

(a) Using the definition of potential energy function,

$$U(r) - U(r_0) = -\int_{r_0}^{r} F\,dr$$

$$U(r) - U(r_0) = -k\int_{r_0}^{r} \frac{dr}{r^2} = k\left[\frac{1}{r}\right]_{r_0}^{r} = k\left[\frac{1}{r} - \frac{1}{r_0}\right]$$

Since at $r = r_0$, $U(r_0) = 0$, therefore, $U(r) = \frac{k}{r} - \frac{k}{r_0}$

(b) Potential energy at $r = \infty$ is $U_\infty = -k/r_0$

Relationship between conservative forces and potential energy: If the point of application of the force undergoes an infinitesimal displacement dr, we can express the infinitesimal change in the potential energy of the system dU as

$$dU = -F_r\,dr$$

Therefore, the conservative force is related to the potential energy function through the relationship

$$F_r = -\frac{dU}{dr}$$

That is, the component of a conservative force in the direction of $\vec{r}$ acting on an object within a system equals the negative derivative of the potential energy of the system with respect to r.

Three-Dimensional Formula for Potential Energy

For only conservative fields, $\vec{F}$ equals the negative gradient of the potential energy.

In Cartesian coordinate system, the components of a force F in the axes x, y and z can be given as the negative space derivative (gradient) of the potential energy measured along the corresponding axes.

$$\vec{F}_x = -\frac{\partial U}{\partial x}\hat{i},\ \vec{F}_y = -\frac{\partial U}{\partial y}\hat{j},\text{ and } \vec{F}_z = -\frac{\partial U}{\partial z}\hat{k}$$

Then, the net force $\vec{F}$ can be given as

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$$

where $\partial U/\partial x$ is the partial derivative of U w.r.t. x (keeping y and z constant), $\partial U/\partial y$ is the partial derivative of U w.r.t. y (keeping x and z constant), and $\partial U/\partial z$ is the partial derivative of U w.r.t. z (keeping x and y constant).

ILLUSTRATION 7.14 The potential energy of configuration changes in x and y directions as $U = kxy$, where k is a positive constant. Find the force acting on the particle of the system as the function of x and y.

Solution. Substituting $U = kxy$ in the expression

$$F = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

we have $\vec{F} = -k(y\hat{i} + x\hat{j})$

Potential Energy Curve

A graph plotted between the potential energy of a particle and its displacement from the center of force is called *potential energy curve.*

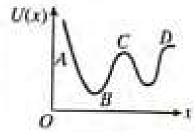


Figure shows a graph of potential energy function $U(x)$ for one dimensional motion.

As we know that negative gradient of the potential energy gives force. Therefore,

$$F_x = -\frac{dU}{dx}$$

Nature of Force

Attractive forc	Repulsive force	Zero force
On increasing x, if U increases, dU/dx is positive, then F is in negative direction, i.e., force is attractive in nature. In graph, this is represented in region BC.	On increasing x, if U decreases, dU/dx is negative, then F is in positive direction, i.e. force is repulsive in nature. In graph, this is represented in region AB.	On increasing x, if U does not change, dU/dx is 0, then F is zero, i.e. no force works on the particle. Point B, C, and D represent the point of zero force or these points can be termed as position of equilibrium.

We can easily check equation $F_r = -dU/dr$ for the two examples already discussed.

In the case of the deformed spring, $U_s = \frac{1}{2}kx^2$. Therefore,

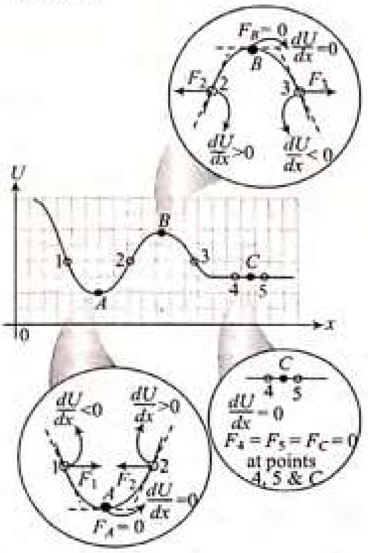
$$F_s = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

which corresponds to the restoring force in the spring (Hooke's law). The gravitational potential energy function is $U_g = mgy$, it follows $F_g = -\frac{dU}{dy} = \frac{-d(mg\,y)}{dy} = -mg$

We now see that U is an important function because a conservative force can be derived from it.

Stability

If the potential energy of a particle changes with a distance along an axis, x and the graph of potential energy U versus x is given, we can tell many important facts of the motion of the particles moving under the influence of the conservative forces by analyzing U–x graph.



We assume an arbitrary U–x graph as shown in the figure. Here we should not confuse x as horizontal—it may be horizontal, vertical. It means we consider the variation of potential energy of a particle in x direction

We can see that, U is minimum at A and maximum at B.

Let $U_{min} = U_A$ and $U_{max} = U_B$.

Hence, the slope of U–x graph, that is dU/dx, must be zero at A and B. Furthermore, $dU/dx = 0$ at C because U remains constant near C. Since $F = -dU/dx = 0$ at the points A, B, and C, the net force acting on the particle (under consideration) at these points is zero. In other words, the particle is in equilibrium at A, B, and C.

Let us now enquire more about the nature of equilibrium by just looking at the U–x graph.

Stable Equilibrium

The particle is in equilibrium at A, we call it "stable equilibrium." This is the sufficient and necessary condition for "oscillations" of an object.

At stable equilibrium, $\dfrac{dU}{dx} = 0$ and $\dfrac{d^2U}{dx^2} > 0$.

Unstable Equilibrium

The particle is in "unstable equilibrium" at B.

At stable equilibrium, $\dfrac{dU}{dx} = 0$ and $\dfrac{d^2U}{dx^2} > 0$

Neutral Equilibrium

Finally, let us come the point C. At C, $F = 0$ as discussed earlier and the particle is in equilibrium at C. Let us displace the particle slowly in either side to (4) and (5). Since at both (4) and (5); $dU/dx = 0$, we can say that the particle experiences no force when it is displaced near C. In other words, the particle will remain at rest (or move with constant velocity) at the points (4) and (5). Then, we can say that the particle is simultaneously neutral and it is in equilibrium because no net force acts on the particle when it undergoes a displacement near C. Hence, at C, the particle is said to be in "neutral equilibrium."

ILLUSTRATION 7.15 The potential energy of a particle in a certain field has the form $U = (a/r^2) - (b/r)$, where a and b are positive constants and r is the distance from the center of the field. Find the value of r_0 corresponding to the equilibrium position of the particle; examine whether this position is stable.

Solution. $U(r) = \dfrac{a}{r^2} - \dfrac{b}{r}$

$$\text{Force} = -\frac{dU}{dr} = -\left(\frac{-2a}{r^3} + \frac{b}{r^2}\right) = -\frac{(br-2a)}{r^3}$$

At equilibrium, $F = -\dfrac{dU}{dr} = 0$

Hence, $br - 2a = 0$, at equilibrium.

$r = r_0 = \dfrac{2a}{b}$ corresponds to equilibrium.

At stable equilibrium, the potential energy is minimum and at unstable equilibrium, it is maximum.

For minimum potential energy,

$dU/dr = 0$ and $d^2U/dr^2 > 0$ at $r = r_0$.

Let us investigate the second derivative.

$$\frac{d^2U}{dr^2} = \frac{d}{dr}\left(\frac{dU}{dr}\right) = \frac{d}{dr}\left(\frac{-2a}{r^3} + \frac{b}{r^2}\right) = \frac{6a}{r^4} - \frac{2b}{r^3}$$

$$\text{At } r = r_0 = \frac{2a}{b} \quad \Rightarrow \quad \frac{d^2U}{dr^2} = \frac{6a - 2br_0}{r_0^4} = \frac{2a}{r_0^4} > 0$$

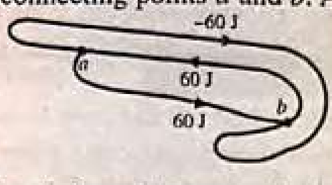
Hence, the potential energy function $U(r)$ has a minimum value at $r_0 = 2a/b$. The system has a stable equilibrium at minimum potential energy state.

NOTE: Conservative force can be defined in three ways as follows:

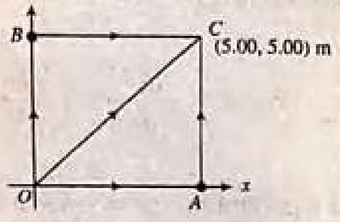
- If the work done by a force on a body depends only upon the initial and final positions of that body, then the force is conservative, e.g., gravitational, electrostatic, $W = K_f - K_i = U(x_i) - U(x_f)$
- A force is conservative if it can be derived from a scalar quantity $U(x)$ by the relation $F(x) = -\Delta U(x)/\Delta x$ or $F = -dU/dx$
- If the work done by a force on a body that has moved in closed path and has come back to its initial position is zero, the force is conservative.

CONCEPT APPLICATION EXERCISE 7.2

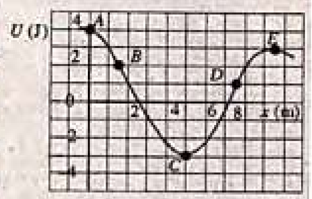
1. The kinetic energy of a body is increased by 21%. What is the percentage increase in the magnitude of linear momentum of the body?
2. Figure shows three paths connecting points a and b. A single force F does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force F conservative?



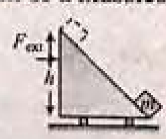
3. A particle moves in the x-y plane in the figure under the influence of a friction force with magnitude 3.00 N and acting in the direction opposite to the particle's displacement. Calculate the work done by the friction force on particle as it moves along the following closed paths: (a) path OA followed by the return path AO, (b) path OA followed by AC and the return path CO, (c) path OC followed by the return path CO, and (d) each of your three answers should be non-zero. What is the significance of this observation?



4. A potential energy function for a two-dimensional force is of the form $U = 3x^2y - 7x$. Find the force that acts at the point (x, y).
5. For the potential energy curve shown in the figure, (a) Determine whether the force F_x is positive, negative, or zero at the five points indicated; (b) Indicate points of stable, unstable, and neutral equilibrium; (c) Sketch the curve for F_x versus x from $x = 0$ to $x = 9.5$ m.



6. A block of mass m is placed at the bottom of a massless smooth wedge which if placed on a horizontal surface. When we push the wedge with a constant force, the block moves up the wedge. Find the work done by the external agent when the block has a speed v and is reaches the top of the wedge.



MECHANICAL ENERGY AND ITS CONSERVATION

Mechanical energy, E, of an object or a system is defined as the sum of kinetic energy K and potential energy U, i.e.,

$$E = K + U$$

We know the work-energy theorem:

$$W_{\text{conservative}} + W_{\text{non-conservative}} + W_{\text{other}} = \Delta K \quad \text{(i)}$$

But we know the definition of the potential energy

$$W_{\text{conservative}} = -\Delta U \quad \text{(ii)}$$

From (i) and (ii),

$$-\Delta U + W_{\text{non-conservative}} + W_{\text{other}} = \Delta K \quad \text{(iii)}$$

$$W_{\text{non-conservative}} + W_{\text{other}} = \Delta K + \Delta U$$

= Change in mechanical energy of the system

If only conservative forces act on the system, then we have $W_{\text{non-conservative}} = 0$ and $W_{\text{other}} = 0$.

Then we have, $\Delta K + \Delta U = 0$ (iv)

or we can say the mechanical energy of the system is constant.

Equation (iv) is a statement of conservation of mechanical energy for an isolated system with no non-conservative forces acting. The mechanical energy in such a system is conserved: the sum of the kinetic and potential energies remains constant.

If there are non-conservative forces acting within the system, mechanical energy is transformed to internal energy. If non-conservative forces act in an isolated system, the total energy of the system is conserved although the mechanical energy is not.

Let us summarize the concepts developed so far in this section:

- Work done on a particle is equal to the change in its kinetic energy.
- Work done on a system by all the (external and internal) forces is equal to the change in its kinetic energy.
- A force is called conservative if the work done by it during a round trip of a system is always zero. The force of gravitation, Coulomb force, force by a spring etc. are conservative. If the work done by it during a round trip is not zero, the force is non-conservative. Friction is an example of non-conservative force.
- The change in the potential energy of a system corresponding to conservative internal forces is equal to negative of the work done by this forces.
- If no external forces act (or the work done by them is zero) and the internal forces are conservative, the mechanical energy of the system remains constant. This is known as the principle of conservation of mechanical energy.
- If some of the internal forces are non-conservative and they do some work, the mechanical energy of the system is not constant.
- If the internal forces are conservative, the work done by the external forces is equal to the change in mechanical energy.

$$W_{\text{external}} = \Delta K + \Delta U$$

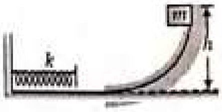
ILLUSTRATION 7.16 A smooth block of mass m is released from rest from a height h. It slides and compresses the spring of stiffness k. Find the maximum compression of the spring.

Solution. Taking block spring a system. No external force is doing work on system and no non-conservative force is present. The mechanical energy should be conserved.

$$\Delta K + \Delta U = 0$$

Let x = maximum compression of the spring.

Here $\Delta U = \Delta U_{sp} + \Delta U_{gr}$



$$\Delta U_{sp} = \frac{k}{2}x^2$$

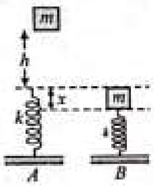
because the spring is deformed from $x = 0$ to $x = x$ and $\Delta U_{gr} = -mgh$ because the block falls down through a vertical distance h. Hence,

$$\Delta U = \frac{k}{2}x^2 - mgh$$

Since, the block will come to rest at the time of maximum compression $\Delta K = 0$. Substituting ΔU and ΔK in the equation $\Delta U + \Delta K = 0$, we have $\frac{k}{2}x^2 - mgh = 0$

Then $x = \sqrt{\frac{2mgh}{k}}$

ILLUSTRATION 7.17 A block of mass m strikes a light pan fitted with a vertical spring after falling through a distance h. If the stiffness of the spring is k, find the maximum compression of the spring.



Solution. Let the maximum compression in the spring be x. The reference level for potential energy is assumed at the position of maximum compression. Applying mechanical energy conservation, $\Delta K + \Delta U = 0$

As block is released from rest and finally comes to rest. Hence, net change in kinetic energy, $\Delta K = 0$.

Net change in potential energy,

$$\Delta U = \Delta U_{gr} + \Delta U_{sp} = [-mg(x+h)] + \left(\frac{1}{2}kx^2\right)$$

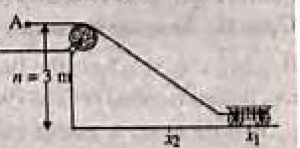
$$0 + [-mg(h+x)] + \left(\frac{1}{2}kx^2 - 0\right) = 0$$

$$x^2 - 2\left(\frac{mg}{k}\right)x - 2\left(\frac{mg}{k}\right)h = 0$$

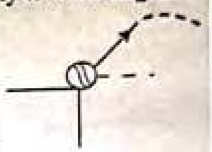
After solving, we get $x = \frac{mg}{k}\left[1 + \sqrt{1 + \frac{2kh}{mg}}\right]$

Concept Application Exercise 7.3

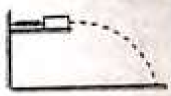
1. Figure shows a light, inextensible string attached to a cart, which can slide along a frictionless horizontal rail aligned along the x axis. The left end of the string is pulled over a pulley of negligible mass and friction and fixed at height $h = 3$ m from the ground level. The cart slides from $x_1 = 3\sqrt{3}$ m to $x_2 = 4$ m and during the move, tension in the string is kept constant at 50 N. Find the change in kinetic energy of the cart in joules. (Use $\sqrt{3} = 1.3$)



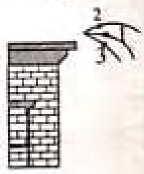
2. A boy throws a ball with initial velocity u at an angle of projection θ from a tower of height H. Neglecting air resistance, find
   (a) How high above the building the ball rises
   (b) Its speed just before it hits the ground.



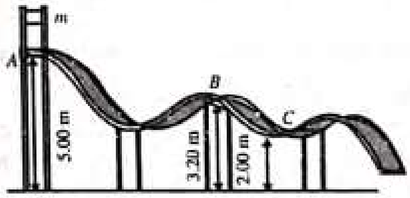
3. A small block of mass 100 g is pressed against a horizontal spring, fixed at one end, to compress the spring through 5.0 cm (see figure). The spring constant is 100 N m^{-1}. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring?



4. Three identical balls are thrown from the top of a building, all with the same initial speed. As shown in the figure, the first ball is thrown horizontally, second above horizontal level, and third at an angle below the horizontal. Neglecting air resistance, rank the speeds of the balls at the instant each hits the ground.



5. A particle of mass $m = 5.00$ kg is released from point A and it slides on the frictionless track shown in the figure. Determine (a) the particle's speed at points B and C and (b) the net work done by the gravitational force as the particle moves from A to C.

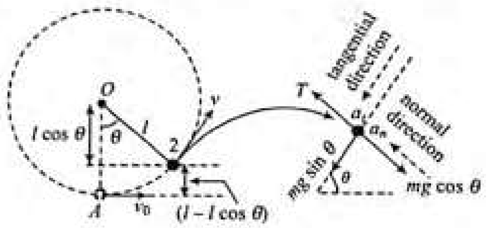


6. A block slides down a curved frictionless track and then up an inclined plane as in the figure. The coefficient of kinetic friction between the block and incline is μ_k. Find the maximum height reached by the block.

7. A block of mass m is released from point (A). The track is frictionless except for the portion between points (B) and (C), which has a length of d_{BC}. The block travels down the track, hits a spring of force constant k, and compresses the spring x from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between (B) and (C).

MOTION IN A VERTICAL CIRCLE

Let us consider the motion of a point mass tied to a string of length l and whirled in a vertical circle. If at any time the body is at angular position θ, as shown in the figure.



Let the particle is given velocity v_0 at lowest position. Using conservation of mechanical energy at A and (2).

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2\right) + mgl(1-\cos\theta) \qquad \text{(i)}$$

$$v^2 = v_0^2 - 2gl(1-\cos\theta)$$

or $$v = \sqrt{v_0^2 - 2gl(1-\cos\theta)} \qquad \text{(ii)}$$

For tangential acceleration at any position on vertical circle,

$$a_t = \frac{mg\sin\theta}{m} = g\sin\theta \qquad \text{(iii)}$$

Normal acceleration, $a_n = \dfrac{v^2}{l}$

$$\Rightarrow \quad a_n = \frac{v^2 - 2gl(1-\cos\theta)}{l} \qquad \text{(iv)}$$

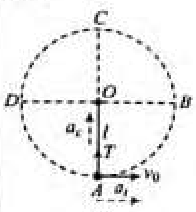
Minimum velocity required at top most position to complete circle is $\sqrt{gR}$, where R is the radius of the circle.

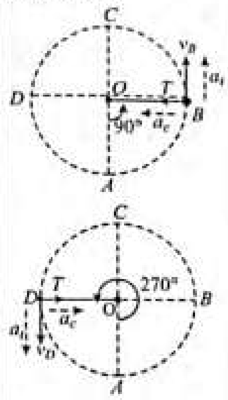
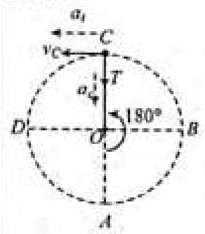
If speed at the lowest point is v_0, then from the conservation of mechanical energy between lowest point and top most point,

$v = v_{top}$ and $v_1 = v_{lowest} = v_0$

$$v_{top} = \sqrt{gl} = \sqrt{v_0^2 - 2gl(1-\cos\theta)} \Rightarrow v_0 \ge \sqrt{5gl}$$

i.e., for looping the loop, minimum velocity at lowest point must be greater than or equal to $\sqrt{5gR}$ where R is the radius of the circle.

If velocity at the lowest point is just enough for looping the loop, the value of various quantities is (True for a point mass attached to a string or a mass moving on a smooth vertical circular track.):	
At A  (a)	Tension at lowest position from (v), $T = \dfrac{m(\sqrt{5gl})^2}{l} + mg\cos 0° = 6mg$ Tangential acceleration, $a_t = g\sin 0° = 0$ Normal acceleration, $a_n = \dfrac{v_0^2}{2} = \dfrac{(\sqrt{5gl})^2}{l} = 5g$

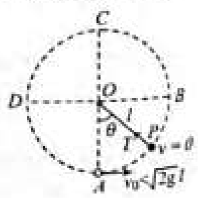
At B and D  (b)	Velocity at B from (i), $v_B = \sqrt{5gl - 2gl(1-\cos 90°)} = \sqrt{3gl}$ Tension at B from (iv), $T_B = \dfrac{mv_B^2}{l} + mg\cos 90°$ $= \dfrac{m(3gl)}{l} + 0 = 3mg$ Tangential acceleration, $\lvert a_t \rvert = g\sin\theta = g\sin 90° = g$ Normal acceleration, $a_n = \dfrac{v_B^2}{l} = 3g$
At C  (c)	Velocity at C from (i) $v_C = \sqrt{5gl - 2gl(1-\cos 180°)} = \sqrt{gl}$ Tension at C from (iv), $T_C = \dfrac{m(\sqrt{gl})^2}{l} + mg\cos 180° = 0$ Tangential acceleration, $\lvert a_t \rvert = g\sin\theta = g\sin 180° = 0$ Normal acceleration, $a_n = \dfrac{v_C^2}{l} = \dfrac{(\sqrt{gl})^2}{l} = g$

CONDITION FOR OSCILLATION OR LEAVING THE CIRCLE

In case of non-uniform circular motion in a vertical plane, if the velocity of body at lowest point is lesser than $\sqrt{5gl}$, the particle will not complete the circle in vertical plane. In this case, the motion of the point mass which depend on whether tension becomes zero before speed becomes zero or vice versa.

Case I: The speed becomes zero before tension

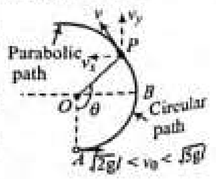
If velocity of the particle at lowest point, $v_0 < \sqrt{2gl}$, the ball never rises above the level of the center O, i.e., the body is confined to move within C and B, ($|\theta| <$ 90°) for this the speed at A.



In this case, tension cannot be zero, since a component of gravity acts radially outwards. Hence, the string will not go slack, and the ball will reverse back as soon as its speed becomes zero. Its motion will be oscillatory motion.

Case II: Tension becomes zero before speed

If $\sqrt{2gl} < v_0 < \sqrt{5gl}$, the ball rises above the level of center O, i.e., it goes beyond point B (90° < θ < 180°).

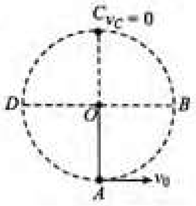


In this case, a component of gravity will always act towards center. Hence, centripetal acceleration or speed will remain non-zero. Hence, tension becomes zero first.

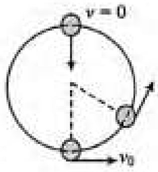
As soon as the tension becomes zero at any point, string will go slack and subsequently, the only force acting on the body is gravity. Hence its subsequent motion will be similar to that of a projectile. In this case, motion is a combination of circular and projectile motion.

Condition for looping the loop in some other cases

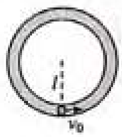
- A mass moving on a smooth vertical circular track.
- A particle attached to a light rod rotated in vertical circle.
  Condition for just looping the loop, velocity $v = 0$ at highest point (even if tension is zero, rod would not slack, and a compressive force can appear in the rod).
  By energy conservation,
  velocity at lowest point $= \sqrt{4gl}$
  $v_{min} = \sqrt{4gl}$ (for completing the circle)



- A bead attached to a ring and rotated.
  Condition for just looping the loop, velocity $v = 0$ at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).

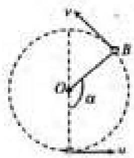


  By energy conservation, velocity at lowest point $= \sqrt{4gl}$
  $v_{min} = \sqrt{4gl}$ (for completing the circle)
- A block rotated between smooth surfaces of a pipe.
  Condition for just looping the loop, velocity $v = 0$ at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).



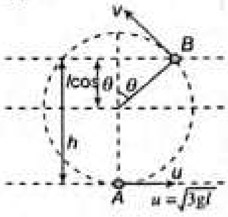
  By energy conservation,
  velocity at lowest point $= \sqrt{4gl}$
  $v_{min} = \sqrt{4gl}$ (for completing the circle)

ILLUSTRATION 7.18 The bob of a pendulum at rest is given a sharp hit to impart a horizontal velocity u where l is the length of the pendulum. Find the angle rotated by the string before it becomes slack.



Solution. Let string slacks at B.

Applying COM, E at A and B, $\Delta U + \Delta K = 0$



$$mgh + \left[\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right] = 0$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -mgh$$

$$v^2 = u^2 - 2g(l + l\cos\theta) \qquad \text{(i)}$$

When string slacks tension in the string becomes zero. The component of the weight in radial direction provide centripetal force at this position. From FBD of bob, we can write

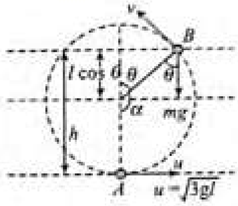
$$\frac{mv^2}{l} = mg\cos\theta$$

$$v^2 = lg\cos\theta \qquad \text{(ii)}$$

From equation (i) and (ii), we get

$$gl\cos\theta = 3gl - 2gl - 2gl\cos\theta$$

$$3\cos\theta = 1 \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$



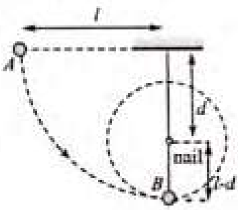
Hence, the angle rotated by the string before it becomes slack is

$$\Rightarrow \quad \alpha = \pi - \theta = \pi - \cos^{-1}\left(\frac{1}{3}\right)$$

ILLUSTRATION 7.19 In the given system, when the ball of mass m is released, it will swing down the dotted arc.

(a) How fast will it reach the lowest point in its swing? A nail is located at a distance d below the point of suspension.

(b) Show that d must at $0.6l$, if the ball is to swing completely around a circle centered along the nail.



(c) If $d = 0.6l$, find the change in tension in the string just after it touches the nail.

Solution.

(a) Radius of the circle centered at nail $= l - d$. To complete the circle centered at nail, the speed at the bottom must be at least $= \sqrt{5g(l-d)}$

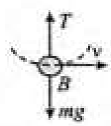
From A to B conserving mechanical energy $\Delta K + \Delta U = 0$:
Loss in GPE = Gain in KE

$$mgl = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gl}$$

(b) To complete the circle: $\sqrt{2gl} = \sqrt{5g(l-d)}$

$$\Rightarrow \quad 5l - 5d = 2l \quad \Rightarrow \quad d = \frac{3}{5}l = 0.6l$$

(c) Just before touching the nail, the ball is moving in a circle of radius l.



$$\Rightarrow \quad T_{before} - mg = \frac{mv^2}{l}$$

$$\Rightarrow \quad T_{before} = mg + \frac{mv^2}{l} = mg + 2mg = 3mg$$

Just after touching the nail, the ball is moving in a circle of radius $(l - d)$.

$$T_{after} - mg = \frac{mv^2}{(l-d)}$$

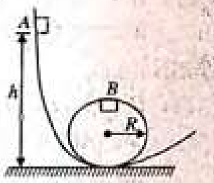
$$\Rightarrow \quad T_{\text{after}} = mg + \frac{mv^2}{l-d} = mg + \frac{m(2gl)}{0.4l}$$

$\Rightarrow$ Tension = $6mg$

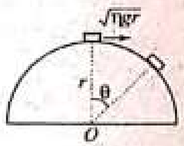
Hence, the tension in the string changes from $3mg$ to $6mg$ as it touches the nail.

Concept Application Exercise 7.4

1. A heavy particle hanging from a fixed point by a light inextensible string of length l is projected horizontally with speed $\sqrt{gl}$. Find the speed of the particle and the inclination of the string to the vertical at the instant when the tension in the string equals the weight of the particle.
2. A ball is attached to a horizontal cord of length l whose other end is fixed. (a) If the ball is released, what will be its speed at the lowest point of its path? (b) A peg is located a distance h directly below the point of attachment of the cord. If $h = 0.75l$, what will be the speed of the ball when it reaches the top of its circular path about the peg?
3. A smooth sphere of radius R is moving in a straight line with an acceleration a. A particle is released from the top of the sphere from rest. Find the speed of the particle when it is at an angular position θ from the initial position relative to the sphere.
4. The block on the loop shown in the figure slides without friction. At what height from A it starts so that it passes against the track at B with a net upward force equal to its own weight? The radius of the loop is R.



5. A particle rests on the top of a smooth hemisphere of radius r. It is imparted a horizontal velocity of $\sqrt{\eta gr}$. Find the angle made by the radius vector joining the particle with the vertical at the instant the particle losses contact with the sphere.



6. A block is released from rest at the top of an inclined plane which later curves into a circular track of radius r as shown in the figure. Find the minimum height h from where it should be released so that it is able to complete the circle.

MECHANICAL POWER

In many cases, it is useful to know not just the total amount of work being done, but how fast the work is done. For example, if you have a machine that can provide only a certain amount of work in a day and you wish to accomplish double of that much work, then you will have to either spend two days for the job or get an additional machine. We define work as the rate at which work is being done. Its defining equation is

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken to do work}}$$

or $\quad P = \dfrac{\Delta W}{\Delta t}$

Power is defined as the line rate of doing work or the time rate at which energy is transferred. Therefore,

$$P = \frac{dW}{dt}$$

where P is the instantaneous power, dW/dt is the rate of doing work.

We can write, $dW = \vec{F}.\,d\vec{r}$

$$\Rightarrow \quad \frac{dW}{dt} = \vec{F}.\frac{d\vec{r}}{dt} \quad \Rightarrow \quad P = \vec{F}\,\vec{v}$$

where $\vec{F}$ is the instantaneous force applied, and $\vec{v}$ is the instantaneous velocity.

Important Points

- Dimension: $[P] = [F]\,[v] = [MLT^{-2}]\,[LT^{-1}]$
  $\therefore \quad [P] = [ML^2T^{-3}]$
- Units: Watt or $J\,s^{-1}$ [SI]
  Erg/s [CGS]
  Practical units: Kilowatt (KW), Megawatt (MW) and Horse power (hp)
  Relations between different units:
  $1 \text{ watt} = 1\,J\,s^{-1} = 10^7 \text{ erg}\,s^{-1}$
  $1 \text{ hp} = 746 \text{ W}$
  $1 \text{ MW} = 10^6 \text{ W}$
  $1 \text{ kW} = 10^3 \text{ W}$
- If work done by the two bodies is same, then
  $$\text{Power} \propto \frac{1}{\text{time}}$$
  i.e., the body which performs the given work in lesser time possesses more power and vice-versa.
- As power = work/time, any unit of power multiplied by a unit of time gives unit of work (or energy) and not power. i.e., kilowatt-hour or watt-day are units of work or energy.
  $$1\,\text{kWh} = 10^3\,\frac{J}{s} \times (60 \times 60\,s) = 3.6 \times 10^6\,J$$
- The slope of work time curve gives the instantaneous power. As $P = dW/dt = \tan\theta$.
- Area under power–time curve gives the work done as

$$P = \frac{dW}{dt}$$

$\therefore \quad W = \int P\,dt$

= Area under P-t curve

ILLUSTRATION 7.20 A car of mass 500 kg moving with a speed 36 kmh in a straight road unidirectionally doubles its speed in 1 min. Find the power delivered by the engine.

Solution. Its initial speed $v_1 = \dfrac{36000}{3600} = 10\,\text{m}\,\text{s}^{-1}$.

If the car doubles its speed, finally its speed becomes $v_2 = 20\ \text{m}\,\text{s}^{-1}$.

Change in KE of the car = $\Delta KE = (1/2)\,mv_2^2 - (1/2)\,mv_1^2$

During $\Delta t = 1$ min = 60 s

the power delivered by the engine,

$$P = \frac{\text{Work done}}{\text{Time taken}} = \frac{|\Delta KE|}{\Delta t}$$

$$= \frac{\frac{1}{2}\times m\left[v_2^2 - v_1^2\right]}{\Delta t}$$

$$= \frac{1/2\times 500\left[20^2 - 10^2\right]}{60} = 1250\ \text{W}$$

ILLUSTRATION 7.21 An automobile of mass m accelerates, starting from rest, while the engine supplies constant power P, its position and instantaneous velocity changes w.r.t. time assuming the automobile starts from rest.

Solution. **Velocity:** As $Fv = P$ = constant, i.e.,

$$m\frac{dv}{dt}v = P \quad \left[\text{as } F = \frac{m\,dv}{dt}\right]$$

or $$\int v\,dv = \int \frac{P}{m}dt$$

By integrating both sides, we get $\dfrac{v^2}{2} = \dfrac{P}{m}t + C_1$

As initially the body is at rest, i.e., $v = 0$ at $t = 0$, so $C_1 = 0$.

$$\therefore \quad v = \left(\frac{2Pt}{m}\right)^{1/2}$$

Position: From the above expression, $v = \left(\dfrac{2Pt}{m}\right)^{1/2}$

or $$\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2} \quad \left[\text{As } v = \frac{ds}{dt}\right]$$

i.e., $$\int ds = \int \left(\frac{2Pt}{m}\right)^{1/2} dt$$

By integrating both sides, we get

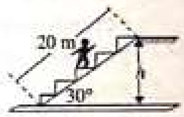
$$s = \left(\frac{2P}{m}\right)^{1/2} \cdot \frac{2}{3}t^{3/2} + C_2$$

Now as at $t = 0$, $s = 0$, so $C_2 = 0$.

$$s = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$$

CONCEPT APPLICATION EXERCISE 7.5

1. One coolie takes 1 min to raise a box through a height of 2 m. Another one takes 30 s for the same job and does the same amount of work. Which one of the two has greater power and which one uses greater energy?
2. A large family uses 8 kW of power. Direct solar energy is incident on the horizontal surface at an average rate of 200 W m^{-2}. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW?
3. An elevator of total mass (elevator + passenger) 1800 kg is moving up with a constant speed of 2 m s^{-1}. A frictional force of 4000 N opposes its motion. Determine the minimum power delivered by the motor to the elevator. Take $g = 10$ m s^{-2}.
4. Two persons of equal weight are running at speeds of 4 m s^{-1} and 5 m s^{-1}, respectively. Both increase their speeds by 1 m s^{-1} in a time span of 10 s. Who does more work? Who develops more power?
5. A 90-kg man runs up an escalator while it is not in operation in 10 s. What is the average power developed by the man. Suppose the escalator is running so that the escalator steps move at a speed of 0.5 m s^{-1}. What is then the power developed by the man as seen by the ground reference if he moves at the same speed relative to the escalator steps as he did when the escalator is not in operation?



6. The electric motor of a model train accelerates the train from rest to 0.620 m s^{-1} in 21.0 ms. The total mass of the train is 875 g. Find the average power delivered to the train during the acceleration.

SOLVED EXAMPLES

1. The potential energy between two atoms in a molecule is given by $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The atom is in stable equilibrium when

(a) $x = \sqrt[6]{\dfrac{11a}{5b}}$ (b) $x = \sqrt[6]{\dfrac{a}{2b}}$

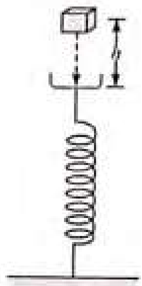
(c) $x = 0$ (d) $x = \sqrt[6]{\dfrac{2a}{b}}$

Sol. (d) Condition for stable equilibrium, $F = -\dfrac{dU}{dx} = 0$

$$\Rightarrow \quad -\frac{d}{dx}\left[\frac{a}{x^{12}}-\frac{b}{x^{6}}\right]=0 \quad \Rightarrow \quad -12ax^{-13}+6bx^{-7}=0$$

$$\Rightarrow \quad \frac{12a}{x^{13}}=\frac{6b}{x^{7}} \Rightarrow \frac{2a}{b}=x^{6} \Rightarrow x=\sqrt[6]{\frac{2a}{b}}$$

2. A block of mass m initially at rest is dropped from a height h on to a spring of force constant k. The maximum compression in the spring is x then



(a) $mgh=\frac{1}{2}kx^2$

(b) $mg(h+x)=\frac{1}{2}kx^2$

(c) $mgh=\frac{1}{2}k(x+h)^2$

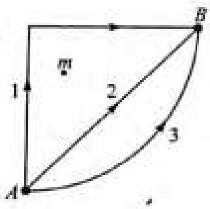
(d) $mg(h+x)=\frac{1}{2}k(x+h)^2$

Sol. (b) Change in gravitational potential energy

= Elastic potential energy stored in compressed spring

$$\Rightarrow \quad mg(h+x)=\frac{1}{2}kx^2$$

3. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3, respectively, (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.



(a) $W_1 > W_2 > W_3$

(b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$

(d) $W_2 > W_1 > W_3$

Sol. (b) Gravitational force is a conservative force and work done against it is a point function, i.e., does not depend on the path.

4. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force equal to $-K/r^2$, where K is a constant. The total energy of the particle is

(a) $\frac{K}{2r}$ (b) $-\frac{K}{2r}$

(c) $-\frac{K}{r}$ (d) $\frac{K}{r}$

Sol. (b) Here $\frac{mv^2}{r}=\frac{K}{r^2}$ $\therefore$ K.E. $=\frac{1}{2}mv^2=\frac{K}{2r}$

$$U=-\int_{\infty}^{r}F.dr=-\int_{\infty}^{r}\left(-\frac{K}{r^2}\right)dr=-\frac{K}{r}$$

Total energy $E=\text{K.E.}+\text{P.E.}=\frac{K}{2r}-\frac{K}{r}=-\frac{K}{2r}$

5. A force $F=-K(yi+xj)$ (where K is a positive constant) acts on a particle moving in the xy-plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and then parallel to the y-axis to the point (a, a). The total work done by the force F on the particles is

(a) $-2Ka^2$ (b) $2Ka^2$

(c) $-Ka^2$ (d) Ka^2

Sol. (c) While moving from $(0,0)$ to $(a, 0)$

Along positive x-axis, $y=0 \Rightarrow \vec{F}=-kx\hat{j}$

i.e. force is in negative y-direction while displacement is in positive x-direction.

$\therefore \quad W_1=0$

Because force is perpendicular to displacement.

Then particle moves from $(a,0)$ to (a, a) along a line parallel to y-axis $(x=+a)$ during this $\vec{F}=-k(y\hat{i}+a\hat{j})$

The first component of force, $-ky\hat{i}$ will not contribute any work because this component is along negative x-direction $(-\hat{i})$ while displacement is in positive y-direction $(a,0)$ to (a,a).

The second component of force *i.e.* $-ka\hat{j}$ will perform negative work

$\therefore \quad W_2=(-ka\hat{j})(a\hat{j})=(-ka)(a)=-ka^2$

So net work done on the particle

$W=W_1+W_2=0+(-ka^2)=-ka^2$

6. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ (b) $t^{3/4}$

(c) $t^{3/2}$ (d) t^2

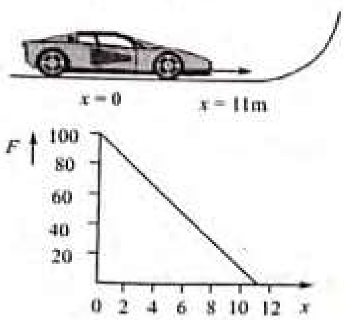
Sol. (c) $P=Fv=mav=m\left(\frac{dv}{dt}\right)v \Rightarrow \frac{P}{m}dt=v\,dv$

$$\Rightarrow \quad \frac{P}{m}\times t=\frac{v^2}{2} \Rightarrow v=\left(\frac{2P}{m}\right)^{1/2}(t)^{1/2}$$

Now $s=\int v\,dt=\int\left(\frac{2P}{m}\right)^{1/2}t^{1/2}dt$

$$\therefore \quad s=\left(\frac{2P}{m}\right)^{1/2}\left[\frac{2t^{3/2}}{3}\right] \Rightarrow s \propto t^{3/2}$$

7. A toy car of mass 5 kg moves up a ramp under the influence of force F plotted against displacement x. The maximum height attained is given by



(a) $y_{max} = 20$ m (b) $y_{max} = 15$ m
(c) $y_{max} = 11$ m (d) $y_{max} = 5$ m

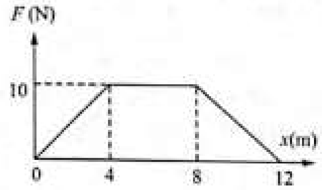
Sol. (c) Work done = Gain in potential energy

Area under curve = mgh

$\Rightarrow \quad \frac{1}{2} \times 11 \times 100 = 5 \times 10 \times h$

$\Rightarrow \quad h = 11$ m

8. A particle of mass 0.1 kg is subjected to a force which varies with distance as shown in the figure. If it starts its journey from rest at $x = 0$, its velocity at $x = 12$ m is



(a) 0 m/s (b) $20\sqrt{2}$ m/s
(c) $20\sqrt{3}$ m/s (d) 40 m/s

Sol. (d) Area between curve and displacement axis

$= \frac{1}{2} \times (12 + 4) \times 10 = 80$ J

In this time, body acquires kinetic energy $= \frac{1}{2}mv^2$

By the law of conservation of energy

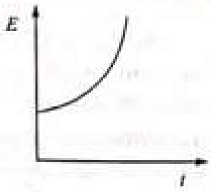
$\frac{1}{2}mv^2 = 80$ J

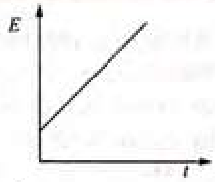
$\Rightarrow \quad \frac{1}{2} \times 0.1 \times v^2 = 80$

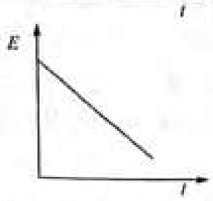
$\Rightarrow \quad v^2 = 1600$

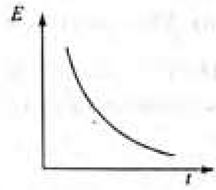
$\Rightarrow \quad v = 40$ m/s

9. A particle is dropped from a height h. A constant horizontal velocity is given to the particle. Taking g to be constant every where, kinetic energy E of the particle w.r.t. time t is correctly shown in

(a)


(b)


(c)


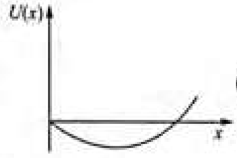
(d)


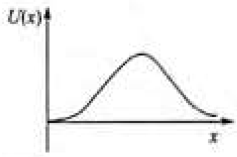
Sol. (a) As particle is projected with some velocity, therefore, its initial kinetic energy will not be zero.

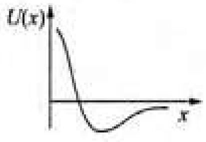
As it moves downward under gravity, then its velocity increases with time K.E. $\propto v^2 \propto t^2$ (As $v \propto t$)

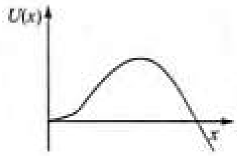
So the graph between kinetic energy and time will be parabolic in nature.

10. A particle which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional from of the potential energy $U(x)$ of the particle is

(a)


(b)


(c)


(d)


Sol. (d) $F = \frac{-dU}{dx} \Rightarrow dU = -F\,dx$

$\Rightarrow \quad U = -\int_0^x (-Kx + ax^3)dx = \frac{kx^2}{2} - \frac{ax^4}{4}$

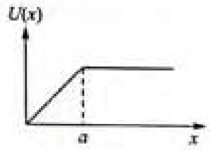
$\therefore$ We get $U = 0$ at $x = 0$ and $x = \sqrt{2k/a}$

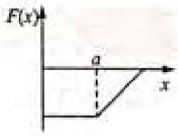
and also U = negative for $x > \sqrt{2k/a}$.

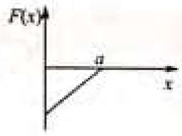
So $F = 0$ at $x = 0$

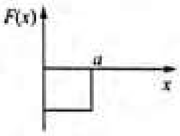
i.e. slope of $U - x$ graph is zero at $x = 0$.

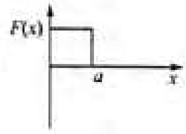
11. The potential energy of a system is represented in the first figure. the force acting on the system will be represented by



(a)


(b)


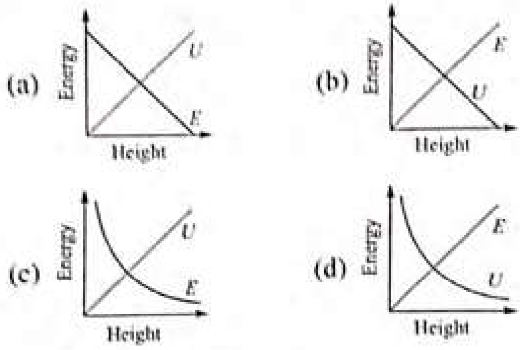
(c)


(d)


Sol. (c) As slope of problem graph is positive and constant upto certain distance and then it becomes zero.

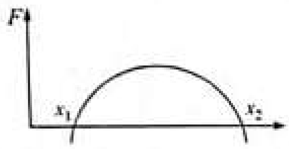
So from $F = \frac{-dU}{dx}$, up to distance a, F = constant (negative) and becomes zero suddenly.

12. Which of the following graphs is correct between kinetic energy (E), potential energy (U) and height (h) from the ground of the particle



***Sol.* (a)** Potential energy increases and kinetic energy decreases when the height of the particle increases it is clear from the graph (*a*).

13. The force acting on a body moving along x-axis varies with the position of the particle as shown in the figure.



The body is in stable equilibrium at

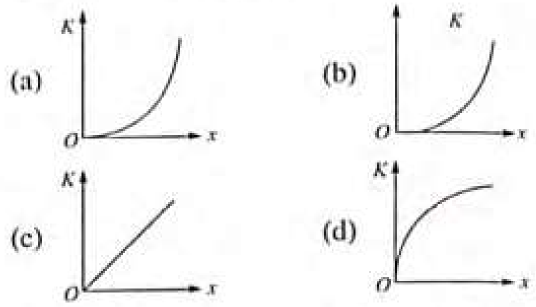
(a) $x = x_1$ (b) $x = ix_2$
(c) both x_1 and x_2 (d) neither x_1 nor x_2

***Sol.* (b)** When particle moves away from the origin, then at position $x = x_1$, force is zero and at $x > x_1$, force is positive (repulsive in nature). So particle moves further and does not return back to original position, i.e., the equilibrium is not stable.

Similarly, at position $x = x_2$, force is zero and at $x > x_2$, force is negative (attractive in nature).

So, particle returns back to original position, i.e., the equilibrium is stable.

14. A body moves from rest with a constant acceleration. Which one of the following graphs represents the variation of its kinetic energy K with the distance travelled x?

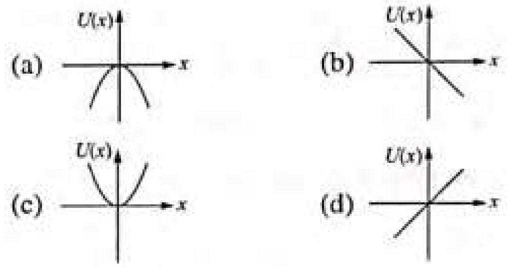


***Sol.* (c)** When body moves under the action of constant force, then kinetic energy acquired by the body K.E. $= F \times S$

$\therefore$ KE $\propto S$ (If F = constant)

So, the graph will be straight line.

15. A particle is placed at the origin and a force $F = kx$ is acting on it (where k is positive constant). If $U(0) = 0$, the graph of $U(x)$ versus x will be (where U is the potential energy function)



***Sol.* (a)** $U = -\int F dx = -\int kx\, dx = -k\dfrac{x^2}{2}$

This is the equation of parabola symmetric to U axis in negative direction.

EXERCISES

Concept of Work and Work Energy Theorem

1. A cart is set rolling across a level table, at the same speed on every trial. If it runs into a patch of sand, the cart exerts on the sand an average horizontal force of 6 N and travels a distance of 6 cm through the sand as it comes to a stop. If instead the cart runs into a patch of gravel on which the cart exerts an average horizontal force of 9 N, how far into the gravel will the cart roll before stopping?

(a) 9 cm (b) 6 cm (c) 4 cm (d) 3 cm

2. If a person is pushing a box with force $\vec{F}$ inside a train moving with constant velocity, the work done in the frame of earth will be:

$\vec{s}_0$ = displacement of the train relative to ground.

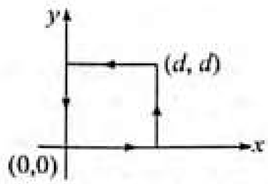
$\vec{s}$ = displacement of the box w.r.t. train.

(a) zero (b) $\vec{F}\cdot(\vec{s}+\vec{s}_0)$ (c) $\vec{F}\cdot\vec{s}$ (d) $\vec{F}\cdot\vec{s}_0$

3. Two springs have their force constant as k_1 and $k_2 (k_1 > k_2)$. When they are stretched by the same force

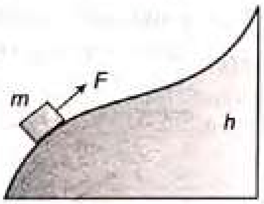
(a) No work is done in case of both the springs
(b) Equal work is done in case of both the springs
(c) More work is done in case of second spring
(d) More work is done in case of first spring

4. The work done by the force $= A(y^2\hat{i} + 2x^2\hat{j})$, where A is a constant and x and y are in meters around the path shown is:

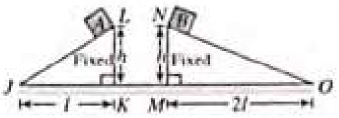


(a) zero (b) $A\,d$ (c) $A\,d^2$ (d) $A\,d^3$

5. If the net work done by external forces on a particle is zero, which of the following statements about the particle must be true?
   (a) Its velocity is zero.
   (b) Its velocity is decreased.
   (c) Its velocity is unchanged.
   (d) Its speed is unchanged.
6. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that
   (a) Its velocity is constant
   (b) Its acceleration is constant
   (c) Its kinetic energy is constant
   (d) It moves in a straight line
7. A ball of mass m moves with speed v and strikes a wall having infinite mass and it returns with same speed then the work done by the ball on the wall is
   (a) Zero (b) mv J (c) m/v J (d) v/m J
8. Is the work required to be done by an external force on an object on a frictionless, horizontal surface to accelerate it from a speed v to a speed $2v$
   (a) equal to the work required to accelerate the object from $t = 0$ to v
   (b) twice the work required to accelerate the object from $v = 0$ to v
   (c) three times the work required to accelerate the object from $v = 0$ to v
   (d) four times the work required to accelerate the object from 0 to v
9. A body of mass m is slowly pulled up the hill by a force F which at each point was directed along the tangent of the trajectory as shown in figure. All surfaces are smooth. Find the work performed by this force.



   (a) mgl (b) $-mgl$ (c) mgh (d) zero
10. An engine pumps water continuously through a hole. Speed with which water passes through the hole nozzle is v, and k is the mass per unit length of the water jet as it leaves the nozzle. Find the rate at which kinetic energy is being imparted to the water.
   (a) $\frac{1}{2}kv^2$ (b) $\frac{1}{2}kv^3$ (c) $\frac{v^2}{2k}$ (d) $\frac{v^3}{2k}$
11. A bus can be stopped by applying a retarding force F when it is moving with speed v on a level road. The distance covered by it before coming to rest is s. If the load of the bus increases by 50% because of passengers, for the same speed and same retarding force, the distance covered by the bus to come to rest shall be
   (a) $1.5s$ (b) $2s$ (c) $1s$ (d) $2.5s$
12. Power supplied to a particle of mass 2 kg varies with time as $P = 3t^2/2$ W. Here t is in second. If the velocity of particle at $t = 0$ is $v = 0$, the velocity of particle at time $t = 2$ s will be
   (a) $1\ \text{ms}^{-1}$ (b) $4\ \text{ms}^{-1}$ (c) $2\ \text{ms}^{-1}$ (d) $2\sqrt{2}\ \text{ms}^{-1}$
13. An object of mass m slides down a hill of arbitrary shape and after travelling a certain horizontal path stops because of friction. The total vertical height descended is h. The friction coefficient is different for different segments for the entire path but is independent of the velocity and direction of motion. The work that a tangential force must perform to return the object to its initial position along the same path is
   (a) mgh (b) $-mgh$ (c) $-2mgh$ (d) $2mgh$
14. A spring is compressed between two toy carts of masses m_1 and m_2. When the toy carts are released, the spring exerts on each toy cart equal and opposite forces for the same small time t. If the coefficients of friction μ between the ground and the toy carts are equal, then the magnitude of displacements of the toy carts are in the ratio
   (a) $\frac{S_1}{S_2} = \frac{m_2}{m_1}$ (b) $\frac{S_1}{S_2} = \frac{m_1}{m_2}$
   (c) $\frac{S_1}{S_2} = \left(\frac{m_2}{m_1}\right)^2$ (d) $\frac{S_1}{S_2} = \left(\frac{m_1}{m_2}\right)^2$
15. A particle is released one by one from the top of two inclined rough surfaces of height h each. The angles of inclination of the two planes are 30° and 60°, respectively. All other factors (e.g., coefficient of friction, mass of block, etc.) are same in both the cases. Let K_1 and K_2 be the kinetic energies of the particle at the bottom of the plane in the two cases. Then
   (a) $K_1 = K_2$ (b) $K_1 > K_2$
   (c) $K_1 < K_2$ (d) Data insufficient
16. A person of mass 70 kg jumps from a stationary helicopter with the parachute open. As he falls through 50 m height, he gains a speed of $20\ \text{m s}^{-1}$. The work done by the viscous air drag is
   (a) 21000 J (b) −21000 J
   (c) −14000 J (d) 14000 J
17. A particle of mass m moves with a variable velocity v, which changes with distance covered x along a straight line as $v = k\sqrt{x}$, where k is a positive constant. The work done by all the forces acting on the particle, during the first t seconds is
   (a) $\frac{mk^4}{t^2}$ (b) $\frac{mk^4t^2}{4}$ (c) $\frac{mk^4t^2}{8}$ (d) $\frac{mk^4t^2}{16}$
18. Two identical blocks A and B are placed on two inclined planes as shown in the figure. Neglect resistance and other friction.



Read the following statements and choose options.

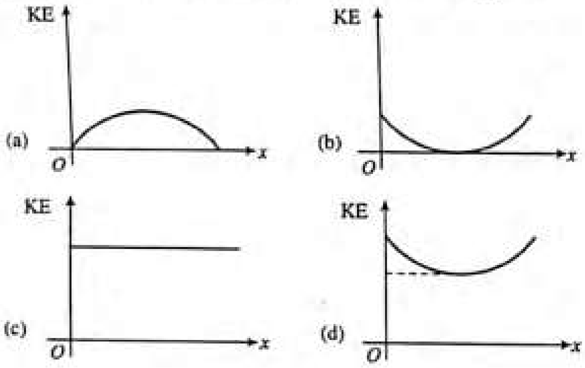
Statement I: The kinetic energy of A on sliding to J will be greater than the kinetic energy of B on sliding to O.

Statement II: The acceleration of A will be greater than acceleration of B when both are released on the inclined plane.

Statement III: The work done by external agent to move the block slowly from position B to O is negative.

(a) Only statement I is true (b) Only statement II is true
(c) Only I and III are true (d) Only II and III are true

19. A projectile is fired with some velocity making certain angle with the horizontal. Which of the following graphs is the best representation for the kinetic energy of a projectile (KE) versus its horizontal displacement (x)?



Concept of Potential Energy and Concept of Conservation of Mechanical Energy

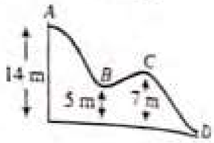
20. In which of the following cases can the work done increases the potential energy?
(a) Both conservative and non-conservative forces
(b) Conservative force only
(c) Non-conservative force only
(d) Neither conservative nor non-conservative forces.

21. Work done by a conservative force on a system is equal to
(a) The change in kinetic energy of the system
(b) The change in potential energy of the system
(c) The change in total mechanical energy of the system
(d) None of the above

22. Which of the following statements is correct?
(a) Kinetic energy of a system can be changed without changing its momentum.
(b) Kinetic energy of a system cannot be changed without changing its momentum.
(c) Momentum of a system cannot be changed without changing its kinetic energy.
(d) A system cannot have energy without having momentum.

23. A heavy weight is suspended from a spring. A person raises the weight till the spring becomes slack. The work done by him is W. The energy stored in the stretched spring was E. What will be the gain in gravitational potential energy?
(a) W (b) E (c) $W+E$ (d) $W-E$

24. Figure shows the vertical section of a frictionless surface. A block of mass 2 kg is released from rest from position A; its KE as it reaches position C is ($g = 10\ \text{m s}^{-2}$)



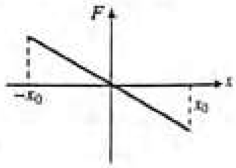
(a) 140 J (b) 180 J (c) 120 J (d) 280 J

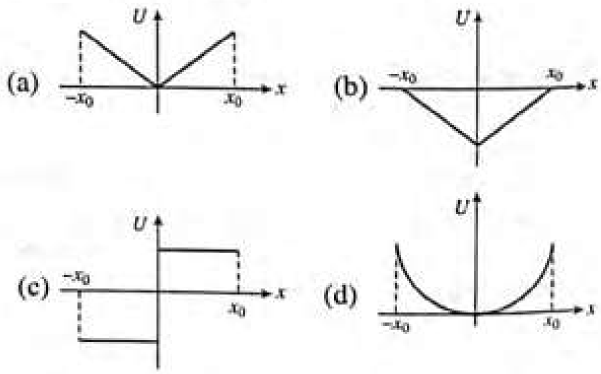
25. A toy gun uses a spring of force constant k. Before being triggered in the upward direction, the spring is compressed by a distance x. If the mass of the shot is m, on being triggered, it will go up to a maximum height of
(a) $\dfrac{kx^2}{mg}$ (b) $\dfrac{x^2}{kmg}$ (c) $\dfrac{kx^2}{2mg}$ (d) $\dfrac{k^2x^2}{mg}$

26. A block of 4 kg mass starts at rest and slides a distance d down a friction less incline (angle 30°) where it runs into a spring of negligible mass. The block slides an additional 25 cm before it is brought to rest momentarily by compressing the spring. The force constant of the spring is 400 N m^{-1}. The value of d is (take $g = 10\ \text{m s}^{-2}$)

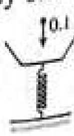
(a) 25 cm (b) 37.5 cm
(c) 62.5 cm (d) None of the above

27. Figure shows a plot of the conservative force F in a unidimensional field. The plot representing the function corresponding to the potential energy (U) in the field is





28. A massless platform is kept on a light elastic spring as shown in the figure. When a particle of mass 0.1 kg is dropped on the pan from a height of 0.24 m, the particle strikes the pan, and the spring is compressed by 0.01 m. From what height should the particle be dropped to cause a compression of 0.04 m?



(a) 0.96 m (b) 2.96 m
(c) 3.96 m (d) 0.48 m

29. The system shown in the figure is released from rest with mass 2 kg in contact with the ground. Pulley and spring are massless, and friction is absent everywhere. The speed of 5 kg block when 2 kg block leaves the contact with the ground is (force constant of the spring $k = 40\ \text{N m}^{-1}$ and $g = 10\ \text{m s}^{-2}$)

(a) $\sqrt{2}\ \text{ms}^{-1}$ (b) $2\sqrt{2}\ \text{ms}^{-1}$
(c) $2\ \text{ms}^{-1}$ (d) $\sqrt{2}\ \text{ms}^{-1}$

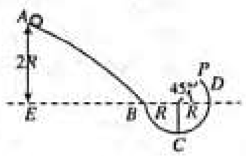
30. When a person stands on a weighing balance, working on the principle of Hooke's law, it shows a reading of 60 kg after a long time and the spring gets compressed by 2.5 cm. If the person jumps on the balance from a height of 10 cm, the maximum reading of the balance will be
(a) 60 kg (b) 120 kg (c) 180 kg (d) 240 kg

31 A particle of mass m is projected at an angle α to the horizontal with an initial velocity u. The work done by gravity during the time it reaches its highest point is
(a) $u^2\sin^2\alpha$ (b) $\frac{mu^2\cos^2\alpha}{2}$
(c) $\frac{mu^2\sin^2\alpha}{2}$ (d) $-\frac{mu^2\sin^2\alpha}{2}$

32. A particle located in a one-dimensional potential field has its potential energy function as $U(x)\ \frac{a}{x^4}-\frac{b}{x^2}$, where a and b are positive constants. The position of equilibrium x corresponds to
(a) $\frac{b}{2a}$ (b) $\sqrt{\frac{2a}{b}}$ (c) $\sqrt{\frac{2b}{a}}$ (d) $\frac{a}{2a}$

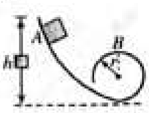
33. A man places a chain (of mass m and length l) on a table slowly. Initially, the lower end of the chain just touches the table. The man brings down the chain by length $l/2$. Work done by the man in this process is
(a) $-mg\frac{l}{2}$ (b) $-\frac{mgl}{4}$ (c) $\frac{-3mgl}{8}$ (d) $-\frac{mgl}{8}$

34. A particle of mass m slides on a frictionless surface $ABCD$, starting from rest as shown in the figure. The part BCD is a circular arc. If it looses contact at point P, the maximum height attained by the particle from point C is



(a) $R\left[2+\frac{1}{2\sqrt{2}}\right]$ (b) $R\left[1+\frac{1}{2\sqrt{2}}\right]R$
(c) $3R$ (d) None of these

35. A mass m starting from A reaches B of a frictionless track. On reaching B, it pushes the track with a force equal to x times its weight, then the applicable relation is



(a) $h = \frac{(x+5)}{2}r$ (b) $h = \frac{x}{2}r$
(c) $h = r$ (d) $h = \left(\frac{x+1}{2}\right)r$

36. A particle of mass m slides along a curved-flat-curved track. The curved portions of the track are smooth. If the particle is released at the top of one of the curved portions, the particle comes to rest at flat portion of length l and of $\mu = \mu_{\text{kinetic}}$ after covering a distance of

(a) $\frac{l}{3\mu}$ (b) $\frac{H}{2\mu_{\text{kinetic}}}$ (c) $\frac{l}{6}$ (d) $\frac{H}{\mu_{\text{kinetic}}}$

Mechanical Power

37. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the same staircase in 20 s. The ratio of the powers developed by them will be
(a) 1 (b) $\frac{4}{3}$ (c) $\frac{16}{9}$ (d) none of these

38. An engine pumps up 100 kg of water through a height of 10 m in 5 s. Given that the efficiency of the engine is 60%, what is the power of the engine? Take $g = 10\ \text{m s}^{-2}$.
(a) 33 kW (b) 3.3 kW (c) 0.33 kW (d) 0.033 kW

39. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain n times water from the same pipe in the same time, by what amount should the power of the motor be increased?
(a) n^2 times (b) n^3 times (c) n times (d) $n^{3/2}$ times

40. The speed v reached by a car of mass m in travelling a distance x, driven with constant power P, is given by
(a) $v = \frac{3xP}{m}$ (b) $v = \left(\frac{3xP}{m}\right)^{1/2}$
(c) $v = \left(\frac{3xP}{m}\right)^{1/3}$ (d) $v = \left(\frac{3xP}{m}\right)^2$

41. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to
(a) $\sqrt{t}$ (b) $t^{3/4}$ (c) $t^{3/2}$ (d) t^2

42. A car drives along a straight level frictionless road by an engine delivering constant power. Then velocity is directly proportional to
(a) t (b) $\frac{1}{\sqrt{t}}$ (c) $\sqrt{t}$ (d) None of these

43. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is
(a) Zero (b) $mk^2r^2t^2$ (c) mk^2r^2t (d) mk^2rt

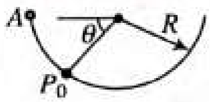
44. In the above question, the average power delivered by gravity is
(a) $-mgu\cos\alpha$ (b) $-mgu\sin\alpha$
(c) $-\dfrac{mgu\cos\alpha}{2}$ (d) $-\dfrac{mgu\sin\alpha}{2}$

45. A particle of mass m moves along a circular path of radius r with a centripetal acceleration a_n changing with time t as $a_n = kt^2$, where k is a positive constant. The average power developed by all the forces acting on the particle during the first t_0 seconds is
(a) $mkrt_0$ (b) $\dfrac{mkrt_0^2}{2}$ (c) $\dfrac{mkrt_0}{2}$ (d) $\dfrac{mkrt_0}{4}$

46. A 500-kg car, moving with a velocity of 36 $\mathrm{km\,h^{-1}}$ on a straight road unidirectionally, doubles its velocity in 1 min. The average power delivered by the engine for doubling the velocity is
(a) 750 W (b) 1050 W (c) 1150 W (d) 1250 W

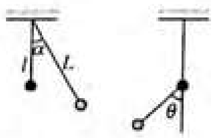
Circular Motion in Vertical Plane

47. A bead of mass m is released from rest at A to move along the fixed smooth circular track as shown in the figure. The ratio of magnitudes of centripetal force and normal reaction by the track on the bead at any point P_0 described by the angle $\theta (\neq 0)$ would



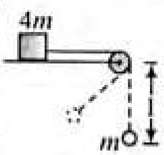
(a) Increase with θ
(b) Decrease with θ
(c) Remain constant
(d) First increase with θ and then decrease

48. A simple pendulum consisting of a mass M attached to a string of length L is released from rest at an angle α. A pin is located at a distance l below the pivot point. When the pendulum swings down, the string hits the pin as shown in the figure. The maximum angle θ which the string makes with the vertical after hitting the pin is



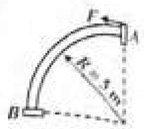
(a) $\cos^{-1}\left[\dfrac{L\cos\alpha + l}{L+l}\right]$ (b) $\cos^{-1}\left[\dfrac{L\cos\alpha + l}{L-l}\right]$
(c) $\cos^{-1}\left[\dfrac{L\cos\alpha - l}{L-l}\right]$ (d) $\cos^{-1}\left[\dfrac{L\cos\alpha - l}{L+l}\right]$

49. Two bodies of masses m and $4m$ are attached to a light string as shown in the figure. A body of mass m hanging from string is executing oscillations with angular amplitude 60°, while other body is at rest on a horizontal surface. The minimum coefficient of friction between mass $4m$ and the horizontal surface is (here pulley is light and smooth)



(a) $\dfrac{1}{4}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{8}$

50. A bead of mass 1/2 kg starts from rest from A to move in a vertical plane along a smooth fixed quarter ring of radius 5 m, under the action of a constant horizontal force $F = 5$ N as shown in the figure. The speed of bead as it reaches point B is



(a) 14.14 $\mathrm{m\,s^{-1}}$ (b) 7.07 $\mathrm{m\,s^{-1}}$
(c) 5 $\mathrm{m\,s^{-1}}$ (d) 25 $\mathrm{m\,s^{-1}}$

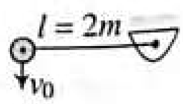
51. A 1-kg stone at the end of 1 m long string is whirled in a vertical circle at a constant speed of 4 $\mathrm{m\,s^{-1}}$. The tension in the string is 6 N when the stone is
(a) At the top of the circle
(b) At the bottom of the circle
(c) Half way down
(d) None of above

52. A stone of mass 1 kg tied to a light inextensible string of length $L = 10/3$ m is whirling in a circular path of radius L in a vertical plane. If the ratio of the maximum tension in the string to the minimum tension is 4 and if g is taken to be 10 $\mathrm{m\,s^{-2}}$, the speed of the stone at the highest point of the circle is
(a) 10 $\mathrm{m\,s^{-1}}$ (b) $5\sqrt{3}$ $\mathrm{m\,s^{-1}}$
(c) $10\sqrt{3}$ $\mathrm{m\,s^{-1}}$ (d) 20 $\mathrm{m\,s^{-1}}$

53. Ball A of mass m, after sliding from an inclined plane, strikes elastically another ball B of same mass at rest. Find the minimum height h so that ball B just completes the circular motion of the surface at C. (All surfaces are smooth.)

(a) $h = \dfrac{5}{2}R$ (b) $h = 2R$
(c) $h = \dfrac{2}{5}R$ (d) $h = 3R$

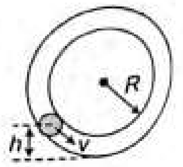
54. A small sphere is given vertical velocity of magnitude $v_0 = 5$ $\mathrm{m\,s^{-1}}$ and it swings in a vertical plane about the end of a massless string. The angle θ with the vertical at which string will break, knowing that it can withstand a maximum tension equal to twice the weight of the sphere, is



(a) $\cos^{-1}\dfrac{2}{3}$ (b) $\cos^{-1}\left(\dfrac{1}{4}\right)$
(c) 60° (d) 30°

55. A heavy particle hanging from a string of length l is projected horizontally with speed $\sqrt{gl}$. The speed of the particle at the point where the tension in the string equals weight of the particle
(a) $\sqrt{2gl}$ (b) $\sqrt{3gl}$ (c) $\sqrt{gl/2}$ (d) $\sqrt{gl/3}$

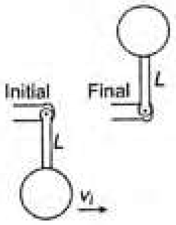
56. With what minimum speed v must a small ball should be pushed inside a smooth vertical tube from a height h so that it may reach the top of the tube? Radius of the tube is R.



(a) $\sqrt{2g(h+2R)}$ (b) $\frac{5}{2}R$

(c) $\sqrt{g(5R-2h)}$ (d) $\sqrt{2g(2R-h)}$

57. A light, rigid rod is 40.0 cm long. Its top end is pivoted on a frictionless, horizontal axle. The rod hangs straight down at rest with a small, massive ball attached to its bottom end. You strike the ball, suddenly giving it a horizontal velocity so that it swings around in a full circle. What minimum speed at the bottom is required to make the ball go over the top of the circle?



(a) 2.5 m/s (b) 4.0 m/s (c) 3.0 m/s (d) 5.0 m/s

58. A particle is moving in the vertical plane. It is attached at one end of a string of length λ whose other end is fixed. The velocity at the lowest point is u. The tension in the string is $\vec{T}$ and velocity of the particle is $\vec{v}$ at any position. Then, which of the following quantity will remain constant.

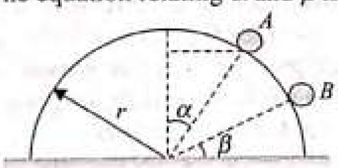
(a) $\vec{T}\cdot\vec{v}$

(b) kinetic energy

(c) Gravitational potential energy

(d) $\vec{T}\times\vec{v}$

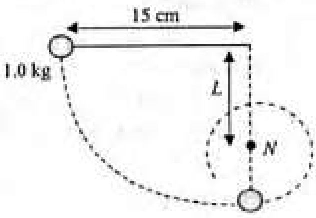
59. A particle moves from rest at A on the surface of a smooth circular cylinder of radius r as shown. At B it leaves the cylinder. The equation relating α and β is



(a) $3\sin\alpha = 2\cos\beta$ (b) $2\sin\alpha = 3\cos\beta$

(c) $3\sin\beta = 2\cos\alpha$ (d) $2\sin\beta = 3\cos\alpha$

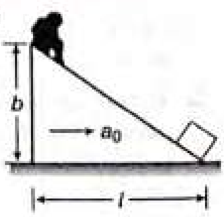
60. A ball weighing 1.0 kg is tied to a string 15 cm long. Initially the ball is held in position such that the string is horizontal. The ball is now released. A nail N is situated vertically below the support at a distance L. The minimum value of L such that the string will be wound round the nail is



(a) 15 cm (b) 4 cm (c) 9 cm (d) None of these

Problems Based on Mixed Concepts

61. A smooth block of mass m moves up from bottom to top of a wedge which is moving with an acceleration a_0. Find the work done by the pseudo force measured by the person sitting at the edge of the wedge.



(a) $ma_0 l$ (b) $ma_0\sqrt{l^2+b^2}$

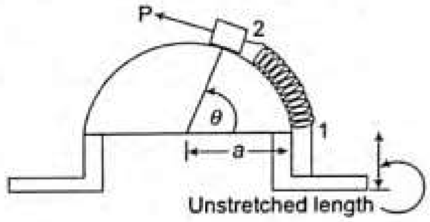
(c) $ma_0\sqrt{l+b}$ (d) $ma_0 l\frac{b}{\sqrt{l^2+b^2}}$

62. A particle is released from the top of two inclined rough surfaces of height 'h' each. The angle of inclination of the two planes are 30° and 60° respectively. All other factors (e.g. coefficient of friction, mass of block etc.) are same in both the cases. Let K_1 and K_2 be the kinetic energies of the particle at the bottom of the plane in two cases. Then

(a) $K_1 = K_2$ (b) $K_1 > K_2$

(c) $K_1 < K_2$ (d) data insufficient

63. A variable force P is maintained tangent to a frictionless cylindrical surface of radius a as shown in figure. By slowly varying this force, a block of weight W is moved and the spring to which it is stretched from position 1 to position 2. The work done by the force P is:



(a) $Wa\sin\theta$ (b) $\frac{1}{2}ka^2\theta^2$

(c) $Wa\sin\theta + ka^2\theta^2$ (d) $Wa\sin\theta + \frac{1}{2}ka^2\theta^2$

64. A ball is thrown from a height of h meter with an initial downward velocity v_0. It hits the ground, loses half of its kinetic energy and bounces back to the same height. The value of v_0 is

(a) $\sqrt{2gh}$ (b) $\sqrt{gh}$ (c) $\sqrt{3gh}$ (d) $\sqrt{2.5gh}$

65. In the situation shown in the adjacent figure, what is the maximum elongation produced in the spring if initially the spring is relaxed (consider all contact surfaces to be smooth)?

(a) $\frac{F}{k}$ (b) $\frac{F}{2k}$ (c) $\frac{2F}{k}$ (d) $\frac{4F}{k}$

66. The kinetic energy of a particle moving along a straight line increases uniformly with respect to the distance travelled by it. The force acting on the particle is (v is the speed of particle at any time)

(a) constant

(b) proportional to v

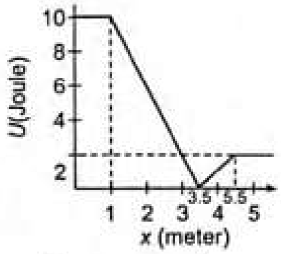
(c) proportional to v^2

(d) inversely proportional to v

67. Two springs A and B are identical except that A is stiffer than B i.e., $k_A > k_B$. If the two springs are stretched by the same force, then

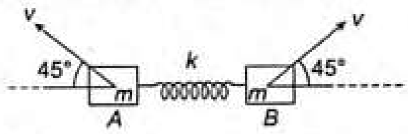
(a) more work is done on B i.e., $W_B > W_A$
(b) more work is done on A, i.e., $W_A > W_B$
(c) work done on A and B are equal
(d) work done depends upon the way in which they are stretched

68. A body with mass 2 kg moves in one direction in the presence of a force which is described by the potential energy graph. If the body is released from rest at $x = 2$ m, then its speed when it crosses $x = 5$ m is



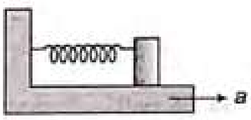
(a) zero (b) 1 ms^{-1} (c) 2 ms^{-1} (d) 3 ms^{-1}

69. Blocks A and B of mass m each are connected with spring of constant k. Both blocks lie on frictionless ground and are imparted horizontal velocity v as shown when spring is unstretched. Find the maximum stretch of spring.



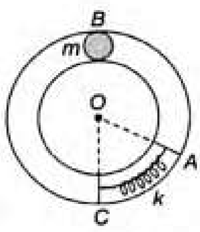
(a) $v\sqrt{\dfrac{m}{k}}$ (b) $v\sqrt{\dfrac{m}{2k}}$ (c) $v\sqrt{\dfrac{2m}{k}}$ (d) None of these

70. Find the maximum compression in the spring, if the lower block is shifted to rightwards with acceleration 'a'. All the surfaces are smooth:



(a) $\dfrac{ma}{2k}$ (b) $\dfrac{2ma}{k}$ (c) $\dfrac{ma}{k}$ (d) $\dfrac{4ma}{k}$

71. In the figure shown, there is a smooth tube of radius 'R', fixed in the vertical plane. A ball 'B' of mass 'm' is released from the top of the tube. B slides down due to gravity and compresses the spring. The end 'C' of the spring is fixed and the end A is free. Initially the line OA makes an angle of 60° with OC and finally it makes an angle of 30° after compression. Find the spring constant of the spring.



Fix smooth tube

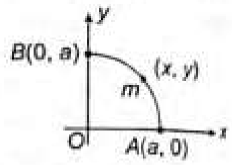
(a) $\dfrac{12mg(2+\sqrt{3})}{\pi^2 R}$ (b) $\dfrac{36mg(2+\sqrt{3})}{\pi^2 R}$

(c) $\dfrac{18mg}{\pi^2 R}$ (d) None of these

72. While running, a person transforms about 0.60 J of chemical energy to mechanical energy per step per kilogram of body mass. If a 60 kg runner transforms energy at a rate of 72 W during a race, how fast is the person running? Assume that a running step is 1.5 m long.

(a) 2.0 m/s (b) 3.0 m/s (c) 2.5 m/s (d) 5.0 m/s

73. A particle of mass 'm' moves along the quarter section of the circular path whose centre is at the origin. The radius of the circular path is 'a'. A force $\vec{F} = y\hat{i} - x\hat{j}$ newton acts on the particle, where x, y denote the coordinates of position of the particle. Calculate the work done by this force in taking the particle from point A $(a, 0)$ to point B $(0, a)$ along the circular path.



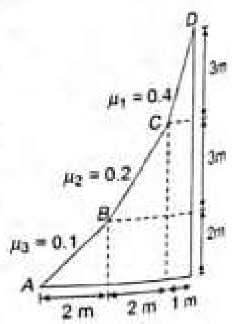
(a) $-\sqrt{2\pi a^2}$ J (b) $-\dfrac{\pi a^2}{2}$J

(c) $-\pi a^2$J (d) $-\dfrac{\sqrt{3}\pi a^2}{2}$J

74. The force exerted by a compression device is given by $F(x) = kx\,(x - l)$ for $0 \le x \le l$, where l is the maximum possible compression, x is the compression and k is a constant. The work required to compress the device by a distance d will be maximum when:

(a) $d = \dfrac{l}{4}$ (b) $d = \dfrac{l}{\sqrt{2}}$ (c) $d = \dfrac{l}{2}$ (d) $d = l$

75. A body of mass 1 kg is shifted from A to D on inclined planes by applying a force slowly such that the block is always in contact with the plane surfaces. Neglecting the jerk experienced at points C and B, total work done by the force is:



(a) 90 J (b) 56 J
(c) 180 J (d) 0 J

ARCHIVES

1. From a building, two balls A and B are thrown such that A is thrown upward and B downward (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then

(a) $v_B > v_A$ (b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses. **(AIEEE 2002)**

2. The speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances in which the two cars are stopped from that instant is

(a) 1 : 1 (b) 1 : 4
(c) 1 : 8 (d) 1 : 16 **(AIEEE 2002)**

3. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(a) 16 J (b) 8 J
(c) 32 J (d) 24 J **(AIEEE 2002)**

4. Two masses of 1 kg and 16 kg are moving with equal kinetic energy. The ratio of magnitude of the linear momentum is
(a) 1 : 2 (b) 1 : 4
(c) $1:\sqrt{2}$ (d) $\sqrt{2}:1$ **(AIEEE 2002)**

5. A car moving with a speed of 50 km/h can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/h, the minimum stopping distance is
(a) 6 m (b) 2 m
(c) 18 m (d) 24 m **(AIEEE 2003)**

6. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to
(a) $t^{1/2}$ (b) $t^{3/4}$
(c) $t^{3/2}$ (d) $t^{1/4}$ **(AIEEE 2003)**

7. A spring of constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is
(a) 6.25 N-m (b) 12.50 N-m
(c) 18.75 N-m (d) 25.00 N-m **(AIEEE 2003)**

8. Consider the following two statements
1. Linear momentum of a system of particles is zero.
2. Kinetic energy of a system of particles is zero.

Then
(a) 1 implies 2 and 2 implies 1
(b) 1 does not imply 2 and 2 does not imply 1
(c) 1 implies 2 but 2 does not imply 1
(d) 1 does not imply 2 but 2 implies 1 **(AIEEE 2003)**

9. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(a) x (b) e^x
(c) x^2 (d) $\log_e x$ **(AIEEE 2004)**

10. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ N is applied over a particle which displaces it from its origin to the point $\vec{r} = (2\hat{i} - \hat{j})$ m. The work done on the particle (in J) is
(a) +10 (b) +7
(c) −7 (d) +13 **(AIEEE 2004)**

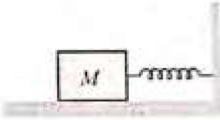
11. A body of mass m accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time t is

(a) $\dfrac{mv_1t^2}{t_1}$ (b) $\dfrac{mv_1^2t}{t_1^2}$ (c) $\dfrac{mv_1t}{t_1}$ (d) $\dfrac{mv_1^2t}{t_1}$ **(AIEEE 2004)**

12. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle; the motion of the particle takes place in a plane. It follows that
(a) its kinetic energy is constant.
(b) its acceleration is constant.
(c) its velocity is constant.
(d) it moves in a straight line. **(AIEEE 2004)**

13. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?
(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J **(AIEEE 2004)**

14. The block of mass M moving on a frictionless horizontal surface collides with a spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is



(a) $\dfrac{ML^2}{k}$ (b) zero (c) $\dfrac{kL^2}{2M}$ (d) $\sqrt{MkL}$ **(AIEEE 2005)**

15. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15$ N/m. What will be its initial acceleration if it is released from a point 20 cm away from the origin?
(a) 10 m/s^2 (b) 5 m/s^2
(c) 15 m/s^2 (d) 3 m/s^2 **(AIEEE 2005)**

16. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is
(a) $10\sqrt{30}$ m/s (b) 10 m/s
(c) 20 m/s (d) 40 m/s **(AIEEE 2005)**

17. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is given by

(a) $\dfrac{1}{2}\dfrac{mv^2}{T^2}t^2$ (b) $\dfrac{1}{2}\dfrac{mv^2}{T^2}t$

(c) $\dfrac{mv^2}{T^2}t^2$ (d) $\dfrac{mv^2}{T^2}t$ **(AIEEE 2005)**

18. A particle of mass 100 g is thrown vertically upward with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is
(a) 1.25 J (b) 0.5 J
(c) −0.5 J (d) −1.25 J **(AIEEE 2006)**

19. The potential energy of a 1 kg particle free to move along the x-axis is given by $V(x) = [(x^4/4) - (x^2/2)]$ J. The total mechanical energy of the particle is 2 J. Then the maximum speed (in m/s) is

(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{2}$ **(AIEEE 2006)**

20. A mass of M kg is suspended by a weightless string. The horizontal force required to displace it until the string makes an angle of 45° with the initial vertical direction is

(a) $\frac{Mg}{\sqrt{2}}$ (b) $Mg(\sqrt{2}-1)$

(c) $Mg(\sqrt{2}+1)$ (d) $Mg\sqrt{2}$ **(AIEEE 2006)**

21. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g = 10$ m/s^2

(a) 20 N (b) 22 N

(c) 4 N (d) 16 N **(AIEEE 2006)**

22. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10000 N/m. The spring compresses by (in cm)

(a) 2.5 (b) 11.0

(c) 8.5 (d) 5.5 **(AIEEE 2007)**

23. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is

(a) zero (b) $\frac{K}{4}$ (c) $\frac{K}{2}$ (d) K **(AIEEE 2007)**

24. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range

(a) 200–500 J (b) 2×10^5–3×10^5 J

(c) 20000–50000 J (d) 2000–5000 J

(AIEEE 2008)

25. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U_{(x=\infty)} - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$ (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

(AIEEE 2010)

26. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement I: If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement II: $k_1 < k_2$

(a) Statement I is false, Statement II is true.

(b) Statement I is true, Statement II is false.

(c) Statement I is true, Statement II is true, Statement II is the correct explanation for statement I.

(d) Statement I is true, Statement II is true, Statement II is not the correct explanation of Statement I.

(AIEEE 2012)

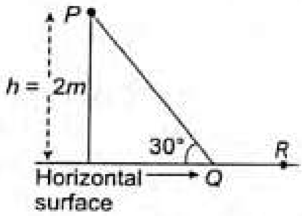
27. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber band by L is,

(a) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (b) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

(c) $aL^2 + bL^3$ (d) $\frac{1}{2}(aL^2 + bL^3)$

(JEE Main 2014)

28. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R.



The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR.

The values of the coefficient of friction μ and the distance $x(= QR)$, are, respectively close to:

(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m

(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

(JEE Main 2016)

29. A person trying to lose weight by burning fat lifts a mass of 10 kg up to a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8$ m s^{-2}.

(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg

(c) 9.89×10^{-3} kg (d) 12.89×10^{-3} kg

(JEE Main 2016)

30. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

(a) 10^{-4} kg m^{-1} (b) 10^{-1} kg m^{-1}s^{-1}

(c) 10^{-3} kg m^{-1} (d) 10^{-3} kg s^{-1}

(JEE Main 2017)

31. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be

(a) 9 J (b) 18 J
(c) 4.5 J (d) 22 J **(JEE Main 2017)**

32. A particle is moving is moving in a circular path of radius a under the action of an attractive potential $U = -k/2r^2$. Its total energy is:

(a) $-\frac{3}{2}\frac{k}{a^2}$ (b) $-\frac{k}{4a^2}$ (c) $\frac{k}{2a^2}$ (d) Zero

(JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (b)
11. (a)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (b)
21. (b)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (c)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (b)	**30.** (d)
31. (d)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (a)	**36.** (d)	**37.** (b)	**38.** (b)	**39.** (b)	**40.** (c)
41. (c)	**42.** (c)	**43.** (c)	**44.** (d)	**45.** (b)	**46.** (d)	**47.** (c)	**48.** (c)	**49.** (c)	**50.** (a)
51. (a)	**52.** (a)	**53.** (a)	**54.** (b)	**55.** (d)	**56.** (d)	**57.** (b)	**58.** (a)	**59.** (c)	**60.** (c)
61. (a)	**62.** (c)	**63.** (d)	**64.** (a)	**65.** (c)	**66.** (a)	**67.** (a)	**68.** (c)	**69.** (a)	**70.** (b)
71. (b)	**72.** (b)	**73.** (b)	**74.** (d)	**75.** (a)					

Archives

1. (b)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (d)	**6.** (c)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (b)
11. (b)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (b)
21. (a)	**22.** (d)	**23.** (b)	**24.** (d)	**25.** (c)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (d)	**30.** (a)
31. (c)	**32.** (d)								

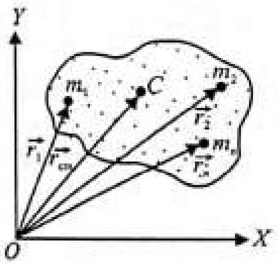
Chapter 8

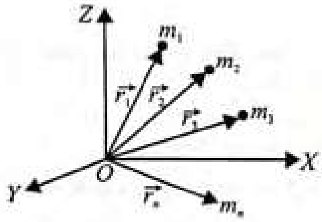
Centre of Mass, Conservation of Linear Momentum and Collision

CENTRE OF MASS OF A SYSTEM OF N DISCRETE PARTICLES

Consider a system of N point masses $m_1, m_2, m_3, \ldots, m_n$ whose position vectors from origin O are given by $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_n$, respectively.

Then, the position vector of the centre of mass C of the system is given by



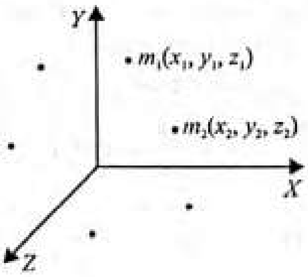


$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{m_1 + m_2 + \cdots + m_n}$$

$$= \frac{\Sigma_{i=1}^{n} m_i\vec{r}_i}{\Sigma_{i=1}^{n} m_i} = \frac{1}{M}\Sigma_{i=1}^{n} m_i\vec{r}_i$$

where $M(=\Sigma_{i=1}^{n} m_i)$ is the total mass of the system.

Consider a system of point masses $m_1, m_2, m_3, \ldots$, located at the coordinates $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$, respectively. The centre of mass of this system of masses is a point whose coordinates are (x_{cm}, y_{cm}, z_{cm}), which are given by



$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + \cdots}{m_1 + m_2 + \cdots}$$

NOTE:

- The centre of mass of two particles lies on the line joining them.
- The centre of mass divides the distance between two particles in the inverse ratio of masses.

  As shown in the previous section

  $$m_1\vec{r}_1' + m_2\vec{r}_2' = 0 \Rightarrow \frac{|\vec{r}_1'|}{|\vec{r}_2'|} = \frac{m_2}{m_1} \quad \text{[taking modulus]}$$

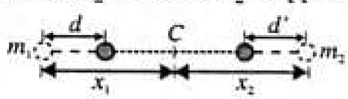
  Thus, the centre of mass is dividing the distance in inverse ratio. Thus, $r' \propto \frac{1}{m}$. Thus, the centre of mass less closer to more massive particle is

  $$m_1r'_1 = m_2(l - r'_1)$$

  $$r_1' = \left(\frac{m_2}{m_1 + m_2}\right)l$$

ILLUSTRATION 8.1 Consider a two-particle system with the particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved so as to keep the centre of mass at the same position?

Solution. Consider the figure. Suppose the distance of m_1 from the centre of mass C is x_1 and that of m_2 from C is x_2. Suppose mass m_2 is moved through a distance d' towards C so as to keep the centre of mass at C.



Then, $m_1x_1 = m_2x_2$ (i)

and $m_1(x_1 - d) = m_2(x_2 - d')$ (ii)

Subtracting Eq. (ii) from Eq. (i), we get

$m_1d = m_2d'$ or $d' = (m_1/m_2)\,d$.

CENTRE OF MASS OF A CONTINUOUS MASS DISTRIBUTION

For a continuous mass distribution, the centre of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation.

$$x_{CM} = \frac{\int x\,dm}{\int dm},\ y_{CM} = \frac{\int y\,dm}{\int dm},\ z_{CM} = \frac{\int z\,dm}{\int dm}$$

$\int dm = M$ (mass of the body)

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\,dm$$

NOTE: If an object has symmetric uniform mass distribution about x-axis, then y-coordinate of CM is zero and vice versa.

The following points regarding centre of mass can be noted.

- For a lamina type (two-dimensional) body with uniform negligible thickness, the formulae for finding the position of centre of mass are as follows:

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$
$$= \frac{\rho A_1 t\vec{r}_1 + \rho A_2 t\vec{r}_2 + \cdots}{\rho A_1 t + \rho A_2 t + \cdots}, (m = \rho A t)$$
$$= \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + \cdots}{A_1 + A_2 + \cdots}$$

  Here, A stands for area and ρ for density.

- If some mass is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained from the following formulae:

  (a) $\vec{r}_{CM} = \dfrac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2}$ or $\vec{r}_{CM} = \dfrac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$

  (b) $x_{CM} = \dfrac{m_1x_1 - m_2x_2}{m_1 - m_2}$ or $x_{CM} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$

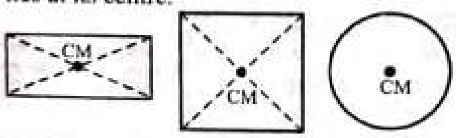
  (c) $y_{CM} = \dfrac{m_1y_1 - m_2y_2}{m_1 - m_2}$ or $y_{CM} = \dfrac{A_1y_1 - A_2y_2}{A_1 - A_2}$

  (d) $z_{CM} = \dfrac{m_1z_1 - m_2z_2}{m_1 - m_2}$ or $z_{CM} = \dfrac{A_1z_1 - A_2z_2}{A_1 - A_2}$

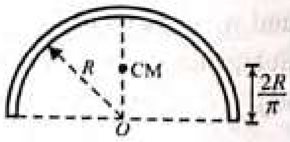
Here, $m_1, A_1, \vec{r}_1, x_1, y_1$ and z_1 are the values for the whole mass while $m_2, A_2, \vec{r}_2, x_2, y_2$ and z_2 are the values for the mass which has been removed.

NOTE: Centre of mass of some well known rigid bodies

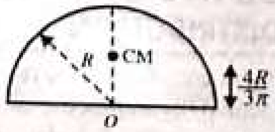
- Centre of mass of a uniform rectangular, square or circular plate lies at its centre.



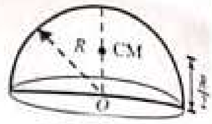
- Centre of mass of a uniform semicircular ring lies at a distance of $h = 2R/\pi$ from its centre, on the axis of symmetry where R is the radius of the ring.



- Centre of mass of a uniform semicircular disc of radius R lies at a distance of $h = 4R/3\pi$ from the centre on the axis of symmetry as shown in the figure.



- Centre of mass of a hemispherical shell of radius R lies at a distance of $h = R/2$ from its centre on the axis of symmetry as shown in the figure.



- Centre of mass of a solid hemisphere of radius R lies at a distance of $3R/8$ from its centre on the axis of symmetry.

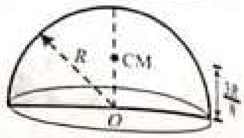
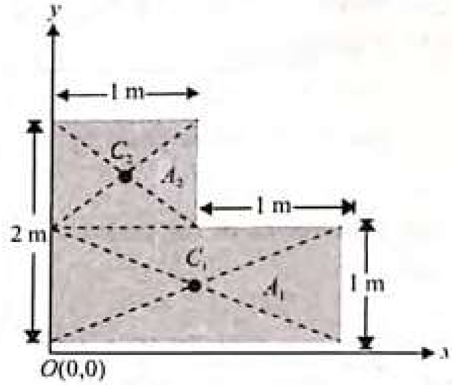


ILLUSTRATION 8.2 Find the centre of mass of a uniform L-shaped lamina (a thin flat plate) with dimensions as shown in the figure. The mass of lamina is 3 kg.



Solution. The plate has uniform density and same thickness everywhere. So its *CM* will coincide with the centroid. Divide the given plate into two parts of area A_1 and A_2 as shown in the figure. We have

$A_1 = 2 \times 1\ \text{m}^2$ with its centroid C_1 (1, 1/2) and

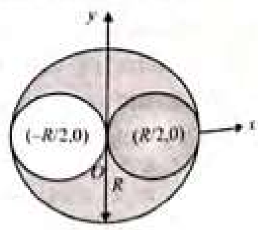
$A_2 = 1 \times 1\ \text{m}^2$ with its centroid C_2 (1/2, 3/2)

The centroid of the whole plate can be defined as

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{2\times 1 + 1\times\frac{1}{2}}{2+1} = \frac{5}{6}\ \text{m}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{2\times\frac{1}{2} + 1\times\frac{3}{2}}{2+1} = \frac{5}{6}\ \text{m}$$

ILLUSTRATION 8.3 Figure shows a uniform disc of radius R, from which a hole of radius $R/2$ has been cut out from left of the centre and is placed on the right of the centre of the disc. Find the CM of the resulting disc.



Solution. Mass of the cut-out disc is

$$m = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$

Let centre of the disc is at the origin of the coordinates. Then we can write the CM of the system as

$$x_{CM} = \frac{M\vec{R} - m\vec{r} + m\vec{r}}{M - m + m} = \frac{M\times 0 - \frac{M}{4}\left(\frac{-R}{2}\right) + \frac{M}{4}\left(\frac{R}{2}\right)}{M - \frac{M}{4} + \frac{M}{4}} = \frac{R}{4}$$

$y_{CM} = 0$

MOTION OF THE CENTRE OF MASS

Motion of Centre of Mass and Conservation of Momentum: Velocity of Centre of Mass of System

$$\vec{v}_{CM} = \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + \cdots + m_n\frac{d\vec{r}_n}{dt}}{M}$$

$$= \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{m_1 + m_2 + \cdots} \Rightarrow \vec{v}_{CM} = \frac{\vec{p}_{total}}{M_{total}}$$

Here, the numerator of the right-hand-side term is the total momentum of the system, i.e., summation of momentum of the individual components (particles) of the system.

Hence, velocity of centre of mass of the system is the ratio of momentum of the system per unit mass of the system.

Acceleration of Centre of Mass of System

$$\vec{a}_{CM} = \frac{m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} + \cdots + m_n\frac{d\vec{v}_n}{dt}}{M}$$

$$= \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n}{M}$$

$$= \frac{\vec{F}_{ext}}{M} = \frac{\text{Net external force}}{M}$$

(Both the action and reaction of an internal force must be within the system. Vector summation will cancel all internal forces and hence net internal force on system is zero.)

$$\therefore \quad \vec{F}_{ext} = M\vec{a}_{CM}$$

where $\vec{F}_{ext}$ is the sum of the 'external' forces acting on the system. The internal forces which the particles exert on one another play absolutely no role in the motion of the centre of mass.

If no external force is acting on a system of particles, the acceleration of centre of mass of the system will be zero. If $\vec{a}_{CM} = 0$, it implies that $\vec{v}_{CM}$ must be a constant and if $\vec{v}_{CM}$ is a constant, it implies that the total momentum of the system must remain constant. It leads to the principle that the total momentum of the system must remain constant. It leads to the principle of conservation of momentum in the absence of external forces.

If $\vec{F}_{ext} = 0$, then $\vec{p}_{total} =$ constant.

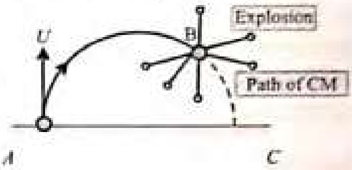
If no external force is acting on the system, net momentum of the system must remain constant.

Centre of Mass at Rest

If $\vec{F}_{ext} = 0$ and $\vec{v}_{cm} = 0$, then centre of mass remains at rest. Individual components of a system may move and have non-zero momentum due to mutual forces (internal), but the net momentum of the system remains zero.

NOTE:

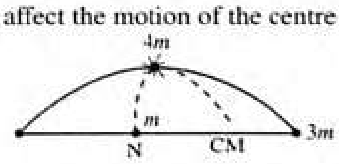
- Net force on a system of particles is equal to net external force.
- Sum of internal forces on particles of a system is zero.
- Centre of mass cannot be accelerated by internal forces.
- External forces equal mass multiplied by acceleration of centre of mass .
- Action point of external force is immaterial in producing acceleration in centre of mass.
- If a projectile explodes in air in different parts, the path of the centre of mass remains unchanged. This is because during explosion no external force (except gravity) acts on the centre of mass. The situation is as shown in the figure.



- Path of centre of mass is ABC, even though the different parts travel in different directions after explosion.
- Suppose a system consists of more than one particle (or bodies). Net external force on the system in a particular direction is zero. If initially the centre of mass of the system is at rest, then obviously the centre of mass will not move along that particular direction even though some particles (or bodies) of the system may move along that direction. The following example will illustrate the above theory.
- For a two particle system, the ratio of velocities, displacements and accelerations of the particles is equal to the inverse ratio of their masses.
- If $\vec{F}_{ext} = 0$, the CM moves with constant velocity. Hence, the CM (but not necessarily other points of the system which may be accelerating) can be treated as an inertial frame (point).
- The net effect of internal forces is zero on the CM whereas it is not zero (in general) for other points of the system. Inconsequence, the CM moves with a constant velocity whereas other points may move with same accelerations when $\vec{F}_{ext} = 0$.

ILLUSTRATION 8.4 A projectile is fired at a speed of 100 m/s at an angle of 37° above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1:3, the smaller coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Solution. Internal forces do not affect the motion of the centre of mass, the centre of mass hits the ground at a position where the original projectile would have landed. The range of the original projectile is



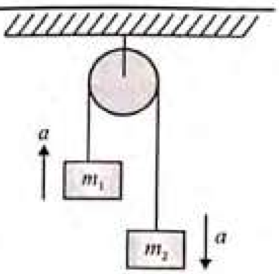
$$x_{CM} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2 \times 10^4 \times \frac{3}{5} \times \frac{4}{5}}{10} \text{ m} = 960 \text{ m}$$

The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at $x = 480$ m. If the heavier block hits the ground at x_2, then

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \Rightarrow 960 = \frac{(m)(480) + (3m)(x_2)}{(m + 3m)}$$

$\therefore \quad x_2 = 1120$ m

ILLUSTRATION 8.5 A pulley fixed to the ceiling carried a thread with bodies of masses m_1 and m_2 attached to its ends. The masses of the pulley and the thread are negligible and friction is absent. Find the acceleration of the centre of mass of this system.



Solution. Let us assume that $m_2 > m_1$. We can see that the masses have equal and opposite acceleration of the same magnitude.

$$a = \frac{m_2 - m_1}{m_2 + m_1} g \text{ and tension in string is } T = \frac{2m_1m_2g}{m_2 + m_1}$$

Taking downward direction as positive, $a_1 = -a$; $a_2 = +a$

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{(m_2 - m_1)}{m_2 + m_1}$$

Substituting for the value of a, we have

$$a_{CM} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)^2 g$$

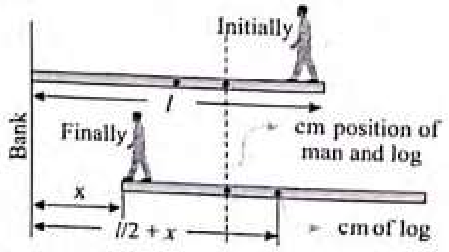
Alternative method:

$$a_{CM} = \frac{F_{ext}}{m_1 + m_2} = \frac{(m_1g + m_2g) - 2T}{m_1 + m_2} = g - \frac{2T}{m_1 + m_2}$$

$$= g - \frac{4m_1m_2g}{(m_2 + m_1)^2} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)^2 g \text{ (downwards)}$$

ILLUSTRATION 8.6 A log of wood of length l and mass M is floating on the surface of a river perpendicular to the banks. One end of the log touches the banks. A man of mass m standing at the other end walks towards the bank. Calculate the displacement of the log when he reaches the nearer end of the log.

Solution. Let PQ be the log of wood. As there is no external force, the centre of mass of man and the log system remains at rest. Let the bank of the river be the origin A. Initially, the man is at point Q.



Let $\quad m$ = mass of man

$\quad M$ = mass of log

x = displacement of log w.r.t. ground (here water)

$$X_{CM} \text{ (initial)} = \frac{m(l) + M\left(\frac{l}{2}\right)}{m + M}$$

$$X_{CM} \text{ (final)} = \frac{m(x) + M\left(\frac{l}{2} + x\right)}{m + M}$$

Now, X_{CM} (initial) = X_{CM} (final)

$$\Rightarrow \quad ml + \frac{Ml}{2} = mx + \frac{Ml}{2} + Mx$$

$$\Rightarrow \quad x = \frac{ml}{m + M}.$$

Hence, the log moves away from the bank through a distance of $\dfrac{ml}{(m+M)}$.

Alternative method: Displacement of the log = $\Delta x_1 = x$

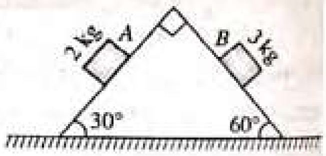
Displacement of the man = $\Delta x_2 = l - x$

Apply $m_1x_1 = m_2x_2 \Rightarrow$ (if the centre of mass remains at the same place)

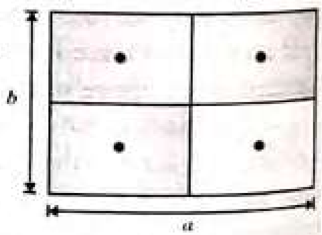
$$\Rightarrow Mx = m(l - x) \Rightarrow x = \frac{ml}{(m + M)}$$

CONCEPT APPLICATION EXERCISE 8.1

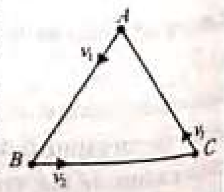
1. Figure shows a fixed wedge on which two blocks of masses 2 kg and 3 kg are placed on its smooth inclined surfaces. When the two blocks are released from rest, find the acceleration of centre of mass of the two blocks.



2. Consider a rectangular plate of dimensions $a \times b$. If the plate is considered to be made up of four rectangles of dimensions $\frac{a}{2} \times \frac{b}{2}$ and we now remove one (the lower right) out of the four rectangles, find the position where the centre of mass of the remaining system lie.



3. There are two masses m_1 and m_2 placed at a distance l apart, let the centre of mass of this system is at a point named C. If m_1 is displaced by l_1 towards C and m_2 is displaced by l_2 away from C, find the distance from C where the new centre of mass will be located.
4. Let there are three equal masses situated at the vertices of an equilateral triangle, as shown in the figure. Now particle A starts with a velocity v_1 towards line AB, particle B starts with the velocity v_2 towards line BC and particle C starts with velocity v_3 towards line CA. Find the displacement of the



centre of mass of the three particles A, B and C after time t. What would it be if $v_1 = v_2 = v_3$?

5. A 30 kg projectile moving horizontally with a velocity $\vec{v}_0 = (120 \text{ m/s})\ \hat{i}$ explodes into two fragments A and B of masses 12 kg and 18 kg, respectively. Taking point of explosion as origin and knowing that 3 s later the position of fragment A is (300 m, 24 m, –48 m), determine the position of fragment B at the same instant.

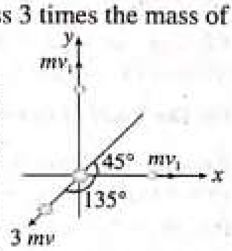
CONSERVATION OF LINEAR MOMENTUM

The total momentum of an isolated system remains constant.

This law is universal, i.e., it applies to both macroscopic and microscopic systems. It holds good even in atomic and nuclear physics where classical mechanics fails. Further it is more generally applicable than the law of 'conservation of mechanical energy' because 'internal forces' are often non-conservative and so mechanical energy is not conserved but momentum is (if $F_{ext} = 0$). Principal applications of conservation of linear momentum are in the field of collisions.

NOTE: Remember that the momentum of an isolated system is conserved. The momentum of one particle within an isolated system is not necessarily conserved because other particles in the system may be interacting with it. Always apply conservation of momentum to an isolated system.

ILLUSTRATION 8.7 A body explodes at rest break up into three parts. If two parts having equal masses fly off perpendicularly to each other with a velocity of 18 m/s, then calculate the velocity of the third part which has a mass 3 times the mass of each part.



Solution. Initially the body is at rest. Initial linear momentum of the body is zero. Let due to explosion the body break up into three parts. There is no external force acting on the body. Hence, linear momentum just after collision also be zero.

Let total mass of the body be $5m$, hence the masses of the broken parts will be m, m and $3m$, respectively.

$$\vec{p}_{\text{initial}} = 0$$

Linear momentum just after explosion

$$\vec{p}_{\text{final}} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = mv_1\hat{i} + mv_1\hat{j} + \vec{p}_3$$

As $\vec{p}_{\text{initial}} = \vec{p}_{\text{final}} \Rightarrow |\vec{p}_3| = \sqrt{2}mv_1 \Rightarrow 3mv = \sqrt{2}mv_1$

$$v = \frac{\sqrt{2}v_1}{3} = \frac{\sqrt{2}\times 18}{3} = 6\sqrt{2}\ \text{ms}^{-1}$$

$\Rightarrow$ $6\sqrt{2}\ \text{ms}^{-1}$ at an angle 135° from the either ball.

NOTE: During explosion, there will not be any change in gravitational potential energy of the body. But kinetic energy of the broken parts of the body will increase due to internal energy provided during explosion. This increase in kinetic energy will be equal to energy released during explosion.

ILLUSTRATION 8.8 A shell is fired from a cannon with a speed of 100 m/s at an angle 60° with the horizontal (positive x-direction). At the highest point of its trajectory, the shell explodes into two equal fragments. One of the fragments moves along the negative x-direction with a speed of 50 m/s. What is the speed of the other fragment at the time of explosion.

Solution. The velocity of the shell at the highest point of trajectory is

$v_M = u \cos\theta = 100 \cos 60° = 50$ m/s

Let v_1 be the speed of the fragment which moves along the negative x-direction and the other fragment has speed v_2, which must be along the positive x-direction. Now from momentum conservation, we have

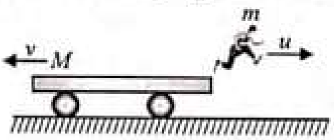
$$mv = \frac{-m}{2}v_1 + \frac{m}{2}v_2 \text{ or } 2v = v_2 - v_1$$

or $v_2 = 2v + v_1 = (2 \times 50) + 50 = 150$ m/s

ILLUSTRATION 8.9 A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If the child jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after its jump.

Solution. Let the car attain a velocity v. The net velocity of the child with respect to the earth will be $u - v$, as u is its velocity with respect to the car.

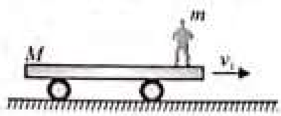
Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero. Hence,

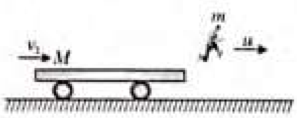


$$m(u - v) = Mv \Rightarrow v = \frac{mu}{m + M}$$

ILLUSTRATION 8.10 A flat car of mass M with a child of mass m is moving with a velocity v_1. The child jumps in the direction of motion of car with a velocity u with respect to the car. Find the final velocities of the child and that of the car after jump.

Solution. This case is similar to the previous example, except now the car is moving before jump. Here, also no external force is acting on the system in horizontal direction, hence momentum remains conserved in this direction. After the jump, car attains a velocity v_2 in the same direction, which is less than v_1, due to backward push of the child for jumping. After the jump, child attains a velocity $u + v_2$ in the direction of motion of car with respect to ground.





According to momentum conservation,

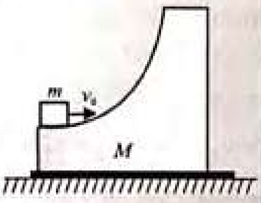
$(M + m)v_1 = Mv_2 + m(u + v_2)$

Velocity of car after jump is $v_2 = \dfrac{(M + m)v_1 - mu}{M + m}$

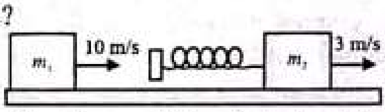
Velocity of child after jump is $u + v_2 = \dfrac{(M+m)v_1 + Mu}{M+m}$

CONCEPT APPLICATION EXERCISE 8.2

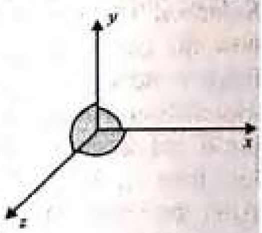
1. A shell is fired from a cannon with a speed of 100 m/s at an angle 30° with the vertical (y-direction). At the highest point of its trajectory, the shell explodes into two fragments of masses in the ratio 1:2. The lighter fragment moves vertically upwards with an initial speed of 200 m/s. What is the speed of the heavier fragment at the time of explosion?
2. A smooth wedge of mass M rests on a smooth horizontal surface. A block of mass m is projected from its lowermost point with velocity v_0. What is the maximum height reached by the block?



3. Two blocks of masses $m_1 = 2$ kg and $m_2 = 5$ kg are moving in the same direction along a frictionless surface with speeds 10 m/s and 3 m/s, respectively, m_2 being ahead of m_1. An ideal spring with $k = 1120$ N/m is attached to the back side of m_2. Find the maximum compression of the spring when the blocks collide. What are the final velocities of the blocks when they separate?



4. A projectile of mass 50 kg is shot vertically upwards with an initial velocity of 100 m/s. After 5 s, it explodes into two fragments, one of which having a mass of 20 kg travels vertically up with a velocity of 150 m/s.
   (a) What is the velocity of the other fragment at that instant?
   (b) Calculate the sum of momenta of fragments 3 s after the explosion. What would have been the momentum of the projectile at this instant if there had been no explosion?
5. A shell of mass 2 kg moving at a rate of 4 m/s suddenly explodes into two equal fragments. The fragments go in directions inclined with the original line of motion with equal velocities. If the explosion imparts 48 J of translational kinetic energy to the fragments, find the velocity and direction of each fragment.



6. A mud ball at rest explodes into three fragments of masses in the ratio 1:2:1. The two equal masses move with velocities $2\hat{i} + 5\hat{j} - 6\hat{k}$ and $-4\hat{i} + 3\hat{j} + 2\hat{k}$. Find the velocity of the third mass.

COLLISION OR IMPACT

Collision is an isolated event in which a strong force acts between two or more bodies for a short time, which results in change in their velocities. In a collision, a relatively large force acts on each colliding particle for a relatively short time. The basic idea of a 'collision' is that the motion of the colliding particles (or of at least one of them) changes rather abruptly and that we can make a relatively clean separation of times that are 'before the collision' and those that are 'after the collision'.

NOTE:

- In a collision, particles may or may not come in physical contact.
- The duration of collision Δt is negligible as compared to the usual time intervals of observation of motion.
- In a collision, the effects of external non-impulsive forces such as gravity are not taken into account as due to small duration of collision (Δt). Average impulsive force responsible for collision is much larger than external forces acting on the system.

The collision is, in fact, a redistribution of total momentum of the particles. Thus, law of conservation of linear momentum is indispensable in dealing with the phenomenon of collision between particles.

Classification of Collisions

On the Basis of Line of Impact

Head-on collision: The velocities of the particles are along the same line before and after the collision.

Oblique collision: The velocities of the particles are along different lines before and after the collision.

On the Basis of Energy

Elastic collision: In an elastic collision, the particles regain their shape and size completely after collision. That is, no fraction of mechanical energy remains stored as deformation potential energy in the bodies. Thus, kinetic energy of a system after collision is equal to kinetic energy of a system before collision. Thus, in addition to the linear momentum, kinetic energy also remains conserved before and after collision.

Inelastic collision: In an inelastic collision, the particles do not regain their shape and size completely after collision. Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles no longer remains conserved. However, in the absence of external forces, law of conservation of linear momentum still holds good.

Perfectly inelastic collision: If velocity of separation just after collision becomes zero, then the collision is perfectly inelastic. Collision is said to be perfectly inelastic if both the particles stick together after collision and move with the same velocity.

NOTE: Actually collision between all real objects is neither perfectly elastic nor perfectly inelastic, its inelastic in nature.

Coefficient of Restitution (e)

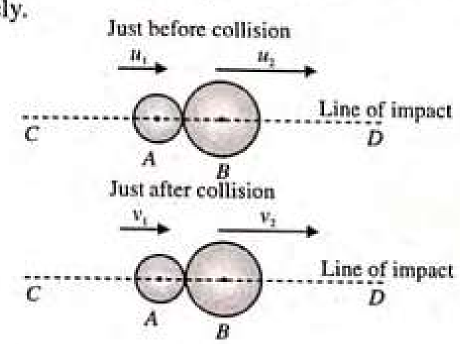
The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

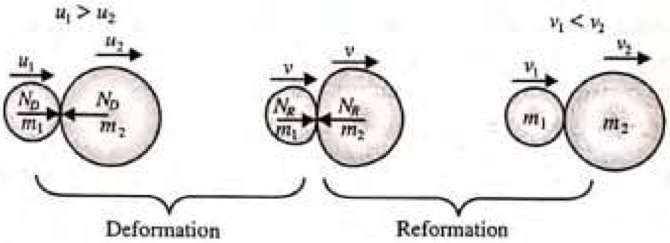
The most general expression for coefficient of restitution is

$$e = \frac{\text{Velocity of separation of points of contact along line of impact}}{\text{Velocity of approach of points of contact along line of impact}}$$

Illustration for Calculation of e

Two smooth balls A and B approach each other such that their centres are moving along the line CD in the absence of external impulsive force. Let the velocities of A and B just before collision be u_1 and u_2, respectively, and the velocities of A and B just after collision be v_1 and v_2, respectively.





Since, momentum is conserved for the system.

$\therefore \quad F_{ext} = 0$

$\Rightarrow \quad m_1u_1 + m_2u_2 = (m_1 + m_2)v = m_1v_1 + m_2v_2$

$$\Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \qquad \text{(i)}$$

Impulse of Deformation

J_D = change in momentum of any one body during deformation

$= m_2(v - u_2)$ for m_2

$= m_1(-v + u_1)$ for m_1

Impulse of Reformation

J_R = change in momentum of any one body during reformation

$= m_2(v_2 - v)$ for m_2

$= m_1(v - v_1)$ for m_1

$$e = \frac{\text{Impulse of reformation } (J_R)}{\text{Impulse of deformation } (J_D)} = \frac{v_2 - v}{v - u_2} = \frac{v_2 - v_1}{u_1 - u_2}$$

[Substituting v from Eq. (i)]

$$e = \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

This is also known as Newton's experimental law.

NOTE: e is independent of shape and mass of the object but depends on the material.

The coefficient of restitution is constant for two particular objects.

(1) For $e = 1$

$\Rightarrow$ Impulse of reformation = Impulse of deformation

$\Rightarrow$ Velocity of separation = Velocity of approach

$\Rightarrow$ Kinetic energy is conserved

$\Rightarrow$ Elastic collision

(2) For $e = 0$

$\Rightarrow$ Impulse of reformation = 0

$\Rightarrow$ Velocity of separation = 0

$\Rightarrow$ Kinetic energy is not conserved

$\Rightarrow$ Perfectly inelastic collision

(3) For $0 < e < 1$

$\Rightarrow$ Impulse of reformation < Impulse of deformation

$\Rightarrow$ Velocity of separation < Velocity of approach

$\Rightarrow$ Kinetic energy is not conserved

$\Rightarrow$ Inelastic collision

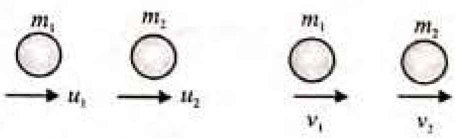
NOTE:

- In case of contact collisions, e is always less than unity.
- u_1, u_2, v_1 and v_2 can be positive, negative or zero.

General Equation for Direct Impact

If u_1, u_2 are the velocities before the impact of the masses m_1, m_2 and v_1, v_2 are the velocities after the impact, then applying conservation of momentum, we get

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ and $v_1 - v_2 = -e(u_1 - u_2)$



Combining these equations, we get

$$v_1 = \frac{m_1 - em_2}{m_1 + m_2}u_1 + \frac{(1+e)m_2}{m_1 + m_2}u_2;$$

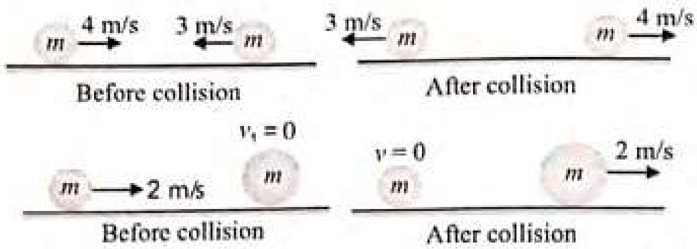
$$v_2 = \frac{(1+e)m_1}{m_2 + m_1}u_1 + \frac{m_2 - em_1}{m_2 + m_1}u_2$$

For a perfectly elastic collision, we can substitute $e = 1$.

Special case: For $e = 1$ and $m_1 = m_2 = m$, we get

$v_1 = u_2$ and $v_2 = u_1$

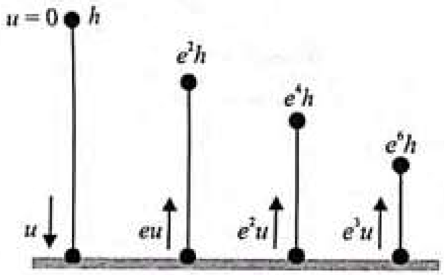
i.e., when two particles of equal mass collide elastically and the collision is head-on, they exchange their velocities, e.g.,



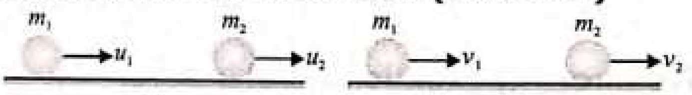
If a body of mass m with initial velocity u strikes on another identical body but at rest and e is the coefficient of restitution, then their velocities v_1 and v_2 after collision will be

$$v_1 = \frac{1-e}{2}u;\ v_2 = \frac{1+e}{2}u$$

If a ball of mass m falls on ground from a vertical height h and rebounds with e as coefficient of restitution between them (see figure), then the upward velocity of the ball after nth collision will be $(e^n u)$ and the maximum height attained by the ball after nth collision will be $(e^{2n}h)$.



Collision in One Dimension (Head-on)



(a) Before collision (b) After collision

$u_1 > u_2,\ v_2 > v_1$

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow (u_1 - u_2)e = (v_2 - v_1)$$

By momentum conservation, we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

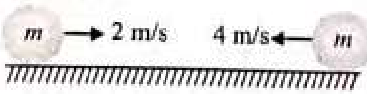
Now, $v_2 = v_1 + e(u_1 - u_2)$

Hence,

$$v_1 = \frac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$$

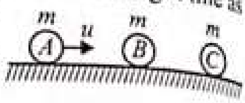
$$v_2 = \frac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2}$$

ILLUSTRATION 8.11 Two identical balls are approaching towards each other on a straight line with velocity 2 m/s and 4 m/s, respectively. Find the final velocities, after elastic collision between them.

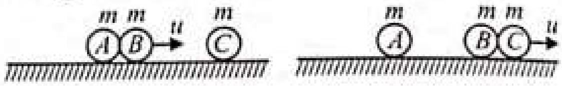


Solution. The two velocities will be exchanged and the final motion is the reverse of the initial motion for both.

ILLUSTRATION 8.12 Three balls A, B and C of same mass 'm' are placed on a frictionless horizontal plane in a straight line as shown. Ball A is moved with velocity u towards the middle ball B. If all the collisions are elastic, find the final velocities of all the balls.



Solution. A collides elastically with B and comes to rest but B starts moving with velocity u.



After a while B collides elastically with C and comes to rest but C starts moving with velocity u.

Therefore, final velocities, $v_A = 0$, $v_B = 0$ and $v_C = u$.

ILLUSTRATION 8.13 Two particles of masses m and $2m$ moving in opposite directions collide elastically with velocity $2v$ and v, respectively. Find their velocities after collision.

Solution. Let the final velocities of m and $2m$ be v_1 and v_2, respectively, as shown in the figure.

By conservation of momentum, we get

$$m(2v) + 2m(-v) = m(v_1) + 2m(v_2)$$

or $\quad 0 = mv_1 + 2mv_2$

or $\quad v_1 + 2v_2 = 0 \qquad$ (i)

and since the collision is elastic,

$$v_2 - v_1 = 2v - (-v)$$

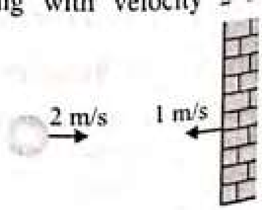
or $\quad v_2 - v_1 = 3v \qquad$ (ii)

Solving the above two equations, we get

$$v_2 = v \text{ and } v_1 = -2v$$

That is, mass $2m$ returns with velocity v while mass m returns with velocity $2v$ in the direction shown in the figure.

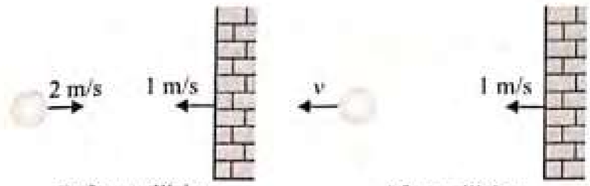
ILLUSTRATION 8.14 A ball is moving with velocity 2 m/s towards a heavy wall moving towards the ball with speed 1 m/s as shown in the figure. Assuming collision to be elastic, find the velocity of the ball immediately after the collision.



Solution. The speed of the wall will not change after the collision. Let v be the velocity of the ball after collision in the direction shown in the figure. Since collision is elastic ($e = 1$).

Speed of separation speed = Speed of approach

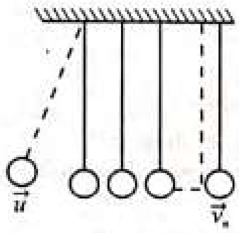
or $\quad v - 1 = 2 + 1$ or $v = 4$ m/s



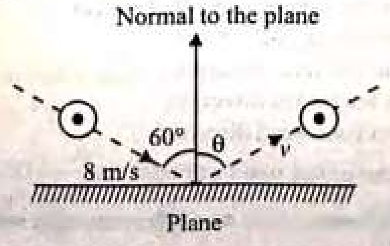
Before collision After collision

Concept Application Exercise 8.3

1. In the figure, there are n identical spheres of mass m suspended with wires of equal length. The spheres are almost in contact with each other. Sphere 1 is pulled aside and released. If sphere 1 strikes sphere 2 with velocity u, find an expression for velocity v_n of the nth sphere immediately after being struck by the one adjacent to it. The coefficient of restitution for all the impacts is e.



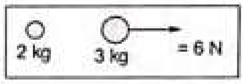
2. A block m_1 strikes a stationary block m_3 inelasticlly. Another block m_2 is kept on m_3. Neglecting the friction between all contacting surfaces, calculate the fractional decrease in KE of the system in collision.
3. A mass m_1 moves with a great velocity. It strikes another mass m_2 at rest in head-on collision. It comes back along its path with low speed after collision. Then find out whether $m_1 < m_2$ or $m_1 > m_2$.
4. A ball of mass 4 kg moving with a velocity of 12 m/s impinges directly on another ball of mass 8 kg moving with velocity of 4 m/s in the same direction. Find their velocities after impact and calculate the loss of KE due to impact if $e = 0.5$.
5. A bullet of mass 2 g travelling at a speed of 500 m/s is fired into a ballistic pendulum of mass 1.0 kg suspended from a cord 1.0 m long. The bullet penetrates the pendulum and emerges with a velocity of 100 m/s. Through what vertical height will the pendulum rise?
6. A spherical imperfectly elastic ball strikes a plane with velocity 8 m/s at an angle of 30° with the plane. Determine the magnitude and direction of the velocity after impact if $e = 0.5$ (neglect gravity).



7. Three balls of masses m_1, m_2 and m_3 are lying in a straight line. The first ball is moved with a certain velocity so that it strikes the second ball directly and itself comes to rest. The second ball collides with the third and is itself reduced to rest. If e is the coefficient of restitution for each ball, write down the relation of m_3 in terms of m_1 and m_2.

Solved Examples

1. A square of side a and uniform thickness is divided into four equal parts. If the upper right part is removed, then find the coordinates of centre of mass of remaining part.



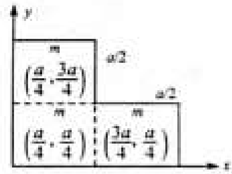
(a) $\left(\frac{5}{12}a, \frac{5}{12}a\right)$ (b) $\left(\frac{7}{12}a, \frac{7}{12}a\right)$

(c) $\left(\frac{1}{4}a, \frac{1}{4}a\right)$ (d) $\left(\frac{1}{3}a, \frac{1}{3}a\right)$

Sol. (a) Let the mass of each part is m. Coordinates of their respective centre of masses are shown in the figure.

$$x_{CM} = \frac{m\frac{a}{4} + m\frac{3a}{4} + m\frac{a}{4}}{m+m+m} = \frac{5a}{12}$$

$$y_{CM} = \frac{m\frac{a}{4} + m\frac{a}{4} + m\frac{3a}{4}}{m+m+m} = \frac{5a}{12}$$



2. A free body of mass 8 kg is travelling at 2 meter per second in a straight line. At a certain instant, the body splits into two equal parts due to internal explosion which releases 16 joules of energy. Neither part leaves the original line of motion finally
   (a) Both parts continue to move in the same direction as that of the original body
   (b) One part comes to rest and the other moves in the same direction as that of the original body
   (c) One part comes to rest and the other moves in the direction opposite to that of the original body
   (d) One part moves in the same direction and the other in the direction opposite to that of the original body

Sol. (b) As the body splits into two equal parts due to internal explosion therefore momentum of system remains conserved, i.e. $8 \times 2 = 4v_1 + 4v_2 \Rightarrow v_1 + v_2 = 4$ ...(i)

By the law of conservation of energy
Initial kinetic energy + Energy released due to explosion = Final kinetic energy of the system

$$\Rightarrow \frac{1}{2}\times 8\times(2)^2 + 16 = \frac{1}{2}4v_1^2 + \frac{1}{2}4v_2^2$$

$$\Rightarrow v_1^2 + v_2^2 = 16 \quad \text{...(ii)}$$

By solving eq. (i) and (ii) we get $v_1 = 4$ and $v_2 = 0$ i.e. one part comes to rest and other moves in the same direction as that of original body.

3. A sphere of mass m, moving with velocity v, enters a hanging bag of sand and stops. If the mass of the bag is M and it is raised by height h, then the velocity of the sphere was

(a) $\frac{M+m}{m}\sqrt{2gh}$ (b) $\frac{M}{m}\sqrt{2gh}$

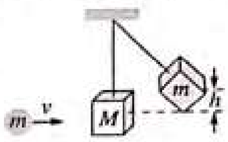
(c) $\frac{m}{M+m}\sqrt{2gh}$ (d) $\frac{m}{M}\sqrt{2gh}$

Sol. (a) By the conservation of linear momentum

Initial momentum of sphere = Final momentum of system

$$mv = (m+M)v_{sys} \quad \text{...(i)}$$

If the system rises up to height h then by the conservation of energy



$$\frac{1}{2}(m+M)v_{sys.}^2 = (m+M)gh \quad \text{...(ii)}$$

$$\Rightarrow v_{sys.} = \sqrt{2gh}$$

Substituting this value in equation (i)

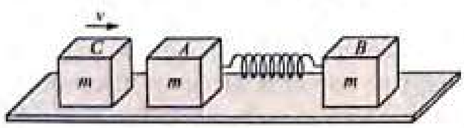
$$v = \left(\frac{m+M}{m}\right)\sqrt{2gh}$$

4. Two identical blocks A and B, each of mass m resting on smooth floor are connected by a light spring of natural length L and spring constant k, with the spring at its natural length. A third identical block 'C' (mass m) moving with a speed v along the line joining A and B collides with A. the maximum compression in the spring is

(a) $v\sqrt{\frac{m}{2k}}$ (b) $m\sqrt{\frac{v}{2k}}$

(c) $\sqrt{\frac{mv}{k}}$ (d) $\frac{mv}{2k}$

Sol. (a) Initial momentum of the system (block C) = mv



After striking with A, the block C comes to rest and now both block A and B moves with velocity V, when compression in spring is maximum.

By the law of conservation of linear momentum

$$mv = (m+m)V \Rightarrow V = \frac{v}{2}$$

By the law of conservation of energy

K.E. of block C = K.E. of system + P.E. of system

$$\frac{1}{2}mv^2 = \frac{1}{2}(2m)V^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\Rightarrow kx^2 = \frac{1}{2}mv^2$$

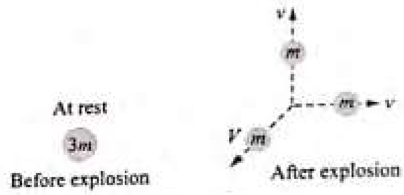
$$\Rightarrow x = v\sqrt{\frac{m}{2k}}$$

5. An object of mass $3m$ splits into three equal fragments. Two fragments have velocities $v\hat{j}$ and $v\hat{i}$. The velocity of the third fragment is

(a) $v(\hat{j}-\hat{i})$ (b) $v(\hat{i}-\hat{j})$

(c) $-v(\hat{i}+\hat{j})$ (d) $\frac{v(\hat{i}+\hat{j})}{\sqrt{2}}$

Sol. (c)



Initial momentum of $3m$ mass = 0 ...(i)

Due to explosion this mass splits into three fragments of equal masses.

Final momentum of system = $m\vec{V} + mv\hat{i} + mv\hat{j}$ ...(ii)

By the law of conservation of linear momentum

$$m\vec{V} + mv\hat{i} + mv\hat{j} = 0 \Rightarrow \vec{V} = -v(\hat{i}+\hat{j})$$

6. A bomb is kept stationary at a point. It suddenly explodes into two fragments of masses 1 g and 3g. The total K.E. of the fragments is 6.4×10^4 J. What is the K.E. of the smaller fragment?

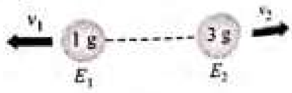
(a) 2.5×10^4 J (b) 3.5×10^4 J

(c) 4.8×10^4 J (d) 5.2×10^4 J

Sol. (c) As the momentum of both fragments are equal therefore $\frac{E_1}{E_2} = \frac{m_2}{m_1} = \frac{3}{1}$ i.e. $E_1 = 3E_2$ ...(i)

According to problem

$E_1 + E_2 = 6.4 \times 10^4$ J ...(ii)



By solving equation (i) and (ii) we get

$E_1 = 4.8 \times 10^4$ J and

$E_2 = 1.6 \times 10^4$ J

7. A body is moving with a velocity v, breaks up into two equal parts. One of the part retraces back with velocity v. Then the velocity of the other part is

(a) v in forward direction

(b) $3v$ in forward direction

(c) v in backward direction

(d) $3v$ in backward direction

Sol. (b) Let the initial mass of body = m

Initial linear momentum $= mv$ ...(i)

When it breaks into equal masses then one of the fragment retrace back with same velocity

$\therefore$ Final linear momentum $= \dfrac{m}{2}(-v) + \dfrac{m}{2}(v_2)$ ...(ii)

By the conservation of linear momentum

$$\Rightarrow mv = \frac{-mv}{2} + \frac{mv_2}{2} \Rightarrow v_2 = 3v$$

i.e. other fragment moves with velocity $3v$ in forward direction.

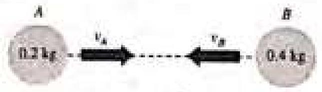
8. Two solid rubber balls A and B having masses 200 and 400 g, respectively, are moving in opposite directions with velocity of A equal to 0.3 m/s. After collision, the two balls come to rest, then the velocity of B is

(a) 0.15 m/s (b) 1.5 m/s

(c) -0.15 m/s (d) None of the above

Sol. (c) Initial linear momentum of system $= m_A\vec{v}_A + m_B\vec{v}_B$

$= 0.2 \times 0.3 + 0.4 \times v_B$



Finally both balls come to rest

$\therefore$ Final linear momentum = 0

By the law of conservation of linear momenum,

$0.2 \times 0.3 + 0.4 \times v_B = 0$

$$\therefore v_B = -\frac{0.2 \times 0.3}{0.4} = -0.15 \text{ m/s}$$

9. A cannon ball is fired with a velocity 200 m/s at an angle of 60° with the horizontal. At the highest point of its flight it explodes into 3 equal fragments, one going vertically upwards with a velocity 100 m/s, the second one falling vertically downwards with a velocity 100 m/s. The third fragment will be moving with a velocity

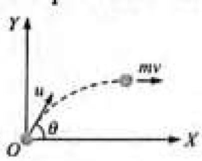
(a) 100 m/s in the horizontal direction

(b) 300 m/s in the horizontal direction

(c) 300 m/s in a direction making an angle of 60° with the horizontal

(d) 200 m/s in a direction making an angle of 60° with the horizontal

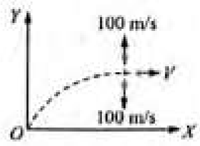
Sol. (b) Momentum of ball (mass m) before explosion at the highest point $= mv\hat{i} = mu\cos 60°\hat{i}$



$= m \times 200 \times \dfrac{1}{2}\hat{i} = 100\, m\hat{i}$ kgms^{-1}

Let the velocity of third part after explosion is V.

After explosion momentum of system



$= \vec{p}_1 + \vec{p}_2 + \vec{p}_3$

$= \dfrac{m}{3} \times 100\hat{j} - \dfrac{m}{3} \times 100\hat{j} + \dfrac{m}{3} \times V\hat{i}$

By comparing momentum of system before and after the explosion

$$\frac{m}{3} \times 100\hat{j} - \frac{m}{3} \times 100\hat{j} + \frac{m}{3}V\hat{i} = 100m\hat{i}$$

$\Rightarrow V = 300$ m/s

10. A lead ball strikes a wall and falls down, a tennis ball having the same mass and velocity strikes the wall and bounces back. Check the correct statement.

(a) The momentum of the lead ball is greater than that of the tennis ball

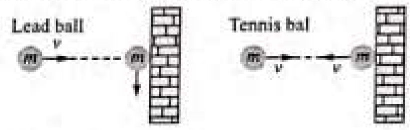
(b) The lead ball suffers a greater change in momentum compared with the tennis ball

(c) The tennis ball suffers a greater change in momentum as compared with the lead ball

(d) Both suffer an equal change in momentum

Sol. (c) Change in the momentum

= Final momentum – initial momentum



For lead ball, $\Delta\vec{p}_{\text{lead}} = 0 - m\vec{v} = -m\vec{v}$

For tennis ball, $\Delta\vec{p}_{\text{tennis}} = -m\vec{v} - m\vec{v} = -2m\vec{v}$

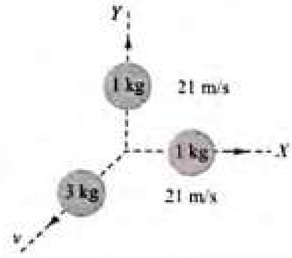
i.e. tennis ball, suffers a greater change in momentum.

11. A body of mass 5 kg explodes at rest into three fragments with masses in the ratio 1 : 1 : 3. The fragments with equal masses fly in mutually perpendicular directions with speeds of 21 m/s. The velocity of the heaviest fragment will be

(a) 11.5 m/s (b) $\dfrac{7}{\sqrt{2}}$ m/s

(c) 7 m/s (d) $7\sqrt{2}$ m/s

Sol. (d)



$p_x = m \times v_x = 1 \times 21$

$= 21$ kg m/s

$p_y = m \times v_y = 1 \times 21$

$= 21$ kg m/s

$\therefore$ Resultant

$= \sqrt{p_x^2 + p_y^2} = 21\sqrt{2}$ kg m/s

The momentum of heavier fragment should be numerically equal to resultant of $\vec{p}_x$ and $\vec{p}_y$.

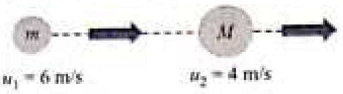
$$3 \times v = \sqrt{p_x^2 + p_y^2} = 21\sqrt{2} \Rightarrow v = 7\sqrt{2} \text{ m/s}$$

12. A particle of mass m moving with horizontal speed 6 m/s as shown in figure. If $m << M$, then for one dimensional elastic collision, the speed of lighter particle after collision will be

(a) 2 m/s in original direction

(b) 2 m/s opposite to the original direction
(c) 4 m/s opposite to the original direction
(d) 4 m/s in original direction

Sol. (a) $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$



Substituting $m_1 = 0$, $v_1 = -u_1 + 2u_2$

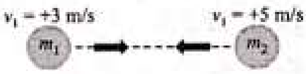
$\Rightarrow v_1 = -6 + 2(4) = 2$ m/s

i.e. the lighter particle will move in original direction with the speed of 2 m/s.

13. Two equal masses m_1 and m_2 moving along the same straight line with velocities +3 m/s and −5 m/s respectively collide elastically. Their velocities after the collision will be respectively

(a) +4 m/s for both (b) −3 m/s and +5 m/s
(c) −4 m/s and +4 m/s (d) −5 m/s and +3 m/s

Sol. (d) As $m_1 = m_2$, therefore after elastic collision velocities of masses get interchanged

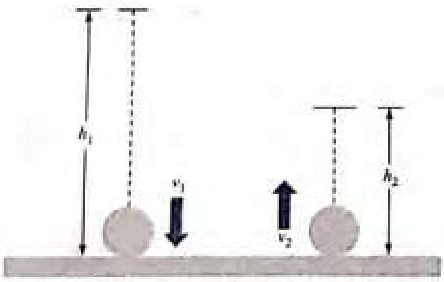


i.e. velocity of mass $m_1 = -5$ m/s
and velocity of mass $m_2 = +3$ m/s

14. A rubber ball is dropped from a height of 5m on a planet where the acceleration due to gravity is not known. On bouncing, it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of

(a) 16/25 (b) 2/5 (c) 3/5 (d) 9/25

Sol. (b) If ball falls from height h_1 and bounces back up to height h_2, then $e = \sqrt{\frac{h_2}{h_1}}$



Similarly, if the velocity of ball before and after collision are v_1 and v_2 respectively, then $e = \frac{v_2}{v_1}$

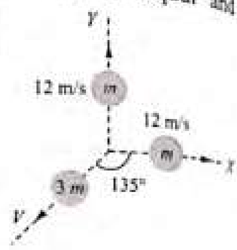
So $\frac{v_2}{v_1} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{1.8}{5}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

i.e. fractional loss in velocity $= 1 - \frac{v_2}{v_1} = 1 - \frac{3}{5} = \frac{2}{5}$

15. A body at rest breaks up into 3 parts. If 2 parts having equal masses fly off perpendicularly each after with a velocity of 12m/s, then the velocity of the third part which has 3 times mass of each part is

(a) $4\sqrt{2}$ m/s at an angle of 45° from each body
(b) $24\sqrt{2}$ m/s at an angle of 135° from each body
(c) $6\sqrt{2}$ m/s at 135° from each body
(d) $4\sqrt{2}$ m/s at 135° from each body

Sol. (d) The momentum of third part will be equal and opposite to the resultant of momentum of rest two equal parts let v is the velocity of third part.



By the conservation of linear momentum

$$3m \times v = m \times 12\sqrt{2}$$

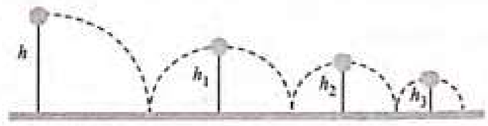
$$\Rightarrow v = 4\sqrt{2} \text{ m/s}$$

16. A particle falls from a height h upon a fixed horizontal plane and rebounds. If e is the coefficient of restitution, the total distance travelled before rebounding has stopped is

(a) $h\left(\frac{1+e^2}{1-e^2}\right)$ (b) $h\left(\frac{1-e^2}{1+e^2}\right)$

(c) $\frac{h}{2}\left(\frac{1-e^2}{1+e^2}\right)$ (d) $\frac{h}{2}\left(\frac{1+e^2}{1-e^2}\right)$

Sol. (a)



Particle falls from height h, then formula for height covered by it in nth rebound is given by

$$h_n = he^{2n}$$

where e = coefficient of restitution, n = no. of rebound.

Total distance travelled by particle before rebounding has stopped.

$$H = h + 2h_1 + 2h_2 + 2h_3 + 2h_n + \ldots$$

$$= h + 2he^2 + 2he^4 + 2he^6 + 2he^8 + \ldots$$

$$= h + 2h(e^2 + e^4 + e^6 + e^8 + \ldots)$$

$$= h + 2h\left[\frac{e^2}{1-e^2}\right] = h\left[1 + \frac{2e^2}{1-e^2}\right] = h\left(\frac{1+e^2}{1-e^2}\right)$$

17. A bag (mass M) hangs by a long thread and a bullet (mass m) comes horizontally with velocity v and gets caught in the bag. Then for the combined (bag + bullet) system

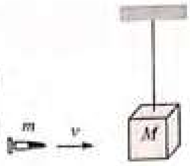
(a) Momentum is $\frac{mvM}{M+m}$

(b) Kinetic energy is $\frac{mv^2}{2}$

(c) Momentum is $\frac{mv(M+m)}{M}$

(d) Kinetic energy is $\frac{m^2v^2}{2(M+m)}$

Sol. (d) Initial momentum $= mv$



Final momentum $= (m+M)V$

By conservation of momentum $mv = (m+M)V$

$\therefore$ Velocity of (bag + bullet) system $V = \dfrac{mv}{M+m}$

$\therefore$ Kinetic energy $= \dfrac{1}{2}(m+M)V^2$

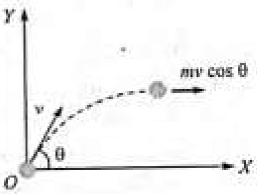
$$= \frac{1}{2}(m+M)\left(\frac{mv}{M+m}\right)^2 = \frac{1}{2}\frac{m^2v^2}{M+m}$$

18. A shell is fired from a cannon with velocity v m/sec at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed in m/s of the other piece immediately after the explosion is

(a) $3v\cos\theta$ (b) $2v\cos\theta$

(c) $\dfrac{3}{2}v\cos\theta$ (d) $\dfrac{\sqrt{3}}{2}v\cos\theta$

Sol. (a) Shell is fired with velocity v at an angle θ with the horizontal.



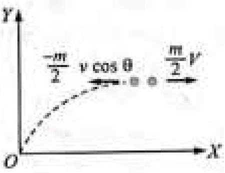
So its velocity at the highest point = horizontal component of velocity $= v\cos\theta$

So momentum of shell before explosion $= mv\cos\theta$

When it breaks into two equal pieces and one piece retrace its path to the canon, then other part moves with velocity V.

So momentum of two pieces after explosion

$$= \frac{m}{2}(-v\cos\theta) + \frac{m}{2}V$$



By the law of conservation of momentum

$$mv\cos\theta = \frac{-m}{2}v\cos\theta + \frac{m}{2}V$$

$\Rightarrow V = 3v\cos\theta$

19. A vessel at rest explodes into three pieces. Two pieces having equal masses fly off perpendicular to one another with the same velocity 30 meter per second. The third piece has three times mass of each of other piece. The magnitude and direction of the velocity of the third piece will be

(a) $10\sqrt{2}$ m/s and 135° from either

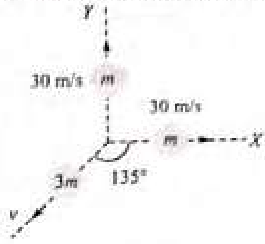
(b) $10\sqrt{2}$ m/s and 45° from either

(c) $\dfrac{10}{\sqrt{2}}$ m/s and 135° from either

(d) $\dfrac{10}{\sqrt{2}}$ m/s and 45° from either

Sol. (a) Let two pieces are having equal mass m and third piece have a mass of $3m$.

According to law of conservation of linear momentum. Since the initial momentum of the system was zero, therefore final momentum of the system must be zero, i.e., the resultant of momentum of two pieces must be equal to the momentum of third piece. We know that if two particle possesses same momentum and angle in between them is 90°, then resultant will be given by $P\sqrt{2} = mv\sqrt{2} = m30\sqrt{2}$



Let the velocity of mass $3m$ is V. So $3mV = 30m\sqrt{2}$

$\therefore V = 10\sqrt{2}$ and angle 135° from either.

(as it is clear from the figure)

20. A set of n identical cubical blocks lies at rest parallel to each other along a line on a smooth horizontal surface. The separation between the near surfaces of any two adjacent blocks is L. The block at one end is given a speed v towards the next one at time $t = 0$. All collisions are completely inelastic. Then

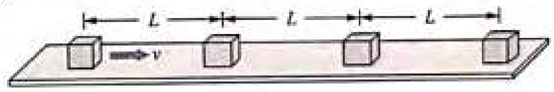
(a) The last block starts moving at $t = \dfrac{(n-1)L}{v}$

(b) The last block starts moving at $t = \dfrac{n(n-1)L}{2v}$

(c) The centre of mass of the system will have a final speed v

(d) The centre of mass of the system will have a final speed $\dfrac{2v}{n}$

Sol. (b)



Since collision is perfectly inelastic so all the blocks will stick together one by one and move in a form of combined mass.

Time required to cover a distance 'L' by first block $= \dfrac{L}{v}$

Now first and second block will stick together and move with $v/2$ velocity (by applying conservation of momentum) and combined system will take time $\dfrac{L}{v/2} = \dfrac{2L}{v}$ to reach up to block third.

Now these three blocks will move with velocity $v/3$ and combined system will take time $\dfrac{L}{v/3} = \dfrac{3L}{v}$ to reach upto the block fourth.

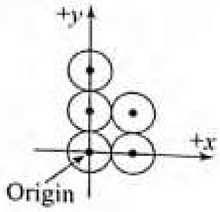
So, total time $= \dfrac{L}{v} + \dfrac{2L}{v} + \dfrac{3L}{v} + \ldots + \dfrac{(n-1)L}{v} = \dfrac{n(n-1)L}{2v}$

and velocity of combined system having n blocks as $\dfrac{v}{n}$.

EXERCISES

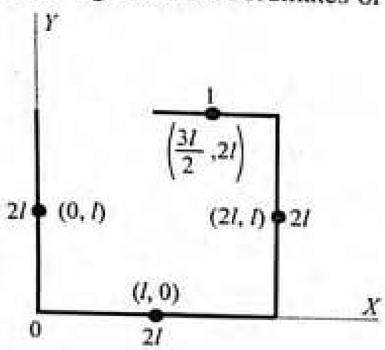
Centre of Mass and Its Motion

1. Five uniform circular plates, each of diameter b and mass m, are laid out in a pattern shown. Using the origin shown, find the y coordinate of the center of mass of the five-plate system.



(a) $b/5$ (b) $b/3$ (c) $4b/5$ (d) $2b/5$

2. A wire of uniform cross-section is bent in the shape shown in the figure. The coordinates of the center of mass of each side are shown in the figure. The coordinates of the center of mass of the system are



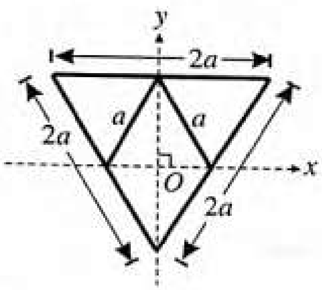
(a) $\left(\frac{15l}{14}, \frac{6l}{7}\right)$

(b) $\left(\frac{15l}{14}, l\right)$

(c) $\left(l, \frac{l}{2}\right)$

(d) (l, l)

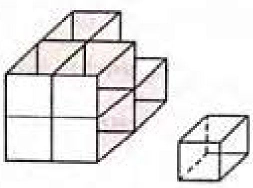
3. The 'y' coordinate of the centre of mass of the system of three rods of length '$2a$' and two rods of length 'a' as shown in the figure is: (Assume all rods to be of uniform density)



(a) $\frac{9a}{8\sqrt{3}}$ (b) $\frac{9a}{16\sqrt{3}}$

(c) zero (d) $\frac{8a}{\sqrt{3}}$

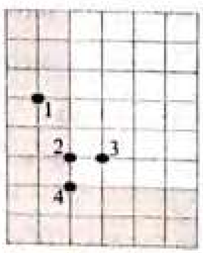
4. Eight solid uniform cubes of edge l are stacked together to form a single cube with center O. One cube is removed from this system. Distance of the centre of mass of remaining 7 cubes from O is



(a) $\frac{7\sqrt{3}l}{16}$ (b) $\frac{\sqrt{3}l}{16}$

(c) $\frac{\sqrt{3}l}{14}$ (d) zero

5. In the figure, the L-shaped shaded piece is cut from a metal plate of uniform thickness. The point that corresponds to the center of mass of the L-shaped piece is



(a) 1 (b) 2 (c) 3 (d) 4

6. A brick of length L is placed on the horizontal floor. The bricks of same length and size are placed on this brick, one above the other by providing a margin of $L/8$ from the edge of the brick placed just below, in the same direction. Find the correct option.

(a) Fifth brick will fall down
(b) Sixth brick alone will fall down
(c) Sixth brick along with fifth brick will fall down
(d) None of these

7. Two semicircular rings of linear mass densities λ and 2λ and of radius 'R' each are joined to form a complete ring. The distance of the center of the mass of complete ring from its centre is

(a) $\frac{3R}{8\pi}$ (b) $\frac{2R}{3\pi}$ (c) $\frac{4R}{21\pi}$ (d) None of these

8. A system of particles is free from any external force. If $\vec{v}$ and $\vec{a}$ be the velocity and acceleration of the centre of mass, then it necessarily follows that

(a) $\vec{v}=0;\ \vec{a}=0$ (b) $\vec{v}\neq 0;\ \vec{a}=0$
(c) $\vec{v}=0;\ \vec{a}\neq 0$ (d) both (a) and (b).

9. The velocity of the CM of a system changes from $\vec{v}_1 = 4\hat{i}$ m/s to $\vec{v}_2 = 3\hat{j}$ m/s during time $\Delta t = 2$ s. If the mass of the system is $m = 10$ kg, the constant force acting on the system is

(a) 25 N (b) 20 N (c) 50 N (d) 5 N

10. A child is sitting at one end of a long trolley moving with a uniform speed v on a smooth horizontal track. If the child starts running towards the other end of the trolley with a speed u (w.r.t. trolley), the speed of the center of mass of the system will

(a) $u+v$ (b) $v-u$ (c) v (d) none of these

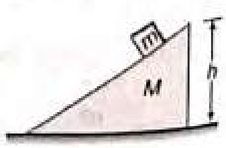
11. A boy of mass m is standing on a block of mass M kept on a rough surface. When the boy walks from left to right on the block, the center of mass (boy + block) of system:

(a) remains stationary (b) shifts towards left
(c) shift towards right
(d) shifts towards right if $M > m$ and towards left if $M < m$.

12. An insulated particle of mass m is moving in a horizontal plane (x-y) along the X-axis. At a certain height above the ground, it suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $Y = +15$ cm. The larger fragment at this instant is at:

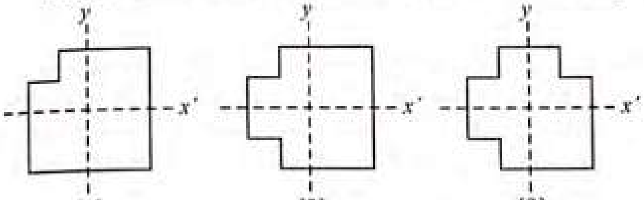
(a) $Y = -5$ cm (b) $Y = +20$ cm
(c) $Y = +5$ cm (d) $Y = -20$ cm

13. A mass m is rest on an inclined plane of mass M which is further resting on a smooth horizontal plane. Now if the mass starts moving the position of C.M. of mass of system will



(a) remains unchanged
(b) change along the horizontal
(c) will move up in the vertical direction
(d) will move down in the vertical direction and changes along the horizontal

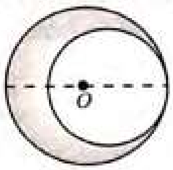
14. A machinist starts with three identical square plates but cuts one corner from one of them, two corners from the second and three corners from the third. Rank the three according to the x-coordinate of their centre of mass, from smallest to largest.



[1] [2] [3]

(a) 3, 1, 2 (b) 1, 3, 2
(c) 3, 2, 1 (d) 1 and 3 tie, then 2

15. A circular plate of uniform thickness has a diameter of 28 cm. A circular portion of diameter 21 cm is removed from the plate as shown. O is the centre of mass of complete plate. The position of centre of mass of remaining portion will shift towards left from O by



(a) 5 cm (b) 9 cm (c) 4.5 cm (d) 5.5 cm

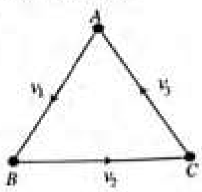
16. A man stands at one end of a boat which is stationary in water. Neglect water resistance. The man now moves to the other end of the boat and again becomes stationary. The centre of mass of the 'man plus boat' system will remain stationary with respect to water
(a) only when the man is stationary initially and finally
(b) only if the man moves without acceleration on the boat
(c) only if the man and the boat have equal masses
(d) in all cases

17. Two particles are shown in the figure. At $t = 0$, a constant force $F = 6$ N starts acting on the 3 kg man.

Find the velocity of the centre of mass of these particles at $t = 5$ s.
(a) 5 m/s (b) 4 m/s (c) 6 m/s (d) 3 m/s

18. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The velocity of the centre of mass
(a) of the box remains constant
(b) of the (box + ball) system remains constant
(c) of the ball remains constant
(d) of the ball relative to the box remains constant

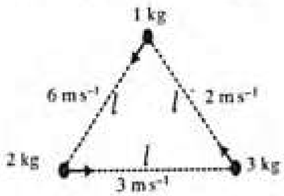
19. Three particles of equal masses are placed at the corners of an equilateral triangle as shown in the figure. Now particle A starts with a velocity v_1 towards line AB, particle B starts with a velocity v_2 towards line BC and particle C starts with velocity v_3 towards line CA. The displacement of CM of three particle A, B and C after time t will be (given if $v_1 = v_2 = v_3$)



(a) zero (b) $\dfrac{v_1 + v_2 + v_3}{3}t$

(c) $\dfrac{v_1 + \dfrac{\sqrt{3}}{2}v_2 + \dfrac{v_3}{2}}{3}t$ (d) $\dfrac{v_1 + v_2 + v_3}{4}t$

20. Three particles of masses 1 kg, 2 kg and 3 kg are situated at the corners of an equilateral triangle move at speed $6\ \text{ms}^{-1}$, $3\ \text{ms}^{-1}$ and $2\ \text{ms}^{-1}$ respectively. Each particle maintains a direction towards the particle at the next corner symmetrically. Find velocity of CM of the system at this instant



(a) $3\ \text{ms}^{-1}$ (b) $5\ \text{ms}^{-1}$ (c) $6\ \text{ms}^{-1}$ (d) zero

Conservation of Linear Momentum

21. An object initially at rest explodes into three fragments A, B and C. The momentum of A is $p\hat{i}$ and that of B is $\sqrt{3}p\,\hat{j}$ where p is a +ve number. The momentum of C will be
(a) $(1+\sqrt{3})p$ in a direction making angle 120° with that of A.
(b) $(1+\sqrt{3})p$ in a direction making angle 150° with that of B.
(c) $2p$ in a direction making angle 150° with that of A.
(d) $2p$ in a direction making angle 150° with that of B.

22. A vessel at rest explodes breaking it into three pieces. Two pieces having equal mass fly off perpendicular to one another with the same speed of 30 m/s. The third piece has three times the mass of each of the other two pieces. What is the direction (w.r.t. the pieces having equal masses) and magnitude of its velocity immediately after the explosion?
(a) $10\sqrt{2}$, 135° (b) $10\sqrt{2}$, 90°
(c) $10\sqrt{2}$, 60° (d) $10\sqrt{2}$, 30°

23. A stationary body of mass 3 kg explodes into three equal pieces. Two of the pieces fly off at right angles to each other. One with a velocity of $2\hat{i}$ m/s and the other with a velocity of $3\hat{j}$ m/s. If the explosion takes place in 10^{-5} s, the average force acting on the third piece in newtons is
(a) $(2\hat{i}+3\hat{j}) \times 10^{-5}$ (b) $-(2\hat{i}+3\hat{j}) \times 10^{5}$
(c) $(3\hat{j}+2\hat{i}) \times 10^{5}$ (d) $-(2\hat{i}+3\hat{j}) \times 10^{-5}$

24. A particle of mass $2m$ is projected at an angle of 45° with the horizontal with a velocity of $20\sqrt{2}$ m/s. After 1 s of explosion, the particle breaks into two equal pieces. As a result of this one part comes to rest. The maximum height from the ground attained by the other part is ($g = 10\ \text{m/s}^2$)
(a) 50 m (b) 25 m (c) 40 m (d) 35 m

25. A bag of mass M hangs by a long massless rope. A bullet of mass m, moving horizontally with velocity u, is caught

in the bag. Then for the combined (bag + bullet) system, just after collision

(a) momentum is $muM/(M + m)$
(b) kinetic energy is $mu^2/2$
(c) momentum is $mu(M + m)/M$
(d) kinetic energy is $m^2u^2/2(M + m)$

26. A stationary body explodes in to four identical fragments such that three of them fly mutually perpendicular to each other, each with same KE(E_0). The energy of explosion will be

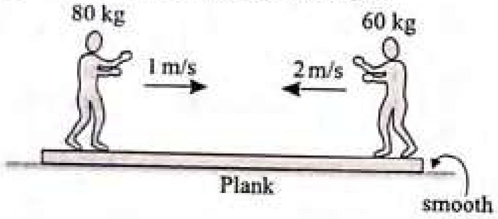
(a) $6E_0$ (b) $3E_0$ (c) $4E_0$ (d) $2E_0$

27. A body of mass 1 kg initially at rest, explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces of equal mass fly off perpendicular to each other with a speed of 15 m s^{-1} each. What is the velocity of the heavier fragment?

(a) $10\sqrt{2}$ m s^{-1} (b) $5\sqrt{3}$ m s^{-1}

(c) $10\sqrt{3}$ m s^{-1} (d) $5\sqrt{2}$ m s^{-1}

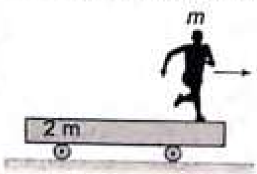
28. Two men of masses 80 kg and 60 kg are standing on a wood plank of mass 100 kg that has been placed over a smooth surface. If both the men start moving toward each other with speeds 1 m/s and 2 m/s respectively. The velocity of the plank by which it starts moving is



(a) $\frac{1}{6}$ m/s towards right (b) $\frac{1}{6}$ m/s towards left

(c) $\frac{2}{5}$ m/s towards right (d) $\frac{2}{5}$ m/s towards left

29. A man is standing on a cart of mass double the mass of the man. Initially cart is at rest on the smooth ground. Now man jumps with relative velocity 'v' horizontally towards right with respect to cart. The work done by man during the process of jumping is

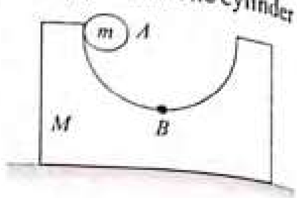


(a) mv^2 (b) $\frac{mv^2}{2}$ (c) $\frac{mv^2}{4}$ (d) $\frac{mv^2}{3}$

30. A gun fires a shell and recoils horizontally. If the shell travels along the barrel with speed v, the ratio of speed with which the gun recoils if (i) the barrel is horizontal (ii) inclined at an angle of 30° with horizontal is

(a) 1 (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

31. A block of mass $M = 2$ kg with a semicircular track of radius $R = 1.1$ m rests on a horizontal frictionless surface. A uniform cylinder of radius $r = 10$ cm and mass $m = 1.0$ kg is released from rest from the top point a. The cylinder slips on the semicircular frictionless track. The speed of the block when the cylinder reaches the bottom of the track at B is ($g = 10$ m/s^2)

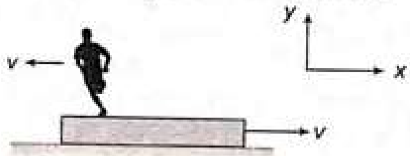


(a) $\sqrt{\frac{10}{3}}$ m/s (b) $\sqrt{\frac{4}{3}}$ m/s (c) $\sqrt{\frac{5}{2}}$ m/s (d) $\sqrt{10}$ m/s

32. A cannon shell moving along a straight line bursts into two parts. Just after the burst one part moves with momentum 40 Ns making an angle 30° with the original line of motion. The minimum momentum of the other part of shell just after the burst is:

(a) 0 N s (b) 10 N s (c) 20 N s (d) 17.32 N s

33. A man and a plank of the same mass are moving with a velocity v along positive x-axis. At the same time man jumps along negative x-axis with a velocity v with respect to ground, then the speed of the plank is:



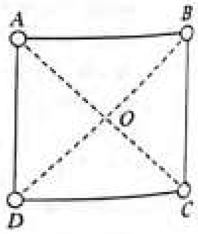
(a) v (b) $2v$ (c) $3v$ (d) none of these

34. Two particles A and B start moving due to their mutual interaction only. If at any time t, $\vec{a}_A$ and $\vec{a}_B$ are their respective accelerations, $\vec{v}_A$ and $\vec{v}_B$ are their respective velocities, and up to that time W_A and W_B are the work done on A and B respectively by the mutual force, m_A and m_B are their masses respectively, then which of the following is always correct?

(a) $\vec{v}_A + \vec{v}_B = 0$ (b) $m_A\vec{v}_A + m_B\vec{v}_B = 0$

(c) $W_A + W_B = 0$ (d) $\vec{a}_A + \vec{a}_B = 0$

35. Four particles A, B, C and D of equal masses are placed at the corners of a square. They move with equal uniform speed v towards the intersection of the diagonals. After collision, A comes to rest, B traces its path back with same speed and C and D move with equal velocities in same direction. What is the velocity of C after collision?



(a) $\frac{2v}{3}$ (b) $2v$ (c) $\frac{v}{2}$ (d) v

36. Two skaters A and B, both of mass 70 kg, are approaching one another over a frictionless fixed surface, each with a speed of 1 m/s with respect to the surface. A carries a bowling ball of mass of 10 kg. Both skaters can throw the ball at 5 m/s relative to themselves. To avoid collision they start throwing the ball back and forth when they are

10 m apart. How many minimum number of throws are required?
(a) 1 (b) 2 (c) 3 (d) 4

37. A man of mass m on an initially stationary boat gets off the boat by jumping to the left in an exactly horizontal direction. Immediately after the jump, the boat of mass M, is observed to be moving to the right at speed v. How much work did the man do during the jump (both on his own body and on the boat)?

(a) $\frac{1}{2}(M+m)v^2$ (b) $\frac{1}{2}\left(M+\frac{M^2}{m}\right)v^2$

(c) $\frac{1}{2}\left(\frac{Mm}{M+m}\right)v^2$ (d) None of these

38. A block of mass $m_1 = 2$ kg slides on a frictionless table with speed of 10 m/s. In front of it, another block of mass $m_2 = 5$ kg is moving with speed 3 m/s in the same direction. A massless spring of spring constant $k = 1120$ N/m is attached on the backside of m_2 as shown. The maximum compression of the spring in cm when the blocks collide is

(a) 25 cm (b) 10 cm (c) 2.5 cm (d) 5 cm

39. A loaded spring gun, initially at rest on a horizontal frictionless surface fires a marble of mass m at an angle of elevation θ. The mass of the gun is M, that of the marble is m and the muzzle velocity of the marble is v_0, then velocity of the gun just after the firing is

(a) $\frac{m v_0}{M}$ (b) $\frac{m v_0 \cos\theta}{M}$

(c) $\frac{m v_0 \cos\theta}{M+m}$ (d) $\frac{m v_0 \cos 2\theta}{M+m}$

40. A particle is projected at an angle of 45°. After 1 second, it breaks into two equal parts. One part stops and other part attains the height of 20 m after the breaking of the particle. Find the velocity of projection ($g = 10$ m/s^2).
(a) 20 m/s (b) $20\sqrt{2}$ m/s (c) $10\sqrt{2}$ m/s (d) 15 m/s

Problems Based on Collision

41. A ball impinges directly on another ball at rest. The first ball is brought to rest by the impact. If half of the kinetic energy is lost by the impact, the value of coefficient of restitution is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

42. A particle of mass m travelling with velocity v and kinetic energy E collides elastically to another particle of mass nm, at rest. What is the fraction of total energy retained by the particle of mass m?

(a) $\left(\frac{n+1}{n}\right)^2$ (b) $\left(\frac{n+1}{n-1}\right)^2$

(c) $\left(\frac{n-1}{n+1}\right)^2$ (d) None of these

43. A ball of mass m moving with speed u undergoes a head-on elastic collision with a ball of mass nm initially at rest. The fraction of the incident energy transferred to the second ball is

(a) $\frac{n}{1+n}$ (b) $\frac{n}{(1+n)^2}$ (c) $\frac{2n}{(1+n)^2}$ (d) $\frac{4n}{(1+n)^2}$

44. In the above question, the ratio of the kinetic energies of the first ball to the second ball after collision is

(a) $\frac{(1-n)^2}{2n}$ (b) $\frac{(1-n)^2}{4n}$ (c) $\frac{(1+n)^2}{2n}$ (d) $\frac{(1+n)^2}{4n}$

45. A ball is let fall from a height h_0. There are n collisions with the earth. If the velocity of rebound after n collisions is v_n and the ball rises to a height h_n, then coefficient of restitution e is given by

(a) $e^n = \sqrt{\frac{h_n}{h_0}}$ (b) $e^n = \sqrt{\frac{h_0}{h_n}}$

(c) $ne = \sqrt{\frac{h_n}{h_0}}$ (d) $\sqrt{ne} = \sqrt{\frac{h_n}{h_0}}$

46. A particle of mass m is moving horizontally with a constant velocity v towards a rigid wall that is moving in opposite direction with a constant speed u. Assuming elastic impact between the particle and wall, the work done by the wall in reflecting the particle is equal to
(a) $(1/2)m(u+v)^2$ (b) $(1/2)m(u+v)$
(c) $(1/2)muv$ (d) None of these

47. A ball collides with a fixed inclined plane of inclination θ after falling through a distance h. If it moves horizontally just after the impact, the coefficient of restitution is
(a) $\tan\theta$ (b) $\tan^2\theta$ (c) $\cot\theta$ (d) $\cot^2\theta$

48. A mass m_1 moves with a great velocity. It strikes another mass m_2 at rest in head-on collision. It comes back along its path with low speed after collision. Then
(a) $m_1 > m_2$ (b) $m_1 < m_2$
(c) $m_1 = m_2$
(d) there is no relation between m_1 and m_2

49. A steel ball of mass 0.5 kg is fastened to a cord 20 cm long and fixed at the far end and is released when the cord is horizontal. At the bottom of its path the ball strikes a 2.5 kg steel block initially at rest on a frictionless surface. The collision is elastic. The speed of the block just after the collision will be.

(a) $\frac{10}{3}$ ms^{-1} (b) $\frac{20}{3}$ ms^{-1} (c) 5 ms^{-1} (d) $\frac{5}{3}$ ms^{-1}

50. Two particles of equal masses moving with same speed collide perfectly inelastically. After the collision the combined mass moves with half of the speed of the individual masses. The angle between the initial momenta of individual particle is
(a) 60° (b) 90° (c) 120° (d) 45°

51. A ball is dropped from a height of 45 m from the ground. The coefficient of restitution between the ball and the

ground is 2/3. What is the distance travelled by the ball in 4th second of its motion. Assume negligible time is spent in rebounding. Let $g = 10$ m s^2.
(a) 5 m (b) 20 m (c) 15 m (d) 10 m

52. A ball falls vertically onto a floor with momentum p, and then bounces repeatedly. If the coefficient of restitution is e, then the total momentum imparted by the ball on the floor till the ball comes to rest is
(a) $p(1+e)$ (b) $\frac{p}{1-e}$ (c) $p\left(1+\frac{1}{e}\right)$ (d) $p\left(\frac{1+e}{1-e}\right)$

53. A body of mass 3 kg moving with a velocity of 4 m/s towards left collides head on with a body of mass 4 kg moving in opposite direction with a velocity of 3 m/s. After collision the two bodies stick together and move with a common velocity which is
(a) zero (b) 12 m/s towards left
(c) 12 m/s towards right (d) $\frac{12}{7}$ m/s towards left

54. A particle of mass m_1 moving with velocity v in a positive direction collides elastically with a mass m_2 moving in opposite direction also at velocity v. If $m_2 >> m_1$, then
(a) the velocity of m_1 immediately after collision is nearly $3v$
(b) the change in momentum of m_1 is nearly $4m_1v$
(c) the change in kinetic energy of m_1 is nearly $4m_1v^2$
(d) all of the above

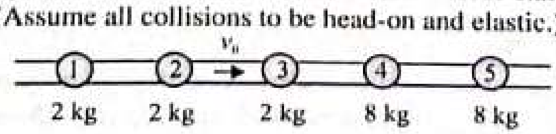
55. A particle of mass m moving with velocity 1 m/s collides perfectly elastically with another stationary particle of mass $2m$. If the incident particle is deflected by 90°, the heavy mass will make an angle θ with the initial direction of m equal to
(a) 60° (b) 45° (c) 15° (d) 30°

56. A smooth sphere is moving on a horizontal surface with velocity vector $2\hat{i}+2\hat{j}$ immediately before it hits a vertical wall. The wall is parallel to $\hat{j}$ vector and the coefficient of restitution between the sphere and the wall is $e = 1/2$. The velocity vector of the sphere after it hits the wall is
(a) $\hat{i}-\hat{j}$ (b) $-\hat{i}+2\hat{j}$ (c) $-\hat{i}-\hat{j}$ (d) $2\hat{i}-\hat{j}$

57. Three balls A, B and C of masses 2 kg, 4 kg and 8 kg, respectively, move along the same straight line and in the same direction, with velocities 4 m/s, 1 m/s and 3/4 m/s. If A collides with B and subsequently B collides with C, find the velocity of ball A and ball B after collision respectively, taking the coefficient of restitution as unity.

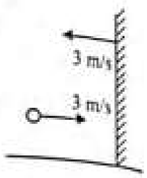
(a) 3 m/s, 9/4 m/s (b) 0 m/s, 3 m/s
(c) 3 m/s, 0 m/s (d) 0 m/s, 0 m/s

58. Five balls are placed one after the other along a straight line as shown in the figure. Initially, all the balls are at rest. Then the second ball has been projected with speed v_0 towards the third ball. Mark the correct statements. (Assume all collisions to be head-on and elastic.)



(a) Total number of collisions in the process is 5.
(b) Velocity of separation between the first and fifth ball after the last possible collision is v_0.
(c) Finally, three balls remain stationary.
(d) All of the above.

59. A highly elastic ball moving at a speed of 3 m/s approaches a wall moving towards it with a speed of 3 m/s. After the collision, the speed of the ball will be

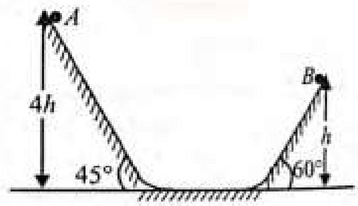


(a) 3 m/s (b) 6 m/s
(c) 9 m/s (d) zero

60. After a totally inelastic collision, two objects of the same mass and same initial speeds are found to move together at half of their initial speeds. The angle between the initial velocities of the objects is
(a) 120° (b) 60° (c) 150° (d) 45°

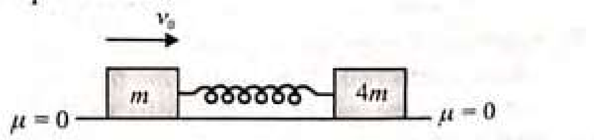
Problems Based on Mixed Concepts

61. Two identical balls A and B are released from the position shown in the figure. They collide elastically with each other on the horizontal portion. The ratio of heights attained by A and B after collision is (neglect friction)



(a) 1:4
(b) 2:1
(c) 4:13
(d) 2:5

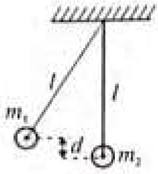
62. Two blocks of masses m and $4m$ lie on a smooth horizontal surface connected with a spring in its natural length. Mass m is given velocity v_0 through an impulse as shown in the figure. Which of the following is not true about subsequent motion?



(a) Kinetic energy is maximum in ground frame and centre of mass (CM) frame simultaneously.
(b) Value of maximum and minimum kinetic energy is same in CM and ground frame.
(c) Minimum kinetic energy is zero in CM frame but non-zero in ground frame.
(d) Maximum and minimum kinetic energy of m in ground frame is, respectively, $\frac{1}{2}mv_0^2$ and zero.

63. A pendulum consists of a wooden bob of mass m and of length l. A bullet of mass m_1 is fired towards the pendulum with a speed v_1. The bullet emerges out of the bob with a speed $v_1/3$ and the bob just completes motion along a vertical circle. Then v_1 is
(a) $\left(\frac{m}{m_1}\right)\sqrt{5gl}$ (b) $\frac{3}{2}\left(\frac{m}{m_1}\right)\sqrt{5gl}$
(c) $\frac{2}{3}\left(\frac{m_1}{m}\right)\sqrt{5gl}$ (d) $\left(\frac{m_1}{m}\right)\sqrt{gl}$

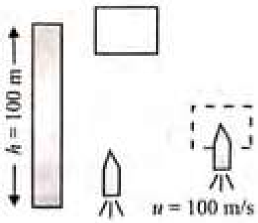
64. Two pendulums each of length l are initially situated as shown in the figure. The first pendulum is released and strikes the second. Assume that the collision is completely inelastic and neglect the mass of the string and any frictional effects. How high does the centre of mass rise after the collision?



(a) $d\left[\frac{m_1}{(m_1+m_2)}\right]^2$ (b) $d\left[\frac{m_1}{(m_1+m_2)}\right]$

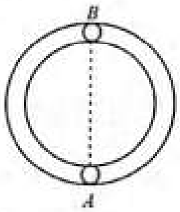
(c) $d\left[\frac{(m_1+m_2)}{m_2}\right]^2$ (d) $d\left[\frac{m_2}{(m_1+m_2)}\right]^2$

65. A wooden block of mass 10 g is dropped from the top of a tower 100 m high. Simultaneously, a bullet of mass 10 g is fired from the foot of the tower vertically upwards with a velocity of 100 m/s. If the bullet is embedded in it, how high will the block rise above the top of tower before it starts falling? ($g = 10$ m/s^2)



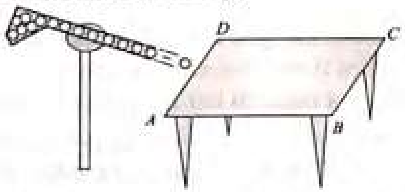
(a) 75 m (b) 85 m (c) 80 m (d) 10 m

66. Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of a diameter. At time $t = 0$, A is projected along the groove and it first impinges on B at time $t = T_1$ and again at time $t = T_2$. If e is the coefficient of restitution, the ratio T_2/T_1 is



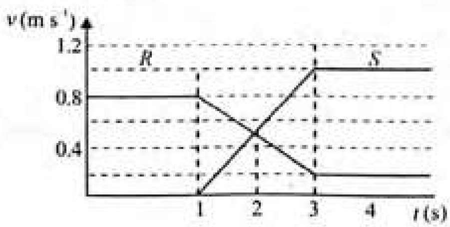
(a) $\frac{2}{e}$ (b) $\frac{(2+e)}{2}$ (c) $\frac{2(e+1)}{e}$ (d) $\frac{(2+e)}{e}$

67. A gun which fires small balls of mass 20 g is firing 20 balls per second on the smooth horizontal table surface $ABCD$. If the collision is perfectly elastic and balls are striking at the centre of table with a speed of 5 m/s at an angle of 60° with the vertical just before collision, then force exerted by one of the legs on ground is (assume total weight of the table is 0.2 kg)



(a) 0.5 N (b) 1 N (c) 0.25 N (d) 0.75 N

68. Figure shows the velocity–time graph for two masses R and S that collided elastically. Which of the following statements is true?



i. R and S moved in the same direction after the collision.
ii. The velocities of R and S were equal at the mid time of the collision.
iii. The mass of R was greater than mass of S.

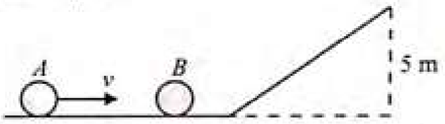
Which of the following is true?

(a) i only (b) ii only
(c) i and ii only (d) i, ii and iii

69. A cracker is thrown into air with a velocity of 10 m/s at an angle of 45° with the vertical. When it is at a height of 0.5 m from the ground, it explodes into a number of pieces which follow different parabolic paths. What is the velocity of centre of mass, when it is at a height of 1 m from the ground? ($g = 10$ m/s^2)

(a) $4\sqrt{5}$ m/s (b) $2\sqrt{5}$ m/s
(c) $5\sqrt{4}$ m/s (d) 10 m/s

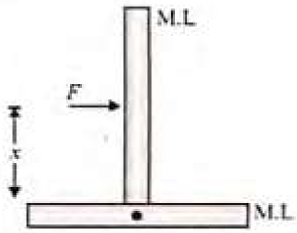
70. Two identical balls, of equal masses A and B, are lying on a smooth surface as shown in the figure. Ball A hits ball B (which is at rest) with a velocity $v = 16$ m/s. What should be the minimum value of coefficient of restitution between A and B so that B just reaches the highest point of inclined plane?



(a) $\frac{2}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

71. An inverted T-shaped object is placed on a smooth horizontal floor as shown in the figure.

A force F is applied on the system as shown in the figure. The value of x so that the system performs pure translational motion is

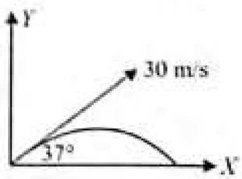


(a) $\frac{L}{4}$ (b) $\frac{3L}{4}$ (c) $\frac{L}{2}$ (d) $\frac{3L}{2}$

72. A particle at rest is constrained to move on a smooth horizontal surface. Another identical particle hits the fractional particle with a velocity v at an angle $\theta = 60°$ with horizontal. If the particles move together, the velocity of the combination just after impact is equal to

(a) v (b) $\frac{v}{2}$ (c) $\frac{\sqrt{3}v}{4}$ (d) $\frac{v}{4}$

73. An object of mass 10 kg is launched from the ground at $t = 0$, at an angle of 37° above the horizontal with a speed of 30 m/s. At some time after its launch, an explosion splits the projectile into two pieces. One piece of mass 4 kg is observed at (105 m, 43 m) at $t = 2$ s. Find the location of second piece at $t = 2$ s.



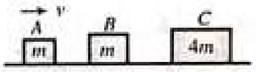
(a) (10, 2) (b) (48, 16)

(c) (10, – 2) (d) Information insufficient

74. A block of mass m starts from rest and slides down a frictionless semi-circular track from a height h as shown. When it reaches the lowest point of the track, it collides with a stationary piece of putty also having mass m. If the block and the putty stick together and continue to slide, the maximum height that the block–putty system could reach is

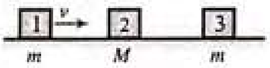
(a) $h/4$ (b) $h/2$
(c) h (d) independent of h

75. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity v to the right. Block B with mass m and block C with mass $4m$ are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of blocks A is



(a) $0.6v$ to the left (b) $1.4v$ to the left
(c) v to the left (d) $0.4v$ to the right

76. Three blocks are placed on smooth horizontal surface and lie on same horizontal straight line. Block 1 and block 3 have mass m each and block 2 has mass M ($M > m$). Block 2 and block 3 are initially stationary, while block 1 is initially moving towards block 2 with speed v as shown. Assume that all collisions are head on and perfectly elastic. What value of M/m ensures that block 1 and block 3 have the same final speed?

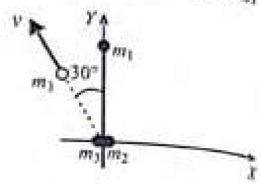


(a) $5+\sqrt{2}$ (b) $5-\sqrt{2}$ (c) $2+\sqrt{5}$ (d) $3+\sqrt{5}$

77. A ball is projected in a direction inclined to the vertical and bounces on a smooth horizontal plane. The range of one rebound is R. If the coefficient of restitution is e, then range of the next rebound is

(a) $R' = eR$ (b) $R' = e^2R$ (c) $R' = \frac{R}{e}$ (d) $R' = R$

78. Three point like equal masses m_1, m_2 and m_3 are connected to the ends of a massless rod of length L which lies at rest on a smooth horizontal plane (see figure). At $t = 0$, an explosion occurs between m_2 and m_3, and as a result, mass m_3 is detached from the rod, and moves with a known velocity v at an angle of 30° with the y-axis. Assume that the masses m_2 and m_3 are unchanged during the explosion.



What is the velocity of the centre of mass of the system consisting of three masses, after the expulsion?

(a) $\frac{v}{4}(\hat{i}-3\hat{j})$ (b) $\frac{v}{4}(-\hat{i}+\sqrt{3}\,\hat{j})$
(c) $-v$ (d) None of these

79. Two objects that are moving along an xy-plane on a frictionless floor collide. Assume that they form a closed, isolated system. The following table gives some of the momentum components (in kilogram meters per second) before and after the collision.

	Before collision		After collision	
Object	P_X	P_Y	P_X	P_Y
A	–4	5	3	a
B	b	–2	4	2

What are the missing values (a, b)?
(a) 10, 11 (b) 1, 11 (c) 5, 7 (d) 6, 4

80. In the arrangements shown in the figure masses of each ball is 1 kg and mass of trolley is 4 kg. In the figure, shell of mass 1 kg moving horizontally with velocity $v = 6\ \text{ms}^{-1}$ collides with the ball and get stuck to it then its maximum deflection of the thread (length 1.5 m) with vertical is

(a) 53° (b) 37° (c) 30° (d) 60°

ARCHIVES

1. Two identical particles move towards each other with velocity $2v$ and v, respectively. The velocity of the centre of mass is

(a) v (b) $\frac{v}{3}$ (c) $\frac{v}{2}$ (d) zero **(AIEEE 2002)**

2. A bomb of mass 9 kg explodes into 2 pieces of mass 3 kg and 6 kg. The velocity of mass 3 kg is 1.6 m/s, the K.E. of mass 6 kg is

(a) 3.84 J (b) 9.6 J
(c) 1.92 J (d) 2.92 J **(AIEEE 2002)**

3. Consider the following two statements.

I. The linear momentum of a system of particles is zero.
II. The kinetic energy of a system of particles is zero.
Then

(a) I implies II and II implies I.
(b) I does not imply II and II does not imply I.
(c) I implies II but II does not imply I.
(d) I does not imply II but II implies I. **(AIEEE 2003)**

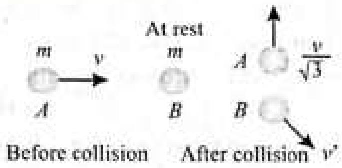
4. A ^{238}U nucleus decays by emitting an alpha particle of speed $v\ \text{ms}^{-1}$. The recoil speed of the residual nucleus is (in ms^{-1})

(a) $-4v/234$ (b) $v/4$
(c) $-4v/238$ (d) $4v/238$ **(AIEEE 2003)**

5. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table?

(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J **(AIEEE 2004)**

6. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision, the first mass moves with velocity $v/\sqrt{3}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision.



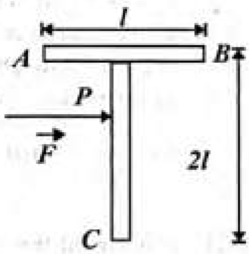
(a) $\frac{v}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}v$

(c) $\sqrt{3}\,v$ (d) v **(AIEEE 2005)**

7. A body A of mass M, while falling vertically downward under gravity, breaks into two parts—a body B of mass $M/3$ and a body C of mass $2M/3$. The centre of mass of B and C taken together shifts compared to that of body A towards

(a) body B
(b) body C
(c) does not shift
(d) depends on the height of breaking **(AIEEE 2005)**

8. A T-shaped object with dimensions shown in the figure is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB such that the object has only translational motion without rotation. Find the location of P with respect to C.



(a) l (b) $\left(\frac{4}{3}\right)l$

(c) $\left(\frac{3}{2}\right)l$ (d) $\left(\frac{3}{2}\right)l$ **(AIEEE 2005)**

9. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4 m/s. The kinetic energy of the other mass is

(a) 192 J (b) 96 J
(c) 144 J (d) 288 J **(AIEEE 2006)**

10. Consider a two-particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position?

(a) $\frac{m_1}{m_2}d$ (b) d

(c) $\frac{m_2}{m_1}d$ (d) $\frac{m_1}{m_1+m_2}d$ **(AIEEE 2006)**

11. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is αR from the centre of the bigger disc. The value of α is

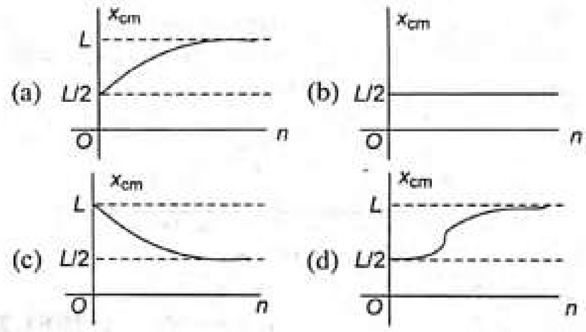
(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$ **(AIEEE 2007)**

12. A block of mass 0.50 kg is moving with a speed of 2.00 m/s on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

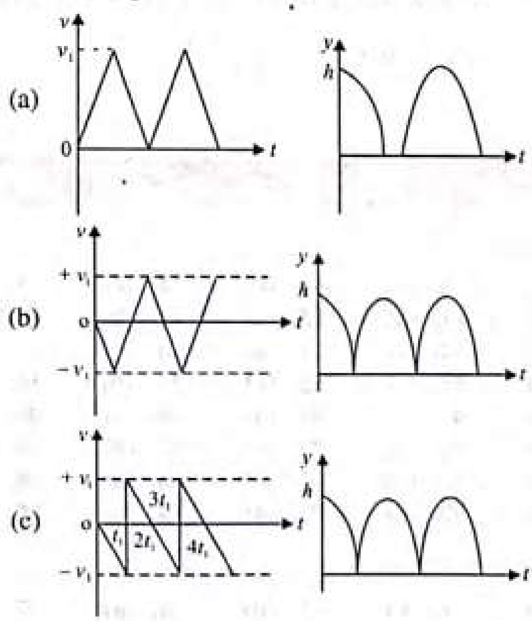
(a) 0.16 J (b) 1.00 J
(c) 0.67 J (d) 0.34 J **(AIEEE 2008)**

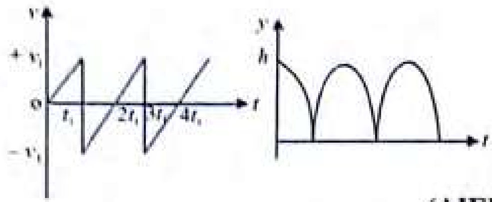
13. A thin rod of length L is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\,(x/L)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n?



(AIEEE 2008)

14. Consider a rubber ball freely falling from a height $h = 4.9$ m on a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be



(d) 

(AIEEE 2009)

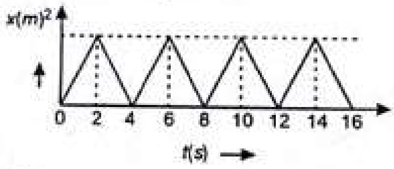
15. **Statement I:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement II: The principle of conservation of momentum holds true for all kinds of collisions.

(a) Statement I is true, statement II is true; statement II is a correct explanation for statement I.
(b) Statement I is true, statement II is true; statement II is not a correct explanation for statement I.
(c) Statement I is false, statement II is true.
(d) Statement I is true, statement II is false.

(AIEEE 2010)

16. The figure shows the position–time $(x-t)$ graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(a) 0.4 Ns (b) 0.8 Ns
(c) 1.6 Ns (d) 0.2 Ns **(AIEEE 2010)**

17. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement I: A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $F=\left(\frac{1}{2}mv^2\right)$ then $f=\left(\frac{m}{M+m}\right)$.

Statement II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(a) Statement I is true, Statement II is true, Statement II is not a correct explanation of Statement I.
(b) Statement I is true, Statement II is false.
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is true, Statement II is a correct explanation of Statement I.

(JEE Main 2013)

18. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to:

(a) 44% (b) 50%
(c) 56% (d) 62% **(JEE Main 2015)**

19. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to

(a) $\frac{h^2}{4R}$ (b) $\frac{3h}{4}$ (c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

(JEE Main 2015)

20. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(a) $\frac{v_0}{\sqrt{2}}$ (b) $\frac{v_0}{4}$ (c) $\sqrt{2}\,v_0$ (d) $\frac{v_0}{2}$

(JEE Main 2018)

21. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is P_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The value of P_d and P_c are respectively.

(a) (0, 1) (b) (0.89, 0.28)
(c) (0.28, 0.89) (d) (0, 0) **(JEE Main 2018)**

ANSWER KEY

Exercises

1. (c)	2. (a)	3. (b)	4. (c)	5. (b)	6. (c)	7. (b)	8. (d)	9. (a)	10. (c)
11. (c)	12. (a)	13. (c)	14. (b)	15. (c)	16. (d)	17. (c)	18. (b)	19. (a)	20. (d)
21. (d)	22. (a)	23. (b)	24. (d)	25. (d)	26. (a)	27. (d)	28. (b)	29. (d)	30. (b)
31. (a)	32. (c)	33. (c)	34. (b)	35. (c)	36. (b)	37. (b)	38. (a)	39. (c)	40. (b)
41. (a)	42. (c)	43. (d)	44. (b)	45. (a)	46. (d)	47. (b)	48. (b)	49. (b)	50. (c)
51. (c)	52. (d)	53. (a)	54. (d)	55. (d)	56. (b)	57. (d)	58. (d)	59. (c)	60. (a)
61. (c)	62. (b)	63. (b)	64. (a)	65. (a)	66. (d)	67. (b)	68. (d)	69. (a)	70. (b)
71. (a)	72. (d)	73. (d)	74. (b)	75. (a)	76. (c)	77. (a)	78. (d)	79. (b)	80. (b)

Archives

1. (c)	2. (c)	3. (d)	4. (a)	5. (b)	6. (b)	7. (c)	8. (b)	9. (d)	10. (a)
11. (None)	12. (c)	13. (a)	14. (c)	15. (a)	16. (b)	17. (c)	18. (c)	19. (b)	20. (c)
21. (b)									

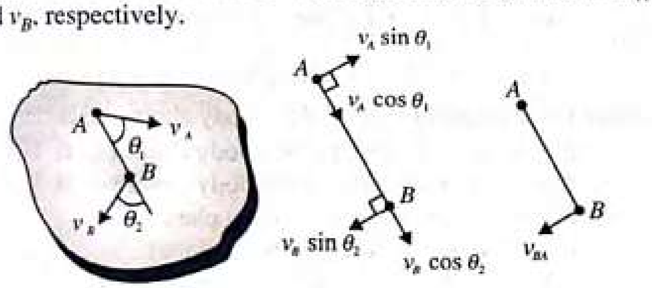
Chapter 9

Rotational Dynamics

RIGID BODY

Rigid body is defined as a system of particles in which distance between each pair of particles remains constant (with respect to time), which means the shape and size do not change during the motion.

For every pair of particles in a rigid body, there is no velocity of separation or approach between the particles. In the figure, velocities of A and B with respect to the ground are v_A and v_B, respectively.



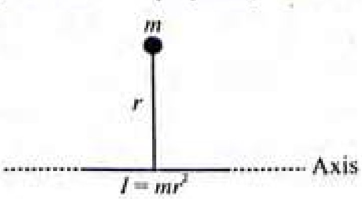
(a) (b) (c)

- If the above body is rigid, $v_A \cos\theta_1 = v_B \cos\theta_2$
- With respect to any particle of the rigid body, the motion of any other particle of that rigid body is circular. v_{BA} = relative velocity of B with respect to A.

MOMENT OF INERTIA

Inertia is a property of a body to resist the change in linear state of motion. It is measured by the mass of the body. The moment of inertia I of a body is a measure of its ability to resist change in its rotational state of motion.

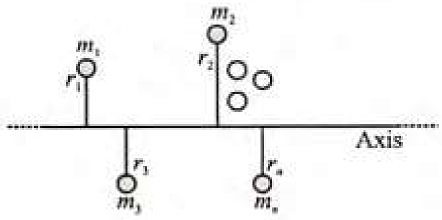
If we compare for rotational motion with corresponding relations for translatory motion, we find that moment of inertia plays the same role in rotatory motion as is played by mass in translatory motion, i.e., if a body has large moment of inertia, it is difficult to start rotation or to stop it if rotating. Large moment of inertia also helps in keeping the motion uniform. This is why stationary engines are provided with fly wheels having large moment of inertia.



Moment of inertia of a particle about an axis is defined as $I = mr^2$ where r is the distance (perpendicular) of the particle from the axis of rotation (see figure).

For a system of particles of masses m_1, m_2, m_3,.... m_n at perpendicular distances $r_1, r_2, r_3, \ldots, r_n$ (see figure), respectively, from the axis of rotation, the moment of inertia is given by

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2 = \Sigma m_i r_i^2$$



Moment of inertia of a rigid body about an axis is given by $I = \int r^2\, dm$ where r is the perpendicular distance of a particle of mass dm of rigid body from the axis of rotation.

NOTE:

- MI is not constant for a body. It depends on the axis of rotation.
- MI depends on the mass of the body. The higher the mass, the higher is the MI.
- MI depends on the distribution of the mass about an axis. The farther the mass is distributed from the axis, the higher will be the MI.

  Moment of inertia does not change if the mass

  (a) is shifted parallel to the axis of the rotation.

  (b) is rotated with constant radius about axis of rotation.

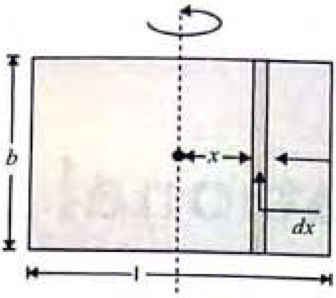
Moment of Inertia of a Uniform Rectangular Lamina

Length, breadth and mass of the lamina are l, b and M, respectively.

(a) Axis in the plane of the lamina passing through its centre and parallel to its breadth

Mass per unit area $= \dfrac{M}{lb}$

Consider a rectangular differential mass element parallel to the breadth at a distance 'x' from the axis of rotation (see figure).



Mass of the strip

$dm = \frac{M}{lb}(bdx) = \frac{M}{l}dx$

Since each particle lying on the differential strip is at the same perpendicular distance 'x' from the axis of rotation, hence the MI of the strip about the given axis is

$$dI = dm\, x^2 = \left(\frac{M}{l}dx\right)x^2$$

Total MI of the entire rectangular lamina can be obtained by integrating the above expression between the limits $-l/2$ to $+l/2$. Thus,

$$I = \int_{-l/2}^{+l/2}\left(\frac{M}{l}x^2\,dx\right) = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{+l/2} = \frac{Ml^2}{12}$$

NOTE: The MI of a square lamina of mass M and side l, about the axes shown in figure are all equal.

$$I_{1,1} = I_{2,2} = I_{3,3} = I_{4,4} = \frac{Ml^2}{12}$$

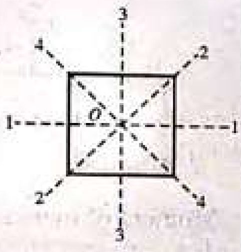
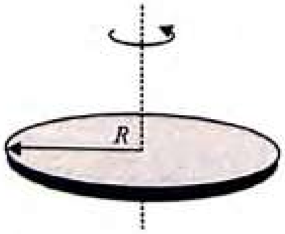


ILLUSTRATION 9.1 Find the moment of inertia of a circular disk or solid cylinder of radius R about the axis through the centre and perpendicular to the flat surface.

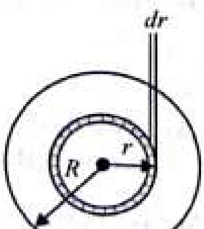


Solution. The figure shows that the appropriate mass element is a circular ring of radius r and width dr.

Its area is $dA = 2\pi r\,dr$ and its mass is $dm = \sigma dA$; where $\sigma = M/A$ is the real mass density.

The moment of inertia of this element is

$dI = dm\,r^2 = 2\pi\sigma r^3 dr$



For the whole body,

$$I = 2\pi\sigma\int_0^R r^3 dr = \frac{1}{2}\pi\sigma R^4$$

The mass of the whole disk or cylinder is

$$M = \sigma A = \sigma\pi R^2, \text{ and so } I = \frac{1}{2}MR^2$$

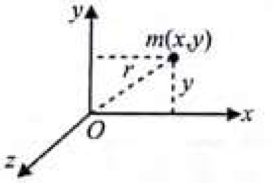
NOTE: The moment of inertia of a cylinder having the same mass and same radius as disc will also be same as MI is independent of the size of the object parallel to its axis of rotation.

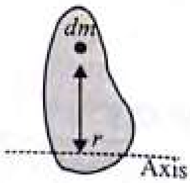
Perpendicular-Axis Theorem

Consider a particle of mass m (see figure) located on the x-y plane at (x, y).

$$I_x = my^2 \text{ and } I_y = mx^2$$

$$\Rightarrow \quad I_x + I_y = my^2 + mx^2 = m(x^2 + y^2) = mr^2 = I_z$$





The distance of the particle from the z-axis $= r$, $I_z = mr^2$.

Now consider a lamina object lying in the x-y plane. Consider a particle of mass dm at (x, y).

$$I_x = y^2\,dm \text{ and } I_y = \int x^2\,dm$$

$$I_x + I_y = \int y^2\,dm + \int x^2\,dm = \int(x^2+y^2)\,dm = \int r^2\,dm = I_z$$

$$I_z = I_x + I_y \qquad \text{(when object is in } x\text{-}y \text{ plane)}$$

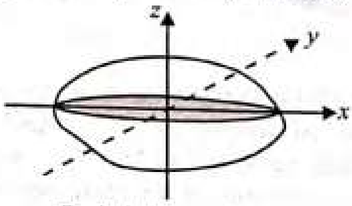
where I_z = moment of inertia of the body about z-axis

I_x = moment of inertia of the body about x-axis

I_y = moment of inertia of the body about y-axis

$I_y = I_x + I_z$ (when object is in x-z plane)

$I_x = I_y + I_z$ (when object is in y-z plane)



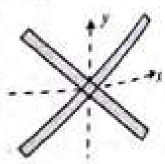
Body is in x-y plane

NOTE: The sum of moments of inertia of a laminar object about two mutually perpendicular axes lying in the plane of lamina is equal to the moment of inertia about an axis normal to the plane of the lamina and passing through the two perpendicular axes.

This theorem is applicable only for a laminar (thin sheet kind of) object.

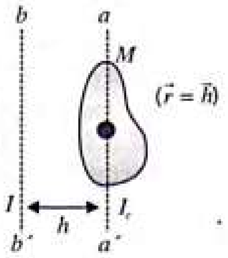
ILLUSTRATION 9.2 Two uniform identical rods each of mass M and length l are joined to form a cross as shown in the figure. Find the moment of inertia of the cross about a bisector shown by dotted lines in the figure.

Solution. Consider the line perpendicular to the plane of the figure through the centre of the cross. The moment of inertia of each rod about this line (say about z-axis) $I_z' = Ml^2/12$. Hence the moments of inertia of the cross $I_z = 2I_z' = Ml^2/6$. The moments of inertia of the cross about the two bisector are equal by symmetry



$I_X = I_Y$ according to the theorem of perpendicular axis, $I_Z = I_X + I_Y$ the moment of inertia of the cross about the bisector is $Ml^2/12$.

Parallel-Axis Theorem



In the figure, bb' is an axis parallel to aa' (an axis passing through centre of mass)

Moment of inertia of the body about centroidal axis (aa'), $I_c = mr^2$

Moment of inertia of the body about an axis parallel to the line bb',

$$I = mr^2 + mh^2$$

ILLUSTRATION 9.3 Find the moment of inertia of a solid cylinder of mass M and radius R about a line parallel to the axis of the cylinder and on the surface of the cylinder.

Solution. The moment of inertia of the cylinder about its axis is $\frac{MR^2}{2}$.

Using parallel axis theorem,

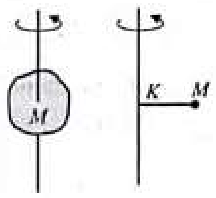
$$I = I_0 + MR^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

Similarly, the moment of inertia of a solid sphere about a tangent is

$$\frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

NOTE:
- Theorem of parallel axes is applicable for any type of rigid body whether it is two dimensional or three dimensional, while the theorem of perpendicular axes is applicable for laminar type or two dimensional bodies only.
- In theorem of perpendicular axes, the point of intersection of the three axes (x, y and z) may be any point on the plane of the body (it may even lie outside the body). This point may or may not be the centre of the mass of the body.

RADIUS OF GYRATION



Radius of gyration (K) of a body about an axis is the effective distance from the axis where the whole mass can be assumed to be concentrated so that the moment of inertia remains the same (see figure). Thus,

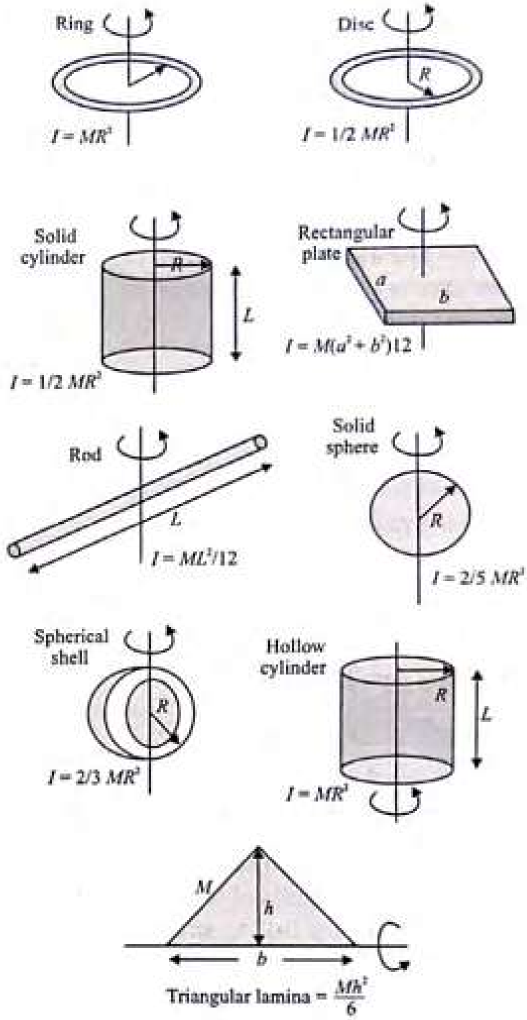
$$I = MK^2 \text{ or } K = \sqrt{\frac{I}{M}}$$

- Radius of gyration of a disc about an axis perpendicular to its plane and passing through its centre of mass is

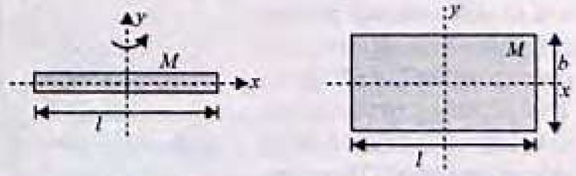
$$K = \sqrt{\frac{\frac{1}{2}MR^2}{M}} = \frac{R}{\sqrt{2}}$$

- Radius of gyration of a solid sphere, $K = \sqrt{\frac{2}{5}}R$
- Radius of gyration of a hollow sphere, $K = \sqrt{\frac{2}{3}}R$

Moment of Inertia of Some Important Cases



NOTE: As moment of inertia of a body having uniform mass distribution is independent of size of the body parallel to axis of rotation. Consider this case:



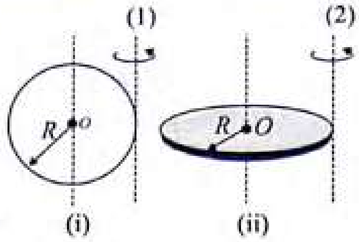
The moment of inertia of a rectangular plate about the y-axis will be same as the moment of inertia of the same mass of the rod about the y-axis. In both the cases $I_y = Ml^2/2$.

We can say directly the moment of inertia of the rectangular plate about x-axis, $Ix = Mb^2/12$.

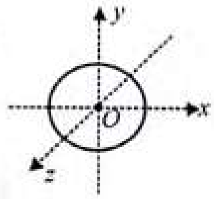
If we are asked to find moment of inertia of the plate about the z-axis, we can use perpendicular axis theorem, i.e.,

$$I_z = I_x + I_y = \frac{Mb^2}{12} + \frac{Ml^2}{12} = \frac{M}{12}(b^2 + l^2)$$

ILLUSTRATION 9.4 Find out the moment of inertia of a ring having uniform mass distribution of mass M and radius R about an axis which is tangent to the ring and (a) in the plane of the ring, (b) perpendicular to the plane of the ring.



Solution. The moment of inertia of a ring about an axis passing through centre and perpendicular to plain of ring is $I_z = MR^2$ because of symmetry we can say $I_x = I_y$ and using perpendicular axis theorem,



$$I_z = I_x + I_y \Rightarrow MR^2 = 2I_x$$

$$\Rightarrow \quad I_x = \frac{MR^2}{2} = I_0 \text{ [in Fig. (i)]}$$

For case (a): Using parallel axis theorem,

$$I_1 = I_o + MR^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$$

For case (b): Using parallel axis theorem,

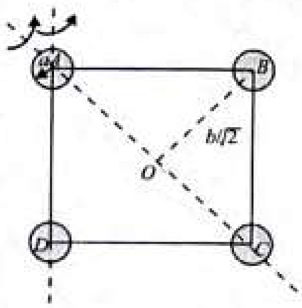
$$I_2 = I_z + MR^2 = MR^2 + MR^2 = 2MR^2$$

ILLUSTRATION 9.5 Four spheres, each of radius a and mass m, are placed with their centres on four corners of a square of side b. Calculate the moment of inertia of the arrangement about any (a) diagonal of the square and (b) any side of the square.

Solution.

(a) Moment of inertia of the arrangement about the diagonal AC:

The moment of inertia of each of spheres A and C about their common diameter $AC = (2/5)ma^2$. The moment of inertia of each of spheres B and D about an axis passing through their centres and parallel to AC is $(2/5)ma^2$. The distance between the axis and AC is $b/\sqrt{2}$, b being side of the square. Therefore, the moment of inertia of each spheres B and D about AC by the theorem of parallel axis is



$$\frac{2}{5}ma^2 + m\left(\frac{b}{\sqrt{2}}\right)^2 = \frac{2}{5}ma^2 + \frac{mb^2}{2}$$

Therefore, the moment of inertia of all the four spheres about diagonal AC is

$$I = 2\left(\frac{2}{5}ma^2\right) + 2\left[\frac{2}{5}ma^2 + \frac{mb^2}{2}\right]$$

(b) The moment of inertia of each of spheres A and D about side AD is $(2/5)\,ma^2$.

The moment of inertia of each of spheres B and C about AD is $(2/5)\,ma^2 + mb^2$.

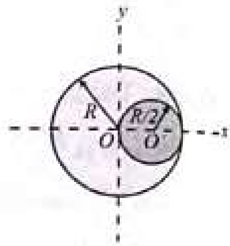
Therefore, the moment of inertia of the whole arrangement about side AD is

$$2\left[\frac{2}{5}ma^2 + \left(\frac{2}{5}ma^2 + mb^2\right)\right] = \frac{2m}{5}\,[4a^2 + 5b^2]$$

ILLUSTRATION 9.6 A circular hole of radius $R/2$ is cut from a circular disc of radius R. The disc lies in the xy-plane and its centre coincides with the origin. If the remaining mass of the disc is M, then

(a) determine the initial mass of the disc, and

(b) determine its moment of inertia about the z-axis.



Solution.

(a) Let σ be the surface mass density of the plate given by

$$\sigma = \frac{M}{\pi R^2 - \pi\left(\frac{R}{2}\right)^2} = \frac{4}{3}\frac{M}{\pi R^2}$$

Now, initial mass of the plate, $m_1 = \sigma\,(\pi R^2) = \frac{4}{3}M$

Mass of the plate removed, $m_2 = \sigma\left(\frac{\pi R^2}{4}\right) = \frac{M}{3}$

The given arrangement may be considered as a combination of a circular plate of mass $(4/3)\,M$ and circular plate of negative mass $(M/3)$.

The moment of inertia of the big disc of radius R and mass m_1 about O and parallel to the z-axis is

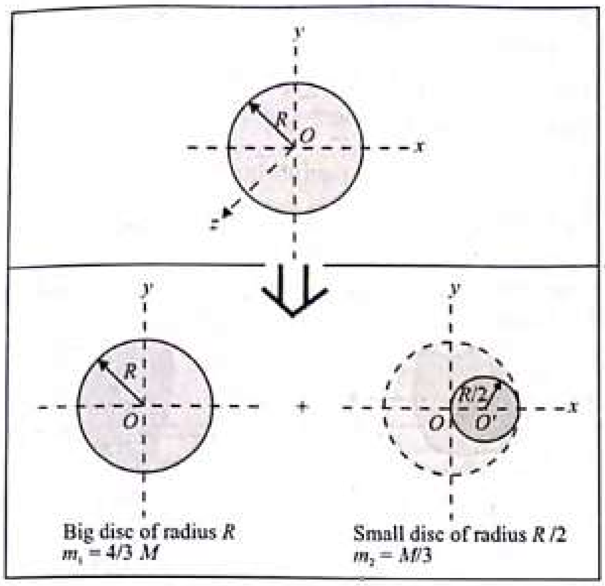
$$I_{O_1} = \frac{m_1R^2}{2}$$

The moment of inertia of the small disc of mass m_2 and radius $R/2$ about O' and parallel to z-axis

$$I_{O_1} = \frac{m_2(R/2)^2}{2} = \frac{m_2R^2}{8}$$

The moment of inertia of the small disc about O (using parallel axis theorem)

$$I_{O_2} = \frac{m_2R^2}{8} + \frac{m_2R^2}{4} = \frac{3m_2R^2}{8}$$



As the part of the disc of mass m_2 is removed from disc m_1, hence the net moment of inertia of structure is

$$I_O = I_{O_1} - I_{O_2}$$

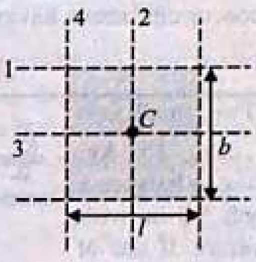
$$I_O = \frac{m_1 R^2}{2} - \frac{3 m_2 R^2}{8}$$

Substituting the values of m_1 and m_2, we get

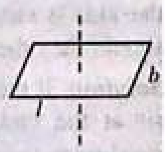
$$I_O = \frac{13}{24} MR^2$$

CONCEPT APPLICATION EXERCISE 9.1

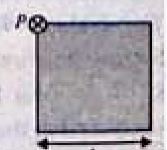
1. In the figure, find moment of inertia of a plate having mass M, length l and width b about axes 1, 2, 3 and 4. Assume that C is the centre and mass is uniformly distributed.



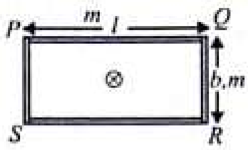
2. Find the moment of inertia of a uniform rectangular plate of mass M and edges of length 'l' and 'b' about its axis passing through the centre and perpendicular to it.



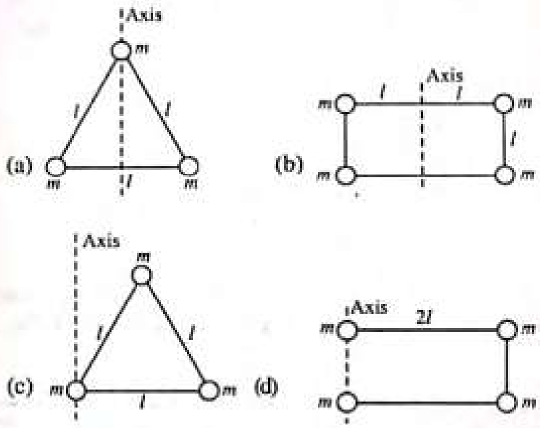
3. Find the moment of inertia of a uniform square plate of mass M and edge of length 'l' about its axis passing through P and perpendicular to it.



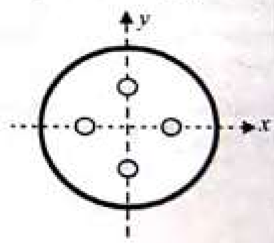
4. Calculate the moment of inertia of a rectangular frame formed by uniform rods having mass m each as shown in the figure about an axis passing through its centre and perpendicular to the plane of frame. Also find moment of inertia about an axis passing through PQ ?



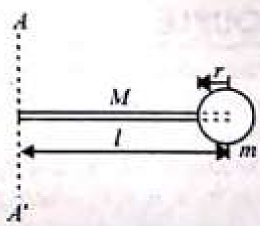
5. Calculate the moment of inertia of each particle in the figure about the indicated axis of rotation.



6. A uniform disc of mass m and radius R has an additional rim of mass m as well as four symmetrically placed masses, each of mass $m/4$, tied at positions $R/2$ from the centre as shown in the figure. What is the total moment of inertia of the disc about an axis perpendicular to the disc through its centre?



7. Find the moment of inertia of a spherical ball of mass m and radius r attached at the end of a straight rod of mass M and length l, if this system is free to rotate about an axis passing through the end of the rod (end of the rod opposite to sphere).



TORQUE

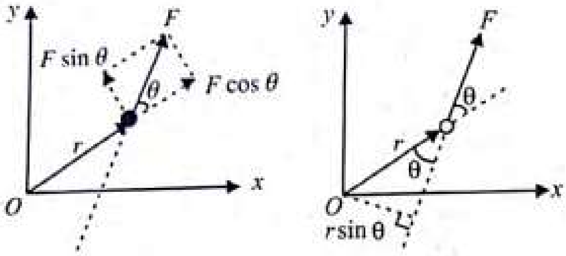
Force causes linear acceleration, whereas torque causes angular acceleration.

It is defined as the moment of force. In other words, the turning ability of a force about an axis is called its torque about that axis.

Consider force $\vec{F}$ which acts through a point whose position vector is r, as shown in the figure.

Its torque about the origin is defined as $\vec{\tau} = \vec{r} \times \vec{F}$.

The direction of the torque can be obtained from the right hand rule and its magnitude is given by $\tau = rF \sin\theta$, where θ is the angle between the vector r and F.



(a) (b)

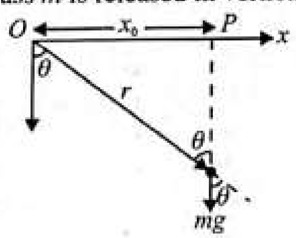
$$\tau = r(F \sin\theta) = rF_\perp$$

where $F_\perp$ is the component of F perpendicular to r.

$$\tau = (r \sin\theta)\,F = r_\perp F$$

where $r_\perp$ is the perpendicular distance from the origin to the line of action of the force. It is also called the lever arm.

ILLUSTRATION 9.7 A particle of mass m is released in vertical plane from a point P at $x = x_0$ on the x-axis. It falls vertically parallel to the y-axis. Find the torque τ acting on the particle at a time t about origin.



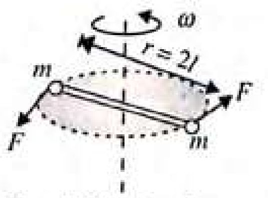
Solution. Torque is produced by the force of gravity.

$$\vec{\tau} = rF \sin\theta \quad \text{or} \quad \tau = r\,mg\,(x_0/r) = mgx_0\,\hat{k}$$

COUPLE

Two equal and opposite forces whose lines of action do not coincide make a couple

Moment of a couple is given by $\tau = r_\perp F$, where $r_\perp$ is the separation of the lines of action of the forces and F is the magnitude of either of the forces.



Two balls each of mass m are fixed at the ends of a rigid body of length $2\,l$.

EQUILIBRIUM OF RIGID BODIES

The conditions of equilibrium for a rigid body are different from that of a particle. Unlike particle such as bodies, rigid bodies have a tendency to rotate, therefore, one has to consider the rotational equilibrium also, in addition to the translational equilibrium.

The equations of equilibrium (in two dimensions) for a rigid body are stated as:

Translational equilibrium: $\sum \vec{F} = 0$

Rotational equilibrium: $\sum \vec{\tau}_c = 0$

Note that for rotational equilibrium of a rigid body, the net torque about its centre of mass must be zero.

Action of two forces on a particle and a rigid body is shown in Figs. (a) and (b).

A particle is in equilibrium under the action of two equal and opposite forces.

Equilibrium
$\Sigma F = 0$
$\Sigma \tau_c = 0$
(a)

No equilibrium
$\Sigma F = 0$
$\Sigma \tau_c \neq 0$
(b)

The body have both translational and rotational equilibria.

A rigid body may or may not be in equilibrium under the action of two equal and opposite forces.

The body is in translational equilibrium but not in rotational equilibrium.

Figures (a) and (b) show equilibrium of a rigid body under the action of three parallel or concurrent forces.

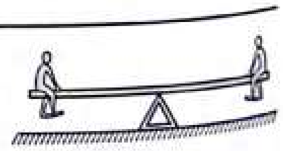
A rigid body may be in equilibrium under the action of three forces if their lines of action are parallel to each other.

Equilibrium
$\Sigma F = 0$
$\Sigma \tau_c = 0$
(a)

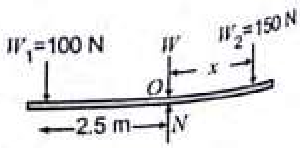
Equilibrium
$\Sigma F = 0$
$\Sigma \tau_c = 0$
(b)

A rigid body may be in equilibrium under the action of three concurrent forces (concurrent means having a common point of application).

ILLUSTRATION 9.8 Two small kids weighing 10 kg and 15 kg, respectively, are trying to balance a seesaw of total length 5.0 m, with the fulcrum at the centre. If one of the kids is sitting at an end, where should the other sit?



Solution. If the kid of mass 15 kg sit at the end, he will produce maximum torque about fulcrum. The sea saw will not be balance for any position of kid of mass 10 kg. It is clear that the 10 kg kid should sit at the end and the 15 kg kid should sit closer to the centre.



Suppose his distance from the centre is x. As the kids are in equilibrium, the normal force between a kid and the seesaw equals the weight of that kid. Considering the rotational equilibrium of the seesaw, the torque of the forces acting on it should add to zero. The forces are as follows:

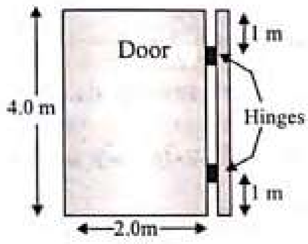
(a) $(15 \text{ kg}) g = 150$ N downwards by the 15 kg kid,
(b) $(10 \text{ kg}) g = 100$ N downwards by the 10 kg kid,
(c) weight of the seesaw, and
(d) the normal force by the fulcrum.

Taking torques about the fulcrum,

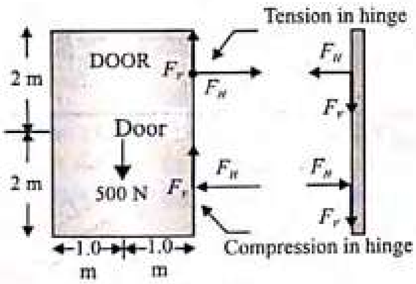
$$150 \times x = 100 \times 2.5 \quad \text{or} \quad x = 1.7 \text{ m}$$

ILLUSTRATION 9.9 The door of an almirah is 4 m high, 1 m wide and weighs 50 kg. The door is supported by two hinges situated at a distance of 1.0 m from the ends. If the magnitude of each forces exerted by the hinges on the door is equal, find this magnitude.

Solution.



(a)



(b) FBD

The free-body diagram of the door is shown in Fig. (b). The forces exerted on the hinges are equal in magnitude. The door is in equilibrium in horizontal as well as in vertical direction, i.e., the horizontal and vertical components of the forces will be equal. For vertical equilibrium of the door, we have

$$\Sigma F_v = 0;\ 2F_v = 500 \Rightarrow F_v = 250 \text{ N}$$

For rotational equilibrium of the door, we have $\Sigma \tau = 0$

Taking moment of all forces acting on the door about lower hinge (may be upper hinge or any other point), we get

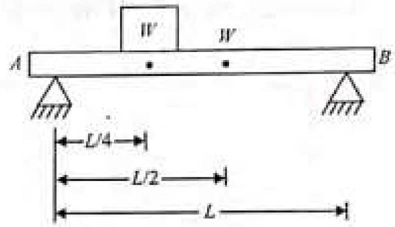
$$500 \times 1.0 - F_H \times 2 = 0 \text{ which gives } F_H = 250 \text{ N}$$

The force on the door exerted by either hinge, in magnitude, is

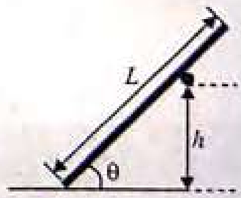
$$F = \sqrt{F_H^2 + F_v^2} = \sqrt{(250)^2 + (250)^2} = 250\sqrt{2} \text{ N}$$

CONCEPT APPLICATION EXERCISE 9.2

1. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point directly above the centre of the face, at a height $3a/4$ above the base. What is the minimum value of F for which the cube begins to tip about an edge?
2. A uniform rod is made to lean between a rough vertical wall and the ground. The coefficient of friction between the rod and the ground is μ_1 and between the rod and the wall is μ_2. Find the angle at which the rod can be leaned without slipping.
3. A beam of weight W supports a block of weight W. The length of the beam is L and weight is at a distance $L/4$ from the left end of the beam. The beam rests on two rigid supports at its ends. Find the reactions of the supports.



4. A uniform ladder of mass 10 kg leans against a smooth vertical wall making an angle of 53° with it. The other end rests on a rough horizontal floor. Find the normal force and the friction force that the floor exerts on the ladder.
5. A uniform ladder of length 10.0 m and mass 16.0 kg is resting against a vertical wall making an angle of 37° with it. The vertical wall is frictionless but the ground is rough. An electrician weighing 60.0 kg climbs up the ladder. If he stays on the ladder at a point 8.00 m from the lower end, what will be the normal force and the force of friction on the ladder by the ground? What should be the minimum coefficient of friction for the electrician to work safely?
6. A uniform rod of length L rests against a smooth roller as shown in the figure. Find the friction coefficient between the ground and the lower end if the minimum angle that the rod can make with the horizontal is θ.



ANALOGUE OF NEWTON'S SECOND LAW OF MOTION FOR PURE ROTATION

Equation analogous to force equation $\Sigma \vec{F} = M\vec{a}_{CM}$ for pure rotation is $\tau = I\alpha$, where τ is the torque acting on the body and α is its angular acceleration about the axis of rotation provided moment of inertial (I) of the rigid body about the axis of

rotation remains constant throughout the motion. Such torque is a three-dimensional vector quantity. In the problems we encounter at this level, rotation takes place in two dimensions and τ and α are taken about the axis of rotation in the third dimension (normal to the plane of rotation) and the torque equation is written as

$$\tau_{\text{about the axis of rotation}} = I_{\text{about the axis of rotation}} \times \alpha_{\text{about the axis of rotation in the sense of } \tau}$$

or $\tau = I\alpha$

The above equation is called 'torque equation'.
The torque equation is, however, more complex than the force equation. Why? It is because,

- Force is an absolute value, whereas torque depends on the point about which it is being calculated.
- Mass is an absolute value for a body, whereas MI can have infinite values for a body. MI is defined only for a particular axis of the body.

Remember the following points about the torque equation:

Torque equation can be applied only about two points in a body. These are as follows:

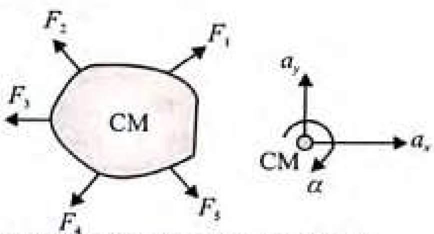
- Centre of mass.
- Point which has zero velocity/acceleration. This point is commonly referred to as the axis of rotation or instantaneous axis of rotation.

Now, remember that the axis of rotation passes through the point and is perpendicular to the surface (valid only for two dimensional bodies).

APPLICATION OF TORQUE EQUATION AND EQUATION OF MOTION COMBINED

If the vector sum of all the forces acting on a rigid body is not equal to zero, then its centre of mass has a linear acceleration. That is

If $\sum F_{\text{net}} \neq 0$, then $a_c \neq 0$



Two dimensional motion of a rigid body

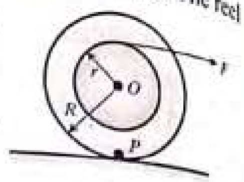
If the vector sum of all the torques about the centre of mass is not equal to zero, then the body rotate about its centre of mass. That is,

If $\sum \tau_c \neq 0$ then $\alpha \neq 0$

According to Newton's second law, for two dimensional motion of a rigid body as shown in the figure, we can state that

- Translational motion: $\sum F_x = ma_x$, $\sum F_y = ma_y$
- Rotational motion: $\sum \tau_c = I_c\alpha$

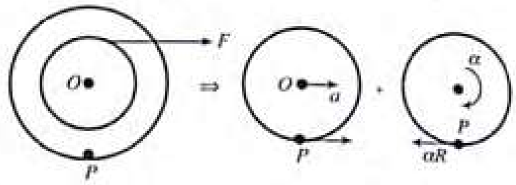
ILLUSTRATION 9.10 A cotton reel of mass m and moment of inertia I is kept at rest on a smooth horizontal surface. The reel has inner and outer radius r and R respectively. A horizontal force F starts acting as shown in figure. Find the



(a) acceleration of the centre of mass of reel.
(b) angular acceleration of the reel
(c) net acceleration of point of contact.

Solution.

(a) From force equation $F = ma \Rightarrow \quad a = \dfrac{F}{m}$ (i)



(b) We can apply torque equation about centre of mass.
From torque equation

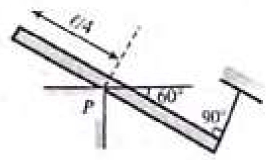
$$Fr = I\alpha \Rightarrow \alpha = \frac{Fr}{I} \quad \text{(ii)}$$

(c) acceleration of 'P' : $\vec{a}_P = \vec{a}_{P,O} + \vec{a}_O$

$$\Rightarrow \quad a_P = (-\alpha R) + a = a - \alpha R = \left(a - \frac{Fr}{I}\right)R$$

Hence acceleration of point of contact will be $a - \left(\dfrac{Fr}{I}\right)R$ in forward direction

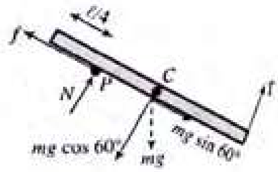
ILLUSTRATION 9.11 A uniform rod of mass m and length l is in equilibrium under the action of constraint forces, gravity and tension in the string. Find the



(a) frictional force acting on the rod.
(b) tension in the string.
(c) normal reaction on the rod.
Now, the string is cut. Find the
(d) angular acceleration of the rod just after the string is cut.
(e) normal reaction on the rod just after the string is cut.

Solution.

(a) Resolving the forces parallel and perpendicular to rod.
Along the length of rod .



$$f = mg\sin 60° = \frac{\sqrt{3}mg}{2}$$

Hence friction force action on rod at support P will be $mg/2$.

(b) for finding tension we can take torque about support P.

$$T\left(\frac{3}{4}l\right) = mg\left(\frac{l}{4}\right) \text{ or } T = \frac{mg}{3}$$

(c) Just before cutting the string the rod is at equilibrium. Considering the forces perpendicular to rod length.

$$N + T = mg\cos 60^\circ \Rightarrow N = \frac{mg}{2} - \frac{mg}{3} = \frac{mg}{6}$$

(d) Just after cutting the string the equilibrium of the rod will disturbed. The rod will have angular acceleration. As the rod does not slip point P will be at rest at the time just after cutting the string.
We can apply torque equation about P: $\tau_P = I_P\alpha$

$$(mg\cos 60^\circ).\frac{l}{4} = \left(\frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2\right)\alpha \Rightarrow \alpha = \frac{6g}{7l}$$

(e) If we apply torque equation about centre of mass

$$N \cdot \frac{l}{4} = \left(\frac{ml^2}{12}\right)\alpha$$

$$N.\frac{l}{4} = \left(\frac{ml^2}{12}\right)\left(\frac{6g}{7l}\right) \Rightarrow N = \frac{2}{7}mg$$

Illustration 9.12 A block of mass m is attached at the end of an inextensible string which is wound over a rough pulley of mass M and radius R [Fig. (a)]. Assume the string does not slide over the pulley. Find the acceleration of the block when released.

Solution. This problem is different from the problems which we have solved in laws of motion where we assumed the string is sliding over the pulley without friction. Here, the pulley is rough and string is not sliding. We will go step by step to analyse the situation.

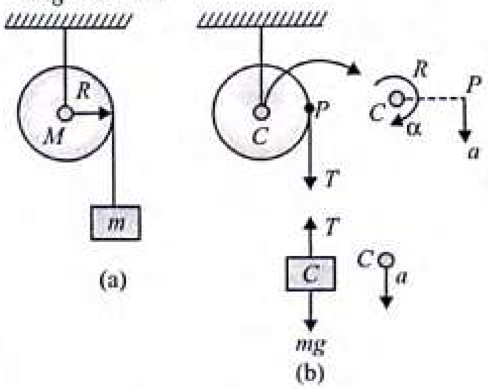
Step I

- We will draw the free-body diagrams of the given situation as in Fig. (b).

Step II

- We should apply Newton's second law for each assumed acceleration.
Equation of motion of the block:

$$mg - T = ma \qquad \text{(i)}$$



Step III

- Formulate the constraint relations, if required.
Constraint relation: $a = \alpha R$ (ii)

Step IV

- Solve the equation to get the desired unknown.
Torque equation for the pulley: $\tau_c = I_c\alpha$

$$TR = \left(\frac{MR^2}{2}\right)\alpha \Rightarrow \alpha = \frac{2T}{MR} \qquad \text{(iii)}$$

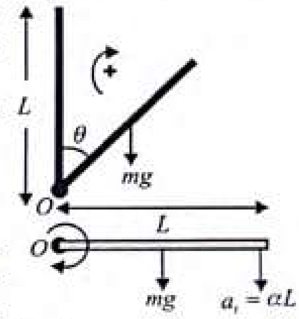
The acceleration of the block is a downwards; hence, acceleration of the point P will also be a downwards.
After solving Eqs. (i), (ii) and (iii), we get

$$a = \frac{2mg}{2m + M}$$

Illustration 9.13 A uniform rod of length L and mass M is pivoted freely at one end and placed in vertical position.
(a) What is the angular acceleration of the rod when it is at an angle θ with the vertical?
(b) What is the tangential linear acceleration of the free end when the rod is horizontal?

Solution. Figure shows the rod at an angle θ to the vertical. If we take torques about the pivot we need not be concerned with the force due to the pivot.
The torque due to the weight about O,

$$\tau_{mg} = \frac{mgL}{2}\sin\theta$$



Using torque equation about O. Here we apply torque equation about fixed axis passing through O.
We can apply torque equation about fixed axis or about centre of mass only.

(a) $\tau = I\alpha \Rightarrow \frac{mgL}{2}\sin\theta = \frac{ML^2}{3}\alpha$

Thus, $\alpha = \frac{3g\sin\theta}{2L}$

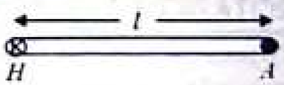
When the rod is horizontal, $\theta = \frac{\pi}{2}$ and $\alpha = \frac{3g}{2L}$

(b) The tangential linear acceleration of the free end is

$$a_t = \alpha L = \frac{3g}{2}$$

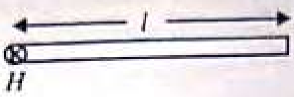
Concept Application Exercise 9.3

1. A uniform rod of mass m and length l can rotate in a vertical plane about a smooth horizontal axis hinged at point H.



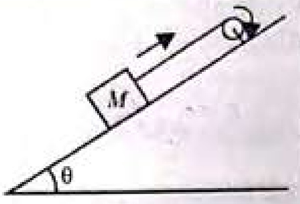
(a) Find angular acceleration α of the rod just after it is released from initial horizontal position from rest?
(b) Calculate the acceleration (tangential and radial) of point A at this moment.

2. A uniform rod of mass m and length l can rotate in a vertical plane about a smooth horizontal axis hinged at point H.

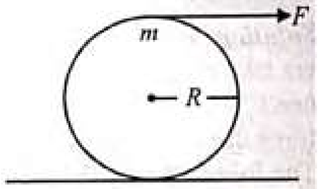


Find the force exerted by the hinge just after the rod is released from rest, from an initial horizontal position?

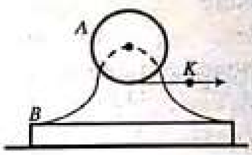
3. A wheel of radius r and moment of inertia I about its axis is fixed at the top of an inclined plane of inclination θ as shown in the figure. A string is wrapped around the wheel and its free end supports a block of mass M which can slide on the plane. Initially, the wheel is rotating at a speed ω in a direction such that the block slides up the plane. How far will the block move before stopping?



4. A cotton reel of mass m, radius R and moment of inertia I is kept on a smooth horizontal surface. If the string is pulled horizontally by a force F, find the (i) acceleration of CM, (ii) angular acceleration of the cotton reel.



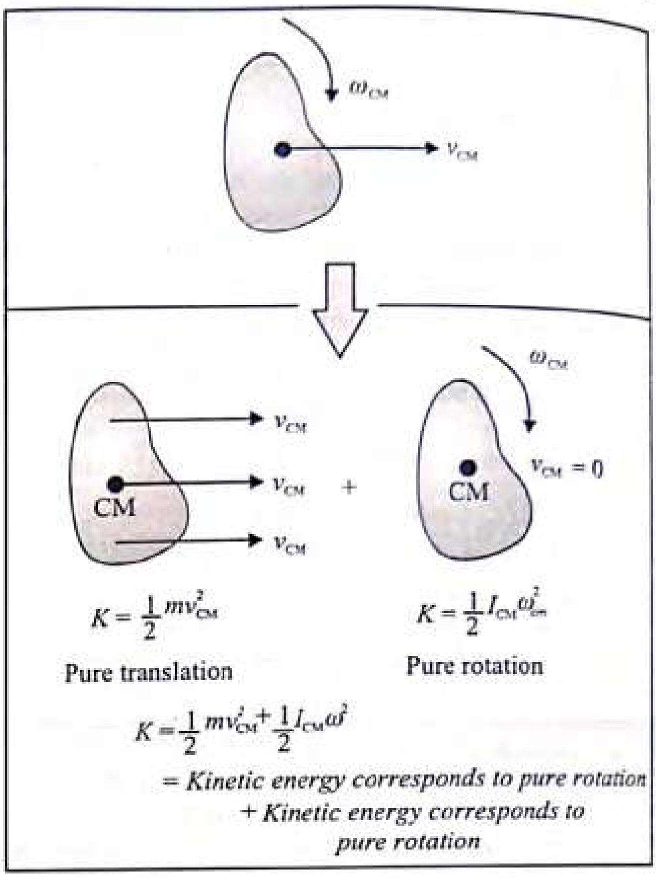
5. A uniform solid cylinder A of mass m_1 can freely rotate about a horizontal axis fixed to a mount of mass m_2. A constant horizontal force F is applied to the end K of a light thread tightly wound on the cylinder. The friction between the mount and the supporting horizontal plane is assumed to be absent. Find the acceleration of the point K.



6. For what value of x, the point P on the rod of length $l = 6$ m has zero acceleration if a force F is applied at the end of rod as shown.

KINETIC ENERGY OF A BODY IN COMBINED ROTATION AND TRANSLATION

Consider a body in combined translational and rotational motion in the lab frame. Suppose in the frame of the centre of mass, the body is making a pure rotation with an angular velocity ω. The centre of mass itself is moving in the lab frame at a velocity $\vec{v}_0$.



NOTE:

- **KE for pure translation:** $K = \dfrac{mv_C^2}{2}$

  As a perfectly translating body has KE formula identical to that of a point mass, we can say that a translating body behaves as a point mass.

- **KE for fixed axis rotation:** $K = \dfrac{1}{2}I_P\omega^2$

  I_P = moment of inertia of body about fixed axis

- **Kinetic energy for combined motion:** $K = \dfrac{m}{2}v_C^2 + \dfrac{1}{2}I_C\omega^2$

  The first term signifies the translational KE of the rigid body which can be understood as the KE of the centre of mass of the body. The second term gives the KE of rotational of the body about the centre of mass. Hence, we can call the term $\frac{1}{2}I_C\omega^2$ as rotational kinetic energy of the rigid body.

  The above formula informs us that, The KE of the centre of mass plus KE about the KE about the CM of a rigid body (or any system of group of particles) gives the total KE of the body:

  $$K_{trans} = \frac{1}{2}Mv_C^2 \text{ and } K_{rot} = \frac{1}{2}I_C\omega^2$$

- If a body is rotating about a fixed axis and spinning (or rotating) about it own axis we write the total kinetic energy of the body as

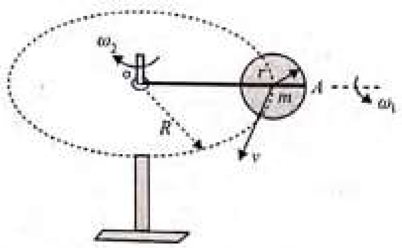
  $$K_{total} = K_{orbital} + K_{spin}$$

ILLUSTRATION 9.14 A uniform sphere of mass m and radius r rolls without sliding over a horizontal plane, rotating about a horizontal axle OA. In the process, the centre of the sphere moves with a velocity v along a circle of radius R. Find the kinetic energy of the sphere.

Solution. Here the motion of the body is orbital and spin combined. The sphere is spining about a horizontal axis and orbiting about a vertical axis.

We write total kinetic energy of the sphere, i.e.,

$$\text{KE} = K_{\text{orbital}} + K_{\text{spin}}$$



The kinetic energy of the sphere due to its rotation about its own axis and its motion along circular path with velocity v.

$$\text{KE} = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

where $I_1 = \frac{2}{5}mr^2$ and $I_2 = \left(\frac{2}{5}mr^2 + mR^2\right)$

$$\omega_1 = \frac{v}{r} \text{ and } \omega_2 = \frac{v}{R}$$

Substituting these values in the above equation, we get

$$\text{KE} = \frac{7mv^2}{10}\left[1 + \frac{2r^2}{7R^2}\right]$$

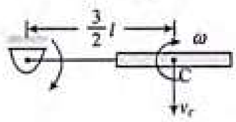
ILLUSTRATION 9.15 A rod of mass m and length l is connected with a light rod of length l. The composite rod is made to rotate with angular velocity ω as shown in the figure. Find the

(a) translational kinetic energy.

(b) rotational kinetic energy.

(c) total kinetic energy of rod.

Solution.



(a) Translational kinetic energy of rod,

$$K_{\text{translational}} = \frac{1}{2}mv_c^2$$

Velocity of centre of mass $v_c = \omega\left(\frac{3}{2}l\right)$

$$\text{Hence } K_{\text{translational}} = \frac{1}{2}m\left[\frac{3\omega l}{2}\right]^2 = \frac{9}{8}m\omega^2 l^2$$

(b) Rotational kinetic energy of rod,

$$K_{\text{rotational}} = \frac{1}{2}I_c\omega^2 = \frac{1}{2}\left[\frac{ml^2}{12}\right]\omega^2 = \frac{1}{24}m\omega^2 l^2$$

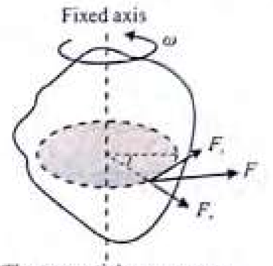
(c) Total kinetic energy

$$K_{\text{total}} = K_{\text{translational}} + K_{\text{rotational}}$$

$$= \frac{9}{8}m\omega^2 l^2 + \frac{1}{24}m\omega^2 l^2 = \frac{7}{6}m\omega^2 l^2$$

ROTATIONAL WORK AND POWER

Figure shows a rigid body rotating about a fixed axis. It is acted on by an external force F that has a radial component F_r and a tangential component F_t. In an infinitesimal time interval dt, the point of application of force moves in a circular arc of length $r = d\theta$. Therefore, the work done is



The tangential component of the force does rotational work

$$dW_{\text{rot}} = F_t\,(rd\theta) = (F_t r)d\theta = \tau d\theta$$

For a finite angular displacement, $W_{\text{rot}} = \int_{\theta_i}^{\theta_f} \tau d\theta$

By definition, the instantaneous power is defined as

$$P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} \Rightarrow P = \tau\omega$$

The rotational analogue of work–kinetic energy theorem may be stated as

$$W_{\text{rot}} = \Delta K_{\text{rot}} = K_f - K_i \quad \text{or} \quad W_{\text{rot}} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

NOTE: The work done by a torque on a rigid body rotating about a fixed axis produces a change in its rotational kinetic energy.

$$W_{\text{rot}} = \int \tau\, d\theta;\ W_{\text{trans}} = \int F dx$$

Power imparted by torque in rotation $P_{\text{rot}} = \tau\omega$; which is equivalent to expression for power in translation motion $P_{\text{trans}} = Fv$.

ILLUSTRATION 9.16 Calculate the torque developed by an airplane engine whose output is 2000 HP at an angular velocity of 2400 rev/min.

Solution. $\omega = 2\pi\,(2400/60) = 80\pi$ rad/s

Work done by torque = (Torque) × (Angular displacement)

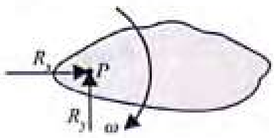
Power = Work done per sec = $\tau\dfrac{\Delta\theta}{\Delta t}$

Power = $\tau\omega \Rightarrow \tau = \dfrac{2000 \times 746}{80\pi} = 5937$ Nm

CONSERVATION OF MECHANICAL ENERGY

When a rigid body moves, the constraint forces are reactions at the pivot. The normal reaction sometimes does not perform work if the point of application of the forces does not move.

For instance, when a rigid body rotates about a fixed axis, the reaction forces R_x and R_y do not perform work.



If a rigid body does not experience any non-conservative force or even though a non-conservative force like force of static friction acts, if it does not perform any work (as described in pure rolling etc), the work–energy equation assumes the form $W_{cons} = \Delta K$.

Since conservative forces perform work at the expense of their potential enegy, we can write $W_{cons} = \Delta U$.

Then, we have $-\Delta U = \Delta K$. This gives $\Delta U + \Delta K = 0$.

WORK–ENERGY THEOREM

In an inertial frame, the work–energy theorem may be stated as

$$W_C + W_{NC} + W_{oth} = \Delta K$$

$$W_{NC} + W_{oth} = \Delta K + \Delta U = \Delta E$$

where W_C is the work done by conservative forces, W_{NC} is the work done by non-conservative forces, W_{oth} is the work done by all other forces which are not included in the above two categories.

NOTE:

- Each term of work includes both the translational and rotational work.
- The kinetic energy also includes both the translational and rotational kinetic energies.

ILLUSTRATION 9.17 The]top in the figure has a moment of inertia equal to 4.00 × 10^{-4} kgm^2 and is initially at rest. It is free to rotate about the stationary axis AA'. A string wrapped around a peg along the axis of the top is pulled in such a manner as to maintain a constant tension of 5.57 N. If the string does not slip while it is unwound from the peg, what is the angular speed of the top after 80.0 cm of string has been pulled off the peg?

Solution. Work done = $F\Delta r$ = (5.57 N) (0.800 m) = 4.46 J

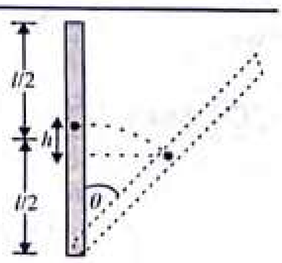
and work = $\Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

(The last term is zero because the top starts from rest.)

Thus, $4.46\text{ J} = \frac{1}{2}(4.00\times10^{-4}\,kg\times m^2)\omega_f^2$ and from this,

ω_f = 149 rad/s.

ILLUSTRATION 9.18 A rod of length l is pivoted about a horizontal, frictionless pin through one end. The rod is released from rest in a vertical position. Find the velocity of the CM of the rod when the rod is inclined at an angle θ from the vertical.



Solution. The fall in position of the CM of the rod, $h = \frac{l}{2}(1 - \cos\theta)$.

In the process, decrease in PE is equal to increase in rotational KE of the rod, so $mgh = \frac{1}{2}I\omega^2$

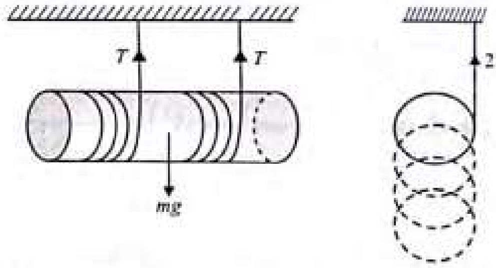
or $\quad mg\frac{l}{2}(1 - \cos\theta) = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$

$\therefore \quad \omega = \sqrt{\frac{3g}{l}(1 - \cos\theta)}$

The velocity of the CM of the rod, $V_{CM} = \omega r$ is

$$\sqrt{\frac{3g}{l}(1 - \cos\theta)} \times \frac{l}{2} = \sqrt{\frac{3gl(1 - \cos\theta)}{4}}$$

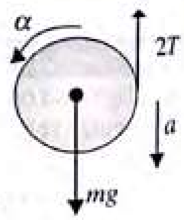
ILLUSTRATION 9.19 A solid cylinder of mass m = 4 kg and radius R = 10 cm has two ropes wrapped around it, one near each end. The cylinder is held horizontally by fixing the two free ends of the cords to the hooks on the ceiling such that both the cords are exactly vertical. The cylinder is released to fall under gravity. Find the linear acceleration of the cylinder.



Solution. **Method 1: Force/Torque Method:**

Let a = linear acceleration and α = angular acceleration of the cylinder.

For the linear motion of the cylinder: $mg - 2T = ma$



For the rotational motion: Net torque = $I\alpha$

$$2TR = (1/2\, mR^2)\alpha$$

Also, the linear acceleration of the cylinder is the same as the tangential acceleration of any point on its surface,

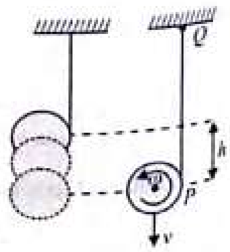
$$a = R\alpha$$

Combining the three equations, we get

$$mg = ma + \frac{m}{2}a \Rightarrow a = \frac{2}{3}g$$

Method 2: Energy Method

The motion of the cylinder is rotational and translational combined. Using conservation of mechanical energy.



$\Delta K + \Delta U = 0$

orLoss in PE = gain in KE

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad \text{(i)}$$

Constraint relation:

Velocity of point P, $v_P = 0 = v - \omega R$

$$\omega = \frac{v}{R} \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR}{2}\right)\left(\frac{v}{R}\right)^2 = \frac{m}{2}\left(v^2 + \frac{v^2}{2}\right)$$

$$2gh = \frac{3}{2}v^2 \Rightarrow v^2 = \frac{4}{3}gh \quad \text{(iii)}$$

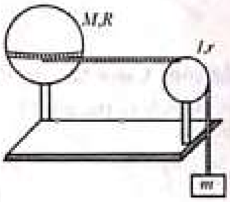
Differentiating Eq. (iii) with respect to time, we get

$$2v\frac{dv}{dt} = \frac{4}{3}g\frac{dh}{dt}$$

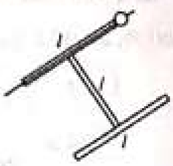
$$2va = \frac{4}{3}gv \Rightarrow a = \frac{2}{3}g$$

CONCEPT APPLICATION EXERCISE 9.4

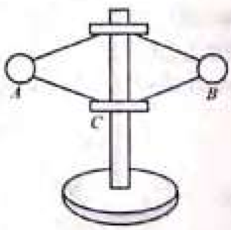
1. A uniform spherical shell of mass M and radius R rotates about a vertical axis on frictionless bearing. A massless cord passes around the equator of the shell, over a pulley of rotational inertia I and radius r and is attached to a small object of mass m that is otherwise free to fall under the influence of gravity. There is no friction of pulley's axle; the cord does not slip on the pulley. What is the speed of the object after it has fallen a distance h from rest? Use work–energy considerations.



2. A uniform rod of mass m and length l is kept vertical with the lower end clamped. It is slightly pushed to let it fall down under gravity. Find its angular speed when the rod is passing through its lowest position. Neglect any friction at the clamp. What will be the linear speed of the free end at this instant?
3. A rigid body is made of three identical thin rods, each with length l, fastened together in the form of letter H. The body is free to rotate about a horizontal axis that runs along the length of one of the arms of H. The body is allowed to fall from rest from a position in which the plane of the H is horizontal. What is the angular speed of the body when the plane of H is vertical?



4. The steel balls A and B have a mass of 500 g each and are rotating about the vertical axis with an angular velocity of 4 rad/s at a distance of 15 CM from the axis. Collar C is now forced down until the balls are at a distance of 5 CM from the axis. How much work must be done to move the collar down?



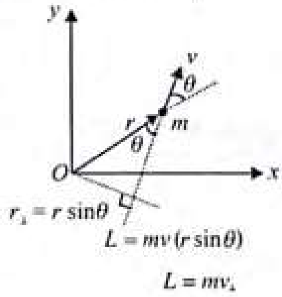
5. A uniform rod smoothly pivoted at one of its ends is released from rest. If it swings in vertical plane, find the maximum speed of the end P of the rod.

ANGULAR MOMENTUM: ROTATIONAL ANALOGUE OF MOMENTUM

The angular momentum of a moving particle about a point is defined as

$$\vec{L} = \vec{r} \times \vec{p} \quad \text{(i)}$$

where $\vec{p}$ is the linear momentum of particle and $\vec{r}$ is its position vector from the point. Regarding angular momentum, it is worth nothing that it is an axial vector (i.e., always perpendicular to the plane of motion) having the dimensions $[ML^2T^{-1}]$.



As torque $(= \vec{r} \times \vec{F})$ is defined as the 'moment of force', angular momentum is also referred sometimes as 'moment of linear momentum'.

For a particle of mass m moving with a velocity $\vec{v}$, the angular momentum $\vec{L}$ about the origin O is defined as

$$\vec{L} = \vec{r} \times m\vec{v}$$

The direction of L vector is given by the right hand rule, and the magnitude is given by

$$L = (mvr\sin\theta) = m(v\sin\theta)r$$
$$= (mv_\perp).\ r \text{ where } v_\perp = v\sin\theta$$

Alternatively, the above expression may be written as

$$L = mv\,(r\sin\theta) = mvr_\perp, \text{ where } r_\perp = r\sin\theta$$

NOTE: If a body is rotating about a fixed axis, we can write angular momentum of the body $L = I\omega$

I = Moment of inertia, about fixed axis

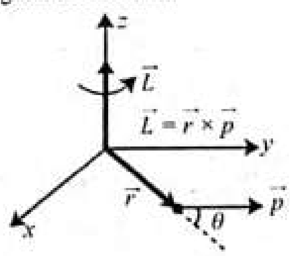
ω = angular velocity of rotation

NOTE: That the above expression is analogous to $p = mv$

$I \Rightarrow m,\ \omega \Rightarrow v,\ L \Rightarrow p$

NOTE:

- Direction of angular momentum.



- We define angular momentum even if the particle is not moving in a circular path. Even a particle moving in a straight line has angular momentum about any axis displaced from the path of the particle.

For rotation about an axis of symmetry, $\vec{\omega}$ and $\vec{L}$ are parallel and along the axis, respectively. The directions of both vectors are given by the right hand rule.

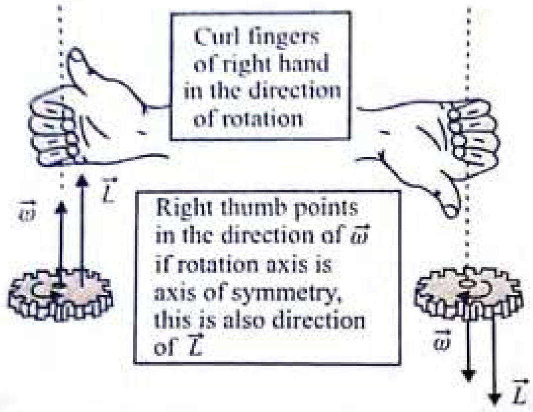
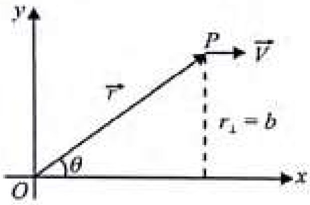


ILLUSTRATION 9.20 A particle of mass m is moving along the line $y = b$, $z = 0$ with constant speed v. Find the angular momentum of the particle about the origin.

Solution. $|\vec{L}| = mvr \sin\theta = mvr_{\perp}\ mvb$

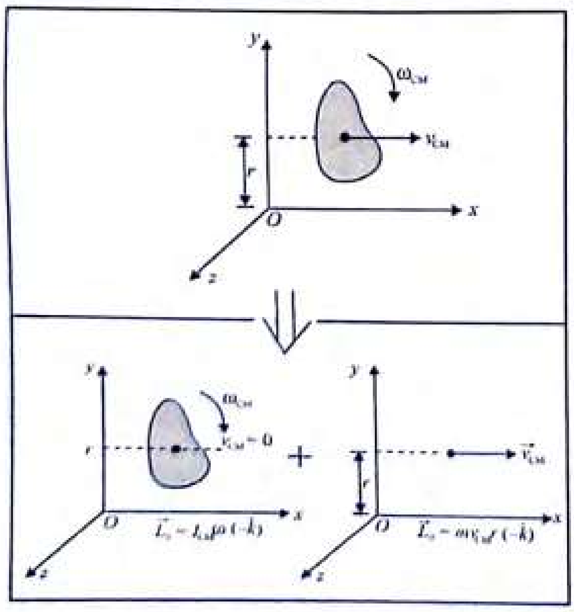
$\therefore \quad |\vec{L}| =$ constant as m, v and b all are constants.

Direction of $\vec{r} \times \vec{v}$ also remains the same. Therefore, angular momentum of the particle about the origin remains constant with due course of time.



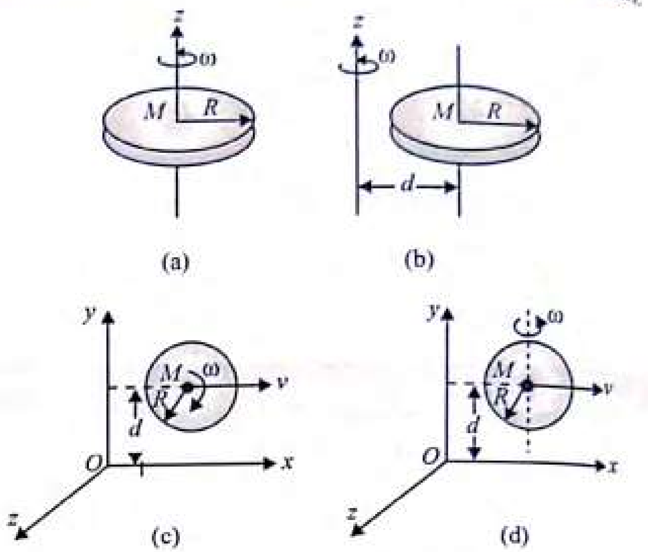
NOTE: In this problem, $|\vec{r}|$ is increasing, θ is decreasing but $r \sin\theta$, i.e., b remains constant. Hence, the angular momentum remains constant.

ANGULAR MOMENTUM OF A RIGID BODY DUE TO TRANSLATION AND ROTATION BOTH



$\vec{L} = I_{CM}\omega(-k) + mv_{CM}r(-k)$

ILLUSTRATION 9.21 Find the angular momentum of a disc about the axis shown in the figure in the following situations.



Solution. **Case (a):** In this case the disc is rotating about fixed axis which is parallel to the z-axis.

Hence, $\vec{L} = I_z(\hat{k}) = \left(\dfrac{MR^2}{2}\right)\omega(\hat{k})$

Direction can be determined using right hand rule.

Case (b): In this case also the disc rotates about fixed axis. We can write angular momentum for this situation.

$$\vec{L} = I_z\omega(\hat{k})$$

Using parallel axis we can find the value of $I_z = \dfrac{MR^2}{2} + Md^2$

Hence, $\vec{L} = \left(\dfrac{MR^2}{2} + Md^2\right)\omega(\hat{k})$

Case (c): In this case the disc is rotating about an axis parallel to the z-axis and centre of mass of disc is also translating.

Hence, angular momentum.

$$\vec{L} = I_{cm}\vec{\omega} + \vec{r} \times m\vec{v}$$

$$= \frac{MR^2}{2}\omega(-\hat{k}) + Mvd(-\hat{k})$$

$$= \left(\frac{MR^2}{2}\omega + Mvd\right)(-\hat{k})$$

Case (d): In this case also the disc is rotating about an axis passing through it centre and parallel to y-axis and centre of mass of disc is translating with velocity v parallel to x-axis.

$$\vec{L} = I_{CM}\omega(\hat{j}) + Mvd(\hat{k})$$

Here $\quad I_{CM} = \dfrac{MR^2}{4}$

Hence, $\quad \vec{L} = \dfrac{MR^2}{4}\omega(\hat{j}) + Mvd(\hat{k})$

LAW OF CONSERVATION OF ANGULAR MOMENTUM

In case of translation motion, we define force as $\vec{F} = \frac{d\vec{p}}{dt}$.

For rotational motion, we define torque.

$$\vec{\tau} = \frac{d\vec{L}}{dt}\left[\equiv \frac{d\vec{p}}{dt} = \vec{F}\right]$$

So if the net torque on a particle (or system) is zero; $\frac{d\vec{L}}{dt} = 0$,

i.e., $\vec{L}$ = constant.

Therefore, angular momentum of a system (may be particle or body) remains constant if the resultant torque acting on it is zero. This principle is called 'law of conservation of angular momentum'.

For a system of particle angular momentum,

$$\vec{L} \Rightarrow \vec{L}_1 + \vec{L}_2 + \cdots$$

Hence, when the total angular momentum is conserved,

$$\vec{L} \Rightarrow \vec{L}_1 + \vec{L}_2 + \cdots = \text{constant}$$

This shows that the angular momentum of an individual particle may change, but their sum remains constant in the absence of an external torque.

ANGULAR IMPULSE

The sum of torques of all external forces acting on a system of particles (or rigid body) about any fixed point is equal to the rate of change in angular momentum about the point.

Since $\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow d\vec{L} = \vec{\tau}\, dt$

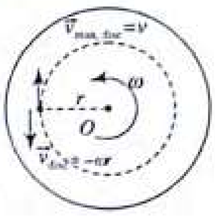
The change in angular momentum of a rigid body during a time interval Δt is given by

$$\int d\vec{L} = \int \vec{\tau}\, dt \Rightarrow \Delta\vec{L} = \int_{t_1}^{t_2} \vec{\tau}\, dt$$

Hence, the angular impulse $\Delta\vec{L} = \int_{t_1}^{t_2} \vec{\tau}\, dt$.

ILLUSTRATION 9.22 A man of mass m stands on a horizontal platform in the shape of a disc of mass M and radius R, pivoted on a vertical axis through its centre about which it can freely rotate. The man starts to move around the centre of the disc in a circle of radius r with a velocity v relative to the disc. Calculate the angular velocity of the disc.

Solution. Since there is no torque acting about the axis of rotation of the disc, so the angular momentum of the system (disc + man) remains constant. Initially it is zero. Suppose ω is the angular velocity of the disc (take positive in the sense of motion of the man). The velocity of the man with respect to the ground observer will be:



$$\vec{v}_{\text{man, disc}} = \vec{v}_{\text{man}} - \vec{v}_{\text{disc}}$$

$$\Rightarrow \quad \vec{v}_{\text{man}} = \vec{v}_{\text{man, disc}} - \vec{v}_{\text{disc}}$$

or $\quad v_{\text{man}} = v - \omega r$

Thus, angular momentum of the man $= m(v - \omega r)r$

And angular momentum of the disc is

$$I_{\text{disc}}\,\omega = \frac{MR^2}{2}\,\omega$$

Initial angular momentum of the system

$$\vec{L}_i = 0$$

Final angular momentum

$$\vec{L}_f = m(v - \omega r) - \frac{MR^2}{2}\omega$$

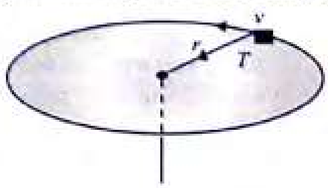
By conservation of angular momentum, we have

$$m(v - \omega r)r = \frac{MR^2}{2}\,\omega$$

After solving, we get $\omega = \dfrac{mvr}{\left(mr^2 + \dfrac{MR^2}{2}\right)}$

ILLUSTRATION 9.23 A small block of mass 4 kg is attached to a cord passing through a hole in a horizontal frictionless surface. The block is originally revolving in a circle of radius 0.5 m about the hole, with a tangential velocity of 4 m/s. The cord is then pulled slowly from below, shortening the radius of the circle in which the block revolves. The breaking strength of the cord is 600 N. What will be the radius of the circle when the cord breaks?

Solution. The tension of the rope is the only net force on the block and it does not exert any torque about the axis of rotation. Hence, the angular momentum of the block about the axis should remain conserved.



$\Rightarrow \quad mvr$ = constant

Let $\quad r_1 = 0.5$ m and $v_1 = 4$ m/s

Let r_2, v_2, T_2 be the radius, velocity and tension, respectively, when the string breaks.

$\Rightarrow \quad T_2 = 600$ N

$mv_1r_1 = mv_2r_2$ and $T_2 = mv_2^2/r_2$

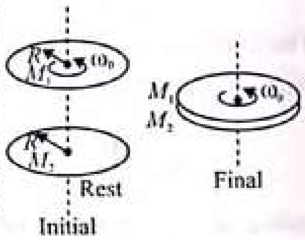
$$\Rightarrow \quad mv_1r_1 = m\sqrt{\frac{r_2T_2}{m}}\, r_2$$

$$r_2 = \left(\frac{mv_1^2 r_1^2}{T_2}\right)^{\frac{1}{3}} = \left(\frac{4\times 16\times 0.25}{600}\right)^{\frac{1}{3}} = \left(\frac{16}{600}\right)^{\frac{1}{3}} = 0.3 \text{ m}$$

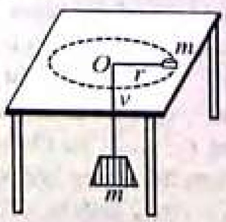
NOTE: The tension in the string is inversely proportional to the cube of the radius.

Concept Application Exercise 9.5

1. A circular disc of mass M_1 and radius R, initially moving with a constant angular speed ω_0 is gently placed coaxially on a stationary circular disc of mass M_2 and radius R, as shown in the figure. There is a frictional force between the two discs.



   (a) If disc M_2 is placed on a smooth surface, then determine the final angular speed of each disc.
   (b) Determine the work done by friction.
   (c) Determine the frictional loss in kinetic energy, i.e., $\Delta K/K_i$.
2. A child of mass m is standing on the periphery of a circular platform of radius R, which can rotate about its central axis. The moment of inertia of the platform is I. Child jumps off from the platform with a velocity u tangentially relative to the platform. Find the angular speed of the platform after the child jumps off.
3. Suppose in the previous problem, the child stays at rest on the platform and one of his friend throws a ball of mass m_1 towards him with a velocity of v from a direction tangential to the platform and the child on the platform catches the ball. Find the angular velocity of the platform after he catches the ball.
4. In the previous problem, initially the platform with the child is rotating with an angular speed ω_1. If the child starts walking along its periphery in opposite direction with speed u relative to the platform, what will be the new angular speed of the platform.
5. A thread is passing through a hole at the centre of a frictionless table. At the upper end a block of mass 0.5 kg is tied and a block of mass 8.0 kg is tied at the lower end which is freely hanging. The smaller mass is rotated on the table with a constant angular velocity about the axis passing through the hole so as to balance the heavier mass. If the mass of the hanging block is changed from 8.0 kg to 1.0 kg, what is the fractional change in the radius and the angular velocity of the smaller mass so that it balances the hanging mass again?



6. A girl jumps from a height h on the end of a see-saw. The see-saw consists of a uniform plank of length l pivoted at its centre. The plank is horizontal before the girl jumps. The mass of the see-saw is twice the mass of the girl. Find the angular velocity of the plank just after the girl jumps on the plank.

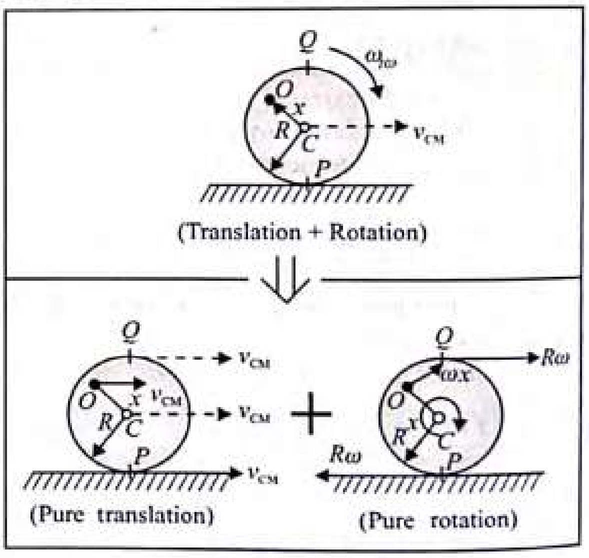
COMBINATION OF TRANSLATION AND ROTATION

When the axis of rotation is not fixed (stationary in space), the motion of a rigid body is considered as combination of motion of centre of mass plus a rotation about centre of mass. The two components of motion are described by

$$\Sigma\vec{F}_{ext} = m\vec{a}_{CM} \text{ and } \Sigma\vec{\tau}_{CM} = I_{CM}\vec{\alpha}$$

Superposition of Translation and Rotation

How the translational motion and rotational motion about the centre of mass are superimposed to get the motion of a rigid body (say a disc of radius R) is illustrated in the following figures.



To get the instantaneous velocity of any point on the rigid body we calculate the instantaneous velocity of that point in pure translation and in pure rotation and add them vectorically.

$$\vec{v}_{net} = \vec{v}_{translation} + \vec{v}_{rotation}$$

Velocity of the Instantaneous Point of Contact P

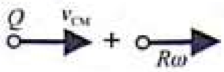
v_P = Velocity of point of contact in pure translation + Velocity of instantaneous point of contact in pure rotation

$v_P = v_{CM}$ (in the forward direction) + $R\omega$ (in the backward direction)

$v_P = v_{CM} - R\omega$, in the forward direction

Velocity of the Instantaneous Topmost Point Q

v_Q = Velocity of the topmost point in pure translation + velocity of point Q in pure rotation



$v_Q = v_{CM}$, in the forward direction + $R\omega$, in the forward direction.

$v_Q = v_{CM} + R\omega$, in the forward direction

Velocity of Point O

v_0 = Velocity of point O in pure translation + instantaneous velocity of point O in pure rotation

ACCELERATION OF DIFFERENT POINTS

Acceleration of Instantaneous Point of Contact P

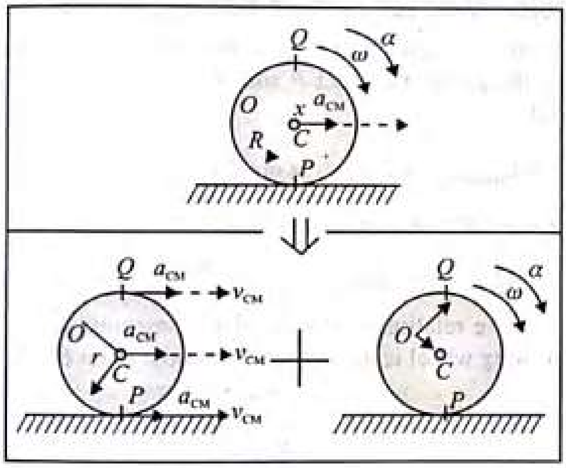
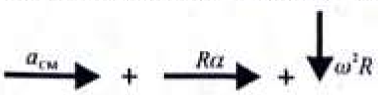


Figure illustrates how the instantaneous accelerations are superimposed. Acceleration of the instantaneous point of contact w.r.t. supporting surface, a_P = acceleration of point of contact in pure translation + acceleration of instantaneous point of contact in pure rotation

= (a_{CM}, in the forward direction) + ($R\alpha$, in the backward direction + $\omega^2 R$, vertically upward towards the centre of mass)

Acceleration of the Instantaneous Topmost Point

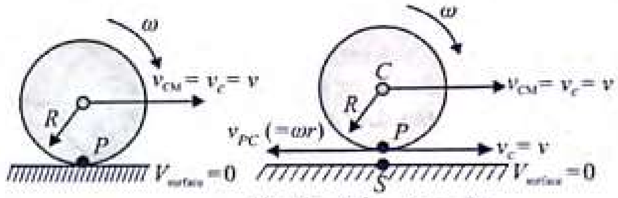
a_Q = Acceleration of the topmost point in pure translation + Acceleration of the instantaneous topmost point in pure rotation



(a_{CM} in the forward direction) + ($R\alpha$, in the forward direction + $\omega^2 R$, vertically downwards towards the centre of mass)

ROLLING

Take a uniform disc of radius R (say), place it vertically on a smooth horizontal surface. Spin the disc and simultaneously push it. In consequence, the disc will rotate with an angular velocity ω and its centre of mass C moves with a velocity v. The velocity of the lowest point P of the disc can be given as



$V_P = V_S = 0$, i.e., $V_{CM} = R\omega$

$$\vec{v}_P = \vec{v}_{PC} + \vec{v}_C$$

where $\quad \vec{v}_{PC} = -R\omega\hat{i}$ and $\vec{v}_C = v\hat{i}$

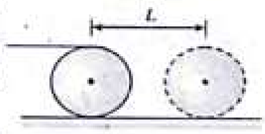
Then, $\quad \vec{v}_P = (v - R\omega)\hat{i}$

The above expression tells us that for different v and ω, the point P will move with different speeds. For suitable velocities v and ω, let us assume that P moves with a velocity which is equal to the velocity of the horizontal surface S, then

$$\vec{v}_P = \vec{v}_S$$

A body in combined motion is said to be rolling over a surface if the surfaces in contact do not slide relative to each other. It means that the relative velocity between the points of contact is zero.

ILLUSTRATION 9.24 A cylindrical drum, pushed along by a board rolls forward on the ground. There is no slipping at any contact. Find the distance moved by the man who is pushing the board, when axis of the cylinder covers a distance L.



Solution. Let v_0 be the linear speed of the axis of the cylinder and ω be its angular speed about the axis. As it does not slip on the ground hence $\omega = v_0/R$. Where R is the radius of the cylinder.

Speed of the topmost point is $v = v_0 + \omega R = 2v_0$

Since time taken by the axis to move a distance L is equal to $t = L/v_0$.

In the same interval of time distance moved by the topmost point is

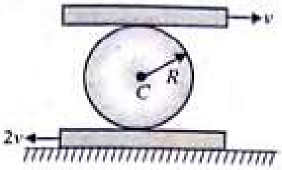
$$S = 2v_0 \times \frac{L}{v_0} = 2L$$

As there is no slipping between any point of contact hence distance moved by the man is $2L$.

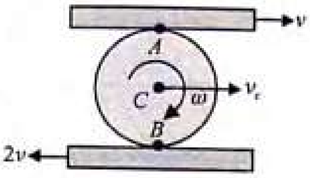
ILLUSTRATION 9.25 A uniform disc of radius R rolls perfectly over two horizontal plank A and B moving with velocities v and $2v$, respectively. Find the

(a) velocity of CM of the disc.

(b) angular velocity of the disc.



Solution. As the disc is rolling points A and B which are on the disc and in contact with the planks will not slide. The velocities of points A and B should be the same as the planks. Let the disc rotate with an angular velocity ω and let the velocity of centre of the mass of the disc be v_c.



Velocity of point A, v_A, $v_c + \omega R = v$ (i)

Velocity of point B, v_B, $\omega R - v_c = 2v$ (ii)

From Eqs. (i) and (ii), $\omega = 3v/R$ and $v_c = -2v$

Hence, velocity of centre of disc will be $2v$ towards the left and the angular velocity will be $3v/R$ the in the clockwise sense.

Solution. Velocity of any point on the circumference of a rolling body,

$$v_p = 2v_{CM}\cos\frac{\theta}{2}$$

For $v_p = v_{CM}$, we have

$$v_{CM} = 2v_{CM}\cos\frac{\theta}{2} \text{ or } \cos\frac{\theta}{2} = \frac{1}{2} \text{ or } \theta = 120°$$

Kinetic Energy of Rolling Wheel

Energy of a Rolling Body

The translational KE of a rolling body is

$$KE_t = \frac{1}{2}mv^2$$

The rotational KE of a rolling body is

$$KE_r = \frac{1}{2}I_0\,\omega^2$$

Since we know that $I_0 = mk^2$ and for rolling $\omega = \dfrac{v}{r}$

$$\Rightarrow \quad KE_r = \frac{1}{2}mk^2\left(\frac{v^2}{r^2}\right) = \frac{1}{2}\frac{k^2}{r^2}mv^2$$

Hence the total KE of the rolling body $KE = KE_t + KE_r$.

Putting $v = r\omega$ for rolling, we obtain the total KE in terms of ω as

$$KE = \frac{1}{2}m(r\omega)^2\left(1+\frac{k^2}{r^2}\right)$$

$$\Rightarrow \quad KE = \frac{1}{2}(mr^2 + mk^2)\omega^2$$

Since mk^2 = MI of a body about its centre of mass $O = I_0$

Hence, $mk^2 + mr^2 = I_0 + mr^2 = I_P$

where I_P = MI of the body about P.

Therefore, we conclude that the combined effects of translation of centre of mass of a body and its rotation about an axis passing through the CM are equivalent to its pure rotation about an axis passing through the point of contact P, of the rolling body.

$$\%\text{ of energy of translation} = \frac{KE_t}{KE}\times 100$$

$$= \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2\left(1+\frac{k^2}{r^2}\right)}\times 100 = \frac{r^2}{k^2+r^2}\times 100$$

$$\text{Similarly, }\%\text{ of energy of rotation} = \frac{k^2}{k^2+r^2}\times 100$$

Angular Momentum of Rolling Wheel

Angular momentum of a rolling wheel about an axis passing through the point of contact P and perpendicular to the plane of wheel:

$$\vec{L} = \vec{L}_{\text{translation}} + \vec{L}_{\text{rotation}} = m(\vec{R}\times\vec{v}_{CM}) + I_{CM}\vec{\omega}$$

or $\quad L = m\omega R^2 + I_{CM}\omega$

or $\quad L = (I_{CM} + mR^2)\omega = I_p\omega$

The above relation of the angular momentum is the same as the rolling wheel is having rotation about point of contact.

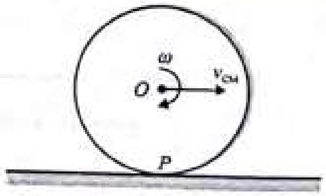
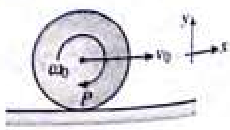


ILLUSTRATION 9.26 A sphere of mass M rolls without slipping on a rough surface with centre of mass has constant speed v_0. If mass of the sphere is m and its radius be R, then find the angular momentum of the sphere about the point of contact.

Solution. Since $\vec{L}_P = \vec{L}_{cm} + \vec{r}\times\vec{p}_{cm}$



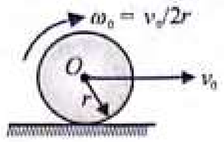
$$= I_{cm}\omega_0(-\hat{k}) + Mv_0R(-\hat{k})$$

Since sphere is in pure rolling motion hence

$$\omega = v_0/R$$

$$\Rightarrow \quad \vec{L}_P = \left[\frac{2}{5}MR^2\left(\frac{v_0}{R}\right) + Mv_0R\right](-\hat{k}) = \frac{7}{5}Mv_0R(-\hat{k})$$

ILLUSTRATION 9.27 A sphere of mass M and radius r shown in the figure slips on a rough horizontal plane. At some instant it has translational velocity v_0 and rotational velocity about the centre $v_0/2r$. Find the translational velocity after the sphere starts pure rolling.



Solution. Velocity of the centre = v_0 and the angular velocity about the centre = $v_0/2r$. Thus, $v_0 > \omega_0 r$. The sphere slips forward and thus the friction by the plane on the sphere will act backward. As the friction is kinetic, its value is $\mu N = \mu Mg$ and the sphere will be decelerated by $a_{CM} = f/M$.

Hence,

$$v(t) = v_0 - \frac{f}{M}t \qquad \text{(i)}$$

This friction will also have a torque $= fr$ about the centre. This torque is clockwise and in the direction of ω_0. Hence, the angular acceleration about the centre will be

$$\alpha = f\frac{r}{(2/5)\,Mr^2} = \frac{5f}{2Mr}$$

and the clockwise angular velocity at time t will be

$$\omega(t) = \omega_0 + \frac{5f}{2Mr}$$

$$= \frac{v_0}{2r} + \frac{5f}{2Mr}t$$

Pure rolling starts when $v(t) = r\omega(t)$,

i.e., $$v(t) = \frac{v_0}{2} + \frac{5f}{2M}t \qquad \text{(ii)}$$

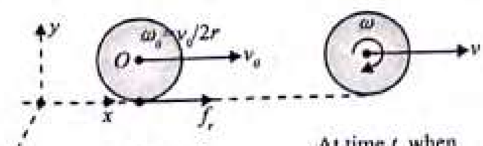
Eliminating t from Eqs. (i) and (ii), we get

$$\frac{5}{2}v(t) + v(t) = \frac{5}{2}v_0 + \frac{v_0}{2}$$

or $$v(t) = \frac{2}{7}\times 3v_0 = \frac{6}{7}v_0$$

Thus, the sphere rolls with translational velocity $6v_0/7$ in the forward direction.

Alternative method: We can solve this problem by using conservation of angular momentum. We can conserve the angular momentum about any point situated on the ground. (As torque of friction force about any point on the ground is zero.)



At $t = 0$ At time t, when rolling starts

Angular momentum about any point O situated on ground at $t = 0$

$$\vec{L}_{\text{initial}} = \vec{L}_{CM} + \vec{r}\times m\vec{v}$$

$$= I_{CM}\omega_0(-\hat{k}) + Mv_0 r(-\hat{k})$$

$$= \left[\frac{2}{5}Mr^2\left(\frac{v_0}{2r}\right) + Mv_0 r\right](-\hat{k})$$

$$\vec{L}_{\text{initial}} = \frac{6}{5}Mv_0 r(-\hat{k}) \qquad \text{(i)}$$

Angular momentum at time t when rolling starts

$$\vec{L}_{\text{final}} = \vec{L}_{\text{cm}} + \vec{r}\times m\vec{v} = \frac{2}{5}mr^2\omega(-\hat{k}) + mvr(-\hat{k})$$

When rolling starts $v = \omega R$ or $\omega = v/r$

Hence, $$\vec{L}_{\text{final}} = \left[\frac{2}{5}mr^2\left(\frac{v}{r}\right) + mvr\right](-\hat{k})$$

$$= \frac{7}{5}mvr(-\hat{k}) \qquad \text{(ii)}$$

Conserving angular momentum, we get

$$\vec{L}_{\text{initial}} = \vec{L}_{\text{final}}$$

$$\frac{6}{5}mv_0 r(-\hat{k}) = \frac{7}{5}mvr(-\hat{k})$$

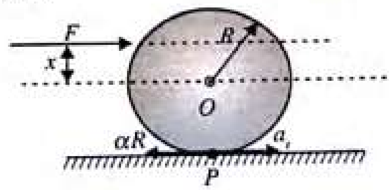
which gives $v = \frac{6}{7}v_0$

Direction of Friction in Case of Translation and Rotation Combined

In case of translation motion we can decide the direction of friction force by direct observation. But the direction of friction cannot be found by direct observation in case of rotational motion, as the body is translating as well as rotating. The direction of friction force is determined after deciding the motion tendency of the point of contact of the body under consideration with the ground.

Let us take an example of analysis of direction of friction in this case.

In the following diagram, a rolling object of mass M and radius R is placed on a rough horizontal surface. A force F is applied as shown.



To find the direction of friction at P, let us consider the surface to be frictionless.

Acceleration of point P due to translation only,

$$a_t = \frac{F}{M}(\rightarrow)$$

Acceleration of point P due to rotation only,

$$a_r = \alpha R = \frac{\tau}{I}R \Rightarrow \frac{FR}{I}x(\leftarrow)$$

Net acceleration of point P is $\vec{a}_P = \vec{a}_t + \vec{a}_r$

$$a_P = \frac{F}{M} - \frac{FRx}{I}(\rightarrow) \qquad \text{(i)}$$

From Eq. (ii), it is clear that motion tendency at point P depends upon both x and I.

Equation (i) can be written as

$$a_P = \frac{F}{M}\left(1 - \frac{Rx}{K^2}\right) \quad \text{(ii)}$$

The motion tendency of point P will be in forward direction if

$$1 - \frac{Rx}{K^2} > 0 \Rightarrow 1 > \frac{Rx}{K^2} \Rightarrow K^2 > Rx \quad \text{(iii)}$$

If this condition is satisfied, friction will act in backward direction.

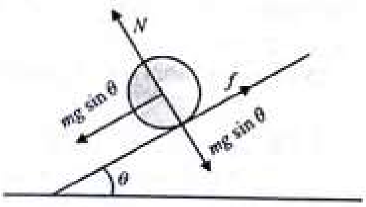
NOTE: We can summerise the situation as

If $K^2 > Rx$: friction will act in backward direction.
If $K^2 = Rx$: no friction will act.
If $K^2 < Rx$: friction will act in forward direction.
If force acts in lower diametric plane, friction will act in backward direction only.

Rolling of a Body on an Inclined Plane

A rigid body of radius of gyration k and radius R rolls (without slipping) down a plane inclined at an angle θ with the horizontal.

When a body is placed on an inclined plane, it tries to slip down and hence a static friction f acts upwards. This friction provides a torque which causes the body to rotate. Let a be the linear acceleration of centre of mass and α be the angular acceleration of the body.



From force diagram: For linear motion parallel to the plane,

$mg \sin \theta - f = ma$

For rotation around the axis through centre of mass, net torque $= I\alpha$.

$\Rightarrow \quad fR = (mk^2)\alpha$

As there is no slipping, the point of contact of the body with the plane is instantaneously at rest.

$\Rightarrow \quad v = R\omega$ and $a = R\alpha$

Solving the following three equations for a and f,

$mg \sin \theta - f = ma$

$\Rightarrow \quad fR = mk^2\alpha$

$a = R\alpha$

$$a = \frac{g \sin \theta}{\left(1 + \frac{k^2}{R^2}\right)} \text{ and } f = \frac{mg \sin \theta}{\left(1 + \frac{R^2}{k^2}\right)}$$

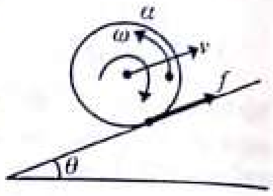
We can also derive the condition for pure rolling (rolling without slipping), to avoid slipping, $f \le \mu_s N$.

$$\frac{g \sin \theta}{1 + \frac{R^2}{k^2}} \le \mu_s mg \cos \theta$$

$$\Rightarrow \quad \mu_s \ge \frac{\tan \theta}{\left(1 + \frac{R^2}{k^2}\right)}$$

This is the condition of μ_s so that the body rolls without slipping.

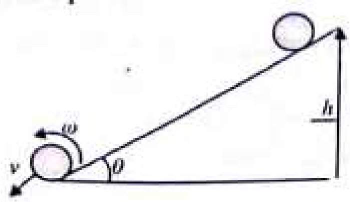
NOTE:



- If a body rolls up an inclined plane the direction of friction force in this case also in upward direction. As body rolls up velocity of centre of mass decreases, hence torque of friction should decrease angular velocity.
- The acceleration of the bodies rolling on inclined plane depends on radius of gyration (k). We can say the bodies with same value of k will have the same acceleration.
- For example, two uniform solid spheres made of different material, unequal mass and unequal radii are released from the same inclined plane if they rolls, they will have same accelerations.
- If a solid sphere and a hollow sphere of the same mass and same radii are released, the hollow sphere will take a longer time and acquires less velocity.
- If a solid sphere, a hollow sphere, a disc and a ring of same mass and radius are released from top of incline and they roll. The solid sphere take the least time while the ring will take maximum time to reach the bottom.
- If all the four objects are released from the top of a smooth incline. All the four bodies will reach the bottom at the same time.
- If all the rolling objects are released on a rough incline, such that friction is not sufficient for pure rolling, still the objects reach the bottom of incline at the same time because acceleration of centre of mass is the same in each case. Each object will have the same velocity of centre of mass but different ω.

ILLUSTRATION 9.28 A solid cylinder rolls down an inclined plane of height h and inclination θ. Calculate its speed at the bottom of the plane using acceleration method and energy method. Also calculate the time taken to reach the bottom.

Solution. Energy Method: Let v and ω be the velocity of the centre of mass and the angular velocity of cylinder, respectively, at the bottom of the plane.



As the cylinder rolls down:

Loss in G PE = Gain in translation KE + gain in rotational KE

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\omega^2$$

As the cylinder is rolling without slipping, $v = R\omega$

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\frac{v^2}{R^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{4gh}{3}}$$

Force/Torque Method: From the result of the last example (using $k^2 = R^2/2$), the acceleration of the cylinder is

$$\frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$$

Using $v^2 = u^2 + 2as$ down the plane (taking downward direction positive)

$$\Rightarrow \quad v = \sqrt{0 + 2a\frac{h}{\sin\theta}}$$

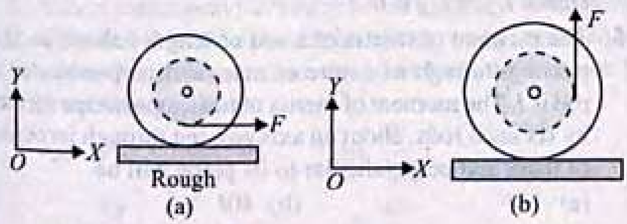
$$= \sqrt{2\left(\frac{2}{3}g\sin\theta\right)\frac{h}{\sin\theta}} = \sqrt{\frac{4gh}{3}}$$

$$\text{Time to reach bottom} = t = \frac{v-u}{a} = \frac{\sqrt{\frac{4gh}{3}} - 0}{\frac{2}{3}g\sin\theta} = \sqrt{\frac{3h}{g}}\frac{1}{\sin\theta}$$

In the forthcoming illustration, we will learn the application of various methods, i.e., force/torque, conservation of mechanical energy, work–energy theorem and concept of angular momentum in problem solving.

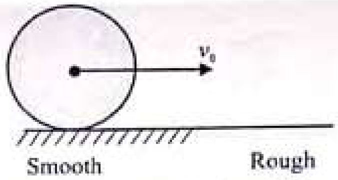
Concept Application Exercise 9.6

1. Figures (a) and (b) show two different arrangements of a spool being pulled with a constant force F in each case. Identify the direction of motion of the centre of mass, and the direction of friction force in each case.



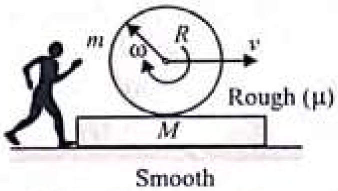
   (a) The CM moves along.............................x-axis.
   (b) The friction force acts along ....x-axis.
   (c) The CM moves along.............................x-axis.
   (d) The friction force acts along ....x-axis.

2. A sphere moving with a velocity v_0 on a smooth surface suddenly enters on a rough horizontal surface as shown in the figure. State which of the following statements are true or false



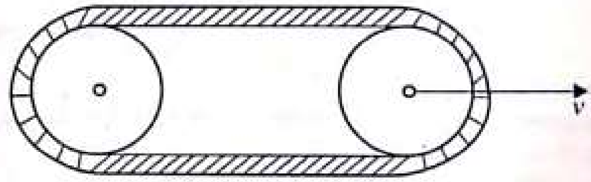
   (a) The sphere loses translational kinetic energy and gains rotational kinetic energy.
   (b) The total energy of the sphere is conserved.
   (c) The angular momentum of the sphere about any point on the surface is conserved.
   (d) The final velocity attained by the centre of mass is $2v_0/3$.

3. A cylinder of mass m and radius R rolls on a stationary plank of mass M. The lower surface of the plank is smooth and the upper surface is sufficiently rough with a coefficient of friction μ. A man is to hold the plank stationary with respect to the ground, as shown the figure.

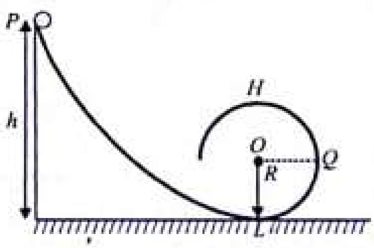


   The force exerted by the man to keep the plank stationary is equal to.......

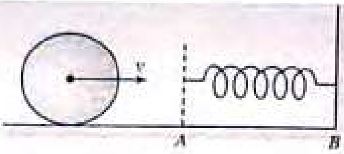
4. Calculate the kinetic energy of a tractor crawler belt of mass m if the tractor moves with a velocity v. There is no slipping. Neglect the size of the wheels.



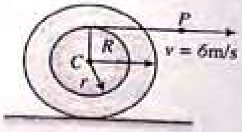
5. A small solid marble of mass M and radius r rolls down along the loop track, without slipping. Find the height h above the base, from where it has to start rolling down the incline such that the sphere just completes the vertical circular loop of radius R.



6. A sphere of mass m and radius R rolls without sliding on a horizontal surface. It collides with a light spring of stiffness K with a kinetic energy E. If the surface (AB) under the spring is smooth, find the maximum compression of the spring.



7. A cotton reel rolls without sliding such that the point P of the string has velocity $v = 6$ m s^{-1}. If $r = 10$ cm and $R = 20$ cm then find the
(a) Velocity of its centre C
(b) Angular velocity of the cotton reel



SOLVED EXAMPLES

1. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = 64 I_X$ (b) $I_Y = 32 I_X$
(c) $I_Y = 16 I_X$ (d) $I_Y = I_X$

Sol. (a) Moment of Inertia of disc $I = \frac{1}{2}MR^2$

$$= \frac{1}{2}(\pi R^2 t\rho)R^2 = \frac{1}{2}\pi t\rho R^4$$

[As $M = V \times \rho = \pi R^2 t\rho$ where t = thickness, ρ = density]

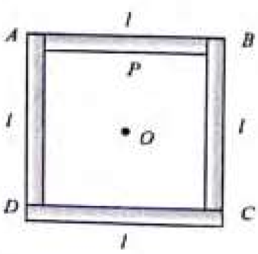
$$\therefore \quad \frac{I_y}{I_x} = \frac{t_y}{t_x}\left(\frac{R_y}{R_x}\right)^4 \quad \text{[If } \rho = \text{constant]}$$

$$\Rightarrow \quad \frac{I_y}{I_x} = \frac{1}{4}(4)^4 = 64 \quad \text{[Given } R_y = 4R_x,\ t_y = \frac{t_x}{4}]$$

$$\Rightarrow \quad I_y = 64 I_x$$

2. Four thin rods of same mass M and same length l, form a square as shown in the figure. Moment of inertia of this system about an axis through centre O and perpendicular to its plane is

(a) $\frac{4}{3}Ml^2$
(b) $\frac{Ml^2}{3}$
(c) $\frac{Ml^2}{6}$
(d) $\frac{2}{3}Ml^2$



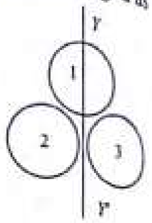
Sol. (a) Moment of inertia of rod AB about point $P = \frac{1}{12}Ml^2$

M.I. of rod AB about point $O = \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{1}{3}Ml^2$

[by the theorem of parallel axis]

and the system consists of 4 rods of similar type so by the symmetry $I_{\text{System}} = \frac{4}{3}Ml^2$.

3. Three rings each of mass M and radius R are arranged as shown in the figure. The moment of inertia of the system about YY' will be



(a) $3MR^2$ (b) $\frac{3}{2}MR^2$
(c) $5MR^2$ (d) $\frac{7}{2}MR^2$

Sol. (d) MI of system about YY': $I = I_1 + I_2 + I_3$

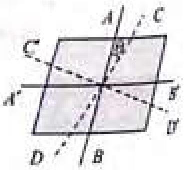
where I_1 = moment of inertia of ring about diameter, $I_2 = I_3$ = MI of inertia of ring about a tangent in a plane

$$\therefore \quad I = \frac{1}{2}mR^2 + \frac{3}{2}mR^2 + \frac{3}{2}mR^2 = \frac{7}{2}mR^2$$

4. Let I be the moment of inertia of an uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(a) I (b) $I\sin^2\theta$
(c) $I\cos^2\theta$ (d) $I\cos^2\frac{\theta}{2}$

Sol. (a) Let I_Z is the moment of inertia of square plate about the axis which is passing through the centre and perpendicular to the plane.



$I_Z = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$
[By the theorem of perpendicular axis]
$I_Z = 2I_{AB} = 2I_{A'B'} = 2I_{CD} = 2I_{C'D'}$
[As AB, $A'B'$ and CD, $C'D'$ are symmetric axis]
Hence $I_{CD} = I_{AB} = I$

5. The moment of inertia of a rod of length l about an axis passing through its centre of mass and perpendicular to rod is I. The moment of inertia of hexagonal shape formed by six such rods, about an axis passing through its centre of mass and perpendicular to its plane will be

(a) $16I$ (b) $40I$
(c) $60I$ (d) $80I$

Sol. (c) Moment of inertia of rod AB about its centre and perpendicular to the length $= \frac{ml^2}{12} = I \Rightarrow ml^2 = 12I$

Now moment of inertia of the rod about the axis which is passing through O and perpendicular to the plane of hexagon $I_{\text{rod}} = \frac{ml^2}{12} + mx^2$

[From the theorem of parallel axes]

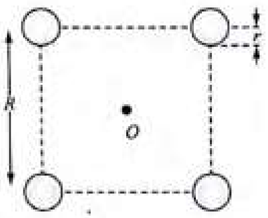
$$= \frac{ml^2}{12} + m\left(\frac{\sqrt{3}}{2}l\right)^2 = \frac{5ml^2}{6}$$

Now the moment of inertia of system,

$$I_{system} = 6 \times I_{rod} = 6 \times \frac{5ml^2}{6} = 5ml^2$$

$I_{system} = 5\,(12\,I) = 60I$ [As $ml^2 = 12I$]

6. Four spheres, each of mass M and radius ω are situated at the four corners of square of side R. The moment of inertia of the system about an axis perpendicular to the plane of square and passing through its centre will be



(a) $\frac{5}{2}M\,(4r^2+5R^2)$

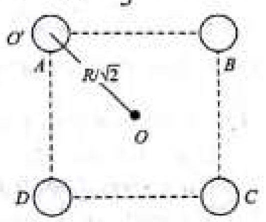
(b) $\frac{2}{5}M\,(4r^2+5R^2)$

(c) $\frac{2}{5}M\,(4r^2+5r^2)$

(d) $\frac{5}{2}M\,(4r^2+5r^2)$

Sol. (b) *MI* of sphere A about its diameter $I_{O'} = \frac{2}{5}Mr^2$

Now *MI* of sphere A about an axis perpendicular to the plane of square and passing through its centre will be



$$I_O = I_{O'} + M\left(\frac{R}{\sqrt{2}}\right)^2$$

$$= \frac{2}{5}Mr^2 + \frac{MR^2}{2} \quad \text{[by the theorem of parallel axis]}$$

Moment of inertia of system (i.e. four sphere)

$$= 4I_0 = 4\left[\frac{2}{5}Mr^2 + \frac{MR^2}{2}\right] = \frac{2}{5}M\left[4r^2+5R^2\right]$$

7. A horizontal heavy uniform bar of weight W is supported at its ends by two men. At the instant, one of the men lets go off his end of the rod, the other feels the force on his hand changed to

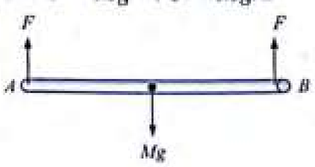
(a) W (b) $\frac{W}{2}$

(c) $\frac{3W}{4}$ (d) $\frac{W}{4}$

Sol. (d) Let the mass of the rod is $M \Rightarrow$ Weight $(W) = Mg$

Initially for the equilibrium $F + F = Mg \Rightarrow F = Mg/2$

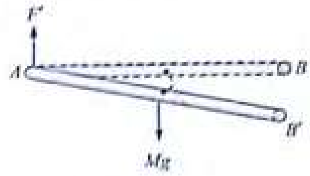
When one man withdraws, the torque on the rod



$$\tau = I\alpha = Mg\frac{l}{2}$$

$$\Rightarrow \frac{Ml^2}{3}\alpha = Mg\frac{l}{2} \quad [\text{As } I = Ml^2/3]$$

$\Rightarrow$ Angular acceleration,



$$\alpha = \frac{3}{2}\frac{g}{l}$$

and linear acceleration,

$$a = \frac{l}{2}\alpha = \frac{3g}{4}$$

Now if the new normal force at A is F' then $Mg - F' = Ma$

$$\Rightarrow \quad F' = Mg - Ma = Mg - \frac{3Mg}{4} = \frac{Mg}{4} = \frac{W}{4}.$$

8. A thin circular ring of mass M and radius R is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(a) $\frac{M\omega}{M+4m}$ (b) $\frac{(M+4m)\omega}{M}$

(c) $\frac{(M-4m)\omega}{M+4m}$ (d) $\frac{M\omega}{4m}$

Sol. (a) Initial angular momentum of ring $= I\omega = MR^2\omega$

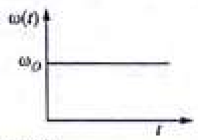
If four objects, each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring, then final angular momentum $= (MR^2 + 4MR^2)\omega'$

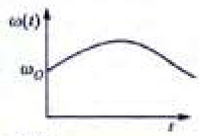
By the conservation of angular momentum,

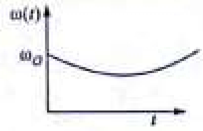
Initial angular momentum = Final angular momentum

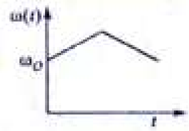
$$MR^2\omega = (MR^2 + 4mR^2)\,\omega' \Rightarrow \omega' = \left(\frac{M}{M+4m}\right)\omega$$

9. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its center. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform $\omega(t)$ will vary with time t as

(a) 

(b) 

(c) 

(d) 

Sol. (b) The angular momentum (L) of the system is conserved, i.e. $L = I\omega$ = constant

When the tortoise walks along a chord, it first moves closer to the centre and then away from the centre. Hence, M.I. first decreases and then increases. As a result, ω will first increase and then decrease. Also the change in ω will be non-linear function of time.

10. Two discs of moment of inertia I_1 and I_2 and angular speeds ω_1 and ω_2 are rotating along collinear axes passing through their centre of mass and perpendicular to their plane. If the two are made to rotate together along the same axis the rotational KE of system will be

(a) $\dfrac{I_1\omega_1+I_2\omega_2}{2(I_1+I_2)}$ (b) $\dfrac{(I_1+I_2)(\omega_1+\omega_2)^2}{2}$

(c) $\dfrac{(I_1\omega_1+I_2\omega_2)^2}{2(I_1+I_2)}$ (d) None of these

Sol. (c) By the law of conservation of angular momentum, $I_1\omega_1+I_2\omega_2=(I_1+I_2)\omega$

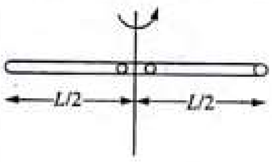
Angular velocity of system, $\omega=\dfrac{I_1\omega_1+I_2\omega_2}{I_1+I_2}$

Rotational kinetic energy

$$=\frac{1}{2}(I_1+I_2)\,\omega^2=\frac{1}{2}(I_1+I_2)\left(\frac{I_1\omega_1+I_2\omega_2}{I_1+I_2}\right)^2$$

$$=\frac{(I_1\omega_1+I_2\omega_2)^2}{2(I_1+I_2)}$$

11. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially, the two beads are at the centre of the rod and the system is rotating with angular velocity ω_0 about an axis perpendicular to the rod and passing through the mid point of the rod (see figure). There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is



(a) ω_0 (b) $\dfrac{M\omega_0}{M+12m}$

(c) $\dfrac{M\omega_0}{M+2m}$ (d) $\dfrac{M\omega_0}{M+6m}$

Sol. (d) Since there are no external forces, therefore, the angular momentum of the system remains constant.

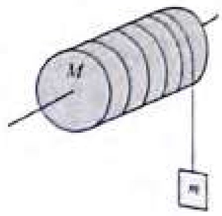
Initially when the beads are at the centre of the rod angular momentum, $L_1=\left(\dfrac{ML^2}{12}\right)\omega_0$ .....(i)

When the beads reach the ends of the rod, then angular momentum, $L_2=\left(m\left(\dfrac{L}{2}\right)^2+m\left(\dfrac{L}{2}\right)^2+\dfrac{ML^2}{12}\right)\omega'$ .....(ii)

Equating (i) and (ii), $\dfrac{ML^2}{12}\omega_0=\left(\dfrac{mL^2}{2}+\dfrac{ML^2}{12}\right)\omega'$

$\Rightarrow \omega'=\dfrac{M\omega_0}{M+6m}$.

12. In the following figure, a body of mass m is tied at one end of a light string and this string is wrapped around the solid cylinder of mass M and radius R. At the moment $t=0$, the system starts moving. If the friction is negligible, angular velocity at time t would be



(a) $\dfrac{mgRt}{(M+m)}$

(b) $\dfrac{2Mgt}{(M+2m)}$

(c) $\dfrac{2mgt}{R(M-2m)}$

(d) $\dfrac{2mgt}{R(M+2m)}$

Sol. (d) We know the tangential acceleration

$$a=\frac{g}{1+\dfrac{I}{mR^2}}=\frac{g}{1+\dfrac{1/2MR^2}{mR^2}}=\frac{2mg}{2m+M}$$

$$\left[\text{As } I=\frac{1}{2}MR^2 \text{ for cylinder}\right]$$

After time t, linear velocity of mass m, $v=u+at=0+\dfrac{2mgt}{2m+M}$

So, angular velocity of the cylinder, $\omega=\dfrac{v}{R}=\dfrac{2mgt}{R(M+2m)}$

13. A number of holes are drilled along a diameter of a disc of radius R. To get minimum time period of oscillations, the disc should be suspended from a horizontal axis passing through a hole whose distance from the centre should be

(a) $\dfrac{R}{2}$ (b) $\dfrac{R}{\sqrt{2}}$

(c) $\dfrac{R}{2\sqrt{2}}$ (d) Zero

Sol. (b) $T=2\pi\sqrt{\dfrac{L}{g}}$ where $L=\dfrac{l^2+k^2}{l}$

Here $k^2=\dfrac{R^2}{2}$

$$\therefore \quad L=\frac{l^2+\dfrac{R^2}{2}}{l}=l+\frac{R^2}{2l}$$

For minimum time period, L should be minimum. Hence

$$\frac{dL}{dl}=0$$

$$\Rightarrow \quad \frac{d}{dl}\left(l+\frac{R^2}{2l}\right)=0$$

$$\Rightarrow \quad 1+\frac{R^2}{2}\left(\frac{-1}{l^2}\right)=1-\frac{R^2}{2l^2}=0\Rightarrow l=\frac{R}{\sqrt{2}}$$

14. A thick walled hollow sphere has outer radius R. It rolls down an inclined plane without slipping and its speed at the bottom is v. If the inclined plane is frictionless and the sphere slides down without rolling, its speed at the bottom is $5v/4$. What is the radius of gyration of the sphere?

(a) $\dfrac{R}{\sqrt{2}}$ (b) $\dfrac{R}{2}$

(c) $\dfrac{3R}{4}$ (d) $\dfrac{\sqrt{3}R}{4}$

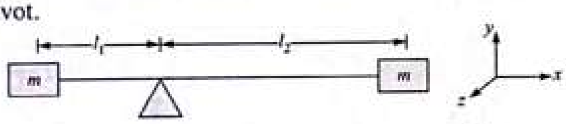
Sol. (c) Case 1: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$

Case 2: $mgh = \frac{1}{2}m\times\left(\frac{5v}{4}\right)^2$

$$\frac{1}{2}mk^2\frac{v^2}{R^2}+\frac{1}{2}mv^2 = \frac{1}{2}m\times\frac{25v^2}{16}$$

$$\frac{k^2}{R^2}+1=\frac{25}{16} \Rightarrow k=\frac{3R}{4}$$

15. Two blocks each of the mass m are attached to the ends of a massless rod which pivots as shown in the figure. Initially, the rod is held in the horizontal position and then released. Calculate the net torque on this system above pivot.



(a) $(ml_2g - ml_1g)\hat{k}$ (b) $(ml_1g - ml_2g)\hat{k}$

(c) $(ml_1g + ml_2g)\hat{k}$ (d) $-(ml_1g + ml_2g)\hat{k}$

Sol. (b) Torque due to the weight of left m:

$\vec{\tau}_1 = mg\,l_1\,\hat{k}$ ($\because$ this is clock wise)

Torque due to weight of right m:

$\vec{\tau}_2 = -mg\,l_2\,\hat{k}$ ($\because$ this is clockwise)

So net torque: $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 = (mgl_1 - mgl_2)\hat{k}$

16. Given a uniform disc of mass M and radius R. A small disc of radius $R/2$ is cut from this disc in such a way that the distance between the centres of the two discs is $R/2$. Find the moment of inertia of the remaining disc about a diameter of the original disc perpendicular to the line connecting the centres of the two discs.

(a) $3MR^2/32$ (b) $5MR^2/16$

(c) $11MR^2/64$ (d) None of these

Sol. (c) Mass of cut disc: $m_1 = M/4$

Moment of inertia of original disc about axis 1:

$$I = \frac{1}{4}MR^2$$

Moment of Inertia of small disc about axis 1:

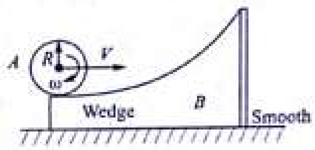
$$I' = \frac{1}{4}m_1\left(\frac{R}{2}\right)^2 + m_1\left(\frac{R}{2}\right)^2$$

or $I' = \dfrac{5}{16}m_1R^2 = \dfrac{5}{64}MR^2$

Required moment of inertia:

$$I - I' = \frac{1}{4}MR^2 - \frac{5}{64}MR^2 = \frac{11}{64}MR^2$$

17. In the figure, a cylinder A is initially rolling with velocity v on the horizontal surface of the wedge B (of same mass as A). All surfaces are smooth and B has no initial velocity. Then maximum height reached by cylinder on the wedge will be

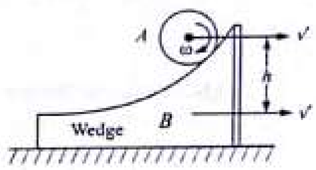


(a) $v^2/4g$ (b) v^2/g

(c) $v^2/2g$ (d) $v^2/8g$

Sol. (a) At the maximum height, vertical velocity of cylinder is zero. But horizontal velocity of the wedge and cylinder will be same.

In the absence of friction between the cylinder and the wedge surface, angular velocity of cylinder remains constant.



From energy conservation:

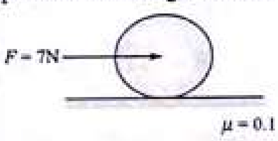
$$\frac{1}{2}mv^2+\frac{1}{2}I\omega^2 = \frac{1}{2}mv'^2+\frac{1}{2}I\omega^2+\frac{1}{2}mv'^2+mgh \quad ..(i)$$

By the conservation of linear momentum

$$mv = 2mv' \Rightarrow v' = v/2 \quad ...(ii)$$

From (i) and (ii): $h = v^2/4g$

18. A solid sphere of mass 10 kg is placed on a rough surface having coefficient of friction $\mu = 0.1$. A constant force $F = 7$ N is applied along a line passing through the centre of the sphere as shown in the figure. The value of frictional force on the sphere is



(a) 1 N (b) 2 N

(c) 3 N (d) 7 N

Sol. (b) Let f the be frictional force towards left for pure rolling.

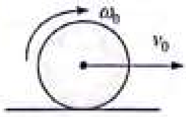
$$a = R\times\alpha$$

where $a = \dfrac{F-f}{m} = \dfrac{7-f}{10}$

$$\alpha = \frac{\tau}{I} = \frac{fR}{I} = \frac{fR}{\frac{2}{5}mR^2} = \frac{5f}{2mR}$$

Hence, $\frac{7-f}{10}=\frac{5f}{20} \Rightarrow f = 2\text{ N}$

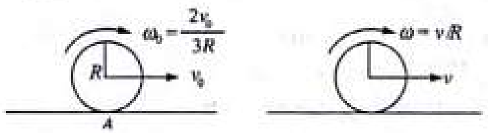
19. A solid sphere of radius R starts rotating on rough horizontal surface with translational velocity v_0 and initial angular velocity $\omega_0 = 2v_0/3R$. The sphere starts pure rolling after some time t. Find the angle by which sphere rotates upto the instant at which pure rolling starts, if v is the translational velocity at pure rolling. Assume uniformly accelerated motion upto start of pure rolling.



(a) $\frac{33}{38}\frac{vt}{R}$ (b) $\frac{37}{38}\frac{vt}{R}$

(c) $\frac{31}{38}\frac{vt}{R}$ (d) $\frac{29}{37}\frac{vt}{R}$

Sol. *(a)* $\omega_0 = \frac{2v_0}{3R}$ or, $v_0 = \frac{3}{2}R\omega_0$



Angular momentum is conserved about A.

$L_1 = L_2$

$\omega_0 = \frac{2v_0}{3R}$ or, $v_0 = \frac{3}{2}R\omega_0$

or $\frac{2}{5}mR^2 \times \frac{2v_0}{3R} + mv_0R = \frac{2}{5}mR^2\frac{v}{R} + mvR$

or $v_0 = \frac{21}{19}v$ .... (i)

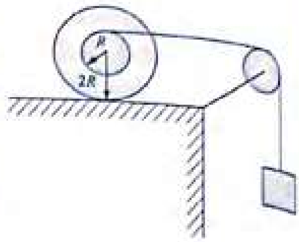
$\omega = \omega_0 + \alpha t \Rightarrow v/R = 2v_0/3R + \alpha t \Rightarrow \alpha = \frac{5v}{19Rt}$

where t is the time in which pure rolling starts.

Pure rolling starts when sphere rotates by an angle,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{2v_0}{3R}t + \frac{1}{2}\cdot\frac{5v}{19Rt}\cdot t^2 = \frac{33}{38}\frac{vt}{R}$$

20. In the figure mass of both spherical body and block is m. Moment of inertia of the spherical body about centre of mass is $2mR^2$. The spherical body rolls on the horizontal surface. There is no slipping at any surfaces in contact. The ratio of kinetic energy of the spherical body to that of block is



(a) $\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

Sol. *(c)* Let v be the linear velocity of centre of mass of the spherical body and ω its angular velocity about centre of mass. Then

$$\omega = \frac{v}{2R}$$

K.E. of spherical body,

$$K_1 = \frac{1}{2}mv^2 + \frac{1}{2}\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}(2mR)^2\left(\frac{v^2}{4R^2}\right)$$

$$= \frac{3}{4}mv^2 \quad ...(i)$$

Speed of the block will be

$$v' = (\omega)(3R) = 3R_\omega = (3R)\left(\frac{v}{2R}\right) = \frac{3}{2}v$$

$\therefore$ K.E. of block $K_2 = \frac{1}{2}mv'^2 = \frac{1}{2}m\left(\frac{3}{2}v\right)^2 = \frac{9}{8}mv^2$ ...(ii)

From equations (i) and (ii), $\frac{K_1}{K_2} = \frac{2}{3}$

EXERCISES

Moment of Inertia

1. Two circular discs A and B are of equal masses and thicknesses but made of metal with densities d_A and d_B $(d_A > d_B)$. If their moments of inertia about an axis passing through their centres and perpendicular to circular faces are I_A and I_B, then

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) $I_A \geq I_B$

2. Four identical rods are joined end to end to form a square. The mass of each rod is M. The moment of inertia of the square about the median line is

(a) $\frac{Ml^2}{3}$ (b) $\frac{Ml^2}{4}$

(c) $\frac{Ml^2}{6}$ (d) $\frac{2Ml^2}{3}$

3. In problem 2, the moment of inertia of the system about an axis passing through the point of intersection of diagonals and perpendicular to the plane of the square is

(a) $\frac{4Ml^2}{3}$ (b) $\frac{13Ml^2}{3}$

(c) $\frac{Ml^2}{6}$ (d) $\frac{13Ml^2}{6}$

4. In problem 2, the moment of inertia of the system about one of the diagonals is

(a) $\frac{2Ml^2}{3}$ (b) $\frac{13Ml^2}{3}$

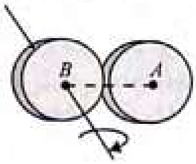
(c) $\frac{Ml^2}{6}$ (d) $\frac{13Ml^2}{6}$

5. Two circular discs are of same thickness. The diameter of A is twice that of B. The moment of inertia of A as compared to that of B is

(a) twice as large (b) four times as large

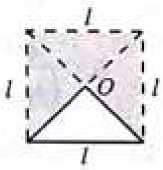
(c) eight times as large (d) 16 times as large

6. Two thin discs, each of mass M and radius r metre, are attached as shown in the figure, to form a rigid body. The rotational inertia of this body about an axis perpendicular to the plane of disc B passing through its centre is



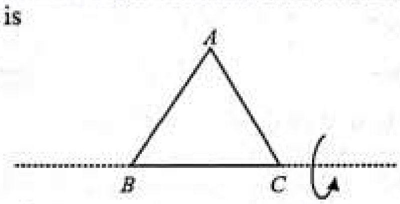
(a) $2Mr^2$ (b) $3Mr^2$ (c) $4Mr^2$ (d) $5Mr^2$

7. An isosceles triangular piece is cut from a square plate of side l. The piece is one-fourth of the square and mass of the remaining plate is M. The moment of inertia of the plate about an axis passing through O and perpendicular to its plane is



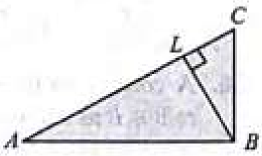
(a) $\frac{Ml^2}{6}$ (b) $\frac{Ml^2}{12}$ (c) $\frac{Ml^2}{24}$ (d) $\frac{Ml^2}{3}$

8. Three identical rods, each of mass m and length l, form an equaliteral triangle. Moment of inertia about one of the sides is



(a) $\frac{ml^2}{4}$ (b) ml^2 (c) $\frac{3ml^2}{4}$ (d) $\frac{2ml^2}{3}$

9. About which axis, the moment of inertia in the given triangular lamina is maximum?



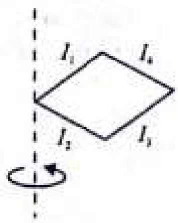
(a) AB

(b) BC

(c) AC

(d) BL

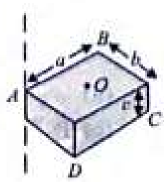
10. The moment of inertia of a system of four rods, each of length l and mass m, about the axis shown is



(a) $\frac{2}{3}ml^2$ (b) $2ml^2$

(c) $3ml^2$ (d) $\frac{8}{3}ml^2$

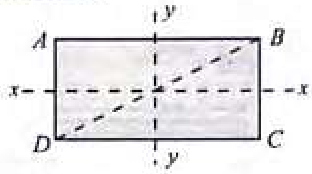
11. Figure shows a uniform solid block of mass M and edge lengths a, b and c. Its M.I. about an axis through one edge and perpendicular (as shown) to the large face of the block is



(a) $\frac{M}{3}(a^2+b^2)$ (b) $\frac{M}{4}(a^2+b^2)$

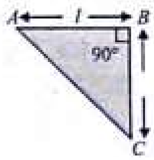
(c) $\frac{7M}{12}(a^2+b^2)$ (d) $\frac{M}{12}(a^2+b^2)$

12. In rectangle $ABCD$, $AB = 2l$ and $BC = l$. Axes xx and yy pass through the centre of the rectangle. The moment of inertia is least about



(a) DB (b) BC (c) xx (d) yy

13. Figure shows a thin metallic triangular sheet ABC. The mass of the sheet is M. The moment of inertia of the sheet about side AC is



(a) $\frac{Ml^2}{18}$ (b) $\frac{Ml^2}{12}$

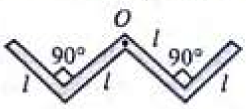
(c) $\frac{Ml^2}{6}$ (d) $\frac{Ml^2}{4}$

14. The moment of inertia of a door of mass m, length $2l$ and width l about its longer side is

(a) $\frac{11ml^2}{24}$ (b) $\frac{5ml^2}{24}$

(c) $\frac{ml^2}{3}$ (d) none of these

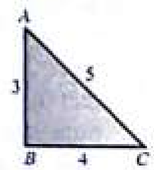
15. A thin rod of length $4l$ and mass $4m$ is bent at the points as shown in the figure. What is the moment of inertia of the rod about the axis passing through point O and perpendicular to the plane of the paper.



(a) $\frac{Ml^2}{3}$ (b) $\frac{10Ml^2}{3}$

(c) $\frac{Ml^2}{12}$ (d) $\frac{Ml^2}{24}$

16. ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC}, I_{CA} are the moments of inertia of the plated about AB, BC and CA respectively. Which one of the following relation is correct?



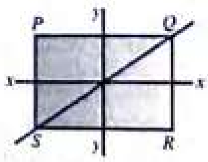
(a) I_{CA} is maximum

(b) $I_{AB} > I_{BC}$

(c) $I_{BC} > I_{AB}$

(d) $I_{AB} + I_{BC} = I_{CA}$

17. In a rectangle $PQRS$, $PQ = 5l$ and $RS = 2l$. Axes xx and yy pass through centre of the rectangle. The moment of inertia is least about

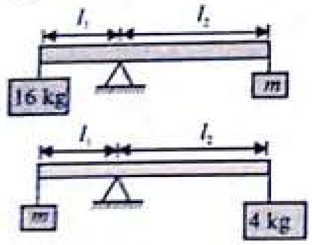


(a) SQ (b) RS

(c) xx (d) yy

Torque and Torque Equation

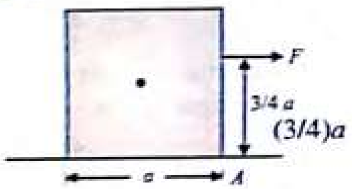
18. In an experiment with a beam balance, an unknown mass m is balanced by two known masses of 16 kg and 4 kg as shown in the figure.



The value of the unknown mass m is

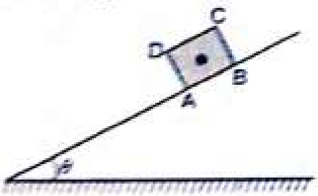
(a) 10 kg (b) 6 kg
(c) 8 kg (d) 12 kg

19. A cube of side a and mass m is to be tilted at point A by applying a force F as shown in the figure. The minimum force required is



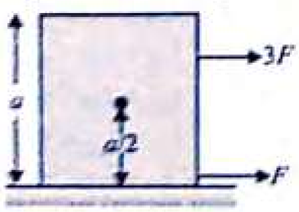
(a) mg (b) $\frac{2}{3}mg$
(c) $\frac{3}{2}mg$ (d) $\frac{3}{4}mg$

20. A cube of side a is placed on an inclined plane of inclination θ. What is the maximum value of θ for which the cube will not topple?



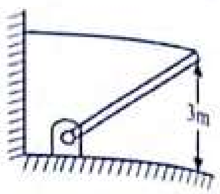
(a) 15° (b) 30° (c) 45° (d) 60°

21. A rectangular block of mass M and height a is resting on a smooth level surface. A force F is applied to one corner as shown in the figure. At what point should a parallel force $3F$ be applied in order that the block shall undergo pure translational motion? Assume normal contact force between the block and surface passes through the centre of gravity of the block.



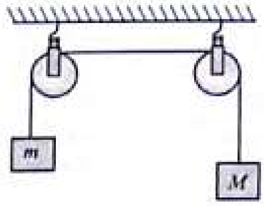
(a) $\frac{a}{3}$ vertically above centre of gravity
(b) $\frac{a}{6}$ vertically above centre of gravity
(c) No such point exists
(d) it is not possible

22. A uniform rod of mass 15 kg is held stationary with the help of a light string as shown in the figure. The tension in the string is



(a) 150 N (b) 225 N
(c) 100 N (d) None of the above

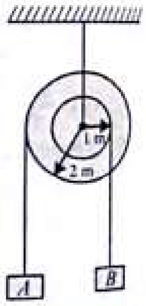
23. Each pulley in the figure has radius r and moment of inertia I. The acceleration of the block is



(a) $\dfrac{(M-m)g}{\left(M+m+\frac{2I}{r^2}\right)}$ (b) $\dfrac{(M-m)g}{\left(M+m-\frac{2I}{r^2}\right)}$

(c) $\dfrac{(M-m)g}{\left(M+m+\frac{I}{r^2}\right)}$ (d) $\dfrac{(M-m)g}{\left(M+m-\frac{I}{r^2}\right)}$

24. In the pulley system shown, if radii of the bigger and smaller pulley are 2 m and 1 m, respectively, and the acceleration of block A is 5 m s^{-2} in the downward direction, the acceleration of block B will be

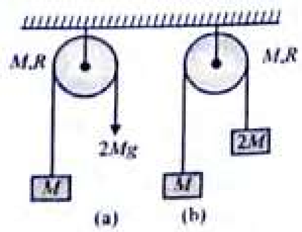


(a) 0 m s^{-2} (b) 5 m s^{-2}
(c) 10 m s^{-2} (d) $\frac{5}{2}$ m s^{-2}

25. A uniform disc of mass M and radius R is mounted on an axle supported in frictionless bearings. A light cord is wrapped around the rim of the disc and a steady downward pull T is exerted on the cord. The angular acceleration of the disc is

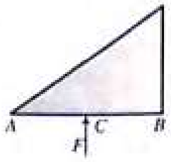
(a) $\frac{T}{MR}$ (b) $\frac{MR}{T}$ (c) $\frac{2T}{MR}$ (d) $\frac{MR}{2T}$

26. A cord is wrapped on a pulley (disk) of mass M and radius R as shown in the figure. To one end of the cord, a block of mass M is connected as shown and to other end in (a) a force of 2 Mg and in (b) a block of mass $2M$. Let angular acceleration of the disk in A and B is α_A and α_B respectively, then (cord is not slipping on the pulley).



(a) $\alpha_A = \alpha_B$ (b) $\alpha_A > \alpha_B$
(c) $\alpha_A < \alpha_B$ (d) None of these

27. A triangular plate of uniform thickness and density is made to rotate about an axis perpendicular to the plane of the paper (a) passing through A, (b) passing through B, by the application of the same force F, at C (mid-point of AB) as shown. Now,

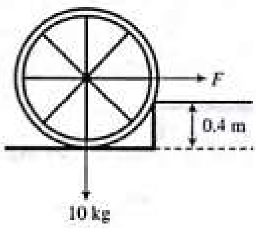


(a) angular acceleration in both the cases is the same
(b) angular acceleration for case (a) is larger
(c) angular acceleration for case (b) is larger
(d) there would be no angular acceleration for case (a)

28. Three children are sitting on a see-saw in such a way that is balances. A 20 kg and a 30 kg boy are on opposite sides at a distance of 2m from the pivot. It the third boy jumps off, thereby destroying balance, then the initial angular acceleration of the board is: (Neglect weight of board)

(a) 0.01 rad s^{-2} (b) 1.0 rad s^{-2}
(c) 10 rad s^{-2} (d) 100 rad s^{-2}

29. Calculate the force F that is applied horizontally at the axle of the wheel which is necessary to raise the wheel over the obstacle of height 0.4 m. Radius of wheel is 1m and mass = 10 kg . F is

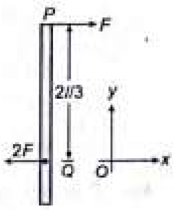


(a) 100 N (b) 66 N
(c) 167 N (d) 133.3 N

30. A string is wrapped around a cylinder of mass M and radius R. The string is pulled vertically upwards to prevent the centre of mass from falling as the cylinder unwinds the string. The tension in the string is

(a) $\frac{Mg}{6}$ (b) $\frac{Mg}{3}$
(c) $\frac{Mg}{2}$ (d) $\frac{2Mg}{3}$

31. Two forces F and $2F$ are applied on a rod of length l and mass m, as shown. The angular acceleration of the rod is



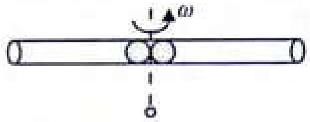
(a) $\frac{8F}{ml}(-\hat{k})$ (b) $\frac{4F}{ml}(\hat{k})$
(c) $\frac{4F}{ml}(-\hat{k})$ (d) $\frac{8F}{ml}(\hat{k})$

32. The mass of a sphere placed on a smooth horizontal surface is distributed non-uniformly. A horizontal force equal to weight of sphere is applied at a distance 'a' from centre of the sphere for a short time. The sphere is rotated by an angle θ, but it behaves as if in stable equilibrium. Find the distance between centre and CG of the sphere:

(a) $a \tan \theta$ (b) $a \cot \theta$
(c) $a \sec \theta$ (d) $a \operatorname{cosec} \theta$

Angular Momentum and Energy of Rigid Body

33. A smooth tube of certain mass is rotated in a gravity-free space and released. The two balls shown in the figure move towards the ends of the tube. For the whole system, which of the following quantities is not conserved.

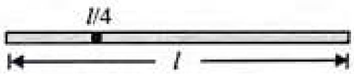


(a) Angular momentum (b) Linear momentum
(c) Kinetic energy (d) Angular speed

34. Two bodies with moments of inertia I_1 and I_2 ($I_1 > I_2$) have equal angular momenta. If their kinetic energies of rotation are E_1 and E_2, respectively, then

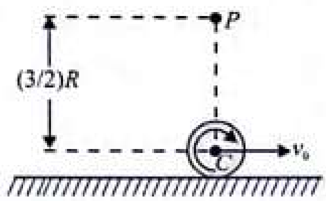
(a) $E_1 = E_2$ (b) $E_1 < E_2$
(c) $E_1 > E_2$ (d) $E_1 \geq E_2$

35. A uniform thin rod of length l and mass m is hinged at a distance $l/4$ from one of the end and released from horizontal position as shown in the figure. The angular velocity of the rod as it passes the vertical position is



(a) $2\sqrt{\frac{5g}{7l}}$ (b) $2\sqrt{\frac{6g}{7l}}$
(c) $\sqrt{\frac{3g}{7l}}$ (d) $2\sqrt{\frac{g}{l}}$

36. A disc of mass M and radius R rolls without slipping on a horizontal surface. If the velocity of its centre is v_0, then the total angular momentum of the disc about a fixed point P at a height $(3/2)R$ above the centre C

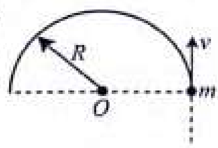


(a) increases continuously as the disc moves away
(b) decreases continuously as the disc moves away
(c) is equal to $2MRv_0$
(d) is equal to MRv_0

37. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is

(a) $2K$ (b) $\frac{K}{2}$
(c) $\frac{K}{4}$ (d) $4K$

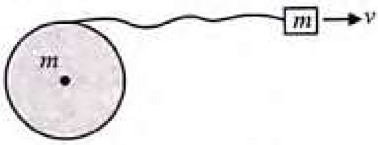
38. A small bead of mass m moving with velocity v gets threaded on a stationary semicircular ring of mass m and radius R kept on a horizontal table. The ring can freely rotate about its centre. The bead comes to rest relative to the ring. What will be the final angular velocity of the system?



(a) $\frac{v}{R}$ (b) $\frac{2v}{R}$

(c) $\frac{v}{2R}$ (d) $\frac{3v}{R}$

39. A block of mass m is attached to a pulley disc of equal mass m and radius r by means of a slack string as shown. The pulley is hinged about its centre on a horizontal table and the block is projected with an initial velocity of 5 m/s. Its velocity when the string becomes taut will be



(a) 3 m/s (b) 2.5 m/s

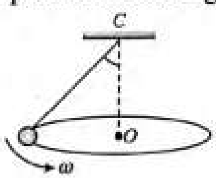
(c) 5/3 m/s (d) 10/3 m/s

40. Two points of a rod move with velocities $3v$ and v perpendicular to the rod and in the same direction separated by a distance 'r'. The angular velocity of the rod is

(a) $\frac{3v}{r}$ (b) $\frac{4v}{r}$

(c) $\frac{5v}{r}$ (d) $\frac{2v}{r}$

41. A conical pendulum consists of a simple pendulum moving in a horizontal circle as shown in the figure. C is the pivot, O the centre of the circle in which the pendulum bob moves and ω the constant angular velocity of the bob. If $\vec{L}$ is the angular momentum about point C, then



(a) $\vec{L}$ is constant

(b) only direction of $\vec{L}$ is constant

(c) only magnitude of $\vec{L}$ is constant

(d) none of the above

42. Two rigid bodies A and B rotate with angular momenta L_A and L_B respectively. The moments of inertia of A and B about the axes of rotation are I_A and I_B respectively. If $I_A = I_B/4$ and $L_A = 5L_B$, Then the ratio of rotational kinetic energy K_A of A to the rotational kinetic energy K_B of B is given by

(a) $\frac{K_A}{K_B} = \frac{25}{4}$ (b) $\frac{K_A}{K_B} = \frac{5}{4}$

(c) $\frac{K_A}{K_B} = \frac{1}{4}$ (d) $\frac{K_A}{K_B} = 100$

43. Two discs, each having moment of inertia 5 kg m^2 about its central axis, rotating with speeds 10 rad s^{-1} and 20 rad s^{-1}, are brought in contact face to face with their axes of rotation coincided. The loss of kinetic energy in the process is

(a) 2 J (b) 5 J (c) 125 J (d) 0 J

44. A boy stands over the centre of a horizontal platform which is rotating freely with a speed of 2 revolutions/s about a vertical axis through the centre of the platform and straight up through the boy. He holds 2 kg masses in each of his hands close to his body. The combined moment of inertia of the system is 1 kg × metre^2. The boy now stretches his arms so as to hold the masses far from his body. In this situation, the moment of inertia of the system increases to 2 kg × metre^2. The kinetic energy of the system in the latter case as compared with that in the previous case will

(a) Remain unchanged (b) Decrease

(c) Increase (d) Remain uncertain

45. A sphere is released on a smooth inclined plane from the top. When it moves down, its angular momentum is

(a) conserved about every point

(b) conserved about the point of contact only

(c) conserved about the centre of the sphere only

(d) conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball

46. A horizontal 90 kg merry-go-round is a solid disk of radius 1.50 m and is started from rest by a constant horizontal force of 50.0 N applied tangentially to the edge of the disk. The kinetic energy of the disk after 3.00 is

(a) 125 J (b) 500 J (c) 250 J (d) 150 J

47. A solid uniform disk of mass m and radius R is pivoted about a horizontal axis through its center and a small body of mass m is attached to the rim of the disk. If the disk is released from rest with the small body, initially at the same level as the centre, the angular velocity when the small body reaches the lowest point of the disk is

(a) $\sqrt{\frac{12g}{13r}}$ (b) $\sqrt{\frac{11g}{12r}}$

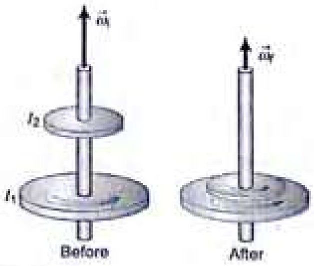
(c) $\sqrt{\frac{12g}{11r}}$ (d) $\sqrt{\frac{7g}{11r}}$

48. A solid sphere of mass m and radius r is released from rest from the given position. If it rolls without sliding on the circular track of radius R, its speed when it reaches its lowest position will be

(a) $\sqrt{\frac{8}{7}g(R-r)}$ (b) $\sqrt{\frac{10}{7}g(R-r)}$

(c) $\sqrt{\frac{10}{9}g(R-r)}$ (d) $\sqrt{\frac{5}{7}g(R-r)}$

49. A disk with moment of inertia I_1 rotates about a frictionless, vertical axle with angular speed ω_1. A second disk, this one having moment of inertia I_2 and initially not rotating, drops onto the first disk (figure). Because of friction between the surfaces, the two eventually reach the same angular speed ω_f. The value of ω_f is



(a) $\frac{I_1+I_2}{I_1}\omega_i$ (b) $\frac{I_1}{I_1+I_2}\omega_i$

(c) $\frac{I_1+I_2}{I_2}\omega_i$ (d) $\frac{I_1}{I_2}\omega_i$

50. A girl of mass M stands on the rim of a frictionless merry-go-round of radius R and rotational inertia I that is not moving. She throws a rock of mass m horizontally in a direction that is tangent to the outer edge of the merry-go-round. The speed of the rock, relative to the ground, is v. Afterward, the linear speed of the girl is

(a) $\frac{mvR^2}{I+MR^2}$ (b) $\frac{(m+M)vR^2}{I+MR^2}$

(c) $\frac{mvR^2}{I+(M+m)R^2}$ (d) $\frac{mvR^2}{I+(M-m)R^2}$

51. A particle is moving along a line $y = x + a$ with a constant velocity v. Find the angular momentum of the particle about the origin.

(a) mva (b) $mva\sqrt{2}$

(c) $\frac{mva}{\sqrt{2}}$ (d) $\frac{mvy}{x\sqrt{2}}$

52. Two men of equal masses stand at opposite ends of the diameter of a turntable disc of a certain mass, moving with constant angular velocity. The two men make their way to the middle of the turntable at equal rates. In doing so

(a) kinetic energy of rotation has increased while angular momentum remains the same.

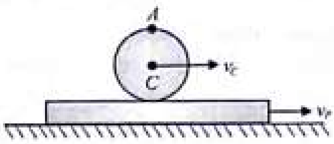
(b) kinetic energy of rotation has decreased while angular momentum remains same.

(c) kinetic energy of rotation has decreased but angular momentum has increased.

(d) both kinetic energy of rotation and angular momentum have decreased.

Problems Based on Rolling

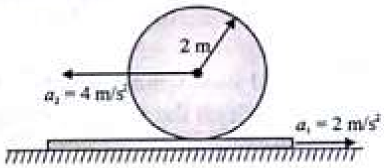
53. In the figure, the velocities are in ground frame and the cylinder is performing pure rolling on the plank, velocity of point 'A' would be



(a) $2v_C$ (b) $2v_C + v_P$

(c) $2v_C - v_P$ (d) $2(v_C - v_P)$

54. In the figure, a sphere of radius 2 m rolls on a plank. The accelerations of the sphere and the plank are indicated. The value of α is



(a) 2 rad/s^2 (b) 4 rad/s^2

(c) 3 rad/s^2 (d) 1 rad/s^2

55. A ring of radius R is first rotated with an angular velocity ω_0 and then carefully placed on a rough horizontal surface. The coefficient of friction between the surface and the ring is μ. Time after which its angular speed is reduced to half is

(a) $\frac{\omega_0\mu R}{2g}$ (b) $\frac{\omega_0 g}{2\mu R}$

(c) $\frac{2\omega_0 R}{\mu g}$ (d) $\frac{\omega_0 R}{2\mu g}$

56. If a spherical ball rolls on a table without slipping, the fraction of its total energy associated with rotation is

(a) $\frac{3}{5}$ (b) $\frac{2}{7}$

(c) $\frac{2}{5}$ (d) $\frac{3}{7}$

57. A force F acts tangentially at the highest point of a disc of mass m kept on a rough horizontal plane. If the disc rolls without slipping, the acceleration of centre of the disc is:

(a) $\frac{2F}{3m}$ (b) $\frac{10F}{7m}$

(c) zero (d) $\frac{4F}{3m}$

58. A body is rolling without slipping on a horizontal plane. The rotational energy of the body is 40% of the total kinetic energy. Identify the body.

(a) Ring (b) Hollow cylinder

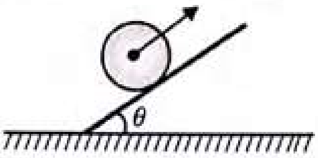
(c) Solid cylinder (d) Hollow sphere

59. A solid sphere rolls down two different inclined planes of the same height but of different inclinations:
(a) the speeds will be same but time of descent will be different
(b) in both cases, the speeds and time of descent will be same
(c) speeds and time of descent both will be different
(d) the speeds will be different but time of descent will be same

60. A body of mass m slides down an smooth incline and reaches the bottom with a velocity v. Now smooth incline surface is made rough and the same mass was in the form of a ring which rolls down this incline, the velocity of the ring at the bottom would have been:
(a) $\sqrt{2}\,v$ (b) v
(c) $\left(\sqrt{\frac{2}{5}}\right)v$ (d) $v/\sqrt{2}$

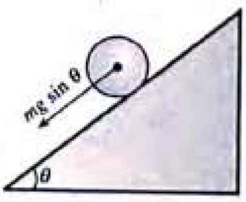
61. A ring, cylinder and solid sphere are placed on the top of a rough incline on which the sphere can just roll without slipping. When all of them are released at the same instant from the same position, then
(a) all of them reach the ground at the same instant
(b) the sphere reaches first and the ring at the last
(c) the sphere reaches first and the cylinder and ring reach together
(d) none of the above

62. A sphere has to purely roll upwards. At an instant when the velocity of sphere is v, frictional force acting on it is



(a) downwards and $\mu mg \cos\theta$
(b) downwards and $\frac{2mg\sin\theta}{7}$
(c) upwards and $\mu mg\cos\theta$
(d) upwards and $\frac{2mg\sin\theta}{7}$

63. A solid cylinder of mass 3 kg is placed on a rough inclined plane of inclination 30°. If $g = 10\ \text{ms}^{-2}$, then the minimum frictional force required for it to roll without slipping down the plane is

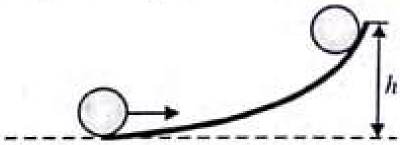


(a) 2 N (b) 5 N
(c) 15 N (d) 18 N

64. A small solid sphere of radius r rolls down an incline without slipping which ends into a vertical loop of radius R. Find the height above the base so that it just loops the loop
(a) $\frac{5}{2}R$ (b) $\frac{5}{2}(R-r)$
(c) $\frac{25}{10}(R-r)$ (d) $\frac{27}{10}R-\frac{17r}{10}$

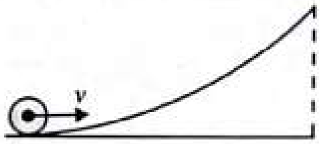
65. A sphere is rolling down an inclined plane without slipping. The ratio of rotational kinetic energy to total kinetic energy is
(a) $\frac{5}{7}$ (b) $\frac{2}{5}$
(c) $\frac{2}{7}$ (d) none of these

66. In the figure shown, a ball without sliding on a horizontal surface. It ascends a curved track up to height h and returns. The value of h is h_1 for sufficiently rough curved track to avoid sliding and is h_2 for smooth curved track, then



(a) $h_1 = h_2$ (b) $h_1 < h_2$
(c) $h_1 > h_2$ (d) $h_2 = 2h_1$

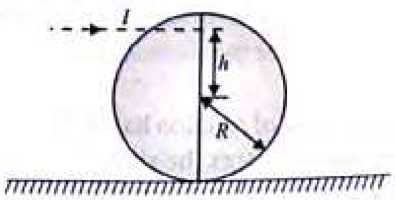
67. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $3v^2/4g$ w.r.t. the initial position. The object is



(a) ring (b) solid sphere
(c) hollow sphere (d) disc

68. A rolling object rolls without slipping down an inclined plane (angle of inclination θ), then the minimum acceleration it can have is
(a) $g\sin\theta$ (b) $\frac{2g\sin\theta}{3}$
(c) $\frac{g\sin\theta}{2}$ (d) zero

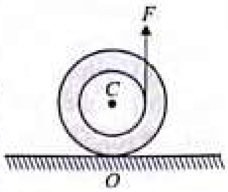
69. A solid sphere rests on a horizontal surface. A horizontal impulse is applied at height h from centre. The sphere starts rolling just after the application of impulse. The ratio h/r will be



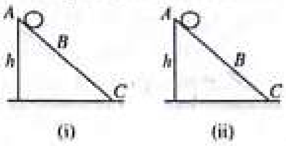
(a) $\frac{1}{2}$ (b) $\frac{2}{5}$
(c) $\frac{1}{5}$ (d) $\frac{2}{3}$

70. A yo-yo is placed on a rough horizontal surface and a constant force F, which is less than its weight, pulls it vertically. Due to this

(a) friction force acts towards left, so it will move towards left
(b) friction force acts towards right, so it will move towards right
(c) it will move towards left, so friction force acts towards left
(d) it will move towards right so friction force acts towards right



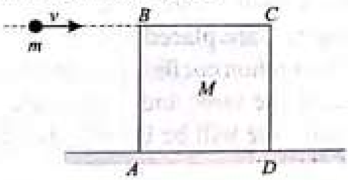
71. In both the figures all other factors are same, except that in Figure (i) AB is rough and BC is smooth while in Figure (ii) AB is smooth and BC is rough. In Figure (i), if a sphere is released from rest it starts rolling. Now consider the Figure (ii), if same sphere is released from top of the inclined plane, what will be the kinetic energy of the sphere on reaching the bottom:



(a) is same in both the cases
(b) is greater in case (i)
(c) is greater in case (ii)
(d) information insufficient

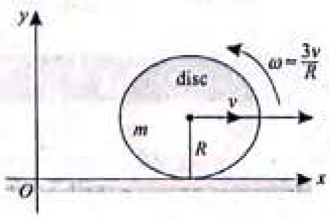
Problems Based on Mixed Concepts

72. A solid cube $ABCD$ of side a and mass M is placed on a rough horizontal surface. A bullet of mass m and speed v is shot at the top of the cube so that cube rotates about point D. The bullet sticks to the cube, $m << M$. Find the angular velocity of the cube.



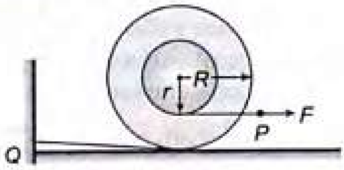
(a) $\frac{6mv}{5Ma}$ (b) $\frac{3Mv}{2ma}$
(c) $\frac{2Mv}{3ma}$ (d) $\frac{2mv}{3Ma}$

73. The angular momentum of the disc which spins with $\vec{\omega} = \frac{3v}{R}\hat{k}$ and its CM moves with a velocity $\vec{v} = v\hat{i}$ about O will be



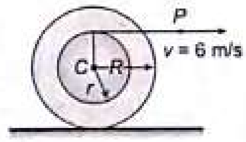
(a) $\frac{3mvR}{2}\hat{k}$ (b) $\frac{mvR}{2}\hat{k}$
(c) $-\frac{3mvR}{2}\hat{k}$ (d) $-\frac{mvR}{2}\hat{k}$

74. Find the acceleration of the body if a force $F = 8$ N pulls the string at P that passes over the body and it is connected by another string to a rigid support at Q. (Take radius of gyration $k = \frac{2}{\sqrt{3}}m$, $R = 2$ m, $r = 1$ m, and mass of the body $m = 3$ kg)



(a) 1 m/s^2 (b) 1.5 m/s^2
(c) 1.2 m/s^2 (d) 2 m/s^2

75. A cotton reel rolls without sliding such that the point P of the string has velocity $v = 6$ m/s. If $r = 10$ cm and $R = 20$ cm, then the velocity of its centre C is

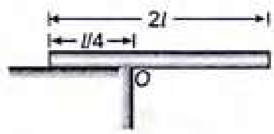


(a) 2.5 m/s (b) 5 m/s
(c) 4m/s (d) 2 m/s

76. A homogenous rod of length $l = \eta x$ and mass M is lying on a smooth horizontal floor. A bullet of mass m hits the rod at a distance x from the middle of the rod at a velocity v_0 perpendicular to the rod and comes to rest after collision. If the velocity of the farther end of the rod just after the impact is in the opposite direction of v_0, then

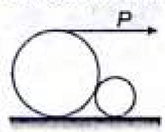
(a) $\eta > 3$ (b) $\eta < 3$
(c) $\eta > 6$ (d) $\eta < 6$

77. One fourth length of a uniform rod of length $2l$ and mass m is placed on a horizontal table and the rod is held horizontally. The rod is released from rest. The normal reaction on the rod as soon as the rod is released will be



(a) $\frac{5mg}{27}$ (b) $\frac{4mg}{27}$
(c) $\frac{mg}{9}$ (d) $\frac{4mg}{19}$

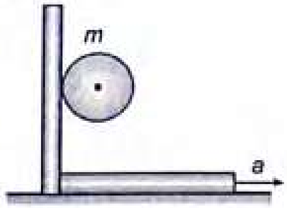
78. Two heavy right circular rollers of radii R and r respectively rest on a rough horizontal plane as shown in figure. The larger roller has a string wound around it to which a horizontal force P can be applied as shown. Assuming that the coefficient of friction μ has the same value for all surfaces of contact and the smaller cylinder should neither roll nor slide. The minimum coefficient of friction so that the larger roller can be pulled over the smaller one is



(a) $\frac{r}{R}$ (b) $\left(\frac{r}{R}\right)^2$

(c) $3\sqrt{\frac{r}{R}}$ (d) $\sqrt{\frac{r}{R}}$

79. A uniform solid sphere rolls down a vertical surface without sliding. If the vertical surface moves with an acceleration $a = \frac{g}{2}$, the minimum coefficient of friction between the sphere and vertical surfaces so as to prevent relative sliding is



(a) 3/7 (b) 4/7

(c) 4/5 (d) 3/5

80. A spherical ball of radius r initially at rest on a rough horizontal surface is hit horizontally at a point at a distance x above the central line. Due to this sharp impulse the centre of the ball acquires a velocity v_c. After some time, the ball will start pure rolling with a velocity equal to

(a) $\frac{5}{7}v_c\left[\frac{x+r}{r}\right]$ (b) $\frac{2}{7}v_c\left[\frac{x+r}{r}\right]$

(c) $\frac{5}{7}v_c\left[\frac{x+r}{x}\right]$ (d) $\frac{2}{7}v_c\left[\frac{x+r}{x}\right]$

81. A uniform rod of length L (in between the supports) and mass m is placed on two supports A and B. The rod breaks suddenly at length $L/10$ from the support B. Find the reaction at support A immediately after the rod breaks.

(a) $\frac{9}{40}mg$ (b) $\frac{19}{40}mg$

(c) $\frac{mg}{2}$ (d) $\frac{9}{20}mg$

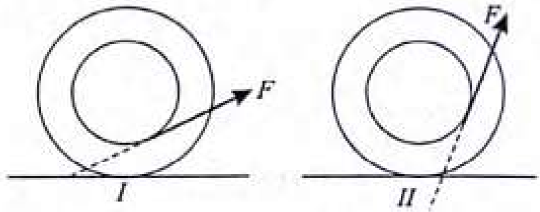
82. A rigid body of radius R, either hollow or solid, lies on a smooth horizontal surface. The body is pulled by a horizontal force acting tangentially from the highest point. The distance travelled by the body in the time in which it makes one full rotation is the same that it will make in one full rotation during pure rolling. The rigid body will be

(a) solid sphere
(b) hollow sphere
(c) circular disc
(d) hollow cylinder

83. A solid sphere is set into rotation at an angular velocity and it is then placed on a rough horizontal surface. The ratio of distances covered by rotational and translational motions up to the start of the pure rolling is (Assume uniformly accelerated motion up to start of pure rolling):

(a) 5/2 (b) 7/2 (c) 7/4 (d) 9/2

84. The string of a step rolling wheel is pulled by applying force F with different lines of action in two situations as shown. The wheel starts rolling without slipping due to application of the force.



(a) The wheel rolls to the right in situation I and to the left in situation II.
(b) The wheel rolls to the left in situation I and to the right in situation II.
(c) The wheel rolls to the right in both situations.
(d) The wheel rolls to the left in both situations.

85. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are the same and not sufficient to allow pure rolling. Least time will be taken in reaching the bottom by

(a) the solid sphere
(b) the hollow sphere
(c) the disc
(d) all will take the same time

86. In the previous question, the smallest kinetic energy at the bottom of the incline will be achieved by

(a) the solid sphere
(b) the hollow sphere
(c) the disc
(d) all will achieve the same kinetic energy

ARCHIVES

1. The initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m each are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

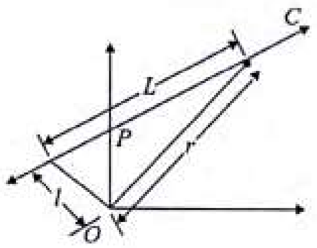
(AIEEE 2002)

2. A solid sphere, a hollow sphere, and a ring are released from the top of an inclined plane (frictionless) so that they slide down the plane. Then the maximum acceleration down the plane is for (no rolling)

(a) solid sphere
(b) hollow sphere
(c) ring
(d) all have the same acceleration **(AIEEE 2002)**

3. The moment of inertia of a circular wire of mass M and radius R about its diameter is
(a) $MR^2/2$ (b) MR^2
(c) $2MR^2$ (d) $MR^2/4$ **(AIEEE 2002)**

4. A particle of mass m moves along a line PC with velocity v as shown. What is the angular momentum of the particle about O?



(a) mvL (b) mvl
(c) mvr (d) zero **(AIEEE 2002)**

5. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin. Then
(a) $\vec{r}\cdot\vec{\tau}=0$ and $\vec{F}\cdot\vec{\tau}\neq 0$
(b) $\vec{r}\cdot\vec{\tau}=0$ and $\vec{F}\cdot\vec{r}=0$
(c) $\vec{r}\cdot\vec{\tau}\neq 0$ and $\vec{F}\cdot\vec{\tau}\neq 0$
(d) $\vec{r}\cdot\vec{\tau}=0$ and $\vec{F}\cdot\vec{\tau}=0$ **(AIEEE 2003)**

6. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is
(a) $I_Y=32I_X$ (b) $I_Y=16I_X$
(c) $I_Y=I_X$ (d) $I_Y=64I_X$
(AIEEE 2003)

7. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is
(a) $\frac{L}{4}$ (b) $2L$
(c) $4L$ (d) $\frac{L}{2}$ **(AIEEE 2003)**

8. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping its mass the same, which one of the following will not be affected?
(a) angular velocity
(b) angular momentum
(c) moment of inertia
(d) rotational kinetic energy **(AIEEE 2004)**

9. A solid sphere A and another hollow sphere B are of the same mass and same outer radii. Their moment of inertia about their diameters are, respectively, I_A and I_B such that (d_A and d_B are the densities)
(a) $I_A<I_B$ (b) $I_A>I_B$
(c) $I_A=I_B$ (d) $\frac{I_A}{I_B}=\frac{d_A}{d_B}$ **(AIEEE 2004)**

10. Which of the following statements is false for a particle moving in a circle with a constant angular speed?
(a) The velocity vector is tangent to the circle.
(b) The acceleration vector is tangent to the circle.
(c) The acceleration vector points towards the centre of the circle.
(d) The velocity and acceleration vectors are perpendicular to each other. **(AIEEE 2004)**

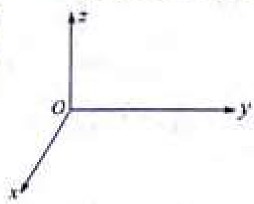
11. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by two particles situated on the inner and outer parts of the ring is
(a) $\frac{R_1}{R_2}$ (b) 1
(c) $\left(\frac{R_1}{R_2}\right)^2$ (d) $\frac{R_2}{R_1}$ **(AIEEE 2005)**

[**Note:** The particles should be of same mass.]

12. The moment of inertia of a uniform semi-circular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is
(a) $\frac{1}{2}Mr^2$ (b) Mr^2
(c) $\frac{2}{5}Mr^2$ (d) $\frac{1}{4}Mr^2$ **(AIEEE 2005)**

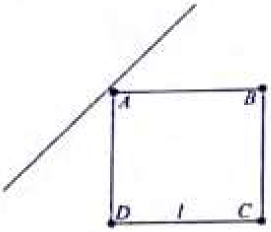
13. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity of
(a) $\frac{\omega m}{m+M}$ (b) $\frac{\omega m}{m+2M}$
(c) $\frac{\omega(m+2M)}{m}$ (d) $\frac{\omega(m-2M)}{m+2M}$
(AIEEE 2006)

14. A force $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point $(1, -1)$ is



(a) $F(\hat{i}+\hat{j})$
(b) $-F(\hat{i}-\hat{j})$
(c) $F(\hat{i}-\hat{j})$
(d) $-F(\hat{i}+\hat{j})$ **(AIEEE 2006)**

15. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is



(a) $3ml^2$ (b) ml^2

(c) $2ml^2$ (d) $\sqrt{3}\, ml^2$ **(AIEEE 2006)**

16. A round uniform body of radius R, mass M, and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is

(a) $\dfrac{g\sin\theta}{1+\dfrac{MR^2}{I}}$ (b) $\dfrac{g\sin\theta}{1-\dfrac{I}{MR^2}}$

(c) $\dfrac{g\sin\theta}{1-\dfrac{MR^2}{I}}$ (d) $\dfrac{g\sin\theta}{1+\dfrac{I}{MR^2}}$

(AIEEE 2007)

17. The angular momentum of a particle rotating with a central force is constant due to

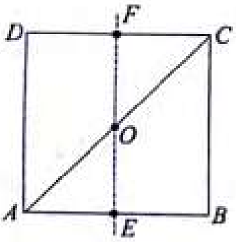
(a) constant linear momentum

(b) zero torque

(c) constant torque

(d) constant force **(AIEEE 2007)**

18. For the given uniform square lamina $ABCD$, whose centre is O,



(a) $I_{AD} = 3I_{EF}$ (b) $I_{AC} = I_{EF}$

(c) $I_{AC} = \sqrt{2}\, I_{EF}$ (d) $\sqrt{2}\, I_{AC} = I_{EF}$

(AIEEE 2007)

19. Consider a uniform square plate of side a and mass M. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(a) $\dfrac{5}{6}Ma^2$ (b) $\dfrac{1}{12}Ma^2$

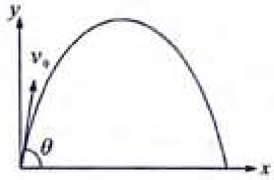
(c) $\dfrac{7}{12}Ma$ (d) $\dfrac{2}{3}Ma^2$ **(AIEEE 2008)**

20. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

(a) $\dfrac{1}{3}\dfrac{l^2\omega^2}{g}$ (b) $\dfrac{1}{6}\dfrac{l\omega}{g}$

(c) $\dfrac{1}{2}\dfrac{l^2\omega^2}{g}$ (d) $\dfrac{1}{6}\dfrac{l^2\omega^2}{g}$ **(AIEEE 2009)**

21. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < (v_0 \sin\theta/g)$, the angular momentum of the particle is



(a) $-mgv_0t^2\cos\theta\,\hat{j}$ (b) $mgv_0t\cos\theta\,\hat{k}$

(c) $-\dfrac{1}{2}mgv_0t^2\cos\theta\,\hat{k}$ (d) $\dfrac{1}{2}mgv_0t^2\cos\theta\,\hat{i}$

(AIEEE 2010)

22. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc

(a) remains unchanged

(b) continuously decreases

(c) continuously increases

(d) first increases and then decreases **(AIEEE 2011)**

23. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(a) $3g/2$ (b) g

(c) $2g/3$ (d) $g/3$ **(AIEEE 2011)**

24. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kgm^2, the number of rotations made by the pulley before its direction of motion is reversed, is:

(a) less than 3

(b) more than 3 but less than 6

(c) more than 6 but less than 9

(d) more than 9 **(AIEEE 2011)**

25. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The

initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

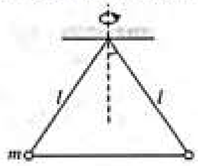
(a) $\frac{r\omega_0}{3}$ (b) $\frac{r\omega_0}{2}$

(c) $r\omega_0$ (d) $\frac{r\omega_0}{4}$ **(JEE Main 2013)**

26. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension,

(a) angular momentum changes in direction but not in magnitude.
(b) angular momentum changes both in direction and in magnitude.
(c) angular momentum is conserved.
(d) angular momentum changes in magnitude but not in direction. **(JEE Main 2014)**

27. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



(a) $\frac{5g}{6}$ (b) g

(c) $\frac{2g}{3}$ (d) $\frac{g}{2}$ **(JEE Main 2014)**

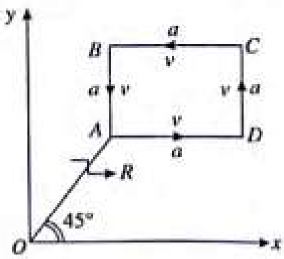
28. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. The moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is

(a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{MR^2}{16\sqrt{2}\pi}$

(c) $\frac{4MR^2}{9\sqrt{3}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

(JEE Main 2015)

29. A particle of mass m is moving along side of a square of side 'a', with a uniform speed v in the x-y plane as shown in the figure:



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(a) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B

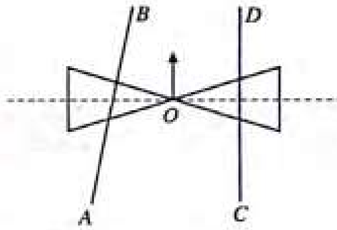
(b) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(c) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(d) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.

(JEE Main 2016)

30. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to:



(a) turn left
(b) turn right
(c) go straight
(d) turn left and right alternately

(JEE Main 2016)

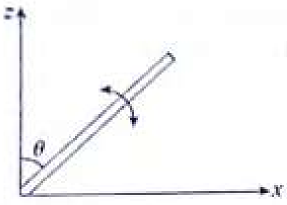
31. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum?

(a) 1 (b) $\frac{3}{\sqrt{2}}$

(c) $\sqrt{\frac{3}{2}}$ (d) $\frac{\sqrt{3}}{2}$

(JEE Main 2017)

32. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is

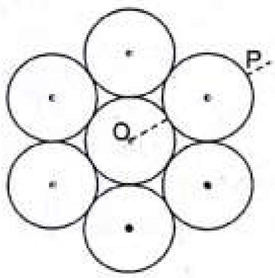


(a) $\frac{3g}{2l}\cos\theta$ (b) $\frac{2g}{3l}\cos\theta$

(c) $\frac{3g}{2l}\sin\theta$ (d) $\frac{2g}{3l}\sin\theta$

(JEE Main 2017)

33. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is

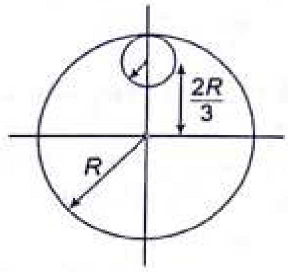


(a) $\frac{181}{2}MR^2$ (b) $\frac{19}{2}MR^2$

(c) $\frac{55}{2}MR^2$ (d) $\frac{73}{2}MR^2$

(JEE Main 2018)

34. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is:



(a) $\frac{37}{9}MR^2$ (b) $4\,MR^2$

(c) $\frac{40}{9}MR^2$ (d) $10\,MR^2$

(JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	**2.** (d)	**3.** (a)	**4.** (a)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (d)
11. (a)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (b)	**20.** (c)
21. (b)	**22.** (c)	**23.** (a)	**24.** (d)	**25.** (c)	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (d)	**30.** (b)
31. (a)	**32.** (d)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (d)	**37.** (b)	**38.** (c)	**39.** (d)	**40.** (d)
41. (c)	**42.** (d)	**43.** (c)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (c)	**48.** (b)	**49.** (b)	**50.** (a)
51. (c)	**52.** (a)	**53.** (c)	**54.** (c)	**55.** (d)	**56.** (b)	**57.** (d)	**58.** (d)	**59.** (a)	**60.** (d)
61. (a)	**62.** (d)	**63.** (b)	**64.** (d)	**65.** (c)	**66.** (c)	**67.** (d)	**68.** (c)	**69.** (b)	**70.** (a)
71. (d)	**72.** (a)	**73.** (b)	**74.** (a)	**75.** (c)	**76.** (d)	**77.** (b)	**78.** (d)	**79.** (b)	**80.** (a)
81. (a)	**82.** (d)	**83.** (d)	**84.** (a)	**85.** (d)	**86.** (b)				

Archives

1. (c)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (b)
11. (a)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (d)	**20.** (d)
21. (c)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (b, d)	**30.** (a)
31. (c)	**32.** (c)	**33.** (a)	**34.** (b)						

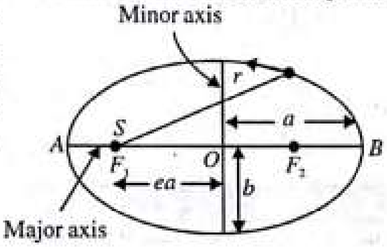
Chapter 10

Gravitation

KEPLER'S LAWS

A heavenly body revolving around a planet in an orbit is called a natural satellite. For example, Moon revolves around the planet earth, so Moon is the satellite of earth. Their motions can be studied with the help of Kepler's laws, as stated in the following:

Law of orbit: Each planet moves around the Sun in an elliptical orbit with the Sun at one of the foci as shown in the figure. The eccentricity of an ellipse is defined as the ratio of the distance SO and AO, i.e., $e = SO/AO$.



$\therefore \quad e = \dfrac{SO}{a}$

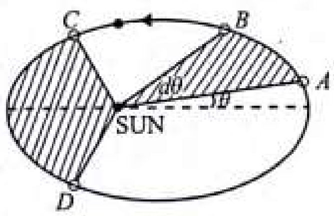
$SO = ea$

The distance of closest approach with the Sun at F_1 is AS. This distance is called perigee. The greatest distance (BS) of the planet from the Sun is called apogee.

Perigee $(AS) = AO - OS = a - ea = a(1 - e)$

Apogee $(BS) = OB + OS = a + ea = a(1 + e)$

Law of areas: The line joining the Sun and a planet sweeps out equal areas in equal intervals of time. A planet takes the same time to travel from A to B as from C to D as shown in the figure (the shaded areas are equal). Naturally, the planet has to move faster from C to D. In other words, areal velocity of a planet is constant at dA/dt = constant. The law of areas is identical with the law of conservation of angular momentum.



$$\text{Areal velocity} = \frac{\text{Area swept}}{\text{Time}}$$

$$= \frac{\frac{1}{2} r\,(r\,d\theta)}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \text{constant}$$

Hence, $\frac{1}{2} r^2 \omega = \text{constant}$

or $\quad mr^2 \omega = \text{constant}$

where $\quad mr^2 = MI$ of the planet relative to Sun $= I$

$I\omega =$ constant or $\vec{L}$ = constant

Law of periods: The square of the time for a planet to complete a revolution about the Sun is proportional to the cube of semimajor axis of the elliptical orbit.

$$T^2 \propto a^3 \qquad \text{(i)}$$

where T = Time period of revolution of the planet a = semi-major axis of the ellipse. The orbits of all the planets except Mercury and Pluto are very close to being circular. Hence, we can take the orbits of planets as sufficiently circular since the semi-major axis becomes the radius of the circle. Hence, Eq. (i) can be written as

$$T^2 = Kr^3$$

ILLUSTRATION 10.1 The mean distance of Mars from the Sun is 1.524 times that of the Earth from the Sun. Find the number of years required for Mars to make one revolution about the Sun.

Solution. For planets revolving around the Sun, $T^2 \propto r^3$

$\Rightarrow \quad \dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$ (T_1 = time period of Mars, T_2 = time period of the Earth)

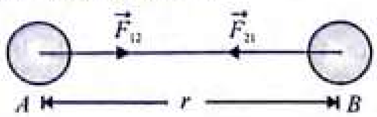
$\Rightarrow \quad \dfrac{T_1}{T_2} = \left(\dfrac{r_1}{r_2}\right)^{3/2}$

$\Rightarrow \quad T_1 = T_2\left(\dfrac{r_1}{r_2}\right)^{3/2} = (1\text{ year})\,(1.524)^{3/2} = 1.88$ years

NEWTON'S LAW OF GRAVITATION

It states that every body in this universe attracts every other body with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

Consider two point mass bodies A and B of masses m_1 and m_2. Let r be the distance between their centres and F be the force of attraction between them.



According to Newton's law of gravitation,

$$F \propto \frac{m_1 m_2}{r^2} \text{ or } F = \frac{G m_1 m_2}{r^2} \quad \text{(ii)}$$

Definition of G

Let $m_1 = m_2 = 1$ and $r = 1$.

From Eq. (ii), $F = G\frac{1\times 1}{1^2} = G$ or $G = F$

Thus universal gravitational constant is equal to the force of attraction acting between two bodies each of unit mass, whose centres are placed unit distance apart.

Gravitational constant is a scalar quantity. Its value is same throughout the universe and is independent of the nature and size of the bodies as well as the nature of the medium between the bodies. The value of G in SI unit is 6.67×10^{11} Nm^{2}kg^{-2} and in CGS system its value is 6.67×10^{-8} dyne cm^2 g^{-2}.

The dimensional formula for G is

$$\frac{Fr^2}{m_1 m_2} = \frac{(MLT^{-2})}{M \times M} = [M^{-1}L^3T^{-2}]$$

NOTE: Newton's law of gravitation holds good for objects lying at very large distances and also at very short distances. It fails when the distance between the objects is less than 10^{-9} m (i.e., of the order of intermolecular distances).

Evidence in Support of Newton's Law of Gravitation

(1) The rotation of earth around the Sun or the rotation of Moon around the earth is explained on the basis of this law.

(2) The prediction about solar and lunar eclipses, made on the basis of this law, always came out to be true.

(3) The formation of tides in the ocean is due to force of attraction between the Moon and ocean water.

(4) The prediction about the orbits and time period of the modern artificial satellites made on the basis of this law are found to be very accurate.

Important Characteristics of Gravitational Force

According to Newton's law of gravitation, the gravitational force of attraction between two given bodies

(1) is independent of nature of intervening medium.

(2) is independent of the presence or absence of other bodies.

(3) is independent of nature and size of the bodies, till their masses remain the same and the distance between their centres is fixed.

(4) forms action and reaction pair, i.e., the gravitational forces are equal in magnitude and opposite in direction and hence obey Newton's third law of motion.

(5) is a central force as it acts along the line joining the centres of the two bodies.

(6) is conservative in nature.

ILLUSTRATION 10.2 The mass of planet Jupiter is 1.9×10^{27} kg and that of the Sun is 1.99×10^{30} kg. The mean distance of Jupiter from the Sun is 7.8×10^{11} m. Calculate the gravitational force which Sun exerts on Jupiter. Assuming that Jupiter moves in circular orbit around the Sun, also calculate the speed of Jupiter. G = 6.67×10^{-11} Nm^2 kg^{-2}.

Solution. Here, $M_j = 1.9 \times 10^{27}$ kg

$$M_s = 1.99 \times 10^{30} \text{ kg}; r = 17.8 \times 10^{11} \text{ m}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} : F = ?$$

Now $$F = \frac{GM_J M_S}{r^2} = \frac{6.67 \times 10^{11} \times 1.9 \times 10^{27} \times 1.99 \times 10^{30}}{(7.8 \times 10^{11})^2}$$

$$= 4.15 \times 10^{23} \text{ N}$$

Since the gravitational pull of the Sun on Jupiter provides the required centripetal force to Jupiter, therefore the velocity of Jupiter v can be given by the relation,

$$F = \frac{M_J v^2}{r}$$

or $$v = \sqrt{\frac{F_r}{M_J}} = \sqrt{\frac{4.15 \times 10^{23} \times 7.8 \times 10^{11}}{1.9 \times 10^{27}}} = 1.3 \times 10^4 \text{ ms}^{-1}$$

ILLUSTRATION 10.3 Gravitational force between two point masses m and M separated by a distance r is F. Now if a point mass $3m$ is placed next to m, what will be the (a) force on M due to m and (b) total force on M?

Solution.

(a) If r is the distance between two point masses m and M, then the gravitational force on m due to mass M is $F = GMm/r^2$. Since the gravitational force between two point masses is independent of the presence of other masses, so if a point mass $3m$ is placed next to m, the force on M due to m is $F = \frac{GMm}{r^2}$.

(b) Total force on the body of mass M is F, i.e.,

$$F = \frac{GM \times (m + 3m)}{r^2} = \frac{4GMm}{r^2} = 4F$$

INERTIAL AND GRAVITATIONAL MASSES

The mass of a body is the quantity of matter possessed by a body. There are two different concepts about the mass of body as discussed below.

Inertial Mass

Inertial mass of a body is related to its inertia of linear motion and is defined by Newton's second law of motion.

Let a body of mass m_i move with acceleration a under the action of an external force F. According to Newton's second law of motion,

$$F = m_i a \text{ or } m_i = \frac{F}{a} \quad \text{(i)}$$

The mass m_i of the body in this sense is the inertial mass of the body. If $a = 1$, then from Eq. (i) $m_i = F$.

Thus, inertial mass of a body is equal to the magnitude of external force required to produce unit acceleration in the body.

In fact, inertial mass of a body is the measure of the ability of the body to oppose the production of acceleration in its motion by an external force.

Properties of Inertial Mass

(1) Inertial mass of a body is proportional to the quantity of matter contained in the body.

(2) Inertial mass of a body is independent of size, shape and state of the body.

(3) Inertial mass of a body does not depend upon the temperature of the body.

(4) Inertial mass of a body is not affected by the presence or absence of other bodies near it.

(5) Inertial mass is conserved when the two bodies combine physically or chemically.

(6) Inertial mass can be added by simple laws of algebra, irrespective of the materials of the bodies.

(7) Inertial mass of a body increases with the speed of the body. When a body moves with a velocity v, its inertial mass m is given by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad \text{(ii)}$$

where m_0 is the rest mass of the body, c is the velocity of light in vacuum. The mass m is affected only when the velocity of the body is comparable with the velocity of light.

Gravitational Mass

Gravitational mass of a body is related to gravitational pull on the body and is defined by Newton's law of gravitation.

If a body of mass m_G is placed on the surface of earth of radius R and mass M, then gravitational pull on the body is given by

$$F = \frac{GMm_G}{R^2} \text{ which gives } m_G = \frac{F}{(GM/R^2)} = \frac{F}{I} \quad \text{(iii)}$$

The mass m_G of the body in this sense is the gravitational mass of the body. The inertia of the body has no effect on the gravitational mass of the body. If $I = 1$, then from Eq. (iii), $m_G = F$. Thus, gravitational mass of a body is defined as the magnitude of gravitational pull experienced by the body in a gravitational field of unit intensity.

The properties of gravitational mass are the same as those of inertial mass except those mentioned below in the comparison of inertial mass and gravitational mass.

Comparison of Inertial and Gravitational Mass

(1) Both inertial and gravitational masses are measured in the same units.

(2) Both are scalar quantities.

(3) Both do not depend on the shape and state of the body.

(4) Gravitational mass of a body is affected by the presence of other bodies near it whereas the inertial mass of a body remains unaffected by the presence of other bodies near it.

(5) The inertial mass of a body is measured by finding its acceleration, produced by the applied force and gravity plays no part in the measurement. On the other hand, the gravitational mass of a body is measured by finding the gravitational pull on it and inertia plays no part in the measurement.

(6) The gravitational mass is measured by spring balance whereas inertial mass is measured by inertial balance.

(7) Both are equivalent to each other.

Concept Application Exercise 10.1

1. A mass M is broken into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum?
2. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, then find the relation between T and R.
3. A planet of mass m moves around the Sun of mass M in an elliptical orbit. The maximum and minimum distance of the planet from the Sun are r_1 and r_2, respectively. Find the relation between the time period of the planet in terms of r_1 and r_2.
4. Suppose the gravitational force varies inversely as the n^{th} power of the distance. Show that the time period of a planet in circular orbit of radius R around the Sun will be proportional to $R^{(n+1)/2}$.
5. The distance of planet Jupiter from the Sun is 5.2 times that of the earth. Find the period of revolution of Jupiter around the Sun.
6. The distance of the two planets from the Sun are 10^{13} m and 10^{12} m, respectively. Find the ratio of time periods and speeds of the two planets.

INTENSITY OF GRAVITATIONAL FIELD

The intensity of the gravitational field of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field. It is always directed towards the centre of gravity of the body whose gravitational

field is considered. Intensity of the gravitational field at a point is a vector quantity and is denoted by $\vec{E}$.

Let M be the mass of a body with centre of gravity at O. Let F be the gravitational force of attraction experienced by a test mass m_0 when placed at P in the gravitational field of the body, such that $OP = x$. According to Newton's law of gravitation,

$$F = MG\, m_0/x^2$$

Intensity of gravitational field at P will be

$$E = F/m_0 = \frac{GMm_0/x^2}{m_0} = \frac{GM}{x^2} \qquad \text{(i)}$$

In vector form,

$$\vec{E} = -\frac{Gm}{x^2}\hat{x} \qquad \text{(ii)}$$

Here the negative sign shows that the gravitational intensity is of attractive force.

It means that the intensity of gravitational field at a point in the gravitational field is equal to the acceleration of test mass placed at that point.

If the gravitational field is due to earth and point P lies on the surface of the earth then $x = R$. Now

$$E = \frac{GM}{R^2} = g$$

where g is the acceleration due to gravity at the surface of earth.

Unit of intensity of gravitational field in SI system is N kg^{-1} or m s^{-2} and in CGS system, it is dyne g^{-1} or cm s^{-2}.

Dimensional formula of gravitational intensity

$$E = F/m_0 = \frac{[MLT^{-2}]}{[M]} = [M^0LT^{-2}]$$

Acceleration Due to Gravity

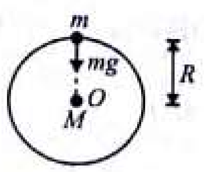
If a force F acting on a body of mass m produces an acceleration a then according to Newton's second law of motion $F = ma$. If the force on the body is due to gravity of earth, then acceleration in the body is called acceleration due to gravity, which is denoted by g, i.e., $a = g$. Then,

$$F = mg \text{ or } g = \frac{F}{m}$$

If $m = 1$, then $g = \frac{F}{1} = F$. Thus, acceleration due to gravity is defined as the force of gravity acting on a unit mass of the body placed on or near the surface of the earth.

Relation Between g and G

Consider earth to be a spherical body of mass M, radius R with centre O. Suppose a body of mass m is placed on the surface of earth, where acceleration due to gravity is g.



The force on the body of mass m, outside the surface of earth is due to earth whose mass M is concentrated at the centre O. Let F be the force of attraction between the body and the earth.

According to Newton's law of gravitation

$$F = \frac{GMm}{R^2}$$

From gravity pull, $F = mg$

$$\therefore \quad mg = \frac{GMm}{R^2} \quad \Rightarrow \quad g = \frac{GM}{R^2} \qquad \text{(i)}$$

From Eq. (i), we note that the value of acceleration due to gravity is independent of mass, shape and size of the body but depends upon mass and radius of the earth.

Mass and Density of Earth

Mass of Earth

From Eq. (i), we find

$$M = \frac{gR^2}{G} \qquad \text{(i)}$$

We know that $R = 6.4 \times 10^6$ m

$$G = 6.66 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

and $\quad g = 9.8 \text{ ms}^{-2}$

Then, $M = \dfrac{9.8 \times (6.4 \times 10^6)^2}{6.66 \times 10^{-11}} = 6.018 \times 10^{24}$ kg

Density of Earth

Consider earth to be a spherical body of radius R. Let ρ be the uniform density of the material of the earth.

Now, density $\rho = \dfrac{\text{Mass}}{\text{Volume}}$

$$= \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\rho = \frac{\frac{3gR^2}{G}}{4\pi R^3} = \frac{3g}{4\pi RG} \qquad \text{(iii)}$$

We know that g = 9.8 ms^{-2}

$$R = 6.4 \times 10^6 \text{ m}$$

$$G = 6.66 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

Hence, $\rho = \dfrac{3 \times 9.8}{4 \times \frac{22}{7} \times 6.4 \times 10^6 \times (6.67 \times 10^{-11})}$

$$= 5.4783 \times 10^3 \text{ kg m}^{-3} \approx 5.5 \times 10^3 \text{ kg m}^{-3}$$

Variation of Acceleration Due to Gravity

The value of acceleration due to gravity changes with height (i.e., altitude), depth, shape of the earth and rotation of earth about its own axis. The effect of each of the above factors on the value of g has been discussed below.

Effect of Altitude

Consider earth to be a sphere of mass M, radius R with centre at O. Let g be the value of acceleration due to gravity at a point A on the surface of earth.

$$g = \frac{GM}{R^2} \quad \text{(iv)}$$

If g' is the acceleration due to gravity at point B, at a height h above the surface of earth, then the force on the body at B is due to earth whose mass M is concentrated at the centre O of earth.

$$g' = \frac{GM}{(R+h)^2} \quad \text{(v)}$$

Dividing Eq. (v) by Eq. (iv), we get

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM} = \frac{R^2}{(R+h)^2} \quad \text{(vi)}$$

$$= \frac{R^2}{R^2(1+\frac{h}{R})^2} = \left(1+\frac{h}{R}\right)^{-2}$$

If $h \ll R$, then h/R is very small compared to 1. Expanding the R.H.S. of the above equation by Binomial theorem and neglecting the square and higher powers of h/R, we get

$$\frac{g'}{g} = 1 - \frac{2h}{R}$$

$$g' = g\left(1 - \frac{2h}{R}\right) \quad \text{(vii)}$$

From Eqs. (vi) and (vii), we note that the value of acceleration due to gravity decreases with height.

NOTE:

- Relation (vi) is used to find the value of acceleration due to gravity at a height h, when h is comparable to the radius of earth R and relation (vii) is used to find g' when h is very small as compared to R.
- With height h, the decrease in the value of g is

  $$g - g' = \frac{2hg}{R}$$

  Therefore, fractional decrease in the value of g is

  $$\frac{g' - g}{g} = \frac{2h}{R}$$

  Therefore, % decrease in the value of g is

  $$\left(\frac{g-g'}{g}\right) \times 100 = \frac{2h}{R} \times 100$$

ILLUSTRATION 10.4 How much above the surface of earth does the acceleration due to gravity reduce by 36% of its value on the surface of earth? Radius of earth = 6400 km.

Solution. Since the acceleration due to gravity reduces by 36%, the value of acceleration due to gravity there is $100 - 36 = 64\%$. It means, $g' = \frac{64}{100} g$. If h is the height of location above the surface of earth, then

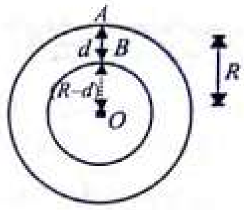
$$g' = g\frac{R^2}{(R+h)^2}$$

or $$\frac{64}{100} g = g\frac{R^2}{(R+h)^2}$$

$$\frac{8}{10} = \frac{R}{R+h} \quad \text{or} \quad 8R + 8h = 10R$$

$$\Rightarrow \quad h = \frac{2R}{8} = \frac{R}{4} = \frac{6.4 \times 10^6}{4} = 1.6 \times 10^6 \text{ m}$$

Effect of Depth



Consider earth to be a homogeneous sphere of radius R and mass M with centre at O. Let g be the value of acceleration due to gravity at a point A on the surface of the earth. Then

$$g = \frac{GM}{R^2}$$

If ρ is uniform density of the material of earth, then

$$M = \frac{4}{3}\pi R^3 \rho$$

$$g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2} = \frac{4}{3}\pi G R \rho \quad \text{(viii)}$$

Let g' be the acceleration due to gravity at the point B at a depth d below the surface of earth. The distance of the point B from the centre of the earth is $(R - d)$. The earth can be supposed to be made of (i) a smaller sphere of radius $(R - d)$ and, (ii) a spherical shell of thickness d.

$$g' = \frac{GM'}{(R-d)^2}$$

and $$M' = \frac{4}{3}\pi (R-d)^3 \rho$$

$$g' = \frac{G \times \frac{4}{3}\pi (R-d)^3 \rho}{(R-d)^2} = \frac{4}{3}\pi G (R-d) \rho \quad \text{(ix)}$$

Dividing Eq. (ix) by Eq. (viii), we get

$$\frac{g'}{g} = \frac{\frac{4}{3}\pi G(R-d)\rho}{\frac{4}{3}\pi G R \pi} = \frac{R-d}{R} = \frac{R}{R} - \frac{d}{R}$$

$$\Rightarrow \quad g' = g\left(1 - \frac{d}{R}\right) \quad \text{(x)}$$

From Eq. (x), we note that the value of acceleration due to gravity decreases with depth.

At the centre of the earth, $d = R$, $g' = g_0$ (say)

From Eq. (x) we get,

$$g' = g\left(1 - \frac{d}{R}\right) \quad \text{(xi)}$$

It means the acceleration due to gravity is zero at the centre of the earth. Therefore, the weight of the body of mass m at the centre of the earth $= mg_0 = 0$, but the mass of the body will not be zero. Thus, the value of acceleration due to gravity is maximum at the earth's surface and becomes zero at the centre of the earth.

NOTE: Decrease in the value of g with depth d is $g - g' = \frac{dg}{R}$.

Therefore, fractional decrease in the value of g with depth d

$$\frac{g-g'}{g} = \frac{d}{R} \quad \text{(xii)}$$

Therefore, % decrease in the value of g

$$\frac{g-g'}{g} \times 100 = \frac{d}{R} \times 100 \quad \text{(xiii)}$$

ILLUSTRATION 10.5 Find the percentage decrease in the weight of the body when taken to a depth of 32 km below the surface of earth. Radius of the earth is 6400 km.

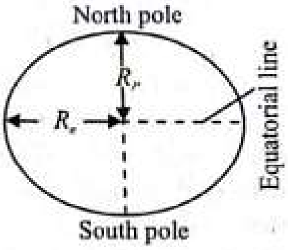
Solution. Here, $d = 32$ km; $R = 6400$ km

Weight of body at depth d is $= mg' = mg\left(1 - \frac{d}{R}\right)$

$$\% \text{ decrease in weight } = \frac{mg - mg'}{mg}$$

$$= \frac{d}{R} \times 100 = \frac{32}{6400} \times 100 = 0.5\%$$

Effect of Shape of Earth



Earth is not a perfect sphere. It is flattened at the poles and bulges out at the equator. Equatorial radius R_e of the earth is about 21 km greater than the polar radius R_P.

Now,

$$g = \frac{GM}{R^2}$$

Since G and M are constants. Therefore,

$$g \propto \frac{1}{R^2}$$

Thus, we conclude that the value of g is least at the equator and maximum at the pole. It means, the value of acceleration due to gravity increases as we go from equator to the pole.

At sea level, the value of g at pole is greater than its value at equator by 1.80 cm s^{-2}.

ILLUSTRATION 10.6 (a) Find the height from the earth's surface where g will be 25% of its value on the surface of earth ($R = 6400$ km). (b) Find the percentage increase in the value of g at a depth h from the surface of earth.

Solution.

(a) At a height h above the earth's surface, we have

$$g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{g}{4} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{2}$$

$$\Rightarrow \quad h = R = 6400 \text{ km}$$

(b) At a depth h below earth's surface, we have

$$g' = g\left(1 - \frac{h}{R}\right) \Rightarrow \frac{g'}{g} = 1 - \frac{4}{6400}$$

$$\frac{g-g'}{g} = \frac{4}{6400} \Rightarrow \frac{\Delta g}{g} = \frac{1}{16}$$

$$\therefore \% \text{ age} = \frac{\Delta g}{g} \times 100 = \frac{1}{16} \% \text{ increase}$$

CONCEPT APPLICATION EXERCISE 10.2

1. If a planet of given density were made larger, its force of attraction for an object on its surface would increase because of the greater distance from the object to the centre of the planet. Which effect predominates?
2. A stone is dropped along the centre of a deep vertical mine shaft. Assume no air resistance but consider the earth's rotation. Will the stone continue along the centre of the shaft? If not, describe the motion.
3. Show that if the earth were not rotating about its axis the value of g at the equator would exceed its present value by 3.36 cm/s^{-2}. Given the radius of earth $= 6.37 \times 10^6$ m and angular speed of earth $= 7.27 \times 10^{-5}$ rad/s^{-1}.
4. How far away from the earth does the acceleration due to gravity become 10% of its value on earth's surface? Radius of earth $= 6.37 \times 10^6$ m.
5. (a) Assuming the earth to be a sphere of uniform density, calculate the value of acceleration due to gravity at a point (i) 1600 km above the earth, (ii) 1600 km below the earth, (b) Also find the rate of variation of acceleration due to gravity above and below the earth's surface. Radius of earth = 6400 km. $g = 9.8$ m/s^2.
6. How much faster than its present speed should the earth (radius 6.37×10^6 m) rotate so that the bodies lying on the equator may fly off into space? (At equator, $g = 9.78$ m/s^{-2}.)

GRAVITATIONAL POTENTIAL

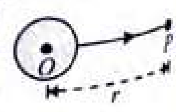
Gravitational potential at a point in a gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

If W is the amount of work done in bringing a body of mass m_0 from infinity to a point P in the gravitational field without acceleration, then gravitational potential at P is $V = W/m_0$

Gravitational potential is a scalar quantity, since work done is a scalar. Unit of gravitational potential at a point in SI system is J kg^{-1} and in cgs system it is erg g^{-1}. Dimensional formula for gravitational potential is given by

$$\frac{W}{m_0} = \frac{[ML^2T^{-2}]}{[M]} = [M^0L^2T^{-2}]$$

Expression for Gravitational Potential at a Point



Let earth to be a perfect sphere of radius R and mass M. The mass of the earth can be supposed to be concentrated at its centre O. Let us calculate the gravitational potential at any point P, where $OP = r$ and $r > R$.

$$V_p = -\frac{GM}{r} \quad \text{(i)}$$

From Eq. (i), we note the following:

(1) The gravitational potential at a point is always negative.

(2) When $r = \infty$, from (1), $V_P = 0$, hence gravitational potential is maximum (= zero) at infinity.

(3) At the surface of earth, $r = R$, therefore $V_P = -GM/R$ at the surface of earth.

The unit of gravitational potential is J kg^{-1}. Dimensional formula of gravitational potential is

$$\frac{\text{Work}}{\text{Mass}} = \frac{[ML^2T^{-2}]}{[M]} = [M^0L^2T^{-2}]$$

Gravitational potential is a scalar field because at any point in space it has only magnitude and no direction.

Relation Between Gravitational Field and Potential

The work done by an external agent to move unit mass from a point to another point in the direction of the field E, slowly through an infinitesimal distance dr = Force by external agent x distance moved = $-Edr$.

Thus, $\quad dV = -Edr \Rightarrow E = \frac{-dV}{dr}$

Therefore, gravitational field at any point is equal to the negative gradient of potential at that point.

GRAVITATIONAL POTENTIAL ENERGY

We know that potential energy of a body at a given position is defined as the energy stored in the body at that position. If the position of a body changes on account of the forces acting on it, then change in its potential energy is equal to the amount of work done on the body by the forces acting on it, for the given change in position. As this work done is independent of the path followed, so the gravitational forces are conservative forces.

When the body is at infinity with respect to another body, the gravitational attraction on the body is zero. Therefore, its potential energy is zero, which is called zero level of potential energy.

If W_0 is the zero level of potential energy of the body and W is the work done in raising the body from the position of zero level potential energy to present location, then PE of the body $= W + W_0$.

If initially the body is at infinity, then the zero level of PE is zero, i.e., $W_0 = 0$. Now, PE of the body = W.

Gravitational potential energy of a body at a point in a gravitational field of another body is defined as the amount of work done in bringing the given body from infinity to that point without acceleration.

Expression for Gravitational Potential Energy

Consider a gravitational field due to earth of mass M and radius R. The mass of the earth can be supposed to be concentrated at its centre O. Let us calculate the gravitational potential energy of the body of mass m placed at point P in the gravitational field, where $OP = r$ and $r > R$. Join OP and produce it onwards. Take two points A and B on this line such that $OA = x$ and $AB = dx$.

The gravitational force of attraction on the body at A will be $F = GMm/x^2$.

Small amount of work done in bringing the body without acceleration through a very small distance AB (= dx) is given by

$$dW = Fdx = \frac{GMm}{x^2}dx$$

Total work done in bringing the body from infinity to point P is given by

$$W = \int_{\infty}^{r} \frac{GMm}{x^2}dx = -GMm\int_{\infty}^{r} x^{-2}dx$$

$$= -GMm\left[\frac{1}{x}\right]_{\infty}^{r} = -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$= \frac{-GMm}{r} \quad \text{(ii)}$$

Since this work done is stored in the body as its gravitational potential energy U, therefore gravitational potential energy of the body of mass m placed at point P in the gravitational field of earth of mass M at distance r will be

$$U = \frac{-GMm}{r} \quad \text{(iii)}$$

From Eqs. (i) and (iii), we note that $U = V_p \times m$.

That is, gravitational potential energy = gravitational potential × mass of the body.

NOTE:

- The gravitational potential energy of a point mass m at a distance r (>R) from the centre of earth is
$$U = mV = -\frac{GMm}{r}$$
- Gravitational potential energy of a point mass m relative to infinity at the surface of earth will be
$$U = -\frac{GMm}{R} \quad (\because r = R)$$
- The potential energy of a point mass m at the centre of earth relative to infinity will be given by
$$U = mV_C = -\frac{3}{2}\frac{GMm}{R} \quad \left[\because V_C = \frac{3}{2}V_S = -\frac{3GM}{2R}\right]$$
The potential energy of a body at the centre of earth is minimum but not zero.
- If there is a gravitational field due to point masses $m_1, m_2, m_3, \ldots$ then gravitational potential energy of a point mass body m_0 at a point in the gravitational field is
$$U = -\left[\frac{Gm_0m_1}{r_{01}} + \frac{Gm_0m_2}{r_{02}} + \frac{Gm_0m_3}{r_{03}} + \cdots\right]$$

ILLUSTRATION 10.7 If g is the acceleration due to gravity on the surface of earth, find the gain in potential energy of an object of mass m raised from the surface of earth to a height equal to the radius R of the earth.

Solution. Let M, R be the mass and radius of the earth. Then

$$g = GM/R^2$$

or $$GM = gR^2 \quad \text{(i)}$$

Potential energy of the object on the surface of earth

$$U_1 = \frac{-GMm}{R}$$

The potential energy of the object at a height equal to radius of the earth is

$$U_2 = \frac{-GMm}{2R}$$

Gain in PE is

$$U_2 - U_1 = -\frac{GMm}{2R} + \frac{GMm}{R}$$

$$= \frac{GMm}{2R} = \frac{(gR^2)m}{2R} = \frac{1}{2} mg\,R$$

ILLUSTRATION 10.8 Two particles of masses m and M are initially at rest at an infinite distance apart. They move towards each other and gain speeds due to gravitational attraction. Find their speeds when the separation between the masses becomes equal to d.

Solution. Let v_1 and v_2 be the speeds of two masses m and M, respectively, when they are at a separation d.

As they approach each other, the kinetic energy increases and GPE decreases. Hence, for the system,

Loss in GPE = Gain in KE

$$\Rightarrow \quad (\text{GPE})_i - (\text{GPE})_f = \text{KE}_f - \text{KE}_i$$

$$\Rightarrow \quad 0 - \left(-\frac{GMm}{d}\right) = \left(\frac{1}{2} mv_1^2 + \frac{1}{2} Mv_2^2\right) - 0$$

$$\frac{GMm}{d} = \frac{1}{2} mv_1^2 + \frac{1}{2} Mv_2^2$$

As there is no external force on this system, its total momentum remains conserved.

$$p_i = p_f$$

$$0 = mv_1 - Mv_2$$

Combining the two equations, we have

$$v_1 = \sqrt{\frac{2GM^2}{d(m+M)}}$$

and $$v_2 = \sqrt{\frac{2GM^2}{d(m+M)}}$$

Escape Speed

The minimum speed required to project a body from the surface of the earth so that it never returns to the surface of the earth is called escape speed. If a velocity greater than the escape velocity is imparted, the body will escape and leave the surface. If a velocity lesser than the escape velocity is given, it will fall back to the surface or be in an orbit. A body thrown with escape speed goes out of the gravitational pull of the earth. Work done to displace the body from the surface of the earth ($r = R_e$) to infinity ($r = \infty$) is given by

$$\int dW = \int_{R_e}^{\infty} \frac{GM_e m}{r^2}\, dr$$

or $$W = GM_e m \int_{R_e}^{\infty} \frac{1}{r^2}\, dr = -GM_e m \left[\frac{1}{r}\right]_{R_e}^{\infty}$$

$$= -GM_e m \left[\frac{1}{\infty} - \frac{1}{R_e}\right] \Rightarrow W = \frac{GM_e m}{R_e}$$

Let v_e be the escape speed of the body of mass m, then kinetic energy of the body is given by

$$\frac{1}{2} mv_e^2 = \frac{GM_e m}{R_e} \Rightarrow v_e = \sqrt{2gR_e} = 11.2 \text{ km s}^{-1}$$

☞ Important Results

1. Escape speed depends on the mass and size of the planet. That is why escape velocity on the Jupiter is more than that on the earth.
2. Escape speed is independent of the mass of the body.
3. Any body thrown upward with escape speed starts moving around the Sun.
   (a) The value of escape speed of a body does not depend upon the mass (m) of the body and its angle of projection from the surface of earth or any other planet.
   (b) The value of escape speed depends upon the mass and radius of the planet from the surface of which the body is to be projected. Clearly, the value of escape speed of a body will be different for different planets.
   (c) For earth, $g = 9.8 \text{ ms}^{-2}$, $R = 6.4 \times 10^6$ m.
   $$\therefore \quad v_e = \sqrt{2gR_e} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$
   $$= 11.2 \times 10^3 \text{ ms}^{-1} = 11.2 \text{ km s}^{-1}$$
   (d) If a body is projected from a planet with a speed v which is smaller than the escape speed v_e (i.e., $v < v_e$) then the body will reach a certain height, may either move in an orbit around the planet or may fall back to the planet.
   (e) If speed of projection (v) of the body from the surface of a planet is greater than the escape speed (v_e) of that planet, the body will escape out from the gravitational field of that planet and will move in the interstaller field of that planet with speed v' which can be obtained using law of conservation of energy, according to which
   $$\frac{1}{2} mv^2 + \left(-\frac{GMm}{R}\right) = \frac{1}{2} mv'^2 + 0$$
   $$v'^2 = v^2 - \frac{2GM}{R} = v^2 - v_e^2 \quad \left[\because v_e = \sqrt{\frac{2GM}{R}}\right]$$
   $$v' = \sqrt{v^2 - v_e^2}$$

NOTE:
- In deriving the expression for escape speed, we have neglected the resistance to the body by earth's atmosphere. Therefore in reality, the value of escape speed is slightly greater than the value calculated from these relations.
- Escape speed of Sun is 618 km/s.

ILLUSTRATION 10.9 The escape speed from earth's surface is 11 km s^{-1}. A certain planet has a radius twice that of earth but its mean density is the same as that of the earth. Find the value of the escape speed from the planet.

Solution. Let M, R, ρ be the mass, radius and density of earth and M, R, ρ be the corresponding values for the planet. Then escape speed from earth's surface is

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8\pi G\rho R^2}{3}}$$

Escape speed from planet's surface is

$$v_e = \sqrt{\frac{8\pi G\rho (2R)^2}{3}} = 2\sqrt{\frac{8\pi G\rho R^2}{3}} = 2v_e = 2 \times 11 = 22 \text{ km/s}$$

ILLUSTRATION 10.10 A spaceship goes into a circular orbit close to the earth's surface. What additional velocity must be imparted to the ship so that it is able to escape the gravitational pull of the earth? ($R = 6400$ km, $g = 9.8$ m s^{-2})

Solution. The orbital velocity in a circular orbit close to the earth is $v = \sqrt{gR}$.

The velocity required to escape $v_e = \sqrt{2gR}$.

Hence additional velocity required is $v_e - v = (\sqrt{2} - 1)\sqrt{gR}$.

Therefore,

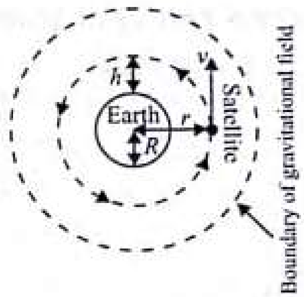
$$v_e - v = 0.414 \times \sqrt{9.8 \times 6400 \times 10^3}$$
$$= 3278.71 \text{ m/s} = 3.278 \text{ km s}^{-1}$$

CONCEPT APPLICATION EXERCISE 10.3

1. The radius of a planet is twice that of the earth but its average density is the same. If the escape speed at the planet and at the earth are v_p and v_e, respectively, then prove that $v_p = 2v_e$.
2. A particle is projected vertically upwards from the surface of the earth (radius R) with a kinetic energy equal to half of the minimum value needed for it to escape. Find the height to which it rises above the surface of earth.
3. What is the escape velocity of the object, if the magnitude of the potential energy per unit mass of the object at the surface of earth is E?
4. A body is released at a distance far away from the surface of the earth. Calculate its speed when it is near the surface of earth. Given $g = 9.8$ m s^{-2}, radius of earth $R = 6.37 \times 10^6$ m.
5. A rocket is launched from the surface of earth with an initial speed of 10 km s^{-1}. How far above the surface of earth would it go? Radius of earth = 6400 km. Neglect atmospheric resistance.
6. Two earth satellites A and B, each of mass m are to be launched into circular orbits about earth's centre at altitudes 6400 km and 19200 km, respectively. The radius of earth is 6400 km. Find (i) the ratio of potential energy of B to that of A, (ii) ratio of kinetic energy of B to that of A and (iii) which one has greater total energy?
7. Calculate the radius of an isolated sphere of density 3.0 g cm^{-3} from the surface of which the escape velocity be 40 m s^{-1}.

ORBITAL SPEED, TIME PERIOD AND HEIGHT OF SATELLITE

Orbital Speed



Orbital speed of a satellite is the minimum speed required to put the satellite into a given orbit around earth. The value of orbital speed is different for different orbits around earth and is independent of the mass of the satellite. Let

M = mass of earth
R = radius of earth
m = mass of the satellite
v = orbital speed of the satellite
h = height of the satellite above the surface of earth
r = radius of the orbit of the satellite = $R + h$

The centripetal force required to keep the satellite in its orbit,

$$F = \frac{mv^2}{r}$$

According to Newton's law of gravitation, gravitational pull acting on the satellite = $GM\,m/r^2$.

In equilibrium, the gravitational pull provides the required centripetal force to the satellite. Therefore,

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}} \qquad \text{(i)}$$

If g is the value of acceleration due to gravity on the surface of earth, then

$$g = \frac{GM}{R^2} \text{ or } GM = gR^2$$

Putting this value in Eq. (i), we get

$$v = \sqrt{\frac{gR^2}{r}} = R\sqrt{\frac{g}{R + h}} \qquad \text{(ii)}$$

It is clear from Eq. (ii) that:

- The orbital speed of a satellite is independent of the mass of the satellite. It decreases with an increase in the radius of orbit or increase in the height of the satellite. It depends upon the mass and radius of the earth/planet around which the revolution of the satellite is taking place.

- The direction of orbital speed of a satellite at an instant is along the tangent to the orbital path of the satellite at that instant.
- When a satellite is orbiting very close to the surface of earth $h \ll R$, then

$$r = R + h \approx R \text{ and } v = v_0 \text{ (say)}$$

From Eq. (ii), $v_0 = R\sqrt{g/R} = \sqrt{gR}$

Substituting $g = 9.8 \text{ ms}^{-2}$

and $R = 6.4 \times 10^6$ m

We get $v_0 = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 \times 10^3 \text{ m s}^{-1}$
$= 7.92 \text{ km s}^{-1}$

Time Period of Satellite

It is the time taken by a satellite to complete one revolution around the earth and is denoted by T. Thus

$$T = \frac{\text{Distance travelled in one revolution}}{\text{Orbital velocity}} = \frac{2\pi r}{v}$$

$$= \frac{2\pi r}{R}\sqrt{\frac{r}{g}} = \frac{2\pi}{R}\sqrt{\frac{r^3}{g}} = \frac{2\pi r}{R}\sqrt{\frac{(R+h)^3}{g}} \quad \text{(iii)}$$

If the earth is supposed to be a sphere of mean density ρ, then the mass of the earth is

$$M = \frac{4}{3}\pi R^3 \rho \text{ and } g = \frac{GM}{R^2} = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right) = \frac{4\pi a R\pi}{3}$$

Substituting this value of g in Eq. (iii), we get

$$T = \frac{2\pi}{R}\sqrt{\frac{3(R+h)^3}{4\pi G R\rho}} = \sqrt{\frac{4\pi^2}{R^2} \times \frac{3(R+h)^3}{4\pi G R \rho}}$$

$$= \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}} \quad \text{(iv)}$$

For a satellite orbiting close to the surface of earth, $h \ll R$.

$\therefore \quad h + R \approx R$

From Eq. (iii), $T = \sqrt{\frac{3\pi}{G\rho}}$

From Eq. (iv), $T = \frac{2\pi}{R}\sqrt{\frac{R^3}{g}} = 2\pi\sqrt{\frac{R}{g}} \quad \text{(v)}$

By substituting $g = 9.8 \text{ ms}^{-2}$ and $R = 6.4 \times 10^6$ m in Eq. (v), we get the value of $T = 5.08 \times 10^3 \text{ s} = 84.6$ min.

It means a satellite orbiting close to the surface of the earth has a time period of revolution about 84.6 min.

It is clear from Eq. (iv) that the period of revolution of a satellite depends only upon its height above the earth's surface. The larger is the height of a satellite above the surface of earth, greater will be its period of revolution.

Altitude or Height of Satellite Above the Earth's Surface

Squaring both sides of Eq. (iii), we get

$$T^2 = \frac{4\pi^2(R+h)^3}{R^2 g} \Rightarrow (R+h)^3 = \frac{T^2R^2g}{4\pi^2}$$

$$R + h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} \Rightarrow h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R \quad \text{(vi)}$$

Knowing the values of T, R and g, the height h of the satellite above the surface of earth can be calculated from Eq. (vi).

Energy of an Orbiting Satellite

The total mechanical energy of a satellite revolving around the earth is the sum of its potential energy (U) and kinetic energy (K).

The potential energy of a satellite is due to its position with respect to earth. It appears because of the gravitational pull acting on the satellite due to earth. If a satellite of mass m is revolving around the earth of mass M, radius R, with orbital velocity v, in an orbit of radius r, then potential energy of the satellite is

$$U = -GMm/r$$

The kinetic energy of a satellite,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{GM}{r}\right) \qquad [\because v = \sqrt{GM/r}]$$

Thus, total mechanical energy of a satellite is $E = U + K$

$$= -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r} = -\frac{GMm}{2r}$$

$$\Rightarrow \qquad E = -\frac{GMm}{2r} = -\frac{GMm}{2(R+h)}$$

If the satellite is orbiting close to earth, then $r = R$. Now, total energy of the satellite is $E = -GMm/2R$.

In the total energy of a circularly orbiting satellite which is negative, the potential energy is negative and magnitude of potential energy is twice the magnitude of the positive kinetic energy. We know that the total energy of a body is positive or zero, when it is at infinity from the earth. Therefore, the satellite will escape to infinity if its total energy becomes zero or positive. Since a satellite is always at finite distance from the earth, its total energy can never be positive or zero.

Binding Energy of a Satellite

The energy required to remove the satellite from its orbit around the earth to infinity is called binding energy of the satellite. Binding energy is equal to negative value of total mechanical energy of a satellite in its orbit. Thus binding energy is

$$-E = \frac{GMm}{2r}$$

ILLUSTRATION 10.11 An artificial satellite revolves round the earth at a height of 1000 km. The radius of the earth is 6.38×10^3 km. Mass of the earth $= 6 \times 10^{24}$ kg; $G = 6.67 \times 10^{-11}$ $\text{Nm}^2\text{kg}^{-2}$. Find the orbital speed and period of revolution of the satellite.

Solution. Here, $h = 1000 \text{ km} = 1000 \times 10^3 \text{ m} = 10^6$ m

$$r = R + h = 6.38 \times 10^6 + 10^6 = 7.38 \times 10^6 \text{ m}$$

Orbital speed,

$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{7.38 \times 10^6}} = 7364 \text{ ms}^{-1}$$

Time period,

$$T = \frac{2\pi r}{v_0} = \frac{2 \times (22/7) \times (7.38 \times 10^6)}{7.364 \times 10^3} = 6297 \text{ s}$$

ILLUSTRATION 10.12 A satellite orbits the earth at a height of 3.6×10^6 m from its surface. Compute its (a) kinetic energy, (b) potential energy, (c) total energy. Mass of the satellite = 500 kg, mass of the earth = 6×10^{24} kg, radius of the earth = 6.4×10^6, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

Solution. Here, $r = R + h = 6.4 \times 10^6 + 3.6 \times 10^6 = 10^7$ m.

Orbital velocity of the satellite around the earth is given by

$$v_0 = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{10^7}}$$

$$= \sqrt{6.67 \times 6 \times 10^6} \text{ m s}^{-1}$$

(a) KE of the satellite = $1/2\, mv_o^2$

$$= \frac{1}{2} \times 500 \times 6.67 \times 6 \times 10^6 = 10^{10} \text{ J}$$

(b) PE of the satellite $= -\frac{GMm}{R+h}$

$$= -\frac{6.67 \times 10^{-11} \times (6 \times 10^{24}) \times 500}{10^7}$$

$$= -2 \times 10^{10} \text{ J}$$

(c) Total energy = KE + PE = $10^{10} - 2 \times 10^{10} = -10^{10}$ J

GEOSTATIONARY OR GEOSYNCHRONOUS SATELLITES

A satellite which appears to be at fixed position at a definite height (i.e., stationary) to an observer on earth is called geostationary satellite. This satellite is also called the geosynchronous satellite as its angular speed is synchronized with the angular speed of the earth about its axis, i.e., this satellite revolves around the earth with the same angular speed in the same direction as is done by earth around its axis.

Obviously, the speed of such a satellite relative to earth is zero. That is why it appears to be stationary w.r.t. any point on the surface of earth. Clearly, $T = 24$ h for a geostationary satellite.

To calculate the height of geostationary satellite, we use the relation

$$h = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{\frac{1}{3}} - R$$

Putting, $R = 6.4 \times 10^6$ m, $g = 9.8$ m^{-2}, $T = 24$ h

$= 24 \times 60 \times 60$ s, we get $h = 3.6 \times 10^7$ m

$= 36000$ km

The orbital speed of such a satellite, using the relation $v = R\, Rg/R + h$ is found to be 3.1 km s^{-1}.

Geostationary satellites are used for the purposes of communication as they act as reflectors of suitable waves carrying the messages. Therefore, they are also called communication satellites. Telstar was the first communication satellite launched by U.S.A. in 1962. Since then, a large number of communication satellites have been put in use by various countries. India has also joined this race. INSAT 2B and INSAT 2C are the communication satellites of India.

Essential Conditions for Geostationary Satellite

(1) A geostationary satellite should be at a height nearly 36000 km above the equator of earth.

(2) Its time period of revolution around the earth should be the same as that of the earth about its axis, i.e., exactly 24 h.

(3) It should revolve in an orbit concentric and coplanar with the equatorial plane, so the plane of the orbit of the satellite is normal to the axis of rotation of the earth.

(4) Its sense of rotation should be the same as that of the earth about its own axis, i.e., from west to east. Its orbital speed is nearly 3.1 km s^{-1}.

CONCEPT APPLICATION EXERCISE 10.4

1. Why is there 'weightlessness' in a satellite?
2. A satellite moves in a circular orbit around the earth at height $R_e/2$ from earth's surface where R_e is the radius of the earth. Calculate its period of revolution. Given $R_e = 6.38 \times 10^6$ m.
3. A small satellite revolves around a planet in an orbit just above planet's surface. Taking the mean density of the planet 8000 kg m^{-3} and $G = 6.67 \times 10^{-11}$ N m^{-2}kg^{-2}, find the time period of the satellite.
4. Two satellites of the same mass are launched in the same orbit around the earth so as to rotate opposite to each other. They collide inelastically and stick together as wreckage. Obtain the total energy of the system before and just after the collision. Describe the subsequent motion of the wreckage.
5. Mars and earth have masses in the ratio 1:11 and radii in the ratio 42:79. Compare
   (a) their densities, assuming them to be spheres of uniform density;
   (b) gravitational field strengths at their surfaces;
   (c) escape velocities from their surfaces;
   (d) periods of their satellites near their surfaces.
6. If a satellite is revolving around a planet of mass M in an elliptical orbit of semi-major axis a. Show that the orbital speed of the satellite when it is at a distance r from the planet will be given by

$$v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$$

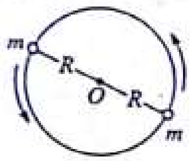
SOLVED EXAMPLES

1. Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $v=\frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (b) $v=\sqrt{\frac{Gm}{2R}}$

(c) $v=\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (d) $v=\sqrt{\frac{4Gm}{R}}$

Sol. (c) Centripetal force provided by the gravitational force of attraction between two particles



i.e. $\frac{mv^2}{R}=\frac{Gm\times m}{(2R)^2}$

$\Rightarrow v=\frac{1}{2}\sqrt{\frac{Gm}{R}}$

2. If the change in the value of g at a height h above the surface of the earth is the same as at a depth x below it, then (both x and h being much smaller than the radius of the earth)

(a) $x=h$ (b) $x=2h$

(c) $x=\frac{h}{2}$ (d) $x=h^2$

Sol. (b) The value of g at the height h from the surface of earth

$$g'=g\left(1-\frac{2h}{R}\right)$$

The value of g at depth x below the surface of earth

$$g'=g\left(1-\frac{x}{R}\right)$$

These two are given equal, hence $\left(1-\frac{2h}{R}\right)=\left(1-\frac{x}{R}\right)$

On solving, we get $x=2h$

3. At what depth below the surface of the earth, acceleration due to gravity g will be half its value 1600 km above the surface of the earth?

(a) 4.2×10^6 m (b) 3.19×10^6 m

(c) 1.59×10^6 m (d) None of these

Sol. (a) Radius of earth $R=6400$ km $\Rightarrow h=\frac{R}{4}$

Acceleration due to gravity at a height h

$$g_h=g\left(\frac{R}{R+h}\right)^2=g\left(\frac{R}{R+\frac{R}{4}}\right)^2=\frac{16}{25}g$$

At depth d, value of acceleration due to gravity

$$g_d=\frac{1}{2}g_h \quad \text{(According to problem)}$$

$$\Rightarrow \quad g_d=\frac{1}{2}\left(\frac{16}{25}\right)g\Rightarrow g\left(1-\frac{d}{R}\right)=\frac{1}{2}\left(\frac{16}{25}\right)g$$

By solving we get $d=4.3\times10^6$m.

4. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2, respectively. Their centres are distance d apart. The minimum velocity with which a particle of mass m should be projected from a point midway between their centres so that it escapes to infinity is

(a) $2\sqrt{\frac{G}{d}(M_1+M_2)}$ (b) $2\sqrt{\frac{2G}{d}(M_1+M_2)}$

(c) $2\sqrt{\frac{Gm}{d}(M_1+M_2)}$ (d) $2\sqrt{\frac{Gm(M_1+M_2)}{d(R_1+R_2)}}$

Sol. (a) Gravitational potential at mid point

$$V=\frac{-GM_1}{d/2}+\frac{-GM_2}{d/2}$$

Now, $PE=m\times V=\frac{-2Gm}{d}(M_1+M_2)$

[m = mass of particle]

So, for projecting particle from mid point to infinity

KE = |PE|

$$\Rightarrow\frac{1}{2}mv^2=\frac{2Gm}{d}(M_1+M_2)$$

$$\Rightarrow v=2\sqrt{\frac{G(M_1+M_2)}{d}}$$

5. A body of mass m kg starts falling from a point $2R$ above the Earth's surface. Its kinetic energy when it has fallen to a point R above the Earth's surface [R: Radius of Earth, M: Mass of Earth, G: Gravitational Constant]

(a) $\frac{1}{2}\frac{GMm}{R}$ (b) $\frac{1}{6}\frac{GMm}{R}$

(c) $\frac{2}{3}\frac{GMm}{R}$ (d) $\frac{1}{3}\frac{GMm}{R}$

Sol. (b) Potential energy, $U=\frac{-GMm}{r}=-\frac{GMm}{R+h}$

$U_{\text{initial}}=-\frac{GMm}{3R}$ and $U_{\text{final}}=-\frac{-GMm}{2R}$

Loss in PE = gain in KE $=\frac{GMm}{2R}-\frac{GMm}{3R}=\frac{GMm}{6R}$

6. v_e and v_p denotes the escape velocity from the earth and another planet having twice the radius and the same mean density as the earth. Then

(a) $v_e=v_p$ (b) $v_e=v_p/2$

(c) $v_e=2v_p$ (d) $v_e=v_p/4$

Sol. (b) $v_e=\sqrt{\frac{2GM}{R}}=R\sqrt{\frac{8}{3}\pi G\rho}$

If mean density is constant, then $v_e\propto R$

$$\frac{v_e}{v_p}=\frac{R_e}{R_p}=\frac{1}{2}\quad\Rightarrow\quad v_e=\frac{v_p}{2}$$

7. A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energy is
   (a) Positive
   (b) Negative
   (c) Zero
   (d) May be positive or negative depending upon its initial velocity

Sol. (b) If missile is launched with escape velocity, then it will escape from the gravitational field and at infinity its total energy becomes zero.
But if the velocity of projection is less than escape velocity, then sum of energies will be negative. This shows that attractive force is working on the satellite.

8. A satellite is moving around the earth with speed v in a circular orbit of radius r. If the orbit radius is decreased by 1%, its speed will
   (a) increase by 1% (b) increase by 0.5%
   (c) decrease by 1% (d) decrease by 0.5%

Sol. (b) $v \propto \frac{1}{\sqrt{r}}$

$$\% \text{ increase in speed} = \frac{1}{2}(\% \text{ decrease in radius})$$

$$= \frac{1}{2}(1\%) = 0.5\%$$

i.e., speed will increase by 0.5%.

9. Two identical satellites are at R and $7R$ away from earth surface, the wrong statement is (R = Radius of earth)
   (a) Ratio of total energy will be 4
   (b) Ratio of kinetic energies will be 4
   (c) Ratio of potential energies will be 4
   (d) Ratio of total energy will be 4 but ratio of potential and kinetic energies will be 2

Sol. (d) Orbital radius of satellites, $r_1 = R + R = 2R$

$$r_2 = R + 7R = 8R$$

$$U_1 = \frac{-GMm}{r_1} \text{ and } U_2 = \frac{-GMm}{r_2}$$

$$K_1 = \frac{GMm}{2r_1} \text{ and } K_2 = \frac{GMm}{2r_2}$$

$$E_1 = \frac{GMm}{2r_1} \text{ and } E_2 = \frac{GMm}{2r_2}$$

$$\therefore \quad \frac{U_1}{U_2} = \frac{K_1}{K_2} = \frac{E_1}{E_2} = 4$$

10. In the following four periods,
   (i) Time of revolution of a satellite just above the earth's surface (T_{st})
   (ii) Period of oscillation of mass inside the tunnel bored along the diameter of the earth (T_{ma})
   (iii) Period of simple pendulum having a length equal to the earth's radius in a uniform field of 9.8 N/kg (T_{sp})
   (iv) Period of an infinite length simple pendulum in the earth's real gravitational field (T_{is})

   (a) $T_{st} > T_{ma}$ (b) $T_{ma} > T_{st}$
   (c) $T_{sp} < T_{is}$ (d) $T_{st} = T_{ma} = T_{sp} = T_{is}$

Sol. (c) (i) $T_{st} = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$

[As $h << R$ and $GM = gR^2$]

(ii) $T_{ma} = 2\pi\sqrt{\frac{R}{g}}$

(iii) $T_{sp} = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}} = 2\pi\sqrt{\frac{R}{2g}}$

[As $l = R$]

(iv) $T_{is} = 2\pi\sqrt{\frac{R}{g}}$ [As $l = \infty$]

11. The magnitudes of the gravitational force at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2, respectively. Then

   (a) $\frac{F_1}{F_2} = \frac{r_2}{r_1}$ if $r_1 < R$ and $r_2 < R$

   (b) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 > R$ and $r_2 > R$

   (c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

   (d) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 < R$ and $r_2 < R$

Sol. (b) $g = \frac{4}{3}\pi\rho G r \;\therefore\; g \propto r$ if $r > R$

$$g = \frac{GM}{r^2} \;\therefore\; g \propto \frac{1}{r^2} \text{ if } r > R$$

If $r_1 < R$ and $r_2 < R$, then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \frac{r_1}{r_2}$

If $r_1 > R$ and $r_2 > R$, then $\frac{F_1}{F_2} = \frac{g_1}{g_2} = \left(\frac{r_2}{r_1}\right)^2$

12. A rocket of mass M is launched vertically from the surface of the earth with an initial speed V. Assuming the radius of the earth to be R and negligible air resistance, the maximum height attained by the rocket above the surface of the earth is

   (a) $R/\left(\frac{gR}{2V^2}-1\right)$ (b) $R\left(\frac{gR}{2V^2}-1\right)$

   (c) $R/\left(\frac{2gR}{V^2}-1\right)$ (d) $R\left(\frac{2gR}{V^2}-1\right)$

Sol. (c) $\Delta K.E. = \Delta U$

$$\Rightarrow \frac{1}{2}MV^2 = GM_eM\left(\frac{1}{R}-\frac{1}{R+h}\right) \quad \text{...(i)}$$

Also $g = \dfrac{GM_e}{R^2}$ ...(ii)

On solving (i) and (ii), $h = \dfrac{R}{\left(\dfrac{2gR}{V^2}-1\right)}$

13. Two bodies of masses m_1 and m_2 are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is

(a) $\left[2G\dfrac{(m_1-m_2)}{r}\right]^{1/2}$ (b) $\left[\dfrac{2G}{r}(m_1+m_2)\right]^{1/2}$

(c) $\left[\dfrac{r}{2G(m_1m_2)}\right]^{1/2}$ (d) $\left[\dfrac{2G}{r}m_1m_2\right]^{1/2}$

Sol. (b) Let velocities of these masses at r distance from each other be v_1 and v_2, respectively.
By conservation of momentum.

$$m_1v_1 - m_2v_2 = 0$$
$$\Rightarrow m_1v_1 = m_2v_2 \quad \text{... (i)}$$

By conservation of energy
change in PE = change in KE

$$\frac{Gm_1m_2}{r} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$\Rightarrow \frac{m_1^2v_1^2}{m_1} + \frac{m_2^2v_2^2}{m_2} = \frac{2Gm_1m_2}{r} \quad \text{...(ii)}$$

On solving equations (i) and (ii),

$$v_1 = \sqrt{\frac{2Gm_2^2}{r(m_1+m_2)}} \text{ and } v_2 = \sqrt{\frac{2Gm_1^2}{r(m_1+m_2)}}$$
$$\therefore v_{app} = |v_1| + |v_2| = \sqrt{\frac{2G}{r}(m_1+m_2)}$$

14. A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space. (v_e is escape velocity and $k < 1$). If air resistance is considered to be negligible, then the maximum height from the centre of earth to which it can go will be (R = radius of earth)

(a) $\dfrac{R}{k^2+1}$ (b) $\dfrac{R}{k^2-1}$

(c) $\dfrac{R}{1-k^2}$ (d) $\dfrac{R}{k+1}$

Sol. (c) Kinetic energy = Potential energy

$$\frac{1}{2}m(kv_e)^2 = \frac{mgh}{1+\frac{h}{R}} \Rightarrow \frac{1}{2}mk^2 2gR = \frac{mgh}{1+\frac{h}{R}}$$

$$\Rightarrow h = \frac{Rk^2}{1-k^2}$$

Height of projectile from the earth's surface = h

Height from the centre, $r = R + h = R + \dfrac{Rk^2}{1-k^2}$

By solving, we get $r = \dfrac{R}{1-k^2}$

15. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius $(1.01)R$. The period of the second satellite is larger than that of the first one by approximately

(a) 0.5% (b) 1.0%
(c) 1.5% (d) 3.0%

Sol. (c) In the problem, the orbital radius is increased by 1%.
Time period of satellite, $T \propto r^{3/2}$
Percentage change in time period

$$= \frac{3}{2}(\% \text{ change in orbital radius}) = \frac{3}{2}(1\%) = 1.5\%$$

16. If the distance between the earth and the sun becomes half its present value, the number of days in a year would have been

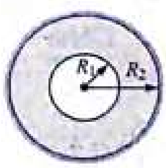
(a) 64.5 (b) 129
(c) 182.5 (d) 730

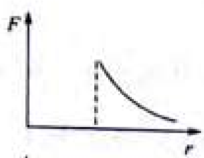
Sol. (b) According to Kepler's third law, the ratio of the squares of the periods of any two planets revolving about the sun is equal to the ratio of the cubes of their average distances from the sun, i.e.,

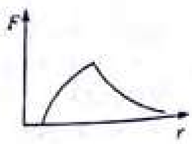
$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left[\frac{r_1}{\frac{1}{2}r_1}\right]^3 = 8 \Rightarrow \frac{T_1}{T_2} = 2\sqrt{2}$$

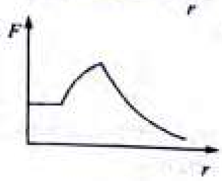
$$\therefore T_2 = \frac{T_1}{2\sqrt{2}} = \frac{365 \text{ days}}{2\sqrt{2}} = 129 \text{ days}$$

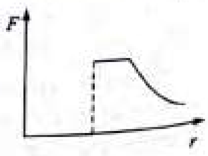
17. A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure. The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as $(0 \le r \le \infty)$



(a)


(b)


(c)


(d)


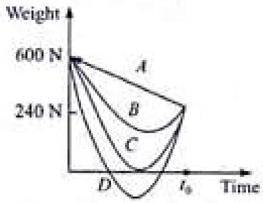
Sol. (b) $F = 0$ when $0 \le r \le R_1$

Because intensity is zero inside the cavity.

F increases when $R_1 \le r \le R_2$

$F \propto \frac{1}{r^2}$ when $r > R_2$

18. Suppose, the acceleration due to gravity at the Earth's surface is 10 m/s^2 and at the surface of Mars it is 4.0 m/s^2. A 60-kg passenger goes from the Earth to the Mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of figure best represents the weight (net gravitational force) of the passenger as a function of time?



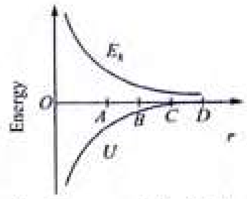
(a) A (b) B
(c) C (d) D

Sol. (c) Initially the weight of the passenger

$= 60 \times 10 = 600$ N

Finally the weight of the passenger $= 60 \times 4 = 240$ N and during the flight in between somewhere its weight will be zero because at that point gravitational pull of Earth and Mars will be equal.

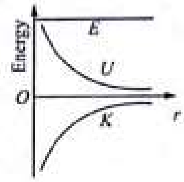
19. The curves for potential energy (U) and kinetic energy (E_k) of a two particle system are shown in figure. At what points the system will be bound?

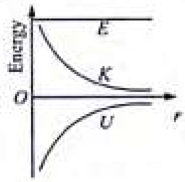


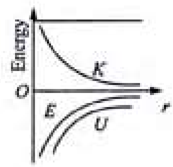
(a) Only at point D (b) Only at point A
(c) At point D and A (d) At points A, B and C

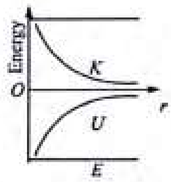
Sol. (d) The system will be bound at points where total energy is negative. In the given curve at point A, B and C, the PE is more than KE

20. The correct graph representing the variation of total energy (E_t), kinetic energy (E_k) and potential energy (U) of a satellite with its distance from the centre of Earth is

(a)


(b)


(c)


(d)


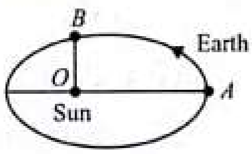
Sol. (c) $U = \frac{-GMm}{r}, K = \frac{GMm}{2r}$ and $E = \frac{-GMm}{2r}$

For a satellite U, K and E varies with r and also U and E remains negative whereas K remains always positive.

EXERCISES

Kepler's Laws of Planetary Motion and Newton's Law of Gravitation

1. The earth moves around the Sun in an elliptical orbit as shown in the figure. The ratio $OA/OB = x$. The ratio of the speed of the earth at B to that at A is nearly



(a) $\sqrt{x}$ (b) x
(c) $x\sqrt{x}$ (d) x^2

2. A planet is revolving in an elliptical orbit around the Sun. Its closest distance from the Sun is r_{min} and the farthest distance is r_{max}. If the velocity of the planet at the distance of the closest approach is v_1 and that at the farthest distance from the Sun is v_2, then v_1/v_2

(a) $\frac{r_{max}}{r_{min}}$ (b) $\frac{r_{min}}{r_{max}}$

(c) $\frac{r_{min}+r_{max}}{r_{max}-r_{min}}$ (d) none

3. Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $r^{5/2}$, then the square of the time period will be proportional to

(a) r^3 (b) r^2
(c) $r^{2.5}$ (d) $r^{3.5}$

4. The gravitational force between two objects is proportional to $1/R$ (and not as $1/R^2$) where R is separation between them, then a particle in circular orbit under such a force would have its orbital speed v proportional to

(a) $\frac{1}{R^2}$ (b) R^0

(c) R^1 (d) $\frac{1}{R}$

5. Two particles of equal mass go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $v = \frac{1}{2R}\sqrt{\left(\frac{1}{Gm}\right)}$ (b) $v = \sqrt{\left(\frac{Gm}{2R}\right)}$

(c) $v = \frac{1}{2}\sqrt{\left(\frac{Gm}{R}\right)}$ (d) $v = \sqrt{\left(\frac{4Gm}{R}\right)}$

6. If three uniform spheres, each having mass M and radius R, are kept in such a way that each touches the other two, the magnitude of the gravitational force on any sphere due to the other two is

(a) $\frac{GM^2}{4r^2}$ (b) $\frac{2GM^2}{r^2}$

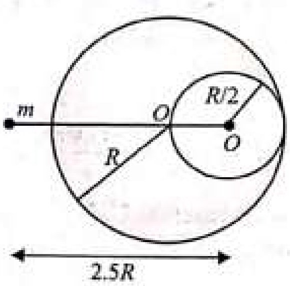
(c) $\frac{2GM^2}{4r^2}$ (d) $\frac{\sqrt{3}GM^2}{4r^2}$

7. The distance between the centres of the Moon and the earth is D. The mass of the earth is 81 times the mass of the Moon. At what distance from the centre of the Earth, the gravitational force will be zero?

(a) $\frac{D}{2}$ (b) $\frac{2D}{3}$

(c) $\frac{4D}{3}$ (d) $\frac{9D}{10}$

8. A solid sphere of radius $R/2$ is cut out of a solid sphere of radius R such that the spherical cavity so formed touches the surface on one side and the centre of the sphere on the other side, as shown. The initial mass of the solid sphere was M. If a particle of mass m is placed at a distance $2.5R$ from the centre of the cavity, then what is the gravitational attraction on the mass m?



(a) $\frac{GMm}{R^2}$ (b) $\frac{GMm}{2R^2}$

(c) $\frac{GMm}{8R^2}$ (d) $\frac{23}{100}\frac{GMm}{R^2}$

9. As observed from the earth, the sun appears to move in an approximate circular orbit. For the motion of another planet like mercury as observed from the earth, this would

(a) be similarly true

(b) not be true because the force between the earth and mercury is not inverse square law

(c) not be true because the major gravitational force on mercury is due to the sun

(d) not be true because mercury is influenced by forces other than gravitational forces

10. In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They

(a) will not move around the sun, since they have very small masses compared to the sun

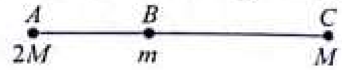
(b) will move in an irregular way because of their small masses and will drift away into outer space

(c) will move around the sun in closed orbits but not obey Kepler's laws

(d) will move in orbits like planets and obey Kepler's laws

11. Particles of masses $2M$, m and M are respectively at points A, B and C with $AB = \frac{1}{2}(BC)$. m is much-much smaller than M and at time $t = 0$, they are all at rest as given in figure.

At subsequent times before any collision takes place.



(a) m will remain at rest

(b) m will move towards M

(c) m will move towards $2M$

(d) m will have oscillatory motion

12. In planetary motion the areal velocity of position vector of a planet depends on angular velocity (ω) and the distance of the planet from sun (r). If so the correct relation for areal velocity is

(a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$

(c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

13. The maximum and minimum distances of a comet from the sun are 8×10^{12} m and 1.6×10^{12} m. If its velocity when nearest to the sun is 60 m/s, what will be its velocity in m/s when it is farthest?

(a) 12 (b) 60

(c) 112 (d) 6

14. Three identical point masses, each of mass 1 kg lie in the x-y plane at points (0, 0), (0, 0.2m) and (0.2m, 0). The net gravitational force on the mass at the origin is

(a) $1.67\times10^{-9}(\hat{i}+\hat{j})$ N

(b) $3.34\times10^{-10}(\hat{i}+\hat{j})$ N

(c) $1.67\times10^{-9}(\hat{i}-\hat{j})$ N

(d) $3.34\times10^{-10}(\hat{i}+\hat{j})$ N

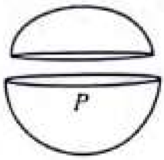
15. A planet moves around the sun. At a given point P, it is closed from the sun at a distance d_1 and has a speed v_1. At another point Q, when it is farthest from the sun at a distance d_2, its speed will be

(a) $\frac{d_1^2 v_1}{d_2^2}$ (b) $\frac{d_2 v_1}{d_1}$

(c) $\frac{d_1 v_1}{d_2}$ (d) $\frac{d_2^2 v_1}{d_1^2}$

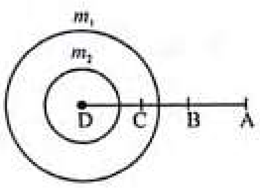
Gravitational Field and Acceleration Due to Gravity

16. A spherical shell is cut into two pieces along a chord as shown in the figure. P is a point on the plane of the chord. The gravitational field at P due to the upper part is I_1 and that due to the lower part is I_2. What is the relation between them?



(a) $I_1 > I_2$ (b) $I_1 < I_2$
(c) $I_1 = I_2$ (d) no definite relation

17. Figure shows two shells of masses m_1 and m_2. The shells are concentric. At which point, a particle of mass m shall experience zero force?



(a) A (b) B
(c) C (d) D

18. If g is same at a height h and at a depth d, then
(a) $R = 2d$ (b) $d = 2h$
(b) $h = d$ (d) none

19. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is
(a) $\frac{4\pi G}{3gR}$ (b) $\frac{3\pi R}{4gG}$
(c) $\frac{3g}{4\pi RG}$ (d) $\frac{\pi R}{12G}$

20. The value of g (acceleration due to gravity) at earth's surface is 10 ms^{-2}. Its value in ms^{-2} at the centre of the earth which is assumed to be a sphere of radius R metre and uniform mass density is
(a) 5 (b) $\frac{10}{R}$
(c) $\frac{10}{2R}$ (d) zero

21. If the radius of the earth decreases by 10%, the mass remaining unchanged, what will happen to the acceleration due to gravity?
(a) Decreases by 19%
(b) Increases by 19%
(c) Decreases by more than 19%
(d) Increases by more than 19%

22. The distances from the centre of the Earth, where the weight of a body is zero and one-fourth that of the weight of the body on the surface of the Earth are (assume R is the radius of the earth)
(a) $0, \frac{R}{4}$ (b) $0, \frac{3R}{4}$
(c) $\frac{R}{4}, 0$ (d) $\frac{3R}{4}, 0$

23. If a man at the equator would weigh (3/5)th of his weight, The angular speed of the earth is
(a) $\sqrt{\frac{2}{5}\frac{g}{R}}$ (b) $\sqrt{\frac{g}{R}}$
(c) $\sqrt{\frac{R}{g}}$ (d) $\sqrt{\frac{2}{5}\frac{R}{g}}$

24. How many hours would make a day if the Earth were rotating at such a high speed that the weight of a body on the equator were zero?
(a) 6.2 h (b) 1.4 h
(c) 28 h (d) 5.6 h

25. If the mass of a planet is 10% less than that of the earth and the radius is 20% greater than that of the earth, the acceleration due to gravity on the planet will be
(a) 5/8 times that on the surface of the earth
(b) 3/4 times that on the surface of the earth
(c) 1/2 times that on the surface of the earth
(d) 9/10 times that on the surface of the earth

26. What should be the angular velocity of rotation of the earth about its own axis so that the weight of a body at the equator reduces to 3/5 or its present value? (Take R as the radius of the earth)
(a) $\sqrt{\frac{g}{3R}}$ (b) $\sqrt{\frac{2g}{3R}}$
(c) $\sqrt{\frac{2g}{5R}}$ (d) $\sqrt{\frac{2g}{7R}}$

27. The value of g at a certain height h above the free surface of the earth is $x/4$ where x is the value of g at the surface of the earth. The height h is
(a) R (b) $2R$
(c) $3R$ (d) $4R$

28. The mass of the moon is 1/81 of the earth but the gravitational pull is 1/6 of the earth. It is due to the fact that
(a) The radius of the moon is 81/6 of the earth
(b) The radius of the earth is $9/\sqrt{6}$ of the moon
(c) Moon is the satellite of the earth
(d) None of the above

29. Acceleration due to gravity on moon is 1/6 of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_e) and moon (ρ_m) is $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$ then radius of moon R_m in terms of R_e will be
(a) $\frac{5}{18}R_e$ (b) $\frac{1}{6}R_e$
(c) $\frac{7}{18}R_e$ (d) $\frac{1}{2\sqrt{3}}R_e$

30. The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is

(a) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$

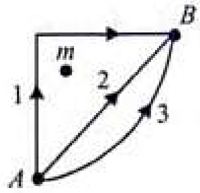
(b) $g_1 : g_2 = R_1R_2 : \rho_1\rho_2$

(c) $g_1 : g_2 = R_1\rho_2 : R_2\rho_1$

(d) $g_1 : g_2 = R_1\rho_1 : R_2\rho_2$

Gravitational Potential, Energy and Escape Velocity

31. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3, respectively, (as shown in the figure) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.



(a) $W_1 > W_2 > W_3$ (b) $W_1 = W_2 = W_3$

(b) $W_1 < W_2 < W_3$ (d) $W_2 > W_1 > W_3$

32. If g is acceleration due to gravity on the Earth's surface, the gain in the potential energy of an object of mass m raised from the surface of Earth to a height equal to the radius R of the earth is

(a) $\frac{1}{2} mgR$ (b) $2mgR$

(c) mgR (d) $\frac{1}{4} mgR$

33. A space ship is launched into a circular orbit close to the surface of the earth. The additional velocity now imparted to the spaceship in the orbit to overcome the gravitational pull is

(a) 11.2 km s^{-1} (b) 8 km s^{-1}

(c) 3.2 km s^{-1} (d) 1.414×8 km s^{-1}

34. A skylab of mass m kg is first launched from the surface of the earth in a circular orbit of radius $2R$ (from the centre of the earth) and then it is shifted from this circular orbit to another circular orbit of radius $3R$. The minimum energy required to place the lab in the first orbit and to shift the lab from first orbit to the second orbit are

(a) $\frac{3}{4}mgR, \frac{mgR}{6}$ (b) $\frac{3}{4}mgR, \frac{mgR}{12}$

(c) mgR, mgR (d) $2mgR, mgR$

35. The gravitational potential due to earth at infinite distance from it is zero. Let the gravitational potential at a point P be -5 J kg^{-1}. Suppose, we arbitrarily assume the gravitational potential at infinity to be $+10$ J kg^{-1}, then the gravitational potential at P will be

(a) -5 J kg^{-1} (b) $+5$ J kg^{-1}

(c) -15 J kg^{-1} (d) $+15$ J kg^{-1}

36. Two bodies of masses M_1 and M_2 are placed at a distance R apart. Then at the position where the gravitational field due to them is zero, the gravitational potential is

(a) $-G\frac{\sqrt{M_1}}{R}$ (b) $-G\frac{\sqrt{M_2}}{R}$

(c) $-(\sqrt{M_1}+\sqrt{M_2})^2\frac{G}{R}$ (d) $-(\sqrt{M_1}-\sqrt{M_2})^2\frac{G}{R}$

37. In the solar system, the Sun is in the focus of the system for Sun–Earth binding system. Then the binding energy for the system will be [given that radius of the earth's orbit round the Sun is 1.5×10^{11} m and mass of the earth $= 6 \times 10^{24}$ kg]

(a) 2.7×10^{33} J (b) 5.4×10^{33} J

(c) 2.7×10^{30} J (d) 5.4×10^{30} J

38. The minimum energy required to launch a m kg satellite from the Earth's surface in a circular orbit at an altitude $2R$, where R is the radius of earth is

(a) $\frac{5}{3} mgR$ (b) $\frac{4}{3} mgR$

(c) $\frac{5}{6} mgR$ (d) $\frac{5}{4} mgR$

39. In order to shift a body of mass m from a circular orbit of radius $3R$ to a higher orbit of radius $5R$ around the earth, the work done is

(a) $\frac{3GMm}{5R}$ (b) $\frac{GMm}{2R}$

(c) $\frac{2}{15}\frac{GMm}{R}$ (d) $\frac{GMm}{5R}$

40. A body of mass m is situated at a distance $4R_e$ above the earth's surface, where R_e is the radius of earth. How much minimum energy be given to the body so that it may escape

(a) mgR_e (b) $2\,mgR_e$

(c) $\frac{mgR_e}{5}$ (d) $\frac{mgR_e}{16}$

41. Assuming earth as a uniform sphere of radius R, if we project a body along the smooth diametrical chute from the centre of earth with a speed v such that it will just reach the earth's surface then v is equal to

(a) $\sqrt{gR}$ (b) $\sqrt{2gR}$

(c) $\sqrt{\frac{gR}{2}}$ (d) none of these

42. In the above question, escape speed from the centre of earth is

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$

(c) $\sqrt{gR}$ (d) $\sqrt{\frac{3gR}{2}}$

43. In question 41, the minimum external work done in shifting a particle from centre of earth to earth's surface is W_1 and that from surface of earth to infinity is W_2, then W_1/W_2 is equal to

(a) 1 : 1 (b) 1 : 2

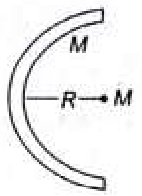
(c) 2 : 1 (d) 1 : 3

44. The escape velocity from the centre of a uniform ring of mass M and radius R is

(a) $\sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{GM}{R}}$

(c) $\sqrt{\frac{GM}{2R}}$ (d) $2\sqrt{\frac{GM}{R}}$

45. The potential energy of interaction between the semi-circular ring of mass M and radius R, and the particle of mass M placed at the centre of curvature of the semi-circular arc is



(a) $-\frac{GM^2}{R}$ (b) $-\frac{2GM^2}{R}$

(c) $-\frac{GM^2}{\pi R}$ (d) none of these

Motion of Satellite

46. Two satellites A and B of masses m_1 and m_2 ($m_1 = 2\,m_2$) are moving in circular orbits of radii r_1 and r_2 ($r_1 = 4r_2$), respectively, around the earth. If their periods are T_A and T_B, then the ratio T_A/T_B is

(a) 4 (b) 16

(c) 2 (d) 8

47. A satellite of mass m is revolving around the earth at height R (radius of the earth) from the earth's surface. Its potential energy will be

(a) mgR (b) $0.67\,mgR$

(c) $-\frac{mgR}{2}$ (d) $0.33\,mgR$

48. A satellite moves around the earth in a circular orbit with speed v. If m is the mass of the satellite, its total energy is

(a) $-\frac{1}{2}mv^2$ (b) $\frac{1}{2}mv^2$

(c) $\frac{3}{2}mv^2$ (d) $\frac{1}{4}mv^2$

49. Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are revolving around the earth in a circular orbit of radii r_1 and r_2 ($r_1 > r_2$), respectively. Which of the following statements is true regarding their speeds v_1 and v_2?

(a) $v_1 = v_2$ (b) $v_1 > v_2$

(c) $v_1 < v_2$ (d) $\frac{v_1}{r_1} = \frac{v_2}{v_2}$

50. Two satellites A and B of the same mass are revolving around the earth in the concentric circular orbits such that the distance of satellite B from the centre of the earth is thrice as compared to the distance of the satellite A from the centre of the earth. The ratio of the centripetal force acting on B as compared to that on A is

(a) $\frac{1}{3}$ (b) 3

(c) $\frac{1}{9}$ (d) $\frac{1}{\sqrt{3}}$

51. A satellite is seen after each 8 hours over equator at a place on the earth when its sense of rotation is opposite to the earth. The time interval after which it can be seen at the same place when the sense of rotation of earth and satellite is same will be

(a) 8 hours (b) 12 hours

(c) 24 hours (d) 6 hours

52. A satellite is moved from one circular orbit around the earth to another of lesser radius. Which of the following statement is true?

(a) The kinetic energy of satellite increases and the gravitational potential energy of satellite-earth system increases.

(b) The kinetic energy of satellite increases and the gravitational potential energy of satellite-earth system decreases.

(c) The kinetic energy of satellite decreases and the gravitational potential energy of satellite-earth system decreases.

(d) The kinetic energy of satellite decreases and the gravitational potential energy of satellite-earth system increases.

53. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is

(a) $\frac{3}{2}v$ (b) $\sqrt{\frac{3}{2}}\,v$

(c) $\sqrt{\frac{2}{3}}\,v$ (d) $\frac{2}{3}v$

54. In the following four periods

(i) Time of revolution of a satellite just above the earth's surface (T_{st})

(ii) Period of oscillation of mass inside the tunnel bored along the diameter of the earth (T_{ma})

(iii) Period of simple pendulum having a length equal to the earth's radius in a uniform field of 9.8 N/kg (T_{sp})

(iv) Period of an infinite length simple pendulum in the earth's real gravitational field (T_{is})

(a) $T_{st} > T_{ma}$ (b) $T_{ma} > T_{st}$

(c) $T_{sp} < T_{is}$ (d) $T_{st} = T_{ma} = T_{sp} = T_{is}$

55. In a satellite if the time of revolution is T, then kinetic energy is proportional to

(a) $\frac{1}{T}$ (b) $\frac{1}{T^2}$

(c) $\frac{1}{T^3}$ (d) $T^{-2/3}$

56. A satellite is revolving around the Earth in a circular orbit with a constant speed. If its speed is made 2 times by supplying energy from an external source then what will be the path of satellite after this?

(a) Straight line (b) Circular

(c) Parabola (d) Hyperbola

57. A satellite can be in a geostationary orbit around earth in an orbit of radius r. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth of radius

(a) $\frac{r}{2}$ (b) $\frac{r}{2\sqrt{2}}$

(c) $\frac{r}{(4)^{1/3}}$ (d) $\frac{r}{(2)^{1/3}}$

58. A satellite in equatorial plane is rotating in the direction of earth's rotation with time interval between its two consecutive appearances over head of an observer as time period of rotation of the earth, T_E. What is the time period of the satellite?

(a) T_E (b) $2T_E$

(c) $\frac{T_E}{2}$ (d) $\frac{2T_E}{3}$

59. A satellite is orbiting with areal velocity v_a. At what height from the surface of the earth, it is rotating, if the radius of earth is R?

(a) $\frac{4v_a^2}{gR^2} - R$ (b) $\frac{2v_a^2}{gR^2} - R$

(c) $\frac{v_a^2}{gR^2} - R$ (d) $\frac{3v_a^2}{gR^2} - R$

60. A spherical planet has uniform density $\frac{\pi}{2} \times 10^4$ kg/m^3. Find out the minimum period for a satellite in a circular orbit around it in seconds. (Use $G = \frac{20}{3} \times 10^{-11} \frac{\text{N-m}^2}{\text{kg}^2}$).

(a) 7500 (b) 3000

(c) 4500 (d) 6000

Problems Based on Mixed Concepts

61. The maximum vertical distance through which a fully dressed astronaut can jump on the earth is 0.5 m. If mean density of the Moon is two-thirds that of the Earth and radius is one quarter that of the Earth, the maximum vertical distance through which he can jump on the Moon and the ratio of the time of duration of the jump on the Moon to hold on the Earth are

(a) 3 m, 6:1 (b) 6 m, 3:1

(c) 3 m, 1:6 (d) 6 m, 1:6

62. Two satellites of the same mass are launched in the same orbit around the earth so as to rotate opposite to each other. If they collide inelastically and stick together as wreckage, the total energy of the system just after collision is

(a) $-\frac{2GMm}{r}$ (b) $-\frac{GMm}{r}$

(c) $\frac{GMm}{2r}$ (d) $\frac{GMm}{4r}$

63. A ball of mass m is fired vertically upwards from the surface of the earth with velocity nv_e, where v_e is the escape velocity and $n > 1$. To what height will the ball rise? Neglecting air resistance, take radius of the earth as R

(a) $\frac{R}{n^2}$ (b) $\frac{R}{(1-n^2)}$

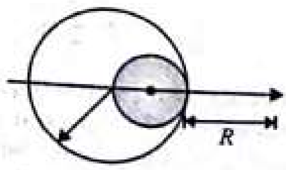
(c) $\frac{Rn^2}{(1-n^2)}$ (d) Rn^2

64. A body is released from a point of distance R' from the centre of earth. Its velocity at the time of striking the earth will be $(R' > R_e)$

(a) $\sqrt{2gR_e}$ (b) $\sqrt{R_e g}$

(c) $\sqrt{2g(R' - R_e)}$ (d) $\sqrt{2gR_e\left(1 - \frac{R_e}{R'}\right)}$

65. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at a distance $2R$ from the centre of the sphere. A spherical cavity of radius $R/2$ is now made in the sphere as shown in the figure. The sphere with the cavity now applies a gravitational force F_2 on the same particle. The ratio F_1/F_2 is



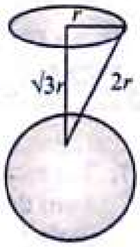
(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{7}{8}$ (d) $\frac{9}{7}$

66. The value of g at a particular point is 10 ms^{-2}. Suppose the earth shrinks uniformly to half of its present size without losing any mass. The value of g at the same point (assuming that the distance of the point from the centre of the earth does not change) will now be

(a) 5 $m\,s^{-2}$ (b) 10 $m\,s^{-2}$
(c) 3 $m\,s^{-2}$ (d) 20 $m\,s^{-2}$

67. A uniform ring of mass m and radius r is placed directly above a uniform sphere of mass M and of equal radius. The centre of the ring is directly above the centre of the sphere at a distance $r\sqrt{3}$ as shown in the figure. The gravitational force exerted by the sphere on the ring will be



(a) $\dfrac{GMm}{8r^2}$ (b) $\dfrac{GMm}{4r^2}$
(c) $\sqrt{3}\dfrac{GMm}{8r^2}$ (d) $\dfrac{GMm}{8r^2\sqrt{3}}$

68. A tunnel is dug along a diameter of the earth. If M_e and R_e are the mass and radius, respectively, of the earth, then the force on a particle of mass m placed in the tunnel at a distance r from the centre is

(a) $\dfrac{GM_em}{R_e^3}r$ (b) $\dfrac{GM_em}{R_e^3r}$
(c) $\dfrac{GM_emR_e^3}{r}$ (d) $\dfrac{GM_em}{R_e^2}r$

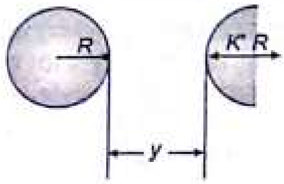
69. A small body of superdense material, whose mass is twice the mass of the earth but whose size is very small compared to the size of the earth, starts from rest at a height $H << R$ above the earth's surface, and reaches the earth's surface in time t. Then t is equal to

(a) $\sqrt{2H/g}$ (b) $\sqrt{H/g}$
(c) $\sqrt{2H/3g}$ (d) $\sqrt{4H/3g}$

70. If a particle of mass m is projected with minimum velocity from the surface of a star with kinetic energy K_1GMm/a and potential energy at surface of the star K_2GMm/a towards the star of same mass m and radius a (K_1 and K_2 are constants) to reach the other star. Find the distance between the centre of the two stars

(a) $\dfrac{2a}{(K_2-K_1)}$ (b) $\dfrac{4a}{(K_2-K_1)}$
(c) $\dfrac{2a}{(K_1-K_2)}$ (d) $\dfrac{a}{(K_1-K_2)}$

71. In a cosmic event, suppose a planet heavier than the earth with mass KM ($K > 1$) and radius $K'R$ ($K' > 1$) passes through a path near the earth (M and R are the mass and radius of earth). At what closest distance from surface of planet, we are in danger of being thrown into space:



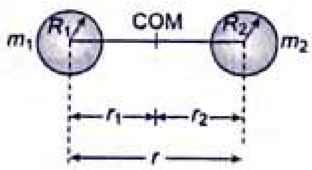
(a) $\left[\dfrac{2KGM}{g}\right]^{1/2}-\dfrac{1}{2}K'R$

(b) $\left[\dfrac{KGM}{2g}\right]^{1/2}-\dfrac{1}{2}K'R$

(c) $\left[\dfrac{KGM}{g}\right]^{1/2}-\dfrac{1}{2}K'R$

(d) $\left[\dfrac{KGM}{g}\right]^{1/2}-K'R$

72. Binary stars of comparable masses rotate under the influence of each other's gravity at a distance $\left[2G/\omega^2\right]^{1/3}$ where ω is the angular velocity of each of the system. If difference between the masses of two stars is 6 units. Find the ratio of the masses of smaller to bigger star.



(a) 4 : 10 (b) 1 : 7
(c) 2 : 8 (d) 3 : 9

73. In the previous problem, if two stars suddenly stop and collide with each other with sum of their radii as 1/10th of the distance r at which they were earlier rotating, find the relative velocity of collapse

(a) $12\sqrt{\dfrac{G}{r}}$ (b) $8\sqrt{\dfrac{G}{r}}$
(c) $10\sqrt{\dfrac{G}{r}}$ (d) $3\sqrt{\dfrac{G}{r}}$

74. Two planets revolve with same angular velocity about a star. The radius of orbit of outer planet is twice the radius of orbit of the inner planet. If T is time period of the revolution of outer planet, find the time in which inner planet will fall into the star. If it was suddenly stopped.

(a) $\dfrac{T\sqrt{2}}{8}$ (b) $\dfrac{T\sqrt{2}}{16}$

(c) $\frac{T\sqrt{2}}{4}$ (d) $\frac{T\sqrt{2}}{32}$

75. A satellite moves eastwards very near the surface of the Earth in equatorial plane with speed (v_0). Another satellite moves at the same height with the same speed in the equatorial plane but westwards. If R = radius of the Earth and ω be its angular speed of the Earth about its own axis. Then find the approximate difference in the two time period as observed on the Earth.

(a) $\frac{4\pi\omega R^2}{v_0^2+R^2\omega^2}$ (b) $\frac{2\pi\omega R^2}{v_0^2-R^2\omega^2}$

(c) $\frac{4\pi\omega R^2}{v_0^2-R^2\omega^2}$ (d) $\frac{2\pi\omega R^2}{v_0^2+R^2\omega^2}$

ARCHIVES

1. The energy required to move a body of mass m from an orbital of radius $2R$ to $3R$ is (where M is the mass of the earth and R is the radius of the earth)

(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{8R}$

(c) $\frac{GMm}{3R^2}$ (d) $\frac{GMm}{6R}$ **(AIEEE 2002)**

2. If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, then the satellite will
   (a) continue to move in its orbit with the same velocity.
   (b) move tangentially to the original orbit with the same velocity.
   (c) become stationary in its orbit.
   (d) move towards the earth. **(AIEEE 2002)**

3. The escape velocity of a body depends upon mass as

(a) m^0 (b) m^1

(c) m^2 (d) m^3 **(AIEEE 2002)**

4. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is

(a) $\frac{mgR}{2}$ (b) $2\,mgR$

(c) mgR (d) $\frac{mgR}{4}$ **(AIEEE 2002)**

5. The time period of a satellite of the earth is 5 h. If the separation between the earth and the satellite is increased to four times the previous value, the new time period will become

(a) 10 h (b) 80 h

(c) 40 h (d) 20 h **(AIEEE 2003)**

6. Two spherical bodies of masses m and $5M$ and radii R and $2R$, respectively, are released in free space with initial separation between their centres equal to $12R$. If they attract each other by gravitational force only, then the distance covered by the smaller body just before collision is

(a) $2.5R$ (b) $4.5R$

(c) $7.5R$ (d) $1.5R$ **(AIEEE 2003)**

7. The escape velocity for a body projected vertically upward from the surface of the earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be

(a) $11\sqrt{2}$ km/s (b) 22 km/s

(c) 11 km/s (d) $\frac{11}{\sqrt{2}}$ km/s

(AIEEE 2003)

8. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is

(a) gx (b) $\left(\frac{gR^2}{R+x}\right)^2$

(c) $\frac{gR^2}{R+x}$ (d) $\frac{gR}{R-x}$ **(AIEEE 2004)**

9. The time period of an earth satellite in circular orbit is independent of
   (a) the mass of the satellite
   (b) radius of its orbit
   (c) both the mass and radius of the orbit
   (d) neither the mass of the satellite nor the radius of its orbit **(AIEEE 2004)**

10. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the earth's surface to a height equal to the radius R of the earth is

(a) $2\,mgR$ (b) $\frac{1}{2}mgR$

(c) $\frac{1}{4}mgR$ (d) mgR **(AIEEE 2004)**

11. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in a circular orbit of radius R around the sun will be proportional to

(a) $R^{(n+1)/2}$ (b) $R^{(n-1)/2}$

(c) R^n (d) $R^{(n-2)/2}$ **(AIEEE 2004)**

12. The average density of the earth
   (a) does not depend on g
   (b) is a complex function of g

(c) is directly proportional to g
(d) is inversely proportional to g **(AIEEE 2005)**

13. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the earth. When both d and h are much smaller than the radius of the earth, which one of the following is correct?
(a) $d=\frac{h}{2}$ (b) $d=\frac{3h}{2}$
(c) $d=2h$ (d) $d=h$ **(AIEEE 2005)**

14. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work done against the gravitational force between them to take the particle far away from the sphere.
(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J
(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J
(AIEEE 2005)

15. If g_E and g_M are the acceleration due to gravity on the moon, respectively, and if Millikan oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electronic charge on moon}}{\text{electronic charge on earth}}$ to be
(a) 0 (b) $\frac{g_E}{g_M}$
(c) $\frac{g_M}{g_E}$ (d) 1 **(AIEEE 2007)**

16. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the velocity from the earth is 11 m/s the escape velocity from the surface of the planet would be
(a) 1.1 km/s (b) 11 km/s
(c) 110 km/s (d) 0.11 km/s
(AIEEE 2007)

17. The height at which the acceleration due to gravity becomes $g/9$ (where g is the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is
(a) $2R$ (b) $\frac{R}{\sqrt{2}}$
(c) $\frac{R}{2}$ (d) $\sqrt{2}R$ **(AIEEE 2008)**

18. Directions: The following question contains statement 1 and statement 2. Of the four choices given, choose the one that best describes the two statements.

Statement 1: For a mass M kept at a center of a cube of side a, the flux of gravitational field passing through its side is $4\pi GM$.

Statement 2: If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given is $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(a) statement 1 is true, but statement 2 is false.
(b) statement 1 is false, but statement 2 is true.
(c) statement 1 is true, statement 2 is true; statement 2 is the correct explanation for statement 1.
(d) statement 1 is true, statement 2 is true; statement 2 is not the correct explanation for statement 1.
(AIEEE 2008)

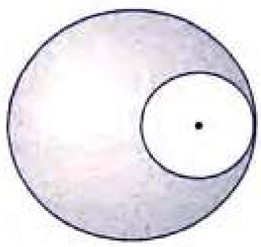
19. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is
(a) zero (b) $-\frac{4Gm}{r}$
(c) $-\frac{6Gm}{r}$ (d) $-\frac{9Gm}{r}$ **(AIEEE 2011)**

20. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km, respectively. The required energy for this work will be
(a) 6.4×10^{11} J (b) 6.4×10^{8} J
(c) 6.4×10^{9} J (d) 6.4×10^{10} J
(AIEEE 2012)

21. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?
(a) $\frac{2GmM}{3R}$ (b) $\frac{GmM}{2R}$
(c) $\frac{GmM}{3R}$ (d) $\frac{5GmM}{6R}$ **(JEE Main 2013)**

22. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is
(a) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (b) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$
(c) $\sqrt{\frac{GM}{R}}$ (d) $\sqrt{2\sqrt{2}\frac{GM}{R}}$
(JEE Main 2014)

23. From a solid sphere of mass M and radius R, a spherical portion of radius $R/2$ is removed, as shown in the figure. Taking gravitational potential $V=0$ at $r=\infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant)



(a) $\dfrac{-GM}{2R}$ (b) $\dfrac{-GM}{R}$

(c) $\dfrac{-2GM}{3R}$ (d) $\dfrac{-2GM}{R}$

(JEE Main 2015)

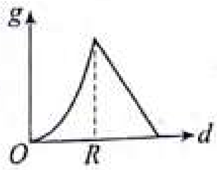
24. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; $h << R$). The minimum increases in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to: (Neglect the effect of atmosphere.)

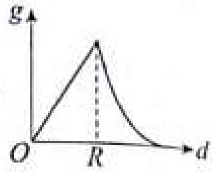
(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$

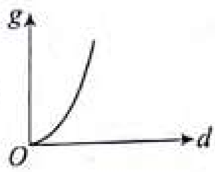
(c) $\sqrt{gR/2}$ (d) $\sqrt{gR}\left(\sqrt{2}-1\right)$

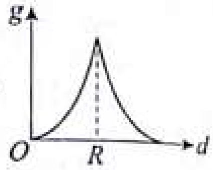
(JEE Main 2016)

25. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius)

(a) 

(b) 

(c) 

(d) 

(JEE Main 2017)

ANSWER KEY

Exercises

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (d)	**9.** (c)	**10.** (d)
11. (c)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (d)
21. (d)	**22.** (a)	**23.** (a)	**24.** (b)	**25.** (a)	**26.** (c)	**27.** (a)	**28.** (b)	**29.** (a)	**30.** (d)
31. (b)	**32.** (a)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (c)	**39.** (c)	**40.** (c)
41. (a)	**42.** (c)	**43.** (b)	**44.** (a)	**45.** (a)	**46.** (d)	**47.** (c)	**48.** (a)	**49.** (c)	**50.** (c)
51. (c)	**52.** (b)	**53.** (c)	**54.** (c)	**55.** (d)	**56.** (d)	**57.** (c)	**58.** (c)	**59.** (a)	**60.** (b)
61. (a)	**62.** (a)	**63.** (c)	**64.** (d)	**65.** (d)	**66.** (b)	**67.** (c)	**68.** (a)	**69.** (c)	**70.** (b)
71. (d)	**72.** (b)	**73.** (a)	**74.** (b)	**75.** (c)					

Archives

1. (d)	**2.** (b)	**3.** (a)	**4.** (c)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (d)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (d)	**20.** (d)
21. (d)	**22.** (b)	**23.** (b)	**24.** (d)	**25.** (b)					

Chapter 11

Elasticity

INTRODUCTION

The elastic behaviour of material plays an important role in engineering design and also constructing bridges, automobiles, ropeways, etc. In this section, we will study the elastic behaviour and the mechanical properties of the solids.

SOME DEFINITIONS

Deforming force: If a force applied on a body produces a change in the normal positions of the molecules of the body, which results in a change in the configuration of the body either in lengths, volume or shape, then the force applied is called deforming force. Thus, a deforming force is the one which when applied changes the configuration of the body.

Elasticity: The property of a material body by virtue of which it regains its original configuration (i.e., shape and size) when the external deforming force is removed is called elasticity.

Perfectly elastic body: A body which regains its original configuration immediately and completely after the removal of deforming force from it is called the perfectly elastic body. Quartz and phosphor bronze are the examples of nearly perfectly elastic bodies.

Plasticity: The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

Perfectly plastic body: A body which does not regain its original configuration at all on the removal of the deforming force, howsoever small the deforming force may be, is called perfectly plastic body. Putty, mud and paraffin wax are the examples of nearly perfectly plastic bodies. In fact, nothing is perfectly elastic or perfectly plastic. Only the degree of elasticity or plasticity differs from body to body.

CAUSE OF ELASTICITY

In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. These forces are known as intermolecular forces. When no deforming force is applied on the body, each molecule of the solid (i.e., body) is in its equilibrium position and the intermolecular forces between the molecules of the solid are maximum.

On applying a deforming force on the body, the molecules either come closer or go far apart from each other. As a result of this, the molecules are displaced from their equilibrium position. In other words, intermolecular forces get changed and restoring forces are developed on the molecules. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium positions and hence the solid (or the body) regains its original form.

STRESS

When a deforming force is applied on a body, it changes the configuration of the body by changing the normal positions of the molecules or atoms of the body. As a result, an internal restoring force comes into play which tends to bring the body back to its initial configuration. This internal restoring force acting per unit area of a deformed body is called stress, i.e.,

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} \qquad \text{(i)}$$

The SI unit of stress is N/m^2 or pascal (denoted by Pa) and in CGS system it is dyn/cm^2. The dimensional formula of stress is $ML^{-1}T^{-2}$.

Types of Stress

There are mainly three types of stresses as discussed in the following.

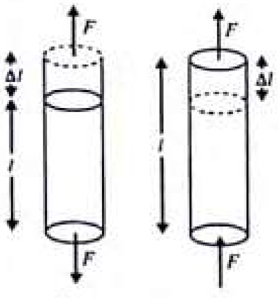
Normal Stress

The intensity of the internal force perpendicular to or normal to the section is called the normal stress or axial stress at a point. That is,

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} \qquad \text{(ii)}$$

If restoring force developed is perpendicular to the cross-sectional area chosen, then the corresponding stress is known as normal stress. It is also known as longitudinal stress. It can be of two types.

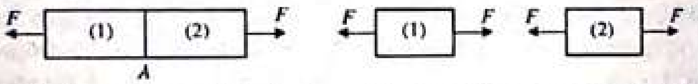
1. **Tensile stress:** If there is an increase in the length or extension of the body in the direction of force applied, the stress set up is called tensile stress.
   When a solid cylinder is stretched by two equal forces applied normal to its cross-sectional area, Figure (a), the restoring force per unit area in the body is called tensile stress.



2. **Compressive stress:** If there is a decrease in the length of the wire or compression of the body due to force applied, the stress set up is called compressive stress. When a solid cylinder is compressed under the action of two equal forces applied normal to its cross-sectional area as shown in Figure (b), then the restoring force per unit area in the body is called compressive stress.

NOTE:

- Tensile or compressive stress can also be termed as longitudinal stress.
- If the two parts of the body on the two sides of cross-sectional area pull each other, then the normal stress is known as tensile stress.



- If the two parts of the body on the two sides of cross-sectional area push each other, then the normal stress is known as compressive stress.

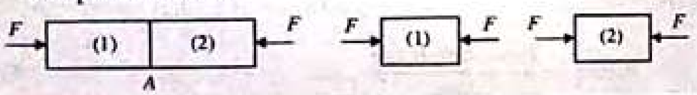
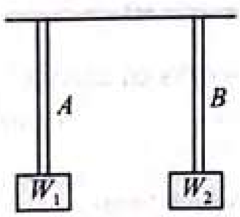


ILLUSTRATION 11.1 Two rods A and B, each of equal length but different materials are suspended from a common support as shown in the figure. The rods A and B can support a maximum load of $W_1 = 600$ N and $W_2 = 6000$ N, respectively. If their cross-sectional areas are $A_1 = 10\text{ mm}^2$ and $A_2 = 1000\text{ mm}^2$, respectively, then identify the stronger material.



Solution. At first sight, it looks that rod B is stronger as it supports a larger load. But we cannot compare strengths without having a common basis of comparison. Since the rods have different cross-sectional area, therefore, the strengths can be compared by determining the load capacity per unit area.

For rod A

$$\sigma_1 = \frac{W_1}{A_1} = \frac{600\text{ N}}{10\text{ mm}^2} = \frac{600}{10\times10^{-6}} = 60\times10^6\text{ Nm}^{-2}$$

For rod B

$$\sigma_2 = \frac{W_2}{A_2} = \frac{6000\text{ N}}{1000\text{ mm}^2} = \frac{6000}{1000\times10^{-6}} = 6\times10^6\text{ Nm}^{-2}$$

Since $\sigma_1 > \sigma_2$, therefore, the material of rod A is stronger than that of rod B.

Shearing Stress or Tangential Stress

When a deforming force, acting tangentially to the surface of a body produces a change in the shape of the body without any change in volume, the stress set up in the body is called tangential stress.

If a tangential force F applied on the top face of area a of a cubical body whose bottom face is rigidly fixed changes the shape of the cubical body without changing its volume, then

Tangential stress = F/a

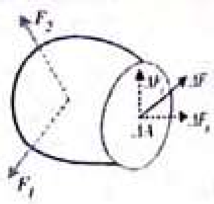
The intensity of the internal force parallel or tangential to the section is called the shear stress or tangential stress at a point. That is,

$$\tau = \lim_{\Delta A\to 0}\frac{\Delta F}{\Delta A} \qquad \text{(iii)}$$

The SI unit of stress is N/m^2 or Pa (pascal).

NOTE:

Sometimes, stress is simply determined by dividing the normal or tangential force with the cross-sectional area.



$$\text{Normal stress} = \frac{F_n}{A} \qquad \text{(iv)}$$

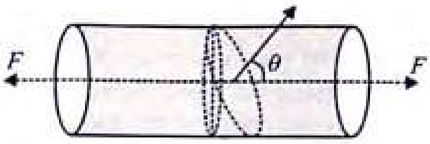
$$\text{Shearing stress} = \frac{F_t}{A} \qquad \text{(v)}$$

As shown in the figure, ΔF_n is the normal component and ΔF_t is the tangential component of internal force ΔF on an infinitesimally small area ΔA of the section of a member loaded in a plane.

- This method does not give stress at all points in the cross-sectional area but it merely gives the average stress.
- When the stress is constant or uniform over the cross section, Eqs. (iv) and (v) define the stress at all points of the cross section.
- A uniform distribution can exist only if the resultant of the applied loads passes through the centroid of the cross section.
- Stresses multiplied by the respective areas on which they act give forces, and it is the sum of these forces at an imaginary cut that keeps a body in equilibrium. That is,

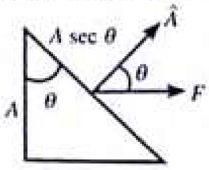
$$F_n = \int \sigma\, dA \text{ and } F_t = \int \tau\, dA$$

ILLUSTRATION 11.2 A bar of cross-section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.



(a) What is the tensile stress on the plane?
(b) What is the shearing stress on this plane?
(c) For what value of θ is the tensile stress maximum?
(d) For what value of θ is the shearing stress maximum?

Solution.



(a) The resolved part of F along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on this plane.

$$\text{Therefore, tensile stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta} = \frac{F}{A}\cos^2\theta$$

($\because$ Area of section $= A\sec\theta$)

(b) $$\text{Shearing stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\sin\theta}{A\sec\theta} = \frac{F}{2A}\sin 2\theta$$

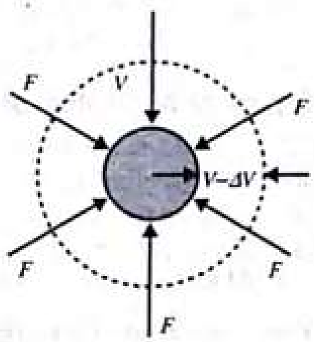
(c) Obviously, tensile stress on the plane is maximum when $\cos^2\theta$ is maximum, that is, $\cos\theta = 1$ or $\theta = 0°$.

(d) Obviously, shearing stress is maximum when $\sin 2\theta$ is maximum, that is

$$\sin 2\theta = 1 \quad \text{or} \quad 2\theta = 90° \quad \text{or} \quad \theta = 45°$$

Volumetric Stress

When a solid body undergoes a change in volume without any change in its geometrical shape on applying the force perpendicular to every point on the surface of body, see figure, the restoring force per unit area in the body is called volumetric stress.



If the deforming forces act on the body from all sides, then corresponding stress developed is known as volume stress. For example, if a block is kept completely immersed in water, hydrostatic pressure acts on all sides of the block. As stress is the ratio of some force and area, it is a kind of pressure.

STRAIN

When a deforming force is applied on a body, there is a change in the configuration of the body. The body is said to be strained or deformed. The ratio of the change in configuration (i.e., shape, length or volume) to the original configuration of the body is called strain. Mathematically, strain is given as

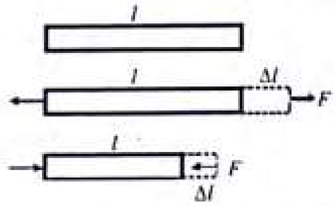
$$\epsilon = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

Strain does not have any unit.

Types of Strain

The change in configuration can be a change either in length, volume or shape of the body. There are three types of strain.

Longitudinal strain: This type of strain is produced when the deforming force causes a change in length of the body. It is defined as the ratio of the change in length to the original length of the body. Consider a wire of length L. The wire is stretched by a force F. Let the change in length of the wire be ΔL. Then longitudinal strain.



$$\epsilon = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L} \qquad \text{(i)}$$

NOTE:

- The above expression measures only the average value of strain. The correct expression for strain at any position is

$$\epsilon = \frac{d\chi}{dL}$$

where $d\chi$ is the differential elongation of the differential length dL.

- Under certain conditions the strain produced in a rod is assumed to be uniform and it can be computed from Eq. (i). These conditions are as follows:
  - The specimen must be of uniform cross section.
  - The material must be homogeneous.
  - The load must be axial so as to produce uniform stress.

  Since strain represents a change in length divided by the original length, strain is a dimensionless quantity. Sometimes, its units are used as metre per metre (m/m). However, it is commonly used in percentage.

ILLUSTRATION 11.3 A steel wire 2 m long is suspended from the ceiling. When a mass is hung at its lower end, the increase in length recorded is 1 cm. Determine the strain in the wire.

Solution. Original length = 2 m

Increase in length = 1 cm = 1×10^{-2} m

$$\text{Longitudinal strain} = \frac{\text{Increase in length}}{\text{Original length}}$$

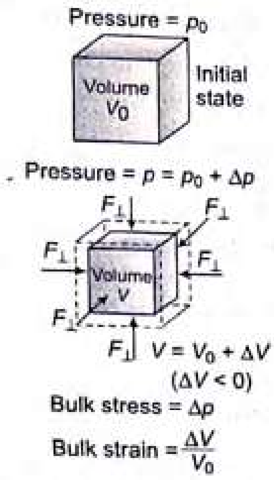
$$= \frac{1\times 10^{-2}\text{ m}}{2\text{ m}} = 5\times 10^{-3}$$

ILLUSTRATION 11.4 A brass rod of length 1 m is fixed to a vertical wall at one end, with the other end keeping free to expand. When the temperature of the rod is increased by 120°C, the length increases by 3 cm. What is the strain?

Solution. After the rod expands to the new length, there are no elastic forces developed internally in it.

So, strain = 0.

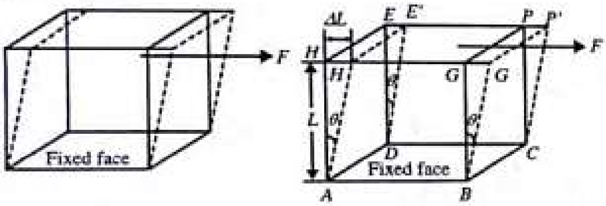
Volume strain: This type of strain is produced when the deforming force produces a change in volume of the body as shown in the figure. It is defined as the ratio of the change in volume to the original volume of the body. If ΔV is change in volume and V is the original volume, then



$$\epsilon_v = \text{Volume strain}$$

$$= \frac{\Delta V}{V}$$

Shear strain: This type of strain is produced when the deforming force causes a change in the shape of the body without changing its volume. It is defined as the angle (θ) through which a face originally perpendicular to the fixed face is turned as shown in the figure.



Consider a cubical body as shown in the figure. Its lower surface $ABCD$ is fixed and a tangential force F is applied on the top surface $EPGH$. Let the vertical planes $AHED$ and $BCPG$ be laterally shifted to positions $AH'E'D$ and $BCP'G'$, respectively, through an angle θ, where $\angle H AH' = 0$. Let $HA = L$ and $HH' = \Delta L$.

$$\text{Shearing strain} = \theta = \frac{\Delta L}{L}$$

Thus, the shearing strain is also defined as the ratio of displacement of a surface under a tangential force to the perpendicular distance of the displaced surface from the fixed surface.

NOTE:

- More precisely, the shearing strain is defined as the angular change between two perpendicular faces of a differential element.
- Strain in a rectangular shaped element is determined from the following parameters:
  - longitudinal deformation d under the action of normal forces F_n
  - shear deformation χ, under the action of tangential forces F_t
- A shearing strain is never accompanied by a change in volume, i.e., in a shear strain, the body gets distorted, resulting only in an alternation in its shape but not in size.
- A change in shape/size, i.e., dimensions need not necessarily imply a strain. For example, if a body is heated to expand, its volume changes. It acquires a new size due to expansion. However, the strain remains zero. Unless and until internal elastic forces operate to bring the body to the original state, no strain exists. When a body is heated, the total energy of molecules increases owing to an increase in the kinetic energy of the molecules. This results in a shift (increment) of the 'equilibrium distance' of molecules and the body acquires a new shape and size in the expanded from, whereby the molecules are in 'zero force' state. Hence, there is no strain. However, if the body is restricted to expand during the process of heating, then the molecules become 'strained', and even if there is no apparent change in dimensions of the body, there is strain. In such a case, strain is measured as the ratio of the change in dimension that would have occurred, had the body been free to expand or contract to the original dimension.

ELASTIC LIMIT

Elastic limit is the upper limit of deforming force up to which, if deforming force is removed, the body regains its original form completely and beyond which if deforming force is increased, the body loses its property of elasticity and gets permanently deformed.

HOOKE'S LAW AND ELASTIC MODULI

Within certain limits (called proportional limit), stress produced in a material is proportional to the strain and the constant of proportionality is the modulus of elasticity. That is,

$$\sigma \propto \epsilon \quad \text{or} \quad \sigma = Y\epsilon \tag{i}$$

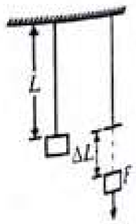
Types of Modulus of Elasticity

Corresponding to the three types of strain, there are three types of modulus of elasticity:

(1) Young's modulus of elasticity (Y)
(2) Bulk modulus of elasticity (K)
(3) Modulus of rigidity (η)

Young's Modulus of Elasticity

It is defined as the ratio of the normal stress to the longitudinal strain. That is,



Young's modulus

$$(Y) = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

Normal stress = F/A

Longitudinal strain = $\Delta L/L$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L} \tag{ii}$$

NOTE:

- Extension is proportional to the deforming force and length of the rod, i.e., greater the force larger the deformation; longer the rod larger the extension.

- Extension is inversely proportional to the cross-sectional area A and the modulus of elasticity, i.e., a thicker rod produces less deformation and a stiffer material produces less deformation. Eq. (ii) may be rearranged as

$$F=\left(\frac{YA}{L}\right)\Delta L \qquad \text{(iii)}$$

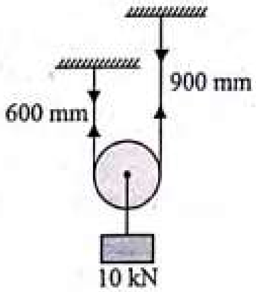
- It indicates that deforming force is proportional to the elongation and the term in the bracket indicates another constant of proportionality called the stiffness constant k. Thus, Eq. (iii) may be written as

$$F=k\Delta L \qquad \text{(iv)}$$

where $k=(YA/L)=$ constant.

- By using Eq. (iv), a rod of uniform cross section may be considered as an elastic spring of stiffness $k=(YA/L)$.
- Longer the rod, lesser the stiffness and thicker the rod, greater the stiffness.

ILLUSTRATION 11.5 A load of 10 kN is supported from a pulley which in turn is supported by a rope of sectional area 1×10^3 mm^2 and modulus of elasticity 10^3 N mm^{-2}, as shown in the figure. Neglecting the friction at the pulley, determine the deflection of the load.



Solution. Let T be the tension in the rope. Then

$2T=10$ kN $\Rightarrow T=5$ kN

Hence, longitudinal stress in the rope is

$$\sigma=\frac{T}{A}=\frac{5\text{ kN}}{10^3\text{ mm}^2}=5\text{N/mm}^2$$

$$\text{Extension in the rope}=\frac{\text{Stress}}{Y}\times L$$

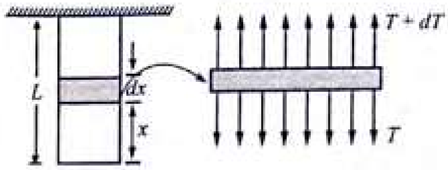
$$=\frac{5\text{ N/mm}^2}{10^3\text{N/mm}^2}\times1500\text{ mm}=7.5\text{ mm}$$

Therefore, deflection of the load, $\delta=\frac{7.5}{2}=3.75$ mm.

ILLUSTRATION 11.6 A rod of uniform cross-sectional area A and length L has a weight W. It is suspended vertically from a fixed support. If the material of the rod is homogeneous and its modulus of elasticity is Y, then determine the total elongation produced in the rod due to its own weight.

Solution. In this case, different parts of the rod does not elongate to the same extent. The element closer to the support elongates more as the stress is higher with respect to the elements closer to the free end of the rod.

To determine total elongation of the rod, let us consider a small element of differential length dx at a distance x from the free end of the rod, as shown in the figure.



The stress at the position of this element is produced by the weight of the rod of length x lying below it, i.e., $(W/L)x$.

Therefore, stress at this section

$$\sigma=\frac{(W/L)x}{A}=\frac{Wx}{AL}$$

The elongation $d\delta$ produced in the differential element dx is

$$d\delta=\frac{\sigma}{Y}dx=\frac{W}{YAL}x\,dx$$

Thus, total elongation produced in the rod can be calculated as

$$\delta=\int d\delta$$

$$\delta=\frac{W}{YAL}\int_0^L x\,dx=\frac{W}{YAL}\left[\frac{x^2}{2}\right]_0^L$$

$$\text{or}\qquad \delta=\frac{1}{2}\frac{WL}{YA} \qquad \text{(v)}$$

NOTE: Equation (v) may be visualized as

$$\delta=\frac{(W/2)L}{YA}$$

which suggests that a rod of uniform mass is equivalent to a massless rod with a concentrated load equal to half weight ($W/2$) of the rod acting at its lower end, as shown in the figure.

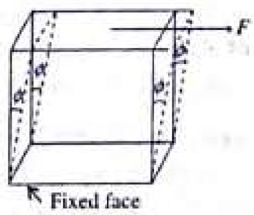
If σ is the density of the material of the rod, then Eq. (v) may be written as

$$\delta=\frac{1}{2}\frac{\rho gL^2}{Y}$$

It indicates that the deflection of the rod is proportional to the square of the length of the rod.

Modulus of Rigidity

It is defined as the ratio of the tangential stress to the shear strain. Let us consider a cube whose lower face is fixed and a tangential force F acts on the upper face whose area is A.



$\therefore$ Tangential stress $=F/A$

If the vertical sides of the cube shifts through an angle ϕ called shear strain, then the modulus of rigidity is given by

$$\eta = \frac{\text{Tangential stress}}{\text{Shear strain}}$$

or $$\eta = \frac{F/A}{\phi} = \frac{F}{A\phi} \quad \text{(vi)}$$

Equation (vi) is an extension of Hooke's law for shearing stress and strain. The constant of proportionality G is called the *shear modulus* or the modulus of rigidity. It is a measure of the resistance to alter the shape of the body. Its units are N m^{-2} or Pa.

ILLUSTRATION 11.7 A 0.05 m cube has its upper face displaced by 0.2 cm by a tangential force of 8 N. Calculate the shearing strain, shearing stress and modulus of rigidity of the material of the cube.

Solution. $l = 5\times10^{-2}\text{ m}$, $\Delta l = 0.2\text{ cm} = 0.2 \times 10^{-2}\text{ m}$, $F = 8\text{ N}$

$$\text{Shearing strain} = \frac{\Delta l}{l} = \frac{0.2}{5} = 0.04$$

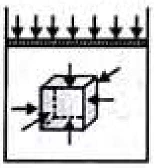
$$\text{Shearing stress} = \frac{F}{l\times l} = \frac{8}{(5\times10^{-2})^2} = 3200\text{ N m}^{-2}$$

Modulus of rigidity,

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{3200}{0.04} = 80000\text{ N m}^{-2} = 8\times10^4\text{ N m}^{-2}$$

Bulk Modulus

If a body is uniformly compressed from all sides then it undergoes a decrease in volume. The figure shows a small metal cube which is uniformly compressed from all sides by placing it inside a liquid column which is compressed externally. If p is the uniform compression on the cube and ΔV is the change in volume V of the cube, then it has been obtained experimentally that $p \propto \Delta V/V$.



A metal placed inside a liquid column is equally compressed from all directions.

Volumetric strain is defined as

$$\epsilon_V = \frac{\Delta V}{V}$$

$$p = -B\left(\frac{\Delta V}{V}\right) \Rightarrow B = -\frac{P}{\Delta V/V} \quad \text{(vii)}$$

The constant of proportionality B is called the bulk modulus of elasticity. Negative sign shows that increase in pressure (p) causes decrease in volume (ΔV).

Compressibility: The reciprocal of bulk modulus of elasticity is called compressibility. Unit of compressibility in SI is $\text{N}^{-1}\text{ m}^2$ or pascal^{-1} (Pa^{-1}).

Bulk modulus of solids is about 50 times that of liquids, and for gases it is 10^{-8} times that of solids.

$K_{\text{solids}} > K_{\text{liquids}} > K_{\text{gases}}$

Isothermal modulus of elasticity of gas $K = P$ (pressure of gas). Adiabatic modulus of elasticity of gas $K = \gamma \times P$ where $\gamma = C_p/C_v$.

NOTE:

- Bulk modulus is different from modulus of elasticity. A material with large bulk modulus is difficult to compress but it does not need to be stiff. For example, water.
- The reciprocal of bulk modulus is defined as compressibility (K)

$$K = (1/B)$$

ILLUSTRATION 11.8 Calculate the density of water near the bottom of the ocean where the pressure is about 500 atm and density of water is 1025 kg/m^3 approximately. Take bulk modulus of water as for water as $2.0\times10^9\text{ N/m}^2$.

Solution. The density of water at the ocean's surface where the pressure is 1 atm is about 1025 kg/m^3. Thus, 1 m^3 of water at the surface has a mass of 1025 kg. Let us calculate the change in volume ΔV of this water when it is at a pressure of 500 atm.

$$\Delta V = \frac{(\Delta p)V}{K}$$

The pressure change Δp is 500 atm – 1 atm = 499 atm, which is equivalent to $499\times10^5\text{ N m}^{-2}$. Thus, the volume change ΔV is

$$\therefore \quad \Delta V = \frac{(499\times10^5\text{N/m}^2)(1\text{ m}^3)}{2.0\times10^9\text{N/m}^2} = 0.025\text{ m}^3$$

The density of the water is then

$$\rho = \frac{m}{V - \Delta V}$$

$$= \frac{1025\text{ kg}}{1.00\text{ m}^3 - 0.025\text{ m}^3} = \frac{1025\text{ kg}}{0.9775\text{ m}^3} = 1051\text{ kg m}^{-3}$$

The water is compressed from only 2% to 3% at the bottom of the ocean where the pressure is several hundred atmosphere. Liquids and solids are not easily compressed.

ILLUSTRATION 11.9 Bulk modulus for rubber is 9.8×10^8 N m^{-2}. To what depth should a rubber ball be taken in a lake so that its volume is decreased by 0.1%?

Solution. $K = \dfrac{(\Delta P)V}{\Delta V}$ or $\Delta P = \dfrac{K\Delta V}{V}$

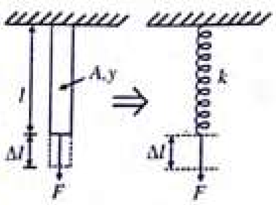
Let h be the required depth and the density of water be ρ. Then

$$h\rho g = \frac{K\Delta V}{V}$$

or $$h = \frac{K\Delta V}{\rho g V} = \frac{9.8\times10^8\times0.1}{10^3\times9.8\times100} = 100\text{ m}$$

ANALOGY OF ROD AS A SPRING

A thin wire can be imagined as a parallel combination of arrays of molecular springs. When we pull a wire, we really pull the springs. Let us take an elastic rod of length l, area of cross-section A and modulus of elasticity y and apply a force F as shown. If the rod elongates under the action of force Δl then we can write.



Young's modulus,

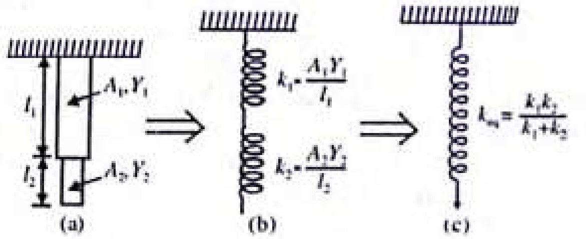
$$Y = \frac{\text{Stress}}{\text{Strain}} \Rightarrow Y = \frac{Fl}{A\Delta l} \quad \text{or} \quad F = \frac{AY}{l}\Delta l$$

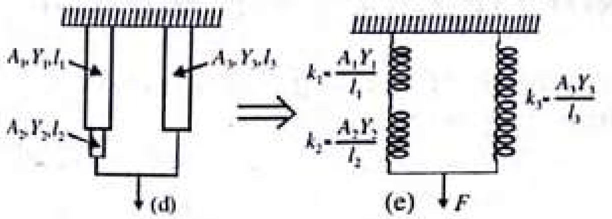
$AY/l = k$ (constant) depends on type of material and geometry of rod. Now,

$$F = k\Delta l \Rightarrow F \propto \Delta l \qquad \text{(i)}$$

If we compare the relation in Eq. (i) with spring force ($F_{spring} = kx$) we can say k in Eq. (i) is equivalent to force constant of the spring. Hence we can write,

$$k = \frac{Ay}{l} = \text{equivalent spring constant.}$$





For the system of rods shown in Figure (a), the replaced spring system is shown in Figure (b) (two springs in series). Figure (c) represents the equivalent spring system. Figure (d) represents another combination of rods and their replaced spring system.

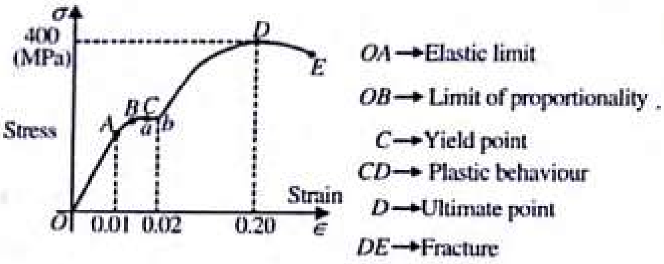
NOTE: Since $k_{eq} = YA/l$, we have $k_{eq}\, l = YA$. It means that the product of stiffness of an elastic wire and its length is always constant. If we cut the wire into two equal parts, the stiffness of each part will be double that of the original wire. Following the same logic we can say that if a spring is cut into N equal parts, each part will have stiffness N times the stiffness of the original spring. Hence, kl = constant, where k = stiffness and l = natural length of the body. It is also equally valid for the springs.

If F is the stretching force applied on the bar, then the same force is transmitted longitudinally to each bar; however, stresses induced will be different and hence elongations of each bar will also be different. Figure (e) shows the free-body diagram of the bar. Stress in the bar is given by

$$\sigma_1 = F/A_1$$

STRESS–STRAIN DIAGRAM

When a wire is stretched by a load, it is seen that for small value of load, the extension produced in the wire is proportional to the load. On removing the load, the wire returns to its original length. The wire regains its original dimensions only when load applied is less or equal to a certain limit. This limit is called elastic limit. Thus, elastic limit is the maximum stress on whose removal, the bodies regain their original dimensions. As shown in the figure, this type of behaviour is represented by OA portion of the graph. As we increase the stress further, a point B is reached where the increase in strain is proportional to the stress. On reaching point B, we remove the stress. Then the wire does not regain its original dimension.



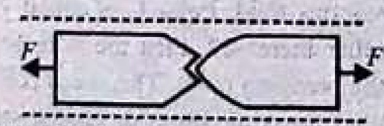
Stress–strain graph for a mild steel rod

As we go beyond point B, even for a very small increase in stress, the strain produced is very large. This type of behaviour is observed around point C and at this stage the wire begins to flow like a viscous fluid. Point C is called yield point. If the stress is further increased, then the wire breaks off at a point D called the breaking point. The stress corresponding to this point is called breaking stress or tensile strength of the material of the wire. A material for which the plastic range CD is relatively high is called ductile material. These materials get permanently deformed before breaking. The materials for which plastic range is relatively small are called brittle materials. These materials break as soon as the elastic limit is crossed.

The stress–strain diagram is used to study the properties of materials under the action of external forces. In these stress-strain diagrams, conventionally the ordinate scale is used for stresses (σ) and the abscissa for strains (ϵ).

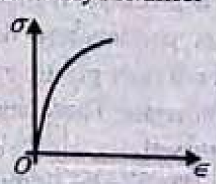
Important Points

- Breaking stress = breaking force/area of cross section.
- Breaking stress is constant for a material.
- Breaking force depends upon the area of the section of the wire of a given material.
- The working stress is always kept lower than that of a breaking stress so that 'safety factor = breaking stress/working stress' may have a large value.
- Breaking strain = elongation or compression/original dimension.
- Breaking strain is constant for material.
- In the yield region, strain is 15–20 times those that takes place up to the proportional limit occurring during yielding.
- A study of stress–strain diagrams shows that the yield point is so close to the elastic limit that for most purposes the two may be taken as one.
- As the deformation is increased further beyond the yield region, the curve moves upward and attains its maximum at a point U (called the ultimate stress) after which the curve moves downward to point B at which the material fails or breaks down.
- The breaking stress is lower than the ultimate stress because the breaking stress is computed by dividing the breaking load by the original cross-sectional area which, although convenient, is incorrect. This error is caused by a phenomenon known as necking. After the point U, the material stretches very rapidly and simultaneously narrows down, as shown in the figure, so that the breaking load is distributed over a smaller area. If the breaking area is measured after failure occurs, and divided into the breaking load, the result is a truer value of the actual failure stress, although this is commonly taken as the maximum stress of the material.



Necking of the mild-steel bar beyond the ultimate point.

- The stress–strain diagrams differ considerably for different materials. The steeper curve indicates a stiffer material; a taller graph indicates a stronger material; a long graph parallel to the ϵ-axis indicates a ductile material and the absence of yield point or plastic zone refers to a brittle material.



Stress–strain graph of a brittle material.

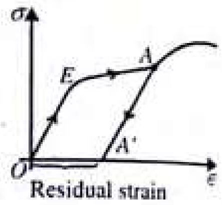
- Even for the same material, the stress–strain graphs differ, depending on the temperature at which the test is conducted, the speed of the test, etc.
- Breaking strain is constant for material.

ELASTICITY AND PLASTICITY

The property by which a body returns to its original shape after removal of the deforming force (or load) is called elasticity. It has been observed experimentally that some bodies return to their original states immediately after the removal of external load while some other bodies take longer time to return back.

The delay in returning to the original state by an elastic body after the removal of external load is called elastic aftereffect. Quartz and phosphor bronze are free from elastic aftereffects.

Plasticity is the property of a body by which it shows a permanent deformation which is not recovered after the removal of external load. In the stress–strain graph as shown in the figure, the region of plastic deformation lies on the right of the elastic limit. If the body is loaded to a stress in the plastic region and then unloaded as shown in the figure, the unloading curve does not retrace the loading curve but it returns back along a different path. The area enclosed by the loop $OEAA'$ gives the energy lost per unit volume of the body. The loop is also called the hysteresis loop.



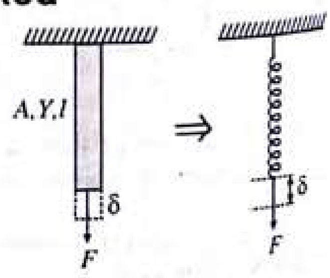
A specimen is loaded to a point A in the plastic region and then unloaded. The specimen shows permanent or residual strain.

On the contrary, if the body is loaded in the elastic region and then unloaded, it returns back by retracing the same path. It indicates that the internal forces between the particles of the body act as conservative forces.

ENERGY STORED IN A DEFORMED BODY

Elastic Potential Energy Stored in a Stretched Wire or in a Rod

We can replace a rod with a springs as shown in the figure. Strain energy stored in equivalent spring is



$$U = \frac{1}{2}k\delta^2 = \frac{1}{2}\left(\frac{AY}{L}\right)\delta^2$$

where $\delta = \dfrac{Fl}{AY}$

$\therefore$ Strain energy $U = \dfrac{1}{2}\dfrac{AY}{l}\dfrac{F^2l^2}{A^2Y^2}$

$$= \frac{1}{2}\frac{F^2 l}{AY} \quad \text{or} \quad U = \frac{1}{2}F\delta$$

This equation can be rearranged as

$$U = \frac{1}{2}\frac{F^2}{A^2}\cdot\frac{lA}{Y} \quad (lA = \text{volume of the rod, } F/A = \text{stress})$$

$$U = \frac{1}{2}\frac{(\text{Stress})^2}{Y}.\text{Volume}$$

Again $U = \frac{1}{2}\frac{F}{A} \times \frac{F}{AY} \times Al \quad (\text{Strain} = \frac{F}{AY})$

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

Again $U = \frac{1}{2}\frac{F^2}{A^2Y^2}AlY$

$$U = \frac{1}{2}Y(\text{Strain})^2 \times \text{Volume}$$

$\therefore$ Strain energy density $u = \frac{\text{Strain energy}}{\text{Volume}} = \frac{1}{2}\frac{(\text{Stress})^2}{Y}$

$$u = \frac{1}{2}Y(\text{Strain})^2 = \frac{1}{2}\text{Stress} \times \text{Strain}$$

$$u = \frac{U}{AL} = \frac{\rho^2}{2Y} = \in\frac{1}{2}Y^2 = \frac{1}{2}\in Y^2$$

where $\in = \delta/L$ and $\rho = F/A$.

1. The area under the stress–strain diagram gives the potential energy (or strain energy) stored per unit volume of the specimen.
2. If stress applied is not constant we can write energy stored in any elastic rod

$$U = \frac{1}{2Y}\int \rho^2 dV$$

where ρ = stress and dV = change in volume of rod.

NOTE: The potential energy stored in the rod in terms of stiffness constant k is given by

$$U = \frac{1}{2}k\delta^2$$

where $k = YA/L$.

The potential energy stored in the rod in terms of the maximum stretching force is

ILLUSTRATION 11.10 A catapult consists of two parallel rubber strings, each of lengths 10 cm and cross-sectional area 10 mm^2. When stretched by 5 cm, it can throw a stone of mass 100 gm to a vertical height of 25 m. Determine Young's modulus of elasticity of rubber.

Solution.

A stretched catapult has elastic potential energy stored in it (Strain energy stored in both the rubber strings)

$$U = \left(\frac{1}{2}\frac{YAl^2}{L}\right) \times 2$$

This energy, when imparted to the stone, it flies off a height 20 m. Energy possessed by the stone = mgh.

Now,

$$U = mgh \Rightarrow \frac{YAl^2}{L} = mgh$$

$$Y = \frac{mghL}{Al^2} = \frac{1\times10^{-1}\times10\times25\times10^{-1}}{10\times10^{-6}\times(5\times10^{-2})^2}$$

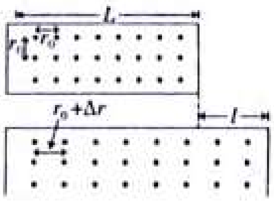
$\therefore \quad Y = 10 \times 10^8$ N/m^2

INTERATOMIC FORCE CONSTANT

The interatomic force F' developed between the atoms is directly proportional to the change in distance Δr between them.

$$F' = k\Delta r \qquad \text{(i)}$$

where k is interatomic force constant. Let L be the length of a wire and r_0 be the interatomic distance between the atoms of the wire. Suppose, when a force F is applied, the length of the wire increases by l, and the distance between its atoms increases from r_0 to $r_0 + \Delta r$. Then longitudinal strain,



$$\frac{l}{L} = \frac{\Delta r}{r_0} \qquad \text{(ii)}$$

If A is the area of cross section of the wire, it will have A/r_0^2 chains of atoms. Therefore, the interatomic force (force acting on chain) is

$$F' = \frac{\text{External force}}{\text{Number of chains}} = \frac{F}{A/r_0^2} = \frac{Fr_0^2}{A}$$

Interatomic force constant is

$$k = \frac{F'}{\Delta r} = \frac{Fr_0^2/A}{\Delta r} = \frac{F}{A}\frac{r_0^2}{\Delta r} \qquad \text{[using Eq. (i)]}$$

But $r_0/\Delta r = L/l$. Hence,

$$k = \frac{F}{A}\frac{L}{l}r_0 = \frac{F/A}{l/L}r_0 \qquad \text{[using Eq. (ii)]}$$

But by definition,

$$\frac{F/A}{l/L} = Y \text{ (Young's modulus of the material of the wire)}$$

$\Rightarrow \quad k = Yr_0$

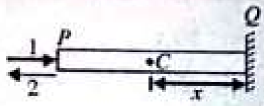
Thus, interatomic force constant k is equal to the product of Young's modulus of the material of the wire and the normal distance r_0 between the atoms of the wire.

☞ Important Points

- Breaking stress depends on the material of the wire. It does not depend upon the diameter or length of the wire.
- Breaking load depends on the area of cross section of the wire.
- The strain persists even when the stress is removed and thus lags behind the stress.
- Steel is highly elastic material.
- Steel is more elastic than rubber.
- Young's modulus decreases with rise in temperature.
- A liquid has zero modulus of rigidity.
- Rubber remains elastic even for a strain of the order of thousand.
- A rise in temperature results, in general, in weakening of elastic properties and a fall in temperature results in strengthening them.
- Young's modulus is independent of length or radius of wire.
- Solids are the most elastic and gases are the least elastic.
- Under tensile stress, the restoring forces are caused by interatomic attraction.
- Under compressional stress, the restoring force is due to the interatomic repulsion.
- Force of repulsion, not the force of attraction, is responsible in the interatomic or intermolecular potential for the formation of a solid.
- Solids have all the three types of elasticity whereas liquids and gases have only volume elasticity, i.e., only bulk modulus is applicable to them. Gases, however, have two types of bulk moduli, viz. isothermal and adiabatic.
- Young's modulus is defined for solids but not for liquids and gases. The value of Young's modulus increases on mixing the impurity in the solid and decreases on increasing the temperature of the body.
- The area under the stress-strain graph represents the work done per unit volume.

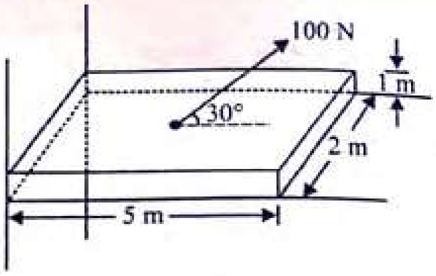
Concept Application Exercise 11.1

1. A horizontal force of magnitude F acts at the end P of a uniform rigid rod which is welded at point Q. In each case 1 and 2, as shown in the figure, find the reaction force acting at a point C at a distance x from the fixed end Q of the rod.

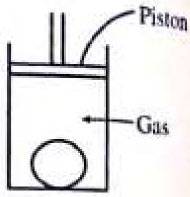


2. A rubber ball of bulk modulus B is taken to a depth h of a liquid of density ρ. Find the fractional change in the radius of the ball.

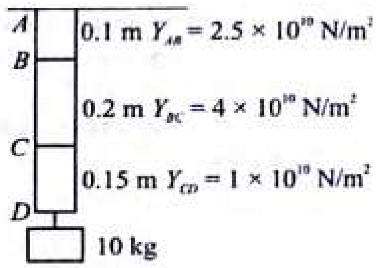
3. Find out longitudinal stress and tangential stress on a fixed block.



4. Find out bulk stress on the spherical object of radius $10/\pi$ cm if area and mass of piston are 50 cm^2 and 50 kg, respectively, for a cylinder filled with gas as shown in the figure.



5. Three rods of uniform area of cross section $A = 10^{-7}$ m^2 are arranged as shown in the figure. Find out the shift in point B, C and D.



6. Find the depth of lake at which density of water is 1% greater than that at the surface. Given compressibility $k = 50 \times 10^{-6}$ atm^{-1}.

Solved Examples

1. A wire of length L and radius r is rigidly fixed at one end. On stretching the other end of the wire with a force F, the increase in its length is l. If another wire of same material but of length $2L$ and radius $2r$ is stretched with a force of $2F$, the increase in its length will be

(a) l (b) $2l$

(c) $\frac{l}{2}$ (d) $\frac{l}{4}$

Sol. (a) $l = \frac{FL}{AY} = \frac{FL}{\pi r^2 Y} \Rightarrow l \propto \frac{FL}{r^2}$ (Y = constant)

$$\therefore \frac{l_2}{l_1} = \frac{F_2}{F_1} \times \frac{L_2}{L_1}\left(\frac{r_1}{r_2}\right)^2 = 2 \times 2 \times \left(\frac{1}{2}\right)^2 = 1$$

$\therefore l_2 = l_1$, i.e., increment in its length will be l.

2. A copper wire and a steel wire of the same diameter and length are connected end to end and a force is applied,

which stretches their combined length by 1 cm. The two wires will have

(a) Different stresses and strains
(b) The same stress and strain
(c) The same strain but different stresses
(d) The same stress but different strains

Sol. (d) Stress $= \dfrac{\text{Force}}{\text{Area}}$

In the present case, force applied and area of cross-section of wires are same, therefore stress has to be the same.

$$\text{Strain} = \frac{\text{Stress}}{Y}$$

Since the Young's modulus of steel wire is greater than the copper wire, therefore, strain in case of steel wire is less than that in case of copper wire.

3. A steel ring of radius r and cross-section area A is fitted on to a wooden disc of radius R $(R > r)$. If Young's modulus be E, then the force with which the steel ring is expanded is

(a) $AE\dfrac{R}{r}$ (b) $AE\left(\dfrac{R-r}{r}\right)$

(c) $\dfrac{E}{A}\left(\dfrac{R-r}{A}\right)$ (d) $\dfrac{Er}{AR}$

Sol. (b) Initial length (circumference) of the ring $= 2\pi r$
Final length (circumference) of the ring $= 2\pi R$
Change in length $= 2\pi R - 2\pi r$

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{2\pi(R-r)}{2\pi r} = \frac{R-r}{r}$$

Now Young's modulus, $E = \dfrac{F/A}{l/L} = \dfrac{F/A}{(R-r)/r}$

$$\therefore F = AE\left(\frac{R-r}{r}\right)$$

4. The coefficient of linear expansion of brass and steel are α_1 and α_2. If we take a brass rod of length l_1 and steel rod of length l_2 at 0°C, their difference in length $(l_2 - l_1)$ will remain the same at a temperature if

(a) $\alpha_1 l_2 = \alpha_2 l_1$ (b) $\alpha_1 l_2^2 = \alpha_2 l_1^2$

(c) $\alpha_1^2 l_1 = \alpha_2^2 l_2$ (d) $\alpha_1 l_1 = \alpha_2 l_2$

Sol. (d) $L_2 = l_2(1 + \alpha_2 \Delta\theta)$ and $L_1 = l_1(\alpha_1 \Delta\theta)$

$\Rightarrow (L_2 - L_1) = (l_2 - l_1) + \Delta\theta(l_2\alpha_2 - l_1\alpha_1)$

Now $(L_2 - L_1) = (l_2 - l_1)$ so, $l_2\alpha_2 - l_1\alpha_1 = 0$

5. A 5 m long aluminium wire ($Y = 7\times10^{10}$ N/m^2) of diameter 3 mm supports a 40 kg mass. In order to have the same elongation in a copper wire ($Y = 12\times10^{10}$ N/m^2) of the same length under the same weight, the diameter should now be (in mm).

(a) 1.75 (b) 1.5
(c) 2.5 (d) 5.0

Sol. (c) $l = \dfrac{FL}{\pi r^2 Y} \Rightarrow r^2 \propto \dfrac{1}{Y}$ (F, L and l are constant)

$$\frac{r_2}{r_1} = \left(\frac{Y_1}{Y_2}\right)^{1/2} = \left(\frac{7\times10^{10}}{12\times10^{10}}\right)^{1/2}$$

$$\Rightarrow r_2 = 1.5\times\left(\frac{7}{12}\right)^{1/2} = 1.145 \text{ mm} \Rightarrow \text{dia} = 2.29 \text{ mm}$$

6. The length of an elastic string is a metre when the longitudinal tension is 4 N and b metre when the longitudinal tension is 5 N. The length of the string in metre when the longitudinal tension is 9 N is

(a) $a - b$ (b) $5b - 4a$

(c) $2b - \dfrac{1}{4}a$ (d) $4a - 3b$

Sol. (b) Let L be the original length of the wire and K be the force constant of wire.
Final length = Initial length + Elongation

$$L' = L + \frac{F}{K}$$

For first condition $a = L + \dfrac{4}{K}$ ...(i)

For second condition $b = L + \dfrac{5}{K}$ ...(ii)

By solving equations (i) and (ii), we get

$$L = 5a - 4b \text{ and } K = \frac{1}{b-a}$$

Now when the longitudinal tension is 9 N, length of the string

$= L + \dfrac{9}{K} = 5a - 4b + 9(b-a) = 5b - 4a.$

7. A pan with set of weights is attached with a light spring. When disturbed, the mass-spring system oscillates with a time period of 0.6 s. When some additional weights are added, then time period is 0.7s. The extension caused by the additional weights is approximately given by

(a) 1.38 cm (b) 3.5 cm
(c) 1.75 cm (d) 2.45 cm

Sol. (b) $2\pi\sqrt{\dfrac{m}{k}} = 0.6$ ...(i) and $2\pi\sqrt{\dfrac{m+m'}{k}} = 0.7$ ...(ii)

Dividing (ii) by (i), we get $\left(\dfrac{7}{6}\right)^2 = \dfrac{m+m'}{m} = \dfrac{49}{36}$

$$\frac{m+m'}{m} - 1 = \frac{49}{36} - 1 \Rightarrow \frac{m'}{m} = \frac{13}{36} \Rightarrow m' = \frac{13m}{36}$$

Also $\dfrac{k}{m} = \dfrac{4\pi^2}{(0.6)^2}$

Desired extension $= \dfrac{m'g}{k} = \dfrac{13}{36}\times\dfrac{mg}{k}$

$$= \frac{13}{36} \times 10 \times \frac{0.36}{4\pi^2} = 3.5 \text{ cm}$$

8. The area of cross section of a steel wire ($Y = 2.0 \times 10^{11}$ N/m^2) is 0.1 cm^2. The force required to double its length will be

(a) 2×10^{12} N (b) 2×10^{11} N
(c) 2×10^{10} N (d) 2×10^{6} N

Sol. (d) When the length of wire is doubled, then $l = L$ and strain = 1 $\therefore$ Y = strain = $\frac{F}{A}$

$\therefore$ Force $= Y \times A = 2 \times 10^{11} \times 0.1 \times 10^{-4} = 2 \times 10^{6}$ N

9. A rubber cord catapult has cross-sectional area 25 mm^2 and initial length of rubber cord is 10 cm. It is stretched to 5 cm and then released to project a missile of mass 5 gm. Taking $Y_{rubber} = 5 \times 10^8$ N/m^2 velocity of projected missile is

(a) 20 ms^{-1} (b) 100 ms^{-1}
(c) 250 ms^{-1} (d) 200 ms^{-1}

Sol. (c) Potential energy stored in the rubber cord catapult will be converted into kinetic energy of mass.

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{YAl^2}{L} \Rightarrow v = \sqrt{\frac{YAl^2}{mL}}$$

$$= \sqrt{\frac{5\times10^8 \times 25\times10^{-6} \times (5\times10^{-2})^2}{5\times10^{-3} \times 10\times10^{-2}}} = 250 \text{ m/s}$$

10. A uniform cube is subjected to volume compression. If each side is decreased by 1%, then bulk strain is

(a) 0.01 (b) 0.06
(c) 0.02 (d) 0.03

Sol. (d) If side of the cube is L, then $V = L^3 \Rightarrow \frac{dV}{V} = 3\frac{dL}{L}$

$\therefore$ % change in volume = 3 × (% change in length)
= 3 × 1% = 3%

$\therefore$ Bulk strain $\frac{\Delta V}{V} = 0.03$

11. A ball falling in a lake of depth 200 m shows 0.1% decrease in its volume at the bottom. What is the bulk modulus of the material of the ball?

(a) 19.6×10^{8} N/m^2 (b) 19.6×10^{-10} N/m^2
(c) 19.6×10^{10} N/m^2 (d) 19.6×10^{-8} N/m^2

Sol. (a) $B = \frac{\Delta p}{\Delta V/V} = \frac{h\rho g}{0.1/100} = \frac{200\times10^3 \times 9.8}{1/1000}$

$= 19.6 \times 10^8$ N/m^2

12. A 5-metre long wire is fixed to the ceiling. A weight of 10 kg is hung at the lower end and is 1 metre above the floor. The wire was elongated by 1 mm. The energy stored in the wire due to stretching is

(a) Zero (b) 0.05 J
(c) 100 J (d) 500 J

Sol. (b) $W = \frac{1}{2} \times F \times l$

$$= \frac{1}{2}mgl = \frac{1}{2} \times 10 \times 10 \times 1 \times 10^{-1} = 0.05 \text{ J}$$

13. A brass rod of cross-sectional area 1 cm^2 and length 0.2 m is compressed lengthwise by a weight of 5 kg. If Young's modulus of elasticity of brass is 1×10^{11} N/m^2 and g = 10 m/s, then increase in the energy of the rod will be

(a) 10^{-5} J (b) 2.5×10^{-5} J
(c) 5×10^{-5} J (d) 2.5×10^{-4} J

Sol. (b) $U = \frac{1}{2} \times \frac{(\text{Stress})^2}{Y} \times \text{Volume}$

$$= \frac{1}{2} \times \frac{F^2 \times A \times L}{A^2 \times Y}$$

$$= \frac{1}{2} \times \frac{F^2 L}{AY} = \frac{1}{2} \times \frac{(50)^2 \times 0.2}{1\times10^{-4} \times 1\times10^{11}} = 2.5 \times 10^{-5} \text{ J}$$

14. If one end of a wire is fixed with a rigid support and the other end is stretched by a force of 10 N, then the increase in length is 0.5 mm. The ratio of the energy of the wire and the work done in displacing it through 1.5 mm by the weight is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

Sol. (c) Work done in stretching a wire

$$W = \frac{1}{2}Fl = \frac{1}{2} \times 10 \times 0.5 \times 10^{-3} = 2.5 \times 10^{-3} \text{ J}$$

Work done to displace it through 1.5 mm

$W = F \times l = 5 \times 10^{-3}$ J

The ratio of above two work = 1 : 2

15. The ratio of Young's modulus of the material of two wires is 2 : 3. If the same stress is applied on both, then the ratio of elastic energy per unit volume will be

(a) 3 : 2 (b) 2 : 3
(c) 3 : 4 (d) 4 : 3

Sol. (a) Energy per unit volume = $\frac{(\text{Stress})^2}{2Y}$

$$\frac{E_1}{E_2} = \frac{Y_2}{Y_1} \text{ (Stress is constant)} \Rightarrow \frac{E_1}{E_2} = \frac{3}{2}$$

16. Two rods of different materials having coefficients of linear expansion α_1, α_2 and Young's moduli Y_1 and Y_2 respectively, are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equally provided, $Y_1 : Y_2$ is equal to

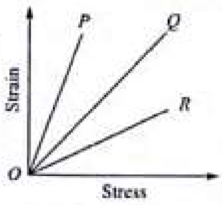
(a) 2 : 3 (b) 1 : 1
(c) 3 : 2 (d) 4 : 9

Sol. (c) Thermal stress = $Y\alpha\,\Delta\theta$.

If thermal stress and rise in temperature are equal, then $Y \propto \frac{1}{\alpha}$

$\Rightarrow \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$

17. The strain-stress curves of three wires of different materials are shown in the figure. P, Q and R are the elastic limits of the wires. The figure shows that



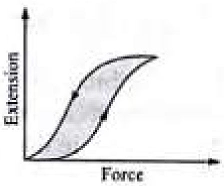
(a) Elasticity of wire P is maximum
(b) Elasticity of wire Q is maximum
(c) Tensile strength of R is maximum
(d) None of the above is true

Sol. (d) As stress is shown on x-axis and strain on y-axis

So we can say that $Y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\text{Slope}}$

So the elasticity of wire P is minimum and of wire R is maximum.

18. The diagram below shows a force-extension graph for a rubber band. Consider the following statements:



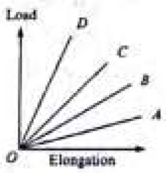
I. It will be easier to compress this rubber than expand it.
II. Rubber does not return to its original length after it is stretched.
III. The rubber band will get heated if it is stretched and released.

Which of these can be deduced from the graph?

(a) III only (b) II and III
(c) I and III (d) I only

Sol. (a) Area of hysterisis loop gives the energy loss in the process of stretching and unstretching of rubber band and this loss will appear in the form of heating.

19. The load-versus-elongation graph for four wires of the same material is shown in the figure. The thickest wire is represented by the line

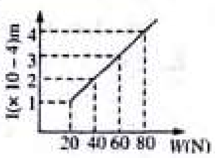


(a) OD (b) OC
(c) OB (d) OA

Sol. (a) $l = \frac{FL}{AY}$ $\therefore$ $l \propto \frac{1}{r^2}$ (Y, L and F are constant)

i.e. for the same load, the thickest wire will show minimum elongation. So, graph D represents the thickest wire.

20. The adjacent graph shows the extension (Δl) of a wire of length 1m suspended from the top of a roof at one end with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire.



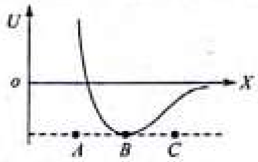
(a) 2×10^{11} N/m^2 (b) 2×10^{-11} N/m^2
(c) 3×10^{-12} N/m^2 (d) 2×10^{-13} N/m^2

Sol. (a) From the graph, $l = 10^{-4}$m, $F = 20$ N.

$A = 10^{-6}$ m^2, $L = 1$ m

$\therefore\ Y = \frac{FL}{Al} = \frac{20 \times 1}{10^{-6} \times 10^{-4}} = 20 \times 10^{10} = 2 \times 10^{11}\,\text{N/m}^2$

21. The potential energy U between two molecules as a function of the distance X between them has been shown in the figure. The two molecules are



(a) attracted when x lies between A and B and are repelled when X lies between B and C
(b) attracted when x lies between B and C and are repelled when X lies between A and B
(c) attracted when they reach B
(d) repelled when they reach B

Sol. (b) $F = -\left(\frac{dU}{dx}\right)$.

In the region BC, slope of the graph is positive.

$\therefore$ F = negative, i.e., force is attractive in nature.

In the region AB, slope of the graph is negative.

$\therefore$ F = positive, i.e., force is repulsive in nature.

22. To break a wire of one meter length, minimum 40 kg-wt is required. Then the wire of the same material of double radius and 6 m length will require breaking weight

(a) 80 kg-wt (b) 240 kg-wt
(c) 200 kg-wt (d) 160 kg-wt

Sol. (d) Breaking force = Breaking stress × Area of cross section of wire

$\therefore$ Breaking force $\propto r^2$ (Breaking distance is constant)

If radius becomes doubled, then breaking force will become 4 times i.e. $40 \times 4 = 160$ kg wt

23. The mass and length of a wire are M and L, respectively. The density of the material of the wire is d. On applying the force F on the wire, the increase in length is l. Then the Young's modulus of the material of the wire will be

(a) $\frac{Fdl}{Ml}$ (b) $\frac{FL}{Mdl}$

(c) $\frac{FMl}{dl}$ (d) $\frac{FdL^2}{Ml}$

Sol. (d) $Y = \frac{FL}{Al} = \frac{FdL^2}{Ml}$

As M = Volume × Density = $A = \times L - d \Rightarrow A = \frac{M}{Ld}$

24. Two exactly similar wires of steel and copper are stretched by equal forces. If the difference in their elongations is 0.5 cm, the elongation (l) of each wire is
Y_s(steel) $= 2.0 \times 10^{11}$ N/m^2
Y_c(copper) $= 1.2 \times 10^{11}$ N/m^2
(a) $l_s = 0.75$ cm, $l_c = 1.25$ cm
(b) $l_s = 1.25$ cm, $l_c = 0.75$ cm
(c) $l_s = 0.25$ cm, $l_c = 0.75$ cm
(d) $l_s = 0.75$ cm, $l_c = 0.25$ cm

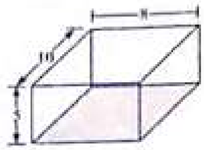
Sol. (a) $l \propto \dfrac{1}{Y} \Rightarrow \dfrac{Y_s}{Y_c} = \dfrac{l_c}{l_s} \Rightarrow \dfrac{l_c}{l_s} = \dfrac{2\times10^{11}}{1.2\times10^{11}} = \dfrac{5}{3}$ ...(i)

Also $l_c - l_s = 0.5$ ...(ii)

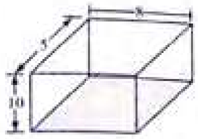
On solving (i) and (ii), we get

$l_c = 1.25$ cm and $l_s = 0.75$ cm

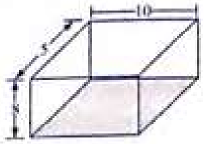
25. A rectangular block of size 10 cm × 8 cm × 5 cm is kept in three different positions P, Q and R in turn as shown in the figure. In each case, the shaded area is rigidly fixed and a definite force F is applied tangentially to the opposite face to deform the block. The displacement of the upper face will be



(P)



(Q)



(R)

(a) Same in all the three cases
(b) Maximum in P position
(c) Maximum in Q position
(d) Maximum in R position

Sol. (d) $\eta = \dfrac{F/A}{x/L} \Rightarrow x = \dfrac{L}{\eta} \times \dfrac{F}{A}$

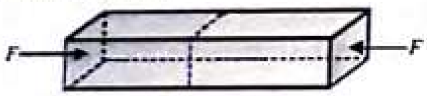
If η and F are constant, then $x \propto \dfrac{L}{A}$

For maximum displacement, area at which force applied should be minimum and vertical side should be maximum. This is given in the R position of rectangular block.

EXERCISES

Stress, Strain and Elasticity

1. In the figure shown, forces of equal magnitude are applied to the two ends of a uniform rod. Consider A as the cross-sectional area of the rod. For this situation, mark out the incorrect statements.



(a) The rod is in compressive stress.
(b) The numerical value of stress developed in the rod is equal to F/A.
(c) The stress is defined as internal force developed at any cross section per unit area.
(d) None of the above.

2. A nylon rope 2 cm in diameter has a breaking strength of 1.5×10^5 N. The breaking strength of a similar rope 1 cm in diameter is
(a) 0.375×10^5 N (b) 2×10^5 N
(c) 6×10^5 N (d) 9×10^4 N

3. The breaking stress for a substance is 10^6 N/m^2. What length of the wire of this substance should be suspended vertically so that the wire breaks under its own weight? (Given: density of material of the wire $= 4 \times 10^3$ kg/m^3 and $g = 10$ ms^{-2})
(a) 10 m (b) 15 m
(c) 25 m (d) 34 m

4. The ratio of diameters of two wires of same material is n:1. The length of each wire is 4 m. On applying the same load, the increases in the length of the thin wire will be ($n > 1$)
(a) n^2 times (b) n times
(c) $2n$ times (d) $(2n + 1)$ times

5. The dimensions of four wires of the same material are given below. In which wire the increase in the length will be maximum?
(a) Length 100 cm, diameter 1 mm
(b) Length 200 cm, diameter 2 mm
(c) Length 300 cm, diameter 3 mm
(d) Length 50 cm, diameter 0.5 mm

6. Two wires of the same material and length are stretched by the same force. Their masses are in the ratio 3:2. Their elongations are in the ratio
(a) 3:2 (b) 9:4
(c) 2:3 (d) 4:9

7. Two wires of the same length and same material but radii in the ratio of 1:2 are stretched by unequal forces to produce equal elongation. The ratio of the two forces is
(a) 1:1 (b) 1:2
(c) 1:3 (d) 1:4

8. When a weight of 5 kg is suspended from a copper wire of length 30 m and diameter 0.5 mm, the length of the wire increases by 2.4 cm. If the diameter is doubled, the extension produced is
(a) 1.2 cm (b) 0.6 cm
(c) 0.3 cm (d) 0.15 cm

9. The length of a wire is increased by 1 mm on the application of a given load. In a wire of the same material, but of length and radius twice that of the first, on application of the same load, extension is
(a) 0.25 mm (b) 0.5 mm
(c) 2 mm (d) 4 mm

10. A cube is shifted to a depth of 100 m is a lake. The change in volume is 0.1%. The bulk modulus of the material is nearly
(a) 10 Pa (b) 10^4 Pa
(c) 10^7 Pa (d) 10^9 Pa

11. Young's modulus of rubber is 10^4 N/m^2 and area of cross section is 2 cm^{-2}. If force of 2×10^5 dyn is applied along its length, then its initial l becomes
(a) $3l$ (b) $4l$
(c) $2l$ (d) None of these

12. When a certain weight is suspended from a long uniform wire, its length increases by 1 cm. If the same weight is suspended from another wire of the same material and length but having a diameter half of the first one, the increases in length will be
(a) 0.5 cm (b) 2 cm
(c) 4 cm (d) 8 cm

13. Two wires of the same material have lengths in the ratio 1:2 and their radii are in the ratio $1:\sqrt{2}$. If they are stretched by applying equal forces, the increase in their lengths will be in the ratio
(a) $\sqrt{2}:2$ (b) $2:\sqrt{2}$
(c) 1:1 (d) 1:2

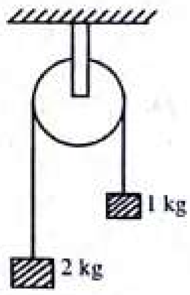
14. Two identical wires of iron and copper with their Young's modulus in the ratio 3:1 are suspended at same level. They are to be loaded so as to have the same extension and hence level. Ratio of the weight is
(a) 1:3 (b) 2:1
(c) 3:1 (d) 4:1

15. One end of uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If s is the area of cross section of the wire, the stress in the wire at a height $(3L/4)$ from its lower end is
(a) $\dfrac{W_1}{s}$ (b) $\left[W_1 + \dfrac{W}{4}\right]/s$
(c) $\left[W_1 + \dfrac{3W}{4}\right]/s$ (d) $\dfrac{W_1 + W}{s}$

16. A wire is stretched 1 mm by a force of 1 kN. How far would a wire of the same material and length but of four times that diameter be stretched by the same force?
(a) $\dfrac{1}{2}$ mm (b) $\dfrac{1}{4}$ mm
(c) $\dfrac{1}{8}$ mm (d) $\dfrac{1}{16}$ mm

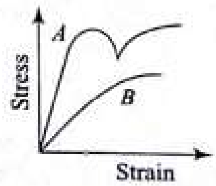
17. A copper bar of length L and area of cross section A is placed in a chamber at atmospheric pressure. If the chamber is evacuated, the percentage change in its volume will be (compressibility of copper is 8×10^{-12} m^2/N and 1 atm = 10^5 N/m)
(a) 8×10^{-7} (b) 8×10^{-5}
(c) 1.25×10^{-4} (d) 1.25×10^{-5}

18. Two blocks of masses 1 kg and 2 kg are connected by a metal wire going over a smooth pulley as shown in the figure. The breaking stress of the metal is $(40/3\pi) \times 10^6$ N/m^2. If $g = 10$ m s^{-12}, then what should be the minimum radius of the wire used if it is not to break?



(a) 0.5 mm (b) 1 mm
(c) 1.5 mm (d) 2 mm

19. The diagram shows stress vs. strain curve for the materials A and B. From the curves we infer that



(a) A is brittle but B is ductile
(b) A is ductile and B is brittle
(c) Both A and B are ductile
(d) Both A and B are brittle

20. A 900 kg elevator hangs by a steel cable for which the allowable stress is 1.15×10^8 N/m^2. What is the minimum diameter required if the elevator accelerates upward at 1.5 m/s^2. Take $g = 10$ m/s^2
(a) $\dfrac{6\times10^{-2}}{\sqrt{5\,\pi}}$ m (b) $\dfrac{6\times10^{-2}}{\sqrt{10\,\pi}}$ m
(c) $\dfrac{3\times10^{-2}}{\sqrt{10\,\pi}}$ m (d) $\dfrac{3\times10^{-2}}{\sqrt{5\,\pi}}$ m

Energy Stored in a Deformed Body

21. Two wires of the same material and length but diameters in the ratio 1:2 are stretched by the same force. The potential energy per unit volume for the two wires when stretched will be in the ratio

(a) 16:1 (b) 4:1
(c) 2:1 (d) 1:1.

22. An elastic material of Young's modulus Y is subjected to a stress S. The elastic energy stored per unit volume of the material is

(a) $\frac{SY}{2}$ (b) $\frac{S^2}{2Y}$
(c $\frac{S}{2Y}$ (d) $\frac{2S}{Y}$

23. What amount of work is done in increasing the length of a wire through unity?

(a) $\frac{YL}{2A}$ (b) $\frac{YL^2}{2A}$
(c) $\frac{YA}{2L}$ (d) $\frac{YL}{A}$

24. When the load on a wire is slowly increased from 3 to 5 kg wt, the elongation increases from 0.61 to 1.02 mm. The work done during the extension of wire is

(a) 0.16 J (b) 0.016 J
(c) 1.6 J (d) 16 J

25. A long wire hangs vertically with its upper end clamped. A torque of 8 Nm applied to the free end twists it through 45°. The potential energy of the twisted wire is

(a) π J (b) $\frac{\pi}{2}$ J
(c) $\frac{\pi}{4}$ J (d) $\frac{\pi}{8}$ J

26. A long elastic spring is stretched by 2 cm and its potential energy is U. If the spring is stretched by 10 cm, the P.E., will be

(a) $5U$ (b) $25U$
(c) $U/5$ (d) $U/20$

27. If the work done by stretching a wire by 1 mm is 2 J, the work necessary for stretching another wire of the same material but with the radius of cross section and half the length by 1 mm is

(a) $\frac{1}{4}$ J (b) 4 J
(c) 8 J (d) 16 J

28. If the potential energy of a spring is V on stretching it by 2 cm, then its potential energy when it is stretched by 10 cm will be

(a) $V/25$ (b) $5V$
(c) $V/5$ (d) $25V$

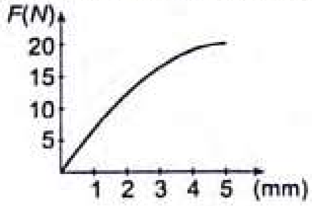
29. Two wires of the same diameter and of the same material have the length l and $2l$. If the force F is applied on each, the ratio of the work done in the two wires will be

(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 1 : 1

30. When a 4 kg mass is hung vertically on a light spring that obeys Hooke's law, the spring stretches by 2 cm. The work required to be done by an external agent in stretching this spring by 5 cm will be ($g = 9.8$ metres/sec^2)

(a) 4.900 J (b) 2.450 J
(c) 0.495 J (d) 0.245 J

31. The force (F)-extension (Δl), graph shows that the strain energy stored in the material under test, for an extension of 4 mm, is greater than which of the following values?



(a) 80 mJ (b) 60 mJ
(c) 40 mJ (d) None of these

32. The length of a rod is 20 cm and area of cross-section 2 cm^2. The Young's modulus of the material of wire is 1.4×10^{11} N/m^2. If the rod is compressed by 5 kg-wt along its length, then increase in the energy of the rod in joules will be

(a) 8.57×10^{-6} (b) 22.5×10^{-4}
(c) 9.8×10^{-5} (d) 45.0×10^{-5}

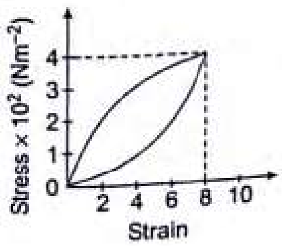
33. When a force is applied on a wire of uniform cross-sectional area 3×10^{-6} m^2 and length 4 m, the increase in length is 1 mm. Energy stored in it will be ($Y = 2 \times 10^{11}$ N/m^2)

(a) 6250 J (b) 0.177 J
(c) 0.075 J (d) 0.150 J

34. The work per unit volume to stretch the length by 1% of a wire with cross sectional area of 1 mm^2 will be. [$Y = 9 \times 10^{11}$ N/m^2]

(a) 9×10^{11} J (b) 4.5×10^{7} J
(c) 9×10^{7} J (d) 4.5×10^{11} J

35. A rubber of volume 2000 cc is alternately subjected to tension and released. The figure shows the stress-strain curve of rubber. Each curve is a quadrant of an ellipse. The amount of energy lost as heat per cycle per unit volume will be



(a) $\left(\frac{\pi}{2}-1\right)\times 16\times 10^2$ J (b) $\left(\frac{\pi}{4}-1\right)\times 8\times 10^2$ J

(c) $\left(\frac{\pi}{4}-1\right)\times 32\times 10^2$ J (d) $\left(\frac{\pi}{2}-1\right)\times 32\times 10^2$ J

Problems Based on Mixed Concepts

36. A wire of cross section A is stretched horizontally between two clamps located $2l$ m apart. A weight W kg is suspended from the mid-point of the wire. If the mid-point sags vertically through a distance $x < 1$ the strain produced is

(a) $\frac{2x^2}{l^2}$ (b) $\frac{x^2}{l^2}$

(c) $\frac{x^2}{2l^2}$ (d) None of these

37. If in the above question, the Young's modulus of the material is Y, the value of extension x is

(a) $\left(\frac{Wl}{YA}\right)^{1/3}$ (b) $\left(\frac{YA}{Wl}\right)^{1/3}$

(c) $\frac{1}{l}\left[\frac{WA}{Y}\right]^{1/3}$ (d) $l\left[\frac{W}{YA}\right]^{1/3}$

38. Two rods of different materials having coefficients of linear expansion α_1 and α_2 and Young's moduli, Y_1 and Y_2, respectively, are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of rods. If $\alpha_1/\alpha_2 = 2/3$, then the thermal stresses developed in the two rods are equal, provided Y_1/Y_2 is equal to

(a) 2:3 (b) 1:1

(c) 3:2 (d) 4:9

39. When the tension in a metal wire is T_1, its length is l_1. When the tension is T_2, its length is l_2. The natural length of wire is

(a) $\frac{T_2}{T_1}(l_1+l_2)$ (b) $T_1l_1 + T_2l_2$

(c) $\frac{l_1T_2 - l_2T_1}{T_2 - T_1}$ (d) $\frac{l_1T_2 + l_2T_1}{T_2 + T_1}$

40. A small but heavy block of mass 10 kg is attached to a wire 0.3 m long. Its breaking stress is 4.8×10^7 N/m^2. The area of the cross section of the wire is 10^{-6} m^2. The maximum angular velocity with which the block can be rotated in the horizontal circle is

(a) 4 rad/s (b) 8 rad/s

(c) 10 rad/s (d) 32 rad/s

41. A ball falling in a lake of depth 200 m shows a decrease of 0.1% in its volume at the bottom. The bulk modulus of the elasticity of the material of the ball is (take $g = 10$ m s^{-12})

(a) 10^9 N/m^2 (b) 2×10^9 N/m^2

(c) 3×10^9 N/m^2 (d) 4×10^9 N/m^2

42. A wire is suspended vertically from a rigid support. When loaded with a steel weight in air, the wire extends by 16 cm. When the weight is completely immersed in water, the extension is reduced to 14 cm. The relative density of the material of the weight is

(a) 2 g/cm^3 (b) 6 g/cm^3

(c) 8 g/cm^3 (d) 16 g/cm^3

43. Two bars A and B of circular cross section, same volume and made of the same material, are subjected to tension. If the diameter of A is half that of B and if the force applied to both the rod is the same and it is in the elastic limit, the ratio of extension of A to that of B will be

(a) 16 (b) 8

(c) 4 (d) 2

44. A uniform cylindrical wire is subjected to a longitudinal tensile stress of 5×10^7 N/m^2. Young's modulus of the material of the wire is 2×10^{11} N/m^2. The volume change in the wire is 0.02%. The fractional change in the radius is

(a) 0.25×10^{-4} (b) 0.5×10^{-4}

(c) 1.0×10^{-4} (d) 1.5×10^{-4}

45. A material has normal density ρ and bulk modulus K. The increase in the density of the material when it is subjected to an external pressure P from all sides is

(a) $P/\rho K$ (b) $K/\rho P$

(c) $\rho P/K$ (d) $\rho K/P$

46. The length of a steel cylinder is kept constant by applying pressure at its two ends. When the temperature of rod is increased by 100°C from its initial temperature, the increase in pressure to be applied at its ends is

($Y_{steel} = 2 \times 10^{11}$ N/m^2,
$\alpha_{steel} = 11 \times 10^{-6}$/°C, 1 atm = 10^5 N/m^2.)

(a) 22×10^7 atm (b) 2.2×10^3 atm

(c) zero (d) 4.3×10^3 atm

47. Maximum excess pressure inside a thin-walled steel tube of radius r and thickness Δr ($<< r$), so that the tube would not rupture would be (breaking stress of steel is σ_{max})

(a) $\sigma_{max} \times \frac{r}{\Delta r}$ (b) $\sigma_{max} \times \frac{\Delta r}{r}$

(c) σ_{max} (d) $\sigma_{max} \times \frac{\Delta 2r}{r}$

48. A 5-kg rod of square cross section 5 cm on a side and 1 m long is pulled along a smooth horizontal surface by a force applied at one end. The rod has a constant acceleration of 2 m s^{-12}. Determine the elongation in the rod. (Young's modulus of the material of the rod is 5×10^3 N/m^9).

(a) Zero, as for elongation to be there, equal and opposite forces must act on the rod

(b) Non-zero but cannot be determine from the given situation

(c) 0.4 μm

(d) 16 μm

49. The elastic limit of an elevator cable is 2×10^9 N/m^2. The maximum upward acceleration that an elevator of mass 2×10^3 kg can have when supported by a cable whose cross-sectional area is 10^{-4} m^2, provided the stress in cable would not exceed half of the elastic limit would be

(a) $10\ \text{m s}^{-12}$ (b) $50\ \text{m s}^{-12}$
(c) $40\ \text{m s}^{-12}$ (d) Not possible to move up

50. A wire can sustain the weight of 20 kg before breaking. If the wire is cut into two equal parts, each part can sustain a weight of

(a) 10 kg (b) 20 kg
(c) 40 kg (d) 35 kg

51. A mass m is hanging from a wire of cross-sectional area A and length L. Y is Young's modulus of wire. An external force F is applied on the wire which is then slowly further pulled down by Δx from its equilibrium position. Find the work done by the force F the wire exerts on the mass.

(a) $mg\Delta x + \frac{YA}{2L}(\Delta x)^2$ (b) $mg\Delta x + \frac{YA(\Delta X)^2}{L}$

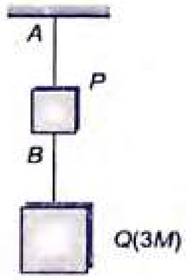
(c) $mg\frac{\Delta x}{2} + \frac{YA(\Delta x)^2}{L}$ (d) $mg\frac{\Delta x}{2} + \frac{YA(\Delta x)^2}{2L}$

52. In the previous problem, find the work done by the gravity, if Δl is the elongation.

(a) $mg\Delta l$ (b) $mg\Delta x$

(c) $\frac{mg(\Delta l)^2}{\Delta x}$ (d) $\frac{mg(\Delta x)^2}{\Delta l}$

53. Wires A and B are connected with blocks P and Q, as shown. The ratio of lengths, radii and Young's modulus of wires A and B are r, $2r$ and $3r$ respectively (r is a constant). Find the mass of block P if ratio of increase in their corresponding lengths is $\frac{1}{6r^2}$. The mass of the block Q is $3M$.



(a) M (b) $3M$
(c) $6M$ (d) $9M$

ARCHIVES

1. A wire is suspended vertically. One of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is

(a) 0.1 J (b) 0.2 J
(c) 10 J (d) 20 J **(AIEEE 2003)**

2. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(a) 6.25 N-m (b) 12.50 N-m
(c) 18.75 N-m (d) 25.00 N-m

(AIEEE 2003)

3. A wire fixed at the upper end stretches by length l when a force F is applied. The work done in stretching is

(a) $\frac{F}{2l}$ (b) Fl

(c) $\frac{2F}{l}$ (d) $\frac{Fl}{2}$ **(AIEEE 2004)**

4. If S is stress and Y is Young's modulus of the material of a wire, the energy stored in the wire per unit volume is

(a) $\frac{S}{2Y}$ (b) $\frac{2Y}{S^2}$

(c) $\frac{S^2}{2Y}$ (d) $2S^2Y$ **(AIEEE 2005)**

5. A wire is elongated by l millimetre when a load W is hung from it. If the wire goes over a pulley and two weights (W each) are hung at the two ends, the elongation of the wire will be (in mm)

(a) zero (b) $\frac{l}{2}$

(c) l (d) $2l$ **(AIEEE 2006)**

6. Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying a force F, how much force is needed to stretch wire 2 by the same amount?

(a) F (b) $4F$
(c) $6F$ (d) $9F$ **(AIEEE 2009)**

7. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is

(For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

(a) 2.2×10^7 Pa (b) 2.2×10^6 Pa
(c) 2.2×10^8 Pa (d) 2.2×10^9 Pa

(JEE Main 2014)

8. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's

modulus of the material of the wire is Y, then $1/Y$ is equal to (g = gravitational acceleration)

(a) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (b) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

(c) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (d) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(JEE Main 2015)

9. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

(a) 81 (b) $\frac{1}{81}$

(c) 9 (d) $\frac{1}{9}$ **(JEE Main 2017)**

10. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere,

(a) $\frac{mg}{Ka}$ (b) $\frac{Ka}{mg}$

(c) $\frac{Ka}{3\,mg}$ (d) $\frac{mg}{3\,Ka}$

(JEE Main 2018)

ANSWER KEY

Exercises

1. (d)	2. (a)	3. (c)	4. (a)	5. (d)	6. (c)	7. (d)	8. (b)	9. (b)	10. (d)
11. (c)	12. (c)	13. (c)	14. (c)	15. (c)	16. (d)	17. (b)	18. (b)	19. (b)	20. (b)
21. (a)	22. (b)	23. (c)	24. (b)	25. (a)	26. (b)	27. (d)	28. (d)	29. (a)	30. (b)
31. (c)	32. (a)	33. (c)	34. (b)	35. (d)	36. (c)	37. (d)	38. (c)	39. (c)	40. (a)
41. (b)	42. (c)	43. (a)	44. (a)	45. (c)	46. (b)	47. (b)	48. (c)	49. (c)	50. (b)
51. (a)	52. (b)	53. (c)							

Archives

1. (a)	2. (c)	3. (d)	4. (c)	5. (c)	6. (d)	7. (c)	8. (a)	9. (c)	10. (d)

Chapter 12

Fluid Mechanics, Surface Tension and Viscosity

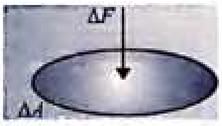
PRESSURE

While considering the equilibrium of solids we usually think in terms of forces exerted by one body on another body. But in case of liquids, we usually speak in terms of pressure instead of forces. If a uniform force F is exerted normal to an area A, then the pressure P is defined as

$$P = \frac{F}{A}$$

NOTE: The area A in the above expression is the actual area of contact. In case of solids, the actual area of contact between two surfaces is less than the apparent area of contact. Therefore, the concept of pressure is of no use in case of solids. On the contrary, the contact between a liquid and a solid surface or between two liquid surfaces is rather a smoother one, that is, the actual area of contact is equal to the apparent area of contact. Therefore, one can easily use the concept of pressure.

If the applied force F is not uniform, then the pressure at a point is defined as the intensity of force at that point. That is,



$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

Pressure is a scalar quantity. Pressure acts normal to a surface and it is always compressive in nature. Therefore, only its magnitude is required for its complete description. The SI unit of pressure is N/m^2 and is also called pascal (Pa). The other common units of pressure are atmosphere and bar.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} \Rightarrow 1 \text{ bar} = 10^5 \text{ Pa}$$

NOTE:

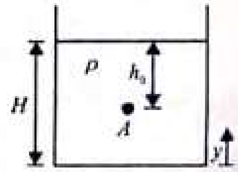
- The liquid pressure is the same on all points at the same depth.
- The total (or absolute) pressure at a depth h below the free liquid surface is more than the outside atmospheric pressure by an amount $\rho g h$.
- The pressure at the bottom of the container will not depend upon the shape or size of the container.
- **Pressure increases with depth linearly:** The rate of increase of pressure with depth is equal to the specific weight of the liquid at that point.

  The specific weight is defined as the weight per unit volume, i.e., ρg. Thus,

  $$\frac{dp}{dy} = \rho g$$

- **Pressure decreases with height linearly:** The rate of decrease of pressure with height is constant and is given by

  $$\frac{dp}{dy} = -\rho g$$



The pressure at the point A is obtained as follows.

$$dp = -\rho g\, dy$$

$$\Rightarrow \quad \int dp = -\rho g \int dy$$

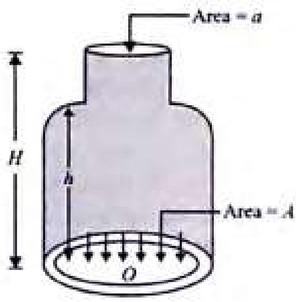
$$\Rightarrow \quad p = -\rho g y + c$$

Now, at $y = H$, $p = p_0$. Therefore, $c = p_0 + \rho g H$

Thus, $p = \rho g(H - y) + p_0$

From the figure, since $H - y = h_0$ $\therefore$ $p = \rho g H + p_0$

A liquid may exert thrust greater than its weight: Consider a cylindrical vessel of base area A and height h, with a thin cylindrical stem of cross-sectional area 'a'. If a liquid of density ρ is poured in it to a height $H(H > h)$, the thrust exerted by the liquid on the base of the vessel = pressure at the base due to liquid column × base area.



$$F_{\text{base}} = (\rho g H)A \tag{i}$$

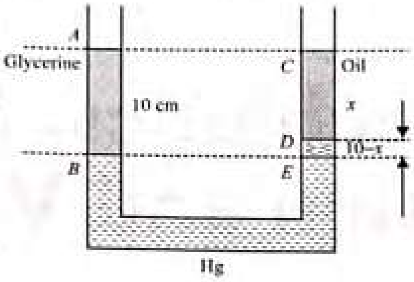
The weight of the liquid contained in the vessel is given by

$$\text{Volume} \times \text{density} \times g = [AH - (A - a)(H - h)]\rho g \tag{ii}$$

$$W_{\text{liquid}} = [Ah - a(H - h)]\rho g \tag{iii}$$

Comparing Eqs. (i) and (ii) it is evident that the force exerted by the liquid on the base of the vessel exceeds the weight of the liquid.

ILLUSTRATION 12.1 A vertical U tube of uniform cross section contains mercury in both of its arms. A glycerine (d = 1.3 g/cm^3) column of length 10 cm is introduced into one of the arms. Oil of density 0.8 g/cm^3 is poured in the other arm until the upper surfaces of the oil and glycerine are in the same horizontal level. Find the length of oil column. Density of mercury is 13.6 g/cm^3.



Solution. Let the length of oil column = $CD = x$ cm.
Length of glycerine = AB = 10 cm

$\therefore \quad DE = (10 - x)$ cm

Equating pressure at horizontal level BE in both arms, we have

$$P_{\text{oil}} + P_{\text{Hg}} = P_{\text{glycerine}}$$

$$xd_{\text{oil}}g + (10 - x)\rho_{\text{Hg}}g = 10d_{\text{glycerine}}g$$

or $$x(0.8) + (10 - x)13.6 = 10(1.3) \Rightarrow x = \frac{136 - 13}{12.8} = 9.6 \text{ cm}$$

PASCAL'S LAW

Pressure applied to a confined liquid is transmitted equally undiminished to every part of the liquid and to all the walls of the containing vessel, and is always exerted at right angles to the walls.

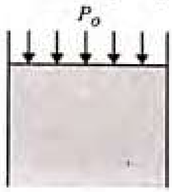
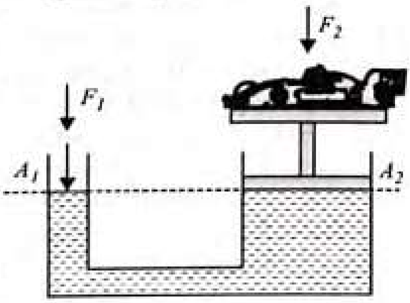


Figure shows a liquid confined by solid boundaries from the four sides and the base. At the free surface the atmosphere is exerting a pressure p_0 on this confined liquid which is transmitted to every part of the liquid.

This principle is used in a hydraulic jack or a lift where a heavy load can be lifted up by a small force. In the figure, the pressure due to a small force F_1 applied to a piston of area A_1 is transmitted to the larger piston of area A_2. The pressure at the two pistons is the same because they are at the same level.

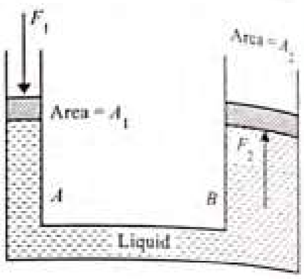


$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = \left(\frac{A_2}{A_1}\right)F_1$$

Thus, a small force F_1 acting on a small area A_1 results in a larger force F_2 acting on a larger area A_2.

ILLUSTRATION 12.2 In a simple hydraulic press, the cross-sectional area of the two cylinders is 5×10^{-4} m^2 and 10^{-2} m^2, respectively. A force of 20 N is applied at the small plunger.

(a) What is the pressure produced in the cylinders?
(b) What is the thrust exerted on the large plunger?
(c) How much work is done by the operator, if the smaller plunger moves down 0.1 m?



Solution. In a hydraulic press, a force F_1 applied to the smaller plunger creates a pressure F_1/A_1 in the liquid and this pressure is transmitted equally throughout the liquid and acts on the larger plunger. The thrust acting on the larger plunger upwards due to this pressure is

$$F_2 = A_2\left(\frac{F_1}{A_1}\right)$$

Hence, the thrust on A_2 is magnified by A_2/A_1 times

(a) $\dfrac{F_1}{A_1} = \dfrac{20}{5 \times 10^{-4}} = 40000 \text{ N/m}^2$

(b) $F_2 = F_1 \times \left(\dfrac{A_2}{A_1}\right) = 20 \times \dfrac{10^{-2}}{5 \times 10^{-4}} = 400 \text{ N}$

(c) Work done by force $F_1 = F_1 d_1 = 20 \times 0.1 = 2$ J

NOTE: The displacement d_2 of the larger plunger = $d_1 A_1/A_2$. The displacement of the larger plunger is less.

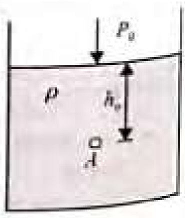
Measurement of Pressure

Absolute Pressure and Gauge Pressure

Absolute pressure is the total pressure at a point including the contribution of the liquid as well as that of the atmosphere. Gauge pressure is the pressure measured at a point relative to the local atmospheric pressure. In other words, it does not include the contribution of atmospheric pressure. Absolute pressure and gauge pressure are related as

$$p_{\text{absolute}} = p_{\text{gauge}} + p_{\text{atm}}$$

Consider a point A at a depth h from the free surface of a liquid of density ρ, as shown in the figure. The gauge pressure at the point A is $p_{\text{gauge}} = \rho gh$ and the absolute pressure is $p_{\text{absolute}} = \rho gh + p_0$.



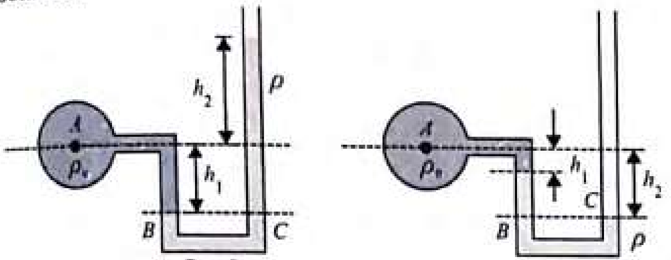
NOTE:

- Absolute pressure is always positive and is never equal to zero.

- Gauge pressure may be positive, zero or negative.
- Positive gauge pressure means that pressure at a point is more than the atmospheric pressure.
- Negative gauge pressure means that pressure at a point is less than the atmospheric pressure.
- Practically, the exact value of absolute pressure cannot be measured due to the uncertainty in the value of atmospheric pressure. The atmospheric pressure varies from place to place and at a place from time to time.

ILLUSTRATION 12.3 A container A is having some fluid of density θ_0. It is connected with a U-tube manometer shown in the figure. Find the pressure of the container in both the cases given in the figure.

Solution.



Case I Case II

Case I

Let the gauge pressure at A be p_A. Starting from container move along the manometer by a height h_1 downwards. The change in pressure will be $+\rho_0 g h_1$. The pressure at B and C will be the same. To move to the open end, we have to move by a distance $(h_1 + h_2)$ upward. Hence, change in pressure will be $-\rho g(h_1 + h_2)$. Finally, we reach the top, i.e., open to atmosphere. Hence, the manometric equation of the gauge pressure will be

$$p_A + \rho_0 gh_1 - \rho g\,(h_1 + h_2) = 0$$

$$\Rightarrow \quad p_A = \rho g(h_1 + h_2) - \rho_0 gh_1$$

In terms of absolute pressure at A, the manometric equation will be

$$p'_A + \rho_0\, gh_1 - \rho g(h_1 + h_2) = p_0$$

$$p'_A = (h_1 + h_2) - \rho_0\, gh_1 + p_0$$

Case II

Gauge pressure at A, $\quad p_A + \rho_0\, gh_1 + \rho g(h_2 - h_1) = 0$

Absolute pressure at A, $p'_A + \rho_0\, gh_1 + \rho g(h_2 - h_1) = p_0$

NOTE:

- Pressures at B and C are equal.
- Absolute pressure is always greater than or equal to zero, while gauge pressure can be negative also.
- In the same liquid, pressure will be same at all points at the same level. For example, in the figure, $P_1 \neq P_2$, $P_3 = P_4$, $P_5 = P_6$.

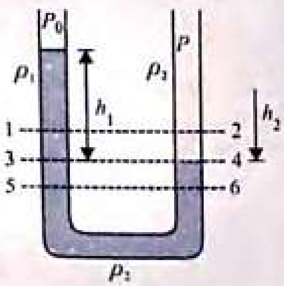
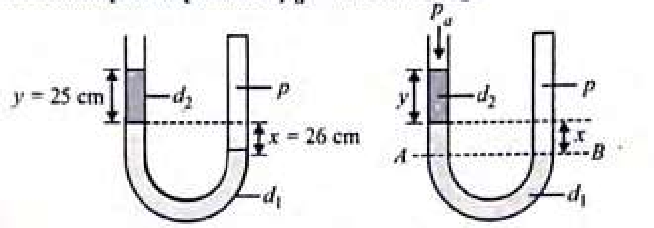


ILLUSTRATION 12.4 In a given U-tube open at left end and closed at right end, if the pressure above the liquid in the right arm is p, find the value of p. Given $d_2 = 2 \times 13.6$ g/cm^3, $d_1 = 13.6$ g/cm^3. Atmospheric pressure $p_a = 76$ cm of Hg.



Solution. The line AB is passing through the same liquid; hence, the pressure in the liquid at the same level is the same, i.e., at A and B, the pressures will be equal. The manometric equation is

$$p_a + d_2 yg + d_1 xg = p$$

$$\Rightarrow \quad p_a + 13.6 \times 2 \times 0.25 \times g + 13.6 \times 26 \times g = p$$

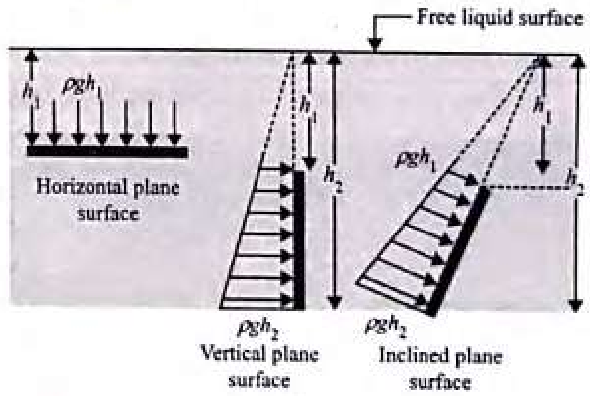
$$p_a + 13.6 \times g/100[50 + 26] = p$$

$$\Rightarrow \quad p = 2 \times 13.6 \times g \times \frac{76}{100} \qquad [\because p_a = 13.6 \times g \times 76/100]$$

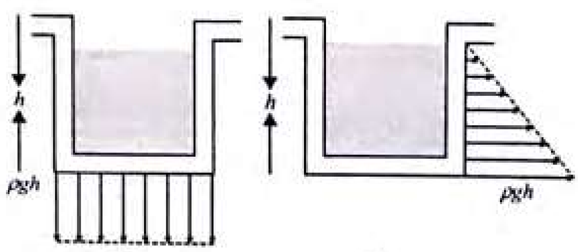
$$= 2p_a$$

Pressure Diagrams and Force on Boundaries

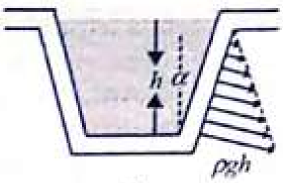
A pressure diagram is a graphical representation of the variation of the pressure intensity over a surface. Such a diagram may be prepared by plotting to some convenient scale the pressure intensities at various points on the surface. Net force (total pressure) as well as point of application of force (centre of pressure) for a plane surface wholly submerged in a static liquid, either vertically or inclined, may also be determined by drawing a pressure diagram.



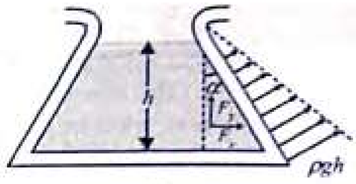
(a)



(b) (c)

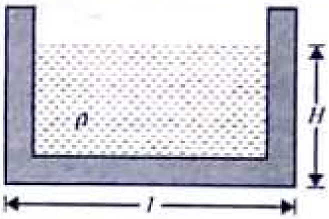


(d)

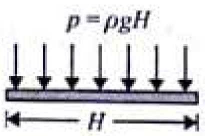


(e)

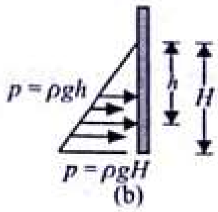
ILLUSTRATION 12.5 Consider a regular tank of size $(l \times b)$ filled with a liquid of density ρ to a height H as shown in the figure. Find the force at the base and on the walls of the tank.



Solution. Whenever a liquid comes in contact with solid boundaries it exerts a force on it. The force on the boundary may be obtained by integrating the pressure over the entire area of the boundary. The variations of liquid pressure acting at the base and at the wall are shown in Figs. (a) and (b), respectively.



(a)



(b)

1. **Force at the base:** Since the pressure is uniform at the base, force acting at the base is given by

   $F = p \times$ (area of the base)

   Since $p = \rho gH$, therefore

   $F = \rho gH(lb) = \rho g(lbH)$

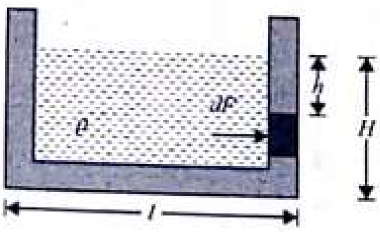
   Since $lbH = V$ (volume of the liquid), so

   $F = \rho gV =$ weight of the liquid inside the tank

NOTE: The force acting at the base per unit width (F/b) is equal to the area of the pressure diagram. That is,

$$\frac{F}{b} = pl = \rho gHl$$

2. **Force acting on the vertical wall:** Pressure acting on the vertical wall is not uniform but increases linearly with depth. Pressure at a depth h from the free surface is $p = \rho gh$.



Force dF acting on a differential element of height dh is

$$dF = p(bdh) = (\rho gh)(bdh) = \rho gbhdh$$

The total force is

$$F = \rho gb\int_0^H hdh = \frac{1}{2}\rho gbH^2$$

The total force acting per unit width of the vertical wall is

$$\frac{F}{b} = \frac{1}{2}\rho gH^2$$

NOTE: The force acting per unit width is equal to the area of the pressure diagram shown in the figure.

$$\frac{F}{b} = \frac{1}{2}\text{(base)(height)} = \frac{1}{2}(\rho gH)(H) = \frac{1}{2}\rho gH^2$$

Hence, force per unit width of an immersed surface is equal to the area of the pressure diagram on the surface.

The point of application (or the centre of force) of the total force from the free surface is given by $h_C = \frac{1}{F}\int_0^H hF$.

Here, $\int_0^H h\,dF = \int_0^H h(\rho gbh\,dh) = \rho gb\int_0^H h^2\,dh = \frac{1}{3}\rho gbH^3$

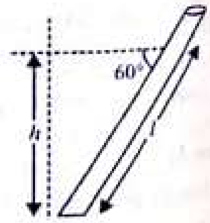
$$F = \frac{1}{2}\rho gbH^2 \Rightarrow h_C = \frac{2}{3}H$$

The total force acts at a depth $(2/3)H$ from the force diagram.

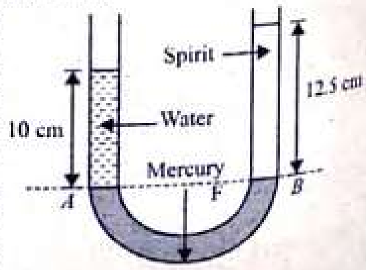
NOTE: The total force acts through the centroid of the pressure diagram, at a depth $2H/3$ from the top of the liquid level.

CONCEPT APPLICATION EXERCISE 12.1

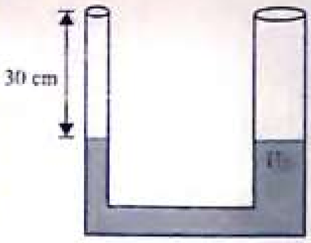
1. What is the absolute pressure on a swimmer 10 m below the surface of a lake? Take atmospheric pressure 1×10^5 N/m^2.
2. What will be the length of mercury column in a barometer tube when the atmospheric pressure is 75 cm of mercury and the tube is inclined at an angle of 60° with the horizontal direction?



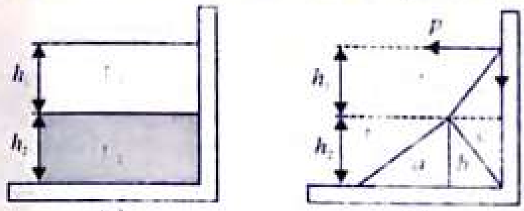
3. A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in the same level with 10 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?



4. A U-tube in which the cross-sectional area of the limb on the left is one-third of the limb on the right contains mercury (density 13.6 g/cm^3). The level of mercury in the narrow limb extends to a distance of 30 cm from the upper end of the tube. What will be the rise in the level of mercury in the right limb, if the left limb is filled to the top with water (neglect surface tension effects)?

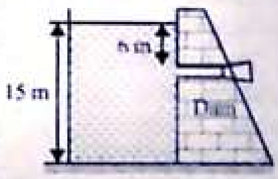


5. Figure (b) shows the variation of pressure (p) with depth (h) along a vertical wall supporting two liquids of different densities as shown in Fig. (a). Identify the correct variation of pressure in the liquid of density ρ_2.



(a) (b)

6. Water stands at a depth of 15 m behind a reservoir dam. A horizontal pipe 4 cm in diameter passes through the dam 6 m below the surface of water as shown. There is a plug which secures the pipe opening. Then find the friction between the plug and pipe wall.



FLUID IN UNIFORMLY ACCELERATING MOTION

Liquid Subjected to Constant Horizontal Acceleration

Consider a liquid in a tank which is moving on a horizontal surface with constant acceleration a. The free surface of the liquid takes the shape as shown in the figure. Consider a cylinder of liquid of length l and area of cross section A. The force on the left face of the cylinder is $F_1 = P_1A$ and force on the right face of the cylinder is $F_2 = P_2A$. Here $P_1 = \rho g y_1$ and $P_2 = \rho g y_2$. Mass of the liquid cylinder is $m = A\,l\rho$. Using Newton's second law for the liquid cylinder,

$$F_1 - F_2 = ma$$

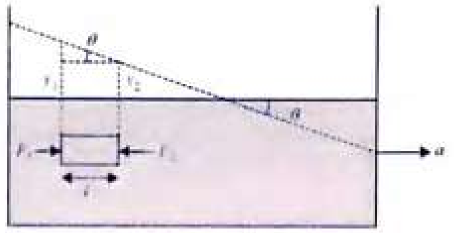
or $$P_1A - P_2A = ma \text{ or } (\rho g y_1 - \rho g y_2)A = Al\rho a$$

or $$\frac{y_1 - y_2}{l} = \frac{a}{g}$$

From the figure,

$$\frac{y_1 - y_1}{l} = \tan\theta$$

$$\therefore \quad \tan\theta = \frac{a}{g}$$

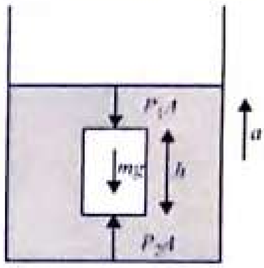


The negative sign shows that *pressure increases in a direction opposite to the direction of acceleration.*

Fluid Subjected to Constant Vertical Acceleration

Pressure difference when liquid is accelerating in vertical direction

Consider a cylindrical element of height h and area A. The force on the top face of the element is P_1A and force on the bottom face is P_2A. If a is the acceleration of the liquid, then



$P_2A - (mg + P_1A) = ma$

Here m is the mass of the element of the liquid which is equal to $hA\rho$.

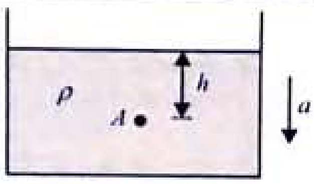
Thus, we have $P_2A - (hA\rho g + P_1A) = (hA\rho)\,a$

After simplification, we get $P_2 - P_1 = \rho(g + a)h = \rho g_{eff}h$.

COMPARISON OF AN ACCELERATED FLUID WITH AN ACCELERATED PENDULUM

Constant downward acceleration ($a < g$)

The free surface remains horizontal (see figure). Pressure decreases at every point below the surface. At the point A,

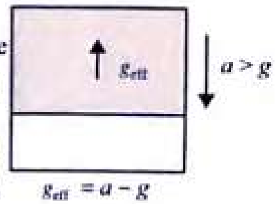


$$P = \rho(g - a)h\ \rho g_{eff}h$$

Pressure becomes zero everywhere

Constant downward acceleration ($a > g$)

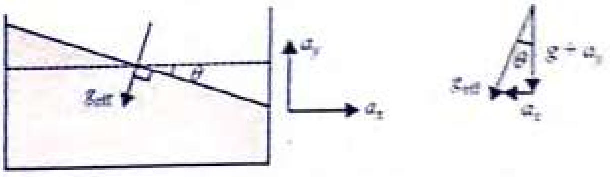
In this case, the fluid occupies the upper part of the container as shown in the figure.



Fluid Subjected to Combined Horizontal and Vertical Acceleration

In general, when the fluid has both horizontal and vertical acceleration, the inclination of the free surface with the horizontal may be obtained as

$$\tan\theta = \frac{dy}{dx} = \frac{dp/dx}{dp/dy} = \frac{a_x}{g+a_y}$$



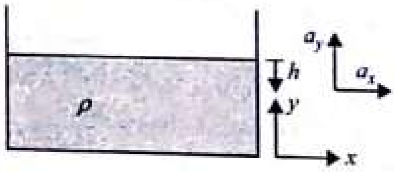
NOTE: In a non-accelerating fluid, pressure increases with depth and decreases with height:

$$\frac{dp}{dh} = \rho g \Rightarrow \frac{dp}{dy} = -\rho g$$

But when a fluid is subjected to a vertical acceleration, the above two equations may be modified as

Variation with depth: $\dfrac{dp}{dh} = \rho(g+a_y)$

Variation with height: $\dfrac{dp}{dy} = -\rho(g+a_y)$



A container accelerated in the vertical direction.

a_y is positive for upward acceleration and a_y is negative for downward acceleration.

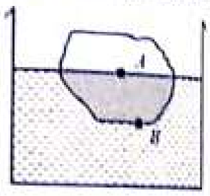
3. A barometer kept in an elevator accelerating upwards reads 76 cm of Hg. What will be the possible air pressure inside the elevator?

4. A barometer kept in an elevator reads 76 cm when it is at rest. What will be the barometric reading when the elevator accelerates upwards?

5. A container shown in the figure is accelerated horizontally with acceleration a. Find the minimum value of a at which liquid just starts spilling out.

ARCHIMEDES' PRINCIPLE

Consider an irregularly shaped object at rest within a fluid. Liquid exerts a thrust on sides of the solid surface in contact with it due to its pressure. Because the liquid pressure increases with depth, the force exerted by the liquid on the surface of the object is greater for those portions which are more deeply immersed. In the figure, liquid thrust at point B is greater than that at A. The net effect on the entire body is an upward or lifting force which is called the buoyant force.



The magnitude of this upward buoyant force (B) is given by Archimedes' principle. We have $B = v\rho g$, where v is the volume occupied by the solid inside the liquid and ρ is the density of the liquid.

ILLUSTRATION 12.6 A piece of ice is floating in water. What will happen to the level of water when all ice melts? What will happen if the beaker is filled not with water but with liquid (a) denser than water, and (b) lighter than water?

Solution. If M grams of ice is floating in a liquid of density σ_L, then for its equilibrium, weight of the ice = thrust

or $\quad Mg = V_D\sigma_L g$

So the volume of the liquid displaced by the floating ice is

$$V_D = (M/\sigma_L) \qquad \text{(i)}$$

Now if M grams of ice melts completely, water formed will have mass M grams (as mass is conserved). Now if σ_W is the density of water, the volume of water formed will be

$$V_F = (M/\sigma_W) \qquad \text{(ii)}$$

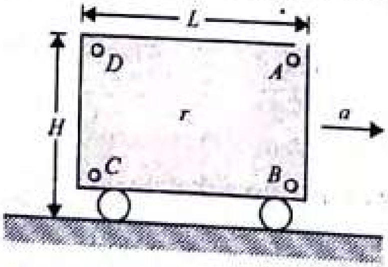
Here the liquid is water, i.e., $\sigma_L = \sigma_W$; so water displaced by floating ice is equal to the water formed by melting of the whole ice, and hence the level of water will remain unchanged.

The following two points may be noted:

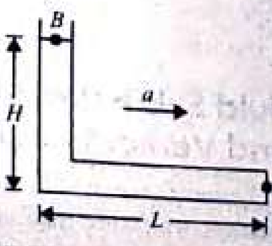
(a) If $\sigma_L > \sigma_W$, then $M/\sigma_L < M/\sigma_W$, i.e., $V_D < V_F$
i.e., water displaced by the floating ice will be lesser than water formed and so the level of the liquid in the beaker will rise.

CONCEPT APPLICATION EXERCISE 12.2

1. A rectangular box is completely filled with a liquid of density ρ, as shown in the figure. The box is accelerated horizontally with a constant acceleration a. Determine the gauge pressures at the four points A, B, C and D.

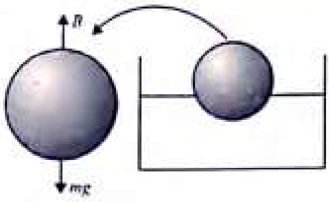


2. Figure shows an L-shaped tube filled with a liquid to a height H. What should be the horizontal acceleration a of the tube so that the pressure at the point A becomes atmospheric.



(b) If $\sigma_L < \sigma_W$, $M/\sigma_L > M/\sigma_W$, i.e., $V_D > V_F$
i.e., water displaced by the floating ice will be more than water formed and so the level of the liquid in the beaker will fall.

FLOATING BODY



Whenever the buoyant force on a body equals its weight, the net force acting on the body becomes zero. Hence, the body floats in equilibrium. Let a body of total volume V and density d be floating on a liquid of density ρ. Let v be the volume of the body inside the liquid.

Then $B = mg$ (for a floating body)

or $\quad v\rho g = Vdg$

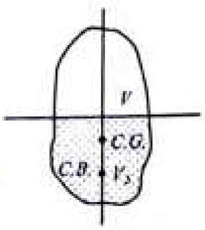
NOTE:

- When a body is in air, the net downward force on it is due to earth's gravity only. This net force $= mg = Vdg$. This force is known as weight of the body.
- When the same body is immersed in a liquid of density σ, the net downward force on it is $= mg - B$.
- Buoyant force $= Vdg - v\rho g$.
- This downward force is also known as apparent weight in the liquid.
- When a body is immersed in a liquid, it feels lighter due to buoyant force. Hence, the buoyant force is also known as loss in weight ($= v\rho g$).
- Buoyant force in accelerating fluids: Suppose a body is dipped inside a liquid of density ρ_L placed in an elevator moving with an acceleration. The buoyant force F in this case becomes $F = V\rho_L g_{\text{eff}}$. Here, $g_{\text{eff}} = |\vec{g} - \vec{a}|$.

CENTRE OF BUOYANCY

Centre of gravity of the displaced fluid is called buoyant centre or centre of buoyancy. Buoyant force is applied on buoyant centre.

Important Results Regarding Floating Bodies



- The net force on the floating object should be zero, i.e., weight of the object is same as the buoyant force on it.
- Let density of the solid be ρ_s, density of the fluid be ρ_l, submersed volume of the floating object be V_s and total volume of the object be V. Then

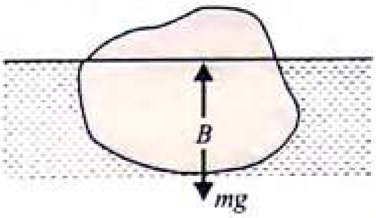
$$W = B \Rightarrow V\rho_s g = V_s\rho_l g \Rightarrow \frac{V_s}{V} = \frac{\rho_s}{\rho_l}$$

- The net torque on the floating object should be zero, i.e., centre of gravity of the object and buoyant centre should lie on the same vertical line.

ILLUSTRATION 12.7 What fraction of an iceberg lies beneath the surface of the sea? Density of sea water (ρ) = 1.028 $\times 10^3$ kg/m^3, density of ice (d) = 0.917 $\times 10^3$ kg/m^3.

Solution. For floating conditions, we know

Weight of the iceberg = Buoyant force



Let V be the total volume of the iceberg and v be the volume inside the sea water. Then

$Vdg = v\rho g$

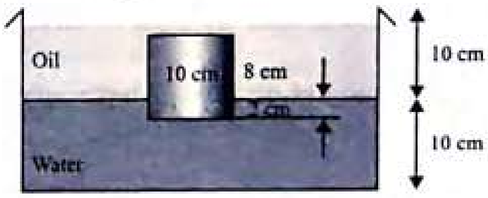
$$\frac{v}{V} = \frac{d}{\rho} \Rightarrow \frac{v}{V} = \frac{0.917\times10^3}{1.028\times10^3} \Rightarrow \frac{v}{V} = 0.892$$

About 89.2% of the iceberg remains inside.

ILLUSTRATION 12.8 A cubical block of wood 10 cm along each side floats at the interface between an oil and water with its lowest surface 2 cm below the interface. If the heights of oil and water columns are 10 cm each and $\rho_{\text{oil}} = 0.8$ g/cc, find (a) the mass of the block, (b) the liquid pressure at the lower surface of the block.

Solution.

(a) For the floating cube,



Weight of the cube = Buoyant force

$$\Rightarrow \quad mg = B_{\text{oil}} + B_{\text{water}}$$

$$= 8\times10^2 \times \rho_{\text{oil}} g + 2\times10^2 \rho_w \times g$$

$$\Rightarrow \quad m = 800\times0.8 + 200\times1$$

$$= 640 + 200 = 840 \text{ g}$$

(b) The pressure at the lower surface of the cube is

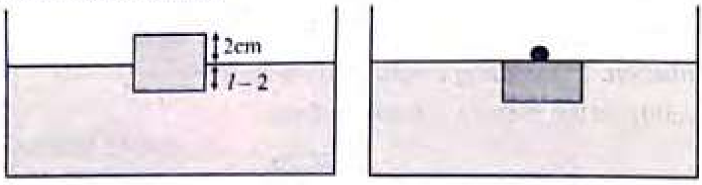
$$P_{\text{oil}} + P_w = \frac{10}{100}\times(0.8\times10^3)9.8 + \frac{2}{100}\times(1+10^3)9.8$$

$$= 980 \text{ N/m}^2$$

(Here we have used $h_{\text{oil}} = 10$ cm and $h_w = 2$ cm above the lower surface.)

ILLUSTRATION 12.9 A cube of wood supporting a 200 g mass just floats in the water. When the mass is removed, the cube rises by 2 cm. What is the size of the cube?

Solution. The cube 'just floats' means that it is fully under water while floating.



Let m be the mass of the cube. Then

$$(m+200)g = l^3 \rho g \quad \text{(i)}$$

When the mass is removed,

$$mg = (l-2)l^2 \rho g \quad \text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we have

$$200 = \rho\,[l^3 - (l-2)\,l^2]$$

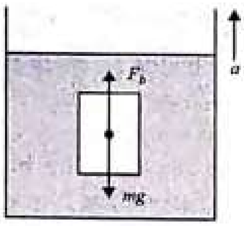
$\Rightarrow$ $l = 10$ cm (using $\rho = 1$ g/cc of water)

Buoyant Force in Accelerating Fluid

Suppose the body is submerged into a liquid of density ρ, which is accelerating upwards. If a is the acceleration of the liquid, then

$$F_b - mg = ma$$

or $\quad F_b = m(g+a)$



Here m is the mass of the displaced liquid, which is equal to $V\rho$. Therefore,

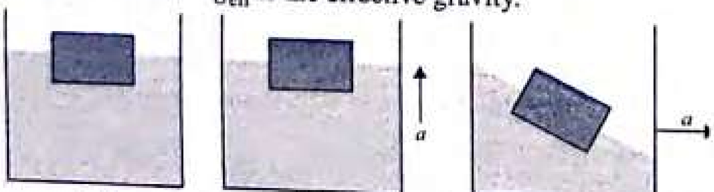
$$F_b = \rho V(g+a)$$

Expression for the Buoyant Force for an Accelerated Fluid

Instead of taking gravity 'g', we take the effective gravity g_{eff}. Then

$$F_B = \rho V g_{\text{eff}}$$

where ρ is the density of the fluid, V is the volume of the displaced fluid and g_{eff} is the effective gravity.



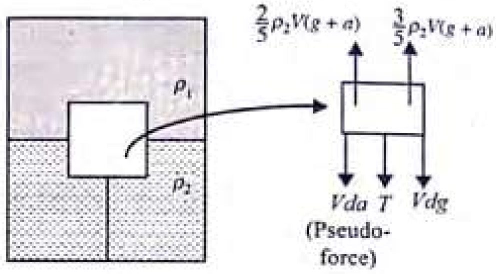
(a) (b) (c)

Figure (a) shows that a body floats on the surface of a stationary liquid. In a vertically accelerated liquid, the depth of submergence remains constant. In a horizontally accelerated liquid, the body aligns itself along the free surface or normal to the effective gravity. The depth of submergence remains constant.

ILLUSTRATION 12.10 A vessel contains two immiscible liquids of density $\rho_1 = 1000$ kg/m^3 and $\rho_2 = 1500$ kg/m^3. A solid block of volume $V = 10^{-3}$ m^3 and density $d = 800$ kg/m^3 is tied to one end of a string and the other end is tied to the bottom of the vessel. The block is immersed with two-fifths of its volume in the liquid of lower density. The entire system is kept in an elevator which is moving upwards with an acceleration of $a = g/2$. Find the tension in the string.

Solution. We will analyse this problem from the reference frame of elevator. Total buoyant force on the block is

$$F_B = \left(\frac{2}{5}V\rho_2 + \frac{3}{5}V\rho_1\right)(g+a)$$



(a) (b)

From the condition of equilibrium,

$$F_B = T + Vd(g+a)$$

$$\Rightarrow T = F_B - Vd(g+a) = (g+a)V\left[\frac{2}{5}\rho_2 + \frac{3}{5}\rho_1 - d\right]$$

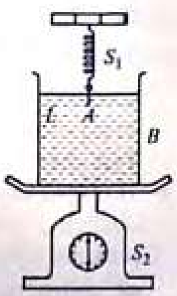
$$= 15\times10^{-3}\left[\frac{2}{5}\times1500 + \frac{3}{5}\times1000 - 800\right] = 6\text{ N}$$

CONCEPT APPLICATION EXERCISE 12.3

1. A beaker exactly full of water has an ice piece floating on it. As the cube melts what happens to the water level if (a) the cube contains an air bubble, (b) the cube contains (i) a lead piece and (ii) a cork piece.
2. A boat containing some piece of material is floating in a pond. What will happen to the level of water in the pond if on unloading the pieces in the pond, the piece (a) floats, (b) sinks?
3. A body of density ρ floats with volume V_1 of its total volume V immersed in a liquid of density ρ_1 and with the remainder of volume V_2 immersed in another liquid of density ρ_2, where $\rho_1 > \rho_2$. Find the volume immersed in two liquids (V_1 and V_2).
4. An iron casting has a number of cavities in it. It weighs 6000 N in air and 4000 N in water. Determine the total volume of all the cavities in the casting. The density of iron (without cavities) is 8.0 g/cm^3, $\rho_w = 1$ g/cm^3.
5. A necklace weighs 50 g in air, but it weighs 46 g in water. Assume that copper is mixed with gold to prepare the necklace. Find how much copper is present in it. (Specific gravity of gold is 20 and that of copper is 10.)
6. A block of ice of area A and thickness 0.5 m is floating in fresh water. In order to just support a man of 100 kg,

find the area A. (The specific gravity of ice is 0.917 and density of water is 1000 kg/m^3.)

7. As the figure shows, S_1 and S_2 are spring balances. Block A is hanging from spring balance S_1 and immersed in liquid L which is contained in beaker B. The mass of beaker B is 1 kg and mass of liquid L is 1.5 kg. Balances S_1 and S_2 reads 2.5 kg and 7.5 kg, respectively. What will be the readings of S_1 and S_2 when block A is pulled up out of the liquid. Find the reading of S_1 and S_2?



HYDRODYNAMICS

Types of Flow

Steady and Unsteady Flow

A flow is said to be steady if the velocity, pressure and density at any point in the flow do not change with time, i.e.,

$$\frac{dv}{dt}=0,\ \frac{dp}{dt}=0 \text{ and } \frac{d\rho}{dt}=0$$

In an unsteady flow, the velocity at a point in the flow varies with time, i.e., $\frac{dv}{dt}\neq 0$.

Uniform and Non-uniform Flow

A flow is said to be *uniform* at any instant of time if the velocity (both in magnitude and in direction) does not vary along the direction of flow. In a non-uniform flow, velocity varies in the direction of flow.

Visualization of Flow

The fluid flow pattern may be visualized in terms of pathlines and streamlines.

Pathline: It is a line drawn such that it describes the trajectory of a given fluid particle as it moves with the passage of time. The concept of the pathline in fluids is identical to the trajectory of a solid body. Tangent drawn at a point on a pathline gives the direction of velocity at that point at the time when the particle passes that point.

Important properties of a pathline:

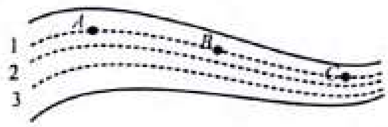
- A pathline can intersect itself at different times.
- Pathlines are the history lines of individual fluid particles over a period of time.

Streamline: It is the most common graphical concept of flow visualization. It is a line drawn such that a tangent at every point of it is in the direction of velocity at that instant.

Unlike pathlines, a streamline cannot intersect itself; furthermore, two streamlines cannot meet or intersect at a point. This is because instantaneously fluid can have a unique velocity at a point.

Streamline Flow

If every point of a steadily flowing liquid follows exactly the same path that has been followed by the particles preceding it, the flow is said to be streamlined. The path is known as 'streamline'. In the figure, the paths 1, 2, 3 are streamlines. If a liquid follows the path ABC, particles following it move along the same path.



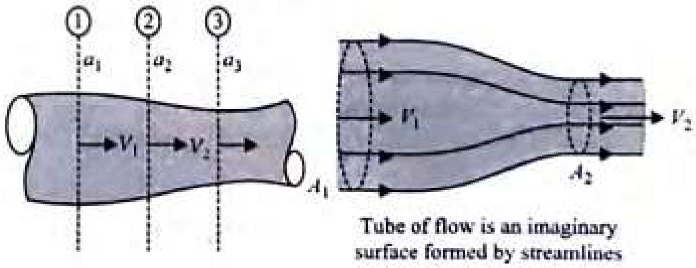
Turbulent Flow

When the velocity exceeds a certain critical value, the nature of flow becomes complicated. Random, irregular, local currents (called vortices) develop throughout the fluids. The resistance to the flow increases tremendously. This type of flow is called 'turbulent flow'.

PRINCIPLE OF CONTINUITY: THE CONTINUITY EQUATION

The continuity equation is a mathematical expression of the law of conservation of mass in fluid dynamics.

It states that 'in a steady flow, the amount (mass) of liquid entering a section is the same as that leaving another section in a given time interval'. If a_1, a_2, a_3, etc., are the cross-sectional areas of a pipe (carrying a liquid in a steady flow) at various sections, and v_1, v_2, v_3, etc., are the corresponding velocities (average), with ρ_1, ρ_2, ρ_3, etc., as the densities, then the mass of liquid flowing past these sections in time Δt would be $\rho_1 v_1 a_1 \Delta t$, $\rho_2 v_2 a_2 \Delta t$, $\rho_3 v_3 a_3 \Delta t$, etc. Since there is no accumulation of the liquid within the pipe, so



Tube of flow is an imaginary surface formed by streamlines

$$\rho_1 v_1 a_1 \Delta t = \rho_2 v_2 a_2 \Delta t = \rho_3 v_3 a_3 \Delta t = \cdots$$

$$\Rightarrow \quad \rho_1 v_1 a_1 = \rho_2 v_2 a_2 = \rho_3 v_3 a_3 = \cdots$$

In case of incompressible liquids, $\rho_1 = \rho_2 = \rho_3 = \cdots$

$$\therefore \quad v_1 a_1 = v_2 a_2 = v_3 a_3 = \cdots$$

The above equation is known as the 'equation of continuity',

$$A_1 v_1 = A_2 v_2$$

ILLUSTRATION 12.11 A broad pipe having a radius 10 cm branches into two pipes of radii 5 cm and 3 cm. If the velocity of flowing water in the pipe of radius 3 cm is 5 cm/s, determine the velocities of water in the remaining two pipes. Given that the rate of discharge through the main branch is 600π cm^3/s.

Solution. Consider any three sections (1), (2) and (3) in the three pipes of different radii as shown in the figure. If v_1 and v_2 be the velocities of water at sections (1) and (2), respectively, then

$$a_1 = \pi(10)^2 \text{ cm}^2,\ a_2 = \pi(5)^2 \text{ cm}^2 \text{ and } a_3 = \pi(3^2) \text{ cm}^2$$

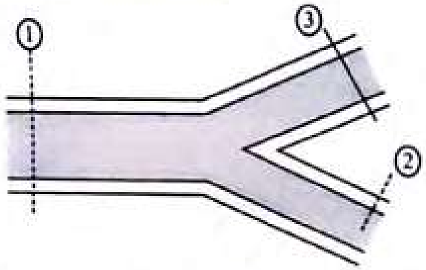
$$v_1 = ?,\ v_2 = ?,\ v_3 = 5 \text{ cm/s}$$

The rates of discharge through the three pipes are

$$a_1 v_1 = 100\pi v_1 \text{ cm}^3/\text{s}$$

$$a_2 v_2 = 25\pi v_2 \text{ cm}^3/\text{s}$$

$$a_3 v_3 = 9\pi \times 5 = 45\pi \text{ cm}^3/\text{s}$$



Now, $Q = a_1 v_1 = a_2 v_2 + a_3 v_3 \Rightarrow 600\pi = 100\pi v_1 = 25\pi v_2 + 45\pi$

Solving, we get $v_1 = 6$ cm/s and $v_2 = 22.2$ cm/s

BERNOULLI'S EQUATION

According to Bernoulli's theorem, the sum total of these three kinds of energies, kinetic energy, potential energy and pressure energy remains constant along a streamline in a steady flow of an ideal fluid. That is,

$$\frac{1}{2}\rho v^2 + \rho g y + p = \text{constant}$$

An ideal fluid is inviscous and incompressible. In other words, there is no dissipation of energy due to internal friction between adjacent layers of the fluid and density of this fluid remains constant. In other words, the sum of total energy per unit volume (pressure + kinetic + potential) is constant for an ideal fluid.

Other Forms of Bernoulli's Equation

If the kinetic, potential and pressure energies are expressed in terms of per unit mass or per unit weight, then the Bernoulli's equation may be written as

$$\frac{p}{\rho} + \frac{v^2}{2} + gy = \text{constant} \qquad \text{(per unit mass)}$$

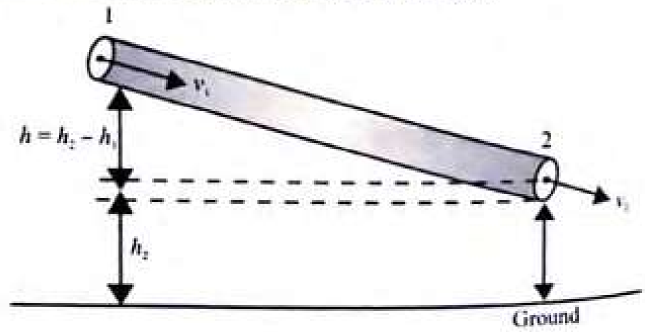
$$\frac{p}{\rho g} + \frac{v^2}{2g} + y = \text{constant} \qquad \text{(per unit weight)}$$

The Bernoulli's equation in the per unit weight form is the most commonly used expression. Each term in this equation has the dimensions of length. The term $v^2/2g$ is known as the velocity head, $p/\rho g$ is known as the pressure head and y is known as the potential head.

Applications and Limitations of Bernoulli's Equation

- The flow is assumed to be steady, i.e., there is no change in pressure, velocity and density of the fluid at any point with respect to time.
- However, in the problems of unsteady flow with gradually changing conditions, Bernoulli's equation can be applied without appreciable error. For example, a problem of emptying a large tank can be solved by applying Bernoulli's equation.
- The flow is assumed to be incompressible. Since liquids are incompressible, Bernoulli's equation can be applied to all liquids. However, it can be applied to the problems of gas flow when there is little variation in pressure, velocity and temperature so that density of gas can be assumed to be constant.
- The flow is assumed to be irrotational. The irrotationality means the net angular momentum at any point in the fluid flow is zero.
- The fluid is assumed to be ideal, i.e., the energy loss due to friction is assumed to be absent.

ILLUSTRATION 12.12 A liquid moves in a smooth tube of uniform cross section. Find the pressure P_2 at point 2 of the tube if the pressure at point 1 of the extended tube is P_1 and the vertical distance between points 1 and 2 is h.



Solution. Since the cross section of the tube is uniform,

$$A_1 = A_2 = A$$

Then, applying equation of continuity at points 1 and 2, we have

$$v_1 = v_2 \qquad (i)$$

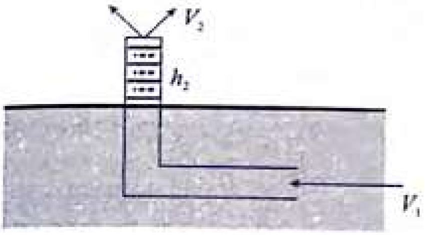
Substituting $v_1 = v_2 (= v)$ in Bernoulli's equation, we have

$$P_1 + \frac{1}{2}\rho v^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v^2 + \rho g h_2$$

This gives $P_1 + \rho g h_1 = P_2 + \rho g h_2$

Since $h_1 - h_2 = h$, we have $P_2 = P_1 + \rho g h$

ILLUSTRATION 12.13 A bent tube is lowered into a water stream as shown in the figure. The velocity of the stream is V_1. The closed upper end is located at a height h_0 and has a small orifice. To what height h will the liquid get spurt?



Solution. $\frac{1}{2}\rho V_1^2 + P_0 + 0 = \frac{1}{2}\rho V_2^2 + P_0 + \rho g h_0$

$\Rightarrow \quad V_2^2 = V_1^2 - 2gh_0$

If the liquid reaches a height h, then $V_2^2 = 2gh$,

$\Rightarrow \quad 2gh = V_1^2 - 2gh_0 \Rightarrow h = \frac{V_1^2}{2g} - h_0$

VENTURI METER

This is an instrument for measuring the rate of flow of liquids and gases (fluids). The instrument is connected horizontally in the tube in which the rate of flow is to be measured. We have

$$P_A - P_B = h\rho g$$

where h is the difference of heights of liquids of density r in vertical tubes. The velocities of fluid at A and B are v_1 and v_2, respectively.

For liquids:

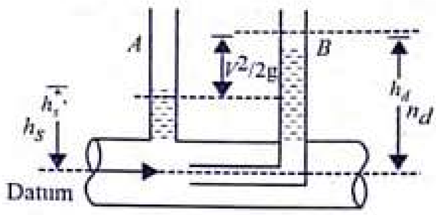
$$P_A + \rho\frac{v_1^2}{2} = P_B + \rho\frac{v_2^2}{2}$$

$$\Rightarrow \quad v_2^2 - v_1^2 = \frac{2}{\rho}(P_A - P_B) = \frac{2}{\rho}h\rho g \quad \Rightarrow v_2^2 - v_1^2 = 2hg$$

$$\Rightarrow \quad \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2hg \quad (\because Q = Av) \quad \Rightarrow Q = A_1A_2\sqrt{\frac{2hg}{A_1^2 - A_2^2}}$$

STATIC PRESSURE AND DYNAMIC PRESSURE

The static pressure at a point in a fluid flow may be obtained by connecting a single-tube manometer perpendicular to the direction of the fluid flow as shown by tube A in the figure. The height h_s to which the liquid rises in it with respect to the reference level (datum) is a measure of the static pressure in the fluid at a point on the axis. That is,



$$p_s = \rho g h_s \quad \text{or} \quad \frac{p_s}{\rho g} = h_s$$

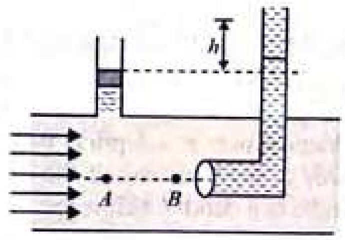
The height h_s is also called the static pressure head. When a tube bent at right angle is inserted in the fluid flow such that the open end faces against the direction of fluid flow as shown by tube B in the figure. At the open end of the tube the velocity of the fluid suddenly reduces to zero and the kinetic energy gets converted into pressure energy. The liquid in tube B rises to a higher level than that in tube A. The difference in the levels of liquid is a measure of velocity of the fluid. Tube B measures the dynamic pressure and the height h_d of the liquid in the tube is called the dynamic head, which is equal to the sum total of the static pressure head and velocity head.

That is, $h_d = h_s + \frac{v^2}{2g}$.

PITOT TUBE

Pitot tube is used to measure the speed of flow of a fluid. The cross section of tube at B is perpendicular to the direction of flow and tube is at rest, so that at B, velocity of fluid becomes zero, i.e., $V_B = 0$. Let V be the velocity of fluid at A and P_A and P_B are pressures at A and B. Then $P_B - P_A = h\rho g$, where ρ is density of liquid in vertical tubes.

For liquids: If points A and B are in the same horizontal line and area of is the same at A and B, gravitational head is the same and the pressure difference is due to difference in velocity. Applying Bernoulli's theorem at A and B, we have



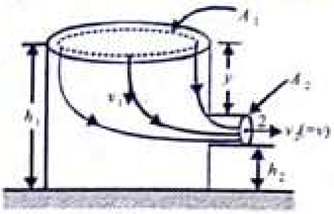
$$P_A + \frac{1}{2}\rho V^2 = P_B + 0$$

$$\Rightarrow \quad \frac{2(P_B - P_A)}{\rho} = V^2 \Rightarrow V = \sqrt{\frac{2(P_B - P_A)}{\rho}} \quad \Rightarrow V = \sqrt{2gh}$$

VELOCITY OF EFFLUX: TORRICELLI'S THEOREM

The liquid comes out of a hole made at a depth y. The areas of the base and hole are A_1 and A_2, respectively. Let us calculate the velocity of efflux.

Choose two points 1 and 2 at the free surface of liquid and at just outside the hole, respectively. Since the points 1 and 2 are exposed to atmosphere, $P_1 = P_2 = P_{atm}$. We have done this to get a mathematical advantage. However, you can take any other point inside the tube instead of taking at the free surface of the liquid. Equation of continuity gives



$$A_1 v_1 = A_2 v_2 \qquad \text{(i)}$$

$$P_{atm} + \frac{\rho v_1^2}{2} + \rho g h_1 = P_{atm} + \frac{\rho v_2^2}{2} + \rho g h_2$$

Substituting $h_1 - h_2 = y$, we have

$$v_2 = \sqrt{v_1^2 + 2gy} \qquad \text{(ii)}$$

Substituting $v_1 = (A_2/A_1)v_2$ from Eq. (i) in Eq. (ii), we have

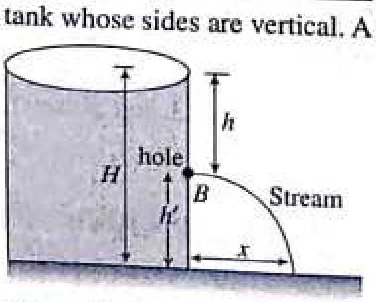
$$v_2 = \sqrt{\frac{2gy}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

NOTE:

- If area of opening is much lesser than area of cross section of tank ($A_2 << A_1$), then velocity of efflux
  $$v = \sqrt{2gh}$$
- which is the same speed that an object would acquire in falling from rest through a distance h under gravity.
- The velocity of efflux is the velocity of escaping liquid relative to the container (but not necessarily relative to ground when the container moves).

HORIZONTAL RANGE OF THE ESCAPING LIQUID

Water stands at a depth H in a tank whose sides are vertical. A hole is made in one of the walls at a depth h below the water surface. Let container remains at rest.



When the liquid stream emerges out of the hole, it goes along a parabolic path. So time taken by water to fall through a height of $H - h$ is given as

$$H - h = 0 \times t + 1/2\, gt^2 \Rightarrow t = \sqrt{\frac{2(H-h)}{g}}$$

Horizontal range is given by

$$V \times t = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = \sqrt{4h(H-h)} = 2\sqrt{hh'}$$

Horizontal range is given by

$$x = \sqrt{4h(H-h)}$$

$$= \sqrt{4hH - 4h^2 + H^2 - H^2} = \sqrt{H^2 - (H^2 - 4hH + 4h^2)}$$

$$= \sqrt{H^2 - (2h - H)^2}$$

For the range to be maximum, $(2h - H)^2 = 0$

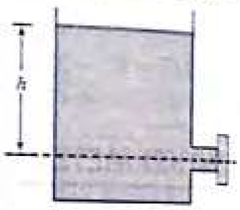
$\Rightarrow \quad 2h - H = 0 \Rightarrow h = H/2$

So the maximum range $x = 2\sqrt{hh'}$.

The formula $= 2\sqrt{hh'}$ tells us if we make two holes at equal vertical distances from top and bottom, both liquids jets will strike the same spot (but not simultaneously).

FORCE OF REACTION DUE TO EJECTION OF LIQUID

A cylindrical vessel has an opening of cross-sectional area 'a' near the bottom. A disc is held against the opening to prevent liquid of density ρ from coming out. If the height of the liquid above opening is h. Let us analyse the force on the disc in this situation.



The disc experiences a hydrostatic pressure from the liquid inside the vessel. Pressure at the level of the disc is

$$P_1 = P_{atm} + \rho g h$$

The air pressure on the outside of disc is $P_2 = P_{atm}$.

The net outward force $= (p_2 - p_1)a = \rho g h a$.

Now the disc is moved a short distance away in horizontal direction. The liquid comes out, strikes the disc inelastically and drops vertically downward. The water in this case will impart impulsive (inpact) force on the disc.

When the disc is moved away, the liquid moves out with speed $v = \sqrt{2gh}$.

The (mass per second), i.e, the rate of mass coming out of the opening is given by $dm/dt = \rho a v = \rho a\sqrt{2gh}$.

Momentum per second imparted by water rightward is given by

$$\frac{dm}{dt}v = (\rho a v)v = \rho a v^2 = 2\rho g h a$$

The change in momentum per second after striking the disc is

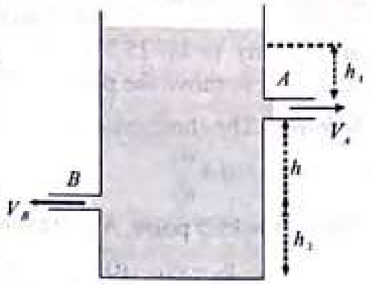
$$\Delta \vec{P} = \vec{P}_f - \vec{P}_i \Rightarrow 0 - (\rho a v^2) = -\rho a v^2 = -2\rho g h a$$

Taking the rightward direction as positive, the force on the liquid is towards the left and its reaction with disc is towards the right. The force obtained is twice the hydrostatic force. The force due to atmospheric pressure is cancelled out on the both sides.

NOTE: If the velocity of a liquid (or gas) of density ρ coming out through an opening of area of cross section a is v, then there will be a thrust force $F = \rho a v^2$ due to liquid coming out the opening. The direction of the thrust force will be just opposite to the velocity direction.

ILLUSTRATION 12.14 On the opposite sides of a wide vertical vessel filled with water, two identical holes are opened, each having cross-sectional area 'a'. The height difference between them is equal to h. Find the resultant force of reaction of water flowing out of vessel.

Solution. Let A and B be two openings, each of area 'a' the velocity of water coming out of opening are V_A and V_B, respectively. Volume of water emerging per second through A is aV_A.



Volume of water emerging per second through B is aV_B. Force of reaction due to water coming out at A is given by

F_A = Rate of change of momentum per second at A

$$(aVA\rho)V_A = a\rho V_A^2$$

Similarly, force of reaction at B is given by $F_B = a\rho V_B^2$

Therefore, net force on vessel is

$$F = F_B - F_A = a\rho\left(V_B^2 - V_A^2\right) \quad \text{(i)}$$

Now, applying Bernoulli's theorem at A and B, we get

$$p_a + \frac{1}{2}\rho V_A^2 + (h_2 + h)\rho g = p_0 + \frac{1}{2}\rho V_B^2 + \rho g h_2 \quad \text{(ii)}$$

where p_a is atmospheric pressure.

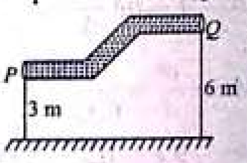
$$\therefore \quad V_B^2 - V_A^2 = 2gh$$

Substituting this value in Eq. (i), we get $F = a\rho 2gh$

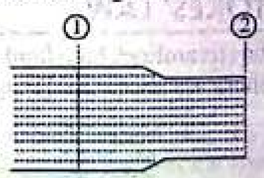
or $F = 2\rho gah$

CONCEPT APPLICATION EXERCISE 12.4

1. A non-viscous liquid of constant density 500 kg/m^3 flows in a variable cross-sectional tube. The area of cross section of the tube at two points P and Q at heights of 3 m and 6 m are 2×10^{-3} m^3 and 4×10^{-3} m^3, respectively. Find the work done per unit volume by the forces of gravity as the fluid flows from point P to Q.



2. A horizontal pipe of varying cross-sectional area carries water flowing at a constant rate. The pressure at sections (1) and (2) are 50 cm and 20 cm of Hg. If the velocity of section (1) is 5 m/s, what is the velocity of section (2). Also determine the ratio of areas at sections (1) and (2) (as shown in the figure.



3. The speed of flow past the lower surface of a wing of an aeroplane is 50 ms^{-1}. What speed of flow over the upper surface will give a dynamic lift of 1000 Pa? Density of air = 1.3 kg m^{-3}.
4. A gardener holds a pipe of inside cross-sectional area 3.60 cm^2 at a height of 1.50 m from ground. The opening at the nozzle of pipe has cross-sectional area 0.250 cm^2. Water velocity in the segment of the hose that lies on the ground is 50.0 cm/s.
   (a) What is the velocity of water leaving the nozzle?
   (b) What is the thrust force experienced by the gardener due to flow of water through nozzle.
   (c) What is the water pressure in the hose on the ground?
5. If water flows horizontally through a pipe of varying cross section and the pressure of water equals 10 cm of Hg at a point where the velocity of flow is 40 cm/s, what is the pressure at another point where the velocity of flow is 50 cm/s?

VISCOSITY

When a solid body slides over another solid body, a frictional force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional force'.

The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.

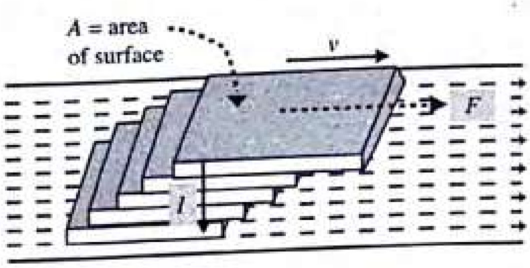
If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down due to gravity, the relative motion between its layers is opposed strongly.

Newton's Formula

The basic formula for the frictional (or viscous force) F in a liquid was first suggested by Newton. He observed that larger the area A of the surface of liquid considered, larger was the frictional force F; larger the gradient of velocity dv/dy across the layers of liquid, larger was the frictional force F. That is,

$$F \propto A\frac{dv}{dy}$$

Consider a fluid confined between two parallel plates, with the lower plate stationary and the upper plate moving with a velocity v_0, as shown in the figure. It has been proved experimentally that for all real fluids possessing any viscosity, the fluid particles in immediate contact with any solid surface move with the velocity of the surface itself. Thus, the topmost layer of the liquid moves with velocity v_0 and the bottommost layer remains stationary. And all the intermediate layers acquire velocities $0 < v < v_0$ as indicated by the lengths of the arrows in the figure.



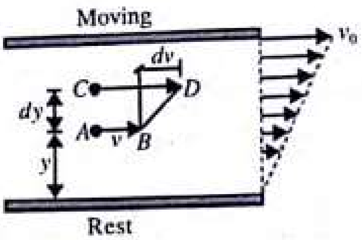


Figure shows a layer AB of the liquid at a distance y from stationary plate which is moving with a velocity v parallel to the plate. An adjacent layer CD at a distance $y + dy$ moves with a velocity $v + dv$. The velocity gradient between the two layers is $dv{\cdot}dy$. Due to relative motion between the adjacent layers of the liquid there exists a frictional resistance, which according to Newton is given by

$$F \propto A\frac{dv}{dy} \quad \text{or} \quad F = -\eta A\frac{dv}{dy} \qquad \text{(i)}$$

η is called coefficient of viscosity. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.

It is important to note that solids resist relative motion between its adjacent layers to maintain their shapes. The resistance offered by solids in maintaining their shapes is measured in terms of their modulus of rigidity G.

NOTE: The coefficient of viscosity g is analogous to the modulus of rigidity of solids. The shear stress of fluids is proportional to the rate of shear strain and that in case of solids is proportional to the shear strain.

Units of Coefficient of Viscosity

From the definition, we have $\eta = \dfrac{F}{A(\Delta v_x/\Delta y)}$

$\therefore$ Dimensions of $\eta = \dfrac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \dfrac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$

Its unit is kg/(m s).

In CGS system, the unit of coefficient of viscosity is dyn s cm^{-2} and is called poise. In SI the unit of coefficient of viscosity is N s m^{-2} and is called decapoise.

$$1 \text{ decapoise} = 1 \text{ N s m}^{-2} = (10^5 \text{ dyn}) \times \text{s} \times (10^2 \text{ cm})^{-2}$$
$$= 10 \text{ dyn s cm}^{-2} = 10 \text{ poise}$$

Effect of Temperature on Viscosity

Experiments show that η of a liquid decreases sharply with increase in its temperature and becomes zero at its boiling point. On the other hand, η of gases increase with temperature.

The viscosity of liquids decreases with increase in temperature and increases with the decrease in temperature. That is, $\eta \propto 1/\sqrt{T}$. On the other hand, the value of viscosity of gases increases with the increase in temperature and vice versa. That is, $\eta \propto \sqrt{T}$.

ILLUSTRATION 12.15 A plate of area 100 cm^2 is placed on the upper surface of castor oil, 2 mm thick. Taking the coefficient of viscosity to be 15.5 poise, calculate the horizontal force necessary to move the plate with a velocity 3 cm s^{-1}.

Solution. The (horizontal tangential) viscous force is given by

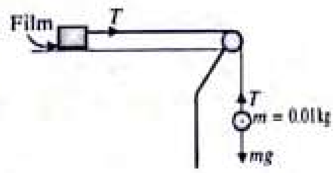
$$F = -\eta A\frac{dv}{dy}$$

Given η = 15.5 poise, A = 100 cm^2

$$\frac{dv}{dx} = \frac{v_2 - v_1}{x_2 - x_1} = \frac{(0-3)\text{cm/s}}{(2-0)\text{ mm}} = -\frac{3}{0.2}\text{s}^{-1} = -15\text{ s}^{-1}$$

$$F = -15.5 \times 100 \times (-15) = 2.235 \times 10^4 \text{ dyn} = 0.2325 \text{ N}$$

ILLUSTRATION 12.16 A metal plate of area 0.10 m^2 is connected to a 0.01 kg mass via a string that passes over an ideal pulley (considered to be frictionless), as shown in the figure. A liquid with a film thickness of 3.0 mm is placed between the plate and the table. When released, the plate moves to the right with a constant speed of 0.085 m s^{-1}. Find the coefficient of viscosity of the liquid.



Solution. As the metal plate moves with constant velocity,

$$mg - T = 0 \quad \Rightarrow T = mg$$

If F is the viscous force on the plate, then

$$F = T = mg = 0.01 \times 9.8 = 9.8 \times 10^{-2} \text{ N}$$

By Newton's law of viscosity,

$$F = \eta A\frac{dv}{dy}$$

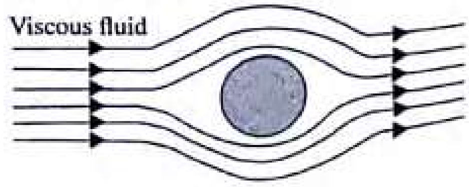
where $\dfrac{dv}{dy} = \dfrac{0.085}{0.3\times10^{-3}} = 28.3$ s

$$\Rightarrow \quad 9.8\times10^{-2} = \eta\times0.10\times28.3$$

$$\Rightarrow \quad \eta = 3.4\times10^{-3} \text{ N s/m}^2$$

STOKES' LAW

The streamlines for a fluid flowing slowly past a stationary solid sphere are shown in the figure.



When the sphere moves slowly as compared to the fluid, the pattern is similar but the streamlines then flow in such a way that the apparent motion of the fluid particles is as seen by someone on the moving sphere. In this latter case, it is known that the layer of fluid in contact with the sphere moves with it, thus creating a velocity gradient between this layer and other layers of fluid. Viscous forces are thereby brought into play and constitute the resistance experienced by the moving sphere. If we make the plausible assumption that the viscous retarding force F depends on the size of the body, the velocity with which it moves, the viscosity of the fluid and mass density of the fluid, then an expression can be derived for F by the method of dimensions. Thus,

$$F = k\eta^x v^y r^z$$

where x, y and z are the indices to be found and k is a dimensionless constant. The dimensional equation is

$$[MLT^{-2}] = [ML^{-1}T^{-1}]^x\,[LT^{-1}]^y[L]^z$$

Equating indices of M, L and T on both sides, we have

$$1 = x$$

$$1 = -x + y + z$$

$$-2 = -x - y$$

Solving, we get $x = 1$, $y = 1$ and $z = 1$. Hence $F = k\eta vr$

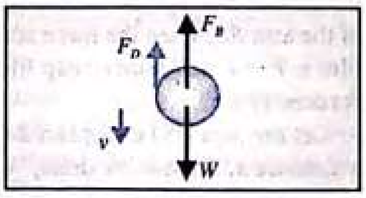
A detailed treatment, first done by Stokes, gives $k = 6\pi$ and so $F = 6\pi\eta vr$.

This expression, called Stokes' law, only holds for steady motion in a fluid of infinite extent (otherwise the walls and bottom of the vessel affect the resisting force).

TERMINAL VELOCITY

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity. Consider a small ball of radius r, which is gently dropped into the liquid of infinite extent. As the steel ball falls, it experiences three forces: the force of gravity W; the buoyant force F_B; the drag force F_D.

The free-body diagram of the ball is shown in the figure. If ρ_0 be the density of the body and ρ be the density of the liquid, then



Free-body diagram of a ball falling under gravity in a viscous liquid.

$$W = \frac{4}{3}\pi r^3 \rho_0 g$$

and $$F_B = \frac{4}{3}\pi r^3 \rho g$$

The magnitude of the drag force is $F_D = 6\pi\eta rv$.

As the ball falls under gravity, its net weight $W - F_B$ is opposed by liquid resistance F_D. Initially, the drag force is small and as the body gains velocity the drag force also increases and at a particular velocity, called the terminal velocity, the magnitude of drag force becomes equal to the effective weight of the body. At this moment, the net force on the ball becomes zero and it moves with a constant velocity. That is,

$$F_D = W - F_B$$

or $$6\pi\eta rv = \frac{4}{3}\pi r^3(\rho_0 - \rho)g$$

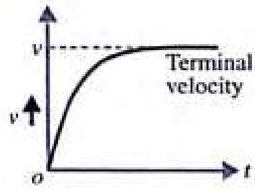
$$\Rightarrow \quad v = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho)g \qquad \text{(i)}$$

If we plot the variation of velocity of the falling sphere with time, we obtain a graph as shown in the figure. Following points can be noted:

(1) Initially the velocity of the ball increases at a fast rate.

(2) The rate of increase of velocity decreases with time.

(3) Finally, the rate of increase of velocity becomes zero and the sphere acquires a terminal velocity.



Variation of velocity of sphere with time.

Important Results

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

ILLUSTRATION 12.17 Spherical particles of pollen are shaken up in water and allowed to settle. The depth of water is 2×10^{-2} m. What is the diameter of the largest particles remaining in suspension one hour later? Density of pollen $= 1.8 \times 10^3$ kg m^{-3}, viscosity of water $= 1 \times 10^{-2}$ poise and density of water $= 1 \times 10^{-5}$ kg m^{-3}.

Solution. Terminal velocity, $v = \dfrac{2r^2}{9}\dfrac{(\rho - \sigma)g}{\eta}$ (i)

But we know $v = \dfrac{s}{t}$

$$\therefore \quad \frac{s}{t} = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \Rightarrow r^2 = \frac{9s}{2t}\frac{\eta}{(\rho - \sigma)g}$$

Given $s = 2 \times 10^{-2}$ m, $t = 1$ h $= 3600$ s

$$\therefore \quad \eta = 1 \times 10^{-2}\ \text{poise} = 1 \times 10^{-3}\ \text{kg m}^{-1}\text{s}^{-1}$$

Substituting given values, we get

$$r^2 = \frac{9}{2} \times \frac{2 \times 10^{-2}}{3600} \times \frac{1 \times 10^{-3}}{(1.8 \times 10^3 - 1 \times 10^3) \times 10}$$

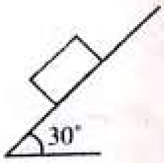
$$= \frac{9}{36} \times \frac{1}{8} \times 10^{-10} = \frac{1}{32} \times 10^{-10}$$

$$\therefore \quad r = \sqrt{\frac{100}{32}} \times 10^{-6}\ \text{m} = 1.77 \times 10^{-6}\ \text{m}$$

Diameter, $D = 2r = 2 \times 1.77\ \mu\text{m} = 3.54\ \mu\text{m}$

CONCEPT APPLICATION EXERCISE 12.5

1. A man is rowing a boat with a constant velocity 'v_0' in a river. The contact area of boat is 'A' and coefficient of viscosity is η. The depth of river is 'D'. Find the force required to row the boat.
2. A cubical block (of side 2 m) of mass 20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m s^{-1}. Find out the thickness of the layer of liquid (take $g = 10 \text{ m s}^{-2}$).

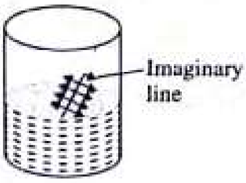


3. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} kg/(m s), what will be the terminal velocity of the drop? Density of water = $1.0 \times 10^3 \text{ kg/m}^2$ and $g = 9.8$ N/kg. Density of air can be neglected.
4. A metallic sphere of radius 1.0×10^{-3} m and density $1.0 \times 10^4 \text{ kg/m}^3$ enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given: coefficient of viscosity of water = $1.0 \times 10^{-3} \text{ N s/m}^2$, $g = 10 \text{ m s}^{-12}$ and density of water = $1.0 \times 10^3 \text{ kg/m}^3$.
5. Find the minimum force required to drag a hard polythene plate of area 2 m^2 on a thin film of oil of thickness 0.25 cm and $\eta = 15$ poise. Assume the speed of the plate is 10 cm s^{-1}.
6. A force of 3.14 N is required to drag a sphere of radius 4 cm with a speed of 5 m s^{-1} in a medium in gravity free space. Find the coefficient of the viscosity of the medium.

SURFACE TENSION

Surface tension is a property of liquid at rest by virtue of which a liquid surface gets contracted to a minimum area and behaves like a stretched membrane.

Surface tension of a liquid is measured by force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line as shown in the figure. Mathematically, surface tension is defined as

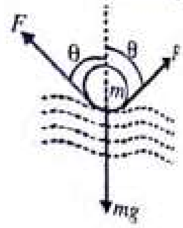


$$T = \frac{\text{Total force on either side of the imaginary line } (F)}{\text{Length of the line } (l)}$$

In CGS system the unit of surface tension is dyn/cm (dyn cm^{-1}) and in SI system its units is Nm^{-1}.

ILLUSTRATION 12.18 Find the maximum possible mass of a greased needle floating on water surface.

Solution. Let the mass of the needle be m. As the liquid surface is distorted, the surface tension forces acing on both sides of the needle make an angle θ, say, with vertical. Since the forces acting on the needle are F, F and mg, resolving the forces vertically for its equilibrium, we have



$$\sum F_y = F\cos\theta + F\cos\theta - mg = 0$$

This gives $m = \dfrac{2F\cos\theta}{g}$

where $F = Tl$

Then $m = \dfrac{2Tl\cos\theta}{g}$

For m to be maximum, $\cos\theta = 1$

Hence, $m_{\max} = \dfrac{2Tl}{g}$

Surface Energy

We know that the molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to be done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface energy.

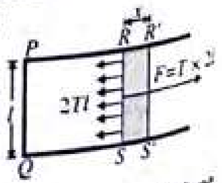
Unit of surface energy is erg cm^{-2} in CGS system and J m^{-2} in SI system. Dimensional formula of surface energy is $[ML^0T^{-2}]$. Surface energy depends on number of surfaces, e.g., a liquid drop is having one liquid air surface while bubble is having two liquid air surfaces.

Relation Between Surface Tension and Surface Energy

Consider a rectangular frame $PQSR$ of wire, whose arm RS can slide on the arms PR and QS. If this frame is dipped in a soap solution, then a soap film is produced in the frame $PQSR$ in the figure. Due to surface tension (T), the film exerts a force on the frame (towards the interior of the film). Let l be the length of the arm RS. Then the force acting on the arm RS towards the film is $F = T \times 2l$ (since soap film has two surfaces, the length is taken twice).

Let the arm RS be displaced to a new position $R'S'$ through a distance x. Then work done, $W = Fx = 2Tlx$

Increase in potential energy of the soap film is give by



$EA = 2Elx$ = work done in increasing the area (ΔW)

where E = surface energy of the soap film per unit area.

According to the law of conservation of energy, the work done must be equal to the increase in the potential energy.

i.e., $2Tlx = 2Elx$

or $T = E = \dfrac{\Delta W}{A}$

Thus, surface tension is numerically equal to surface energy or work done per unit increase of surface area.

ILLUSTRATION 12.19 A film of water is formed between two straight parallel wires each 10 cm long and at separation 0.5 cm. Calculate the work required to increase 1 mm distance between the wires. Surface tension of water = 72×10^{-3} N/m.

Solution. Initial surface area = 2 × length × separation

$$= 2 \times 10 \text{ cm} \times 0.5 \text{ cm}$$

$$= 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2$$

Final surface area = $2 \times 10 \text{ cm} \times (0.5 + 0.1)$ cm

$$= 2 \times 10 \times 0.6 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2$$

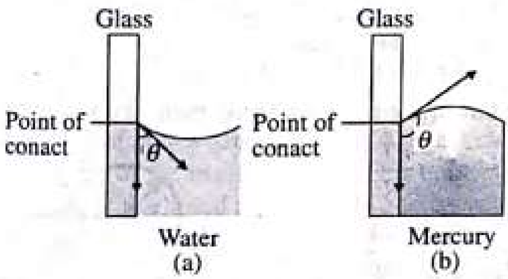
The required work,

$$W = T\,\Delta A = 72 \times 10^{-3} \times (12 \times 10^{-4} - 10 \times 10^{-4}) \text{ J}$$

$$= 72 \times 10^{-3} \times 2 \times 10^{-4} = 144 \times 10^{-7} \text{ J}$$

Angle of Contact

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact. Those liquids, which wet the walls of the container (say in case of water and glass), have meniscus concave upwards and their value of angle of contact is less than 90° (also called acute angle). However, those liquids, which do not wet the walls of the container (say in case of mercury and glass), have meniscus convex upwards and their value of angle of contact is greater than 90° (also called obtuse angle). The angle of contact of mercury with glass is about 140°, whereas the angle of contact of water with glass is about 8°. But, for pure water, the angle of contact θ with glass is taken as 0°.



Since the weight of the molecule is negligible as compared to F_1 and F_2, so it can be neglected. Thus, there are only two forces (F_1 and F_2) acting on the liquid molecules. These forces are inclined at an angle of 135°.

The resultant force represented by AC will depend upon the values of F_1 and F_2. Let the resultant force make an angle α with F_1.

EXCESS PRESSURE INSIDE A LIQUID DROP AND A BUBBLE

On the concave face of a curved surface there is always an excess pressure over the convex face of the surface. The magnitude of excess pressure can be obtained by studying the formation of air and soap bubbles.

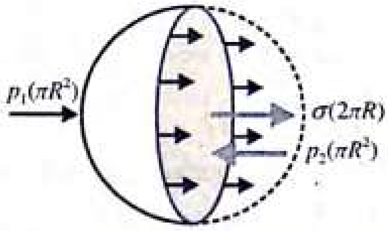
Excess Pressure in an Air Bubble in Liquid

Figure shows one-half cross section of an air bubble formed inside liquid. It is in equilibrium under the action of three forces:

(1) due to external pressure p_1

(2) due to internal pressure p_2

(3) due to surface tension of the liquid σ



The cross section of an air bubble of radius R.

If R is the radius of the air bubble, then the forces due to external and internal pressures are $p_1(\pi R^2)$ and $p_2(\pi R^2)$, respectively. Since the surface tension acts around the circumference of the bubble, therefore, the force of surface tension is $\sigma(2\pi R)$.

Thus, from the condition of equilibrium,

$$p_2(\pi R^2) = p_1(\pi R^2) + T(2\pi R)$$

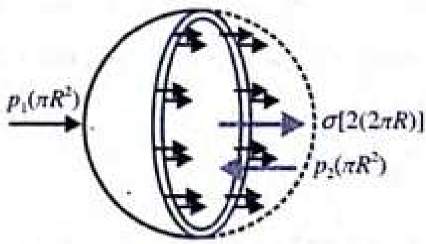
or $$p_2 - p_1 = \frac{2T}{R}$$

NOTE:

- A curved surface always maintains a pressure difference, which depends on the radius of the curved surface and surface tension of the liquid.
- The concave side of the surface always possesses greater pressure. If the angle of contact is zero, then the excess pressure is given by $\Delta p = 2T/R$.

Excess Pressure in a Soap Bubble

A soap bubble forms two liquid surfaces in contact with air, one inside the bubble and the other outside the bubble. Figure shows one-half cross section of the soap bubble. By considering its equilibrium, we get



The cross section of an air bubble of radius R

$$p_2(\pi R^2) = p_1(\pi R^2) + T[2(2\pi R)]$$

$$\Rightarrow \quad p_2 - p_1 = \frac{4T}{R}$$

Excess Pressure on Curved Surfaces in General

The pressure on the concave side (whatever be on its left or right) is always greater than the pressure on the convex side, such that

$$p_{\text{concave}} - p_{\text{convex}} = T\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

where σ = surface tension.

This formula is same as what we derived earlier, i.e.,

$$p_i - p_0 = \frac{2T}{R} = T\left(\frac{1}{R}+\frac{1}{R}\right)$$

(1) For a single spherical surface, e.g. drops, cavities, etc.,

$R_1 = R_2 = R$(say)

Then $p_i - p_0 = T\left(\frac{1}{R}+\frac{1}{R}\right) = \frac{2T}{R}$

(2) For spherical film, e.g., soap bubble,

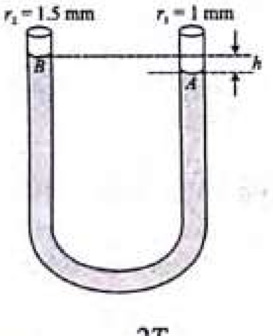
$$p_i - p_0 = (2T)\left(\frac{1}{R}+\frac{1}{R}\right) = \frac{4T}{R}$$

because it has two free surfaces.

(3) For a single cylindrical surface, $R_1 = R$, $R_2 = \infty$

$$\therefore \quad p_i - p_0 = (T)\left(\frac{1}{R}+\frac{1}{\infty}\right) = \frac{T}{R}$$

ILLUSTRATION 12.20 Calculate the difference (h) in water levels in two communicating capillary tubes of radius 1 mm and 1.5 mm. Surface tension of water = 0.07 Nm^{-1}.



Solution. Pressure at $A = p_0 + \frac{2T}{r_1}$

(Since pressure on concave side is greater than that on convex side)

Pressure at $B = p_0 - \frac{2T}{r_2}$

Therefore, pressure difference $= 2T\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$

Let this pressure difference correspond to h units of the liquid. Then,

$$2T\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \rho g h \quad \Rightarrow h = \frac{2T}{\rho g}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\therefore \quad h = \frac{2\times 0.07}{1000\times 9.8}\left(\frac{1}{1\times 10^{-3}}-\frac{1}{1.5\times 10^{-3}}\right) = 4.76\,\text{mm}$$

ILLUSTRATION 12.21 A vessel filled with air under pressure p_0 contains a soap bubble of diameter d. The air pressure have been reduced n-fold, and the bubble diameter increased r-fold isothermally. Find the surface tension of the soap-water solution.

Solution. For the air enclosed in the bubble we have

$$pV = nRT$$

$$\Rightarrow \quad \left(p_0 + \frac{8T}{d}\right)\frac{\pi}{6}d^3 = \left(\frac{p_0}{n}+\frac{8T}{rd}\right)\frac{\pi}{6}r^3 d^3$$

$$\Rightarrow \quad p_0 + \frac{8T}{d} = \frac{p_0 r^3}{n} + \frac{8Tr^2}{d}$$

$$\Rightarrow \quad p_0\left(1-\frac{r^3}{n}\right) = \frac{8T}{d}\left(r^2-1\right) \Rightarrow T = \frac{1}{8}p_0 d \times \frac{\left(1-\frac{r^3}{n}\right)}{(r^2-1)}$$

ILLUSTRATION 12.22 Two separate air bubbles (radii 0.002 cm and 0.004 m) formed of the same liquid (surface tension 0.07 N/m) come together to form a double bubble. Find the radius and the sense of curvature of the internal film surface common to both the bubbles.

Solution. Let R be the radius of curvature of common surface when bubbles A and B of radii R_A and R_B coalesce. The excess pressure in A and B are $4T{\cdot}R_A$ and $4T{\cdot}R_B$, respectively.

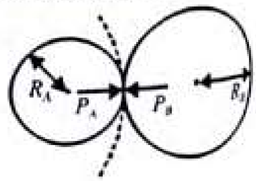
$$\therefore \quad p_A = \frac{4T}{R_A} \quad \text{and} \quad p_B = \frac{4T}{R_B}$$

If R is radius of common interface, then we must have

$$p_A - p_B = \frac{4T}{R}$$

or

$$\frac{4T}{R_A} - \frac{4T}{R_B} = \frac{4T}{R}$$



This gives

$$R = \frac{R_A R_B}{R_B - R_A} = \frac{0.002\times 0.004}{0.004-0.002} = 0.004\text{ m}$$

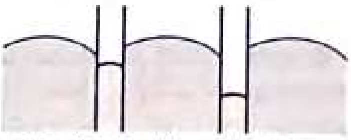
As the excess pressure is always towards concave surface and pressure in smaller bubble is greater than that in larger bubble, the common surface is concave towards the centre of the smaller bubble.

CAPILLARITY

If a narrow glass tube open at both ends is pushed in water as shown in the figure, water rises in the tube to a height above its surface. The narrower the tube, the greater is the height to which water rises. This phenomenon is known as capillarity.

When the same capillary tubes are placed in mercury, the liquid is depressed below the outside level. The depression increases as the diameter of the capillary tube decreases. A close observation of the capillary phenomenon reveals the following facts.

Capillary action with water. Narrower the tube, greater the height.



Capillary action with mercury. Narrower the tube, greater the depression in the tube.

The free surface (meniscus) of the liquid which rises in the capillary tube is concave upward. The meniscus of liquid which falls in the capillary tube is convex upward.

Wetting and Non-Wetting Liquids

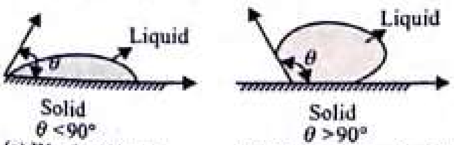
The liquids which rise in a capillary are called wetting liquids; and the liquids which fall in the capillary are called non-wetting liquids. The wetting and non-wetting action of a liquid can be explained in terms of the cohesive and adhesive forces.

Adhesion is the attractive force between the molecules of solids and liquids or between the molecules of two different liquids.

Cohesion is the attractive force among the molecules of the same liquid.

If the adhesive forces are stronger than the cohesive forces then the liquids wet the solid surface, as water wets the surface of glass. If the cohesive forces are stronger than the adhesive forces, then the liquids do not wet the solid surface, as mercury does not wet the surface of glass.

The wetting and non-wetting action of a liquid can also be explained in terms of the angle of contact. If a tangent is drawn on the meniscus of the liquid at the line of contact between the liquid and the surface, as shown in the figure, then the angle of contact is defined as the angle between the tangent and the solid surface measured through the liquid.

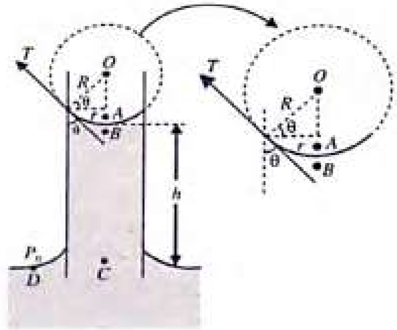


(a) Wetting liquid (b) Non-wetting liquid

If the angle of contact is acute, i.e., $\theta < 90°$, then the liquid wets the surface. If the angle of contact is obtuse, i.e., $\theta > 90°$, then the liquid does not wet the surface. The angle of contact between water and clean glass is zero, but when the glass is not clean the angle of contact may be about 8°.

Alternative method:

Let us take four points A, B, C and D as shown in the figure. A and B are the points just above and just below the meniscus, respectively.



Now, we have $P_A = P_B + \dfrac{2T}{R}$

where R = radius of curvature of liquid meniscus. Substituting $P_A = P_0$, we have

$$P_B = P_0 - \frac{2T}{R} \quad \text{(i)}$$

If C is the point at the bottom of the excess liquid column, we have

$$P_C = P_B + \rho g h$$

where h = height of the excess liquid column.

Substituting $P_C = P_D = P_0$, we have

$$P_B = P_0 - \rho g h \quad \text{(ii)}$$

Using Eqs. (i) and (ii), we have

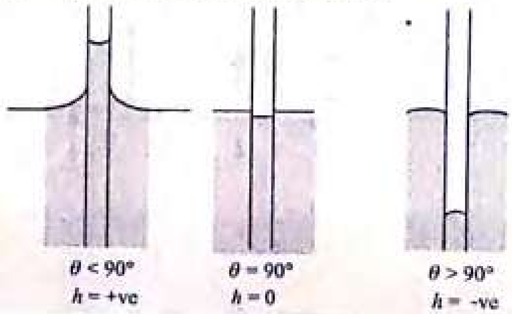
$$P_0 - \rho g h = P_0 - \frac{2T}{R}$$

This gives $h = \dfrac{2T}{\rho g R}$ where $R = \dfrac{r}{\cos\theta}$

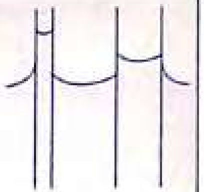
Then $h = \dfrac{2T\cos\theta}{\rho g r}$

Important Results

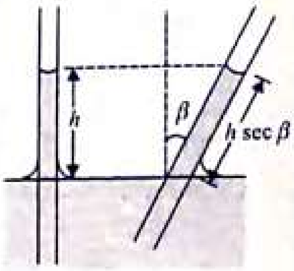
- h depends on surface tension T, angle of contact θ, radius of the tube, density ρ of liquid and acceleration due to gravity (or effective g). Hence the capillary action depends on the nature of liquid and solid (capillary tube) in contact, temperature of the surroundings.
- If $\theta < 90°$, h is positive (wetting) which signifies capillary rise and concave meniscus; if $\theta = 90°$, $h = 0$ (neither wetting nor non-wetting): plane meniscus; if $\theta > 90°$, h = negative (non-wetting) which signifies capillary fall and convex meniscus.



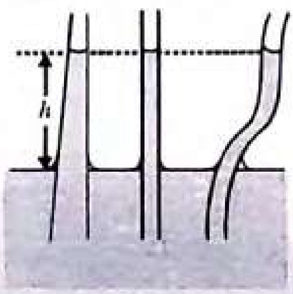
- Since T, ρ and θ are constant at a given temperature, for any given liquid and capillary tube, $hr = c$; this tells us that capillary rise (or fall) will be greater in thinner tubes and vice versa.



- Since T, ρ, θ, g and R are constant for a given set of capillary and liquid, we have h = constant. It means that if we change the angle β of orientation from vertical, the capillary rise h (or capillary fall) remains constant even though the length of the excess liquid in the tube increases from h to $h \sec \beta$.



- If the radius of the liquid meniscus remains constant, the weight (but not length) of the liquid column remains the same in capillary tubes of different shapes and sizes.



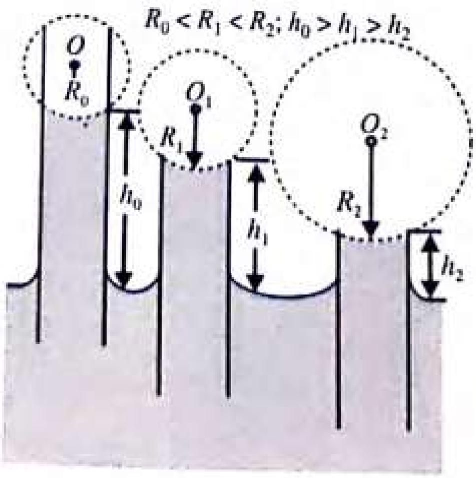
Rise of Liquid in a Capillary of Insufficient Length

The expression of capillary rise is given as

$$h = \frac{2T\cos\theta}{\rho g r}$$

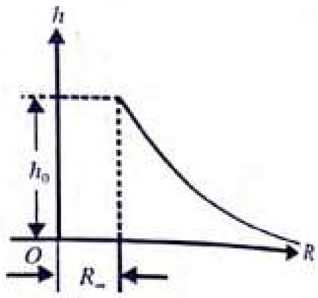
Substituting $r/\cos\theta = R$ (radius of curvature of the liquid meniscus), we have

$$hR = \frac{2T}{\rho g} = \text{Constant}$$



Since T, ρ and g are constant, we have $hR = C$ (constant) $= h_0R_0$ (say), where h = free height of the liquid and R = radius of curvature of free meniscus of liquid.

If we slowly push the capillary tube vertically down (into) the liquid, the height of the liquid column remains the same till the length of the tube above the liquid surface is equal to $2T \cos\theta/(\rho g r)(= h)$. Therefore, the liquid meniscus gets flatt-ened gradually by increasing its radius of curvature to obey the law hR = constant.



$h \propto \frac{1}{R}$ where R = radius of curvature of the liquid meniscus

It means that $hr = h_1r_1 = h_2r_2 = \cdots$

When the capillary tube just sinks, i.e., $h = 0$, we can see that the meniscus becomes flat, radius of curvature $r \to \infty$.

ILLUSTRATION 12.23 A vertical capillary with inside diameter 0.50 mm is submerged into water so that the length of its part emerging outside the water surface is equal to 25 mm. Find the radius of curvature of the meniscus. Surface tension of water is 73×10^{-3} N/m.

Solution. In the capillary tube, the water should rise to a height

$$h = \frac{2T}{r\rho g}$$

Here $T = 73 \times 10^{-3}$ N·m

$$r = \frac{0.50 \text{ mm}}{2} = 0.25 \times 10^{-3} \text{ m}$$

$$\rho = 10^3 \text{ kg/m}^3$$

$$\therefore \quad h = \frac{2 \times 73 \times 10^{-3}}{0.25 \times 10^{-3} \times 10^3 \times 9.8} = 59 \times 10^{-3} \text{ m} = 59 \text{ mm}$$

Now $h > h'$, i.e., length is outside water surface.
Therefore, radius of meniscus > radius of capillary r.
If R is the radius of meniscus, then we have

$$\frac{2T}{R} = h'\rho g$$

or $$R = \frac{2T}{h'\rho g}$$

Here $h' = 25 \text{ mm} = 25 \times 10^{-3}$ m

$$\therefore \quad R = \frac{2 \times 73 \times 10^{-3}}{25 \times 10^{-3} \times 10^3 \times 9.8} = 0.6 \times 10^{-3} \text{ m} = 0.6 \text{ mm}$$

CONCEPT APPLICATION EXERCISE 12.6

1. A mercury drop of radius R is sprayed into n droplets of equal size. Calculate the energy expanded if surface tension of mercury is T.

2. If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by

$$\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$$

where T is the surface tension of water and J is the mechanical equivalent of heat.

3. A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate the excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid.

4. A drop of water of volume 0.05 cm^3 is pressed between two glass plates, as a consequence of which it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyn/cm, find the normal force required to separate out the two glass plates in newton.

5. A glass tube of circular cross section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Given: outer radius of the tube is 0.14 cm, mass of weighted tube is 0.2 g, surface tension of water 73 dyn/cm and $g = 980$ cm s^{-2}.

6. If a 5 cm long capillary tube with 0.1 mm internal diameter, open at both ends, is slightly dipped in water having surface tension 75 dyn/cm, state whether: (i) water will rise halfway in the capillary, (ii) water will rise up to the upper end of capillary, (iii) water will overflow out of the upper end of capillary. Explain your answer.

SOLVED EXAMPLES

1. A hemispherical bowl just floats without sinking in a liquid of density 1.2 × 10^{3}kg/m^3. If the outer diameter and the density of the bowl are 1 m and 2 × 10^4 kg/m^3, respectively, then the inner diameter of the bowl will be
(a) 0.94 m (b) 0.97 m
(c) 0.98 m (d) 0.99 m

Sol. (c) Weight of the bowl = mg

$$= V\rho g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3-\left(\frac{d}{2}\right)^3\right]\rho g$$

where D = Outer diameter
d = Inner diameter
ρ = Density of bowl

Weight of the liquid displaced by the bowl

$$V\sigma g = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3\sigma g$$

where σ is the density of the liquid.

For floatation: $\frac{4}{3}\pi\left(\frac{D}{2}\right)^3\sigma g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3-\left(\frac{d}{2}\right)^3\right]\rho g$

$$\Rightarrow \left(\frac{1}{2}\right)^3\times 1.2\times 10^3 = \left[\left(\frac{1}{2}\right)^3-\left(\frac{d}{2}\right)^3\right]2\times 10^4$$

By solving, we get $d = 0.98$ m.

2. A concrete sphere of radius R has a cavity of radius r which is packed with sawdust. The specific gravities of concrete and sawdust are respectively 2.4 and 0.3 for this sphere to float with its entire volume submerged under water. Ratio of mass of concrete to mass of sawdust will be
(a) 8 (b) 4
(c) 3 (d) Zero

Sol. (b) Let specific gravities of concrete and saw dust are ρ_1 and ρ_2, respectively.
According to principle of floatation weight of whole sphere = upthrust on the sphere

$$\frac{4}{3}\pi(R^3-r^3)\rho_1 g+\frac{4}{3}\pi r^3\rho_2 g = \frac{4}{3}\pi R^3\times 1\times g$$

$$\Rightarrow R^3\rho_1 - r^3\rho_1 + r^3\rho_2 = R^3$$

$$\Rightarrow R^3(\rho_1-1) = r^3(\rho_1-\rho_1)$$

$$\Rightarrow \frac{R^3}{r^3} = \frac{\rho_1-\rho_2}{\rho_1-1}$$

$$\Rightarrow \frac{R^3-r^3}{r^3} = \frac{\rho_1-\rho_2-\rho_1+1}{\rho_1-1}$$

$$\Rightarrow \frac{(R^3-r^3)\rho_1}{r^3\rho_2} = \left(\frac{1-\rho_2}{\rho_1-1}\right)\frac{\rho_1}{\rho_2}$$

$$\Rightarrow \frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1-0.3}{2.4-1}\right)\times\frac{2.4}{0.3} = 4$$

3. A hollow sphere of volume V is floating on water surface with *half* immersed in it. What should be the minimum volume of water poured inside the sphere so that the sphere now sinks into the water
(a) $V/2$ (b) $V/3$ (c) $V/4$ (d) V

Sol. (a) When body (sphere) is half immersed, then
Upthrust = Weight of sphere

$$\Rightarrow \frac{V}{2}\times\rho_{\text{liq}}\times g = V\times\rho\times g \;\therefore\; \rho = \frac{\rho_{\text{liq}}}{2}$$

When body (sphere) is fully immersed then,
Upthrust = Wt. of sphere + Wt. of water poured in sphere

$$\Rightarrow V\times\rho_{\text{liq}}\times g = V\times\rho\times g + V'\times\rho_{\text{liq}}\times g$$

$$\Rightarrow V\times\rho_{\text{liq}} = \frac{V\times\rho_{\text{liq}}}{2}+V\times\rho_{\text{liq}} \;\Rightarrow V' = \frac{V}{2}$$

4. A ball whose density is 0.4 × 10^3 kg/m^3 falls into water from a height of 9 cm. To what depth does the ball sink?
(a) 9 cm (b) 6 cm
(c) 4.5 cm (d) 2.25 cm

Sol. (b) The velocity of ball before entering the water surface:

$v=\sqrt{2gh}=\sqrt{2g\times 9}$

When ball enters into water, due to the upthrust of water, the velocity of ball decreases (or retarded).

$$\text{Retardation, } a=\frac{\text{Apparent weight}}{\text{Mass of ball}}$$

$$=\frac{V(\rho-\sigma)g}{V\rho}=\left(\frac{\rho-\sigma}{\rho}\right)g=\left(\frac{0.4-1}{0.4}\right)\times g=-\frac{3}{2}g$$

If h be the depth upto which the ball sinks. Then

$$0-v^2=2\times\left(-\frac{3}{2}g\right)\times h\Rightarrow 2g\times 9=3gh$$

$\therefore h=6\text{ cm}$

5. A cylindrical tank has a hole of 1 cm^2 in its bottom. If the water is allowed to flow into the tank from a tube above it at the rate of 70 cm^3/s, then the maximum height up to which water can rise in the tank is

(a) 2.5 cm (b) 5 cm
(c) 10 cm (d) 0.25 cm

Sol. (a) The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second.

Volume of water flowing out per second $=Av=A\sqrt{2gh}$ ...(i)
Volume of water flowing in per second = 70 cm^3/sec ...(ii)
From (i) and (ii), we get

$$A\sqrt{2gh}=70\Rightarrow 1\times\sqrt{2gh}=70$$

$$\Rightarrow 1\times\sqrt{2\times 980\times h}=70$$

$$\therefore\ h=\frac{4900}{1960}=2.5\text{cm}.$$

6. A square plate of 0.1 m side moves parallel to a second plate with a velocity of 0.1 m/s, both plates being immersed in water. If the viscous force is 0.002 N and the coefficient of viscosity is 0.01 poise, distance between the plates in m is

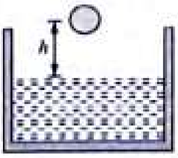
(a) 0.1 (b) 0.05
(c) 0.005 (d) 0.0005

Sol. (d) $A=(0.1)^2=0.01\text{m}^2$, $\eta=0.01$ poise $=0.001$ decapoise (M.K.S. unit), $dv=0.1$ m/s and $F=0.002$ N

$$F=\eta A\frac{dv}{dx}$$

$$\therefore\ dx=\frac{\eta A dv}{F}=\frac{0.001\times 0.01\times 0.1}{0.002}=0.0005\text{m}$$

7. A ball of radius r and density ρ falls freely under gravity through a distance h before entering water. Velocity of ball does not change even on entering water. If viscosity of water is η, the value of h is given by



(a) $\frac{2}{9}r^2\left(\frac{1-\rho}{\eta}\right)g$ (b) $\frac{2}{81}r^2\left(\frac{\rho-1}{\eta}\right)g$

(c) $\frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2g$ (d) $\frac{2}{9}r^4\left(\frac{\rho-1}{\eta}\right)^2g$

Sol. (c) Velocity of ball when it strikes the water surface

$$v=\sqrt{2gh} \quad\text{...(i)}$$

Terminal velocity of ball inside the water

$$v=\frac{2}{9}r^2g\frac{(\rho-1)}{\eta} \quad\text{...(ii)}$$

Equating (i) and (ii), we get $\sqrt{2gh}=\frac{2}{9}\frac{r^2g}{\eta}(\rho-1)$

$$\Rightarrow h=\frac{2}{81}r^4\left(\frac{\rho-1}{\eta}\right)^2g$$

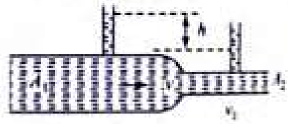
8. A liquid flows through a horizontal tube. The velocities of the liquid in the two sections, which have areas of cross-section A_1 and A_2, are v_1 and v_2 respectively. The difference in the levels of the liquid in the two vertical tubes is h. Following observations are taken

(i) The volume of the liquid flowing through the tube in unit time is A_1v_1

(ii) $v_2-v_1=\sqrt{2gh}$

(iii) $v_2^2-v_1^2=2gh$

(iv) The energy per unit mass of the liquid is the same in both sections of the tube



The correct observations is/are

(a) (i) and (ii) (b) (i), (ii) and (iii)
(c) (i) and (iv) (d) (i), (iii) and (iv)

Sol. (d) According to the equation of continuity, the volume of liquid flowing through the tube in unit time remains constant. i.e., $A_1v_1=A_2v_2$. Hence, option (a) is correct.
According to Bernoulli's theorem,

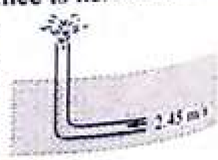
$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_1-P_2=\frac{1}{2}\rho\left(v_2^2-v_1^2\right)\Rightarrow h\rho g=\frac{1}{2}\rho\left(v_2^2-v_1^2\right)$$

$$\therefore\ v_2^2-v_1^2=2gh$$

Hence, option (c) is correct.
Also, according to Bernoulli's theorem, option (d) is correct.

9. An L-shaped tube with a small orifice is held in a water stream as shown in the figure. The upper end of the tube is 10.6 cm above the surface of water. What will be the height of the jet of water coming from the orifice? Velocity of water stream is 2.45 m/s.



(a) Zero (b) 20.0 cm
(c) 10.6 cm (d) 40.0 cm

Sol. (b) According to Bernoulli's theorem, $h = \frac{v^2}{2g}$

$$\Rightarrow h = \frac{(2.45)^2}{2\times10} = 0.314 = 31.4 \text{ cm}$$

$\therefore$ Height of jet coming from orifice $= 31.4 - 10.6$
$= 20.8$ cm

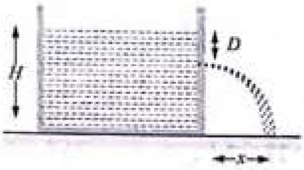
10. A tank is filled with water up to a height H. Water is allowed to come out of a hole P in one of the walls at a depth D below the surface of water. Express the horizontal distance x in terms of H and D

(a) $x = \sqrt{D(H-D)}$

(b) $x = \sqrt{\frac{D(H-D)}{2}}$

(c) $x = 2\sqrt{D(H-D)}$

(d) $x = 4\sqrt{D(H-D)}$



Sol. (c) Time taken by water to reach the bottom

$$t = \sqrt{\frac{2(H-D)}{g}}$$

and velocity of water coming out of hole, $v = \sqrt{2gD}$

$\therefore$ Horizontal distance covered $x = v \times t$

$$= \sqrt{2gD} \times \sqrt{\frac{2(H-D)}{g}} = 2\sqrt{D(H-D)}$$

11. A streamlined body falls through air from a height h on the surface of a liquid. If d and $D(D > d)$ represents the densities of the material of the body and liquid respectively, then the time after which the body will be instantaneously at rest, is

(a) $\sqrt{\frac{2h}{g}}$ (b) $\sqrt{\frac{2h}{g}\cdot\frac{D}{d}}$

(c) $\sqrt{\frac{2h}{g}\cdot\frac{d}{D}}$ (d) $\sqrt{\frac{2h}{g}}\left(\frac{d}{D-d}\right)$

Sol. (d) Upthrust – weight of body = apparent weight

$VDg - Vdg = Vda,$

where a = retardation of body

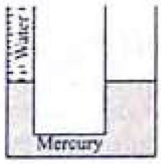
$$\therefore a = \left(\frac{D-d}{d}\right)g$$

The velocity gained after fall from h height in air, $v = \sqrt{2gh}$

Hence, time to come in rest,

$$t = \frac{v}{a} = \frac{\sqrt{2gh}\times d}{(D-d)g} = \sqrt{\frac{2h}{g}} \times \frac{d}{(D-d)}$$

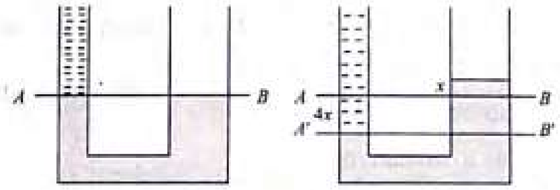
12. A U-tube in which the cross-sectional area of the limb on the left is one quarter, the limb on the right contains mercury (density 13.6 g/cm^3). The level of mercury in the narrow limb is at a distance of 36 cm from the upper end of the tube. What will be the rise in the level of mercury in the right limb if the left limb is filled to the top with water?



(a) 1.2 cm (b) 2.35 cm
(c) 0.56 cm (d) 0.8 cm

Sol. (c) If the rise of level in the right limb be x cm. the fall of level of mercury in left limb be $4x$ cm because the area of cross section of right limb is 4 times as that of left limb.

$\therefore$ Level of water in left limb is $(36 + 4x)$ cm.

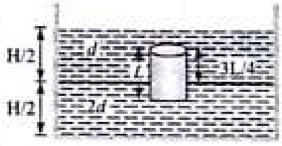


Now equating pressure at interface of Hg and water (at $A'B'$)

$$(36 + 4x) \times 1 \times g = 5x \times 13.6 \times g$$

By solving, we get $x = 0.56$ cm.

13. A homogeneous solid cylinder of length $L(L > H/2)$. Cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure P_0. Then density D of solid is given by



(a) $\frac{5}{4}d$ (b) $\frac{4}{5}d$

(c) d (d) $\frac{d}{5}$

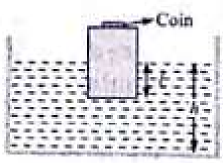
Sol. (a) Weight of cylinder = Upthrust due to both liquids

$$V \times D \times g = \left(\frac{A}{5}\times\frac{3}{4}L\right)\times d \times g + \left(\frac{A}{5}\times\frac{L}{4}\right)\times 2d \times g$$

$$\Rightarrow \left(\frac{A}{5}\times L\right)\times D \times g = \frac{A\times L\times d\times g}{4} \Rightarrow \frac{D}{5} = \frac{d}{4}$$

$$\therefore D = \frac{5}{4}d$$

14. A wooden block, with a coin placed on its top, floats in water as shown in the figure. the distance l and h are shown there. After some time, the coin falls into the water. Then



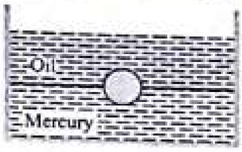
(a) l decreases and h increases
(b) l increases and h decreases
(c) Both l and h increase
(d) Both l and h decrease

Sol. (d) As the block moves up with the fall of coin, l decreases. Similarly, h will also decrease because when the coin is in water, it displaces water equal to its own volume only.

15. A vessel contains oil (density = 0.8 gm/cm^3) over mercury (density = 13.6 gm/cm^3). A homogeneous sphere floats with half of its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm/cm^3 is

(a) 3.3 (b) 6.4 (c) 7.2 (d) 12.8

Sol. (c) As the sphere floats in the liquid. Therefore, its weight will be equal to the upthrust force on it.



Weight of sphere $= \frac{4}{3}\pi R^3 \rho g$ (i)

Upthrust due to oil and mercury

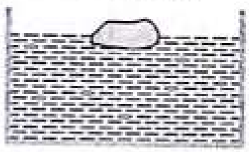
$$= \frac{2}{3}\pi R^3 \times \sigma_{oil} g + \frac{2}{3}\pi R^3 \sigma_{Hg} g \quad \text{(ii)}$$

Equating (i) and (ii)

$$\frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 \times 0.8g + \frac{2}{3}\pi R^3 \times 13.6\, g$$

$$\Rightarrow 2\rho = 0.8 + 13.6 = 14.4 \Rightarrow \rho = 7.2$$

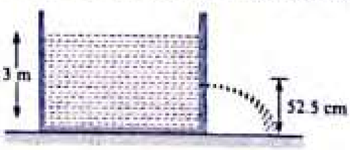
16. A body floats in a liquid contained in a beaker. The whole system as shown falls freely under gravity. The upthrust on the body due to the liquid is



(a) Zero
(b) Equal to the weight of the liquid displaced
(c) Equal to the weight of the body in air
(d) Equal to the weight of the immersed position of the body

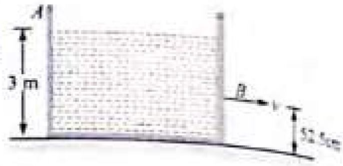
Sol. (a) Upthrust $= V\rho_{\text{liquid}}(g - a)$
where a = downward acceleration and
V = volume of liquid displaced
But for free fall, $a = g \Rightarrow$ Upthrust = 0

17. Water is filled in a cylindrical container to a height of 3 m. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. The square of the speed of the liquid coming out from the orifice is (g = 10 m/s^2)



(a) 50 m^2/s^2 (b) 50.5 m^2/s^2
(c) 51 m^2/s^2 (d) 52 m^2/s^2

Sol. (a) Let A = cross-section of tank
a = cross-section hole
V = velocity with which level decreases
v = velocity of efflux



From equation of continuity,

$$av = AV \Rightarrow V = \frac{av}{A}$$

By using Bernoulli's theorem for energy per unit volume
Energy per unit volume at point A = Energy per unit volume at point B

$$P + \rho gh + \frac{1}{2}\rho V^2 = P + 0 + \frac{1}{2}\rho v^2$$

$$\Rightarrow v^2 = \frac{2gh}{1-\left(\frac{a}{A}\right)^2} = \frac{2\times 10\times(3-0.525)}{1-(0.1)^2} = 50(\text{m/s})^2$$

18. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both the holes are the same. Then R is equal to

(a) $2\pi L$ (b) $\frac{L}{\sqrt{2\pi}}$
(c) L (d) $\frac{L}{2\pi}$

Sol. (b) Velocity of efflux when the hole is at depth h,

$$v = \sqrt{2gh}$$

Rate of flow of water from square hole

$$Q_1 = a_1 v_1 = L^2\sqrt{2gy}$$

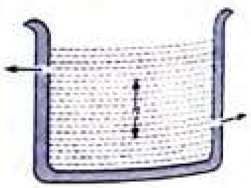
Rate of flow of water from circular hole

$$Q_2 = a_2 v_2 = \pi R^2 \sqrt{2g(4y)}$$

According to problem, $Q_1 = Q_2$

$$\Rightarrow L^2\sqrt{2gy} = \pi R^2\sqrt{2g(4y)} \Rightarrow R = \frac{L}{\sqrt{2\pi}}$$

19. There are two identical small holes of area of cross-section a on the opposite sides of a tank containing a liquid of density ρ. The difference in height between the holes is h. Tank is resting on a smooth horizontal surface. Horizontal force which will has to be applied on the tank to keep it in equilibrium is

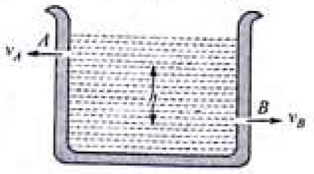


(a) $gh\rho a$ (b) $\frac{2gh}{\rho a}$
(c) $2\rho agh$ (d) $\frac{\rho gh}{a}$

Sol. (c) Net force (reaction) $= F = F_B - F_A = \dfrac{dp_B}{dt} - \dfrac{dp_A}{dt}$

$= av_B\rho \times v_B - av_A\rho \times v_A$

$\therefore\ F = a\rho\left(v_B^2 - v_A^2\right)$ ...(i)



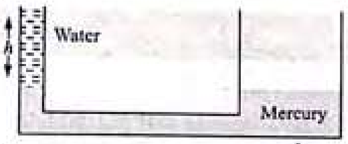
According to Bernoulli's theorem

$$p_A + \frac{1}{2}\rho v_A^2 + \rho gh = p_B + \frac{1}{2}\rho v_B^2 + 0$$

$$\Rightarrow \frac{1}{2}\rho\left(v_B^2 - v_A^2\right) = \rho gh \Rightarrow v_B^2 - v_A^2 = 2gh$$

From equation (i), $F = 2a\rho gh$

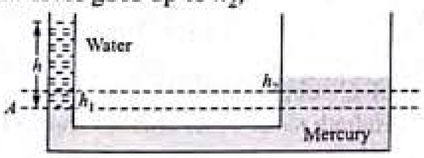
20. Two communicating vessels contain mercury. The diameter of one vessel is n times larger than the diameter of the other. A column of water of height h is poured into the left vessel. The mercury level will rise in the right-hand vessel (s = relative density of mercury and ρ = density of water) by



(a) $\dfrac{n^2h}{(n+1)^2 s}$ (b) $\dfrac{h}{(n^2+1)s}$

(c) $\dfrac{h}{(n+1)^2 s}$ (d) $\dfrac{h}{n^2 s}$

Sol. (b) If the level in narrow tube goes down by h_1, then in wider tube the level goes up to h_2,



Now, $\pi r^2 h_1 = \pi(nr)^2 h_2 \Rightarrow h_1 = n^2 h_2$

Now, pressure at point A = pressure at point B

$$h\rho g = (h_1 + h_2)\rho' g$$

$$\Rightarrow h = (n^2h_2 + h_2)sg \left(\text{As } s = \frac{\rho'}{\rho}\right) \Rightarrow h_2 = \frac{h}{(n^2+1)s}$$

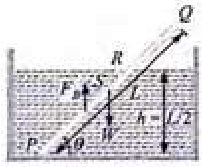
21. A uniform rod of density ρ is placed in a wide tank containing a liquid of density $\rho_0 (\rho_0 > \rho)$. The depth of liquid in the tank is half the length of the rod. The rod is in equilibrium, with its lower end resting on the bottom of the tank. In this position the rod makes an angle θ with the horizontal

(a) $\sin\theta = \dfrac{1}{2}\sqrt{\rho_0/\rho}$ (b) $\sin\theta = \dfrac{1}{2}\cdot\dfrac{\rho_0}{\rho}$

(c) $\sin\theta = \sqrt{\rho/\rho_0}$ (d) $\sin\theta = \rho_0/\rho$

Sol. (a) Let $L = PQ$ = length of rod

$\therefore\ SP = SQ = \dfrac{L}{2}$



Weight of rod, $W = Al\rho g$, acting At point S

And force of buoyancy,

$F_B = Al\rho_0 g$, $[l = PR]$

which acts at mid-point of PR.

For rotational equilibrium,

$$Al\rho_0 g \times \frac{l}{2}\cos\theta = AL\rho g \times \frac{L}{2}\cos\theta$$

$$\Rightarrow \frac{l^2}{L^2} = \frac{\rho}{\rho_0} \Rightarrow \frac{l}{L} = \sqrt{\frac{\rho}{\rho_0}}$$

From figure, $\sin\theta = \dfrac{h}{l} = \dfrac{L}{2l} = \dfrac{1}{2}\sqrt{\dfrac{\rho_0}{\rho}}$

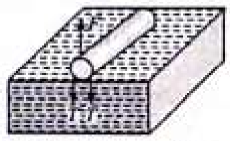
22. The maximum force, in addition to the weight required to pull a wire of 5.0 cm long from the surface of water at temperature 20°C is 728 dynes. The surface tension of water is

(a) 7.28 N/cm (b) 7.28 dyne/cm

(c) 72.8 dyne/cm (d) 7.28×10^2 dyne/cm

Sol. (c) $T = \dfrac{F}{2l} = \dfrac{728}{2 \times 5}$

$\therefore\ T = 72.8$ dyne/cm



23. A thin metal disc of radius r floats on water surface and bends the surface downwards along the perimeter making an angle θ with vertical edge of the disc. If the disc displaces a weight of water W and surface tension of water is T, then the weight of metal disc is

(a) $2\pi rT + W$ (b) $2\pi rT\cos\theta - W$

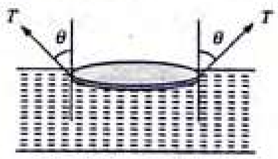
(c) $2\pi rT\cos\theta + W$ (d) $W - 2\pi rT\cos\theta$

Sol. (c) Weight of metal disc = total upward force

= upthrust force + force due to surface tension

= weight of displaced water + $T\cos\theta\,(2\pi r)$

$= W + 2\pi rT\cos\theta$

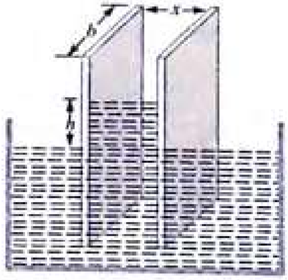


24. Two parallel glass plates are dipped partly in the liquid of density 'd' keeping them vertical. If the distance between the plates is 'x', surface tension for liquids is T and angle of contact is θ, then rise of liquid between the plates due to capillary will be

(a) $\dfrac{T\cos\theta}{xd}$ (b) $\dfrac{2T\cos\theta}{xdg}$

(c) $\dfrac{2T}{xdg\cos\theta}$ (d) $\dfrac{T\cos\theta}{xdg}$

Sol. (b) Let the width of each plate is b and due to surface tension, liquid will rise upto height h. Then upward force due to surface tension $= 2Tb\cos\theta$ ...(i)



Weight of the liquid that rises in between the plates

$= Vdg = (bxh)dg$ ...(ii)

Equating (i) and (ii), we get $2T\cos\theta = bxhdg$

$$h = \frac{2T\cos\theta}{xdg}$$

25. A soap bubble of radius R is blown. After heating the solution a second bubble of radius $2R$ is blown. The work required to blow the second bubble in comparison to that required for the first bubble is
(a) Double
(b) Slightly less than double
(c) Slightly less than four times
(d) Slightly more than four times

Sol. (c) Work done to form a bubble of radius R

$W_1 = 8\pi R^2 T_1$

Work done to form a bubble of radius $2R$

$W_2 = 8\pi(2R)^2 T_2 = 32\,\pi R^2 T_2 \;\therefore\; \frac{W_1}{W_2} = \frac{T_1}{4T_2}$

If surface tension of soap solution is same, then

$W_2 = 4W_1$

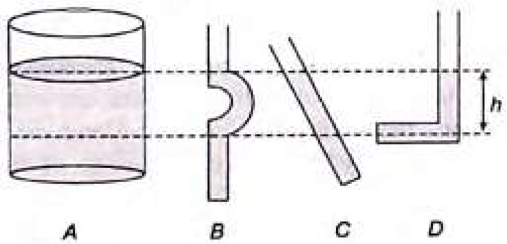
But in the problem the temperaturew of solution is increased. So its surface tension decreases.

$\therefore\; W_2 < 4W_1$

EXERCISES

Fluid Statics, Buoyancy and Floatation

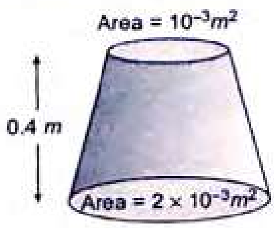
1. Figure shows four containers of olive oil. The pressure at depth h is



(a) least in B and C both (b) greatest in A
(c) greatest in D (d) equal in all the containers

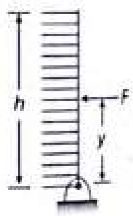
2. An isosceles triangular plates of base 3 m and altitude 3 m is immersed in oil vertically with its base coinciding with the free surface of the oil of relative density 0.8. Determine the total thrust.
(a) 24 kN (b) 48 kN
(c) 36 kN (d) None of these

3. A uniformly tapering vessel is filled with a liquid of density 900 kg/m^3. The force that acts on the base of the vessel due to the liquid is ($g = 10$ ms^{-2})



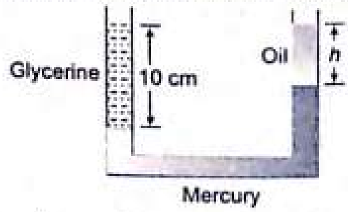
(a) 3.6 N (b) 7.2 N
(c) 9.0 N (d) 14.4 N

4. A smooth gate is kept in equilibrium by applying a horizontal force. What is the value of y so that no horizontal reaction force acts at the pivot?



(a) $\frac{h}{3}$ (b) $\frac{h}{6}$
(c) $\frac{2h}{3}$ (d) zero

5. A vertical U-tube of uniform cross-section contains mercury in both sides of its arms as shown below.

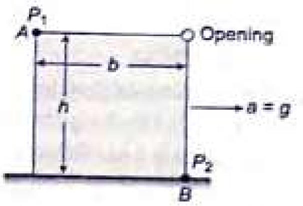


Gylcerine column of 10 cm is introduced into one of its arms. Oil of density 0.8 g/cc is poured into the other arm until the upper surface of the oil and glycerine are at the same horizontal level.

[$\rho_{Hg} = 13.6$ g/cc, $\rho_{glycerine} = 1.3$ g/cc]

The length of oil column is
(a) 10.4 cm (b) 8.2 cm
(c) 7.2 cm (d) 9.6 cm

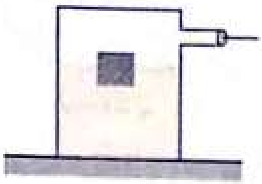
6. A closed rectangular vessel completely filled with a liquid of density ρ moves with an acceleration $a = g$. The value of the pressure difference at point A and B i.e., $(P_1 - P_2)$ is



(a) $\rho g(b-h)$ (b) $\rho gh/2$

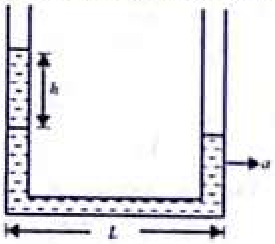
(c) ρgh (d) $\dfrac{\rho g(b+h)}{2}$

7. A block of wood is floating in water in a closed vessel as shown in the figure. The vessel is connected to an air pump. When more air is pushed into the vessel, the block of wood floats with (neglect compressibility of water)



(a) larger part in the water
(b) smaller part in the water
(c) same part in the water
(d) at some instant it will sink

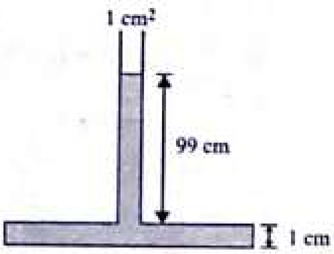
8. When at rest, a liquid stands at the same level in the tubes as shown in the figure. But as indicated, a height difference h occurs when the system is given an acceleration a towards the right. Then h is equal to



(a) $\dfrac{aL}{2g}$ (b) $\dfrac{gL}{2a}$

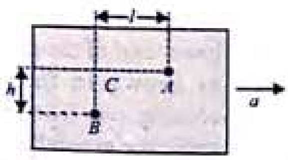
(c) $\dfrac{gL}{a}$ (d) $\dfrac{aL}{g}$

9. A tube 1 cm^2 in cross section is attached to the top of a vessel 1 cm high and of cross section 100 cm^2. Water is poured into the system filling it to a depth of 100 cm above the bottom of the vessel as shown in the figure. Take $g = 10\ m\,s^{-2}$. Find the correct statement.



(a) The force exerted by water against the bottom of the vessel is 100 N.
(b) The weight of water in the system is 1.99 N.
(c) Both (a) and (b) are correct.
(d) Neither (a) nor (b) is correct.

10. A sealed tank containing a liquid of density ρ moves with horizontal acceleration a as shown in the figure. The difference in pressure between two points A and B will be



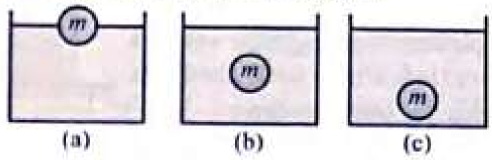
(a) $h\rho g$ (b) $l\rho g$

(c) $h\rho g - l\rho a$ (d) $h\rho g + l\rho a$

11. Water rises to a height of 2 cm in a capillary tube. If the tube is tilted 60° from the vertical, water will rise in the tube to a length of.

(a) 4.0 cm (b) 2.0 cm
(c) 1.0 cm (d) water will not rise at all

12. Three identical vessels A, B and C contain same quantity of liquid. In each vessel balls of different densities but same masses are placed. In vessel A, the ball is partly submerged; in vessel B, the ball is completely submerged but floating and in vessel C, the ball has sunk to the base. If F_A, F_B and F_C are the total forces acting on the base of vessels A, B and C, respectively, then



(a) $F_A = F_B = F_C$ (b) $F_A < F_B < F_C$
(c) $F_A = F_B < F_C$ (d) $F_A < F_B < F_C$

13. An ornament weighing 36 g in air weighs only 34 g in water. Assuming that some copper is mixed with gold to prepare the ornament, find the amount of copper in it. Specific gravity of gold is 19.3 and that of copper is 8.9.
(a) 2.2 g (b) 4.4 g (c) 1.1 g (d) 3.6 g

14. We have two different liquids A and B whose relative densities are 0.75 and 1.0, respectively. If we dip solid objects P and Q having relative densities 0.6 and 0.9 in these liquids, then
(a) P floats in A and Q sinks in B
(b) P sinks in A and Q floats in B
(c) P floats in B and Q sinks in A
(d) P sinks in B and Q floats in A

15. A solid uniform ball of volume V floats on the interface of two immiscible liquids (see the figure). The specific gravity of the upper liquid is ρ_1 and that of lower one is ρ_2 and the specific gravity of ball is ρ ($\rho_1 < \rho < \rho_2$). The fraction of the volume of the ball in the upper liquid is

(a) $\dfrac{\rho_2}{\rho_1}$ (b) $\dfrac{\rho_2-\rho}{\rho_2-\rho_1}$

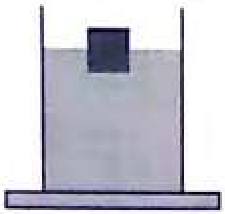
(c) $\dfrac{\rho-\rho_1}{\rho_2-\rho_1}$ (d) $\dfrac{\rho_1}{\rho_2}$

16. A beaker containing water is placed on the platform of a spring balance. The balance reads 1.5 kg. A stone of mass 0.5 kg and density $10^4\ kg/m^3$ is immersed in water without touching the walls of the beaker. What will be the balance reading now?
(a) 2 kg (b) 2.5 kg (c) 1 kg (d) 3 kg

17. A block of silver of mass 4 kg hanging from a string is immersed in a liquid of relative density 0.72. If relative density of silver is 10, then tension in the string will be
(a) 37.12 N (b) 42 N
(c) 73 N (d) 21 N

18. A vessel contains oil (density = 0.8 g/cm^3) over mercury (density = 13.6 g/cm^3). A uniform sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of sphere in g/cm^3 is
(a) 3.3 (b) 6.4 (c) 7.2 (d) 12.8

19. A cylindrical block is floating (partially submerged) in a vessel containing water. Initially, the platform on which the vessel is mounted is at rest. Now the platform along with the vessel is allowed to fall freely under gravity. As a result, the buoyancy force

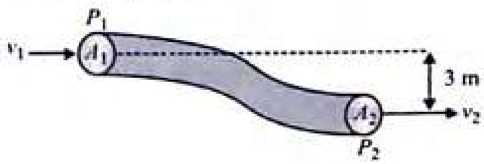


(a) becomes zero (b) decreases
(c) increases (d) information is insufficient

20. An iceberg is floating partially immersed in sea water. The density of sea water is 1.03 g cm^{-3} and that of ice is 0.92 g cm^{-3}. The approximate percentage of total volume of iceberg above the level of sea water is
(a) 8 (b) 11
(c) 34 (d) 89

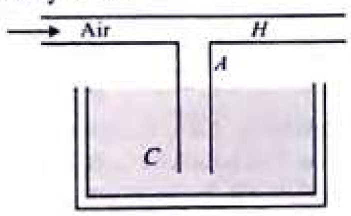
Fluid Dynamics

21. An ideal fluid flows in the pipe as shown in the figure. The pressure in the fluid at the bottom P_2 is the same as it is at the top P_1. If the velocity of the top v_1 = 2 m/s. Then the ratio of areas A_1:A_2 is



(a) 2:1 (b) 4:1
(c) 8:1 (d) 4:3

22. Figure shows a capillary tube C dipped in a liquid that wets it. The liquid rises to a point A. If we blow air through the horizontal tube H, what will happen to the liquid column in the capillary tube?

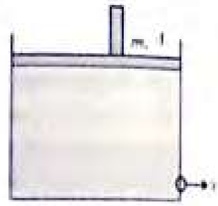


(a) Level will rise above A
(b) Level will fall below A
(c) Level will remain at A
(d) It is difficult to predict

23. A tank is filled with water of density 10^3 kg/m^3 and oil of density 9×10^3 kg/m^3. The height of water layer is 1 m and that of the oil layer is 4 m. The velocity of efflux from an opening in the bottom of the tank is
(a) $\sqrt{85}$ m/s (b) $\sqrt{88}$ m/s
(c) $\sqrt{92}$ m/s (d) $\sqrt{98}$ m/s

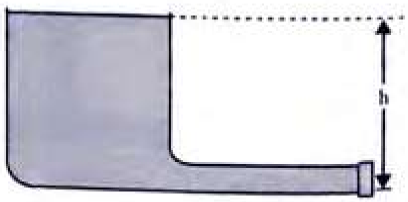
24. A hole is made at the bottom of a tank filled with water (density = 10^3 kg/m^3). If the total pressure at the bottom of the tank is 3 atm (1 atm = 10^5 N/m^2), then the velocity of efflux is
(a) $\sqrt{400}$ m/s (b) $\sqrt{200}$ m/s
(c) $\sqrt{600}$ m/s (d) $\sqrt{500}$ m/s

25. A cylindrical vessel contains a liquid of density ρ up to height h. The liquid is closed by a piston of mass m and area of cross section A. There is a small hole at the bottom of the vessel. The speed v with which the liquid comes out of the hole is



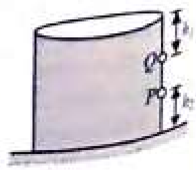
(a) $\sqrt{2gh}$ (b) $\sqrt{2\left(gh+\frac{mg}{\rho A}\right)}$
(c) $\sqrt{2\left(gh+\frac{mg}{A}\right)}$ (d) $\sqrt{2gh+\frac{mg}{A}}$

26. The opening near the bottom of the vessel shown in the figure has an area A. A disc is held against the opening to keep the liquid from running out. Let F_1 be the net force on the disc applied by liquid and air in this case. Now the disc is moved away from the opening a short distance. The liquid comes out and strikes the disc inelastically. Let F_2 be the force exerted by the liquid in this condition. Then F_1/F_2 is



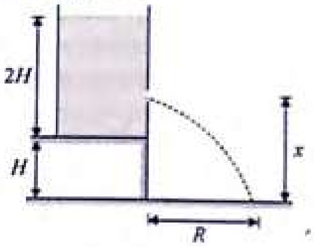
(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{1}$ (d) $\frac{1}{4}$

27. In a cylindrical water tank there are two small holes Q and P on the wall at a depth of h_1 from the upper level of water and at a height of h_2 from the lower end of the tank, respectively, as shown in the figure. Water coming out from both the holes strike the ground at the same point. The ratio of h_1 and h_2 is



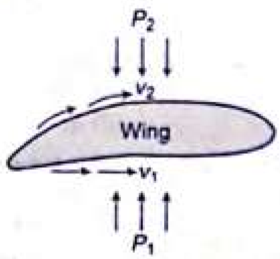
(a) 1 (b) 2
(c) >1 (d) <1

28. A tank is filled up to a height $2H$ with a liquid and is placed on a platform of height H from the ground. The distance x from the ground where a small hole is punched to get the maximum range R is



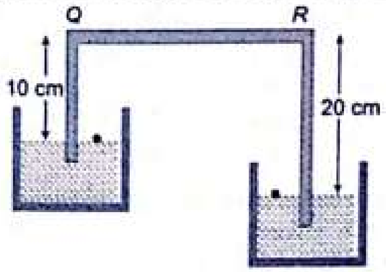
(a) H (b) $1.25H$
(c) $1.5H$ (d) $2H$

29. The speed of flow past the lower surface of a wing of an aeroplane is 50 ms^{-1}. What speed of flow over the upper surface will give a dynamic lift of 1000 Pa? Density of air = 1.3 kg m^{-3}.



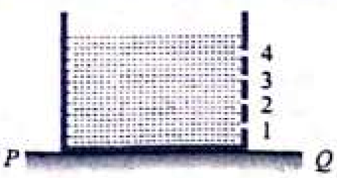
(a) 25.55 m/s (b) 63.55 m/s
(c) 13.25 m/s (d) None

30. A siphon in use is demonstrated in the following figure. The density of the liquid flowing in siphon is 1.5 gm/cc. The pressure difference between the point P and S will be



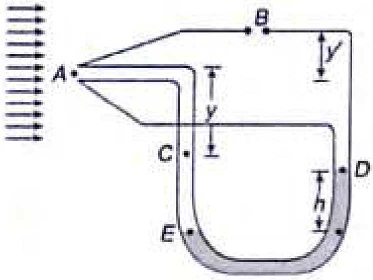
(a) 10^5 N/m (b) 2×10^5 N/m
(c) Zero (d) Infinity

31. A cylindrical vessel of 90 cm height is kept filled up to the brim. It has four holes 1, 2, 3 and 4 which are respectively at heights of 20 cm, 30 cm, 45 cm and 50 cm from the horizontal floor PQ. The water falling at the maximum horizontal distance from the vessel comes from



(a) Hole number 4 (b) Hole number 3
(c) Hole number 2 (d) Hole number 1

32. A Pitot tube is shown in figure. Wind blows in the direction shown. Air at inlet A is brought to rest, whereas its speed just outside of opening B is unchanged. The U tube contains mercury of density ρ_m. Find the speed of wind with respect to Pitot tube. Neglect the height difference between A and B and take the density of air as ρ_a.



(a) $\sqrt{\dfrac{2(\rho_m+\rho_a)gh}{\rho_a}}$ (b) $\sqrt{\dfrac{2(\rho_m-\rho_a)gh}{\rho_a}}$

(c) $\sqrt{\dfrac{(\rho_m-\rho_a)gh}{\rho_a}}$ (d) $\sqrt{\dfrac{(\rho_m+\rho_a)gh}{\rho_a}}$

33. A water tank of height 10 m, completely filled with water is placed on a level ground. It has two holes one at 3 m and the other at 7 m form its base. The water ejecting from
(a) both the holes will fall at the same spot
(b) upper hole will fall farther than that from the lower hole
(c) upper hole will fall closer then that from the lower hole
(d) more information is required.

34. Two holes are made in the side of the tank such that the jets of water flowing out of them meet at the same point on the ground. If one hole is at a height of 3 cm above the bottom, then the distance of the other hole from the top surface of water is

(a) $\dfrac{3}{2}$ cm (b) $\sqrt{6}$ cm

(c) $\sqrt{3}$ cm (d) 3 cm

Surface Tension and Viscosity

35. A spherical liquid drop of radius R is divided into eight equal droplets. If the surface tension is T, then the work done in this process will be
(a) $2\pi R^2T$ (b) $3\pi R^2T$
(c) $4\pi R^2T$ (d) $2\pi RT^2$

36. If T is surface tension of soap solution, the amount of work done in blowing a soap bubble from a diameter D to a diameter $2D$ is
(a) $2\pi D^2T$ (b) $4\pi D^2T$
(c) $6\pi D^2T$ (d) $8\pi D^2T$

37. A water drop is divided into eight equal droplets. The pressure difference between inner and outer sides of the big drop
(a) will be the same as for smaller droplet
(b) will be half of that for smaller droplet
(c) will be one-fourth of that for smaller droplet
(d) will be twice of that for smaller droplet

38. The velocity of small ball of mass M and density d_1 = when dropped a container filled with glycerine becomes constant after some time. If the density glycerine is d_2, the viscous force acting on ball is
(a) $\frac{Md_1 g}{d_2}$ (b) $Mg\left(1-\frac{d_2}{d_1}\right)$
(c) $\frac{M(d_1+d_2)}{g}$ (d) $M\, d_1 d_2$

39. Two soap bubbles, one of radius 50 mm and the other of radius 80 mm, are brought in contact so that they have a common interface. The radius of the curvature of the common interface is
(a) 0.003 m (b) 0.133 m
(c) 1.2 m (d) 8.9 m

40. A glass rod of radius r_1 is inserted symmetrically into a vertical capillary tube of radius r_2 such that their lower ends are at the same level. The arrangement is now dipped in water. The height to which water will rise into the tube will be (σ = surface tension of water, ρ = density of water)
(a) $\frac{2\sigma}{(r_2-r_1)\rho g}$ (b) $\frac{\sigma}{(r_2-r_1)\rho g}$
(c) $\frac{2\sigma}{(r_2+r_1)\rho g}$ (d) $\frac{2\sigma}{(r_2^2+r_1^2)\rho g}$

41. A large number of droplets, each of radius a, coalesce to form a bigger drop of radius b. Assume that the energy released in the process is converted into the kinetic energy of the drop. The velocity of the drop is (σ = surface tension, ρ = density)
(a) $\left[\frac{\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$ (b) $\left[\frac{2\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$
(c) $\left[\frac{3\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$ (d) $\left[\frac{6\sigma}{\rho}\left(\frac{1}{a}-\frac{1}{b}\right)\right]^{1/2}$

42. A straw 6 cm long floats on water. The water film on one side has surface tension of 50 dyn/cm. On the other slide, camphor reduces the surface tension to 40 dyn/cm. The resultant force acting on the straw is
(a) $(50\times 6-40\times 6)$ dyn (b) 10 dyn
(c) $\left(\frac{50}{6}-\frac{40}{6}\right)$ dyn (d) 90 dyn

43. A ring is cut from a platinum tube 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from the pan of a balance, so that it comes in contact with the water in a glass vessel. If an extra 3.103 gf is required to pull it away from water, the surface tension of water is
(a) 72 dyn/cm (b) 70.80 dyn/cm
(c) 63.35 dyn/cm (d) 60 dyn/cm

44. The angle of contact between glass and water is 0° and water (surface tension 70 dyn/cm) rises in a glass capillary up to 6 cm. Another liquid of surface tension 140 dyn/cm, angle of contact 60° and relative density 2 will rise in the same capillary up to
(a) 12 cm (b) 24 cm
(c) 3 cm (d) 6 cm

45. Work W is required to form a bubble of volume V from a given solution. What amount of work is required to be done to form a bubble of volume $2V$?
(a) W (b) $2W$
(c) $2^{1/3}W$ (d) $4^{1/3}W$

46. The surface energy of a liquid drop is E. It is sprayed into 1000 equal droplets. Then its surface energy becomes
(a) $1000E$ (b) $100E$
(c) $10E$ (d) E

47. In the previous question, the work done in spraying is
(a) $999E$ (b) $99E$
(c) $9E$ (d) E

48. A loop of 6.28 cm long thread is put gently on a soap film in a wire loop. The film is pricked with a needle inside the soap film enclosed by the thread. If the surface tension of soap solution is 0.030 N/m, then the tension in the thread is
(a) 1×10^{-4} N (b) 2×10^{-4} N
(c) 3×10^{-4} N (d) 4×10^{-4} N

49. A marble of mass x and diameter $2r$ is gently released in a tall cylinder containing honey. If the marble displaces mass y ($< x$) of the liquid, then the terminal velocity is proportional to
(a) $x+y$ (b) $x-y$
(c) $\frac{x+y}{r}$ (d) $\frac{x-y}{r}$

50. A small metal ball of diameter 4 mm and density 10.5 g/cm^3 in dropped in glycerine of density 1.5 g/cm^3. The ball attains a terminal velocity of 8/cm s^{-1}. The coefficient of viscosity of glycerine is
(a) 4.9 poise (b) 9.8 poise
(c) 98 poise (d) 980 poise

51. A capillary tube is attached horizontally to a constant pressure head arrangement. If the radius of the capillary tube is increased by 10%, then the rate of flow of the liquid shall change nearly by
(a) +10% (b) 46%
(c) −10% (d) −40%

52. A sphere of brass released in a long liquid column attains a terminal speed v_0. If the terminal speed is attained by a sphere of marble of the same radius and released in the same liquid is $n\, v_0$, then the value of n will be

(Given: The specific gravities of brass, marble and liquid are 8.5, 2.5 and 0.8, respectively)

(a) $\frac{5}{17}$ (b) $\frac{17}{77}$

(c) $\frac{11}{31}$ (d) $\frac{17}{5}$

53. Between a plate of area 100 cm^2 and another plate of area 100 m^2 there is a 1 mm, thick layer of water, if the coefficient of viscosity of water is 0.01 poise, then the force required to move the smaller plate with a velocity 10 $cm\,s^{-1}$ with reference to large plate is
(a) 100 dyn (b) 10^4 dyn
(c) 10^6 dyn (d) 10^9 dyn

54. A river 10 m deep is flowing at 5 $m\,s^{-1}$. The shearing stress between horizontal layers of the river is ($\eta = 10^{-3}$ SI units)
(a) $10^{-3}\,N/m^2$ (b) $0.8\times10^{-3}\,N/m^2$
(c) $0.5\times10^{-3}\,N/m^2$ (d) $1\,N/m^2$

55. A spherical ball falls through viscous medium with terminal velocity v. If this ball is replaced by another ball of the same mass but half the radius, then the terminal velocity will be (neglect the effect of buoyancy.)
(a) v (b) $2v$
(c) $4v$ (d) $8v$

56. A solid sphere falls with a terminal velocity of 20 $m\,s^{-1}$ in air. If it is allowed to fall in vacuum,
(a) terminal velocity will be 20 $m\,s^{-1}$
(b) terminal velocity will be less than 20 $m\,s^{-1}$
(c) terminal velocity will be greater than 20 $m\,s^{-1}$
(d) no terminal velocity will be attained

57. The density of water at the surface of ocean is ρ. If the bulk modulus of water is B, then the density of ocean water at depth, when the pressure is αp_0 and p_0 is the atmospheric pressure, is

(a) $\frac{\rho B}{B-(\alpha-1)p_0}$ (b) $\frac{\rho B}{B+(\alpha-1)p_0}$

(c) $\frac{\rho B}{B-\alpha p_0}$ (d) $\frac{\rho B}{B+\alpha p_0}$

58. Water rises to a height h in a capillary tube of cross-sectional area A. The height to which water will rise in a capillary tube of cross-sectional area $4A$ will be
(a) h (b) $h/2$
(c) $h/4$ (d) $4h$

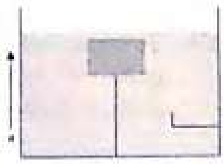
Problems Based on Mixed Concepts

59. A cubical block of wood of side a and density ρ floats in water of density 2ρ. The lower surface of the cube just touches the free end of a massless spring of force constant k fixed at the bottom of the vessel. The weight W put over the block so that it is completely immersed in water without wetting the weight is

(a) $a(a^2\rho g + k)$ (b) $a(a\rho g + 2k)$

(c) $a\left(\frac{a\rho g}{2} + 2k\right)$ (d) $a\left(a^2\rho g + \frac{k}{2}\right)$

60. The tension in a string holding a solid block below the surface of a liquid (where $\rho_{\text{liquid}} > \rho_{\text{block}}$) as in shown in the figure is T_0 when the system is at rest.



Then what will be the tension in the string if the system has upward acceleration a?

(a) $T_0\left(1-\frac{a}{g}\right)$ (b) $T_0\left(1+\frac{a}{g}\right)$

(c) $T_0\left(\frac{a}{g}-1\right)$ (d) $\frac{a}{g}T_0$

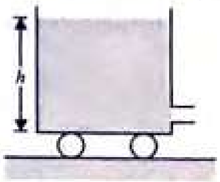
61. A wooden rod of a uniform cross section and of length 120 cm is hinged at the bottom of the tank which is filled with water to a height of 40 cm. In the equilibrium position, the rod makes an angle of 60° with the vertical. The centre of buoyancy is located on the rod at a distance (from the hinge) of
(a) 20 cm (b) 40 cm
(c) 60 cm (d) 75 cm

62. A small ball of density ρ is immersed in a liquid of density σ ($>\rho$) to a depth h and released. The height above the surface of water up to which the ball will jump is

(a) $\left(\frac{\sigma}{\rho}-1\right)h$ (b) $\left(\frac{\rho}{\sigma}-1\right)h$

(c) $\left(\frac{\rho}{\sigma}+1\right)h$ (d) $\left(\frac{\sigma}{\rho}+1\right)h$

63. A container filled with liquid up to height h is placed on a smooth horizontal surface. The container is having a small hole at the bottom. As the liquid comes out from the hole, the container moves in a backward direction with acceleration a and finally, when all the liquid is drained out, it acquires a velocity v. Neglect mass of the container. In this case



(a) both a and v depend on h
(b) only a depends on h
(c) only v depends on h
(d) neither a nor v depends on h

64. A hollow sphere has a small hole in it. On lowering the sphere in a tank of water, it is observed that water enters into the hollow sphere at a depth of 40 cm below the surface. Surface tension of water is 7×10^{-2} N/m. The diameter of the hole is

(a) $\frac{1}{28}$ mm (b) $\frac{1}{21}$ mm

(c) $\frac{1}{14}$ mm (d) $\frac{1}{7}$ mm

65. Two soap bubbles of radii a and b combine to form a single bubble of radius c. If P is the external pressure, then the surface tension of the soap solution is

(a) $\frac{P(c^3+a^3+b^3)}{4(a^2+b^2-c^2)}$ (b) $\frac{P(c^3-a^3-b^3)}{4(a^2+b^2-c^2)}$

(c) $Pc^3-4a^2-4b^2$ (d) $Pc^2-2a^2-3b^2$

66. A thin square plate of side 5 cm is suspended vertically from a balance so that lower side just dips into water with side to surface. When the plate is clean ($\theta = 0°$), it appears to weigh 0.044 N. But when the plate is greasy ($\theta = 180°$), it appears to weigh 0.03 N. The surface tension of water is

(a) 3.5×10^{-2} N/m (b) 7.0×10^{-2} N/m

(c) 14.0×10^{-2} N/m (d) 1.08 N/m

67. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be

(a) 20 cm (b) 4 cm

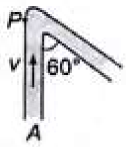
(c) 10 cm (d) 8 cm

68. Neglecting the density of air, the terminal velocity obtained by a raindrop of radius 0.3 mm falling through the air of viscosity 1.8×10^{-5} N/m^2 will be

(a) 10.9 m/s (b) 8.3 m/s

(c) 9.2 m/s (d) 7.6 m/s

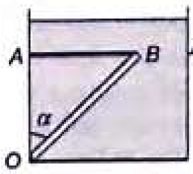
69. Water (density ρ) is flowing through the uniform tube of cross-sectional area A with a constant speed v as shown in the figure. The magnitude of force exerted by the water on the curved corner of the tube is (neglect viscous forces)



(a) $\sqrt{3}\,\rho A v^2$ (b) $2\rho A v^2$

(c) $\sqrt{2}\rho A v^2$ (d) $\frac{\rho A v^2}{\sqrt{2}}$

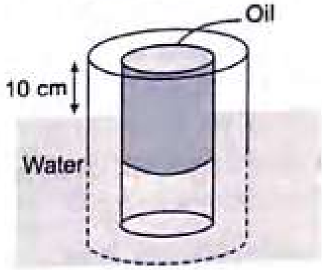
70. A uniform rod OB of length 1 m, cross-sectional area 0.012 m2 and relative density 2.0 is free to rotate about O in vertical plane. The rod is held with a horizontal string AB which can withstand a maximum tension of 45 N. The rod and string system is kept in water as shown in figure. The maximum value of angle α which the rod can make with vertical without breaking the string is

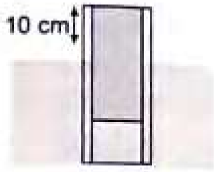


(a) 45° (b) 37°

(c) 53° (d) 60°

71. A tube with both ends open floats vertically in water. Oil with a density 800 kg/m^3 is poured into the tube. The tube is filled with oil up to the top end while in equilibrium. The portion out of the water is of length 10 cm. The length of the tube is:





(a) 50 cm (b) 60 cm

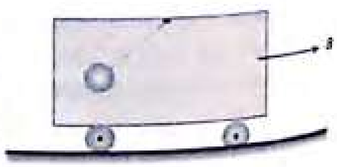
(c) 90 cm (d) 100 cm

72. One end of a long iron chain of linear mass density λ is fixed to a sphere of mass m and specific density 1/3 while the other end is free. The sphere along with the chain is immersed in a deep lake. If specific density of iron is 7, the height h above the bed of the lake at which the sphere will float in equilibrium is (Assume that the part of the chain lying on the bottom of the lake exerts negligible force on the upper part of the chain):

(a) $\frac{16}{7}\frac{m}{\lambda}$ (b) $\frac{7m}{3\lambda}$

(c) $\frac{5m}{2\lambda}$ (d) $\frac{8m}{3\lambda}$

73. Figure shows a closed container completely filled with an ideal liquid of density ρ. In the liquid there is a spherical body of volume V and density σ attached to a string whose other end is attached to the roof of the container. The container is accelerating with an acceleration 'a' towards right. The force exerted by the liquid on the spherical body when it is in equilibrium with respect to the liquid, will be:



(a) $V\rho\sqrt{a^2+g^2}+V\sigma a$ (b) $V\rho g+V\sigma a$

(c) $V\rho(g+a)$ (d) $V\rho\sqrt{a^2+g^2}$

ARCHIVES

1. A U-tube is partially filled with water. Oil, which does not mix with water, is next poured into one side, until water rises by 25 cm on the other side. If the density of the oil is 0.8 g/cc, the oil level will stand higher than the water level by
   (a) 6.25 cm (b) 18.75 cm
   (c) 12.50 cm (d) 25.00 cm **(AIEEE 2002)**

2. An object of weight W and density ρ is dipped in a fluid of density ρ_1. Its apparent weight will be
   (a) $W(\rho - \rho_1)$ (b) $W\left(1 - \frac{\rho_1}{\rho}\right)$
   (c) $\frac{(\rho - \rho_1)}{W}$ (d) $W(\rho_1 - \rho)$ **(AIEEE 2002)**

3. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in m/s) through a small hole on the side wall of the cylinder near its bottom is
   (a) 10 (b) 20
   (c) 25.5 (d) 5 **(AIEEE 2002)**

4. If two soap bubbles of different radii are connected by a tube,
   (a) air flows from the bigger bubble to the smaller bubble till their sizes become equal.
   (b) air flows from the bigger bubble to the smaller bubble till their sizes are interchanged.
   (c) air flows from the smaller bubble to the bigger.
   (d) there is no flow of air. **(AIEEE 2004)**

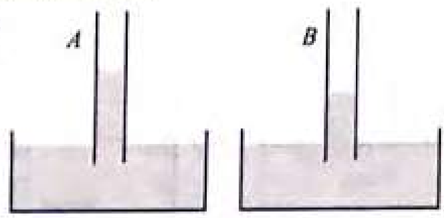
5. A spherical ball of radius R is falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the ball is
   (a) directly proportional to R but inversely proportional to v.
   (b) directly proportional to both radius R and velocity v.
   (c) inversely proportional to both radius R and velocity v.
   (d) inversely proportional to R but directly proportional to velocity v. **(AIEEE 2004)**

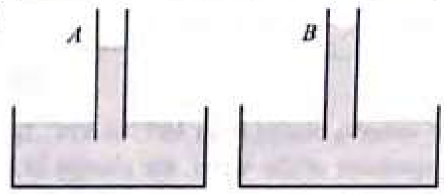
6. A 20-cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of the water column in the capillary tube will be
   (a) 20 cm (b) 4 cm
   (c) 10 cm (d) 8 cm **(AIEEE 2005)**

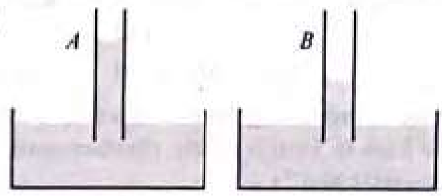
7. If the terminal speed of a sphere of gold (of density 19.5 kg/m^3) is 0.2 m/s in a viscous liquid, find the terminal speed of a sphere of silver (of density 10.5 kg/m^3) of the same size in the same liquid.
   (a) 0.4 m/s (b) 0.133 m/s
   (c) 0.1 m/s (d) 0.2 m/s **(AIEEE 2006)**

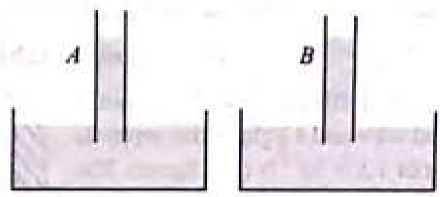
8. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 $(\rho_2 < \rho_1)$. Assume that the liquid applied a viscous force on the ball proportional to the square of its speed v, i.e., $F_{\text{viscous}} = -kv^2\ (k > 0)$. The terminal speed of the ball is
   (a) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$ (b) $\frac{Vg\rho_1}{k}$
   (c) $\sqrt{\frac{Vg\rho_1}{k}}$ (d) $\frac{Vg(\rho_1 - \rho_2)}{k}$
   (AIEEE 2008)

9. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

(a)


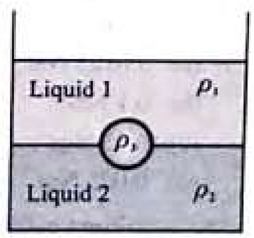
(b)


(c)


(d)


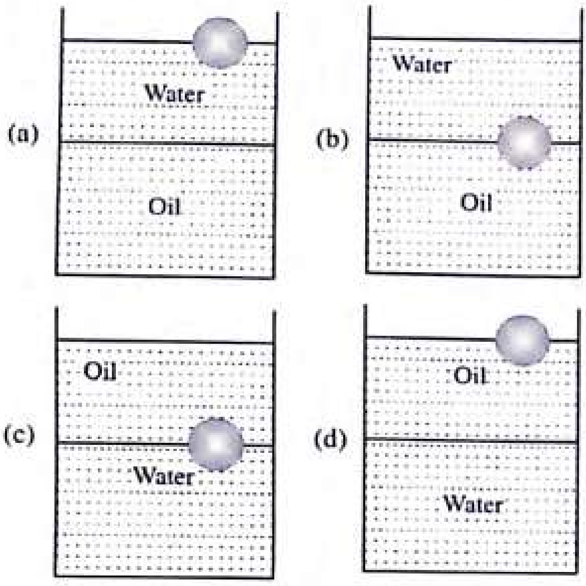
(AIEEE 2008)

10. A jar is filled with two non-mixing liquids having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It attains equilibrium in the position as shown in the figure.



Which of the following is true for ρ_1, ρ_2, and ρ_3?
   (a) $\rho_3 < \rho_1 < \rho_2$ (b) $\rho_1 > \rho_3 > \rho_2$
   (c) $\rho_1 < \rho_2 < \rho_3$ (d) $\rho_1 < \rho_3 < \rho_2$
   (AIEEE 2008)

11. A ball is made of a material of density ρ, where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?



(AIEEE 2010)

12. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K)

(a) 4.2 kJ (b) 8.4 kJ
(c) 84 kJ (d) 2.1 kJ **(AIEEE 2011)**

13. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly. (Surface tension of soap solution = 0.03 Nm^{-1})

(a) 4π mJ (b) 0.2π mJ
(c) 2π mJ (d) 0.4π mJ **(AIEEE 2011)**

14. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is

(a) 0.0125 Nm^{-1}
(b) 0.1 Nm^{-1}
(c) 0.05 Nm^{-1}
(d) 0.025 Nm^{-1} **(AIEEE 2012)**

15. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is

(a) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (b) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$
(c) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$ (d) $\frac{Mg}{k}$ **(JEE Main 2013)**

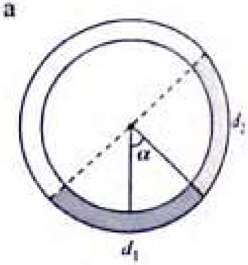
16. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

(a) $\sqrt{\frac{T}{\rho L}}$ (b) $\frac{T}{\rho L}$
(c) $\frac{2T}{\rho L}$ (d) $\frac{\rho L}{T}$ **(JEE Main 2013)**

17. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)

(a) 38 cm (b) 6 cm
(c) 16 cm (d) 22 cm **(JEE Main 2014)**

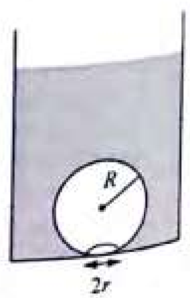
18. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio d_1/d_2 is



(a) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (b) $\frac{1+\sin\alpha}{1-\cos\alpha}$
(c) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (d) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(JEE Main 2014)

19. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, the value of r just before bubbles detach is (density of water is ρ_w)



(a) $R^2\sqrt{\frac{\rho_w g}{T}}$ (b) $R^2\sqrt{\frac{3\rho_w g}{T}}$

(c) $R^2\sqrt{\frac{\rho_w g}{6T}}$ (d) $R^2\sqrt{\frac{\rho_w g}{3T}}$

(JEE Main 2014)

20. The following observations were taken for determining surface tension T of water by capillary method:

Diameter of capillary, $D = 1.25 \times 10^{-2}$ m

Rise of water, $h = 1.45 \times 10^{-2}$ m

Using $g = 9.80$ m/s^2 and the simplified relation $T = \frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to

(a) 2.4% (b) 10%

(c) 0.15% (d) 1.5%

(JEE Main 2017)

ANSWER KEY

Exercises

1. (d)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (d)	**6.** (a)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (d)
11. (a)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (b)
21. (b)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (c)
31. (b)	**32.** (b)	**33.** (a)	**34.** (d)	**35.** (c)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (b)	**40.** (a)
41. (d)	**42.** (a)	**43.** (a)	**44.** (c)	**45.** (d)	**46.** (c)	**47.** (c)	**48.** (c)	**49.** (d)	**50.** (b)
51. (b)	**52.** (b)	**53.** (a)	**54.** (c)	**55.** (b)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (d)	**60.** (b)
61. (b)	**62.** (a)	**63.** (d)	**64.** (c)	**65.** (b)	**66.** (b)	**67.** (a)	**68.** (a)	**69.** (a)	**70.** (b)
71. (d)	**72.** (b)	**73.** (d)							

Archives

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (d)
11. (c)	**12.** (b)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (c)	**17.** (c)	**18.** (a)	**19.** None	**20.** (d)

Chapter 13

Thermal Expansion, Calorimetry and Transmission of Heat

TEMPERATURE

Temperature is defined by zeroth law of thermodynamics, which states that when two bodies A and B are separately in thermal equilibrium with a third body C, then A and B are also in thermal equilibrium with each other (thermal equilibrium implies equality of temperature). Temperature is a scalar quantity which is a property of all thermodynamic systems such that the equality of temperature is necessary and sufficient for thermal equilibrium.

1. Temperature is one of the seven fundamental quantities with dimension $[\theta]$.
2. It is a scalar physical quantity with SI unit kelvin.
3. When heat is given to a body and its state does not change, the temperature of the body rises and if heat is taken from a body its temperature falls, i.e., temperature can be regarded as the effect of cause 'heat'.
4. According to the kinetic theory of gases, temperature (macroscopic physical quantity) is a measure of average translational kinetic energy of a molecule (microscopic physical quantity).
   Temperature $\propto$ kinetic energy [As $E = 3RT/2$]
5. Although the temperature of a body can be raised without limit, it cannot be lowered without limit and theoretically limiting low temperature is taken to be zero of the kelvin scale.

As temperature is measured by the value of the thermodynamic property of a substance, i.e., a property which varies *linearly* with the temperature, two fixed points are needed to define a temperature scale. These two fixed points in modern thermometry are taken as

1. Triple point of water, i.e., the state of water where the liquid, solid and vapour phases of water coexist in equilibrium. It is characterized by unique values of temperature and pressure.
2. On this scale, the other fixed point may be taken as the absolute zero.

We then need to assign some numbers to these two fixed points. The lowest temperature may be taken as zero. The triple point of water on Celsius scale is 0.01°C. Thus, the absolute temperature T for triple point of water will be given by

$$T = T_c + 273.15 = 0.01 + 273.15 = 273.16 \text{ K}$$

$\therefore$ $T = 273.16$ K (triple point of water)

Thermometry

An instrument used to measure the temperature of a body is called a thermometer.

The linear variation in some physical property of a substance with change of temperature is the basic principle of thermometry and these properties are defined as thermometric property (x) of the substance. x may be

(i) length of liquid in capillary;
(ii) pressure of gas at constant volume;
(iii) volume of gas at constant pressure and
(iv) resistance of a given platinum wire.

In old thermometry, two arbitrarily fixed points ice and steam point (freezing point and boiling point at 1 atm) are taken to define the temperature scale. In Celsius scale, freezing point of water is assumed to be 0°C while boiling point 100°C and the temperature interval between these is divided into 100 equal parts.

So, if the thermometric property at temperatures 0°C, 100°C and T_c°C is x_0, x_{100} and x, respectively, then by linear variation ($y = mx + c$) we can say that

$$0 = ax_0 + b \quad \text{(i)}$$

$$100 = ax_{100} + b \quad \text{(ii)}$$

$$T_c = ax + b \quad \text{(iii)}$$

From these equations $\dfrac{T_c - 0}{100 - 0} = \dfrac{x - x_0}{x_{100} - x_0}$

$$\therefore \quad T_c = \frac{x - x_0}{x_{100} - x_0} \times 100°\text{C}$$

In modern thermometry instead of two fixed points only one reference point is chosen (triple point of water 273.16 K at which ice, water and water vapours coexist).

So, if the values of thermometric property at 0 K, 273.16 K and T_K K are 0, x_{Tr} and x, respectively, then by linear variation ($y = mx + c$) we can say that

$$0 = a \times 0 + b \quad \text{(i)}$$

$$273.16 = a \times x_{T_r} + b \quad \text{(ii)}$$

$$T_k = a \times x + b \quad \text{(iii)}$$

From these equations $\frac{T_1}{273.16}=\frac{x}{x_{T_r}}$

$$\therefore \quad T_K = 273.16\left[\frac{x}{x_{T_r}}\right] \text{K}$$

Measurement of Temperature

There are different systems of measurement of temperature. The lower fixed point (LFP) and the upper fixed point (UFP) in any system of units are corresponding to freezing point and boiling point of water at 1 atm.

For different system of units the LFP and UFP are given as

System of units	Units	Lower fixed point (LFP)	Upper fixed point (UFP)	Different UFP − LFP
Degree celsius (centigrade)	°C	0°C	100°C	100
Kelvin scale (SI unit)	K	273.15 K	373.5 K	100
Fahrenheit	°F	32°F	212°F	180

Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - \text{lower fixed point (LFP)}}{\text{Upper fixed point (UFP)} - \text{lower fixed point(LFP)}} = \text{Constant for all scales}$$

The relation between Celsius (C), Kelvin (K), Fahrenheit (F) and any other new scale θ is

$$\frac{C-0}{100}=\frac{F-32}{180}=\frac{K-273}{100}=\frac{\theta-\theta_0}{n} \quad \text{(i)}$$

where n is the number of divisions between ice point and steam point on the new scale and θ_0 is the ice point on it.

A convenient way to change one scale to another is to remember the freezing and boiling points of water in each form:

$$T_{\text{freeze}} = 32.0°\text{F} = 0°\text{C} = 273.15 \text{ K}$$

$$T_{\text{boil}} = 212°\text{F} = 100°\text{C}$$

To convert from Fahrenheit to Celsius, subtract 32 (the freezing point) and then adjust the scale by the liquid range of the water.

$$\text{Scale factor} = \frac{(100-0)°\text{C}}{(212-32)°\text{F}} = \frac{5°\text{C}}{9°\text{F}}$$

A Kelvin is the same size change as a degree celsius, but the Kelvin scale takes its zero point at absolute zero, instead of the freezing point of water. Therefore, to convert from Kelvin to Celsius, subtract 273.15 K from given Kelvin temperature.

ILLUSTRATION 13.1 Two ideal gas thermometers A and B use oxygen and hydrogen, respectively. The following observations are made:

Temperature	Pressure thermometer A	Pressure thermometer B
Triple point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of sulphur	1.797×10^5 Pa	0.287×10^5 Pa

(a) What is the absolute temperature of normal melting point of sulphur as read by thermometer A and B?

(b) What do you think is the reason for slightly different answers from A and B?

Solution.

(a) For thermometer A,

$$T_{tr} = 273 \text{ K}, \quad P_{tr} = 1.250 \times 10^5 \text{ Pa}$$

We have $$T = \frac{P}{P_{tr}} \times T_{tr} = \frac{1.797\times10^5}{1.250\times10^5}\times 273 = 392.46 \text{K}$$

For thermometer B,

$$T_{tr} = 273 \text{ K}, \quad P_{tr} = 0.200 \times 10^5 \text{ Pa}$$

We have $$T = \frac{P}{P_{tr}} \times t_{tr} = \frac{0.287\times10^5\times273}{0.200\times10^5} = 391.75 \text{K}$$

(b) The slight difference in the temperatures as read by two thermometers is due to the fact that oxygen and hydrogen do not behave like an ideal gas.

CALORIMETRY

This is the branch of heat transfer that deals with the measurement of heat. The heat is usually measured in calories or kilocalories.

One Calorie

One calorie is the quantity of heat required to raise the temperature of 1 g of water by 1°C.

Mechanical Equivalent of Heat (J)

According to Joule, work may be converted into heat and vice versa. The ratio of work done (W) to heat produced (Q) by that work without any wastage is always constant.

$$W/Q = \text{constant}$$

This constant is called *mechanical equivalent of heat* (J). The value of this constant is taken as 4.18 J/cal.

ILLUSTRATION 13.2 What is the change in potential energy (in calories) of a 10 kg mass after 41.8 m fall?

Solution. Change in potential energy

$$\Delta U = mgh = 10 \times 10 \times 41.8 = 4180 \text{ J}$$

If mechanical work/energy (W) in joule produces the same temperature change as heat (H), we write,

$$W = JH$$

where J = 4.18 is called mechanical equivalent of heat. J is expressed in joule/calorie.

Hence, equivalent calories will be

$$H = \frac{\Delta U}{J} = \frac{4180}{4.18} = 1000 \text{ cal}$$

Thermal Capacity and Water Equivalent

1. **Thermal capacity:** It is defined as the amount of heat required to raise the temperature of the whole body (mass m) through 1°C or 1 K.

$$\text{Thermal capacity} = C = \frac{Q}{\Delta T}$$

The value of thermal capacity of a body depends upon the nature of the body and its mass.

Dimension: $[ML^2T^{-2}\theta^{-1}]$; unit: cal/°C (practical) J/K (SI)

2. **Water equivalent:** Water equivalent of a body is defined as the mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by W.

If m = mass of the body, c = specific heat of body,

ΔT = rise in temperature.

Then heat given to body

$$\Delta Q = mc\Delta T \quad \text{(i)}$$

If same amount of heat is given to W grams of water and its temperature also rises by ΔT.

Then heat given to water

$$\Delta Q = W \times 1 \times \Delta T \quad [\text{As } c_{water} = 1] \quad \text{(ii)}$$

From Eqs. (i) and (ii), $\Delta Q = mc\Delta T = W \times 1 \times \Delta T$

∴ Water equivalent $(W) = mc$ grams

Unit: kg (SI); dimension: $[ML^\circ T^\circ]$

NOTE:

- Unit of thermal capacity is J/kg while unit of water equivalent is kg.
- Thermal capacity of the body and its water equivalent are numerically equal.
- If thermal capacity of a body is expressed in terms of mass of water, it is called water equivalent of the body.

Specific Heat

Gram specific heat: When heat is given to a body and its temperature increases, the heat required to raise the temperature of unit mass of a body through 1°C (or K) is called specific heat of the material of the body.

If Q heat changes the temperature of mass m by ΔT

$$\text{Specific heat } c = \frac{Q}{m\Delta T}$$

Units: cal/g × °C (practical), J/kg × K (SI); dimension: $[L^2T^{-2}\theta^{-1}]$

☞ Important Points

1. Specific heat for hydrogen is maximum (3.5 cal/g × °C) and for water, it is 1 cal/g × °C.
   For all other substances, the specific heat is less than 1 cal/g × °C and it is minimum for radon and actinium (≈ 0.022 cal/g × °C).
2. Specific heat of a substance also depends on the state of the substance, i.e., solid, liquid or gas.
   For example, $C_{ice} = 0.5$ cal/g°C (solid),
   $C_{water} = 1$ cal/g × °C (liquid) and
   $C_{steam} = 0.47$ cal/g × °C (gas)
3. The specific heat of a substance when it melts or boils at constant temperature is infinite.
   As $\quad C = \dfrac{Q}{m\Delta T} = \dfrac{Q}{m \times 0} = \infty \quad$ (As $\Delta T = 0$)
4. The specific heat of a substance when it undergoes adiabatic changes is zero.
   As $\quad C = \dfrac{Q}{m\Delta T} = \dfrac{0}{m\Delta T} = 0 \quad$ (As $Q = 0$)
5. Specific heat of a substance can also be negative. Negative specific heat means that in order to raise the temperature, a certain quantity of heat is to be withdrawn from the body.
   For example, specific heat of saturated vapours.

ILLUSTRATION 13.3 A 60-kg boy running at 5.0 m/s while playing basketball falls down on the floor and skids along on his leg until he stops. How many calories of heat are generated between his leg and the floor? Assume that all this heat energy is confined to a volume of 2.0 cm³ of his flesh. What will be temperature change of the flesh? Assume c = 1.0 cal/g °C and $\rho = 950$ kg/m³ for flesh.

Solution. Kinetic energy of boy $K = \frac{1}{2}mv^2 = \frac{1}{2}(60)(5)^2 = 750$ J

Changing this energy in joule into calorie

$$Q_1 = \frac{K}{J} = \frac{750}{4.2} = 179 \text{ J} \quad \text{(i)}$$

As this kinetic energy changes into heat energy (Q_2) and this heat is confined to flesh of boy,

$$Q_2 = mc\Delta T = (\rho V)\, c\Delta T \quad \text{(ii)}$$

As $\quad Q_1 = Q_2$, hence from Eqs. (i) and (ii)

$$179 = 0.95 \times 2 \times \Delta T \Rightarrow \Delta T = 94°\text{C}$$

Latent Heat

- When a substance changes from one state to another state (say from solid to liquid or liquid to gas or from liquid to solid or gas to liquid) then energy is either absorbed or liberated. This heat energy is called latent heat.

- No change in temperature is involved when the substance changes its state. That is, phase transformation is an isothermal change. Ice at 0°C melts into water at 0°C. Water at 100°C boils to form steam at 100°C.
- The amount of heat required to change the state of the mass m of the substance is written as: $\Delta Q = mL$, where L is the latent heat. Latent heat is also called as heat of transformation.
- Unit: cal/g or J/kg and dimension: $[L^2T^{-2}]$.
- Any material has two types of latent heats.

i. Latent heat of fusion: The latent heat of fusion is the heat energy required to change 1 kg of the material in its solid state at its melting point to 1 kg of the material in its liquid state. It is also the amount of heat energy released when at melting point 1 kg of liquid changes to 1 kg of solid. For water at its normal freezing temperature or melting point (0°C), the latent heat of fusion (or latent heat of ice) is

$$L_F = L_{ice} \approx 80 \text{ cal/g} \approx 6 \text{ kJ/mol} \approx 336 \text{ kJ/kg}$$

ii. Latent heat of vapourization: The latent heat of vapourization is the heat energy required to change 1 kg of the material in its liquid state at its boiling point to 1 kg of the material in its gaseous state. It is also the amount of heat energy released when 1 kg of vapour changes into 1 kg of liquid. For water at its normal boiling point or condensation temperature (100°C), the latent heat of vapourization (latent heat of steam) is

$$L_V = L_{steam} \approx 540 \text{ cal/g} \approx 40.8 \text{ kJ/mol} \approx 2260 \text{ kJ/kg}$$

NOTE:

- In the process of melting or boiling, heat supplied is used to increase the internal potential energy of the substance and also in doing work against external pressure while internal kinetic energy remains constant. This is the reason that internal energy of steam at 100°C is more than that of water at 100°C.
- It is more painful to get burnt by steam rather than by boiling water at same temperature. This is so because when 1 g of steam at 100°C gets converted to water at 100°C, then it gives out 536 cal of heat. So, it is clear that steam at 100°C has more internal energy than water at 100°C (i.e., boiling of water).
- In case of change of state if the molecules come closer, energy is released and if the molecules move apart, energy is absorbed.
- Latent heat of vapourization is more than the latent heat of fusion. This is because when a substance gets converted from liquid to vapour, there is a large increase in volume. Hence, more amount of heat is required. But when a solid gets converted to a liquid, then the increase in volume is negligible. Hence, very less amount of heat is required. So, latent heat of vapourization is more than the latent heat of fusion.
- After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So, in the mountains, when snow falls, one does not feel too cold, but when ice melts, he feels too cold.
- There is more shivering effect of ice cream on teeth as compared to that of water (obtained from ice). This is because when ice cream melts down, it absorbs large amount of heat from teeth.

Principle of Calorimetry

When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

Heat lost = Heat gained

i.e., the principle of calorimetry represents the law of conservation of heat energy.

Heat lost by A = Heat gained by B

or $\quad m_1c_1(T_1 - T) = m_2c_2(T - T_2)$

(where T = temperature of equilibrium)

$$\therefore \quad T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

i. If bodies are of same material $c_1 = c_2$, then $T = \dfrac{m_1T_1 + m_2T_2}{m_1 + m_2}$

ii. If bodies are of same mass $(m_1 = m_2)$, then $T = \dfrac{T_1c_1 + T_2c_2}{c_1 + c_2}$

iii. If bodies are of same material and of equal masses $(m_1 = m_2, c_1 = c_2)$, then $T = \dfrac{T_1 + T_2}{2}$.

ILLUSTRATION 13.4 Determine the final result when 200 g of water and 20 g of ice at 0°C are in a calorimeter having a water equivalent of 30 g and 50 g of steam is passed into it at 100°C.

Solution. When steam is passed, the final temperature can be 0°C, between 0°C and 100°C, or 100°C.

We will consider all three possibilities.

Case I

Final temperature = 0°C

In this case, all the steam condenses and then cools down to 0°C.

Heat given out by steam

$$= 50 \times 540 + 50 \times 1 \times (100 - 0) = 32000 \text{ cal}$$

Mass of ice which will melt by this heat $= \dfrac{32000}{80} = 400$ g

But there is only 20 g of ice in the calorimeter.

Hence final temperature cannot be 0°C.

Case II

Final temperature = θ and $0 < \theta < 100$

Heat lost by steam = heat gained by (ice + water + calorimeter)

$\Rightarrow \quad 50 \times 540 + 50 \times 1 \times (100 - \theta)$

$$= 20 \times 80 + (20 + 200 + 30) \times 1 \times (\theta - 0)$$

$\Rightarrow \quad \theta = 101.3°C$

The assumption $(0 < \theta < 100)$ is proved to be wrong. Hence, the final temperature cannot be between 0°C and 100°C

$\Rightarrow$ The final temperature will be 100°C.

Case III

Let m = mass of steam condensed.

Heat lost by steam = heat gained by ice to melt + heat gained by (water + water + calorimeter) to reach 100°C

$\Rightarrow \quad m(540) = 20 \times 80 + (20 + 200 + 30) \times (100 - 0)$

$\Rightarrow \quad m = 26600/540 \approx 49$ g

$\Rightarrow$ 49 g of steam gets condensed and the final temperature is 100°C.

Heating Curve

If heat is supplied at constant rate to a given mass m of a solid, P and a graph is plotted between temperature and time, the graph is as shown in figure and is called heating curve. From this curve it is clear that

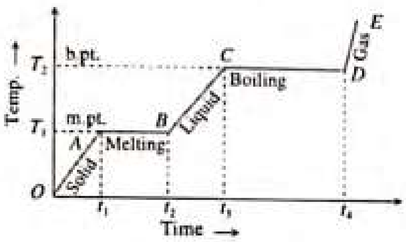
- In the region OA temperature of solid is changing with time, so,

  $Q = mc_s \Delta T$

  or $\quad P\Delta t = mc_s \Delta T \qquad$ (as $Q = P\Delta t$)

  But as $(\Delta T/\Delta t)$ is the slope of temperature–time curve

  $c_S \propto$ (1/slope of line OA)



  i.e., specific heat (or thermal capacity) is inversely proportional to the slope of temperature–time curve.

- In the region AB temperature is constant, so it represents change of state, i.e., melting of solid with melting point T_1. At A melting starts and at B all solid is converted into liquid. So between A and B substance is partly solid and partly liquid. If L_F is the latent heat of fusion.

  $$Q = mL_F \quad \text{or} \quad L_F = \frac{P(t_2 - t_1)}{m} \qquad [\text{as } Q = P(t_2 - t_1)]$$

  or $\quad L_F \propto$ length of line AB

  i.e., latent heat of fusion is proportional to the length of line of zero slope. (In this region specific heat $\to \infty$)

- In the region BC temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC, i.e.,

  $c_L \propto$ (1/slope of line BC)

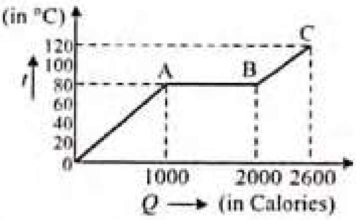
- In the region CD temperature is constant, so it represents the change of state, i.e., boiling with boiling point T_2. At C all substance is in liquid state while at D in vapour state and between C and D partly liquid and partly gas. The length of line i is proportional to latent heat of vapourization, i.e.,

  $L_V \propto$ Length of line CD

  (In this region specific heat $\to \infty$)

- The line DE represents gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

ILLUSTRATION 13.5 A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph.



If the relative specific heat capacity of the solid substance is 0.5, find from the graph (i) the mass of the substance; (ii) the specific latent heat of the melting process and (iii) the specific heat of the substance in the liquid state.

Specific heat capacity of water = 1000 cal/kg/K

Solution. 1000 cal of heat raises the temperature of the substance from 0°C to 80°C.

$\therefore \quad 1000 = m(1000 \times 0.5) \times 80$

($\because$ specific heat = relative sp. heat × sp. heat of water)

or $\quad m = 0.025$ kg

Latent heat = 200 × 5 = 1000 cal

($\because$ 1 div reads 200 cal) = $0.025 \times L$

$\therefore \quad L = 40000$ cal/kg

In the liquid state temperature rises from 80°C to 120°C, that is, by 40°C after absorbing 600 cal.

$\therefore \quad 0.025\, s \times 40 = 600 \quad$ or $\quad s = 600$ cal/kg/K

CONCEPT APPLICATION EXERCISE 13.1

1. Ice of mass 600 g and at a temperature of −10°C is placed in a copper vessel heated to 350°C. The resultant mixture is 550 g of ice and water. Find the mass of the vessel. The specific heat capacity of copper (c) = 100 cal/kg-K.
2. When a small ice crystal is placed in overcooled water it begins to freeze instantaneously.
   (i) What amount of ice is formed from 1 kg of water overcooled to −8°C? L of water = 336×10^3 J/kg and s of water = 4200 J/kg/K.
   (ii) What should be the temperature of the overcooled water in order that all of it be converted into ice at 0°C?
3. An electric heater whose power is 54 W is immersed in 650 cm^3 water in a calorimeter. In 3 min the water is heated by 3.4°C. What part of the energy of the heater passes out of the calorimeter in the form of radiant energy?
4. An ice cube whose mass is 50 g is taken from a refrigerator where its temperature was −10°C. If no heat is gained or lost from outside, how much water will freeze onto the cube if it is dropped into a beaker containing water at 0°C? Latent heat of fusion = 80 kcal/kg, specific heat capacity of ice = 500 cal/kg/K.

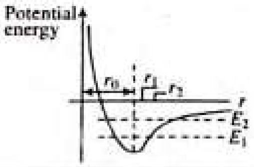
5. Equal volumes of three liquids of densities ρ_1, ρ_2 and ρ_3, specific heat capacities c_1, c_2 and c_3 and temperatures t_1, t_2 and t_3, respectively, are mixed together. What is the temperature of the mixture? Assume no changes in volume on mixing.
6. Victoria Falls in Africa is 122 m in height. Calculate the rise in temperature of the water if all the potential energy lost in the fall is converted into heat.
7. Equal masses of three liquids A, B and C are taken. Their initial temperatures are 10°C, 25°C and 40°C, respectively. When A and B are mixed the temperature of the mixture is 19°C. When B and C are mixed, the temperature of the mixture is 35°C. Find the temperature if all three are mixed.

THERMAL EXPANSION

Most materials expand when their temperature is increased. Railroad tracks, bridges all have some means of compensating for thermal expansion. When a homogeneous object expands, the distance between any two points on the object increases. Figure shows a block of metal with a hole in it. **The expanded object is like a photographic enlargement.** That is the hole expands in the same proportion as the metal; it does not get smaller.

Almost all solids and liquids expand as their temperature increases. Gases also expand if allowed. Solids can change in length, area or volume, while liquids change in their volumes.

The same steel ruler two different temperatures. When it expands, the scale, the numbers, the thickness, and the diameters of the circle and circular hole are all increased by the same factor. (The expansion has been exaggerated for clarity.)



Thermal expansion arises because the well is not symmetrical about the equilibrium position r_0. As the temperature rise the energy of the atom increases. The average position when the energy is E_2 is not the same as that when the energy is E_1

- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.
- Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimension (volume expansion) while liquids and gases usually suffers change in volume only.
- The coefficient of linear expansion of the material of a solid is defined as the increase in its length per unit length per unit rise in its temperature.

$$\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$$

Similarly, the coefficient of superficial expansion

$$\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$$

and coefficient of volume expansion

$$\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

The value of α, β and γ depends upon the nature of material. All have dimension $[\theta^{-1}]$ and unit per °C.

- As $\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}$, $\beta = \frac{\Delta A}{A} \times \frac{1}{\Delta T}$ and $\gamma = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$

$\therefore \quad \Delta L = L\alpha\,\Delta T$, $\Delta A = A\beta\,\Delta T$ and $\Delta V = V\gamma\,\Delta T$

Final length $L' = L + \Delta L = L(1 + \alpha\,\Delta T)$ (i)

Final area $A' = A + \Delta A = A(1 + \beta\,\Delta T)$ (ii)

Final volume $V' = V + \Delta V = V(1 + \gamma\Delta T)$ (iii)

- If L is the side of square plate and it is heated by temperature ΔT, then its side becomes L'.

  The initial surface area $A = L^2$ and final surface area $A' = L'^2$

- $$\frac{A'}{A} = \left(\frac{L'}{L}\right)^2 = \left(\frac{L(1+\alpha\,\Delta T)}{L}\right)^2 = (1+\alpha\,\Delta T)^2 = (1+2\alpha\,\Delta T)$$

(using Binomial theorem)

or $\quad A' = A(1 + 2\alpha\Delta T)$

Comparing with Eq. (ii), we get $\beta = 2\alpha$

Similarly, for volumetric expansion

$$\frac{V'}{V} = \left(\frac{L'}{L}\right)^3 = \left(\frac{L(1+\alpha\Delta T)}{L}\right)^3 = (1+\alpha\Delta T)^3 = (1+3\alpha\Delta T)$$

(using Binomial theorem)

or $\quad V' = V(1 + \gamma\Delta T)$

Comparing with Eq. (iii), we get $\gamma = 3\alpha$

So $\quad \alpha : \beta : \gamma = 1:2:3$

ILLUSTRATION 13.6 A metal rod A of 25 cm length expands by 0.05 cm when its temperature is raised from 0°C to 100°C. Another rod B of a different metal of length 40 cm expands by 0.04 cm for the same rise in temperature. A third rod C of 50 cm length made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the length of each portion of composite rod C.

Solution. From the given data for rod A, we have

$$\Delta L = \alpha_A L \Delta T$$

or $\quad \alpha_A = \frac{\Delta L}{L\Delta T} = \frac{0.05}{25 \times 100} = 2 \times 10^{-5}/°C$

For rod B, we have $\quad \Delta L = \alpha_B L \Delta T$

or $\alpha_B = \frac{\Delta L}{L\Delta T} = \frac{0.04}{40 \times 100} = 10^{-5}/°C$

If rod C is made of segments of rod A and B of lengths l_1 and l_2, respectively, then we have at 0°C

$l_1 + l_2 = 50$ cm (i)

At $T = 50°C$ $l'_1 + l'_2 = 50.03$ cm

Thus $\alpha_A l_1 \Delta T + \alpha_B l_2 \Delta T = 0.03$ cm

or $2 \times 10^{-5} \times l_1 \times 50 + 10^{-5} \times l_2 \times 50 = 0.03$ cm

or $2l_1 + l_2 = \frac{0.03}{50} \times 10^5 = 60$ cm (ii)

Solving Eqs. (i) and (ii), we get $l_1 = 10$ cm and $l_2 = 40$ cm.

Variation of Density with Temperature

i. Suppose ρ_0 is the density of the substance at 0°C and at any temperature t, it becomes ρ_t. As mass of the substance remains constant at any temperature, we have

$\rho_0 V_0 = \rho_t V_t$

Here V_0 and V_t are the volumes of the substance at 0°C and t°C, respectively.

Also $V_t = V_0(1 + \gamma t)$

$\therefore \rho_0 V_0 = \rho_t \{V_0(1 + \gamma t)\}$

or $\rho_t = \frac{\rho_0}{(1+\gamma t)}$

For small value of γ, we can approximate it as

$\rho_t = \rho_0(1 - \gamma t)$

ii. If ρ_1 and ρ_2 are the densities at t_1 and t_2, respectively, then we can write

$\rho_1 V_1 = \rho_2 V_2$

or $\rho_1 V_0(1 + \gamma t_1) = \rho_2 V_0(1 + \gamma t_2)$

or $\rho_1 = \rho_2 \frac{(1+\gamma t_2)}{(1+\gamma t_1)}$

$= \rho_2(1 + \gamma t_2)(1 - \gamma t_1)$

$= \rho_2[1 + \gamma(t_2 - t_1)]$

(neglecting γ_2 on being small)

$\therefore \gamma = \frac{\rho_1 - \rho_2}{\rho_2(t_2 - t_1)}$

Expansion of Liquid

Liquids also expand on heating just like solids. Since liquids have no shape of their own, they suffer only volume expansion. If the liquid of volume V is heated and its temperature is raised by $\Delta\theta$ then

$V'_L = V(1 + \gamma_L \Delta\theta)$

(γ_L = coefficient of real expansion or coefficient of volume expansion of liquid)

As liquid is always taken in a vessel for heating, if a liquid is heated, the vessel also gets heated and it also expands.

$V'_S = V(1 + \gamma_S \Delta\theta)$

(γ_S = coefficient of volume expansion for solid vessel)

So, the change in volume of liquid relative to vessel

$V'_L - V'_S = V(\gamma_L - \gamma_S)\Delta\theta$

$\Delta V_{app} = V\gamma_{app}\Delta\theta$ ($\gamma_{app} = \gamma_L - \gamma_s$ = Apparent coefficient of volume expansion for liquid)

$\gamma_L > \gamma_s$	$\gamma_{app} > 0$	ΔV_{app} = positive	Level of liquid in vessel will rise on heating.
$\gamma_L < \gamma_s$	$\gamma_{app} < 0$	ΔV_{app} = negative	Level of liquid in vessel will fall on heating.
$\gamma_L = \gamma_s$	$\gamma_{app} = 0$	$\Delta V_{app} = 0$	Level of liquid in vessel will remain same.

ILLUSTRATION 13.7 A 250 cm^3 glass bottle is completely filled with water at 50°C. The bottle and water are heated to 60°C. How much water runs over if:

(a) the expansion of the bottle is neglected;

(b) the expansion of the bottle is included? Given the coefficient of areal expansion of glass $\alpha = 1.2 \times 10^{-5}$/K and $\gamma_{water} = 60 \times 10^{-5}$/°C.

Solution. Water overflow = (Final volume of water) − (Final volume of bottle)

(a) If the expansion of bottle is neglected:

Water overflow $= 250(1 + \gamma_l\theta) - 250$

$= 250 \times 60 \times 10^{-5} \times 10$

$\Rightarrow$ Water overflow = 1.5 cm^3

(b) If the bottle (glass) expands:

Water overflow

= (final volume of water) − (final volume of glass)

$= 250(1 + \gamma_l\theta) - 250(1 + \gamma_g\theta)$

$= 250(\gamma_l - \gamma_g)\theta$, where $\gamma_g = 3/2\beta = 1.8 \times 10^{-5}/°C$

$= 250(58.2 \times 10^{-5}) \times (60 - 50)$

Water overflow = 1.455 cm^3

Effect of Temperature on Upthrust

The thrust on volume V of a body in a liquid of density σ is given by $Th = V\sigma g$

Now with rise in temperature by $\Delta\theta$ °C, due to expansion, volume of the body will increase while density of liquid will decrease according to the relations $V' = V(1 + \gamma_S\Delta\theta)]$ and $\sigma' = \sigma/(1 + \gamma_L\Delta\theta)$

So the thrust will become $Th' = V'\sigma' g$

$\therefore \frac{Th'}{Th} = \frac{V'\sigma' g}{V\sigma g} = \frac{(1+\gamma_S\Delta\theta)}{(1+\gamma_L\Delta\theta)}$

and apparent weight of the body W_{app} = actual weight − thrust

As $\gamma_1 < \gamma_2$, therefore, $Th' < Th$ with rise in temperature thrust also decreases and apparent weight of body increases.

ILLUSTRATION 13.8 A solid floats in a liquid at 20°C with 75% of it immersed. When the liquid is heated to 100°C, the same solid floats with 80% of it immersed in the liquid. Calculate the coefficient of expansion of the liquid. Assume the volume of the solid to be constant.

Solution. Let m be the mass of the solid and V its volume. By the law of flotation

Weight of floating object = Buoyant force

In Case I: $mg = \left(\frac{3}{4}V\right)\rho_{20}g$

where ρ_{20} = density of liquid at 20°C

In Case II: $mg = \left(\frac{80}{100}V\right)\rho_{100}g$.

where ρ_{100} = density of liquid at 100°C

Considering both the cases

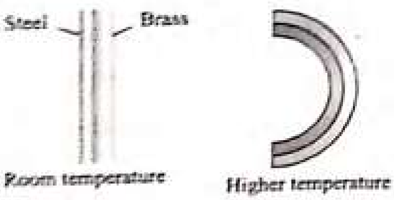
$$\Rightarrow \quad \frac{3}{4}\rho_{20} = \frac{4}{5}\rho_{100} \quad \Rightarrow \frac{3}{4}\frac{\rho_0}{1+\gamma \times 20} = \frac{4}{5}\frac{\rho_0}{1+\gamma \times 100}$$

After solving we get

$$\gamma = \frac{1}{1180} = 8.47 \times 10^{-4}/°C$$

Applications of Thermal Expansion

1. **Bi-metallic strip:** Two strips of equal lengths but of different materials (different coefficient of linear expansion) when join together, is called 'bi-metallic strip' and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater α on outer side, i.e., convex side.



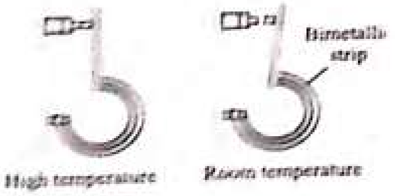
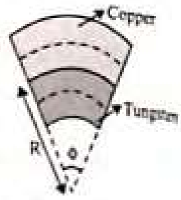


ILLUSTRATION 13.9 A copper and a tungsten plate having a thickness δ = 2 mm each are riveted together so that at 0°C they form a flat bimetallic plate. Find the average radius of curvature of this plate at t = 200°C. The coefficients of linear expansion for copper and tungsten are $\alpha_{Cu} = 1.7 \times 10^{-5}$/K and $\alpha_W = 0.4 \times 10^{-5}$ K, respectively.

Solution. The average length of copper plate at a temperature T = 200°C is $l_C = l_0(1+\alpha_C T)$.



where l_0 is the length of copper plate at 0°C. The length of the tungsten plate is $l_t = l_0(1+\alpha_t T)$

We shall assume that the edges of plates are not displaced during deformation and that an increase in the plate thickness due to heating can be neglected.

From figure we have

$$l_C = \phi(R+\delta/2) \quad \Rightarrow \quad l_t = \phi(R-\delta/2)$$

Consequently,

$$\phi(R+\delta/2) = l_0(1+\alpha_C T) \qquad \text{(i)}$$

$$\phi(R-\delta/2) = l_0(1+\alpha_t T) \qquad \text{(ii)}$$

To eliminate the unknown quantities, ϕ and l_0, we divide Eq. (i) by Eq. (ii) term-wise:

$$\Rightarrow \quad \frac{(R+\delta/2)}{(R-\delta/2)} = \frac{(1+\alpha_C T)}{(1+\alpha_t T)}$$

$$\Rightarrow \quad R = \delta\frac{[2+(\alpha_c+\alpha_t)T]}{[2(\alpha_c-\alpha_t)T]}$$

$$\Rightarrow \quad R = \frac{\delta}{(\alpha_C-\alpha_t)T}$$

neglecting $(\alpha_C+\alpha_t)$ in numerator. Substituting the values in above relation, we get: R = 0.769 m.

2. **Effect of temperature on the time period of a simple pendulum:** A pendulum clock keeps proper time at temperature θ. If temperature is increased to θ' $(> \theta)$ then due to linear expansion, length of pendulum and hence its time period will increase.

If l_0 be the length of the pendulum, at θ°C, then its time period

$$T_0 = 2\pi\sqrt{\frac{l_0}{g}} \qquad \text{(i)}$$

At any temperature increment $\Delta\theta$, the time period of the pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Here, $l = l_0(1+\alpha\Delta\theta)$

$$\therefore \quad T = 2\pi\sqrt{\frac{l_0(1+\alpha\Delta\theta)}{g}} = 2\pi\sqrt{\frac{l_0}{g}}(1+\alpha\Delta\theta)^{1/2}$$

$$= T_0(1+\alpha\Delta\theta)^{1/2} \qquad \text{(ii)}$$

$$= T_0\left(1+\frac{\alpha\Delta\theta}{2}\right) \quad \text{or} \quad \frac{T}{T_0}-1=\frac{\alpha\Delta\theta}{2}$$

$$\frac{T-T_0}{T_0}=\frac{\alpha\Delta\theta}{2} \quad \text{or} \quad \frac{\Delta T}{T_0}=\frac{\alpha\Delta\theta}{2}$$

$$\therefore \quad \Delta T=\left(\frac{\alpha\Delta\theta}{2}\right)T_0$$

NOTE:

- Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time.
- Loss of time in a time period $\Delta T = \frac{1}{2}\alpha\Delta\theta T_0$.
- $\therefore$ Loss of time in any given time interval t can be given by $\Delta t = \frac{1}{2}\alpha\Delta\theta t$
- The clock will lose time, i.e., it will become slow if $\theta' > \theta$ (in summer).
- It will gain time, i.e., it will become fast if $\theta' < \theta$ (in winter).
- The gain or loss in time is independent of time period T and depends on the time interval t.
- Time lost by the clock in a day (t = 86400 s)
- $\Delta t = \frac{1}{2}\alpha\Delta\theta t = \frac{1}{2}\alpha\Delta\theta\,(86400) = 43200\,\alpha\Delta\theta$ s
- Since coefficient of linear expansion (α) is very small for invar, pendulums are made of invar to show the correct time in all seasons.

ILLUSTRATION 13.10 A pendulum clock loses 12 s a day if the temperature is 40°C and goes fast by 4 s a day if the temperature is 20°C. Find the temperature at which the clock will show correct time and the coefficient of linear expansion of the metal of the pendulum shaft.

Solution. Let T be the temperature at which the clock is correct.

Time lost per day = 1/2 α (rise in temperature) × 86400

$$\Rightarrow \quad 12 = 1/2\alpha(40-T)\times 86400 \qquad \text{(i)}$$

Time gained per day = 1/2 α (drop in temperature) × 86400

$$\Rightarrow \quad 4 = 1/2\alpha(T-20)\times 86400 \qquad \text{(ii)}$$

Adding Eqs. (i) and (ii), we get

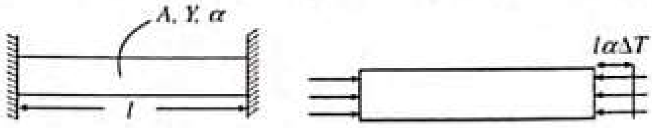
$$32 = 86400\alpha(40-20) \Rightarrow \alpha = 1.85\times 10^{-5}/°C$$

Dividing Eq. (i) by Eq. (ii), we get

$$12(T-20) = 4(40-T) \quad \Rightarrow T = 25°C$$

$\Rightarrow$ Clock shows correct time at 25°C.

3. **Thermal stress in a rigidly fixed rod:** When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports.



If temperature of rod is increased by ΔT, then change in length

$$\Delta l = l\alpha\,\Delta T$$

$$\text{strain} = \frac{\Delta l}{l} = \alpha\Delta T$$

But due to rigid support, there is no strain. Supports provide force or stresses to keep the length of rod same

$$Y = \frac{\text{stress}}{\text{strain}}$$

Thermal stress = Y (strain) = $Y\alpha\Delta T$

$$\frac{F}{A} = Y\alpha\Delta T \quad F = AY\alpha\,\Delta T$$

ILLUSTRATION 13.11 A rod of length 2 m is at a temperature of 20°C. Find the free expansion of the rod, if the temperature is increased to 50°C, then find stress produced when the rod is (i) fully prevented to expand, (ii) permitted to expand by 0.4 mm. $Y = 2\times 10^{11}$ N/m^2; $\alpha = 15\times 10^{-6}$/°C.

Solution. Free expansion of the rod = $\alpha L\Delta\theta$

$= 15\times 10^{-6}$/°C × 2 m × (50 − 20)°C

$= 9\times 10^{-4}$ m = 0.9 mm

(i) If the expansion is fully prevented,

$$\text{then strain} = \frac{9\times 10^{-4}}{2} \Rightarrow 4.5\times 10^{-4}$$

$\therefore$ Temperature stress = strain × Y

$= 4.5\times 10^{-4}\times 2\times 10^{11}$

$= 9\times 10^{7}$ N/m^2

(ii) If 0.4 mm expansion is allowed, then length restricted to expand = 0.9 − 0.4 = 0.5 mm

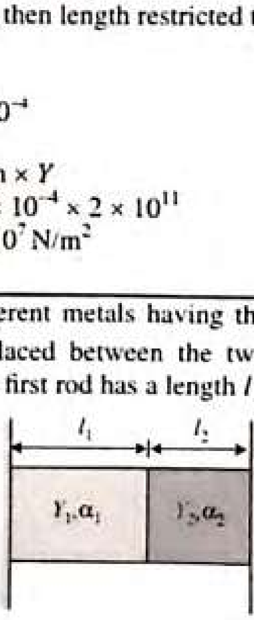
$$\therefore \quad \text{Strain} = \frac{5\times 10^{-4}}{2} = 2.5\times 10^{-4}$$

$\therefore$ Temperature stress = strain × Y

$= 2.5\times 10^{-4}\times 2\times 10^{11}$

$= 5\times 10^{7}$ N/m^2

ILLUSTRATION 13.12 Two rods of different metals having the same area of cross section A are placed between the two massive walls as shown in figure. The first rod has a length l_1, coefficient of linear expansion α_1 and Young's modulus Y_1. The corresponding quantities for second rod are l_2, α_2 and Y_2. The temperature of both the rods is now raised by t°C.



(i) Find the force with which the rods act on each other (at higher temperature) in terms of given quantities.

(ii) Also find the length of the rods at higher temperature.

Solution.

(i) Let t°C = increase in the temperature.

Increase in length of first rod = $l_1\alpha_1 t$

Increase in length of second rod = $l_2\alpha_2 t$

∴ Total extension in length due to rise in temperature

$$= l_1\alpha_1 t + l_2\alpha_2 t = (l_1\alpha_1 + l_2\alpha_2)t \quad \text{(i)}$$

Since the walls are rigid, this increase in length will not happen. This will be compensated by equal and opposite forces F. F producing decrease in the lengths of the rods due to elasticity.

∴ Decrease in length of first rod = $\dfrac{F \times l_1}{Y_1 \times A}$

And decrease in length of second rod = $\dfrac{F \times l_2}{Y_2 \times A}$

∴ Total decrease in length due to elastic force

$$= \frac{F}{A}\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right) \quad \text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{F}{A}\left(\frac{l_1}{Y_1} + \frac{l_2}{Y_2}\right) = (l_1\alpha_1 + l_2\alpha_2)t$$

or $$F = \frac{A(l_1\alpha_1 + l_2\alpha_2)t}{\left(\dfrac{l_1}{Y_1} + \dfrac{l_2}{Y_2}\right)} \quad \text{(iii)}$$

(ii) Length of the first rod = original length + increase in length due to temperature − decrease in length due to force

$$= \left(l_1 + l_1\alpha_1 t - \frac{F}{A}\frac{l_1}{Y_1}\right)$$

and length of second rod = $l_2 + l_2\alpha_2 t - \dfrac{F}{A}\dfrac{l_2}{Y_2}$

∴ The total length is same = $l_1 + l_2$ at all temperatures.

θ = temperature at the time of measurement and θ_C = temperature at the time of calibration.

Concept Application Exercise 13.2

1. A steel rod is 3.000 cm at 25°C. A brass ring has an interior diameter of 2.992 cm at 25°C. At what common temperature will the ring just slide on to the rod?
2. A clock with a metallic pendulum gains 5 s each day at a temperature of 15°C and loses 10 s each day at a temperature of 30°C. Find the coefficient of thermal expansion of the pendulum metal.
3. The design of some physical instrument requires that there be a constant difference in length of 10 cm between an iron rod and a copper cylinder laid side by side at all temperatures. Find their lengths.
   ($\alpha_{Fe} = 11 \times 10^{-6}$ °C^{-1}, $\alpha_{Cu} = 17 \times 10^{-6}$ °C^{-1})
4. A metal rod of 30 cm length expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal of length 45 cm expands by 0.045 cm for the same rise in temperature. A composite rod C made by joining A and B end to end expands by 0.040 cm when its length is 45 cm and it is heated from 0°C to 50°C. Find the length of each portion of the composite rod.
5. A brass scale is graduated at 10°C. What is the true length of a zinc rod which measures 60.00 cm on this scale at 30°C?
   Coefficient of linear expansion of brass = 18×10^{-6} K^{-1}.
6. A long horizontal glass capillary tube open at both ends contains a mercury thread 1 m long at 0°C. Find the length of the mercury thread, as read on this scale, at 100°C.
   (Given, $\gamma_{mercury} = 18.2 \times 10^{-5}$ K^{-1} and $\alpha_{glass} = 9 \times 10^{-6}$ K^{-1})

TRANSMISSION OF HEAT

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any of the following modes.

Conduction	Convection	Radiation
Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place	Each particle absorbing heat is mobile	Heat flows without any intervening medium in the form of electromagnetic waves
Medium is necessary for conduction	Medium is necessary for convection	Medium is not necessary for radiation
It is a slow process	It is also a slow process	It is a very fast process
Path of heat flow may be zig-zag	Path may be zig-zag or curved	Path is a straight line
Conduction takes place in solids	Convection takes place in fluids	Radiation takes place in gaseous and transparent media
The temperature of the medium increases through which heat flows	In this process also the temperature of medium increases	There is no change in the temperature of the medium

Conduction

The process of transmission of heat energy in which the heat is transferred from one particle to other without dislocation of the particles from their equilibrium position is called conduction.

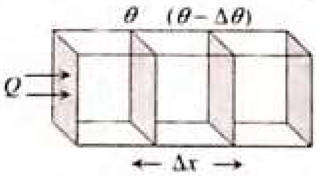
i. Conduction is a process which is possible in all states of matter.

ii. In solids only conduction takes place.

iii. In non-metallic solids and fluids the conduction takes place only due to vibrations of molecules; therefore, they are poor conductors.

iv. In metallic solids free electrons carry the heat energy; therefore, they are good conductors of heat.

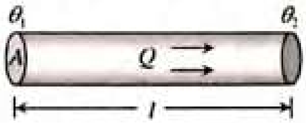
1. **Temperature gradient:** The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient.
If the temperature of two isothermal surfaces be θ and $(\theta - \Delta\theta)$ and the perpendicular distance between them be Δx, then temperature gradient $= \dfrac{(\theta - \Delta\theta) - \theta}{\Delta x} = \dfrac{-\Delta\theta}{\Delta x}$



The negative sign shows that temperature θ decreases as the distance x increases in the direction of heat flow.
Unit: K/m (SI); dimension: $[L^{-1}\theta]$

2. **Coefficient of thermal conductivity:** If L be the length of the rod, A the area of cross section and θ_1 and θ_2 be the temperatures of its two faces, then the amount of heat flowing from one face to the other face in time t is

$Q \propto A$
$\propto (\theta_1 - \theta_2)$
$\propto t$
$\propto \dfrac{1}{l}$



In combined form

$$Q \propto \frac{A(\theta_1 - \theta_2)t}{l}$$

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$

where K is coefficient of thermal conductivity of material of rod. It is the measure of the ability of a substance to conduct heat through it.
This relation can also be expressed as

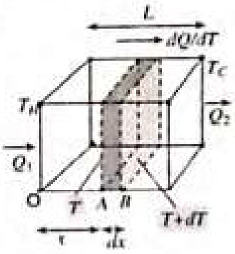
$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{l}$$

If $A = 1\ \text{m}^2$, $(\theta_1 - \theta_2) = 1°\text{C}$, $t = 1$ s and $l = 1$ m, then $Q = K$.
Thus, thermal conductivity of a material is the amount of heat flowing per second during steady state through its rod of length 1 m and cross section 1 m^2 with a unit temperature difference between the opposite faces.

i. Units: cal/cm-s°C (in CGS), kcal/m-s-K (in MKS) and W/m-K (in SI)
ii. Dimension: $[MLT^{-3}\theta^{-1}]$
iii. The magnitude of K depends only on nature of the material.
iv. For perfect conductors, $K = \infty$ and for perfect insulators, $K = 0$
v. Substances in which heat flows quickly and easily are known as good conductors of heat. They possess large thermal conductivity due to large number of free electrons. Example: silver, brass, etc.
vi. Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons. Example: glass, wood, etc.
vii. The thermal conductivity of pure metals decreases with rise in temperature but for alloys thermal conductivity increases with increase of temperature.
viii. Human body is a bad conductor of heat (but it is a good conductor of electricity).

Consider a slab of face area A, lateral thickness L, whose faces have temperatures T_H and T_C $(T_H > T_C)$.

Now consider two cross sections in the slab at positions A and B separated by a lateral distance of dx. Let temperature of face A be T and that of face B be $T + dT$. Then the rate at which heat crosses the area A of the slab at position x in time t is given by



$$\frac{dQ}{dt} = -KA\frac{dT}{dx} \qquad \text{(i)}$$

Here K is a constant depending on the material of the slab and is named thermal conductivity of the material, and the quantity $\left(\dfrac{dT}{dx}\right)$ is called temperature gradient. The (−) sign in equation shows heat flows from high to low temperature (dT is a negative quantity).

ILLUSTRATION 13.13 An ordinary refrigerator is thermally equivalent to a box of corkboard 90 mm thick and 5.6 m^2 in inner surface area. When the door is closed, the inside wall is kept, on the average, 22.2°C below the temperature of the outside wall. If the motor of the refrigerator runs 15% of the time while the door is closed, at what rate must heat be taken from the interior while the motor is running? The thermal conductivity of corkboard is $k = 0.05$ W/mK.

Solution. Consider a time interval Δt during which the door is closed. As approximation, take the heat conduction to be steady over Δt.
Then the rate of heat into the box is

$$\frac{\Delta Q}{\Delta t} = kA\left(\frac{\Delta T}{\Delta x}\right) = (0.05)(5.6)\left(\frac{22.2}{0.090}\right) = 69.1\ \text{W}$$

To remove this heat, the motor must, since it runs only for a time $(0.15)\,\Delta t$, cause heat to leave at the rate $69.1/0.15 = 460$ W.

ILLUSTRATION 13.14 Water is being boiled in flat bottom kettle placed on a stove. The area of the bottom is 3000 cm^2 and the thickness is 2 mm. If the amount of steam produced is 1 g/min, calculate the difference of temperature between the inner and outer surface of the bottom. K for the material of kettle is 0.5 cal/°C/s/cm, and the latent heat of steam is 540 cal/g.

Solution. Mass of steam produced $= \dfrac{dm}{dt} = \dfrac{1}{60}$ g/s

Heat transferred per second

$$= \frac{dH}{dt} = L\frac{dm}{dt} \Rightarrow \frac{dH}{dt} = 540 \times \frac{1}{60} \text{ cal/s} = 9 \text{ cal/s}$$

Area = 3000 cm^2; K = 0.5 cal/°C/s/cm

θ = temperature difference

d = thickness = 2 mm = 0.2 cm

$$\frac{dH}{dt} = \frac{KA\theta}{d} \Rightarrow L\frac{dm}{dt} = \frac{KA\theta}{d}$$

$$\Rightarrow \quad 9 = \frac{0.5 \times 3000 \times \theta}{0.2} \Rightarrow \theta = 1.2 \times 10^{-3}\text{°C}$$

3. **Thermometric conductivity or diffusivity:** It is a measure of rate of change of temperature (with time) when the body is not in steady state (i.e., in variable state). The thermometric conductivity or diffusivity is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material.

Thermal capacity per unit volume = $\frac{mc}{V} = \rho c$ (as ρ is density of substance)

$\therefore$ Diffusivity (D) = $\frac{K}{\rho c}$

Unit: m^2/s; dimension: $[L^2T^{-1}]$

4. **Thermal resistance:** The thermal resistance of a body is a measure of its opposition to the flow of heat through it. It is defined as the ratio of temperature difference to the heat current (= rate of flow of heat).
Now, temperature difference = $(\theta_1 - \theta_2)$ and heat current,

$$H = \frac{Q}{t}$$

$\therefore$ Thermal resistance

$$R = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{Q/t} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/l} = \frac{l}{KA}$$

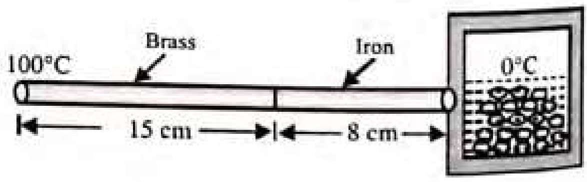
Unit: °C × s/cal or K × s/kcal; dimension: $[M^{-1}L^{-2}T^{3}\theta]$

Electrical Analogy for Thermal Conduction

It is an important fact to appreciate that there exists an exact similarity between thermal and electrical conductivities of a conductor.

Electrical conduction	Thermal conduction
Electric charge flows from higher potential to lower potential	Heat flows from higher temperature to lower temperature
The rate of flow of charge is called the electric current, i.e., $I = \frac{dq}{dt}$	The rate of flow of heat may be called heat current i.e., $H = \frac{dQ}{dt}$
The relation between the electric current and the potential difference is given by Ohm's law, i.e., $I = \frac{V_1 - V_2}{R}$ where R is the electrical resistance of the conductor	Similarly, the heat current may be related with the temperature difference as $H = \frac{\theta_1 - \theta_2}{R}$ where R is the thermal resistance of the conductor
The electrical resistance is defined as $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$, where ρ = resistivity and σ = electrical conductivity	The thermal resistance may be defined as $R = \frac{l}{KA}$. where K = Thermal conductivity of conductor
$\frac{dq}{dt} = I = \frac{V_1 - V_2}{R} = \frac{\sigma A}{l}(V_1 - V_2)$	$\frac{dQ}{dt} = H = \frac{\theta_1 - \theta_2}{R} = \frac{KA}{l}(\theta_1 - \theta_2)$

ILLUSTRATION 13.15 One end of a uniform brass rod 15 cm long and 20 cm^2 cross-sectional area is kept at 100°C. The other end is at prefect contact with an iron rod of identical cross section, but length 8 cm. The lateral surface of the composite rod is surrounded by a heat insulator and the free end of the iron rod is kept in ice at 0°C. If 684 g of ice melts in 1 h, determine the thermal conductivity of iron. Thermal conductivity of brass = 0.25 cal/s-cm°C and latent heat of fusion of ice = 80 cal/g.



Solution. Let the thermal conductivity of iron be K = cal/s-cm-°C.

The thermal resistance of brass rod

$$R_1 = \frac{l_1}{K_1 A} = \frac{15\text{cm}}{\frac{0.25 \text{ cal}}{\text{s-cm°C}} \times 20 \text{ cm}^2} = 3\text{s-°C/cal}$$

And that of iron rod

$$R_2 = \frac{l_2}{K_2 A} = \frac{8 \text{ cm}}{\frac{K \text{ cal}}{\text{s-cm°C}} \times 20 \text{ cm}^2} = \frac{2}{5K} \text{ s-°C/cal}$$

Since, the two rods are arranged in series, their effective

thermal resistance is given by

$$R = R_1 + R_2 = \left(3 + \frac{2}{5K}\right)\text{s-}^\circ\text{C/cal}$$

Now, rate of heat flow through the rods $= \dfrac{\Delta Q}{\Delta t}$

But $\dfrac{\Delta Q}{\Delta t} = \dfrac{(684\ \text{g})}{3600\ \text{s}}\left(80\ \dfrac{\text{cal}}{\text{g}}\right) = 15.2\ \text{cal/s}$

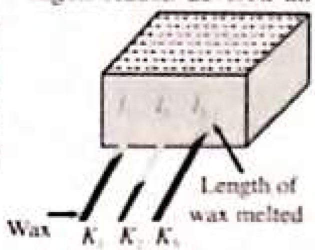
Since $\dfrac{\Delta Q}{\Delta t} = \dfrac{(T_1 - T_2)}{R}$ $\qquad R = \dfrac{(T_1 - T_2)}{(\Delta Q / \Delta t)}$

$$\left(3 + \frac{2}{5K}\right)\text{s}^\circ\text{C/cal} = \frac{(100-0)^\circ\text{C}}{15.2\ \text{cal/s}}$$

$$K = 111.8 \times 10^{-3}\ \text{cal/s-cm-}^\circ\text{C}$$

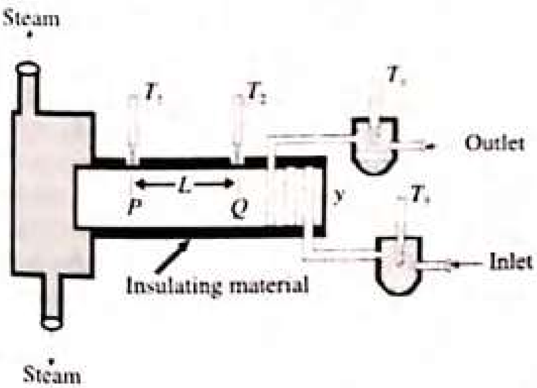
Determination of Thermal Conductivity

1. **Ingen-Hausz experiment:** Ingen-Hausz devised an experiment to compare the thermal conductivities of the metals. If l_1, l_2, …, are the lengths of wax melted on the metal rods, then the ratio of thermal conductivities is



$K_1 : K_2 : K_3 : \ldots = l_1^2 : l_2^2 : l_3^2 : \ldots$

2. **Searle's experiment:** It is a method of determination of K of a metallic rod. Here we are not much interested in the detailed description of the experimental setup. We will only understand its essence, which is the essence of solving many numerical problems.



In this experiment a temperature difference $(T_1 - T_2)$ is maintained across a rod of length L and area of cross section A. If the thermal conductivity of the material of the rod is K, then the amount of heat transmitted by the rod from the hot end to the cold end in time t is given by

$$Q = \frac{KA(T_1 - T_2)t}{L} \qquad \text{(i)}$$

In Searle's experiment, this heat reaching the other end is utilized to raise the temperature of certain amount of water flowing through pipes circulating around the other end of the rod. If temperature of the water at the inlet is T_3 and at the outlet is T_4, then the amount of heat absorbed by water is given by

$$Q = mc(T_4 - T_3) \qquad \text{(ii)}$$

where m is the mass of the water which has absorbed this heat and temperature is raised and c is the specific heat of the water

Equating (i) and (ii), K can be determined, i.e.,

$$K = \frac{mc(T_4 - T_3)l}{A(T_1 - T_2)t}$$

NOTE: In numerical problems, we may have the situation where the amount of heat travelling to the other end may be required to do some other work, e.g., it may be required to melt the given amount of ice. In that case Eq. (i) will have to be equated to mL.

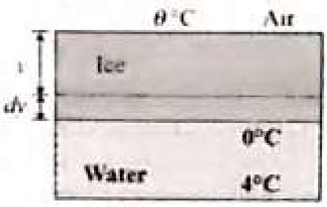
i.e., $\qquad mL = \dfrac{KA(T_1 - T_2)t}{l}$

GROWTH OF ICE ON LAKE

Water in a lake starts freezing if the atmospheric temperature drops below 0°C. Let y be the thickness of ice layer in the lake at any instant t and atmospheric temperature be $-\theta$ °C. The temperature of water in contact with lower surface of ice will be zero. If A is the area of lake, heat escaping through ice in time dt is

$$dQ_1 = \frac{KA[0 - (-\theta)]dt}{y}$$

Now, suppose the thickness of ice layer increases by dy in time dt, due to escaping of above heat. Then



$$dQ_2 = mL = \rho(dy\,A)L$$

As $dQ_1 = dQ_2$, hence, rate of growth of ice will be $(dy/dt) = (K\theta/\rho L y)$

So, the time taken by ice to grow to a thickness y is

$$t = \frac{\rho L}{K\theta}\int_0^y y\,dy = \frac{\rho L}{2K\theta}y^2$$

If the thickness is increased from y_1 to y_2, then time taken

$$t = \frac{\rho L}{K\theta}\int_{y_1}^{y_2} y\,dy = \frac{\rho L}{2K\theta}(y_2^2 - y_1^2)$$

i. Take care and do not apply a negative sign for putting values of temperature in formula and also do not convert it to absolute scale.
ii. Ice is a poor conductor of heat; therefore, the rate of increase of thickness of ice on ponds decreases with time.
iii. It follows from the above equation that time taken to double and triple the thickness, will be in the ratio of $t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2$, ie., $t_1 : t_2 : t_3 :: 1 : 4 : 9$

iv. The time intervals to change the thickness from 0 to y, from y to $2y$ and so on will be in the ratio
$\Delta t_1 : \Delta t_2 : \Delta t_3 :: (1^2 - 0^2):(2^2 - 1^2):(3^2 : 2^2)$;
$\Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$

RADIATION

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

Precisely it is electromagnetic energy transfer in the form of electromagnetic wave through any medium. It is possible even in vacuum.

For example, the heat from the sun reaches the earth through radiation.

Reflectance, Absorptance and Transmittance

When thermal radiations (Q) fall on a body, they are partly reflected, partly absorbed and partly transmitted.

1. **Reflectance or reflecting power (r):** It is defined as the ratio of the amount of thermal radiations reflected (Q_r) by the body in a given time to the total amount of thermal radiations incident on the body in that time.
2. **Absorptance or absorbing power (a):** It is defined as the ratio of the amount of thermal radiations absorbed (Q_a) by the body in a given time to the total amount of thermal radiatons incident on the body in that time.
3. **Transmittance or transmitting power (t):** It is defined as the ratio of the amount of thermal radiations transmitted (Q_t) by the body in a given time to the total amount of thermal radiations incident on the body in that time.

From the above definitions $r = \frac{Q_r}{Q}$, $a = \frac{Q_a}{Q}$ and $t = \frac{Q_t}{Q}$. By adding we get

$$r + a + t = \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = \frac{(Q_r + Q_a + Q_t)}{Q} = 1$$

$\therefore \quad r + a + t = 1$

(a) r, a and t all are the pure ratios; so, they have no unit and dimension.
(b) For perfect reflector, $r = 1$, $a = 0$ and $t = 0$.
(c) For perfect absorber, $a = 1$, $r = 0$ and $t = 0$ (perfectly black body).
(d) For perfect transmitter, $t = 1$, $a = 0$ and $r = 0$.
(e) If body does not transmit any heat radiation, $t = 0$ $\therefore r + a = 1$ or $a = 1 - r$.
So, if r is more, a is less and vice versa. It means good reflectors are bad absorbers.

4. **Monochromatic emittance or spectral emissive power:** For a given surface it is defined as the radiant energy emitted per second per unit area of the surface with in a unit wavelength around λ.

$$\text{Spectral emissive power } (E_\lambda) = \frac{\text{Energy}}{\text{Area} \times \text{time} \times \text{wavelength}}$$

Unit: $\frac{\text{J}}{\text{m}^2 \times \text{s} \times \text{Å}}$; dimension: $[ML^{-1}T^{-3}]$

5. **Total emittance or total emissive power:** It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all possible wavelengths.

$$E = \int_0^\infty E_\lambda d\lambda$$

Unit: $\frac{\text{J}}{\text{m}^2 \times \text{s}}$ or $\frac{\text{Watt}}{\text{m}^2}$; dimension: $[MT^{-3}]$

6. **Monochromatic absorptance or spectral absorptive power:** It is defined as the ratio of the amount of the energy absorbed in a certain time to the total heat energy incident upon it in the same time, both in the unit wavelength interval. It is dimensionless and unitless quantity. It is represented by a_λ.
7. **Total absorptance or total absorpting power:** It is defined as the total amount of thermal energy absorbed per unit time, per unit area of the body for all possible wavelengths.

$$a = \int_0^\infty a_\lambda d\lambda$$

It is also unitless and dimensionless quantity.

8. **Emissivity (e):** Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body ($E_{\text{practical}}$) to the total emissive power of a perfect black body (E_{black}) at that temperature.

$$e = \frac{E_{\text{practical}}}{E_{\text{black}}}$$

$e = 1$ for perfectly black body but for practical bodies emissivity (e) lies between zero and one ($0 < e < 1$).

9. **Perfectly black body:** A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it. As a perfectly black body neither reflects nor transmits any radiation, the absorptance of a perfectly black body is unity, i.e., $t = 0$ and $r = 0$, therefore, $a = 1$

We know that the colour of an opaque body is the colour (wavelength) of radiation reflected by it. As a black body reflects no wavelength so, it appears black, whatever be the colour of radiations incident on it.

When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. For example, temperature of the sun is very high (6000 K approx.). It emits all possible radiation so it is an example of black body.

PREVOST'S THEORY OF HEAT EXCHANGE

1. Everybody emits heat radiations at all finite temperatures (Except 0 K) as well as it absorbs radiations from the surroundings.
2. Exchange of energy along various bodies takes place via radiation.

3. The process of heat exchange among various bodies is a continuous phenomenon.
4. If the amount of radiation absorbed by a body is greater than that emitted by it then the temperature of body increases and it appears hotter.
5. If the amount of radiation absorbed by a body is less than that emitted by it, then the temperature of the body decreases and consequently the body appears colder.
6. If the amount of radiation absorbed by a body is equal to that emitted by the body, then the body will be in thermal equilibrium and the temperature of the body remains constant.
7. At absolute zero temperature (0 K or –273°C) this law is not applicable because at this temperature the heat exchange among various bodies ceases.

KIRCHHOFF'S LAW

The ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

Experimentally a very good approximation to a black body is provided by a cavity enclosed by high temperature opaque walls regardless of the composition of the material of its interior walls. Thus, if $a_{\text{practical}}$ and $E_{\text{practical}}$ represent the absorptive and emissive power of a given surface, while a_{black} and E_{black} for a perfectly black body, then according to law

$$\frac{E_{\text{practical}}}{a_{\text{practical}}}=\frac{E_{\text{black}}}{a_{\text{black}}}$$

But for a perfectly black body, $a_{\text{black}} = 1$

So $\dfrac{E_{\text{practical}}}{a_{\text{practical}}}=E_{\text{black}}$

If emissive and absorptive powers are considered for a particular wavelength λ,

$$\left(\frac{E_\lambda}{a_\lambda}\right)_{\text{practical}}=(E_\lambda)_{\text{black}}$$

Now since $(E\lambda)_{\text{black}}$ is constant at a given temperature, according to this law if a surface is a good absorber of a particular wavelength it is also a good emitter of that wavelength.

This in turn implies that a good absorber is a good emitter (or radiator).

ILLUSTRATION 13.16 An electric heater of surface area 200 cm² emits radiant energy of 60 kJ at time interval of 1 min. Determine its emissive power. If its emissivity be 0.45, what would be the radiant energy emitted by a black body in one hour, identical to the electrical heater in all respects?

Solution. Given $\Delta Q = 60$ kJ; $A = 200\times10^{-4}$ m² and $\Delta t = 60$ s

Using, emissive power $E=\left(\dfrac{\Delta Q}{\Delta t}\right)\dfrac{1}{A}$, we have

$$E=\frac{60\,\text{kJ}}{200\times10^{-4}\,\text{m}^2\times60\,\text{s}}=50\ \text{kW/m}^2$$

∴ Emissivity (of a surface)

$$e=\frac{\text{emissive power (of that surface)}}{\text{emissive power (of a black body)}}$$

∴ Emissive power of a black body

$$=\frac{50}{0.45}\,\text{kW/m}^2=111.11\ \text{kW/m}^2$$

∴ Heat radiated $\Delta Q = EA\Delta t$

$$=\left(111.11\frac{\text{kW}}{\text{m}^2}\right)(200\times10^{-4}\,\text{m}^2)(60\times60\,\text{s})=8000\,\text{kJ}$$

STEFAN'S LAW

According to it the radiant energy emitted by a perfectly black body per unit area per second (i.e. emissive power of black body) is directly proportional to the fourth power of its absolute temperature, i.e.,

$$E\propto T^4 \quad\text{or}\quad E=\sigma T^4$$

where σ is a constant called Stefan's constant having dimension $(MT^{-3}\theta^{-4})$ and value $5.67\times10^{-8}\,\text{W/m}^2\text{K}^4$.

(i) If e is the emissivity of the body then $E=e\sigma T^4$.

(ii) If Q is the total energy radiated by the body then
$$E=\frac{Q}{A\times t}=e\sigma T^4 \quad\Rightarrow Q=Ate\sigma T^4$$

(iii) If a body at temperature T is surrounded by a body at temperature T_0, then Stefan's law may be put as
$$E=e\sigma(T^4-T_0^4)$$

(iv) Cooling by radiation: If a body at temperature T is in an environment of temperature $T_0(<T)$, the body is losing as well as receiving heat; so the net rate of loss of energy
$$\frac{dQ}{dt}=eA\sigma(T^4-T_0^4) \qquad \text{(i)}$$
Now if m is the mass of body and c its specific heat, the rate of loss of heat at temperature T must be
$$\frac{dQ}{dt}=mc\frac{dT}{dt} \qquad \text{(ii)}$$
From Eqs. (i) and (ii), $mc\dfrac{dT}{dt}=eA\sigma(T^4-T_0^4)$

∴ Rate of fall of temperature or rate of cooling
$$\frac{dT}{dt}=\frac{eA\sigma}{mc}(T^4-T_0^4) \qquad \text{(iii)}$$
i.e., when a body cools by radiation the rate of cooling depends on

(a) Nature of radiating surface, i.e., greater the emissivity, faster will be the cooling.
(b) Area of radiating surface, i.e., greater the area of radiating surface, faster will be the cooling.
(c) Mass of radiating body, i.e.. greater the mass of radiating body slower will be the cooling.

(d) Specific heat of radiating body, i.e., greater the specific heat of radiating body slower will be cooling.
(e) Temperature of radiating body, i.e., greater the temperature of body faster will be cooling.
(f) Temperature of surrounding, i.e., greater the temperature of surrounding slower will be cooling.

ILLUSTRATION 13.17 The operating temperature of an incandescent bulb (with a tungsten filament) of power 60 W is 3000 K. If the surface area of the filament be 25 mm^2, find its emissivity e.

Solution. Given $T = 3000$ K; Power $= \dfrac{dQ}{dt} = 60$ W

Area $A = 25\times10^{-6}\,\text{m}^2$, $e = ?$

Using Eq. (i), we have, total radiation lost per second

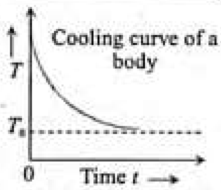
$$= \frac{dQ}{dt} = Ae\sigma T^4$$

$$60\text{ W} = (25\times10^{-6}\,\text{m}^2)e\left(5.67\times\frac{10^{-8}}{\text{m}^2-\text{K}^4}\right)(3000)^4\,\text{K}^4$$

$$60 = 114.82\,e \quad \text{or} \quad e = 0.52$$

NEWTON'S LAW OF COOLING

If in case of cooling by radiation the temperature T of body is not very different from that of surrounding



Then $\dfrac{dT}{dt} \propto \Delta T$ or $\dfrac{dT}{dt} \propto T - T_0$

i.e., if the temperature of body is not very different from surrounding, rate of cooling is proportional to temperature difference between the body and its surrounding. This law is called Newton's law of cooling.

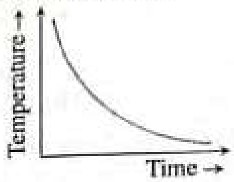
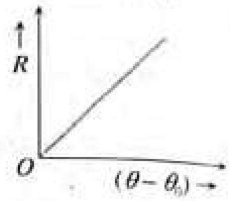
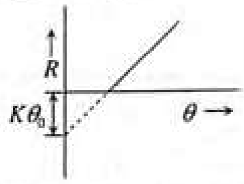
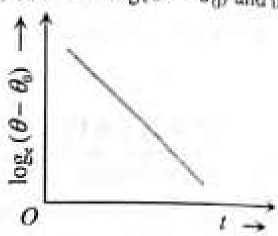
1. Practical examples:
   i. Hot water loses equal amount of heat in smaller duration as compared to moderate warm water.
   ii. Adding milk in hot tea reduces the rate of cooling.
2. Greater the temperature difference between body and its surrounding greater will be the rate of cooling.
3. If $\theta = \theta_0, \dfrac{dQ}{dt} = 0$, i.e., a body can never be cooled to a temperature lesser than its surrounding by radiation.
4. If a body cools by radiation from θ_1°C to θ_2°C in time t, then $\dfrac{d\theta}{dt} = \dfrac{\theta_1-\theta_2}{t}$ and $\theta_{av} = \dfrac{\theta_1+\theta_2}{2}$

   Then Newton's law of cooling becomes

   $$\left[\frac{\theta_1-\theta_2}{t}\right] = K\left[\frac{\theta_1+\theta_2}{2} - \theta_0\right]$$

   This form of law helps in solving numerical problems.

5. **Cooling curves:**

Curve between temperature of body θ and time.  $\theta - \theta_0 = Ae^{-kt}$, which indicates temperature decreases exponentially with increasing time.	Curve between rate of cooling (R) and temperature difference between body (θ) and surrounding (θ_0)  $R \propto (\theta - \theta_0)$. This is a straight line passing through origin.
Curve between the rate of cooling (R) and body temperature (θ).  $R = K(\theta - \theta_0) = K\theta - K\theta_0$ This is a straight line intercept R-axis at $-K\theta_0$	Curve between $\log(\theta - \theta_0)$ and time  As $\dfrac{d\theta}{dt} \propto -(\theta-\theta_0) \Rightarrow \dfrac{d\theta}{(\theta-\theta_0)} = -Kdt$ Integrating $\log_e(\theta-\theta_0) = -Kt + C$ $\log_e(\theta-\theta_0) = -Kt + \log_e A$ This is a straight line with negative slope.

6. **Determination of specific heat of a liquid:** If volume, radiating surface area, nature of surface, initial temperature and surrounding of water and given liquid are equal and they are allowed to cool down (by radiation), then rate of loss of heat and fall in temperature of both will be same.

   i.e., $\left(\dfrac{dQ}{dt}\right)_{\text{water}} = \left(\dfrac{dQ}{dt}\right)_{\text{liquid}}$

   $$(ms+W)\frac{(\theta_1-\theta_2)}{t_1} = (m_1s_1+W)\frac{(\theta_1-\theta_2)}{t_2}$$

   or $$\left[\frac{ms+W}{t_1}\right] = \left[\frac{m_1s_1+W}{t_2}\right]$$

   (where W = waterequivalent of calorimeter)
   If densities of water and liquid are ρ and ρ', respectively, then $m = V\rho$ and $m' = V\rho'$.

ILLUSTRATION 13.18 A body cools down from 60°C to 55°C in 30 s. Using Newton's law of cooling, calculate the time taken by same body to cool down from 55°C to 50°C. Assume that the temperature of surrounding is 45°C.

Solution. Assume that a body cools down from temperature θ_i to θ_f in t seconds, and θ_c is the temperature of surroundings. Applying Newton's law of cooling,

According to Newton's law of cooling

$\dfrac{d\theta}{dt} = -K(\theta-\theta_0)$; $\theta_0 = 45$°C; Integrating both sides $\int\dfrac{d\theta}{\theta-45} = -\int dt$

(K is constant)

from $t = 0$ s to $t = 30$ s, θ changes from 60°C to 55°C.

$$\int_{60}^{55} \frac{d\theta}{\theta-45} = -K\int_{0}^{30} dt \quad \Rightarrow \ln\left(\frac{50-45}{60-45}\right) = -K(30-0) \qquad \text{(i)}$$

$$\int_{55}^{50} \frac{d\theta}{\theta-45} = -K\int_{30}^{t} dt \quad \Rightarrow \ln\left(\frac{50-45}{60-45}\right) = -K(t-30) \qquad \text{(ii)}$$

Divide Eq. (ii) by Eq. (i) to set:

$$\frac{t-30}{30} = \frac{\ln\frac{50-45}{55-45}}{\ln\frac{55-45}{60-45}} \Rightarrow \frac{t-30}{30} = \frac{\ln 2}{\ln 3/2}$$

$$\Rightarrow \quad t = 30\left(1+\frac{\ln 2}{\ln 3/2}\right) = 81.28 \text{ s}$$

time from $\theta = 55$°C to $\theta = 50$°C is $(t - 30) = (81.28 - 30)$
$= 51.28$ s.

DISTRIBUTION OF ENERGY IN THE SPECTRUM OF BLACK BODY

Langley and later on Lummer and Pringsheim investigated the distribution of energy amongst the different wavelengths in the thermal spectrum of a black body radiation. The results obtained are shown in figure. From these curves it is clear that

(1) At a given temperature energy is not uniformly distributed among different wavelengths.

(2) At a given temperature intensity of heat radiation increases with wavelength, reaches a maximum at a particular wavelength and with further increase in wavelength it decreases.

(3) With increase in temperature wavelength λ_m corresponding to most intense radiation decreases in such a way that $\lambda_m \times T$ = constant [Wien's law].

(4) For all wavelengths an increase in temperature causes an increase in intensity.

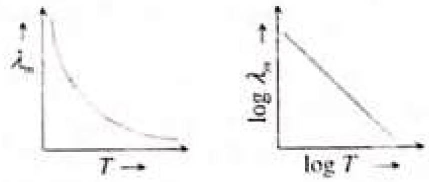
(5) The area under the curve $= \int E_\lambda d\lambda$ will represent the total intensity of radiation at a particular temperature. This area increases with rise in temperature of the body. It is found to be directly proportional to the fourth power of absolute temperature of the body, i.e.,

$$E = \int E_\lambda d\lambda \propto T^4 \quad \text{[Stefan's law]}$$

WIEN'S DISPLACEMENT LAW

When a body is heated it emits radiations of all wavelengths. However, the intensity of radiations of different wavelengths is different.

According to Wien's law the product of wavelengths corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant, i.e., $\lambda_m T = b$ = constant where b is Wien's constant and has value 2.89×10^{-3} m-K.



This law is of great importance in 'Astrophysics' as through the analysis of radiations coming from a distant star, by finding λ_m, the temperature of the star $T(= b/\lambda_m)$ is determined.

ILLUSTRATION 13.19 A hot black body emits the energy at the rate of 16 J m^{-2} s^{-1} and its most intense radiation corresponds to 20,000 Å. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000 Å, then find the value of energy radiated in Jm^{-2} s^{-1}.

Solution. Wien's displacement law is

$$\lambda_m T = b \quad \text{i.e.} \quad T \propto \frac{1}{\lambda_m}$$

Here, λ_m becomes half, so temperature doubles.

Also $\quad e = \sigma T^4$

$$\Rightarrow \quad \frac{e_1}{e_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$\Rightarrow \quad e_2 = \left(\frac{T_1}{T_2}\right)^4 e_1 = (2)^4 \times 16 = 16 \times 16 = 256 \text{ J m}^{-2}\text{ s}^{-1}$$

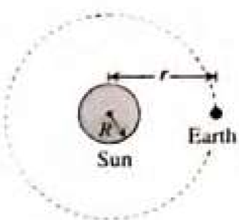
TEMPERATURE OF THE SUN AND SOLAR CONSTANT

If R is the radius of the sun and T its temperature, then the energy emitted by the sun per second through radiation in accordance with Stefan's law will be given by

$$P = eA\sigma T^4 = 4\pi R^2 \sigma T^4$$

[for sun $e = 1$]

In reaching earth, this energy will spread over a sphere of radius r (= average distance between sun and earth); so the intensity of solar radiation at the surface of earth (called solar constant S) will be given by



$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$

$$\text{i.e.,} \quad T = \left[\left(\frac{r}{R}\right)^2 \frac{S}{\sigma}\right]^{1/4} = \left[\left(\frac{1.5\times10^8}{7\times10^5}\right)^2 \times \frac{1.4\times10^3}{5.67\times10^{-8}}\right]^{1/4} = 5800 \text{ K}$$

As $\quad r = 1.5\times10^8$ km, $R = 7\times10^5$ km, $S = 2\dfrac{\text{cal}}{\text{cm}^2\text{ min}} = 1.4\dfrac{\text{kW}}{\text{m}^2}$

and $\sigma = 5.67\times10^{-8}\dfrac{W}{m^2K^4}$

This result is in good agreement with the experimental value of temperature of sun, i.e., 6000 K.

The difference in the two values is attributed to the fact that sun is not a perfectly black body.

ILLUSTRATION 13.20 The intensity of solar radiation just outside the earth's atmosphere is measured to be 1.4 kW/m^2. If the radius of the sun 7×10^8 m, while the earth–sun distance is 150×10^6 km, then find

i. the intensity of solar radiation at the surface of the sun,
ii. the temperature at the surface of the sun assuming it to be a black body,
iii. the most probable wavelength in solar radiation.

Solution. Assuming the sun to be a 'blackbody' at a temperature T_0, we can write,

W = intensity of solar radiation on the sun's surface $= \sigma T_0^4$,

where σ is the Stefan–Boltzmann constant.

i. The radiation emitted from the solar surface per unit time is spread over the surface of a sphere having a radius equal to earth–sun distance where it is received on the earth (just outside the atmosphere)

$$W\times 4\pi R_S^2 = I_0\times 4\pi D_{SE}^2$$

where D_{SE} is the distance between the sun and the earth, and I_0 is the intensity outside the earth's atmosphere.

$$I_0 = W\times\left(\frac{R_S}{D_{SE}}\right)^2$$

Now, $R_S = 7\times10^8\,\text{m}, D_{SE} = 150\times10^9\,\text{m}$

and $I_0 = 1.4\times10^3\ \text{W/m}^2$

$$\therefore\quad 1.4\times10^3 = W\times\left(\frac{7\times10^8}{150\times10^9}\right)^2 = W\times\frac{49}{225}\times10^{-4}$$

or $W = 6.4\times10^7\ \text{W/m}^2$

ii. Assuming the sun to be a black body,

$$6.4\times10^7 = \sigma T_0^4 = (5.67\times10^{-8})T_0^4$$

$$\therefore\quad T_0^4 = \frac{6.4}{5.67}\times10^{15}$$

or $T_0 = 0.58\times10^4\,\text{K} = 5800\ \text{K}$

iii. Using Wien's displacement law,

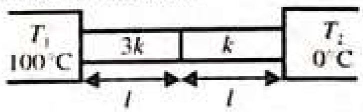
$$\lambda_{mp}T_0 = 0.29\ \text{cm-K} = 2.9\times10^{-3}\,\text{m-K}$$

or $\lambda_{mp} = \dfrac{2.0\times10^{-3}}{5800} = 5\times10^{-7}\,\text{m} = 5000\ \text{Å}$

CONCEPT APPLICATION EXERCISE 13.3

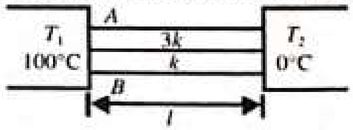
1. An electric heater is used in a room of total wall area 137 m^2 to maintain a temperature of +20°C inside it, when the outside temperature is −10°C. The walls have three different layers of materials. The innermost layer is of wood of thickness 2.5 cm, the middle layer is of cement of thickness 1.0 cm and the outermost layer is of brick of thickness 25.0 cm. Find the power of electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are 0.125, 1.5 and 1.0 W/m-°C, respectively.

2. Two rods A and B of same length and cross-sectional area are connected in series and a temperature difference of 100°C is maintained across the combination as shown in figure. If the thermal conductivity of the rod A is $3k$ and that of rod B is k, then



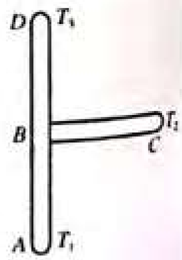
   (i) determine the thermal resistance of each rod.
   (ii) determine the heat current flowing through each rod.
   (iii) determine the heat current flowing through each rod.
   (iv) plot the variation of temperature along the length of the rod.

3. Two conductors A and B given in previous problem are connected in parallel as shown in figure.



   (i) Determine the equivalent thermal resistance.
   (ii) Determine the heat current in each rod.

4. Three rods AB, BC and BD having thermal conductivities in the ratio 1 : 2 : 3 and lengths in the ratio 2 : 1 :1 are joined as shown in figure. The ends A, C and D are at temperatures T_1, T_2 and T_3, respectively. Find the temperature of the junction B. Assume steady state.



5. A cube and a sphere of equal edge and radius, made of the same substance are allowed to cool under identical conditions. Determine which of the two will cool at a faster rate.

6. A spherical ball of radius 1 cm coated with a metal having emissivity 0.3 is maintained at 1000 K temperature and suspended in a vacuum chamber whose walls are maintained at 300 K temperature. Find rate at which electrical energy is to be supplied to the ball to keep its temperature constant.

SOLVED EXAMPLES

1. If there are no heat losses, the heat released by the condensation of x gm of steam at 100°C into water at 100°C can be used to convert y gm of ice at 0°C into water at 100°C. Then the ratio $y : x$ is nearly

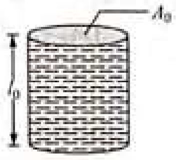
(a) 1 : 1 (b) 2.5 : 1
(c) 2 : 1 (d) 3 : 1

Sol. (d) Heat released to convert x gm of steam at 100°C to water at 100°C is $x \times 540$ cals.

If y gm of ice is converted from 0°C to water at 100°C it requires heat $y \times 80 + y \times 1 \times 100 = 180\,y$

$\therefore x \times 540 = 180y$ or $\frac{y}{x} = \frac{540}{180} = \frac{3}{1}$

2. The figure shows a glass tube (linear co-efficient of expansion is α) completely filled with a liquid of volume expansion co-efficient γ. On heating length of the liquid column does not change. Choose the correct relation between γ and α



(a) $\gamma = \alpha$ (b) $\gamma = 2\alpha$
(c) $\gamma = 3\alpha$ (d) $\gamma = \frac{\alpha}{3}$

Sol. (b) When length of the liquid column remains constant, then the level of liquid moves down with respect to the container, thus γ must be less than 3α.

Now we can write $V = V_0(1 + \gamma\Delta T)$

Since $V = Al_0 = [A_0(1 + 2\alpha\Delta T)]l_0 = V_0(1 + 2\alpha\Delta T)$

Hence $V_0(1 + \gamma\Delta T) = V_0(1 + 2\alpha\Delta T) \Rightarrow \gamma = 2\alpha$.

3. A piece of metal weight 46 gm in air, when it is immersed in the liquid of specific gravity 1.24 at 27°C it weighs 30 gm. When the temperature of liquid is raised to 42°C the metal piece weight 30.5 gm, specific gravity of the liquid at 42°C is 1.20, then the linear expansion of the metal will be

(a) 3.316×10^{-5}/°C (b) 2.316×10^{-5}/°C
(c) 4.316×10^{-5}/°C (d) None of these

Sol. (b) Loss of weight at 27°C is

$= 46 - 30 = 16 = V_1 \times 1.24\,\rho_l \times g$ ...(i)

Loss of weight at 42°C is

$= 46 - 30.5 = 15.5 = V_2 \times 1.2\,\rho_l \times g$ ...(ii)

Now dividing (i) by (ii), we get $\frac{16}{15.5} = \frac{V_1}{V_2} \times \frac{1.24}{1.2}$

But $\frac{V_1}{V_2} = 1 + 3\alpha(t_2 - t_1) = \frac{15.5 \times 1.24}{16 \times 1.2} = 1.001042$

$\Rightarrow 3\alpha(42° - 27°) = 0.001042 \Rightarrow \alpha = 2.316 \times 10^{-5}$/°C.

4. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. The mass of the steam condensed in kg is

(a) 0.130 (b) 0.065
(c) 0.260 (d) 0.135

Sol. (a) Heat is lost by steam in two stages: (i) for change of state from steam at 100°C to water at 100°C is $m \times 540$ (ii) to change water at 100°C to water at 80°C is $m \times 1 \times (100 - 80)$, where m is the mass of steam condensed. Total heat lost by steam is $m \times 540 + m \times 20 = 560m$ (cals) Heat gained by calorimeter and its contents is

$= (1.1 + 0.02) \times (80 - 15) = 1.12 \times 65$ cals.

using Principle of calorimetery, Heat gained = heat lost

$\therefore 560\,m = 1.12 \times 65\ m = 0.130$ gm

5. 2 kg of ice at –20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 kcal/kg per °C and 0.5 kcal/kg/°C while the latent heat of fusion of ice is 80 kcal/kg.

(a) 7 kg (b) 6 kg
(c) 4 kg (d) 2 kg

Sol. (b) Initially ice will absorb heat to raise its temperature to 0°C. then its melting takes place.

If m_i = initial mass of ice, m_i' = Mass of ice that melts and m_W = Initial mass of water

By law of mixture, heat gained by ice = heat lost by water

$\Rightarrow \quad m_i \times c \times (20) + m_i' \times L = m_W C_W[20]$

$\Rightarrow \quad 2 \times 0.5(20) + mi' \times 80 = 5 \times 1 \times 20 \Rightarrow mi' = 1$ kg

So final mass of water = Initial mass of water + Mass of ice that melts = 5 + 1 = 6 kg.

6. A lead bullet at 27°C just melts when stopped by an obstacle. Assuming that 25% of heat is absorbed by the obstacle, then the velocity of the bullet at the time of striking (M.P. of lead = 327°C, specific heat of lead = 0.03 cal/gm°C, latent heat of fusion of lead = 6 cal/gm and J = 4.2 J/cal)

(a) 410 m/s (b) 1230 m/s
(c) 307.5 m/s (d) None of the above

Sol. (a) If mass of the bullet is m gm, then total heat required for bullet to just melt down

$Q_1 = m\,c\,\Delta\theta + m\,L = m \times 0.03\,(327 - 27) + m \times 6$

$= 15\,m$ cal $= (15m \times 4.2)$J

Now when bullet is stopped by the obstacle, the loss in its mechanical energy $= \frac{1}{2}(m \times 10^{-3})v^2$ J

(As m gm $= m \times 10^{-3}$ kg)

As 25% of this energy is absorbed by the obstacle,

The energy absorbed by the bullet

$$Q_2 = \frac{75}{100} \times \frac{1}{2}mv^2 \times 10^{-3} = \frac{3}{8}mv^2 \times 10^{-3}\text{ J}$$

Now the bullet will melt if $Q_2 \geq Q_1$

i.e. $\frac{3}{8}mv^2 \times 10^{-3} \geq 15m \times 4.2 \Rightarrow v_{min} = 410$ m/s

7. The temperature of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C and when B and C are mixed is 23°C. The temperature when A and C are mixed is
(a) 18.2°C (b) 22°C
(c) 20.2°C (d) 25.2°C

Sol. (c) Heat gain = heat lost

$$C_A(16-12)=C_B(19-16) \Rightarrow \frac{C_A}{C_C}=\frac{15}{16}=\frac{5}{4}$$

and $C_B(23-19)=C_C(28-23) \Rightarrow \frac{C_A}{C_C}=\frac{5}{4}$

$$\Rightarrow \frac{C_A}{C_C}=\frac{15}{16} \qquad ...(i)$$

If θ is the temperature when A and C are mixed then,

$$C_A(\theta-12)=C_C(28-\theta) \Rightarrow \frac{C_A}{C_C}=\frac{28-\theta}{\theta-12} \qquad ...(ii)$$

On solving equation (i) and (ii), $\theta = 20.2$°C.

8. In an industrial process 10 kg of water per hour is to be heated from 20°C to 80°C. To do this steam at 150°C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°C. How many kg of steam is required per hour?
(Specific heat of steam = 1 calorie per gm°C, Latent heat of vaporisation = 540 cal/gm)
(a) 1 gm (b) 1 kg
(c) 10 gm (d) 10 kg

Sol. (b) Suppose m kg steam required per hour
Heat released by steam in following three steps

(i) When 150°C steam $\xrightarrow[Q_1]{}$ 100°C steam

$$Q_1 = mc_{Steam}\,\Delta\theta = m \times 1\,(150-100) = 50\,m \text{ cal}$$

(ii) When 100°C steam $\xrightarrow[Q_2]{}$ 100°C water

$$Q_2 = mL_V = m \times 540 = 540\,m \text{ cal}$$

(iii) When 100°C water $\xrightarrow[Q_3]{}$ 90°C water

$$Q_3 = mc_W\,\Delta\theta = m \times 1 \times (100-90) = 10\,m \text{ cal}$$

Hence total heat given by the steam $Q = Q_1 + Q_2 + Q_3$ = 600 mcal ... (i)

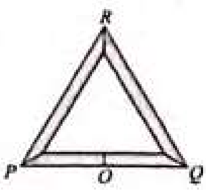
Heat taken by 10 kg water

$$Q' = mc_W\Delta\theta = 10 \times 10^3 \times 1 \times (80-20) = 600 \times 10^3 \text{ cal}$$

Hence $Q = Q' \Rightarrow 600\text{ m} = 600 \times 10^3$

$$\Rightarrow m = 10^3 \text{ gm} = 1\text{kg}.$$

9. Three rods of equal length l are joined to form an equilateral triangle PQR. O is the mid point of PQ. Distance OR remains same for small change in temperature. Coefficient of linear expansion for PR and RQ is same i.e. α_2 but that for PQ is α_1. Then



(a) $\alpha_2 = 3\alpha_1$
(b) $\alpha_2 = 4\alpha_1$
(c) $\alpha_1 = 3\alpha_2$
(d) $\alpha_1 = 4\alpha_2$

Sol. (d) $(OR)^2 = (PR)^2 - (PO)^2 = l^2 - \left(\frac{l}{2}\right)^2$

$$[l(1+\alpha_2 t)]^2 - \left[\frac{1}{2}(1+\alpha_1 t)\right]^2$$

$$l^2 - \frac{l^2}{4} = l^2(1+a_2^2 t^2 + 2\alpha_2 t) - \frac{l^2}{4}(1+a_1^2 t^2 + 2\alpha_1 t)$$

Neglecting $\alpha_2^2 t^2$ and $\alpha_1^2 t^2$

$$0 = l^2(2\alpha_2 t) - \frac{l^2}{4}(2\alpha_1 t) \Rightarrow 2\alpha_2 = \frac{2\alpha_1}{4} \Rightarrow ; \alpha_1 = 4\alpha_2$$

10. A rod of length 20 cm is made of metal. It expands by 0.075cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal B having the same length expands by 0.045 cm for the same change in temperature. A third rod of the same length is composed of two parts, one of metal A and the other of metal B. This rod expands by 0.060 cm for the same change in temperature. The portion made of metal A has the length
(a) 20 cm (b) 10 cm
(c) 15 cm (d) 18 cm

Sol. (b) $\Delta L = L_0\,\alpha\Delta\theta$

$$\text{Rod } A\text{: } 0.075 = 20 \times \alpha_A \times 100 \Rightarrow \alpha_A = \frac{75}{2} \times 10^{-6}/°C$$

$$\text{rod } B\text{: } 0.045 = 20 \times \alpha_B \times 100 \Rightarrow \alpha_B = \frac{45}{2} \times 10^{-6}/°C$$

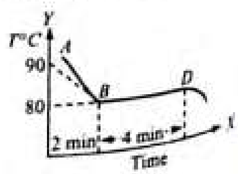
For composite rod: x cm of A and $(20-x)$ cm of B, we have

$$0.060 = x\,\alpha_A \times 100 + (20-x)\,\alpha_B \times 100$$

$$= x\left[\frac{75}{2}\times 10^{-6} \times 100 + (20-x) \times \frac{45}{2} \times 10^{-6} \times 100\right]$$

On solving, we get $x = 10$ cm.

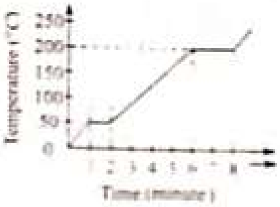
11. The figure given below shows the cooling curve of pure wax material after heating. It cools from A to B and solidifies along BD. If L and C are respective values of latent heat and the specific heat of the liquid wax, the ratio L/C is
(a) 40
(b) 80
(c) 100
(d) 20



Sol. (d) Let the quantity of heat supplied per minute be Q.
Then quantity of heat supplied in 2 min = $mC(90-80)$
In 4 min, heat supplied = $2m\,C(90-80)$

$$2mC(90-80) = mL \Rightarrow \frac{L}{C} = 20$$

12. A student takes 50gm wax (specific heat = 0.6 kcal/kg°C) and heats it till it boils. The graph between temperature and time is as follows. Heat supplied to the wax per minute and boiling point are, respectively



(a) 500 cal, 50°C
(b) 1000 cal, 100°C
(c) 1500 cal, 200°C
(d) 1000 cal, 200°C

Sol. (c) Since specific heat = 0.6 kcal/gm × °C
= 0.6 cal/gm ×°C

From graph it is clear that in a minute, the temperature is raised from 0°C to 50°C.

⇒ Heat required for a minute = 50 × 0.6 × 50 = 1500 cal

Also from graph, Boiling point of wax is 200°C.

13. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surrounding is

(a) 5°C (b) 10°C
(c) 15°C (d) 20°C

Sol. (b) According to Newton's law of cooling,

$$\frac{\theta_1-\theta_2}{t} = K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$$

In the first case, $\dfrac{(60-50)}{10} = K\left[\dfrac{60+50}{2}-\theta_0\right]$

$1 = K(55-\theta)$ ...(i)

In the second case, $\dfrac{(50-42)}{10} = K\left[\dfrac{50+42}{2}-\theta_0\right]$

$0.8 = K(46-\theta_0)$ ....(ii)

Dividing (i) by (ii), we get $\dfrac{1}{0.8} = \dfrac{55-\theta}{46-\theta}$

or $46-\theta_0 = 44-0.8\,\theta_0 \Rightarrow \theta_0 = 10°C$

14. A bucket full of hot water cools from 75°C to 70°C in time T_1, from 70°C to 65°C in time T_2 and from 65°C to 60°C in time T_3, then

(a) $T_1 = T_2 = T_3$ (b) $T_1 > T_2 > T_3$
(c) $T_1 < T_2 < T_3$ (d) $T_1 > T_2 < T_3$

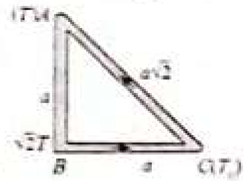
Sol. (c) According to Newton's law of cooling
Rate of cooling ∝ Mean temperature difference

$$\Rightarrow \frac{\text{Fall in temperature}}{\text{Time}} \propto \left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)$$

$$\because \left(\frac{\theta_1+\theta_2}{2}\right)_1 > \left(\frac{\theta_1+\theta_2}{2}\right)_2 > \left(\frac{\theta_1+\theta_2}{2}\right)_3$$

$$\Rightarrow T_1 < T_2 < T_3$$

15. Three rods of identical area of cross-section and made from the same metal form the sides of an isosceles triangle ABC, right angled at B. The points A and B are maintained at temperatures T and $\sqrt{2}T$, respectively. In the steady state, the temperature of the point C is T_C. Assuming that only heat conduction takes place, T_C/T is equal to



(a) $\dfrac{1}{(\sqrt{2}+1)}$ (b) $\dfrac{3}{(\sqrt{2}+1)}$

(c) $\dfrac{1}{2(\sqrt{2}-1)}$ (d) $\dfrac{1}{\sqrt{3}(\sqrt{2}-1)}$

Sol. (b) $\because T_B > T_A \Rightarrow$ Heat will flow B to A via two paths (i) B to A (ii) and along BCA as shown.
Rate of flow of heat in path BCA will be same

i.e. $\left(\dfrac{Q}{t}\right)_{BC} = \left(\dfrac{Q}{t}\right)_{CA}$

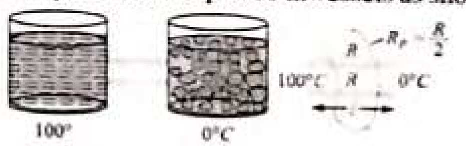
$$\Rightarrow \frac{k(\sqrt{2}T-T_C)A}{a} = \frac{k(T_C-T)A}{\sqrt{2}a}$$

$$\Rightarrow \frac{T_C}{T} = \frac{3}{1+\sqrt{2}}$$

16. Two identical conducting rods are first connected independently to two vessels, one containing water at 100°C and the other containing ice at 0°C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 g/s be the rate of melting of ice in two cases, respectively. The ratio of q_1/q_2 is

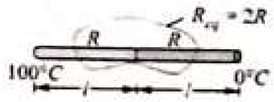
(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{1}$ (c) $\dfrac{4}{1}$ (d) $\dfrac{1}{4}$

Sol. (c) Initially the rods are placed in vessels as shown below.



$$\frac{Q}{t} = \frac{(\theta_1-\theta_2)}{R} \Rightarrow \left(\frac{Q}{t}\right)_1 = \frac{mL}{t} = q_1L = \frac{(100-0)}{\frac{R}{2}} \quad \text{... (i)}$$

Finally when rods are joined end to end as shown,



$$\Rightarrow \left(\frac{Q}{t}\right)_2 = \frac{mL}{t} = q_2L = \frac{(100-0)}{2R} \quad \text{... (ii)}$$

From equations (i) and (ii), $\dfrac{q_1}{q_2} = \dfrac{4}{1}$

17. Three discs A, B and C having radii 2m, 4m, and 6m respectively are coated with carbon black on their other surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_a, Q_b, and Q_c, respectively,

(a) Q_a is maximum (b) Q_b is maximum

(c) Q_c is maximum (d) $Q_a = Q_b = Q_c$

Sol. (b) Radiated power $P = Ae\sigma T^4 \Rightarrow P \propto AT^4$

From Wein's law, $\lambda_m T$ = constant $\Rightarrow T \propto \dfrac{1}{\lambda_m}$

$$P \propto \frac{A}{(\lambda_m)^4} \propto \frac{r^2}{(\lambda_m)^4}$$

$$\Rightarrow \quad Q_A : Q_B : Q_C = \frac{2^2}{(300)^4} : \frac{4^2}{(400)^4} : \frac{6^2}{(500)^4}$$

$\therefore$ Q_B will be maximum.

18. A solid copper sphere (density ρ and specific heat capacity c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. The time required (in µs) for the temperature of the sphere to drop to 100 K is

(a) $\dfrac{72}{7}\dfrac{r\rho c}{\sigma}$ (b) $\dfrac{7}{72}\dfrac{r\rho c}{\sigma}$

(c) $\dfrac{27}{7}\dfrac{r\rho c}{\sigma}$ (d) $\dfrac{7}{27}\dfrac{r\rho c}{\sigma}$

Sol. (b) $\dfrac{dT}{dt} = \dfrac{\sigma A}{mcJ}(T^4 - T_0^4)$ [In the given problem, fall in temperature of body $dT = (200 - 100) = 100\ K$, temp. of surrounding $T_0 = 0$ K, Initial temperature of body $T = 200$ K]

$$\frac{100}{dt} = \frac{\sigma 4\pi r^2}{\frac{4}{3}\pi r^3 \rho c J}(200^4 - 0^4)$$

$$\Rightarrow \quad dt = \frac{r\rho c J}{48\sigma}\times 10^{-6}\,\text{s} = \frac{r\rho c}{\sigma}.\frac{4.2}{48}\times 10^{-6}$$

$$= \frac{7}{80}\frac{r\rho c}{\sigma}\,\mu\text{s} = \frac{7}{72}\frac{r\rho c}{\sigma}\,\mu\text{s} \quad [\text{As } J = 4.2]$$

19. One end of a copper rod of uniform cross-section and of length 3.1 m is kept in contact with ice and the other end with water at 100°C. At what point along it's length should a temperature of 200°C be maintained so that in steady state, the mass of ice melting be equal to that of the steam produced in the same interval of time. Assume that the whole system is insulated from the surroundings. Latent heat of fusion of ice and vaporisation of water are 80 cal/gm and 540 cal/gm respectively

(a) 40 cm from 100°C end

(b) 40 cm from 0°C end

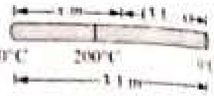
(c) 125 cm from 100°C end

(d) 125 cm from 0°C end

Sol. (a) Rate of flow of heat is given by $\dfrac{dQ}{dt} = \dfrac{\Delta\theta}{l/KA}$

also $\dfrac{dQ}{dt} = L\dfrac{dm}{dt}$ (where L = Latent heat)

$\Rightarrow \dfrac{dm}{dt} = \dfrac{KA}{l}\left(\dfrac{\Delta\theta}{L}\right)$



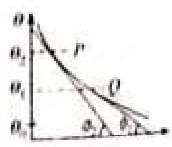
Let the desire point is at a distance x from water at 100°C

$\because$ Rate of ice melting = Rate at which steam is being produced

$$\Rightarrow \left(\frac{dm}{dt}\right)_{\text{Steam}} = \left(\frac{dm}{dt}\right)_{\text{Ice}} \Rightarrow \left(\frac{\Delta\theta}{Ll}\right)_{\text{Steam}} = \left(\frac{\Delta\theta}{Ll}\right)_{\text{Ice}}$$

$$\Rightarrow \frac{(200-100)}{540\times x} = \frac{(200-0)}{80(3.1-x)} \Rightarrow x = 0.4\text{ m} = 40\text{ cm}$$

20. A body cools in a surrounding which is at a constant temperature of θ_0. Assume that it obeys Newton's law of cooling. Its temperature θ is plotted against time t. Tangents are drawn to the curve at the points $P(\theta = \theta_2)$ and $Q(\theta = \theta_1)$. These tangents meet the time axis at angles of ϕ_2 and ϕ_1, as shown



(a) $\dfrac{\tan\phi_2}{\tan\phi_1} = \dfrac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$ (b) $\dfrac{\tan\phi_2}{\tan\phi_1} = \dfrac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$

(c) $\dfrac{\tan\phi_1}{\tan\phi_2} = \dfrac{\theta_1}{\theta_2}$ (d) $\dfrac{\tan\phi_1}{\tan\phi_2} = \dfrac{\theta_2}{\theta_1}$

Sol. (b) For θ-t plot, rate of cooling $= \dfrac{d\theta}{dt}$ = slope of the curve

At P, $\dfrac{d\theta}{dt} = \tan\phi_2 = k(\theta_2 - \theta_0)$, where k = constant.

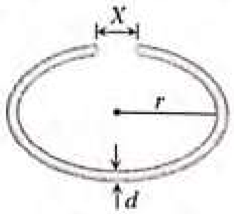
At Q, $\dfrac{d\theta}{dt} = \tan\phi_1 = k(\theta_1 - \theta_0)$

$$\Rightarrow \frac{\tan\phi_2}{\tan\phi_1} = \frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$$

EXERCISES

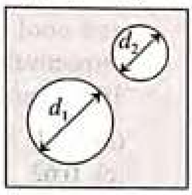
Thermal Expansion

1. A cylindrical metal rod of length L_0 is shaped into a ring with a small gap as shown. On heating the system



(a) x decreases, r and d increase
(b) x and r increase, d decreases
(c) x, r and d all increase
(d) Data insufficient to arrive at a conclusion

2. Two holes of unequal diameters d_1 and d_2 $(d_1 > d_2)$ are cut in a metal sheet. If the sheet is heated



(a) Both d_1 and d_2 will decrease
(b) Both d_1 and d_2 will increase
(c) d_1 will increase, d_2 will decrease
(d) d_1 will decrease, d_2 will increase

3. The coefficient of linear expansion of an inhomogeneous rod changes linearly from α_1 to α_2 from one end to the other end of the rod. The effective coefficient of linear expansion of rod is

(a) $\alpha_1 + \alpha_2$ (b) $\frac{\alpha_1+\alpha_2}{2}$
(c) $\sqrt{\alpha_1\alpha_2}$ (d) $\alpha_1 - \alpha_2$

4. An iron tyre is to be fitted onto a wooden wheel 1.0 m in diameter. The diameter of the tyre is 6 mm smaller than that of wheel. The tyre should be heated so that its temperature increases by a minimum of (coefficient of volume expansion of iron is 3.6×10^{-5}/°C)

(a) 167°C (b) 334°C
(c) 500°C (d) 1000°C

5. The absolute coefficient of expansion of a liquid is 7 times that the volume coefficient of expansion of the vessel. Then the ratio of absolute and apparent expansion of the liquid is

(a) $\frac{1}{7}$ (b) $\frac{7}{6}$
(c) $\frac{6}{7}$ (d) None of these

6. A solid whose volume does not change with temperature floats in a liquid. For two different temperatures t_1 and t_2 of the liquid, fractions f_1 and f_2 of the volume of the solid remain submerged in the liquid. The coefficient of volume expansion of the liquid is equal to

(a) $\frac{f_1-f_2}{f_2t_1-f_1t_2}$ (b) $\frac{f_1-f_2}{f_1t_1-f_2t_2}$
(c) $\frac{f_1+f_2}{f_2t_1+f_1t_2}$ (d) $\frac{f_1+f_2}{f_1t_1+f_2t_2}$

7. A clock with a metal pendulum beating seconds keeps correct time at 0°C. If it loses 12.5 s a day at 25°C, the coefficient of linear expansion of metal of pendulum is

(a) $\frac{1}{86400}$/°C (b) $\frac{1}{43200}$/°C
(c) $\frac{1}{14400}$/°C (d) $\frac{1}{28800}$/°C

8. A wire of length L_0 is supplied heat to raise its temperature by T. If γ is the coefficient of volume expansion of the wire and Y is Young's modulus of the wire then the energy density stored in the wire is

(a) $\frac{1}{2}\gamma^2T^2Y$ (b) $\frac{1}{3}\gamma^2T^2Y^3$
(c) $\frac{1}{18}\frac{\gamma^2T^2}{Y}$ (d) $\frac{1}{18}\gamma^2T^2Y$

9. Span of a bridge is 2.4 km. At 30°C a cable along the span sags by 0.5 km. Taking $\alpha = 12 \times 10^{-6}$/°C, change in length of cable for a change in temperature from 10°C to 42°C is

(a) 9.9 m
(b) 0.099 m
(c) 0.99 m
(d) 0.4 km

10. A brass wire 2 m long at 27°C is held taut with negligible tension between two rigid supports. If the wire is cooled to a temperature of –33°C, then the tension developed in the wire, its diameter being 2 mm, will be (coefficient of linear expansion of brass = 2.0×10^{-5}/°C and Young's modulus of brass = 0.91×10^{11} Pa)

(a) 3400 N (b) 34 kN
(c) 0.34 kN (d) 6800 N

11. The loss in weight of a solid when immersed in a liquid at 0°C is W_0 and at t°C is W. If cubical coefficients of expansion of the solid and the liquid are γ_S and γ_L, respectively, then W is equal to

(a) $W_0[1 + (\gamma_S - \gamma_L)t]$ (b) $W_0[1 - (\gamma_S - \gamma_L)t]$
(c) $W_0[(\gamma_S - \gamma_L)t]$ (d) $W_0t/(\gamma_S - \gamma_L)$

12. A pendulum clock having copper rod keeps correct time at 20°C. It gains 15 s per day if cooled to 0°C. The coefficient of linear expansion of copper is

(a) 1.7×10^{-4}/°C (b) 1.7×10^{-5}/°C
(c) 3.4×10^{-4}/°C (d) 3.4×10^{-5}/°C

13. A glass flask is filled up to a mark with 50 cc of mercury at 18°C. If the flask and contents are heated to 38°C, how much mercury will be above the mark (α for glass is 9×10^{-6}/°C and coefficient of real expansion of mercury is 180×10^{-6}/°C)?
(a) 0.85 cc (b) 0.46 cc
(c) 0.153 cc (d) 0.05 cc

14. A flask of volume 10^3 cc is completely filled with mercury at 0°C. The coefficient of cubical expansion of mercury is 180×10^{-6}/°C and heat of glass is 40×10^{-6}/°C. If the flask is now placed in boiling water at 100°C, how much mercury will overflow?
(a) 7 cc (b) 14 cc
(c) 21 cc (d) 28 cc

15. An aluminium measuring rod, which is correct at 5°C measures the length of a line as 80 cm at 45°C. If thermal coefficient of linear expansion of aluminium is 2.50×10^{-5}/°C, the correct length of the line is:
(a) 80.08 cm (b) 79.92 cm
(c) 81.12 cm (d) 79.62 cm

16. A 1 L glass flask contains some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. What is the volume of mercury in this flask if coefficient of linear expansion of glass is 9×10^{-6}/°C while of volume expansion of mercury is 1.8×10^{-4}/°C?
(a) 50 cc (b) 100 cc
(c) 150 cc (d) 200 cc

Calorimetry

17. Work done in converting 1 g of ice at –10°C into steam at 100°C is
(a) 3045 J (b) 6056 J
(c) 721 J (d) 6 J

18. The specific heat of a substance varies with temperature t(°C) as
$c = 0.20 + 0.14t + 0.023t^2$ (cal/g/°C)
The heat required to raise the temperature of 2 g of substance from 5°C to 15°C will be
(a) 24 cal (b) 56 cal
(c) 82 cal (d) 100 cal

19. 50 g of copper is heated to increase its temperature by 10°C. If the same quantity of heat is given to 10 g of water, the rise in its temperature is (specific heat of copper = 420 J/kg/°C)
(a) 5°C (b) 6°C
(c) 7°C (d) 8°C

20. Two liquids A and B are at 32°C and 24°C. When mixed in equal masses the temperature of the mixture is found to be 28°C. Their specific heats are in the ratio of
(a) 3:2 (b) 2:3
(c) 1:1 (d) 4:3

21. A beaker contains 200 g of water. The heat capacity of the beaker is equal to that of 20 g of water. The initial temperature of water in the beaker is 20°C. If 440 g of hot water at 92°C is poured in it, the final temperature (neglecting radiation loss) will be nearest to
(a) 58°C (b) 68°C
(c) 73°C (d) 78°C

22. Heat is required to change 1 kg of ice at –20°C into steam. Q_1 is the heat needed to warm the ice from –20°C to 0°C, Q_2 is the heat needed to melt the ice, Q_3 is the heat needed to warm the water from 0°C to 100°C and Q_4 is the heat needed to vapourize the water. Then
(a) $Q_4 > Q_3 > Q_2 > Q_1$ (b) $Q_4 > Q_3 > Q_1 > Q_2$
(c) $Q_4 > Q_2 > Q_3 > Q_1$ (d) $Q_4 > Q_2 > Q_1 > Q_3$

23. 250 g of water and equal volume of alcohol of mass 200 g are replaced successively in the same calorimeter and cool from 606°C to 55°C in 130 s and 67 s, respectively. If the water equivalent of the calorimeter is 10 g, then the specific heat of alcohol in cal/g-°C is
(a) 1.30 (b) 0.67
(c) 0.62 (d) 0.985

24. A 2 g bullet moving with a velocity of 200 m/s is brought to a sudden stoppage by an obstacle. The total heat produced goes to the bullet. If the specific heat of the bullet is 0.03 cal/g-°C, the rise in its temperature will be
(a) 158.0°C (b) 15.80°C
(c) 1.58°C (d) 0.1580°C

25. 10 g of ice at –20°C is dropped into a calorimeter containing 10 g of water at 10°C; the specific heat of water is twice that of ice. When equilibrium is reached, the calorimeter will contain
(a) 20 g of water
(b) 20 g of ice
(c) 10 g ice and 10 g of water
(d) 5 g ice and 15 g of water

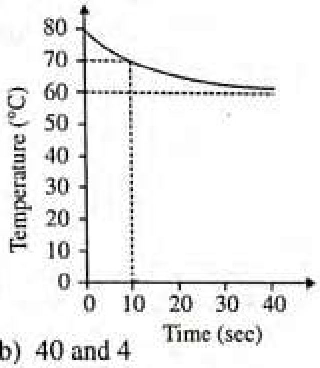
26. Which of the following, when mixed, would raise the temperature of 20 g of water at 30°C most?
(a) 20 g of water at 40°C
(b) 40 g of water at 35°C
(c) 10 g of water at 50°C
(d) 4 g of water at 80°C

27. A kettle with 2 L water at 27°C is heated by operating coil heater of power 1 kW. The heat is lost to the atmosphere at constant rate 160 J/s, when it is open. In how much time will water be heated to 77°C (sp. heat of water = 4.2 kJ/kg) with the lid open?
(a) 8 min 20 s (b) 6 min 2 s
(c) 14 min (d) 7 min

28. An earthen pitcher loses 1 g of water per minute due to evaporation. If the water equivalent of pitcher is 0.5 kg and the pitcher contains 9.5 kg of water, calculate the time required for the water in the pitcher to cool to 28°C from its original temperature of 30°C. Neglect radiation effects. Latent heat of vapourization of water in this range of temperature is 580 cal/g and specific heat of water is 1 k cal/g C°.
(a) 38.6 min (b) 30.5 min
(c) 34.5 min (d) 41.2 min

29. 5 g of water at 30°C and 5 g of ice at –20°C are mixed together in a calorimeter. Find the final temperature of the mixture. Assume water equivalent of calorimeter to be negligible, sp. heats of ice and water are 0.5 and 1 cal/g C°, and latent heat of ice is 80 cal/g.
(a) 0°C (b) 10°C
(c) –30°C (d) >10°C

30. A vessel contains M grams of water at a certain temperature and water at certain other temperature is passed into it at a constant rate of m g/s. The variation of temperature of the mixture with time is shown in figure. The values of M and m are, respectively (the heat exchanged after a long time is 800 cal)

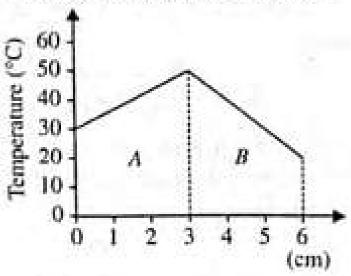


(a) 40 and 2 (b) 40 and 4
(c) 20 and 4 (d) 20 and 2

Transmission of Heat

31. The coefficients of thermal conductivity of copper, mercury and glass are K_c, K_m and K_g, respectively, such that $K_c > K_m > K_g$. If the same quantity of heat is to flow per second per unit area of each and corresponding temperature gradients are, X_c, X_m and X_g, respectively, then
(a) $X_c = X_m = X_g$ (b) $X_c > X_m > X_g$
(c) $X_c < X_m < X_g$ (d) $X_m < X_c < X_g$

32. The temperatures across two different slabs A and B are shown in the steady state (as shown in figure). The ratio of thermal conductivities of A and B is



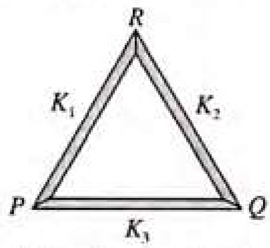
(a) 2:3 (b) 3:2 (c) 1:1 (d) 5:3

33. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. If the temperature difference between the outer and the inner surface of the shell is not to exceed T, then the thickness of the shell should not be less than
(a) $\dfrac{2\pi R^2 KT}{P}$ (b) $\dfrac{4\pi R^2 KT}{P}$
(c) $\dfrac{\pi R^2 KT}{P}$ (d) $\dfrac{\pi R^2 KT}{4P}$

34. There are three thermometers–one in contact with the skin of the man, other in between the vest and the shirt and third in between the shirt and coat. The readings of the thermometers are 30°C, 25°C and 22°C, respectively. If the vest and the shirt are of the same thickness, the ratio of their thermal conductivities is
(a) 9:25 (b) 25:9 (c) 5:3 (d) 3:5

35. Two rods of same length and material transfer a given amount of heat in 12 s, when they are joined end to end. But when they are joined lengthwise, they will transfer same heat in same conditions in
(a) 24 s (b) 3 s (c) 1.5 s (d) 48 s

36. Three rods of same dimensions are arranged as shown in figure. They have thermal conductivities K_1, K_2 and K_3. The points P and Q are maintained at different temperatures for the heat to flow at the same rate along PRQ and PQ. Which of the following options is correct?

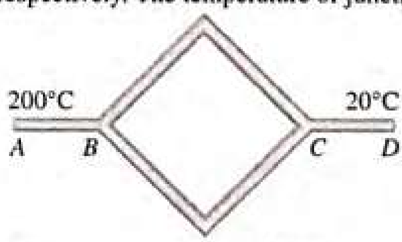


(a) $K_3 = \dfrac{1}{2}(K_1 + K_2)$
(b) $K_3 = K_1 + K_2$
(c) $K_3 = \dfrac{K_1 K_2}{K_1 + K_2}$
(d) $K_3 = 2(K_1 + K_2)$

37. The coefficient of thermal conductivity of copper is nine times that of steel. In the composite cylindrical bar shown in figure, what will be the temperature at the junction of copper and steel?

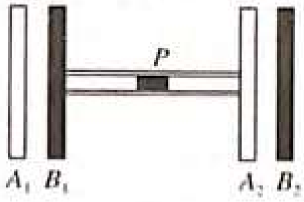
(a) 75°C
(b) 67°C
(c) 33°C
(d) 25°C

38. Six identical conducting rods are joined as shown in figure. Points A and D are maintained at temperatures 200°C and 20°C, respectively. The temperature of junction B will be



(a) 120°C (b) 100°C
(c) 140°C (d) 80°C

39. Two plates identical in size, one of black and rough surface (B_1) and the other smooth and polished (A_2) are interconnected by a thin horizontal pipe with a mercury pellet at the centre. Two more plates A_1 (identical to A_2) and B_2 (identical to B_1) are heated to the same temperature and placed closed to the plates B_1, and A_2 as shown in figure. The mercury pellet



(a) moves to the right
(b) moves to the left
(c) remains stationary
(d) starts oscillating left and right

40. The rates of cooling of two different liquids put in exactly similar calorimeters and kept in identical surroundings are the same if
(a) The masses of the liquids are equal
(b) Equal masses of the liquids at the same temperature are taken
(c) Different volumes of the liquids at the same temperature are taken
(d) Equal volumes of the liquids at the same temperature are taken

41. Two hollow spheres of different materials, one with double the radius and one-fourth wall thickness of the other, are filled with ice. If the times taken for complete melting of ice in the larger to the smaller one are in the ratio of 25 : 16, then their corresponding thermal conductivities are in the ratio
(a) 4:5 (b) 5:4
(c) 8:25 (d) 25:8

42. The wavelength of maximum energy released during an atomic explosion was 2.93×10^{-10} m. Given that Wien's constant is 2.93×10^{-3} m-K, the maximum temperature attained must be of the order of
(a) 10^{-7} K (b) 10^{7} K
(c) 10^{-13} K (d) 5.86×10^{7} K

43. The rectangular surface of area 8 cm × 4 cm of a black body at a temperature of 127°C emits energy at the rate of E per second. If the length and breadth of the surface are each reduced to half of the initial value and the temperature is raised to 327°C, the rate of emission of energy will become
(a) $\frac{3}{8}E$ (b) $\frac{81}{16}E$
(c) $\frac{9}{16}E$ (d) $\frac{81}{64}E$

44. A solid copper cube of edges 1 cm is suspended in an evacuated enclosure. Its temperature is found to fall from 100°C to 99°C in 100 s. Another solid copper cube of edges 2 cm, with similar surface nature, is suspended in a similar manner. The time required for this cube to cool from 100°C to 99°C will be approximately
(a) 25 s (b) 50 s
(c) 200 s (d) 400 s

45. A sphere, a cube and a thin circular plate are made of same substance and all have same mass. These are heated to 200°C and then placed in a room. Then the
(a) Temperature of sphere drops to room temperature at last
(b) Temperature of cube drops to room temperature at last
(c) Temperature of thin circular plate drop to room temperature at last
(d) Temperatures of all the three drop to room temperature at the same time

46. A sphere and a cube of same material and same volume are heated up to same temperature and allowed to cool in the same surroundings. The ratio of the amounts of radiations emitted in equal time intervals will be
(a) 1:1 (b) $\frac{4\pi}{3}:1$
(c) $\left(\frac{\pi}{6}\right)^{1/3}:1$ (d) $\frac{1}{2}\left(\frac{4\pi}{3}\right)^{2/3}:1$

47. A bucket full of hot water cools from 75°C to 70°C in time T_1, from 70°C to 65°C in time T_2 and from 65°C to 60°C in time T_3, then
(a) $T_1 = T_2 = T_3$ (b) $T_1 > T_2 > T_3$
(c) $T_1 < T_2 < T_3$ (d) $T_1 > T_2 < T_3$

48. A cup of tea cools from 80°C to 60°C in 1 min. The ambient temperature is 30°C. In cooling from 60°C to 50°C it will take
(a) 30 s (b) 60 s (c) 90 s (d) 48 s

49. A body takes T minutes to cool from 62°C to 61°C when the surrounding temperature is 30°C. The time taken by the body to cool from 46°C to 45.5°C is
(a) Greater than T minutes
(b) Equal to T minutes
(c) Less than T minutes
(d) None of these

50. Hot water cools from 60°C to 50°C in the first 10 min and to 42°C in the next 10 min. The temperature of the surrounding is
(a) 5°C (b) 10°C
(c) 15°C (d) 20°C

51. A body cools in 7 min from 60°C to 40°C. What will be its temperature after the next 7 min? The temperature of surroundings is 10°C.
(a) 28°C (b) 25°C
(c) 30°C (d) 22°C

52. A body cools from 50°C to 49°C in 5 s. How long will it take to cool from 40°C to 39.5°C? Assume the temperature

of surroundings to be 30°C and Newton's law of cooling to be valid:

(a) 2.5 s (b) 10 s (c) 20 s (d) 5 s

53. A liquid takes 5 min to cool from 80°C to 50°C. How much time will it take to cool from 60°C to 30°C? The temperature of the surrounding is 20°C.
(a) 5 min (b) 9 min
(c) 4 min (d) 12 min

Problems Based on Mixed Concepts

54. An ice box used for keeping eatables cool has a total wall area of 1 m^2 and a wall thickness of 5.0 cm. The thermal conductivity of the ice box is $K = 0.01$ J/m°C. It is filled with large amount of ice at 0°C along with eatables on a day when the temperature is 30°C. The latent heat of fusion of ice is 334×10^3 J/kg. The amount of ice melted in one day is (1 day = 86,400 s)
(a) 776 g (b) 7760 g
(c) 11520 g (d) 1552 g

55. A wire is made by attaching two segments together end to end. One segment is made of aluminium and the other is steel. The effective coefficient of linear expansion of the two segment wire is 19×10^{-6}/(°C). The fraction length of aluminium is [linear coefficients of thermal expansion of aluminium and steel are 23×10^{-6}/(°C) and 12×10^{-6}/(°C), respectively]
(a) $\frac{5}{11}$ (b) $\frac{6}{11}$
(c) $\frac{7}{11}$ (d) $\frac{8}{11}$

56. A block of wood is floating in water at 0°C. The temperature of water is slowly raised from 0°C to 10°C. With the rise in temperature the volume of block V above water level will
(a) increase
(b) decrease
(c) first increase and then decrease
(d) first decrease and then increase

57. An incandescent lamp consuming $P = 54$ W is immersed into a transparent calorimeter containing $V = 10^3$ cm^3 of water. In 420 s the water is heated by 4°C. The percentage of the energy consumed by the lamp that passes out of the calorimeter in the form of radiant energy is
(a) 81.5% (b) 26%
(c) 40.5% (d) 51.5%

58. A thread of liquid is in a uniform capillary tube of length L, as measured by a ruler. The temperature of the tube and thread of liquid is raised by ΔT. If γ be the coefficient of volume expansion of the liquid and α be the coefficient of linear expansion of the material of the tube, then the increase ΔL in the length of the thread, again measured by the ruler will be
(a) $\Delta L = L(\gamma - \alpha)\,\Delta T$ (b) $\Delta L = L(\gamma - 2\alpha)\,\Delta T$
(c) $\Delta L = L(\gamma - 3\alpha)\,\Delta T$ (d) $\Delta L = L\gamma\Delta T$

59. A mass m of lead shot is placed at the bottom of a vertical cardboard cylinder that is 1.5 m long and closed at both ends. The cylinder is suddenly inverted so that the shot falls 1.5 m. If this process is repeated quickly 100 times, assuming no heat is dissipated or lost, the temperature of the shot will increase by (specific heat of lead = 0.03 cal/g°C)
(a) 0 (b) 5°C
(c) 7.3°C (d) 11.3°C

60. An iron rocket fragment initially at −100°C enters the earth's atmosphere almost horizontally and quickly fuses completely in atmospheric friction. Specific heat of iron is 0.11 kcal/kg °C its melting point is 1535°C and the latent heat of fusion is 3 kcals/kg. The minimum velocity with which the fragment must have entered the atmosphere is
(a) 0.45 km/s (b) 1.32 km/s
(c) 2.32 km/s (d) zero

61. A liquid of density 0.85 g/cm^3 flows through a calorimeter at the rate of 8.0 cm^3/s. Heat is added by means of a 250 W electric heating coil and a temperature difference of 15°C is established in steady-state conditions between the inflow and the outflow points of the liquid. The specific heat for the liquid will be
(a) 0.6 kcal/kgK (b) 0.3 kcal/kgK
(c) 0.5 kcal/kgK (d) 0.4 kcal/kgK

62. Ice at 0°C is added to 200 g of water initially at 70°C in a vacuum flask. When 50 g of ice has been added and has all melted, the temperature of flask and contents is 40°C. When a further 80 g of ice is added and has all melted, the temperature of whole becomes 10°C. Neglecting heat lost to surroundings the latent heat of fusion of ice is
(a) 80 cal/g (b) 90 cal/g
(c) 70 cal/g (d) 540 cal/g

63. One end of a copper rod of uniform cross section and of length 1.5 m is kept in contact with ice and the other end with water at 100°C. At what point along its length should a temperature of 200°C be maintained so that in steady state, the mass of ice melting be equal to that of the steam produced in same interval of time? Assume that the whole system is insulated from surroundings. Latent heat of fusion of ice and vapourization of water are 80 cal/g and 540 cal/g, respectively.

(a) 8.59 cm from ice end
(b) 10.34 cm from water end
(c) 10.34 cm from ice end
(d) 8.76 cm from water end

64. A steel ball of mass 0.1 kg falls freely from a height of 10 m and bounces to a height of 5.4 m from the ground. If the dissipated energy in this process is absorbed by the ball, the rise in its temperature is (specific heat of steel = 460 K/kg °C, $g = 10\ m/s^2$)
(a) 0.01°C (b) 0.1°C
(c) 1°C (d) 1.1°C

65. The earth receives on its surface radiation from the sun at the rate of 1400 W/m². The distance of the centre of the sun from the surface of the earth is 1.5×10^{11} m and the radius of the sun is 7.0×10^8 m. Treating sun as a black body, it follows from the above data that its surface temperature is
(a) 5801 K (b) 10^6 K
(c) 50.1 K (d) 5801°C

66. A vessel is partly filled with a liquid. Coefficients of cubical expansion of material of the vessel and liquid are γ_V and γ_L, respectively. If the system is heated, then volume unoccupied by the liquid will necessarily
(a) remain unchanged if $\gamma_V = \gamma_L$
(b) increase if $\gamma_V = \gamma_L$
(c) decrease if $\gamma_V = \gamma_L$
(d) none of the above

67. An electrically heated coil is immersed in a calorimeter containing 360 g of water at 10°C. The coil consumes energy at the rate of 90 W. The water equivalent of calorimeter and coil is 40 g. The temperature of water after 10 min is
(a) 4.214°C (b) 42.14 °C
(c) 30°C (d) None of these

68. A room at 20°C is heated by a heater of resistance 20 ohm connected to 200 V mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1 m² and thickness 0.2 cm. Calculate the temperature outside. Thermal conductivity of glass is 0.2 cal/m C° s and mechanical equivalent of heat is 4.2 J/cal.
(a) 13.69°C (b) 15.24°C
(c) 17.85°C (d) 19.96°C

ARCHIVES

1. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K, respectively. The ratio of the energy radiated per second by the first sphere to that by the second is
(a) 1 : 1 (b) 16 : 1
(c) 4 : 1 (d) 1 : 9 **(AIEEE 2002)**

2. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, and n is equal to
(a) one (b) two
(c) three (d) four **(AIEEE 2002)**

3. If mass energy equivalence is taken into account, when water is cooled to form ice, the mass of water should
(a) increase (b) remain unchanged
(c) decrease (d) first increase then decrease **(AIEEE 2002)**

4. Amount of heat required to raise the temperature of a body through 1k is called its
(a) Water equivalent (b) Thermal capacity
(c) Entropy (d) Specific heat **(AIEEE 2002)**

5. Infrared radiation is detected by
(a) Spectrometer (b) Pyrometer
(c) Nanometer (d) Photometer **(AIEEE 2002)**

6. Which of the following is the example of ideal black body?
(a) Kajal (b) Black board
(c) A pin hole in a box (d) None of these **(AIEEE 2002)**

7. The earth radiates in the infrared region of the spectrum. The spectrum is correctly given by
(a) Wien's law
(b) Rayleigh–Jeans law
(c) Planck's law of radiation
(d) Stefan's law of radiation **(AIEEE 2003)**

8. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on the earth to what it was previously will be
(a) 4 (b) 16
(c) 32 (d) 64 **(AIEEE 2003)**

9. The temperatures of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thicknesses x and $4x$, respectively, are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state, is

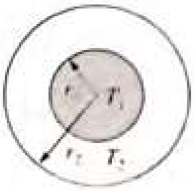
$$\left[\frac{A(T_2 - T_1)K}{x}\right] f,$$

where f is equal to

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$ **(AIEEE 2004)**

10. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres is proportional to



(a) $\frac{r_1 r_2}{(r_1 - r_2)}$ (b) $(r_2 - r_1)$

(c) $(r_2 - r_1)(r_1 r_2)$ (d) $\ln\left(\frac{r_2}{r_1}\right)$ **(AIEEE 2005)**

11. Assuming the sun to be a spherical body of radius R at a temperature of T kelvin, evaluate the total radiant power incident on the earth at a distance r from the sun.

(a) $r_0^2 R^2 \sigma \frac{T^4}{4\pi r^2}$ (b) $R^2 \frac{\sigma T^4}{r^2}$

(c) $4\pi r_0^2 R^2 \frac{\sigma T^4}{r^2}$ (d) $\pi r_0^2 R^2 \frac{\sigma T^4}{r^2}$

(AIEEE 2006)

12. This question contains Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1: The temperature dependence of resistance is usually given as $R = R_0(1 + \alpha\Delta t)$. The resistance of a wire changes from 100 Ω to 150 Ω when its temperature is increased from 27°C to 227°C. This implies that $\alpha = 2.5 \times 10^{-3}$/°C.

Statement 2: $R = R_i(1 + \alpha\Delta T)$ is valid only when the change in the temperature ΔT is small and $\Delta R = (R - R_0) << R_0$.

(a) Statement 1 is true but statement 2 is false.

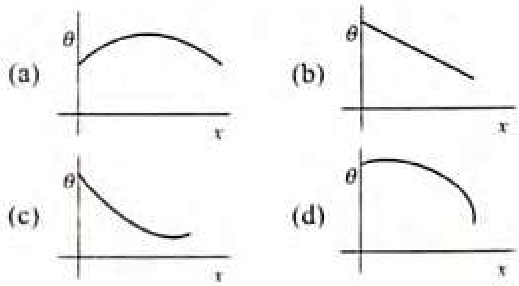
(b) Statement 1 is true, statement 2 is true; statement 2 is the correct explanation for statement 1.

(c) Statement 1 is true, statement 2 is true; statement 2 is not the a correct explanation for statement 1.

(d) Statement 1 is false but statement 2 is true.

(AIEEE 2009)

13. A long metallic bar is carrying heat from one of its ends to the other end under steady state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figures.



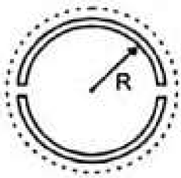
(AIEEE 2009)

14. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/Kg/K)

(a) 4.2 kJ (b) 8.4 kJ

(c) 84 kJ (d) 2.1 kJ **(AIEEE 2011)**

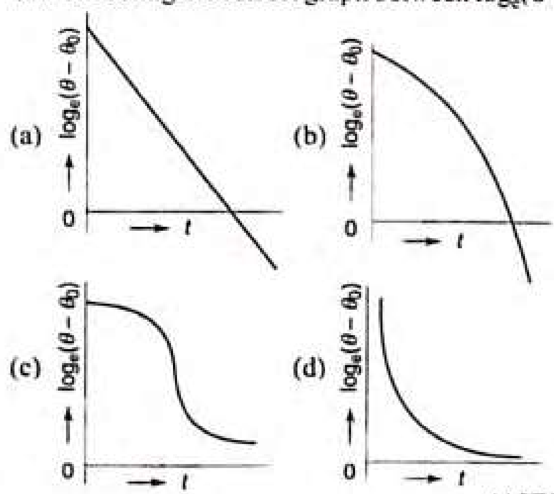
15. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is



(a) $2\pi SY\alpha\Delta T$ (b) $SY\alpha\Delta T$

(c) $\pi SY\alpha\Delta T$ (d) $2SY\alpha\Delta T$ **(AIEEE 2012)**

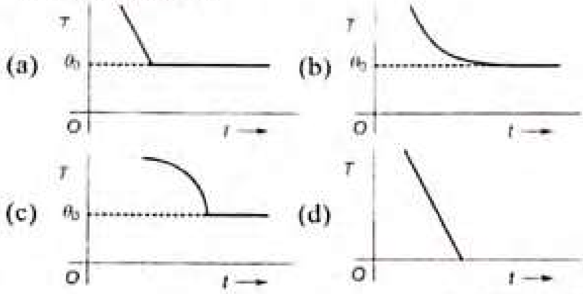
16. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t



(AIEEE 2012)

17. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the

graph between the temperature T of the metal and time t will be closest to



(JEE Main 2013)

18. Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. The end of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. The rate of heat flow through copper rod is

(a) 4.8 cal/s (b) 6.0 cal/s
(c) 1.2 cal/s (d) 2.4 cal/s

(JEE Main 2014)

19. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is:

(For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

(a) 2.2×10^7 Pa (b) 2.2×10^6 Pa
(c) 2.2×10^8 Pa (d) 2.2×10^9 Pa

(JEE Main 2014)

20. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:

(a) 25 °C ; $\alpha = 1.85 \times 10^{-5}$ /°C
(b) 60 °C ; $\alpha = 1.85 \times 10^{-4}$ /°C
(c) 30 °C ; $\alpha = 1.85 \times 10^{-3}$ /°C
(d) 55 °C ; $\alpha = 1.85 \times 10^{-2}$ /°C

(JEE Main 2016)

21. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and a is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by

(a) $\dfrac{3\alpha}{PK}$ (b) $3PK\alpha$

(c) $\dfrac{P}{3\alpha K}$ (d) $\dfrac{P}{\alpha K}$ **(JEE Main 2017)**

22. A copper ball of mass 100 g is at a temperature T. It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by (Given: room temperature = 30°C, specific heat of copper = 0.1 cal/g°C)

(a) 1250°C (b) 825°C
(c) 800°C (d) 885°C **(JEE Main 2017)**

ANSWER KEY

Exercises

1. (c)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (d)	**9.** (c)	**10.** (c)
11. (a)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (a)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (a)	**20.** (c)
21. (b)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (c)	**26.** (d)	**27.** (a)	**28.** (c)	**29.** (a)	**30.** (b)
31. (c)	**32.** (b)	**33.** (b)	**34.** (d)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (c)	**39.** (c)	**40.** (d)
41. (c)	**42.** (b)	**43.** (d)	**44.** (c)	**45.** (a)	**46.** (c)	**47.** (c)	**48.** (d)	**49.** (b)	**50.** (b)
51. (a)	**52.** (b)	**53.** (b)	**54.** (d)	**55.** (c)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (b)
61. (a)	**62.** (b)	**63.** (b)	**64.** (b)	**65.** (a)	**66.** (b)	**67.** (b)	**68.** (b)		

Archives

1. (a)	**2.** (a)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (d)	**10.** (a)
11. (d)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (d)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (a)
21. (c)	**22.** (d)								

Chapter 14

Kinetic Theory of Gases and Thermodynamics

KINETIC THEORY OF GASES

In gases, the intermolecular forces are very weak and the molecules may fly apart in all directions. So, the gas is characterized by the following properties:

(1) It has no shape and size and can be obtained in a vessel of any shape or size.
(2) It expands indefinitely and uniformly to fill the available space.
(3) It exerts pressure on its surroundings.

Assumption of Kinetic Theory of Gases

Kinetic theory of gases relates the macroscopic properties of gases (such as pressure, temperature, etc.) to the microscopic properties of the gas molecules (such as speed, momentum, kinetic energy of molecule, etc.).

Actually, the kinetic theory of gases attempts to develop a model of the molecular behaviour which should result in the observed behaviour of an ideal gas. It is based on the following assumptions:

(1) Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical, but are different from those of another gas.
(2) The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
(3) Their molecular size is negligible in comparison to intermolecular distance (10^{-9} m).
(4) The volume of molecules is negligible in comparison to the volume of a gas. (The volume of molecules is only about 0.014% of the volume of the gas.)
(5) The speed of gas molecules lies between zero and infinity (very high speed).
(6) The number of molecules moving with most probable speed is maximum.
(7) The gas molecules keep on colliding among themselves as well as with the walls of the containing vessel. These collisions are perfectly elastic (i.e., the total energy before collision = total energy after the collision).
(8) The molecules move in a straight line with constant speeds between successive collisions.
(9) The distance covered by the molecules between two successive collisions is known as free path and mean of all free paths is known as mean free path.
(10) The time spent in a collision between two molecules is negligible in comparison to the time between two successive collisions.
(11) The number of collisions per unit volume in a gas remains constant.
(12) No attractive or repulsive force acts between gas molecules.
(13) Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.
(14) The molecules constantly collide with the walls of the container due to which their momenta change. The change in momentum is transferred to the walls of the container. Consequently, pressure is exerted by gas molecules on the walls of the container.
(15) The density of a gas is constant at all points of the container.

Important Points

i. Relation between pressure, volume, mass and temperature

$$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 \quad \text{or} \quad P \propto \frac{(mN)T}{V} \qquad (\text{as } v_{rms}^2 \propto T)$$

- If volume and temperature of a gas are constant, then $P \propto mN$, i.e., pressure $\propto$ (mass of gas). That is, if mass of a gas is increased, number of molecules and hence number of collision per second will also increase, i.e., pressure will increase.
- If mass and temperature of a gas are constant, then $P \propto (1/V)$, i.e., if volume decreases, the number of collisions per second will increase due to lesser effective distance between the walls resulting in greater pressure.
- If mass and volume of gas are constant, then $P \propto (v_{rms})^2 \propto T$. That is, if temperature increases, the mean square speed of gas molecules will increase and as gas molecules are moving faster, they will collide with the walls more often with greater momentum resulting in greater pressure. Therefore,

$$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 = \frac{1}{3}\frac{M}{V}v_{rms}^2$$

(As $M = mN$ = total mass of the gas)

$$\therefore \quad P = \frac{1}{3}\rho v_{rms}^2 \qquad (\text{as } \rho = M/V)$$

ii. Relation between pressure and kinetic energy

Kinetic energy = $1/2\, M\, v_{rms}^2$

Therefore, kinetic energy per unit volume

$$E = (1/2)(M/V)v_{rms}^2 = (1/2)\rho v_{rms}^2 \quad \text{(i)}$$

We know

$$P = \frac{1}{3}\rho v_{rms}^2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $P = 2/3E$.

That is, the pressure exerted by an ideal gas is numerically equal to two-thirds of the mean kinetic energy of translation per unit volume of the gas.

ILLUSTRATION 14.1 (a) Calculate (i) root-mean-square speed and (ii) the mean energy of 1 mol of hydrogen at STP (given that density of hydrogen is 0.09 kg /m^3).

(b) Given that the mass of a molecule of hydrogen is 3.34×10^{-27} kg, calculate Avogadro's number.

(c) Calculate Boltzmann's constant.

Solution.

(a) (i) We know that the pressure of a gas is given as

$$P = \frac{1}{3}\rho v_{rms}^2$$

$$v_{rms} = \sqrt{\left(\frac{3P}{\rho}\right)} = \sqrt{\left(\frac{3 \times 0.76 \times 13.6 \times 10^3 \times 9.8}{0.09}\right)}$$

$$= 1837 \text{ m/s} = 1.837 \text{ km/s}$$

(ii) Kinetic energy $= \frac{1}{2}mv_{rms}^2$

Here $M = 2 \text{ g} = 2 \times 10^{-3}$ kg

or KE $= \frac{1}{2} \times 2 \times 10^{-3} \times (1837)^2 = 3374.56$ J

(b) Mass of one molecule of $H_2 = 3.34 \times 10^{-27}$ kg

Molecular mass of hydrogen $= 2 \times 10^{-3}$ kg

Avogadro's number N_A, which is the number of molecules in one gram molecule of hydrogen, is given by

$$N_A = \frac{2 \times 10^{-3}}{3.34 \times 10^{-27}} = 5.988 \times 10^{23} \text{ molecules}$$

(c) We know that

$k = R/N_A = 8.3/(5.988 \times 10^{23}) = 1.37 \times 10^{-23}$ J/mol K

Ideal Gas Equation

A gas that strictly obeys the gas laws is called perfect or an ideal gas. The size of the molecule of an ideal gas is zero, i.e., each molecule is a point mass with no dimension. There is no force of attraction or repulsion amongst the molecules of the gas. All real gases are not perfect gases. However, at extremely low pressure and high temperature, the gases such as hydrogen, nitrogen and helium, are nearly perfect gases.

The equation that relates the pressure (P), volume (V) and temperature (T) of the given state of an ideal gas is known as gas equation.

Ideal gas equations	
For 1 mole or N_A molecule or M grams or 22.4 L of gas	$PV = RT$
For μ moles of gas	$PV = \mu RT$
For 1 molecule of gas	$PV = (R/N_A)T = kT$
For N molecules of gas	$PV = NkT$
For 1 g of gas	$PV = (R/M)T = rT$
For n grams of gas	$PV = nrT$

Various Speeds of Gas Molecules

The motion of molecules in a gas is characterized by any of the following three speeds, such as root-mean-square speed, most probable speed and average speed.

1. **Root-mean-square speed:** It is defined as the square root of mean of squares of the speed of different molecules, i.e.,

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + v_4^2 + \cdots}{N}}$$

(i) From the expression for pressure of ideal gas,

$$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2$$

$$v_{rms} = \sqrt{\frac{3PV}{mN}} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3P}{\rho}}$$

(as ρ = mass of gas / V)

(ii) $$v_{rms} = \sqrt{\frac{3PV}{\text{Mass of gas}}} = \sqrt{\frac{3\mu RT}{\mu M}} = \sqrt{\frac{3RT}{M}}$$

(as M is the molecular weight of gas; $PV = \mu RT$ and mass of gas $= \mu M$)

(iii) $$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3N_A kT}{N_A m}} = \sqrt{\frac{3kT}{m}}$$

(as $M = N_A m$ and $R = N_A k$)

Therefore, root-mean-square velocity

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$$

Important Points

- With rise in temperature, rms speed of gas molecules increases as $v_{rms} \propto \sqrt{T}$.
- With the increase in molecular weight, rms speed of gas molecule decreases as $v_{rms} \propto 1/\sqrt{M}$. For example, rms speed of hydrogen molecules is four times that of oxygen molecules at the same temperature.
- rms speed of gas molecules is of the order of km/s. For example, at NTP for hydrogen gas

$(v_{rms}) = \sqrt{3RT/M} = \sqrt{(3 \times 8.31 \times 273)/(2 \times 10^{-3})} = 1840$ m/s

- rms speed of gas molecules is $\sqrt{3/\gamma}$ times that of speed of sound in gas.

  As $v_{rms} = \sqrt{\frac{3RT}{M}}$ and $v_s = \sqrt{\frac{\gamma RT}{M}}$

  $\therefore \quad v_{rms} = \frac{3}{\gamma} v_s$
- rms speed of gas molecules does not depend on the pressure of gas (if temperature remains constant) because $P \propto \rho$ (Boyle's law). If pressure is increased n times, then density will also increase by n times hence v_{rms} remains constant.
- Moon has no atmosphere because v_{rms} of gas molecules is more than escape velocity (v_e) on moon. A planet or satellite will have atmosphere only and only if $v_{rms} < v_e$
- At $T = 0$, $v_{rms} = 0$, i.e., the rms speed of molecules of a gas is zero at 0 K. This temperature is called absolute zero.

2. **Most probable speed:** The particles of a gas have a range of speeds. This is defined as the speed which is possessed by the maximum fraction of total number of molecules of the gas. For example, if speeds of 10 molecules of a gas are 1, 2, 2, 3, 3, 3, 4, 5, 6, 6 km/s, then the most probable speed is 3 km/s, as the maximum fraction of total molecules possess this speed.

   Most probable speed $v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$

3. **Average speed:** It is the arithmetic mean of the speed of molecules in a gas at given temperature, i.e.,

   $$v_{av} = \frac{v_1 + v_2 + v_3 + v_4 + \cdots}{N}$$

   According to kinetic theory of gases

   Average speed $v_{av} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8}{\pi}\frac{RT}{M}} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$

NOTE:

- $v_{rms} > v_{av} > v_{mp}$
- $v_{rms} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} = \sqrt{3} : \sqrt{2.5} : \sqrt{2}$

Kinetic Energy of Ideal Gas

Molecules of ideal gases possess only translational motion. So they possess only translational kinetic energy.

Quantity of gas	Kinetic energy
Kinetic energy of a gas molecule ($E_{molecule}$)	$= \frac{1}{2}mv_{rms}^2 = \frac{1}{2}m\left(\frac{3kT}{m}\right) = \frac{3}{2}kT$ $\left(\text{as } v_{rms} = \sqrt{\frac{3kT}{m}}\right)$
Kinetic energy of 1 mole (M gram) gas (E_{mole})	$= \frac{1}{2}Mv_{rms}^2 = \frac{1}{2}M\frac{3RT}{M} = \frac{3}{2}RT$ $\left(\text{as } v_{rms} = \sqrt{\frac{3RT}{M}}\right)$
Kinetic energy of 1g of gas (E_{gram})	$= \frac{3}{2}\frac{R}{M}T = \frac{3}{2}\frac{kN_A}{mN_A}T$ $= \frac{3}{2}\frac{k}{m}T = \frac{3}{2}rT$

Here m, mass of each molecule; M, molecular weight of gas; N_A, Avogadro number $= 6.023 \times 10^{23}$.

Important Points

- Kinetic energy per molecule of gas does not depend upon the mass of the molecule but only depends upon the temperature of the gas.

  As $E = \frac{3}{2}kT$ or $E \propto T$, i.e., molecules of different gases such as He, H_2 and O_2 at same temperature will have same translational kinetic energy, though their rms speeds are different. $\left[v_{rms} = \sqrt{\frac{3kT}{m}}\right]$
- Kinetic energy per mole of gas depends only upon the temperature of gas.
- Kinetic energy per gram of gas depends upon the temperature as well as molecular weight (or mass of one molecule) of the gas.

  $E_{gram} = \frac{3}{2}\frac{k}{m}T \qquad \therefore E_{gram} \propto \frac{T}{m}$

  From the above expressions, it is clear that higher the temperature of the gas, more will be the average kinetic energy possessed by the gas molecules. At $T = 0$, $E = 0$, i.e., at absolute zero the molecular motion stops.

ILLUSTRATION 14.2 A glass bulb of volume 400 cm^3 is connected to another bulb of volume 200 cm^3 by means of a tube of negligible volume. The bulbs contain dry air and are both at a common temperature and pressure of 20°C and 1.000 atm, respectively. The larger bulb is immersed in steam at 100°C and the smaller in melting ice at 0°C. Find the final common pressure.

Solution. Let n_1 and n_2 denote the number of moles of gas in the large and small bulbs, in the final configuration, respectively. Denoting the final temperatures by T_1 and T_2 and the final pressure by P_f, the ideal gas law implies that

$$p_f V_1 = n_1 RT_1 \quad \text{(i)}$$

and $$p_f V_2 = n_2 RT_2 \quad \text{(ii)}$$

where p_0, $V_0 = V_1 + V_2$ and T_0 are the initial pressure, volume, and temperature, respectively.

Using Eqs. (i) and (ii) in the equation $n_1 + n_2 = n$, we get

$$\frac{p_f V_1}{RT_1} + \frac{p_f V_2}{RT_2} = \frac{p_0 V_0}{RT_0}$$

Solving for P_f, we obtain $p_f = \dfrac{p_0 V_0}{T_0\left(\dfrac{V_1}{T_1} + \dfrac{V_2}{T_2}\right)}$

Inserting the numerical values; $p_0 = 1.00$ atm,

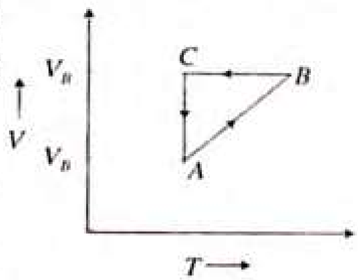
$T_0 = 293.15$ K ($T_0 = 20°$C),

$T_1 = 373.15$ K ($t_1 = 100°$C),

and $T_2 = 237.15$ K ($t_2 = 0°$C), we find

$$p_f = \frac{(1.00)(600)}{(293.15)\left[\left(\frac{400}{373.15}\right) + \left(\frac{200}{273.15}\right)\right]} = 1.13\text{atm}$$

ILLUSTRATION 14.3 A cyclic process $ABCA$ shown in V–T diagram (figure) is performed with a constant mass of an ideal gas. Show the same process on a P–V diagram.



Solution.

According to given diagram, we notice the following:

(a) For line AB, $V = aT$ and both V and T increase so the gas equation $PV = \mu RT$ in the light of above yields $P(aT) = \mu RT$, i.e., $P = (\mu R/a)$ = constant, with V increasing.

So in P–V diagram, line AB will be a straight line parallel to the V-axis (with V increasing)

(b) For line BC, V = constant and T decreasing; so, the gas equation $PV = \mu RT$ in the light above yields.

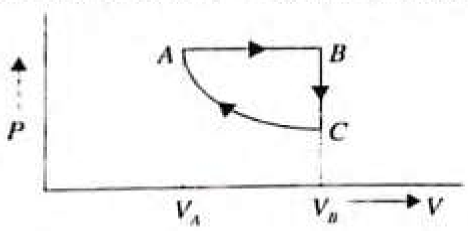
$P = \dfrac{\mu RT}{\text{Constant}}$, where $P \propto T$ with T decreasing

i.e., along line BC, P decreases with V = constant. So in P–V diagram, line BC will be a straight line parallel to the P-axis (with P decreasing).

(c) $P = \dfrac{\mu RT}{V}$, where PV = constant with V decreasing

So in P–V curve line, CA will be a hyperbola (with P increasing).

The complete cycle on P–V diagram is shown in figure.



DEGREE OF FREEDOM

Degree of freedom is the minimum number of variables required to completely specify the state of system.

For thermodynamic system (moving particles), these are the total number of independent terms of energy. The independent possible motions are translational, rotational and vibrational, so there are three types of degrees of freedom.

i. **Translational degrees of freedom:** The maximum number of translational degrees of freedom can be three. These are $1/2mv_x^2$, $1/2mv_y^2$, $1/2mv_z^2$.

ii. **Rotational degrees of freedom:** The maximum number of rotational degrees of freedom can be three. These are $1/2I_x\omega_x^2$, $1/2I_y\omega_y^2$, $1/2I_z\omega_z^2$.

iii. **Vibrational degrees of freedom:** Their numbers depend on atoms in the molecule and their arrangement. These degrees of freedom are considered at a very high temperature only.

NOTE: At room temperature only translational and rotational degrees of freedom are taken into account.

i. **Monatomic gas:** It has been assumed that the molecules of a gas are negligible in size, so moment of inertia and hence rotational kinetic energy of monatomic gas molecules about the axis passes through itself will be zero. The degrees of freedom of monatomic gas molecules are due to three independent translation along the x-, y- and z-axis.

ii. **Diatomic gas:** In diatomic gases, the molecules are assumed to be in the shape of dumbbells; two atoms of negligible size are at some separation. In addition to translation motion, the molecule can rotate about an axis, so the degrees of freedom of diatomic gas molecules are due to translation and due to rotation. If the line joining the two atoms (particles) is taken as the z-axis, then moment of inertia and hence rotational kinetic energy about the z-axis becomes zero. The molecule has three degrees of freedom of translation and two degrees of rotation.

NOTE: If vibrational degrees of freedom are taken into account, then the total number of degrees of freedom of diatomic molecule becomes seven.

Triatomic or polyatomic gas: A non-linear molecule has non-zero moment of inertia about any axis, so there are three rotational degrees of freedom. A total number of degrees of freedom are six.

NOTE:

- At high temperature, in case of diatomic or polyatomic molecules, the atoms within the molecule may also vibrate with respect to each other. In such cases, the molecule will have an additional degrees of freedom due to vibrational motion.
- An object that vibrates in one dimension has two additional degree's of freedom. One for the potential energy and one for the kinetic energy of vibration.

- A diatomic molecule that is free to vibrate (in addition to translation and rotation) will have 7 (2 + 3 + 2) degrees of freedom.
- Though an atom in a solid has no degree of freedom for translational and rotational motion, due to vibration along 3 axes, it has 3 × 2 = 6 degrees of freedom.
- When a diatomic or polyatomic gas dissociates into atoms, it behaves as monatomic gas whose degrees of freedom are changed accordingly.

LAW OF EQUIPARTITION OF ENERGY

For any system in thermal equilibrium, the total energy is equally distributed among its various degrees of freedom. And the energy associated with each molecule of the system per degree's of freedom of the system is $1/2kT$, where k (Boltzmann's constant) $= 1.38 \times 10^{-23}$ J/K, T = absolute temperature of the system.

If the system possesses number of degrees of freedom f, then we study equipartition of energy as given below.

Total energy associated with each molecule	$(f/2)kT$
Total energy associated with N molecules	$N(f/2)kT$
Total energy associated with each mole	$(f/2)RT$
Total energy associated with μ mole	$(\mu f/2)RT$
Total energy associated with each gram	$(f/2)rT$
Total energy associated with M_0 gram	$M_0(f/2)rT$

ILLUSTRATION 14.4 Calculate (a) the average kinetic energy of translation of an oxygen molecule at 27°C, (b) the total kinetic energy of an oxygen molecule at 27°C, (c) the total kinetic energy in joule of one mole of oxygen at 27°C. Given Avogadro's number $= 6.02 \times 10^{23}$ and Boltzmann's constant $= 1.38 \times 10^{-23}$ J/(mol-K).

Solution.

(a) An oxygen molecule has three translational degrees of freedom, thus the average translational kinetic energy of an oxygen molecule at 27°C is given as

$$E_T = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$$

$$= 6.21 \times 10^{-21} \text{ J/molecule}$$

(b) An oxygen molecule has total five degrees of freedom, hence its total kinetic energy is given as

$$E_T = \frac{5}{2}kT = \frac{5}{2} \times 1.38 \times 10^{-23} \times 300$$

$$= 10.35 \times 10^{-21} \text{ J/molecule}$$

(c) Total kinetic energy of one mole of oxygen is its internal energy, which can be given as

$$U = \frac{f}{2}\mu RT = \frac{5}{2} \times 1 \times 8.314 \times 300 = 6235.5 \text{ J/mole}$$

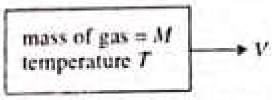
ILLUSTRATION 14.5 A light container having a diatomic gas enclosed with in is moving with velocity v. Mass of the gas is M and number of moles is n.

(a) What is the kinetic energy of gas w.r.t. centre of mass of the system?

(b) What is K.E. of gas w.r.t. ground?

Solution.

The kinetic energy of gas w.r.t. centre of mass of the system is the kinetic energy of molecules due to their random motion. This will be the internal energy of the gas.



(a) $\text{K.E.} = \frac{5}{2}nRT$

(b) Kinetic energy of gas w.r.t. ground = Kinetic energy of centre of mass w.r.t. ground + Kinetic energy of gas w.r.t. centre of mass.

$$\text{K.E.} = \frac{1}{2}MV^2 + \frac{5}{2}nRT$$

SPECIFIC HEAT OR SPECIFIC HEAT CAPACITY

It characterizes the nature of the substance in response to the heat supplied to the substance. Specific heat can be defined by two following ways: gram specific heat and molar specific heat.

Gram Specific Heat

Gram specific heat of a substance may be defined as the amount of heat required to raise the temperature of unit mass of the substance by unit degree.

Gram specific heat $c = \dfrac{\Delta Q}{m\Delta T}$

Units: $\dfrac{\text{cal}}{\text{g} \times {}^\circ\text{C}}; \dfrac{\text{cal}}{\text{g} \times \text{K}}; \dfrac{\text{J}}{\text{kg} \times \text{K}}$; dimension: $(L^2T^{-2}\theta^{-1})$

Molar Specific Heat

Molar specific heat of a substance may be defined as the amount of heat required to raise the temperature of 1 g mole of the substance by a unit degree, it is represented by C.

$$C = \frac{\Delta Q}{\mu \Delta T}$$

Units: $\dfrac{\text{cal}}{\text{mole} \times {}^\circ\text{C}}, \dfrac{\text{cal}}{\text{mole} \times \text{K}}$ or $\dfrac{\text{J}}{\text{mole} \times \text{K}}$

☞ Important Points

- $C = Mc = (M/m)(\Delta Q/\Delta T) = (1/\mu)(\Delta Q/\Delta T)$ (as $\mu = m/M$)
  It means that molar specific heat of the substance is M times the gram specific heat, where M is the molecular weight of that substance.
- Specific heat for hydrogen is maximum, i.e., $C = 3.5(\text{cal/g}^\circ\text{C})$.
- In liquids, water has maximum specific heat $C = 1(\text{cal/g}^\circ\text{C})$.
- Specific heat of a substance also depends on the state of substance, i.e., solid, liquid or gas. For example,
  $$C_{\text{ice}} = 0.5\frac{\text{cal}}{\text{g}^\circ\text{C}},\quad C_{\text{water}} = 1\frac{\text{cal}}{\text{g}^\circ\text{C}},\quad C_{\text{steam}} = 0.47\frac{\text{cal}}{\text{g}^\circ\text{C}}$$
- Specific heat also depends on the conditions of the experiment, i.e., the way in which heat is supplied to the body. In general, experiments are made either at constant volume or at constant pressure.
  In case of solids and liquids, due to small thermal expansion, the difference in measured values of specific heats is very small and is usually neglected. However, in case of gases, specific heat at constant volume is quite different from that at constant pressure.

Specific Heat of Gases

In case of gases, heat energy supplied to a gas is spent not only in increasing the temperature of the gas but also in expanding the gas against atmospheric pressure.

Hence, specific heat of a gas, which is the amount of heat energy required to raise the temperature of 1 g of gas through a unit degree, shall not have a single or unique value.

Consider a gas enclosed in a cylinder which is fitted with an airtight and frictionless piston.

i. If the gas is compressed suddenly and no heat is supplied from outside, i.e., $\Delta Q = 0$, the temperature of the gas increases on the account of compression

$$\therefore \quad C = \frac{\Delta Q}{m(\Delta T)} = 0, \quad \text{i.e., } C = 0$$

Such a process is called 'adiabatic'.

ii. If the gas is heated and allowed to expand at such a rate that the rise in temperature due to heat supplied is exactly equal to the fall in temperature due to expansion of the gas, i.e., $\Delta T = 0$

$$\therefore \quad C = \frac{\Delta Q}{m(\Delta T)} = \frac{\Delta Q}{0} = \infty, \quad \text{i.e. } C = \infty$$

Such a process is called 'isothermal'.

iii. If the rate of expansion of the gas was slow, the fall in temperature of the gas due to expansion would be smaller than the rise in temperature of the gas due to heat supplied. Therefore, there will be some net rise in temperature of the gas, i.e., ΔT will be positive.

$$\therefore \quad C = \frac{\Delta Q}{m(\Delta T)} = \text{positive}, \quad \text{i.e., } C = \text{positive}$$

iv. If the gas is heated and allowed to expand very fast, the fall of temperature of the gas due to expansion would be greater than the rise in temperature due to heat supplied. Therefore, there will be some net fall in temperature of the gas, i.e., ΔT will be negative.

$$\therefore \quad C = \frac{\Delta Q}{m(-\Delta T)} = \text{negative}, \quad \text{i.e., } C = \text{negative}$$

Hence, the specific heat of gas can have any positive value ranging from zero to infinity. Further, it can even be negative. The exact value depends upon the mode of heating the gas. Out of many values of specific heat of a gas, two are of special significance:

1. **Specific heat of a gas at constant volume (c_v):** The specific heat of a gas at constant volume is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1 K when its volume is kept constant, i.e.,

$$c_v = \frac{(\Delta Q)_v}{m\Delta T}$$

If instead of unit mass, 1 mole of gas is considered, the specific heat is called molar specific heat at constant volume and is represented by C_v.

$$C_v = Mc_v = \frac{M(\Delta Q)_v}{m\Delta T} = \frac{1}{\mu}\frac{(\Delta Q)_v}{\Delta T} \qquad (\text{as } \mu = m/M)$$

2. **Specific heat of a gas at constant pressure (c_p):** The specific heat of a gas at constant pressure is defined as the quantity of heat required to raise the temperature of unit mass of gas through 1 K when its pressure is kept constant, i.e.,

$$c_p = \frac{(\Delta Q)_p}{m\Delta T}$$

If instead of unit mass, 1 mole of gas is considered, the specific heat is called molar specific heat at constant pressure and is represented by C_p.

$$C_p = Mc_p = \frac{M(\Delta Q)_p}{m\Delta T} = \frac{1}{\mu}\frac{(\Delta Q)_p}{\Delta T} \qquad (\text{as } \mu = m/M)$$

Mayer's Formula

Out of two principle specific heats of a gas, C_p is more than C_v because in case of C_v, volume of gas is kept constant and heat is required only for raising the temperature of 1 g mole of the gas through 1°C or 1 K.

No heat, whatsoever, is spent in the expansion of the gas. It means that the heat supplied to the gas increases its internal energy only, i.e., (i)

$$(\Delta Q)_v = \Delta U = \mu C_v \Delta T$$

while in case of C_p, the heat is used in two ways:

i. In increasing the temperature of the gas by ΔT,
ii. In doing work (ΔW), due to expansion at constant pressure.

Therefore, (ii)

$$(\Delta Q)_p = \Delta U + \Delta W = \mu C_p \Delta T$$

From Eqs. (i) and (ii), we get

$$\mu C_p \Delta T - \mu C_v \Delta T = \Delta W$$

$\Rightarrow \quad \mu\Delta T(C_p - C_v) = P\Delta V$ (for constant P, $\Delta W = P\Delta V$)

$$\Rightarrow \quad C_p - C_v = \frac{P\Delta V}{\mu\Delta T}$$

(from $PV = \mu RT$. At constant pressure $P\Delta V = \mu R\Delta T$)

$$\Rightarrow \quad C_p - C_v = R$$

This relation is called the Mayer's formula and shows that $C_p > C_v$, i.e., molar specific heat at constant pressure is greater than that at constant volume.

Specific Heat and Kinetic Energy for Different Gases

		Monatomic	Diatomic	Triatomic non-linear	Triatomic linear
Atomicity	A	1	2	3	3
Restriction	B	0	1	3	2
Degree of freedom	$f = 3A - B$	3	5	6	7
Molar specific heat at constant volume	$C_v = (f/2)R = R/(\gamma - 1)$	$3/2R$	$5/2R$	$3R$	$7/2R$
Molar specific heat at constant pressure	$C_p = [(f/2) + 1]\, R = (\gamma/\gamma - 1)R$	$5/2R$	$7/2R$	$4R$	$9/2R$
Ratio of C_p to C_v	$\gamma = C_p/C_v = 1 + (2/f)$	$5/3 = 1.66$	$7/5 = 1.4$	$4/3 = 1.33$	$9/7 = 1.28$
Kinetic energy of 1 mole	$E_{mole} = (f/2)RT$	$(3/2)RT$	$(5/2)RT$	$3RT$	$(7/2)RT$
Kinetic energy of 1 molecule	$E_{molecule} = (f/2)kT$	$(3/2)kT$	$(5/2)kT$	$3kT$	$(7/2)kT$
Kinetic energy of 1 g	$E_{gram} = (f/2)rT$	$(3/2)rT$	$(5/2)rT$	$3rT$	$(7/2)rT$

ILLUSTRATION 14.6 An ideal gas has a specific heat at constant pressure $C_P = (5/2)\, R$. The gas is kept in a closed vessel of volume $8.3 \times 10^{-3}\ \text{m}^3$ at a temperature of 300 K and a pressure of $1.6 \times 10^6\ \text{N/m}^2$. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of the gas ($R = 8.3$ J/mol K).

Solution. As here volume of gas remains constant, we have

$$(\Delta Q)_V = \mu C_V \Delta T$$

Here, $C_V = C_P - R = (5/2)R - R = (3/2)R$

$$\mu = \frac{PV}{RT} = \frac{1.6\times10^6 \times 8.3\times10^{-3}}{8.3\times300} = \frac{16}{3}$$

$$2.49\times10^4 = \frac{16}{3}\times\frac{3}{2}\times8.3\Delta T$$

i.e., $\Delta T = 375$ or $T_2 - T_1 = 375$

i.e., $T_2 = (375 + 300)1 = 675$ K

Now as for a given mass of an ideal gas at constant volume,

$$P \propto T$$

$$(P_2/P_1) = (T_2/T_1)$$

$$P_2 = \frac{675}{300}\times1.6\times10^6 = 3.6\times10^6\ \text{N/m}^2$$

GASEOUS MIXTURE

If two non-reactive gases are enclosed in a vessel of volume V. In the mixture μ_1 moles of one gas are mixed with μ_2 moles of another gas. If N_A is Avogadro's number then number of molecules of first gas $N_1 = \mu_1 N_A$ and number of molecules of second gas $N_2 = \mu_2 N_A$.

i. Total number of moles $\mu = (\mu_1 + \mu_2)$.

ii. If M_1 is the molecular weight of first gas and M_2 that of second gas. Then molecular weight of mixture will be

$$M = \frac{\mu_1 M_1 + \mu_2 M_2}{\mu_1 + \mu_2}$$

iii. Specific heat of the mixture at constant volume will be

$$C_{V_{mix}} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2} = \frac{\mu_1\left(\frac{R}{\gamma_1 - 1}\right) + \mu_2\left(\frac{R}{\gamma_2 - 1}\right)}{\mu_1 + \mu_2}$$

$$= \frac{R}{\mu_1 + \mu_2}\left(\frac{\mu_1}{\gamma_1 - 1} + \frac{\mu_2}{\gamma_2 - 1}\right)$$

$$\therefore \quad C_{V_{mix}} = \frac{R}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}\left(\frac{m_1/M_1}{\gamma_1 - 1} + \frac{m_2/M_2}{\gamma_2 - 1}\right)$$

iv. Specific heat of the mixture at constant pressure will be

$$C_{P_{mix}} = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2}}{\mu_1 + \mu_2}$$

$$\Rightarrow \quad C_{P_{mix}} = \frac{\mu_1\left(\frac{\gamma_1}{\gamma_1 - 1}\right)R + \mu_2\left(\frac{\gamma_2}{\gamma_2 - 1}\right)R}{\mu_1 + \mu_2}$$

$$= \frac{R}{\mu_1 + \mu_2}\left[\mu_1\left(\frac{\gamma_1}{\gamma_1 - 1}\right) + \mu_2\left(\frac{\gamma_2}{\gamma_2 - 1}\right)\right]$$

$$\therefore \quad C_{P_{mix}} = \frac{R}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}\left[\frac{m_1}{M_1}\left(\frac{\gamma_1}{\gamma_1 - 1}\right) + \frac{m_2}{M_2}\left(\frac{\gamma_2}{\gamma_2 - 1}\right)\right]$$

Illustration 14.7 How much heat energy should be added to the gaseous mixture consisting of 1 g of hydrogen and 1 g of helium to raise its temperature from 0°C to 100°C

(a) at constant volume,

(b) at constant pressure ($R = 2$ cal/mol K)?

Solution. As hydrogen is diatomic and has molecular weight 2,

$$(C_V)_H = \frac{5}{2}R \quad \text{and} \quad \mu_H = \frac{1}{2}$$

While He is monatomic and has molecular weight 4,

$$(C_V)_{mix} = \frac{3}{2}R \quad \text{and} \quad \mu_{He} = \frac{1}{4}$$

So, by conservation of energy, we get

$$(C_V)_{mix} = \frac{\mu(C_V)_1 + \mu_2(C_V)_2}{\mu_1 + \mu_2} = \frac{\frac{1}{2}\times\frac{5}{2}R + \frac{1}{4}\times\frac{3}{2}R}{\frac{1}{2}+\frac{1}{4}}$$

$$= \frac{13}{8}\times\frac{4}{3}R = \frac{13}{6}R$$

(a) So

$$Q_V = \mu C_V \Delta T = \left(\frac{1}{2}+\frac{1}{4}\right)\times\frac{13}{6}\times 2\times(100-0) = 325 \text{ cal}$$

(b) Now as $(C_P)_{mix} = (C_V)_{mix} + R = \frac{13}{6}R + R = \frac{19}{6}R$

$$Q_P = \mu C_P \Delta T = \left(\frac{1}{2}+\frac{1}{4}\right)\times\frac{19}{6}\times 2\times(100-0) = 475 \text{ cal}$$

Concept Application Exercise 14.1

1. A thermally insulated vessel with gaseous nitrogen at a temperature of 27°C moves with velocity 100 m/s^{-1}. How much (in percentage) and in what way will the gas pressure change if the vessel is brought to rest suddenly?
2. Find the molar mass and the number of degrees of freedom of molecules in a gas if its heat capacities are $C_v = 650$ J kg^{-1} K^{-1} and $C_p = 910$ J kg^{-1} K^{-1}.
3. At 127°C and 1.00×10^{-2} atm pressure, the density of a gas is 1.24×10^{-2} kg m^{-3}.
   (a) Find v_{rms} for the gas molecules.
   (b) Find the molecular weight of the gas and identify it.
4. The mass of a gas molecule can be computed from the specific heat at constant volume. Take $C_v = 0.075$ kcal kg^{-1} k^{-1} for argon and calculate
   (a) the mass of an argon atom
   (b) the atomic weight of argon.
5. In a certain gas, 2/5th of the energy of the molecules is associated with the rotation of the molecules, and the rest is associated with the motion of their centres of mass. What is the average translational energy of one such molecule when the temperature is 27°C? How much energy is required to raise the temperature by 1°C?
6. If the water molecules in 1 g of water were distributed uniformly over the surface of the earth, how many molecules would there be in 1 m^2 of the earth's surface (radius of the earth = 6400 km)?

THERMODYNAMICS

Thermodynamics is a branch of science which deals with exchange of heat energy between bodies and conversion of the heat energy into mechanical energy and vice versa.

Thermodynamic Variables and Equation of State

A thermodynamic system can be described by specifying its pressure, volume, temperature, internal energy and the number of moles. These parameters are called thermodynamic variables. The relation between the thermodynamic variables (P, V, T) of the system is called equation of state.

For μ moles of an ideal gas, equation of state is $PV = \mu RT$ and for 1 mole of an it ideal gas is $PV = RT$.

Thermodynamic Equilibrium

When the thermodynamic variables attain a steady value, i.e., they are independent of time, the system is said to be in the state of thermodynamic equilibrium. For a system to be in thermodynamic equilibrium, the following conditions must be fulfilled.

Mechanical equilibrium: There is no unbalanced force between the system and its surroundings.

Thermal equilibrium: There is a uniform temperature in all parts of the system and is same as that of surrounding.

Chemical equilibrium: There is a uniform chemical composition throughout the system and the surrounding.

Thermodynamic Process

The transaction of heat between the surroundings and a thermodynamic system can be achieved in several ways, effecting the work done by or on it, and/or allowing an alternation in its internal energy. Each of the ways executed is said to be a process (thermodynamic). Obviously, there is no limitation to the variety of thermodynamic processes.

The process of change of state of a system involves change of thermodynamic variables such as pressure P, volume V and temperature T of the system. This is known as thermodynamic process. Some important processes are (i) isothermal process, (ii) adiabatic process, (iii) isobaric process, (iv) isochoric (isovolumic process), (v) cyclic and non-cyclic process and (vi) reversible and irreversible process.

ZEROTH LAW OF THERMODYNAMICS

If systems A and B are separately in thermal equilibrium with a third system C, then A and B are also in thermal equilibrium with each other.

i. The zeroth law leads to the concept of temperature. All bodies in thermal equilibrium must have a common property which has the same value for all of them. This property is called the temperature.

ii. The zeroth law came to light long after the first and second laws of thermodynamics had been discovered and numbered. Because the concept of temperature is fundamental to those two laws, the law that establishes temperature as a valid concept should have the lowest number. Hence, it is called zeroth law.

FIRST LAW OF THERMODYNAMICS

Consider a thermodynamic system at a certain thermodynamic state with a certain external pressure bearing on its surface (due to surrounding). Let an infinitely small quantity of heat dQ be imparted to it externally. In the general case, the temperature of the system increases and it also expands.

An increase in temperature signifies an increase in the randomness and intensity of molecular motion, i.e., an increase in the kinetic energy of the molecules, which is the modern concept of increase in temperature, according to the kinetic theory's interpretation. Let the increase in internal kinetic energy be dK.

The increased volume of the system also denotes some work done against the intermolecular forces of attraction, which is the work of dispersion, and is stored in the system itself as an increased internal potential energy. Let it be denoted by dU'.

The increase in volume also involves an appreciable amount of work done against the external forces, the work of expansion often spoken of as external work. Let it be denoted by dW.

Ignoring, any other type of changes (electrical, chemical, hysterical, etc.) by the law of conservation of energy, and the principle of the equivalence of heat and work, the heat energy balance equation can be written as

$$dQ = dK + dU' + dW$$

Now, $dK + dU'$ is evidently the sum total of the increase in the internal kinetic and potential energy of the system, which can be more briefly stated as the increase in the internal energy of the system and can be denoted by dU.

The above equation can be rewritten

$$dQ = dU + dW \qquad \text{(i)}$$

Equation (i) establishes the first law of thermodynamics, which states that the heat imparted to a body is, in general, expanded in two ways. A part of it is used to increase its internal energy and the remaining is utilized to do work against external pressure.

While substituting the values of dU, dQ and dW in Eq. (i) the following sign convention should be adopted:

1. If heat is imparted or supplied to a system, then dQ is positive; if heat is extracted from a system, then dQ is negative.
2. If there is an increase in volume, i.e., dV is positive, then the system does work against the external pressure bearing upon the system's surface and accordingly dW is positive and work is said to be done by the system; on the other hand, if there is a decrease in volume, i.e., dV is negative, then dW is negative and work is said to have been done on the system.
3. $dU = U_f - U_i$, where U_i and U_f are the initial and final internal energies of the system, respectively. If the internal energy increases (i.e., $U_f > U_i$), then dU is positive, while if the internal energy decreases during the process (i.e., $U_f < U_i$), then dU is negative.
4. For an ideal gas, internal energy depends on temperature only.

Quantities Involved in First Law of Thermodynamics

Heat (Q)

It is the energy that is transferred between a system and its environment because of the temperature difference between them. Heat always flows from a body at higher temperature to a body at lower temperature till their temperatures become equal.

Important Points

- Heat is a form of energy, so it is a scalar quantity with dimension (ML^2T^{-2}).
- Unit: joule (SI), calorie (practical unit), and 1 calorie = 4.2 J.
- Heat is a path dependent quantity, i.e., heat required to change the temperature of a given gas at a constant pressure is different from that required to change the temperature of same gas through same amount at constant volume.
- For solids and liquids: $Q = mL$ (for change in state) and $Q = mc\Delta T$ (for change in temperature)
  For gases when heat is absorbed and temperature changes:
  $Q_V = \mu C_V \Delta T$ (for constant volume)
  $Q_P = \mu C_P \Delta T$ (for constant pressure)

The total work done by the gas, while its volume change from V_1 to V_2, can be obtained by adding up (integrating) the small works dW performed by the gas in stages during each element change in volume dV. Thus, the total work done will be

$$W = \int dW = \int_{V_1}^{V_2} P dV$$

If the pressure P remains constant during the process of change in volume, then

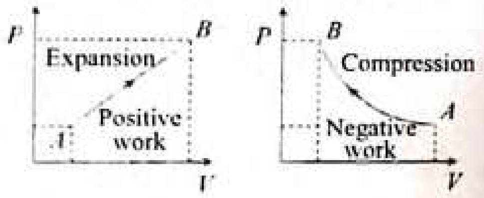
$$W = P\int_{V_1}^{V_2} dV$$

(Pressure being constant can be taken out of the integrating sign)

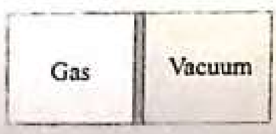
$$W = P[V]_{V_1}^{V_2} = P(V_2 - V_1) = P\Delta V$$

Important Points

- From $\Delta W = P\Delta V = P(V_f - V_i)$
  ΔW = positive if $V_f > V_i$, i.e., system expands against some external force.
  ΔW = negative if $V_f < V_i$, i.e., system contracts because of some external force exerted by the surrounding.
- In P–V diagram or indicator diagram, the area under P–V curve represents work done.
  W = area under P–V diagram
  It is positive if volume increases (for expansion).
  It is negative if volume decreases (for compression).
- In a cyclic process, work done is equal to the area under the cycle.
  It is positive if the cycle is clockwise.
  It is negative if the cycle is anticlockwise.

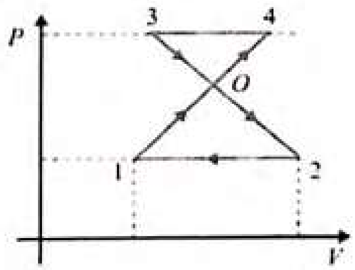


- If the gas expands in such a way that the other side of the piston is vacuum, then work done by the gas will be zero.



As $W = P\Delta V = 0$ (here $P = 0$)

ILLUSTRATION 14.8 Determine the work done by an ideal gas doing $1 \to 4 \to 3 \to 2 \to 1$.
Given $P_1 = 10^5$ Pa, $P_0 = 3 \times 10^5$ Pa, $P_3 = 4 \times 10^5$ Pa and $V_2 - V_1 = 10$ L.



Solution. From figure,

$$\frac{V_4 - V_3}{V_2 - V_1} = \frac{P_3 - P_0}{P_0 - P_1} \Rightarrow \frac{V_4 - V_3}{10} = \frac{4\times10^5 - 3\times10^5}{3\times10^5 - 10^5}$$

$$V_4 - V_3 = 5\text{ L}$$

Now, work done

$$W = \left(\frac{1}{2}\times10\times2\times10^5 - \frac{1}{2}\times5\times1\times10^5\right)\times10^{-3} = 750\text{ J}$$

Internal Energy (U)

Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

The energy due to molecular motion is called internal kinetic energy U_K and energy due to molecular configuration is called internal potential energy U_P. Therefore, total internal energy

$$U = U_K + U_P$$

i. For an ideal gas, as there is no molecular attraction $U_p = 0$, i.e., internal energy of an ideal gas is totally kinetic and is given by $U = U_K = (3/2)\mu RT$ and change in internal energy $U = (3/2)\mu R\Delta T$

ii. In case of gases, whatever may be the process

$$\Delta U = \mu\frac{f}{2}R\Delta T = \mu C_v\Delta T$$

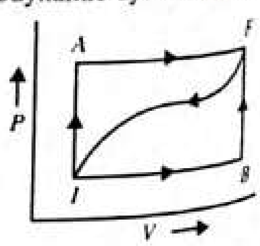
$$= \mu\frac{R}{(\gamma - 1)}\Delta T = \frac{\mu R(T_f - T_i)}{\gamma - 1}$$

$$= \frac{\mu RT_f - \mu RT_i}{\gamma - 1} = \frac{(P_fV_f - P_iV_i)}{\gamma - 1}$$

iii. Change in internal energy does not depend on the path of the process. So it is called a point function, i.e., it depends only on the initial and final states of the system, i.e., $U = U_f = U_i$

iv. Change in internal energy in a cyclic process is always zero as for cyclic process $U_f = U_i$. Therefore, $\Delta U = U_f = U_i = 0$

ILLUSTRATION 14.9 When a thermodynamic system is taken from an initial state I to a final state F along the path IAF, as shown in figure, the heat energy absorbed by the system is $Q = 55$ J and the work done by the system is $W = 25$ J. If the same system is taken along the path IBF, the value of $Q = 35$ J.



(a) Find the work done along the path IBF.
(b) If $W = -15$ J for the curved path FI, how much heat energy is lost by the system along this path?
(c) If $U_I = 10$ J, what is U_F?
(d) If $U_B = 20$ J, what is Q for the process BF and IB?

Solution. The first law of thermodynamics states that

$$Q = \Delta U + W \quad \text{or} \quad Q = (U_f - U_i) + W$$

Here U_i and U_f are the internal energies in the initial and the final state, respectively. Given that for path IAF, $Q = 55$ J and $W = 25$ J. Therefore,

$$\Delta U = U_f - U_i = Q - W = 55 - 25 = 30 \text{ J}$$

The internal energy is independent of the path; it depends only on the initial and final states of the system. Thus, the internal energy between I and F states is 30 J irrespective of the path followed by the system.

(a) For path IBF, $Q = 35$ J and $\Delta U = 30$ J.
Therefore, $W = Q - \Delta U = 35 - 30 = 5$ J

(b) $W = -15$ J, but $\Delta U = -30$ J.
Therefore, $Q = W + \Delta U = -15 - 30 = -45$ J.

(c) Given $U_i = 10$ J.
Therefore, $U_f = \Delta U + U_i = 30 + 10 = 40$ J.

(d) The process BF is isochoric, i.e., the volume is constant. Hence, $W = 0$. Therefore, $Q = (\Delta U)_{BF} = U_F - U_B = 40 - 20 = 20$ J. The process IB is isobaric (constant pressure). Therefore, $Q = (Q)_{IBF} - (Q)_{BF} = 35 - 20 = 15$ J.

☞ Important Points

- First law of thermodynamics makes no distinction between work and heat as according to it the internal energy (and hence temperature) of a system may be increased by adding either heat to it or doing work on it or both.
- ΔQ and ΔW are the path functions, but ΔU is the point function.
- In the above equation all three quantities ΔQ, ΔU and ΔW must be expressed either in Joule or in calorie.
- Sign conventions are as follows:

ΔQ	Positive	When heat is supplied to a system
	Negative	When heat is drawn from the system
ΔW	Positive	When work is done by the gas (expansion)
	Negative	When work is done on the gas (compression)
ΔU	Positive	When temperature increases, internal energy increases
	Negative	When temperature decreases, internal energy decreases

- Limitation: First law of thermodynamics does not indicate the direction of heat transfer. It does not tell anything about the conditions under which heat can be transformed into work and also it does not indicate as to why the total heat energy cannot be converted into mechanical work continuously.

ILLUSTRATION 14.10 Gaseous hydrogen contained initially under standard conditions in a sealed vessel of volume $V = 5$ L was cooled by $\Delta T = 55$ K. Find how much the internal energy of the gas will change and what amount of heat will be lost by the gas.

Solution. By the first law of thermodynamics,

$$Q = \Delta U + W$$

Here $\Delta W = 0$ as the volume remains constant.

$$\therefore \quad Q = \Delta U$$

Now $$\Delta U = \frac{m}{M} C_v (-\Delta T)$$

$$= -\frac{m}{M}\frac{R}{\gamma - 1}\Delta T \qquad \left(\text{as } C_v = \frac{R}{\gamma - 1}\right)$$

We have $$p_0 V = \frac{m}{M} R T_0$$

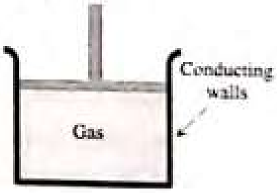
$$\Rightarrow \quad \Delta U = -\frac{1.013 \times 10^5 \times (5 \times 10^{-3}) \times 55}{273(1.4 - 1)} = -255 \text{ J}$$

Isothermal Process

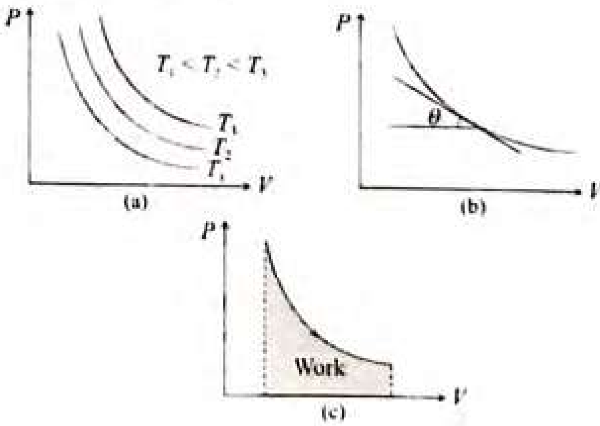
When a thermodynamic system undergoes a physical change in such a way that its temperature remains constant, then the change is known as isothermal change.

In this process, P and V change but T = constant, i.e., change in temperature $\Delta T = 0$

1. **Essential condition for isothermal process:**
   i. The walls of the container must be perfectly conducting to allow free exchange of heat between the gas and its surrounding.



   ii. The process of compression or expansion should be so slow so as to provide time for the exchange of heat.
   Since these two conditions are not fully realized in practice, no process is perfectly isothermal.
2. **Equation of state:** From ideal gas equation $PV = \mu RT$, we observe that if temperature remains constant, then PV = constant, i.e., in all isothermal process Boyle's law is obeyed.
   Hence, equation of state is PV = constant.
3. **Indicator diagram:**



i. Curves obtained on PV graph are called isotherms and they are hyperbolic in nature.

ii. Slope of isothermal curve: By differentiating PV = constant, we get

$$PdV + VdP = 0 \Rightarrow P\,dV = -VdP \Rightarrow \frac{dP}{dV} = -\frac{P}{V}$$

$$\therefore \quad \tan\theta = \frac{dP}{dV} = -\frac{P}{V}$$

iii. Area between the isotherm and volume axis represents the work done in isothermal process.
If volume increases W = positive area under curve and if volume decreases W = negative area under curve

4. Specific heat: Specific heat of gas during isothermal change is infinite.

As $\quad C = \dfrac{Q}{m\Delta T} = \dfrac{Q}{m \times 0} \to \infty \qquad$ (as $\Delta T = 0$)

5. Isothermal elasticity: For isothermal process, PV = constant
Differentiating both sides we get

$PdV + VdP = 0$

$\Rightarrow \quad PdV = -VdP$

$$\Rightarrow \quad P = \frac{dP}{-dV/V} = \frac{\text{Stress}}{\text{Strain}} = E_\theta$$

$\therefore \quad E_\theta = P$, i.e., isothermal elasticity is equal to pressure.
At NTP, isothermal elasticity of gas = atmospheric pressure = 1.01×10^5 N/m^2

6. Work done in isothermal process:

$$W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} \frac{\mu RT}{V} dV \qquad (\text{as } PV = \mu RT)$$

$$W = \mu RT \log_e\left(\frac{V_f}{V_i}\right) = 2.303\mu RT \log_{10}\left(\frac{V_f}{V_i}\right)$$

or $$W = \mu RT \log_e\left(\frac{P_i}{P_f}\right) = 2.303\mu RT \log_{10}\left(\frac{P_i}{P_f}\right)$$

7. First law of thermodynamics in isothermal process:

$Q = \Delta U + W \quad$ but $\quad \Delta U \propto \Delta T$

$\therefore \quad \Delta U = 0 \qquad$ (as $\Delta T = 0$)

$\therefore \quad \Delta U = W$, i.e., heat supplied in an isothermal change is used to do work against the external surrounding, or if the work is done on the system, then equal amount of heat energy will be liberated by the system.

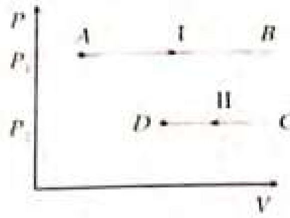
Isobaric Process

When a thermodynamic system undergoes a physical change in such a way that its pressure remains constant, the change is known as isobaric process.

In this process, V and T change but P remains constant. Hence, Charles' law is obeyed in this process.

1. Equation of state: From ideal gas equation, we get $PV = \mu RT$
If pressure remains constant, then $V \propto T$
or $\quad V_1/T_1 = V_2/T_2$ = Constant

2. Indicator diagram: In figure, graph I represents isobaric expansion, and graph II represents isobaric compression. Slope of indicator diagram $dP/dV = 0$.



3. Specific heat: Specific heat of gas during isobaric process

$$C_p = \left(\frac{f}{2} + 1\right)R$$

4. Bulk modulus of elasticity:

$$K = \frac{\Delta P}{-\Delta V/V} = 0 \qquad (\text{as } \Delta P = 0)$$

5. Work done in isobaric process:

$$W = \int_{V_i}^{V_f} PdV = P\int_{V_i}^{V_f} dV = P[V_f - V_i] \qquad (\text{as } P = \text{constant})$$

$$W = P(V_f - V_i) = \mu R[T_f - T_i] = \mu R\Delta T$$

6. First law of thermodynamics in isobaric process:

$$\Delta U = \mu C_v \Delta T = \mu \frac{R}{(\gamma - 1)}\Delta T \quad \text{and} \quad W = \mu R\Delta T$$

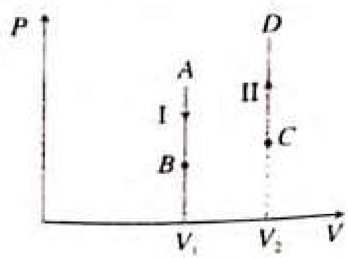
and $\Delta Q = \mu C_p \Delta T$

Isochoric or Isometric Process

When a thermodynamic process undergoes a physical change in such a way that its volume remains constant, the change is known as isochoric process.

In this process, P and T changes, but V = constant. Hence Gay-Lussac's law is obeyed in this process.

1. Equation of state: From ideal gas equation, we get $PV = \mu RT$. If volume remains constant, then $P \propto T$ or $P_1/T_1 = P_2/T_2$ = constant.

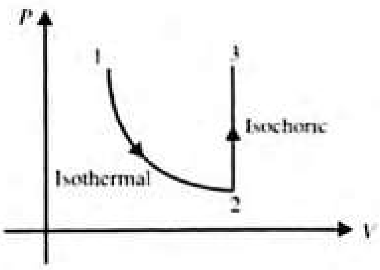


2. **Indicator diagram:** In figure, graphs I and II represent isometric decrease in pressure at volume V_1 and isometric increase in pressure at volume V_2, respectively, and slope of indicator diagram $dP/dV = \infty$.
3. **Specific heat:** Specific heat of gas during isochoric process $C_v = (f/2)R$.
4. **Bulk modulus of elasticity:** $K = \Delta P/(-\Delta V/V)$
$= \Delta P/0 = \infty$.
5. **Work done in isobaric process:** $W = P\Delta V = P[V_f - V_i]$ (as $V = 0$).
$\therefore \quad W = 0$
6. **From the first law of thermodynamics in isochoric process:**
$Q = \Delta U + W = \Delta U$ (as $W = 0$)

$$Q = \mu C_v \Delta T = \mu \frac{R}{\gamma - 1}\Delta T = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

ILLUSTRATION 14.11 Six grams of hydrogen gas at a temperature of 273 K was isothermally expanded to five times its initial volume and then isochorically heated so that the pressure in the final state becomes equal to that in the initial state. Find the total amount of heat absorbed by the gas during the entire process.

Solution. The processes represent on the P–V diagram by the curve 1 to 2 (isothermal) and the line 2 to 3 (isochoric). Here $P_1 = P_3$



For the process 1 to 2 (isothermal)

$$T_2 = T_1,\ V_2 = 5V_1$$

$$Q = \Delta U + W = 0 + W$$

$$\Rightarrow \quad Q = 2.303\, nRT_1 \log\left(\frac{V_2}{V_1}\right)$$

$$Q = 2.303 \times \left(\frac{6}{2}\right) \times 8.3 \times 273 \times \log 5 = 10942.4\text{J}$$

State 1 and state 3 are at the same pressure.

$$\Rightarrow \quad \frac{V_1}{T_1} = \frac{V_3}{T_3} \quad \Rightarrow \quad T_3 = \frac{V_3}{V_1}T_1 = 5 \times 273 = 1365\text{ K}$$

Process 2 to 3: $Q = \Delta U + W$

$$Q = nC_v\Delta T + 0$$

$$= \frac{6}{2} \times \frac{5}{2} R \times (T_3 - T_2)$$

$$= 7.5 \times 8.3 \times (1365 - 273)\text{J} = 67977\text{J}$$

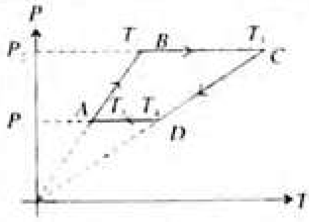
Thus total heat absorbed $= \Delta H_{1\to 2} + \Delta H_{2\to 3} = 78919.4$ J

$\therefore$ Net heat added is $\left[-\frac{1}{2}\frac{RT_0}{\gamma - 1}\right] + \left[\frac{1}{2}\frac{RT_0}{\gamma - 1} + \frac{1}{2}RT_0\right] = \frac{1}{2}RT_0$

This is the heat added when the number of moles is one. When there are 2 moles, heat added is

$$2 \times \frac{1}{2}RT_0 = RT_0 = 8.3 \times 300 = 2490\text{ J}$$

ILLUSTRATION 14.12 P–T curve of a cyclic process is shown. Find out the work done by the gas in the given process if number of moles of the gas are n.



Solution. Since path AB and CD are isochoric therefore work done during AB and CD is zero. Path BC and DA are isobaric.

Hence

$$W_{BC} = nR\Delta T = nR(T_3 - T_2)$$

$$W_{DA} = nR(T_1 - T_4)$$

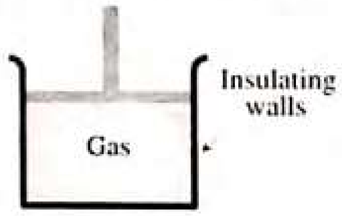
$$\text{Total work done} = W_{BC} + W_{DA} = nR(T_1 + T_3 - T_4 - T_2)$$

Adiabatic Process

When a thermodynamic system undergoes a change in such a way that no exchange of heat takes place between it and the surroundings, the process is known as adiabatic process.

In this process P, V and T change, but $Q = 0$.

1. **Essential conditions for adiabatic process:**



i. There should not be any exchange of heat between the system and its surroundings. All walls of the container and the piston must be perfectly insulating.

ii. The system should be compressed or allowed to expand suddenly so that there is no time for the exchange of heat between the system and its surroundings.

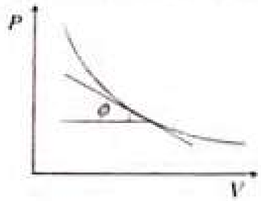
Since, these two conditions are not fully realized in practice, no process is perfectly adiabatic.

3. **First law of thermodynamics in adiabatic process:**
$Q = \Delta U + W$
but for adiabatic process $\Delta Q = 0 \qquad \therefore \quad \Delta U + W = 0$
If ΔW = positive, then ΔU = negative so temperature decreases, i.e., adiabatic expansion produces cooling.
If W = negative, then ΔU = positive so temperature increases, i.e., adiabatic compression produces heating.
4. **Equation of state:** As in case of adiabatic change, the first law of thermodynamics reduces to
$\Delta U + W = 0$, i.e., $dU + W = 0$ (i)
$\Rightarrow \quad PV^{\gamma} = \text{constant}$ (iv)
Equation (iv) is called equation of state for adiabatic change and can also be rewritten as

$TV^{\gamma-1} = \text{constant}$ (as $P = (mRT/V)$) (v)

and $\dfrac{T^{\gamma}}{(P^{\gamma-1})} = \text{constant}$ $\left(\text{as } V = \dfrac{\mu RT}{P}\right)$ (vi)

5. **Indicator diagram:**
   i. Curve obtained on PV graph are called adiabatic curve.
   ii. Slope of adiabatic curve:
   From $PV^{\gamma} = \text{constant}$.
   By differentiating, we get



$$dPV^{\gamma} + P\gamma V^{\gamma-1}dV = 0$$

$$\frac{dP}{dV} = -\gamma\frac{PV^{\gamma-1}}{V^{\gamma}} = -\gamma\left(\frac{P}{V}\right)$$

Therefore, slope of adiabatic curve $\tan\phi = -\gamma(P/V)$.
But we also know that slope of isothermal curve $\tan\phi = -P/V$.

So. $$\frac{\text{Slope of adiabatic curve}}{\text{Slope of isothermal curve}} = \frac{-\gamma(P/V)}{-(P/V)} = \gamma = \frac{C_p}{C_v} > 1$$

6. **Specific heat:** Specific heat of a gas during adiabatic change is zero.

$$C = \frac{Q}{(m\Delta T)} = \frac{0}{(m\Delta T)} = 0 \qquad (\text{as } Q = 0)$$

7. **Adiabatic elasticity:** For adiabatic process,
   $PV^{\gamma} = \text{constant}$.
   Differentiating both sides, we get

$$dPV^{\gamma} + P\gamma V^{\gamma-1}dV = 0$$

$$\gamma P = \frac{dP}{-dV/V} = \frac{\text{Stress}}{\text{Strain}} = E_{\phi}$$

$$E_{\phi} = \gamma P$$

i.e., adiabatic elasticity is γ times that of pressure but we know isothermal elasticity $E_{\theta} = P$

So $$\frac{E_{\phi}}{E_{\theta}} = \frac{\text{Adiabatic elasticity}}{\text{Isothermal elasticity}} = \frac{\gamma P}{P} = \gamma$$

i.e., the ratio of two elasticities of gases is equal to the ratio of two specific heats.

9. **Work done in adiabatic process:**

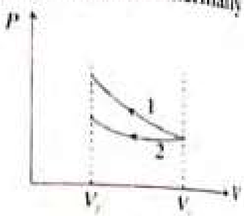
$$W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} \frac{K}{V^{\gamma}}dV \qquad \left(\text{as } P = \frac{K}{V^{\gamma}}\right)$$

$$W = \frac{[P_fV_f - P_iV_i]}{(1-\gamma)}$$

and also $$W = \frac{[P_iV_i - P_fV_f]}{(\gamma-1)} = \frac{\mu R(T_i - T_f)}{(\gamma-1)}$$

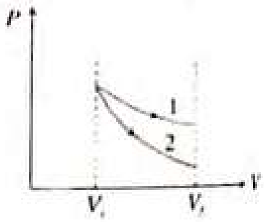
10. **Free expansion:** Free expansion is adiabatic process in which no work is performed on or by the system. Consider two vessels placed in a system which is enclosed with thermal insulation (asbestos-covered). One vessel contains a gas and the other is evacuated. The two vessels are connected by a stopcock. When suddenly the stopcock is opened, the gas rushes into the evacuated vessel and expands freely. The process is adiabatic as the vessels are placed in thermal insulating system ($dQ = 0$). Moreover, the walls of the vessel are rigid and hence no external work is performed ($dW = 0$).
    Now according to the first law of thermodynamics $dU = 0$, if U_i and U_f be the initial and final internal energies of the gas, then
    $U_f - U_i = 0$ (as $U_f = U_i$)
    Thus, the final and initial energies are equal in free expansion.

11. **Comparison between isothermal and adiabatic process:**
    i. **Compression:** If a gas is compressed isothermally and adiabatically from volume V_i to V_f, then from the slope of the graph it is clear that graph 1 (figure) represents adiabatic process, whereas graph 2 represents isothermal process.



Work done	$W_{\text{adiabatic}} > W_{\text{isothermal}}$
Final pressure	$P_{\text{adiabatic}} > P_{\text{isothermal}}$
Final temperature	$T_{\text{adiabatic}} > T_{\text{isothermal}}$

ii. **Expansion:** If a gas expands isothermally and adiabatically from volume V_i to V_f, then from the slope of the graph it is clear that graph 1 (figure) represents isothermal process, whereas graph 2 represents adiabatic process.



Work done	$W_{\text{isothermal}} > W_{\text{adiabatic}}$
Final pressure	$P_{\text{isothermal}} > P_{\text{adiabatic}}$
Final temperature	$T_{\text{isothermal}} > T_{\text{adiabatic}}$

Polytropic Process

A process $PV^{\rho} = \text{constant}$ is called polytropic process, in which $\rho \neq 1$ or γ.

i. **Work done in polytropic process:** As we have calculated in adiabatic process, here also the work done is

$$W = \frac{\nu R}{(r-1)}[T_i - T_f] = \frac{-nR}{(r-1)}[T_f - T_i] = -\frac{nR\Delta T}{(r-1)}$$

For one mole of a gas, $n = 1$

$$W = -\frac{-R\Delta T}{(r-1)}$$

ii. **Specific heat:** If C is the molar specific heat, then heat required to increase the temperature of one mole of a gas by ΔT is

$$Q = C\Delta T$$

From the first law of thermodynamics

$$Q = \Delta U + W$$

or $\quad C\Delta T = C_v\Delta T - \dfrac{R\Delta T}{(r-1)}$

$\therefore \quad C = C_v - \dfrac{R}{(r-1)} = \dfrac{R}{(\gamma-1)} - \dfrac{R}{(r-1)}$

ILLUSTRATION 14.13 An ideal gas ($C_P/C_V = \gamma$) is taken through a process in which the pressure and the volume vary as $P = aV^b$. Find the value of b for which the specific heat capacity in the process is zero.

Solution. Given that $P = aV^b$

or $\quad PV^{-b} = a$

Comparing with PV^γ = constant, we have

$\gamma = -b$

We know that $C = C_v - \dfrac{R}{r-1}$

Here, $C = 0 = C_v - \dfrac{R}{\gamma-1}$

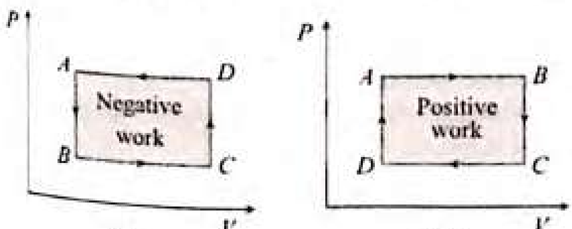
$\therefore \quad 0 = \dfrac{R}{\gamma-1} = \dfrac{R}{-b-1}$

or $\quad b = -\gamma$

Cyclic and Non-Cyclic Processes

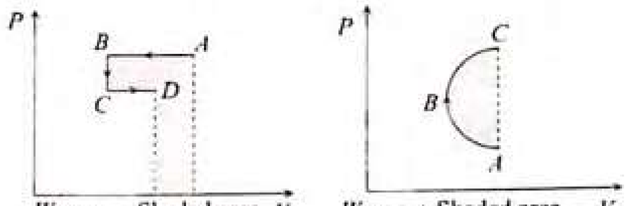
A cyclic process consists of a series of changes that return the system back to its initial state. In a non-cyclic process, the series of changes involved do not return the system back to its initial state.

1. In case of cyclic process as $U_f = U_i$. Therefore
   $\Delta U = U_f - U_i = 0$
   i.e., change in internal energy for cyclic process is zero and also $\Delta U \propto \Delta T$
   $\therefore \quad \Delta T = 0$, i.e., temperature of system remains constant.
2. From the first law of thermodynamics
   $Q = \Delta U + W$
   $Q = W$, i.e., heat supplied is equal to the work done by the system (as $\Delta U = 0$).
3. For cyclic process, P–V graph is a closed curve and area enclosed by the closed path represents the work done.



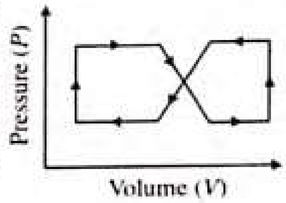
(a) (b)

If the cycle is clockwise work done is positive and if the cycle is anticlockwise work done is negative.



W_{ABCD} = − Shaded area (a) $\quad$ W_{ABC} = + Shaded area (b)

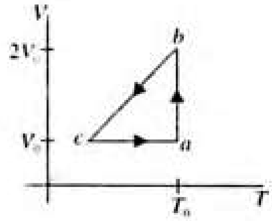
4. Work done in non-cyclic process depends upon the path chosen or the series of changes involved and can be calculated by the area covered between the curve and volume axis on P–V diagram.



Here, it is worth mentioning that the loop can be of any possible shape. Further, it can ever intersect.

The working substance in heat engines, meant to draw a continuous supply of work at the cost of heat, is subjected to cyclic process repeatedly.

ILLUSTRATION 14.14 A sample of an ideal gas has pressure p_0, volume V_0 and temperature T_0. It is isothermally expanded to twice its original volume. It is then compressed at constant pressure to have the original volume V_0. Finally, the gas is heated at constant volume to get the original temperature. (a) Show the process in a V-T diagram; (b) Calculate the heat absorbed in the process.



Solution.

(a) The V–T diagram for the process is shown in figure. The initial state is represented by the POINT a. In the first step, it is isothermally expanded to a volume $2V_0$. This shown by ab. Then the pressure is kept constant and the gas is compressed to the volume V_0. From the ideal gas equation, V/T is constant at constant pressure. Hence, the process is shown by a line bc which passes through the origin. At POINT c, the volume is V_0. In the final step, the gas is heated at constant volume to a temperature T_0. This is shown by ca. The final state is the same as the initial state.

(b) The process is cyclic so that the change in internal energy is zero. The heat supplied is, therefore, equal to the work done by the gas. The work done during ab is

$$W_1 = nRT_0 \ln\frac{2V_0}{V_0} = nRT_0 \ln 2 = p_0V_0 \ln 2$$

Also from the ideal gas equation,

$$p_a V_a = p_b V_b \quad \text{or,} \quad p_b = \frac{p_a V_a}{V_b} = \frac{p_0 V_0}{2V_0} = \frac{p_0}{2}$$

In the step bc, the pressure remains constant. Hence the work done is, $W_2 = \frac{p_0}{2}(V_0 - 2V_0) = -\frac{p_0 V_0}{2}$

In the step ca, the volume remains constant and so the work done is zero. The net work done by the gas in the cyclic process is

$W = W_1 + W_2$

$= p_0V_0 [\ln 2 - 0.5] = 0.193\, p_0V_0.$

Hence, the heat supplied to the gas is $0.193\, p_0V_0$.

Efficiency of a Cyclic Process

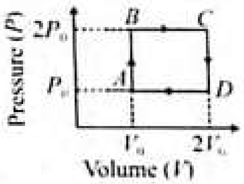
When a system is subjected to a cyclic process, heat is applied during some part of the process, while heat is abstracted during other part.

Evidently, the net heat applied will be the work done by the system ($\therefore Q = W$). However, the gross heat applied will be more than that of the net heat.

Efficiency (η) of a cycle is defined as the ratio of the work performed (net heat given) to the gross heat supplied to the system per cycle. Thus,

$$\eta = \frac{\text{Work done per cycle}}{\text{Gross heat supplied per cycle}}$$

ILLUSTRATION 14.15 Figure shows the indicator diagram corresponding to n moles of an ideal gas taken along the cyclic process $ABCDA$. Find the efficiency of the cycle.



Solution. Evidently, work done by the gas during the cyclic process is

$$W = (2P_0 - P_0)(2V_0 - V_0) \text{ unit} = P_0V_0 \text{ unit}$$

For the process A to B which is isochoric, the heat supplied will be given by

$$Q_1 = \Delta U + W = nC_v\Delta T + 0$$

$$= n\left(\frac{3R}{2}\right)\left[\frac{V_0(2P_0 - P_0)}{nR}\right] \qquad (\because nR\Delta T = V\Delta P)$$

$$= \frac{3}{2}P_0V_0 \text{ unit}$$

And for the process B to C which is isobaric, the heat absorbed will be given by

$$Q_2 = nC_p\Delta T$$

$$= n\left(\frac{5R}{2}\right)\left[\frac{2P_0(2V_0 - V_0)}{nR}\right] \qquad (\because nR\Delta T = P(\Delta V))$$

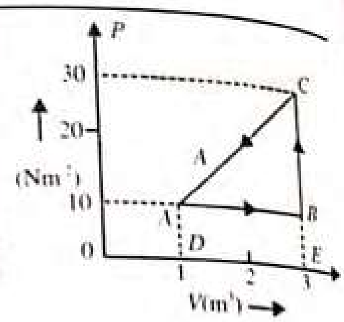
$$= 5P_0V_0 \text{ unit}$$

For the processes C to D and D to A, the heat given will be obviously negative, which implies that heat is abstracted from the system. Therefore, efficiency (η) of the cycle

$$= \frac{\text{Work done per cycle}}{\text{Total heat given per cycle}} \times 100$$

$$= \frac{P_0V_0}{\left(\frac{3P_0V_0}{2}\right) + 5P_0V_0} = \frac{2}{13} \times 100 = \frac{200}{13}\% = 15.38\%$$

ILLUSTRATION 14.16 An ideal gas is taken round a cyclic thermodynamic process $ABCA$ as shown in figure. If the internal energy of the gas at point A is assumed zero while at B it is 50 J. The heat absorbed by the gas in the process BC is 90 J.



(a) What is the internal energy of the gas at point C?

(b) How much heat energy is absorbed by the gas in the process AB?

(c) Find the heat energy rejected or absorbed by the gas in the process CA.

(d) What is the net work done by the gas in the complete cycle $ABCA$?

Solution. Given that $U_A = 0$, $U_B = 50$ J and $Q_{BC} = 90$ J.

Also $P_A = P_B = 10$ Nm^{-2}, $P_C = 300$ Nm^{-2}, $V_A = 1$ m^3

and $V_B = V_C = 3$ m^3

(a) In process BC, as volume of gas remains constant, work done by gas in this process is zero. Thus

$W_{BC} = 0$

Heat absorbed by the gas is $Q_{BC} = 90$ J. From the first law of thermodynamics,

$(\Delta U)_{BC} = U_C - U_B = Q_{BC} - W_{BC} = 90\text{ J} - 0 = 90\text{ J}$

$U_C = (\Delta U)_{BC} + U_B = 90\text{ J} + 50\text{ J} = 140\text{ J}$

(b) In process AB, we have

$(\Delta U)_{AB} = U_B - U_A = 50 - 0 = 50$ J

Work done is given as

W_{AB} = area under AB in P–V diagram

= area of rectangle $ABED$

$= AB \times AD = (3\text{ m}^3 - 1\text{ m}^3) \times 10\text{ Nm}^{-2}$

$= 20$ J

Thus, heat absorbed by the system is

$Q_{AB} = (\Delta U)_{AB} + W_{AB} = 50 + 20 = 70$ J

(c) For process CA

$(\Delta U)_{CA} = U_A - U_C = 0 - 140 = -140$ J

Work done is given as

W_{CA} = area $ACED$

= area of triangle ACB + area of rectangle $ABED$

$= 1/2 \times AB \times BC + AB \times AD$

$= 1/2 \times (3 - 1)\text{ m}^3 \times (30 - 10)\text{Nm}^{-2} + 20$

$= 20 + 20 = 40$ J

In this process, the volume decreases, the work is done on the gas. Hence, the work done is negative.

Thus, $W_{CA} = -40$ J.

Thus heat rejected by the gas is

$$Q_{CA} = (\Delta U)_{CA} + W_{CA} = -140 - 40 = -180 \text{ J}$$

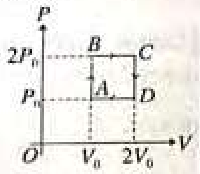
(d) Net work done in the complete cyclic process $ABCA$ is

$$W = \text{area of triangle } ABC = \frac{1}{2} \times 2 \times 20 = 20 \text{ J}$$

As the cycle is anticlockwise, net work is done on the gas.

CONCEPT APPLICATION EXERCISE 14.2

1. A cylindrical vessel of 28 cm diameter contains 20 g of nitrogen compressed by a piston supporting a weight of 75 kg. The temperature of the gas is 17°C. What work will the gas do if it is heated to a temperature of 250°C? What amount of heat should be supplied? To what distance will the weight be raised? The process should be assumed to be isobaric; the heating of the vessel as well as the external pressure is negligible.
2. One mole of oxygen is heated at constant pressure starting at 0°C. How much heat energy must be added to the gas to double its volume?
3. An ideal gas expands from an initial temperature T_1 to a final temperature T_2. Prove that the work done by the gas is $C_v(T_1 - T_2)$.
4. An ideal gas whose adiabatic exponent equals γ expands so that the amount of heat transferred to it is equal to the decrease of its internal energy. Find
   (a) the molar heat capacity of the gas, and
   (b) the T–V equation for the process.
5. One mole of argon expands polytropically, the polytropic constant being 1.5, that is, the process proceeds according to the law $pV^{1.5}$ = constant. In the process, its temperature changes by $\Delta T = -26$ K. Find
   (a) the amount of heat obtained by the gas,
   (b) the work performed by the gas.
6. Find out the work done in the given graph. Also draw the corresponding T–V curve and P–T curve.



SECOND LAW OF THERMODYNAMICS

First law of thermodynamics merely explains the equivalence of work and heat. It does not explain why heat flows from bodies at higher temperatures to those at lower temperatures. It cannot tell us why the converse is possible. It cannot explain why the efficiency of a heat engine is always less than unity.

It is also unable to explain why cool water on stirring gets hotter whereas there is no such effect on stirring warm water in a beaker. Second law of thermodynamics provides answers to these questions. Statement of this law is as follows:

Clausius statement: It is impossible for a self-acting machine to transfer heat from a colder body to a hotter one without the aid of an external agency.

From Clausius statement, it is clear that heat cannot flow from a body at low temperature to one at higher temperature unless work is done by an external agency. This statement is in fair agreement with our experiences in different branches of physics. For example, electrical current cannot flow from a conductor at lower electrostatic potential to that at higher potential unless an external work is done. Similarly, a body at a lower gravitational potential level cannot move up to higher level without work done by an external agency.

Kelvin's statement: It is impossible for a body or system to perform continuous work by cooling it to a temperature lower than the temperature of the coldest one of its surroundings. A Carnot engine cannot work if the source and sink are at the same temperature because work done by the engine will result into cooling the source and heating the surroundings more and more.

Kelvin-Planck's statement: It is impossible to design an engine that extracts heat and fully utilises into work without producing any other effect.

From this statement it is clear that any amount of heat can never be converted completely into work. It is essential for an engine to return some amount of heat to the sink. An engine essentially requires a source as well as sink. The efficiency of an engine is always less than unity because heat cannot be fully converted into work.

CARNOT ENGINE

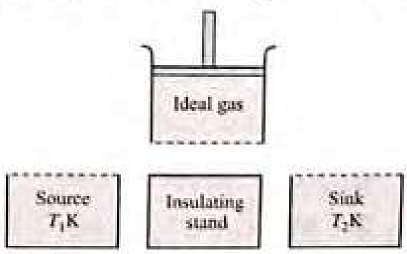
Carnot designed a theoretical engine which is free from all the defects of a practical engine. This engine cannot be realised in actual practice, however, this can be taken as a standard against which the performance of an actual engine can be judged. It consists of the following parts

(i) A cylinder with perfectly non-conducting walls and a perfectly conducting base containing a perfect gas as working substance and fitted with a non-conducting frictionless piston

(ii) A source of infinite thermal capacity maintained at constant higher temperature T_1.

(iii) A sink of infinite thermal capacity maintained at constant lower temperature T_2.

(iv) A perfectly non-conducting stand for the cylinder.



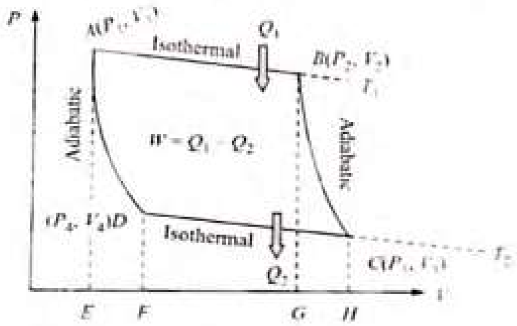
Carnot cycle: As the engine works, the working substance of the engine undergoes a cycle known as Carnot cycle. The Carnot cycle consists of the following four strokes

(i) First stroke (Isothermal expansion) (curve AB):
The cylinder containing ideal gas as working substance allowed to expand slowly at this constant temperature T_1.
Work done = Heat absorbed by the system

$$W_1 = Q_1 = \int_{V_1}^{V_2} P\,dV = RT_1 \log_e\left(\frac{V_2}{V_1}\right) = \text{Area } ABGE$$

(ii) Second stroke (Adiabatic expansion) (curve BC):
The cylinder is then placed on the non conducting stand and the gas is allowed to expand adiabatically till the temperature falls from T_1 to T_2.

$$W_2 = \int_{V_2}^{V_3} P\,dV = \frac{R}{(\gamma-1)}[T_1 - T_2] = \text{ Area } BCHG$$



(iii) Third stroke (Isothermal compression) (curve CD):
The cylinder is placed on the sink and the gas is compressed at constant temperature T_2.
Work done = Heat released by the system

$$W_3 = Q_2 = -\int_{V_3}^{V_4} P\,dV = -RT_2 \log_e \frac{V_4}{V_3} = RT_2 \log_e \frac{V_3}{V_4}$$

$$= \text{Area } CDFH$$

(iv) Fourth stroke (adiabatic compression) (curve DA):
Finally the cylinder is again placed on non-conducting stand and the compression is continued so that gas returns to its initial stage.

$$W_4 = -\int_{V_4}^{V_1} P\,dV = -\frac{R}{\gamma-1}(T_2 - T_1) = \frac{R}{\gamma-1}(T_1 - T_2)$$

$$= \text{Area } ADFE$$

Efficiency of Carnot cycle: The efficiency of engine is defined as the ratio of work done to the heat supplied i.e.

$$\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1}$$

Net work done during the complete cycle

$$W = W_1 + W_2 + (-W_3) + (-W_4)$$

$$= W_1 - W_3 = \text{Area } ABCD \qquad [\text{As } W_2 = W_4]$$

$$\eta = \frac{W}{Q_1} = \frac{W_1 - W_3}{W_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{W_3}{W_1} = 1 - \frac{Q_2}{Q_1}$$

or $$\eta = 1 - \frac{RT_2 \log_e (V_3 / V_4)}{RT_1 \log_e (V_2 / V_1)}$$

Since points B and C lie on same adiabatic curve

$$\therefore \quad T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad \text{...(i)}$$

Also point D and A lie on the same adiabatic curve

$$\therefore \quad T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad \text{...(ii)}$$

From (i) and (ii) $\frac{V_3}{V_2} = \frac{V_4}{V_1}$ or $\frac{V_3}{V_4} = \frac{V_2}{V_1}$

$$\Rightarrow \quad \log_e\left(\frac{V_3}{V_4}\right) = \log_e\left(\frac{V_2}{V_1}\right)$$

So efficiency of Carnot engine $\eta = 1 - \frac{T_2}{T_1}$

(i) Efficiency of a heat engine depends only on temperatures of source and sink and is independent of all other factors.
(ii) All reversible heat engines working between same temperatures are equally efficient and no heat engine can be more efficient than Carnot engine (as it is ideal).
(iii) As on Kelvin scale, temperature can never be negative (as 0 K is defined as the lowest possible temperature) and T_1 and T_2 are finite, efficiency of a heat engine is always lesser than unity, i.e., whole of heat can never be converted into work which is in accordance with second law.

NOTE:

The efficiency of an actual engine is much lesser than that of an ideal engine. Actually the practical efficiency of a steam engine is about (8–15)% while that of a petrol engine is 40%. The efficiency of a diesel engine is maximum and is about (50–55)%.

Carnot theorem: The efficiency of Carnot's heat engine depends only on the temperature of source (T_1) and temperature of sink (T_2), *i.e.*, $\eta = 1 - \frac{T_2}{T_1}$.

Carnot stated that no heat engine working between two given temperatures of source and sink can be more efficient than a perfectly reversible engine (Carnot engine) working between the same two temperatures. Carnot's reversible engine working between two given temperatures is considered to be the most efficient engine.

ENTROPY

Mathematical definition of entropy and its relation to the second law of thermodynamics.

Any thermodynamics system has a state function S, called the entropy. By this is meant that S- like p, V and U is always the same for the system when it is in a given equilibrium state. Entropy may be defined as follows. Let a system at absolute T undergo an infinitesimal reversible process in which it absorbs heat ΔQ. Then, the change in entropy of the system is given by

$$\Delta S = \frac{\Delta Q}{T} \text{ or } dS = \frac{\Delta Q}{T} \quad \text{for infinitesimals}$$

Note that dQ is not the differential of a true function. Entropy will have the units J/K.

The Clausius equation, $dS = dQ/T$, holds only for reversible processes. However, since S is a state function, the entropy change accompanying an irreversible process can be calculated by integrating dQ/T along the path of an arbitrary reversible process connecting the initial and final states.

The importance of the entropy function is exhibited in the following form of the second law of thermodynamics. In any process, the total entropy of the system and its surroundings increases or (in a reversible process) does not change. The second law applies to the system alone if the system is isolated; that is, if it in no way interacts with its surroundings.

ILLUSTRATION 14.17 An ideal gas is confined to a cylinder by a piston. The piston is slowly pushed in so that the gas temperature remains at 20° C. During the compression, 730 J of work is done on the gas. Find the entropy change of the gas.

Solution. The first law tells us that $\Delta Q = \Delta U + \Delta W$. Because the process was isothermal, the internal energy of the ideal gas did not change. Therefore, $\Delta U = 0$ and $\Delta Q = \Delta W = -730$ J. (Because the gas was compressed, the gas did negative work, hence the minus sign). Now we can write

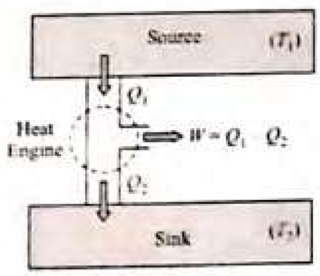
$$\Delta S = \frac{\Delta Q}{T} = \frac{-730 T}{293 K} = -2.49 \text{ J/K}$$

Note that the entropy change is negative. Disorder of the gas decreased as it was pushed into a smaller volume.

HEAT ENGINE

Heat engine is a device which converts heat into work continuously through a cyclic process.

The essential parts of a heat engine are



Source: It is a reservoir of heat at high temperature and infinite thermal capacity. Any amount of heat can be extracted from it.

Working substance: Steam, petrol *etc.*

Sink: It is a reservoir of heat at low temperature and infinite thermal capacity. Any amount of heat can be given to the sink.

The working substance absorbs heat Q_1 from the source, does an amount of work W, returns the remaining amount of heat to the sink and comes back to its original state and there occurs no change in its internal energy.

By repeating the same cycle over and over again, work is continuously obtained.

The performance of heat engine is expressed by means of "efficiency" η which is defined as the ratio of useful work obtained from the engine to the heat supplied to it.

$$\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1}$$

But by first law of thermodynamics for cyclic process $\Delta U = 0$.

$\therefore \quad \Delta Q = \Delta W$, so $W = Q_1 - Q_2$

$$\therefore \quad \eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

A perfect heat engine is one which converts all heat into work i.e. $W = Q_1$ so that $Q_2 = 0$ and hence $\eta = 1$.

But practically efficiency of an engine is always less than 1.

Refrigerator or Heat Pump

A refrigerator or heat pump is basically a heat engine run in reverse direction.

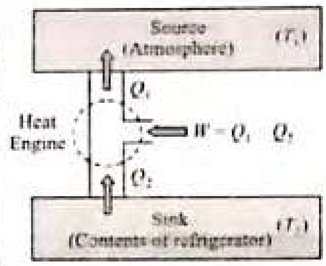
It essentially consists of three parts

Source: At higher temperature T_1.

Working substance: It is called refrigerant liquid ammonia and freon works as a working substance.

Sink: At lower temperature T_2.

The working substance takes heat Q_2 from a sink (contents of refrigerator) at lower temperature, has a net amount of work done W on it by an external agent (usually compressor of refrigerator) and gives out a larger amount of heat Q_1 to a hot body at temperature T_1 (usually atmosphere). Thus, it transfers heat from a cold to a hot body at the expense of mechanical energy supplied to it by an external agent. The cold body is thus cooled more and more.



The performance of a refrigerator is expressed by means of "coefficient of performance" β which is defined as the ratio of the heat extracted from the cold body to the work needed to transfer it to the hot body.

$$\text{i.e.} \quad \beta = \frac{\text{Heat extracted}}{\text{work done}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\therefore \quad \beta = \frac{Q_2}{Q_1 - Q_2}$$

A perfect refrigerator is one which transfers heat from cold to hot body without doing work

i.e. $\quad W = 0$ so that $Q_1 = Q_2$ and hence $\beta = \infty$

Carnot Refrigerator

For Carnot refrigerator, $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2}$

$$\therefore \quad \frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2} \text{ or } \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

So, coefficient of performance, $\beta = \dfrac{T_2}{T_1 - T_2}$

where T_1 = temperature of surrounding and T_2 = temperature of cold body.

It is clear that $\beta = 0$ when $T_2 = 0$, i.e. the coefficient of performance will be zero if the cold body is at the temperature equal to absolute zero.

Relation Between Coefficient of Performance and Efficiency of Refrigerator

We know

$$\beta = \frac{Q_2}{Q_1 - Q_2} \text{ or } \beta = \frac{Q_2/Q_1}{1 - Q_2/Q_1} \quad \text{(i)}$$

But the efficiency

$$\eta = 1 - \frac{Q_2}{Q_1} \text{ or } \frac{Q_2}{Q_1} = 1 - \eta \quad \text{(ii)}$$

From (i) and (ii), we get $\beta = \dfrac{1-\eta}{\eta}$

ILLUSTRATION 14.18 A Carnot-type engine is designed to operate between 480 and 300 K. Assuming that the engine actually produces 1.2 kJ of mechanical energy per kilocalorie of heat absorbed, compare the actual efficiency with the theoretical maximum efficiency.

Solution. Maximum efficiency $= \dfrac{T_h - T_c}{T_h} = \dfrac{480-300}{480} = 37.5\%$

Actual efficiency $= \dfrac{\text{Energy output}}{\text{Energy input}} = \dfrac{1.2}{1 \times 4.184} = 28.7\%$

ILLUSTRATION 14.19 What is the maximum amount of work that a Carnot engine can perform per kilocalorie of heat input if it absorbs heat at 427°C and exhausts heat at 177°C?

Solution. Efficiency $= \dfrac{Q_h - Q_c}{Q_h} = \dfrac{T_h - T_c}{T_h}$

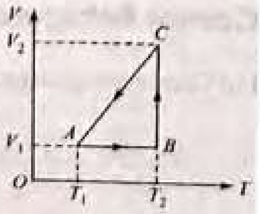
Q_h = (1 kcal) (4.184 kJ/kcal) = 4.184 kJ

Efficiency $= \dfrac{Q_h - Q_c}{4.184 \text{ kJ}} = \dfrac{700 \text{ K} - 450 \text{ K}}{700 \text{ K}}$

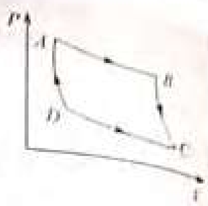
Then $W = Q_h = 1.49$ kJ

CONCEPT APPLICATION EXERCISE 14.3

1. Find the coefficient of performance of a Carnot refrigerator working between 30°C and 0°C.
2. A Carnot engine working between 300 K and 600 K has work output of 800 J per cycle. What is amount of heat energy supplied to the engine from source per cycle.



3. Carnot cycle (reversible) of a gas represented by a Pressure–Volume curve is shown in the diagram. Consider the following statements



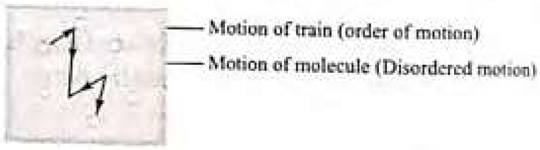
   I. Area $ABCD$ = Work done on the gas
   II. Area $ABCD$ = Net heat absorbed
   III. Change in the internal energy in cycle = 0

   Which of these are correct?
4. A Carnot engine takes 103 kcal of heat from a reservoir at 627°C and exhausts it to a sink at 27°C. Find the efficiency of the engine.

SOLVED EXAMPLES

1. A gas is enclosed in a closed pot. On keeping this pot in a train moving with high speed, the temperature of the gas
   (a) will increase
   (b) will decrease
   (c) will remain the same
   (d) will change according to the nature of the gas

Sol. (c) Temperature of the gas is concerned only with it's disordered motion. It is no way concerned with it's ordered motion.



2. Two spherical vessel of equal volume, are connected by a narrow tube. The apparatus contains an ideal gas at one atmosphere and 300 K. Now if one vessel is immersed in a bath of constant temperature 600 K and the other in a bath of constant temperature 300 K. Then the common pressure will be

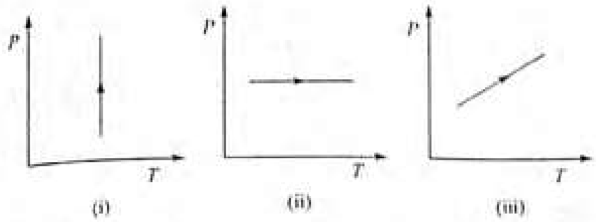
   (a) 1 atm (b) $\dfrac{4}{5}$ atm
   (c) $\dfrac{4}{3}$ atm (d) $\dfrac{3}{4}$ atm

Sol. (c) $\mu = \mu_1 + \mu_2$

$$\frac{P(2V)}{RT_1} = \frac{P'V}{RT_1} + \frac{P'V}{RT_2} \Rightarrow \frac{2P}{RT_1} = \frac{P'}{R}\left[\frac{T_2 + T_1}{T_1 T_2}\right]$$

$$P' = \frac{2PT_2}{(T_1 + T_2)} = \frac{2 \times 1 \times 600}{(300 + 600)} = \frac{4}{3} \text{ atm}$$

3. Pressure versus temperature graphs of an ideal gas are as shown in figure. Choose the wrong statement



(a) Density of gas is increasing in graph (i)
(b) Density of gas is decreasing in graph (ii)
(c) Density of gas is constant in graph (iii)
(d) None of these

Sol. (c) $\rho = \frac{PM}{RT}$

Density ρ remains constant when P/T or volume remains constant.

In graph (i), pressure is increasing at constant temperature hence volume is decreasing so density is increasing. In graphs (ii) and (iii) volume is increasing. Hence, density is decreasing. Note that volume would has been constant in case the straight line in graph (iii) had passed through origin.

4. A cylinder of fixed capacity 44.8 litre contains a monatomic gas at standard temperature and pressure. The amount of heat required to cylinder by 10°C will be.
(R= universal gas constant)
(a) R (b) $10R$
(c) $20R$ (d) $30R$

Sol. (d) As we know, 1 mol of any ideal gas at STP occupies a volume of 22.4 litres.

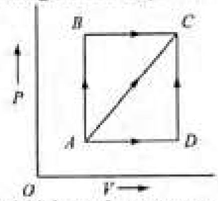
Hence, number of moles of gas, $\mu = \frac{44.8}{22.4} = 2$

Since the volume of cylinder is fixed,

Hence $Q_v = \mu\omega\Delta T$

$$= 2\times\frac{3}{2}R\times 10 = 30R \quad \left(\because (C_V)_{mono} = \frac{3}{2}R\right)$$

5. A thermodynamic process is shown in the following figure. The pressures and volumes corresponding to some points in the figure are:
$P_A = 3\times 10^4$ Pa,
$V_A = 2\times 10^{-3}$ m^3,
$P_B = 8\times 10^4$ Pa,
$V_D = 5\times 10^{-3}$ m^3.



In the process AB, 600 J of heat is added to the system and in process BC, 200 J of heat is added to the system. The change in internal energy of the system in the process AB would be
(a) 560 J (b) 800 J
(c) 60 J (d) 640 J

Sol. (a). No work is done along the path AB, because the process is isochoric.

$\therefore$ Work done $= P_B(V_D - V_A)$
$= 8\times 10^4(5\times 10^{-3} - 2\times 10^{-3}) = 240$ J

$Q_{AC} = Q_{AB} + Q_{BC} = 600 + 200 = 800$ J

$\therefore$ $(\Delta U)_{AC} = Q - W = 800 - 240 = 560$ J

6. One mole of an ideal monatomic gas requires 210 J heat to raise the temperature by 10 K, when heated at constant temperature. If the same gas is heated at constant volume to raise the temperature by 10 K then heat required is
(a) 238 J (b) 126 J
(c) 210 J (d) 350 J

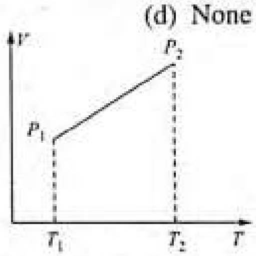
Sol. (b) $Q_P = \mu C_P\Delta T$ and $Q_v = \mu C_v\Delta T$

$$\Rightarrow \quad \frac{Q_V}{Q_P} = \frac{C_V}{C_P} = \frac{\frac{3}{2}R}{\frac{5}{2}R} = \frac{3}{5}$$

$$\left[\because (C_V)_{mono} = \frac{3}{2}R, (C_P)_{mono} = \frac{5}{2}R\right]$$

$$\Rightarrow \quad Q_V = \frac{3}{5}\times(\Delta Q)_P = \frac{3}{5}\times 210 = 126\,\text{J}$$

7. From the following V-T diagram, we can conclude
(a) $P_1 = P_2$ (b) $P_1 > P_2$
(c) $P_1 < P_2$ (d) None of these



Sol. (b) In case of given graph, V and T are related as $V = aT - b$, where a and b are constants.

From ideal gas equation, $PV = \mu RT$,

we find $P = \frac{\mu RT}{aT - b} = \frac{\mu R}{a - b/T}$

Since $T_2 > T_1$, therefore $P_2 < P_1$.

8. Certain amount of an ideal gas are contained in a closed vessel. The vessel is moving with a constant velocity v. The molecular mass of gas is M. The rise in temperature of the gas when the vessel is suddenly stopped is ($\gamma = C_p/C_v$)

(a) $\frac{Mv^2}{2R(\gamma+1)}$ (b) $\frac{Mv^2(\gamma-1)}{2R}$

(c) $\frac{Mv^2}{2R(\gamma+1)}$ (d) $\frac{Mv^2}{2R(\gamma+1)}$

Sol. (b) If m is the total mass of the gas then its kinetic energy $= \frac{1}{2}mv^2$

When the vessel is suddenly stopped, then total kinetic energy will increase the temperature of the gas. Hence,

$$\frac{1}{2}mv^2 = \mu C_v \Delta T = \frac{m}{M}C_v \Delta T \left[\text{as } C_v = \frac{R}{\gamma - 1}\right]$$

$$\Rightarrow \quad \frac{m}{M}\frac{R}{\gamma-1}\Delta T = \frac{1}{2}mv^2 \Rightarrow \Delta T = \frac{Mv^2(\gamma-1)}{2R}.$$

9. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is mg. The gas in each cylinder is now allowed to expand isothermally to the same final volume $2V$. The changes in the pressure in A and B are found to be ΔP and $1.5\Delta P$, respectively. Then
(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_A = 3m_B$

Sol. (c) Process is isothermal. Therefore, T = constant

$\left(P \propto \frac{1}{V}\right)$ volume is increasing, therefore pressure will decrease

In chamber A:

$$\Delta P = P_i - P_f = \frac{\mu_A RT}{V} - \frac{\mu_A RT}{2V} = \frac{\mu_A RT}{2V} \quad .....(i)$$

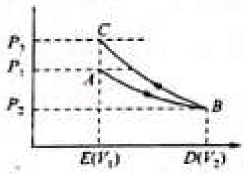
In chamber B :

$$\Delta P = P_i - P_f = \frac{\mu_B RT}{V} - \frac{\mu_B RT}{2V} = \frac{\mu_B RT}{2V} \quad .....(ii)$$

From equations (i) and (ii), $\frac{\mu_A}{\mu_B} = \frac{1}{1.5} = \frac{2}{3}$

$$\Rightarrow \quad \frac{m_A/M}{m_B/M} = \frac{2}{3} \Rightarrow 3m_A = 2m_B.$$

10. An ideal gas expands isothermally from a volume V_1 to V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then
(a) $P_3 > P_1$, $W > 0$
(b) $P_3 < P_1$, $W < 0$
(c) $P_3 > P_1$, $W < 0$
(d) $P_3 = P_1$, $W = 0$



Sol. (c) From graph, it is clear that $P_3 > P_1$.
Since area under adiabatic process ($BCED$) is greater than that of isothermal process ($ABDE$), therefore, net work done

$$W = W_1 + (-W_A)$$

$$\because \quad W_A > W_1 \Rightarrow W < 0$$

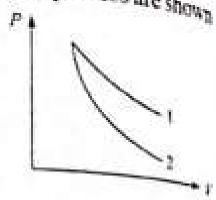
11. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is
(a) 30 K (b) 18 K
(c) 50 K (d) 42 K

Sol. (d) In both cylinders A and B, the gases are diatomic ($\gamma = 1.4$). Piston A is free to move i.e. it is isobaric process. Piston B is fixed i.e. it is isochoric process. If same amount of heat ΔQ is given to both, then

$$Q_{\text{isobaric}} = Q_{\text{isochoric}} \Rightarrow \mu C_P(\Delta T)_A = \mu C_v(\Delta T)_B$$

$$\Rightarrow \quad (\Delta T)_B = \frac{C_P}{C_v}(\Delta T)_A = \gamma(\Delta T)_A = 1.4 \times 30 = 42\text{ K}$$

12. P-V plots for two gases during adiabatic process are shown in the figure. Plots 1 and 2 should correspond respectively to
(a) He and O_2 (b) O_2 and He
(c) He and Ar (d) O_2 and N_2

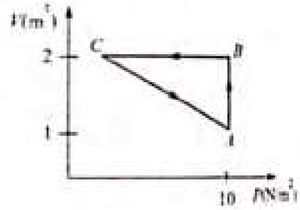


Sol. (b) In adiabatic process, slope of PV-graph.

$$\frac{dp}{dV} = -\gamma\frac{P}{V} \Rightarrow |\text{Slope}| \propto \gamma$$

From the given graph $(\text{Slope})_2 > (\text{Slope})_1 \Rightarrow \gamma_2 > \gamma_1$
Therefore, 1 should correspond to O_2 ($\gamma = 1.4$) and 2 should correspond to He ($\gamma = 1.66$)

13. An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is
(a) -5 J
(b) -10 J
(c) -15 J
(d) -20 J



Sol. (a) For cyclic process. Total work done $= W_{AB} + W_{BC} = W_{CA}$

$$W_{AB} = P\Delta V = 10(2-1) = 10\text{ J and } \Delta W_{BC} = 0$$

(as V = constant)

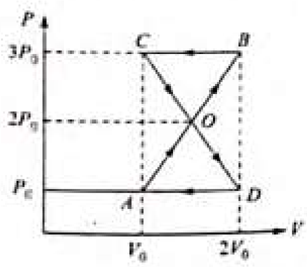
From FLOT, $Q = \Delta U + W$

$\Delta U = 0$ (Process $ABCA$ is cyclic)

$$\Rightarrow \quad Q = W_{AB} + W_{BC} + W_{CA}$$

$$\Rightarrow \quad 5 = 10 + 0 + \Delta W_{CA} \Rightarrow \Delta W_{CA} = -5\text{ J}$$

14. A thermodynamic system undergoes cyclic process $ABCDA$ as shown in figure. The work done by the system is

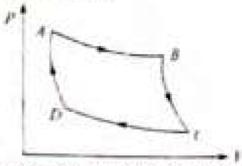


(a) P_0V_0 (b) $2P_0V_0$
(c) $\frac{P_0V_0}{2}$ (d) Zero

Sol. (d) $W_{BCOB} = -$ Area of triangle $BCO = -\frac{P_0V_0}{2}$

$$W_{AODA} = +\text{ Area of triangle } AOD = +\frac{P_0V_0}{2}$$

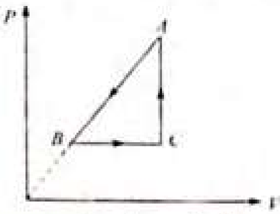
15. The P-V graph of an ideal gas cycle is shown here as below. The adiabatic process is described by



(a) AB and BC
(b) AB and CD
(c) BC and DA
(d) BC and CD

Sol. (c) AD and BC represent adiabatic process (more slope). AB and DC represent isothermal process (less slope).

16. The P-V diagram of a cyclic process $ABCA$ is as shown in the figure. Choose the correct statement.

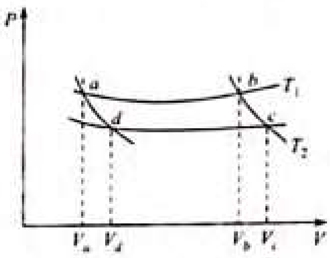


(a) $Q_{A\to B}$ = negative
(b) $U_{B\to C}$ = positive
(c) W_{CAB} = negative
(d) All of these

Sol. (d) During process A to B, pressure and volume both are decreasing. Therefore, temperature and, hence, internal energy of the gas will decrease ($T \propto PV$) or $\Delta U_{A\to B}$ negative. Further $\Delta W_{A\to B}$ is also negative as the volume of the gas is decreasing. Thus, $\Delta Q_{A\to B}$ is negative.

In process B to C, pressure of the gas is constant while volume is increasing. Hence, temperature should increase or $\Delta U_{B\to C}$ = positive. During C to A, volume is constant while pressure is increasing. Therefore, temperature and hence, internal energy of the gas should increase or $\Delta U_{C\to A}$= positive. During process CAB, volume of the gas is decreasing. Hence, work done by the gas is negative.

17. In the following P-V diagram, two adiabatics cut two isothermals at temperatures T_1 and T_2 (figure). The value of V_a/V_d will be



(a) $\frac{V_b}{V_c}$

(b) $\frac{V_c}{V_b}$

(c) $\frac{V_d}{V_a}$

(d) V_bV_c

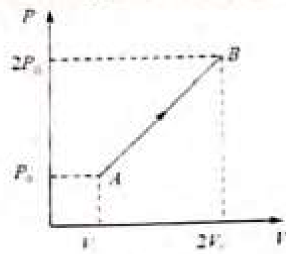
Sol. (a) For adiabatic process, $T_1V_b^{\gamma-1}$ = Constant

For bc curve, $T_1V_b^{\gamma-1} = T_2V_c^{\gamma-1}$ or $\frac{T_2}{T_1} = \left(\frac{V_b}{V_c}\right)^{\gamma-1}$ .....(i)

For ad curve, $T_1V_a^{\gamma-1} = T_2V_d^{\gamma-1}$ or $\frac{T_2}{T_1} = \left(\frac{V_a}{V_d}\right)^{\gamma-1}$ .....(ii)

From equation (i) and (ii), $\frac{V_b}{V_c} = \frac{V_a}{V_d}$

18. The P-V diagram of 2 gm of helium gas for a certain process $A\to B$ is shown in the figure. what is the heat given to the gas during the process $A\to B$



(a) $4P_0V_0$
(b) $6P_0V_0$
(c) $4.5P_0V_0$
(d) $2P_0V_0$

Sol. (b) Change in internal energy from $A \to B$ is

$$\Delta U = \frac{f}{2}\mu R\Delta T = \frac{f}{2}(P_fV_f - P_iP_i)$$

$$= \frac{3}{2}(2P_0 \times 2V_0 - P_0 \times V_0) = \frac{9}{2}P_0V_0$$

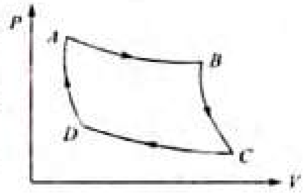
Work done in process $A \to B$ is equal to the area covered by the graph with volume axis, i.e.,

$$W_{A\to B} = \frac{1}{2}(P_0 + 2P_0)\times(2V_0 - V_0) = \frac{3}{2}P_0V_0$$

Hence, $Q = \Delta U + W = \frac{9}{2}P_0V_0 + \frac{3}{2}P_0V_0 = 6P_0V_0$

19. Carnot cycle (reversible) of a gas represented by a Pressure–Volume curve is shown in the diagram. Consider the following statements:

I. Area $ABCD$ = Work done on the gas
II. Area $ABCD$ = Net heat absorbed
III. Change in the internal energy in cycle = 0



Which of these are correct?

(a) I only (b) II only
(c) II and III (d) I, II and III

Sol. (c) Work done by the gas (as cyclic process is clockwise)
$\therefore$ ΔW = Area $ABCD$
So from the first law of thermodynamics ΔQ (net heat absorbed) = ΔW = Area $ABCD$

20. A Carnot engine absorbs an amount Q of heat from a reservoir at an abosolute temperature T and rejects heat to a sink at a temperature of $T/3$. The amount of heat rejected is

(a) $Q/4$ (b) $Q/3$ (c) $Q/2$ (d) $2Q/3$

Sol. (b) $\because \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

where Q_1= heat absorbed, Q_2 = heat rejected

$$\Rightarrow 1 - \frac{T/3}{T} = \frac{W}{Q_1} \Rightarrow \frac{2}{3} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\Rightarrow \frac{2}{3} = 1 - \frac{Q_2}{Q_1} \Rightarrow \frac{Q_2}{Q_1} = \frac{1}{3} \Rightarrow Q_2 = \frac{Q_1}{3} = \frac{Q}{3}$$

EXERCISES

Kinetic Theory of Gases

1. If pressure of a gas contained in a closed vessel is increased by 0.4% when heated by 1°C, the initial temperature must be
(a) 250 K (b) 250°C
(c) 2500 K (d) 25°C

2. The capacity of a vessel is 3 L. It contains 6 g oxygen, 8 g nitrogen and 5 g CO_2 mixture at 27°C. If $R = 8.31$ J/mol K, then the pressure in the vessel in N/m^2 will be (approx.)
(a) 5×10^5 (b) 5×10^4
(c) 10^6 (d) 10^5

3. Energy of all molecules of a monatomic gas having a volume V and pressure P is 3/2 PV. The total translational kinetic energy of all molecules of a diatomic gas at the same volume and pressure is
(a) 1/2 PV (b) 3/2 PV
(c) 5/2 PV (d) 3 PV

4. A monatomic gas expands at constant pressure on heating. The percentage of heat supplied that increases the internal energy of the gas and that is involved in the expansion is
(a) 75%, 25% (b) 25%, 75%
(c) 60%, 40% (d) 40%, 60%

5. Certain amount of an ideal gas is contained in a closed vessel. The vessel is moving with a constant velocity v. The molecular mass of gas is M. The rise in temperature of the gas when the vessel is suddenly stopped is ($\gamma = C_P/C_V$)
(a) $\dfrac{Mv^2(\gamma-1)}{2R(\gamma+1)}$ (b) $\dfrac{Mv^2(\gamma-1)}{2R}$
(c) $\dfrac{Mv^2}{2R(\gamma+1)}$ (d) $\dfrac{Mv^2}{2R(\gamma-1)}$

6. One mole of a diatomic gas undergoes a process $P = P_0/[1 + (V/V_0)^3]$ where P_0 and V_0 are constants. The translational kinetic energy of the gas when $V = V_0$ is given by
(a) $5P_0V_0/4$ (b) $3P_0V_0/4$
(c) $3P_0V_0/2$ (d) $5P_0V_0/2$

7. Two identical containers A and B have frictionless pistons. They contain the same volume of an ideal gas at the same temperature. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to double the initial volume. The change in the pressure in A and B, respectively, is Δp and 1.5 Δp. Then
(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$
(c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$

8. The pressure P, volume V and temperature T of a gas in the jar A and the other gas in the jar B at pressure $2P$, volume $V/4$ and temperature $2T$, then the ratio of the number of molecules in the jar A and B will be
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 4 : 1

9. The root mean square speed of the molecules of a diatomic gas is v. When the temperature is doubled, the molecules dissociate into two atoms. The new root mean square speed of the atom is
(a) $\sqrt{2}v$ (b) v
(c) $2v$ (d) $4v$

10. The molecules of a given mass of a gas have a rms velocity of 200 m/sec at 27°C and 1.0×10^5 N/m^2 pressure. When the temperature is 127°C and pressure is 0.5×10^5 N/m^2 the rms velocity in m/sec will be
(a) $\dfrac{100\sqrt{2}}{3}$ (b) $100\sqrt{2}$
(c) $\dfrac{400}{\sqrt{3}}$ (d) None of these

11. Two molecules of a gas have speeds of 9×10^6 ms^{-1} and 1×10^6 ms^{-1} respectively. What is the root mean square speed of these molecules?
(a) $\sqrt{21} \times 10^6$ m/s (b) $\sqrt{41} \times 10^6$ m/s
(c) 8.5×10^6 m/s (d) $\sqrt{17} \times 10^6$ m/s

12. The molecules of a given mass of a gas have root mean square speeds of 100 ms^{-1} at 27°C and 1.00 atmospheric pressure. What will be the root mean square speeds of the molecules of the gas at 127°C and 2.0 atmospheric pressure?
(a) $\dfrac{150}{\sqrt{3}}$ m/s (b) $\dfrac{125}{\sqrt{3}}$ m/s
(c) $\dfrac{200}{\sqrt{3}}$ m/s (d) $100\sqrt{3}$ m/s

13. The r.m.s. speed of particle of mass 5×10^{-17} kg in their random motion in air at NTP will be (Boltzmann's constant) $K = 1.38 \times 10^{-23}$ J/K:
(a) 15×10 m/s (b) 15×10^{-3} m/s
(c) 10×10^{-2} (d) 1.5×10^{2} m/s

14. Calculate the ratio of the mean free paths of the molecules of two gases having molecular diameters 1 Å and 2 Å. The gases may be considered under identical conditions of temperature, pressure and volume.
(a) 2 : 1 (b) 3 : 1
(b) 4 : 3 (d) 4 : 1

15. Calculate the number of degrees of freedom of molecules of hydrogen in 1 cc of hydrogen gas at NTP.
(a) 2.344×10^{16} (b) 1.344×10^{20}
(c) 2.124×10^{21} (d) 4.212×10^{20}

Ideal Gas Equation and Specific Heat Capacity of Gases

16. Pressure versus temperature graph of an ideal gas of equal number of moles of different volumes is plotted as shown in figure. Choose the correct alternative.

(a) $V_1 = V_2, V_3 = V_4$ and $V_2 > V_3$
(b) $V_1 = V_2, V_3 = V_4$ and $V_2 < V_3$
(c) $V_1 = V_2 = V_3 = V_4$
(d) $V_4 > V_3 > V_2 > V_1$

17. Two gases occupy two containers A and B; the gas in A, of volume $0.10\ m^3$, exerts a pressure of 1.40 MPa and that in B, of volume $0.15\ m^3$, exerts a pressure 0.7 MPa. The two containers are joined by a tube of negligible volume and the gases are allowed to intermingle. Then if the temperature remains constant, the final pressure in the container will be (in MPa)
(a) 0.70 (b) 0.98
(c) 1.40 (d) 2.10

18. A closed vessel contains 8 g of oxygen and 7 g of nitrogen. The total pressure is 10 atm at a given temperature. If now oxygen is absorbed by introducing a suitable absorbent, the pressure of the remaining gas in atm will be
(a) 2 (b) 10 (c) 4 (d) 5

19. Forty calories of heat is needed to raise the temperature of 1 mol of an ideal monatomic gas from 20°C to 30°C at a constant pressure. The amount of heat required to raise its temperature over the same interval at a constant volume ($R = 2$ cal $mol^{-1}K^{-1}$) is
(a) 20 cal (b) 40 cal
(c) 60 cal (d) 80 cal

20. For a gas, the difference between the two specific heats is 4150 J/kg K. What is the specific heat at constant volume of gas if the ratio of specific heats is 1.4?
(a) 8475 J/kg-K (b) 5186 J/kg-K
(c) 1660 J/kg-K (d) 10375 J/kg-K

21. The specific heat at constant volume for the monatomic argon is 0.075 kcal/kg-K, whereas its gram molecular specific heat is $C_v = 2.98$ cal/mol/K. The mass of the argon atom is (Avogadro's number $= 6.02 \times 10^{23}$ molecules/mol)
(a) 6.60×10^{-23} g (b) 3.30×10^{-23} g
(c) 2.20×10^{-23} g (d) 13.20×10^{-23} g

22. The temperature of 5 mol of a gas which was held at constant volume was changed from 100°C to 120°C. The change in internal energy was found to be 80 J. The total heat capacity of the gas at constant volume will be equal to
(a) 8 JK^{-1} (b) 0.8 JK^{-1}
(c) 4 JK^{-1} (d) 0.4 JK^{-1}

23. The density of a polyatomic gas in standard conditions is 0.795 kgm^{-3}. The specific heat of the gas at constant volume is
(a) 930 $J\text{-}kg^{-1}K^{-1}$ (b) 1400 $J\text{-}kg^{-1}K^{-1}$
(c) 1120 $J\text{-}kg^{-1}K^{-1}$ (d) 1600 $J\text{-}kg^{-1}K^{-1}$

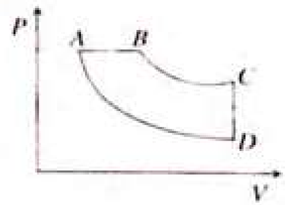
24. The value of $C_p - C_v = 1.00\ R$ for a gas in state A and $C_p - C_v = 1.06\ R$ in another state. If P_A and P_B denote the pressure and T_A and T_B denote the temperatures in the two states, then
(a) $P_A = P_B, T_A > T_B$ (b) $P_A > P_B, T_A = T_B$
(c) $P_A < P_B, T_A > T_B$ (d) $P_A = P_B, T_A < T_B$

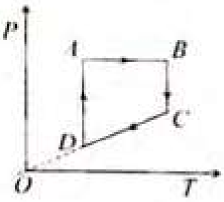
25. If 2 moles of diatomic gas and 1 mole of monatomic gas are mixed, then the ratio of specific heats for the mixture is
(a) $\frac{7}{3}$ (b) $\frac{5}{4}$
(c) $\frac{19}{13}$ (d) $\frac{15}{19}$

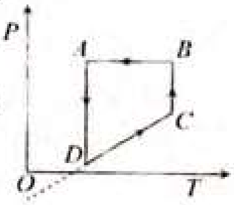
26. Twenty-two grams of CO_2 at 27°C is mixed with 16 g of O_2 at 37°C. The temperature of the mixture is about
(a) 31.5°C (b) 27°C
(c) 37°C (d) 30.5°C

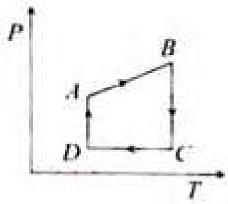
27. A gas mixture consists of 2 mol of oxygen and 4 mol of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is
(a) $4RT$ (b) $15RT$
(c) $9RT$ (d) $11RT$

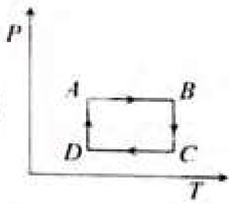
28. A cyclic process $ABCD$ is shown in the following P–V diagram. Which of the following curves represents the same process?



(a)


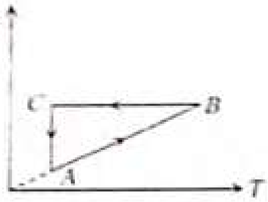
(b)


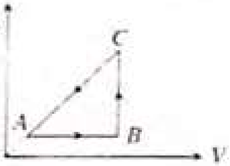
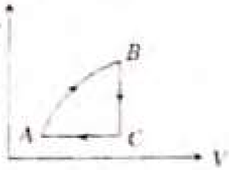
(c)


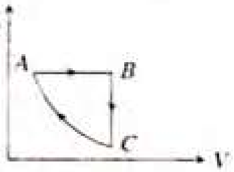
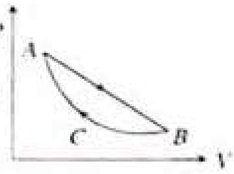
(d)


29. An ideal gas expands in such a manner that its pressure and volume can be related by equation PV^2 = constant. During this process, the gas is
(a) heated (b) cooled
(c) neither heated nor cooled
(d) first heated and then cooled

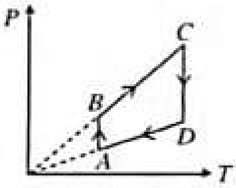
30. A cyclic process $ABCA$ is shown in the V–T diagram. Process on the P–V diagram is



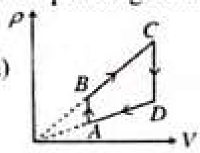
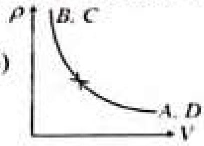
(a)  (b) 

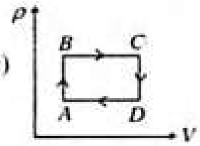
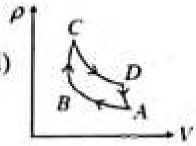
(c)  (d) 

31. Pressure versus temperature graph of an ideal gas is as shown in figure.

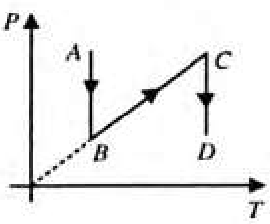


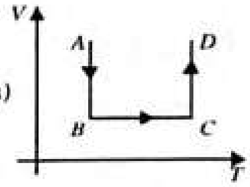
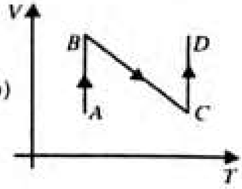
Corresponding density (ρ) versus volume (V) graph will be

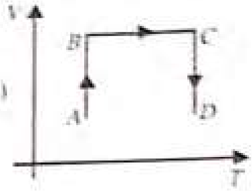
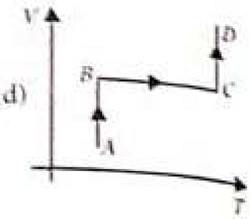
(a)  (b) 

(c)  (d) 

32. P–T diagram is shown in figure. Choose the corresponding V–T diagram.



(a)  (b) 

(c)  (d) 

First Law of Thermodynamics, Internal Energy and Work Done

33. A gas is heated at a constant pressure. The fraction of heat supplied used for external work is

(a) $\dfrac{1}{\gamma}$ (b) $\left(1-\dfrac{1}{\gamma}\right)$

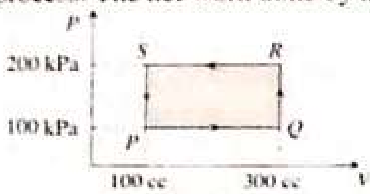
(c) $\gamma-1$ (d) $\left(1-\dfrac{1}{\gamma^2}\right)$

34. The average degrees of freedom per molecule for a gas are 6. The gas performs 25 J of work when it expands at constant pressure. The heat absorbed by gas is

(a) 75 J (b) 100 J

(c) 150 J (d) 125 J

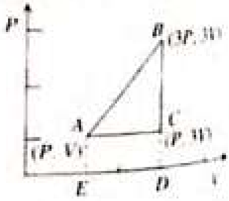
35. A thermodynamic system is taken through the cycle $PQRSP$ process. The net work done by the system is



(a) 20 J (b) −20 J

(c) 400 J (d) −374 J

36. An ideal gas is taken around $ABCA$ as shown in the above P–V diagram. The work done during a cycle is



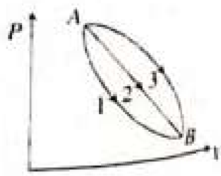
(a) $2PV$

(b) PV

(c) $1/2PV$

(d) Zero

37. An ideal gas of mass m in a state A goes to another state B via three different processes as shown in figure. If Q_1, Q_2 and Q_3 denote the heat absorbed by the gas along the three paths, then



(a) $Q_1 < Q_2 < Q_3$

(b) $Q_1 < Q_2 = Q_3$

(c) $Q_1 = Q_2 > Q_3$

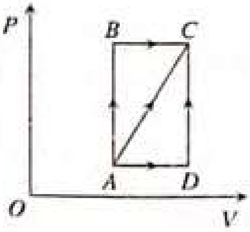
(d) $Q_1 > Q_2 > Q_3$

38. The relation between the internal energy U and adiabatic constant γ is

(a) $U=\dfrac{PV}{\gamma-1}$ (b) $U=\dfrac{PV^{\gamma}}{\gamma-1}$

(c) $U = \frac{PV}{\gamma}$ (d) $U = \frac{\gamma}{PV}$

39. A thermodynamic process is shown in figure. The pressures and volumes corresponding to some points in the figure are:



$P_A = 3 \times 10^4$ Pa,
$P_B = 8 \times 10^4$ Pa and
$V_A = 2 \times 10^{-3}$ m^3,
$V_D = 5 \times 10^{-3}$ m.

In process *AB*, 600 J of heat is added to the system and in process *BC*, 200 J of heat is added to the system. The change in internal energy of the system in process *AC* would be

(a) 560 J (b) 800 J
(c) 600 J (d) 640 J

40. If R = universal gas constant, the amount of heat needed to raise the temperature of 2 mol of an ideal monatomic gas from 273 K to 373 K when no work is done is

(a) $100R$ (b) $150R$ (c) $300R$ (d) $500R$

41 During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p/C_v for the gas is

(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) 2 (d) $\frac{5}{3}$

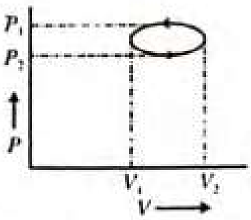
42. An ideal gas at 27 °C is compressed adiabatically to 8/27 of its original volume. If $\gamma = 5/3$, then the rise in temperature is

(a) 450 K (b) 375 K
(c) 225 K (d) 405 K

43. A thermally insulated container is divided into two parts by a screen. In one part the pressure and temperature are P and T for an ideal gas filled. In the second part, it is vacuum. If now a small hole is created in the screen, then the temperature of the gas will

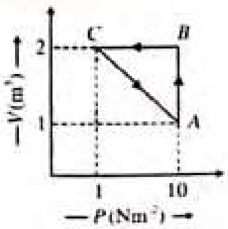
(a) decrease (b) increase
(c) remain same (d) none of these

44. In the given elliptical P–V diagram,



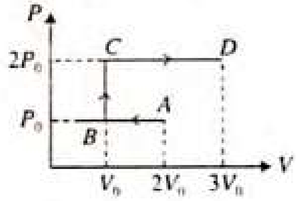
(a) the work done is positive
(b) the change in internal energy is non-zero
(c) the work done $= -(\pi/4)(P_2 - P_1)(V_2 - V_1)$
(d) the work done $= \pi(V_1 - V_1)^2 - \pi(P_1 - P_1)^2$

45. An ideal gas is taken through $A \to B \to C \to A$, as shown in figure. If the net heat supplied to the gas in the cycle is 55 J, the work done by the gas in the process $C \to A$ is



(a) –5 J (b) –10 J
(c) –15 J (d) –20 J

46. P–V diagram of an ideal gas is as shown in figure. Work done by the gas in the process *ABCD* is



(a) $4P_0V_0$ (b) $2P_0V_0$
(c) $3P_0V_0$ (d) P_0V_0

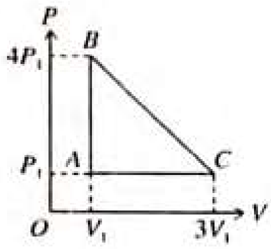
47. One mole of an ideal gas at temperature T_1 expands according to the law (P/V) = constant. Find the work done when the final temperature becomes T_2.

(a) $R(T_2 - T_1)$ (b) $(R/2)(T_2 - T_1)$
(c) $(R/4)(T_2 - T_1)$ (d) $PV(T_2 - T_1)$

48. Two moles of an ideal gas at 300 K were cooled at constant volume so that the pressure is reduced to half the initial value. Then as a result of heating at constant pressure, the gas expands till it attains the original temperature. Find the total heat absorbed by the gas, if R is the gas constant.

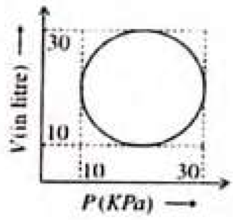
(a) $150R$ J (b) $300R$ J
(c) $75R$ J (d) $100R$ J

49. An ideal gas is taken around the cycle *ABCA* shown in P–V diagram. The net work done by the gas during the cycle is equal to



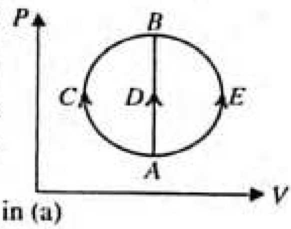
(a) $12P_1V_1$
(b) $6P_1V_1$
(c) $3P_1V_1$
(d) P_1V_1

50. Heat energy absorbed by a system in going through a cyclic process shown in figure is



(a) $10^7\pi$ J (b) $10^4\pi$ J
(c) $10^2\pi$ J (d) $10^{-3}\pi$ J

51. One mole of an ideal gas is taken from state A to state B by three different processes (a) *ACB*, (b) *ADB* and (c) *AEB* as shown in the P–V diagram. The heat absorbed by the gas is



(a) greater in process (b) than in (a)
(b) the least in process (b)
(c) the same in (a) and (c)
(d) less in (c) than in (b)

52. Find the amount of work done to increase the temperature of 1 mol of an ideal gas by 30°C if it is expanding under the condition $V \propto T^{2/3}$.

(a) 166.2 J (b) 136.2 J
(c) 126.2 J (d) none of these

Second Law of Thermodynamics

53. A Carnot engine has the same efficiency between 800 K and 500 K and x K to 600 K. The value of x is
(a) 1000 K (b) 960 K
(c) 846 K (d) 754 K

54. A Carnot's engine is made to work between 200°C and 0°C first and then between 0°C and –200°C. The ratio of efficiencies of the engine in the two cases is
(a) 1.73 : 1 (b) 1 : 1.73
(c) 1 : 1 (d) 1 : 2

55. Efficiency of a Carnot engine is 50% when temperature of outlet is 500 K. In order to increase efficiency up to 60% keeping temperature of intake the same what is temperature of outlet
(a) 200 K (b) 400 K
(c) 600 K (d) 800 K

56. An ideal heat engine working between temperature T_1 and T_2 has an efficiency η, the new efficiency if both the source and sink temperature are doubled, will be
(a) $\frac{\eta}{2}$ (b) η
(c) 2η (d) 3η

57. An ideal refrigerator has a freezer at a temperature of –13°C. The coefficient of performance of the engine is 5. The temperature of the air (to which heat is rejected) will be
(a) 325°C (b) 325K
(c) 39°C (d) 320°C

58. In a mechanical refrigerator, the low temperature coils are at a temperature of –23°C and the compressed gas in the condenser has a temperature of 27°C. The theoretical coefficient of performance is
(a) 5 (b) 8 (c) 6 (d) 6.5

59. An engine is supposed to operate between two reservoirs at temperature 727°C and 227°C. The maximum possible efficiency of such an engine is
(a) 1/2 (b) 1/4
(c) 3/4 (d) 1

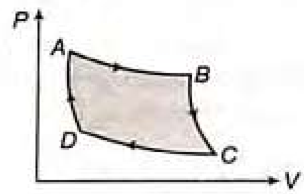
60. An ideal gas heat engine operates in Carnot cycle between 227°C and 127°C. It absorbs 6×10^4 cals of heat at higher temperature. Amount of heat converted to work is
(a) 2.4×10^4 cal (b) 6×10^4 cal
(c) 1.2×10^4 cal (d) 4.8×10^4 cal

61. Two Carnot engines A and B are operated in succession. The first one, A receives heat from a source at $T_1 = 800$ K and rejects to sink at T_2K. The second engine B receives heat rejected by the first engine and rejects to another sink at $T_3 = 300$ K. If the work outputs of two engines are equal, then the value of T_2 is
(a) 100 K (b) 300 K
(c) 550 K (d) 700 K

62. A reversible engine converts one-sixth of the heat input into work. When the temperature of the sink is reduced by 62°C, the efficiency of the engine is doubled. The temperatures of the source and sink are
(a) 80°C, 37°C (b) 95°C, 28°C
(c) 90°C, 37°C (d) 99°C, 37°C

63. A Carnot engine whose low temperature reservoir is at 7°C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees should the temperature of the high temperature reservoir be increased
(a) 840 K (b) 280 K
(c) 560 K (d) 380 K

64. Carnot cycle (reversible) of a gas represented by a Pressure–Volume curve is shown in the diagram.
Consider the following statements:
I. Area $ABCD$ = Work done on the gas
II. Area $ABCD$ = Net heat absorbed
III. Change in the internal energy in cycle = 0
Which of these are correct



(a) I only (b) II only
(c) II and III (d) I, II and III

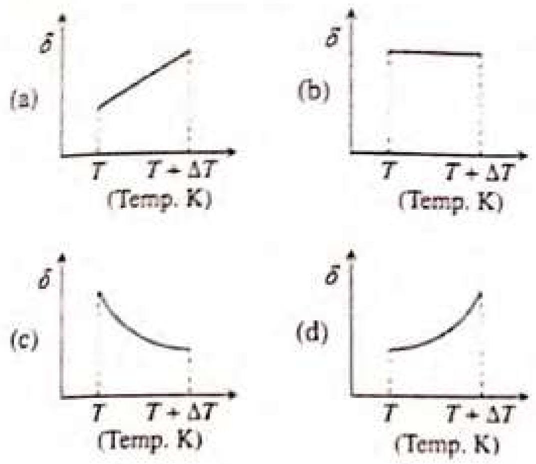
65. A motor cycle engine delivers a power of 10 kW, by consuming petrol at the rate of 2.4 kg/hour. If the calorific value of petrol is 35.5 MJ/kg, the rate of heat rejection by the exhaust is
(a) 5.5 kW (b) 13.7 kW
(c) 11.2 kW (d) 9.7 kW

66. A heat engine receives 50 kcal of heat from the source per cycle, and operates with an efficiency of 20%. The heat rejected by engine to the sink per cycle is
(a) 40 kcal (b) 25 kcal
(c) 30 kcal (d) 50 kcal

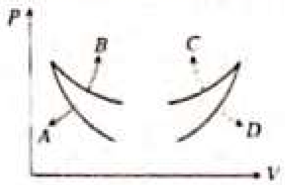
67. A Carnot's engine operates with an efficiency of 40% with its sink at 27°C. By what amount should the temperature of the source be increased with an aim to increase the efficiency by 10%?
(a) 50 K (b) 150 K
(c) 80 K (d) 100 K

Problems Based on Mixed Concepts

68. An ideal gas is initially at a temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \Delta V / V\Delta T$ varies with temperature as

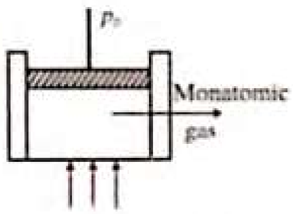


69. Four curves A, B, C and D are drawn in figure for a given amount of gas. The curves which represent adiabatic and isothermal changes are



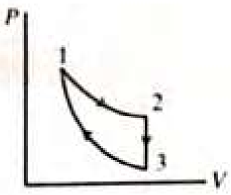
(a) C and D, respectively
(b) D and C, respectively
(c) A and B, respectively
(d) B and A, respectively

70. If 50 cal of heat is supplied to the system containing 2 mol of an ideal monatomic gas, the rise in temperature is (R = 2 cal/mol-K)



(a) 50 K (b) 5 K
(c) 10 K (d) 20 K

71. Three processes compose a thermodynamic cycle shown in the accompanying P–V, diagram of an ideal gas.



Process 1→2 takes place at constant temperature, during this process 60 J of heat enters the system.

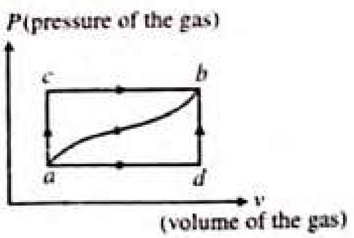
Process 2→3 takes place at constant volume. During this process 40 J of heat leaves the system.

Process 3→1 is adiabatic.

What is the change in internal energy of the system during process 3→1?

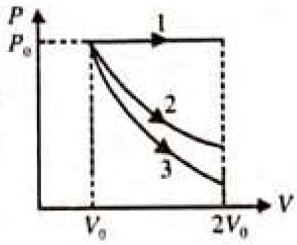
(a) −40 J (b) −20 J
(c) +20 J (d) +40 J

72. When an ideal gas is taken from state a to b, along a path acb, 84 kJ of heat flows into the gas and the gas does 32 kJ of work. The following conclusions are drawn. Mark the one which is not correct.



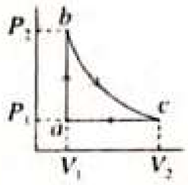
(a) If the work done along the path adb is 10.5 kJ, the heat that will flow into the gas is 62.5 kJ.
(b) When the gas is returned from b to a along the curved path, the work done on the gas is 21 kJ, and the system absorbs 73 kJ of heat.
(c) If $U_a = 0$, $U_d = 42$ kJ, and the work done along the path adb is 10.5 kJ then the heat absorbed in the process ad is 52.5 kJ.
(d) If $U_a = 0$, $U_d = 42$ kJ, heat absorbed in the process db is 10 kJ.

73. A gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric process, process 2 is isothermal process and process 3 is adiabatic. Let ΔU_1, ΔU_2 and ΔU_3 be the change in internal energy of the gas in these three processes. Then



(a) $\Delta U_1 > \Delta U_2 > \Delta U_3$
(b) $\Delta U_1 < \Delta U_2 < \Delta U_3$
(c) $\Delta U_2 > \Delta U_1 > \Delta U_3$
(d) $\Delta U_2 < \Delta U_3 < \Delta U_1$

74. Carbon monoxide is carried around a closed cyclic process abc, in which bc is an isothermal process, as shown in figure. The gas absorbs 7000 J of heat as its temperature is increased from 300 K to 1000 K in going from a to b. The quantity of heat ejected by the gas during the process ca is



(a) 4200 J (b) 500 J
(c) 9000 J (d) 9800 J

75. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are Q_1 = 5960 J, Q_2 = −5585 J, Q_3 = −2980 J, Q_4 = 3645 J, respectively. The corresponding works involved are W_1 = 2200 J, W_2 = −825 J, W_3 = −1100 J and W_4, respectively. The value of W_4 is

(a) 1315 J (b) 275 J
(c) 765 J (d) 675 J

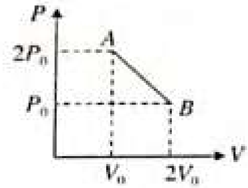
76. Argon gas is adiabatically compressed to half its volume. If P, V and T represent the pressure, volume and temperature of the gaseous system, respectively, at any stage, then the correct equation representing the process is

(a) $TV^{2/5}$ = constant (b) $VP^{5/3}$ = constant
(c) $TP^{-2/5}$ = constant (d) $PT^{2/3}$ = constant

77. Three samples of the same gas A, B and C ($\gamma = 3/2$) initially have equal volume. Now the volume of each sample is doubled. The process is adiabatic for A, isobaric for B and isothermal for C. If the final pressures are equal for all the three samples, the ratio of their initial pressures is

(a) $2\sqrt{2}:2:1$ (b) $2\sqrt{2}:1:2$
(c) $\sqrt{2}:1:2$ (d) $2:1:\sqrt{2}$

78. n moles of an ideal gas undergoes a process $A \to B$ as shown in figure. Maximum temperature of the gas during the process is:



(a) $\frac{3P_0V_0}{2nR}$ (b) $\frac{9P_0V_0}{4nR}$

(c) $\frac{9P_0V_0}{2nR}$ (d) $\frac{9P_0V_0}{nR}$

79. The relation between internal energy U, pressure P and volume V of a gas in an adiabatic process is

$U = a + bPV$

where a and b are constants. What is the effective value of adiabatic constant γ?

(a) $\frac{a}{b}$ (b) $\frac{b+1}{b}$

(c) $\frac{a+1}{a}$ (d) $\frac{b}{a}$

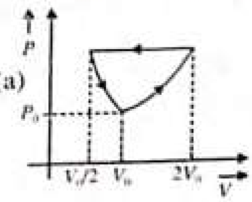
80. A diatomic ideal gas is heated at constant volume until the pressure is doubled and again heated at constant pressure until the volume is doubled. The average molar heat capacity for the whole process is

(a) $\frac{13R}{6}$ (b) $\frac{19R}{6}$

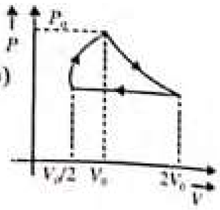
(c) $\frac{23R}{6}$ (d) $\frac{17R}{6}$

81. One mole of an ideal gas at pressure P_0 and temperature T_0 is expanded isothermally to twice its volume and then compressed at constant pressure to $(V_0/2)$ and the gas is brought back to original state by a process in which $P \propto V$ (pressure is directly proportional to volume). The correct temperature of the process is

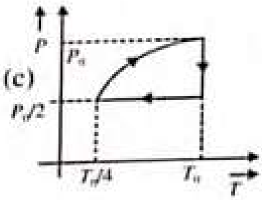
(a)



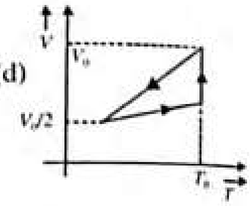
(b)



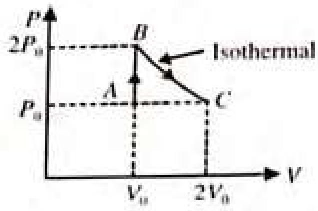
(c)



(d)



82. A diatomic ideal gas undergoes a thermodynamic change according to the P–V diagram shown in figure. The total heat given to the gas is nearly



(a) $2.5P_0V_0$ (b) $1.4P_0V_0$

(c) $3.9P_0V_0$ (d) $1.1P_0V_0$

ARCHIVES

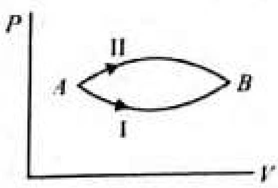
1. Two thermally insulated vessels, 1 and 2, are filled with air at temperature (T_1, T_2), volumes (V_1, V_2), and pressures (P_1, P_2), respectively. If the valve joining the two vessels is opened, the temperature inside the vessel at equilibrium will be

(a) $T_1 + T_2$ (b) $(T_1 + T_2)/2$

(c) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$ (d) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2T_2}$

(AIEEE 2004)

2. A system goes from A to B via two processes I and II as shown in the figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II, respectively, then



(a) $\Delta U_2 > \Delta U_1$

(b) $\Delta U_2 < \Delta U_1$

(c) relation between ΔU_1 and ΔU_2 cannot be determined

(d) $\Delta U_1 = \Delta U_2$ **(AIEEE 2005)**

3. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_p/C_v of the mixture is

(a) 1.54 (b) 1.62

(c) 1.4 (d) 1.59 **(AIEEE 2005)**

4. The work done in compressing 1 kg-mol of a gas adiabatically is 146 kJ. in this process, the temperature of the gas increases by 7°C. The gas is (R = 8.3 J/mol-K)

(a) diatomic

(b) triatomic

(c) a mixture of monatomic and diatomic

(d) monatomic **(AIEEE 2006)**

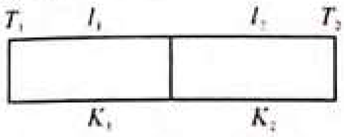
5. Two rigid boxes containing different ideal gases are placed on a table. Box A contains 1 mol of nitrogen at temperature T_0, while box B contains 1 mol of helium at temperature $(7/3)T_0$. The boxes are then put in thermal contact with each other and heat flows between them until

the gases reach a common final temperature. (Ignore the heat capacity of boxes.) Then the final temperature of gases, T_f, in terms of T_0 is

(a) $T_f = \frac{7}{3}T_0$ (b) $T_f = \frac{3}{7}T_0$

(c) $T_f = \frac{3}{2}T_0$ (d) $T_f = \frac{5}{2}T_0$ **(AIEEE 2006)**

6. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2, respectively. The temperature at the interface of the two sections is



(a) $(K_2l_2T_1 + K_1l_1T_2)/(K_1l_1 + K_2l_2)$
(b) $(K_2l_1T_1 + K_1l_2T_2)/(K_2l_1 + K_1l_2)$
(c) $(K_1l_2T_1 + K_2l_1T_2)/(K_1l_2 + K_2l_1)$
(d) $(K_1l_1T_1 + K_2l_2T_2)/(K_1l_1 + K_2l_2)$ **(AIEEE 2007)**

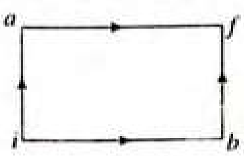
7. If C_P and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume, respectively, then

(a) $C_P - C_v = R/28$ (b) $C_P - C_v = R/14$
(c) $C_P - C_v = R$ (d) $C_P - C_v = 28R$ **(AIEEE 2007)**

8. A Carnot engine, having an efficiency of $h = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at a lower temperature is

(a) 99 J (b) 90 J
(c) 1 J (d) 100 J **(AIEEE 2007)**

9. When a system is taken from state i to state f along the path iaf, it is found that $Q = 50$ cal and $W = 20$ cal. Along the path ibf, $Q = 36$ cal. W along the path ibf is



(a) 6 cal (b) 16 cal
(c) 66 cal (d) 14 cal **(AIEEE 2007)**

10. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(a) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$ (b) $\frac{p_1V_1T_1 + p_2V_2T_2}{p_1V_1 + p_2V_2}$

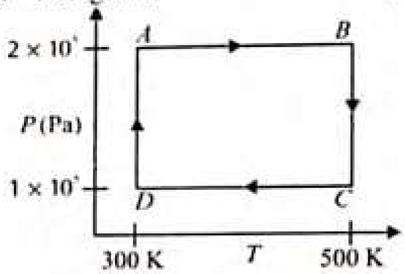
(c) $\frac{p_1V_1T_2 + p_2V_2T_1}{p_1V_1 + p_2V_2}$ (d) $\frac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_1 + p_2V_2T_2}$ **(AIEEE 2008)**

11. One kilogram of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal motion?

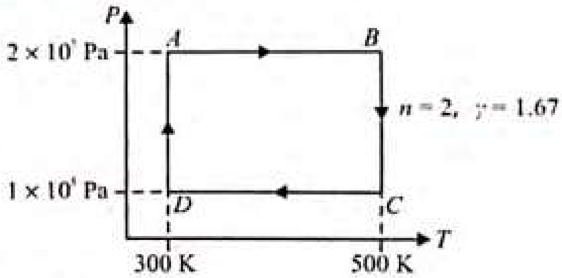
(a) 3×10^4 J (b) 5×10^4 J
(c) 6×10^4 J (d) 7×10^4 J **(AIEEE 2009)**

For Questions 12–14

Two moles of helium gas is taken over the cycle $ABCDA$, as shown in the P–T diagram. **(AIEEE 2009)**



12. Assuming the gas to be ideal, the work done on the gas in taking it from A to B is



(a) $200R$ (b) $300R$
(c) $400R$ (d) $500R$

13. The work done on the gas in taking it from D to A is

(a) $-414R$ (b) $+414R$
(c) $-690R$ (d) $+690R$

14. The net work done on the gas in the cycle $ABCDA$ is

(a) zero (b) $276R$
(c) $1076R$ (d) $1904R$

15. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle, the volume of the gas increases from V to $32V$, the efficiency of the engine is

(a) 0.5 (b) 0.75
(c) 0.99 (d) 0.25 **(AIEEE 2010)**

16. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2

and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is

(a) $\frac{(T_1+T_2+T_3)}{3}$

(b) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$

(c) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

(d) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ **(AIEEE 2011)**

17. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats g. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(a) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ K (b) $\frac{(\gamma-1)}{2\gamma}Mv^2$ K

(c) $\frac{\gamma Mv^2}{2R}$ K (d) $\frac{(\gamma-1)}{2R}Mv^2$ K

(AIEEE 2011)

18. A Carnot engine operating between temperatures T_1 and T_2 has efficiency 1/6. When T_2 is lowered by 62 K, its efficiency increases to 1/3. Then T_1 and T_2 are, respectively,

(a) 372 K and 310 K (b) 372 K and 330 K

(c) 330 K and 268 K (d) 310 K and 248 K

(AIEEE 2011)

19. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be

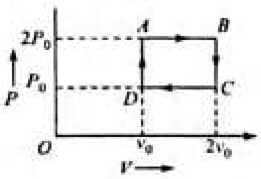
(a) efficiency of carnot engine cannot be made larger than 50%

(b) 1200 K

(c) 750 K

(d) 600 K **(AIEEE 2012)**

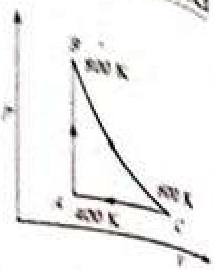
20. The above P–V diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat extracted from the source in a single cycle is



(a) $\left(\frac{13}{2}\right)P_0V_0$ (b) $\left(\frac{11}{2}\right)P_0V_0$

(c) $4P_0V_0$ (d) P_0V_0 **(JEE Main 2013)**

21. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in the figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement:



(a) The change in internal energy in the process AB is $-350R$.

(b) The change in internal energy in the process BC is $-500R$.

(c) The change in internal energy in whole cyclic process is $250R$.

(d) The change in internal energy in the process CA is $700R$. **(JEE Main 2014)**

22. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u=\frac{U}{V}\propto T^4$ and pressure $P=\frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is

(a) $T\propto e^{-R}$ (b) $T\propto e^{-3R}$

(c) $T\propto\frac{1}{R}$ (d) $T\propto\frac{1}{R^3}$ **(JEE Main 2015)**

23. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases, body is brought from initial temperature 100°C to final temperature 200°C. Entropy changes of the body in the two cases, respectively, is

(a) ln 2, 4 ln 2 (b) ln 2, ln 2

(c) ln 2, 2 ln 2 (d) 2 ln 2, 8 ln 2

(JEE Main 2015)

24. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is $\left(\gamma=\frac{C_P}{C_V}\right)$

(a) $\frac{3\gamma+5}{6}$ (b) $\frac{3\gamma-5}{6}$

(c) $\frac{\gamma+1}{2}$ (d) $\frac{\gamma-1}{2}$ **(JEE Main 2015)**

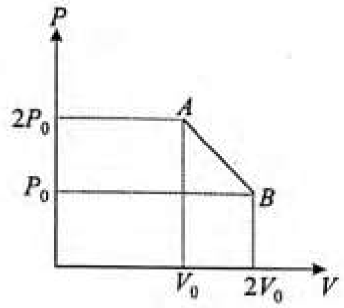
25. An ideal gas undergoes a quasi-static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively):

(a) $n = \frac{C_P}{C_V}$ (b) $n = \frac{C - C_P}{C - C_V}$

(c) $n = \frac{C_P - C}{C - C_V}$ (d) $n = \frac{C - C_V}{C - C_P}$

(JEE Main 2016)

26. 'n' moles of an ideal gas undergoes a process $A \to B$ as shown in the figure. The maximum temperature of the gas during the process will be:



(a) $\frac{9P_0V_0}{4nR}$ (b) $\frac{3P_0V_0}{2nR}$

(c) $\frac{9P_0V_0}{2nR}$ (d) $\frac{9P_0V_0}{nR}$

(JEE Main 2016)

27. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be

(a) 2.5×10^{25} (b) -2.5×10^{25}

(c) -1.61×10^{23} (d) 1.38×10^{23}

(JEE Main 2017)

28. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is

(a) $a = 14b$ (b) $a = 28b$

(c) $a = \frac{1}{14}b$ (d) $a = b$ **(JEE Main 2017)**

29. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume $2V$. Calculate (i) the final temperature of the gas and (ii) change in its internal energy.

(a) (i) 195 K (ii) 2.7 kJ

(b) (i) 189 K (ii) 2.7 kJ

(c) (i) 195 K (ii) –2.7 kJ

(d) (i) 189 K (ii) –2.7 kJ

(JEE Main 2018)

30. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly

(a) 4.70×10^2 N/m^2 (b) 2.35×10^3 N/m^2

(c) 4.70×10^3 N/m^2 (d) 2.35×10^2 N/m^2

(JEE Main 2018)

ANSWER KEY

Exercises

1. (a)	2. (a)	3. (b)	4. (c)	5. (b)	6. (b)	7. (c)	8. (d)	9. (c)	10. (c)
11. (b)	12. (c)	13. (b)	14. (d)	15. (b)	16. (a)	17. (b)	18. (d)	19. (a)	20. (d)
21. (a)	22. (c)	23. (b)	24. (c)	25. (c)	26. (a)	27. (d)	28. (a)	29. (b)	30. (c)
31. (b)	32. (d)	33. (b)	34. (b)	35. (b)	36. (a)	37. (a)	38. (a)	39. (a)	40. (c)
41. (a)	42. (b)	43. (c)	44. (c)	45. (a)	46. (c)	47. (b)	48. (b)	49. (c)	50. (c)
51. (d)	52. (a)	53. (b)	54. (b)	55. (b)	56. (b)	57. (c)	58. (a)	59. (a)	60. (c)
61. (c)	62. (d)	63. (d)	64. (c)	65. (b)	66. (a)	67. (d)	68. (c)	69. (c)	70. (b)
71. (d)	72. (b)	73. (a)	74. (d)	75. (c)	76. (c)	77. (b)	78. (b)	79. (b)	80. (b)
81. (c)	82. (c)								

Archives

1. (c)	2. (d)	3. (b)	4. (c)	5. (c)	6. (c)	7. (a)	8. (b)	9. (a)	10. (a)
11. (b)	12. (c)	13. (a)	14. (b)	15. (b)	16. (b)	17. (d)	18. (a)	19. (c)	20. (a)
21. (b)	22. (c)	23. (b)	24. (c)	25. (b)	26. (a)	27. (b)	28. (a)	29. (d)	30. (b)

Chapter 15

Oscillation and Simple Harmonic Motion

PERIODIC MOTION

Periodic motion is motion of an object that regularly returns to a given position after a fixed time interval. With a little thought, we can identify several types of periodic motions in everyday life. Your car returns to the driveway each afternoon. You return to the dinner table each night to eat. A bumped chandelier swings back and forth, returning to the same position at a regular rate. The earth returns to the same position in its orbit around the sun each year, resulting in the variation among the four seasons.

If T is the period of motion after which it repeats itself, then the frequency f of the periodic motion is the number of cycles performed in 1 s and it is given as $f = 1/T$.

Units of f are s^{-1} or per second. A special name is given to the unit of frequency, hertz (Hz) after the discoverer of radio waves.

1 Hz = 1 cycle per second

Oscillations

An oscillation is a spherical type of periodic motion in which a particle moves to and fro about a fixed point called mean position of particle. Oscillations are commonly seen in general life in our surroundings. As discussed, in all types of oscillations, about which the particle can oscillate. This is the position where the particle is in equilibrium, that is, net force on the particle at this position is zero. If particle is displaced from the mean position.

Periodic Function

If a particle moves along x-axis, its position depends upon time t. We express this fact mathematically by writing

$x = f(t)$ or $x(t)$

There are certain motions that are repeated at equal intervals of time. By this we mean that particle is found at the same position moving in the same direction with the same velocity and acceleration, after each period of time. Let T be the interval of time in which motion is repeated. Then

$x(t) = x(t + T)$

where T is the minimum change in time. The function that repeats itself is known as a periodic function. During the period, its values may remain finite. Such functions are bound functions. Periodic motion of a particle is also bound because it must not go to infinity and return back in one finite period.

ILLUSTRATION 15.1 Find the period of the function.

$y = \sin \omega t + \sin 2\omega t + \sin 3\omega t$

Solution. The given function can be written as

$$y = y_1 + y_2 + y_3$$

Here $y_1 = \sin \omega t$, $T_1 = 2\pi/\omega$

$$y_2 = \sin 2\omega t,\ T_2 = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

and $y_3 = \sin 3\omega t$, $T_3 = 2\pi/3\omega$

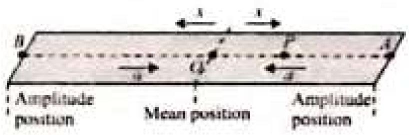
$$T_1 = 2T_2 \quad \text{and } T_1 = 3T_3$$

So, the time period of the given function is T_1 or $2\pi/\omega$.

Because in time $T = 2\pi/\omega$, first function completes one oscillation, the second function two oscillations and the third, three.

SIMPLE HARMONIC MOTION

It is a special case of oscillatory motion in which the acceleration of the vibrating particle (or body), at any position directly as its displacement from a fixed point (which may or may not lie along the line of motion) and is always directed towards that fixed point. Thus, simple harmonic motion (abbreviated as SHM) is a case of variable acceleration; however, the variation takes place in a regular and periodic fashion.



Suppose a particle P performs oscillatory motion between two fixed points A and B. Let O be the mid-point of A and B. If the particle P oscillates about point O in such a way that its acceleration 'a' at any position when its displacement from O

is x can be mathematically expressed as $a \propto x$ and is directed towards O, then its motion will be SHM. Here, the point 'O' is known as mean or stable or equilibrium or neutral position, and in case of 'simple' harmonic motion, the maximum displacement of the particle on either side of the mean position is the same, i.e., $OA = OB$.

Now, taking the motion of the particle to be along the x-axis, SHM can be mathematically expressed as

$$a \propto x \quad \text{or} \quad \vec{a} = (-\omega^2)\vec{x} \qquad \text{(i)}$$

Here, the negative sign stands for the fact that the direction of acceleration is opposite to that of displacement, and ω^2 is a positive constant.

Again, if m is the mass of the oscillating particle (or body), then multiplying both side of Eq. (i) by m, we have

$$m\vec{a} = (-m\omega^2)\vec{x} \quad \text{or} \quad F = (-k)\vec{x} \qquad \text{(ii)}$$

where $m\vec{a} = \vec{F}$ is the force acting on the particle and $k = m\omega^2$ is a positive constant.

Equations (i) and (ii) can equally be used to show that a given motion is SHM.

NOTE:

- The oscillation amplitude of the particle must be very small as compared to its surrounding dimensions (dimensions of bodies with which it can interact).
- During oscillation the acceleration of the particle toward mean position, due to net restoring forces, must be directly proportional to its displacement for mean position.

Equation of Simple Harmonic Motion

The differential equation of simple harmonic motion is given by $d^2x/dt = \omega^2 x$.

The necessary and sufficient condition for a motion to be simple harmonic is that the net restoring force (or torque) must be linear, i.e., $F = ma = -kx$ (where k is a constant). Thus,

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad \text{(iii)}$$

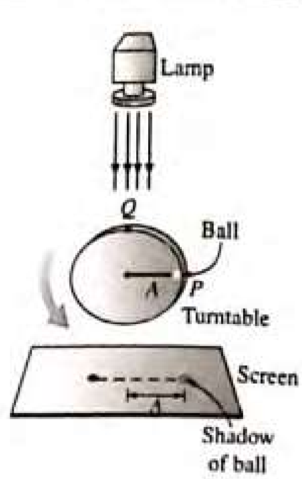
where x is the instantaneous displacement. Multiplying both sides by dx/dt and integrating with respect to t, we get

$$x = A\sin(\omega t + \phi) \qquad \text{(iv)}$$

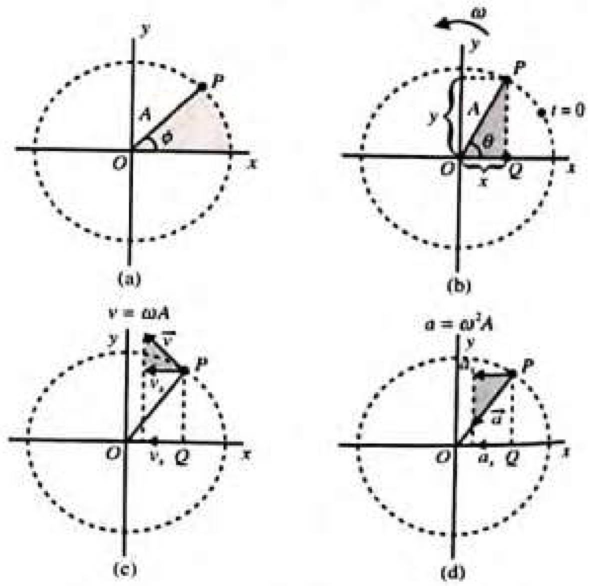
Here ω is called angular frequency and ϕ is called initial phase or epoch constant, whose value depends upon initial conditions.

Comparing Simple Harmonic Motion with Uniform Circular Motion

In this section, we explore this interesting relationship between these two types of motion. Figure is a view of an experimental arrangement that shows this relationship. A ball is attached to the rim of a turntable of radius A, which is illuminated from the side by a lamp. The ball casts a shadow on a screen. As the turntable rotates with constant angular speed, the shadow of the ball moves back and forth in simple harmonic motion.



Consider a particle located at point P on the circumference of a circle of radius A as in Figure (a) with the line OP making an angle ϕ with the x-axis at $t = 0$; we call this circle a reference circle for comparing simple harmonic motion with uniform circular motion, and we choose the position of P at $t = 0$ as our reference position. If the particle moves along the circle with constant angular speed ω until OP makes an angle ϕ with the x-axis as in Figure (b), at some time $t > 0$ the angle between OP and the x-axis is $\theta = \omega t + \phi$. As the particle moves along the circle, the projection of P on the x-axis, labelled as point Q, moves back and forth along the x-axis between the limits $x = \pm a$. Notice that points P and Q always have the same x-coordinate. From the right triangle OPQ, we see that this x-coordinate is



$$x(t) = A\cos(\omega t + \phi) \qquad \text{(i)}$$

This expression is the same as Eq. (i) and shows that the point Q moves with simple harmonic motion along the x-axis. Therefore, simple harmonic motion along a straight line can be represented by the projection of uniform circular motion along a diameter of a reference circle.

If we take projection of point P on y-axis, the y-coordinate is given by

$$y(t) = A\sin(\omega t + \phi) \qquad \text{(ii)}$$

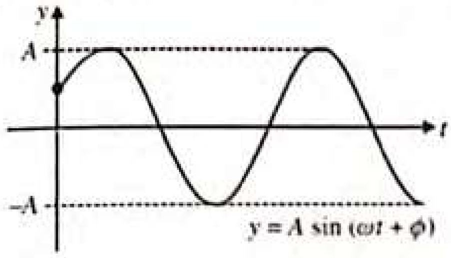
This geometric interpretation shows that the time interval for one complete revolution of point P on the reference circle is equal to the period of motion T for simple harmonic motion between $x = \pm a$. That is, the angular speed ω of P is the same as the angular frequency ω of simple harmonic motion along the x-axis (which is why we use the same symbol).

We may choose any position of the particle as initial position by pressing the stop watch on. It means, from any position of the particle you may start calculating time; for this, we can use $y = A \sin(\omega t + \phi)$ or $y = A\cos(\omega t + \phi)$.

If you start calculating time from extreme position use $y = A\cos\omega t$.

If you start calculating time from mean (stable equilibrium) position then use $y = A\sin\omega t$.

If you choose to start calculation of time when the particle stays in any intermediate position, the variation of displacement 'y' with time can be given as 'sin' or 'cosine' function.



The displaceemnt is always measured from mean position whereas time t can be calculated from any position of the oscillating particle.

If we differentiate Eq. (i) or (ii) twice, we get differential equation of SHM.

Differentiating Eq. (i), we get

$$\frac{dy}{dt} = A\omega\cos(\omega t + \phi) \qquad \text{(iii)}$$

Again differentiating Eq. (iii),

$$\frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t + \phi) = -\omega^2 y\text{, i.e., } \frac{d^2y}{dt^2} + \omega^2 y = 0$$

This is the basic equation of SHM.

ILLUSTRATION 15.2 Identify which of the following functions represent simple harmonic motion.

i. $y = Ae^{i\omega t}$
ii. $y = ae^{-\omega t}$
iii. $y = a\sin^2\omega t$
iv. $y = a\sin\omega t + b\cos\omega t$
v. $y = \sin\omega t + \cos 2\omega t$

Solution.

i. According to given equation in problem, differentiating with respect to time, we get $\frac{dy}{dt} = iA\omega e^{i\omega t}$.

Differentiating again with respect to time, we get

$$\frac{d^2y}{dt^2} = -\omega^2 Ae^{i\omega t} = -\omega^2 y \qquad [\text{as } y = Ae^{i\omega t}]$$

Thus, we have $\frac{d^2y}{dt^2} + \omega^2 y = 0$

This is the basic differential equation of SHM.

ii. Given $y = ae^{-\omega t}$

$$\frac{dy}{dt} = -\omega ae^{-\omega t}$$

and $$\frac{d^2y}{dt^2} = \omega^2 ae^{-\omega t} = \omega^2 y$$

or $$\frac{d^2y}{dt^2} \propto y$$

Hence, the function is not S.H.M.

iii. $y = a\sin^2\omega t$

The function $y = a\sin^2\omega t$ is harmonic.

To become simple harmonic $\frac{d^2y}{dt^2} \propto -y$

Here, $\frac{dy}{dt} = 2a\omega\sin\omega t\cos\omega t$

$$\frac{d^2y}{dt^2} = 2a\omega^2[\cos^2\omega t - \sin^2\omega t]$$

$$= 2a\omega^2[1 - 2\sin^2\omega t] = 2a\omega^2\left[1 - \frac{2y}{a}\right]$$

The function is not simple harmonic.

iv. $y = a\sin\omega t + b\cos\omega t$

The function $y = a\sin\omega t + b\cos\omega t$ is simple harmonic.

Because $\frac{dy}{dt} = \omega a\cos\omega t - \omega b\sin\omega t$

$$\frac{d^2y}{dt^2} = -\omega^2 a\sin\omega t - \omega^2 b\cos\omega t$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -\omega^2 y$$

This is the basic differential equation of SHM.

v. $y = \sin\omega t + \cos 2\omega t$

The function $y = \sin\omega t + \cos 2\omega t$ is not simple harmonic.

Because $\frac{dy}{dt} = \omega\cos\omega t - 2\omega\sin 2\omega t$

$$\frac{d^2y}{dt^2} = -\omega^2\sin\omega t - 4\omega^2\cos 2\omega t$$

$$= -\omega^2[\sin\omega t + \cos 2\omega t]$$

$$\frac{d^2y}{dt^2} \neq -\omega^2 y$$

The function is not simple harmonic.

Velocity and Acceleration in SHM

We can write the equation of motion of a particle performing SHM is

$$y = A\sin(\omega t + \phi) \qquad \text{(i)}$$

Differentiatintg y with respect to 't' given velocity of the particle

$$v = \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$$

From Eq. (i), we can write $\sin(\omega t + \phi) = \frac{y}{A}$ and we know that

$$\cos(\omega t + \phi) = \sqrt{1 - \sin^2(\omega t + \phi)}$$

$$= \sqrt{1 - \frac{y^2}{A^2}} = \frac{1}{A}\sqrt{A^2 - y^2}$$

Substituting the value of $\cos(\omega t + \phi)$ in Eq. (i), we get

$$v = A\omega\left[\frac{1}{A}\sqrt{A^2 - y^2}\right] = \omega\sqrt{A^2 - y^2} \qquad \text{(ii)}$$

Differentiating Eq. (ii) again w.r.t. time,

$$a = \frac{dv}{dt} = -A\omega^2 \sin\omega t = -\omega^2 y$$

NOTE:

For a particle performing SHM

- Velocity of a particle is maximum at mean position and is zero at amplitude positions.
- Acceleration of a particle is zero at mean position and maximum at amplitude positions.
- We have differential equation of SHM as

$$a = -\omega^2 y \Rightarrow \frac{d^2 y}{dt^2} = -\omega^2 y$$

$$v\frac{dv}{dy} = -\omega^2 y \quad \Rightarrow v\,dv = -\omega^2 y\,dy$$

- Integrating both sides $\int v\,dv = -\omega^2 \int y\,dy$

$$\frac{v^2}{2} = -\omega^2\frac{y^2}{2} + \text{constant}$$

$$v^2 + \omega^2 y^2 = \text{constant} \qquad \text{(i)}$$

- the above equation is also an important equation to express SHM.

ILLUSTRATION 15.3 A particle executes SHM with an amplitude 8 cm and a frequency 10 s^{-1}. Assuming the particle to be at a displacement 4 cm initially, in the positive direction, determine its displacement equation and the maximum velocity and acceleration.

Solution. Given $a = 8$ cm and $\omega = 2\pi n = 20\pi$ rad/s

Let the phase constant be ϕ.

The displacement equation can be written as

$x = 8\sin(20\pi t + \phi)$

Given, at $t = 0$; $x = 4$ cm, therefore

$$4 = 8\sin(20\pi(0) + \phi) \quad \Rightarrow \sin(\phi) = \frac{1}{2} \quad \Rightarrow \phi = \frac{\pi}{6}$$

$$\text{Displacement equation } x = 8\sin\left(20\pi t + \frac{\pi}{6}\right)$$

Differentiating the above equation w.r.t. time 't'

$$\frac{dx}{dt} = v = 160\pi\cos\left(20\pi t + \frac{\pi}{6}\right)$$

$\therefore \quad v_{max} = \pm 160\pi$ cm/s $\qquad$ [when $\cos(20\pi t + \pi/6) = \pm 1$]

Differentiating once again w.r.t. time t.

$$\frac{d^2x}{dt^2} = f = -3200\pi^2\sin\left(20\pi t + \frac{\pi}{6}\right)$$

$$f_{max} = \pm 3200\pi^2$$

Phase and Phase Difference in SHM

Phase: We have seen earlier that the displacement, velocity and acceleration of a particle executing SHM vary periodically with the angle $(\omega t + \phi)$ associated with the sine or cosine term. Knowing this angle, we can be sure of its position as well as state of motion as to how and where it is oscillating. This angle $(\omega t + \phi)$ is known as the 'phase' of the oscillating particle. Since the phase is a time dependent factor, it will be more worthwhile to speak in terms of instantaneous phase (i.e., phase at any instant).

Taking the displacement equation as $x = A\sin(\omega t + \phi)$, we have

at $\omega t + \phi = 0$; $\quad x = 0$ (mean position)

at $\omega t + \phi = \pi/2$; $\quad x = A$ (amplitude position)

at $\omega t + \phi = \pi$; $\quad x = 0$ mean position

at $\omega t + \phi = 3\pi/2$; $\quad x = -A \quad$ and so on

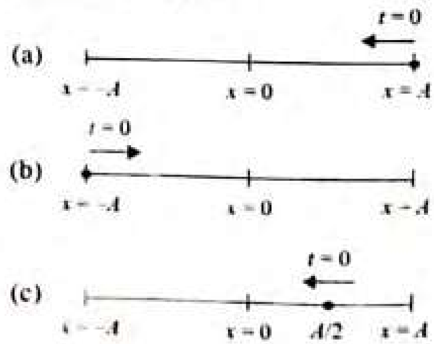
Notice that as time varies indefinitely, phase keeps on changing, 0 to 2π; the reason being $(\omega t + \phi) = (\omega t + \phi) + 2n\pi$, where $n \in I$.

Phase constant (Epoch): The phase of a particle executing SHM, initially (i.e., at the instant when time was reckoned), is known as the initial phase or phase constant or epoch.

Since instantaneous phase $= \omega t + \phi$, so initial phase can be obtained by putting $t = 0$

$\therefore \quad$ Phase constant $= \phi$

ILLUSTRATION 15.4 Write the equation of SHM for the situations shown below:



Solution.

(a) We know general equation of SHM is

$x = A\sin(\omega t + \phi)$

At $t = 0$, $x = +A$

$A = A\sin(\phi) \Rightarrow \sin\phi = 1$; Hence, $\phi = \pi/2$

$$x = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cos\omega t$$

(b) $x = A\sin(\omega t + \phi)$

At $t = 0$, $x = -A$

$-A = A\sin\phi \Rightarrow \sin\phi = -1$; Hence, $\phi = \frac{3\pi}{2}$

$$x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$$

$$x = -A\cos\omega t$$

(c) $x = A\sin(\omega t + \phi)$

At $t = 0$, $x = \frac{A}{2}$

$\frac{A}{2} = A\sin(\phi) \Rightarrow \sin\phi = \frac{1}{2}$. Hence, $\phi = \frac{\pi}{6}, \frac{5\pi}{6}$

The particle is moving towards the mean position and in negative direction, i.e., its velocity is negative. The value of ϕ should be selected such that $\cos\phi$ should be negative.

Velocity, $v = A\omega\cos(\omega t + \phi)$

At $t = 0$, $v = -$ve

$v = A\omega\cos\phi$

Hence $\phi = \frac{5\pi}{6}$

$$x = A\sin\left(\omega t + \frac{5\pi}{6}\right)$$

Phase Difference in Two SHMs

Same Phase SHMs

Two particles execute SHM in such a way that their oscillations are exactly parallel to each other or their phase difference during oscillation is zero, i.e., they are said to be in same phase. We have seen that this happens when both SHMs are started at same time with same angular frequency. This can happen also when the time lag between starting of the two SHMs is T or an integral multiple of time period of SHM. Because if one starts at $t = 0$ and other starts at $t = T$, in this duration the first particle will complete its the first oscillation and is going to start its second oscillations and the second particle will start in synchronization with the first. Hence, the two oscillation will still be parallel or in same phase. The phase difference in the two SHMs will be 2π. Not only this even if the time lag in starting of the two SHMs is $2T, 3T, \ldots, nT$ or the phase difference in the two SHMs is $4\pi, 6\pi, 8\pi, \ldots, 2n\pi$, then also these SHMs can be treated in same phase.

Thus phase difference in two SHMs of same phase is

$$\phi = 2\pi, 4\pi, 6\pi \ldots 2n\pi$$

Opposite Phase SHMs

Two SHMs are said to be in opposite phase when their oscillations are antiparallel. This happens when two SHMs of same angular frequency start with a time lag of $T/2$ and phase difference among the two SHMs is π. By analyzing the situation it can also be stated that the same thing also happens when the time lag in starting of the two SHMs is $3T/2, 5T/2, \ldots, (2n+1)T/2$ or the phase difference between the two is $3\pi, 5\pi, \ldots, (2n+1)\pi$.

Thus phase difference in two SHMs of opposite phase is

$$\phi = \pi, 3\pi\ 5\pi \ldots, (2n+1)\pi$$

Relation between Phase Difference and Time Difference

Let the phase of a particle at time t_1 be ϕ_1, then

$$\phi_1 = \omega t_1 + \phi \quad \text{(i)}$$

Let the phase of the particle at time t_2 changes to ϕ_2, then

$$\phi_2 = \omega t_2 + \phi \quad \text{(ii)}$$

Subtracting Eq. (i) from Eq. (ii), phase difference becomes

$$\phi_2 - \phi_1 = \omega(t_2 - t_1)$$

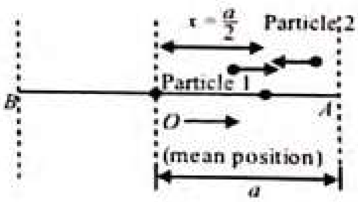
or $$\Delta\phi = \omega\,\Delta t = \frac{2\pi}{T}\Delta t$$

Putting $\Delta t = T$ and $\omega = 2\pi/T$, we have $\Delta\phi = 2\pi$

Thus, a phase difference of 2π is equivalent to a time difference of T. Similarly, a phase difference of π is equivalent to a time difference of $T/2$, and so on.

ILLUSTRATION 15.5 Two particles execute SHM with same frequency and amplitude along the same straight line. They cross each other, at a point midway between the mean and the extreme position. Find the phase difference between them.

Solution. **Method 1**



Let a be the amplitude and ϕ_1 and ϕ_2 their respective phases at any instant, when they cross each other.

Then their displacement equations can be written as

$$x = a\sin(\text{phase})$$

For the two particles we have

$$\frac{a}{2} = a\sin\phi_1 \quad \text{and} \quad \frac{a}{2} = a\sin\phi_2$$

$\Rightarrow$ Taking the least value, $\phi_1 = \frac{\pi}{6}$

and taking the next possible value $\phi_2 = \frac{5\pi}{6}$. Moreover, from $\vec{v} = a\omega\cos(\text{phase})$.

If $\vec{v}_1$ and $\vec{v}_2$ are the respective velocities of the two particles.

The particles cross each other their velocities should be in opposite directions. If we take velocity of the particle 1 positive then velocity of particle should be negative. The velocity of the particle 2 will be negative when it has phase $\phi_2 = 5\pi/6$.

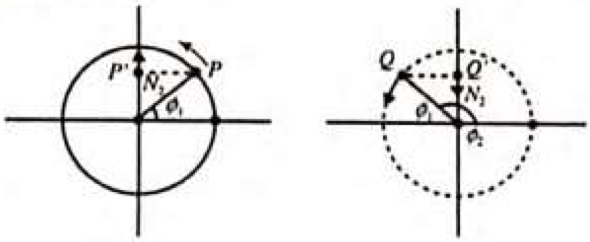
Therefore, the required phase difference $= \phi_2 - \phi_1 = \dfrac{2\pi}{3}$ rad

NOTE: The phase difference can in general be

$$\frac{2\pi}{3} + 2n\pi;\ n \in I^{+}$$

Method 2

Figures (a) and (b) show that two particles P' and Q' in SHM along with their corresponding particles in circular motion. Let P' moves in upward direction when crossing Q' at $A/2$ as shown in Figure (a).



(a) (b)

At this instant, phase of P' is

$$\phi_1 = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \quad \text{(i)}$$

Similarly as shown in Figure (b), particle Q' moves in downward direction (opposite P') at $A/2$, this implies its circular motion particle is in second quadrant thus its phase angle is

$$\phi_2 = \pi - \phi_1 = \pi - \sin^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \quad \text{(ii)}$$

As both are oscillating at same angular frequency, their phase difference remains constant which can be given from Eqs. (i) and (ii), as $\Delta\phi = \phi_2 - \phi_1 = \dfrac{5\pi}{6} - \dfrac{\pi}{6} = \dfrac{2\pi}{3}$

NOTE: In a particular position we have one displacement, one acceleration but two velocities, one is away from mean position other is towards mean position. It means, we have two different phases at a position. Hence the phase is not only decided by the position but it is also decided by the velocity of the particle.

Energy of a Body in SHM

The total energy of a harmonic oscillator consists of two parts, potential energy (PE) and kinetic energy (KE). The former being due to its displacement from the mean position and later due to its velocity. Since the position and velocity of the harmonic oscillator are continuously changing, PE and KE also change but their sum, i.e., the total energy (TE) must have the same value at all times.

Simple harmonic motion is defined by the equation

Let $U(x)$be the potential energy of the system when the displacement is x. The change in potential energy corresponding to a force is negative of the work done by this force,

$$U(x) - U(0) = -W = \frac{1}{2}kx^2$$

Let us choose the potential energy to be zero when the particle is at the centre of oscillation $x = 0$. Then

$$U(0) = 0 \text{ and } U(x) = \frac{1}{2}kx^2$$

This expression for potential energy is same as that for a spring and has been used so far in this chapter.

As $\omega = \sqrt{\dfrac{k}{m}},\quad k = m\omega^2$

We can write

$$U(x) = \frac{1}{2}m\omega^2 x^2 \quad \text{(i)}$$

The displacement and the velocity of a particle executing a simple harmonic motion are given by $x = A\sin(\omega t)$

and $v = A\omega\cos(\omega t)$

The potential energy at time t is, therefore,

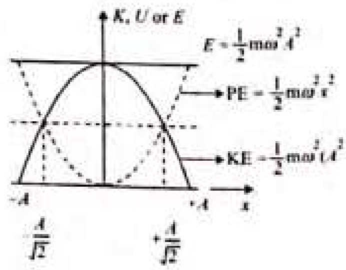
$$U = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t)$$

$$= \frac{m\omega^2 A^2}{4}(1 - \cos 2\omega t)$$

and the kinetic energy at time t is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t)$$

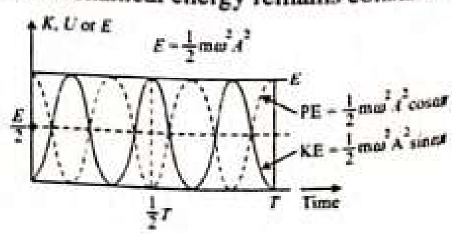
$$= \frac{m\omega^2 A^2}{4}(1 + \sin 2\omega t)$$



The total mechanical energy at time t is

$$E = U + K = \frac{1}{2}m\omega^2 A^2\left[\sin^2(\omega t) + \cos^2(\omega t)\right] = \frac{1}{2}m\omega^2 A^2 \quad \text{(ii)}$$

We see that the total mechanical energy at time t is independent of t. Thus, the mechanical energy remains constant as expected.



NOTE: We can write total mechanical energy of a particle/body performing SHM as

KE + PE = constant

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant} \Rightarrow v^2 + \frac{k}{m}x^2 = \text{constant} \quad \text{(i)}$$

Here k is the equivalent force constant of oscillation ($k = m\omega^2$)

We have differential equation of SHM as

$$\frac{d^2x}{dt^2} = -\omega^2 x \text{ or } v^2 = \omega^2 x^2 = \text{constant} \quad \text{(ii)}$$

If we compare Eqs. (i) and (ii), both equations are same.

ILLUSTRATION 15.6 A particle of mass 0.2 kg executes simple harmonic motion along a path of length 0.2 m at the rate of 600 oscillations per minute. Assume at $t = 0$ the particle starts SHM in positive direction. Find the kinetic and potential energies in joules when the displacement is $x = A/2$, where A stands for the amplitude.

Solution. Given that f = 600 oscillations/min, $m = 0.2$ kg

$$A = \text{Amplitude} = 1/2 \times \text{Length of path} = 0.1 \text{ m} = \frac{1}{10}\text{m}$$

As $\quad f = \dfrac{1}{T} = \dfrac{600}{60} = 10 \text{ Hz} \Rightarrow \omega = 2\pi f = 20\pi$ rad/s

The magnitude of velocity of a particle performing SHM is given as $v = \omega\sqrt{A^2 - x^2}$

At $x = A/2 = \dfrac{1}{20}$ m, the velocity at this position

$$v_{x=A/2} = 20\pi\sqrt{\left(\frac{1}{10}\right)^2 - \left(\frac{1}{20}\right)^2} = \sqrt{3}\pi \text{ m/s}$$

Hence, kinetic energy at this position

$$k_{x=A/2} = \frac{1}{2}mv^2_{x=A/2} = \frac{1}{2}(0.2)(\sqrt{3}\pi)^2 = \frac{3\pi^2}{10}\text{ J}$$

We can write potential energy at any position x as

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(m\omega^2)x^2$$

Hence,

$$U_{x=A/2} = \frac{1}{2}(0.2)(20\pi)^2\left(\frac{1}{20}\right)^2 = \frac{\pi^2}{10}\text{ J}$$

ILLUSTRATION 15.7 A particle of mass m is located in a unidimensional potential field where potential energy of the particle depends on the coordinates x as: $U(x) = U_0(1 - \cos Ax)$; U_0 and A are constants.

Find the period of small oscillations that the particle performs about the equilibrium position.

Solution. The equation of potential energy of the particle as a function of position is given as

$$U_x = U_0(1 - \cos Ax) \quad \text{(i)}$$

Hence, force acting on the particle is given as

$$F_x = -\frac{dU_x}{dx} = U_0 A \sin Ax \quad \text{(ii)}$$

So, for the equilibrium of the body the force given by the above equation must be zero and d^2U/dx^2 should be negative. So, $F = 0$ situations are given by $0 = -U_0 A \sin Ax$

$$x = 0, \quad x = \frac{\pi}{A} \quad \text{(iii)}$$

Since $\quad \dfrac{d^2U}{dx^2} = -U_0A^2\cos Ax$

So, at $x = 0$, $\dfrac{d^2U}{dx^2} = -U_0A^2\cos 0° = -U_0A^2$

i.e., negative

So, $x = 0$ satisfies the condition for stable equilibrium situation. Now, if the particle is given a very small displacement from $x = 0$ position, then the small force arising due to this small displacement is given as: $\Delta F = -U_0 A \sin(A\Delta x)$

$$\Delta F = -U_0A^2\Delta x \quad (\because \sin(A\Delta x) \approx A\Delta x)$$

If the acceleration of the particle is a, then $ma = -U_0A^2\Delta x$

$$a = -\left(\frac{U_0A^2}{m}\right)\Delta x \quad \text{(iv)}$$

$$\Rightarrow \quad a \propto -\Delta x$$

So, from the above relation it is clear that the acceleration is proportional and opposite to the displacement. So the particle performs SHM. But for any SHM,

$$a = -\omega^2\Delta x \quad \text{(v)}$$

So, from Eqs. (iv) and (v), we get

$$\omega^2 = \frac{U_0A^2}{m} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{U_0A^2}}$$

Superposition of Two SHMs

In same direction and of same frequency

Let two particles are preforming SHM with same angular frequency and in same direction.

$$x_1 = A_1 \sin \omega t$$
$$x_2 = A_2 \sin(\omega t + \phi)$$

Then resultant displacement

$$x = x_1 + x_2$$
$$= A_1 \sin \omega t + A_2 \sin(\omega t + \phi) = A \sin(\omega t + \phi')$$

where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

and $\quad \phi' = \tan^{-1}\left[\dfrac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right]$

If $\phi = 0$, both SHMs are in phase and $A = A_1 + A_2$

If $\phi = \pi$, both SHMs are out of phase and $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHMs of this case can also be found by phasor diagram.

In same direction but are of different frequencies

Let in this case angular frequencies be ω_1 and ω_2.

$$x_1 = A_1 \sin \omega_1 t \Rightarrow x_2 = A_2 \sin \omega_2 t$$

then resultant displacement

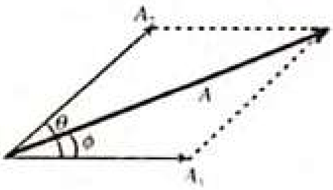
$$x = x_1 + x_2 = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$$

This resultant motion is not SHM.

Superposition of SHMs Along the Same Direction (Using Phasor Diagram)

If two or more SHMs are along the same line, their resultant can be obtained by vector addition by making a phasor diagram.

1. Amplitude of SHM is taken as length (magnitude) of vector.
2. Phase difference between the vectors is taken as the angle between these vectors. The magnitude of resultant of vector gives resultant amplitude of SHM and angle of resultant vector gives phase constant of resultant SHM.



For example, $x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin (\omega t + \theta)$

If equation of resultant SHM is taken as $x = A \sin (\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\theta}$$

$$\tan\phi = \frac{A_2 \sin\theta}{A_1 + A_2 \cos\theta}$$

Illustration 15.8 If the displacement of a moving point at any time is given by an equation of the form $y(t) = a\cos\omega t + b\sin\omega t$, show that the motion is simple harmonic. If $a = 3$m, $b = 4$m and $w = 2$; determine the period, amplitude, maximum velocity and maximum acceleration.

Solution. The particle is moving along the y-axis.

$$y(t) = a \cos \omega t + b \sin \omega t$$

$$y = \sqrt{a^2 + b^2}\sin(\omega t + \phi_0)$$

where $\tan\phi_0 = a/b \Rightarrow y = \sqrt{a^2+b^2}\sin(\omega t + \tan^{-1} a/b)$

Comparing with $y = a\sin(\omega t + \phi)$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi \text{ s}$$

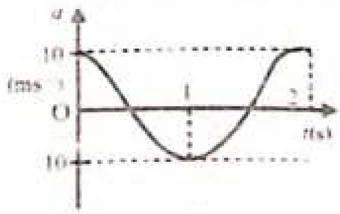
$$A = \sqrt{a^2+b^2} = \sqrt{3^2+4^2} = 5 \text{ m}$$

$$v_{max} = \omega A = 2 \times 5 = 10 \text{ m/s}$$

$$\Rightarrow \quad a_{max} = \omega^2 A = 4 \times 5 = 20 \text{ m/s}^2$$

Concept Application Exercise 15.1

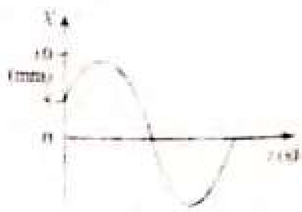
1. The acceleration versus time graph of a particle executing SHM is shown in the following figure



(A)

   i. Plot the displacement versus time graph
   ii. The frequency of oscillation is ______
   iii. The displacement amplitude is ______
   iv. At $t = 0$, the velocity of the particle is ______
   v. The kinetic energy of the particle is maximum at $t =$ ______ and $t =$ ______
   vi. The potential energy is maximum at $t =$ ______ $t =$ ______ and $t =$ ______

2. The equation of motion of a particle started at $t = 0$ is given by $x = 5 \sin (20t + \pi/3)$ where x is in centimetres and t is in second. When does the particle
   (a) first come to rest ______
   (b) first have zero acceleration ______
   (c) first have maximum speed ______
3. A particle in SHM has a period of 4 s. It takes time t_1 to start from mean position and reach half the amplitude. In another case it takes a time t_2 to start from extreme position and reach half the amplitude. Find the ratio t_1/t_2.
4. A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.
5. Figure shows the displacement–time graph of a particle executing SHM. If the time period of oscillation is 2 s, find the equation of motion.



6. A body executing SHM has its velocity 10 cm/sec and 7 cm/sec when its displacement from the mean position are 3 cm and 4 cm, respectively. Calculate the length of the path.

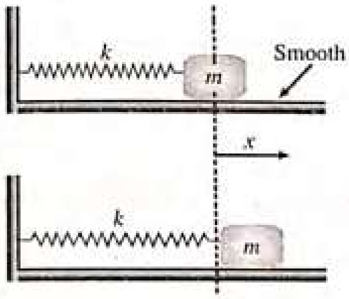
SPRING–MASS SYSTEM

Case 1: Spring–mass system oscillating on a smooth horizontal surface

Let us find out the time period of a spring–mass system oscillating on a smooth horizontal surface as shown in the figure. The SHM will be about relaxed position of spring.

At the equilibrium position the spring is relaxed. When the block is displaced through a distance x towards right, it experiences a net restoring force $F = -kx$ towards left.

The negative sign shows that the restoring force is always opposite to the displacement. That is, when x is positive, F is negative, the force is directed to the left. When x is negative, F is positive, the force always tends to restore the block to its equilibrium position $x = 0$.



$$F = -kx$$

Applying Newton's second law,

$$F = m\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Comparing the above equation with $a = d^2x/dt^2 = -\omega^2 x$, we get

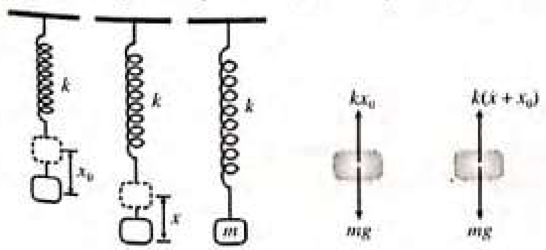
$$\omega^2 = \frac{k}{m} \text{ or } T = 2\pi\sqrt{\frac{m}{k}}$$

NOTE: Time period is independent of the amplitude. For a given spring constant, the period increases with the mass of the block that means more massive block oscillates more slowly. For a given block, the period decreases as k increases. A stiffer spring produces quicker oscillations.

Case 2: Spring–mass system oscillating in a vertical plane

A block of mass m attached to a spring of stiffness k oscillates in a vertical plane. Let us find the period of oscillation of a vertical spring–mass system.

Let x_0 be the deformation in the spring in equilibrium. Then $kx_0 = mg$. When the block is further displaced by x, the net restoring force is given by $F = -[k(x + x_0) - mg]$



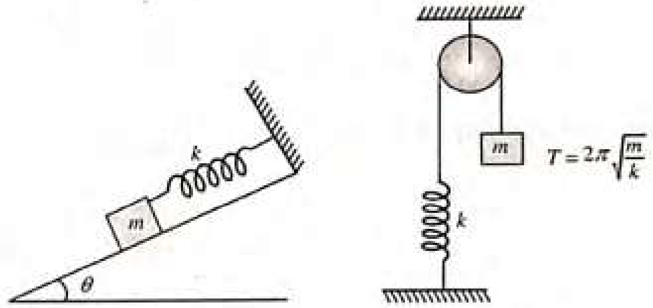
Using second law of motion,

$$m\frac{d^2x}{dt^2} = -kx \Rightarrow \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

$$\text{Thus, } \omega^2 = \frac{k}{m} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

NOTE: Gravity does not influence the time period of the spring–mass system; it merely changes the equilibrium position.

As we have seen that the time period of block–spring system depends only on the mass of the block and force constant of the spring, so the time period in the following cases is

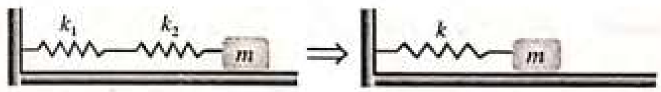


(a) (b)

Series and Parallel Combination of Springs

(a) **Series combination of springs:** When two springs are joined in series, the equivalent stiffness of the combination may be obtained as

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$



(b) **Parallel combination of springs:** When two springs are joined in parallel, the equivalent stiffness of the combination is given by

$$k = k_1 + k_2$$

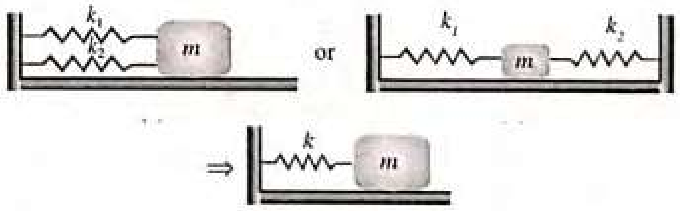
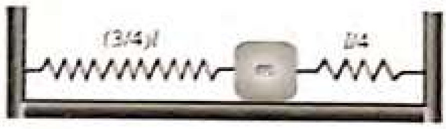


ILLUSTRATION 15.9 A spring of stiffness constant k and natural length l is cut into two parts of length $3l/4$ and $l/4$, respectively, and an arrangement is made as shown in figure. If the mass is slightly displaced, find the time period of oscillation.

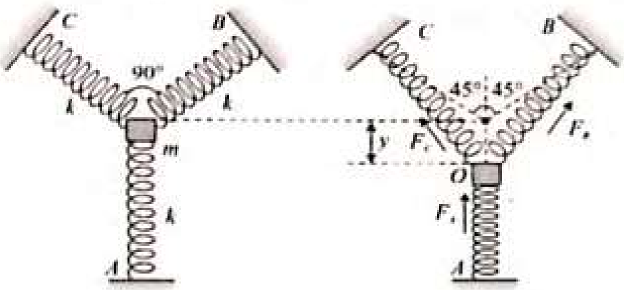


Solution. The stiffness of a spring is inversely proportional to its length. Therefore the stiffness of each part is

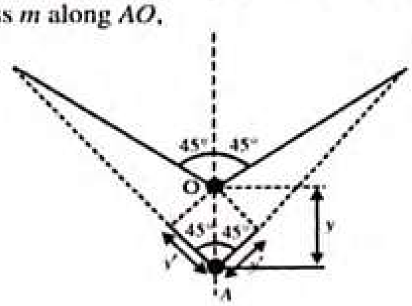
$$k_1 = \frac{4}{3}k \quad \text{and} \quad k_2 = 4k$$

Time period, $T = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}} = 2\pi\sqrt{\dfrac{3m}{16k}} = \dfrac{\pi}{2}\sqrt{\dfrac{3m}{k}}$

Illustration 15.10 A particle of mass m placed on smooth horizontal surface is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass m is pushed slightly against spring A and released, find the time period of oscillations.



Solution. When the particle of mass m at O is pushed by y in the direction of A, spring A will be compressed by y while B and C will be stretched by $y' = y\cos 45°$; so the total restoring force on mass m along AO.



$$RF = F_A + F_B\cos 45° + F_C\cos 45°$$

$$= ky + 2(ky')\cos 45°$$

$$= ky + 2k(y\cos 45°)\cos 45°$$

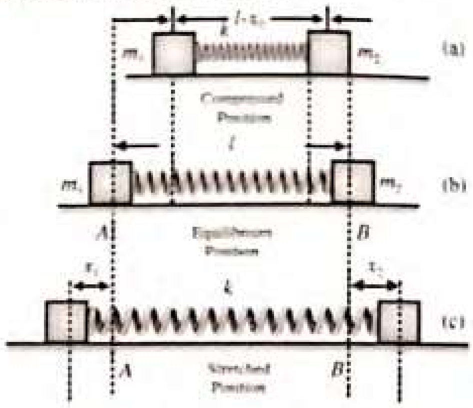
$$F = (2k)y \Rightarrow ma = -(2k)y$$

Hence $a = -\left(\dfrac{2k}{m}\right)y$ as compared with $a = -\omega^2 y$

We get $\omega^2 = \dfrac{2k}{m}$ which gives $T = 2\pi\sqrt{\dfrac{m}{2k}}$

OSCILLATION OF A TWO-PARTICLE SYSTEM TWO-BODY PROBLEM

Two blocks of masses m_1 and m_2 are connected with a spring of natural length l and spring constant k. The system is lying on a frictionless horizontal surface. Initially spring is compressed by a distance x_0 as shown in figure.



The two blocks will perform SHM about their equilibrium position. We will discuss in this section about the oscillation of the system. If we release the blocks.

(a) **Time period of the blocks:** Here both the blocks will be in equilibrium at the same time when spring is in its natural length. Let A and B be equilibrium positions of blocks m_1 and m_2 as shown in figure.

Let at any time during oscillations, blocks are at a distance of x_1 and x_2 from their equilibrium positions (A and B) [Figure (c)]

As no external force is acting on the spring–block system, the displacement of centre of mass of the system, $\Delta r_{CM} = 0$

$\therefore \quad (m_1 + m_2)\Delta x_{CM} = m_1 x_1 - m_2 x_2 \Rightarrow m_1 x_1 = m_2 x_2$

For first particle, force equation can be written as

$$k(x_1 + x_2) = m_1\frac{d^2x_1}{dt^2} \quad \text{or,} \quad k\left(x_1\frac{m_1}{m_2} + x_1\right) = m_1 a_1$$

or, $a_1 = \dfrac{k(m_1 + m_2)}{m_1 m_2}x_1 \qquad \because \omega^2 = \dfrac{k(m_1 + m_2)}{m_1 m_2}$

Using vector we can write $\vec{a}_1 = -\dfrac{k(m_1 + m_2)}{m_1 m_2}\vec{x}_1$

'–' sign is included as $\vec{a}_1$ and $\vec{x}_1$ are opposite to each other.

Hence, $T = 2\pi\sqrt{\dfrac{m_1 m_2}{k(m_1 + m_2)}} = 2\pi\sqrt{\dfrac{\mu}{K}}$

where $\mu = m_1 m_2/(m_1 + m_2)$ which is known as reduced mass.

Similarly time period of 2nd particle can be found. Both will have the same time period.

(b) **Amplitude of the particles:** Let the amplitude of blocks be A_1 and A_2.

$m_1 A_1 = m_2 A_2$

By energy conservation, $\frac{1}{2}k(A_1+A_2)^2 = \frac{1}{2}kx_0^2$

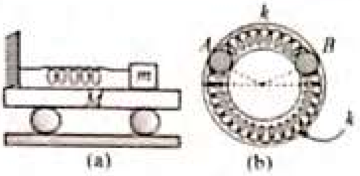
or $A_1 + A_2 = x_0$

or $A_1 + \frac{m_1}{m_2}aA_1 = x_0$

or $A_1 = \frac{m_2 x_0}{m_1 + m_2}$

Similarly, $A_2 = \frac{m_1 x_0}{m_1 + m_2}$

ILLUSTRATION 15.11 Figures (a) and (b) represent spring-block system. If m is displaced slightly, find the time period of oscillation of the system.



(a) (b)

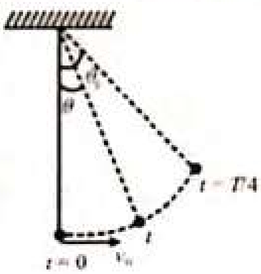
Solution. Both the cases are as follows:

Case (a) Reduced mass of the system	Case (b) Reduced mass of the system
$\mu = \left(\frac{mM}{m+m}\right)$ $\therefore T = 2\pi\sqrt{\frac{\mu}{k}}$	$\mu = \frac{mm}{m+m} = \frac{m}{2}$ and $k_e = k + k = 2k$ $\therefore T = 2\pi\sqrt{\frac{\mu}{k_e}}$

ANALYSIS OF ANGULAR SHM

In case of a simple pendulum a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support.

When the particle is at its extreme position, its angular displacement θ_0 can be regarded as the angular amplitude of oscillations. As the displacement of bob from its mean position is given as



$$x = A\sin(\omega t + \alpha) \quad \text{[general equation of SHM]} \qquad \text{(i)}$$

If θ and θ_0 are angular displacement and angular amplitude of bob, respectively, we have $\theta = \frac{x}{l}$ and $\theta_0 = \frac{A}{l}$

Thus, general equation of SHM of bob in angular form can be given by substituting the values of x and in Eq. (i) as

$$\theta = \theta_0 \sin(\omega t + \phi_0) \qquad \text{(ii)}$$

Using the above equation we can find the angular velocity of the body which in angular SHM is

$$\dot{\theta} = \frac{d\theta}{dt} = \theta_0\omega\cos(\omega t + \phi_0) \qquad \text{(iii)}$$

$$= \omega\sqrt{\theta_0^2 - \theta^2} \qquad \text{(iv)}$$

NOTE: Here we represent angular velocity $\frac{d\theta}{dt}$ by $\dot{\theta}$, not v, as the notation ω is already being used for angular frequency of body in SHM.

Similarly angular acceleration of body is given as

$$\alpha = \frac{d^2\theta}{dt^2} = -\theta_0\omega^2\sin(\omega t + \phi_0) \qquad \text{(v)}$$

Thus restoring torque on body is given as

$$\tau_R = -I\alpha = -I\omega^2\theta \qquad \text{(vi)}$$

Thus we can state that in angular SHM, angular acceleration of the body and the restoring torque on the body are directly proportional to the angular displacement of body from its mean position and are directed toward mean position. Similarly basic differential equation for angular SHM can be written as

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \qquad \text{(vii)}$$

In terms of angular velocity and angular displacement we can write above equation as $(\dot{\theta})^2 + \omega^2(\theta)^2 = \text{constant}$

$\dot{\theta}$ = angular velocity

θ = angular displacement

OSCILLATION OF SIMPLE PENDULUM

As Angular SHM

Length of the simple pendulum is the distance between the point of suspension and the centre of mass of the suspended mass.

Consider the bob when string deflects through a small angle θ.

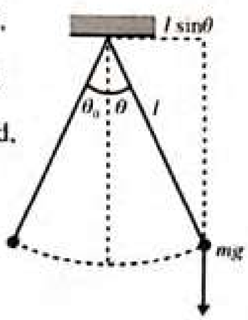
Forces acting on the bob are tension T in the string and weight mg of the bob.

Torque on the bob about point O is

$$\tau = \tau_{mg} + \tau_T = mgl\sin\theta + 0$$

$$\tau = mgl\theta \quad (\text{as } \theta \text{ is very small}) \qquad \text{(i)}$$

Since MI of the bob about point O is $I = ml^2$,



$$\tau = ml^2\frac{d^2\theta}{dt^2} \qquad \text{(ii)}$$

As torque τ and θ are oppositely directed, From Eqs. (i) and (ii), we get

$$ml^2\frac{d^2\theta}{dt^2} = -mgl\theta$$

$$\frac{d^2\theta}{dt^2} = -(g/l)\theta$$

Comparing with the equation $d^2x/dt^2 = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{g}{l}}$$

Since $T = 2\pi/\omega \quad \Rightarrow \quad T = 2\pi\sqrt{\dfrac{l}{g}}$

Equivalent force constant (k_{eq}) for simple pendulum: Finally substituting $T = mg$ from Eq. (ii) in Eq. (i), we have

$$\vec{F}_{net} = -\left(\frac{T}{l}\right)x = \frac{-mg}{l}\vec{x}$$

Frequency and time period: Comparing the above equation with $F_{net} = -kx$, the effective spring constant is $k_{eff} = mg/l$.

As we know $k_{eq} = m\omega^2$

This gives $\quad \omega = \sqrt{\dfrac{k_{eff}}{m}} = \sqrt{\dfrac{g}{l}}$

Then the time period of the simple pendulum is $T = 2\pi/\omega$, where $\omega = \sqrt{g/l}$

This gives $T = 2\pi\sqrt{\dfrac{l}{g}}$

NOTE:

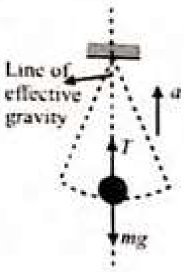
- In simple pendulum, we can find the value of equivalent force constant of oscillation $k_{eq} = T/l$, where T is the tension in the string at equilibrium position of the bob and l is the length of the string.
- Even in case of simple pendulum in accelerated frame, we can use the same approach for calculating the equivalent force of oscillation. In this case, T is the tension in the string at equilibrium position with respect to accelerated frame.

Time Period of Simple Pendulum in Accelerating Reference Frame

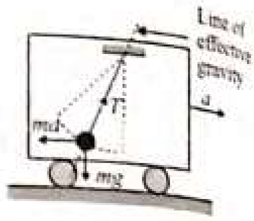
Time period of simple pendulum when point of suspension is accelerating

1. **Acceleration up or retardation down**
   From free body diagram of bob
   $T - mg = ma$, i.e., $T = m(g + a)$
   $$k_{eq} = \frac{T}{l} = \frac{m(g+a)}{l}$$
   Hence $\omega = \sqrt{\dfrac{k_{eq}}{m}} \quad \Rightarrow T = 2\pi\sqrt{\dfrac{l}{g_{eff}}} = 2\pi\sqrt{\dfrac{l}{(g+a)}}$

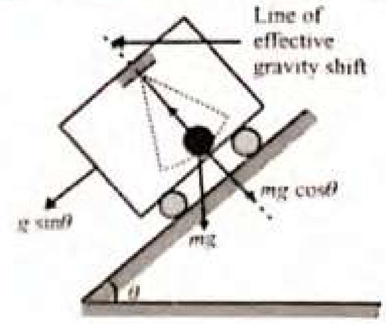
2. **Acceleration down or retardation up**
   $$k_{eq} = \frac{T}{l} = \frac{m(g-a)}{l}$$
   $$\omega = \sqrt{\frac{k_{eq}}{m}}$$
   $$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{(g-a)}}$$



3. **Pendulum is accelerating horizontally**
   $$k_q = \frac{T}{l} = \frac{m\sqrt{a^2+g^2}}{l}$$
   $$\omega = \sqrt{\frac{k_q}{m}}$$
   $$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{g^2+a^2}}$$



4. **Pendulum accelerating down an incline**

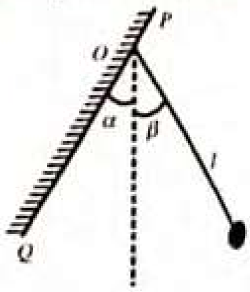


$$T = m\sqrt{g^2 + (g\sin\theta)^2 + 2g(g\sin\theta)\cos(90^\circ+\theta)}$$

$$= mg\cos\theta; \quad \Rightarrow T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{g\cos\theta}}$$

$$k_{eq} = \frac{T}{l} = \frac{mg\cos\theta}{l}$$

ILLUSTRATION 15.12 A ball is suspended by a thread of length l at the point O on an incline wall as shown. The inclination of the wall with the vertical is (α) the thread is displaced through a small angle α away from the vertical and (b) the ball is released. Find the period of oscillation of pendulum. Consider both cases.



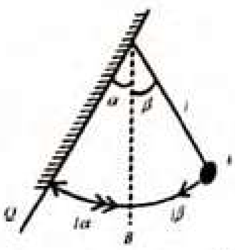
(a) $\alpha > \beta$ (b) $\alpha < \beta$

Assume that any impact between the wall and the ball is elastic.

Solution.

(a) If $\alpha > \beta$, the ball does not collide with the wall and it performs full oscillations like simple pendulum. Thus,

Period $= 2\pi\sqrt{\dfrac{l}{g}}$



(b) If $\alpha < \beta$, the ball collides with the wall and rebounds with same speed; the motion of ball from A to Q is one part of a simple pendulum time period of ball $= 2\,(t_{AQ})$.

Consider A as the starting point ($t = 0$), equation of motion is $x(t) = A\cos\omega t$.

$x(t) = l\beta\cos\omega t$, because amplitude $= A = l\beta$.

Time from A to Q is the time t when x becomes $-l\alpha$.

$$\Rightarrow -l\alpha = l\beta\cos\omega t \quad \Rightarrow t = t_{AQ} = 1/\omega\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$$

The return path from Q to A will involve the same time interval.

Hence time period of ball $= 2t_{AQ}$

$$= \frac{2}{\omega}\cos^{-1}\left(-\frac{\alpha}{\beta}\right) = 2\sqrt{\frac{l}{g}}\cos^{-1}\left(\frac{-\alpha}{\beta}\right)$$

$$= 2\pi\sqrt{\frac{l}{g}} - 2\sqrt{\frac{l}{g}}\cos^{-1}\left(\frac{\alpha}{\beta}\right)$$

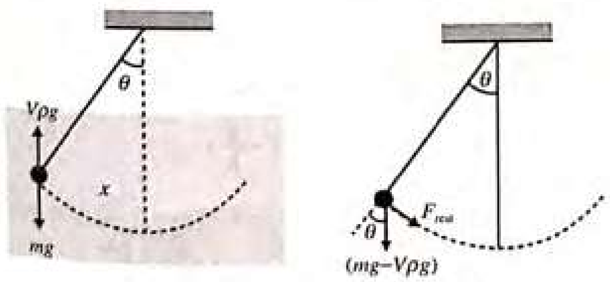
Simple Pendulum in a Liquid

If we immerse a simple pendulum in a liquid, the bob of the pendulum will experience buoyant force in upward direction in addition to the other forces such as gravity and tension. To find the k_{eff}, we need to find the net force on the bob near (or at) the equilibrium position. Dividing F_{net} by the small displacement x of the bob from the equilibrium position, we can find $k_{eff}\left(= \frac{F_{net}}{x} = \frac{T}{l}\right)$. After finding k_{eff}, substitute k_{eff} in the formula $\omega = \sqrt{k_{eff}/m}$ to find frequency (or time period). We will explain the above idea through the following illustration.

ILLUSTRATION 15.13 Derive an expression for the angular frequency of small oscillation of the bob of a simple pendulum when it is immersed in a liquid of density θ. Assume the density of the bob as ρ and length of the string as l.

***Solution.* Method 1**

The bob of the pendulum experiences buoyant force due to liquid $= V\rho g$, in addition to gravitational force. Thus net force on the bob $= (mg - V\rho g)$.



(a) (b)

For small displacement x of the bob, restoring force

$$F_{rest} = (mg - V\rho g)\sin\theta = -(mg - V\rho g)\frac{x}{l}$$

and acceleration $= -\left(g - \frac{V\rho g}{m}\right)\frac{x}{l}$

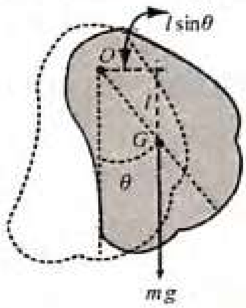
On comparing with standard equation of SHM, $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{\left(g - \frac{V\rho g}{m}\right)}{l}} = \sqrt{\frac{g}{l}\left(1 - \frac{\rho}{\sigma}\right)}$$

and $$T = 2\pi\sqrt{\frac{l}{g\left(1 - \frac{\rho}{\sigma}\right)}}$$

PHYSICAL PENDULUM (COMPOUND PENDULUM)

Any rigid body suspended from a fixed support constitutes a physical pendulum. Consider the situation when a body is displaced through a small angle θ. Torque on the body about O is given by



$$\tau = mgl\sin\theta \qquad \text{(i)}$$

where l is the distance between point of suspension and centre of mass of the body.

If I is the MI of the body about O, then $\tau = I\alpha$ (ii)

From Eqs. (i) and (ii), we get

$$I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$$

as θ and $d^2\theta/dt^2$ are oppositely directed.

$$\Rightarrow \frac{d^2\theta}{dt^2} = \frac{-mgl}{I}\theta, \text{ since } \theta \text{ is very small.}$$

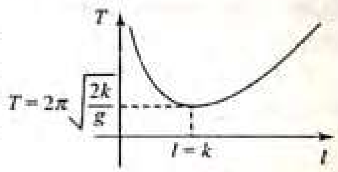
Comparing with the equation $d^2\theta/dt^2 = -\omega^2\theta$, we get

$$\omega = \sqrt{\frac{mgl}{I}} \qquad \Rightarrow T = 2\pi\sqrt{\frac{I}{mgl}}$$

NOTE:

Time period, $T = 2\pi\sqrt{\frac{I}{mgl}}$; $I = I_{CM} + ml^2$

where I_{CM} is moment of inertia relative to the axis which passes from the center of mass and parallel to the axis of oscillation.



Graph of T vs l

$$T = 2\pi\sqrt{\frac{I_{CM} + ml^2}{mgl}},$$

where $I_{CM} = mk^2$

k is gyration radius (about axis passing from centre of mass)

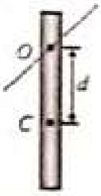
$$T = 2\pi\sqrt{\frac{mk^2 + ml^2}{mgl}} = 2\pi\sqrt{\frac{k^2 + l^2}{lg}} = 2\pi\sqrt{\frac{L_{eq}}{g}}$$

$L_{eq} = \frac{k^2}{l} + l$ = equivalent length of simple pendulum;

T is minimum when $l = k$.

$$T_{min} = 2\pi\sqrt{\frac{2k}{g}}$$

Illustration 15.14 What is the period of a pendulum formed by pivoting a metre stick so that it is free to rotate about a horizontal axis passing through the 75 cm mark?



Solution. Let m be the mass and l be the length of the stick.

$l = 100$ cm

The distance of the point of suspension from centre of gravity is $d = 25$ cm

Moment of inertia about a horizontal axis through O is

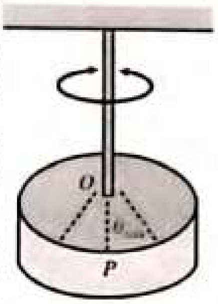
$$I = I_c + md^2 \quad \Rightarrow I = \frac{ml^2}{12} + md^2$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{ml^2}{12} + md^2}{mgd}}$$

$$T = 2\pi\sqrt{\frac{l^2 + 12d^2}{12\,gd}} = 2\pi\sqrt{\frac{l^2 + 12(0.25)^2}{12 \times 9.8 \times 0.25}} = 1.53 \text{ s}$$

TORSIONAL PENDULUM

Figure shows a rigid object suspended by a wire attached at the top to a fixed support. When the object is twisted through some angle θ, the twisted wire exerts on the object a restoring torque that is proportional to the angular position. That is, $\tau = -k\theta$, where k (Greek letter kappa) is called the torsion constant of the support wire. The value of k can be obtained by applying a known torque to twist the wire through a measureable angle θ. Applying Newton's second law for rotational motion, we find that

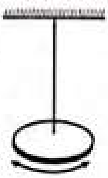


$$\tau = -k\theta = I\frac{d^2\theta}{dt^2} \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{k}{I}\theta$$

Again, this result is the equation of motion for a simple harmonic oscillator with $\omega = \sqrt{k/I}$ and a period $T = 2\pi\sqrt{I/k}$.

This system is called a torsional pendulum. There is no small angle restriction in this situation as long as the elastic limit of the wire is not exceeded.

Illustration 15.15 A uniform disc of radius 5.0 cm and mass 200 g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire.



Solution. The situation is shown in figure. The moment of inertia of the disc about the wire is

$$I = \frac{mr^2}{2} = \frac{(0.200 \text{ kg})(5.0 \times 10^{-2} \text{ m})^2}{2}$$

$$= 2.5 \times 10^{-4} \text{ kg-m}^2$$

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{k}} \quad \text{or} \quad k = \frac{4\pi^2 I}{T^2}$$

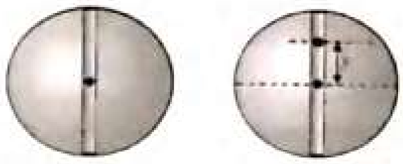
$$= \frac{4\pi^2(2.5 \times 10^{-4} \text{ kg-m}^2)}{(0.20 \text{ s})^2} = 0.25 \text{ kg-m}^2/\text{s}^2$$

MOTION OF A BALL IN A TUNNEL THROUGH THE EARTH

Case I: If the tunnel is along a diameter and the ball is released from the surface. If the ball at any time is at a distance y from the centre of the earth, the restoring force will act on the ball due to gravitation between ball and earth.

Acceleration of the particle at the distance y from the centre of earth.

$$a = -GMy/R^3 = -(GM)y/R^3$$



(a) (b)

Furthermore as $g = GM/R^2$

$$a = \frac{-(gR^2)y}{R^3} = -\frac{g}{R}y$$

Comparing with $a = -\omega^2 y$

$$\omega^2 = \frac{g}{R} \quad \text{or} \quad \omega = \sqrt{\frac{g}{R}} \quad \Rightarrow T = 2\pi\sqrt{\left(\frac{R}{g}\right)} = 84.6 \text{ min}$$

Case II: If the tunnel is along a chord and ball is released from the surface. If the ball at any time is at a distance x from the centre of tunnel, acceleration of the particle at the distance y from the centre of earth

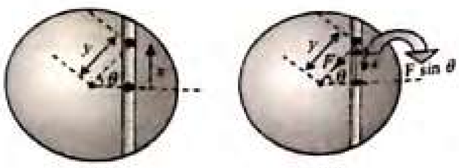
$$a = -GMy/R^3 = -(GM)y/R^3$$

Furthermore, as $g = GM/R^2$

$$a = -(gR^2)y/R^3$$

This acceleration will be towards the centre of earth. The component of acceleration towards the centre of earth

$$a' = a\sin\theta = \left(-\frac{g}{R}y\right)\left(\frac{x}{y}\right) = -\frac{g}{R}x$$



(a) (b)

Comparing with $a = -\omega^2 y$

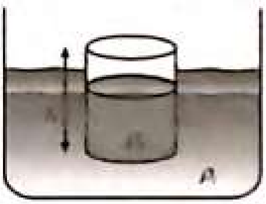
$$\omega^2 = \frac{g}{R} \Rightarrow \omega = \sqrt{\frac{g}{R}} \Rightarrow T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ min}$$

Note:
Time period of oscillation is same in both the cases wheather the tunnel is along a distance or along the chord.

OSCILLATION OF A FLOATING BODY IN A LIQUID

A floating body is in a stable equilibrium. When it is displaced up and released, it accelerates down and when it is pushed down and released, it accelerates up. It means, a floating body experiences a net (resting) force towards its stable equilibrium position. Hence, a floating body oscillates when displaced up or down from its mean position.

ILLUSTRATION 15.16 Consider a solid cylinder of density θ_s, cross-sectional area A and height h floating in a liquid of density θ_l, as shown in figure ($\theta_l > \theta_s$). It is depressed slightly and allowed to oscillate vertically. Find the frequency of small oscillations.



Solution. At equilibrium the net force on the cylinder is zero in the vertical direction.

$F_{net} = B - W = 0$; B = the buoyancy and W = the weight of the cylinder.

When the cylinder is depressed slightly by x, the buoyancy increases from B to $B + \delta B$, where $\delta B = |x|\rho_l Ag$

The weight W remains the same.

Therefore the net force, $F_{net} = B + \delta B - W = \delta B = |x|\rho_l Ag$

The equation of motion is, therefore,

$$\rho_s Ah\frac{d^2x}{dt^2} = -x\rho_l Ag$$

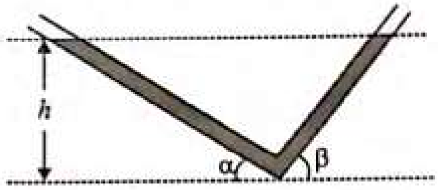
The minus sign takes into account the fact that x and restoring force are in opposite directions. Therefore

$$\frac{d^2x}{dt^2} = -x\frac{\rho_l g}{\rho_s h}$$

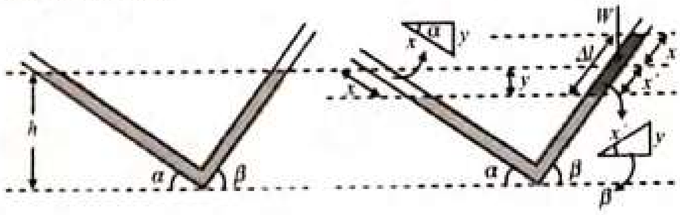
and the angular frequency, ω, is $\omega = \sqrt{\dfrac{g\rho_l}{h\rho_s}}$

OSCILLATION OF A LIQUID COLUMN IN U-TUBE

ILLUSTRATION 15.17 A V-shaped glass tube of uniform cross section is kept in a vertical plane as shown. A liquid is poured in the tube. In equilibrum the level of liquid in both limbs of tube are equal. Find the angular frequency of small oscillations of liquid.



Solution. Let us displace the liquid of left limb by a distance x, the liquid of right limb will rise relative to equilibrum line by same distance x.



Excess length of the liquid on the right limb

$$\Delta l = x + x' \quad \text{(i)}$$

From the figure,

$$y = x\sin\alpha = x'\sin\beta \quad \text{(ii)}$$

From Eq. (ii)

$$x' = \frac{x\sin\alpha}{\sin\beta} \quad \text{(iii)}$$

from Eqs. (i) and (iii), $\Delta l = \left(\dfrac{\sin\alpha + \sin\beta}{\sin\beta}\right)x$

This excess length Δl in right limb will provide restoring force to the liquid in tube.

Weight of the excess liquid column

$$W = \rho A \cdot \Delta l g = \left(\frac{\sin\alpha + \sin\beta}{\sin\beta}\right)\rho Axg$$

ρ is the density of liquid and A is the area of cross section of tube. The component of W along the tube will be equal to restoring force

$$F = W\sin\beta = \rho(\sin\alpha + \sin\beta)Axg$$

This force helps the entire liquid to restore its original position by moving it with an acceleration $a = F/m$; m = mass of total liquids

$$a = \frac{\rho(\sin\alpha + \sin\beta)Axg}{m}$$

Here m = mass of liquid in the tube = $A\rho l$

Length of total liquid column,

$$l = \frac{h}{\sin\alpha} + \frac{h}{\sin\beta} = h\left[\frac{\sin\alpha + \sin\beta}{\sin\alpha \cdot \sin\beta}\right]$$

Hence $a = \dfrac{\rho(\sin\alpha + \sin\beta)Axg}{A\rho\left[h\dfrac{(\sin\alpha + \sin\beta)}{\sin\alpha \cdot \sin\beta}\right]}$

$$\vec{a} = -\left(\frac{g\sin\alpha\cdot\sin\beta}{h}\right)\cdot\vec{x} \quad \begin{bmatrix}\text{with proper}\\ \text{vector sign}\end{bmatrix}$$

Compare with $\vec{a} = -\omega^2\vec{x}$

which gives

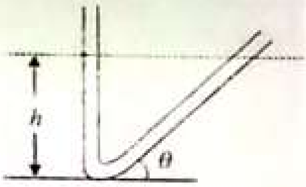
$$\omega = \sqrt{\frac{g\sin\alpha\cdot\sin\beta}{h}} \Rightarrow T = 2\pi\sqrt{\frac{h}{g\sin\alpha\cdot\sin\beta}}$$

NOTE: If we modify the tube as shown in the figure, the different time periods will be

In Figure (a), $\sin\alpha = 1$,

$\sin\beta = \sin\theta$

Hence $T = 2\pi\sqrt{\frac{h}{g\sin\theta}}$



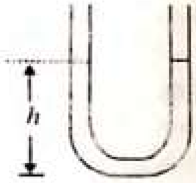
(a)

In Figure (b), $\alpha = \beta = 90°$

Hence $T = 2\pi\sqrt{\frac{h}{g}}$

For an arbitary tube if α and β are the angles made by the targets (with horizontal) drawn at the tube at the free surface of the liquid in both limbs. The time period is given by

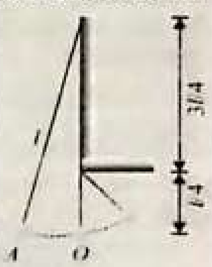
$$T = 2\pi\frac{\sqrt{h}}{g\sin\alpha\cdot\sin\beta}$$



(b)

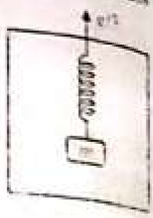
Concept Application Exercise 15.2

1. A pendulum has a period T for small oscillations. An obstacle is placed directly below the pivot, so that only the lowest one-fourth of the string can follow the pendulum bob when it swings to the right of its resting position as shown. The pendulum is released from rest at a certain point. Assuming that the angle between the moving string and the vertical stays small throughout the motion, find the time it takes to return to that point.

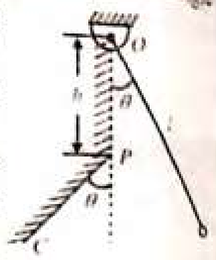


2. A horizontal string-mass system of mass M executes oscillatory motion of amplitude a_0 and time period T_0. When mass M is passing through its equilibrium position another mass m is placed on it such that both move together. Find the new amplitude and time period.

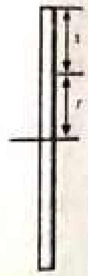
3. A spring of spring constant 200 N/m has a block of mass 1 kg hanging at its one end and from the other end the spring is attached to a ceiling of an elevator. The elevator rises upwards with an acceleration of $g/3$. When acceleration is suddenly ceased, then what should be the angular frequency and elongation during the time when the elevator is accelerating?



4. A simple pendulum of length l swings from a small angle θ. Its swinging is constrained by the smooth inclined planes OP and PC. Assuming elastic collision of the bob with the plane PC, find
   (a) angular amplitude for the motion of the bob in the left hand side of its mean position.
   (b) time period for a complete cycle of motion of the bob.



5. A uniform rod of length l is pivoted at a distance x from the top of the rod. Neglecting friction, find the
   (a) value of x for minimum period of oscillation.
   (b) minimum period of oscillation of the rod.



6. The period of oscillation of a spring pendulum is T. If the spring is cut into four equal parts, then find the time period corresponding to each part.

FREE, FORCED AND RESONANT VIBRATIONS

Free vibrations: A system is said to execute free oscillations if on being displaced or disturbed from its position of equilibrium it is left to oscillate itself without outside interference or under any external force.

The time period of such oscillations is determined by the dimensions and elastic and inertial (mass) properties of the system. This time period is called natural or free time period and the corresponding frequency is called natural frequency. It may be denoted by v_0.

The natural frequency is given by v_0.

$$v_0 = \frac{1}{2\pi}\left[\frac{\text{Elasticity factor}}{\text{Inertia factor}}\right]^{1/2}$$

The frequency of the free vibrations is a constant depending on the nature of the oscillatior.

Forced vibrations: When a system is compelled to oscillate with a frequency other than its natural frequency, it is said to execute forced oscillations.

For example, sound boxes of musical instruments execute forced vibrations, due to the periodic force of the vibrating strings

When a periodic force acts on an oscillator, then to start with it tends to oscillate with a case the oscillator executes vibration with the frequency of external periodic force.

Forced oscillations are of two types: non-resonant and resonant.

Non-resonant oscillations: When the natural frequency of the oscillator is different than the external periodic force acting on it, the forced oscillations are called non-resonant.

The amplitude of non-resonant oscillations is not very large. The transfer of energy to the oscillator is also small.

Resonant vibrations: The phenomenon of producing oscillatory motion in a system by the influence of an external periodic force having the same frequency as the natural frequency of the system is called resonance.

In such a case the oscillator is said to be in resonance (with external periodic force) if the amplitude of oscillation tends to be large and the transfer of energy to the oscillator is also large. The oscillator vibrates in phase with external periodic force.

THEORY OF FORCED OSCILLATIONS

Suppose a simple harmonic oscillator is subjected to external periodic force $F_0 \sin \omega_0 t$. Let the spring constant of the oscillator be $-kx$. The net force on the oscillator is:

$$F = F_0 \sin \omega_0 t - kx$$

If M be the mass of the oscillator, then

$$F = M\frac{d^2x}{dt} = F_0 \sin\omega_0 t - kx$$

where $\frac{d^2x}{dt^2}$ is the acceleration of oscillation. This gives:

$$\frac{d^2x}{dt^2} = \frac{F_0}{M}\sin\omega_0 t - \frac{k}{M}x \qquad ...(i)$$

Here $\left[\frac{k}{M}\right]^{\frac{1}{2}} = \omega$, the natural angular frequency of the oscillator.

On solving Eq. (i), we find: $x = \frac{F_0}{M(\omega^2 - \omega_0^2)}\sin\omega_0 t$

The amplitude of the forced oscillations is given by:

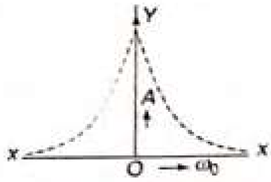
$$A = \frac{F_0}{M(\omega^2 - \omega_0^2)} \qquad (\because \sin \omega_0 t = 1, \text{ the max value})$$

From this expression it is clear that

- The amplitude of forced oscillations depends on the amplitude of applied force (F_0), inertia factor M and frequency of the external periodic force ω_0.
- It varies inversely as the difference of the squares of the natural angular frequency of the external periodic force and the angular frequency of the applied force.
- In principle, at resonance, $\omega = \omega_0$. Therefore, $A = \infty$. However, in actual practice, the amplitude may not be infinite. But it certainly varies with time.
- Amplitude of forced oscillation does not vary with time. The frequency of forced oscillations is the same as that of the external periodic force.

- The resonance frequency is given by:

$$\nu_0 = \frac{\omega}{2\pi} = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

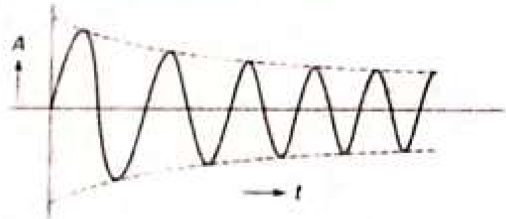


The variation of amplitude of forced oscillations with the frequency of the external periodic force is shown in the figure. For the value of frequency of ω_0 the amplitude 'A' is larger than that at any other frequency for a given system.

Resonance is a special case of forced oscillations. The applied periodic force is such that it helps to increase the amplitude of the system at each step of its oscillation. The result is that the system or the body oscillates with large amplitude.

DAMPED OSCILLATIONS

The external frictional force on an oscillatory system continuously dissipates the energy of the system. The energy of the system may be converted into heat, light, sound, etc. The amplitude of such an oscillator continuously decreases and ultimately the oscillator comes to rest.



The oscillations of a system under the influence of external frictional force are called damped oscillations. When we set a pendulum in oscillation in air, a frictional force due to the air acts on the bob. So, the oscillations are damped and the amplitude may decrease to zero as shown in the figure above. Suppose, an oscillator having mass M and force constant k is acted upon by a resistive force F_0. It is found that the resistive force is proportional to the velocity (v) of the oscillator. That is, $F_d = bv$, where b is a constant and is called the damping factor. If x be the instantaneous displacement of the oscillator, then

$$M\frac{d^2x}{dt^2} = kx - bv$$

But $v = dx/dt$. Therefore $\frac{d^2x}{dt^2} = \frac{k}{M} - \frac{b}{M}\frac{dx}{dt}$

Solving this equation, we find $x = Ae^{-\frac{b}{2M}t}\cos(\omega' t + \phi)$

where $\omega' = \left[\frac{k}{M} - \left(\frac{b}{2M}\right)^2\right]^{1/2}$

The amplitude of damped oscillations is given by: $A_d = Ae^{\frac{-b}{2M}T}$ where A is the amplitude of undamped oscillations. The frequency of the damped oscillations is less than that of the

natural oscillations. If ω be the natural angular frequency, then $\omega = [k/M]^{\frac{1}{2}}$. And, the frequency of the damped oscillations is (from the above equation)

$$\omega' = \left[\omega^2 - \left(\frac{b}{2M}\right)^2\right]^{1/2}$$

SOLVED EXAMPLES

1. Two particles executes S.H.M. of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, and each time their displacement is half of their amplitude. The phase difference between them is
(a) 30° (b) 60°
(c) 90° (d) 120°

Sol. (d) $y = a\sin(\omega t + \phi_0)$. According to the question

$$y = \frac{a}{2} \Rightarrow \frac{a}{2} = a\sin(\omega t + \phi_0)$$

$$\Rightarrow (\omega t + \phi_0) = \phi = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

Physical meaning of $\phi = \frac{\pi}{6}$: Particle is at point P and it is going towards B

Physical meaning of $\phi = \frac{5\pi}{6}$: Particle is at point P and it is going towards O

So phase difference, $\Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3} = 120°$

2. The displacement of a particle varies with time as $x = 12\sin\omega t - 16\sin^3\omega t$ (in cm). If its motion is S.H.M., then its maximum acceleration is
(a) $12\omega^2$ (b) $36\omega^2$
(c) $144\omega^2$ (d) $\sqrt{192}\,\omega^2$

Sol. (b) $x = 12\sin\omega t - 16\sin^3\omega t = 4[3\sin\omega t - 4\sin^3\omega t]$
$= 4[\sin 3\omega t]$ (By using $\sin3\theta = 3\sin\theta - 4\sin^3\theta$)
$\therefore A_{max} = (3\omega)^2 \times 4 = 36\omega^2$

3. A linear harmonic oscillator of force constant 2×10^6 N/m times and amplitude 0.01 m has a total mechanical energy of 160 joules. Its
(a) Maximum potential energy is 100 J
(b) Maximum K.E. is 160 J
(c) Maximum P.E. is 160 J
(d) Minimum P.E. is zero

Sol. (c) Harmonic oscillator has some initial elastic potential energy and amplitude of harmonic variation of energy is $\frac{1}{2}ka^2 = \frac{1}{2} \times 2 \times 10^6 (0.01)^2 = 100$ J

This is the maximum kinetic energy of the oscillator. Thus, $K_{max} = 100$ J

This energy is added to initial elastic potential energy may give maximum mechanical energy to have value 160 J.

4. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is
(a) Proportional to $\frac{1}{\sqrt{a}}$ (b) Independent of a
(c) Proportional to $\sqrt{a}$ (d) Proportional to $a^{3/2}$

Sol. (a) $U = k|x|^3 \Rightarrow F = -\frac{dU}{dx} = -3k|x|^2$ ...(i)

Also, for SHM, $x = a\sin\omega t$ and $\frac{d^2x}{dt^2} + \omega^2 x = 0$

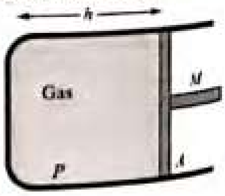
$$\Rightarrow \text{Acceleration } a = \frac{d^2x}{dt^2} = -\omega^2 x$$

$$\Rightarrow F = ma = m\frac{d^2x}{dt^2} = -m\omega^2 x \quad \text{...(ii)}$$

From eqs. (i) and (ii), we get $\omega = \sqrt{\frac{3kx}{m}}$

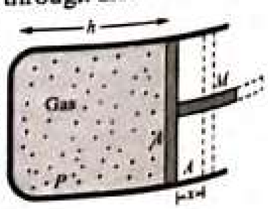
$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3kx}} = 2\pi\sqrt{\frac{m}{3k(a\sin\omega t)}} \Rightarrow T \propto \frac{1}{\sqrt{a}}.$$

5. A cylindrical piston of mass M slides smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is disturbed from its equilibrium position, it oscillates simple harmonically. The period of oscillation will be



(a) $T = 2\pi\sqrt{\left(\frac{Mh}{PA}\right)}$ (b) $T = 2\pi\sqrt{\left(\frac{MA}{Ph}\right)}$
(c) $T = 2\pi\sqrt{\left(\frac{M}{PAh}\right)}$ (d) $T = 2\pi\sqrt{MPhA}$

Sol. (a) Let the piston be displaced through distance x towards left, then volume decreases, pressure increases. If ΔP is increase in pressure and ΔV is decrease in volume, then considering the process to take place gradually (i.e. isothermal)



$$P_1V_1 = P_2V_2$$

$\Rightarrow PV = (P + \Delta P)(V - \Delta V)$

$\Rightarrow PV = PV + \Delta PV - P\Delta V - \Delta P\Delta V$

$\Rightarrow \Delta P.V - P.\Delta V = 0$ (neglecting $\Delta P.\Delta V$)

$\Delta P(Ah) = P(Ax) \Rightarrow \Delta P = \dfrac{P.x}{h}$

This excess pressure is responsible for providing the restoring force (F) to the piston of mass M.

Hence $F = \Delta P.A = \dfrac{PAx}{h}$

Comparing it with $|F| = kx \Rightarrow k = M\omega^2 = \dfrac{PA}{h}$

$$\Rightarrow \omega = \sqrt{\frac{PA}{Mh}} \Rightarrow T = 2\pi\sqrt{\frac{Mh}{PA}}$$

6. A particle is performing simple harmonic motion along x-axis with amplitude 4 cm and time period 1.2 sec. The minimum time taken by the particle to move from x = 2 cm to x = + 4 cm and back again is given by
   (a) 0.6 sec (b) 0.4 sec
   (c) 0.3 sec (d) 0.2 sec

Sol. (b) Time taken by particle to move from $x = 0$ (mean position) to $x = 4$ (extreme position) $= \dfrac{T}{4} = \dfrac{1.2}{4} = 0.3$ s

Let t be the time taken by the particle to move from $x = 0$ to $x = 2$ cm

$$y = a\sin\omega t \Rightarrow 2 = 4\sin\frac{2\pi}{T}t \Rightarrow \frac{1}{2} = \sin\frac{2\pi}{1.2}t$$

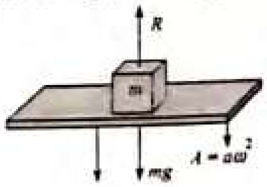
$$\Rightarrow \frac{\pi}{6} = \frac{2\pi}{1.2}t \Rightarrow t = 0.1\text{ s.}$$

Hence time to move from $x = 2$ to $x = 4$ will be equal to 0.3 - 0.1 = 0.2 s

Hence total time to move from $x = 2$ to $x = 4$ and back again = $2 \times 0.2 = 0.4$ sec

7. A horizontal platform with an object placed on it is executing S.H.M. in the vertical direction. The amplitude of oscillation is 3.92×10^{-3} m . What must be the least period of these oscillations, so that the object is not detached from the platform?
   (a) 0.1256 sec (b) 0.1356 sec
   (c) 0.1456 sec (d) 0.1556 sec

Sol. (a) By drawing free body diagram of object during the downward motion at extreme position, for equilibrium of mass



$Mg - R = mA$ (A = Acceleration)

For critical condition $R = 0$

so $mg = mA \Rightarrow mg = ma\omega^2$

$$\Rightarrow \omega = \sqrt{g/a} = \sqrt{\frac{9.8}{3.92\times10^{-3}}} = 50$$

$$\Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1256\text{sec}$$

8. A particle executes simple harmonic motion (amplitude = A) between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2. Then
   (a) $T_1 < T_2$ (b) $T_1 > T_2$
   (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

Sol. (a) Using $x = A\sin\omega t$

For $x = A/2$, $\sin\omega T_1 = 1/2 \Rightarrow T_1 = \dfrac{\pi}{6\omega}$

For $x = A$, $\sin\omega(T_1 + T_2) = 1 \Rightarrow T_1 + T_2 = \dfrac{\pi}{2\omega}$

$$\Rightarrow T_2 = \frac{\pi}{2\omega} - T_1 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega}\text{ i.e. } T_1 < T_2$$

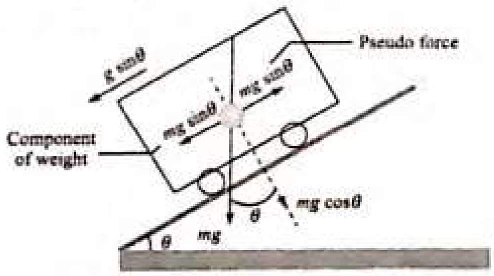
9. The metallic bob of a simple pendulum has the relative density ρ. The time period of this pendulum is T. If the metallic bob is immersed in water, then the new time period is given by
   (a) $T\dfrac{\rho - 1}{\rho}$ (b) $T\dfrac{\rho}{\rho - 1}$
   (c) $T\sqrt{\dfrac{\rho - 1}{\rho}}$ (d) $T\sqrt{\dfrac{\rho}{\rho - 1}}$

Sol. (d) When the bob is immersed in water, its effective weight $= \left(mg - \dfrac{m}{\rho}g\right) = mg\left(\dfrac{\rho - 1}{\rho}\right) \therefore g_{eff} = g\left(\dfrac{\rho - 1}{\rho}\right)$

$$\frac{T'}{T} = \sqrt{\frac{g}{g_{eff}}} \Rightarrow T_1 = 2\pi\sqrt{\frac{m}{K}}$$

10. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down an inclined plane of inclination α, is given by
   (a) $2\pi\sqrt{\dfrac{L}{g\cos\alpha}}$ (b) $2\pi\sqrt{\dfrac{L}{g\sin\alpha}}$
   (c) $2\pi\sqrt{\dfrac{L}{g}}$ (d) $2\pi\sqrt{\dfrac{L}{g\tan\alpha}}$

Sol. (a) See the following force diagram.



Vehicle is moving down the frictionless inclined surface so, it's acceleration is g sinθ sin. Since vehicle is accelerating, a pseudo force $m(g\sin\theta)$ will act on bob of pendulum which cancel the sinθ component of weight of the bob.

Hence net force on the bob is $F_{net} = mg\cos\theta$ or net acceleration of the bob is $g_{eff} = g\cos\theta$

$\therefore$ Time period $T = 2\pi\sqrt{\dfrac{l}{g_{eff}}} = 2\pi\sqrt{\dfrac{l}{g\cos\theta}}$

11. The displacement y of a particle executing periodic motion is given by $y = 4\cos^2(t/2)\sin(1000t)$. This expression may be considered to be a result of the superposition of .... independent harmonic motions
(a) Two (b) Three
(c) Four (d) Five

Sol. (b) $y = 4\cos^2\left(\dfrac{t}{2}\right)\sin 1000\,t$

$\Rightarrow y = 2(1 + \cos t)\sin 1000t$

$\Rightarrow y = 2\sin 1000t + 2\cos t \sin 1000t$

$\Rightarrow y = 2\sin 1000t + \sin 999t + \sin 1001\,t$

It is a sum of three S.H.M.

12. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then
(a) The resultant amplitude is $\sqrt{2}a$
(b) The phase of the resultant motion relative to the first is 90°
(c) The energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion
(d) The resulting motion is not simple harmonic

Sol. (c) Let simple harmonic motions be represented by

$y_1 = a\sin\left(\omega t - \dfrac{\pi}{4}\right)$; $y_2 = a\sin\omega t$ and

$y_3 = a\sin\left(\omega t + \dfrac{\pi}{4}\right)$. On superimposing, resultant SHM

will be $y = a\left[\sin\left(\omega t - \dfrac{\pi}{4}\right) + \sin\omega t + \sin\left(\omega t + \dfrac{\pi}{4}\right)\right]$

$= a\left[2\sin\omega t\cos\dfrac{\pi}{4} + \sin\omega t\right]$

$= a[\sqrt{2}\sin\omega t + \sin\omega t] = a(1+\sqrt{2})\sin\omega t$

Resultant amplitude $= (1+\sqrt{2})a$

Energy is S.H.M. $\propto$ (Amplitude)^2

$$\therefore \frac{E_{\text{Resultant}}}{E_{\text{Single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$$

$$\Rightarrow E_{\text{Resultant}} = (3+2\sqrt{2})E_{\text{Single}}$$

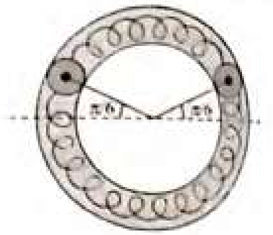
13. A simple pendulum has time period T_1. The point of suspension is now moved upward according to equation $y = kt^2$ where $k = 1$ m/sec^2. If new time period is T_2 then ratio $\dfrac{T_1^2}{T_2^2}$ will be
(a) 2/3 (b) 5/6
(c) 6/5 (d) 3/2

Sol. (c) $y = kt^2 \Rightarrow \dfrac{d^2y}{dt^2} = a_y = 2K = 2 \Rightarrow 1 = 2\text{m/s}^2$ ($\because k = 1$ m/s^2)

Now, $T_1 = 2\pi\sqrt{\dfrac{l}{g}}$ and $T_2 = 2\pi\sqrt{\dfrac{l}{(g+a_y)}}$

Dividing, $\dfrac{T_1}{T_2} = \sqrt{\dfrac{g+a_y}{g}} \Rightarrow \sqrt{\dfrac{6}{5}} \Rightarrow \dfrac{T_1^2}{T_2^2} = \dfrac{6}{5}$

14. Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π m and force constant 0.1N/m. Initially both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle and released from rest. The frequency of oscillation of the ball B is



(a) π Hz
(b) $\dfrac{1}{\pi}$ Hz
(c) 2π Hz
(d) $\dfrac{1}{2\pi}$ Hz

Sol. (b) As here two masses are connected by two springs, this problem is equivalent to the oscillation of a reduced mass m_r of a spring of effective spring constant.

$$T = 2\pi\sqrt{\frac{m_r}{K_{eff}}}$$

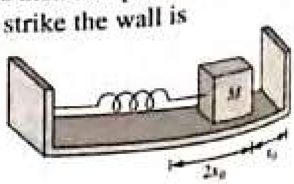
Here $m_r = \dfrac{m_1m_2}{m_1+m_2} = \dfrac{m}{2}$

$\Rightarrow k_{eff} = k_1 + k_2 = 2k$

$\therefore n = \dfrac{1}{2\pi}\sqrt{\dfrac{k_{eff}}{m_r}} = \dfrac{1}{2\pi}\sqrt{\dfrac{2k}{m}\times 2}$

$= \dfrac{1}{\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{\pi}\sqrt{\dfrac{0.1}{0.1}} = \dfrac{1}{\pi}$ Hz

15. One end of a spring of force constant k is fixed to a vertical wall and the other to a block of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the block. The spring is then compressed by $2x_0$ and released. The time taken to strike the wall is



(a) $\dfrac{1}{6}\pi\sqrt{\dfrac{k}{m}}$

(b) $\sqrt{\dfrac{k}{m}}$

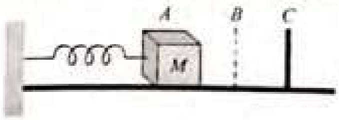
(c) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$

(d) $\frac{\pi}{4}\sqrt{\frac{k}{m}}$

Sol. (c) The total time from A to C

$$t_{AC} = t_{AB} + t = (T/4) + t_{BC}$$

where T = time period of oscillation of spring mass system



t_{BC} can be obtained from, $BC = A\ B\sin(2\pi/T)t_{BC}$

Putting $\frac{BC}{AB} = \frac{1}{2}$ we obtain $t_{BC} = \frac{T}{12}$

$$\Rightarrow \quad t_{AC} = \frac{T}{4} + \frac{T}{12} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}.$$

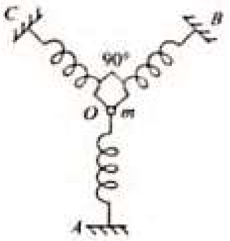
16. A particle of mass m is attached to three identical springs A, B and C each of force constant k a shown in figure. If the particle of mass m is pushed slightly against the spring A and released then the time period of oscillations is

(a) $2\pi\sqrt{\frac{2m}{k}}$

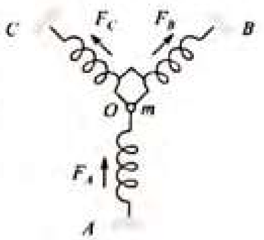
(b) $2\pi\sqrt{\frac{m}{2k}}$

(c) $2\pi\sqrt{\frac{m}{k}}$

(d) $2\pi\sqrt{\frac{m}{3k}}$



Sol. (b) When the particle of mass m at O is pushed by y in the direction of A The spring A will be compressed by y while spring B and C will be stretched by $y' = y\cos 45°$. So that the total restoring force on the mass m along OA.

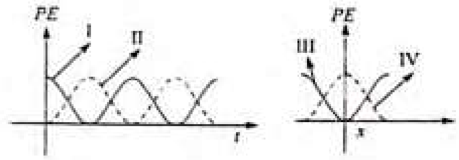


$$F_{net} = F_A + F_B\cos 45° + F_C\cos 45°$$
$$= ky + 2ky'\cos 45°$$
$$= ky + 2k(y\cos 45°)\cos 45°$$
$$= 2ky$$

Also $F_{net} = k'y \Rightarrow k'y = 2ky \Rightarrow k' = 2k$

$$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

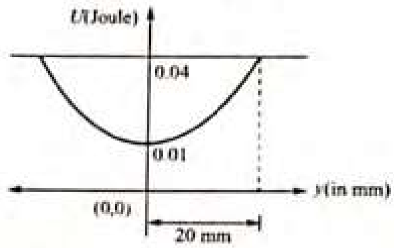
17. For a particle executing S.H.M. the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (P.E.) as a function of time t and displacement x



(a) I, III (b) II, IV
(c) II, III (d) I, IV

Sol. (a) Potential energy is minimum (in this case zero) at mean position ($x = 0$) and maximum at extreme position ($x = \pm A$) At time $t = 0$, $x = A$, hence potential should be maximum. Therefore graph I is correct. Further in graph III. Potential energy is minimum at $x = 0$, hence this is also correct.

18. The variation of potential energy of harmonic oscillator is as shown in figure. The spring constant is



(a) 1×10^2 N/m (b) 150 N/m
(c) 0.667×10^2 N/m (d) 3×10^2 N/m

Sol. (b) Total potential energy = 0.04 J

Resting potential energy =0.01 J

Maximum kinetic energy =(0.04–0.01) = 0.03 J

$$0.03\text{ J} = \frac{1}{2}m\,\omega^2 a^2 = \frac{1}{2}ka^2$$

$$0.03 = \frac{1}{2} \times k \times \left(\frac{20}{1000}\right)^2$$

$k = 0.06 \times 2500$ N/m = 150 N/m.

19. Two pendulums have time periods T and $5T/4$. They start S.H.M. at the same time from the mean position. What will be the phase difference between them after the bigger pendulum has complete one oscillation?

(a) 45° (b) 90°
(c) 60° (d) 30°

Sol. (b) As $\frac{5T}{4} = T + \frac{T}{4}$

By the time, the bigger pendulum makes one full oscillation, the smaller pendulum will make $\left(1 + \frac{1}{4}\right)$ oscillation. The bigger pendulum will be in the mean position and the smaller one will be in the positive extreme position. Thus, phase difference = 90°

20. A man weighing 60 kg stands on the horizontal platform of a spring balance. The platform starts executing simple

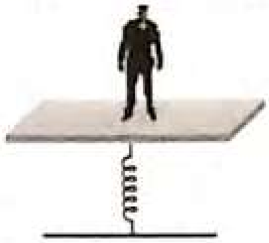
harmonic motion of amplitude 0.1 m and frequency $\frac{2}{\pi}$ Hz. Which of the following statement is correct?

(a) The spring balance reads the weight of man as 60 kg

(b) The spring balance reading fluctuates between 60 kg. and 70 kg

(c) The spring balance reading fluctuates between 50 kg and 60 kg

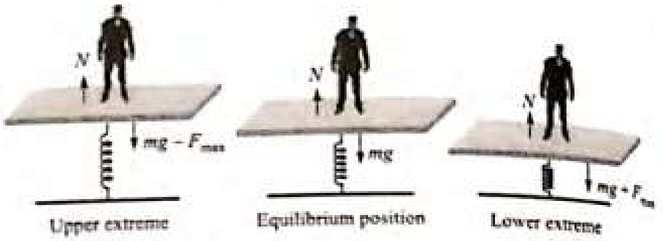
(d) The spring balance reading fluctuates between 50 kg and 70 kg



Sol. (d) The maximum force acting on the body executing simple harmonic motion is

$$m\omega^2 a = m \times (2\pi f)^2 a = 60 \times \left(2\pi \times \frac{2}{\pi}\right)^2 \times 0.1 \ N$$

$$= 60 \times 16 \times 0.1 = 96 \text{ N} = \frac{96}{9.8} = 10 kgf$$ and this force is towards mean position.



The reaction of the force on the platform away from the mean position. It reduces the weight of man on upper extreme i.e. net weight = (60 – 10) kgf.

This force adds to the weight at lower extreme position i.e. net weight becomes = (60 + 10) kgf.

Therefore, the reading the weight recorded by spring balance fluctuates between 50 kgf and 70 kgf.

EXERCISES

Problems Based on Basic Theory

1. A particle is moving in a circle with uniform speed. Its motion is
   (a) not periodic
   (b) periodic and simple harmonic
   (c) periodic but not simple harmonic
   (d) none of the above

2. As the expression is involving sine function, which of the following equations does not represent a simple harmonic motion?
   (a) $y = a \sin \omega t$
   (b) $y = a \cos \omega t$
   (c) $y = a \sin \omega t + b \cos \omega t$
   (d) $y = a \tan \omega t$

3. Two particles P and Q describe SHM of same amplitude a, same frequency f along the same straight line. The maximum distance between the two particles is $a\sqrt{2}$. The phase difference between the particle is
   (a) zero (b) $\frac{\pi}{2}$
   (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

4. A particle executing SHM of amplitude 4 cm and $T = 4$ sec. The time taken by it to move from positive extreme position to half the amplitude is
   (a) 1 sec (b) 1/3 sec
   (c) 2/3 sec (d) $\sqrt{3/2}$ sec

5. The equation of motion of a particle executing simple harmonic motion is $a + 16\pi^2 x = 0$. In this equation, a is the linear acceleration in m/s^2 of the particle at a displacement x in meter. The time period in simple harmonic motion is
   (a) $\frac{1}{4}$ sec (b) $\frac{1}{2}$ sec
   (c) 1 sec (d) 2 sec

6. A particle executes SHM of amplitude A and time period T. The distance travelled by the particle in the duration its phase changes from $\frac{\pi}{12}$ to $\frac{5\pi}{12}$.
   (a) $\frac{1}{\sqrt{2}}A$ (b) $\sqrt{\frac{3}{2}}A$
   (c) $\frac{2}{\sqrt{3}}A$ (d) $\sqrt{\frac{2}{3}}A$

7. The phase difference between the displacement and acceleration of a particle executing simple harmonic motion is
   (a) zero (b) $\pi/2$
   (c) π (d) 2π

8. The phase difference between two particles executing SHM of the same amplitudes and frequency along same straight line while passing one another when going in opposite directions with equal displacement from their respective stating point is $2\pi/3$. If the phase of one particle

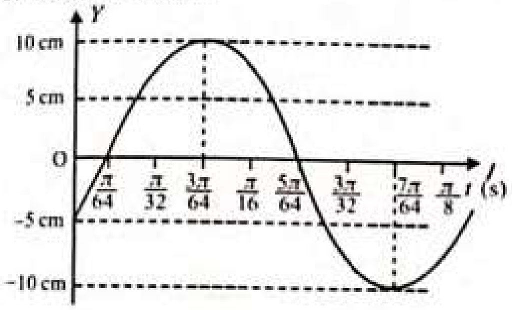
is $\pi/6$, find the displacement at this instant, if amplitude is A

(a) $A/3$ (b) $2A/3$
(c) $3A/4$ (d) $A/2$

9. Time period (T) and amplitude (A) are same for two particles which undergo SHM along the same line. At one particular instant, one particle is at phase $\frac{3\pi}{2}$ and other is at phase zero. While moving in the same direction. Find the time at which they will cross each other:

(a) $4T/2$ (b) $3T/8$
(c) $3T/4$ (d) $3T/7$

10. The diagram below shows a sinusoidal curve. The equation of the curve will be



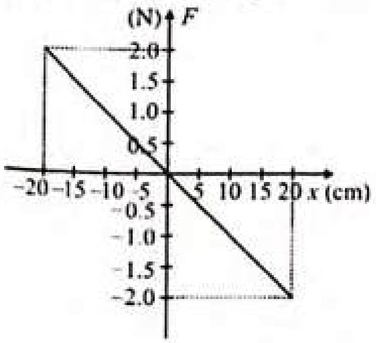
(a) $y = 10\sin\left(16t - \frac{\pi}{3}\right)$ cm

(b) $y = 10\sin\left(16t + \frac{\pi}{3}\right)$ cm

(c) $y = 10\sin\left(16t - \frac{\pi}{4}\right)$ cm

(d) $y = 10\cos\left(16t + \frac{\pi}{4}\right)$ cm

11. Figure shows the variation of force acting on a particle of mass 400 g executing simple harmonic motion. The frequency of oscillation of the particle is



(a) $4\ s^{-1}$ (b) $(5/2\pi)\ s^{-1}$
(c) $(1/8\pi)\ s^{-1}$ (d) $(1/2\pi)\ s^{-1}$

12. Two simple harmonic motions are represented by equations

$y_1 = 4\sin(10t + \phi)$

$y_2 = 5\cos 10t$

What is the phase difference between their velocities?

(a) ϕ (b) $-\phi$

(c) $\left(\phi + \frac{\pi}{2}\right)$ (d) $\left(\phi - \frac{\pi}{2}\right)$

13. Two particles move parallel to the x-axis about the origin with same amplitude 'a' and frequency ω. At a certain instant they are found at a distance $a/3$ from the origin on opposite sides but their velocities are in the same direction. What is the phase difference between the two?

(a) $\cos^{-1}\frac{7}{9}$ (b) $\cos^{-1}\frac{5}{9}$

(c) $\cos^{-1}\frac{4}{9}$ (d) $\cos^{-1}\frac{1}{9}$

14. A small mass executes linear SHM about O with amplitude a and period T. Its displacement from O at time $T/8$ after passing through O is

(a) $a/8$ (b) $a/2\sqrt{2}$
(c) $a/2$ (d) $a/\sqrt{2}$

15. Time period of a particle executing SHM is 8 s. At $t = 0$, it is at the mean position. The ratio of the distance covered by the particle in the 1st second to the 2nd second is

(a) $\frac{1}{\sqrt{2}+1}$ (b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}+1$

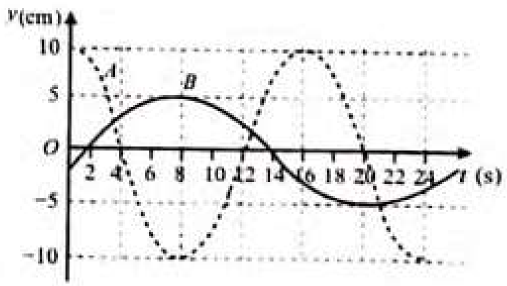
16. A particle moves with a simple harmonic motion in a straight line. In the first second starting from rest it travels a distance a and in the next second it travels a distance b in the same direction. The amplitude of the motion is

(a) $\frac{2a^2}{3b-a}$ (b) $\frac{3a^2}{3a-b}$

(c) $\frac{2a^2}{3a-b}$ (d) $\frac{3a^2}{3b-a}$

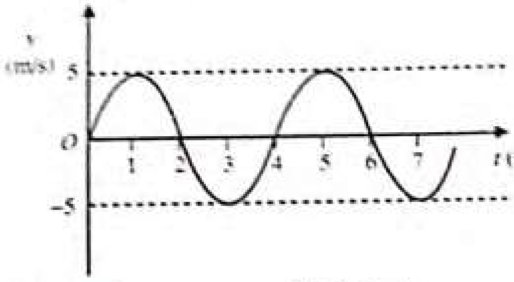
Velocity, Acceleration and Energy of SHM

17. The following figure shows the displacement versus time graph for two particles A and B executing simple harmonic motions. The ratio of their maximum velocities is



(a) 3:1 (b) 1:3 (c) 1:9 (d) 9:1

18. The variation of velocity of a particle executing SHM with time is shown in figure. The velocity of the particle when a phase change of $\pi/6$ takes place from the instant it is at one of the extreme positions will be



(a) 3.53 m/s (b) 2.5 m/s
(c) 4.330 m/s (d) None of these

19. In problem 18, the displacement of the particle from the mean position corresponding to the instant mentioned is

(a) $\frac{5}{\pi}$ m (b) $\frac{5\sqrt{3}}{\pi}$ m

(c) $\frac{10\sqrt{3}}{\pi}$ m (d) $\frac{5\sqrt{3}}{2\pi}$ m

20. In problem 18, the acceleration of the particle is

(a) $\frac{5\sqrt{3}\pi}{2}$ m/s^2 (b) $\frac{5\pi^2}{2}$ m/s^2

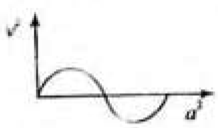
(c) $\frac{5\sqrt{3}\pi}{4}$ m/s^2 (d) $5\sqrt{3}\pi$ m/s^2

21. In problem 18, the maximum displacement and acceleration of the particle are respectively:

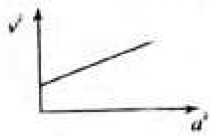
(a) $\frac{10}{\pi}$ m and 5π m/s^2 (b) $\frac{5}{\pi}$ m and $\frac{5\pi}{2}$ m/s^2

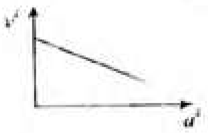
(c) $\frac{10}{\pi}$ m and $\frac{5\pi}{2}$ m/s^2 (d) $\frac{5}{\pi}$ m and $\frac{5\pi}{4}$ m/s^2

22. A graph of the square of the velocity against the square of the acceleration of a given simple harmonic motion is

(a)


(b)


(c)


(d)


23. A particle perfroms SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance $a/2$ from the extreme position is

(a) a/T (b) $2a/T$
(c) $3a/T$ (d) $a/2T$

24. A particle executing harmonic motion is having velocities v_1 and v_2 at distances is x_1 and x_2 from the equilibrium position. The amplitude of the motion is

(a) $\sqrt{\frac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 + v_2^2}}$ (b) $\sqrt{\frac{v_1^2x_1^2 - v_2^2x_2^2}{v_1^2 + v_2^2}}$

(c) $\sqrt{\frac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 - v_2^2}}$ (d) $\sqrt{\frac{v_1^2x_2^2 + v_2^2x_1^2}{v_1^2 + v_2^2}}$

25. The potential energy of a particle executing SHM along the x-axis is given by $U = U_0 - U_0 \cos ax$. What is the period of oscillation?

(a) $2\pi\sqrt{\frac{ma}{U_0}}$ (b) $2\pi\sqrt{\frac{U_0}{ma}}$

(c) $\frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$ (d) $2\pi\sqrt{\frac{m}{aU_0}}$

26. A particle executing SHM of amplitude 'a' has a displacement $a/2$ at $t = T/4$ and a negative velocity. The epoch of the particle is

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) π (d) $\frac{5\pi}{3}$

27. A particle is executing SHM according to the equation $x = A\cos\omega t$. Average speed of the particle during the interval $0 \le t \le \frac{\pi}{6\omega}$.

(a) $\frac{\sqrt{3}A\omega}{2}$ (b) $\frac{\sqrt{3}A\omega}{4}$

(c) $\frac{3A\omega}{\pi}$ (d) $\frac{3A\omega}{\pi}(2-\sqrt{3})$

28. A body is executing Simple Harmonic Motion. At a displacement x its potential energy is E_1 and at a displacement y its potential energy is E_2. The potential energy E at displacement $(x + y)$ is

(a) $\sqrt{E} = \sqrt{E_1} - \sqrt{E_2}$ (b) $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$

(c) $E = E_1 + E_2$ (d) $E = E_1 - E_2$

29. An object of mass 0.2 kg executes simple harmonic along X-axis with frequency of $\frac{25}{\pi}$ Hz. At the position x = 0.04 m, the object has kinetic energy of 0.5 J and potential energy of 0.4 J. The amplitude of oscillation in meter is equal to

(a) 0.05 (b) 0.06
(c) 0.01 (d) None of these

30. A vertical mass-spring system executes simple harmonic oscillations with a period of 2 s. A quantity of this system which exhibits simple harmonic variation with a period of 1 s is

(a) velocity

(b) potential energy
(c) phase difference between acceleration and displacement
(d) difference between kinetic energy and potential energy

31. A body executes simple harmonic motion. The potential energy (PE), kinetic energy (KE) and total energy (TE) are measured as a function of displacement x. Which of the following statement is true?
(a) TE is zero when $x = 0$
(b) PE is maximum when $x = 0$
(c) KE is maximum when $x = 0$
(d) KE is maximum when x is maximum

Spring Particle System and Simple Pendulum

32. A body at the end of a spring executes SHM with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two spring in series is T, then
(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
(c) $\frac{1}{T} = \frac{1}{t_1} + \frac{1}{t_2}$ (d) $\frac{1}{T^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$

33. Two identical springs are attached to a small block P. The other ends of the springs are fixed at A and B. When P is in equilibrium the extension of top spring is 20 cm and extension of bottom spring is 10 cm. The period of small vertical oscillations of P about its equilibrium position is (use $g = 9.8$ m/s^2)

(a) $\frac{2\pi}{7}$ sec (b) $\frac{\pi}{7}$ sec
(c) $\frac{2\pi}{5}$ sec (d) none of these

34. Two equal masses are suspended separately by two springs of constants k_1 and k_2. If their oscillations satisfy the condition that their maximum velocities are equal, then the ratio of the amplitudes of the oscillations of the masses respectively is
(a) $\frac{k_1}{k_2}$ (b) $\frac{k_2}{k_1}$
(c) $\sqrt{\frac{k_1}{k_2}}$ (d) $\sqrt{\frac{k_2}{k_1}}$

35. A spring of spring constant k is cut into n equal parts, out of which r parts are placed in parallel and connected with mass M as shown in figure. The time period of oscillatory motion of mass M is
(a) $T = 2\pi\sqrt{\frac{M}{nrk}}$ (b) $T = 2\pi\sqrt{\frac{nrM}{k}}$
(c) $T = 2\pi\sqrt{\frac{rM}{nk}}$ (d) $T = 2\pi\sqrt{\frac{nM}{rk}}$

36. Three masses 700 g, 500 g and 400 g are suspended at the end of a spring a shown and are in equilibrium. When the 700 g mass is removed, the system oscillates with a period of 3 seconds, when the 500 g mass is also removed, it will oscillate with a period of

(a) 1 s (b) 2 s
(c) 3 s (d) $\sqrt{\frac{12}{5}}$ s

37. Two simple pendulums whose lengths are 100 cm and 121 cm are suspended side by side. Their bobs are pulled together and then released. After how many minimum oscillations of the longer pendulum, will the two be in phase again
(a) 11 (b) 10
(c) 21 (d) 20

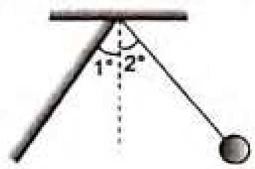
38. Two simple pendulum first of bob mass M_1 and length L_1 second of bob mass M_2 and length L_2. $M_1 = M_2$ and $L_1 = 2L_2$. If these vibrational energy of both is same. Then which is correct
(a) Amplitude of B greater than A
(b) Amplitude of B smaller than A
(c) Amplitudes will be same
(d) None of these

39. A simple pendulum hung from the ceiling of a train moving at constant speed has a period T. If the train starts accelerating or decelerating, then what will be the effect on time period of pendulum?
(a) Decreases only when train accelerates
(b) Decreases only when train decelerates
(c) Decreases in both cases
(d) Increases in both cases

40. Two simple pendulums of length 1 m and 16 m respectively are both given small displacement in the same direction at the same instant. They will be again in phase after the shorter pendulum has completed n oscillations. The value of n is
(a) 1/3 (b) 2/3
(c) 1 (d) 4/3

41. Two pendulums of different lengths are in phase at the mean position at a certain instant. The minimum time after which they will be again in phase is $5T/4$, where T is the time period of shorter pendulum. Find the ratios of lengths of the two pendulums.
(a) 1 : 16 (b) 1 : 4
(c) 1 : 2 (d) 1 : 25

42. A simple pendulum of length 1 m is allowed to oscillate with amplitude 2°. It collides elastically with a wall inclined at 1° to the vertical. Its time period will be: (use $g = \pi^2$)
(a) 2/3 sec
(b) 4/3 sec
(c) 2 sec
(d) none of these



43. A block of mass 1 kg hangs without vibrating at the end of a spring whose force constant is 200 N/m and which is attached to the ceiling of an elevator. The elevator is rising with an upward acceleration of $g/3$ when the acceleration suddenly ceases. The angular frequency of the block after the acceleration ceases is

(a) 13 rad/s (b) 14 rad/s
(c) 15 rad/s (d) None of these

44. A vertical spring carries a 5 kg body and is hanging in equilibrium, an additional force is applied so that the spring is further stretched. When released from this position, it performs 50 complete oscillations in 25 s, with an amplitude of 5 cm. The additional force applied is

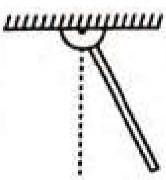
(a) 80 N (b) $80\,\pi^2$ N
(c) $4\,\pi^2$ N (d) 4 N

45. Frequency of a particle executing SHM is 10 Hz. The particle is suspended from a vertical spring. At the highest point of its oscillation the spring is unstretched. Maximum speed of the particle is ($g = 10$ m/s^2)

(a) 2π m/s (b) π m/s
(c) $1/\pi$ m/s (d) $1/2\pi$ m/s

Compound Pendulum and Superposition of SHM

46. A metre stick swinging in vertical plane about a fixed horizontal axis passing through its one end undergoes small oscillation of frequency f_0. If the bottom half of the stick were cut off, then its new frequency of small oscillation would become

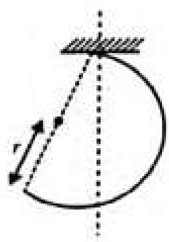


(a) f_0 (b) $\sqrt{2}f_0$ (c) $2f_0$ (d) $2\sqrt{2}f_0$

47. A physical pendulum is positioned so that its centre of gravity is above the suspension point. When the pendulum is released it passes the point of stable equilibrium with an angular velocity ω. The period of small oscillations of the pendulum is

(a) $\dfrac{4\pi}{\omega}$ (b) $\dfrac{2\pi}{\omega}$ (c) $\dfrac{\pi}{\omega}$ (d) $\dfrac{\pi}{2\omega}$

48. A uniform semicircular ring having mass m and radius r is hanging at one of its ends freely as shown in figure. The ring is slightly disturbed so that it oscillates in its own plane. The time period of oscillation of the ring is



(a) $2\pi\sqrt{\dfrac{r}{g\left(1+1/\pi^2\right)}}$

(b) $2\pi\sqrt{\dfrac{r}{g\left(1-4/\pi^2\right)^{1/2}}}$

(c) $2\pi\sqrt{\dfrac{r}{g\left(1-2/\pi^2\right)^{1/2}}}$

(d) $2\pi\sqrt{\dfrac{2r}{g\left(1+4/\pi\right)^{1/2}}}$

49. A system of two identical rods (L–shaped) of mass m and length l are resting on a peg P as shown in the figure. If the system is displaced in its plane by a small angle, find the period of oscillations.

(a) $2\pi\sqrt{\dfrac{\sqrt{2}\,l}{3g}}$ (b) $2\pi\sqrt{\dfrac{2l}{3g}}$

(c) $2\pi\sqrt{\dfrac{2\sqrt{2}\,l}{3g}}$ (d) $3\pi\sqrt{\dfrac{l}{3g}}$

50. A uniform square plate of side 'a' is hinged at one of its corners. It is suspended such that it can rotate about horizontal axis. The time period of small oscillation about its equilibrium position.

(a) $2\pi\sqrt{\dfrac{2a}{3g}}$ (b) $2\pi\sqrt{\dfrac{\sqrt{2}a}{3g}}$

(c) $2\pi\sqrt{\dfrac{2\sqrt{2}a}{3g}}$ (d) $2\pi\sqrt{\dfrac{2\sqrt{2}a}{g}}$

51. A particle is acted simultaneously by mutually perpendicular simple harmonic motion $x = a\cos\omega t$ and $y = a\sin\omega t$. The trajectory of motion of the particle will be

(a) an ellipse (b) a parabola
(c) a circle (d) a straight line

52. A particle is executing a motion in which its displacement as a function of time is given by
$x = 3\sin(5\pi t + \pi/3) + \cos(5\pi t + \pi/3)$ where x is in m and t is in s. Then the motion is

(a) simple harmonic with time period 0.2 s
(b) simple harmonic with time period 0.4 s
(c) simple harmonic with amplitude 3 m
(d) not a simple harmonic but a periodic motion

53. Two particles A and B execute simple harmonic motion according to the equations $y_1 = 3\sin\omega t$ and $y_2 = 4\sin\left(\omega t + \dfrac{\pi}{2}\right) + 3\sin\omega t$. The phase difference between them is

(a) $\dfrac{\pi}{2}$ (b) $\tan^{-1}\left(\dfrac{4}{3}\right)$

(c) $\tan^{-1}\left(\dfrac{3}{4}\right)$ (d) None of these

54. A charged particle is deflected by two mutually perpendicular oscillating electric fields such that the displacement of the particle due to each one of them is given by $x = A\sin(\omega t)$ and $y = A\sin\left(\omega t + \frac{\pi}{6}\right)$ respectively. The trajectory followed by the charged particle is
(a) a circle with equation $x^2 + y^2 = A^2$
(b) a straight line with equation $y = \sqrt{3}x$
(c) an ellipse with equation $x^2 + y^2 - xy = \frac{3}{4}A^2$
(d) an ellipse with equation $x^2 + y^2 - \sqrt{3}xy = \frac{1}{4}A^2$

55. Four types of oscillatory systems: a simple pendulum; a physical pendulum; a torsional pendulum and a spring–mass system, each of same time period is taken to the Moon. If time periods are measured on the moon, which system or systems will have it unchanged?
(a) only spring–mass system
(b) spring–mass system and torsional pendulum
(c) spring–mass system and physical pendulum
(d) none of these

56. A particle is subjected to two simple harmonic motions in the same direction having equal amplitude and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, what is the phase difference between the two simple harmonic motions?
(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{\sqrt{3}}$ (d) $\frac{2\pi}{\sqrt{3}}$

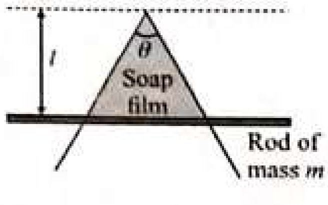
57. Three simple harmonic motions of equal amplitudes A and equal time periods along the same line combine. The phase of the second motion is 60° ahead of the first and phase of the third motion is 60° ahead of the second. The amplitude of resultant motion is
(a) $3A$ (b) $2\sqrt{2}\,A$
(c) $\sqrt{3}\,A$ (d) $2A$

Problems Based on Mixed Concepts

58. The string of a simple pendulum is replaced by a uniform rod of length L and mass M while the bob has a mass m. It is allowed to make small oscillations. Its time period is
(a) $2\pi\sqrt{\left(\frac{2M}{3m}\right)\frac{L}{g}}$ (b) $2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$
(c) $2\pi\sqrt{\left(\frac{M+m}{M+3m}\right)\frac{L}{g}}$ (d) $2\pi\sqrt{\left(\frac{2m+M}{3(M+2m)}\right)\frac{L}{g}}$

59. A cork floating on the pond water executes a simple harmonic motion, moving up and down over a range of 4 cm. The time period of the motion is 1 s. At $t = 0$, the cork is at its lowest position of oscillation, the position and velocity of the cork at $t = 10.5$ s, would be
(a) 2 cm above the mean position, 0 m/s
(b) 2 cm below the mean position, 0 m/s
(c) 1 cm above the mean position, $2\sqrt{3}\,\pi$ m/s up
(d) 1 cm below the mean position, $2\sqrt{3}\,\pi$ m/s up

60. A wire is bent at an angle θ. A rod of mass m can slide along the bended wire without friction as shown in figure. A soap film is maintained in the frame kept in a vertical position and the rod is in equilibrium as shown in the figure. If rod is displaced slightly in vertical direction, then the time period of small oscillation of the rod is

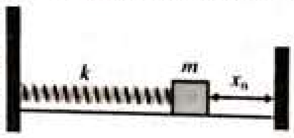


(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{l\cos\theta}{g}}$
(c) $2\pi\sqrt{\frac{l}{g\cos\theta}}$ (d) $2\pi\sqrt{\frac{l}{g\tan\theta}}$

61. A solid right circular cylinder of weight 10 kg and cross-sectional area 100 cm² is suspended by a spring, where $k = 1$ kg/cm, and hangs partially submerged in water of density 1000 kg/m³ as shown in figure. What is its period when it makes simple harmonic vertical oscillations? (Take $g = 10$ m/s²)

(a) 0.6 s (b) 1 s
(c) 1.5 s (d) 2.2 s

62. One end of a spring of force constant k is fixed to a vertical wall and the other to a body of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the body. The spring is then compressed by $3x_0$ and released. The time taken to strike the wall from the instant of release is (given $\sin^{-1}(1/3) = (\pi/9)$)



(a) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$ (b) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$
(c) $\frac{\pi}{4}\sqrt{\frac{m}{k}}$ (d) $\frac{11\pi}{9}\sqrt{\frac{m}{k}}$

63. A particle is moving along the x-axis under the influence of a force given by $F = -5x + 15$. At time $t = 0$, the particle is located at $x = 6$ and is having zero velocity. It takes 0.5 seconds to reach the origin for the first time. The equation of motion of the particle can be represented by

(a) $x = 3 + 3\cos \pi t$ (b) $x = 3\cos \pi t$
(c) $x = 3 + 3\sin \pi t$ (d) $x = 3 + 3\cos (2\pi t)$

64. A particle moves along a straight line to follow the equation $ax^2 + bv^2 = k$, where a, b and k are constants and x and v are x-coordinate and velocity of the particle respectively. Find the amplitude.

(a) $\sqrt{\frac{k}{b}}$ (b) $\sqrt{\frac{b}{k}}$
(c) $\sqrt{\frac{a}{k}}$ (d) $\sqrt{\frac{k}{a}}$

65. A particle starts its SHM on a line at initial phase of $\pi/6$. It reaches again the point of start after time t. It crosses yet another point P on the same line at successive intervals $2t$ and $3t$ respectively. Find the amplitude of the motion, if the particle crosses the point of start at speed 2 m/s:

(a) $\frac{8t}{\pi}$ (b) $\frac{4t}{\pi}$
(c) $\frac{2t}{\pi}$ (d) $\frac{t}{\pi}$

66. A particle of mass m is executing oscillations about the origin on the X-axis with amplitude A. Its potential energy is given as $U(x) = bx^4$, where β is a positive constant. The x-coordinate of the particle, where the potential energy is one-third of the kinetic energy, is

(a) $\pm\frac{A}{2}$ (b) $\pm\frac{A}{\sqrt{2}}$
(c) $\pm\frac{A}{3}$ (d) $\pm\frac{A}{\sqrt{3}}$

67. A constant force produces maximum velocity v on the block connected to the spring of force constant k as shown in the figure. When the force constant of spring becomes $4k$, the maximum velocity of the block is (block is at rest when spring is relaxed):

(a) $v/4$ (b) $2v$
(c) $v/2$ (d) v

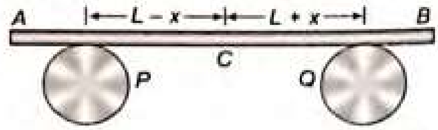
68. An air chamber of volume V, has a long of cross-sectional area A. A ball of mass m is fitted smoothly in the neck. The bulk modulus of air is B. If the ball is pressed down slightly and released, the time period of its oscillation is

(a) $2\pi\sqrt{\frac{mV}{2BA^2}}$ (b) $\pi\sqrt{\frac{2mV}{BA^2}}$
(c) $2\pi\sqrt{\frac{mV}{BA^2}}$ (d) $\frac{\pi}{2}\sqrt{\frac{m}{BA^2}}$

69. From the variation of potential energy in the direction of small oscillation of a simple pendulum, find the effective spring constant for the simple pendulum, where m is mass of the bob, l is length of the simple pendulum.

(a) $\frac{mg}{l}$ (b) $\frac{mg}{2l}$
(c) $\frac{2mg}{l}$ (d) $\frac{mg}{\sqrt{2}l}$

70. Initially cylindrical drums P and Q were placed at equal distance L from center of mass C of the rough rod AB in horizontal position. The drums were spinning in opposite directions with angular velocities. The rod is displaced by distance x towards left and released so that it performs SHM. If difference in reactions at Q and P is $\frac{mgx}{L}$, where m is the mass of the rod, find the time period of oscillations, if m is the coefficient of friction:



(a) $2\pi\sqrt{\frac{L}{g}}$ (b) $2\pi\sqrt{\frac{(L-x)}{g}}$
(c) $2\pi\sqrt{\frac{(L+x)}{\mu g}}$ (d) $2\pi\sqrt{\frac{L}{\mu g}}$

ARCHIVES

1. If a spring has time period T and is cut into n equal parts, then the time period of each part will be

(a) $T\sqrt{n}$ (b) $\frac{T}{\sqrt{n}}$
(c) nT (d) T **(AIEEE 2002)**

2. In a simple harmonic oscillator, at the mean position,

(a) kinetic energy is minimum and potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum and potential energy is minimum
(d) both kinetic and potential energies are minimum **(AIEEE 2002)**

3. A child swinging on a swing in sitting position stands up, then the time period of the swing will

(a) increase

(b) decrease
(c) remain the same
(d) increase if the child is long and decrease if the child is short **(AIEEE 2002)**

4. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes simple harmonic motion (SHM) of time period T. If the mass is increased by m, the time period becomes $5T/3$. Then the ratio m/M is
(a) 3/5 (b) 25/9
(c) 16/9 (d) 5/3 **(AIEEE 2003)**

5. The length of a simple pendulum executing SHM is increased by 21%. The percentage increase in the time period of the pendulum of increased length is
(a) 11% (b) 21%
(c) 42% (d) 10% **(AIEEE 2003)**

6. The displacement of a particle varies according to the relation $X = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is
(a) −4 (b) 4
(c) $4\sqrt{2}$ (d) 8 **(AIEEE 2003)**

7. A body executes SHM. The potential energy (PE), the kinetic energy (KE), and the total energy (TE) are measured as a function of displacement X. Which of the following statements is true?
(a) KE is maximum when $X = 0$.
(b) TE is zero when $X = 0$.
(c) KE is maximum when X is maximum.
(d) PE is maximum when $X = 0$. **(AIEEE 2003)**

8. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitudes of A and B is
(a) $\sqrt{k_1/k_2}$ (b) k_1/k_2
(c) $\sqrt{k_2/k_1}$ (d) k_2/k_1 **(AIEEE 2003)**

9. The bob of a simple pendulum executes SHM in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting the frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m^3. What relationship between t and t_0 is true?
(a) $t = t_0$ (b) $t = 4t_0$
(c) $t = 2t_0$ (d) $t = t_0/2$ **(AIEEE 2004)**

10. A particle at the end of a spring executes SHM with a period t_1. While the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then
(a) $T = t_1 + t_2$ (b) $T^2 = t_1^{-2} + t_2^{-2}$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^2 = t_1^2 + t_2^2$
(AIEEE 2004)

11. The total energy of a particle executing SHM is
(a) proportional to x
(b) proportional to $x^{1/2}$
(c) independent of x
(d) proportional to x^2, where x is the displacement from the mean position **(AIEEE 2004)**

12. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos \omega t$ ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to
(a) $\dfrac{m}{\omega_0^2 - \omega^2}$ (b) $\dfrac{1}{m(\omega_0^2 - \omega^2)}$
(c) $\dfrac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\dfrac{m}{\omega_0^2 + \omega^2}$ **(AIEEE 2004)**

13. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then
(a) $\omega_1 = \omega_2$
(b) $\omega_1 > \omega_2$
(c) $\omega_1 < \omega_2$, when damping is small and $\omega_1 > \omega_2$ when damping is large
(d) $\omega_1 < \omega_2$ **(AIEEE 2004)**

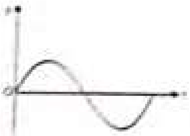
14. The function $\sin^2(\omega t)$ represents
(a) a periodic motion, but not SHM with a period $2\pi/\omega$
(b) a periodic motion, but not SHM with a period π/ω
(c) an SHM with a period $2\pi/\omega$
(d) an SHM with a period π/ω **(AIEEE 2005)**

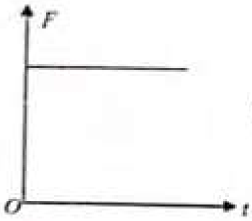
15. Two SHMs are represented by the equations $y_1 = 0.15$ m $[100\pi t + (\pi/3)]$ and $y = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is
(a) $-\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{3}$
(c) $-\dfrac{\pi}{3}$ (d) $\dfrac{\pi}{6}$ **(AIEEE 2005)**

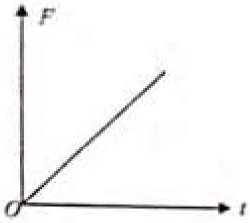
16. If an SHM is represented by $\dfrac{d^2X}{dt^2} + \alpha x = 0$, its time period is
(a) $\dfrac{2\pi}{\alpha}$ (b) $\dfrac{2\pi}{\sqrt{\alpha}}$
(c) $2\pi\alpha$ (d) $2\pi\sqrt{\alpha}$ **(AIEEE 2005)**

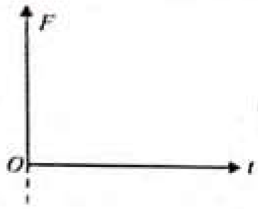
17. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would
(a) first increase and then decrease to the original value
(b) first decrease and then increase to the original value
(c) remain unchanged.
(d) increase towards a saturation value **(AIEEE 2005)**

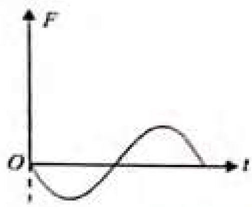
18. The displacement-time graph of a particle executing SHM is shown below. The corresponding force–time graph of the particle is



(a)


(b)


(c)


(d)


(AIEEE 2006)

19. The maximum velocity of a particle executing SHM with an amplitude 7 mm is 4.4 m/s. The period of oscillation is

(a) 0.01 s (b) 10 s
(c) 0.1 s (d) 100 s **(AIEEE 2006)**

20. Starting from origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy?

(a) $\frac{1}{6}$ s (b) $\frac{1}{4}$ s
(c) $\frac{1}{3}$ s (d) $\frac{1}{12}$ s **(AIEEE 2006)**

21. A coin is placed on a horizontal platform which undergoes vertical SHM of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with platform for the first time

(a) at the mean position of platform
(b) for an amplitude g/ω^2
(c) for an amplitude g^2/ω^2
(d) at the highest position of the platform

(AIEEE 2006)

22. Two springs of force constants K_1 and K_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both K_1 and K_2 are made four times their original values, the frequency of oscillation becomes

(a) $f/4$ (b) $4f$
(c) $2f$ (d) $f/2$ **(AIEEE 2007)**

23. A particle of mass m executes SHM with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is

(a) $\frac{1}{4}ma^2\nu^2$ (b) $4\pi^2 ma^2\nu^2$
(c) $2\pi^2 ma^2\nu^2$ (d) $\pi^2 ma^2\nu^2$ **(AIEEE 2007)**

24. The displacement of an object attached to a spring and executing SHM is given by $x = 2 \times 10^{-2} \cos \pi t$ m. The time at which the maximum speed first occurs is

(a) 0.75 s (b) 0.125 s
(c) 0.25 s (d) 0.5 s **(AIEEE 2007)**

25. A point mass oscillates along the x-axis according to the law $x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then

(a) $A = x_0\omega^2, \delta = \frac{\pi}{4}$ (b) $A = x_0\omega^2, \delta = -\frac{\pi}{4}$
(c) $A = x_0\omega^2, \delta = \frac{3\pi}{4}$ (d) $A = x_0, \delta = -\frac{\pi}{4}$

(AIEEE 2007)

26. If x, v, and a denote the displacement, velocity, and acceleration of a particle executing SHM of time period T, then which of the following does not change with time?

(a) $a^2T^2 + 4\pi^2v^2$ (b) $\frac{aT}{x}$
(c) $aT + 2\pi v$ (d) $\frac{aT}{v}$ **(AIEEE 2009)**

27. A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of (A_1/A_2) is

(a) $\frac{M}{M+m}$ (b) $\frac{M+m}{M}$
(c) $\left(\frac{M}{M+m}\right)^{1/2}$ (d) $\left(\frac{M+m}{M}\right)^{1/2}$

(AIEEE 2011)

28. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is:

(a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$ **(AIEEE 2011)**

29. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and cylinder have equal cross sectional area A. When the

piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency

(a) $\frac{1}{2\pi}\frac{V_0MP_0}{A^2\gamma}$ (b) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$ (d) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0M}$

(JEE Main 2013)

30. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s itwill decrease to α times its original magnitude, where α equals.

(a) 0.81 (b) 0.729

(c) 0.6 (d) 0.7 **(JEE Main 2013)**

31. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels $2a$, in same direction, then:

(a) amplitude of motion is $4a$

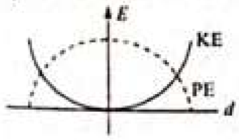
(b) time period of oscillations is 6τ

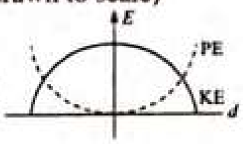
(c) amplitude of motion is $3a$

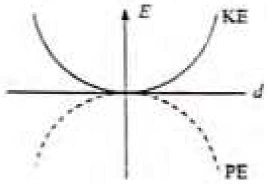
(d) time period of oscillations is 8τ **(JEE Main 2014)**

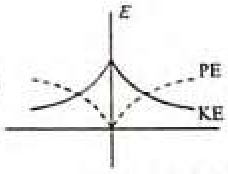
32. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly?

(Graphs are schematic and not drawn to scale)

(a) 

(b) 

(c) 

(d) 

(JEE Main 2015)

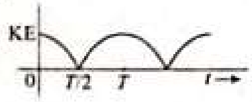
33. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is

(a) $\frac{A}{3}\sqrt{41}$ (b) $3A$

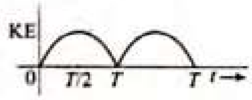
(c) $A\sqrt{3}$ (d) $\frac{7A}{3}$

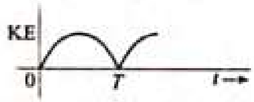
(JEE Main 2016)

34. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like

(a) 

(b) 

(c) 

(d) 

(JEE Main 2017)

35. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avogadro number = 6.02×10^{23} gm mole^{-1})

(a) 5.5 N/m (b) 6.4 N/m

(c) 7.1 N/m (d) 2.2 N/m

(JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	2. (d)	3. (b)	4. (c)	5. (b)	6. (a)	7. (c)	8. (d)	9. (b)	10. (c)
11. (b)	12. (d)	13. (a)	14. (d)	15. (d)	16. (c)	17. (a)	18. (b)	19. (b)	20. (c)
21. (c)	22. (d)	23. (c)	24. (c)	25. (c)	26. (a)	27. (d)	28. (b)	29. (b)	30. (b)
31. (c)	32. (b)	33. (b)	34. (d)	35. (a)	36. (b)	37. (b)	38. (b)	39. (c)	40. (d)
41. (d)	42. (b)	43. (b)	44. (c)	45. (d)	46. (b)	47. (a)	48. (d)	49. (c)	50. (c)
51. (c)	52. (b)	53. (b)	54. (d)	55. (b)	56. (a)	57. (d)	58. (b)	59. (a)	60. (a)
61. (a)	62. (d)	63. (d)	64. (d)	65. (a)	66. (b)	67. (c)	68. (c)	69. (a)	70. (d)

Archives

1. (b)	2. (c)	3. (b)	4. (c)	5. (d)	6. (c)	7. (a)	8. (c)	9. (c)	10. (d)
11. (c)	12. (b)	13. (a)	14. (b)	15. (a)	16. (b)	17. (a)	18. (d)	19. (a)	20. (d)
21. (d)	22. (c)	23. (d)	24. (d)	25. (c)	26. (b)	27. (d)	28. (b)	29. (b)	30. (b)
31. (b)	32. (b)	33. (d)	34. (b)	35. (c)					

Chapter 16

Wave and Acoustics

PROGRESSIVE WAVES

Wave motion is defined as a form of disturbance transferred from one point to another involving transfer of energy but no transfer of matter. Mechanical waves require a medium for propagation, whereas electromagnetic waves can travel in vacuum also (do not require any medium also called non-mechanical waves).

MECHANICAL WAVES

A mechanical wave can be produced and propagated only in those material media which possess *elasticity* and *inertia*. A wave originates due to the displacement of some portion of an elastic medium from its normal position, causing it to oscillate about an equilibrium position, so the medium should posses elasticity in order that it has a tendency to come back to its original position which is a necessary condition for disturbance to be transmitted from one layer to next and hence the wave to travel. The medium should also have inertia in order that it can store energy and transport it further.

TYPE OF WAVES

Transverse Waves

A travelling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a *transverse wave*.

For transverse wave propagating in a taut string, when the displacement of a particle is maximum above the line of mean position, the particle is said to be at crest of the wave, and when the displacement of a particle is maximum below the mean position it is said to be at the trough of the wave.

A particle either at the crest or at the trough has a tendency to move towards the mean position.

- A particle at the crest or the trough has zero velocity, and the distance of the particle from the mean position is termed as amplitude of the wave.
- Distance between two consecutive crests/troughs is equal to the wavelength of the wave.
- Distance between a consecutive pair of crest and trough is half the wavelength of the wave.

Longitudinal Waves

It is a kind of wave motion in which the individual particles of a medium execute periodic motion about their mean position along the direction of propagation of wave.

Sound waves are example of longitudinal waves. The disturbance in a sound wave is a series of high-pressure and low-pressure regions that travels through air.

A longitudinal wave moves by the phenomena of compression and rarefaction in the medium.

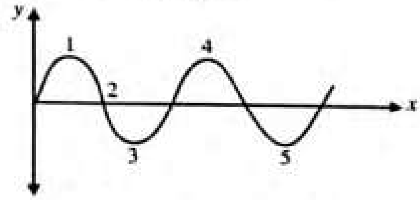
In compression, distance between any two consecutive particles of medium is less than their normal distance. Therefore, density is more than the normal density. In a rarefaction, distance between any two consecutive particles of medium is more than the normal distance, therefore, density is less than the normal density.

Distance between two consecutive compressions or rarefactions is equal to the wavelength of the wave. Longitudinal waves can propagate through any state of matter.

WAVE PARAMETERS

Phase

Phase defines the position (in terms of distance from mean position) and velocity of a particle oscillating under the influence of a wave. The particles of the medium which are in the same state of motion (at the same displacement from their respective mean positions moving in the same direction) are said to be in phase or differing in phase by $2n\pi$, where $n = 1, 2, 3, \ldots$ and the particles of which state of motion are exactly opposite (displacements from the mean position and velocities are exactly opposite) are said to be out of phase or differing in phase by $n\pi$ where $n = 1, 3, 5, \ldots$

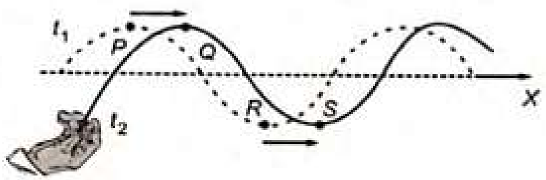


- Oscillations of the particles 1 and 4 are in phase.
- Oscillations of the particles 1 and 2 are not in phase.
- Oscillations of the particles 3 and 5 are in phase.
- Oscillations of the particles 4 and 5 are out of phase.

Wave Speed

Wave speed is the distance travelled by the wave in unit time. In wave motion, the disturbance created at a point of space travels and has a speed. The distance covered per unit time by the wave is known as wave speed.

If we plot the disturbance against position along wave motion at a fixed time, the graph so obtained is called waveform. For a string wave, it is a snapshot of string shape at some moment of time.



The figure shows waveforms at two nearby moments of time t_1 and t_2. The disturbance (say, peak of string) at P has moved to Q in time $t_2 - t_1$. The displacement of wave is $\overrightarrow{PQ}$ in time $\Delta t = t_2 - t_1$. Then

$$v = \text{wave speed} = \frac{PQ}{t_2 - t_1} = \left(\frac{RS}{t_2 - t_1}\right)$$

We see that waveform of t_1 has just translated by PQ, acquiring the new position at t_2.

Intensity of the Wave

Intensity of the wave is the energy transmitted per unit area per second in the form of the wave in the direction of the propagation of the wave by the source. This energy is carried forward by the medium particles which while oscillating transfer the energy to the next particles.

In our further discussion, we assume that as a wave proceeds forward, it does so without any dissipation of energy, i.e., neglecting the effects of air drag, internal resistances, etc. which cause loss of energy as wave progresses. In other words, the amplitude of a simple progressive wave remains same as it progresses forward.

Illustration 16.1 A sinusoidal wave is travelling along a rope. The oscillator that generates wave completes 40.0 vibrations in 30.0 s. Also, a given 'maximum' travels 425 cm along the rope in 10.0 s. What is the wavelength of the wave?

Solution. The given information comprises things that can be measured directly in laboratory. A high speed photograph would reveal the wavelength which can be computed from $v = f\lambda$.

The frequency is

$$f = \frac{40.0 \text{ waves}}{30.0 \text{ s}} = 1.33 \text{ s}^{-1}$$

And the wave speed is

$$v = \frac{425 \text{cm}}{10.0 \text{s}} = 42.5 \text{cm/s}$$

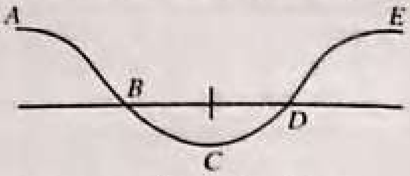
Since $v = \lambda f$, the wavelength is

$$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm}}{1.33 \text{ s}^{-1}} = 0.319 \text{ m}$$

If we turn up the oscillator to a higher frequency, the wavelength would get shorter because speed is a constant factor. It is determined just by the properties of the string itself.

Concept Application Exercise 16.1

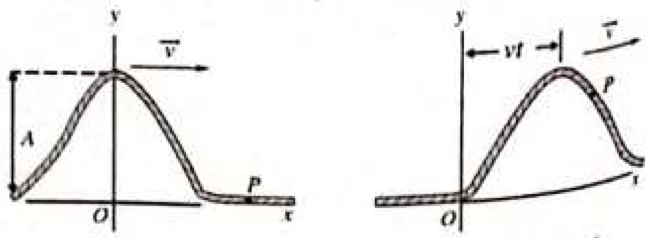
1. Which parts of the curve in the figure shown represent compression and rarefaction for a longitudinal wave?



2. A longitudinal wave is produced on a toy slinky. The wave travels at a speed of 30 cm/s and the frequency of the wave is 20 Hz. What is the minimum separation between two consecutive compressions of the slinky?
3. A narrow pulse (for example, a short pip by a whistle) is sent across a medium. If the pulse rate is 1 after every 20 s (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz?
4. Waves are generated on a water surface. Calculate the phase difference between two points A and B, when
   (i) A and B lie on the same wave front at a distance of 2λ between them
   (ii) A and B lie on successive crests separated by 1 m
   (iii) A and B lie on successive troughs separated by 1.5 m

WAVE FUNCTION

A function of one variable is such as $x(t)$ is sufficient to describe the motion of a point mass moving along a line. To give a mathematical description of an extended moving object such as a moving pulse, functions which depend on two variables such as x and t are required. Functions which can mathematically represent a moving wave pulse are called wave function.



(a) Pulse at $t = 0$ (b) Pulse at time t

A one-dimensional pulse travelling to the right with a speed v. (a) At $t = 0$, the shape of the pulse is given by $y = f(x)$. (b) At some later time t, the shape remains unchanged and the vertical position of any element of the medium is given by $y = f(x - vt)$

In general, then we can represent the transverse position y for all positions and times, measured in a stationary frame with the origin at O, as

$$y(x, t) = f(x - vt) \qquad \text{(i)}$$

Similarly, if the pulse travels to the left, the transverse position of elements of the string is described by

$$y(x, t) = f(x + vt) \qquad \text{(ii)}$$

The function y, sometimes called the *wave function*, depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read 'y as a function of x and t.'

ILLUSTRATION 16.2 At $t = 0$, transverse pulse in a wire is described by the function

$$y = \frac{6}{x^2 + 3}$$

where x and y are in metres. Write the function $y(x, t)$ that describes this pulse if it is travelling in the positive x-direction with a speed of 4.50 m/s.

Solution. At $t = 0$, the wave pulse looks like a bump centred at $x = 0$. As time goes on, the wave function will be function of t as well as x. The point about which the bump is centred will be $x_0 = 4.5t$.

We obtain a function of the same shape by writing

$$y(x, t) = 6/[(x - x_0)^2 + 3]$$

where the centre of the pulse is at $x_0 = 4.5\,t$. Thus we have

$$y(x, t) = \frac{6}{(x - 4.5t)^2 + 3}$$

Note that for y to stay constant as t increases, x must increase by $4.5\,t$, as it describes the wave moving at 4.5 m/s.

NOTE: In general, we can cause any waveform to move along the x-axis at a velocity n, by substituting $(x - n_s t)$ for x in the wave function $y(x)$ at $t = 0$. A wave function that depends on t through $(x + n_s t)$ describes a wave moving in the negative x-direction.

General Expression for a Sinusoidal Wave

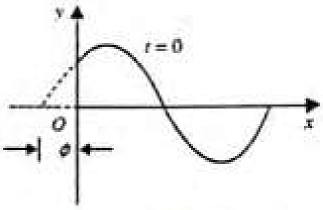
The general equation of simple harmonic progressive wave is given by

$$y = A \sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)$$

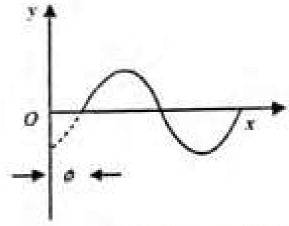
If an initial phase ϕ exists at origination point of wave, equation is changed to $y = A\sin\left[2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right) + \phi\right]$

Positive and Negative Initial Phase Constants

In general, the equation of a harmonic wave travelling along the positive x-axis is expressed as $y = A \sin (kx - \omega t \pm \phi)$, where ϕ is called the initial phase constant. It determines the initial displacement of the particle at $x = 0$ when $t = 0$.



Positive initial phase constant
$y = A \sin(kx - \omega t + \phi)$
The sine curve starts from the left of the origin.



Negative initial phase constant
$y = A \sin(kx - \omega t - \phi)$
The sine curve starts from the right of the origin.

Change in Phase with Time for a Constant x (at a Fixed Point in the Medium)

$$[\phi]_{t_1} = 2\pi\left(\frac{t_1}{T} - \frac{x}{\lambda}\right) + \phi;\ [\phi]_{t_2} = 2\pi\left(\frac{t_2}{T} - \frac{x}{\lambda}\right) + \phi$$

(For the wave travelling in positive x-direction)

$$\Delta\phi = \phi_{t_2} - \phi_{t_1} = \frac{2\pi}{T} \times (t_2 - t_1) = \frac{2\pi}{T} \times \Delta t \quad \Rightarrow \Delta\phi = \frac{2\pi \times \Delta t}{T}$$

$$\text{Phase difference} = \frac{2\pi}{T} \times \text{Time difference}$$

For $\Delta t = T$, i.e., after one time period,

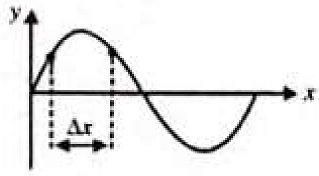
$$\Delta\phi = \frac{2\pi}{T} \times T = 2\pi$$

A phase change of 2π ($2n\pi$ in general) gives the points oscillating in phase.

Variation of Phase with Distance

At a given instant of time $t = t$, phase at $x = x_1$,

$$[\phi]_{x_1} = 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right) + \phi$$



For the wave travelling in positive x-direction and phase at $x = x_2$,

$$[\phi]_{x_2} = 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right) + \phi$$

$$\Rightarrow \quad \Delta\phi = [\phi]_{x_1} - [\phi]_{x_2} = \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi}{\lambda}\Delta x$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

i.e., Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

For $\Delta x = n\lambda$, $\Delta\phi = n2\pi$, where $n = 1, 2, 3, \ldots$

Points separated by distance $n\lambda$ where n is an integer are in same phase (at any instant of time).

For $\Delta x = m\frac{\lambda}{2}$, where $m = 1, 3, 5, 7, \ldots$

$$\Delta\phi = m \times \pi$$

Points separated by distance $m(\lambda/2)$, where m is an odd integer are out of phase (at any instant of time).

ILLUSTRATION 16.3 A wave travelling along X-axis is given by

$y = 2(\text{mm})\sin(3t - 6x + \pi/4)$

where x is in centimetres and t in second. Write the phases and, hence, the find the phase difference between them at $t = 0$ for two points on X-axis, $x = x_1 = \pi/3$ cm and $x = x_2 = \pi/2$ cm.

Solution. At $t = 0$

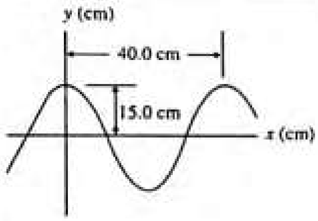
$$y = \sin(-6x + \pi/4)$$

Phase at x_1: $\phi = \left(-6 \times \frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{7}{4}\pi$ and that at

x_2 is $\left(-6 \times \frac{\pi}{2} + \frac{\pi}{4}\right) = \frac{11}{4}\pi$

The phase difference is $\left|\frac{7}{4}\pi - \frac{11}{4}\pi\right| = \pi$; the disturbances at the two points are out of phase.

ILLUSTRATION 16.4 A sinusoidal wave travelling in the positive x-direction has an amplitude of 15 cm, wavelength 40 cm and frequency 8 Hz. The vertical displacement of the medium at $t = 0$ and $x = 0$ is also 15 cm, as shown in figure.



(a) Find the angular wave number, period, angular frequency and speed of the wave.

(b) Determine the phase constant ϕ, and write a general expression for the wave function.

Solution.

(a) Wave number $k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40 \text{ cm}} = \frac{\pi}{20}$ rad/cm

Time period $T = \frac{1}{f} = \frac{1}{8}$ s

Angular frequency $\omega = 2\pi f = 16\pi$ rad/s

Speed of wave $v = f\lambda = 320$ cm/s

(b) It is given that $A = 15$ cm

and also $y = 15$ cm at $x = 0$, and $t = 0$

then using $y = A\sin(\omega t - kx + \phi)$

$15 = 15\sin\phi \Rightarrow \sin\phi = 1$

or $\phi = \frac{\pi}{2}$ rad

Therefore, the wave function is

$$y = A\sin\left(\omega t - kx + \frac{\pi}{2}\right)$$

$$= (15 \text{ cm})\sin\left[(16\pi \text{ rad s}^{-1})t - \left(\frac{\pi \text{ rad}}{20 \text{ cm}}\right)x + \frac{\pi}{2}\right]$$

SINUSOIDAL WAVES ON STRINGS

The wave function of a wave at $t = 0$ can be written as

$$y = A\sin(kx - \omega t) \quad \text{(i)}$$

We can use this expression to describe the motion of any element of the string. An element at point P (or any other element of the string) moves only vertically, and so its x coordinate remains constant. Therefore, the transverse speed v_y (not to be confused with the wave speed v) and the transverse acceleration a_y of elements of the string are

$$v_y = \left.\frac{dy}{dt}\right]_{x=\text{constant}} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t) \quad \text{(ii)}$$

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t) \quad \text{(iii)}$$

These expressions incorporate partial derivatives because y depends on both x and t. In the operation $\partial y/\partial t$, for example, we take a derivative with respect to t while holding x constant. The maximum values of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

$$v_{y,\max} = \omega A \quad \text{(iv)}$$

$$a_{y,\max} = \omega^2 A \quad \text{(v)}$$

The transverse speed and transverse acceleration of elements of the string do not reach their maximum values simultaneously. The transverse speed reaches to its maximum value (ωA) when $y = 0$, whereas the magnitude of the transverse acceleration reaches its maximum value ($\omega^2 A$) when $y = \pm A$. Finally, Eqs. (iv) and (v) are identical in mathematical form to the corresponding equations for simple harmonic motion.

Differentiating Eq. (i) with respect to displacement of wave (x), we will get the slope of displacement curve of wave as

$$\frac{\partial y}{\partial x} = Ak\cos(kx - \omega t) \quad \text{(vi)}$$

From Eqs. (ii) and (vi), we have

$$\frac{\partial y}{\partial x} = -\frac{k}{\omega}\frac{\partial y}{\partial t} = -\frac{1}{v}\frac{\partial y}{\partial t}$$

$$\frac{\partial y}{\partial t} = -v\frac{\partial y}{\partial x} \quad \Rightarrow v_p = -v\frac{dy}{dx}$$

NOTE: Particle velocity = – (wave velocity) slope of wave curve

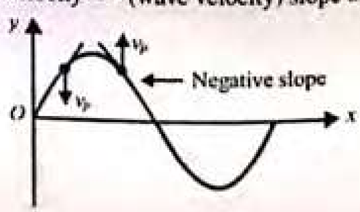


ILLUSTRATION 16.5 If the displacement relation for a particle in a wave is given by $y = 5\sin\left(\frac{t}{0.04} - \frac{x}{4}\right)$, determine the maximum speed of the particle in SI units.

Solution. Particle speed $v_p = \left|\frac{\partial y}{\partial t}\right|$

$$v_p = \left|5 \times \frac{1}{0.04}\cos\left(\frac{t}{0.04} - \frac{x}{4}\right)\right| \text{ms}^{-1}$$

The maximum particle speed is, thus,

$$\left|\frac{\partial y}{\partial t}\right|_{max} = \frac{5}{0.04}\text{ms}^{-1} = 125 \text{ ms}^{-1}$$

ILLUSTRATION 16.6 A plane progressive wave is given by $x = (40 \text{ cm})\cos(50\pi t - 0.02\pi y)$ where y is in centimetres and t in seconds. What will be the particle velocity at $y = 25$ cm in time $t = 1/200$s?

Solution. Given that $x = 40\cos(50\pi t - 0.02\pi y)$

Therefore, particle velocity can be given as

$$v_p = \frac{dx}{dt} = (40 \times 50\pi)[-\sin(50\pi t - 0.02\pi y)]$$

Putting $x = 25$ cm and $t = \frac{1}{200}$ s.

$$v_p = -(2000\pi \text{ cm/s})\sin\left[50\pi\left(\frac{1}{200}\right) - 0.02\pi(25)\right]$$

$$= 10\pi\sqrt{2} \text{ m/s}$$

SPEED OF WAVES ON STRING

If the tension in the string is T and its mass per unit length is μ (Greek letter mu), the wave speed, will be

$$v = \sqrt{\frac{T}{\mu}}$$

ILLUSTRATION 16.7 A transverse wave of wavelength 50 cm is travelling towards +ve x-axis along a string whose linear density is 0.05 g/cm. The tension in the string is 450 N. At $t = 0$, the particle at $x = 0$ is passing through its mean position with an upward velocity. Form an equation describing the wave. The amplitude of the wave is 2.5 cm.

Solution. Let the wave be described by:

$$y(x,t) = A\sin(kx - \omega t + \phi_0) \quad \text{and} \quad A = 2.5 \text{ cm}$$

where $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.5} = 4\pi$

$$y(0,0) = 0 \Rightarrow A\sin\phi_0 = 0$$

which gives $\phi_0 = 0, \pi$ (i)

we also have

$$\left(\frac{dy}{dt}\right)_{t=0} > 0 \Rightarrow -A\omega\cos\phi_0 > 0 \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $\phi_0 = \pi$

Velocity of transverse wave in the string is given by

$$\sqrt{\frac{T}{\mu}} = \sqrt{\frac{450}{0.05\times10^{-1}}} = 300 \text{ m/s}$$

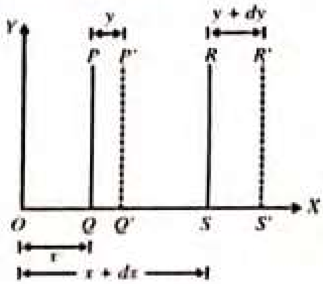
$$\Rightarrow \quad \omega = 2\pi f = \frac{2\pi c}{\lambda} = \frac{2\pi(300)}{0.5} = 1200\pi \text{ rad/s}$$

Using all the quantities, the equation is

$$y = 2.5\sin(4\pi x - 1200\pi t + \pi) \text{ cm}$$

INTERPRETATION OF dy/dx IN LONGITUDINAL WAVES AND TRANSVERSE WAVES

Consider a longitudinal wave travelling along the positive x-axis. Let us choose two layers of medium PQ and RS with planes normal to the direction of wave propagation, at distance x and $x + dx$ from the origin O. After the wave passes through these sections, the layers get displaced from their mean positions, say by y and $y + dy$ respectively. (as shown in figure) if a be the cross-sectional area of the layer, then, the initial volume of the cylinder $PQRS = a(dx)$



and final volume = volume of cylinder $P'Q'R'S'$

$$= a[x + dx + y + dy) - (x + y)]$$

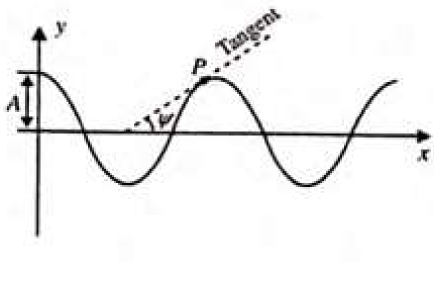
$$= a(dx + dy)$$

$\therefore$ Change in volume $= a\,dy$

$$\therefore \quad \text{Volume strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{a\,dy}{a\,dx} = \frac{dy}{dx}$$

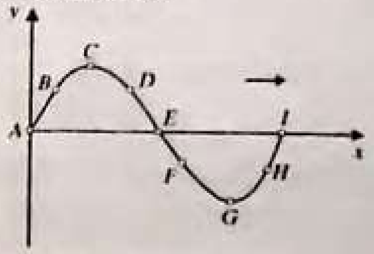
Hence, in case of longitudinal waves dy/dx represents the volume strain.

In case of transverse waves, the term dy/dx has its usual geometrical meaning. If a 'snapshot' be taken, of the particles of a medium at any instant, in which a simple harmonic wave is travelling, then the shape of the waveform will be a sine curve (as shown in figure) dy/dx, at the instant for any particle say P) represents the slope of the curve at that point. Thus, if ψ is be the angle made by the geometrical tangent drawn to the curve at that point, with the positive direction of the x-axis, then $dy/dx = \tan\psi$.



Concept Application Exercise 16.2

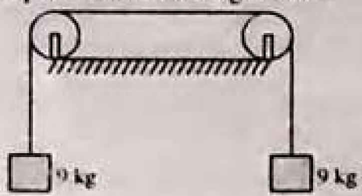
1. A transverse wave is travelling along a string in the positive x-axis. The figure shows the photograph of the wave at an instant. Find



(a) the points moving upward ______
(b) the points moving downwards ______
(c) the points which have zero velocity ______
(d) the points which have maximum velocity ____

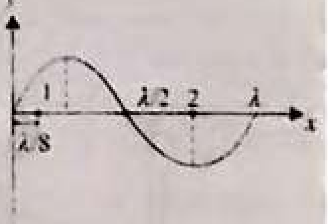
2. The equation of transverse wave travelling on a rope is given by
$y = 5 \sin (4.0t - 0.02x)$
where y and x are in centimetres and time t in seconds
(i) Calculate the amplitude, frequency, velocity, and wavelength.
(ii) Calculate the maximum transverse speed of a particle in the rope.

3. A uniform wire passes over two pulleys and 9 kg weight is suspended at each end. The length of wire between the pulleys is 1.5 m and its mass is 12.0 g. Find the frequency of vibration with which the wire vibrates in two loops, the middle point of wire being at rest.



4. The following equation gives the displacement y at time t for a particle at a distance x:
$y = 0.01 \sin 500\pi (t - x/30)$
where all are in SI units.
Find (i) the wavelength, (ii) the speed of the wave, (iii) the velocity amplitude of the particles of the medium and (iv) the acceleration amplitude of the particles of the medium.

5. What is the phase difference between the particles 1 and 2 located as shown in figure?



6. A circular loop of string rotates about its axis on a frictionless horizontal plane at a uniform rate so that the tangential speed of any particle of the string is v. If a small transverse disturbance is produced at a point of the loop, with what speed (relative to the string) will this disturbance travel on the string?

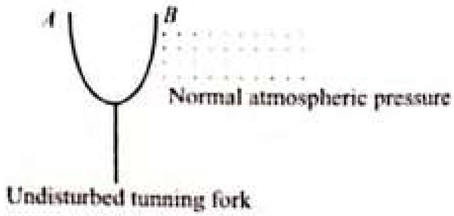
SOUND WAVES

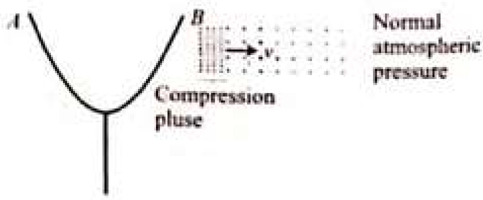
Sound waves travel through any material medium with a speed that depends on the properties of the medium. As sound waves travel through air, the elements of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. The mathematical description of sinusoidal sound waves is very similar to that of sinusoidal waves on strings.

Propagation of Sound Waves

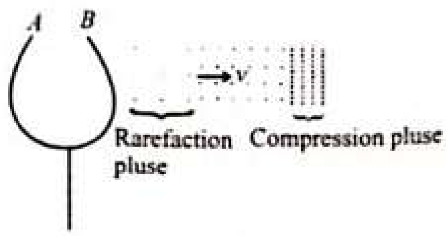
Consider a tuning fork producing sound waves. When prong B moves outward towards right it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a *compression pulse* and it travels away from the prong with the speed of sound.

After producing the compression pulse, prong B reverses its motion and moves inward. This drags away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called a *rarefaction pulse*. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.





If the prongs vibrate in SHM, the pressure variation in the layer close to the prong also varies simple harmonically and hence increase in pressure above normal value can be written as



$$\delta P = \delta P_0 \sin \omega t$$

where δP_0 is the maximum increase in pressure above normal value.

As this disturbance travels towards right with wave velocity v, the excess pressure at any position x at time t will be given

by
$$\delta P = \delta P_0 \sin[\omega(t - x/v)]$$
Using $\Delta p = \delta P$, $\Delta p_{max} = \delta P_0$, the above equation of sound wave can be written as
$$\Delta p = \Delta p_{max} \sin[\omega(t - x/v)]$$

Illustration 16.8 The equation of a sound wave in air is given by $\Delta p = (0.02) \sin[(3000)t - (9.0)x]$, where all variables are in SI units. (a) Find the frequency, wavelength and the speed of sound wave in air. (b) If the equilibrium pressure of air is 1.01×10^5 N/m^2, what are the maximum and minimum pressures at a point as the wave passes through that point?

Solution.

(a) Comparing with the standard form of a travelling wave,
$$\Delta p = \Delta p_{max} \sin[\omega(t - x/v)]$$
we see that $\omega = 3000$ s^{-1}. The frequency is
$$f = \frac{\omega}{2\pi} = \frac{3000}{2\pi} \text{ Hz}$$
Also from the same comparison,
$$\omega/v = 9.0 \text{ m}^{-1}$$
or $$v = \frac{\omega}{9.0 \text{ m}^{-1}} = \frac{3000 \text{ s}^{-1}}{9.0 \text{ m}^{-1}} = \frac{1000}{3} \text{ m/s}$$
The wavelength is
$$\lambda = \frac{v}{f} = \frac{1000/3 \text{ m/s}}{3000/2\pi \text{ Hz}} = \frac{2\pi}{9} \text{ m}$$

(b) The pressure amplitude is $\Delta p_{max} = 0.02$ N/m^2. Hence, the maximum and minimum pressures at a point in the wave motion will be $1.01 \times 10^5 \pm 0.02$ N/m^2.

Speed of Sound Waves

The speed of sound waves in a medium depends on the compressibility and density of the medium. If the medium is a liquid or a gas and has bulk modulus B and density ρ, the speed of sound waves in that medium is
$$v = \sqrt{\frac{B}{\rho}} \quad \text{(i)}$$
It is interesting to compare this expression with the equation for the speed of transverse waves on a string, $v = \sqrt{T/\mu}$. In both cases, the wave speed depends on an elastic property of the medium (bulk modulus B or string tension T) and on an inertial property of the medium (ρ or μ). In fact, the speed of all mechanical waves follows an expression of the general form
$$v = \sqrt{\frac{\text{Elastic property}}{\text{Inertial property}}}$$
For longitudinal sound waves in solid rod of material, for example, the speed of sound depends on Young's modulus Y and the density ρ.

Speed of Sound: Newton's Formula

On the basis of observations, Newton obtained a formula for speed of sound in air as
$$v = \sqrt{\frac{P}{\rho}} \quad \text{(i)}$$
where P is the isothermal elasticity of the air. He argued that when sound propagates through air, the temperature of air remains constant. By Newton's formula the speed of sound at one atmosphere is
$$V = \sqrt{\frac{1.013 \times 10^5}{1.29}} = 280 \text{ m/s}$$
This value is less than experimental value of 332 m/s. Hence Newton's formula requires some correction, which was made by Laplace in 1816.

Laplace's Correction

French scientist Laplace pointed out that when sound propagates in air the heat of the medium remains constant instead of its temperature. So he replaced isothermal elasticity by adiabatic elasticity B_{ad}. The corrected formula is
$$v = \sqrt{\frac{B_{ad}}{\rho}}$$
For adiabatic change, PV^{γ} = constant
Differentiating both sides, we get
$$P(\gamma V^{\gamma-1})\,dV + V^{\gamma}dP = 0$$
$$\Rightarrow \quad \gamma P dV + V dP = 0 \quad \text{or} \quad \frac{dP}{\left(\frac{-dV}{V}\right)} = \gamma P$$
We have $$\frac{dP}{\left(\frac{-dV}{V}\right)} = B_{ad} \quad \therefore \quad B_{ad} = \gamma P \quad \text{(i)}$$
where $\gamma = C_p/C_v$ is the ratio of specific heats. Hence Laplace's formula for the speed of sound in air (gas) is
$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{(ii)}$$
For air γ= 7/4, so the speed of sound in air at STP will be
$$v = \sqrt{\gamma}\sqrt{\frac{P}{\rho}} = \sqrt{\frac{7}{5}} \times 280 = 332 \text{ m/s}$$
This value is in very close agreement with the experimental value.

Factors Affecting Speed of Sound in Gas

Effect of Pressure

The speed of sound in a gas is given by
$$v = \sqrt{\frac{\gamma P}{\rho}}$$

We know that $PV = nRT = \frac{m}{M}RT$

At constant temperature, $P\Delta V = \frac{\Delta m}{M}RT$

$$\therefore \quad P = \frac{\Delta m}{\Delta V}\frac{RT}{M} \quad \text{or} \quad P = \rho\frac{RT}{M}$$

or $\quad \frac{P}{\rho} = \text{constant}$

Therefore, with the change in pressure, the density also changes in such proportion, so that p/ρ remains constant. Hence pressure has no effect on the speed of sound in a gas.

Effect of Density

For two gases of densities ρ_1 and ρ_2 at same pressure with ratios of specific heats γ_1 and γ_2,

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{\rho_2}{\rho_1}}$$

Effect of Temperature

We have, $\frac{P}{\rho} = \frac{RT}{M}$

$$\therefore \quad v = \sqrt{\frac{\gamma RT}{M}} \quad \text{Clearly,} \quad v \propto \sqrt{T}$$

Hence the speed of sound in a gas is proportional to the square root of its absolute temperature.

ILLUSTRATION 16.9 Calculate the velocity of sound in air at NTP. The density of air at NTP is 1.29 g/L. Assume air to be diatomic with $\gamma = 1.4$. Also calculate the velocity of sound in air at 27°C.

Solution. Velocity of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4 \times 1.013 \times 10^5 \text{ N/m}^2}{1.29 \text{ kg/m}^3}} = 331.6 \text{ m/s}$$

$$\left(\text{using } \frac{P}{\rho} = \frac{R}{M}T\right)$$

We can see that the velocity of sound is proportional to the square root of absolute temperature. Hence,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \quad \Rightarrow v_2 = v_1\sqrt{\frac{T_2}{T_1}} = 331.6\sqrt{\frac{273+27}{273}} = 347.6 \text{ m/s}$$

ILLUSTRATION 16.10 Taking the composition of air to be 75% of nitrogen and 25% of oxygen by weight, calculate the velocity of sound through air.

Solution. The molecular weight of a mixture is given by

$$\frac{m_1 + m_2 + m_3 + \cdots}{M} = \frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3} + \cdots$$

$$\therefore \quad \frac{75+25}{M} = \frac{75}{28} + \frac{25}{32}$$

or $\quad M = 28.9$ g

$$\therefore \quad c = \sqrt{\gamma\frac{RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 1000 \times 273}{28.9}} = 331.3 \text{ m/s}$$

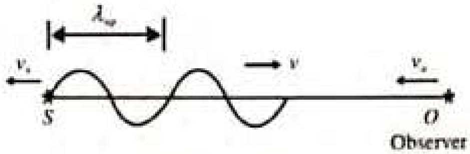
Here $\gamma = 1.4 \rightarrow$ as both the gases are diatomic.

DOPPLER EFFECT

When a car at rest on a road sounds its high frequency horn and you are also standing on the road nearby, you will hear the sound of same frequency it is sounding but when the car approaches you with its horn sounding, the pitch (frequency) of its sound seams to drop as the car passes. This phenomenon was first described by the Austrian scientist Christien Doppler and is called the Doppler effect.

Moving Source and Moving Observer

Let us consider the situation when both source and observer are moving in same direction as shown in figure at speeds v_s and v_0, respectively.



In this case, the apparent wavelength emitted by the source behind it is given as

$$\lambda_{ap} = \frac{v + v_S}{n_0} \quad \text{...(i)}$$

Now, this wavelength will approach the observer at relative speed $v + v_0$. Thus the apparent frequency of sound heard by the observer is given as

$$n_{ap} = \frac{v + v_o}{\lambda_{ap}} = n_0\left(\frac{v + v_o}{v + v_s}\right) \quad \text{...(ii)}$$

By looking at the expression of apparent frequency given by Eq. (ii), we can easily develop a general relation for finding the apparent frequency heard by a moving observer due to moving source as

$$n_{ap} = n_0\left[\frac{v \pm v_o}{v \pm v_s}\right] \quad \text{...(iii)}$$

Here + and – signs are chosen according to the direction of motion of source and observer. The sign convention related to the direction of motion can be stated as follows:

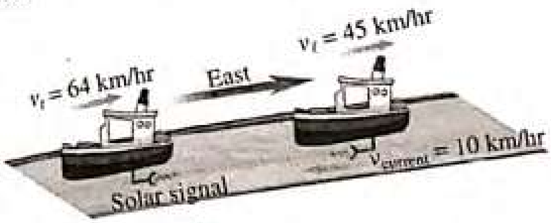
i. For both source and observer, v_0 and v_s are taken in Eq. (iii) with –ve sign if they are moving in the direction of v, i.e., the direction of propagation of sound from source to observer.

ii. For both source and observer, v_o and v_s are taken in Eq. (iii) with +ve sign if they are moving in the direction

opposite to v, i.e., opposite to the direction of propagation of sound from source to observer.

ILLUSTRATION 16.11 Two ships are moving along a line due east. The trailing ship has a speed relative to land-based observation point of 64.0 km/h, and the leading ship has a speed of 45.0 km/h relative to that point. The two ships are in a region of the ocean where the current is moving uniformly due west at 10.0 km/h. The trailing ship transmits a sonar signal at a frequency of 1200.0 Hz. What frequency is monitored by the leading ship? Use 1520 m/s as the speed of sound in ocean water.

Solution.



When the observer is moving in front of and in the same direction as the source, the Doppler equation becomes

$$f' = \left(\frac{v+(-v_l)}{v-v_t}\right)f$$

where v_l and v_t are the speeds of the leading and trailing ships measured relative to the medium in which the sound is propagated. In this case the ocean current is opposite to the direction of travel of the ships and

$$v_l = 45.0 \text{ km/h} - (-10.0 \text{ km/h}) = 55.0 \text{ km/h} = 15.3 \text{ m/s}$$
$$v_t = 64.0 \text{ km/h} - (-10.0 \text{ km/h}) = 74.0 \text{ km/h} = 20.6 \text{ m/s}$$

Therefore,

$$f' = (1200 \text{ Hz})\left(\frac{1520 \text{ m/s} - 15.3 \text{ m/s}}{1520 \text{ m/s} - 20.6 \text{ m/s}}\right) = 1204 \text{ Hz}$$

Doppler Effect in Reflected Sound

When a car is moving towards a stationary wall as shown in figure. If the car sounds a horn, wave travels towards the wall and is reflected form the wall. When the reflected wave is heard by the driver, it appears to be of relatively high pitch. If we wish to measure the frequency of reflected sound then the problem must be handled in two steps.

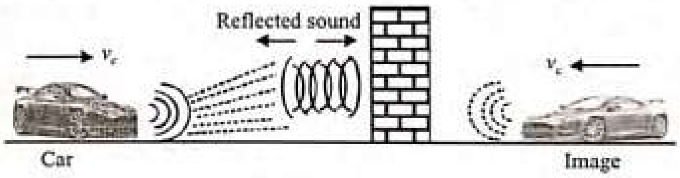
First, we treat the stationary wall as a stationary observer and car as a moving source of sound of frequency n_0. In this case, the frequency received by the wall is given as

$$n_1 = n_0\left(\frac{v}{v-v_c}\right)$$

Now, the wall reflects this frequency and behaves like a stationary source of sound of frequency n_1 and car (driver) behaves like a moving observer moving with velocity v_c. Here the apparent frequency heard by the car driver can be given as

$$n_{ap} = n_1\left(\frac{v+v_c}{v}\right)$$
$$= n_0\left(\frac{v}{v-v_c}\right)\times\left(\frac{v+v_c}{v}\right) = n_0\left(\frac{v+v_c}{v-v_c}\right)$$

Same problem can also be solved in a different manner by using method of sound images. In this procedure, we assume the image of the sound source behind the reflector. In previous example, we can explain this by a situation shown in figure.



Here we assume that the sound which is reflected by the stationary wall is coming from the image of car which is at

Some Important Cases of Doppler Effect

Situation	Apparent frequency	Apparent wavelength
$S \xrightarrow{v_s}$ $\quad O \xrightarrow{v_o}$ $\quad \xrightarrow{} v$	$f' = f\left(\frac{v-v_o}{v-v_s}\right)$	$\lambda' = \frac{(v-v_s)}{f} = \lambda\left(\frac{v-v_s}{v}\right)$
$\xrightarrow{} v$ $\quad \xrightarrow{v_s}$ $\quad \xleftarrow{v_o}$	$f' = f\left(\frac{v+v_o}{v-v_s}\right)$	$\lambda' = \lambda\left(\frac{v-v_s}{v}\right)$
$\xrightarrow{} v$ $\quad \xleftarrow{v_s}$ $\quad \xrightarrow{v_o}$	$f' = f\left(\frac{v-v_o}{v+v_s}\right)$	$\lambda' = \lambda\left(\frac{v+v_s}{v}\right)$
$\xrightarrow{} v$ $\quad \xleftarrow{v_s}$ $\quad \xleftarrow{v_o}$	$f' = f\left(\frac{v+v_o}{v+v_s}\right)$	$\lambda' = \lambda\left(\frac{v+v_s}{v}\right)$

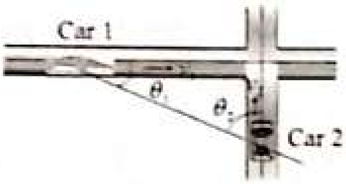
the back of it and coming towards it with velocity v_r. Now the frequency of sound heard by car driver can directly be given as

$$n_{ap} = n_0 \left[\frac{v + v_r}{v - v_r}\right]$$

This method of images for solving problem of Doppler effect is very convenient but is used only for velocities of source and observer which are very small compared to the speed of sound and it should not be used frequently when the reflector of sound is moving.

Doppler's Effect When Source and Observer are not in Same Line of Motion

Consider the situation shown in figure. Two cars 1 and 2 are moving along perpendicular roads at speeds v_1 and v_2. When car 1 sounds a horn of frequency n_0, it emits sound in all directions and say car 2 is at the position, shown in figure when it receives the sound. In such cases, we use velocity components of the cars along the line joining the source and observer. Thus, the apparent frequency of sound heard by car 2 can be given as



$$n_{ap} = n_0 \left[\frac{v + v_2 \cos\theta_2}{v - v_1 \cos\theta_1}\right] \qquad \text{(xiii)}$$

ILLUSTRATION 16.12 A train approaching a railway crossing at a speed of 120 km/h sounds a short whistle at frequency 640 Hz when it is 300 m away from the crossing. The speed of sound in air is 340 m/s. What will be the frequency heard by a person standing on a road perpendicular to the track through the crossing at a distance of 400 m from the crossing?

Solution. The observer A is at rest with respect to the air and the source is travelling at a velocity of 120 km/h, i.e., (100/3) m/s. As is clear from the figure, the person receives the sound of the whistle in a direction BA making an angle θ with the track where $\cos\theta = 300/500 = 3/5$. The component of the velocity of the source (i.e., of the train) along the direction AB is (100/3) × (3/5) m/s = 20 m/s. As the source is approaching the person with this component, the frequency heard by the observer is

$$n' = \frac{v}{v - u\cos\theta}\, n = \frac{340}{340 - 20} \times 640 \text{ Hz} = 680 \text{ Hz}$$

NOTE: Suppose a source of sound moves towards a stationary observer with a speed v'. Then the observed frequency is

$$f' = f\left(\frac{v}{v - v'}\right)$$

Now if the observer moves towards the stationary source with the same speed v', then the observed frequency is

$$f'' = f'\left(\frac{v + v'}{v}\right)$$

From the above equation, it is clear that the observed frequency in two cases is different, although the relative speed between them is same. For this season, the Doppler effect in sound is said to be asymmetric. However, the Doppler effect in light is symmetric. This is because the observed frequency or wavelength depends only on relative speed between source and observer.

CONCEPT APPLICATION EXERCISE 16.3

1. A source of sound with frequency 1000 Hz moves at right angles to a wall with a velocity u = 17 cm/s. Two stationary receivers R_1 and R_2 are located on a straight line coinciding with the path of the source in the following succession: R_1 → source → R_2 → wall. Which receiver registers beating and what is the beat frequency? The velocity of sound is c = 340 m/s.
2. A whistle of frequency 540 Hz rotates in a horizontal circle of radius 2 m at an angular speed of 15 rad/s. What is the lowest and the highest frequency heard by a listener a long distance away at rest with respect to the centre of the circle? (Velocity of sound in air is c = 330 m/s.)
3. A bat flies perpendicularly towards a wall with a speed of 6 m/s, emitting sound of frequency 450 kHz. What is the frequency of the wave reflected from the wall that it will hear? Given, c = 340 m/s.
4. The ratio of the apparent frequencies of a car when approaching and receding a stationary observer is 11:9. What is the speed of the car, if the velocity of sound in air is 330 m/s?
5. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad/s in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle (speed of sound is 330 m/s).
6. A siren emitting a sound of frequency 2000 Hz moves away from you towards a cliff at a speed of 8 m/s.
   (a) What is the frequency of the sound you hear coming directly from the siren.
   (b) What is the frequency of sound you hear reflected off the cliff. Speed of sound in air is 330 m/s.

SUPERPOSITION AND INTERFERENCE

If two or more travelling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave function of the individual waves.

In other words, the wave function $y(x, t)$ that describes the resulting motion in this situation is obtained by adding the two wave functions for the two separate waves:

$y(x, t) = y_1(x, t) + y_2(x, t)$

Waves that obey this principle are called linear waves. In the case of mechanical wave, linear waves are generally characterized by having amplitudes much smaller than their wavelengths. Waves that violate the superposition principle are called non-linear waves and are often characterized by large amplitudes. In this book, we deal only with linear waves.

One consequence of the superposition principle is that two travelling waves can pass through each other without being destroyed or even altered. For instance, when two pebbles are thrown into a pond and hit the surface at different locations, the expanding circular surface waves from the two locations do not destroy each other but rather pass through each other. The resulting complex pattern can be viewed as two independent sets of expanding circles.

Superposition of Sinusoidal Waves

Let us now apply the principle of superposition to two sinusoidal waves travelling in the same direction in a linear medium. If the two waves are travelling to the right and have the same frequency, wavelength, and amplitude but differ in phase, we can express their individual wave functions as

$$y_1 = A\sin(kx - \omega t),\quad y_2 = A\sin(kx - \omega t + \phi)$$

where, as usual $k = 2\pi/\lambda$, $\omega = 2\pi f$, and ϕ is the phase constant. Hence, the resultant wave function y is

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

To simplify this expression, we use the trigonometric identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

Letting $a = kx - \omega t$ and $b = kx - \omega t + \phi$, we find that the resultant wave function y reduces to

$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

The result has several important features.

The resultant wave function y is also sinusoidal and has the same frequency and wavelength as the individual waves because the sine function incorporates the same values of k and ω that appear in the original wave functions.

ILLUSTRATION 16.13 Two travelling sinusoidal waves described by the wave functions

$$y_1 = (5.00\text{ m})\sin[\pi(4.00x - 1200t)]$$

and

$$y_2 = (5.00\text{ m})\sin[\pi(4.00x - 1200t - 0.250)]$$

where x, y_1 and y_2 are in metres and t is in seconds. (a) what is the amplitude of the resultant wave? (b) What is the frequency of resultant wave?

Solution. Graphs for these wave functions are like continuous sine curves. Think of water waves produced by continuously vibrating pencils on the water surface and seen through the side wall of an aquarium.

The pair of waves travelling in the same direction, perhaps separately created, combine to give a single travelling wave.

We can represent the waves symbolically as

$$y_1 = A_0\sin(kx - \omega t)\quad\text{and}\quad y_2 = A_0\sin(kx - \omega t - \phi)$$

with $A_0 = 5.00$ m, $\omega = 1200\pi\ \text{s}^{-1}$ and $\phi = 0.250\pi$ rad

According to the principle of superposition, the resultant wave function has the form

$$y = y_1 + y_2 = 2A_0\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t - \frac{\phi}{2}\right)$$

(a) with amplitude $A = 2A_0\cos\left(\frac{\phi}{2}\right)$

$$= 2(5.00)\cos\left(\frac{\pi}{8.00}\right) = 9.24\text{ m}$$

(b) and frequency $\phi = \frac{\omega}{2\pi} = \frac{1200\pi}{2\pi} = 600$ Hz

Interference of Waves

Mathematical Analysis of Interference

Consider two harmonic waves of same frequency (coherent waves). Suppose A_1 and A_2 be the amplitudes of the waves and ϕ is the phase difference between them. It is assumed that the waves are plane and move almost along a line. Thus wave equations are

$$y_1 = A_1\sin(kx - \omega t) \quad \text{(i)}$$

or $$y_2 = A_2\sin(kx - \omega t + \phi) \quad \text{(ii)}$$

here $\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$

$$y = y_1 + y_2$$
$$= A_1\sin(kx - \omega t) + [A_2\sin(kx - \omega t)\cos\phi + A_2\cos(kx - \omega t)\sin\phi]$$
$$= (A_1 + A_2\cos\phi)\sin(kx - \omega t) + A_2\sin\phi\cos(kx - \omega t)$$

Let $A_1 + A_2\cos\phi = R\cos\theta$ (iii)

and $A_2\sin\phi = R\sin\theta$ (iv)

Squaring and adding Eqs. (iii) and (iv), we get

$$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi \quad \text{(v)}$$

As $I \propto A^2$, $\therefore I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$ (vi)

Also $$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi} \quad \text{(vii)}$$

The resultant wave becomes

$$y = R\sin[(kx - \omega t) + \theta] \quad \text{(viii)}$$

clearly the resultant wave has the same frequency and speed as the interfering waves

Types of Interferences

i. **Constructive interference:** For constructive interference the resultant intensity will be greater than the sum of the individual intensities of the waves. For maximum intensity using Eq. (vi), we get

$$\cos\phi = 1$$

or $\phi = 2\pi n$, when $n = 0, 1, 2, 3, \ldots$

As 2π phase difference is equal to a path difference λ, so

$\Delta x = n\lambda$

Thus from Eq. (v) $R_{max} = A_1 + A_2$

ii. Destructive interference: If two waves are moving to the medium such that they are out of phase. The resultant amplitude of the wave may be less than the sum of individual amplitudes of the waves. For minimum amplitude using Eq. (v)

$\cos\phi = -1$

or $\phi = (2n-1)\pi$, where $n = 1, 2, 3, \ldots$

and $\Delta x = (2n-1)\lambda/2$

Thus from Eq. (v)

$R_{min} = A_1 - A_2$

The ratio of maximum to minimum intensities

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} \quad \text{(ix)}$$

NOTE: The distance between maximum and next minima = l/2.

Conditions of sustained interference: Mathematically, interference phenomenon can take place between two waves of same frequency and different amplitudes. But for observable interference, the amplitude of the waves should be equal. In this case,

$A_1 = A_2 = A$

$\therefore \quad R^2 = A^2 + A^2 + 2AA\cos\phi$

or $\quad R^2 = 2A^2(1 + \cos\phi) = 2A^2 \times 2\cos^2\dfrac{\phi}{2}$

$$R^2 = 4A^2\cos^2\frac{\phi}{2} \quad \text{(x)}$$

Write $R^2 = I$ and $4A^2 = I_0$

$$\therefore \quad I = I_0\cos^2\frac{\phi}{2} \quad \text{(xi)}$$

From Eq. (xi), the maximum intensity is $4A^2$ and minimum intensity is zero. In the phenomenon of interference, the energy is not destroyed but is only redistributed from the positions of minimum intensity to those of maximum intensity.

ILLUSTRATION 16.14 Two identical sources of sound S_1 and S_2 produce intensity I_0 at a point P equidistant from each source.

i. Determine the intensity of each source at the point P.
ii. If the power of S_1 is reduced to 64% and phase difference between the two sources is varied continuously, then determine the maximum and minimum intensities at the point P.
iii. If the power of S_1 is recued by 64%, then determine the maximum and minimum intensities at the point P.

Solution.

i. Both the sources produce maximum at the point P.

Thus, $I_{max} = I_0 = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Since the sources are identical, therefore, $I_1 = I_2 = I$.

$I_0 = 4I$ or $I = I_0/4$

ii. Now $I_1 = 0.64\,I = 0.16\,I_0$

And $I_2 = I = 0.25\,I_0$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 0.81 I_0$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0.01 I_0$

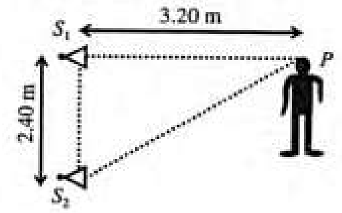
iii. Now $I_1 = (1 - 0.64)I_0 = 0.36\,I = 0.09 I_0$

And $I_2 = I = 0.25\,I_0$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 0.64 I_0$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0.04 I_0$

ILLUSTRATION 16.15 Two stereo speakers S_1 and S_2 are separated by a distance of 2.40 m. A person (P) is at a distance of 3.20 m directly in front of one of the speakers as shown in figure. Find the frequencies in the audible range (20–20,000 Hz) for which the listener will hear a minimum sound intensity. Speed of sound in air = 320 m/s.



Solution. Waves from S_1 to S_2 reach at point P.

Therefore, the path difference between waves reaching from S_1 to S_2 will be $\Delta = S_2P - S_1P$

Given $S_1P = 3.20$ m

$S_2P = \sqrt{(2.40)^2 + (3.20)^2} = 4.0$ m

$\therefore \quad \Delta = 4.0 - 3.20 = 0.80$

For minima, path difference

$$\Delta = (2r-1)\frac{\lambda}{2};\ r = 1, 2, 3, \ldots$$

We have $\lambda = v/n$, where n is the frequency

$$\Delta = (2r-1)\frac{v}{2n}$$

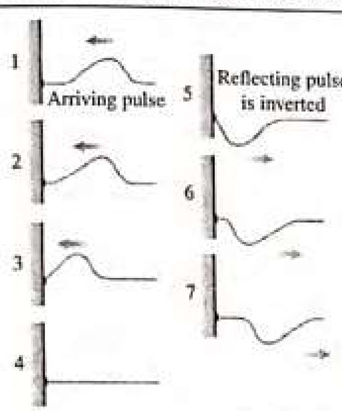
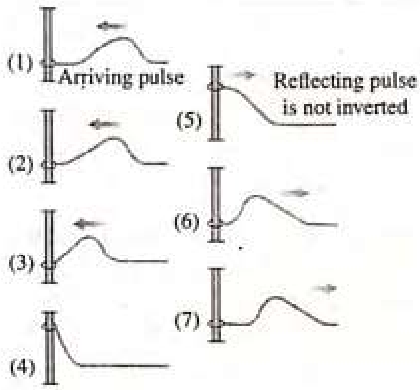
$$0.80 = (2r-1)\frac{v}{2n}$$

Frequency $n = \dfrac{(2r-1)v}{2 \times 0.80} = (2r-1)\dfrac{320}{1.6}$ Hz $= (2r-1)200$ Hz

Therefore, the required frequencies are $n = (2r - 1) \times 200$ with $r = 1, 2, \ldots 50$.

Putting the values of r in the above equation, we get
n = 200 Hz, 600 Hz, 1000 Hz, 1400 Hz, 1800 Hz, ..., 19800 Hz

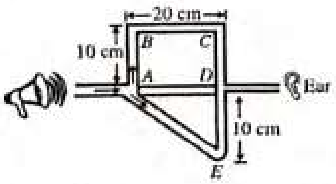
REFLECTION OF WAVES AT FIXED AND FREE ENDS

Reflection of Waves—Fixed End	Reflection of Waves—Free End
• When the wave reaches the fixed end it exerts an upward pull on the end. According to Newton's third law, the fixed point exerts an equal and opposite force downward on the string. Thus, there is phase change of π for the reflected wave, the wave is inverted as shown in the figure. • Whenever a travelling wave reaches a boundary, some or all of the wave is reflected. • When it is reflected from a fixed end, the wave is inverted.	• If the end of the string is free to move vertically, the free end overshoots twice the amplitude as shown in figure. • When a travelling wave reaches a boundary, all or part of it is reflected. • When reflected from a free end, the pulse is not inverted.
	

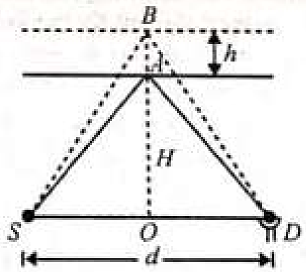
CONCEPT APPLICATION EXERCISE 16.4

1. Find the resultant amplitude and the phase difference between the resultant wave and the first wave, in the event the following waves interfere at a point;
$y_1 = (3\text{ cm})\sin\omega t$,
$$y_2 = (4\text{ cm})\sin\left(\omega t + \frac{\pi}{2}\right);\ y_3 = (5\text{cm})\sin(\omega t + \pi)$$

2. Figure shows a tube structure in which signal is sent from one end and is received at the other end. The frequency of the sound source can be varied electronically between 2000 and 5000 Hz. Find the frequencies at which maxima of intensity are detected. The speed of sound in air 340 m/s.



3. A source and a detector D of high frequency waves are a distance d apart on the ground. Maximum signal is received at D when the reflecting layer is at a height H. When the layer rises a distance h, no signal is detected at D. Neglecting absorption in the atmosphere, find the relation between d, h, H, and the wavelength λ of the waves.



4. Two waves have the same frequency. The first has intensity I_0. The second has intensity $4I_0$ and lags behind the first in phase by $\pi/2$. When they meet, find the resultant intensity, and the phase relationship of the resultant wave with the first wave.

5. Determine the amplitude of the resultant motion when two sinusoidal waves of same frequency, travelling in the same direction are combined. Their amplitudes are 3.0 cm and 4.0 cm and they differ in phase by $\pi/2$ radians.

6. In a large room, a person receives direct sound waves from a source 120 m away. He also receives waves from the same source which reach him after being reflected from the 5-m high ceiling at a point halfway between them. For which wavelengths will these two sound waves interfere constructively?

STATIONARY WAVES

In this situation, two identical waves travel in opposite directions in the same medium as shown in figure. These waves combine in accordance with the wave in interference model.

We can analyse such a situation by considering wave functions for two transverse sinusoidal waves having the same amplitude, frequency, and wavelength but travelling in opposite directions in the same medium. Consider two waves

$$y_1 = A\sin(kx - \omega t) \quad \text{and} \quad y_2 = A\sin(kx + \omega t)$$

where y_1 represents a wave travelling in the $+x$-direction and y_2 represents one travelling in the $-x$-direction. Adding these two functions gives the resultant wave function y.

$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

When we use the trigonometric identity

$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$,

this expression reduces to

$$y = (2A\sin kx)\cos\omega t \qquad \text{(i)}$$

Notice that Eq. (i) does not contain a function $kx - \omega t$. Therefore, it is not an expression for a single travelling wave. When you observe a standing wave, there is no sense of motion in the direction of propagation of either original wave. comparing Eq. (i) with equation $x(t) = A\cos(\omega t + \phi)$, we see that it describes a special kind of simple harmonic motion. Every element of the medium oscillates in simple harmonic motion with the same angular frequency ω (according to the $\cos\omega t$ factor in the equation). The amplitude of the simple harmonic motion of a given element (given by the factor $2A\sin kx$, the coefficient of the cosine function) depends on the location x of the element in the medium. However, the amplitude of the simple harmonic motion of an element of the medium has a minimum value of zero when x satisfies the condition $\sin kx = 0$, that is, when

$kx = 0, \pi, 2\pi, 3\pi$

because $k = 2\pi/\lambda$, these values for kx give

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots = \frac{n\lambda}{2}$$

$$n = 0, 1, 2, 3, \ldots \qquad \text{(ii)}$$

These points of zero amplitude are called nodes.

The element of the medium with the greatest possible displacement from equilibrium has an amplitude of $2A$. The positions in the medium at which this maximum displacement occurs are called antinodes. The antinodes are located at positions for which the coordinate x satisfies the condition $\sin kx = \pm 1$, that is, when

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

Therefore, the positions of the antinodes are given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots = \frac{n\lambda}{4}$$

$n = 1, 3, 5, \ldots$

Characteristics of Stationary Waves

i. In a stationary wave, the disturbance does not move in any direction. The conditions of crests and troughs merely appear and disappear in fixed positions to be followed by opposite conditions after every $T/2$.

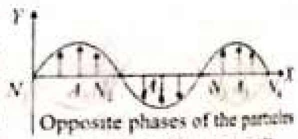
ii. All the particles of the medium, except those at nodes, execute simple harmonic motion with the particle of the wave about their mean position.

iii. During the formation of a stationary wave, the medium is broken into loops between equally spaced points called nodes which remain at rest and in between them are points of maximum displacement called antinodes.

iv. The amplitude of the particles are different at different points. The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.

v. The maximum velocity is different at different points. Its value is zero at the nodes and gradually increases towards the antinodes. All the particles attain their maximum velocities simultaneously when they pass through their mean positions.

vi. All the particles in a particular segment between two nodes vibrate in the same phase but the particles in the neighbouring segments vibrate in opposite phases, as shown in figure.



Opposite phases of the particles in consecutive segments

vii. The energy becomes alternately wholly potential and wholly kinetic twice in each cycle. It is wholly potential when particles are at their positions of maximum displacement and wholly kinetic when the particle pass through the mean positions.

viii. A stationary wave has the same wavelength and time period as the component waves.

ix. The distance between two consecutive nodes and antinodes is $\lambda/2$. The distance between nodes and next antinode is $\lambda/4$.

ILLUSTRATION 16.16 Two travelling waves of equal amplitudes and equal frequencies move in opposite direction along a string. They interfere to produce a standing wave having the equation.

$$y = A\cos kx \sin\omega t$$

in which $A = 1.0$ mm, $k = 1.57\ \text{cm}^{-1}$ and $\omega = 78.5\ \text{s}^{-1}$. (a) Find the velocity and amplitude of the component travelling waves. (b) Find the node closest to the origin in the region $x > 0$. (c) Find the antinode closest to the origin in the region $x > 0$. (d) Find the amplitude of the particle at $x = 2.33$ cm.

Solution.

(a) The standing wave is formed by the superposition of the waves

$$y_1 = \frac{A}{2}\sin(\omega t - kx) \quad \text{and} \quad y_2 = \frac{A}{2}\sin(\omega t + kx)$$

The wave velocity (magnitude) of either of the waves is

$$v = \frac{\omega}{k} = \frac{78.5\ s^{-1}}{1.57\ cm^{-1}} = 50\ cm/s; \quad \text{Amplitude} = 0.5\ mm$$

(b) For a node, $\cos kx = 0$.
The smallest positive x satisfying this relation is given by

$$kx = \pi/2$$

or $$x = \frac{\pi}{2k} = \frac{3.14}{2 \times 1.57\ cm^{-1}} = 1\ cm$$

(c) For an antinode, $|\cos kx| = 1$.
The smallest positive x satisfying this relation is given by

$$kx = \pi$$

or, $$x = \frac{\pi}{k} = 2\ cm$$

(d) The amplitude of vibration of the particle at x is given by $|A \cos kx|$. For the given point,

$$kx = (1.57\ cm^{-1})\ (2.33\ cm)$$

$$= 1.57\left[2 + \frac{1}{3}\right] = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

Thus, the amplitude will be

$$(1.0\ mm)\ |\cos(\pi + \pi/6)| = \frac{\sqrt{3}}{2}\ mm = 0.86\ mm$$

Standing Waves in a String Fixed at Both Ends

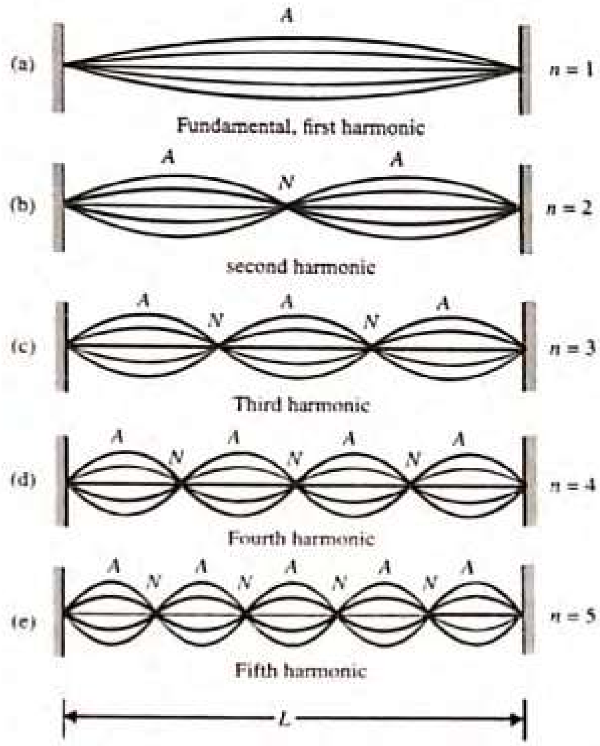
The first normal mode that is consistent with these requirements, shown in Figure (a), has nodes at its ends and one antinode in the middle. This normal mode is the longest wavelength mode that is consistent with our boundary conditions. The first normal mode occurs when the wavelength $\lambda_1 = 2L$. The section of a standing wave from one node to the next node is called a loop. In the first normal mode, the string is vibrating in one loop. In the second normal mode (Figure (b)) the string vibrates in two loops. In this case, the wavelength λ_2 is equal to the length of the string, as expressed by $\lambda_2 = L$. The third normal mode (Figure (c)) corresponds to the case in which $\lambda_3 = 2L/3$ and our string vibrates in three loops. In general, the wavelengths of the various normal modes for a string of length L fixed at both ends are

$$\lambda_n = \frac{2L}{n}; \quad n = 1, 2, 3, \ldots \tag{iv}$$

where the index n refers to the nth normal mode of oscillation. These modes are the possible modes of oscillation for the string. The actual modes that are excited on a string are discussed shortly.

The natural frequencies associated with the modes of oscillation are obtained from the relationship $f = v/\lambda$, where the wave speed v is same for all frequencies. Using Eq. (iv), we find that the natural frequencies f_n of the normal modes are

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L}; \quad n = 1, 2, 3 \tag{v}$$



These natural frequencies are also called the quantized frequencies associated with the vibrating string fixed at both ends.

Because $v = \sqrt{T/\mu}$ for waves on a string, where T is the tension in the string and μ is its linear mass density, we can also express the natural frequencies of a taut string as

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}; \quad n = 1, 2, 3, \ldots \tag{vi}$$

The lowest frequency ϕ_1, which corresponds to $n = 1$, is called either the fundamental frequency or the first harmonic and is given by

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}; \quad n = 1, 2, 3, \ldots \tag{vii}$$

The frequencies of the remaining normal modes are integer multiples of the fundamental frequencies of normal modes that exhibit an integer multiple relationship such as this form a harmonic series, and the normal modes are called harmonics. The fundamental frequency f_1 is the frequency of the first harmonic, the frequency $f_2 = 2f_1$ is the frequency of the second harmonic, and the frequency $f_n = nf_1$ is the frequency of the nth harmonic.

Sonometer

It is a device used to measure velocity of transverse mechanical wave in a stretched metal wire. The principle of sonometer is based on resonance of string vibrations. Working oscillations are induced in a clamped string by an external source like a tuning fork or an oscillator and the corresponding oscillations

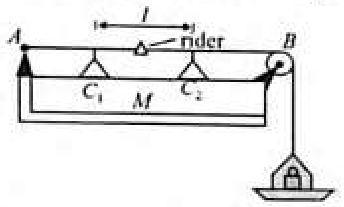
in string will become stronger when resonance takes place, i.e., the frequency of oscillation of source matches with any of the harmonic of the string vibration.

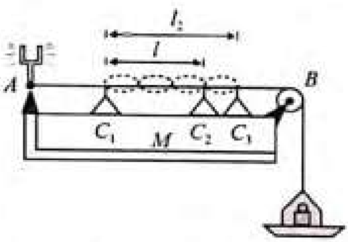
Figure shows basic structure and setup of a sonometer. It consists of a wooden box M on which a wire AB is stretched, by a hanging weight as shown in the figure. On sonometer box there are two clamps C_1 and C_2 placed which can slide under the wire to change the length of wire between the clamps.

When an oscillating turning fork is placed in contact with the sonometer wire as shown in figure some oscillations are transferred to the wire. If tension in wire is T and n_0 is the frequency of turning fork, the wavelength of wave in wire is

$$\lambda = \frac{v}{n_0} = \frac{1}{n_0}\sqrt{\frac{T}{\mu}} \qquad \text{(i)}$$

When the length between clamps is an integral multiple of $\lambda/2$ then stationary waves are established in the portion of wire between C_1 and C_2. To adjust this, clamp C_1 is fixed and C_2 is displaced so that a small rider (a piece of paper) on wire start jumping violently on wire and falls indicating that the oscillation amplitude of wire is increasing and stationary waves are established. Let in this situation the length between clamps be l_1. Now again C_2 is displaced away from C_1 so that again resonance is obtained. This will happen again when the clamp reaches the position C_3 and when next node of stationary waves is present as shown in figure. Let this length be l_2.





So we can say that if l_1 and l_2 are the two successive resonance lengths then we have

$$l_2 - l_1 = \frac{\lambda}{2}$$

So wavelength of wave is $\lambda = 2(l_2 - l_1)$.

As frequency n_0 of oscillating source is known, we can find the velocity of wave in wire as

$$v = n_0\lambda$$
$$= 2n_0(l_2 - l_1) \qquad \text{(ii)}$$

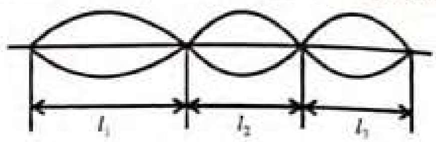
Equation (ii) gives the partially measured value of velocity of transverse waves in a stretched wire. This can be compared with the theoretical value of v given by $\sqrt{T/\mu}$.

ILLUSTRATION 16.17 The length of a sonometer wire between two fixed ends is 1.10 m. Where should the two bridges be placed to divide the wire into three segments whose fundamental frequencies are in the ratio of 1:2:3?

Solution. Let l_1, l_2, l_3 be the lengths of three segments. Given $l_1 + l_2 + l_3 = 1.10$

From relation $n = \dfrac{1}{2l}\sqrt{\left(\dfrac{T}{\mu}\right)}$

If tension T and mass per unit length μ are fixed, then $n \propto 1/l$



so nl = constant

$$n_1 l_1 = n_2 l_2 = n_3 l_3$$

$$\therefore \quad l_1 : l_2 : l_3 = \frac{1}{n_1} : \frac{1}{n_2} : \frac{1}{n_3} = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = \frac{6:3:2}{6} = 6:3:2$$

$\therefore \quad l_1 = 6k,\ l_2 = 3k,\ l_3 = 2k,\ k$ being a constant.

From Eq. (i)

$$6k + 3k + 2k = 1.10 \quad \text{or} \quad 11k = 1.10$$

$$\therefore \quad k = 1.10/11 = 0.1$$

$$\therefore \quad l_1 = 6 \times 0.1 = 0.6 \text{ m}$$
$$l_2 = 3 \times 0.1 = 0.3 \text{ m}$$
$$l_3 = 2 \times 0.1 = 0.2 \text{ m}$$

Therefore the bridges must be placed at distance 0.6 m and $(0.6 + 0.3) = 0.9$ m

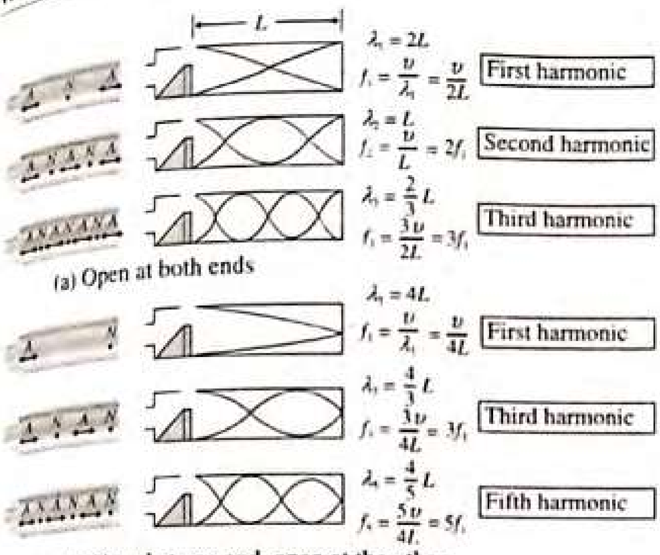
Standing Waves in Air Columns

The waves under boundary conditions model can also be applied to sound waves in a column of air such as that inside an organ pipe. Standing waves are the result of interference between longitudinal sound waves travelling in opposite directions.

In a pipe closed at one end, the closed end is a displacement node because the rigid barrier at this end does not allow longitudinal motion of the air. Because the pressure wave is 90° out of phase with the displacement wave, the closed end of an air column corresponds to a pressure antinode (that is, a point of maximum pressure variation).

The open end of an air column is approximately a displacement antinode and a pressure node. We can understand why no pressure variation occurs at an open end by noting that the end of the air column is open to the atmosphere; therefore, the pressure at this end must remain constant at atmospheric pressure.

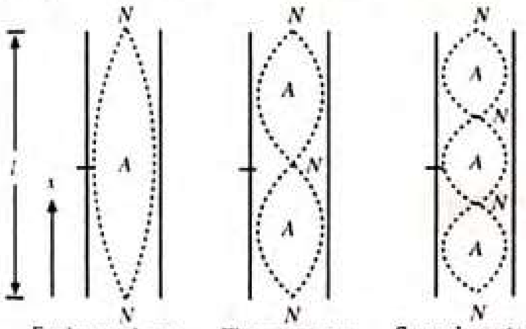
The first three normal modes of oscillation of a pipe open at both ends are shown in Figure (a). Notice that both ends are displacement antinodes (approximately). In the first normal mode, the standing wave extends between two adjacent antinodes, which is distance of half a wavelength. Therefore, the wavelength is twice the length of the pipe, and the fundamental frequency is $f_1 = v/2L$. As Figure (a) shows the frequencies of the higher harmonics are $2f_1, 3f_1, \ldots$



(b) Closed at one end, open at the other

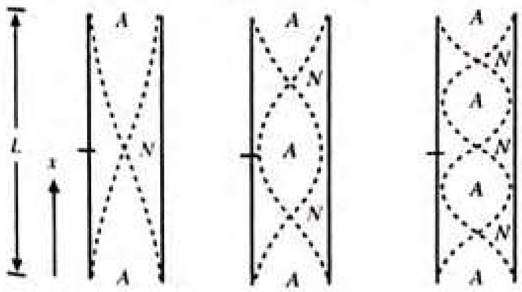
In a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency.

Because all harmonics are present and the fundamental frequency is given by the same expression as that for a string, we can express the natural frequencies of oscillation as



Fundamental note First overtone Second overtone

Various modes of vibration showing pressure nodes and antinodes.



Fundamental note First overtone Second overtone

Various modes of vibration showing displacement nodes and antinodes

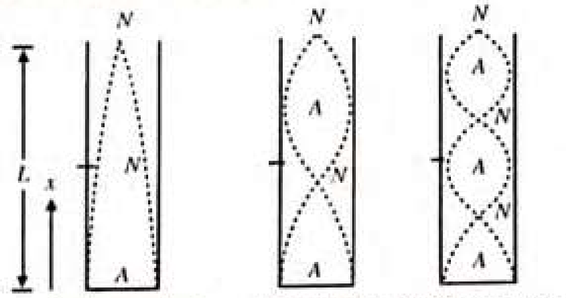
$$f_n = n\frac{v}{2L} \quad \text{where } n = 1, 2, 3, \ldots \tag{i}$$

In a pipe closed at one end, the natural frequencies of oscillation form a harmonic series that includes only odd integral multiples of the fundamental frequency.

We express this result mathematically as

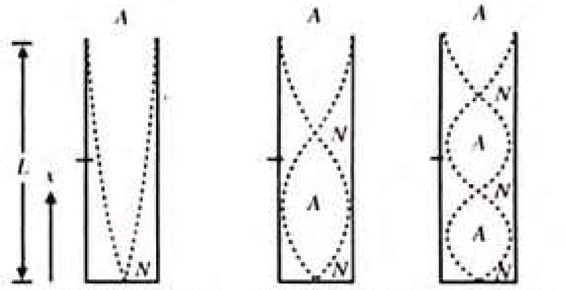
$$f_n = n\frac{v}{4L}; \quad n = 1, 3, 5, \ldots \tag{ii}$$

It is interesting to investigate what happens to the frequencies of instruments based on air columns and strings during a concert as the temperature rises. The sound emitted by a flute, for example, becomes sharp (increase in frequency) as the flute warms up because the speed of sound increases in the increasingly warmer air inside the flute (consider Eq. (i)). The sound produced by a violin becomes flat (decrease in frequency) as the string thermally expand and the expansion causes their tension to decrease.



Fundamental note First overtone Second overtone

Various modes of vibration of closed organ pipe showing pressure nodes and antinodes.



Fundamental tone First overtone Second overtone

Various modes of vibration of closed organ pipe showing displacement nodes and antinodes

ILLUSTRATION 16.18 Find the number of possible natural oscillations of air column in a pipe frequencies of which lie below $\nu_0 = 1250$ Hz. The length of the pipe is $l = 85$ cm. The velocity of sound is $v = 340$ m/s. Consider two cases

i. the pipe is closed from one end,
ii. the pipe is open from both ends.

Solution.

First Method

i. **Pipe is closed from one end:** An air column in a pipe closed from one end oscillates only odd harmonics [1st harmonic (fundamental mode), 3rd harmonic (1st overtone), 5th harmonic (2nd overtone), 7th harmonic (3rd overtone) etc.]

Fundamental frequency

$$= \frac{v}{4l} = \frac{340}{4 \times \frac{85}{100}} = 100 \text{ Hz}$$

Other modes of oscillation are:

3rd harmonic frequency $= 3 \times 100 = 300$ Hz

5th harmonic frequency = 5 × 100 = 500 Hz
7th harmonic frequency = 7 × 100 = 700 Hz
9th harmonic frequency = 9 × 100 = 900 Hz
11th harmonic frequency = 11 × 100 = 1100 Hz
13th harmonic frequency = 13 × 100 = 1300 Hz

Only those natural oscillations are to be counted frequencies of which lie below v_0 = 1250 Hz, the harmonics till 11th harmonic are to be counted.

Since number of possible natural oscillations
= 1 (1st harmonic) + 1 (3rd harmonic) + 1 (5th harmonic) + 1(7th harmonic) + 1(9th harmonic) + 1 (11th harmonic) = 6

Second Method

All the frequencies possible are integral multiples of fundamental frequency which is 100 Hz. Using the fact that integer which is multiplied by fundamental frequency is the number of harmonic itself you get, highest harmonic predicted = [12.50/100] where [x] represents greatest integer less than or equal to x = [12.5] = 12.

Now for closed pipe, only odd harmonics are possible and the highest harmonic possible = 11th. The possible harmonics are 1, 3, 5, 7, 9, 11 which are six in number.

ii. Pipe opened from both ends: Fundamental frequency

$$=\frac{V}{2l}=\frac{340}{2\times 85}\times 100=200 \text{ Hz}$$

Frequency of the other natural modes of oscillational are:
2nd harmonic frequency = 2 × 200 = 400 Hz
3rd harmonic frequency = 3 × 200 = 600 Hz
4th harmonic frequency = 4 × 200= 800 Hz
5th harmonic frequency = 5 × 200 = 1000 Hz
6th harmonic frequency = 6 × 200 = 1200 Hz
7th harmonic frequency = 7 × 200 = 1400 Hz

You have to count only those harmonics whose frequencies are below 1250 Hz. All the harmonics till 6th harmonic are possible, and obviously they are six in number.

Third Method

Fundamental frequency = 200 Hz
Frequencies possible = n × fundamental frequency
= n × 200 [n is 1, 2, ...]

Maximum value of n = [12.50/200] = 6 ([x] represents greater than or equal to x)

Now n is also equal to the number of harmonic for which frequency is being calculated, highest harmonic possible = 6th.

As all harmonics are possible in case of open tube, harmonics possible are 1st, 2nd, 3rd, 4th, 5th and 6th.

Number of harmonics possible in this case = 6.

BEATS: INTERFERENCE IN TIME

The interference phenomena we have studied so far involve the superposition of two or more waves having the same frequency.

Now let's consider another type of interference, one that results from the superposition of two waves having slightly different frequencies. In this case, when the two waves are observed at a point in space, they are periodically in and out of phase. That is, there is a temporal (time) alternation between constructive and destructive interference. As a consequence, we refer to this phenomenon as interference in time or temporal interference. For example, if two tuning forks of slightly different frequencies are struck, one hears a sound of periodically varying amplitude. This phenomenon is called beating.

Beating is the periodic variation in amplitude at a given point due to the superposition of two waves having slightly different frequencies.

The number of amplitude maxima one hears per second, or the beat frequency, equals the difference in frequency between the two sources as we shall show below.

$$f_{\text{beat}} = |f_1 - f_2| \quad \text{(iii)}$$

For instance, if one tuning fork vibrates at 438 Hz and a second one vibrates at 442 Hz, the resultant sound wave of the combination has a frequency of 440 Hz (the musical note A) and a beat frequency of 4 Hz. A listener would hear a 440 Hz sound wave go through an intensity maximum four times every second.

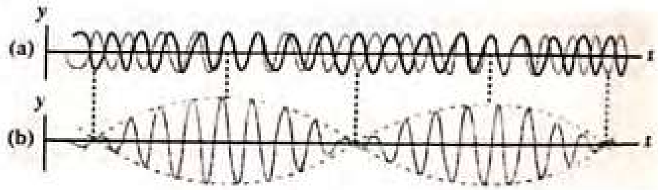


ILLUSTRATION 16.19 Two identical piano strings of length 0.750 m are each tuned exactly to 440 Hz. The tension in one of the strings is then increased by 1.0%. If they are now struck, what is the beat frequency between the fundamental of the two strings?

Solution. As the tension in one of the strings is changed, its fundamental frequency changes. Therefore, when both strings are played, they will have different frequencies and beats will be heard.

We must combine our understanding of the waves under boundary conditions model for strings with our new knowledge of beats.

Set up a ratio of the fundamental frequencies of the two strings using equation

$$\frac{f_2}{f_1}=\frac{(v_2/2L)}{(v_1/2L)}=\frac{v_2}{v_1}$$

Use equation $v=\sqrt{T/\propto}$ to substitute for the wave speeds on the strings.

$$\frac{f_2}{f_1}=\frac{\sqrt{T_2/\mu}}{\sqrt{T_1/\mu}}=\sqrt{\frac{T_2}{T_1}}$$

Incorporate that the tension in one string is 1.0% larger than the other; that is, $T_2 = 1.010\ T_1$.

$$\frac{f_2}{f_1}=\sqrt{\frac{1.010T_1}{T_1}}=1.005$$

Solve for the frequency of the tightened string:

$f_2 = 1.005 f_1 = 1.005$ (440 Hz) = 442 Hz

Find the beat frequency using Eq. (iii)

$f_{beat} = 442$ Hz − 440 Hz = 2 Hz

Notice that a 1.0% mistuning in tension leads to an easily audible beat frequency of 2 Hz. A piano tuner can use beats to tune a stringed instrument by 'beating' a note against a reference tone of known frequency. The tuner can then adjust the string tension until the frequency of the sound it emits equals the frequency of the reference tone. The tuner does so by tightening or loosening the string until the beats produced by it and the reference source become too infrequent to notice.

ILLUSTRATION 16.20 Wavelength of two notes in air is (90/175) m and (90/173) m, respectively. Each of these notes produces 4 beats/s with a third note of a fixed frequency. Calculate the velocity of sound in air.

Solution. Given: $\lambda_1 = 90/175$ m and $\lambda_2 = 90/173$ m

If f_1 and f_2 are the corresponding frequencies and v is the velocity of sound in air, we have

$$v = f_1\lambda_1 \quad \text{and} \quad v = f_2\lambda_2$$

$$f_1=\frac{v}{\lambda_1} \quad \text{and} \quad f_2=\frac{v}{\lambda_2}$$

Since, $\lambda_1 < \lambda_2$, we must have $f_1 > f_2$.

If f is the frequency of the third note, then

$$f_1 - f = 4 \quad \text{and} \quad f - f_2 = 4$$

$$\Rightarrow \quad f_1 - f_2 = 8$$

$$\frac{v}{\lambda_1}-\frac{v}{\lambda_2}=8$$

$$v\left[\frac{175}{90}-\frac{173}{90}\right]=8$$

which gives, $v = 360$ m/s

CONCEPT APPLICATION EXERCISE 16.5

1. Two tuning forks A and B produce 4 beats/s when sounded together. A resonates to 32.4 cm of stretched wire and B is in resonance with 32 cm of the same wire. Determine the frequencies of the two tuning forks.
2. A glass tube of length 1.5 m is filled completely with water; the water can be drained out slowly at the bottom of the tube. Find the total number of resonance obtained, when a tuning fork of frequency 606 Hz is put at the upper open end of the tube. $v_{sound} = 340$ m/s.
3. Calculate the speed of sound in a gas in which two waves of wavelengths 50 cm and 50.5 cm produce 6 beats/s.
4. A stationary wave is given by

   $$y = 5\sin\frac{\pi x}{3}\cos 40\pi t$$

   where x and y are in cm and t is in second.
   (a) What are the amplitude and velocity of the component wave whose superposition can give rise to this vibration?
   (b) What is the distance between the nodes?
   (c) What is the velocity of a particle of the string at the position $x = 1.5$ cm when $t = 9/8$ s?
5. An open organ pipe has a fundamental frequency of 300 Hz. The first overtone of a closed organ pipe has the same frequency as the first overtone of the open pipe. Find length of each pipe. The velocity of sound in air = 350 m/s.
6. A tuning fork A is in resonance with an air column 32 cm long and closed at one end. When the length of this column is increased by 1 cm, it is in resonance with another fork B. When A and B are sounded together, they produce 40 beats in 5 s. Find their frequencies.

SOLVED EXAMPLES

1. A wave travelling in positive X-direction with $A = 0.2$ m has a velocity of 360 m/sec. if $\lambda = 60$ m then correct expression for the wave

   (a) $y=0.2\sin\left[2\pi\left(6t+\frac{x}{60}\right)\right]$

   (b) $y=0.2\sin\left[\pi\left(6t+\frac{x}{60}\right)\right]$

   (c) $y=0.2\sin\left[2\pi\left(6t-\frac{x}{60}\right)\right]$

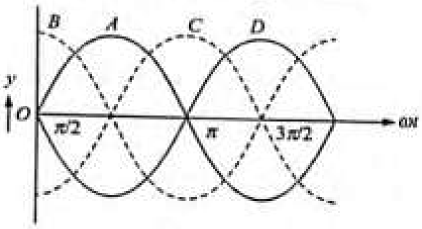
   (d) $y=0.2\sin\left[\pi\left(6t-\frac{x}{60}\right)\right]$

Sol. (c) A wave travelling in positive x-direction may be represented as $y = A\sin\frac{2\pi}{\lambda}(vt - x)$.

On putting values $y = 0.2\sin\frac{2\pi}{60}(360t - x)$

$$\Rightarrow \quad y=0.2\sin 2\pi\left(6t-\frac{x}{60}\right)$$

2. The figure shows four progressive waves A, B, C, and D with their phases expressed with respect to the wave A. It can be concluded from the figure that



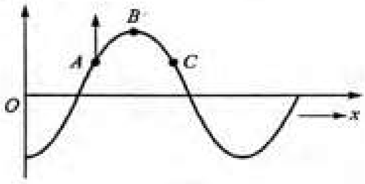
(a) The wave C is ahead by a phase angle of $\pi/2$ and the wave B lags behind by a phase angle of $\pi/2$
(b) The wave C lags behind by a phase angle of $\pi/2$ and the wave B is ahead by a phase angle of $\pi/2$
(c) The wave C is ahead by a phase angle of π and the wave B lags behind by a phase angle of π
(d) The wave C lags behind by a phase angle of π and the wave B ahead by a phase angle of π

Sol. (b) Equation of A, B, C and D are

$$y_A = A\sin\omega t,\ y_B = A\sin(\omega t + \pi/2)$$
$$y_C = A\sin(\omega t - \pi/2),\ y_D = A\sin(\omega t - \pi)$$

It is clear that wave C lags behind by a phase angle of $\pi/2$ and the wave B is ahead by a phase angle at $\pi/2$.

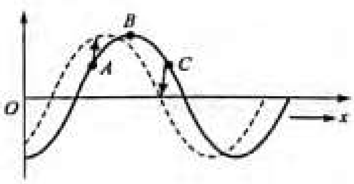
3. A wave is travelling along a string. At an instant, shape of the string is as shown in figure. At this instant, point A is moving upwards.



(i) The wave is travelling to the right
(ii) Displacement amplitude of the wave is equal to displacement of B at this instant
(iii) At this instant velocity of C is also directed upwards
(iv) Phase difference between A and C may be equal to $\dfrac{\pi}{2}$

Which of the following statements are correct?
(a) (i) and (iii) (b) (ii) and (iv)
(c) (i),(iii) and (iv) (d) (i) and (iv)

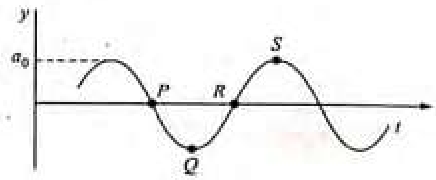
Sol. (b) Since A is moving upwards, therefore, after an elemental time interval the wave will be as shown dotted in following figure. It means, the wave is travelling leftward. Therefore, (a) is wrong.



Displacement amplitude of the wave means maximum possible displacement of medium particles due to propagation of the wave, which is equal to the displacement at B at the instant shown in figure. Hence (b) is correct.
From figure, it is clear that C is moving downwards at this instant. Hence (c) is wrong.

The phase difference between two points will be equal to $\dfrac{\pi}{2}$ if distance between them is equal to $\dfrac{\lambda}{4}$. Between A and C, the distance is less than $\dfrac{\lambda}{2}$ It may be equal to $\dfrac{\lambda}{4}$. Hence, phase difference between these two points may be equal to $\dfrac{\pi}{2}$.

4. A wave motion has the function $y = a_0\sin(\omega t - kx)$. The graph in figure shows how the displacement y at a fixed point varies with time t. Which one of the labelled points shows a displacement equal to that at the position $x = \dfrac{\pi}{2k}$ at time $t = 0$



(a) P (b) Q
(c) R (d) S

Sol. (b) At $t = 0$ and $x = \dfrac{\pi}{2k}$. The displacement

$$y = a_0\sin\left(\omega x_0 - k\times\frac{\pi}{2x}\right) = -a_0\sin\frac{\pi}{2} = -a_0$$

from graph. Point of maximum displacement (a_0) in negative direction is Q.

5. The diagram below shows an instantaneous position of a string as a transverse progressive wave travels along it from left to right

Which one of the following correctly shows the direction of the velocity of the points 1, 2 and 3 on the string

	1	2	3
(a)	→	→	→
(b)	→	←	→
(c)	↓	↓	↓
(d)	↓	↑	↓

Sol. (d) Particle velocity $(v_p) = -v\times$ Slope of the graph at that point
At point 1 : Slope of the curve is positive, hence particle velocity is negative or downward (↓)

At point 2 : Slope negative, hence particle velocity is positive or upwards (↑)

At point 3 : Again slope of the curve is positive, hence particle velocity is negative or downward (↓)

6. Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2, the higher and the lower initial tensions in the strings, then it could be said that while making the above change in tension
(a) T_2 was decreased (b) T_2 was increased
(c) T_1 was increased (d) T_1 was kept constant

Sol. (b) Using $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$;

As $T_1 > T_2 \Rightarrow n_1 > n_2$ giving $n_1 - n_2 = 6$

The beat frequency of 6 will remain fixed when

(i) n_1 remains same but n_2 is increased to a new value $(n_2' - n_2 = 12)$ by increasing tension T_2.

(ii) n_2 remains same but n_1 is decreased to a new value $(n_1 - n_1' = 12)$ by decreasing tension T_1.

7. A wire of density 9×10^3 kg/m^3 is stretched between two clamps 1 m apart and is subjected to an extension of 4.9×10^{-4} m. The lowest frequency of transverse vibration in the wire is ($Y = 9 \times 10^{10}$ N/m^2)
(a) 40 Hz (b) 35 Hz
(c) 30 Hz (d) 25 Hz

Sol. (b) For wire if

M = mass, ρ = density, A = Area of cross section

V = volume, l = length, Δl = change in length

Then mass per unit length $m = \frac{M}{l} = \frac{Al\rho}{l} = A\rho$

And Young's modulus of elasticity $Y = \frac{T/A}{\Delta l/l}$

$$\Rightarrow \quad T = \frac{Y\Delta l A}{l}.$$

Hence lowest frequency of vibration

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2l}\sqrt{\frac{Y\left(\frac{\Delta l}{l}\right)A}{A\rho}} = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{l\rho}}$$

$$\Rightarrow \quad n = \frac{1}{2\times 1}\sqrt{\frac{9\times 10^{10} \times 4.9\times 10^{-4}}{1\times 9\times 10^3}} = 35 \text{ Hz}$$

8. An open pipe is in resonance in its 2nd harmonic with tuning fork of frequency f_1. Now it is closed at one end. If the frequency of the tuning fork is increased slowly from f_1 then again a resonance is obtained with a frequency f_2. If in this case the pipe vibrates n^{th} harmonics then
(a) $n = 3, f_2 = \frac{3}{4}f_1$ (b) $n = 3, f_2 = \frac{5}{4}f_1$
(c) $n = 5, f_2 = \frac{5}{4}f_1$ (d) $n = 5, f_2 = \frac{3}{4}f_1$

Sol. (c) Open pipe resonance frequency $f_1 = \frac{2v}{2L}$

Closed pipe resonance frequency $f_2 = \frac{nv}{4L}$

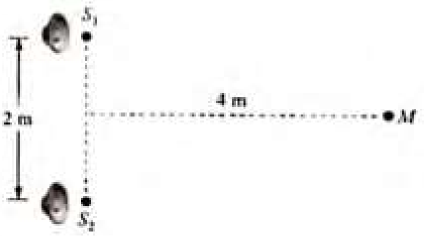
$$f_2 = \frac{n}{4}f_1 \qquad \text{(where } n \text{ is odd and } f_2 > f_1)$$

$\therefore \quad n = 5$

9. Two speakers connected to the same source of fixed frequency are placed 2.0 m apart in a box. A sensitive microphone placed at a distance of 4.0 m from their midpoint along the perpendicular bisector shows maximum response. The box is slowly rotated until the speakers are in line with the microphone. The distance between the midpoint of the speakers and the microphone remains unchanged. Exactly five maximum responses are observed in the microphone in doing this. The wavelength of the sound wave is
(a) 0.2 m (b) 0.4 m
(c) 0.6 m (d) 0.8 m

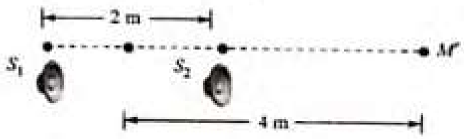
Sol. (b) Initially $S_1M = S_2M$

$\Rightarrow$ Path Difference $(\Delta x) = S_1M - S_2M = 0$.



Finally when the box is rotated

Path Difference $= S_1M' - S_2M' \Rightarrow \Delta x = 5 - 3 = 2$ m



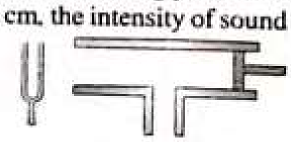
For maxima

Path Difference = (Even multiple) $\frac{\lambda}{2} \Rightarrow \Delta x = (2n)\frac{\lambda}{2}$

For 5 maximum responses

$$\Rightarrow \quad 2 = 2(5)\frac{\lambda}{2}\left\{\because \Delta x = (2n)\frac{\lambda}{2}\right\} \Rightarrow \lambda = \frac{2}{5} = 0.4m.$$

10. Vibrating tuning fork of frequency n is placed near the open end of a long cylindrical tube. The tube has a side opening and is fitted with a movable reflecting piston. As the piston is moved through 8.75 cm, the intensity of sound changes from a maximum to minimum. If the speed of sound is 350 m/s. Then n is



(a) 500 Hz (b) 1000 Hz
(c) 2000 Hz (d) 4000 Hz

Sol. (b) When the piston is moved through a distance of 8.75 cm, the path difference produced is 2×8.75 cm = 17.5 cm.

This must be equal to $\frac{\lambda}{2}$ for maximum to change to minimum.

$$\therefore \quad \frac{\lambda}{2} = 17.5 \text{ cm} \Rightarrow \lambda = 35 \text{ cm} = 0.35 \text{ m}$$

$$\text{So.} \quad v = n\lambda \Rightarrow n = \frac{v}{\lambda} = \frac{350}{0.35} = 1000 \text{Hz}$$

11. If $n_1, n_2, n_3 \ldots$ are the frequencies of segments of a stretched string, the frequency n of the string is given by

(a) $n = n_1 + n_2 + n_3 + \ldots$ (b) $n = \sqrt{n_1 \times n_2 \times n_3 \times \ldots\ldots}$

(c) $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \ldots$ (d) None of these

Sol. (c) For a vibrating string

$$n_1 l_1 = n_2 l_2 = n_3 l_3 = \text{constant} = k \text{ (say)} = nl$$

Also $l_1 + l_2 + l_3 + l_4 + \ldots = 1$

$$\frac{k}{n_1} + \frac{k}{n_2} + \frac{k}{n_3} + \frac{k}{n_4} + \ldots = \frac{k}{n} \Rightarrow \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \ldots$$

12. A stone is hung in air from a wire which is stretched over a sonometer. The bridges of the sonometer are L cm apart when the wire is in unison with a tuning fork of frequency N. When the stone is completely immersed in water, the length between the bridges is l cm for re-establishing unison, the specific gravity of the material of the stone is

(a) $\frac{L^2}{L^2 + l^2}$ (b) $\frac{L^2 - l^2}{L^2}$

(c) $\frac{L^2}{L^2 - l^2}$ (d) $\frac{L^2 - l^2}{L^2}$

Sol. (c) Frequency of vib. is stretched string $n = \frac{1}{2(\text{Length})}\sqrt{\frac{T}{m}}$

When the stone is completely immersed in water, length changes but frequency doesn't ($\because$ unison reestablished)

$$\text{Hence length} \propto \sqrt{T} \Rightarrow \frac{L}{l} = \sqrt{\frac{T_{air}}{T_{water}}} = \sqrt{\frac{V\rho g}{V(\rho - 1)g}}$$

(Density of stone = ρ and density of water = 1)

$$\Rightarrow \quad \frac{L}{l} = \sqrt{\frac{\rho}{\rho - 1}} \Rightarrow \rho = \frac{L^2}{L^2 - l^2}$$

13. A tuning fork of frequency 340 Hz is vibrated just above the tube of 120 cm height. Water is poured slowly in the tube. What is the minimum height of water necessary for the resonance (speed of sound in the air = 340 m/sec)

(a) 15 cm (b) 25 cm
(c) 30 cm (d) 45 cm

Sol. (d) Because the tuning fork is in resonance with air column in the pipe closed at one end, the frequency is $n = \frac{(2N-1)v}{4l}$

where $N = 1, 2, 3 \ldots$ corresponds to different mode of vibration putting $n = 340$Hz, $v = 340$ m/s, the length of air column in the pipe can be

$$l = \frac{(2N-1)340}{4 \times 340} = \frac{(2N-1)}{4} \text{m} = \frac{(2N-1) \times 100}{4} \text{cm}$$

For $N = 1, 2, 3, \ldots$ we get l = 25 cm, 75 cm, 125 cm ...

As the tube is only 120 cm long, length of air column after water is poured in it may be 25 cm or 75 cm only, 125 cm is not possible, the corresponding length of water column in the tube will be (120 – 25) cm = 95 cm or (120 – 75) cm = 45 cm. Thus minimum length of water column is 45 cm.

14. An organ pipe is closed at one end has fundamental frequency of 1500 Hz. The maximum number of overtones generated by this pipe which a normal person can hear is:

(a) 14 (b) 13
(c) 6 (d) 9

Sol. (c) Critical hearing frequency for a person is 20,000 Hz.

If a closed pipe vibration in N^{th} mode then frequency of vibration $n = \frac{(2N-1)v}{4l} = (2N-1)n_1$

(where n_1 fundamental frequency of vibration)

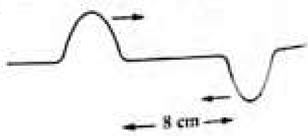
Hence $20{,}000 = (2N-1) \times 1500 \Rightarrow N = 7.1 = 7$

Also, in closed pipe

Number of over tones = (No. of mode of vibration) –1

= 7 – 1 = 6.

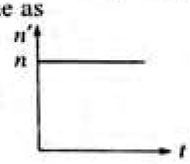
15. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be

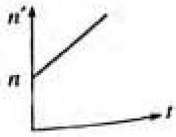


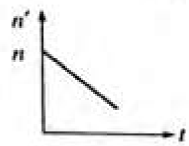
(a) Zero
(b) Purely kinetic
(c) Purely potential
(d) Partly kinetic and partly potential

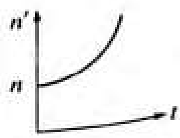
Sol. (b) After 2 sec the pulses will overlap completely. The string becomes straight and therefore does not have any potential energy and its entire energy must be kinetic.

16. An observer starts moving with uniform acceleration a toward a stationary sound source emitting a whistle of frequency n. As the observer approaches source, the apparent frequency, heard by the observer varies with time as

(a)


(b)


(c)


(d)


Sol. (b) For observer approaching a stationary source

$n' = \frac{v+v_0}{v}.n$ and given $v_0 = at \Rightarrow n' = \left(\frac{an}{v}\right)t + n$

this is the equation of straight line with positive intercept n and positive slope $\left(\frac{n}{v}\right)$.

17. The difference between the apparent frequency of a source of sound as perceived by an observer during its approach and recession is 2% of the natural frequency of the source. If the velocity of sound in air is 300 m/sec, the velocity of the source is (It is given that velocity of source << velocity of sound)

(a) 6 m/sec (b) 3 m/sec
(c) 1.5 m/sec (d) 12 m/sec

Sol. (b) When the source approaches the observer

Apparent frequency $n' = \frac{v}{v-v_s}\;n = n\left[\frac{1}{1-\frac{v_s}{v}}\right]$

$$= n\left[1-\frac{v_s}{v}\right]^{-1} = n\left[1+\frac{v_s}{v}\right]$$

(Neglecting higher powers because $v_s << v$)

When the source recedes the observed apparent frequency

$n'' = n\left[1-\frac{v_s}{v}\right]$

Given $n'-n'' = \frac{2}{100}n$, $v = 300\ m/sec$

$\therefore \quad \frac{2}{100}n = n\left[1+\frac{v_s}{v}\right] - n\left[1-\frac{v_s}{v}\right] = n\left[2\frac{v_s}{v}\right]$

$\Rightarrow \quad \frac{2}{100} = 2\frac{v_s}{v} \Rightarrow v_s = \frac{v}{100} = \frac{300}{100} = 3\ \text{m/sec}$

18. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to the left with a speed v. The speed of sound in the medium is c, then study following statements

(i) The frequency of the reflected wave is $\frac{f(c+v)}{c-v}$

(ii) The wavelength of the reflected wave is $\frac{c(c-v)}{f(c+v)}$

(iii) The number of waves striking the surface per second is $\frac{f(c+v)}{c}$

(iv) The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{f\,v}{c-v}$

Correct statements are

(a) (i),(ii) and (iii) (b) (i),(ii) and (iv)
(c) (ii),(iii) and (iv) (d) (i),(iii) and (iv)

Sol. (a) Number of waves striking the surface per second (or the frequency of the waves reaching surface of the moving target) $n' = \frac{(c+v)}{\lambda} = \frac{f(c+v)}{c}$

Now these waves are reflected by the moving target (Which now act as a source). Therefore apparent frequency of reflected second $n'' = \left(\frac{c}{c-v}\right)n' = f\left(\frac{c+v}{c-v}\right)$

The wavelength of reflected wave $= \frac{c}{n''} = \frac{c(c-v)}{f(c+v)}$

The number of beats heard by stationary listener

$$= n'' - f = f\left(\frac{c+v}{c-v}\right) - f = \frac{2fv}{(c-v)}$$

Hence option (a) is correct.

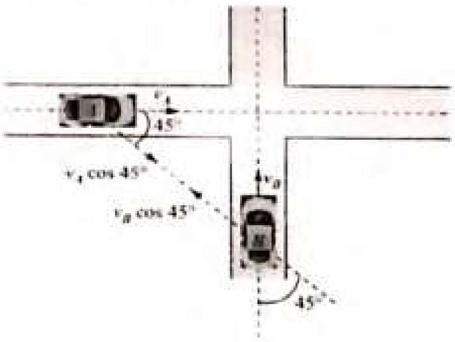
19. Two cars are moving on two perpendicular roads towards a crossing with uniform speeds of 72 km/hr and 36 km/hr. If first car blows horn of frequency 280 Hz, then the frequency of horn heard by the driver of second car when line joining the cars make 45° angle with the roads; will be

(a) 321 Hz (b) 298 Hz
(c) 289 Hz (d) 280 Hz

Sol. (b) Here $v_A = 72\text{km/hr} = 20\text{m/sec}$

$v_B = 36\text{km/hr} = 10\text{m/sec}$

$$n' = n\left(\frac{v + v_B\cos 45^\circ}{v - v_A\cos 45^\circ}\right)$$



$\Rightarrow \quad n' = 280\left(\frac{340+10/\sqrt{2}}{340-20/\sqrt{2}}\right) = 298\,\text{Hz}$

20. A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observes any beats

(a) 33 m/s (b) 22 m/s
(c) Zero (d) 11 m/s

Sol. (b)

n_1 = Frequency of the police car horn observer heard by motorcyclist

n_2 = Frequency of the siren heard by motorcyclist
v_2 = Speed of motor cyclist

$$n_1 = \frac{330 - v}{330 - 22} \times 176 \,;\, n_2 = \frac{330 + v}{330} \times 165$$

$\because \quad n_1 - n_2 = 0 \Rightarrow v = 22$ m/s.

EXERCISES

Progressive Waves and Sound Waves

1. The equation of a wave travelling on a string is

$$y = 4 \sin \frac{\pi}{2}\left(8t - \frac{x}{8}\right)$$

if x and y are in centimetres, then velocity of wave is
(a) 64 cm/s in –ve x-direction
(b) 32 cm/s in –ve x-direction
(c) 32 cm/s in +ve x-direction
(d) 64 cm/s in +ve x-direction

2. A wave is represented by the equation

$$y = 7 \sin\left(7\pi t - 0.04\pi x + \frac{\pi}{3}\right)$$

x is in metres and t is in seconds. The speed of the wave is
(a) 175 m/s (b) 49 π m/s
(c) 49/π m/s (d) 0.28 π m/s

3. The path difference between the two waves

$$y_1 = a_1 \sin\left(\omega t \frac{2\pi x}{\lambda}\right) \text{and } y_2 = a_2 \cos\left(\omega t \frac{2\pi x}{\lambda} + \phi\right) \text{ is}$$

(a) $\frac{\lambda}{2\pi}\phi$ (b) $\frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$
(c) $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$ (d) $\frac{2\pi}{\lambda}(\phi)$

4. If $x = a \sin[\omega t + \pi/6]$ and x' cos ωt, then what is the phase difference between the two waves?
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{2}$ (d) π

5. A transverse wave is described by the equation

$$y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

The maximum particle velocity is four times the wave velocity if
(a) $\lambda = \frac{\pi y_0}{4}$ (b) $\lambda = \frac{\pi y_0}{2}$
(c) $\lambda = \pi y_0$ (d) $\lambda = 2\pi y_0$

6. A simple harmonic wave is represented by the relation

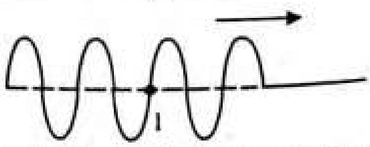
$$y(x,t) = a_0 \sin 2\pi\left(vt - \frac{x}{\lambda}\right)$$

If the maximum particle velocity is three times the wave velocity, the wavelength λ of the wave is
(a) $\pi a_0/3$ (b) $2\pi a_0/3$
(c) πa_0 (d) $\pi a_0/2$

7. Small amplitude progressive wave in a stretched string has a speed of 100 cm/s, and frequency 100 Hz. The phase difference between two points 2.75 cm apart on the string, in radians, is
(a) 0 (b) $11\pi/2$
(c) $\pi/4$ (d) $3\pi/8$

8. The mathematical form of three travelling waves are given by
y_1 = (2 cm) sin (3x – 6t)
y_2 = (3 cm) sin (4x – 12t)
and y_3 = (4 cm) sin (5x – 11t) of these waves.
(a) Wave 1 has greatest wave speed and greatest maximum transverse string speed
(b) Wave 2 has greatest wave speed and wave 1 has greatest maximum transverse string speed
(c) Wave 3 has greatest wave speed and wave 1 has greatest maximum transverse string speed
(d) Wave 2 has greatest wave speed and wave 3 has greatest maximum transverse string speed.

9. A transverse wave on a string travelling along +ve x-axis has been shown in the figure below:



The mathematical form of the shown wave is

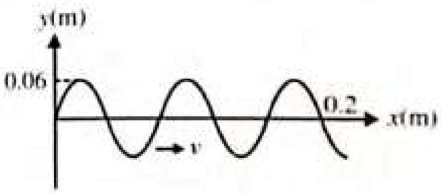
$$y = (3.0 \text{ cm}) \sin\left[2\pi \times 0.1\, t - \frac{2\pi}{100} x\right]$$

where t is in seconds and x is in centimetres. Find the total distance travelled by the particle at (1) in 10 min 15 s measured from the instant shown in the figure and direction of its motion at the end of this time.
(a) 6 cm, in upward direction

(b) 6 cm, in downward direction
(c) 738 cm, in upward direction
(d) 732 cm, in upward direction

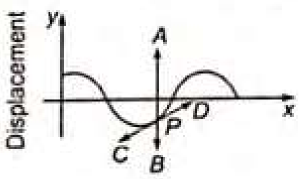
10. A sinusoidal wave travelling in the positive direction on stretched string has amplitude 20 cm, wavelength 1.0 m and wave velocity 5.0 m/s. At $x = 0$ and $t = 0$ it is given that $y = 0$ and $\partial y/\partial t < 0$. Find the wave function $y(x, t)$.
(a) $y(x,t) = (0.02\text{ m})\sin\left[\left(2\pi\text{ m}^{-1}\right)x + \left(10\pi\text{ s}^{-1}\right)t\right]\text{m}$
(b) $y(x,t) = (0.02\text{ m})\cos\left[\left(10\pi\text{s}^{-1}\right)t + \left(2\pi\text{m}^{-1}\right)x\right]\text{ m}$
(c) $y(x,t) = (0.02\text{ m})\sin\left[\left(2\pi\text{ m}^{-1}\right)x - \left(10\pi\text{ s}^{-1}\right)t\right]\text{m}$
(d) $y(x,t) = (0.02\text{ m})\sin\left[\left(\pi\text{ m}^{-1}\right)x + \left(5\pi\text{ s}^{-1}\right)t\right]\text{m}$

11. For the wave shown in figure, write the equation of this wave if its position is shown at $t = 0$. Speed of wave is $v = 300$ m/s.



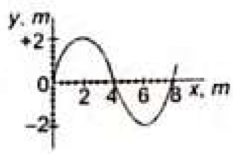
(a) $y = (0.06\text{ m})\cos[(78.5\text{ m}^{-1})x + (23562\text{ s}^{-1})t]$ m
(b) $y = (0.06\text{ m})\sin[(78.5\text{ m}^{-1})x - (23562\text{ s}^{-1})t]$ m
(c) $y = (0.06\text{ m})\sin[(78.5\text{ m}^{-1})x + (23562\text{ s}^{-1})t]$ m
(d) $y = (0.06\text{ m})\cos[(78.5\text{ m}^{-1})x - (28562\text{ s}^{-1})t]$ m

12. The figure below is a representation of a simple harmonic progressive wave, progressing in the negative X-axis, at a given instant. The direction of the velocity of the particle at stage P in the figure is best represented by the arrow.



(a) $\overrightarrow{PA}$ (b) $\overrightarrow{PB}$
(c) $\overrightarrow{PC}$ (d) $\overrightarrow{PD}$

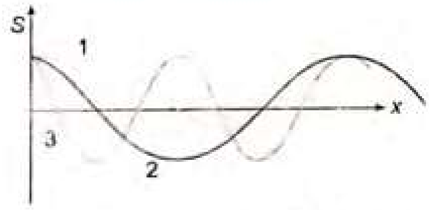
13. The graph shows a wave at $t = 0$ travelling to the right with a velocity of 4 m/s. The equation that best represents the wave is



(a) $y(x, t) = 2\sin(\pi x/4 - \pi t)$ metres
(b) $y(x, t) = 2\sin(16\pi x - 8\pi t)$ metres
(c) $y(x, t) = 2\sin(\pi x/4 + \pi t)$ metres
(d) $y(x, t) = 4\sin(\pi x/4 - \pi t)$ metres

14. The velocity of sound in a gas at temperature 27°C is v then in the same gas its velocity will be $2v$ at temperature:
(a) 54°C (b) 327°C
(c) 927°C (d) 108°C

15. Figure shown is a graph, at a certain time t, of the displacement function $S(x,t)$ of three sound waves 1, 2 and 3 as marked on the curves that travel along x-axis through air. If P_1, P_2 and P_3 represent their pressure amplitudes respectively, then correct relation between them is



(a) $P_1 > P_2 > P_3$ (b) $P_3 > P_2 > P_1$
(c) $P_1 = P_2 = P_3$ (d) $P_2 > P_3 > P_1$

16. The equation of displacement due to a sound wave is $s = s_0 \sin^2(\omega t - kx)$. If the bulk modulus of the medium is B, then the equation of pressure variation due to that sound is
(a) $Bks_0 \sin(2\omega t - 2kx)$
(b) $-Bks_0 \sin(2\omega t - 2kx)$
(c) $Bks_0 \cos^2(\omega t - kx)$
(d) $-Bks_0 \cos^2(\omega t - kx)$

17. The average density of Earth's crust 10 km beneath the surface is 2.7 gm/cm^3. The speed of longitudinal seismic waves at that depth is 5.4 km/s. The bulk modulus of Earth's crust considering its behaviour as fluid at that depth, is
(a) 7.9×10^{10} Pa (b) 5.6×10^{10} Pa
(c) 7.9×10^7 Pa (d) 1.46×10^7 Pa

18. Two identical sounds S_1 and S_2 reach at a point P in phase. The resultant loudness at point P is n dB higher than the loudness of S_1. The value of n is
(a) 2 (b) 4
(c) 5 (d) 6

19. In expressing sound intensity, we take 10^{-19} W m^{-2}, as the reference level. For ordinary conversation, the intensity level is about 10^{-6} W m^{-2}. Expressed in decibel, this is
(a) 10^6 (b) 6
(c) 60 (d) $\log_e(10^6)$

20. A two-fold increase in intensity of a wave implies an increase of (Given: $\log_{10} 2 = 0.3010$.)
(a) 2 dB (b) 10 dB
(c) 3.01 dB (d) 0.5 dB

Doppler Effect

21. When a source moves away from a stationary observer, the frequency is 6/7 times the original frequency. Given: speed of sound = 330 m/s. The speed of the source is
(a) 40 m/s (b) 55 m/s
(c) 330 m/s (d) 165 m/s

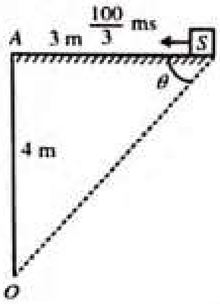
22. An engine running at speed $v/10$ sounds a whistle of frequency 600 Hz. A passenger in a train coming from the opposite side at speed $v/15$ experiences this whistle to be of frequency f. If v is speed of sound in air and there is no wind, f is nearest to

(a) 711 Hz (b) 630 Hz
(c) 580 Hz (d) 510 Hz

23. The apparent frequency of the whistle of an engine changes in the ratio of 6:5 as the engine passes a stationary observer. If the velocity of sound is 330 m/s, then the velocity of the engine is

(a) 3 m/s (b) 30 m/s
(c) 0.33 m/s (d) 660 m/s

24. A source of sound S is travelling at 100/3 m/s along a road, towards a point A. When the source is 3 m away from A, a person standing at a point O on a road perpendicular to AS hears a sound of frequency f'. The distance of O from A at that time is 4 m. If the original frequency is 640 Hz, then the value of f' is (velocity of sound is 340 m/s)

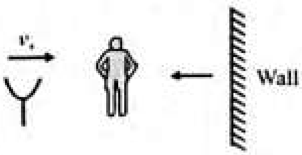


(a) 620 Hz
(b) 680 Hz
(c) 720 Hz
(d) 840 Hz

25. A source of sound is travelling with a velocity of 30 m/s towards a stationary observer. If actual frequency of source is 1000 Hz and the wind is blowing with velocity 20 m/s in a direction at 60° with the direction of motion of source, then the apparent frequency heard by observer is (speed of sound is 340 m/s)

(a) 1011 Hz (b) 1000 Hz
(c) 1094 Hz (d) 1086 Hz

26. A tuning fork of frequency 380 Hz is moving towards a wall with a velocity of 4 m/s. Then the number of beats heard by a stationary listener between direct and reflected sounds will be (velocity of sound in air is 340 m/s)



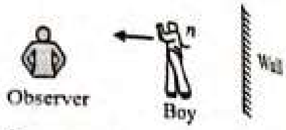
(a) 0 (b) 5
(c) 7 (d) 10

27. A sound wave of frequency n travels horizontally to the right with speed c. It is reflected from a broad wall moving to the left with speed v. The number of beats heard by a stationary observer to the left of the wall is

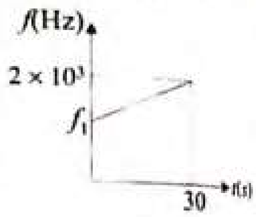
(a) zero (b) $\dfrac{n(c+v)}{c-v}$
(c) $\dfrac{nv}{c-v}$ (d) $\dfrac{2nv}{c-v}$

28. A boy is walking away from a wall at a speed of 1.0 m/s in a direction at right angles to the wall. The boy blows a whistle steadily. An observer towards whom the boy is moving hears 4 beats/s. If the speed of sound is 340 m/s, the frequency of whistle is

(a) 480 Hz
(b) 680 Hz
(c) 840 Hz
(d) 1000 Hz

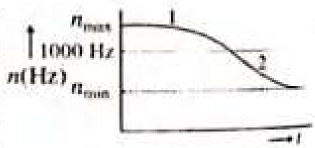


29. A source of sound of frequency f_1 is placed on the ground. A detector placed at a height is released from rest on this source. The observed frequency f(Hz) is plotted against time t(sec). The speed of sound in air is 300 m/s. Find f_1 (g = 10 m/s^2).



(a) 0.5×10^3 Hz (b) 1×10^3 Hz
(c) 0.25×10^3 (d) 0.2×10^3 Hz

30. A stationary observer receives a sound from a sound of frequency f_0 moving with a constant velocity v_s = 30 m/s. The apparent frequency varies with time as shown in figure. Velocity of sound v = 300 m/s. Then which of the following is incorrect?



(a) The minimum value of apparent frequency is 889 Hz.
(b) The natural frequency of source is 1000 Hz.
(c) The frequency–time curve corresponds to a source moving at an angle to the stationary observer.
(d) The maximum value of apparent frequency is 1111 Hz.

31. Two passenger trains moving with a speed of 108 km/hour cross each other. One of them blows a whistle whose frequency is 750 Hz. If sound speed is 330 m/s, then passengers sitting in the other train, after trains cross each other will hear sound whose frequency will be

(a) 900 Hz (b) 625 Hz
(c) 750 Hz (d) 800 Hz

32. A table is revolving on its axis at 5 revolutions per second. A sound source of frequency 1000 Hz is fixed on the table at 70 cm from the axis. The minimum frequency heard by a listener standing at a distance from the table will be (speed of sound = 352 m/s)

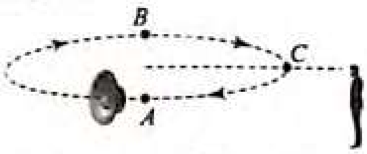
(a) 1000 Hz (b) 1066 Hz
(c) 941 Hz (d) 352 Hz

33. Two sirens situated one kilometer apart are producing sound of frequency 330 Hz. An observer starts moving

from one siren to the other with a speed of 2 m/s. If the speed of sound be 330 m/s, what will be the beat frequency heard by the observer?

(a) 8 (b) 4
(c) 6 (d) 1

34. A small source of sound moves on a circle as shown in the figure and an observer is standing on O. Let n_1, n_2 and n_3 be the frequencies heard when the source is at A, B and C respectively. Then



(a) $n_1 > n_2 > n_3$ (b) $n_2 > n_3 > n_1$
(c) $n_1 = n_2 > n_3$ (d) $n_2 > n_1 > n_3$

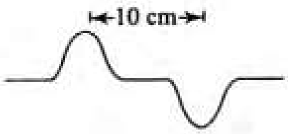
Superposition of Waves

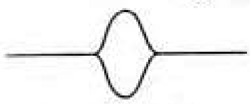
35. Equation of motion in the same direction is given by $y_1 = A\sin(\omega t - kx)$, $y_2 = A\sin(\omega t - kx - \theta)$. The amplitude of the medium particle will be

(a) $2A\cos\frac{\theta}{2}$ (b) $2A\cos\theta$
(c) $\sqrt{2}A\cos\frac{\theta}{2}$ (d) $1.2f, 1.2\lambda$

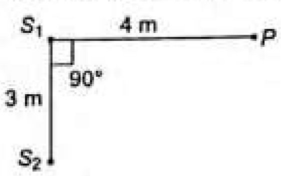
36. Two pulses travel in mutually opposite directions in a string with a speed of 2.5 cm/s as shown in the figure. Initially the pulses are 10 cm apart. What will be the state of the string after two seconds?

(a) 

(b) 

(c)
(d) 

37. S_1 and S_2 are two coherent sources of sound of frequency 110 Hz each. They have no initial phase difference. The intensity at a point P due to S_1 is I_0 and due to S_2 is $4I_0$. If the velocity of sound is 330 m/s then the resultant intensity at P is



(a) I_0 (b) $9I_0$
(c) $3I_0$ (d) $8I_0$

38. The wavelength of the waves arriving at P from two coherent sources S_1 and S_2 is 4 m, while intensity of each wave is I_0. The resultant intensity at P is $2\,I_0$. Find the minimum value of S_2P.

(a) $\frac{x\sqrt{5}}{2}$ (b) $2x$
(c) $(x + 1)$ (d) $(x + \sqrt{2})$

39. At a point, beats frequency of n Hz is observed. It means:
(a) medium particles, at that point, are vibrating with frequency n Hz
(b) amplitude of vibrations changes simple harmonically with frequency n Hz at that point only
(c) at that, zero intensity is observed $2n$ times per second
(d) none of the above

40. When beats are produced by two progressive waves of nearly the same frequency, which one of the following is correct?
(a) The particles vibrate simple harmonically, with the frequency equal to the difference in the component frequencies.
(b) The amplitude of vibration at any point changes simple harmonically with a frequency equal to the difference in the frequencies of the two waves.
(c) The frequency of beats depends upon the position, where the observer is.
(d) The frequency of beats changes as the time progresses.

41. Wavelengths of two notes in air are 1 m and $\frac{1}{164}$ m. Each note produces 1 beat/s with a third note of a fixed frequency. The speed of sound in air is
(a) 330 m/s (b) 340 m/s
(c) 350 m/s (d) 328 m/s

42. The equation of a stationary wave is given by
$y = 6\sin(\pi x/3)\cos 40\pi t$
where y and x are given in cm and time t in second, then the amplitude of progressive wave is
(a) 6 cm (b) 3 cm
(c) 12 cm (d) 2 cm

43. In question 42, the wavelength of the component progressive wave is
(a) 6 cm (b) 3 cm
(c) 12 cm (d) 2 cm

44. In question 42, the time period of the component progressive wave is
(a) 20 sec (b) 40 sec
(c) (1/20) sec (d) (1/10) sec

45. In question 42, the frequency of the component progressive wave is
(a) 20 Hz (b) 40 Hz
(c) (1/20) Hz (d) (1/10) Hz

46. In question 42, the speed of the component progressive wave is
(a) 20 cm/sec (b) 6 cm/sec
(c) 120 cm/sec (d) 40 cm/sec

47. In question 46, the separation between two consecutive antinodes is:
(a) 3 cm (b) 6 cm
(c) 12 cm (d) 20 cm

48. In question 42, the phase difference between two points on opposite sides of an antinode with a separation of 1 cm between them is
(a) zero radian (b) $(\pi/3)$ radian
(c) $(\pi/2)$ radian (d) π radian

49. The equation of displacement of two waves are given as

$$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right); \quad y_2 = 5[\sin 3\pi t + \sqrt{3}\cos 3\pi t]$$

Then what is the ratio of their amplitudes
(a) 1 : 2 (b) 2 : 1
(c) 1 : 1 (d) None of these

50. On sounding tuning fork A with another tuning fork B of frequency 384 Hz, 6 beats are produced per second. After loading the prongs of A with wax and then sounding it again with B, 4 beats are produced per second. What is the frequency of the tuning fork A.
(a) 388 Hz (b) 80 Hz
(c) 378 Hz (d) 390 Hz

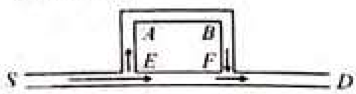
51. The equation of a stationary wave is $y = 0.8\cos\left(\frac{\pi x}{20}\right)$ $\sin 200\,\pi t$ where x is in cm and t is in s. The separation between consecutive nodes will be
(a) 20 cm (b) 10 cm
(c) 40 cm (d) 30 cm

52. Which of the following statements is correct for stationary waves?
(a) Nodes and antinodes are formed in case of stationary transverse wave only
(b) In case of longitudinal stationary wave, compressions and rarefactions are obtained in place of nodes and antinodes respectively
(c) Suppose two plane waves, one longitudinal and the other transverse having same frequency and amplitude are travelling in a medium in opposite directions with the same speed, by superposition of these waves, stationary waves cannot be obtained
(d) None of the above

53. A sound wave of wavelength 0.40 m enters the tube at S. The smallest radius r of the circular segment to hear minimum at detector D must be

(a) 1.75 m (b) 0.175 m
(c) 0.93 m (d) 9.3 m

54. A sound wave starting from source S, follows two paths $SEFD$ and $SEABFD$. If $AB = l$, $AE = BF = 0.6\ l$ and wavelength of wave is $\lambda = 11$ m. If maximum sound is heard at D, then minimum value of length l is



(a) 11 m (b) 6 m
(c) 2.5 m (d) 5 m

55. The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\frac{t}{2}\sin 1000t$$

How many independent harmonic motions may be considered to superpose to result this expression:
(a) two (b) three
(c) four (d) five

Vibration of String and Organ Pipes

56. A sonometer wire of length l vibrates in fundamental mode when excited by a tuning fork of frequency 416 Hz. If the length is doubled keeping other things same, the string will
(a) vibrate with a frequency of 416 Hz
(b) vibrate with a frequency of 208 Hz
(c) vibrate with a frequency of 832 Hz
(d) stop vibrating

57. A sonometer wire, 100 cm in length has fundamental frequency of 330 Hz. The velocity of propagation of transverse waves along the wire is
(a) 330 m/s (b) 660 m/s
(c) 115 m/s (d) 990 m/s

58. A wire of length 'l' having tension T and radius 'r' vibrates with fundamental frequency 'f'. Another wire of the same metal with length '$2l$' having tension $2T$ and radius $2r$ will vibrate with fundamental frequency:
(a) f (b) $2f$
(c) $\frac{f}{2\sqrt{2}}$ (d) $\frac{f}{2}\sqrt{2}$

59. When the string of a sonometer of length L between the bridges vibrates in the first overtone, the amplitude of vibration is maximum at
(a) $L/2$
(b) $(L/4)$ and $(3L/4)$
(c) $(L/6)$, $(3L/6)$ and $(5L/6)$
(d) $\frac{L}{8}, \frac{3L}{8}, \frac{5L}{8}, \frac{7L}{8}$

60. A string is under tension so that its length is increased by $1/n$ times its original length. The ratio of fundamental frequency of longitudinal vibrations and transverse vibrations will be
(a) $1:n$ (b) $n^2:1$
(c) $\sqrt{n}:1$ (d) $n:1$

61. A stretched string of length 1 m fixed at both ends, having a mass of 5×10^{-4} kg is under a tension of 20 N. It is plucked at a point situated at 25 cm from one end. The stretched string would vibrate with a frequency of

(a) 400 Hz (b) 100 Hz
(c) 200 Hz (d) 256 Hz

62. A closed organ pipe and an open organ pipe have their first overtones identical in frequency. Their lengths are in the ratio
(a) 1:2 (b) 2:3
(c) 3:4 (d) 4:5

63. Two organ pipes, both closed at one end, have lengths l and $l+\Delta l$. Neglect end correction. If the velocity of sound in air is v, then the number of beats / s is
(a) $\frac{v}{4l}$ (b) $\frac{v}{2l}$
(c) $\frac{v}{4l^2}\Delta l$ (d) $\frac{v}{2l^2}\Delta l$

64. A closed organ pipe has a frequency 'n'. If its length is doubled and radius is halved, its frequency nearly becomes.
(a) halved (b) doubled
(c) trebled (d) quadrupled

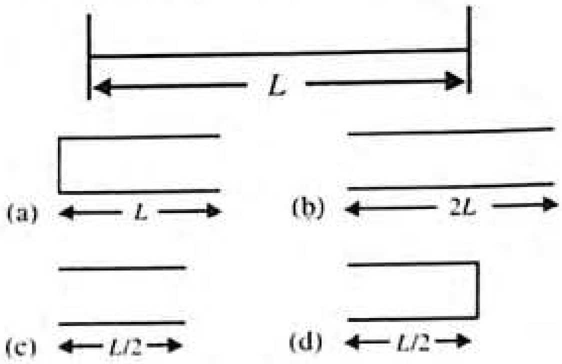
65. An organ pipe A closed at one end vibrating in its fundamental frequency and another pipe B open at both ends is vibrating in its second overtone are in resonance with a given tuning fork. The ratio of length of pipe A to that of B is
(a) 1:2 (b) 3:8
(c) 2:3 (d) 1:6

66. An open and a closed pipe have same length. The ratio of frequency of their nth overtone is
(a) $\frac{n+1}{2n+1}$ (b) $\frac{2(n+1)}{2n+1}$
(c) $\frac{n}{2n+1}$ (d) $\frac{n+1}{2n}$

67. A closed organ pipe and an open organ pipe of same length produce 2 beats when they are set into vibration simultaneously in their fundamental mode. The length of the open organ pipe is now halved and of the closed organ pipe is doubled; the number of beats produced will be
(a) 8 (b) 7
(c) 4 (d) 2

68. An open organ pipe of length l is sounded together with another open organ pipe of length $l+x$ in their fundamental tones. Speed of sound in air is v. The beat frequency heard will be ($x \ll l$)
(a) $\frac{vx}{4l^2}$ (b) $\frac{vl^2}{2x}$
(c) $\frac{vx}{2l^2}$ (d) $\frac{vx^2}{2l}$

69. Figure shows a stretched string of length L and pipes of length L, $2L$, $L/2$ and $L/2$ in options (a), (b), (c) and (d) respectively. The string's tension is adjusted until the speed of waves on the string equals the speed of sound waves in the air. The fundamental mode of oscillation is then set up on the string. In which pipe will the sound produced by the string cause resonance?



70. An ideal organ pipe resonates at successive frequencies of 50 Hz, 150 Hz, 250 Hz, etc.(speed of sound = 340 m/s) The pipe is
(a) open at both ends and of length 3.4 m
(b) open at both ends and of length 6.8 m
(c) closed at one end, open at the other, and of length 1.7 m
(d) closed at one end, open at the other, and of length 3.4 m

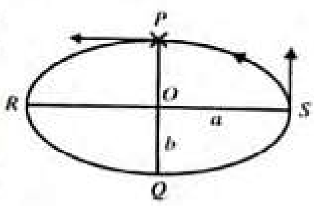
Problems Based on Mixed Concepts

71. The linear density of a vibrating string is 10^{-4} kg/m. A transverse wave is propagating on the string, which is described by the equation $y = 0.02 \sin(x + 30t)$, where x and y are in metres and time t in seconds. Then tension in the string is
(a) 0.09 N (b) 0.36 N
(c) 0.9 N (d) 3.6 N

72. Two blocks of masses 40 kg and 20 kg are connected by a wire that has a linear mass density of 1 g/m. These blocks are being pulled across horizontal frictionless floor by horizontal force F that is applied to 20 kg block. A transverse wave travels on the wave between the blocks with a speed of 400 m/s (relative to the wire). The mass of the wire is negligible compared to the mass of the blocks. The magnitude of F is
(a) 160 N (b) 240 N
(c) 320 N (d) 400 N

73. A source of sound produces waves of wavelength 60 cm when it is stationary. If the speed of sound in air is 320 m/s and source moves with speed 20 m/s, the wavelength of sound in the forward direction will be nearest to
(a) 56 cm (b) 60 cm
(c) 64 cm (d) 68 cm

74. A train is moving in an elliptical orbit in anticlockwise sense with a speed of 110 m/s. Guard is also moving in the given direction with same speed as that of train. The



ratio of the length of major and minor axes is 4/3. Driver blows a whistle of 1900 Hz at P, which is received by guard at S. The frequency received by guard is (velocity of sound $v = 330$ m/s)

(a) 1900 Hz (b) 1800 Hz
(c) 2000 Hz (d) 1500 Hz

75. In sports meet the timing of a 200 m straight dash is recorded at the finish point by starting an accurate stop watch on hearing the sound of starting gun fired at the starting point. The time recorded will be more accurate

(a) in winter (b) in summer
(c) in all seasons (d) none of these

76. Length of a string of density ρ and Young's modulus Y under tension is increased by $1/n$ times of its original length. If the velocity of transverse and longitudinal vibrations of the string is same, find the value of such velocity.

(a) $\sqrt{\dfrac{Y}{\rho n}}$ (b) $\sqrt{\dfrac{Y}{\rho n^{1/2}}}$
(c) $\sqrt{\dfrac{Y}{\rho}}$ (d) $\sqrt{\dfrac{Y}{\rho n^{3/2}}}$

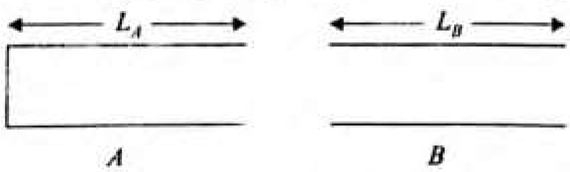
77. Source and observer start moving simultaneously along x and y-axis, respectively. The speed of source is twice the speed of observer, v_0. If the ratio of observed frequency to the frequency of the source is 0.75, find the velocity of sound.

(a) $\dfrac{11}{\sqrt{5}}v_0$ (b) $\dfrac{17}{\sqrt{5}}v_0$
(c) $\dfrac{16}{\sqrt{5}}v_0$ (d) $\dfrac{19}{\sqrt{5}}v_0$

78. A string under a tension of 100 N, emitting its fundamental mode, gives 5 beats/s with a tuning fork. When the tension is increased to 121 N, again 5 beats/s are heard. The frequency of the fork is

(a) 105 Hz (b) 95 Hz
(c) 210 Hz (d) 190 Hz

79. Two pipes are submerged in sea water, arranged as shown in figure. Pipe A with length $L_A = 1.5$ m and one open end, contains a small sound source that sets up the standing wave with the second lowest resonant frequency of that pipe. Sound from pipe A sets up resonance in pipe B, which has both ends open. The resonance is at the second lowest resonant frequency of pipe B. The length of the pipe B is



(a) 1 m (b) 1.5 m
(c) 2 m (d) 3 m

80. Equations of stationary and a traveling waves are as follows:

$y_1 = a \sin kx \cos \omega t$ and $y_2 = a \sin (\omega t - kx)$

The phase difference between two points $x_1 = \dfrac{\pi}{3k}$ and $x_2 = \dfrac{3\pi}{2k}$ are ϕ_1 and ϕ_2 respectively for the two waves. The ratio (ϕ_1/ϕ_2) is

(a) 1 (b) $\dfrac{5}{6}$
(c) $\dfrac{3}{4}$ (d) $\dfrac{6}{7}$

81. Two waves of wavelengths λ and $(\lambda + \Delta\lambda)$ produce 6 beats per second. If $\dfrac{\Delta\lambda}{\lambda} = \dfrac{\lambda + \Delta\lambda}{51}$, find the speed of sound wave in a gas in which these waves produce beats.

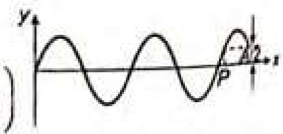
(a) 316 m/s (b) 310 m/s
(c) 306 m/s (d) 326 m/s

82. The equations of a travelling and stationary waves are $y_1 = a \sin (\omega t - kx)$ and $y_2 = a \sin kx \cos \omega t$. The phase differences between two points $x_1 = \dfrac{\pi}{4k}$ and $x_2 = \dfrac{4\pi}{3k}$ are ϕ_1 and ϕ_2 respectively for two waves, where k is the wave number. The ratio of ϕ_1/ϕ_2 is

(a) 6/7 (b) 16/3
(c) 12/13 (d) 13/12

83. The position of a wave (of wavelength λ)

$y(x,t) = A \sin 2\pi\left(\dfrac{x}{\lambda} - 3750\,t\right)$



is shown at $t = 0$, find x-coordinate of point P in metres if the wave speed is 300 m/sec:

(a) $\dfrac{27\lambda}{12}$ (b) $\dfrac{29\lambda}{12}$
(c) $\dfrac{25\lambda}{12}$ (d) $\dfrac{31\lambda}{12}$

84. A simple harmonic wave of amplitude 2 units is travelling along positive x-axis. At a given instant, displacement for two different particle on the wave is 1 unit and $\sqrt{2}$ unit. If the wavelength is λ, find the distance between them.

(a) $\dfrac{\lambda}{12}$ (b) $\dfrac{\lambda}{8}$
(c) $\dfrac{\lambda}{24}$ (d) $\dfrac{\lambda}{16}$

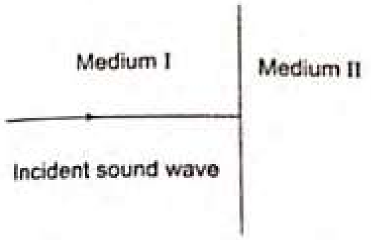
85. A rightward moving wave $y(x, t) = A \sin (kx - \omega t + \phi)$ has initial condition $y = 0$ at $x = 0$ and $t = 0$. Find the value of x and t at $\phi = (1/2)\,n\pi$ for which particle acceleration will have the same value as for initial condition and $\phi = 2n\pi$.

$n = 0, 2,...$ if λ and f are wave length and frequency respectively.

(a) $\lambda, \frac{1}{f}$ (b) $\frac{\lambda}{4}, \frac{2}{f}$

(c) $2\lambda, \frac{1}{f}$ (d) Both (a) and (c)

86. A sound wave propagating along x-axis, in medium I of density $\rho_1 = 1.5$ kg/m^3 is transmitted to a medium II of density $\rho_2 = 3$ kg/m^3 as shown.



The equation of excess pressure developed by wave in medium I and that in medium II respectively are

$$p_1 = 4\times10^{-2}\cos\omega\left(t - \frac{x}{400}\right) \quad \text{(in SI units)}$$

$$p_2 = 3\times10^{-2}\cos\omega\left(t - \frac{x}{1200}\right) \quad \text{(in SI units)}$$

Then the ratio of intensity of transmitted wave I_2 (wave in medium II) to the intensity of incident wave I_1 (wave in medium I), that is, $\frac{I_2}{I_1}$ is

(a) $\frac{3}{4}$ (b) $\frac{9}{16}$

(c) $\frac{3}{32}$ (d) None of these

ARCHIVES

1. A wave $y = a\sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation of the unknown wave is

(a) $y = a\sin(\omega t + kx)$ (b) $y = a\sin(\omega t - kx)$

(c) $y = -a\sin(\omega t + kx)$ (d) $y = -a\sin(\omega t - kx)$

(AIEEE 2002)

2. A tuning fork produces 4 beats per second with another fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats per second. The unknown frequency is

(a) 286 cps (b) 284 cps

(c) 292 cps (d) 290 cps **(AIEEE 2002)**

3. Tube A has both ends open, while tube B has one end closed; otherwise, they are identical. The ratio of the fundamental frequencies of tubes A and B is

(a) 1 : 2 (b) 2 : 1

(c) 1 : 4 (d) 4 : 1 **(AIEEE 2002)**

4. The length of a string tied to two rigid supports is 40 cm. The maximum length (wavelength in cm) of a stationary wave produced on it is

(a) 20 (b) 80

(c) 40 (d) 120 **(AIEEE 2002)**

5. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg weight between two rigid supports 1 metre apart. The wire passes at its middle point between the poles of a permanent magnet, and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is

(a) 25 Hz (b) 50 Hz

(c) 100 Hz (d) 200 Hz **(AIEEE 2003)**

6. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(a) 256 + 5 Hz (b) 256 + 2 Hz

(c) 256 – 2 Hz (d) 256 – 5 Hz

(AIEEE 2003)

7. The displacement y of a particle in a medium can be expressed as

$$y = -6\sin\left(100t + 20x + \frac{\pi}{4}\right)m$$

where t is in seconds and x in metres. The speed of the wave is

(a) 2000 m/s (b) 50 m/s

(c) 20 m/s (d) 5 m/s **(AIEEE 2004)**

8. The displacement y of a particle in a medium can be expressed as: $y = 10^{-6}\sin(100t + 20x + \pi/4)m$, where t is in second and x in meter. The speed of wave is

(a) 2000 m/s (b) 5 m/s

(c) 20 m/s (d) 5 π m/s **(AIEEE 2004)**

9. An observer moves towards a stationary source of sound with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency?

(a) 20% (b) 5%

(c) 0.5% (d) zero **(AIEEE 2005)**

10. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now some tape is attached to the prong of fork 2. When the tuning forks are sounded again, 6 beats per second are heard.

If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2?

(a) 204 Hz (b) 196 Hz
(c) 202 Hz (d) 200 Hz **(AIEEE 2005)**

11. Two simple harmonic motions are represented by the equations

$y_1 = 0.1\,[\sin 100\pi t + (\pi/3)]$ and $y_2 = 0.1\cos \pi t$.

The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{-\pi}{3}$
(c) $\dfrac{\pi}{3}$ (d) $\dfrac{-\pi}{6}$ **(AIEEE 2005)**

12. A string is stretched between fixed points departed by 75 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then the lowest resonant frequency for this string is

(a) 105 Hz (b) 1.05 Hz
(c) 1050 Hz (d) 10.5 Hz **(AIEEE 2006)**

13. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with velocity v metres per second. The velocity of sound in air is 300 m/s. If a person can hear frequencies up to 10000 Hz, the maximum value of v up to which he can hear the whistle is

(a) $15\sqrt{2}$ m/s (b) $\dfrac{15}{\sqrt{2}}$ m/s
(c) 15 m/s (d) 30 m/s **(AIEEE 2006)**

14. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of

(a) 10000 (b) 10
(c) 100 (d) 1000 **(AIEEE 2007)**

15. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, he measures the column length to be x centimetre for the second resonance. Then

(a) $18 > x$ (b) $x > 54$
(c) $54 > x > 36$ (d) $36 > x > 18$

(AIEEE 2008)

16. The speed of sound in oxygen at a certain temperature is 460 m/s. The speed in helium at the same temperature will be (assume both gases to be ideal)

(a) 460 m/s (b) 500 m/s
(c) 650 m/s (d) 330 m/s **(AIEEE 2008)**

17. A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos (\alpha x - \beta t)$. If the wavelength and time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

(a) $\alpha = 25.00\pi,\ \beta = \pi$ (b) $\alpha = \dfrac{0.08}{\pi},\ \beta = \dfrac{2.0}{\pi}$
(c) $\alpha = \dfrac{0.04}{\pi},\ \beta = \dfrac{1.0}{\pi}$ (d) $\alpha = 12.50\pi,\ \beta = \dfrac{\pi}{2.0}$

(AIEEE 2008)

18. A motor cycle starts from rest and accelerates along a straight path at 2 m/s^2. At the starting point of the motor cycle, there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound is 330 m/s.)

(a) 49 m (b) 98 m
(c) 147 m (d) 196 m **(AIEEE 2009)**

19. Three sound waves of equal amplitudes have frequencies $(f - 1)$, f, and $(f + 1)$. They superpose to give beats. The number of beats produced per second will be

(a) 4 (b) 3
(c) 2 (d) 1 **(AIEEE 2009)**

20. The equation of a wave on a string of linear mass density 0.04 kg/m is given by

$$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$$

The tension in the string is

(a) 4.0 N (b) 12.5 N
(c) 0.5 N (d) 6.25 N **(AIEEE 2010)**

21. The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)}$

This represents a

(a) wave moving in $+x$ direction with speed $\sqrt{\dfrac{a}{b}}$

(b) wave moving in $-x$ direction with speed $\sqrt{\dfrac{b}{a}}$

(c) standing wave of frequency $\sqrt{b}$

(d) standing wave of frequency $\dfrac{1}{\sqrt{b}}$ **(AIEEE 2011)**

22. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now

(a) f (b) $f/2$
(c) $3f/4$ (d) $2f$ **(AIEEE 2012)**

23. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 and 2.2×10^{11} N/m^2 respectively?

(a) 178.2 Hz (b) 200.5 Hz
(c) 770 Hz (d) 188.5 Hz **(JEE Main 2013)**

24. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.
(a) 6 (b) 4
(c) 12 (d) 8 **(JEE Main 2014)**

25. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to:
(a) 6% (b) 12%
(c) 18% (d) 24% **(JEE Main 2015)**

26. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is: (take $g = 10\ ms^{-2}$)
(a) $2\pi\sqrt{2}$ s (b) 2 s
(c) $2\sqrt{2}$ s (d) $\sqrt{2}$ s **(JEE Main 2016)**

27. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now:
(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$
(c) $2f$ (d) f **(JEE Main 2016)**

28. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3\ kg/m^3$ and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?
(a) 7.5 kHz (b) 5 kHz
(c) 2.5 kHz (d) 10 kHz **(JEE Main 2018)**

ANSWER KEY

Exercises

1. (d)	2. (a)	3. (b)	4. (a)	5. (b)	6. (b)	7. (b)	8. (d)	9. (c)	10. (c)
11. (b)	12. (a)	13. (a)	14. (c)	15. (b)	16. (a)	17. (a)	18. (d)	19. (c)	20. (c)
21. (b)	22. (a)	23. (b)	24. (b)	25. (c)	26. (a)	27. (d)	28. (b)	29. (b)	30. (a)
31. (b)	32. (c)	33. (b)	34. (b)	35. (a)	36. (c)	37. (c)	38. (c)	39. (d)	40. (b)
41. (a)	42. (b)	43. (a)	44. (c)	45. (a)	46. (c)	47. (a)	48. (a)	49. (c)	50. (d)
51. (a)	52. (c)	53. (b)	54. (d)	55. (b)	56. (a)	57. (b)	58. (c)	59. (b)	60. (c)
61. (c)	62. (c)	63. (c)	64. (a)	65. (d)	66. (b)	67. (b)	68. (c)	69. (b)	70. (c)
71. (a)	72. (b)	73. (a)	74. (b)	75. (b)	76. (b)	77. (c)	78. (a)	79. (c)	80. (d)
81. (c)	82. (c)	83. (c)	84. (c)	85. (d)	86. (c)				

Archives

1. (c)	2. (c)	3. (b)	4. (b)	5. (b)	6. (d)	7. (d)	8. (b)	9. (a)	10. (b)
11. (d)	12. (a)	13. (c)	14. (c)	15. (b)	16. (None)	17. (a)	18. (b)	19. (c)	20. (d)
21. (b)	22. (a)	23. (a)	24. (a)	25. (b)	26. (c)	27. (d)	28. (b)		

Chapter 17

Electric Charge and Field

ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. It is known that every atom is electrically neutral, containing as many electrons as the number of protons in the nucleus. Charged particles can be created by disturbing the neutrality of an atom. Loss of electrons gives positive charge (as $n_p > n_e$) and gain of electrons gives negative charge (as $n_e > n_p$) to a particle. When an object is negatively charged, it gains electrons and therefore its mass increases negligibly. Similarly, on charging a body with positive electricity its mass decreases. Change in the mass of the object is equal to $n \times m_e$, where n is the number of electrons transferred and m_e is the mass of electron.

The rate of flow of electric charge is called electric current, i.e., $i = dQ/dt$ or $dQ = idt$.

Hence, SI unit of charge is ampere × second = coulomb (C). Smaller SI units are mC, μC, and nC (1 mC = 10^{-3} C, 1 μC = 10^{-6} C, 1 nC = 10^{-9} C). CGS unit of charge is stat coulomb or esu. Dimensional formula of charge is $[Q] = [AT]$.

Properties of Charge

Charge is transferable: If a charged body is put in contact with an uncharged body, the uncharged body becomes charged due to the transfer of electrons from one body to the other.

Charge is always associated with mass: Charge cannot exist without mass though mass can exist without charge.

Charge is conserved: Charge can neither be created nor be destroyed. Some important observations regarding this law are as follows:

- The net charge remains constant even though it may not be zero.
- If the universe is considered as a whole, conservation of charge means that the net charge of the universe is constant.
- This principle does not prohibit the creation and destruction of charged particles. However, because of charge conservation, charged particles are created and destroyed only in pairs with equal and opposite charges.
- The law is also true for relativistic motion which means that the total electric charge of an isolated system is relativistically invariant.
- On the basis of the evidence on the macroscopic scale, the principle is largely speculative, but there is ample justification for it on the microscopic scale.

CHARGING OF A BODY

It is possible to give a net charge to an object by different methods, all of which are said to involve electrostatic charging.

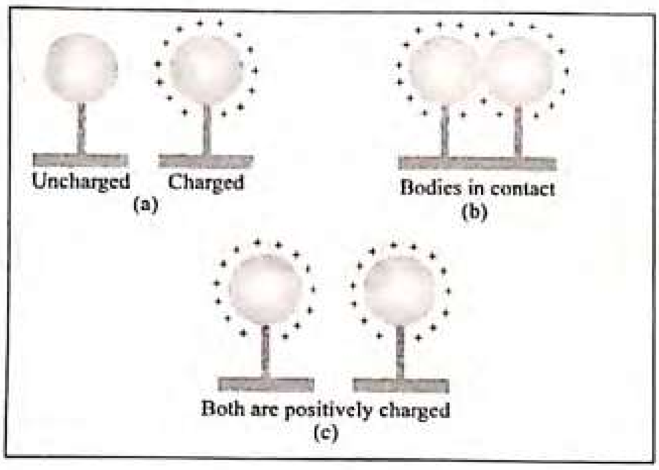
Charging by Friction

When certain insulators are rubbed with cloth or fur, they become electrically charged by a transfer of charge (i.e., electrons). As the two objects are rubbed together, one object loses electrons while the other gains electrons; that is, there is transfer of electrons from one object to the other. The object that gains electrons becomes negatively charged, while the object that loses electrons has an excess of positive charge. Hence it is positively charged. The transfer of charge is due to the contact between the materials, and the amount of charge transferred depends, as we might expect, on the nature of these materials.

Because of conservation of charge, each electron adds negative charge to the silk and an equal positive charge is left on the glass rod.

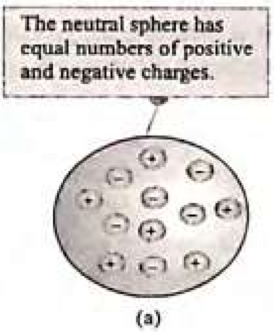
Charging by Conduction

Charging by conduction requires contact between two objects (see figure). Take two conductors, one charged and the other uncharged. Bring the conductors in contact with each other. The charge (whether positive/negative) under its own repulsion will spread over both the conductors. Thus, the conductors will be charged with the same sign. This is called charging by conduction (through contact).



Charging by Induction

Induction is a process by which a charged body can be used to create other charged bodies without touching them or losing its own charge. If a charged body is brought near a neutral body, the charged body attracts opposite charge and repels similar charge present on the neutral body. If the neutral body is now earthed, the like charge is neutralized by the flow of charge from earth, leaving unlike charge on the body. Now the earthing and the charging body are removed leaving the initially neutral body charged. The whole process is shown step by step in figure.



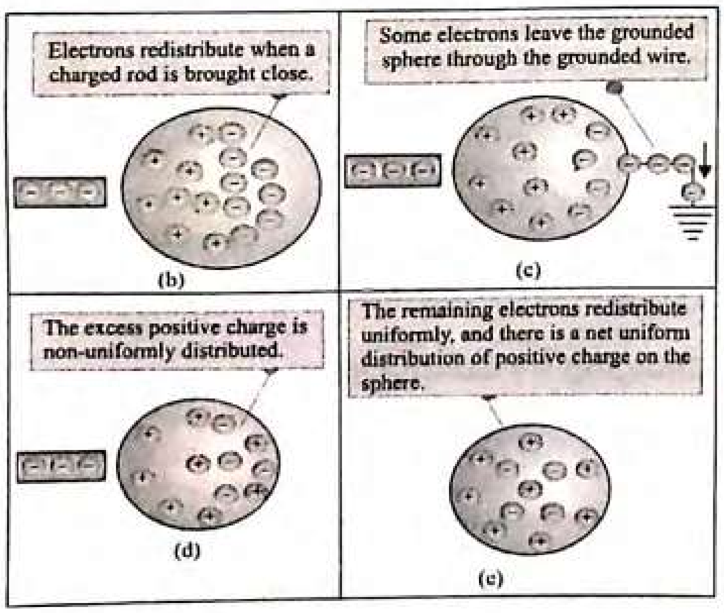


ILLUSTRATION 17.1 Calculate the total positive (or negative) charge on a 3.11 g copper penny. Given Avogadro's number = $6.023 \times 10^{23}\ g^{-1}\ mol^{-1}$; for copper, atomic number = 29 and atomic mass = 63.5.

Solution. The number of atoms in the penny is

$$\left[\frac{6.02 \times 10^{23}}{63.5}\right]3.11 = 0.295 \times 10^{23}$$

Since each copper atom contains 29 protons (and 29 electrons), the total number of positive (or negative) charges in the penny is $n = 29(0.295 \times 10^{23}) = 8.56 \times 10^{23}$.

Total charge (positive or negative) on the penny is

$$q = ne = (8.56 \times 10^{23})(1.6 \times 10^{-19}\ C)$$
$$= 1.37 \times 10^5\ C$$

CONCEPT APPLICATION EXERCISE 17.1

1. How many megacoulombs of positive (or negative) charge are present in 2.0 mol of neutral hydrogen gas.
2. A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.
   (a) Estimate the number of electrons transferred (from which to which)?
   (b) Is there a transfer of mass from wool to polythene?
3. Two identical conducting spheres, one having an initial charge $+Q$ and the other initially uncharged, are brought into contact.
   (a) What is the new charge on each sphere?
   (b) While the spheres are in contact, a positively charged rod is moved close to one sphere, causing a redistribution of the charges on the two spheres, so the charge on the sphere closest to the rod has a charge $-Q$. What is the charge on the other sphere?
4. Two identical conducting spheres are charged by induction and then separated by a large distance; sphere-1 has charge $+Q$ and sphere-2 has charge $-Q$. A third sphere is initially uncharged. If sphere-3 is touched to sphere-1 and separated and then touched to sphere-2 and separated, what is the final charge on each of the three spheres?
5. In 1 g of a solid, there are 5×10^{21} atoms. If one electron is removed from each of 0.01% atoms of the solid, find the charge gained by the solid (given that electronic charge is 1.6×10^{-19} C).

COULOMB'S LAW

Charles Coulomb measured the magnitude of electric forces between charged objects using the torsion balance. From Coulomb's experiments, we can generalize the properties

of the electric force (sometimes called electrostatic force) between two stationary charged particles.

We use the term point charge to refer to a charged particle of zero size. The electrical behavior of electrons and protons is very well described by modeling them as point charges. From experimental observations, Coulomb showed that the electric force between two stationary charged particles

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
- is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign. Thus,

$$F_e = k_e \frac{q_1 q_2}{r^2}$$

where k_e is a constant called the Coulomb constant. The value of the Coulomb constant depends on the choice of units. The SI unit of charge is coulomb (C). The Coulomb constant k_e in SI units has the value $k_e = 9 \times 10^9$ Nm^2 C^{-2}. This constant is also written in the form $k_e = 1/4\pi\varepsilon_0$, where the constant ε_0 (Greek letter epsilon) is known as the permittivity of free space and has the value $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}. Now we can write

$$F_e = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Important Points

- The permittivity of a given medium is the measure of the fact how strongly a medium is influenced by external electric field. If an externally applied field has stronger polarizing effect on the medium, it has high permittivity.
- If two charges are placed in any medium other than vacuum or air, the force between two charges decreases due to the polarization of the medium. Thus, the resultant force on a charge gets reduced by a factor k known as the dielectric constant of the medium or relative permittivity of the medium. Thus $\varepsilon/\varepsilon_0 = k = \varepsilon_r$, where ε is the permittivity of the medium, ε_0 is the permittivity of vacuum, and ε_r is relative permittivity. Thus,

$$|\vec{F}| = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2}$$

For vacuum, $\varepsilon_r = 1$, and for conductivity medium, $\varepsilon_r = \infty$.

- Coulomb force between two charges is an action-reaction pair, which is conservative in nature and is a central force. It acts along the line joining two point charges.
- Coulomb's law is valid only for point charges. If the size of an object is very small as compared to the separation, then they are considered point charges.
- The force between two point charges is independent of the presence or absence of any other charges. Due to the presence of the surrounding medium the resultant force changes because of polarization of the molecules of the medium.
- Coulomb's law is not valid for distances less than 10^{-15} m.
- Electrostatic forces are comparatively stronger than gravitational forces.
- Coulomb's law is similar to Newton's gravitational law and both obey inverse square law.
- Coulomb's law obeys Newton's third law, i.e., the forces exerted by the two charges on each other are equal and opposite.
- Electrostatic force is a conservative force.

COULOMB'S LAW IN VECTOR FORM

Let q_1 and q_2 be two like charges placed at points A and B, respectively, in vacuum. $\vec{r}_1$ is the position vector of point A, and $\vec{r}_2$ is the position vector of point B. Let $\vec{r}_{21}$ be vector from A to B, then

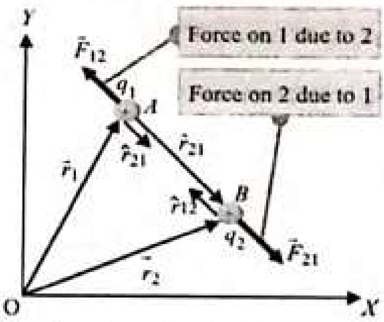
$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

and $\quad |\vec{r}_{21}| = r = |\vec{r}_2 - \vec{r}_1|$

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{r} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

From figure it is clear that $\vec{F}_{21}$ and $\hat{r}_{12}$ are in the same direction, so

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \frac{\vec{r}}{r} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}}{r^3} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$



The above equations give the Coulomb's law in vector form. As we know that charges apply equal and opposite forces on each other, we have $\vec{F}_{12} = -\vec{F}_{21}$

or $$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3}$$

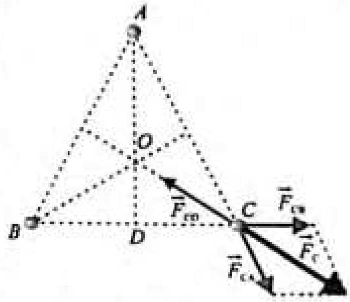
We can also write in terms of unit vector notation:

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$

where $\hat{r}_{12}$ is a unit vector directed toward q_1 from q_2. This force of Coulomb's law is illustrated in figure. For three different point charge distributions, we have $\hat{r}_{12} = -\hat{r}_{21}$. So

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r^2}(-\hat{r}_{21}) = -\frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r^2}\hat{r}_{21} = -\vec{F}_{21}$$

ILLUSTRATION 17.2 Three charges of equal magnitude q are placed at the vertices of an equilateral triangle of side l. How can the system of charges be placed in equilibrium?



Solution. To keep the system in equilibrium, the net force experienced by charges at A, B, and C should be zero. For this, another charge of opposite sign should be placed at the centroid of the triangle. Let this charge be $-Q$.

$$AD = l\cos 30° = \frac{l\sqrt{3}}{2}$$

$$AO = \frac{2}{3}AD = \frac{l}{\sqrt{3}} = CO$$

$$2|\vec{F}_{CA}|\cos 30° = |\vec{F}_{CO}|$$

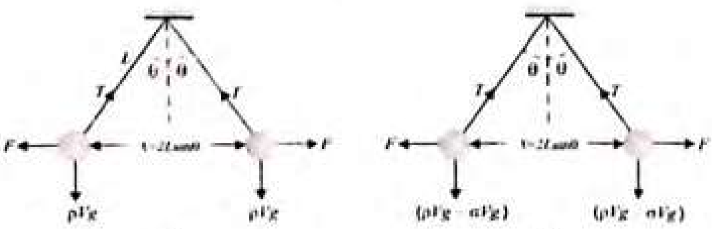
$$|\vec{F}_{CB}| = |\vec{F}_{CA}|$$

$$2\frac{1}{4\pi\varepsilon}\frac{q^2}{l^2}\frac{\sqrt{3}}{2} = \frac{1}{4\pi\varepsilon}\frac{Qq}{(l/\sqrt{3})^2} \text{ or } Q = \frac{q}{\sqrt{3}}$$

ILLUSTRATION 17.3 Two identical balls, each having a density ρ, are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with the vertical. Now, both the balls are immersed in a liquid. As a result, the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid.

Solution. Let V be the volume of each ball, then the mass of each ball is $m = \rho V$. When the balls are in air, from the previous problem,

$$F = mg\tan\theta = \rho Vg\tan\theta \qquad \text{(i)}$$



(a) (b)

When the balls are suspended in liquid, the Coulombic force is reduced to $F' = F/K$ and apparent weight = weight − upthrust, i.e.,

$$W' = \rho Vg - \sigma Vg$$

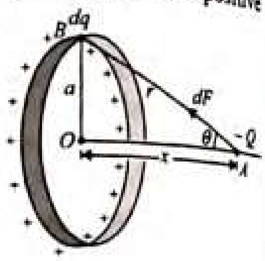
According to the problem, angle θ is unchanged. Therefore,

$$F' = W'\tan\theta = (\rho Vg - \sigma Vg)\tan\theta \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{F}{F'} = K = \frac{\rho Vg}{\rho Vg - \sigma Vg} = \frac{\rho}{\rho - \sigma}$$

ILLUSTRATION 17.4 A thin fixed ring of radius a has a positive charge q uniformly distributed over it. A particle of mass m, having a negative charge Q, is placed on the axis at a distance of x ($x << a$) from the center of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillation.



Solution. The force on the point charge Q due to the element dq of the ring is

$$dF = \frac{1}{4\pi\varepsilon_0}\frac{dqQ}{r^2} \text{ along } AB$$

For every element of the ring, there is a diametrically opposite element. The components of forces along the axis will add up, while those perpendicular to it will cancel each other. Hence, the net force on the charge is $-Q$; negative sign shows that this force will be toward the center of ring.

$$F = \int dF\cos\theta = \cos\theta\int dF = \frac{x}{r}\int\frac{1}{4\pi\varepsilon_0}\left[-\frac{Qdq}{r^2}\right]$$

So
$$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qx}{r^3}\int dq = -\frac{1}{4\pi\varepsilon_0}\frac{Qqx}{(a^2+x^2)^{3/2}} \qquad \text{(i)}$$

$$\left[\text{as } r = (a^2+x^2)^{1/2} \text{ and } \int dq = q\right]$$

As the restoring force is not linear, the motion will be oscillatory. However, if $x \ll a$ so that $x^2 \ll a^2$, then

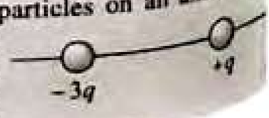
$$F = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^3}x = -kx \text{ with } k = \frac{Qq}{4\pi\varepsilon_0 a^3}$$

Thus, the restoring force will become linear and so the motion is simple harmonic with time period

$$T = \frac{2\pi}{\omega}2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{qQ}}$$

CONCEPT APPLICATION EXERCISE 17.2

1. Figure shows two charge particles on an axis. The charges are free to move. At one point, however, a third charged particle can be



placed such that all three particles are in equilibrium.
(a) Is that point to the left of the first two particles, to their right, or between them?
(b) Should the third particle be positively or negatively charged?
(c) Is the equilibrium stable or unstable?

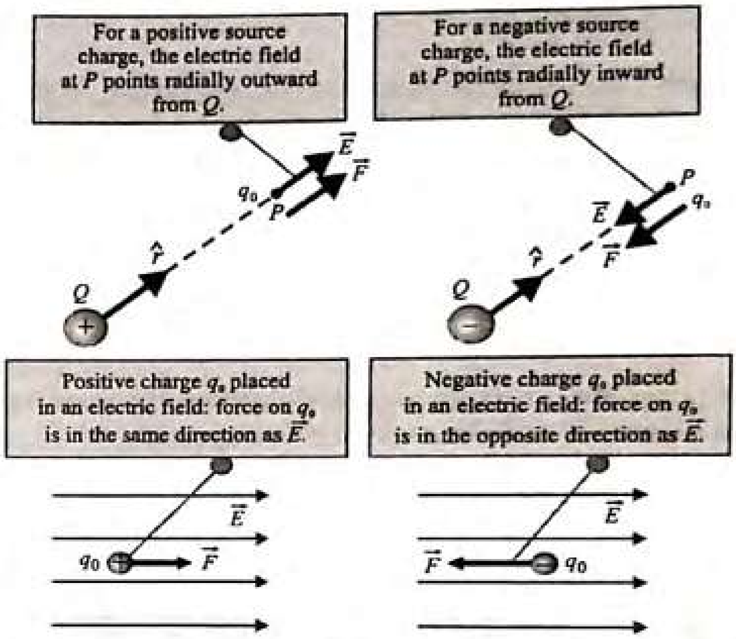
2. The force between two point electric charges kept at a distance d apart in air is F. If these charges are kept at the same distance in water, the force between the charges is F'. The ratio F'/F is equal to ________
3. Two small balls, each having charge q, are suspended by two insulating threads of equal length L from a hook in an elevator. The elevator is freely falling. Calculate the angle between the two threads and tension in each thread.
4. Suppose we have a large number of identical particles, very small in size. Any two of them at 10 cm separation repel with a force of 3×10^{-10} N.
   (a) If one of them is at 10 cm from a group (of very small size) of n others, how strongly do you expect it to be repelled?
   (b) Suppose you measure the repulsion and find it 6×10^{6} N How many particles were there in the group?
5. The electrostatic force of repulsion between two positively charged ions carrying equal charge is 3.7×10^{-9} N when these are separated by a distance of 5 Å. How many electrons are missing from each ion?
6. Two fixed point charges $+4e$ and $+e$ units are separated by a distance a. Where should a third point charge be placed for it to be in equilibrium?
7. Two insulated identically sized charged copper spheres A and B have their centers separated by a distance of 50 cm. A third sphere of the same size but uncharged is brought in contact with the first, then in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?
8. Two small balls having equal positive charge Q (coulomb) on each are suspended by two insulating strings of equal length L (meter) from a hook fixed to a stand. The whole set-up is taken in a satellite into space where there is no gravity (state of weightlessness).
   (i) What is the angle between the two strings?
   (ii) What is the tension in each string?

ELECTRIC FIELD

We define electric field as the space around a charge in which its influence can be felt by any other charged particle. An electric field is said to exist in the region of space around a charged object, the source charge. When another charged object—the test charge—enters this electric field, an electric force acts on it.

As an example, consider figure, which shows a small positive test charge q_0 placed near a second object carrying a much greater positive charge Q. We define the electric field due to the source charge at the location of the test charge to be the electric force on the test charge per unit charge. To be more specific, the electric field vector $\vec{E}$ at a point in space is defined as the electric force $\vec{F}_e$ acting on a positive test charge q_0 placed at that point divided by the test charge:

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$$



The direction of $\vec{E}$ will be same as that of $\vec{F}$. The magnitude of the test charge is kept small because otherwise it may disturb the original charge distribution and then we will get electric field due to disturbed configuration and not original.

The unit of electric field is NC^{-1} (newton per coulomb). The dimensional formula of electric field is

$$\frac{\text{Force}}{\text{Charge}} = \frac{MLT^{-2}}{\text{ampere} \times \text{time}} = \frac{MLT^{-2}}{AT} = [MLT^{-3}A^{-1}]$$

A Point Charge in an Electric Field

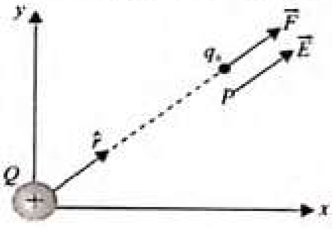
What happens if a point charge q_0 is placed at any point in an electric field produced by some other stationary charges? Let this electric field be $\vec{E}$. Charge q_0 will experience a force; let this force be $\vec{F}$. Then the value of electric field at that point must be

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{or} \quad \vec{F} = q_0\vec{E}$$

This is the force on q_0 by E. q_0 has no contribution in $\vec{E}$. A charged particle is not affected due to its own field. It means a charge particle can experience force due to the field produced by other charge particles, but not due to the field produced by itself.

ELECTRIC FIELD DUE TO AN ISOLATED POINT CHARGE

Consider a point charge Q placed at the origin O. To find the electric intensity at a point P, distant r from O, place a test charge q_0 at P. According to Coulomb's law, the force exerted on q_0 by Q is



$$\vec{F} = k_e \frac{Qq_0}{r^2}\hat{r}$$

where $\hat{r}$ is a unit vector from O to P, i.e., from Q to q_0. If $\vec{E}$ is the electric field created by Q at P, then by definition

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \lim_{q_0 \to 0} \frac{k_e \frac{Qq_0}{r^2}\hat{r}}{q_0}$$

$$= k_e \frac{Q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{r}$$

If Q is positive, $\vec{E}$ is directed radially outward from it and if Q is negative, $\vec{E}$ is directed toward it. In magnitude,

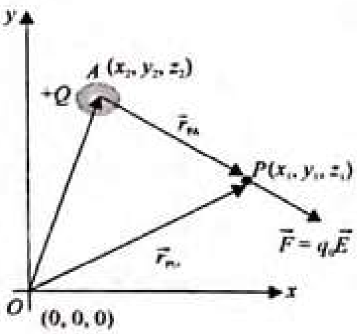
$$E = k_e \frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

ELECTRIC FIELD DUE TO A POINT CHARGE IN VECTOR NOTATION

Electric force on test charge $+q_0$ in the direction of position vector $\vec{r}_{PA}$ is given by $\vec{r}_{PA} = \vec{r}_{PO} - \vec{r}_{AO}$

$$\vec{F}_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^3}(\vec{r}_{PA})$$

$$\vec{E} = \frac{\vec{F}_E}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}(\vec{r}_{PA})$$



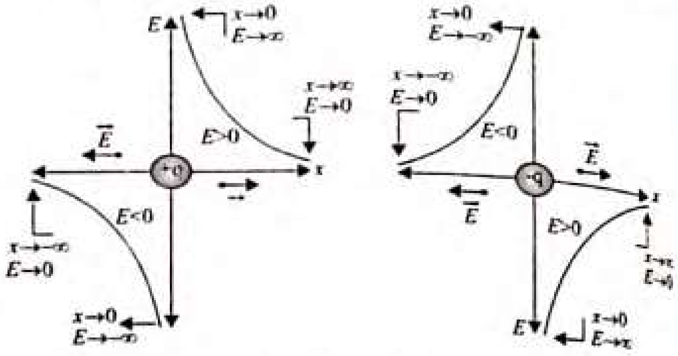
$\vec{E}$ is the electric field intensity vector at position P due to a point charge placed at the position A.

$$\vec{r}_{PA} = \vec{r}_{PO} - \vec{r}_{AO}$$

If $\quad \vec{r}_{PO} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r}_{AO} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$, then

$$\frac{Q\{(x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j} + (z_1 - z_2)\hat{k}\}}{\pi\varepsilon\ \{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2\}^{3/2}}$$

Graphical Variation of $\vec{E}$ on x-axis Due to a Point Charge



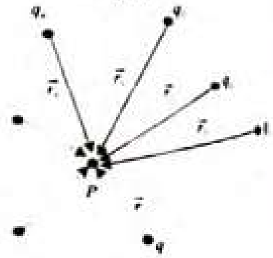
Electric Field Intensity Due to a Group of Charges

Using the principle of superposition, the net field at point P is given by

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\hat{r}_1 + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n^2}\hat{r}_n$$

$$= \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$



In terms of position vectors,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

ILLUSTRATION 17.5 Two point charges of $+5 \times 10^{-19}$ C and $+20 \times 10^{-19}$ C are separated by a distance of 2 m. Find the point on the line joining them at which electric field intensity is zero.

Solution. Here, $q_1 = +5 \times 10^{-19}$ C, $q_2 = +20 \times 10^{-19}$ C. If P is the point (distant x from q_1), where electric field intensity is zero, then

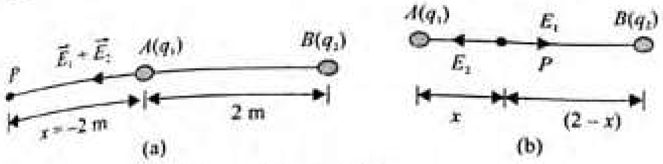
$$|E_1| = |E_2|$$

or $\quad k_e \dfrac{q_1}{x^2} = k_e \dfrac{q_2}{(2-x)^2}$

or $\quad \dfrac{(2-x)^2}{x^2} = \dfrac{q_2}{q_1} = \dfrac{20 \times 10^{-19}\,\text{C}}{5 \times 10^{-19}\,\text{C}} = 4$

or $\quad \dfrac{(2-x)}{x} = \pm 2$

Hence, $x = -2$ m or $x = 2/3$ m. If $x = -2$ m, the point P lies to the left of A. In this case, since $\vec{E}_1$ and $\vec{E}_2$ are in the same direction, the two positive charges produce a resultant electric field $\vec{E}$, where $\vec{E} = \vec{E}_1 + \vec{E}_2$ as shown in Figure (a). Hence, this is not a feasible situation.



Thus, as shown in Figure (b), $x = 2/3$ m.

ILLUSTRATION 17.6 The positions of two point charges q_1 and q_2 are $\vec{r}_1$ and $\vec{r}_2$, respectively. Find the position of the point where the net field is zero due to these charges.

Solution. At P, let the net field be zero.

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = 0 \tag{i}$$

As we know,

$$\vec{E}_1 = \frac{q_1(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0\,|\vec{r}-\vec{r}_1|^3} \tag{ii}$$

and $$\vec{E}_2 = \frac{q_2(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0\,|\vec{r}-\vec{r}_2|^3} \tag{iii}$$

Substituting $\vec{E}_1$ and $\vec{E}_2$ from Eqs. (ii) and (iii) in Eq. (i), we have

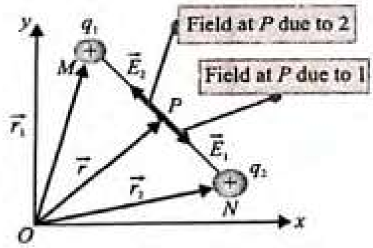
$$\frac{q_1(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|^3} + \frac{q_2(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^3} = 0$$

or $$q_1(\vec{r}-\vec{r}_1) + q_2(\vec{r}-\vec{r}_2)\cdot\frac{|\vec{r}-\vec{r}_1|^3}{|\vec{r}-\vec{r}_2|^3} = 0 \tag{iv}$$

Since $E = 0$ at P,

$$|\vec{E}_1| = |\vec{E}_2|$$

$$\Rightarrow \quad \frac{q_1}{4\pi\varepsilon_0 r_{P_1}^2} = \frac{q_2}{4\pi\varepsilon_0 r_{P_2}^2}$$



where $\vec{r}_{P_1} = \vec{r} - \vec{r}_1$

and $\vec{r}_{P_2} = \vec{r} - \vec{r}_2$. Therefore,

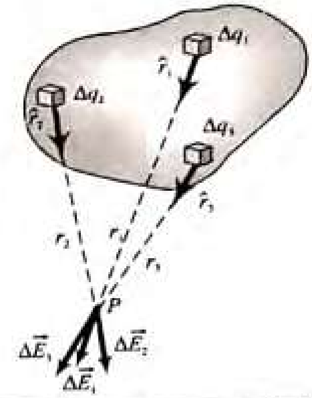
$$\frac{q_1}{|\vec{r}-\vec{r}_1|^2} = \frac{q_2}{|\vec{r}-\vec{r}_2|^2} \quad \text{or} \quad \frac{|\vec{r}-\vec{r}_1|}{|\vec{r}-\vec{r}_2|} = \frac{\sqrt{q_1}}{\sqrt{q_2}} \tag{v}$$

Substituting the value of $|\vec{r}-\vec{r}_1|/|\vec{r}-\vec{r}_2|$ from Eq. (v) in Eq. (iv), we get

$$q_1(\vec{r}-\vec{r}_1) + q_2(\vec{r}-\vec{r}_2)\left(\frac{q_1}{q_2}\right) = 0 \quad \text{or} \quad \vec{r} = \frac{\vec{r}_1\sqrt{q_2}+\vec{r}_2\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}$$

ELECTRIC FIELD DUE TO CONTINUOUS DISTRIBUTION OF CHARGE

A system of closely spaced electric charges forms a continuous charge distribution. To find the field of a continuous charge distribution, we divide the charge into infinitesimal charge elements. Each infinitesimal charge element is then considered a point charge, and the electric field $d\vec{E}$ is determined due to this charge at a given point. The net field at the given point is the summation of fields of all the elements, i.e., $\vec{E} = \int d\vec{E}$.

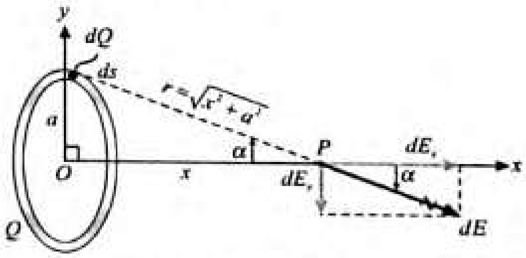


Linear charge distribution	Surface charge distribution	Volume charge distribution
In this distribution, charge is distributed on a line, *for example* charge on a wire, charge on a ring, etc. The linear charge density is given by $\lambda = \frac{\text{Charge}}{\text{Length}} = \frac{Q}{2\pi R}$	In this distribution, charge is distributed on the surface, *for example* charge on a conducting sphere, charge on a sheet, etc. The surface charge density is given by $\sigma = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{4\pi R^2}$	In this distribution, charge is distributed in the whole volume of the body, *for example* a solid uniformly charged object, say a solid charged sphere. The volume charge density is given by $\rho = \frac{\text{Charge}}{\text{Volume}} = \frac{Q}{\frac{4}{3}\pi R^3}$

FIELD OF RING CHARGE

A ring-shaped conductor with radius a carries a total charge Q uniformly distributed around it. Let us calculate the electric field at a point P that lies on the axis of the ring at a distance x from its center.

$$\vec{E} = E_x\hat{i} = \frac{1}{4\pi\varepsilon_0}\frac{xQ}{(x^2+a^2)^{3/2}}\hat{i} \tag{i}$$



Electric field is directed away from positively charged ring. For $x = 0$, $E = 0$, this conclusion may be arrived at by the symmetry consideration. At a large distance from the ring, the electric field will be zero. Hence, it should have certain maximum value between $x = 0$ and $x = \infty$ (or $x = -\infty$). If we maximize Eq. (i), we can get the value of x_m as well as E_{max}. For the maximum value of E_x, we get

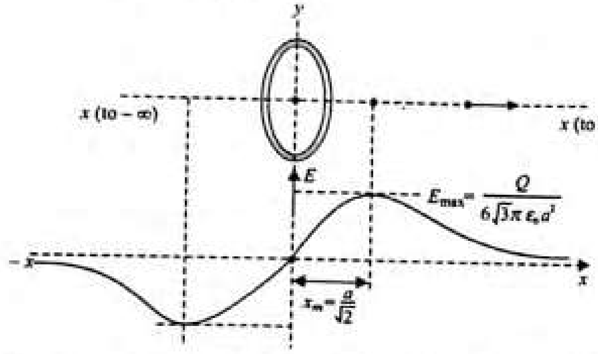
$$\frac{d}{dx}\left\{\frac{1}{4\pi\varepsilon_0}Q\frac{x}{(x^2+a^2)^{3/2}}\right\} = 0$$

or $$\frac{(x^2+a^2)^{3/2}(1)-x\frac{3}{2}(x^2+a^2)^{1/2}(2x)}{(x^2+a^2)^3}=0$$

or $$(x^2+a^2)-3x^2=0 \text{ or } x=\pm\frac{a}{\sqrt{2}}$$

The maximum value of the electric field is

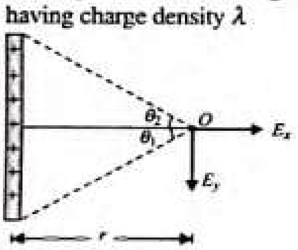
$$E_{a(\max)}=\frac{1}{4\pi\varepsilon_0}\left(\frac{2Q}{3\sqrt{3}\,a^2}\right)$$



Electric field is in the direction along the line that divides the charge distribution symmetrically. Some cases of summetry are as follows:

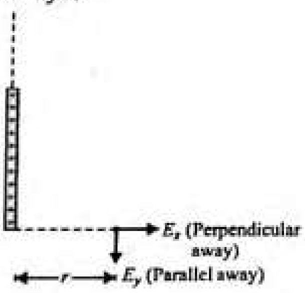
Some Useful Results

A charged rod of fixed length having charge density λ



$$E_x=\frac{\lambda}{4\pi\varepsilon_0 r}(\sin\theta_1+\sin\theta_2)$$
$$E_y=\frac{\lambda}{4\pi\varepsilon_0 r}(\cos\theta_1-\cos\theta_2)$$

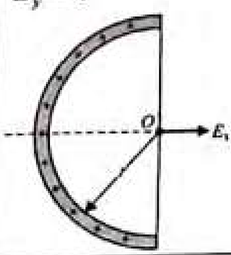
Semi-infinite rod having charge density λ



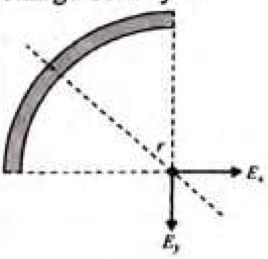
$$E_x=\frac{\lambda}{4\pi\varepsilon_0 r},\quad E_y=\frac{\lambda}{4\pi\varepsilon_0 r}$$

Semicircular ring having charge density λ

$$E_x=\frac{\lambda}{2\pi\varepsilon_0 r}$$
$$E_y=0$$

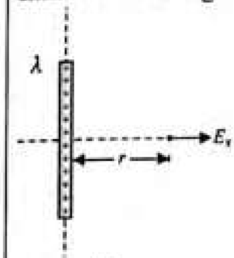


Quarter circular ring having charge density λ



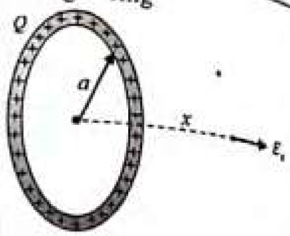
$$E_x=\frac{\lambda}{4\pi\varepsilon_0 r},\quad E_y=\frac{\lambda}{4\pi\varepsilon_0 r}$$

Infinite line charge



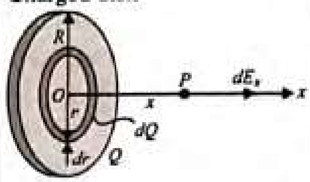
$$E_x=\frac{\lambda}{2\pi\varepsilon_0 r}$$
$$E_y=0$$

Charged ring



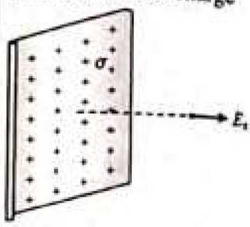
$$E_x=\frac{kQx}{(x^2+a^2)^{3/2}}$$
$$E_y=0$$

Charged disk



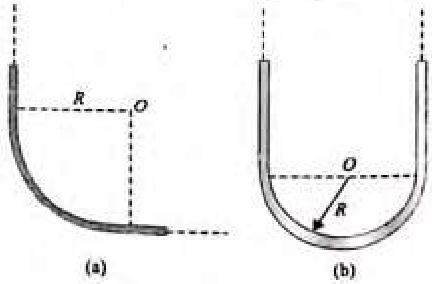
$$E_x=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$
$$E_y=0$$

Infinite sheet of charge



$$E_x=\frac{\sigma}{2\varepsilon_0},\quad E_y=0$$

ILLUSTRATION 17.7 A long wire with a uniform charge density λ is bent in two configurations shown in Figure (a) and (b). Determine the electric field intensity at point O.



Solution.

(a) For Figure (a), field due to segment 1 is

$$\vec{E}_1=\left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{i}+\left[-\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{j}$$

Field due to segment 2 is

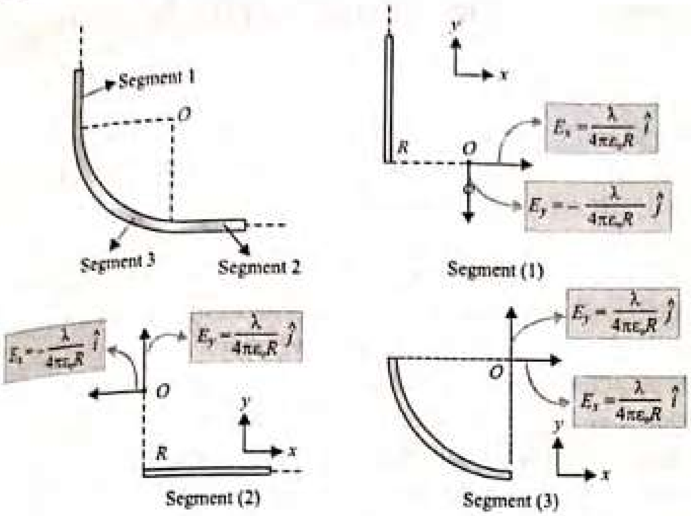
$$\vec{E}_2=\left[-\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{i}+\left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{j}$$

Field due to quarter shape wire segment 3 is

$$\vec{E}_3=\left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{i}+\left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{j}\ (\because \theta_1=90^\circ, \theta_2=0^\circ)$$

The resultant field is the superposition of the fields due to each part, i.e.,

$$\vec{E}=\vec{E}_1+\vec{E}_2+\vec{E}_3 \qquad \text{(i)}$$



Substituting the values of $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ in Eq. (i), we get

$$\vec{E} = \left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{i} + \left[\frac{\lambda}{4\pi\varepsilon_0 R}\right]\hat{j}$$

$$|\vec{E}| = \left[\left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2 + \left(\frac{\lambda}{4\pi\varepsilon_0 R}\right)^2\right]^{1/2} = \frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 R}$$

Here, $E_x = E_y = \dfrac{\lambda}{4\pi\varepsilon_0 R}$

Hence, the resultant field will make an angle of 45° with the axis.

(b) For Figure (b), field due to segment 1 is

$$\vec{E}_1 = \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}]$$

Field due to segment 2 is

$$\vec{E}_{x_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{i}$$

$$\vec{E}_{y_2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\hat{j}$$

$$\vec{E}_2 = -\frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}]$$

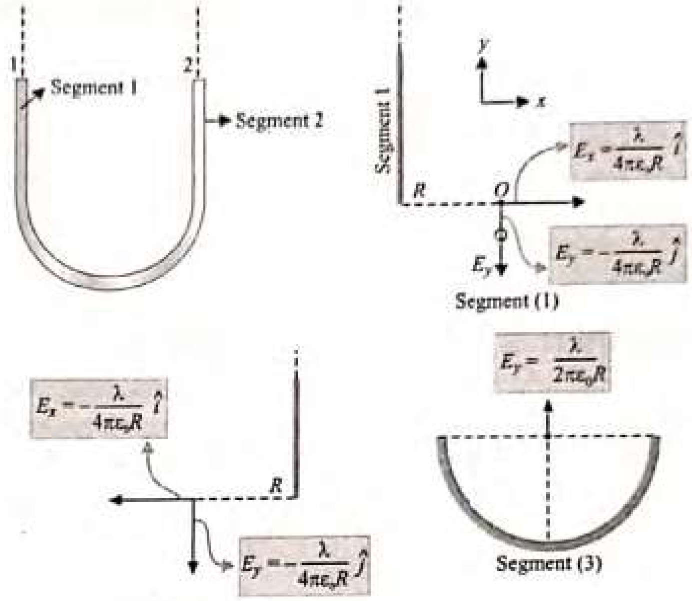
Field due to segment 3 is

$$\vec{E}_3 = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j}$$

From the principle of superposition of electric fields,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

$$= \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} - \hat{j}] - \frac{\lambda}{4\pi\varepsilon_0 R}[\hat{i} + \hat{j}] + \frac{\lambda}{2\pi\varepsilon_0 R}\hat{j} = 0$$



Segment (2)

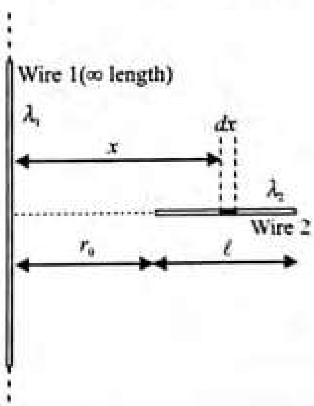
Hence, the net field is zero.

ILLUSTRATION 17.8 A segment of a charged wire of length l, charge density λ_2, and an infinitely long charged wire, charge density λ_1, lie in a plate at right angles to each other. The separation between the wires is r_0. Determine the force of interaction between the wires.

Solution. Electric field near a long wire is given by the expression

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The second wire lies in the nonuniform field of the first wire. Each element of the second wire experiences different magnitude of field. Therefore, we consider differential element dx, and charge $dQ = \lambda_2 dx$, at a distance x from the long wire. The force acting on this element dF is



$$dF = EdQ = \left[\frac{\lambda_1}{2\pi\varepsilon_0 x}\right]\lambda_2 dx$$

The force acting on each element depends on x, the separation between wire 1 and 2. Integrating the expression for dF in the limits $x = r_0$ to $x = r_0 + l$, we obtain

$$F = \int_{r_0}^{r_0+\ell} \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\frac{dx}{x} = \frac{\lambda_1\lambda_2}{2\pi\varepsilon_0}\ln\left[1 + \frac{\ell}{r_0}\right]$$

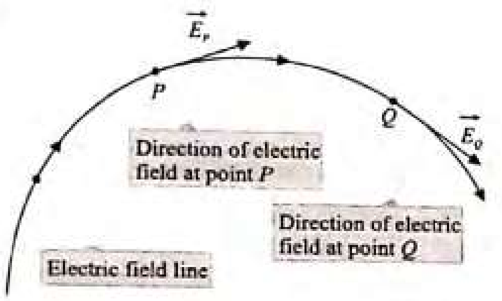
LINES OF FORCE

We have defined the electric field mathematically. Let us now explore a means of visualizing the electric field in a pictorial

representation. A convenient way of visualizing electric field patterns is to draw lines, called electric field lines. The lines of force provide a nice idea to visualize the pattern of electric field in a given space. We assume that the space around a charged body is filled with some lines known as electric lines of force. *These lines of force are drawn in space in such a way that the tangent to the line at any point gives the direction of the electric field at that point.* It has been found quite convenient to visualize the electric field in terms of lines of force.

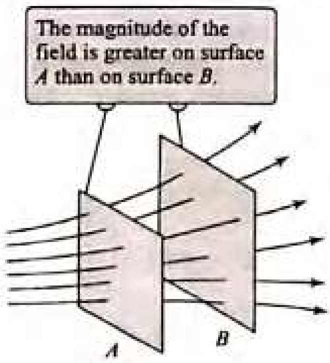
Properties of Electric Lines of Force

The electric field vector $\vec{E}$ is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector. The direction of the line is that of the force on a positive test charge placed in the field.



The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Therefore, the field lines are close together where the electric field is strong and far apart where the field is weak.

No field line originates or terminates in the space surrounding a charge. Every field line is a continuous and smooth curve originating from a positive charge and ending on a negative charge. If one type of charge is in excess, some lines will begin or end infinitely far away.



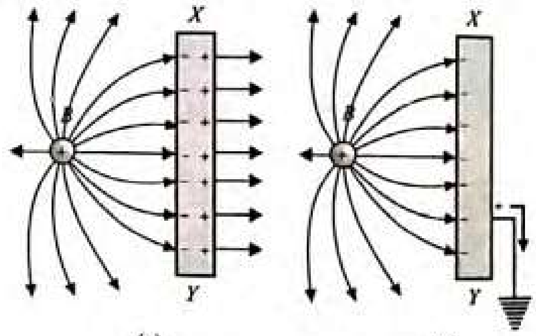
Two electric lines of force never cross each other, because if they do, intensity will have two directions at the point of intersection, which is absurd.

Field lines do not pass through the conductor. This indicates that the *electric field within the conductor is zero.* The field lines are perpendicular to the surface of the conductor. Had it been not so, there would have been a component of the field along the surface of the conductor and a current would flow through it. But no current flows in such an *electrostatic situation. Thus, the electric field just outside the surface of the conductor is perpendicular to its surface.*

ELECTROSTATIC SHIELDING (OR SCREENING)

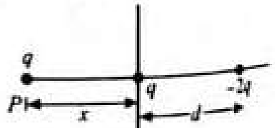
Electrostatic screening is the process of limiting the electric field to a certain region of space.

An earthed conductor acts as a screen (or a shield) against the electric field. If we place a metal screen XY near a charged body B, the resultant electrostatic field due to electrostatic induction is as shown in Figure (a). But when the conductor XY is earthed, the free induced positive charge on it flows to the earth and with it the lines of force emanating from it disappear as shown in Figure (b). Thus, the region to the right of XY is shielded from the field due to B. It is to be noted that the shielding surface may or may not be continuous. Continuous shielding surfaces, called Faraday cages, are hollow conductors and are based on the fact that electric field inside a hollow conductor is zero. To protect people and dielectric instruments from external electric field, we enclose them in such cages and leave them unearthed. An apparatus generating high voltage is also enclosed in such a cage to prevent its field from spreading out. But such a cage is earthed.



(a) (b)

ILLUSTRATION 17.9 Charges $+q$ and $-2q$ are fixed at distance d apart as shown in figure.



i. Sketch roughly the pattern of electric field lines, showing the position of neutral point.

ii. Where should a charge particle q be placed so that it experiences no force?

Solution. Let the net force on q at P be zero. Then,

$$\frac{kq^2}{x^2} = \frac{kq\,2q}{(d+x)^2} \text{ or } x = \frac{d}{\sqrt{2}-1}$$

P is the neutral point where electric field will be zero.

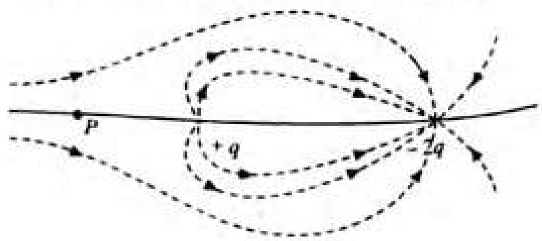
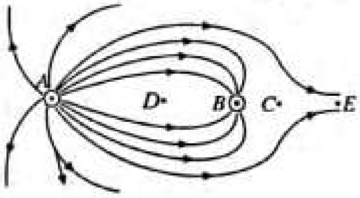


ILLUSTRATION 17.10 The field lines for two point charges are shown in figure.



i. Is the field uniform?
ii. Determine the ratio q_A/q_B.
iii. What are the sign of q_A and q_B?
iv. Apart from infinity, where is the neutral point?
v. If q_A and q_B are separated by a distance $10(\sqrt{2}-1)$ cm, find the position of neutral point.
vi. Where will the lines meet which are coming from A and are not meeting at q_B?
vii. Will a positive charge follow the line of force if free to move?

Solution.

i. No
ii. The number of lines coming from or coming to a charge is proportional to the magnitude of the charge, so

$$\frac{q_A}{q_B}=\frac{12}{6}=2$$

iii. q_A is positive and q_B is negative.
iv. C is the other neutral point.
v. For neutral point

$$E_A=E_B \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_A}{(l+x)^2}=\frac{1}{4\pi\varepsilon_0}\frac{q_B}{x^2}$$

or $$\left(\frac{l+x}{x}\right)^2=\frac{q_A}{q_B}=2 \text{ or } x=10\text{ cm}$$

vi. At infinity.
vii. No. As lines of force are curved, the direction of velocity and acceleration will be different. Hence, a charge cannot follow strictly the lines of force. Also to move on some curved path, centripetal force is required, whereas the lines of force will provide only tangential force.

MOTION OF CHARGED PARTICLE IN UNIFORM ELECTRIC FIELD

When a charged particle is placed in an electric field, it experiences an electrical force. If this is the only force on the particle, it must be the net force. The net force will cause the particle to accelerate according to Newton's second law. So

$$\vec{F}_e=q\vec{E}=m\vec{a}$$

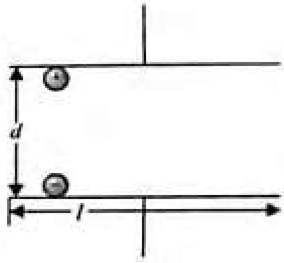
If $\vec{E}$ is uniform, then $\vec{a}$ is constant and $\vec{a}=q\vec{E}/m$. If the particle has a positive charge, its acceleration is in the direction of the field. If the particle has a negative charge, its acceleration is in the direction opposite to the electric field. Since the acceleration is constant, the kinematic equations can be used.

ILLUSTRATION 17.11 A uniform electric field E exists between two metal plates one negative and other positive. The plate length is l and the separation of the plates is d.

i. An electron and a proton start from the negative plate and positive plate, respectively, and go to opposite plates. Which one of them wins this race?
ii. An electron and a proton start moving parallel to the plates toward the other end from the midpoint of the separation of plates at one end of the plates. Which of the two will have greater deviation when they come out of the plates if they start with the
 (a) same initial velocity,
 (b) same initial kinetic energy, and
 (c) same initial momentum.

Solution.

i. $a_e=\frac{eE}{m_e}, a_p=\frac{eE}{m_p}; d=\frac{1}{2}at^2$



$$t=\sqrt{\frac{2d}{a}} \text{ or } t=\sqrt{\frac{2md}{eE}}$$

Therefore, we have

$$\frac{t_e}{t_p}=\sqrt{\frac{m_e}{m_p}}$$

As $m_e<m_p$, so $t_e<t_p$. Hence, electron will take less time, i.e., the electron wins the race.

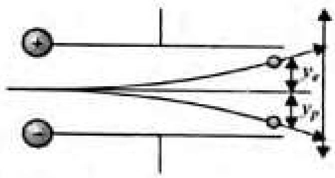
ii. Time to cross the plates is $t=l/u$. Deviation is

$$y=\frac{1}{2}at^2=\frac{1}{2}\frac{eE}{m}\left(\frac{l}{u}\right)^2$$

or $$\frac{y_e}{y_p}=\frac{m_p}{m_e}\left(\frac{u_p}{u_e}\right)^2 \qquad \text{(i)}$$

(a) If $u_p=u_e$, then

$$\frac{y_e}{y_p}=\frac{m_p}{m_e}$$



As $m_p>m_e$, so $y_e>y_p$. Hence, the deviation of the electron will be more.

(b) From Eq. (i),

$$\frac{y_e}{y_p}=\left(\frac{m_pu_p^2}{m_eu_e^2}\right)=1 \qquad \text{(as given)}$$

Hence, the deviation of both the electron and the proton will be the same.

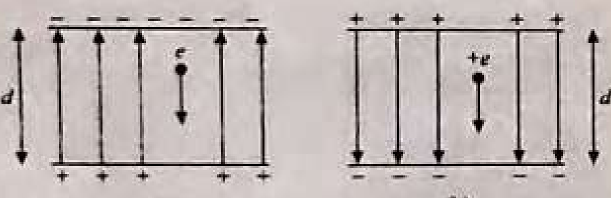
(c) From Eq. (i),

$$\frac{y_e}{y_p}=\left(\frac{m_pu_p}{m_eu_e}\right)^2\frac{m_e}{m_p}=\frac{m_e}{m_p}$$

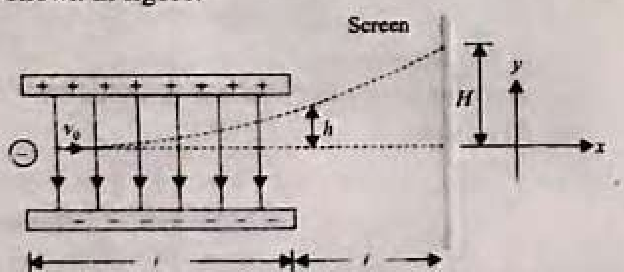
As $m_e<m_p$, so $y_e<y_p$. Hence, the deviation of proton will be more.

Concept Application Exercise 17.3

1. Two identical positive charges are fixed on the y-axis, at equal distances from the origin O. A particle with a negative charge starts on the x-axis at a large distance from O, moves along the x-axis, passes through O, and moves far away from O on the other side. Its acceleration a is taken as positive along its direction of motion. Plot acceleration a of the particle against its x-coordinate.
2. Three identical positive charges Q are arranged at the vertices of an equilateral triangle. The side of the triangle is a. Find the intensity of the field at the vertex of a regular tetrahedron of which the triangle is the base.
3. Two point charges $+5 \times 10^{-19}$ C and $+20 \times 10^{-19}$ C are separated by a distance of 2 m. The electric field intensity will be zero at a distance d = ______ from the charge of 5×10^{-19} C.
4. An electron (mass m_e) falls through a distance d in a uniform electric field of magnitude E. The direction of the field is reversed keeping its magnitude unchanged, and a proton (mass m_p) falls through the same distance. If the times taken by the electron and the proton to fall the distance d is t_{electron} and t_{proton}, respectively, then the ratio $t_{\text{electron}}/t_{\text{proton}}$ = ______



5. An electron moving in a gravitational free space enters a uniform electric field E with an initial velocity v_0 as shown in figure.



(a) Find the deflection distance h in the field.
(b) Find an expression for the velocity of electron when it just emerges from the field.
(c) Find an expression for the total deflection distance H at a vertical screen placed at a distance l from the region of uniform field. (Assume that the field abruptly ends outside the field.)

ELECTRIC DIPOLE

An electric dipole is a system of two equal and opposite point charges separated by a very small and finite distance. Figure shows an electric dipole consisting of two equal and opposite point charges $-q$ and $+q$ separated by a small distance $2l$. The strength of an electric dipole is measured by a vector quantity known as electric dipole moment. Its magnitude is equal to the product of the magnitude of either charge and the distance between the two charges, i.e., $p = 2ql$. The direction of p is from the negative charge to the positive charge. In the SI system of units, p is measured in coulomb-meter.

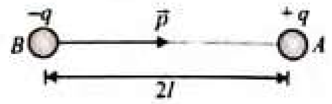


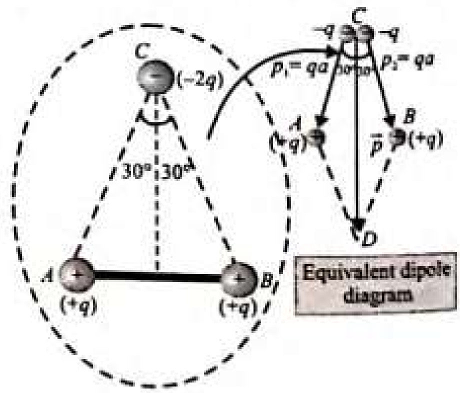
ILLUSTRATION 17.12 Three charges $+q$, $+q$, and $-2q$ are placed at the vertices of an equilateral triangle. What is the dipole moment of the system?

Solution. Charge $-q$ at C and $+q$ at A form one dipole, and the remaining charge $-q$ at C and $+q$ at B form another dipole (figure).

Dipole moment along CA is qa, and dipole moment along CB is qa. So the net dipole moment is

$$p = \sqrt{(qa)^2 + (qa)^2 + 2(qa)(qa)\cos 60^\circ} = \sqrt{3}qa$$

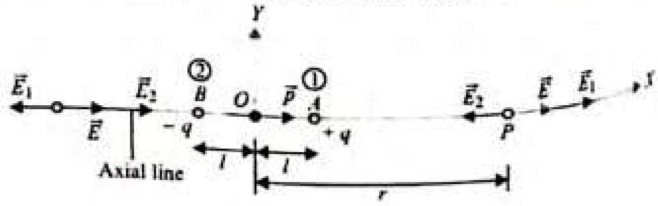
$\vec{p}$ acts along CD, which bisects the angle (60°) at C.



ELECTRIC FIELD DUE TO A DIPOLE

Electric Field Intensity Due to an Electric Dipole at a Point on the Axial Line

A line passing through the negative and positive charges of the electric dipole is called the axial line of the electric dipole.



$$\vec{E} = -\frac{1}{4\pi\varepsilon_0} \times \frac{\vec{p}}{r^3}$$

NOTE: The direction of the electric field E is opposite to the direction of $\vec{p}$, i.e., antiparallel to the axis of dipole from the positive charge toward the negative charge.

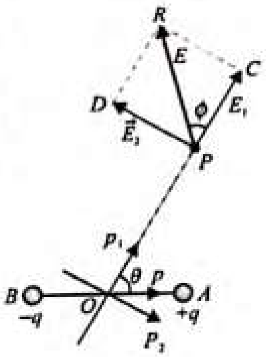
ELECTRIC FIELD INTENSITY DUE TO A SHORT DIPOLE AT SOME GENERAL POINT

Let AB be a short electric dipole of dipole moment $\vec{p}$ (directed from B to A). We are interested to find the electric field at some general point P. The distance of the observation point P with respect to midpoint O of the dipole is r, and the angle made by the line OP with the axis of the dipole is θ.

We know that the dipole moment of a dipole is a vector quantity. It can be resolved into two rectangular components $\vec{p}_1$ and $\vec{p}_2$ as shown in figure, so that $\vec{p} = \vec{p}_1 + \vec{p}_2$. The magnitudes of $\vec{p}_1$ and $\vec{p}_2$ are $p_1 = p\cos\theta$ and $p_2 = p\sin\theta$.

It is clear from the figure that point P lies on the axial line of the dipole with moment $\vec{p}_1$. Hence, the magnitude of the electric field intensity $\vec{E}_1$ at P due to $\vec{p}_1$ is

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{2p\cos\theta}{r^3} \text{ (along the direction } OC) \quad \text{(i)}$$



Similarly, P lies on the equatorial line of dipole with moment $\vec{p}_2$. Hence, the magnitude of electric field intensity $\vec{E}_2$ at P due to $\vec{p}_2$ is

$$E_2 = \frac{1}{4\pi\varepsilon_0} \frac{p\sin\theta}{r^3} \text{ (opposite to } p_2) \quad \text{(ii)}$$

Hence, the resultant intensity at P is $\vec{E} = \vec{E}_1 + \vec{E}_2$.

The magnitude of $\vec{E}$ is $E = \sqrt{E_1^2 + E_2^2}$ (as $\vec{E}_1$ and $\vec{E}_2$ are mutually perpendicular).

$$E = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

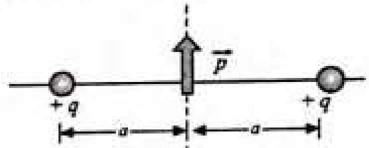
$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{1 + 3\cos^2\theta}$$

If the resultant field intensity vector $\vec{E}$ makes an angle ϕ with the direction of $\vec{E}_1$, then

$$\tan\phi = \frac{E_2}{E_1} = \frac{p\sin\theta / 4\pi\varepsilon_0 r^3}{2p\cos\theta / 4\pi\varepsilon_0 r^3} = \frac{1}{2}\tan\theta$$

ILLUSTRATION 17.13 What is the force on a dipole of dipole moment p placed as shown in figure.



Solution. Force on any q by dipole is

$$F = qE_{\text{dipole}} = \frac{q}{4\pi\varepsilon_0}\frac{p}{a^3} \text{ downward}$$

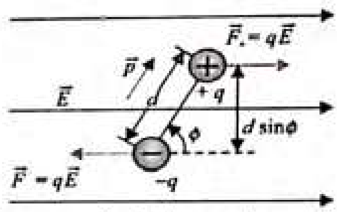
So from Newton's third law, force on the dipole due to both charges is

$$2F = \frac{qp}{2\pi\varepsilon_0 a^3} \text{ upward}$$

DIPOLE IN A UNIFORM ELECTRIC FIELD

Torque

When a dipole is placed in a uniform field as shown in figure, the net force on it is $F_R = q\vec{E} + (-q)\vec{E} = 0$.



Hence, the net force on a dipole is zero in a uniform electric field. While the torque is

$$\tau = qE \times d\sin\phi = pE \times \sin\phi \text{ (as } p = qd)$$

$$\therefore \quad \vec{\tau} = \vec{p} \times \vec{E} \text{ (by electric field)}$$

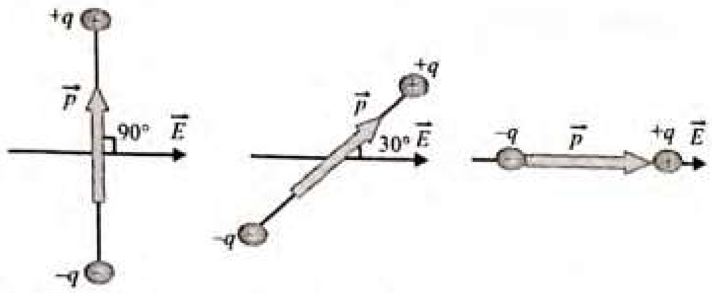
and $\quad \vec{\tau} = \vec{E} \times \vec{p}$ (if the dipole is in equilibrium)

From the expression, it is clear that the couple acting on a dipole is maximum $(= pE)$ when the dipole is perpendicular $(\phi = 90°)$ to the field and minimum $(= 0)$ when the dipole is parallel $(\phi = 0°)$ or antiparallel $(\phi = 180°)$ to the field.

By applying a torque, the electric field tends to align a dipole in its own direction.

ILLUSTRATION 17.14 An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$. Write the expression for the torque $\vec{\tau}$ experienced by the dipole. Identify two pairs of perpendicular vectors in the expression. Show diagrammatically the orientation of the dipole in the field for which the torque is (i) maximum, (ii) half the maximum value, and (iii) zero.

Solution. As $\vec{\tau} = \vec{p} \times \vec{E}$, the pair of perpendicular vectors are (i) $\vec{\tau}$ and $\vec{p}$ and (ii) $\vec{\tau}$ and $\vec{E}$. The required orientations of the dipole in the electric field are shown in figure.



$\tau = pE \sin 90°$
$= pE$ = maximum (τ_{max})

$\tau = pE \sin 30°$
$= (1/2)pE = 1/2\,(\tau_{max})$

$\tau = pE \sin 0°$
$= 0$

ELECTRIC DIPOLE IN A NONUNIFORM ELECTRIC FIELD

If an electric dipole is placed in a nonuniform electric field, there is a net force on the dipole in addition to the torque tending to align the dipole with the field. The net force depends on

- the orientation of the dipole with respect to the electric field,
- the dipole moment of the dipole, and
- how rapidly the field varies in space.

Qualitative Discussion

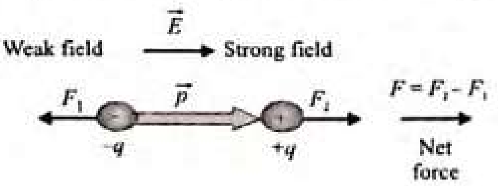
To clarify the basic ideas, let us consider only the simplest situation where

- the electric field has a constant direction and
- its magnitude increases steadily as we move in the direction of the electric field.

Further, consider the following two cases:

When $\vec{p}$ is parallel to $\vec{E}$, i.e., $\vec{p}\uparrow\uparrow\vec{E}$

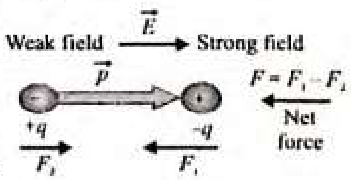
Since the force F_2 acting on $+q$ (lying in the strong field) is greater than the force F_1 acting on $-q$ (lying in the weak field), the net force $F = F_2 - F_1$ is in the direction of increasing $\vec{E}$, i.e., toward the region of the strong field as shown in figure.



(a)

When $\vec{p}$ is antiparallel to $\vec{E}$, i.e., $\vec{p}\uparrow\downarrow\vec{E}$

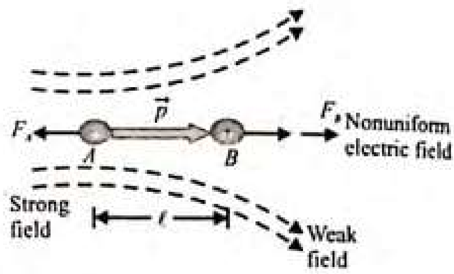
Since the force F_1 acting on $-q$ (placed in the strong field) is greater than the force F_2 acting on $+q$ (placed in the weak field), the net force $F = F_1 - F_2$ is in the direction of decreasing field, i.e., toward the region of the weak field as shown in figure.



Thus, when $\vec{p}\uparrow\uparrow\vec{E}$, the dipole moves toward the region of strong electric field (i.e., in the direction of increasing $\vec{E}$) and when $\vec{p}\uparrow\downarrow\vec{E}$, the dipole moves toward the region of weak field (i.e., in the direction of decreasing $\vec{E}$).

Quantitative Discussion

An electric dipole in a nonuniform electric field is subject to the action of a resultant force because the negative and the positive charges ($-q$ and $+q$) of the dipole are located at different points in the field, having different field strengths as shown in figure.



The resultant force is

$$F = F_A - F_B = qE_A - qE_B = q\ell\left[\frac{E_A - E_B}{\ell}\right] \quad \text{(i)}$$

But $q\ell = p$ is the dipole moment (l being the length of the dipole) and $(E_A - E_B)/\ell = \Delta E/\Delta\ell$ is field strength gradient (i.e., the quantity showing the change in field strength per unit length). Thus, from Eq. (i),

$$F = p\frac{\Delta E}{\Delta\ell} \quad \text{(ii)}$$

In differential form

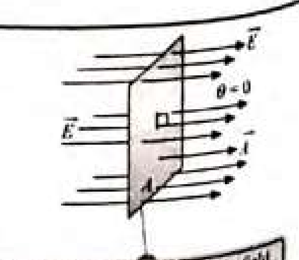
$$|\vec{F}| = \left|p\frac{d\vec{E}}{dx}\right|$$

where $d\vec{E}/dx$ is the gradient of the field in the x-direction.

This force pulls the dipole into the region with a stronger field. It is due to this reason that a charged body attracts light objects such as pieces of paper, dust, pieces of foil, etc. By the action of the field, these objects first acquire a dipole moment and then pulled to a region where the field strength is greater, i.e., closer to the electrified body.

ELECTRIC FLUX

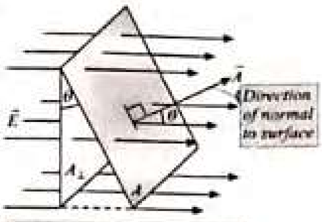
The field lines penetrate a rectangular surface of area A, whose plane is oriented perpendicular to the field. As we know that the number of lines per unit area (in other words, the *line density*) is proportional to the magnitude of the electric field, the total number of



Surface face on to electric field $\vec{E}$ and $\vec{A}$. Parallel angle between $\vec{E}$ and $\vec{A}$ is $\theta = 0$; flux $\Phi_E = \vec{E}\cdot\vec{A} = EA$

lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field E and surface area A perpendicular to the field is called the **electric flux** Φ_E (uppercase Greek letter phi) given by

$$\Phi_E = EA \quad \text{(i)}$$



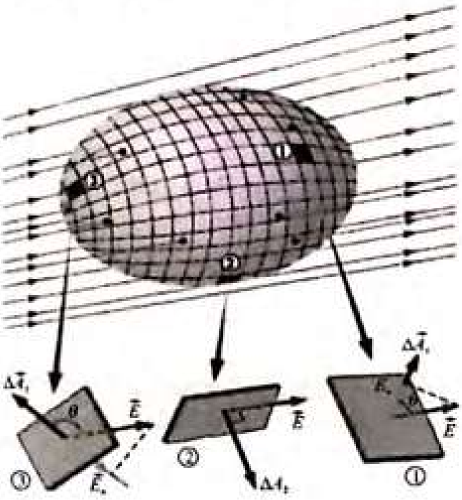
The number of field lines that go through area A is the same as the number of lines that go through area $A_\perp$

From the SI units of E and A, we see that Φ_E has the units of Nm^2C^{-1}. Electric flux is proportional to the number of electric field lines penetrating some surface.

If the area of each element approaches zero, the number of elements approaches infinity and the sum is replaced by an integral. Therefore, the general definition of electric flux is

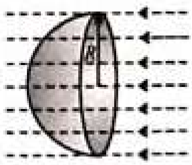
$$\Phi_E \equiv \int_{\text{Surface}} \vec{E}\cdot d\vec{A} \quad \text{(ii)}$$

Equation (ii) is a surface integral, which means it must be evaluated over the surface in question. In general, the value of Φ_E depends both on the field pattern and on the surface.



We are often interested in evaluating the flux through a closed surface, defined as a surface that divides space into an inside and an outside region so that one cannot move from one region to the other without crossing the surface. The surface of a sphere, for example, is a closed surface.

ILLUSTRATION 17.15 Find out the flux through the curved surface of a hemisphere of radius R if it is placed in a uniform electric field E as shown in figure.



Solution. The number of electric lines passing through the base of the hemisphere is the same as that of the lines passing through the hemisphere.

The flux associated with the base of the hemisphere is $\phi = -E\pi R^2$ (negative as it is the incoming flux). Hence, the same amount of flux will be associated with the curved surface, but the sign of the flux will be positive as it is as outgoing flux. Hence,

$$\phi_{\text{curve}} = E\pi R^2$$

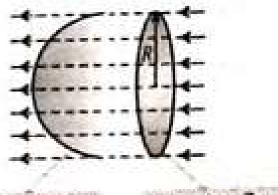
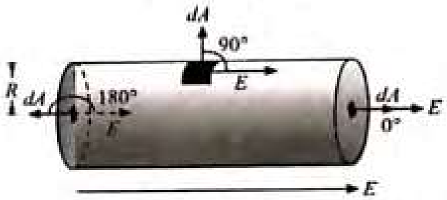


ILLUSTRATION 17.16 Consider a cylindrical surface of radius R and length l in a uniform electric field E. Compute the electric flux if the axis of the cylinder is parallel to the field direction.

Solution. We can divide the entire surface into three parts, right and left plane faces and curved portion of its surface. Hence, the surface integral consists of the sum of the three terms:

$$\phi_E = \oint \vec{E}\cdot\overrightarrow{dA} = \int_{\text{left end}} \vec{E}\cdot\overrightarrow{dA} + \int_{\text{right end}} \vec{E}\cdot\overrightarrow{dA} + \int_{\text{curved}} \vec{E}\cdot\overrightarrow{dA}$$



All the area elements on the left end and electric field E are at an angle of 180°.

$$(\phi_E)_{\text{left end}} = \oint_{\text{left end}} \vec{E}\cdot\overrightarrow{dA} = \oint_{\text{left end}} E\,dA\cos 180°$$

$$= -E\oint_{\text{left end}} dA = -E\pi R^2$$

Note that E is constant over the entire plane surface of left end; therefore, we take it out from the integral.

Similarly, all the area elements on the right end are parallel to electric field E, i.e., angle is 0°.

$$(\phi_E)_{\text{right end}} = \oint_{\text{right end}} \vec{E}\cdot\overrightarrow{dA} = \oint_{\text{right end}} E\,dA\cos 0°$$

$$= +E\oint_{\text{right end}} dA = E\pi R^2$$

Finally, at every point on the curved surface the area vectors are perpendicular to the direction of the electric field. Thus,

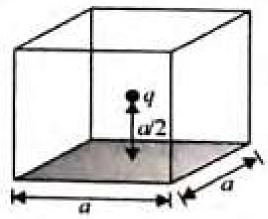
$$(\phi_E)_{\text{curved}} = \oint_{\text{curved surface}} \vec{E}\cdot\overrightarrow{dA} = \oint_{\text{curved surface}} E\,dA(\cos 90°) = 0$$

$$\text{Total flux} = (\phi_E)_{\text{right end}} + (\phi_E)_{\text{left end}} + (\phi_E)_{\text{curved surface}}$$

$$= (+E\pi R^2) + (-E\pi R^2) + 0 = 0$$

Hence, we see that in a uniform field the flux through a closed surface is zero. This is true for any shape of closed surface.

ILLUSTRATION 17.17 A point charge q is placed at a distance $a/2$ from the center of a square of side a as shown in figure. Calculate the electric flux passing through the square.

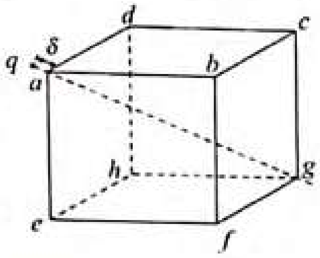


Solution.

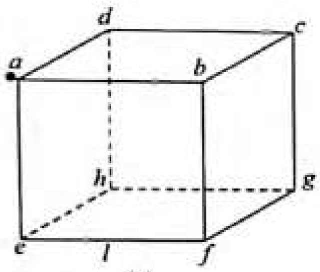
- This problem can be solved by symmetry consideration and Gauss's law.
- We can enclose the charged particle by a cube of side '*a*' and keep the particle at the center of the cube.
- The total flux passing through the close cube is $\phi = q/\varepsilon_0$.
- All the six surfaces are symmetrical with respect to charge, hence they will have equal contribution of the flux. So, flux through any one face is $\phi' = \phi/6 = q/6\varepsilon_0$.

ILLUSTRATION 17.18 In figure, a charge q is placed at a distance $\delta \to 0$ near one of the corners of a cube of edge l on a line of symmetry along diagonal.

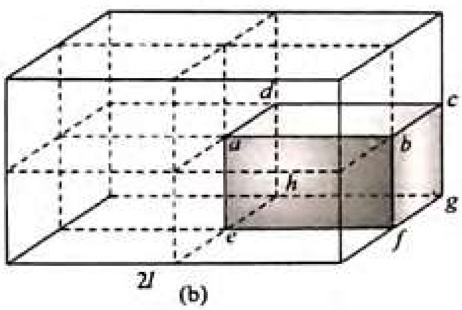
1. What is the flux through each of the faces containing the point a?
2. What is the flux through the other three faces?



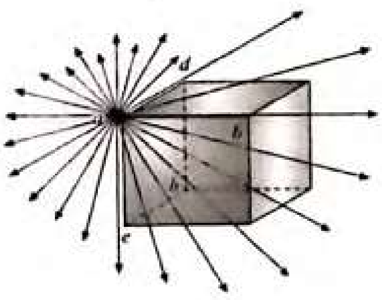
Solution. Use of symmetry consideration may be useful in problems of flux calculation.



(a)



(b)



(c)

- We can imagine a charged particle placed at the center of a cube of side $2l$.
- The flux enclosed with the cube is $\phi = q/\varepsilon_0$.
- The flux passing through one of the faces of the cube is $\phi' = \phi/6 = q/6\varepsilon_0$.
- Hence, the flux passing through the face *bcgf* is $\phi'/4 = q/24\varepsilon_0$.
- Each of the faces (*efgh*), (*bcgf*), and (*dcgh*) is symmetrical with respect to charge. Hence, the flux passing through each of the face is $q/24\varepsilon_0$.
- The electric field lines for the faces (*efgh*), (*bcgf*) and (*dcgh*) are away from the faces. Hence, the flux associated with each of the faces will be positive (i.e., $+ q / 24\varepsilon_0$). Hence, the total flux through these sides is

$$\frac{3 \times q}{24\varepsilon_0} = \frac{q}{8\varepsilon_0}.$$

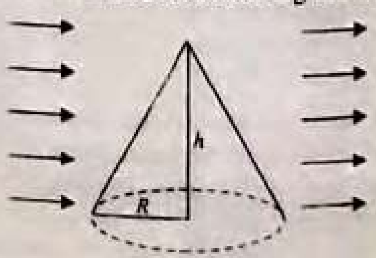
- As $\delta \to 0$, we can say the faces (*abcd*), (*abfe*) and (*adhe*) are also symmetrical about charge. Charge is slightly outside the cube.
- The number of electric field lines passing through the faces, which do not contain the point a, is same as the number of electric field lines passing through the faces containing the point a.
- Hence, the same amount of flux will pass through the faces containing the point a, i.e., $q/8\varepsilon_0$.
- The electric field lines are toward the faces containing the point a. Hence, the flux will be negative, i.e., $\phi'' = -q/8\varepsilon_0$.
- Hence, the flux through each of the faces containing the point '*a*' will be $\phi''/3 = q/24\varepsilon_0$.

Your Task: Repeat Illustration 17.18 if the charge is exactly at the corner of the cube given.

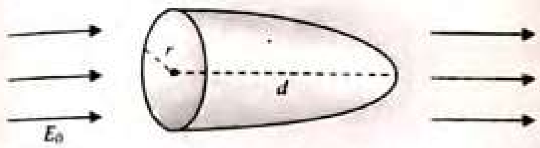
CONCEPT APPLICATION EXERCISE 17.4

1. A point charge Q is located just above the center of the flat face of a hemisphere of radius R as in figure. What is the flux
   (a) through the curved surface, and
   (b) through the flat face?
   (c) Repeat parts (a) and (b) if the charge is exactly at the center.

2. In figure, a cone lies in a uniform electric field E. Determine the electric flux entering the cone.



3. A uniform electric field $a\hat{i} + b\hat{j}$ intersects a surface of area A. What is the flux through this area if the surface lies (a) in the yz plane, (b) in the xz plane, (c) in the xy plane?
4. (a) A point charge q is located at distance d from an infinite plane. Determine the electric flux through the plane due to the point charge.
   (b) A point charge q is located at a very small distance from the center of a very large square on the line perpendicular to the square passing through its center. Determine the approximate electric flux through the square due to the point charge.
5. Calculate the total electric flux through the paraboloidal surface due to a uniform electric field of magnitude E_0 in the direction shown in figure.



6. Consider a closed surface of arbitatry shape as shown in figure. Suppose a single charge Q_1 is located at some point within the surface and second charge Q_2 is located outside the surface.

   (a) What is the total flux passing through the surface due to charge Q_1?
   (b) What is the total flux passing through the surface due to charge Q_2?

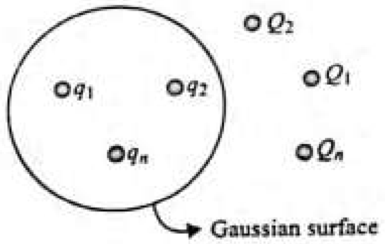
GAUSS'S LAW

Gauss's law is an alternative to Coulomb's law. Although completely equivalent to Coulomb's law, Gauss' law provides a different way to express the relationship between electric charge and electric field.

Gauss's law states that the total electric flux through a closed surface is proportional to the total electric charge enclosed within the surface. This law is useful in calculating field caused by charge distributions that have various symmetry properties. Mathematically, Gauss's law can be written as

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

One thing is to note that the electric field appearing in Gauss's law is the resultant electric field due to all the charges present inside as well as outside the given closed surface. On the other hand, the charges q_{in} appearing in the law are only the charges contained within the closed surface. The contribution of the charges outside the closed surface in producing the flux is zero. A surface on which Gauss's law is applied is sometimes called the Gaussian surface (figure).



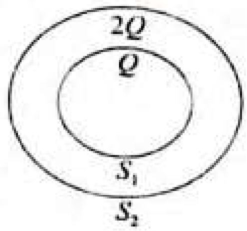
Caution: Remember that the closed surface in Gauss's law is imaginary; there need not be any material object at the position of the surface.

Important Points

Some important points on Gauss's law are as follows:

- The net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number of lines leaving the surface minus the number of lines entering the surface. If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative.
- If no net flux passes through any closed surface, E may not be zero, but the flux of $\vec{E}$ must be zero and the net charge enclosed must be zero.
- In Gauss's law, $\vec{E}$ is the net field due to all charges present inside and outside the Gaussian surface.
- No net flux is contributed by the charges present outside the Gaussian surface because the number of E-lines (flux) coming into is equal to the number of E-lines going out of the surface.
- The net flux is contributed only by the charges inside (or enclosed by) the Gaussian surface.
- If we change the configuration by displacing the charges inside the Gaussian surface, the electric field at any point may change leaving the net flux passing through the Gaussian surface unchanged.
- However, any change in configuration of the charges outside a Gaussian surface may change E at a point, but the net flux remains the same as external charges contribute nothing to the total flux.

ILLUSTRATION 17.19 S_1 and S_2 are two hollow concentric spheres enclosing charges Q and $2Q$, respectively, as shown in figure.



i. What is the ratio of the electric flux through S_1 and S_2?
ii. How will the electric flux through sphere S_1 change if a medium of dielectric constant 5 is introduced in the space inside S_1 in place of air.

Solution.

i. Flux through S_1 is

$$\phi_1 = \frac{Q}{\varepsilon_0}$$

Flux through S_2 is

$$\phi_2 = \frac{Q+2Q}{\varepsilon_0} = \frac{3Q}{\varepsilon_0}$$

Thus,

$$\frac{\phi_1}{\phi_2} = \frac{Q/\varepsilon_0}{3Q/\varepsilon_0} = \frac{1}{3} = 1:3$$

ii. As $E_m = \dfrac{E}{\varepsilon_r}$ and $\varepsilon_r = 5$, so $E_m = \dfrac{E}{5}$

ϕ_1' (changed flux through S_1)

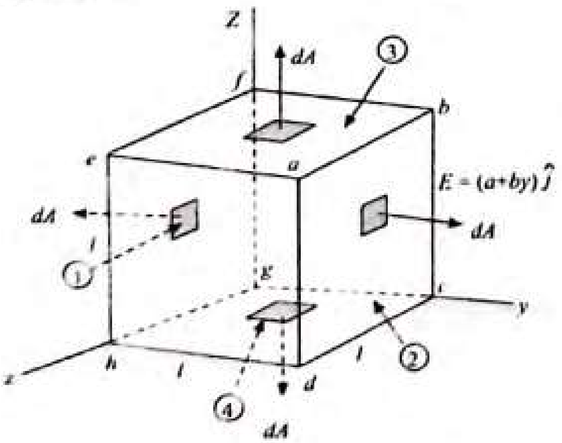
$$= \oint \vec{E}_m \cdot d\vec{S} = \oint \vec{E} \cdot d\vec{S}$$

$$= \frac{1}{5}\phi_1 \quad \left(\text{as} \oint \vec{E} \cdot d\vec{S} = \phi_1\right)$$

$$= \frac{Q}{5\varepsilon_0} \quad \left(\text{as } \phi_1 = \frac{Q}{\varepsilon_0}\right)$$

ILLUSTRATION 17.20 A cube of side l has one corner at the origin of coordinates and extends along the positive x-, y- and z-axes. Suppose that the electric field in this region is given by $\vec{E} = (a + by)\hat{j}$. Determine the charge inside the cube (a and b are some constants).

Solution. The faces *adhe*, *bcgf*, *cdhg*, and *abfe* will contribute zero flux because the area vector is normal to the electric field for these faces.



Flux through face *efgh* is

$$\phi_1 = \int \vec{E} \cdot d\vec{A} = a(\hat{j}) \cdot l^2(-\hat{j}) = -al^2$$

The field at the face *efgh* (that lies in the yz-plane, $y = 0$) is $\vec{E} = a\hat{j}$ and the area vector is $l^2(-\hat{j})$ (direction outward normal). Flux through face *abcd* is

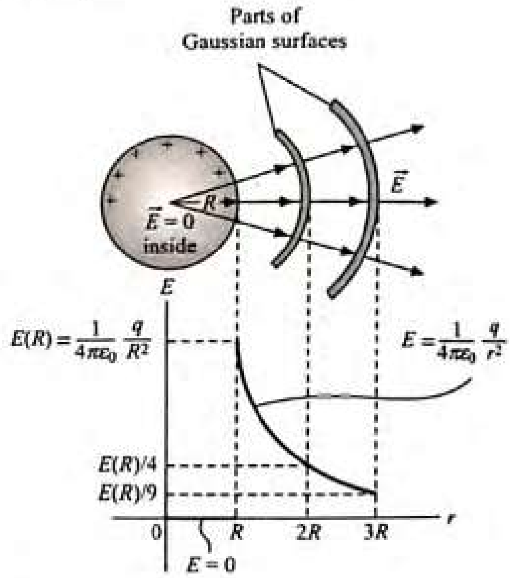
$$\phi_2 = (a + bl)\hat{j} \cdot l^2\hat{j} = (al^2 + bl^3) \quad (\text{for this } y = l)$$

Net flux through the cube is $\phi_1 + \phi_2 = bl_3$. From Gauss's law

$$\phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \text{ or } Q_{\text{enclosed}} = \varepsilon_0\phi_E = \varepsilon_0 bl^3$$

FIELD OF A CHARGED CONDUCTING SPHERE

We place positive charge q on a solid conducting sphere with radius R (as shown in figure). All the charges must be on the surface of the sphere.



Electric Field Outside the Sphere

We first consider the field outside the conductor, so we choose $r > R$. The entire conductor is within the Gaussian surface, so the enclosed charge is q. The area of the Gaussian surface is $4\pi r^2$; $\vec{E}$ is uniform over the surface and perpendicular to it at each point. The flux integral $\oint E_\perp dA$ in Gauss's law is therefore $E(4\pi r^2)$ which gives

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(outside a charged conducting sphere)}$$

This expression for the field at any point outside the sphere ($r > R$) is the same as for a point charge; the field due to the charged sphere is the same as if the entire charge were concentrated at its centre. Just outside the surface of the sphere, where $r = R$, we get

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

(at the surface of a charged conducting sphere)

Electric Field Inside the Sphere

We know that extra charge on a conductor lies on its outer surface. So there is no charge inside the Gaussian surface, i.e., $q_{in} = 0$ (figure). Therefore,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} = 0$$

$\Rightarrow \quad E 4\pi r^2 = 0$

$\Rightarrow \quad E = 0$

Hence, at a point inside the sphere, electric field is zero.

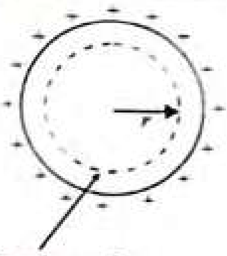


ILLUSTRATION 17.21 A point charge $+Q$ is placed at the center of an uncharged spherical conducting shell of inner radius a and outer radius b.

i. Find the electric field for $r < a$.

ii. What is the magnitude and sign of the induced charge q' on the inner shell surface?

iii. What is the electric field at points $r > b$?

iv. What is the surface charge on the outer surface of the conductor?

Solution.

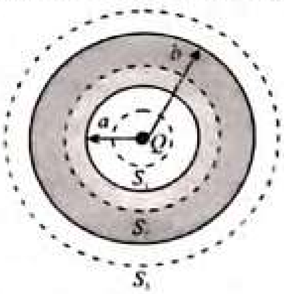
i. Consider a Gaussian surface S_1 of radius $r < R$ inside the cavity, centered on charge Q. From Gauss's law,

$$\phi_E = \oint E \cdot dA = E(4\pi r^2) = \frac{+Q}{\varepsilon_0}$$

From this we find the electric field to be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

ii. Consider a Gaussian surface S_2 inside the conducting material. We do not know if there is a charge on the inside surface of the conductor or not. We assume that the charge is q'; if q' is zero, the result of Gauss's law will show it. Because the Gaussian surface is inside the conductor, the electric field is zero. From Gauss's law,



$$\oint \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{enclosed}}{\varepsilon_0} = 0$$

or $\quad \dfrac{Q_{enclosed}}{\varepsilon_0} = \dfrac{Q + q'}{\varepsilon_0} = 0 \quad$ or $q' = -Q$

There is a charge on the inside surface of the conductor. The total charge induced on the inside surface of the cavity is the negative of the charge placed at its center.

iii. For E $(r > b)$, consider a Gaussian surface S_3. From Gauss's law,

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

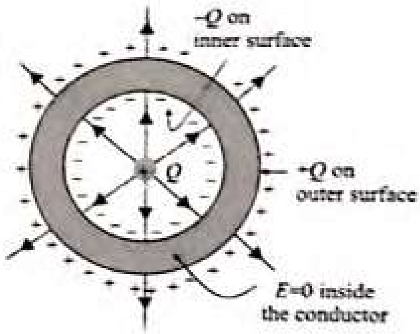
or $\quad E(4\pi r^2) = \dfrac{+Q}{\varepsilon_0}$

or $\quad E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

It was stated in the problem that the conducting sphere has no net charge. Consequently, the total charge inside our Gaussian surface S_3 is the sum of charge $+Q$ and induced charges $-Q$ on the inner surface of the conductor and $+Q$ on the surface. Once more we can see that the field outside the sphere is the same as for a point charge. The conducting sphere has no shielding effect at all.

However, such a conducting shield prevents electrostatic fields from charges outside the shell from entering it.

iv. The conducting shell has no net charge, yet there is a surface charge $-Q$ on its surface. Because the net charge on the shell is zero and no charge can reside inside a conductor, there must be $+Q$ on the outer surface of the conductor.



FIELD OF A LINE CHARGE

There is no flux through the ends because $\vec{E}$ lies in the plane of the surface. To find the flux through the side walls, note that $\vec{E}$ is perpendicular to the surface at each point; by symmetry, E has the same value everywhere on the walls (curved surface). The area of the side wall is $2\pi rl$. Hence, the total flux Φ_E through the entire cylinder is the sum of the flux through the side wall, which is $(E)(2\pi rl)$, and the zero flux through the two ends. Finally, we need the total enclosed charge, which is the charge per unit length multiplied by the length of wire inside the Gaussian surface, or $Q_{encl} = \lambda l$. From Gauss's law, we get

$$\Phi_E = (E)(2\pi rl) = \frac{\lambda l}{\varepsilon_0} \text{ and } E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

(field of an infinite line of charge)

We have assumed that λ is positive. If it is negative, $\vec{E}$ is directed radially inward toward the line of charge, and in the above expression for the field magnitude E, we must interpret λ as the magnitude (absolute value) of the charge per unit length.

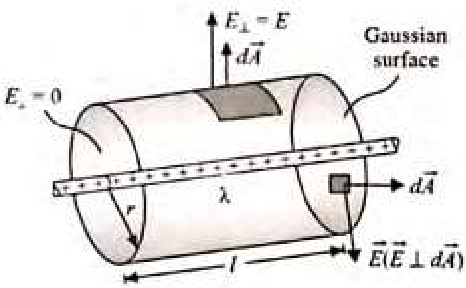


ILLUSTRATION 17.22 A long, straight wire is surrounded by a hollow metal cylinder whose axis coincides with that of the wire. The wire has a charge per unit length of λ, and the cylinder has a net charge per unit length of 2λ. From this information, use Gauss's law to find (i) the charge per unit length on the inner and outer surfaces of the cylinder and (ii) the electric field outside the cylinder, a distance r from the axis.

Solution.

i. **Inside surface:** Consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the Gaussian surface must be zero; so the charge/length on the inner surface of the cylinder is $-\lambda$.

Outside surface: The total charge on the metal/cylinder is

$$2\lambda l = q_{in} + q_{out} \quad \text{or} \quad q_{out} = 2\lambda l - q_{in},$$

where $q_{in} = -\lambda l$

$$q_{out} = 2\lambda l + \lambda l = 3\lambda l$$

So the outside charge/length is $q_{out}/l = 3\lambda$

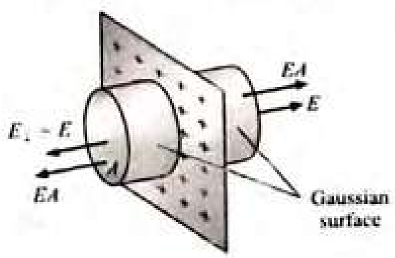
ii. Electric field out side the cylinder $E = \dfrac{\lambda_{net}}{2\pi\varepsilon_0 r} = \dfrac{3\lambda}{2\pi\varepsilon_0 r}$

radially outward

FIELD OF AN INFINITE PLANE SHEET OF CHARGE

To take advantage of these symmetry properties, we use a cylinder as our Gaussian surface with its axis perpendicular to the sheet of charge, with ends of area A (figure).

The charged sheet passes through the middle of the cylinder's length, so the cylinder is perpendicular to the surface; hence, the flux through each end is EA. Because $\vec{E}$ is perpendicular to the charged sheet, it is parallel to the curved side wall of the cylinder, and there is no flux through these wall.



The total flux integral in Gauss's law is then $2EA$ (EA from each end and zero from the side wall). The net charge within the Gaussian surface is the charge per unit area multiplied by the sheet area enclosed by the surface, or $Q_{encl} = \sigma A$. Hence, according to the Gauss's law

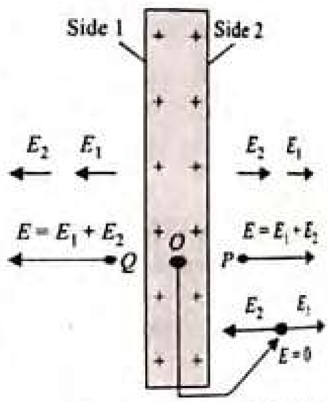
$$2EA = \frac{\sigma A}{\varepsilon_0}$$

or $$E = \frac{\sigma}{2\varepsilon_0} \quad \text{(field of an infinite sheet of charge)}$$

Electric Field Due to a Charged Isolated Conducting Plate

The electric field due to a charged isolated conducting plate is twice the field due to a plane sheet of charge. This is due to the reason that in the case of the sheet, the same charge is present on both its sides.

The above result can be obtained very easily from the principle of superposition of fields. The electric field at P is due to two sheets of charge, one on each surface of the plate.



Further, the fields due to both the sheets are perpendicular to the plate and are in the same direction.

Thus, E is the electric field (E_1) at P due to the sheet of charge on side 1 plus the electric field (E_2) at P due to the sheet of charge on side 2 of the plate, i.e.,

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Same is true for a point Q on the left side of the plate.

For a point inside the plate,

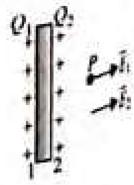
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

or $$E = E_1 - E_2 = 0 \quad (\text{as } \vec{E}_1 \text{ and } \vec{E}_2 \text{ are equal and opposite})$$

ILLUSTRATION 17.23 If an isolated infinite plate contains a charge Q_1 on one of its surfaces and a charge Q_2 on its other surface, then prove that electric field intensity at a point in front of the plate will be $Q/2A\varepsilon_0$, where $Q = Q_1 + Q_2$.

Solution. Let us take a point P right of the plate.

The electric field at P is the vector sum of the field due to both the surfaces. Hence,



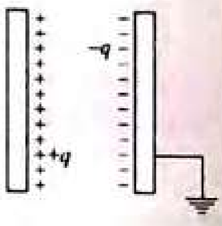
$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n}$$

$$= \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$$

Important Point

- The illustration shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces.
- Facing surfaces have equal but opposite nature of charges with magnitude half the difference of the charges on different plates, i.e., $(q_1 - q_2)/2$ in surface (2) and $(q_2 - q_1)/2$ or $-(q_1 - q_2)/2$ in surface (3).
- Outer surfaces always have equal charges of magnitude half the summation of charges, i.e., $(q_1 + q_2)/2$ in each surfaces (1) and (4).
- If we have this type of charge distribution, then the electric field inside any metal plate will be zero.



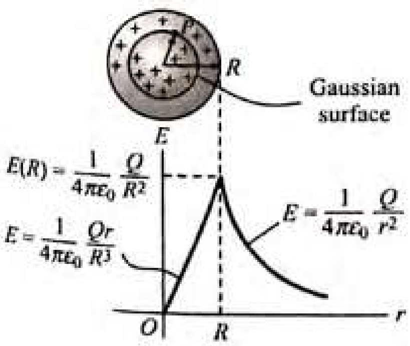
- The charge appearing on surfaces (2) and (3) is called bounded charge, and the charge appearing on surfaces (1) and (4) is called free charge.
- If we join the second plate (right plate) with ground, the charge appearing on surface (4) will go to the earth.
- Any metal plate or object connected to the earth need not have zero charge. If the conducting body is isolated and connected to earth, then it will have no charge. If the conducting body connected to earth has any charged object near it, then the body will not have zero charge.

FIELD OF A UNIFORMLY CHARGED SPHERE

Positive electric charge Q is distributed uniformly throughout the volume of an insulating sphere with radius R.

Electric Field Inside the Sphere

From symmetry, the magnitude E of the electric field has the same value at every point on the Gaussian surface, and the direction of $\vec{E}$ is radial at every point on the surface. Hence, the total electric flux through the Gaussian surface is the product of E and the total area of the surface $A = 4\pi r^2$, that is, $\Phi_E = 4\pi r^2 E$.



The amount of charge enclosed within the Gaussian surface depends on the radius r. Let us first find the field magnitude inside the charged sphere of radius R; so we choose $r < R$.

The volume charge density ρ is the charge Q divided by volume of the entire charged sphere of radius R, i.e.,

$$\rho = \frac{Q}{4\pi R^3/3}$$

$$Q_{encl} = \rho V_{encl} = \left(\frac{Q}{4\pi R^3/3}\right)\left(\frac{4}{3}\pi r^3\right) = Q\frac{r^3}{R^3}$$

Then using Gauss's law, we get

$$\phi = \int E dA = \frac{Q_{encl}}{\varepsilon_0}$$

$$E\int dA = E4\pi r^2 = \frac{Q}{\varepsilon_0}\frac{r^3}{R^3}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \text{ (field inside a uniformly charged sphere)}$$

The field magnitude is proportional to the distance r of the field point from the center of the sphere (figure).

At the center ($r = 0$), $E = 0$.

Electric field in terms of charge density (at inside point) is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\left(\rho\frac{4}{3}\pi R^3\right)r}{R^3} = \frac{\rho r}{3\varepsilon_0}$$

$\Rightarrow$ $$E = \frac{\rho r}{3\varepsilon_0} \text{ (field inside a uniformly charged sphere)}$$

Electric Field Outside the Charged Sphere

We use a spherical Gaussian surface of radius $r > R$. This surface encloses the entire charged sphere, so $Q_{encl} = Q$ and Gauss's law gives

$$E4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \text{ (field outside a uniformly charged sphere)}$$

For any spherically symmetric charged body, the electric field outside the body is the same; however, the entire charge is concentrated at the center.

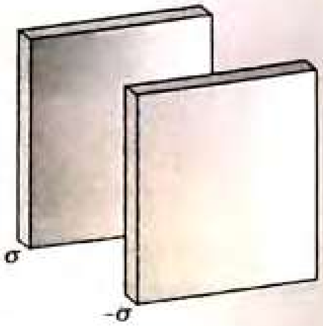
CONCEPT APPLICATION EXERCISE 17.5

1. A small conducting spherical shell with inner radius a and outer radius b is concentric with a larger conducting sphereical shell with inner radius c and outer radius d. The inner shell has total charge $+2q$ and the outer shell has charge $+4q$.
   (a) Make a plot of the magnitude of the electric field versus r.
   (b) Calculate the electric field (magnitude and direction in terms of q) and the distance r from the common

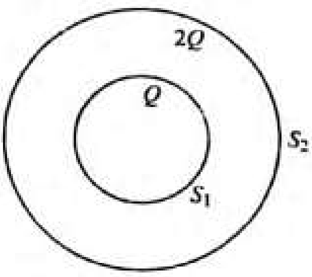
centre of the two shell for (i) $r < a$; (ii) $a < r < b$; (iii) $b < r < c$; (iv) $c < r < d$; (v) $r > d$. Show your result in a graph with radial component of $\vec{E}$ as function of r.

(c) What is the total charge on the
  i. inner surface of the small shell;
  ii. outer surface of the small shell;
  iii. inner surface of the large shell;
  iv. outer surface of the large shell?

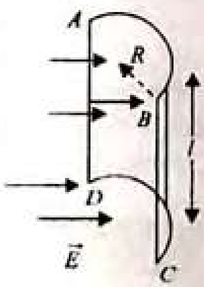
2. Two infinite, non-conducting sheets of charge are parallel to each other, as shown in figure. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets.



3. S_1 and S_2 are two hollow concentric spheres enclosing charge Q and $2Q$, respectively, as shown in figure. What is the ratio of the electric flux leaving through the surface of S_1 and S_2?



4. A hollow half cylinder surface of radius R and length l is placed in a uniform electric field $\vec{E}$. Electric field is acting perpendicular on the plane $ABCD$. Find the flux through the curved surface of the hollow cylindrical surface.



5. Consider two concentric conducting spheres. The outer sphere is hollow and initially has a charge $-7Q$ on it. The inner sphere is solid and has a charge $+2Q$ on it.
  (a) How much charge is on the outer surface and inner surface of the outer sphere.
  (b) If a wire is connected between the inner and outer sphere, after electrostatic equilibrium is established how much total charge is on the outer sphere? How much charge is on the outer surface and inner surface of the outer sphere? Does the electric field at the surface of the inside sphere change when the wire is connected?
  (c) We return to original condition in (a). We now connect the outer sphere to ground with a wire and then disconnect it. How much total charge will be on the outer sphere? How much charge will be on the inner surface and outer surface of the outer sphere?

SOLVED EXAMPLES

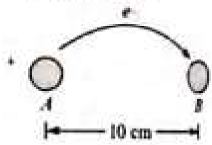
1. Two copper balls, each weighing 10 g are kept in air 10 cm apart. If one electron from every 10^6 atoms is transferred from one ball to the other, the coulomb force between them is (atomic weight of copper is 63.5)

(a) 2.0×10^{10} N (b) 2.0×10^4 N
(c) 2.0×10^8 N (d) 2.0×10^6 N

Sol. (c) Number of atoms in given mass $= \dfrac{10}{63.5} \times 6.02 \times 10^{23}$

$= 9.48 \times 10^{22}$

Transfer of electron between balls



$$= \frac{9.48 \times 10^{22}}{10^6}$$

$= 9.48 \times 10^{16}$

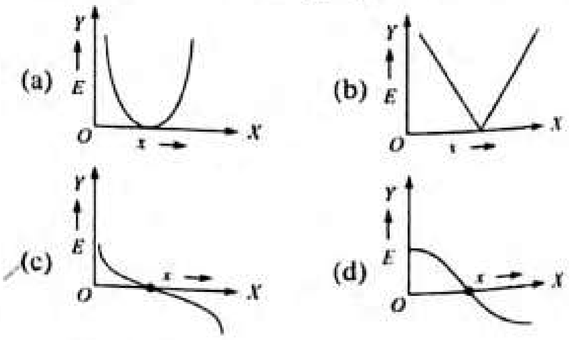
Hence magnitude of charge gained by each ball.

$$Q = 9.48 \times 10^{16} \times 1.6 \times 10^{-19} = 0.015 \text{ C}$$

Force of attraction between the balls

$$F = 9 \times 10^9 \times \frac{(0.015)^2}{(0.1)^2} = 2 \times 10^8 \text{ N}.$$

2. Two identical point charges are placed at a separation of d. P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E, E is plotted against x for values of x from close to zero to slightly less than d. Which of the following represents the resulting curve



Sol. (c) At mid point, $E = 0$

Before mid point, E is positive. This is maximum near the charge and decreases towards mid point.

After mid point, E is negative, The curve crosses x-axis at $x = d/2$. From centre to end, E decreases.

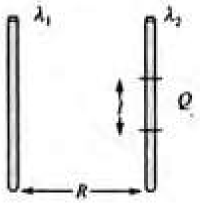
The variation is shown by curve (c).

3. Two infinitely long parallel wires having linear charge densities λ_1 and λ_2 respectively are placed at a distance of R metres. The force per unit length on either wire will be $\left(k = \dfrac{1}{4\pi\varepsilon_0}\right)$

(a) $k\dfrac{2\lambda_1\lambda_2}{R^2}$ (b) $k\dfrac{2\lambda_1\lambda_2}{R}$

(c) $k\dfrac{\lambda_1\lambda_2}{R^2}$ (d) $k\dfrac{\lambda_1\lambda_2}{R}$

Sol. (b) Force on l length of the wire 2 is



$$F_2 = QE_1 = (\lambda_2 l)\frac{2k\lambda_1}{R}$$

$$\Rightarrow \quad \frac{F_2}{l} = \frac{2k\lambda_1\lambda_2}{R}$$

Also $\dfrac{F_1}{l} = \dfrac{F_2}{l} = \dfrac{F}{l} = \dfrac{2k\lambda_1\lambda_2}{R}$

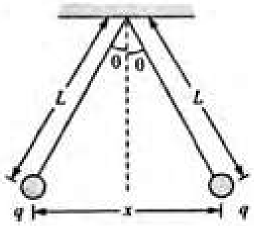
4. In the given figure two tiny conducting balls of identical mass m and identical charge q hang from non-conducting threads of equal length L. Assume that θ is so small that $\tan\theta \approx \sin\theta$, then for equilibrium x is equal to

(a) $\left(\dfrac{q^2L}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

(b) $\left(\dfrac{qL^2}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

(c) $\left(\dfrac{q^2L^2}{4\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

(d) $\left(\dfrac{q^2L}{4\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$



Sol. (a) In equilibrium

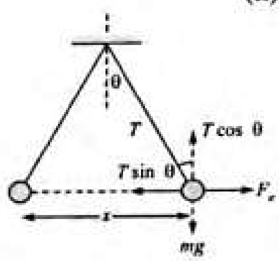
$F_e = T\sin\theta$ (i)

$mg = T\cos\theta$ (ii)

$$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\varepsilon_o x^2 \times mg}$$

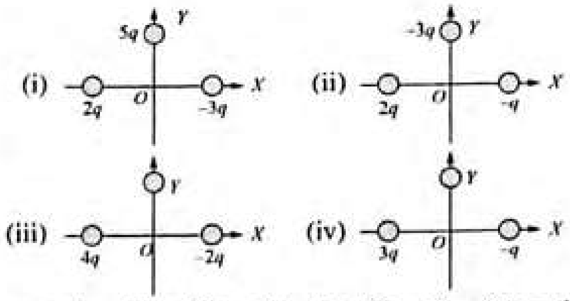
also $\tan\theta \approx \sin\theta = \dfrac{x/2}{L}$

Hence $\dfrac{x}{2L} = \dfrac{q^2}{4\pi\varepsilon_o x^2 \times mg}$



$$\Rightarrow \quad x^3 = \frac{2q^2L}{4\pi\varepsilon_o mg} \Rightarrow x = \left(\frac{q^2L}{2\pi\varepsilon_o mg}\right)^{1/3}$$

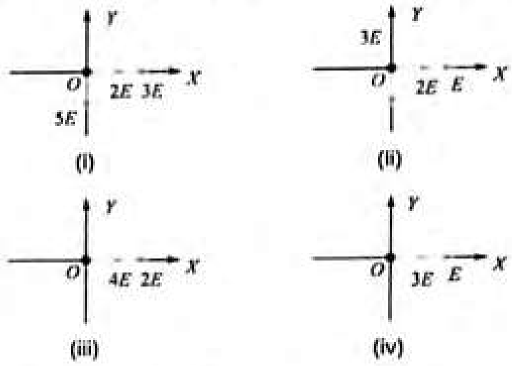
5. In the following four situations charged particles are at equal distance from the origin. Arrange them as per the magnitude of the net electric field at origin greatest first



(a) (i) > (ii) > (iii) > (iv) (b) (ii) > (i) > (iii) > (iv)

(c) (i) > (iii) > (ii) > (iv) (d) (iv) > (iii) > (ii) > (i)

Sol. (c) If electric field due to charge $|q|$ at origin is E then electric field due to charges $|2q|$, $|3q|$, $|4q|$ and $|5q|$ are respectively $2E$, $3E$, $4E$ and $5E$



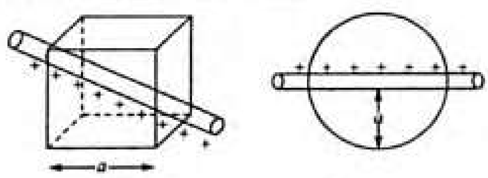
$$E_{(i)} = \sqrt{(5E)^2 + (5E)^2} = 5\sqrt{2}E$$

$$E_{(ii)} = \sqrt{(3E)^2 + (3E)^2} = 3\sqrt{2}E$$

$$E_{(iii)} = 4E + 2E = 6E \text{ and } E_{(iv)} = 3E + E = 4E$$

$$\Rightarrow \quad E_{(i)} > E_{(iii)} > E_{(ii)} > E_{(iv)}$$

6. A linear charge having linear charge density λ penetrates a cube diagonally and then it penetrates a sphere diametrically as shown. What will be the ratio of flux coming out of cube and sphere



(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{\sqrt{3}}$

(c) $\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{1}{1}$

Sol. (c) Flux coming out of the cube

$$\phi_1 = \frac{\lambda . a\sqrt{3}}{\varepsilon_0} \qquad \text{(i)}$$

and from sphere

$$\phi_2 = \frac{\lambda \cdot 2a}{\varepsilon_0} \qquad \text{(ii)}$$

$\therefore \quad \frac{\phi_1}{\phi_2} = \frac{\sqrt{3}}{2}$

7. Two charges each equal to $\eta q (\eta^{-1} < \sqrt{3})$ are placed at the corners of an equilateral triangle of side a. The electric field at the third corner is E_3
(a) $E_3 = E_0$ (b) $E_3 < E_0$ (c) $E_3 > E_0$ (d) $E_3 < E_0$
where $(E_0 = q/4\pi\varepsilon_0 a^2)$

Sol. *(c)* $E_1 = \frac{\eta q}{4\pi\varepsilon_0 a^2}, E_2 = \frac{\eta q}{4\pi\varepsilon_0 a^2}$

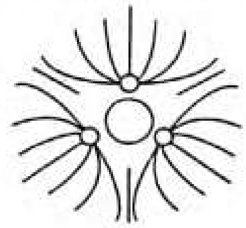
Therefore $\vec{E} = \vec{E}_1 + \vec{E}_2$

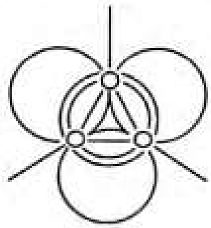
$$= \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 60^o} = \frac{\sqrt{3}\eta q}{4\pi\varepsilon_0 a^2}$$

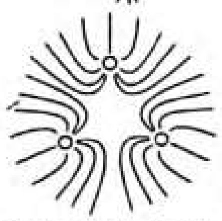
Since $\eta^{-1} < \sqrt{3}$, $1 < \sqrt{3}\eta$, $\sqrt{3}\eta > 1$.

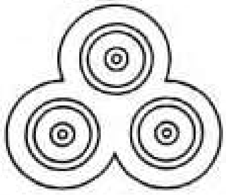
$$\Rightarrow \quad \frac{\sqrt{3}\eta q}{4\pi\varepsilon_0 a^2} > \frac{q}{4\pi\varepsilon_0 a^2} \Rightarrow E_3 > E_0 \left(E_0 = \frac{q}{4\pi\varepsilon_0 a^2} \right)$$

8. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in

(a) 

(b) 

(c) 

(d) 

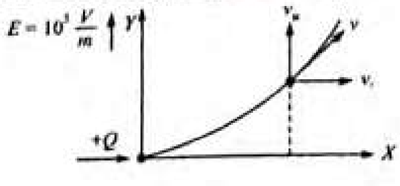
Sol. *(c)* Option (a) shows lines of force starting from one positive charge and terminating at another. Option (b) has one line of force making closed loop. Option (d) shows all lines making closed loops. All these are not correct. Only option (c) is correct.

9. There is a uniform electric field of strength 10^3 V/m along y-axis. A body of mass 1g and charge 10^{-6}C is projected into the field from origin along the positive x-axis with a velocity 10 m/s. Its speed in m/s after 10 s is (Neglect gravitation)

(a) 10 (b) $5\sqrt{2}$

(c) $10\sqrt{2}$ (d) 20

Sol. *(c)* Body moves along the parabolic path.



For vertical motion: By using $v = u + at$

$$\Rightarrow \quad v_y = 0 + \frac{QE}{m} \cdot t = \frac{10^{-6} \times 10^3}{10^{-3}} \times 10 = 10 \text{ m/sec}$$

For horizontal motion. Its horizontal velocity remains the same i.e. after 10 sec, horizontal velocity of body $v_x = 10$ m/sec.

Velocity after 10 sec $v = \sqrt{v_x^2 + v_y^2} = 10\sqrt{2}$ m/sec

10. An electric dipole is situated in an electric field of uniform intensity E whose dipole moment is p and moment of inertia is I. If the dipole is displaced slightly from the equilibrium position, then the angular frequency of its oscillations is

(a) $\left(\frac{pE}{I}\right)^{1/2}$ (b) $\left(\frac{pE}{I}\right)^{3/2}$

(c) $\left(\frac{I}{pE}\right)^{1/2}$ (d) $\left(\frac{p}{IE}\right)^{1/2}$

Sol. *(a)* When dipole is given a small angular displacement θ about its equilibrium position, the restoring torque will be

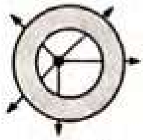
$$\tau = -pE\sin\theta = -pE\theta \text{ (as } \sin\theta = \theta)$$

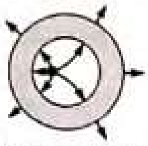
or $\quad I\frac{d^2\theta}{dt^2} = -pE\theta$ (as $\tau = I\alpha = I\frac{d^2\theta}{dt^2}$)

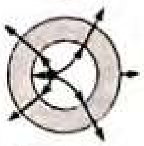
or $\quad \frac{d^2\theta}{dt^2} = -\omega^2\theta$ with $\omega^2 = \frac{pE}{I} \Rightarrow \omega = \sqrt{\frac{pE}{I}}$

11. A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces?

(a) 

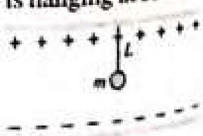
(b) 

(c) 

(d) 

Sol. *(c)* Electric field is perpendicular to the equipotential surface and is zero everywhere inside the metal.

12. A small sphere carrying a charge 'q' is hanging in between two parallel plates by a string of length L. Time period of pendulum is T_0. When parallel plates are charged, the time period changes to T. The ratio T/T_0 is equal to

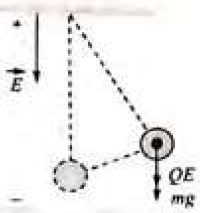


(a) $\left(\frac{g + \frac{qE}{m}}{g}\right)^{1/2}$ (b) $\left(\frac{g}{g + \frac{qE}{m}}\right)^{3/2}$

(c) $\left(\frac{g}{g + \frac{qE}{m}}\right)^{1/2}$ (d) None of these

Sol. (c)
Net downward force $mg' = mg + qE$

$\Rightarrow$ Effect acceleration $g' = \left(g + \frac{qE}{m}\right)$



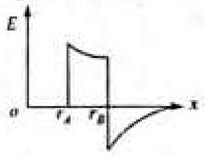
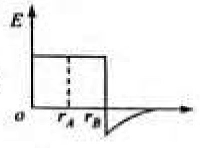
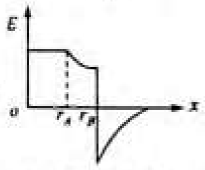
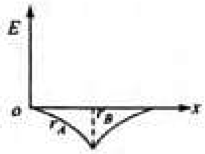
Hence time period

$$T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\left(g + \frac{qE}{m}\right)}}$$

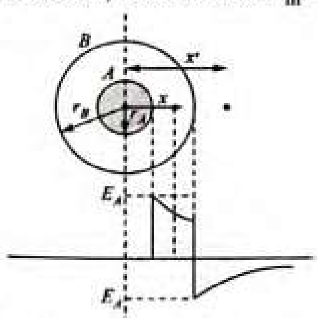
$$T_0 = 2\pi\sqrt{\frac{l}{g}}$$

So, $$\frac{T}{T_0} = \sqrt{\frac{g}{g + \frac{qE}{m}}}$$

13. Two concentric conducting thin spherical shells A, and B having radii r_A and r_B ($r_B > r_A$) are charged to Q_A and $-Q_B$ ($|Q_B| > |Q_A|$). The electrical field along a line, (passing through the centre) is

(a) 
(b) 
(c) 
(d) 

Sol. (a) Inside the shell A, electric field $E_{in} = 0$



At the surface of shell A,

$$E_A = \frac{kQ_A}{r_A^2} \longrightarrow \text{(a fixed positive value)}$$

Between the shells A and B, at a distance x from the common centre

$$E = \frac{k.Q_A}{x^2} \longrightarrow \text{(as } x \text{ increases } E \text{ decreases)}$$

At the surface of shell B,

$$E_B = \frac{k.(Q_A - Q_B)}{r_B^2} \longrightarrow$$

(a fixed negative value because $|Q_A| < |Q_B|$).

Outside the both shells, at a distance x from the common centre

$E_{out} = \frac{k(Q_A - Q_B)}{x'^2} \longrightarrow$ (as x' increases negative value of E_{out} decreases and it becomes zero at $x = x$)

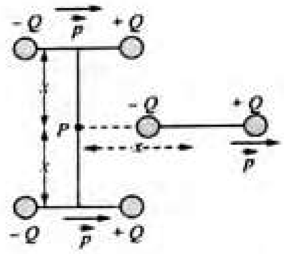
14. Three identical dipoles are arranged as shown below. What will be the net electric field at P $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$?

(a) $\frac{k.p}{x^3}$

(b) $\frac{2kp}{x^3}$

(c) Zero

(d) $\frac{\sqrt{2}\,kp}{x^3}$

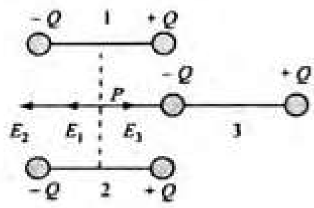


Sol. (c) Point P lies at equatorial positions of dipole 1 and 2 and axial position of dipole 3.

Hence field at P

due to dipoles 1

$$E_1 = \frac{k.p}{x^3} \text{ (towards left)}$$



due to dipole 2

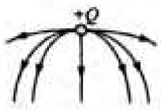
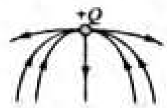
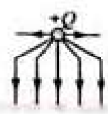
$$E_2 = \frac{k.p}{x^2} \text{ (towards left)}$$

due to dipole 3

$$E_3 = \frac{k.(2p)}{x^3} \text{ (towards right)}$$

So net field at P will be zero.

15. A charge Q is fixed at a distance d in front of an infinite metal plate. The lines of force are represented by

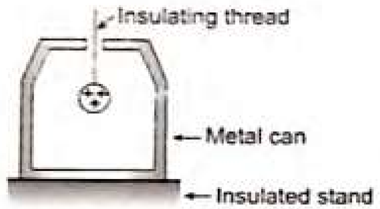
(a) 
(b) 
(c) 
(d) 

Sol. (a) Metal plate acts as an equipotential surface, therefore the field lines should enter normal to the surface of the metal plate.

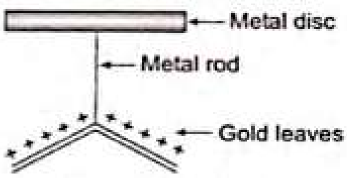
EXERCISES

Electric Charge and Coulomb's Law

1. A charged metallic ball is lowered into an insulated metal can.

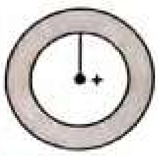


The ball is made to touch bottom of the can, then it is placed on the disc of electroscope shown below. Final observation must be



(a) leaves of electroscope diverges
(b) leaves of electroscope converges
(c) leaves of electroscope remains unaffected
(d) leaves of electroscope oscillates

2. An electrically isolated hollow (initially uncharged), conducting sphere has a small positively charged ball suspended by an insulating rod from its inside surface, see diagram.



This causes the inner surface of the sphere to become negatively charged. When the ball is centered in the sphere the electric field outside the conducting sphere is
(a) zero
(b) the same as if the sphere wasn't there
(c) twice what it would be if the sphere wasn't there
(d) equal in magnitude but opposite in direction to what it would be if the sphere wasn't there

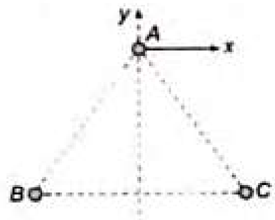
3. A copper sphere of mass 2.0 g contains about 2×10^{22} atoms. The charge on the nucleus of each atom is 29 e (e = electronic charge). The mass of an electron is 9.11×10^{-31} kg. How much mass will the sphere lose or gain if it is a given a charge of +2 μC?
(a) Loss a mass of 1.13×10^{-14} g
(b) Gain a mass of 1.13×10^{-14} g
(c) Neither gain nor loss any mass
(d) Change in mass will depend upon the kinetic energy of the electrons

4. Two identical balls having like charges and placed at a certain distance apart repel each other with a certain force. They are brought in contact and then moved apart to a distance equal to half their initial separation. The force of repulsion between them increases 4.5 times in comparison with the initial value. The ratio of the initial charges of the balls is
(a) 2 (b) 3
(c) 4 (d) 6

5. Two identical simple pendulums, A and B, are suspended from the same point. The bobs are given positive charges, with A having more charge than B. They diverge and reach equilibrium, with A and B making angles θ_1 and θ_2 with the vertical respectively. Which of the following is correct?
(a) $\theta_1 > \theta_2$
(b) $\theta_1 < \theta_2$
(c) $\theta_1 = \theta_2$
(d) The tension in A is greater than that in B

6. Two copper balls, each weighing 10 g are kept in air 10 cm apart. If one electron from every 10^6 atoms is transferred from one ball to the other, the coulomb force between them is (atomic weight of copper is 63.5)
(a) 2.0×10^{10} N (b) 2.0×10^4 N
(c) 2.0×10^8 N (d) 2.0×10^6 N

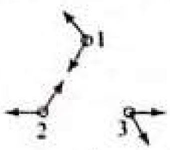
7. Three similar charges $+q$ are placed on 3 corners of an equilateral triangle ABC of side a. How many minimum charges should be placed on a circle of radius a with the centre at A so that resultant force on the charge placed at the centre is $\dfrac{q^2}{4\pi\varepsilon_0 a^2}$ along x-axis?

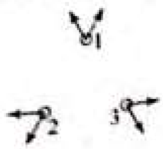


(a) 4 (b) 6
(c) 3 (d) Any number

8. Two identical conducting spheres having unequal positive charges q_1 and q_2 separated by distance r. If they are made to touch each other and then separated again to the same distance. The electrostatic force between the spheres in this case will be (neglect induction of charges)
(a) less than before (b) same as before
(c) more than before (d) zero

9. Which of the following four figures correctly show the forces that three charged particles exert on each other?

(i) 

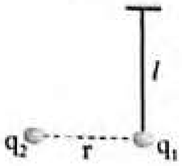
(ii) 

(iii) 

(iv) 

(a) all of the above (b) none of the above
(c) ii, iii (d) ii, iii and iv

10. An isolated charge q_1 of mass m is suspended freely by a thread of length l. Another charge q_2 is brought near it ($r >> l$). When q_1 is in equilibrium, tension in thread will be



(a) mg
(b) $>mg$
(c) $<mg$
(d) none of these

11. Three charges $+Q_1$, $+Q_2$, and q are placed on a straight line such that q is somewhere in between $+Q_1$ and $+Q_2$. If this system of charges is in equilibrium, what should be the magnitude and sign of charge q?

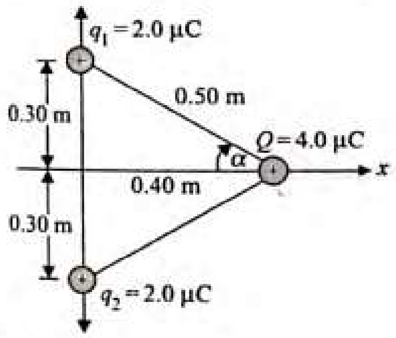
(a) $\dfrac{Q_1Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$, positive

(b) $\dfrac{Q_1+Q_2}{2}$, positive

(c) $\dfrac{Q_1Q_2}{\left(\sqrt{Q_1}+\sqrt{Q_2}\right)^2}$, negative

(d) $\dfrac{Q_1+Q_2}{2}$, negative

12. In figure, two equal positive point charges $q_1 = q_2 = 2.0$ μC interact with a third point charge $Q = 4.0$ μC. The magnitude, as well as direction, of the net force on Q is



(a) 0.23 N in the $+x$-direction
(b) 0.46 N in the $+x$-direction
(c) 0.23 N in the $-x$-direction
(d) 0.46 N in the $-x$-direction

13. Three positive charges of equal magnitude q are placed at the vertices of an equilateral triangle of side l. How can the system of charges be placed in equilibrium?
(a) by placing a charge $Q = -q/\sqrt{3}$ at the centroid of the triangle
(b) by placing a charge $Q = q/\sqrt{3}$ at the centroid of the triangle
(c) by placing a charge $Q = q$ at a distance l from all the three charges
(d) by placing a charge $Q = -q$ above the plane of the triangle at a distance l from all the three charges

14. Three identical spheres, each having a charge q and radius R, are kept in such a way that each touches the other two. The magnitude of the electric force on any sphere due to the other two is

(a) $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{R}\right)^2$ (b) $\dfrac{\sqrt{3}}{4\pi\varepsilon_0}\left(\dfrac{q}{R}\right)^2$

(c) $\dfrac{\sqrt{3}}{16\pi\varepsilon_0}\left(\dfrac{q}{R}\right)^2$ (d) $\dfrac{\sqrt{5}}{16\pi\varepsilon_0}\left(\dfrac{q}{R}\right)^2$

15. Five point charges, each of value $+q$, are placed on five vertices of a regular hexagon of side L. The magnitude of the force on a point charge of value $-q$ coulomb placed at the center of the hexagon is

(a) $\dfrac{1}{\pi\varepsilon_0}\left(\dfrac{q}{L}\right)^2$ (b) $\dfrac{2}{\pi\varepsilon_0}\left(\dfrac{q}{L}\right)^2$

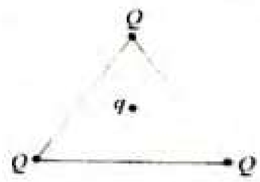
(c) $\dfrac{1}{2\pi\varepsilon_0}\left(\dfrac{q}{L}\right)^2$ (d) $\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{L}\right)^2$

16. It is required to hold equal charges q in equilibrium at the corners of a square. What charge when placed at the center of the square will do this?

(a) $-\dfrac{q}{2}(1+2\sqrt{2})$ (b) $\dfrac{q}{2}(1+2\sqrt{2})$

(c) $\dfrac{q}{4}(1+2\sqrt{2})$ (d) $-\dfrac{q}{4}(1+2\sqrt{2})$

17. Three charges (each Q) are placed at the three corners of an equilateral triangle. A fourth charge q is placed at the center of the triangle. The ratio $|q/Q|$ so as to make the system in equilibrium is



(a) 1:3
(b) 1:$\sqrt{3}$
(c) $\sqrt{3}$:1
(d) 2:$\sqrt{3}$

18. An electric charge q exerts a force F on a similar electric charge q separated by a distance r. A third charge $q/4$ is placed midway between the two charges. Now, the force F will
(a) become $F/3$ (b) become $F/9$
(c) become $F/27$ (d) remain F

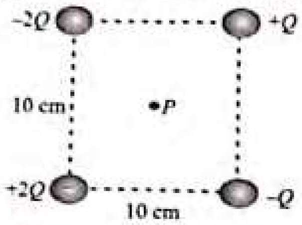
Electric Field

19. A positively charged ball hangs from a long silk thread. Electric field at a certain point (at the same horizontal level of ball) due to this charge is E. Let us put a positive test charge q_0 at this point and measure F/q_0 on this charge. Then, E
(a) $> F/q_0$ (b) $< F/q_0$
(c) $= F/q_0$ (d) none of these

20. A point charge of 100 μC is placed at $3\hat{i}+4\hat{j}$ m. Find the electric field intensity due to this charge at a point located at $9\hat{i}+12\hat{j}$ m.

(a) 8000 Vm^{-1} (b) 9000 Vm^{-1}
(c) 2250 Vm^{-1} (d) 4500 $V m^{-1}$

21. Four electrical charges are arranged on the corners of a 10 cm square as shown. What would be the direction of the resulting electric field at the center point *P*?

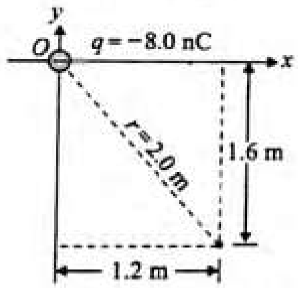


(a) → (b) ↑
(c) ← (d) ↓

22. Two charges $Q_1 = 18$ μC and $Q_2 = -2$ μC are separated by a distance *R*, and Q_1 is on the left of Q_2. The distance of the point where the net electric field is zero is

(a) between Q_1 and Q_2 (b) left of Q_1 at *R*/2
(c) right of Q_2 at *R* (d) right of Q_2 at *R*/2

23. A point charge $q = -8.0$ nC is located at the origin. The electric field (in NC^{-1}) vector at the point $x = 1.2$ m, $y = -1.6$ m, as shown in figure, is



(a) $-14.4\hat{i}+10.8\hat{j}$ (b) $-14.4\hat{i}-10.8\hat{j}$
(c) $-10.8\hat{i}+14.4\hat{j}$ (d) $-10.8\hat{i}-14.4\hat{j}$

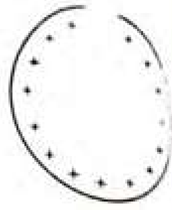
24. A positive point charge 50 μC is located in the plane *xy* at a point with radius vector $\vec{r}_0 = 2\hat{i}+3\hat{j}$. The electric field vector $\vec{E}$ at a point with radius vector $\vec{r} = 8\hat{i}-5\hat{j}$, where r_0 and *r* are expressed in meter, is

(a) $(1.4\hat{i}-2.6\hat{j})$ kNC^{-1} (b) $(1.4\hat{i}+2.6\hat{j})$ kNC^{-1}
(c) $(2.7\hat{i}-3.6\hat{j})$ kNC^{-1} (d) $(2.7\hat{i}+3.6\hat{j})$ kNC^{-1}

25. Five point charges, $+q$ each, are placed at the five vertices of a regular hexagon. The distance of the center of the hexagon from any of the vertices is *a*. The electric field at the center of the hexagon is

(a) $\dfrac{q}{4\pi\varepsilon_0 a^2}$ (b) $\dfrac{q}{8\pi\varepsilon_0 a^2}$
(c) $\dfrac{q}{16\pi\varepsilon_0 a^2}$ (d) zero

26. A ring of charge with radius 0.5 m has 0.002 m gap. If the ring carries a charge of +1 C, the electric field at the center is

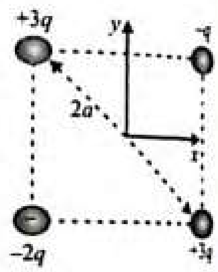


(a) 7.5×10^7 NC^{-1}
(b) 7.2×10^7 NC^{-1}
(c) 6.2×10^7 NC^{-1}
(d) 6.5×10^7 NC^{-1}

27. Three equal charges, each $+q$, are placed on the corners of an equilateral triangle. The electric field intensity at the centroid of the triangle is

(a) kq/r^2 (b) $3kq/r^2$
(c) $\sqrt{3}kq/r^2$ (d) zero

28. Four point charges are placed at the corners of a square with diagonal 2*a* as shown. What is the total electric field at the center of the square?



(a) kq/a^2 at an angle 45° above the +*x*-axis
(b) kq/a^2 at an angle 45° below the −*x*-axis
(c) $3kq/a^2$ at an angle 45° above the −*x*-axis
(d) $3kq/a^2$ at an angle 45° below the +*x*-axis

29. The maximum electric field at a point on the axis of a uniformly charged ring is E_0. At how many points on the axis will the magnitude of the electric field be $E_0/2$.

(a) 1 (b) 2
(c) 3 (d) 4

30. *A* and *B* are two points on the axis and the perpendicular bisector, respectively, of an electric dipole. *A* and *B* are far away from the dipole and at equal distances from it. The fields at *A* and *B* are $\vec{E}_A$ and $\vec{E}_B$. Then

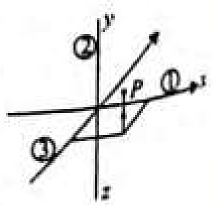
(a) $\vec{E}_A = \vec{E}_B$

(b) $\vec{E}_A = 2\vec{E}_B$

(c) $\vec{E}_A = -2\vec{E}_B$

(d) $|\vec{E}_B| = \frac{1}{2}|\vec{E}_A|$, and $\vec{E}_A$ is perpendicular to $\vec{E}_B$

31. Find the electric field vector at *P* (*a*, *a*, *a*) due to three infinitely long lines of charges along the *x*-, *y*- and *z*-axes, respectively. The charge density, i.e., charge per unit length of each wire is λ.



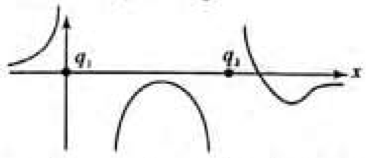
(a) $\frac{\lambda}{3\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

(b) $\frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

(c) $\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

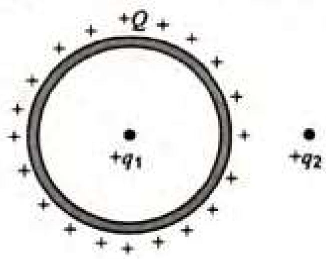
(d) $\frac{\sqrt{2}\lambda}{\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$

32. Two charges q_1 and q_2 are kept on the x-axis, and the electric field at different points on the x-axis is plotted against x. Choose the correct statement about the nature and magnitude of q_1 and q_2.



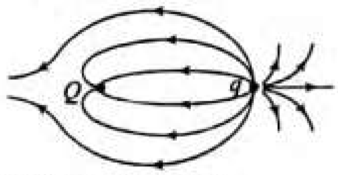
(a) q_1 is positive, q_2 is negative; $|q_1| > |q_2|$
(b) q_1 is positive, q_2 is negative; $|q_1| < |q_2|$
(c) q_1 is negative, q_2 is positive; $|q_1| > |q_2|$
(d) q_1 is negative, q_2 is positive; $|q_1| < |q_2|$

33. A thin metallic spherical shell contains a charge Q on its surface. A point charge q_1 is placed at the center of the shell, and another charge q_2 is placed outside the shell. All the three charges are positive. Then the force on charge q_1 is



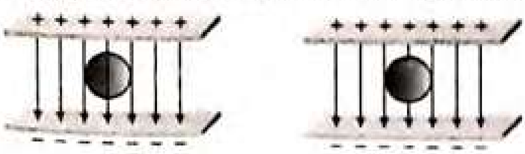
(a) toward right
(b) toward left
(c) zero
(d) none of these

34. The lines of force of the electric field due to two charges q and Q are sketched in the figure. State if



(a) Q is positive and $|Q| > |q|$
(b) Q is negative and $|Q| > |q|$
(c) q is positive and $|Q| < |q|$
(d) q is negative and $|Q| < |q|$

35. An uncharged sphere of metal is placed in between two charged plates as shown. The lines of force look like

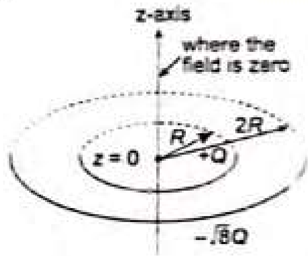


A B

C D

(a) A (b) B
(c) C (d) D

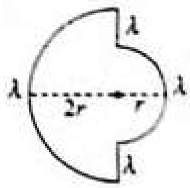
36. Two concentric rings, one of radius R and total charge $+Q$ and the second of radius $2R$ and total charge $-\sqrt{8}\,Q$, lie in x-y plane (i.e., $z = 0$ plane). The common centre of rings lies at origin and the common axis coincides with z-axis. The charge is uniformly distributed on both rings. At what distance from origin is the net electric field on z-axis zero?



(a) $\frac{R}{2}$ (b) $\frac{R}{\sqrt{2}}$

(c) $\frac{R}{2\sqrt{2}}$ (d) $\sqrt{2}\,R$

37. Two semicircular rings lying in the same plane of uniform linear charge density λ have radii r and $2r$. They are joined using two straight uniformly charged wires of linear charge density λ and length r as shown in the figure. The magnitude of electric field at common centre of semi-circular rings is



(a) $\frac{1}{4\pi\varepsilon_o}\frac{3\lambda}{2r}$

(b) $\frac{1}{4\pi\varepsilon_o}\frac{\lambda}{2r}$

(c) $\frac{1}{4\pi\varepsilon_o}\frac{2\lambda}{r}$

(d) $\frac{1}{4\pi\varepsilon_o}\frac{\lambda}{r}$

38. A positively charged particle moving along x-axis with a certain velocity enters a uniform electric field directed along positive y-axis. Its

(a) vertical velocity changes but horizontal velocity remains constant
(b) horizontal velocity changes but vertical velocity remains constant
(c) both vertical and horizontal velocities change
(d) neither vertical nor horizontal velocity changes

39. An electron of mass m_e initially at rest moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p also initially at rest takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio of t_2/t_1 is nearly equal to

(a) 1 (b) $(m_p/m_e)^{1/2}$
(c) $(m_e/m_p)^{1/2}$ (d) 1836

40. An electron falls through a small distance in a uniform electric field of magnitude 2×10^4 NC^{-1}. The direction of the field is reversed keeping the magnitude unchanged and a proton falls through the same distance. The time of fall will be

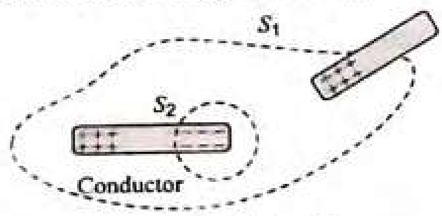
(a) same in both cases
(b) more in the case of an electron
(c) more in the case of proton
(d) independent of charge

Electric Flux and Gauss Law

41. Under what conditions can the electric flux ϕ_E be found through a closed surface?

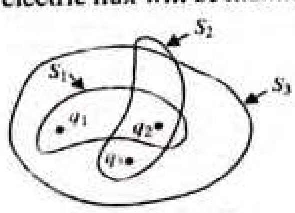
(a) If the magnitude of the electric field is known everywhere on the surface.
(b) If the total charge inside the surface is specified.
(c) If the total charge outside the surface is specified.
(d) Only if the location of each point charge inside the surface is specified.

42. Charge on an originally uncharged conductor is separated by holding a positively charged rod very closely nearby, as shown in figure. Assume that the induced negative charge on the conductor is equal to the positive charge q on the rod. Then the flux through surface S_1 is



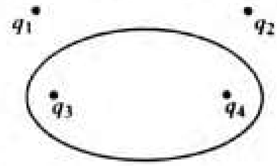
(a) zero (b) q/ε_0
(c) $-q/\varepsilon_0$ (d) none of these

43. Three charges $q_1 = 1 \times 10^{-6}$ C, $q_2 = 2 \times 10^{-6}$ C, and $q_3 = -3 \times 10^{-6}$ C have been placed as shown in figure. Then the net electric flux will be maximum for the surface



(a) S_1 (b) S_2
(c) S_3 (d) same for all three

44. Figure shows four charges q_1, q_2, q_3, and q_4 fixed in space. Then the total flux of the electric field through a closed surface S, due to all the charges, is



(a) not equal to the total flux through S due to q_3 and q_4
(b) equal to the total flux through S due to q_3 and q_4
(c) zero if $q_1 + q_2 = q_3 + q_4$
(d) twice the total flux through S due to q_3 and q_4 if $q_1 + q_2 = q_3 + q_4$

45. If the flux of the electric field through a closed surface is zero, then

(a) the electric field must be zero everywhere on the surface
(b) the total charge inside the surface must be zero
(c) the electric field must be uniform throughout the closed surface
(d) the charge outside the surface must be zero

46. In a region of space, the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$. The electric flux through a surface of area 100 units in the xy plane is

(a) 800 units (b) 300 units
(c) 400 units (d) 1500 units

47. The electric flux from a cube of edge l is ϕ. If an edge of the cube is made $2l$ and the charge enclosed is halved, its value will be

(a) 4ϕ (b) 2ϕ
(c) $\phi/2$ (d) ϕ

48. In a certain region of space, there exists a uniform electric field of value $2 \times 10^2 \hat{k}$ Vm^{-1}. A rectangular coil of dimension 10 cm × 20 cm is placed in the xy plane. The electric flux through the coil is

(a) zero (b) 30 Vm
(c) 40 Vm (d) 50 Vm

49. Consider two concentric spherical surfaces S_1 with radius a and S_2 with radius $2a$, both centered at the origin. There is a charge $+q$ at the origin and there are no other charges. Compare the flux ϕ_1 through S_1 with the flux ϕ_2 through S_2.

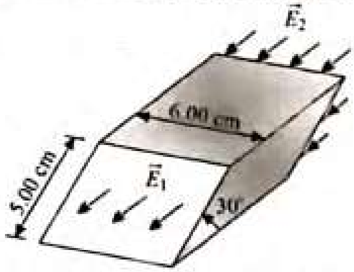
(a) $\phi_1 = 4\phi_2$ (b) $\phi_1 = 2\phi_2$
(c) $\phi_1 = \phi_2$ (d) $\phi_1 = \phi_2/2$

50. Eight charges, 1 μC, -7 μC, -4 μC, 10 μC, 2 μC, -5 μC, -3 μC, and 6 μC, are situated at the eight corners of a cube of side 20 cm. A spherical surface of radius 80 cm encloses this cube. The center of the sphere coincides with the center of the cube. Then, the total outgoing flux from the spherical surface (in units of Vm) is

(a) $36\pi \times 10^3$ (b) $684\pi \times 10^3$
(c) zero (d) none of these

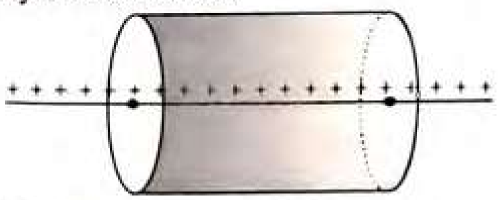
51. A flat, square surface with sides of length L is described by the equations
$x = L, 0 \le y \le L, 0 \le z \le L$
The electric flux through the square due to a positive point charge q located at the origin ($x = 0, y = 0, z = 0$) is
(a) $\frac{q}{4\varepsilon_0}$ (b) $\frac{q}{6\varepsilon_0}$
(c) $\frac{q}{24\varepsilon_0}$ (d) $\frac{q}{48\varepsilon_0}$

52. The electric field $\vec{E}_1$ at one face of a parallelopiped is uniform over the entire face and is directed out of the face. At the opposite face, the electric field $\vec{E}_2$ is also uniform over the entire face and is directed into that face (as shown in figure). The two faces in question are inclined at 30° from the horizontal, $\vec{E}_1$ and $\vec{E}_2$ (both horizontal) have magnitudes of $2.50 \times 10^4\ NC^{-1}$ and $7.00 \times 10^4\ NC^{-1}$, respectively. Assuming that no other electric field lines cross the surfaces of the parallelopiped, the net charge contained within is



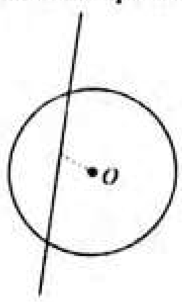
(a) $-67.5\ \varepsilon_0 C$ (b) $37.5\ \varepsilon_0 C$
(c) $105\ \varepsilon_0 C$ (d) $-105\ \varepsilon_0 C$

53. Consider an infinite line charge having uniform linear charge density and passing through the axis of a cylinder. What will be the effect on the flux passing through the curved surface if the portions of the line charge outside the cylinder is removed.



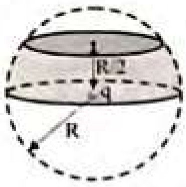
(a) decreases (b) increases
(c) remains same (d) cannot say

54. A uniformly charged and infinitely long line having a liner charge density λ is placed at a normal distance y from a point O. Consider a sphere of radius R with O as the center and $R > y$. Electric flux through the surface of the sphere is



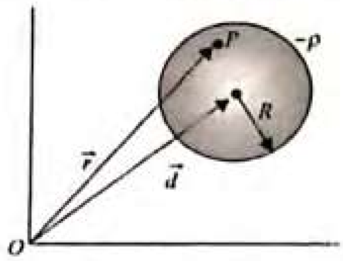
(a) zero
(b) $\frac{2\lambda R}{\varepsilon_0}$
(c) $\frac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$
(d) $\frac{\lambda\sqrt{R^2 + y^2}}{\varepsilon_0}$

55. Flux passing through the shaded surface of a sphere when a point charge q is placed at the center is (radius of the sphere is R)



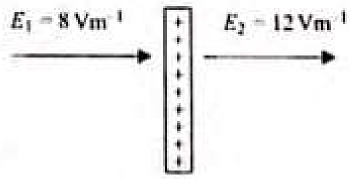
(a) q/ε_0 (b) $q/2\varepsilon_0$
(c) $q/4\varepsilon_0$ (d) zero

56. A nonconducting sphere of radius R is filled with uniform volume charge density $-\rho$. The center of this sphere is displaced from the origin by $\vec{d}$. The electric field $\vec{E}$ at any point P having position vector inside the sphere is



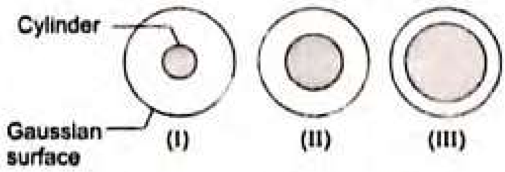
(a) $\frac{\rho}{3\varepsilon_0}\vec{d}$
(b) $\frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{d})$
(c) $\frac{\rho}{3\varepsilon_0}(\vec{d} - \vec{r})$
(d) $\frac{\rho}{3\varepsilon_0}(\vec{r})$

57. The electric field on two sides of a large charged plate is shown in figure. The charge density on the plate in SI units is given by (ε_0 is the permittivity of free space in SI units)



(a) $2\varepsilon_0$ (b) $4\varepsilon_0$
(c) $10\varepsilon_0$ (d) zero

58. Figure shows, in cross section, three solid cylinders, each of length L and uniform charge Q. Concentric with each cylinder is a cylindrical Gaussian surface, with all three surfaces having the same radius. Rank the Gaussian surfaces according to the electric field at any point on the surface, greatest first.



(a) I → II → III (b) II → I → III
(c) III → II → I (d) all tie

59. A charge q is placed at some distance along the axis of a uniformly charged disc of surface charge density σ. The flux due to the charge q through the disc is ϕ. The electric force on charge q exerted by the disc is

(a) $\sigma\phi$ (b) $\dfrac{\sigma\phi}{4\pi}$

(c) $\dfrac{\sigma\phi}{2\pi}$ (d) $\dfrac{\sigma\phi}{3\pi}$

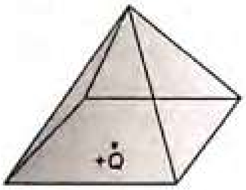
60. A point charge $+Q$ is positioned at the centre of the base of a square pyramid as shown. The flux through one of the four identical upper faces of the pyramid is

(a) $\dfrac{Q}{16\varepsilon_0}$

(b) $\dfrac{Q}{4\varepsilon_0}$

(c) $\dfrac{Q}{8\varepsilon_0}$

(d) None of these



Problems Based on Mixed Concepts

61. Two particles of masses in the ratio 1 : 2, with charges in the ratio 1 : 1, are placed at rest in a uniform electric field. They are released and allowed to move for the same time. The ratio of their kinetic energies will be finally

(a) 2 : 1 (b) 8 : 1

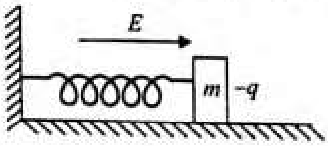
(c) 4 : 1 (d) 1 : 4

62. An oil drop, carrying six electronic charges and having a mass of 1.6×10^{-12} g, falls with some terminal velocity in a medium. What magnitude of vertical electric field is required to make the drop move upward with the same speed as it was formerly moving downward with? Ignore buoyancy.

(a) 10^5 NC^{-1} (b) 10^4 NC^{-1}

(c) 3.3×10^4 NC^{-1} (d) 3.3×10^5 NC^{-1}

63. A block of mass m containing a net negative charge $-q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant k. If the horizontal electric field E parallel to the spring is switched on, then the maximum compression of the spring is



(a) $\sqrt{qE/k}$ (b) $2qE/k$

(c) qE/k (d) zero

64. A thin glass rod is bent into a semicircle of radius r. A charge $+Q$ is uniformly distributed along the upper half, and a charge $-Q$ is uniformly distributed along the lower half, as shown in figure. The electric field E at P, the center of the semicircle, is

(a) $\dfrac{Q}{\pi^2\varepsilon_0 r^2}$ (b) $\dfrac{2Q}{\pi^2\varepsilon_0 r^2}$

(c) $\dfrac{4Q}{\pi^2\varepsilon_0 r^2}$ (d) $\dfrac{Q}{4\pi^2\varepsilon_0 r^2}$

65. A system consists of a thin charged wire ring of radius r and a very long uniformly charged wire oriented along the axis of the ring, with one of its ends coinciding with the center of the ring. The total charge on the ring is q, and the linear charge density on the straight wire is λ. The interaction force between the ring and the wire is

(a) $\dfrac{\lambda q}{4\pi\varepsilon_0 r}$ (b) $\dfrac{\lambda q}{2\sqrt{2}\pi\varepsilon_0 r}$

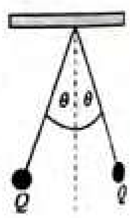
(c) $\dfrac{2\sqrt{2}\lambda q}{\pi\varepsilon_0 r}$ (d) $\dfrac{4\lambda q}{\pi\varepsilon_0 r}$

66. A particle of mass m and charge $-q$ moves diametrically through a uniformly charged sphere of radius R with total charge Q. The angular frequency of the particle's simple harmonic motion, if its amplitude $< R$, is given by

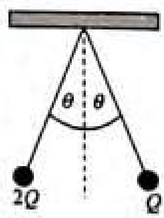
(a) $\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{mR}}$ (b) $\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{mR^2}}$

(c) $\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{mR^3}}$ (d) $\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{m}{qQ}}$

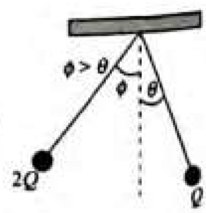
67. Two pith balls each with mass m are suspended from insulating threads. When the pith balls are given equal positive charge Q, they hang in equilibrium as shown. We now increase the charge on the left pith ball from Q to $2Q$ while leaving its mass essentially unchanged. Which of the following diagrams best represents the new equilibrium configuration?



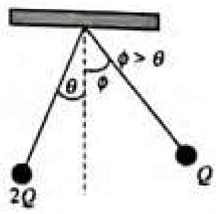
(a)



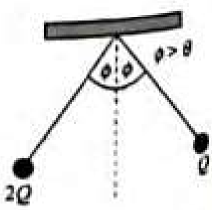
(b)



(c)

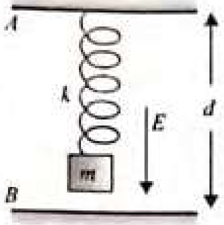


(d)



68. A block of mass m is suspended vertically with a spring of spring constant k. The block is made to oscillate

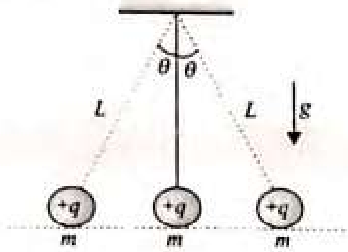
in a gravitation field. Its time period is found to be T. Now the space between the plates is made gravity free, and an electric field E is produced in the downward direction. Now the block is given a charge q. The new time period of oscillation is



(a) T (b) $T + 2\pi\sqrt{\dfrac{qE}{md}}$

(c) $2\pi\sqrt{\dfrac{qE}{md}}$ (d) none of the above

69. Three identical point charges, each of mass m and charge q, hang from three strings as shown in figure. The value of q in terms of m, L, and q is



(a) $q = \sqrt{(16/5)\pi\varepsilon_0 mg L^2 \sin^2\theta \tan\theta}$

(b) $q = \sqrt{(16/15)\pi\varepsilon_0 mg L^2 \sin^2\theta \tan\theta}$

(c) $q = \sqrt{(15/16)\pi\varepsilon_0 mg L^2 \sin^2\theta \tan\theta}$

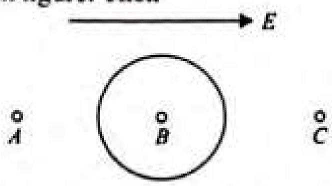
(d) none of these

70. A spherical shell of radius R = 1.5 cm has a charge q = 20 μC uniformly distributed over it. The force exerted by one half over the other half is

(a) zero (b) 10^{-2} N

(c) 500 N (d) 2000 N

71. A dielectric in the form of a sphere is introduced into a homogeneous electric field. A, B, and C are three points as shown in figure. Then



(a) intensity at A increases while that at B and C decreases

(b) intensity at A and B decreases, whereas intensity at C increases

(c) intensity at A and C increases and that at B decreases

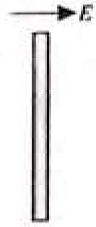
(d) intensity at A, B, and C decreases

72. A sphere of radius R carries charge such that its volume charge density is proportional to the square of the distance from the center. What is the ratio of the magnitude of the electric field at a distance $2R$ from the center to the magnitude of the electric field at a distance of $R/2$ from the center?

(a) 1 (b) 2

(c) 4 (d) 8

73. An uncharged conducting large plate is placed as shown. Now an electric field E toward right is applied. Find the induced charge density on the right surface of the plate.



(a) $-\varepsilon_0 E$

(b) $\varepsilon_0 E$

(c) $-2\varepsilon_0 E$

(d) $2\varepsilon_0 E$

74. An uncharged aluminum block has a cavity within it. The block is placed in a region where a uniform electric field is directed upward. Which of the following is a correct statement describing conditions in the interior of the block's cavity?

(a) The electric field in the cavity is directed upward.

(b) The electric field in the cavity is directed downward.

(c) There is no electric field in the cavity.

(d) The electric field in the cavity is of varying magnitude and is zero at the exact center.

75. Figure shows a uniformly charged hemisphere of radius R. It has a volume charge density ρ. If the electric field at a point $2R$, above its center is E, then what is the electric field at the point $2R$ below its center?

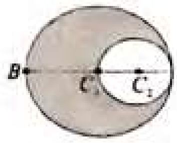
(a) $\rho R/6\varepsilon_0 + E$

(b) $\rho R/12\varepsilon_0 - E$

(c) $-\rho R/6\varepsilon_0 + E$

(d) $\rho R/12\varepsilon_0 + E$

76. A positively charged sphere of radius r_0 carries a volume charge density ρ (figure). A spherical cavity of radius $r_0/2$ is then scooped out and left empty. C_1 is the center of th esphere and C_2 that of the cavity. What is the direction and magnitude of the electric field at point B?



(a) $\dfrac{17\rho r_0}{54\varepsilon_0}$ left (b) $\dfrac{\rho r_0}{6\varepsilon_0}$ left

(c) $\dfrac{17\rho r_0}{54\varepsilon_0}$ right (d) $\dfrac{\rho r_0}{6\varepsilon_0}$ right

77. Two charged particles (M, $+Q$) and (m, $-q$) are placed in a gravity free space where a uniform electric field E exists. After the particles are released, they stay at a constant distance from each other. What is this distance? Neglect Gravitational interaction. k is electrostatic constant.

(a) $\sqrt{\dfrac{kqQ}{E(q+Q)}}$ (b) $\sqrt{\dfrac{(M+m)kQq}{E(qm+QM)}}$

(c) $\sqrt{\dfrac{(M+m)kQq}{E(qM+Qm)}}$ (d) not possible

78. Four point positive charges are held at the vertices of a square in a horizontal plane. Their masses are 1 kg, 2 kg, 3 kg and 4 kg. Another point positive charge of mass 10 kg is kept on the axis of the square. The weight of this fifth charge is balanced by the electrostatic force due to those four charges. If the four charges on the vertices are released such that they can freely move in any direction (vertical, horizontal etc.) then the acceleration of the centre of mass of the four charges immediately after the release is (use $g = 10\ \text{m/s}^2$)

(a) $10\ \text{m/s}^2$ downwards (b) $20\ \text{m/s}^2$ downwards
(c) zero (d) $10\ \text{m/s}^2$ upwards

79. A soap bubble (surface tension = T) is charged to a maximum surface density of charge = σ. When it is just going to burst? Its radius R is given by

Dependence on surface tension

(a) $R = \dfrac{\sigma^2}{8\,\varepsilon_0 T}$ (b) $R = 8\,\varepsilon_0 \dfrac{T}{\sigma^2}$

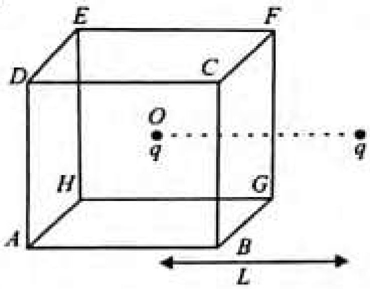
(c) $R = \dfrac{\sigma}{\sqrt{8\,\varepsilon_0 T}}$ (d) $R = \dfrac{\sqrt{8\,\varepsilon_0 T}}{\sigma}$

80. Two identical spheres of same mass and specific gravity (which is the ratio of density of a substance and density of water) 2.4 have different charges of Q and $-3Q$. They are suspended from two strings of same length l fixed to points at the same horizontal level, but distant l from each other. When the entire set up is transferred inside a liquid of specific gravity 0.8, it is observed that the inclination of each string in equilibrium remains unchanged. Then the dielectric constant of the liquid is

(a) 2 (b) 3
(c) 1.5 (d) None of these

ARCHIVES

1. A charged particle q is placed at the centre O of a cube of length L ($ABCDEFGH$). Another same charge q is placed at a distance L from O. Then the electric flux through $ABCD$ is



(a) $\dfrac{q}{4\pi\varepsilon_0 L}$ (b) zero

(c) $\dfrac{q}{2\pi\varepsilon_0 L}$ (d) $\dfrac{q}{3\pi\varepsilon_0 L}$ **(AIEEE 2002)**

2. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium, then the value of q is

(a) $\dfrac{Q}{2}$ (b) $\dfrac{-Q}{2}$

(c) $\dfrac{Q}{4}$ (d) $\dfrac{-Q}{4}$ **(AIEEE 2002)**

3. A simple pendulum of period T has a metal bob which is negatively charged. If it is allowed to oscillate above a positively charged metal plate, its period will

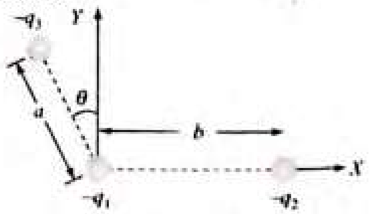
(a) remain equal to T (b) less than T
(c) greater than T (d) infinite **(AIEEE 2002)**

4. If three charges are placed at the vertices of equilateral triangle of charge 'q' each. What is the net potential energy, if the side of equilateral triangle is l cm?

(a) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{l}$ (b) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{2q^2}{l}$

(c) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{3q^2}{l}$ (d) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{4q^2}{l}$ **(AIEEE 2002)**

5. Three charges $-q_1$, $+q_2$, and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to



(a) $\dfrac{q_2}{b^2} - \dfrac{q_3}{a^2}\cos\theta$ (b) $\dfrac{q_2}{b^2} + \dfrac{q_3}{a^2}\sin\theta$

(c) $\dfrac{q_2}{b^2} + \dfrac{q_3}{a^2}\cos\theta$ (d) $\dfrac{q_2}{b^2} - \dfrac{q_3}{a^2}\sin\theta$

(AIEEE 2003)

6. If the electric flux entering and leaving an enclosed surface, respectively, are ϕ_1 and ϕ_2, the electric charge inside the surface will be

(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $(\phi_1 + \phi_2)/\varepsilon_0$
(c) $(\phi_2 - \phi_1)/\varepsilon_0$ (d) $(\phi_1 + \phi_2)\varepsilon_0$ **(AIEEE 2003)**

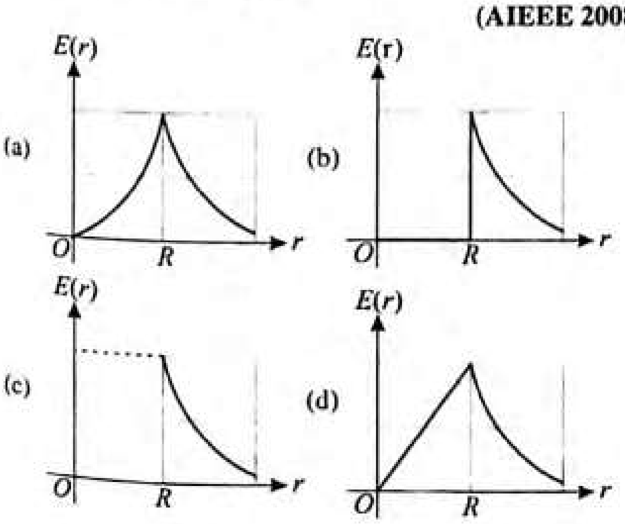
7. Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F when kept apart at some distance. A third spherical conductor, having the same radius as that of B but uncharged, is brought in contact with B and then in contact with C and finally removed away from both. The new force of repulsion between B and C is
(a) $F/4$ (b) $3F/4$
(c) $F/8$ (d) $3F/8$ **(AIEEE 2004)**

8. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is placed at its centre. If the system is in equilibrium, the value of q is
(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$
(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$
(AIEEE 2004)

9. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $= L$, respectively. The location of a point on the x-axis at which the net electric field due to these two point charges is zero is
(a) $2L$ (b) $L/4$
(c) $8L$ (d) $4L$ **(AIEEE 2005)**

10. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience
(a) a translational force only in the direction of the field
(b) a translational force only in a direction normal to the direction of the field
(c) a torque as well as translational force
(d) a torque only **(AIEEE 2006)**

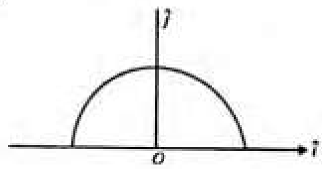
11. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < \infty$, where r is the distance from the center of the shell?
(AIEEE 2008)



12. Let $P(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point p inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is
(a) 0 (b) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$
(c) $\frac{Q r_1^2}{4\pi\varepsilon_0 R^4}$ (d) $\frac{Q r_1^2}{3\pi\varepsilon_0 R^4}$ **(AIEEE 2009)**

13. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then the Q/q equals
(a) $-2\sqrt{2}$ (b) -1
(c) 1 (d) $-\frac{1}{\sqrt{2}}$ **(AIEEE 2009)**

14. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field E at the centre O is



(a) $\frac{q}{4\pi^2\varepsilon_0 r^2}j$ (b) $-\frac{q}{4\pi^2\varepsilon_0 r^2}j$
(c) $-\frac{q}{2\pi^2\varepsilon_0 r^2}j$ (d) $\frac{q}{2\pi^2\varepsilon_0 r^2}j$ **(AIEEE 2010)**

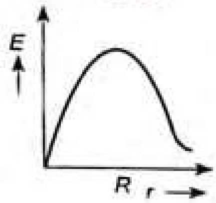
15. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ up to $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance r $(r < R)$ from the origin is given by
(a) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (b) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$
(c) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$
(AIEEE 2010)

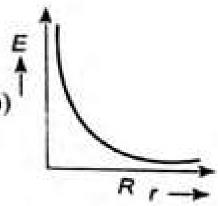
16. Two identically charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g/cm^3, the angle remains the same. If the density of the material of the sphere is 16 g/cm^3, the dielectric constant of the liquid is
(a) 4 (b) 3
(c) 2 (d) 1 **(AIEEE 2010)**

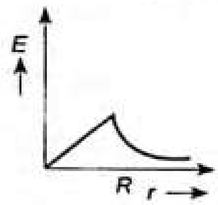
17. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart becuase of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

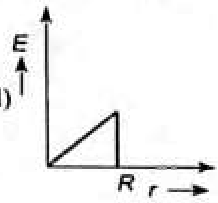
(a) $v \propto x^{-1/2}$ (b) $v \propto x^{-1}$
(c) $v \propto x^{1/2}$ (d) $v \propto x$ **(AIEEE 2011)**

18. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be

(a)


(b)


(c)


(d)


(AIEEE 2012)

19. This questions has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describe the two statements.

An insulating solid sphere of radius R has a unioformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinite is zero.

Statement 1: When a charge 'q' is take from the centre of the surface of the sphere its potential energy changes by $q\rho/3\varepsilon_0$

Statement 2: The electric field at a distance r $(r < R)$ from the centre of the sphere is $\rho r/3\varepsilon_0$

(a) Statement 1 is true, statement 2 is true; statement 2 is not the correct explanation of statement 1.
(b) Statement 1 is true statement 2 is false.
(c) Statement 1 is false statement 2 is true.
(d) Statement 1 is true, statement 2 is true, statement 2 is the correct explanation of statement 1.

(AIEEE 2012)

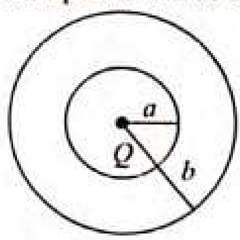
20. Two charges, each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given a small displacement $(y << a)$ along the y-axis, the net force acting on the particle is proportional to

(a) $-y$ (b) $\frac{1}{y}$
(c) $-\frac{1}{y}$ (d) y **(JEE Main 2013)**

21. Let $[\varepsilon_0]$ denote the dimensional formula of the permitivity of vacuum. If M = mass, L = length, T = time and A = electric current, then:

(a) $[\varepsilon_0] = [M^{-1}L^{-3}T^{4}A^{2}]$
(b) $[\varepsilon_0] = [M^{-1}L^{2}T^{1}A^{-2}]$
(c) $[\varepsilon_0] = [M^{-1}L^{2}T^{1}A]$
(d) $[\varepsilon_0] = [M^{-1}L^{-3}T^{2}A]$ **(JEE Main 2013)**

22. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is



(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$
(c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$ **(JEE Main 2016)**

23. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is

(a) 60° (b) 90°
(c) 30° (d) 45° **(JEE Main 2017)**

ANSWER KEY

Exercises

1. (b)	2. (b)	3. (a)	4. (a)	5. (c)	6. (c)	7. (c)	8. (c)	9. (c)	10. (b)
11. (c)	12. (b)	13. (a)	14. (c)	15. (d)	16. (d)	17. (b)	18. (d)	19. (a)	20. (b)
21. (b)	22. (d)	23. (c)	24. (c)	25. (a)	26. (b)	27. (d)	28. (b)	29. (d)	30. (c)
31. (b)	32. (c)	33. (c)	34. (c)	35. (c)	36. (d)	37. (d)	38. (a)	39. (b)	40. (c)
41. (b)	42. (b)	43. (a)	44. (b)	45. (b)	46. (b)	47. (c)	48. (c)	49. (c)	50. (c)
51. (c)	52. (a)	53. (a)	54. (c)	55. (c)	56. (c)	57. (b)	58. (d)	59. (a)	60. (c)
61. (a)	62. (c)	63. (b)	64. (a)	65. (a)	66. (c)	67. (d)	68. (a)	69. (a)	70. (d)
71. (c)	72. (b)	73. (b)	74. (c)	75. (b)	76. (a)	77. (c)	78. (b)	79. (b)	80. (c)

Archives

1. (None)	2. (d)	3. (b)	4. (c)	5. (b)	6. (a)	7. (d)	8. (b)	9. (a)	10. (c)
11. (c)	12. (c)	13. (a)	14. (c)	15. (b)	16. (c)	17. (a)	18. (c)	19. (c)	20. (d)
21. (a)	22. (a)	23. (a)							

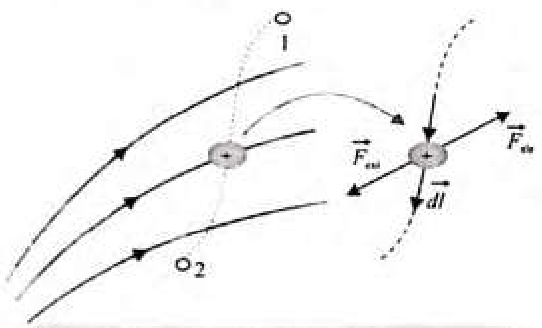
Chapter 18

Electric Potential and Capacitance

WORK DONE TO MOVE A CHARGE IN AN ELECTROSTATIC FIELD

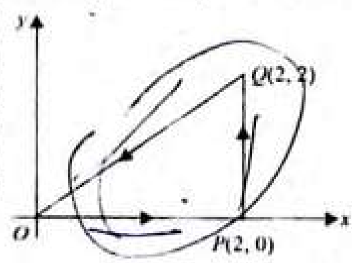
Let us consider an arbitrary electric field due to any charged object. If we move a test charge $+q$ from position 1 to position 2 (see figure) in this electrostatic field, at each position of the test charge, it will experience an electrostatic force $\vec{F}_{el} = q\vec{E}$. To move the test charge slowly, we must pull it against the electric force (field) with a force $\vec{F}_{ext} = -q\vec{E}$, opposite to the electric field $\vec{E}$. The work done by the external agent in shifting the test charge along the dashed line from 1 to 2 is

$$W_{ext} = \int_1^2 \vec{F}_{ext}\,\vec{dl} = \int_1^2 (-q\vec{E})\cdot\vec{dl} = -q\int_1^2 \vec{E}\cdot\vec{dl}$$



The external agent does a work $W = -q\int_1^2 \vec{E}\cdot\vec{dl}$ in transporting the test charge q slowly from position 1 to position 2 in the static electric field E.

ILLUSTRATION 18.1 A charge particle $q = -10$ μC is carried along OP and PQ and then back to O along QO as shown in figure, in an electric field $\vec{E} = (x + 2y)\hat{i} + 2x\hat{j}$. Find the work done by an external agent in (i) each path and (ii) the round trip.



Solution.

i. $W = -q\int \vec{E}\cdot d\vec{\ell} = -q(\int E_x dx + \int E_y dy)$

In Path $O \to P$: Since the displacement is along the x-axis for the path OP,

$$W_{O\to P} = -q\int E_x dx = -(-10\times10^{-6})\int_0^2 (x+2y)dx,$$

where $y = 0$

or $\quad W_{O\to P} = +20\times10^{-6}$ J

In Path $P \to Q$: Since the displacement is along y-axis for path PQ,

$$W_{P\to Q} = -q\int E_y d_y = -q\left(2\int_0^2 x dy\right) \text{ where } x = 2$$

$$= -2(-10\times10^{-6})\int_0^2 2dy = 80\times10^{-6}\text{ J}$$

In Path $Q \to O$: Since the displacement $d\ell = dx\hat{i} + dy\hat{j}$ along QO,

$$W_{Q\to O} = -q\int(E_x dx + E_y d_y)$$

$$= -q\int_2^0 (x+2y)dx + 2xdy$$

$$= -q\left[\int_2^0 (x+2y)dx + 2\int_2^0 xdy\right]$$

where the equation of the path QO is given as $y = x$

or $\quad W_{Q\to O} = -q\left[\int_2^0 (x+2x)dx + 2\int_2^0 ydy\right]$

$$= -q\left(\frac{3x^2}{2}\Big|_2^0 + y^2\Big|_2^0\right)$$

$$= -(-10\times10^{-6})\left\{\frac{-3\times4}{2} - 4\right\} = -100\times10^{-6}\text{ J}$$

ii. Then, the total work done in the round trip is

$$W = W_{O\to P} + W_{P\to Q} + W_{Q\to O} = 0$$

In the above example, $\oint \vec{E}\cdot d\vec{\ell} = 0$. Hence, the field is conservative and static.

POTENTIAL AND POTENTIAL DIFFERENCE

Potential

We learnt know that in slowly bringing a point charge from point 1 to point 2, the work done by the external agent is independent of the path followed by the charge, which can be given by

$$W_{ext} = q\int_{1}^{2} \vec{E}\cdot d\vec{l}$$

If we choose the initial point at infinity and the final point at P, the above expression will be

$$W_{ext} = -q\int_{\infty}^{P} \vec{E}\cdot d\vec{l}$$

The potential at any point P is given by

$$V = \frac{W_{ext}(\infty \to P)}{q} = -\int_{\infty}^{P} \vec{E}\cdot d\vec{l}$$

Then, the field expression for potential is

$$V_P = -\int_{\infty}^{P} \vec{E}\cdot d\vec{l}$$

The potential at any observation point P of a static electric field is defined as *the work done by the external agent (or negative of work done by the electrostatic field) in slowly bringing a unit positive point charge from infinity to the observation point.*

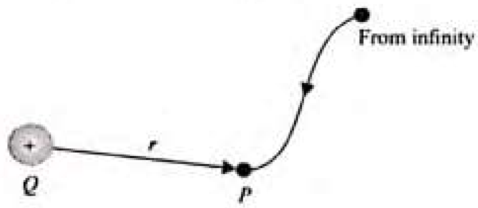
The potential at a point is more if the external agent does more work to shift the charge from infinity to the given point and vice versa. In this way, we define potential as the external work done by unit charge or roughly potential energy per unit charge.

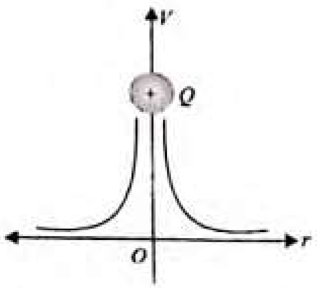
Potential Due to a Point Charge

The definition of a potential is given by the expression

$$V = \int_{\infty}^{P} \vec{E}\cdot d\vec{l} = \int_{\infty}^{P} E\cdot dl\cos\theta$$

$$= \int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2}(-dr) = \frac{Q}{4\pi\varepsilon_0 r}$$





If $r \to 0$, $V \to \infty$.
If $r \to \infty$, $V \to 0$.

Hence, V varies hyperbolically. If Q is positive, V is positive and vice versa.

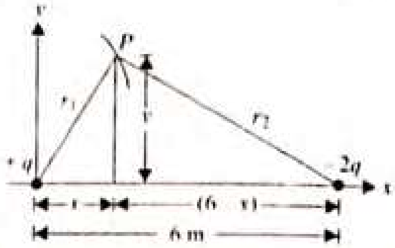
ILLUSTRATION 18.2 The electric field in a region is given by $\vec{E} = \frac{A}{x^3}\hat{i}$. Write an expression for the potential in the region assuming the potential at infinity to be zero.

Solution. As $E = A/x^3$, potential in the region is

$$V = -\int_{\infty}^{x} \vec{E}\cdot d\vec{x} = -\int_{\infty}^{x}\left(\frac{A}{x^3}\hat{i}\right)\cdot(dx\hat{i})$$

$$= -A\int_{\infty}^{x} x^{-3}dx = -A\left[\frac{x^{-2}}{-2}\right]_{\infty}^{x} = \frac{A}{2x^2}$$

ILLUSTRATION 18.3 Two electric charges q and $-2q$ are placed at a distance 6 m apart on a horizontal plane. Find the locus of point on this plane where the potential has a value zero.



Solution. If point P is at a distance r_1 from point charge $+q$ and r_2 from charge $-2q$ (see figure), then

$$V_P = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{+q}{r_1} + \frac{-2q}{r_2}\right]$$

According to the problem, $V_P = 0$, i.e.,

$$\frac{q}{r_1} - \frac{2q}{r_2} = 0$$

or $\quad r_2 = 2r_1 \quad$ (i)

But if the charge q is assumed to be situated at the origin, then

$$r_1^2 = x^2 + y^2 \text{ and } r_2^2 = (6-x)^2 + y^2$$

So substituting these values of r_1 and r_2 in Eq. (i), we get

$$(x+2)^2 + y^2 = 16$$

So the locus of the point P is a circle with radius 4 m and center (−2, 0).

Potential Difference

The difference in potentials between any two points 1 and 2 can be defined as the work done by an external agent in slowly shifting a unit positive charge between these points (from 1 to 2).

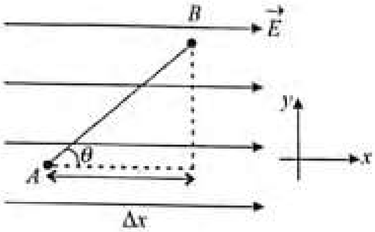
$$\frac{W_{ext(1\to 2)}}{q} = V_2 - V_1$$

where $\quad W_{ext(1\to 2)} = -\int_{1}^{2} \vec{E}\cdot d\vec{l}$

or $\quad V_2 - V_1 (= \Delta V) = -\int_{1}^{2} \vec{E}\cdot d\vec{l}$

If $W_{ext} > 0$, $V_2 > V_1$; point 2 is at higher potential than point 1.
If $W_{ext} = 0$, $V_2 = V_1$; point 2 is at same potential as at point 1.
If $W_{ext} < 0$, $V_2 < V_1$; point 2 is at lesser potential than point 1.

Potential Difference in a Uniform Electric Field



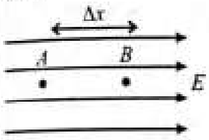
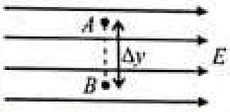
$$V_B - V_A = -\vec{E}\cdot\vec{AB}$$

$\Rightarrow$ $= |E|\,|AB|\cos\theta$

$= -|E|\Delta x = -E\Delta x$

where Δx is the distance between A and B along electric field.

We can also say that $|E| = \Delta V/\Delta x$.

Special cases	
CASE I	CASE II
Line AB is parallel to electric field.  $V_A - V_B = E\Delta x$ and $V_B - V_A = -E\Delta x$	Line AB is perpendicular to electric field.  $V_A - V_B = 0$ or $V_A = V_B$

Important Points

- In the direction of electric field, potential always decreases.
- The difference of potential between two points is called potential difference. It is also called voltage.
- Potential difference is a scalar quantity. Its S.I. unit is also volt.
- If V_A and V_B are the potential of two points A and B, then work done by an external agent in taking the charge q from A to B is

$$(W_{ext})_{AB} = \Delta U_{AB} = (U_B - U_A) = q(V_B - V_A)$$

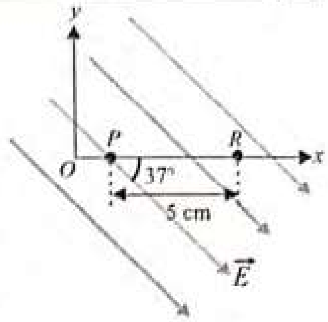
or $\quad (W_{el})_{AB} = q(V_A - V_B) \qquad$ (i)

- Potential difference between two points is independent of reference point.
- Potential at a point is in fact potential difference between the potential at the point and infinity.
- There is another way to interpret Eq. (i). Let W'_{AB} be the work done by the electric force in displacing charge q_0 from A to B. Since external force is equal and opposite to the electrostatic force, $W_{AB} = -W'_{AB} = W'_{BA}$. Thus,

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{W'_{BA}}{q_0}$$

- In uniform electric field $\Delta V = V_2 - V_1 = -\vec{E}\cdot\Delta\vec{x}$

ILLUSTRATION 18.4 A uniform field of magnitude $\vec{E} = 2000\ \text{NC}^{-1}$ is directed $\theta = 37°$ below the horizontal (see figure).



i. Find the potential difference between P and R.

ii. If we define the reference level of potential so that potential at R is $V_R = 500$ V, what is the potential at P?

Solution.

i. Here $E = 2000\ \text{NC}^{-1}$, $\theta = 37°$, $V_R = 500$ V

$$\Delta\ell = PR = 5\text{ cm} = 5\times10^{-2}\text{ m}$$

$$V_R - V_P = \vec{E}\cdot\Delta\vec{l} = -Ed\ell\cos\theta$$

$$= -(2000\ \text{NC}^{-1})(5\times10^{-2}\text{ m})\cos 37°$$

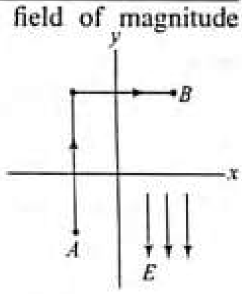
$$V_R - V_P = -80\text{ V}$$

Thus, $V_P - V_R = \vec{E}\cdot\Delta\vec{l} = 80$ V

ii. As $V_P - V_R = 80$ V and $V_R = 500$ V,

$$V_P = 500\text{ V} + 80\text{ V} = 580\text{ V}$$

ILLUSTRATION 18.5 A uniform electric field of magnitude $325\ \text{Vm}^{-1}$ is directed in the negative y-direction in figure. The coordinates of point A are (–0.2 m, –0.3 m) and those of point B are (0.4 m, 0.5 m). Calculate the potential difference $V_B - V_A$ along the path shown in the figure.



Solution. We can define the potential difference between the points A and B.

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$$

$$= -\int_A^B (E_x\hat{i} + E_y\hat{j} + E_z\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

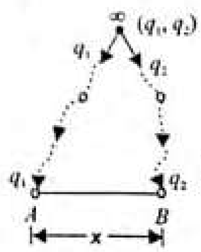
$$= -\left[E_x\int_{x_A}^{x_B} dx + E_y\int_{y_A}^{y_B} dy + E_z\int_{z_A}^{z_B} dz\right]$$

$$= -[E_x(x_B - x_A) + E_y(y_B - y_A) + E_z(z_B - z_A)]$$

$$= -[(-325)^5\{(0.5)-(-0.3)^3\}] = 325\times0.8 = 260\text{ V}$$

$$V_B - V_A = +260\text{ V}$$

Energy of Two Charges



- As initially there is no charge (i.e., electric field is zero), work done in bringing q_1 from ∞ to q_1 (A) is $W_1 = 0$.
- Potential at B due to charge q_1 at A is

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{x}$$

- Work done in bringing q_2 from infinity to B is

$$W_2 = q_2(V_B - 0)$$

$$= q_2\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x}$$

Total work in assembling q_1 and q_2 at A and B from infinity is

$$W = W_1 + W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x}$$

This work done becomes the potential energy (U) of the system of two charges. Thus,

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{x} \quad \text{(i)}$$

NOTE: Eq. (i) is true for any sing of q_1 and q_2. If both q_1 and q_2 are either positive or negative (i.e., $q_1q_2 > 0$), U is positive. This implies that the system is free (as the electrostatic force between them is repulsive) and a positive amount of work is required to be done against this force to bring the charges from infinity to their present locations. But if q_1 and q_2 are of opposite sings (i.e., $q_1q_2 < 0$), U is negative. This means that the system is bound (as the electrostatic force between them is attractive).

NOTE: In case of discrete distribution charges,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \cdots\right] = \frac{1}{2}\frac{1}{(4\pi\varepsilon_0)}\sum_{i \neq j}\frac{q_iq_j}{r_{ij}}$$

[½ is used as each term in summation will appear twice]

POTENTIAL ENERGY OF CHARGES IN AN EXTERNAL ELECTRIC FIELD

Potential Energy of Single Charge in an External Field

Let V be the potential at a point P due to anexternal field. Work done by an external agent in bringing the charge q from infinity to P is

$$W = q(V - 0) = qV$$

This work done becomes the potential energy (U) of the charge (q) when placed at a point P in the external field, i.e.,

$$U = qV \quad \text{(i)}$$

NOTE:

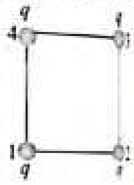
- Since $U = q_0V$, we may define the potential energy of charge q_0 at any point in the field to be equal to the amount of work done by external agent in bringing the charge q_0 from infinity to that point.
- In the discussion above, we have assumed that the charged particle, which is moving in equilibrium, will change its kinetic energy as it moves from point to point. Thus, if a particle of mass m and q_0 has potential energy U_A and kinetic energy K_A at point A and U_B and K_B at B, the conservation of mechanical energy requires (in the absence of work done by any external force acting of the charge)

$$U_A + K_B = U_B + K_B$$

Since $U_A = q_0V_A$ and $U_B = q_0V_B$, we may write

$$q_0(V_A - V_B) = K_B - K_A$$

ILLUSTRATION 18.6 Four identical point charges q are placed at four corners of a square of side a. Find the potential energy of the charge system



Solution.

Method I (using direct formula)

$$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

$$= \frac{kq^2}{a} + \frac{kq^2}{a\sqrt{2}} + \frac{kq^2}{a} + \frac{kq^2}{a} + \frac{kq^2}{a\sqrt{2}} + \frac{kq^2}{a}$$

$$= \left[\frac{4kq^2}{a} + \frac{2kq^2}{a\sqrt{2}}\right] = \frac{2kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{q^2}{2\pi\varepsilon_0 a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

Method II [using $U = (U_1 + U_2 + \cdots)$]

U_1 is the total potential energy of charge at corner 1 due to all other charges; U_2 is the total potential energy of charge at corner 2 due to all other charges; U_3 is the total potential energy of charge at corner 3 due to all other charges; U_4 is the total potential energy of charge at corner 4 due to all other charges.

Since due to symmetry,

$$U_1 = U_2 = U_3 = U_4$$

$$U_{\text{net}} = \frac{U_1 + U_2 + U_3 + U_4}{2}$$

$$= \frac{1}{2}\cdot 4\cdot\left[\frac{kq^2}{a} + \frac{kq^2}{a} + \frac{kq^2}{\sqrt{2}a}\right] = \frac{2kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$$

ILLUSTRATION 18.7 A proton moves with a speed u directly toward a free proton originally at rest. Find the distance of closest approach for the two protons. Given that mass of the proton is m and charge of the proton is $+e$.

Solution. Since the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion, the other particle will also start moving. So the velocity of the first particle will decrease while that of the other particle will increase, and at the closest approach both will move with the same velocity. So, if v is the common velocity of each particle at the closest approach, as no external forces are acting on them (the system of particles), the linear momentum of the system of particles will be conserved. By conservation of momentum, we get

$$mu = mv + mv,$$

i.e., $v = \frac{u}{2}$

And by conservation of energy, we have

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

So, $r = \dfrac{e^2}{\pi\varepsilon_0 mu^2}$ (as $v = u/2$)

EQUIPOTENTIAL SURFACE

For a given charge distribution, locus of all points or regions for which the electric potential has a constant value are called *equipotential regions*. Such equipotentials can be surfaces, volumes, or lines. The following points about equipotential surfaces should be noted.

- Lines of force always intersect an equipotential perpendicularly.
- No two equipotential surfaces can intersect each other. If they do so, there will be two values of electric potential at the point of intersection, which is not possible.
- A charged conductor of any shape is an equipotential surface. If it were not so, there would be a flow of charge from one end to another along the conductor.
- Equipotential surfaces are crowded together in a region of strong field, whereas they are relatively far apart where the field is weak. If there are two equipotential surfaces having a potential difference dV and lying at a distance dr from each other, then

$$E = -\frac{dV}{dr} \text{ or } dr = -\frac{dV}{E}$$

- If a charge is moved from one point to the other over an equipotential surface, work done will be zero as

$$W_{AB(\text{by us})} = U_{BA} = q(V_B - V_A) = 0 \qquad [\text{as } V_B = V_A]$$

- Work has to be done to move a charge from one equipotential surface to another.
- For fixed value of dV, $dr \propto 1/E$. Thus, dr (i.e., the spacing between two equipotential surface) decreases as E increases (i.e., in a stronger field). Thus, the intensity of the electric field is inversely proportional to the distance between the adjacent equipotential surfaces. This, in fact, means that the closer the equipotential surfaces, the greater is the field intensity at the given point, where the equipotential surfaces near the isolated charge are crowded and are closer to each other showing that the field is stronger nearer the charge. As we move away from the charge, this crowding of surface decreases denoting thereby a decrease in the intensity of the field.
- The charge resides on the outer surface of a conductor, and the electric field inside the conductor is zero. Therefore, the potential difference between any two points inside the conductor is zero.

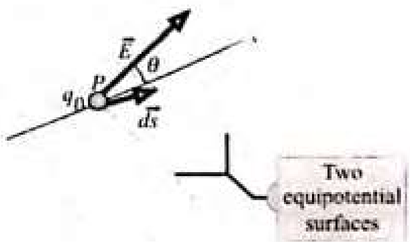
That is, all points in the inner region are at the same potential. We state this fact by saying that the interior region of any charged conductor is an equipotential volume. That is, no work will be done in moving charges in such a volume.

FINDING ELECTRIC FIELD FROM ELECTRIC POTENTIAL

We know in an electric field $\Delta U = -W$, where W is the work done by electric field. So

$$U_f - U_i = -\int_i^f q\vec{E}\cdot d\vec{s} \quad \text{or} \quad \frac{U_f - U_i}{q} = -\int_i^f \vec{E}\cdot d\vec{s}$$

or $$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$$



A test charge q_0 moves a distance ds from one equipotential surface to another. (The separation between the surfaces has been exaggerated for clarity.) The displacement ds makes an angle θ with the direction of the electric field E.

We can write

$$dV = -\vec{E}\cdot d\vec{s} = E\cos\theta \cdot ds \qquad \text{(i)}$$

$$E\cos\theta = -\frac{dV}{ds}$$

$E\cos\theta$ is the component of E in the direction of ds

$$E_s = -\frac{dV}{ds} \qquad \text{(ii)}$$

The y- and z- components of E are related to the corresponding derivatives of V in the same way, so we have

$$E_x = -\frac{\partial V}{\partial x};\ E_y = -\frac{\partial V}{\partial y};\ E_z = -\frac{\partial V}{\partial z}$$

(components of E in terms of V)

The equation states that the negative of the rate of change of potential with position in any direction is the component of E in that direction. The minus sign implies that E points in the direction of decreasing V.

The electric field intensity in uniform electric field is

$$|E| = \frac{\Delta V}{\Delta d}$$

where ΔV is the potential difference between two points. Δd is the effective distance between the two points (projection of the displacement along the direction of electric field).

ILLUSTRATION 18.8 The potential at any point is given by $V = x(y^2 - 4x^2)$. Calculate the Cartesian components of the electric field at the point.

Solution. Here $V = x(y^2 - 4x^2)$, so

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}[xy^2 - 4x^3] = -[y^2 - 12x^2]$$

or $\quad E_x = [12x^2 - y^2]$

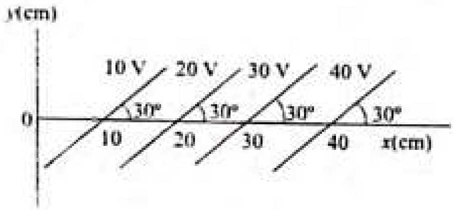
Similarly,

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}[x(y^2 - 4x^2)]$$

or $\quad = -\dfrac{\partial}{\partial y}[xy^2 - 4x^3] = -2xy$

$E_z = -0$ because V does not depend upon the z-coordinate.

ILLUSTRATION 18.9 Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field?



Solution. Here we can say that the electric field will be perpendicular to equipotential surfaces. Also

$$|\vec{E}| = \frac{\Delta V}{\Delta d}$$

where ΔV = potential difference between two equipotential surfaces.

Δd = perpendicular distance between two equipotential surfaces.

$$|\vec{E}| = \frac{10}{(10 \sin 30^\circ) \times 10^{-2}} = 200 \text{ Vm}^{-1}$$

Now there are two perpendicular directions (1 or 2) as shown in figure, but since we know that but since we know that electric potential decreases in the direction of electric field, so the correct direction is 2.

Hence, $E = 200 \text{ Vm}^{-1}$, making an angle 120° with the x-axis.

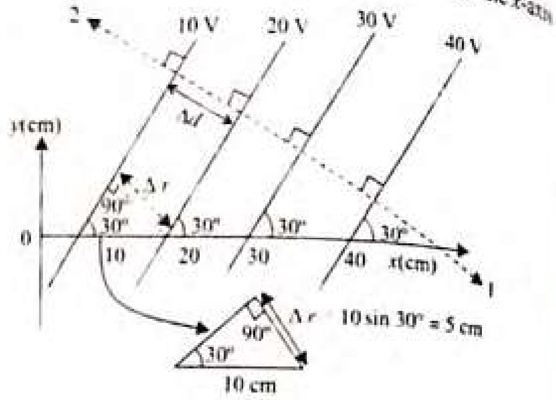
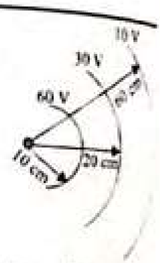


ILLUSTRATION 18.10 Referring to the spherical equipotential lines in figure, find i. $\vec{E} = f(r)$. ii. $\vec{E}$ -pattern.



Solution.

(i) For the first equipotential

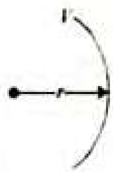
$$60(\text{V}) \times \frac{10}{100}(\text{m}) = 6 \text{ Vm}$$

For the second equipotential

$$30(\text{V}) \times \frac{20}{100}(\text{m}) = 6 \text{ Vm}$$

For the third equipotential

$$10(\text{V}) \times \frac{60}{100}(\text{m}) = 6 \text{ Vm}$$



We can understand that the product of potential (V) and radial distance (r) of equipotential lines [as in figure] is equal to 6 Vm^{-1}. Hence, general relation of potential (V) and radial distance (r) can be written as

$$V = \frac{6}{r} \text{Vm}^{-1}$$

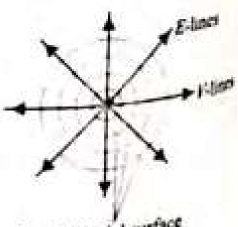
Since $\vec{E} = -(\partial V)/(\partial r)\hat{r}$, substituting $V = 6/r$, we have

$$\vec{E} = -\frac{\partial}{\partial r}\left(-\frac{6}{r}\right)\hat{r} \text{ Vm}^{-1}$$

or $\quad \vec{E} = \dfrac{6}{r^2}\hat{r} \text{ Vm}^{-1}$

(ii) Since $\vec{E}$ is directed in $\hat{r}$-direction and obeys inverse square law, E must be outward [as shown in figure].

The above E-field must be caused by a positive point charge.

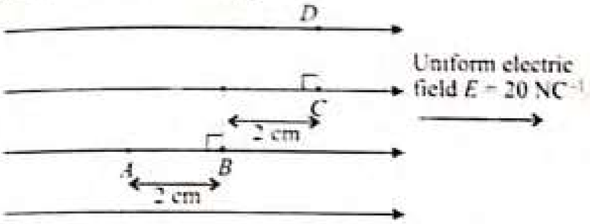


Concept Application Exercise 18.1

1. When charge 10 μC is shifted from infinity to a point 'P' in an electric field, it is found that work done by electrostatic forces is 10 μJ. If the charge is doubled and taken again from infinity to the same point without accelerating it, then find the amount of work done by electric field and against electric field.

2. A charge 3 μC is released at rest from a point P where electric potential is 20 V. Find its kinetic energy when it reaches infinity.

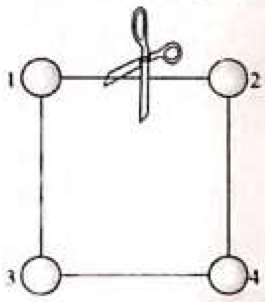
3. Find out the following:



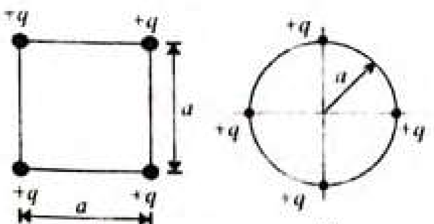
(a) $V_A - V_B$ (b) $V_B - V_C$
(c) $V_C - V_A$ (d) $V_D - V_C$
(e) $V_A - V_D$

(f) Arrange the order of potential for points A, B, C, and D.

4. Four balls, each with mass m, are connected by four nonconducting strings to form a square with side a, as shown in figure. The assembly is placed on a horizontal frictionless nonconducting surface. Balls 1 and 2 each have charge q, and balls 3 and 4 are uncharged. Find the maximum speed of balls 1 and 2 after the string connecting them is cut.



5. Consider the configuration of a system of four charges each of value $+q$. Find the work done by external agent in changing the configuration of the system from Figure (a) to Figure (b).



(a) (b)

6. The electric potential in a region is represented as $V = 2x + 3y - z$. Obtain expression for electric field strength.

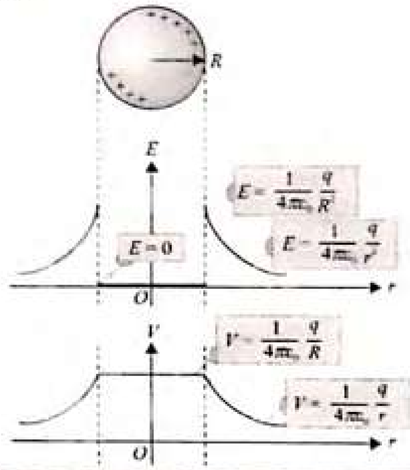
ELECTRIC POTENTIAL OF SOME CONTINUOUS CHARGE DISTRIBUTIONS

Charged Conducting Sphere

A solid conducting sphere of radius R has a total charge q. At all points outside the sphere the field is the same at equal distance from center of the sphere, as if the sphere were removed and replaced by a point charge q. We take $V = 0$ at infinity, as we did for a point charge.

Then the potential at a point outside the sphere at a distance r from its center is the same as the potential due to a point charge q at the center. We have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



Electric field magnitude E and potential V at points inside and outside a positively charged spherical conductor.

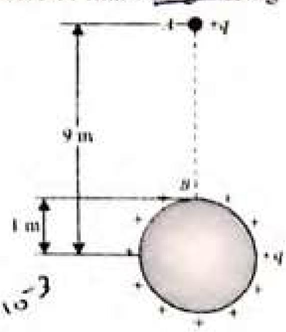
The potential at the surface of the sphere is

$$V_{surface} = \frac{q}{4\pi\varepsilon_0 R}$$

> Inside the sphere, $\vec{E}$ is zero everywhere; otherwise, charge would move within the sphere. Hence, if a test charge moves from any point to any other point inside the sphere, no work is done on that charge. This means that the potential is the same at every point inside the sphere and is equal to its value $q/4\pi\varepsilon_0 R$ at the surface.

NOTE: The variation of electric field and potential for conducting shell is same as conducting sphere.

ILLUSTRATION 18.11 A very small sphere of mass 80 g having a charge q is held at a height of 9 m vertically above the center of a fixed conducting sphere of radius 1 m, carrying an equal charge q. When released, it falls until it is repelled back just before it comes in contact with the sphere as shown in figure. Calculate the charge q. [$g = 10\ ms^{-2}$]



$\frac{kq^2}{9}$

$m: 80 \times 10^{-3}$
$h: 9m$
$R: 1m$

Solution. Here both electric and gravitational potential energies are changing, and for an external point, a charged sphere behaves as if whole of its charge were concentrated at its center. Applying conservation of energy between initial and final positions, we have

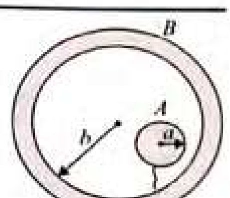
$$\frac{1}{4\pi\varepsilon_0}\frac{qq}{9}+mg\times 9=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{1}+mg\times 1$$

(as kinetic energy is zero at both locations)

or $$q^2=\frac{80\times10^{-3}\times10}{10^9}$$

or $$q=20\sqrt{2}\,\mu C$$

ILLUSTRATION 18.12 A metal sphere A of radius a is charged to potential V. What will be its potential if it is enclosed by a spherical conducting shell B of radius b and the two are connected by a wire?



Solution. If the charge on the sphere of radius a is q, then

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

i.e., $q=(4\pi\varepsilon_0 a)V$

Now, when sphere A is enclosed by spherical conductor B and the two are connected by a wire, charge will reside on the outer surface of B and so the potential of B will be

$$V_B=\frac{1}{4\pi\varepsilon_0}\frac{q}{b}=\frac{1}{4\pi\varepsilon_0}\frac{4\pi\varepsilon_0 a}{b}V=\frac{a}{b}V$$

Now as sphere A is inside B, so its potential is

$$V_A=V_B=\frac{a}{b}(V) \qquad [V \text{ as } a<b]$$

NONCONDUCTING SOLID SPHERE

Outside the Sphere

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

i.e., at the surface, potential is

$$V_S=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

Inside the Sphere

Inside the sphere,

$$E_{\text{inside}}=\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r$$

or $$\frac{dV_{\text{inside}}}{dr}=-E_{\text{inside}}$$

or $$dV_{\text{inside}}=-E_{\text{inside}}\,dr$$

or $$\int_{V_S}^{V} dV_{\text{inside}}=-\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\int_R^r r\,dr$$

or $$V-V_S=-\frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}\left[\frac{r^2}{2}\right]_R^r$$

Substituting

$$V_S=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

We get

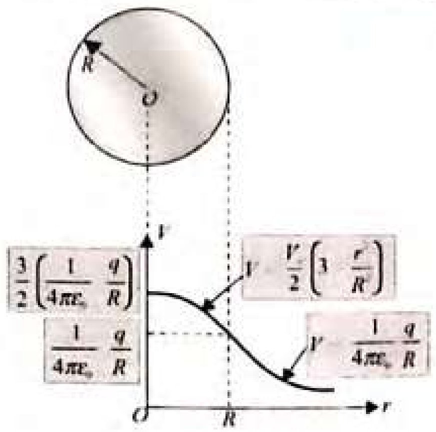
$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\left[\frac{3}{2}-\frac{1}{2}\frac{r^2}{R^2}\right]=\frac{V_S}{2}\left[3-\frac{r^2}{R^2}\right]=\frac{q}{8\pi\varepsilon_0 R}\left[3-\frac{r^2}{R^2}\right]$$

At the center $r=0$ and

$$V_C=\frac{3}{2}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R}\right)=\frac{3}{2}V_S$$

That is, potential at the center is 1.5 times the potential at surface. Thus, for a uniformly charged nonconducting sphere, we have the following formula for potential:

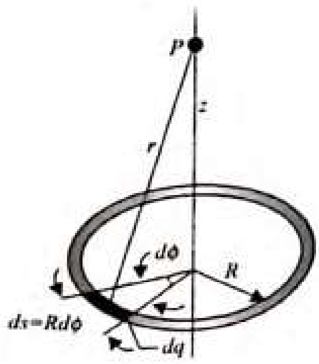
$$V_{\text{outside}}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r};\ V_{\text{surface}}=\frac{1}{4\pi\varepsilon_0}\frac{q}{R};\ V_{\text{inside}}=\frac{q}{8\pi\varepsilon_0 R}\left[3-\frac{r^2}{R^2}\right]$$



A RING OF CHARGE

Figure shows a uniform ring of positive charge. The contribution to the potential at point P on its axis due to the charge element $dq=\lambda\,ds=\lambda R\,d\phi$ is

$$dV=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r}=\frac{1}{4\pi\varepsilon_0}\frac{\lambda R\,d\phi}{\sqrt{R^2+z^2}}$$

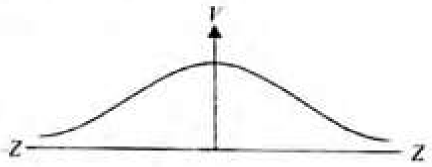


Integrating around the ring, we note that R and z both remain constant. The variable of integration is ϕ, which ranges from 0 to 2π. Thus,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{\sqrt{R^2+z^2}}\int_0^{2\pi} d\phi = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\lambda R}{\sqrt{R^2+z^2}}$$

Total charge on the ring is $q = 2\pi\lambda R$. Distance of point under consideration from the ring is $r = \sqrt{R^2+z^2}$.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



where r is the distance from ring to point under consideration.

At $z = 0$,

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$: maximum potential is at the center of ring

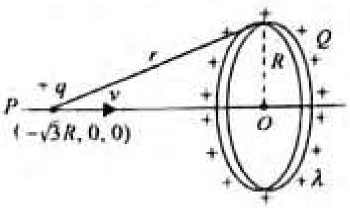
and at $z \to \infty$, $V = 0$.

ILLUSTRATION 18.13 A circular ring of radius R with uniform positive charge density λ per unit length is located in the yz plane with its center at the origin O. A particle of mass m and positive charge q is projected from the point $p(-\sqrt{3}R, 0, 0)$ on the negative x-axis directly toward O, with initial speed v. Find the smallest (nonzero) value of the speed such that the particle does not return to P?

Solution. As the electric field at the center of a ring is zero, the particle will not come back due to repulsion if it crosses the center (figure), i.e.,

$$\frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} > \frac{1}{4\pi\varepsilon_0}\frac{qQ}{R}$$

But here, $Q = 2\pi R\lambda$ and $r = \sqrt{\left(\sqrt{3}R\right)^2 + R^2} = 2R$



So $\frac{1}{2}mv^2 > \frac{1}{4\pi\varepsilon_0}\frac{2\pi R\lambda q}{R}\left[1-\frac{1}{2}\right]$ or $v > \sqrt{\frac{\lambda q}{2\varepsilon_0 m}}$.

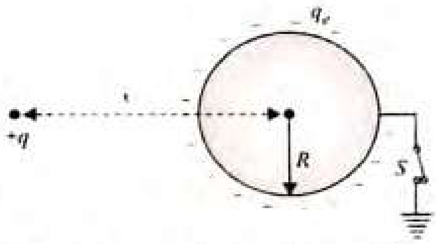
Therefore, $v_{min} = \sqrt{\frac{\lambda q}{2\varepsilon_0 m}}$

EARTHING OF A CONDUCTOR

Earthing means connecting a conductor with earth. Earth is an infinite resource and a sink of charges, so the potential of earth does not change and it is assumed to be zero. After earthing, the charges on conductors vary, and so the potential of the conductor becomes zero.

Consider a solid uncharged conducting sphere shown in figure. A point charge q is placed at a distance x from the center of the sphere. Here due to q, the potential on the sphere is

$$V = \frac{kq}{x}$$

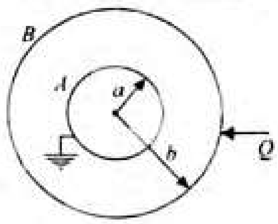


The charge is induced on the sphere due to the point charge q, but the potential at the center due to the induced charges on the sphere is zero. If we close the switch S, earth supplies a charge q_e to the sphere to make the net potential zero. Thus, the final potential on the sphere can be taken as

$$V = \frac{kq}{x} + \frac{kq_e}{R} = 0 \quad \text{or} \quad q_e = -\frac{qR}{x}$$

Earth supplied a negative charge to nullify positive potential on it due to q.

ILLUSTRATION 18.14 Consider two concentric spherical metal shells of radii a and b, where $b > a$. The outer shell has charge Q, but the inner shell has no charge. Now, the inner shell is grounded. This means that the inner shell will come at zero potential and that electric field lines leave the outer shell and go to infinity, but other electric field lines leave the outer shell and end on the inner shell.

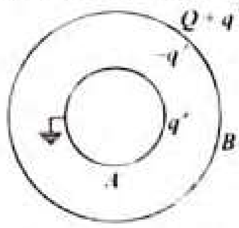


i. Find the charge on the inner shell.

ii. Find the potential of the outer sphere.

Solution.

i. When an object is connected to earth (grounded), its potential is reduced to zero. Let q' be the charge on A after it is earthed as shown in figure.



The charge q' on A induces $-q'$ on the inner surface of B and $+q'$ on the outer surface of B. In equilibrium, the charge distribution is as shown in figure.

Potential of inner sphere = Potential due to charge on

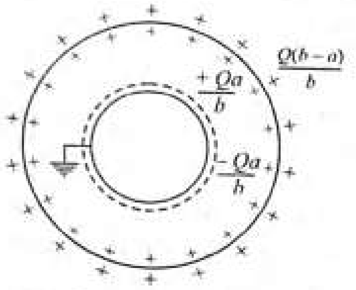
A + Potential due to charge on $B = 0$

$$V'_A = \frac{q'}{4\pi\varepsilon_0 a} - \frac{q'}{4\pi\varepsilon_0 b} + \frac{Q+q'}{4\pi\varepsilon_0 b} = 0$$

or $\quad q' = -Q\left(\frac{a}{b}\right)$

This implies that a charge $+Q(a/b)$ has been transferred to the earth leaving negative charge on A.

Final charge distribution will be as shown in figure.



As $b > a$, so charge on the outer surface of the outer shell will be positive $\left(\frac{Q(b-a)}{b} > 0\right)$.

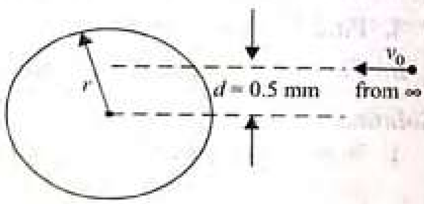
ii. Potential of outer surface V_B = Potential due to charge on A + Potential due to charge on B

$$V_B = V_{a,\text{out}} + V_{b,\text{both surface}} = \frac{1}{4\pi\varepsilon_0}\frac{q'}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

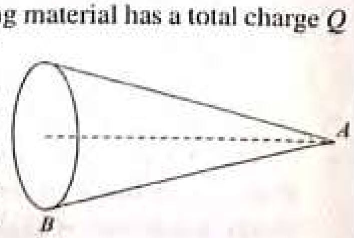
$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(-Q\frac{a}{b}\right)}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = \frac{Q(b-a)}{4\pi\varepsilon_0 b^2}$$

Concept Application Exercise 18.2

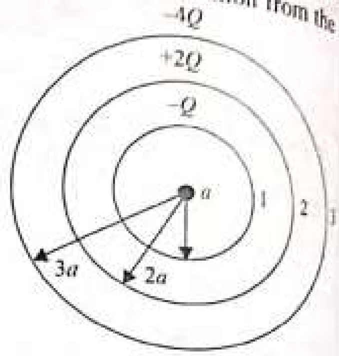
1. A particle of mass 1 kg and charge 1/3 μC is projected toward a non-conducing fixed spherical shell of radius r = 1 mm having the same charge uniformly distributed on its surface. Find the minimum initial velocity of projection required if the particle just grazes the shell.



2. A cone made of insulating material has a total charge Q spread uniformly over its sloping surface. Calculate the energy required to take a test charge q from infinity to apex A of cone. The slant length is L.



3. Three concentric spherical conductors of radii a, $2a$, and $3a$ have charges $-Q$, $+2Q$, and $-4Q$, respectively. If r is the distance of the point under consideration from the center of the spheres, then find the electric field and potential due to the given configuration, for the values
   (i) $r < a$
   (ii) $a < r < 2a$
   (iii) $2a < r < 3a$
   (iv) $r > 3a$

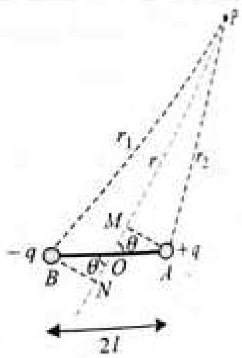


4. In the previous problem, if we connect the conductors 1 and 2, 2 and 3, 1 and 3, 1 and ground, 2 and ground, and 3 and ground, separately, find the charge flown in each case.

5. Electrically charged drops of mercury fall from an altitude h into a spherical metal vessel of radius R, in the upper part of which there is a small opening. The mass of each drop is m, and the charge on the drop is Q. What will be the number n of the last drop that can still enter the sphere?

POTENTIAL DUE TO AN ELECTRIC DIPOLE

Suppose an electric dipole consists of two charges $-q$ and $+q$ at B and A, respectively, separated by a distance $2l$ (as shown in figure). We have to calculate the electric potential at any point P where $OP = r$ and $\angle AOP = \theta$. Let $BP = r_1$ and $AP = r_2$.



Draw AM perpendicular to PO and BN perpendicular to PO produced backward, then $OM = ON$. In ΔOMA,

$$\frac{OM}{OA} = \cos\theta$$

$\therefore \quad OM = OA\cos\theta = \ell\cos\theta = ON$

Hence, $r_1 = BP \approx NP = OP + ON = r + l\cos\theta \quad$ (i)

and $\quad r_1 = AP \approx MP = OP - OM = r - l\cos\theta \quad$ (ii)

Therefore, potential at point P due to both the charges $-q$ (at B) and $+q$ (at A) can be written as

$$V = \frac{-q}{4\pi\varepsilon_0}\frac{1}{r_1} + \frac{q}{4\pi\varepsilon_0}\frac{1}{r_2} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - \ell\cos\theta} - \frac{1}{r + \ell\cos\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r + \ell\cos\theta - r + \ell\cos\theta}{r^2 - \ell^2\cos^2\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{2\ell\cos\theta}{r^2 - \ell^2\cos^2\theta}$$

i.e., $$= \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2 - \ell^2\cos^2\theta} \qquad \{\because \; p = q(2l)\}$$

Special cases

- If $l << r$, then electric dipole is very short and the potential is given by
  $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$
- If P lies on the axial line, we have $\theta = 0°$, so
  $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos 0°}{r^2 - \ell^2\cos^2 0°} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2 - \ell^2}$$
- ($\because \cos 0° = 1$)
- If $l << r$, then electric dipole is very short ; and
  $$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \text{ (for a point on axial line)}$$
- If P lies on the equatorial line, $\theta = 90°$, so that $V = 0$.

WORK DONE IN ROTATING AN ELECTRIC DIPOLE IN A UNIFORM ELECTRIC FIELD

Suppose an electric dipole of dipole moment $p(= q\ 2l)$ is rotated in a uniform electric field E through an angle θ from its stable equilibrium position. The work done by an external agent in rotating the dipole further from this position through a very small angle $d\theta$ is

$$dW = \text{couple} \times \text{angular displacement} = (pE\sin\theta)\,d\theta$$

This work will be done on the dipole by external agent as it is being rotated against its natural tendency (which is to align itself along the direction of the electric field). Hence, the work done in rotating the dipole through the angle θ from its equilibrium position is

$$W = \int_0^\theta pE\sin\theta\,d\theta = pE[-\cos\theta]_0^\theta$$

$$= pE[-\cos\theta + \cos 0] = pE[1 - \cos\theta]$$

$$\Rightarrow \quad W_{\text{external}} = pE(1 - \cos\theta)$$

POTENTIAL ENERGY OF AN ELECTRIC DIPOLE IN A UNIFORM ELECTRIC FIELD

Let us assume that V_+ and V_- are the potentials at the points where $+q$ and $-q$ are placed. The potential energy possessed by the dipole is equal to the work done by an external agent to bring the charges from infinity to the given points.

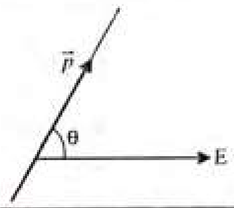
A dipole $\vec{p}$ in an electric field possesses potential energy U.

$$U = W_{ext} = +qV_+ + (-q)(V_-)$$
$$= q(V_+ - V_-) = q\Delta V$$
$$\Delta V = \vec{E}\cdot\Delta\vec{l} = \vec{E}\cdot\vec{l}$$
$$= q(-\vec{E}\cdot\vec{l}) = -q\vec{l}\cdot\vec{E}$$

where

$$q\vec{l} = \vec{p}$$
$$U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$$

where θ is the angle between $\vec{p}$ and $\vec{E}$.



θ is the angle between $\vec{p}$ and $\vec{E}$.

NOTE: When a dipole rotates from an initial orientation θ_i to another orientation θ_f, the work W done on the dipole by the electric field is

$$W = -\Delta U = -(U_f - U_i) = -pE(\cos\theta_i - \cos\theta_f)$$

where u_f and u_i are calculated with Eq. (iv). If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field, that is,

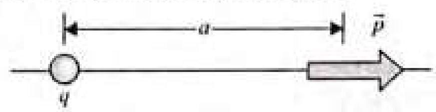
$$W_a = -W = (U_f - U_i) = -pE(\cos\theta_f - \cos\theta_i)$$

ILLUSTRATION 18.15 Two point charges of 3.2×10^{-19} C and -3.2×10^{-19} C are separated from each other by 2.4×10^{-10} m. The dipole is situated in a uniform electric field of intensity 4×10^5 Vm^{-1}. Calculate the work done in rotating the dipole by 180°.

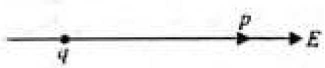
Solution. Work done in rotating the dipole by angle θ is

$$W = pE(1 - \cos\theta) \qquad (\text{Here } \theta = 180°)$$
$$= pE[1 - (-1)] = 2pE = 2qd\,E$$
$$= 2 \times 3.2 \times 10^{-19} \times 2.4 \times 10^{-10} \times 4 \times 10^5 \text{ J}$$
$$= 61.44 \times 10^{-24} \text{ J}$$

ILLUSTRATION 18.16 What is the potential energy of the charge and dipole system shown in figure?



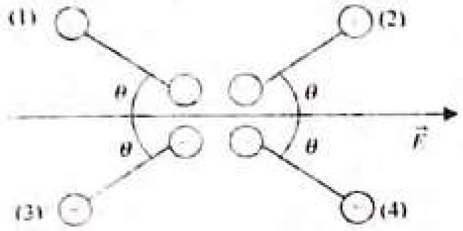
Solution. $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$ (figure)



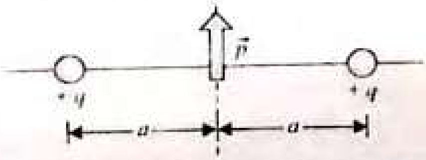
$$U = -pE\cos\theta = -\frac{pq}{4\pi\varepsilon_0 a^2} \qquad (\theta = 0°)$$

Concept Application Exercise 18.3

1. An electric dipole consists of two opposite charges each of magnitude 1 mC separated by 2 cm. The dipole is placed in an external uniform field of 10^5 NC^{-1} intensity. Find the
   (a) maximum torque exerted by the field on the dipole, and
   (b) work done in rotating the dipole through 180° starting from the position $\theta = 0°$.
2. Figure shows four orientations of an electric dipole in an external electric field. Rank the orientations according to the
   (a) magnitude of the torque on the dipole, and
   (b) potential energy of the dipole, greatest first.



3. (a) In question 3, if the dipole rotates from orientation 1 to orientation 2, is the work done on the dipole by the field positive, negative, or zero?
   (b) If, instead, the dipole rotates from the orientation 1 to orientation 4, is the work done by the field more than, less than, or the same as in (a)?
4. The potential energies associated with four orientations of an electric dipole in an electric field are (1) $-5U_0$, (2) $-7U_0$, (3) $3U_0$, and (4) $5U_0$, where U_0 is positive. Rank the orientations according to the
   (a) angle between the electric dipole moment and the electric field $\vec{E}$, and
   (b) magnitude of the torque on the electric dipole, greatest first.
5. What is the potential energy of dipole with charge particles system as shown in figure.



CAPACITANCE

Isolated Conductor

An electrical conductor (such as metals) holds its charge at its surface. When an isolated spherical conductor of radius R is given with a charge Q, its potential is given by

$$V = \frac{kQ}{R}$$

This tells us that the potential of an isolated spherical conductor is directly proportional to its charge. This hold good for an isolated conductor of any shape and size.

Since $V \propto Q$, the ratio of the charge given to an isolated conductor and its potential rise, that is, Q/V is a constant, which is defined as the capacitance of the conductor denoted by C. So $C = Q/V$.

If $V = 1$ V, $C = Q$. Hence, we define the capacitance of an isolated conductor as the charge required to rise the potential of the conductor by 1 V. In the SI system, the unit of capacitance is farad.

$$1(\text{F}) = \frac{1 \text{ coulomb (C)}}{1 \text{ volt (V)}} = 1 \text{ CV}^{-1}$$

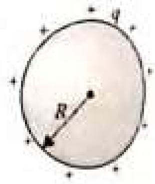
In practice, the following smaller units of capacitance are also used:

1 microfarad (μF) = 10^{-6} farad

1 micro microfarad (μμF) = 10^{-12} farad

1 μμF is also known as 1 picofarad (pF)

Capacitance of a Spherical Conductor or Capacitor



As we have already said that a single conductor can also act as a capacitor, here we will find the capacitance of a single isolated sphere. For this, let a charge q be given to a spherical conductor of radius R, then potential on it is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

The other conductor is supposed to be at infinity, whose potential will be taken as zero. So the potential difference between the sphere and the conductor at infinity becomes $V - 0 = V$. Then capacitance is

$$C = \frac{q}{V} = 4\pi\varepsilon_0 R$$

Thus, the capacitance of a spherical conductor is $C = 4\pi\varepsilon_0 R$.

NOTE: The earth is a spherical conductor of radius $R = 6.4 \times 10^6$ m. The capacitance of earth is

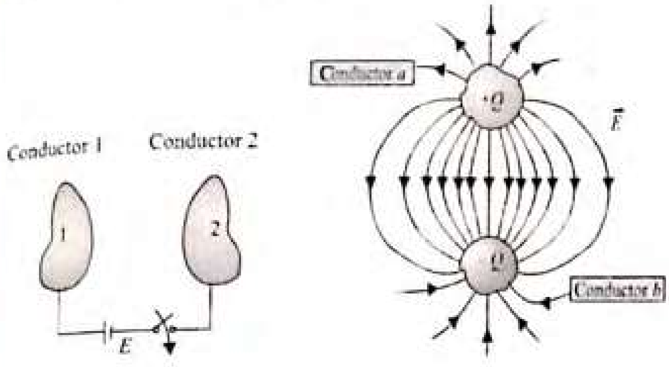
$$C = \left(\frac{1}{9 \times 10^9}\right)(6.4 \times 10^6) = 711 \times 10^{-6}\text{ F} = 711\ \mu\text{F}$$

Therefore, we can say that farad is a big unit. Thus, no body in the universe can have a capacitance of 1 F.

System of Conductors

The potential difference between the conductors is $V = V_+ - V_-$, where V_+ and V_- are the potentials of the positive

and the negative conductors, respectively. Since, $|V_+|$ and $|V_-|$ vary linearly with the charge Q, we have $V \propto Q$.



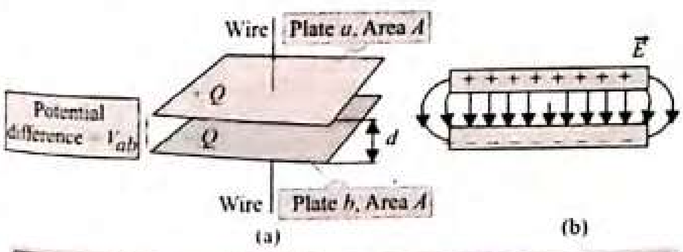
Then the ratio of Q and V, that is, Q/V is a constant, which can be defined as the capacitance of the system of two conductors. The ratio of the amount of charge supplied by the battery to each conductor and the potential difference between the conductors is defined as the capacitance of the system of two conductors. Thus,

$$C = \frac{Q}{V_+ - V_-} = \frac{\text{Charge flown through the battery}}{\text{Rise in potential difference}}$$

PARALLEL PLATE CAPACITOR

A parallel plate capacitor consists of two large plates placed parallel to each other with a separation d smaller in comparison to the length and breadth of the plates.

In an ideal capacitor, electric field resides in the region within the plates. No electric field is outside the plates (neglecting fringing effect for ideal case). So the entire energy resides within the capacitor and no energy is, therefore, outside the capacitor. Electric field is directed from the positive plate to the negative plate in such a way that the lines emerge perpendicularly from the positive plate and terminate perpendicularly to the negative plate.



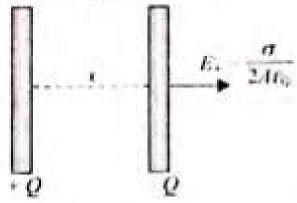
(a) A charged parallel plate capacitor.
(b) When the separation of the plates is small compared to their size, the fringing of the electric field $\vec{E}$ at the edges is slight.

Electric field between the plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Therefore, potential difference between the plates is

$$V = Ed = \frac{Qd}{A\varepsilon_0}$$



and capacitance is

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

FORCE BETWEEN THE PLATES OF A PARALLEL PLATE CAPACITOR

Consider a parallel plate capacitor with plate area A. Suppose a positive charge $+Q$ is given to one plate and a negative charge $-Q$ to the other plate. The electric field due to the positive plate is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

at all points if the plate is large. The negative charge $-Q$ on the other plate finds itself in the field of this positive charge. Therefore, the force on this plate is

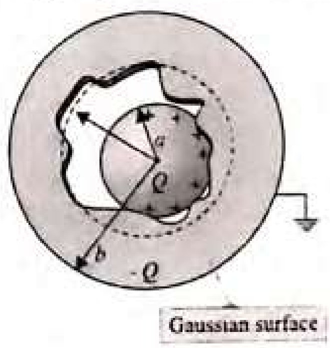
$$F = EQ = \frac{Q^2}{2A\varepsilon_0}.$$

This force will be attractive.

SPHERICAL CAPACITOR

A spherical capacitor consists of two concentric spherical conducing shells of radii a and b, say $b > a$. The outer shell is earthed. Place a charge $+Q$ on the inner shell. It will reside on the outer surface of the shell. A charge $-Q$ will be induced on the inner surface of the outer shell. A charge $+Q$ will flow from the outer shell to earth.

Consider a Gaussian spherical surface of radius r such that $a < r < b$. From Gauss's law, the electric field at distance $r > a$ is



$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

The potential difference is

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{r} = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr$$

Since $V_b = 0$, we have

$$V_a = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q(b-a)}{4\pi\varepsilon_0 ab}$$

Therefore, capacitance is

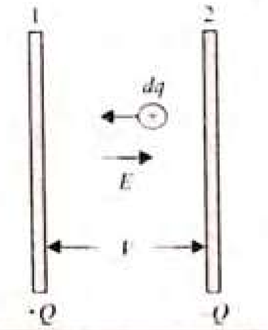
$$C = \frac{Q}{V_a - V_b} = \frac{Q}{V_a} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

ENERGY STORED IN A CHARGED CONDUCTOR OR CAPACITOR

The charging of a capacitor is associated with a continuous pumping of positive charge from conductor 2 to conductor 1 by the battery (see figure). If a charge $+dq$ is brought from conductor 2 to conductor 1 against the electric field E of the charged conductors, the work done by the battery is

$$dW = (Ed)\,dq = V dq,$$

where $V = \frac{q}{C} = \frac{q}{C}dq$



A work $dW = q/C\,dq$ is done in shifting the charge dq from plate 2 to plate 1

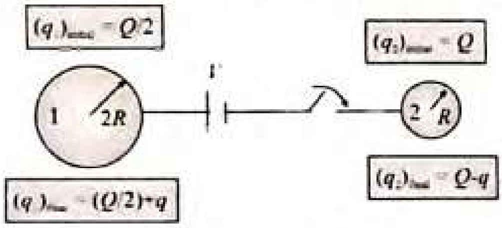
Then the total work done by the battery in sending a charge Q is

$$W = \int dW = \int \frac{q\,dq}{C} = \frac{Q}{C}$$

The work is done at the expense of the chemical energy of the battery, which is stored in the capacitor in the form of electrostatic field energy U. Then, $U = Q^2/2C$. Putting $Q = CV$, the other two expressions of energy can be written as

$U = QV/2 = CV^2/2.$

ILLUSTRATION 18.17 There are two spheres of radii R and $2R$ having charges Q and $Q/2$, respectively. These two spheres are connected with a cell of emf V volts as shown in figure. When the switch is closed, find the final charge on each sphere.



Solution. When the switch is closed, the potential difference between the spheres should be V. Let q charges flow from the sphere of radius R.

$$\frac{(q_1)_{final}}{C_1} - \frac{(q_2)_{final}}{C_2} = V$$

$$C_1 = 4\pi\varepsilon_0(2R),\ C_2 = 4\pi\varepsilon_0 R$$

Then

$$\frac{\left(\frac{Q}{2}+q\right)}{4\pi\varepsilon_0(2R)} - \frac{(Q-q)}{4\pi\varepsilon_0(R)} = V$$

or $$\frac{Q}{2} + q - 2Q + 2Q = 4\pi\varepsilon_0(2R)V$$

or $$q = \frac{8\pi\varepsilon_0 RV}{3} + \frac{Q}{2}$$

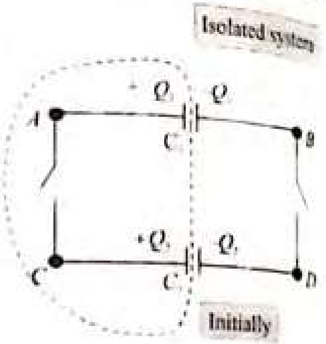
So the final charges on each of the spheres are

$$Q_1 = Q - q = \frac{-8\pi\varepsilon_0 RV}{3} + \frac{Q}{2}$$

and $$Q_2 = \frac{Q}{2} + q = Q + \frac{8\pi\varepsilon_0 RV}{3}$$

Distribution of Charges on Connecting Two Charged Capacitors

Two capacitors C_1 and C_2 are connected as shown in figure.



Common potential:

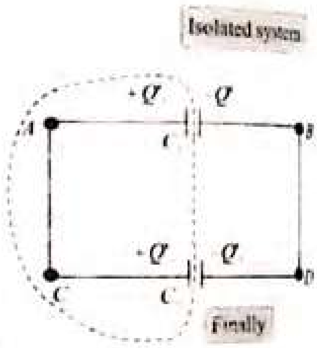
By charge conservation of plates A and C before and after connection,

$$Q_1 + Q_2 = C_1V + C_2V$$

So common potential is

$$V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1 + C_2V_2}{C_1+C_2}$$

$$= \frac{\text{Total charge}}{\text{Total capacitance}}$$



Also

$$Q_1' = C_1V = \frac{C_1}{C_1+C_2}(Q_1+Q_2)$$

$$Q_2' = C_2V = \frac{C_2}{C_1+C_2}(Q_1+Q_2)$$

Heat loss during redistribution:

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$$

The loss of energy is in the form of heating in the wire.

NOTE:

i. When plates of similar charges are connected with each other (+ with + and − with −), put all values (Q_1, Q_2, V_1 and V_2) with positive sign.

ii. When plates of opposite polarity are connected with each other (+ with −), take the charge and potential of one of the plates to be negative.

ILLUSTRATION 18.18 Two capacitors C_1 and C_2 are charged seperately to potentials 20 V and 10 V, respectively. The terminals of capacitors C_1 and C_2 are marked as (A-B) and (C-D), respectively. A is connected with C and B is connected with D.

i. Find the final potential difference across each capacitors

ii. Find the final charge in both capacitors.

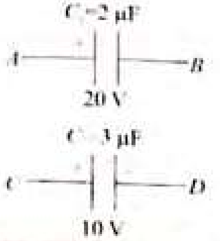
iii. How much heat is produced in the circuit

Solution. i. Initial charge in capacitor C_1 is

$$(Q_1)_{initial} = C_1V_1 = 2 \times 20 = 40\,\mu C$$

Initial charge in capacitor C_2 is

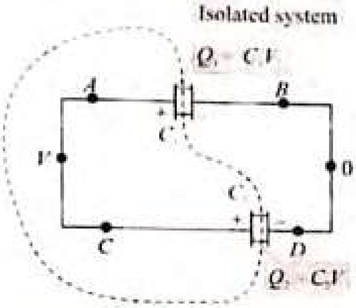
$$(Q_2)_{initial} = C_2V_2 = 3 \times 10 = 30\,\mu C$$



Initially capacitors C_1 and C_2 are separately charged.

Let the potential of B and D be zero and the common potential difference across the capacitors be V, then the potentials at A and C will be V.

From figure, it is clear that the left plates of capacitors C_1 and C_2 are forming an isolated system, i.e., they are not connected from outside. From charge conservation,



Finally capacitors C_1 and C_2 are connected together.

$$C_1V + C_2V = 3V + 2V = 40 + 30$$

$$5V = 70 \text{ or } V = 14\text{ V}$$

ii. Final charge in capacitor C_1 is $(Q_1)_{final} = 2 \times 14 = 28\,\mu C$

Final charge in capacitor C_2 is

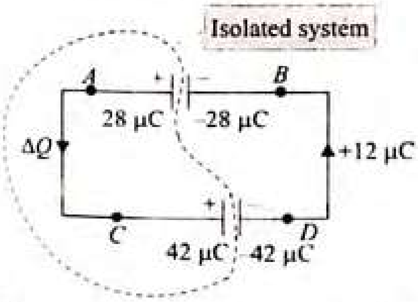
$$(Q_2)_{final} = 3 \times 14 = 42\,\mu C$$

The charge flowing in the circuit in the direction from A to C is

$$\Delta Q = 40 - 28 = 12\,\mu C$$

Now final charges on each plate are shown in figure.

iii. Heat produced in the circuit is



$$H = \left[\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right] - \left[\frac{1}{2}(C_1 + C_2)V^2\right]$$

$$= \left[\frac{1}{2}\times 2\times(20)^2 + \frac{1}{2}\times 3\times(10)^2\right] - \left[\frac{1}{2}\times 5\times(14)^2\right]$$

$$= 400 + 150 - 490 = 550 - 490 = 60\,\mu J$$

DIELECTRIC

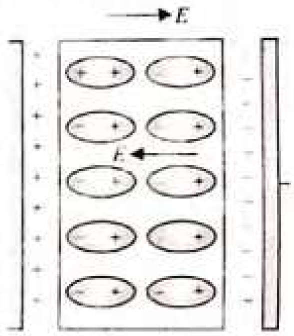
A dielectric is a nonconductor up to a certain value of the field depending upon its nature. If the field exceeds the limiting value, called dielectric strength, a dielectric loses its insulating property and begins to conduct. Dielectrics are of two types:

NOTE: In general, any nonconducting material can be called a dielectric, but nonconducting materials having nonpolar molecules are referred to as dielectric because an induced dipole moment is created in the nonpolar molecule.

Polarization of a dielectric slab: A dielectric slab can be polarized by inducing equal and opposite charges on the two faces of the dielectric by the application of an electric field. Suppose a dielectric slab is inserted between the plates of a capacitor as shown in figure.

Induced electric field inside the dielectric is E_i; hence, this induced electric field decreases the main field E to $E - E_i$ i.e. new electric field between the plates will be $E' = E - E_i$.

Dielectric constant: After placing a dielectric slab in an electric field, the net field is decreased in that region. If E is the original electric field and E' is the reduced electric field, $E/E' = K$, where K is called dielectric constant. K is also known as relative permittivity (ε_r) of the material. The value of K is always greater than one. For vacuum, there is no polarization and hence $E = E'$ and $K = 1$.



Dielectric breakdown and dielectric strength: If a very high electric field is created in a dielectric, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is known as dielectric breakdown.

The maximum value of an electric field (or potential gradient) that a dielectric material can tolerate without electric breakdown is called its dielectric strength. SI unit of the dielectric strength of a material is Vm^{-1}.

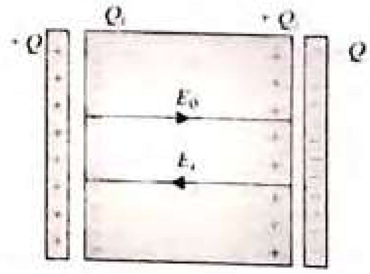
INDUCED CHARGE ON THE SURFACE OF DIELECTRIC

Let K be the dielectric constant, E_0 be the original field in vacuum if the dielectric slab was not there, E_i be the electric field induced in the dielectric slab, and E be the net electric field in the dielectric slab. Therefore,

$$E = E_0 - E_i \qquad \text{(i)}$$

and $\dfrac{E_0}{E} = K$ (by definition of K of ε_r)

or $$E = \frac{E_0}{K} \qquad \text{(ii)}$$



From (i) and (ii),

$$E_0 - E_i = \frac{E_0}{K}$$

or $$E_0K - E_iK = E_0$$

or $$E_0K - E_0 = E_iK$$

or $\quad E_i = \dfrac{K-1}{K} E_0 \qquad$ (iii)

or $\quad Q_i = Q\left(1 - \dfrac{1}{K}\right) \qquad$ (iv)

This is irrespective of the thickness of the dielectric slab, i.e., whether it fills up the entire space between the charged plates or any part of it.

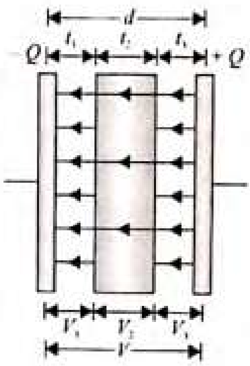
CAPACITY OF PARALLEL PLATE CAPACITOR WITH DIELECTRIC

Suppose that a parallel plate capacitor has a plate area A and a separation d, and a dielectric slab of thickness t and area A is inserted between the plates. Let Q be the charge given to the capacitor plates.

$$\therefore \quad C = \frac{Q}{V_+ - V_-} = \frac{1}{\dfrac{(d-t)}{\varepsilon_0 A} + \dfrac{t}{k\varepsilon_0 A}}$$

$$= \frac{\varepsilon_0 A}{(d-t) + \dfrac{t}{K}}$$

$$= \frac{\varepsilon_0 A}{(d-t) + \dfrac{t}{K}}$$



NOTE:

- The capacitance in the above situation is independent of the position of the dielectric slab with respect to the plates. The capacitance depends upon the thickness of the dielectric slab and the dielectric constant.
- The dielectric constant of the conducting slab (metal plate) is infinity; therefore, the term t/K reduces to zero. If we insert a metal plate of thickness t between the plates of the capacitor having area A and separation d, the capacitance will become $C = \varepsilon_0 A/(d-t)$. Also the capacitance will be independent of the position of the metal plate between the plates of the capacitor.
- If $t << d$, then $C = \varepsilon_0 A/d$. Hence if we place a thin metal plate parallel to the plate of a capacitor, the capacitance of the capacitor remains unchanged.
- If the slab completely fills the space between the plates, then $t = d$, and therefore,
- If the space between the plates is completely filled with a conductor, then $t = d$ and $K = \infty$.

If we place many dielectric slabs parallel to the plate of a capacitor as shown in figure, then the capacitance is given by

$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + \cdots) + \left(\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \cdots\right)}$$

If we introduce a number of dielectric slabs, which completely fill the space between the plates, then

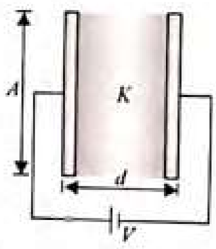
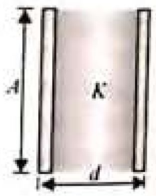
$$d = t_1 + t_2 + t_3 + \ldots t_n$$

Therefore, the capacity of the capacitor will be

$$C = \frac{\varepsilon_0 A}{\left(\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \ldots\right)}$$

EFFECT OF DIELECTRIC ON DIFFERENT PARAMETERS

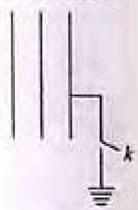
Let the entire space between the plates of a capacitor be filled with a dielectric of dielectric constant K under two conditions:
(i) when the battery remains connected and
(ii) after the battery is disconnected.

If battery remains connected	When battery is disconnected
In this case, the potential difference across the plates will remain same.	In this case, the charge on the plates will remain same, i.e., $q = q_0$, since in an isolated system, charge is conserved.
	
• Capacitance increases, i.e., $C = KC_0$, since capacitance depends upon geometrical factors only.	• Capacitance increases, i.e., $C = KC_0$, since capacitance depends upon geometrical factors only.
• Charge on capacitor is $q = CV = KC_0V_0 = Kq_0$ (Since initially $q_0 = C_0V_0$) Thus, charge increases and becomes K times of previous charge.	• Potential difference between the plates is $V = \dfrac{q}{C} = \dfrac{q_0}{KC_0} = \dfrac{V_0}{K}$ So the potential difference decreases.
• Electric field is $E = \left[\dfrac{V}{d}\right] = \dfrac{V_0}{d} = E_0$ $\left(\text{as } V = V_0 \text{ and } \dfrac{V_0}{d} = E_0\right)$ Thus, electric field remains same.	• Field between the plates is $E = \dfrac{V}{d} = \dfrac{V_0}{Kd} = \dfrac{E_0}{K}$ $\left(\text{as } V = \dfrac{V_0}{K} \text{ and } E_0 = \dfrac{V_0}{d}\right)$ So, the electric field decreases.

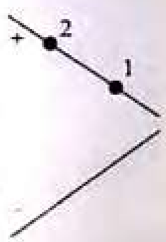
• Energy stored in the capacitor is $U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)(V_0)^2$ $= K\frac{1}{2}C_0V_0^2 = KU_0$ $\left(\text{as } C = KC_0 \text{ and } U_0 = \frac{1}{2}C_0V_0^2\right)$	• Energy stored in the capacitor is $U = \frac{q^2}{2C} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K}$ (as $q = q_0$ and $C = KC_0$)
• Thus, energy increases and becomes K times of previous energy.	• Thus, energy decreases and becomes $1/K$ times of previous energy.

Concept Application Exercise 18.4

1. A capacitor is connected across a battery.
   (a) Why does each plate receive a charge of exactly the same magnitude?
   (b) Is this true even if the plates are of different size?
2. Three identical large metallic plates are placed parallel to each other at a very small separation as shown in figure. The central plate is given a charge Q. What amount of charge will flow to earth when the key is pressed?



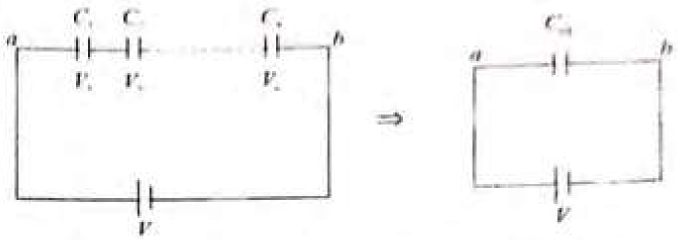
3. The plates of a plane capacitor are drawn apart keeping them connected to a battery. Next, the same plates are drawn apart from the same initial condition keeping the battery disconnected. In which case is more work done?
4. If a small charge q is moved along a closed path in the field between the plates of a parallel plate capacitor, will any work be done by the agent that moves the charge?
5. At which of the two points, 1 or 2, of a charged capacitor with nonparallel plates is the surface charge density greater?
6. A parallel plate air capacitor is connected to a battery. If the plates of the capacitor are pulled farther apart, then state whether the following statements are true or false.



   (a) Strength of the electric field inside the capacitor remains unchanged, if the battery is disconnected before pulling the plates.
   (b) During the process, work is done by the external force applied to pull the plates irrespective of whether the battery is disconnected or not.
   (c) Strain energy in the capacitor decreases if the battery remains connected.

Capacitors Connected in Series

Suppose n capacitors are connected in series as shown in figure. C_{eq} is the equivalent capacitance of this system.



Let a battery V be applied across the combination, then

$$V = V_1 + V_2 + \cdots + V_n$$

or $$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \cdots + \frac{Q}{C_n}$$

or $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

In general

$$\frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$

Thus, *the reciprocal of the equivalent capacitance of a series combination equals the sum of the reciprocals of the individual capacitances.*

Special Case of Two Capacitors in Series

Suppose two capacitors C_1 and C_2 are connected in series and a potential difference V is applied across them as in figure.

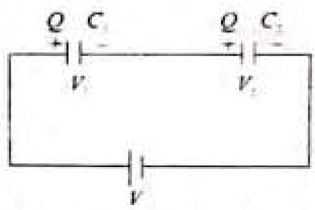
Potentials appearing on the capacitors are V_1 and V_2, respectively. Q is the net charge that flows in circuit, then

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

$$V_1 + V_2 = V \qquad \text{(i)}$$

$$Q = C_1V_1 = C_2V_2$$

or $$\frac{V_1}{V_2} = \frac{C_2}{C_1} \qquad \text{(ii)}$$



It means potentials on the capacitors will be divided in the inverse ratio of their capacitances.

From (i) and (ii),

$$V_1 = \frac{C_2V}{C_1 + C_2}, \quad V_2 = \frac{C_1V}{C_1 + C_2}$$

NOTE:

- In a series combination, the equivalent capacitance is always less than any of the individual capacitance.
- In a series combination, charge on each capacitor is same but potential is different. From $V = q/C$, it can be said that larger is the capacitance, lesser is the potential. We can say that the potential difference across the capacitor is inversely proportional to its capacitance in series combination.

$$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

$$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots} V, \quad V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots} V$$

where $V = V_1 + V_2 + V_3$

CAPACITORS CONNECTED IN PARALLEL

Let n capacitors be connected in parallel as shown in figure. C_{eq} is the equivalent capacitance of this system.

Let the total charge flowing through the battery be q, then

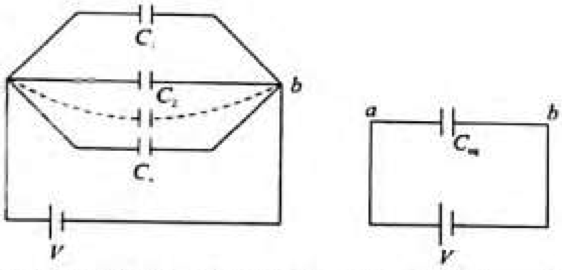
$$q = q_1 + q_2 + \cdots + q_n$$

or $\quad C_{eq}V = C_1V + C_2V + \cdots + C_nV$

or $\quad C_{eq} = C_1 + C_2 + \cdots + C_n$

In general

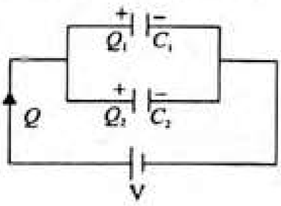
$$C_{eq} = \sum_{n=1}^{n} C_n$$



That is, the equivalent capacitance of a parallel combination equals the sum of the individual capacitances.

Special Case of Two Capacitors in Parallel

Let two capacitors C_1 and C_2 be connected in parallel and a potential difference V is applied across them. Then



$$C_{eq} = C_1 + C_2$$

$$Q_1 + Q_2 = Q \qquad \text{(i)}$$

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \qquad \text{(ii)}$$

From (ii)

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} \qquad \text{(iii)}$$

It means charge will be divided in the direct ratio of the capacitances. From (i) and (iii),

$$Q_1 = \frac{C_1Q}{C_1 + C_2}, Q_2 = \frac{C_2Q}{C_1 + C_2}$$

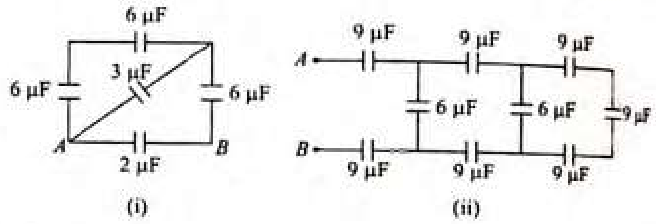
NOTE:

- In a parallel combination, the equivalent capacitance is always greater than any individual capacitance.
- In a parallel combination, the potential on each capacitor is same but charge may be different. From $q = CV$, it can be said that greater is the capacitance, greater is the charge.
- Charge is distributed on the capacitors in the ratio of their capacitances, i.e., $q_1 : q_2 : \ldots : q_n = C_1 : C_2 : \ldots : C_n$

Finding Equivalent Capacitance

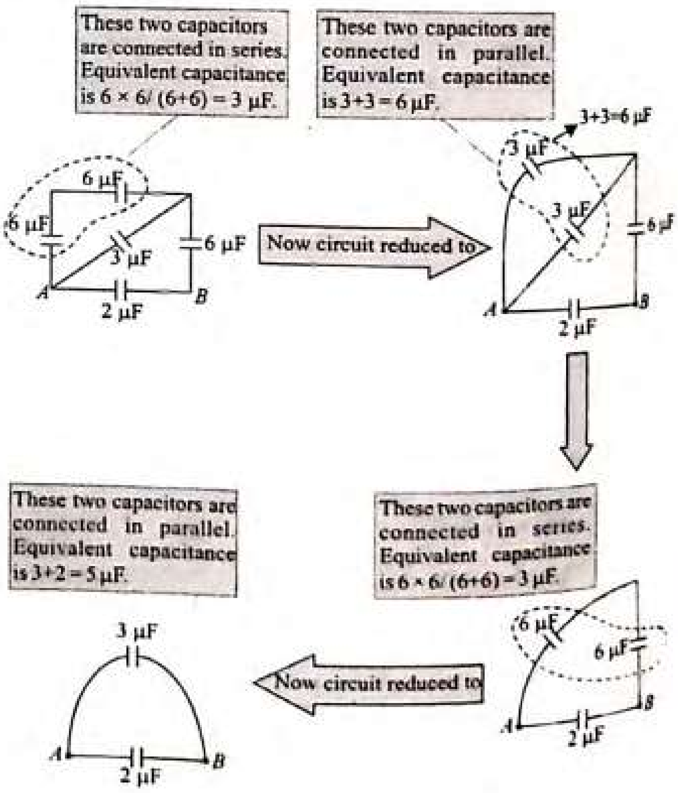
Equivalent capacitance can be found by using the successive reduction method. Let us learn this method through the following illustrations.

ILLUSTRATION 18.19 In figure, different capacitors are arranged. Find the equivalent capacity across the points A and B.



Solution.

(i)



(ii)

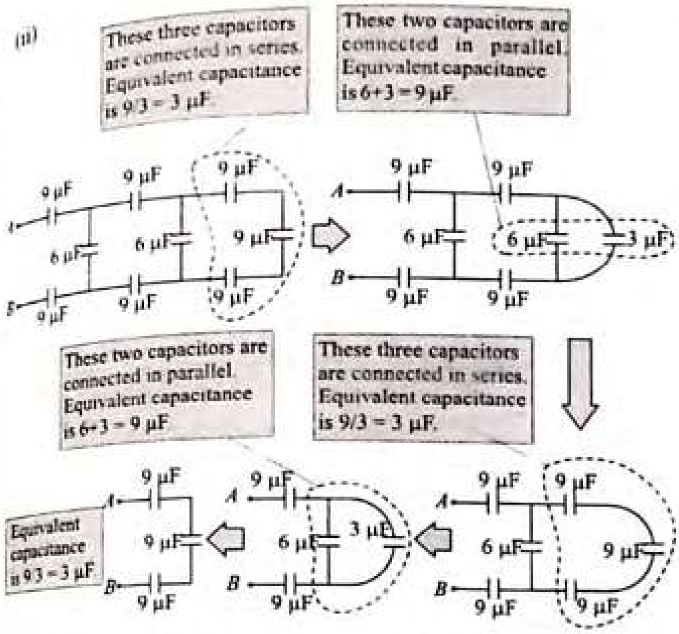
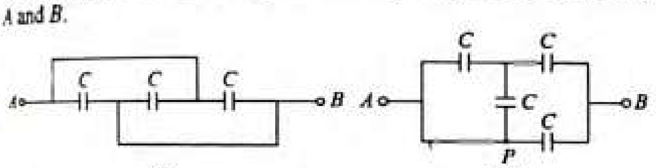


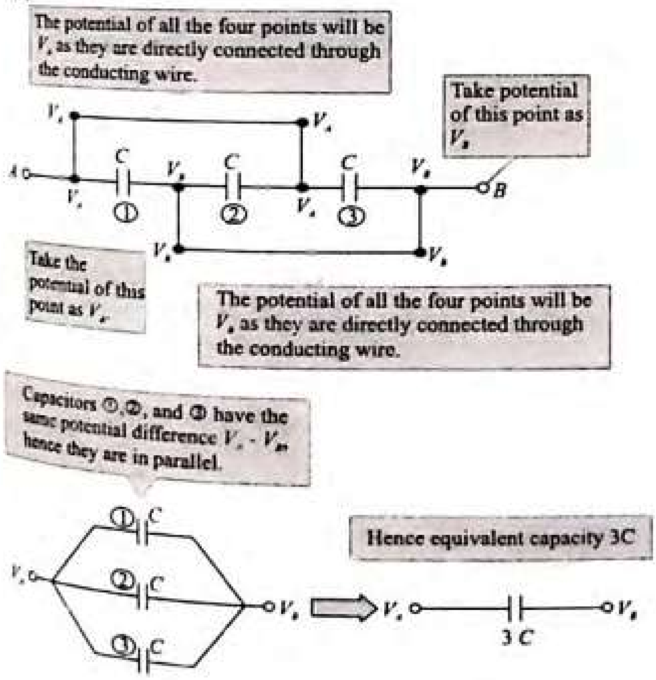
ILLUSTRATION 18.20 Different capacitors are arranged as shown in figure. Find the equivalent capacity across the points A and B.



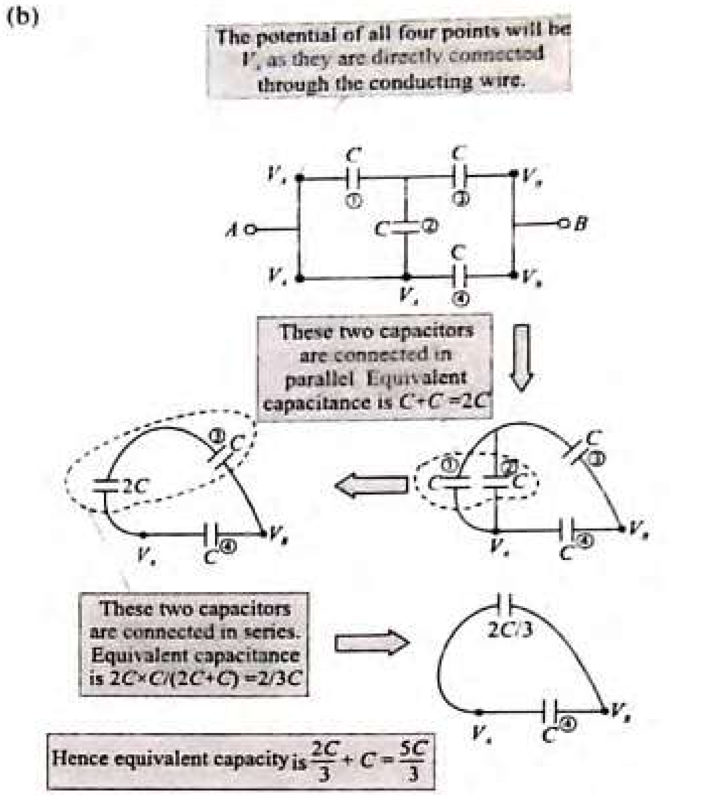
(a) (b)

Solution.

(a)

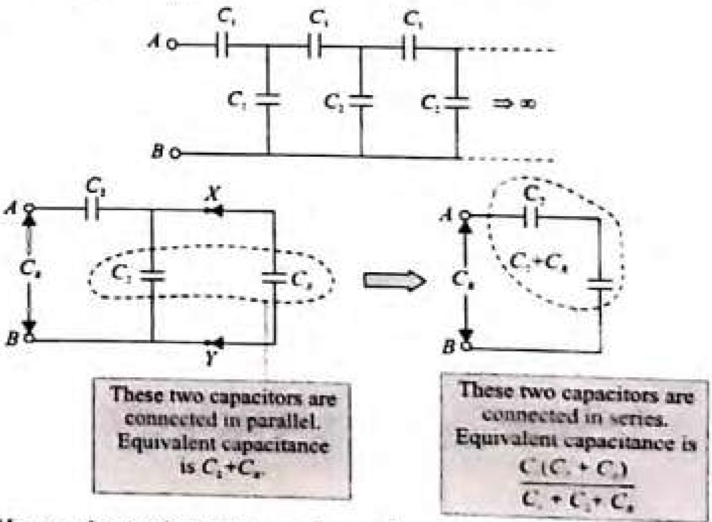


(b)



Infinite Chain of Capacitors

Suppose the effective capacitance between A and B is C_R. Since the network is infinite, even if we remove one pair of capacitors from the chain, the remaining network would still have infinite pairs of capacitors, i.e., effective capacitance between X and Y would also be C_R.



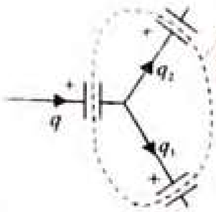
Hence, the equivalent capacitance between A and B is

$$C_{AB} = \frac{C_1(C_2+C_R)}{C_1+C_2+C_R} = C_R \text{ or } C_{AB} = \frac{C_2}{2}\left[\sqrt{1+4\frac{C_1}{C_2}} - 1\right]$$

KIRCHHOFF'S RULES FOR CAPACITORS

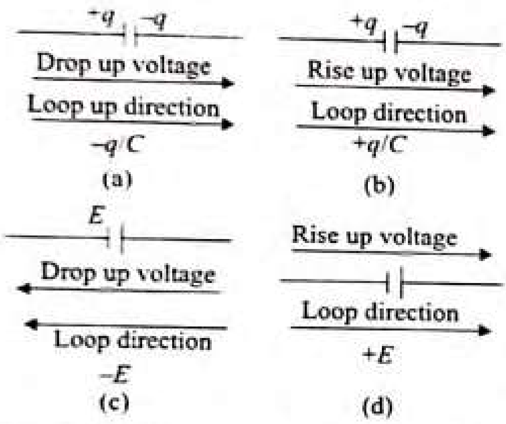
Kirchhoff's rules can be used to determine the potential difference and charge on the plates of a capacitor in any electric circuit. In a circuit with capacitors and batteries, two important rules are involved.

Junction rule: A capacitor circuit obeys the principle of conservation of charge. The incoming charge at any junction is equal to the outgoing charge from the junction, i.e., $q = q_1 + q_2$. In other words, in any isolated system, the net charge is conserved. So in the system shown in figure.



$$-q + q_1 + q_2 = 0 \Rightarrow q = q_1 + q_2$$

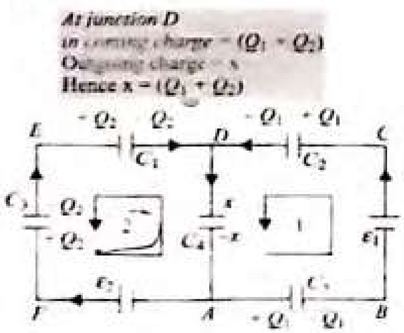
Loop rule: In a closed circuit, the algebraic sum of the rise up and drop up voltages is zero, i.e., $\Sigma V = 0$.



The direction of the loop is not specified and is chosen in a comfortable manner. When we go from a point of higher potential to a point of lower potential in the loop direction, drop up voltage occurs [Figure (a)]. In sign convention, drop up voltage is taken as negative. If we go from a lower potential point to a higher potential point, rise up voltage occurs. In sign convention, rise up of voltage is taken as positive.

Application of Kirchhoff's Loop Rule in a Circuit

If you travel from the positively charged plate of a capacitor or the terminal of a cell to the negatively charged plate of a capacitor or the terminal of a cell, then the sign of V or ε is negative and vice versa.



Applying loop rule in closed loop $ABCDA$

$$-\frac{Q_1}{C_5} + \varepsilon_1 - \frac{Q_1}{C_2} - \frac{Q_1 + Q_2}{C_4} = 0$$

or $$Q_1\left(\frac{1}{C_2} + \frac{1}{C_4} + \frac{1}{C_5}\right) + \frac{Q_2}{C_4} = \varepsilon_1 \quad \text{(i)}$$

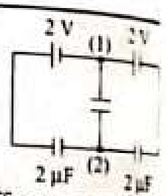
Applying loop rule in closed loop $ADEFA$

$$\frac{Q_1 + Q_2}{C_4} + \frac{Q_2}{C_1} + \frac{Q_2}{C_3} - \varepsilon_2 = 0$$

or $$\frac{Q_1}{C_4} + Q_2\left[\frac{1}{C_1} + \frac{1}{C_3} + \frac{1}{C_4}\right] = \varepsilon_2 \quad \text{(ii)}$$

Now you have two equations and two unknowns (Q_1 and Q_2), which can be found easily by solving the equations.

ILLUSTRATION 18.21 Find the potential difference $V_1 - V_2$ between the points (1) and (2) shown in each part of figure.



Solution.

(a) We first distribute the charges on different capacitors and branches keeping in mind Kirchhoff's junction rule. We can start from any battery. In this case, we start from battery ε_1. Let battery ε_1 supply a charge q. The charge on the left plate of capacitor C_1 will be q as this plate is receiving the charge. Now charge q reaches junction 2 where it is divided into two paths. Let charge x go to capacitor C_3, then the remaining charge $q - x$ will go to capacitor C_2. In junction 1, we can verify Kirchhoff's junction rule, i.e., the incoming charge is equal to the outgoing charge. Now we write the loop equations.

For loop 1

$$2 - \frac{q}{2} - \frac{x}{C} = 0 \quad \text{(i)}$$

For loop 2

$$2 + \frac{x}{C} - \frac{(q - x)}{2} = 0$$

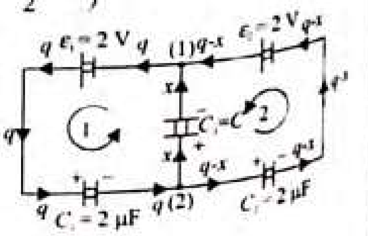
or $$2 + \frac{x}{C} - \frac{q}{2} + \frac{x}{2} = 0 \quad \text{(ii)}$$

From (i) and (ii)

$$\frac{2x}{C} + \frac{x}{2} = 0 \quad \text{or} \quad x\left(\frac{2}{C} + \frac{1}{2}\right) = 0$$

or $$x = 0 \left(\text{since } \frac{2}{C} + \frac{1}{2} \neq 0\right)$$

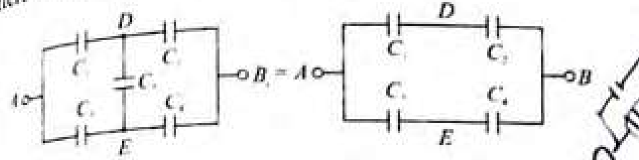
No charge will go to the capacitor connected across (1) and (2). As there is no charge in the capacitor connected

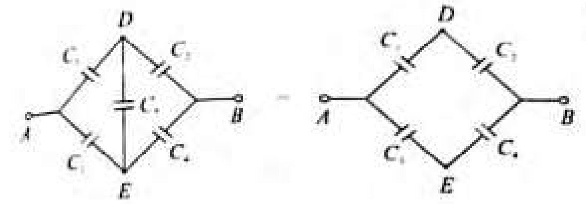


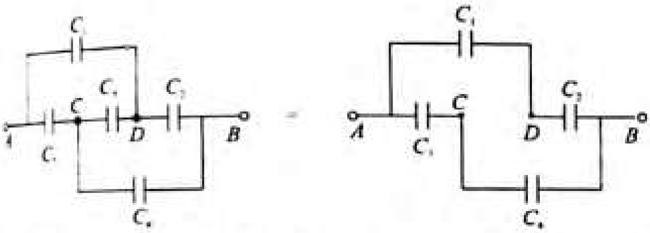
across (1) and (2), the potential difference across (1) and (2) should be zero.

Circuits Based on Wheatstone Bridge

In following circuits if $(C_1/C_2) = (C_3/C_4)$, the potential difference across D and E will be zero.







In all above cases, the equivalent capacity between A and B is

$$C_{AB} = \frac{C_1C_2}{C_1+C_2} + \frac{C_3C_4}{C_3+C_4}$$

Extended Wheatstone Bridge

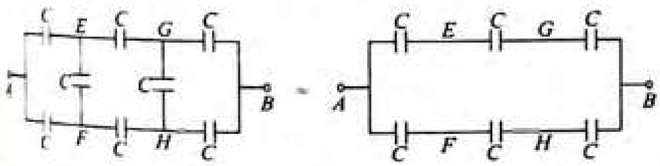
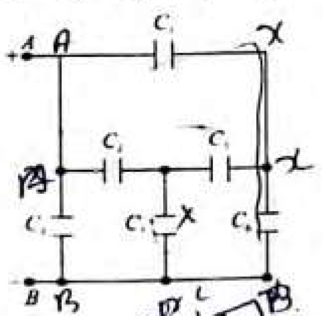


ILLUSTRATION 18.22 Six capacitors $C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C$ are arranged as shown in figure. Determine the equivalent capacitance between A and B.

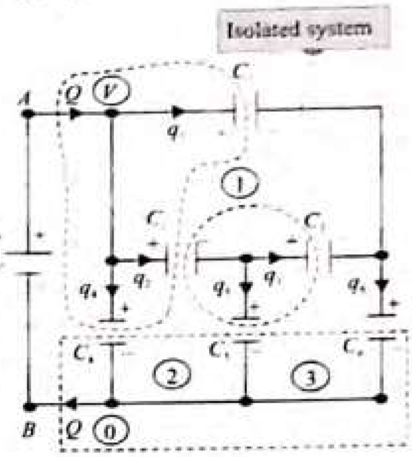


Solution. Let the potential difference across the battery terminals be V and the charge supplied be Q. We have to find the capacitance of the system and the capacitance of a capacitor that would have the same charge Q on its plates as the battery at voltage V. Hence,

$$C_{\text{equivalent}} = \frac{Q}{V_A - V_B} = \frac{Q}{V}$$

The incoming charge Q is equal to the charge received by the plates of the capacitors C_1, C_2, and C_4. Thus,

$$Q = q_1 + q_2 + q_4$$



Similarly, outgoing charge Q is equal, to charge coming out from the plates of the capacitors C_4, C_5, and C_6.

$$Q = q_4 + q_5 + q_6$$

The above equations give

$$q_1 + q_2 = q_5 + q_6 \qquad \text{(i)}$$

In a closed loop, the net potential drop must be zero, according to Kirchhoff's voltage law. Therefore, for loop 1, loop 2, and loop 3, we have

$$1:\ \frac{q_1}{C} - \frac{q_2}{C} - \frac{q_3}{C} = 0 \Rightarrow \left.\begin{array}{l} q_1 = q_2 + q_3 \\ q_1 + q_2 = 2q_2 + q_3 \end{array}\right] \qquad \text{(ii)}$$

$$2:\ \frac{q_2}{C} - \frac{q_4}{C} + \frac{q_5}{C} = 0 \Rightarrow q_4 = q_2 + q_5 \qquad \text{(iii)}$$

$$3:\ \frac{q_3}{C} - \frac{q_5}{C} + \frac{q_6}{C} = 0 \Rightarrow \left.\begin{array}{l} q_5 = q_3 + q_6 \\ q_5 + q_6 = 2q_6 + q_3 \end{array}\right] \qquad \text{(iv)}$$

From the isolated system shown in Figure, we can conclude

$$q_3 + q_5 - q_2 = 0 \qquad \text{(v)}$$

From Eqs. (i), (ii), and (iv), we conclude $q_2 = q_6$ and $q_1 = q_5$. If $q_1 = q_5$, then Eq. (v) becomes

$$q_2 - q_1 = q_3 \qquad \text{(vi)}$$

Solving Eqs. (ii) and (vi), we get

$$q_3 = 0,\ q_1 = q_2 = q_5 = q_6 = \frac{q_4}{2},\ \text{and } Q = 2q_4$$

We can write

$$V = \frac{q_4}{C} = \frac{Q}{2C} \text{ or } \frac{Q}{V} = 2C$$

Hence,

$$C_{\text{equivalent}} = \frac{Q}{V_A - V_B} = \frac{Q}{V} = 2C$$

Alternative method: Connect point a to point b and note the indicated Wheatstone bridge (figure). Now simplify the circuit to obtain the desired result.

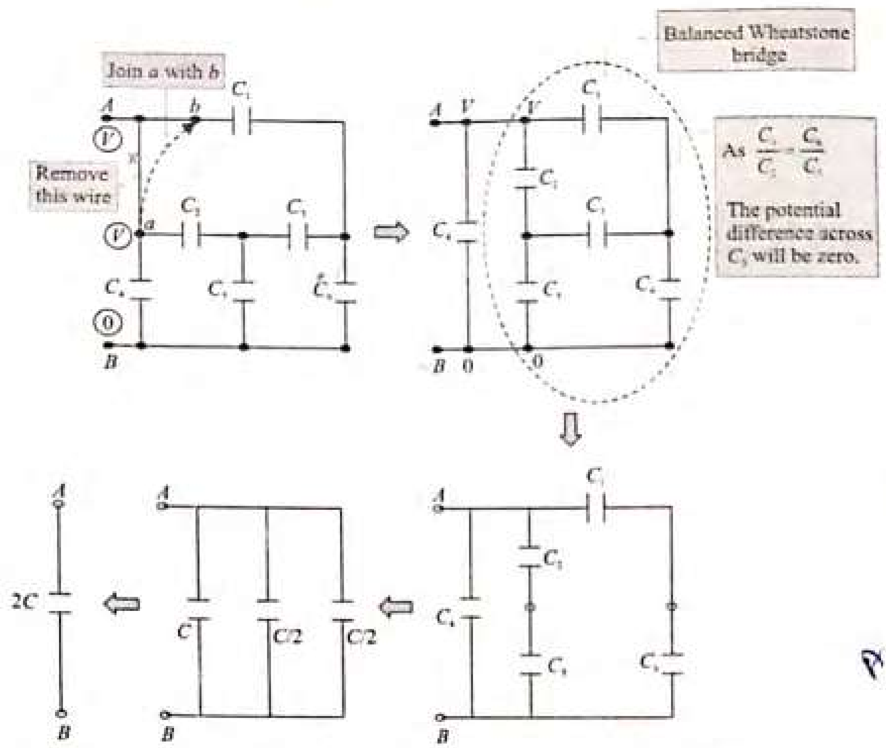
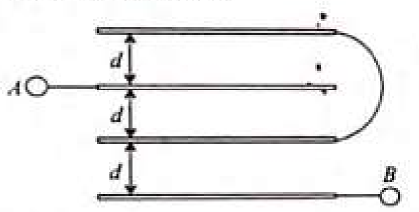
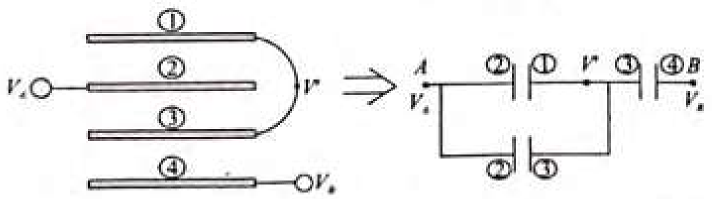


ILLUSTRATION 18.23 Four identical plates, each having area A, are arranged as shown in figure. Find the equivalent capacity of the structure between A and B.



Solution. **Method 1:** Let the potentials of plates A and B be V_A and V_B, respectively. As the plates (1) and (3) (figure) are connected together, they will have a common potential, say V'. Now we will make an equivalent circuit diagram by connecting the different plates across the assumed potential difference.



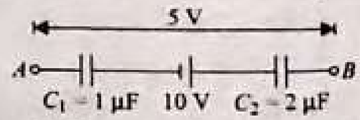
The equivalent capacitance between A and B can easily be calculated as follows:

$$C_{eq} = \frac{2}{3}C = \frac{2}{3}\frac{\varepsilon_0 A}{d}$$

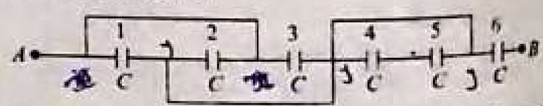
Method 2: Let us assume that a battery is connected between A and B. Let the potentials of A and B be V and 0. Plates (1) and (2) are connected together and both have the same potential V'. Let the charge supplied by the positive terminal of the battery to plate (1) be Q; same charge will come out from plate (4). The outermost surface of the plates will carry no charge.

CONCEPT APPLICATION EXERCISE 18.5

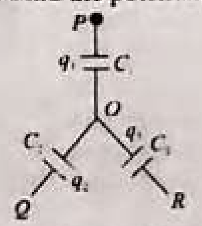
1. A circuit has section AB as shown in figure. The emf of the cell is 10 V, and the capacitors have capacitances $C_1 = 1\ \mu F$ and $C_2 = 2\ \mu F$. The potential difference is $V_{AB} = 5$ V. Find the charges on the capacitors.



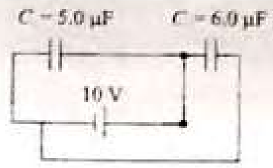
2. Find the equivalent capacitance between points A and B as shown in figure.



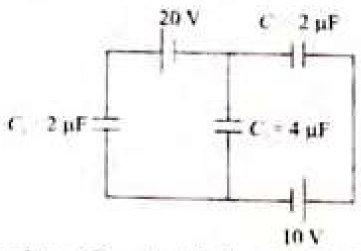
3. Three capacitors C_1, C_2, and C_3 are connected as shown in figure. The potentials of P, Q, and R are V_1, V_2, and V_3, respectively. Find the potential V_0 at the junction O.



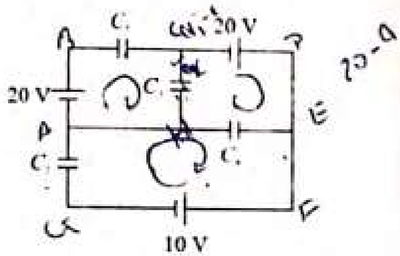
4. Find the charge supplied by the battery in the arrangement as shown in figure.



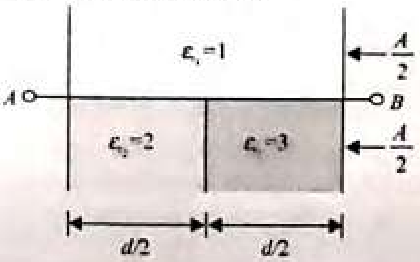
5. Three capacitors C_1, C_2, and C_3 are arranged as shown in figure. Determine the charge on each capacitor.



6. Four capacitors $C_1 = 8$ μF, $C_2 = 2$ μF, $C_3 = 6$ μF, and $C_4 = 6$ μF are arranged as shown in figure. Find the charge on all the capacitors in the circuit.



7. Three dielectrics of relative permittivities $\varepsilon_{r_1} = 1$ $\varepsilon_{r_2} = 2$, and $\varepsilon_{r_3} = 3$ are introduced in a parallel plate capacitor of plate area A and separation d. Find the effective capacitance between A and B.



SOLVED EXAMPLES

1. A non-conducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10}C distributed non-uniformly on its circumference producing an electric field $\vec{E}$ everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -\vec{E}.\vec{dl}$ ($l = 0$ being centre of the ring) in volt is

(a) +2 (b) −1
(c) −2 (d) Zero

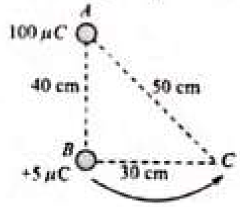
Sol. *(a)* $\int_{-\infty}^{0} -\vec{E}\cdot\vec{dl}$ = potential at centre of non-conducting ring

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r} = \frac{9\times10^9 \times 1.11\times10^{-10}}{0.5} = 2 \text{ volt}$$

2. Two point charges 100 μC and 5 μC are placed at points A and B respectively with $AB = 40$ cm. The work done by external force in displacing the charge 5 μC from B to C, where $BC = 30$ cm, angle $ABC = \pi/2$ and $1/4\pi\varepsilon_0 = 9\times10^9$ Nm^2/C^2

(a) 9 J (b) $\frac{81}{20}$ J
(c) $\frac{9}{25}$ J (d) $-\frac{9}{4}$ J

Sol. *(d)* Work done in displacing charge of 5 μC from B to C is $W = 5 \times 10^{-6}(V_C - V_B)$ where



$$V_B = 9\times10^9 \times \frac{100\times10^{-6}}{0.4} = \frac{9}{4}\times10^6 \text{ V}$$

and $$V_C = 9\times10^9 \times \frac{100\times10^{-6}}{0.5} = \frac{9}{5}\times10^6 \text{ V}$$

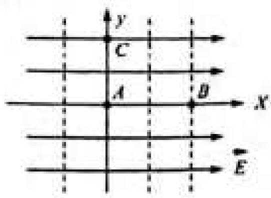
So $$W = 5\times10^{-6}\times\left(\frac{9}{5}\times10^6 - \frac{9}{4}\times10^6\right) = -\frac{9}{4} \text{ J}$$

3. A uniform electric field pointing in positive x-direction exists in a region. Let A be the origin, B be the point on the x-axis at $x = +1$ cm and C be the point on the y-axis at $y = +1$ cm. Then the potentials at the points A, B and C satisfy

(a) $V_A < V_B$ (b) $V_A > V_B$
(c) $V_A < V_C$ (d) $V_A > V_C$

Sol. *(b)* Potential decreases in the direction of electric field. Dotted lines are equipotential lines

$\therefore$ $V_A = V_C$ and $V_A > V_B$



4. Electric potential is given by
$V = 6x - 8xy^2 - 8y + 6yz - 4z^2$
Then electric force acting on 2C point charge placed on origin will be

(a) 2 N (b) 6 N
(c) 8 N (d) 20 N

Sol. (d) $E_x = -\frac{dV}{dx} = -(6-8y^2)$,

$$E_y = -\frac{dV}{dy} = -(-16xy - 8 + 6z)$$

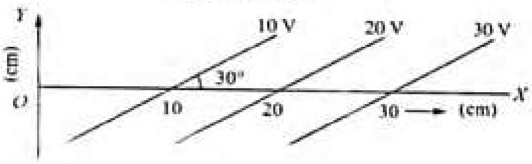
$$E_z = -\frac{dV}{dz} = -(6y - 8z)$$

At origin $x = y = z = 0$ so, $E_x = -6$, $E_y = 8$ and $E_z = 0$

$\Rightarrow$ $E = \sqrt{E_x^2 + E_y^2} = 10$ N/C

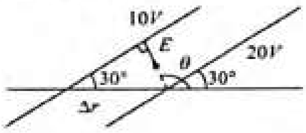
Hence force $F = QE = 2 \times 10 = 20$ N

5. Equipotential surfaces are shown in figure. Then the electric field strength will be



(a) 100 Vm^{-1} along X-axis
(b) 100 Vm^{-1} along Y-axis
(c) 200 Vm^{-1} at an angle 120° with X-axis
(d) 50 Vm^{-1} at an angle 120° with X-axis

Sol. (c) Using $dV = -\vec{E}.d\vec{r}$



$$\Rightarrow \quad \Delta V = -E.\Delta r \cos\theta$$

$$\Rightarrow \quad E = \frac{-\Delta V}{\Delta r \cos\theta}$$

$$\Rightarrow \quad E = \frac{-(20-10)}{10 \times 10^{-2} \cos 120°}$$

$$= \frac{-10}{10 \times 10^{-2}(-\sin 30°)} = \frac{-10^2}{-1/2} = 200 \text{ V/m}$$

Direction of E be perpendicular to the equipotential surface i.e. at 120° with x-axis.

6. If on the concentric hollow spheres of radii r and $R(> r)$ the charge Q is distributed such that their surface densities are same then the potential at their common centre is

(a) $\frac{Q(R^2 + r^2)}{4\pi\varepsilon_0(R+r)}$ (b) $\frac{QR}{R+r}$

(c) Zero (d) $\frac{Q(R+r)}{4\pi\varepsilon_0(R^2+r^2)}$

Sol. (d) $q_1 + q_2 = Q$ and $\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$ (given)

$$q_1 = \frac{Qr^2}{R^2 + r^2} \text{ and } q_2 = \frac{QR^2}{R^2 + r^2}$$

Potential at common centre

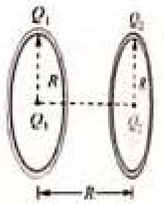
$$\frac{1}{4\pi\varepsilon_0}\left[\frac{Qr^2}{(R^2+r^2)r} + \frac{QR^2}{(R^2+r^2)R}\right] = \frac{Q(R+r)}{4\pi\varepsilon_0(R^2+r^2)}$$

7. Two identical thin rings each of radius R meters are coaxially placed at a distance R meters apart. If Q_1 coulomb and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of other is

(a) Zero (b) $\frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{\sqrt{2}.4\pi\varepsilon_0 R}$

(c) $\frac{q\sqrt{2}(Q_1+Q_2)}{4\pi\varepsilon_0 R}$ (d) $\frac{q(Q_1+Q_2)(\sqrt{2}+1)}{\sqrt{2}.4\pi\varepsilon_0 R}$

Sol. (b) $W = q(V_{O_2} - V_{O_1})$



where $V_{O_1} = \frac{Q_1}{4\pi\varepsilon_0 R} + \frac{Q_2}{4\pi\varepsilon_0 R\sqrt{2}}$

and $V_{O_2} = \frac{Q_2}{4\pi\varepsilon_0 R} + \frac{Q_1}{4\pi\varepsilon_0 R\sqrt{2}}$

$$\Rightarrow \quad V_{O_2} - V_{O_1} = \frac{(Q_2 - Q_1)}{4\pi\varepsilon_0 R}\left[1 - \frac{1}{\sqrt{2}}\right]$$

So, $W = \frac{q.(Q_2 - Q_1)}{4\pi\varepsilon_0 R}\frac{(\sqrt{2}-1)}{\sqrt{2}}$

8. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-3Q$, the new potential difference between the same two surfaces is

(a) V (b) $2V$
(c) $4V$ (d) $-2V$

Sol. (a) In case of a charged conducting sphere

$$V_{\text{inside}} = V_{\text{centre}} = V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{R}, \quad V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

If a and b are the radii of sphere and spherical shell respectively, then potential at their surface will be

$$V_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{a} \text{ and } V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{b}$$

$$\therefore \quad V = V_{\text{sphere}} - V_{\text{shell}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} - \frac{Q}{b}\right]$$

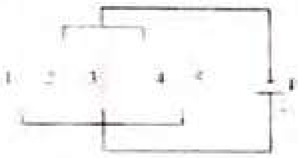
Now when the shell is given charge $(-3Q)$, then the potential will be

$$V'_{sphere} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} + \frac{(-3Q)}{b}\right].$$

$$V'_{shell} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b} + \frac{(-3Q)}{b}\right]$$

$$\therefore \quad V'_{sphere} - V'_{shell} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a} - \frac{Q}{b}\right] = V$$

9. Five identical plates each of area A are joined as shown in the figure. The distance between the plates is d. The plates are connected to a potential difference of V volts. The charge on plates 1 and 4 will be



(a) $\frac{\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$

(b) $\frac{\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$

(c) $\frac{\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$

(d) $\frac{-\varepsilon_0 AV}{d}, \frac{-2\varepsilon_0 AV}{d}$

Sol. (c) The given circuit can be redrawn as follows. All capacitors are identical and each having capacitance $C = \frac{\varepsilon_0 A}{d}$

|Charge on each capacitor| = |Charge on each plate|

$$= \frac{\varepsilon_0 A}{d} V$$

Plate 1 is connected with positive terminal of battery so charge on it will be $+\frac{\varepsilon_0 A}{d}\cdot V$

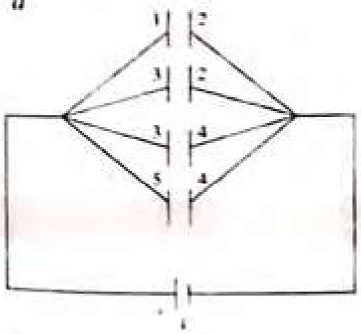
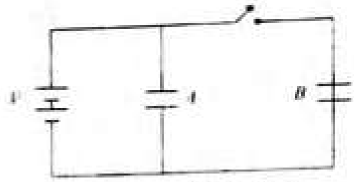


Plate 4 comes twice and it is connected with negative terminal of battery, so charge on plate 4 will be $-\frac{2\varepsilon_0 A}{d}V$

10. Figure given below shows two identical parallel plate capacitors connected to a battery with switch S closed. The switch is now opened and the free space between the plate of capacitors is filled with a dielectric of dielectric constant 3. What will be the ratio of total electrostatic energy stored in both capacitors before and after the introduction of the dielectric



(a) 3 : 1 (b) 5 : 1

(c) 3 : 5 (d) 5 : 3

Sol. (c) Initially potential difference across both the capacitor is same hence energy of the system is

$$U_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2 \quad \text{(i)}$$

In the second case when key K is opened and dielectric medium is filled between the plates, capacitance of both the capacitors becomes $3C$, while potential difference across A is V and potential difference across B is $V/3$ hence energy of the system now is

$$U_2 = \frac{1}{2}(3C)V^2 + \frac{1}{2}(3C)\left(\frac{V}{3}\right)^2 = \frac{10}{6}CV^2$$

So, $\frac{U_1}{U_2} = \frac{3}{5}$

11. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance $2C$ is connected to another battery and is charged to potential difference $2V$. The charging batteries are now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

(a) Zero (b) $\frac{25CV^2}{6}$

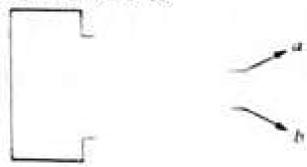
(c) $\frac{3CV^2}{2}$ (d) $\frac{9CV^2}{2}$

Sol. (c) Total charge = $(2C)(2V) + (C)(-V) = 3CV$

$\therefore$ Common potential $= \frac{3CV}{3C} = V$

$\therefore$ Energy $= \frac{1}{2}(3C)(V)^2 = \frac{3}{2}CV^2$

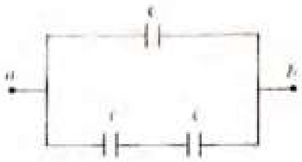
12. Four metallic plates each with a surface area of one side A are placed at a distance d from each other. The plates are connected as shown in the circuit diagram. Then the capacitance of the system between a and b is



(a) $\frac{3\varepsilon_0 A}{d}$ (b) $\frac{2\varepsilon_0 A}{d}$

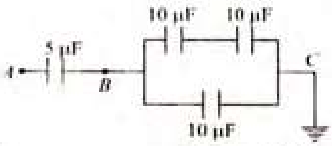
(c) $\frac{2\varepsilon_0 A}{3d}$ (d) $\frac{3\varepsilon_0 A}{2d}$

Sol.(d) The given circuit can be redrawn as follows



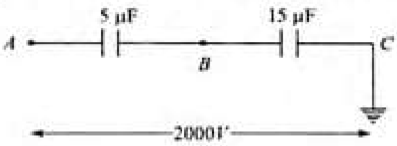
$\Rightarrow \quad C_{eq} = \frac{3C}{2} = \frac{3\varepsilon_0 A}{2d}$

13. In the given circuit if point C is connected to the earth and a potential of +2000 V is given to the point A, the potential at B is



(a) 1500 V (b) 1000 V
(c) 500 V (d) 400 V

Sol. (c) The given circuit can be redrawn as follows



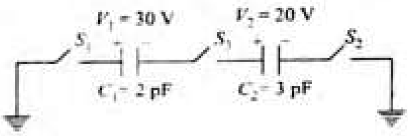
$$(V_A - V_B) = \left(\frac{15}{5+15}\right) \times 2000$$

$\Rightarrow \quad V_A - V_B = 1500\text{V}$

$\Rightarrow \quad 2000 - V_B = 1500\text{V}$

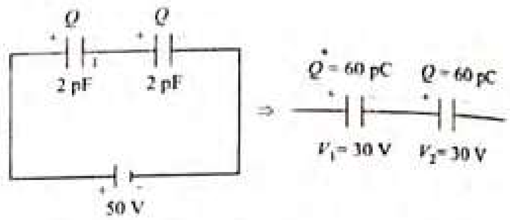
$\Rightarrow \quad V_B = 500\text{V}$

14. For the circuit shown, which of the following statements is true?



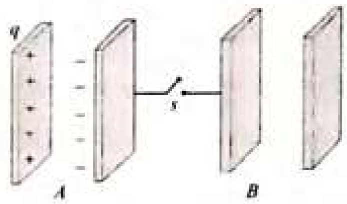
(a) With S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V
(b) With S_3 closed $V_1 = V_2 = 25$ V
(c) With S_1 and S_2 closed $V_1 = V_2 = 0$
(d) With S_1 and S_3 closed, $V_1 = 30$ V, $V_2 = 20$ V

Sol. (d) Charges on capacitors are $Q_1 = 30 \times 2 = 6$ pC and $Q_2 = 20 \times 3 = 60$ pC or $Q_1 = Q_2 = Q$(say)
The situation is situation is similar as the two capacitors in series are first charged with a battery of emf 50 V and then disconnected



Therefore, when S_3 is closed $V_1 = 30$ V and $V_2 = 20$ V

15. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long time after the switch is closed is



(a) Zero (b) $q/2$
(c) q (d) $2q$

Sol. (a) The $\pm q$ charges appearing on the inner surfaces of A are bound charges. As B is uncharged initially, as it is isolated, the charges on A will not be affected on closing the switch S. No charge will flow in to B.

EXERCISES

Electric Potential and Potential Energy

1. Four equal charges Q are placed at the four corners of a square of each side is 'a'. Work done in removing a charge $-Q$ from its centre to infinity is

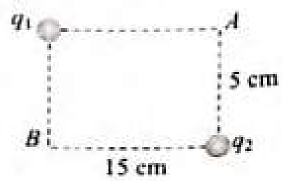
(a) 0 (b) $\frac{\sqrt{2}Q^2}{4\pi\varepsilon_0 a}$

(c) $\frac{\sqrt{2}Q^2}{\pi\varepsilon_0 a}$ (d) $\frac{Q^2}{2\pi\varepsilon_0 a}$

2. Two electric charges 12 μC and –6 μC are placed 20 cm apart in air. There will be a point P on the line joining these charges and outside the region between them, at which the electric potential is zero. The distance of P from –6 μC charge is

(a) 0.10 m (b) 0.15 m
(c) 0.20 m (d) 0.25 m

3. In the rectangle, shown below, the two corners have charges $q_1 = -5$ μC and $q_2 = +2.0$ μC. The work done in moving a charge +3.0 μC from B to A is (take $1/4\pi\varepsilon_0 = 10^{10}$ N.m²/C²)



(a) 2.8 J (b) 3.5 J
(c) 4.5 J (d) 5.5 J

4. A charge $(-q)$ and another charge $(+Q)$ are kept at two points A and B respectively. Keeping the charge $(+Q)$ fixed at B, the charge $(-q)$ at A is moved to another point C such that ABC forms an equilateral triangle of side l. The net work done in moving the charge $(-q)$ is

(a) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{l^2}$

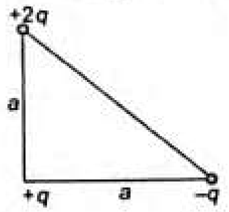
(c) $\frac{1}{4\pi\varepsilon_0}Qql$ (d) Zero

5. If identical charges $(-q)$ are placed at each corner of a cube of side b, then electric potential energy of charge $(+q)$ which is placed at centre of the cube will be

(a) $\frac{8\sqrt{2}q^2}{4\pi\varepsilon_0 b}$ (b) $\frac{-8\sqrt{2}q^2}{\pi\varepsilon_0 b}$

(c) $\frac{-4\sqrt{2}q^2}{\pi\varepsilon_0 b}$ (d) $\frac{-4q^2}{\sqrt{3}\pi\varepsilon_0 b}$

6. Three charges $+q$, $-q$, and $+2q$ are placed at the vertices of a right angled triangle (isosceles triangle) as shown. The net electrostatic energy of the configuration is



(a) $-\frac{kq^2}{a}(\sqrt{2}+1)$ (b) $\frac{kq^2}{a}(\sqrt{2}+1)$

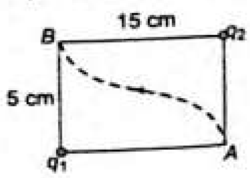
(c) $-\frac{kq^2}{a}(\sqrt{2}-1)$ (d) None of these

7. Three equal charges Q are placed at the three vertices of an equilateral triangle. What should be the value of a charge that when placed at the centroid reduces the interaction energy of the system to zero:

(a) $\frac{-Q}{2}$ (b) $\frac{-Q}{3}$

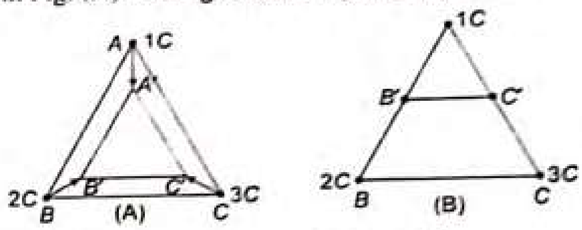
(c) $\frac{-Q}{2\sqrt{3}}$ (d) $\frac{-Q}{\sqrt{3}}$

8. In rectangle shown in the figure, the two corners have charges $q_1 = -5\ \mu C$ and $q_2 = +2.0\ \mu C$ The work done in moving a charge of $+3.0\ \mu C$ from B to A is $\left(\text{take } \frac{1}{4\pi\varepsilon_0} = 10^{10}\right)$



(a) 2.8 J
(b) 3.5 J
(c) 4.5 J
(d) 5.5 J

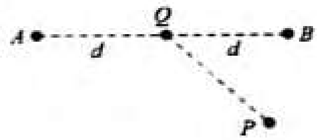
9. Three point charges $1C$, $2C$ and $3C$ are placed at the corners of an equilateral triangle of side 1 m. The work required to move these charges to the corners of a smaller equilateral triangle of side 0.5 m in two different ways as in Fig. (A) and Fig. (B) are W_a and W_b then:



(a) $W_a > W_b$ (b) $W_a < W_b$
(c) $W_a = W_b$ (d) $W_a = 0$ and $W_b = 0$

10. When a 2 μC charge is carried from point A to point B, the amount of work done by the electric field is 50 μJ. What is the potential difference and which point is at a higher potential?
(a) 25 V, B (b) 25 V, A
(c) 20 V, B (d) both are at same potential

11. The work done in taking a unit positive charge from P to A is W_A and from P to B is W_B.
Then



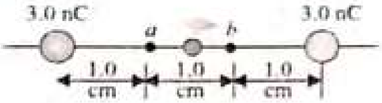
(a) $W_A > W_B$
(b) $W_A < W_B$
(c) $W_A = W_B$
(d) $W_A + W_B = 0$

12. A small conducting sphere of radius a, carrying a charge $+Q$, is placed inside an equal and oppositely charged conducting shell of radius b such that their centers coincide. Determine the potential at a point which is at a distance c from center such that $a < c < b$.
(a) $k(Q/c + Q/b)$ (b) $k(Q/a + Q/b)$
(c) $k(Q/a - Q/b)$ (d) $k(Q/c - Q/b)$

13. Two metal spheres (radii r_1, r_2 with $r_1 < r_2$) are very far apart but are connected by a thin wire. If their combined charge is Q, then what is their common potential?
(a) $kQ/(r_1 + r_2)$ (b) $kQ/(r_1 - r_2)$
(c) $-kQ/(r_1 + r_2)$ (d) $-kQ/r_1r_2$

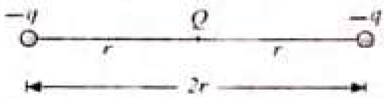
14. An uncharged conductor A is brought near a positively charged conductor B. Then
(a) the charge on B will increase, but the potential of B will not change
(b) the charge on B will not change, but the potential of B will decrease
(c) the charge on B will decrease, but the potential of B will not change
(d) the charge on B will not change, but the potential of B will increase

15. As shown in figure, a dust particle with mass $m = 5.0 \times 10^{-9}$ kg and charge $q_0 = 2.0$ nC starts from rest at point a and moves in a straight line to point b. What is its speed v at point b?



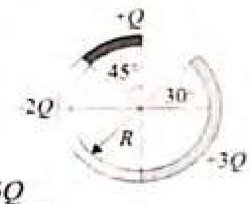
(a) $26\ \text{ms}^{-1}$ (b) $34\ \text{ms}^{-1}$
(c) $46\ \text{ms}^{-1}$ (d) $14\ \text{ms}^{-1}$

16. Charges $-q$, Q, and $-q$ are placed at an equal distance on a straight line. If the total potential energy of the system of three charges is zero, then find the ratio Q/q.



(a) 1/2 (b) 1/4
(c) 2/3 (d) 3/4

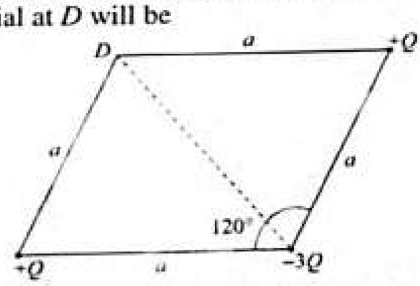
17. Figure shows three circular arcs, each of radius R and total charge as indicated. The net electric potential at the center of curvature is



(a) $\dfrac{Q}{2\pi\varepsilon_0 R}$ (b) $\dfrac{5Q}{12\pi\varepsilon_0 R}$

(c) $\dfrac{3Q}{32\pi\varepsilon_0 R}$ (d) none of these

18. A charge $+Q$ at A (see figure) produces electric field E and electric potential V at D. If we now put charges $-3Q$ and $+Q$ at B and C, respectively, then the electric field and potential at D will be



(a) E and 0 (b) $2E$ and $-V$

(c) $\sqrt{2}\,E$ and $\dfrac{V}{\sqrt{2}}$ (d) $2E$ and V

19. There is an electric field E in the x-direction. If the work done by the electric field in moving a charge of 0.2 C through a distance of 2 m along a line making an angle 60° with the x-axis is 4 J, then what is the value of E?

(a) $\sqrt{3}\ \text{NC}^{-1}$ (b) $4\ \text{NC}^{-1}$
(c) $5\ \text{NC}^{-1}$ (d) $20\ \text{NC}^{-1}$

20. Two identical rings P and Q of radius 0.1 m are mounted coaxially at a distance 0.5 m apart. The charges on the two rings are 2 μC and 4 μC, respectively. The work done in transferring a charge of 5 μC from the center of P to that of Q is

(a) 1.28 J (b) 0.72 J
(c) 0.144 J (d) 2.24 J

21. A point charge q is placed inside a conducting spherical shell of inner radius $2R$ and outer radius $3R$ at a distance of R from the center of the shell. Find the electric potential at the center of the shell.

(a) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{2R}$ (b) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{4q}{3R}$

(c) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{5q}{6R}$ (d) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{3R}$

22. An electron is taken from point A to point B along the path AB in a uniform electric field of intensity $E = 10\ \text{Vm}^{-1}$. Side $AB = 5$ m, and side $BC = 3$ m. Then, the amount of work done on the electron by us is

(a) 50 eV (b) 40 eV
(c) −50 eV (d) −40 eV

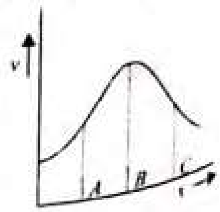
Relation Between Electric Field and Potential

23. Variation in potential is maximum if one goes
(a) along the line of force
(b) perpendicular to the line of force
(c) in any direction
(d) none of these

24. Mark the correct statement:
(a) If E is zero at a certain point, then V should be zero at that point.
(b) If E is not zero at a certain point, then V should not be zero at that point.
(c) If V is zero at a certain point, then E should be zero at that point.
(d) If V is zero at a certain point, then E may or may not be zero.

25. At a point in space, the electric field points toward north. In the region surrounding this point, the rate of change of potential will be zero along
(a) north (b) south
(c) north-south (d) east-west

26. Variation of electrostatic potential along the x-direction is shown in figure. The correct statement about electric field is

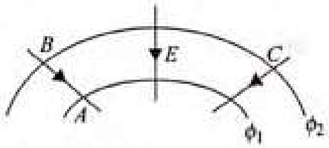


(a) x-component at point B is maximum
(b) x-component at point A is toward positive x-axis
(c) x-component at point C is along negative x-axis

(d) x-component at point C is along positive x-axis

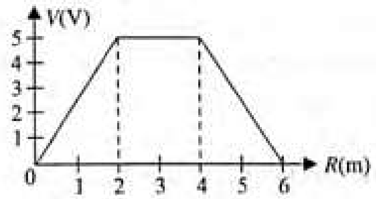
27. The electric field lines are closer together near object A than they are near object B. We can conclude that
(a) the potential near A is greater than the potential near B
(b) the potential near A is less than the potential near B
(c) the potential near A is equal to the potential near B
(d) nothing about the relative potentials near A and B

28. In moving from A to B along an electric field line, the work done by the electric field on an electron is 6.4×10^{-19} J. If ϕ_1 and ϕ_2 are equipotential surfaces, then the potential difference $V_C - V_A$ is



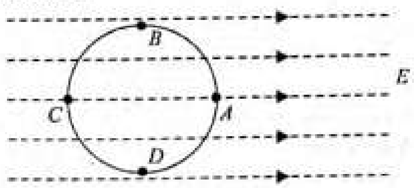
(a) −4 V (b) 4 V
(c) zero (d) 6.4 V

29. The variation of potential with distance R from the fixed point is shown in figure.



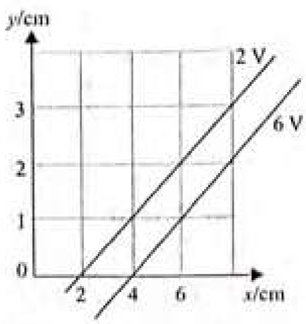
The electric field at $R = 5$ m is
(a) 2.5 Vm^{-1} (b) −2.5 Vm^{-1}
(c) 0.4 Vm^{-1} (d) −0.4 Vm^{-1}

30. The electric field in a region surrounding the origin and along the x-axis is uniform. A small circle is drawn with the center at the origin cutting the axes at points A, B, C, and D having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$, and $(0, -a)$, respectively, as shown in figure. Then the potential is minimum at



(a) A (b) B
(c) C (d) D

31. Figure shows two equipotential lines in the xy plane for an electric field. The scales are marked. The x-component and y-component of the field in the space between these equipotential lines are, respectively,



(a) $+100\ Vm^{-1}, -200\ Vm^{-1}$
(b) $-100\ Vm^{-1}, +200\ Vm^{-1}$
(c) $+200\ Vm^{-1}, +100\ Vm^{-1}$
(d) $-200\ Vm^{-1}, -400\ Vm^{-1}$

32. The potential function of an electrostatic field is given by $V = 2x^2$. Determine the electric field strength at the point (2 m, 0, 3 m).
(a) $\vec{E} = 4\hat{i}\ (NC^{-1})$ (b) $\vec{E} = -4\hat{i}\ (NC^{-1})$
(c) $\vec{E} = 8\hat{i}\ (NC^{-1})$ (d) $\vec{E} = -8\hat{i}\ (NC^{-1})$

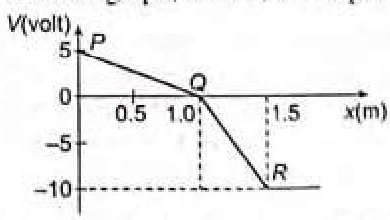
33. The potential V is varying with x as $V = \frac{1}{2}(y^2 - 4x)$ volt. The field at $x = 1$ m, $y = 1$ m, is
(a) $2\hat{i} + \hat{j}\ Vm^{-1}$ (b) $-2\hat{i} + \hat{j}\ Vm^{-1}$
(c) $2\hat{i} - \hat{j}\ Vm^{-1}$ (d) $-2\hat{i} + 2\hat{j}\ Vm^{-1}$

34. An electric field is expressed as $\vec{E} = 2\hat{i} + 3\hat{j}$. Find the potential difference $(V_A - V_B)$ between two points A and B whose position vectors are given by $r_a = \hat{i} + 2\hat{j}$ and $r_B = 2\hat{i} + \hat{j} + 3\hat{k}$.
(a) −1 V (b) 1 V
(c) 2 V (d) 3 V

35. Find the potential V of an electrostatic field $\vec{E} = a(y\hat{i} + x\hat{j})$, where a is a constant.
(a) $axy + C$ (b) $-axy + C$
(c) axy (d) $-axy$

36. Electrical potential V in space as a function of co-ordinates is given by, $V = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$. Then the electric field intensity at (1, 1, 1) is given by
(a) $-(\hat{i} + \hat{j} + \hat{k})$ (b) $\hat{i} + \hat{j} + \hat{k}$
(c) zero (d) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

37. In an electric field region, the electric potential varies along the x axis as shown in the graph. The x components of the electric field in the regions for the intervals PQ and QR as marked in the graph, in N/C, are respectively:



(a) 5.0 along negative x-direction and 20.0 along positive x-direction
(b) 5.0 along positive x-direction and 20.0 along negative x-direction
(c) 5.0 along negative x-direction and 20.0 along negative x-direction
(d) 5.0 along positive x-direction and 20.0 along positive x-direction

38. The potential field of an electric field $\vec{E} = (y\hat{i} + x\hat{j})$ is
(a) $V = -xy$ + constant
(b) $V = -(x + y)$ + constant
(c) $V = -(x^2 + y^2)$ + constant
(d) V = constant

Capacitance and Combination of Capacitors

39. A parallel plate capacitor is connected across a battery. Now, keeping the battery connected, a dielectric slab is inserted between the plates. In this process,
    (a) no work is done
    (b) work is done by the battery, and the stored energy increases
    (c) work is done by the external agent, and the stored energy decreases
    (d) work is done by the battery as well as the external agent, but the stored energy does not change

40. A parallel plate capacitor is charged and then disconnected from the source of potential difference. If the plates of the condenser are then moved farther apart by the use of insulated handle, which of the following is true?
    (a) The charge on the capacitor increases.
    (b) The charge on the capacitor decreases.
    (c) The capacitance of the capacitor increases.
    (d) The potential difference across the plates increases.

41. Seven capacitors, each of capacitance 2 μF, are to be combined to obtain a capacitance of 10/11 μF. Which of the following combinations is possible?
    (a) 2 in parallel, 5 in series
    (b) 3 in parallel, 4 in series
    (c) 4 in parallel, 3 in series
    (d) 5 in parallel, 2 in series

42. A parallel plate capacitor has plates of area A and separation d and is charged to a potential difference V. The charging battery is then disconnected and the plates are pulled apart until their separation is $2d$. What is the work required to separate the plates?
    (a) $2\varepsilon_0 AV^2/d$ (b) $\varepsilon_0 AV^2/d$
    (c) $3\varepsilon_0 AV^2/2d$ (d) $\varepsilon_0 AV^2/2d$

43. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. What is the common potential (in V) and energy lost (in J) after reconnection?
    (a) 100, 6×10^{-6} (b) 200, 6×10^{-5}
    (c) 200, 5×10^{-6} (d) 100, 6×10^{-5}

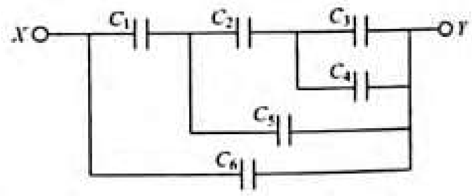
44. Two parallel plate capacitors of capacitances C and $2C$ are connected in parallel and charged to a potential difference V. The battery is then disconnected, and the region between the plates of C is filled completely with a material of dielectric constant K. The common potential difference across the combination becomes
    (a) $\dfrac{2V}{K+2}$ (b) $\dfrac{V}{K+2}$
    (c) $\dfrac{3V}{K+3}$ (d) $\dfrac{3V}{K+2}$

45. A capacitor of capacitance $C_1 = 1$ μF charged up to a voltage $V = 110$ V is connected in parallel to the terminals of a circuit consisting of two uncharged capacitors connected in series and possessing capacitances $C_2 = 2$ μF and $C_3 = 3$ μF. Then, the amount of charge that will flow through the connecting wires is
    (a) 40 μC (b) 50 μC
    (c) 60 μC (d) 110 μC

46. Ten capacitors are joined in parallel and charged with a battery up to a potential V. They are then disconnected from the battery and joined in series. Then, the potential of this combination will be
    (a) 1 V (b) 10 V
    (c) 5 V (d) 2 V

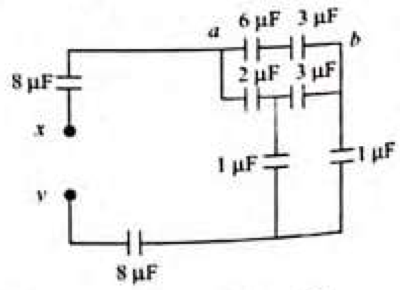
47. Two identical parallel plate capacitors are connected in series and then joined in series with a battery of 100 V. A slab of dielectric constant $K = 3$ is inserted between the plates of the first capacitor. Then, the potential difference across the capacitors will be, respectively,
    (a) 25 V, 75 V (b) 75 V, 25 V
    (c) 20 V, 80 V (d) 50 V, 50 V

48. In the given network of capacitors as shown in figure, given that $C_1 = C_2 = C_6 = 600$ pC and $C_3 = C_4 = C_5 = 300$ pF. The effective capacitance of the circuit between X and Y is



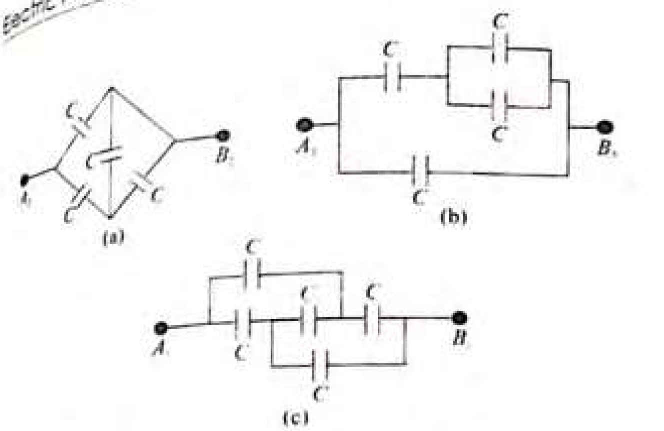
    (a) 450 pF (b) 750 pF
    (c) 600 pF (d) 900 pF

49. In the circuit shown, the effective capacitance between points X and Y is



    (a) 3.33 μF (b) 1 μF
    (c) 0.44 μF (d) none of these

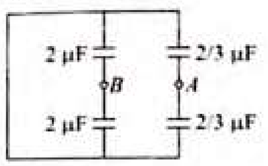
50. In figure, identical capacitors are connected in the following three configurations.



The ratio of the total capacitances in (i), (ii), and (iii), respectively, is

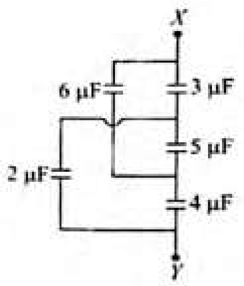
(a) 3 : 5 : 5 (b) 3 : 3 : 5
(c) 5 : 4 : 4 (d) 5 : 5 : 3

51. The equivalent capacitance of the circuit across the terminals A and B is equal to



(a) 0.5 µF (b) 2 µF
(c) 1 µF (d) none of these

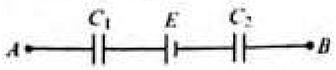
52. The equivalent capacitance between points X and Y in figure is



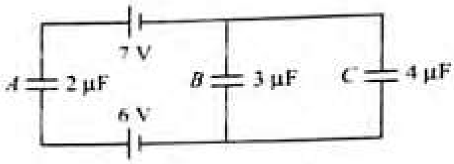
(a) $\frac{6}{5}$ µF (b) 4 µF
(c) $\frac{18}{5}$ µF (d) none of these

53. For section AB of a circuit shown in figure, $C_1 = 1$ µF, $C_2 = 2$ µF, $E = 10$ V, and the potential difference $V_A - V_B = -10$ V. Charge on capacitor C_1 is

(a) 0 µC
(b) 20/3 µC
(c) 40/3 µC
(d) none of these

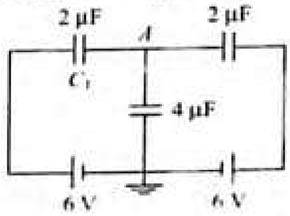


54. Three capacitors A, B, and C are connected in a circuit as shown in figure. What is the charge in µC on the capacitor B?



(a) 1/3 (b) 2/3
(c) 1 (d) 4/3

55. Three capacitors are connected as shown in figure. Then, the charge on capacitor C_1 is



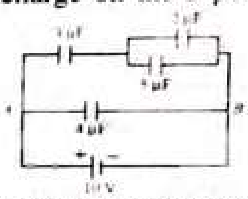
(a) 6 µC (b) 12 µC
(c) 18 µC (d) 24 µC

56. In the above question, the potential of point A is

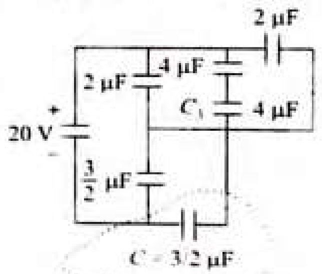
(a) 3 V (b) 6 V
(c) 9 V (d) zero

57. In the circuit shown in figure, the charge on the 5 µF capacitor will be

(a) 4.5 µC
(b) 9 µC
(c) 15 µC
(d) none of these

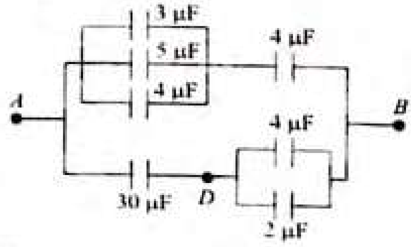


58. In figure, the battery has a potential difference of 20 V. The charge in the capacitor marked as C is



(a) 20 µC (b) 40 µC
(c) 10 µC (d) none of these

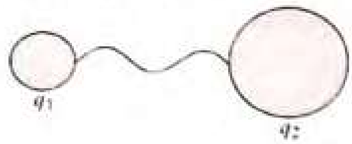
59. Several capacitors are connected as shown in figure. If the charge on the 5 µF capacitor is 120 µC, the potential between points A and D is



(a) 16 V (b) 32 V
(c) 64 V (d) none of these

Problems Based on Mixed Concepts

60. Two spherical conductors of radii R_1 and R_2 are separated by a distance much larger than the radius of either sphere. The spheres are connected by a conducting wire as shown in figure. If the charges on the spheres in equilibrium are q_1 and q_2, respectively, what is the ratio of the field strength at the surfaces of the spheres?



(a) R_2/R_1 (b) R_2^2/R_1^2

(c) R_1/R_2 (d) R_1^2/R_2^2

61. n charged drops, each of radius r and charge q, coalesce to form a big drop of radius R and charge Q. If V is the electric potential and E is the electric field at the surface of a drop, then

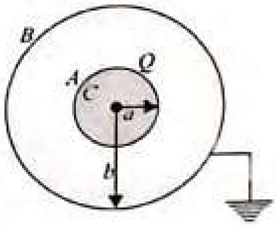
(a) $E_{big} = n^{2/3} E_{small}$ (b) $V_{big} = n^{1/3} V_{small}$

(c) $E_{small} = n^{2/3} E_{big}$ (d) $V_{big} = n^{2/3} V_{small}$

62. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common center of A and B. Study the following statements.

i. The potential at a distance r from C, where $a \le r \le b$, is $\frac{1}{4\pi\varepsilon_0}\left|\frac{Q}{r}\right|$

ii. The potential difference between A and B is $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{a}-\frac{1}{b}\right)$



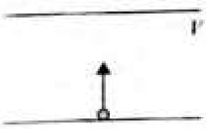
iii. The potential at a distance r from C, where $a \le r \le b$, is $\frac{1}{4\pi\varepsilon_0}Q\left(\frac{1}{r}-\frac{1}{b}\right)$

Which of the following statements are correct?

(a) Only (i) and (ii) (b) Only (ii) and (iii)

(c) Only (i) and (iii) (d) All

63. An electron having charge e and mass m starts from the lower plate of two metallic plates separated by a distance d. If the potential difference between the plates is V, the time taken by the electron to reach the upper plate is given by



(a) $\sqrt{\frac{2md^2}{eV}}$ (b) $\sqrt{\frac{md^2}{eV}}$

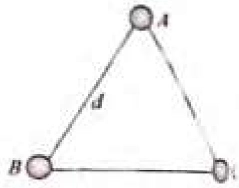
(c) $\sqrt{\frac{md^2}{2eV}}$ (d) $\frac{2md^2}{eV}$

64. There is an infinite straight chain of alternating charges q and $-q$. The distance between the two neighbouring charges is equal to a. Find the interaction energy of any charge with all the other charges.

(a) $-\frac{2q^2}{4\pi\varepsilon_0 a}$ (b) $\frac{2q^2\log_e 2}{4\pi\varepsilon_0 a}$

(c) $-\frac{2q^2\log_e 2}{4\pi\varepsilon_0 a}$ (d) none of these

65. Three identical metallic uncharged spheres A, B, and C each of radius a, are kept at the corners of an equilateral triangle of side d ($d >> a$) as shown in figure. The fourth sphere (of radius a), which has a charge q, touches A and is then removed to a position far away. B is earthed and then the earth connection is removed. C is then earthed. The charge on C is



(a) $\frac{qa}{2d}\left(\frac{2d-a}{2d}\right)$

(b) $\frac{qa}{2d}\left(\frac{2d-a}{d}\right)$

(c) $-\frac{qa}{2d}\left(\frac{d-a}{d}\right)$

(d) $\frac{2qa}{d}\left(\frac{d-a}{2d}\right)$

66. A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge $q = 20$ μC is given to the inner sphere. Find the heat generated in the process when the inner sphere is connected to the shell by a conducting wire.

(a) 12 J (b) 9 J

(c) 24 J (d) zero

67. We have three identical metallic spheres A, B, and C. A is given a charge Q, and B and C are uncharged. The following processes of touching of two spheres are carried out in succession. Each process is carried out with sufficient time.

i. A and B ii. B and C

iii. C and A iv. A and B

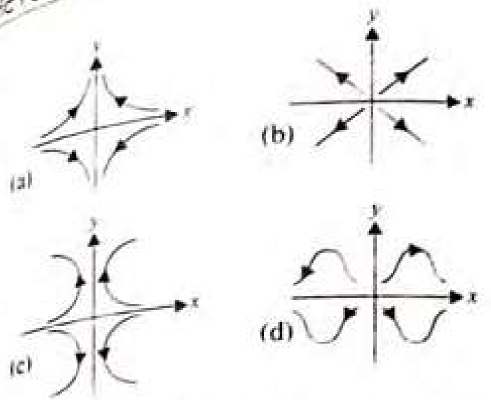
v. B and C

The final charges on the spheres are

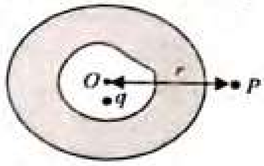
(a) $\frac{11Q}{32}, \frac{5Q}{16}, \frac{11Q}{32}$ (b) $\frac{11Q}{32}, \frac{11Q}{32}, \frac{5Q}{16}$

(c) $\frac{8Q}{8}, \frac{5Q}{16}, \frac{5Q}{16}$ (d) $\frac{5Q}{16}, \frac{11Q}{32}, \frac{11Q}{32}$

68. The potential field depends on the x- and y-coordinates as $V = x^2 - y^2$. The corresponding electric field lines in x-y plane are as



69. The point charge q is within the cavity of an electrically neutral conducting shell whose outer surface has spherical shape. Find the potential V at a point P lying outside the shell at a distance r from the center O of the outer sphere.



(a) $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ (b) $V = \frac{1}{2\pi\varepsilon_0}\frac{q}{r}$

(c) $V = \frac{1}{\pi\varepsilon_0}\frac{q}{r}$ (d) $V = \frac{3}{4\pi\varepsilon_0}\frac{q}{r}$

70. Two concentric conducting spherical shells of radii a_1 and a_2 $(a_2 > a_1)$ are charged to potentials ϕ_1 and ϕ_2, respectively. Find the charge on the inner shell.

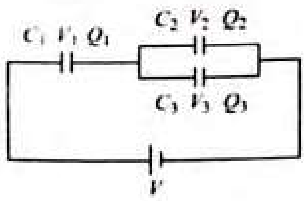
(a) $q_1 = \pi\varepsilon_0\left(\frac{\phi_1}{a_2} - \frac{\phi_2}{a_1}\right)a_1a_2$

(b) $q_1 = 4\pi\varepsilon_0\left(\frac{\phi_1+\phi_2}{a_2+a_1}\right)a_1a_2$

(c) $q_1 = \pi\varepsilon_0\left(\frac{\phi_1-\phi_2}{a_2+a_1}\right)a_1a_2$

(d) $q_1 = 2\pi\varepsilon_0\left(\frac{\phi_1+\phi_2}{a_2-a_1}\right)a_1a_2$

71. In figure, three capacitors C_1, C_2, and C_3 are joined to a battery. With symbols having their usual meaning, the correct conditions will be



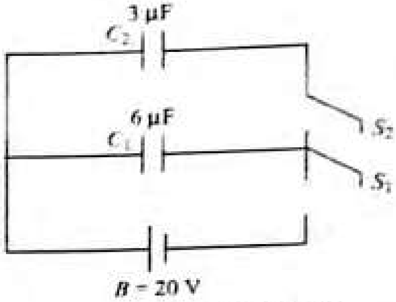
(a) $Q_1 = Q_2 = Q_3$ and $V_1 = V_2 = V_3 + V$

(b) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2 + V_3$

(c) $Q_1 = Q_2 + Q_3$ and $V = V_1 + V_2$

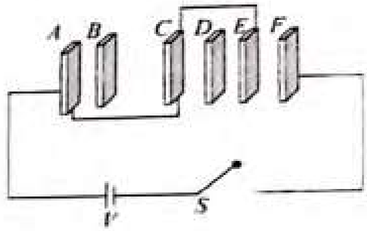
(d) $Q_2 = Q_3$ and $V_2 = V_3$

72. In the circuit shown in figure, $C_1 = 6\ \mu F$, $C_2 = 3\ \mu F$, and battery $B = 20$ V. The switch S_1 is first closed. It is then opened, and S_2 is closed. What is the final charge on C_2?



(a) 120 μC (b) 80 μC
(c) 40 μC (d) 20 μC

73. A, B, C, D, E, and F are conducting plates each of area A, and any two consecutive plates are separated by a distance d. The net energy stored in the system after the switch S is closed is



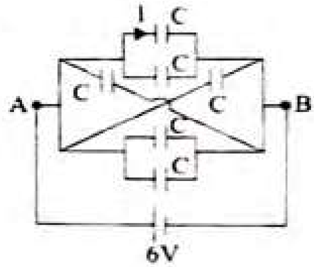
(a) $\frac{3\varepsilon_0 A}{2d}V^2$

(b) $\frac{5\varepsilon_0 A}{12d}V^2$

(c) $\frac{\varepsilon_0 A}{2d}V^2$

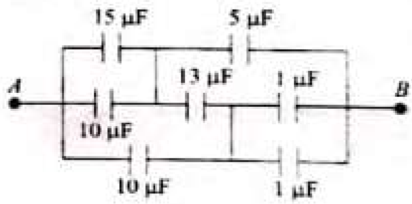
(d) $\frac{\varepsilon_0 A}{d}V^2$

74. Six capacitors each of capacitance 1 μF are connected as shown in figure. Find the charge flowing in direction 1 as shown in the figure will be



(a) 12 μC
(b) 6 μC
(c) 3 μC
(d) none of these

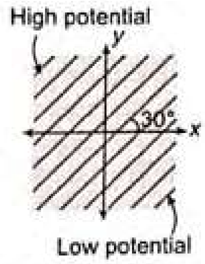
75. Find the equivalent capacitance across A and B.

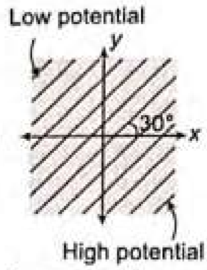


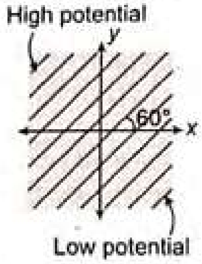
(a) $\frac{35}{6}$ μF (b) $\frac{25}{6}$ μF

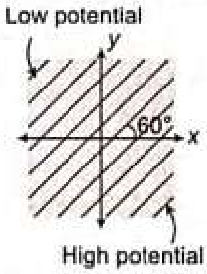
(c) 15 μF (d) none of these

76. The electric field intensity at all points in space is given by $\vec{E} = \sqrt{3}\,\hat{i} - \hat{j}$ volts/metre. The nature of equipotential lines in x-y plane is given by

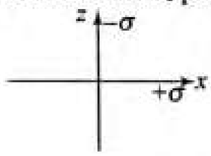
(a) 

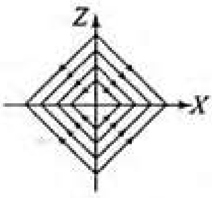
(b) 

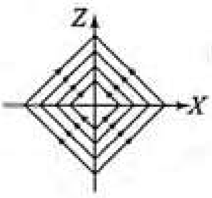
(c) 

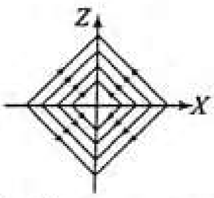
(d) 

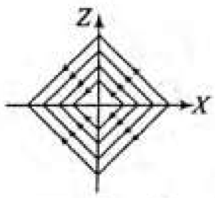
77. Two infinitely large charged planes having uniform surface charge density $+\sigma$ and $-\sigma$ are placed along x-y plane and yz plane respectively as shown in the figure. Then the nature of electric lines of forces in x-z plane is given by:



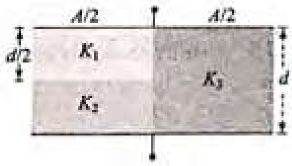
(a) 

(b) 

(c) 

(d) 

78. In the figure, a capacitor is filled with dielectrics. The resultant capacitance is



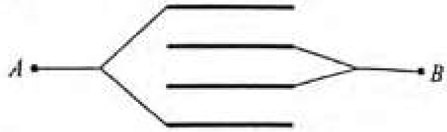
(a) $\dfrac{2\varepsilon_0 A}{d}\left[\dfrac{1}{K_1}+\dfrac{1}{K_2}+\dfrac{1}{K_3}\right]$

(b) $\dfrac{\varepsilon_0 A}{d}\left[\dfrac{1}{K_1}+\dfrac{1}{K_2}+\dfrac{1}{K_3}\right]$

(c) $\dfrac{2\varepsilon_0 A}{d}[K_1+K_2+K_3]$

(d) None of these

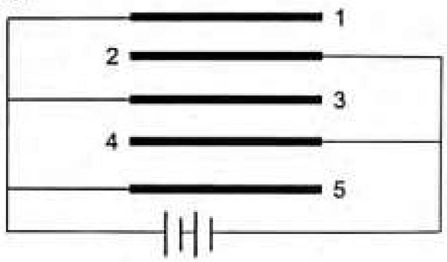
79. Four plates of equal area A are separated by equal distances d and are arranged as shown in the figure. The equivalent capacity is



(a) $\dfrac{2\varepsilon_0 A}{d}$ (b) $\dfrac{3\varepsilon_0 A}{d}$

(c) $\dfrac{3\varepsilon_0 A}{2d}$ (d) $\dfrac{\varepsilon_0 A}{d}$

80. Five identical plates are connected across a battery as follows:



If the charge on plate 1 be $+q$, then the charges on the plates 2,3,4 and 5 are

(a) $-q, +q, +q$

(b) $-2q, +2q, -2q, +q$

(c) $-q, +2q, -2q, +q$

(d) None of the above

ARCHIVES

1. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium, then the value of q is

(a) $\dfrac{Q}{2}$ (b) $\dfrac{-Q}{2}$

(c) $\dfrac{Q}{4}$ (d) $\dfrac{-Q}{4}$ **(AIEEE 2002)**

2. If there are n capacitors in parallel connected to a V volt source, then the energy stored is equal to

(a) CV (b) $\dfrac{1}{2}nCV^2$

(c) CV^2 (d) $\dfrac{1}{2n}CV^2$ **(AIEEE 2002)**

3. The capacitance (in F) of a spherical conductor of radius 1 m is

(a) 1.1×10^{-10} (b) 10^{-6}

(a) 9×10^{-9} (d) 10^{-3} **(AIEEE 2002)**

4. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is

(a) $\frac{2Q}{4\pi\varepsilon_0 R}$ (b) $\frac{2Q}{4\pi\varepsilon_0 R} - \frac{2q}{4\pi\varepsilon_0 R}$

(c) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$ (d) $\frac{(q+Q)^2}{4\pi\varepsilon_0 R}$ **(AIEEE 2003)**

5. The work done in placing a charge of 8×10^{-18} C on a condenser of capacity 100 μF is
(a) 16×10^{-32} J (b) 3.1×10^{-26} J
(c) 4×10^{-10} J (d) 32×10^{-32} J
(AIEEE 2003)

6. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor
(a) decreases (b) remains unchanged
(c) becomes infinite (d) increases **(AIEEE 2003)**

7. A charged oil drop is suspended in a uniform field of 3×10^4 V/m so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge as 9.9×10^{-15} kg and g as 10 m/s^2)
(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C
(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C
(AIEEE 2004)

8. A charged particle q is fired towards another charged particle Q, which is fixed, with a speed v. It approaches Q up to a closest distance r and then returns. If q was given a speed $2v$, the closest distance of approach would be
(a) r (b) $2r$
(c) $r/2$ (d) $r/4$ **(AIEEE 2004)**

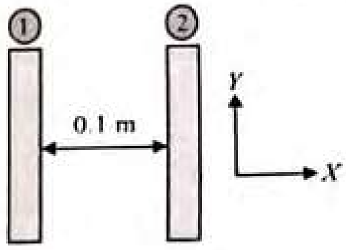
9. Two thin wire rings, each of radius R, are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is
(a) $\frac{qR}{4\pi\varepsilon_0 d^2}$
(b) $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$
(c) zero
(d) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ **(AIEEE 2005)**

10. A parallel plate capacitor is made by stacking n equally spaced plates connected alternately. If the capacitance between any two adjacent plates is C, then the resultant capacitance is
(a) $(n-1)C$ (b) $(n+1)C$
(c) C (d) nC **(AIEEE 2005)**

11. A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass 'm'. If the temperature of the block is raised by 'ΔT', the potential difference 'V' across the capacitance is
(a) $\frac{ms\Delta T}{C}$ (b) $\sqrt{\frac{2ms\Delta T}{C}}$
(c) $\sqrt{\frac{2mC\Delta T}{s}}$ (d) $\frac{mC\Delta T}{s}$ **(AIEEE 2005)**

12. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitude of the electric fields at the surface of spheres A and B is
(a) 4 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 4 **(AIEEE 2006)**

13. Two insulating plates are uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V (i.e., plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is the speed when it hits plate 2?



($e = 1.6 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg)
(a) 2.65×10^6 m/s (b) 7.02×10^{12} m/s
(c) 1.87×10^6 m/s (d) 32×10^{-19} m/s
(AIEEE 2006)

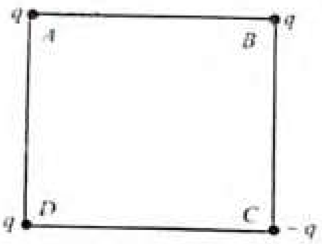
14. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by

$$V(x) = \frac{20}{(x^2-4)} V$$

The electric field E at $x = 4$ μm is given by
(a) 5/3 V/μm and in the positive x-direction.
(b) 10/9 V/μm and in the negative x-direction.
(c) 10/9 V/μm and in the positive x-direction.
(d) 5/3 V/μm and in the negative direction.
(AIEEE 2007)

15. An electric charge 10^{-3} μC is placed at the origin (0, 0) of x-y coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0), respectively. The potential difference between the points A and B will be
(a) zero (b) 2 V
(c) 4.5 V (d) 9 V **(AIEEE 2007)**

16. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C, respectively, then



(a) both $\vec{E}$ and V change.
(b) $\vec{E}$ and V remain unchanged.
(c) $\vec{E}$ changes, V remains unchanged.
(d) $\vec{E}$ remains unchanged, V changes. **(AIEEE 2007)**

17. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

(a) $\dfrac{CV^2(K-1)}{K}$ (b) $(K-1)CV^2$
(c) zero (d) $\dfrac{1}{2}(K-1)CV^2$

(AIEEE 2007)

18. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be

(a) 2 (b) $\dfrac{1}{4}$
(c) $\dfrac{1}{2}$ (d) 1 **(AIEEE 2007)**

19. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $k_2 = 6$ and thickness $2d/3$. The capacitance of the capacitor is now

(a) 20.25 pF (b) 1.8 pF
(c) 45 pF (d) 40.5 pF **(AIEEE 2008)**

20. This question contains Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1: For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement 2: The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement 1 is true, statement 2 is false.
(b) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
(c) Statement 1 is true, statement 2 is true; statement 2 is not the correct explanation for statement 1.
(d) Statement 1 is false, statement 2 is true.

(AIEEE 2009)

21. Two points P and Q are maintained at the potentials of 10 V and − 4 V, respectively. The work done in moving 100 electrons from P to Q is

(a) -19×10^{-17} J (b) 9.60×10^{-17} J
(c) -2.24×10^{-16} J (d) 2.24×10^{-16} J

(AIEEE 2009)

22. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is

(a) $-24\pi a\varepsilon_0 r$ (b) $-6\pi a\varepsilon_0 r$
(c) $-24\pi a\varepsilon_0$ (d) $-6a\varepsilon_0$ **(AIEEE 2011)**

23. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is

(a) $\dfrac{3Q}{4\pi\varepsilon_0 L}$ (b) $\dfrac{Q}{4\pi\varepsilon_0 L \ln 2}$
(c) $\dfrac{Q\ln 2}{4\pi\varepsilon_0 L}$ (d) $\dfrac{Q}{8\pi\varepsilon_0 L}$ **(JEE Main 2013)**

24. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then

(a) $3C_1 = 5C_2$ (b) $3C_1 + 5C_2 = 0$
(c) $9C_1 = 4C_2$ (d) $5C_1 = 3C_2$

(JEE Main 2013)

25. Assume that an electric field $\vec{E} = 30x^2\,\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is

(a) −80 J (b) 80 J
(c) 120 J (d) −120 J **(JEE Main 2014)**

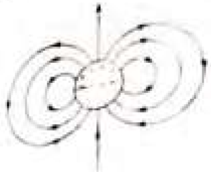
26. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to

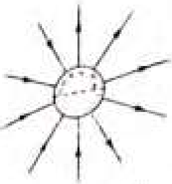
(a) 3×10^4 C/m^2 (b) 6×10^4 C/m^2
(c) 6×10^{-7} C/m^2 (d) 3×10^{-7} C/m^2

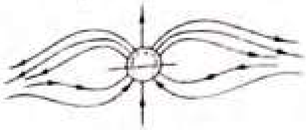
(JEE Main 2014)

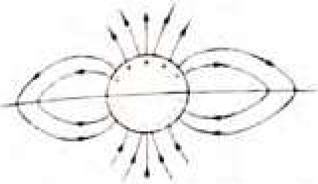
27. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in:

(figures are schematic and not drawn to scale)

(a) 

(b) 

(c) 

(d) 

(JEE Main 2015)

28. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

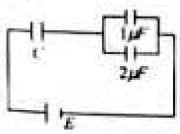
(a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

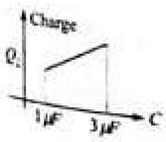
(b) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

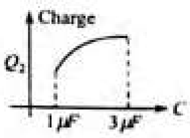
(c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(d) $2R < R_4$ **(JEE Main 2015)**

29. In the given circuit, charge Q_2 on the 2 μF capacitor changes as C is varied from 1 μF to 3 μF. Q_2 as a function of 'C' is given properly by: (figures are drawn schematically and are not to scale)



(a) 

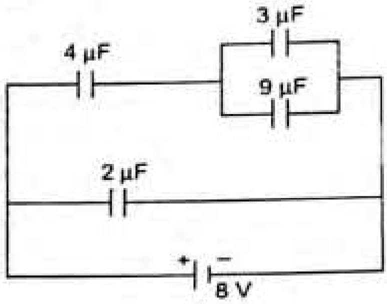
(b) 

(c) 

(d) 

(JEE Main 2015)

30. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal



(a) 240 N/C (b) 360 N/C
(c) 420 N/C (d) 480 N/C

(JEE Main 2016)

31. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is

(a) 24 (b) 32
(c) 2 (d) 16 **(JEE Main 2017)**

32. Three concentric metal shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is

(a) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$ (b) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

(c) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$ (d) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

(JEE Main 2018)

33. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant K = 5/3 is inserted between the plates, the magnitude of the induced charge will be

(a) 0.9 nC (b) 1.2 nC
(c) 0.3 nC (d) 2.4 nC **(JEE Main 2018)**

ANSWER KEY

Exercises

1. (c)	2. (c)	3. (a)	4. (d)	5. (d)	6. (c)	7. (d)	8. (a)	9. (c)	10. (b)
11. (c)	12. (d)	13. (a)	14. (b)	15. (c)	16. (b)	17. (a)	18. (b)	19. (d)	20. (b)
21. (c)	22. (b)	23. (a)	24. (d)	25. (d)	26. (d)	27. (d)	28. (b)	29. (a)	30. (a)
31. (b)	32. (d)	33. (c)	34. (a)	35. (b)	36. (b)	37. (d)	38. (a)	39. (b)	40. (d)
41. (d)	42. (d)	43. (a)	44. (d)	45. (c)	46. (b)	47. (a)	48. (d)	49. (b)	50. (d)
51. (c)	52. (c)	53. (c)	54. (b)	55. (a)	56. (a)	57. (c)	58. (a)	59. (a)	60. (a)
61. (d)	62. (b)	63. (a)	64. (c)	65. (c)	66. (b)	67. (d)	68. (a)	69. (a)	70. (a)
71. (c)	72. (c)	73. (c)	74. (b)	75. (a)	76. (c)	77. (c)	78. (d)	79. (a)	80. (b)

Archives

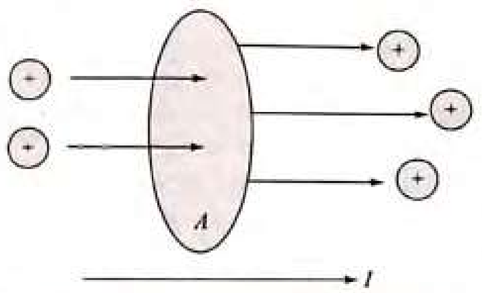
1. (d)	2. (b)	3. (a)	4. (c)	5. (d)	6. (b)	7. (a)	8. (d)	9. (b)	10. (a)
11. (b)	12. (c)	13. (a)	14. (c)	15. (a)	16. (c)	17. (c)	18. (c)	19. (d)	20. (b)
21. (d)	22. (d)	23. (c)	24. (a)	25. (none)	26. (c)	27. (a)	28. (c, d)	29. (b)	30. (c)
31. (b)	32. (c)	33. (b)							

Chapter 19

Electric Resistance and Simple Circuits

ELECTRIC CURRENT

An isolated metallic conductor, say a wire, contains a few electrons moving at random with high speeds. These are called conduction electrons. The rate at which these electrons pass from left to right through an area in a wire is the same as the rate at which they pass from right to left through the same area, i.e., net rate is zero.



Charges in motion through an area A. The time rate at which a charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

To define current mathematically, suppose that the charged particles are moving perpendicular to a surface of area A as shown in figure (this area could be the cross-sectional area of a wire, for example). The current is defined as the rate at which electric charge flows through this surface. If ΔQ is the amount of charge that passes through this area in time interval Δt, then the average current, I_{avg}, over this time interval through this area is the ratio of the charge to the time interval, i.e.,

$$\boxed{I_{avg} = \frac{\Delta Q}{\Delta t}} \quad \text{(i)}$$

From Eq. (i), total charge flowing in a time interval can be found.

Important Points

- The particles flowing through a surface can be charged positively or negatively, or we can have two or more types of particles moving, with charges of both the signs in the flow. Conventionally, we define the direction of the current as the direction of flow of positive charges.
- In a common conductor such as copper, the current is in physical state due to the motion of the negatively charged electrons. Therefore, when we speak of current in such a conductor, the direction of the current is opposite to the direction of flow of electrons.
- On the other hand, if one considers a beam of positively charged protons in a particle accelerator, then the current is in the direction of the motion of the protons.
- In some cases, gases and electrolytes, for example, the current is the result of the flow of both positive and negative charges.
- It is common to refer to a moving charged particle (whether it is positive or negative) as a mobile charge carrier. For example, the charge carriers in a metal are electrons.

Unit of Electric Current

In SI system, the unit of current is ampere (A).

"*The current is said to be 1 A when 1 C of charge flows past any cross section of a conductor every second.*"

Statement of Ohm's Law

"The electric current in any conductor is proportional to the potential difference between its ends, other factors remaining constant."

The ratio of the potential difference to current is termed the resistance of the conductor. Accordingly,

$$\boxed{R = \frac{V}{I} \text{ or } V = IR}$$

where ~~I is the~~ current, V is the potential difference, and R is the resistance. The resistance of the conductor is the opposition offered by the conductor to the flow of electric current passing through it. The resistance R not only depends on the material of the conductor but also on the dimensions of the conductor.

The resistance of an ohmic conducting wire is found to be proportional to its length ℓ and inversely proportional to its cross-sectional area A, i.e.,

$$\boxed{R = \rho\frac{\ell}{A}} \quad \text{(i)}$$

where the constant of proportionality ρ is called the *resistivity* of the material, which has the unit ohm meter (Ωm). To

understand the relationship between resistance and resistivity, we should know that ρ depends on the properties of the material and on temperature. On the other hand, the resistance R of a particular conductor depends on its size and shape as well as on the resistivity of the material.

The inverse of resistivity is defined as *conductivity* σ. Hence, the resistance of an ohmic conductor can be expressed in terms of its conductivity as

$$R = \frac{\ell}{\sigma A}, \quad \text{where } \sigma = \frac{1}{\rho}$$

Resistance and resistivity: Resistivity is a property of a substance, whereas resistance is a property of an object. We have seen similar pairs of variables before. For example, density is a property of a substance, whereas mass is a property of an object. Equation (i) relates resistance to resistivity.

SI Unit of Resistance

The unit of resistance is ohm (Ω).

"*The resistance of a conductor is said to be 1 Ω if a current of 1 A flows through it when the potential difference across its ends is 1 V.*"

Dependence of Resistance on Various Factors

We have $\quad R = \rho \dfrac{\ell}{A}$

Therefore, the relation of resistance with length and area of cross section will be as follows:

$R \propto \ell$ and $R \propto 1/A$, which gives $R \propto \ell/A$.

Change in Resistance

On stretching a wire keeping volume (V) constant

If the length of wire is changed,

$$R \propto \frac{\ell}{A}\frac{\ell}{\ell} = \frac{\ell^2}{V}$$

or $\quad R \propto \ell^2 \quad$ Hence, $\dfrac{R_1}{R_2} = \dfrac{\ell_1^2}{\ell_2^2}$

If the radius of cross section is changed

$$R \propto \frac{\ell}{A}\frac{A}{A} = \frac{V}{A^2} \text{ or } R \propto \frac{1}{A^2}$$

If the radius of the wire is r, then $A \propto r^2$ and $R \propto 1/r^4$.

Hence, $\dfrac{R_1}{R_2} = \dfrac{r_2^4}{r_1^4}$

where R_1 and R_2 are initial and final resistances, ℓ_1 and ℓ_2, are initial and final lengths, and r_1 and r_2 are initial and final radii, respectively.

Effect of percentage change in length of wire

$$\frac{R_2}{R_1} = \frac{\ell^2\left[1 + \frac{x}{100}\right]^2}{\ell^2}$$

where ℓ is the original length and x is the percentage increment in length. If x is quite small (say < 5%), then percentage change in resistance is

$$\frac{\Delta R}{R}\% = \frac{R_2 - R_1}{R_1} \times 100 = \frac{\left(1 + \frac{x}{100}\right)^2 - 1}{1} \cong 2x\%$$

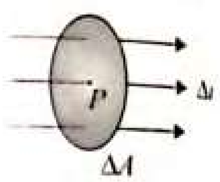
CURRENT DENSITY

Current density at a point is defined as the amount of current flowing per unit area around that point, provided the area is normal to the direction of current. It is denoted by J. So we can write $J = I/A$, where I is current and A is area. The unit of current density is Am^{-2}.

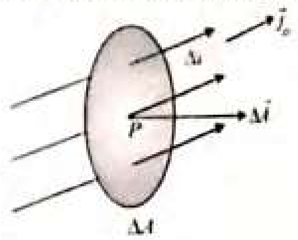
Current density is a vector quantity. So it must have direction also. Its direction at a point is the direction of motion of the positive charge or direction of current at that point.

Average current density: Let through an area ΔA around a point P, a current Δi is passing normal to the area. Then the average current density over the area is given by $J_{av} = \Delta i/\Delta A$.

Current density at a point: Now suppose we want to find current density at point P in the preceding figure. Take an infinitesimally small area dA around point P. Let the current through this small area be di. Then the current density at point P is given by $J = di/dA$.



Current not perpendicular to area: Here we divide the current by the component of area in the direction of the current. Hence, the average current density is given by



$$J_{av} = \frac{\Delta i}{\Delta A \cos\theta}$$

We can also write

$$\Delta i = J_{av}\Delta A\cos\theta = \vec{J}_{av} \cdot \Delta\vec{A}$$

Current density at P: Take an infinitesimally small area dA around point P. Let the current through this small area be di. Then the current density at point P is given by $J = di/(dA\cos\theta)$

We can also write,

$$di = JdA\cos\theta = \vec{J}\cdot d\vec{A} \text{ or } i = \int \vec{J}\cdot d\vec{A}.$$

NOTE: While electric current is a scalar quantity, electric current density is a vector quantity.

In case of conductors,

$$J = \frac{I}{A} = \frac{V}{RA} = \frac{EL}{(\rho L/A)A} = \frac{E}{\rho} = \sigma E$$

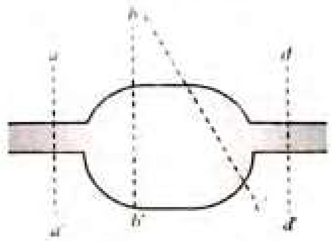
or $\quad \vec{J} = \sigma\vec{E}$

- Current density is proportional to electric field, and direction of current density is the same as that of electric field.

- If electric field is uniform (i.e., constant), current density will be constant.
- If electric field is zero (as in electrostatics inside a conductor), current density and hence current will be zero.

Important Points

- If the current has not reached a steady state, i.e., the flow of charge is not constant, then the current through different cross sections at a particular instant may have different values.
- Electric current may be distributed nonuniformly over the surface through which it passes. Hence, to characterize the current in greater detail, current density vector $\vec{J}$ is introduced.
- Current density, j, tells us how charge flows at a certain point and its direction tells us about the direction of the flow of charge at that point. Current describes how charge flows through an extended object.



Current through all sections would be same in steady state

- The direction of current density is the same as that of the velocity of positive charge or opposite to the direction of the velocity of negative charge.
- Current density can be represented by a similar set of lines known as stream lines. The spacing of the stream lines suggests the value of current density. Narrower stream lines mean more current density, and spaced stream lines mean less current density (figure).

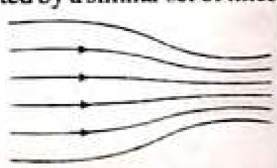

A conductor having nonuniform cross section

ILLUSTRATION 19.1 A wire of mass m, length l, density d, and area of cross section A is stretched in such a way that its length increases by 10% of its original value. Express the changed resistance in percentage.

Solution. Given mass m, length $l_1 = l$, density d, and area of cross section $A_1 = A$. Let ρ be the resistivity and R_1 be the resistance of the wire. Mass of wire m = volume × density $= Al \times d = Al\,d$.

Therefore, area of cross section is $A_1 = m/ld$, and the resistance of the wire is

$$R_1 = \rho\frac{l}{A_1} = \rho\frac{l^2 d}{m} = \left(\frac{\rho d}{m}\right) l^2 = k\,l^2 \quad \text{(i)}$$

Let l_2 be the new length, then

$$l_2 = l + \frac{10}{100}\,l = l + 0.1l = 1.1l$$

Let R_2 be the resistance of the wire after stretching, then

$$R_2 = kl_2^2 \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we have $\dfrac{R_2}{R_1} = \dfrac{l_2^2}{l_1^2} = \dfrac{(1.1)^2 l_1^2}{l_1^2} = 1.21$

or $\quad R_2 = 1.21\,R_1 = R_1 + 0.21\,R_1$

or $\quad R_2 - R_1 = 0.21\,R_1$

Hence, the percentage change in the resistance is

$$\frac{R_2 - R_1}{R_1} \times 100 = 21\%$$

DRIFT VELOCITY

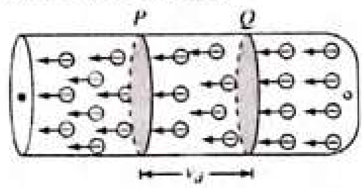
Under the normal conditions of temperature and pressure and without the influence of any external electrostatic field, the motion of free electrons in a conductor is due to the thermal energy.

Now, when this conductor is placed in an external field, these free electrons start experiencing electric force and start moving under the influence of this force (see figure). However, although they are free to move, they are not able to move in a straight line because they encounter other electrons, ions, atoms, or molecules in their way (see figure). Hence, they experience collisions after collisions but are able to drift in a particular direction because of this external field. The drifting of these free electrons over some period of time is called drift velocity.

Relation Between Drift Velocity and Current

Let A be the area of cross section of the conductor, e be the charge on each electron, v_d be the drift velocity, n be the number of free electrons per unit volume, and I be the current, then the total number of electrons between cross sections P and Q, which are v_d distance apart is (volume between P and Q) × n $= Av_d n = nAv_d$. Therefore, the total charge in this volume is $nAv_d e = neAv_d$.

Now, the electron which is present at cross section Q will reach cross section P after 1/sec because P and Q are so selected that the distance between them is v_d, which is the drift speed of the electron. Therefore, $neAv_d$ is the charge that will pass through cross section at P (where P can be any point on the conductor). Hence, this is the electric current that flows through the conductor (see figure).



A is the cross section of conductor. Therefore, $I = neAv_d$. Accordingly current density is $J = I/A = nev_d$.

The equation $J = nev_d$ can be written in vector form as follows: $\vec{J} = nq\vec{v}_d$, where q is the charge of the charge carrier and v_d is the average drift velocity. This equation is correct for both the signs of q. If $q > 0$, $\vec{v}_d$ is in the direction of electric field $\vec{E}$, and $\vec{J}$ is in the direction of $\vec{E}$. If $q < 0$ ($q = -e$ for electrons), as it is in metallic conductor, $\vec{v}_d$ is opposite to $\vec{E}$, and $\vec{J} = -ne\vec{v}_d$ continues to be in the direction of $\vec{E}$.

ILLUSTRATION 19.2 (a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m^2 carrying a current of 1.5 A. Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kgm^{-3}, and its atomic mass is 63.5 u. (b) Compare the drift speed obtained with the speed of propagation of electric field along the conductor, which causes the drift motion.

Solution.

(a) The direction of drift velocity of conduction electrons is opposite to the electric field direction, i.e., the electrons drift in the direction of increasing potential. The drift speed v_d is given by $v_d = (I/neA)$.

Now, $e = 1.6 \times 10^{-19}$ C, $A = 1.0 \times 10^{-7}$ m^2, $I = 1.5$ A, the density of n conduction electrons is equal to the number of atoms per cubic meter (assuming one conduction electron per Cu atom is reasonable from its valence electron count of one). A cubic meter of copper has a mass of 9.0×10^3 kg. Since 6.0×10^{23} copper atoms have a mass of 63.5 g,

$$n = \frac{6.0 \times 10^{23}}{63.5} \times 9.0 \times 10^6 = 8.5 \times 10^{28}$$

which gives

$$v_d = \frac{1.5}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}}$$

$$= 1.1 \times 10^{-3}\ \text{ms}^{-1} = 1.1\ \text{mm s}^{-1}$$

(b) An electric field traveling along the conductor has a speed of an electromagnetic wave, i.e., equal to 3.0×10^8 ms^{-1}. The drift speed is, in comparison, extremely small, smaller by a factor of 10^{-11}.

TEMPERATURE COEFFICIENT OF RESISTIVITY

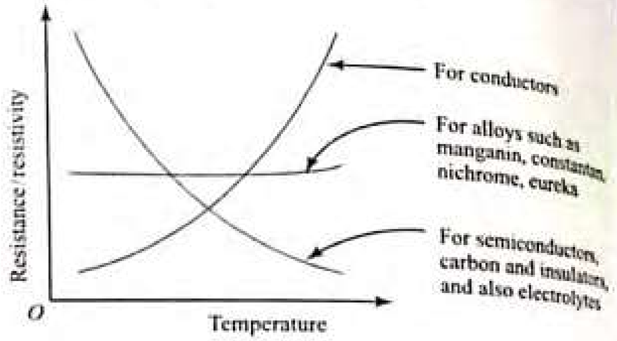
Let us study the equation $R_T = R_{T_0}[1 + \alpha(T - T_0) + \beta(T - T_0)^2]$ where both α and β are called temperature coefficients of resistance, though different in magnitude.

Temperature Coefficients of Resistance

In case of pure metals, β is negligibly small, so the resistance varies linearly with the rise of temperature (see figure).

$$R_T = R_{T_0}[1 + \alpha(T - T_0)]$$

where R_T is the resistance at temperature T, R_{T_0} is the resistance at temperature T_0 (called reference temperature), and α is a constant for a given metal and for a given reference temperature and is called temperature coefficient of resistance. Its unit is per degree temperature ($°\text{C}^{-1}$).



T_0 is some reference temperature, generally either 0°C or 20°C. α is positive for metals; it is 0.004 per degree for Cu, but very small (almost zero) for alloys such as manganin ($\alpha = 0.00001$ per degree), nichrome, constantan, and eureka. It is negative for semiconductors and insulators. Let

$$\rho_T = \rho_0[1 + \alpha(T - T_0)]$$

or $$\rho_T - \rho_0 = \alpha\rho_0(T - T_0)$$

or $$\alpha = \frac{1}{\rho_0} \times \frac{(\rho_T - \rho_0)}{(T - T_0)}$$

or $$\rho = \frac{1}{\rho} \times \frac{d\rho}{dT}$$

ILLUSTRATION 19.3 A resistance R of thermal coefficient of resistivity α is connected in parallel with a resistance $3R$, having thermal coefficient of resistivity 2α. Find the value of α_{eff}.

Solution. The equivalent resistance at 0°C is

$$R_0 = \frac{R_{10}R_{20}}{R_{10} + R_{20}} \quad \text{(i)}$$

The equivalent resistance at t°C is

$$R = \frac{R_1 R_2}{R_1 + R_2} \quad \text{(ii)}$$

But $R_1 = R_{10}(1 + \alpha t)$ (iii)

$R_2 = R_{20}(1 + 2\alpha t)$ (iv)

and $R = R_0(1 + \alpha_{\text{eff}}t)$ (v)

Putting the value of (i), (iii), (iv), and (v) in Eq. (ii), we have

$$\alpha_{\text{eff}} = \frac{5}{4}\alpha$$

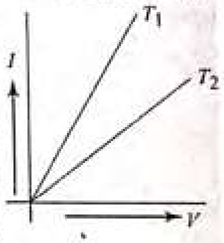
Concept Application Exercise 19.1

1. It is desired to make a 20 Ω coil of wire, which has a zero thermal coefficient of resistance. To do this, a carbon resistor of resistance R_1 is placed in series with an iron resistor of resistance R_2. The proportions of iron and carbon are so chosen that $R_1 + R_2 = 20\ \Omega$ for all temperatures near 20°C. How long are R_1 and R_2?
$\alpha_{carbon} = -0.5 \times 10^{-3}\ {}^\circ C^{-1}$, $\alpha_{iron} = 5 \times 10^{-3}\ {}^\circ C^{-1}$

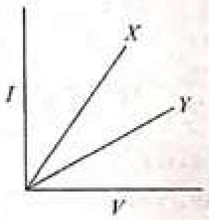
2. The following table gives the length of three copper rods, their diameters, and the potential differences between their ends. Rank the rods according to (a) the magnitude of the electric field within them, (b) the current density within them, and (c) the drift speed of electrons through them, greatest first.

Rod	Length	Diameter	P.D.
1	L	$3d$	V
2	$2L$	d	$2V$
3	$3L$	$2d$	$2V$

3. The V-I graph for a metallic wire at two different temperatures T_1 and T_2 is shown in figure. Which of the two temperatures T_1 and T_2 is higher and why?



4. The voltage–current variation of two metallic wires X and Y at constant temperature is shown in figure. Assuming that the wires have the same length and the same diameter, explain which of the two wires will have larger resistivity.

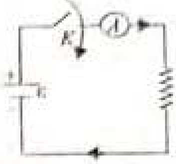


5. The current density across a cylindrical conductor of radius R varies in magnitude according to the equation $J = J_0\left(1 - \frac{r}{R}\right)$ where r is the distance from the central axis. Thus, the current density is a maximum J_0 at that axis ($r = 0$) and decreases linearly to zero at the surface ($r = R$). Calculate the current in terms of J_0 and the conductor's cross-sectional area $A = \pi R^2$.

6. A uniform copper wire of mass 2.23×10^{-3} kg carries a current of 1 A when 1.7 V is applied across it. Calculate its length and area of cross section. If the wire is uniformly stretched to double its length, calculate the new resistance. Density of copper is $8.92 \times 10^3\ \text{kgm}^{-3}$ and resistivity is $1.7 \times 10^{-8}\ \Omega$m.

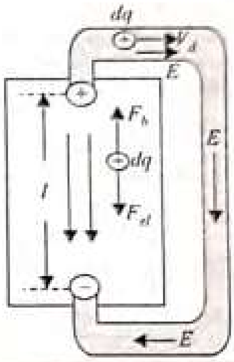
ELECTROMOTIVE FORCE AND POTENTIAL DIFFERENCE

When we close the key, a battery sets up a direct current in the conductor. Each charge carrier moves from positive terminal of the battery by the electric field set up by the battery. The electric field inside the conductor does positive work in driving a positive charge carrier and does equal negative work, while the charge carrier moves back to the positive pole (electtode) from negative pole (electrode) of the battery against the electric field inside the battery because $\oint \vec{E} \cdot \vec{dl} = 0$.



The seat of emf ε does a positive work dW circulating an elementary charge dq; dW/dq

Hence, the electric field cannot take the credit of circulating a charge carrier along the closed loop. Then, it must be the battery that does a positive work in pushing the charges from negative to positive terminal of the battery to set up a permanent potential difference across the terminals.



The elementary charges (positive charge carries) move from the negative to the positive electrode of the battery due to emf against the static electric field inside the battery.

The work done by the battery in pushing the positive charges from its negative terminal to its positive terminal through a distance ℓ with a force F_b is given by

$$W = F_b\ell = F_{el}\ell = qE_{el}\ell = qV_b$$

(F_b is the force acting on the test charge $+q$ inside the battery due to the electrochemical action of the battery.)

Then, the work done per unit charge is

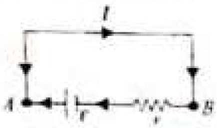
$$\frac{W}{q} = V_b$$

which is called *electromotive force* (emf) of the battery given by $\varepsilon = W_b/q$.

Emf is numerically equal to the work done by the battery in circulating a unit positive charge. A cell (seat of emf) generates a potential difference between its terminals across the circuit, which is numerically equal to its emf when the circuit is open.

INTERNAL RESISTANCE OF A CELL

Real sources in a circuit do not behave in exactly the way we have described; the potential difference across a real source in a circuit is not equal to the emf. The reason is that the charge moving through the material of any real source encounters resistance. We call this the internal resistance of the source, denoted by r. If this resistance behaves according to Ohm's law, r is constant and independent of the current I. As the current moves through r, it experiences an associated drop in potential equal to Ir. Thus, when a current is drawn through a source, the potential difference between the terminal of the source is $V = \varepsilon - Ir$.

Situation	Potential difference (V) across the terminal of cell
Discharging of a battery 	$V_A - \varepsilon + Ir = V_B$ or $V_A - V_B = \varepsilon - Ir$ $V_{AB} = \varepsilon - Ir$ or $V_{AB} < \varepsilon$
Charging of a battery 	$V_A - \varepsilon - Ir = V_B$ or $V_A - V_B = \varepsilon + Ir$ $V_{AB} = \varepsilon + Ir$ or $V_{AB} > \varepsilon$
Battery is open circuited 	$V_{AB} = \varepsilon$ as $I = 0$
Battery is short circuited 	$I = \dfrac{\varepsilon}{r}$ or $\varepsilon = Ir$ $\therefore$ $\varepsilon - Ir = 0$ or $V = 0$

NOTE:

- Emf is independent of the resistance of the circuit and depends upon the nature of electrolyte of the cell, while potential difference depends upon the resistance between the two points of the circuit and current flowing through the circuit.
- Emf is the cause, and potential difference is the effect.

ILLUSTRATION 19.4

(a) After long use, the internal resistance of the storage battery increases to 500 Ω. What maximum current can be drawn from the battery? Assume the emf of the battery to remain unchanged.

(b) If the discharged battery is charged by an external emf source, is the terminal voltage of the battery during charging greater of less than its emf 12 V?

Solution. We have emf of the storage battery as $\varepsilon = 12$ V, internal resistance of the battery as $r = 0.5 \times 10^{-2}$ Ω, and current flowing through the circuit as $I = 90$ A.

(a) We know that $I = \varepsilon/(R + r)$, where R is the external resistance. For I to be maximum (i.e., I_{max}), $R = 0$ (i.e., the battery is to be shorted). Thus,

$$I_{max} = \frac{\varepsilon}{r} = \frac{12V}{500\ \Omega} = 24 \text{ mA}$$

(as after long use, $r = 500$ Ω)

Clearly, the battery can now no longer be used for starting the car.

(b) During charging, the current inside the battery flows in a direction opposite to that when it is discharging.

Clearly, replacing I by $-I$, we get $V = \varepsilon - (-I)r = \varepsilon + Ir$. Hence, V should be greater than ε (= 12 V) during charging.

COMBINATION OF RESISTANCES

Resistances in Series

If resistances are connected as shown in figure such that the current flowing through them is the same, the resistances are said to be in series.

If I is the current flowing through the resistances, then potential drop across each is

$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3$$

Adding,

$$V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3) \quad \text{(i)}$$

But $V_1 + V_2 + V_3 = V$, so we get from Eq. (i)

$$V = I(R_1 + R_2 + R_3)$$

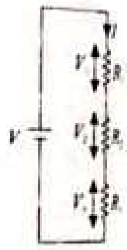
where V is the potential difference across the combinations. Also from Ohm's law

$$V = IR \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $R = R_1 + R_2 + R_3$ where R is known as *equivalent resistance*.

Voltage Divider

In a series circuit, current through each resistor is the same (see figure).



$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

$$I = \frac{V}{R_1 + R_2 + R_3}$$

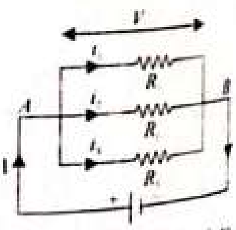
$$V_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$

$$V_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

NOTE: If n identical resistances are connected in series, $R_{eq} = nR$ and potential difference across each resistance is $V' = V/n$.

Resistances in Parallel



If the resistances are connected between the same two points such that the potential drop across each resistance is same, then the resistances are said to be in parallel (figure).

In this case, if V is the potential difference between the points A and B, then

$$V = i_1R_1,\ V = i_2R_2, \text{ and } V = i_3R_3$$

$$\Rightarrow \quad i_1 = \frac{V}{R_1},\ i_2 = \frac{V}{R_2} \text{ and } i_3 = \frac{V}{R_3}$$

Therefore, the total current flowing through the battery is

$$i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

i.e., $i = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

Also by Ohm's law, $i = V/R$, where R is the equivalent resistance between A and B. So we get

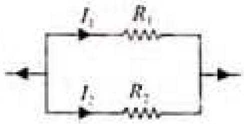
$$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Current Divider for Two Resistances

From figure, we have

$$I = I_1 + I_2 \quad \text{(i)}$$

$$I_1R_1 = I_2R_2 \quad \text{(ii)}$$



On solving Eqs. (i) and (ii), we get

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I; \quad I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I$$

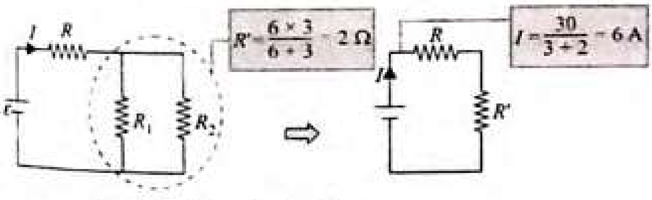
The division of current in the branches of a parallel circuit is inversely proportional to their resistances.

NOTE: Current through any resistance is

$$i = I_{\text{total}} \times \left[\frac{\text{Resistance of opposite branch}}{\text{Total resistance}}\right]$$

ILLUSTRATION 19.5 In the circuit given, find the currents I, I_1, and I_2 in the circuit.

Solution. The resistances R_1 and R_2 are connected in parallel.



$$\therefore \quad I_1 = I\left[\frac{R_2}{R_1 + R_2}\right] = 6\left[\frac{6}{3+6}\right] = 4\text{ A}$$

and $$I_2 = I\left[\frac{R_1}{R_1 + R_2}\right] = 6\left[\frac{3}{3+6}\right] = 2\text{ A}$$

Current Divider for Three Resistances

The division of current in the branches of a parallel circuit is inversely proportional to their resistances (see figure).

$$I = I_1 + I_2 + I_3$$

$$I_1R_1 = I_2R_2 = I_3R_3$$

and

$$R_{eq} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

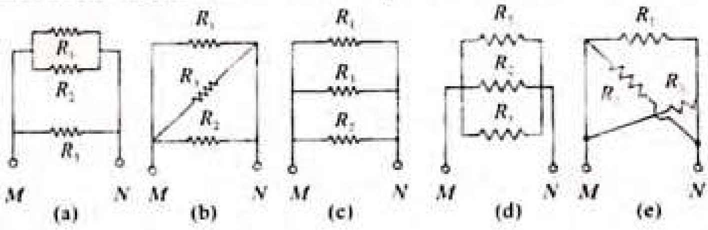
$$I_1 = I\left[\frac{R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_2 = I\left[\frac{R_1R_3}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

$$I_3 = I\left[\frac{R_1R_2}{R_1R_2 + R_2R_3 + R_3R_1}\right]$$

It is easy to remember the expressions for I_1, I_2, and I_3. Notice which resistance is missing in the numerator.

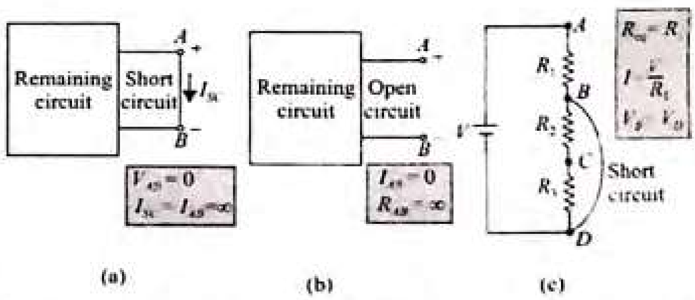
In all parts of figure, the resistances R_1, R_2, and R_3 are connected in parallel between the points M and N.



Short and Open Circuits

When two points of a circuit are commenced together by a conducting wire, they are said to be *short-circuited*. The connecting wire is assumed to have zero resistance. No voltage can exist across a 'short', and the current through it is very large (theoretically infinity).

Two points are said to be *open-circuited* when there is no direct connection between them; a break in the continuity of the circuit exists. Due to this break, the resistance between the two points is infinite and there is no of current between the two points.



Whole of the applied voltage is felt across the 'open', i.e., across terminals A and B.

EQUIPOTENTIAL POINTS

In a current-carrying electrical network, two points are said to be equipotential if they are at the same potential. Between the points 1 and 2, $V_1 = V_2$, if $\Delta V = iR = 0$.

Then we have two cases: if $R = 0$, $\Delta V = 0$ ($i \neq 0$) and if $i = 0$ (R is finite), $\Delta V = 0$. The first case tells that when we

connect any two points by an ideal conductor, the potential difference between them becomes zero. It is called short-circuiting. The second case tells that if we connect any two points by a nonzero resistor and find no current along the resistor, we can call these points equipotential. After finding the equipotential points, join them to a single point to simplify the given circuit.

Electrical Symmetry

If the branches ab and ac have same resistances and same current, same potential will be dropped along them. Hence, the branches ab and ac are electrically symmetrical. In this case, the points b and c are equipotential points. Then you can join these points.

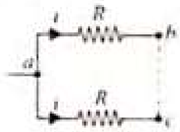
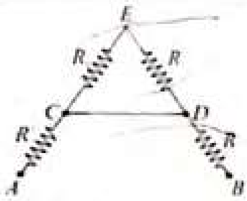


ILLUSTRATION 19.6 Four identical resistances each having value R are arranged as shown in figure. Find the equivalent resistance between A and B.



Solution. Since C and D are connected with a zero resistor, they are equipotential. Then superimpose C and D to obtain the simplified circuit as shown. Since no current flows in the branches CE and ED, cut and then throw them to have

$$R_{AB} = R + R = 2R$$

If we get a closed loop of resistors without any battery, it carries no current. Then remove the total loop to get a simpler circuit or if the current in any branch is zero, remove it.

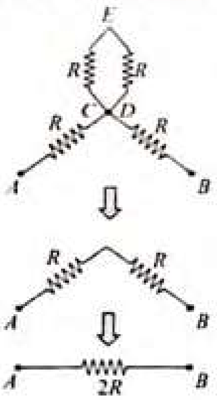
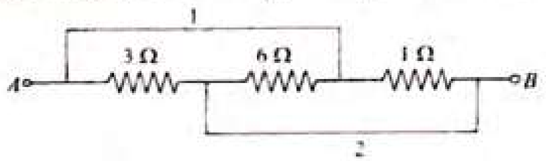


ILLUSTRATION 19.7 Find the R_{AB} in the given network.



Solution. As A, C and B, D are short-circuited, A and C are at the same potential; B and D are at the same potential. Bringing A and C to one point and B and D to one point, we have redrawn the circuit. You can see that all resistors are in parallel. Then,

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} + \frac{1}{2} \quad \text{or} \quad R = 1\ \Omega$$

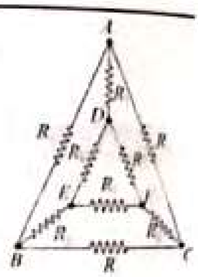
Method of Equipotential Reduction

This method is based on identifying the points of equal potential and connecting them. By doing so the electric resistance network reduces to an arrangement of series and parallel combinations that can be easily solved by the successive reduction method.

Now the question arises how to identify the points of same potential? In this section, we will discuss the method to calculate the equivalent resistance and capacitance using symmetry techniques. There are various kinds of symmetry considerations. The most common are

- parallel axis of symmetry
- perpendicular axis of symmetry
- shifted symmetry or shifted asymmetry
- path symmetry

ILLUSTRATION 19.8 In figure, the resistances are connected as shown. Given $R_1 = 10\ \Omega$ and $R_2 = 20\ \Omega$. Determine the equivalent resistance between points (i) A and D and (ii) B and C.



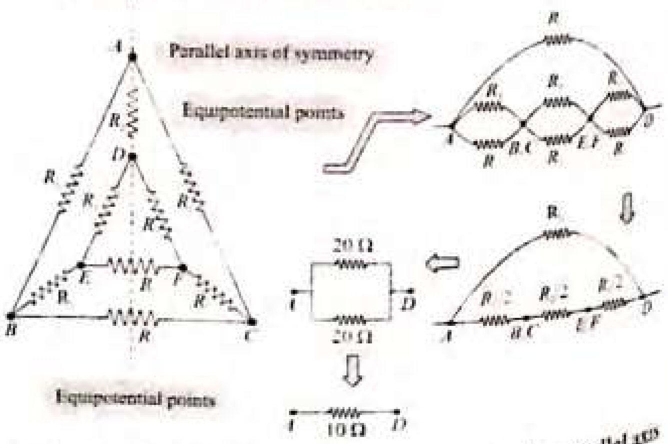
Solution.

(i) Calculation of equivalent resistance between A and D:

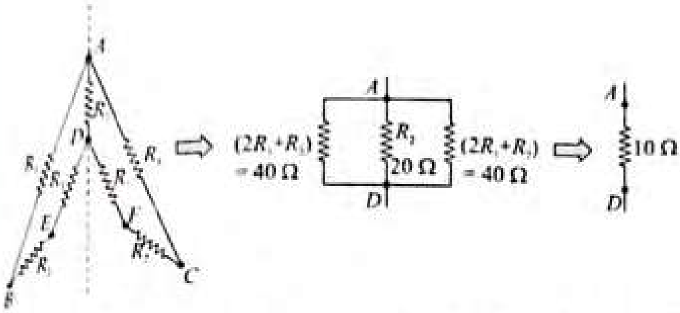
There exists parallel axis of symmetry.

The points across the parallel axis of symmetry can be treated as equipotential points.

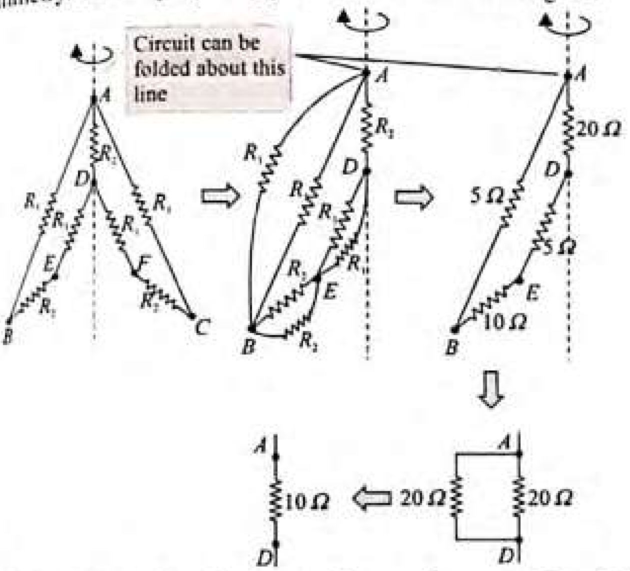
Method 1: Points B and C and E and F are at the same potential, so the circuit can be redrawn as shown in figure. Thus, the equivalent resistance is $10\ \Omega$.



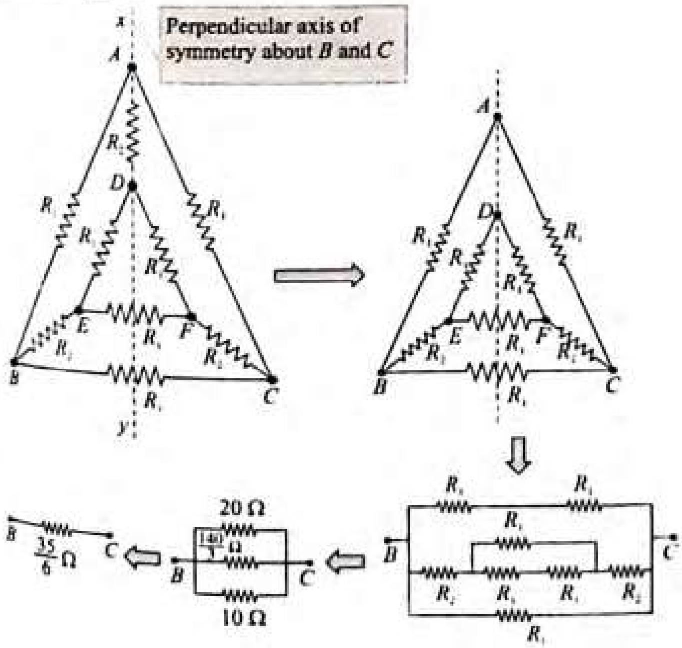
Method 2: The points with charge symmetrical to parallel axis are equipotential points. The resistances connected between these points can be removed. A step-by-step procedure for the calculation of equivalent resistance is shown in figure.



Method 3: We can fold the circuit about the parallel axis of symmetry. The step-by-step procedure is shown in figure.



(ii) Calculation of equivalent resistance between *B* and *C*: There exists perpendicular axis of symmetry about line *x*-*y*. Also points *A* and *D* are lying on the perpendicular axis of symmetry. Hence, points *A* and *D* are equipotential. The resistances connected between *A* and *D* can be removed. The circuit reduces as shown, and the step-by-step procedure is given in figure.



KIRCHHOFF'S LAW FOR ELECTRICAL NETWORKS

Kirchhoff's first law: This law is also known as junction rule or current law (*KCL*). According to it, the algebraic sum of currents meeting at a junction is zero. i.e., $\Sigma i = 0$.

In a circuit, at any junction the sum of the currents entering the junction must equal the sum of the currents leaving the junction, i.e.,

$$i_1 + i_3 = i_2 + i_4$$

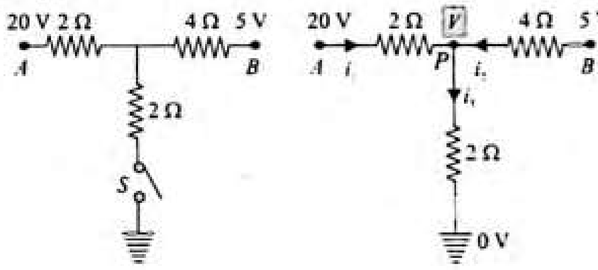
Here it is worthy to note that

i. If a current comes out to be negative, the actual direction of current at the junction is opposite to that assumed. $i_1 + i_2 + i_3 + i_4 = 0$ can be satisfied only if at least one current is negative, i.e., leaving the junction.

ii. This law is simply a statement of conservation of charge as if current reaching a junction is not equal to the current leaving the junction, charge will not be conserved.

ILLUSTRATION 19.9 Figure (a) shows three resistances are connected with switch *S*, initially the switch is open. When the switch *S* is closed find the current passed through it.

Solution. Let *V* be the potential of the junction as shown in Figure (b).



(a) (b)

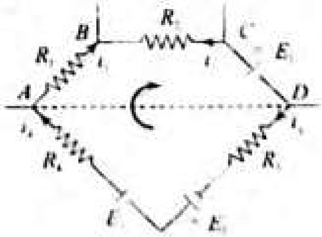
Applying junction law at *P* we have

$$\frac{20 - V}{2} + \frac{5 - V}{4} = \frac{V - 0}{2}$$

or $\quad 40 - 2V + 5 - V = 2V \quad$ or $\quad 5V = 45$ or $V = 9V$

$$\therefore \quad i_3 = \frac{V}{2} = 4.5\,\text{A}$$

Kirchhoff's second law: This law is also known as loop rule or voltage law (*KVL*) and according to it "the algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero", i.e., $\Sigma V = 0$.



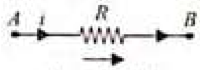
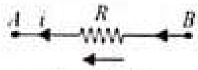
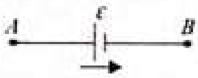
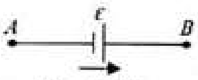
In the following closed loop,

$$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$$

Here it is worthy to note that

- This law represents conservation of energy as if the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.
- If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be $(n - 1)$.

Sign convention for the application of Kirchhoff's law: For the application of Kirchhoff's laws, the following sign conventions are to be considered.

The change in potential in traversing a resistance in the direction of the current is $-iR$, while in the opposite direction it is $+iR$.	 $V_A - iR = V_B$  $V_A + iR = V_B$
The change in potential in traversing an emf source from negative to positive terminal is $+\varepsilon$, while in the opposite direction it is $-\varepsilon$ irrespective of the direction of current in the circuit.	 $V_A - \varepsilon = V_B$  $V_A + \varepsilon = V_B$
The change in potential in traversing a capacitor from the negative terminal to the positive terminal is $+q/C$, while in the opposite direction it is $-q/C$.	 $V_A - q/C = V_B$  $V_A + q/C = V_B$

Guidelines for Applying Kirchhoff's Law

- Starting from the positive terminal of the battery having highest emf, distribute current at various junctions in the circuit in accordance with *junction rule*. It is not always easy to correctly guess the direction of current, so there is no problem if one assumes the wrong direction.
- After assuming the current in each branch, we pick a point and begin to walk (mentally) around a closed loop. As we traverse each resistor, capacitor, inductor, or battery, we must write down the voltage change for that element according to the above sign convention.
- By applying *KVL*, we get one equation, but in order to solve the circuit, we require as many equations as there are unknowns. So we select the required number of loops and apply Kirchhoff's voltage law across each such loop.
- After solving the set of simultaneous equations, we obtain the numerical values of the assumed currents. If any of these values comes out to be negative, it indicates that the particular current is in the opposite direction from the assumed one.

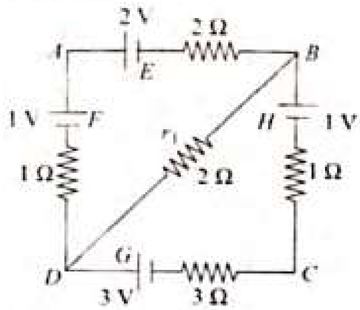
NOTE:

- The number of loops must be selected so that every element of the circuit must be included in at least one of the loops.
- While traversing through a capacitor or battery, we do not consider the direction of current.
- While considering the voltage drop or gain across an inductor, we always assume current to be in increasing function.

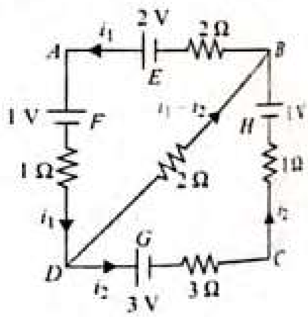
ILLUSTRATION 19.10 In the circuit shown in figure, E, F, G, and H are cells of emf 2, 1, 3, and 1 V, respectively. The resistances 2 Ω, 1 Ω, 3 Ω, and 1 Ω are their respective internal resistances.

i. Find the potential difference between B and D.

ii. Calculate the potential differences across the terminals of each of the cells G and H.



Solution. The circuit with the currents shown is redrawn in figure. Applying the loop law to $BADB$, we get



$$-2i_1 + 2 - 1 - 1i_1 - 2(i_1 - i_2) = 0 \text{ or } -5i_1 + 2i_2 = -1 \quad \text{(i)}$$

Applying the same law to loop $DCBD$, we get

$$-3 - 3i_2 - 1i_2 + 1 + 2(i_1 - i_2) = 0 \text{ or } 2i_1 - 6i_2 = 2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$i_1 = \frac{1}{13}\text{ A}, i_2 = -\frac{4}{13}\text{ A}$$

$$\therefore \quad i_1 - i_2 = \frac{5}{13}\text{ A}$$

i. $V_D - V_B = (2\ \Omega)\left(\frac{5}{13}\text{ A}\right) = \frac{10}{13}\text{ V}$

ii. Potential differences across the cell G is

$$V_D - V_C = 3 + 3i_2 = 3 + 3(-4/13) = 27/13\text{ V}$$

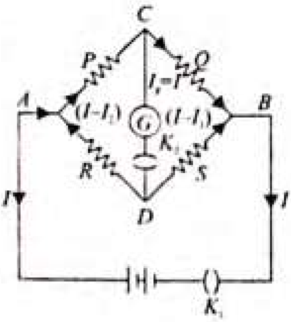
Potential difference across the cell H is

$$V_B - V_C = 1 - i_2 = 1 + 4/13 = 17/13\text{ V}$$

$(\because V = E + ir)$

BALANCED WHEATSTONE BRIDGE

Wheatstone bridge is also known as a meter bridge or slide wire bridge. If four resistances P, Q, R, and S are joined as shown in figure, both the keys (K_1 and K_2) are on and no current flows through the galvanometer (i.e., $I_g = 0$). Then the combination of resistances is called a balanced Wheatstone bridge. So

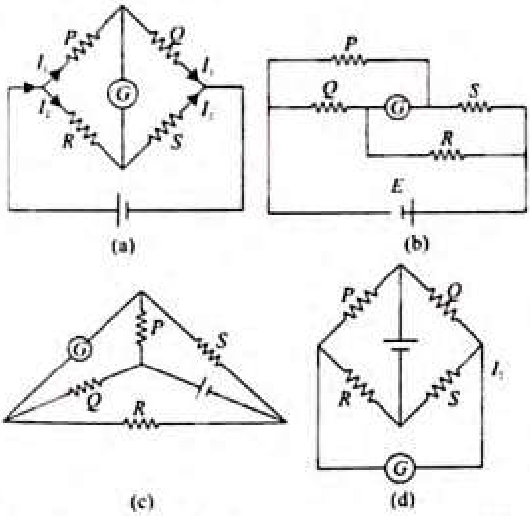


$$\frac{P}{Q} = \frac{R}{S}$$

If S is an unknown resistance, then $S = R \times \dfrac{Q}{P}$

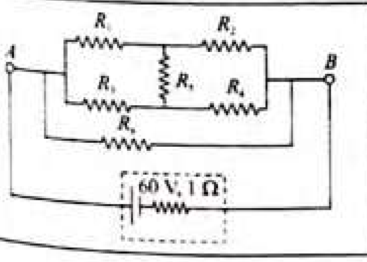
Now if we are given R, P, and Q or R and ratio Q/P, then we can calculate the value of S easily.

The balanced Wheatstone bridge method is an accurate method because we do not have to read out deflection, but we only have to see that the needle remains at dead zero. It is not affected by internal resistance of cells, resistances of galvanometers, etc. This is the principle used in the meter bridge or in the slide-wire bridge. Other circuits that can form Wheatstone bridge are shown in figure.



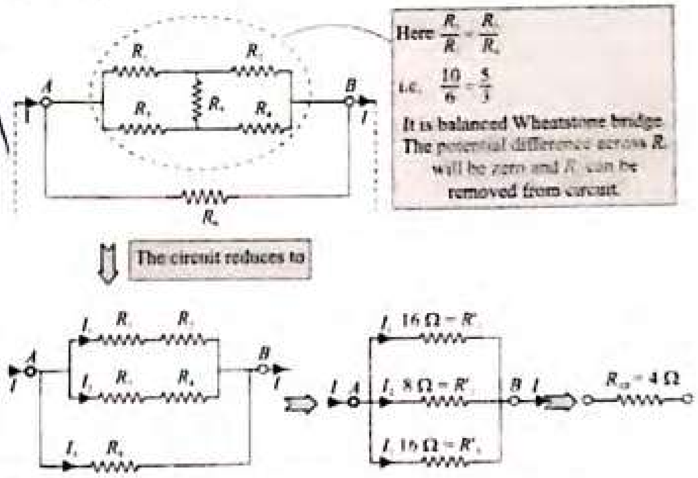
All the four circuits (a), (b), (c), and (d) represent a Wheatstone bridge network.

ILLUSTRATION 19.11 Resistances R_1, R_2, R_3, R_4, R_5, and R_6 are connected with a 6 V battery with internal resistance 1 Ω as shown in figure. Find (a) equivalent resistance and (b) current in each resistance.



$R_1 = 10\ \Omega$
$R_2 = 6\ \Omega$
$R_3 = 5\ \Omega$
$R_4 = 3\ \Omega$
$R_5 = 20\ \Omega$
$R_6 = 16\ \Omega$

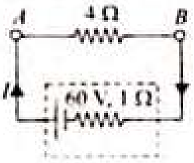
Solution.



Hence, the equivalent resistance between A and B is

$$\frac{1}{R_{AB}} = \frac{1}{16} + \frac{1}{8} + \frac{1}{16} = \frac{4}{16}$$

or $\quad R_{AB} = 4\ \Omega$



Now the circuit reduces to the one shown in figure. The current supplied by the battery is

$$I = \frac{60}{4+1} = 12\ \text{A}$$

As potential difference across R_5 is zero, no current will be in R_5. Now using current distribution law, we can find the current in each branch.

The currents in R_1 and R_2 will be equal.

$$I_1 = I\left(\frac{R_2'R_3'}{R_1'R_2' + R_2'R_3' + R_3'R_1'}\right)$$

$$= 12\left[\frac{8 \times 16}{16 \times 8 + 8 \times 16 + 16 \times 16}\right] = 3\ \text{A}$$

The currents in R_3 and R_4 is

$$I_2 = I\left(\frac{R_1'R_3'}{R_1'R_2' + R_2'R_3' + R_3'R_1'}\right)$$

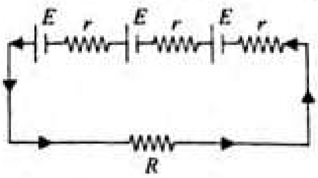
$$= 12\left[\frac{16 \times 16}{16 \times 8 + 8 \times 16 + 16 \times 16}\right] = 6\ \text{A}$$

Hence, current in R_6 is $I_3 = 12 - (3 + 6) = 3$ A.

COMBINATION OF CELLS

Series Grouping

Suppose n cells each of emf E and internal resistance r are connected in series as shown in figure. Then net emf is nE and total resistance is $nr + R$.



Therefore the current in the circuit is

$$i = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{nE}{nr + R}$$

NOTE: If polarity of m cells is reversed, then equivalent emf is $(n - 2m)E$. While total resistance is still $nr + R$.

$$i = \frac{(n-2m)E}{nr + R}$$

- If the same current passes through every resistor in a given branch, irrespective of the presence of sources in that branch, the resistors are in series even though they are not directly connected to each other. Same is true about capacitors.

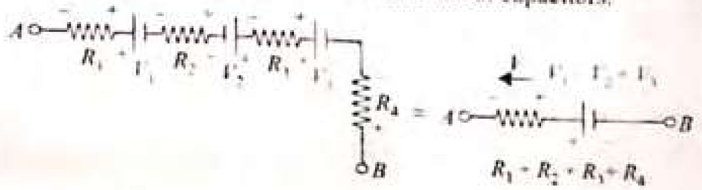
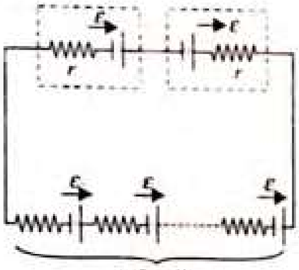


ILLUSTRATION 19.12 n identical cells each of emf 6 V, connected in series with an external resistor of 5 Ω, carry a current of 10 A. If two cells are connected wrongly in series with the same external resistor, the current flowing through the cells will be 6A. Find the value of n and the internal resistance of each cell.

Solution. Let r be the internal resistance of each cell. For series connection, $\varepsilon_{\text{eff}} = n\varepsilon = 6n$.

$$r_{\text{eff}} = nr$$

or $$i = \frac{n\varepsilon}{nr + R} = \frac{6n}{nr + 5} = 10 \quad \text{(i)}$$



n-2 cell
ε_{eff} $n\varepsilon - 2\varepsilon$

For wrong connection of two cells, the effective emf decreases by 4ε because the emfs of two identical cells counteract with each other. Then,

$$\varepsilon'_{\text{eff}} = n\varepsilon - 4\varepsilon = (n-4)\varepsilon = (n-4)6$$

and $$r'_{\text{eff}} = nr$$

or $$i' = \frac{(n-4)\varepsilon}{nr + R} = \frac{(n-4)6}{nr + 5} = 6 \quad \text{(ii)}$$

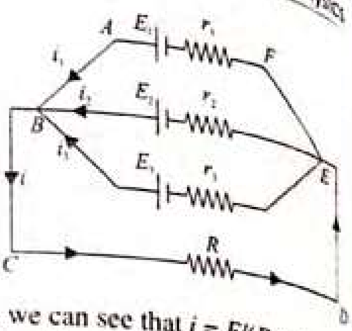
By solving Eqs. (i) and (ii), we have $n = 10$ and $r = 0.1$ Ω.

Parallel Grouping

Case 1: E and r of each cell are different, but still the positive terminals of all cells are connected at one junction while the negative terminals at the other.

$$\therefore \quad i = \frac{\sum\left(\frac{E}{r}\right)}{1 + R\sum\left(\frac{1}{r}\right)} = \frac{E_{\text{eq}}}{R_{\text{eq}}}$$

where $$E_{\text{eq}} = \frac{\sum\left(\frac{E}{r}\right)}{\sum\left(\frac{1}{r}\right)}$$



and $$R_{\text{eq}} = R + \frac{1}{\sum\left(\frac{1}{r}\right)}$$

From the above expression, we can see that $i = E/(R + r/n)$ if n cells of same emf E and internal resistance r are connected in parallel. This is because

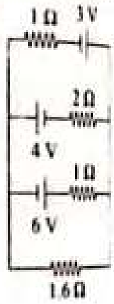
$$\sum\left(\frac{E}{r}\right) = \frac{nE}{r} \text{ and } \sum\left(\frac{1}{r}\right) = \frac{n}{r}$$

$$\therefore \quad i = \frac{nE/r}{1 + nR/r} = \frac{E}{R + r/n}$$

We can also write

$$i = \frac{(E_1/r_1) + (E_2/r_2) + (E_3/r_3)}{1 + R\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)}$$

ILLUSTRATION 19.13 Three cells of emf 3 V, 4 V, and 6 V are connected in parallel. If their internal resistances are 1 Ω, 2 Ω, and 1 Ω, find the ε_{eff}, r_{eff}, and the current in the external load $R = 1.6$ Ω.



Solution. The cells are connected in parallel. The equivalent cell emf is

$$\varepsilon_{\text{eff}} = \frac{\Sigma\varepsilon_i/r_i}{\Sigma 1/r_i} = \frac{\frac{3}{1} + \left(-\frac{4}{2}\right) + \frac{6}{1}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{1}} = 2.8 \text{ V}$$

The internal resistance of the cell is

$$\frac{1}{r_{\text{eff}}} = \Sigma\frac{1}{r_i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1} = \frac{5}{2}$$

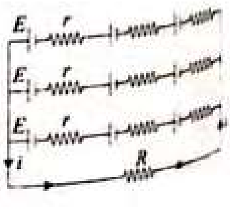
or $$r_{\text{eff}} = \frac{2}{5}\,\Omega$$

The current in the circuit is

$$i = \frac{\varepsilon}{R + r} = \frac{2.8}{\frac{8}{5} + \frac{2}{5}} = 1.4 \text{ A}$$

Mixed Grouping

The situation is shown in figure. There are n identical cells in a row, and number of rows is m. The emf of each cell is E, and internal resistance is r. Treating each row as a single cell of emf nE and internal resistance nr, we have



net emf = nE

total internal resistance = $\dfrac{nr}{m}$

total external resistance = R

Therefore, the current through the external resistance R is

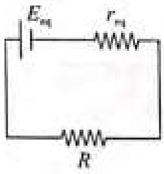
$$i=\frac{nE}{R+\dfrac{nr}{m}}$$

This expression after some rearrangement can also be written as

$$i=\frac{mnE}{(\sqrt{mR}-\sqrt{nr})^2+2\sqrt{mnrR}}$$

From this expression we see that i is maximum when,

or $\sqrt{mR}=\sqrt{nR}$ or $R=\dfrac{nr}{m}$



or total external resistance = total internal resistance.

Thus, we can say that the current and hence the power transferred to the load are maximum when the load resistance is equal to the internal resistance. This is known as *maximum power transfer theorem*.

ILLUSTRATION 19.14 In a mixed grouping of identical cells, five rows are connected in parallel and each row contains 10 cell. This combination sends a current i through an external resistance of 20 Ω. If the emf and internal resistance of each cell is 1.5 V and 1 Ω, respectively, then find the value of i.

Solution. The number of cells in a row is $n = 10$, and the number of such rows is $m = 5$.

$$i=\frac{nE}{\left(R+\dfrac{nr}{m}\right)}=\frac{10\times1.5}{\left(20+\dfrac{10\times1}{5}\right)}=\frac{15}{22}=0.68$$

ILLUSTRATION 19.15 100 cells each of emf 5 V and internal resistance 1 Ω are to be arranged to produce maximum current in a 25 Ω resistance. Each row contains equal number of cells. Find the number of rows.

Solution. Total number of cells is

$mn = 100$. (i)

Current will be maximum when

$R=\dfrac{nr}{m}$ or $25=\dfrac{n\times1}{m}$

$n = 25\,m$ (ii)

From Eqs. (i) and (ii), we get $n = 50$ and $m = 2$.

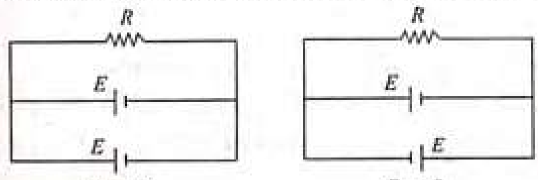
Important Points

- In series grouping of cells, their emfs are additive or subtractive, while their internal resistances are always additive. If dissimilar plates of cells are connected together, their emfs are added to each other, while if their similar plates are connected together, their emfs are subtractive.

$E_{eq} = E_1 + E_2$ and $r_{eq} = r_1 + r_2$

$E_{eq} = E_1 - E_2 (E_1 > E_2)$ and $r_{eq} = r_1 + r_2$

- In series grouping of identical cells, if one cell is wrongly connected, then it will cancel out the effect of two cells. If in the combination of n identical cells (each having emf E and internal resistance r) x cells are wrongly connected, then equivalent emf is $E_{eq} = (n - 2x)E$ and equivalent internal resistance is $r_{eq} = nr$.
- In parallel grouping of two identical cells having no internal resistance, we have the situation in figure.



$E_{eq} = E$ $\quad$ $E_{eq} = 0$

- When two cells of different emf and no internal resistance are connected in parallel, then equivalent emf is indeterminate. Note that connecting a wire with a cell but with no resistance is equivalent to short-circuiting. Therefore, the total current that will be flowing will be infinite.

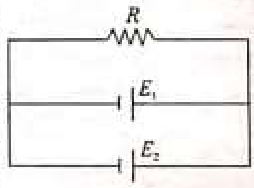
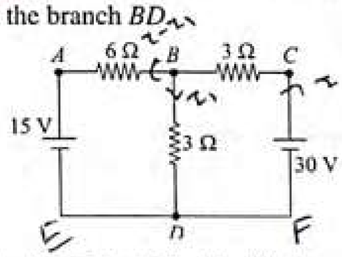
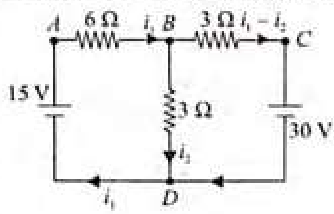


ILLUSTRATION 19.16 In the circuit shown in figure, find the current through the branch BD.



Solution. **Method 1: Using Kirchhoff's law**

The currents in the circuit are assumed as shown in figure.



Applying KVL along the loop $ABDA$, we get

$-6i_1 - 3i_2 + 15 = 0$

or $\quad 2i_1 + i_2 = 5$ (i)

Applying KVL along the loop $BCDB$, we get

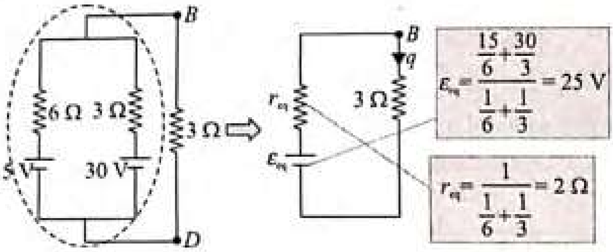
$-3(i_1 - i_2) - 30 + 3i_2 = 0$

or $\quad -i_1 + 2i_2 = 10$ (ii)

Solving Eqs. (i) and (ii) for i_2, we get $i_2 = 5$ A.

Method 2: Using equivalent battery method

We can treat 6 Ω and 3 Ω resistances connected with 15 V and 30 V batteries, respectively, as internal resistances of the batteries. Now we can assume these batteries are connected in parallel, and equivalent battery of these two is connected with resistance connected across B and D.



Hence, current through resistance connected across B and D is

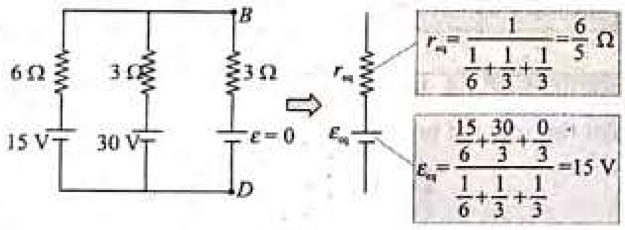
$$i = \frac{20}{2+3} = 5\text{ A}$$

Method 3: Using equivalent battery method

We can assume a battery of zero emf is connected in series with resistance connected across BD. This resistance can be treated as internal resistance of zero emf battery. Now we have three batteries in parallel. The equivalent of these batteries will be a single battery which is the open circuit.

As equivalent battery is an open circuit, the potential difference across each of the battery branch will be 15 V. Hence, current through the resistance connected across BD is

$$i = \frac{15}{3} = 5\text{ A}$$

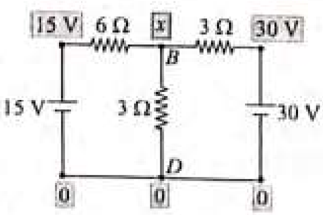


Method 4: Using nodal method

Let us assume the potential of node D zero and the potential of node B x. Then we can assign the potentials at different nodes as shown in figure. At node B,

$$\frac{(x-15)}{6} + \frac{(x-30)}{3} + \frac{(x-0)}{3} = 0$$

or $\quad 5x = 75$ or $x = 15$ V

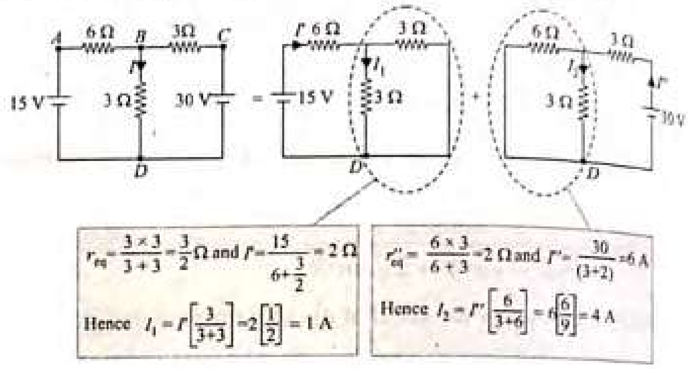


Hence, current through resistance connected across BD is

$$i = \frac{15-0}{3} = 5\text{ A} \quad \text{(from } B \text{ to } D\text{)}$$

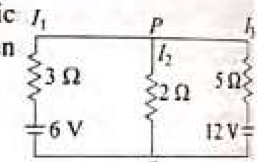
Method 5: Using superposition principle

In superposition method, we can take the effect of one battery at a time. From figure, it is clear that the effect of two batteries can be taken by adding the effect of individual batteries.



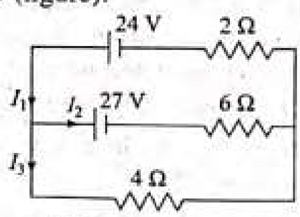
CONCEPT APPLICATION EXERCISE 19.2

1. Calculate the value of the electric currents I_1, I_2, and I_3 in the given electrical network.



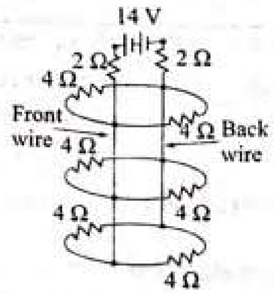
   (a) $I_1 =$ ___
   (b) $I_2 =$ ___
   (c) $I_3 =$ ___

2. Determine the currents I_1, I_2, and I_3 for the network shown below (figure).

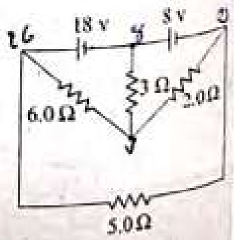


   (a) $I_1 =$ ___ (b) $I_2 =$ ___ (c) $I_3 =$ ___

3. Find the current supplied by the source in figure. The resistors are mounted around in a cylindrical form.

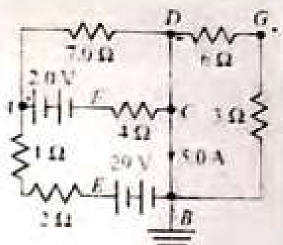


4. Find the current in each resistor of the circuit shown in figure.

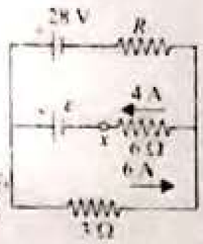


   (a) Current through resistance of 6 Ω is _____.
   (b) Current through resistance of 3 Ω is _____.
   (c) Current through resistance of 2 Ω is _____.
   (d) Current through resistance of 5 Ω is _____

5. Given that a current of 5.0 A passes along the branch from C to B in figure. What is the voltage of points A, D, and G?
(a) V_A = ___
(b) V_D = ___
(c) V_G = ___

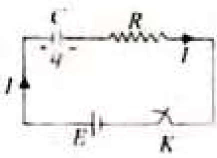


6. In the circuit shown in figure,
(a) find the current in resistor R
(b) find the resistance R
(c) find the unknown emf ε
(d) if the circuit is broken at point x, what is the current in resistor R?



CHARGING AND DISCHARGING OF A CAPACITOR THROUGH A RESISTANCE

Charging: Consider an uncharged capacitor C connected to a resistor R through a battery of emf E. Let switch K be closed at $t = 0$. Let at any time t, charge on the capacitor be q and current in the circuit be I.



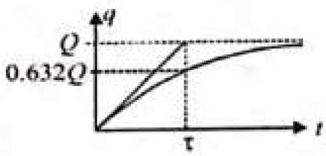
or $\quad q = EC[1 - e^{-t/RC}] = Q(1 - e^{-t/\tau}) \qquad$ (i)

where $Q = EC$ is the maximum charge on capacitor and $\tau = RC$ is the time constant of the circuit. Let us find the charge at $t = \tau$. Putting $t = \tau$ in Eq. (i), we get

$$q = Q(1 - e^{-\tau/\tau}) = Q(1 - e^{-1})$$

$$= Q\left(1-\frac{1}{e}\right)=\frac{Q(e-1)}{e}=0.632Q$$

So time constant (τ) is the time in which charge on the capacitor becomes 63.2% of maximum charge. The variation of charge q with time t is as shown in the given in figure.



Current or rate of charging: Differentiating Eq. (i), we get

$$I=\frac{dq}{dt}=Q\left(0-e^{-t/\tau}\left(-\frac{1}{\tau}\right)\right)=\frac{Q}{\tau}e^{-t/\tau}=\frac{EC}{RC}e^{-t/\tau}$$

$$=\frac{E}{R}e^{-t/\tau}=I_0e^{-t/\tau} \qquad \text{(ii)}$$

where $I_0 = E/R$ is the maximum current in the circuit. Let us find the current at $t = \tau$, putting $t = \tau$ in Eq. (ii), we get

$$I = I_0e^{-\tau/\tau}=\frac{I_0}{e}=0.368I_0$$

So time constant (τ) may also be defined as the time in which current decreases to 36.8% of its initial maximum value. The variation of current I with time t is as shown in figure.

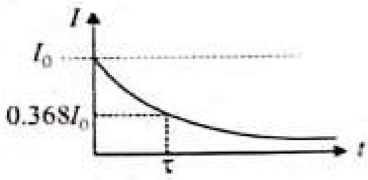


ILLUSTRATION 19.17 An uncharged capacitor is connected to a 15 V battery through a resistance of 10 Ω. It is found that in a time of 2 μs, potential difference across the capacitor becomes 5 V. Find the capacitance of the capacitor. Take ln(1.5) = 0.4.

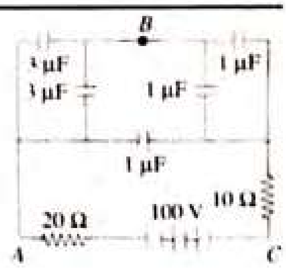
Solution. We know that charge on the capacitor at any time is given by $q = Q(1 - e^{-t/\tau})$ where $Q = EC = 15C$. Here charge q at any time is given by $q = VC$ where V is potential difference across capacitor at that time. Here $V = 5$ V, so $q = 5C$. Putting the values, we get

$$5C = 15C(1 - e^{-t/\tau}) \text{ or } e^{-t/\tau} = 2/3$$

or $\quad \dfrac{t}{\tau}=\ln\left(\dfrac{3}{2}\right)$ or $\dfrac{t}{RC}=\ln\left(\dfrac{3}{2}\right)$

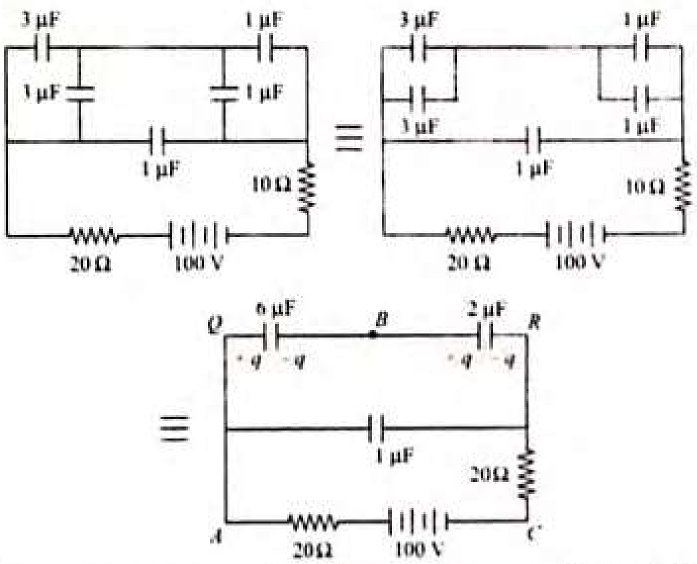
or $\quad C=\dfrac{t}{R\ln(3/2)}=\dfrac{2\times10^{-6}}{10\ln(3/2)}=0.5\ \mu\text{F}$

ILLUSTRATION 19.18 In the diagram shown in figure, find the potential difference between the points A and B and between the points B and C in the steady state.



Solution. Applying Kirchhoff's law in loop $AQBRC$, we get

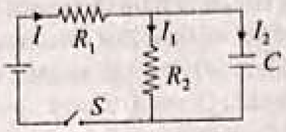
$$-\frac{q}{6}-\frac{q}{2}+100=0 \text{ or } q = 150\ \mu\text{C}$$



Therefore, the potential difference between AB is 150/6 = 25 V and potential difference between BC is 100 − 25 = 75 V.

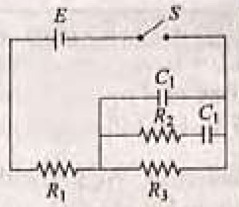
Concept Application Exercise 19.3

1. Consider the circuit shown in figure. If the switch is closed at $t = 0$, then calculate the values of I, I_1, and I_2 at



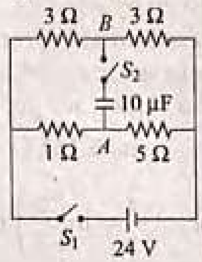
   (a) $t = 0$ (b) $t = \infty$

2. Determine the current through the battery in the circuit shown in figure.



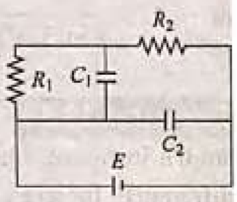
   (a) Immediately after the switch S is closed ______.
   (b) After a long time ______.

3. Consider the network shown in figure; initially, the switch S_1 is closed and S_2 is open.

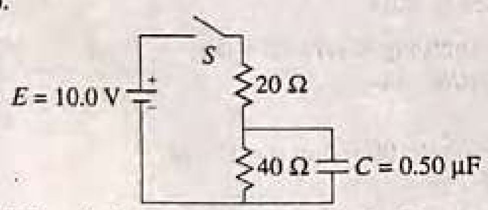


   (a) Calculate $V_A - V_B$.
   (b) When S_2 is also closed, what is $V_A - V_B$
      i. just after closing
      ii. after long time

4. In the circuit shown in figure, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $C_1 = 1\ \mu F$, $C_2 = 2\ \mu F$, and $E = 6$ V. Calculate the charge on each capacitor in the steady state.



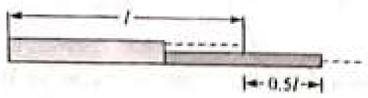
5. In the circuit shown in figure, switch S is closed at time $t = 0$.



   (a) What is the current I_0 leaving the battery at $t = 0$, immediately after the switch is closed?
   (b) What is the current I "long time" later?
   (c) What charge has accumulated on the capacitor after this long time?
   (d) If, finally, switch S is opened again, how long will it take after the switch is opened for the capacitor to lose 80% of its charge?

Solved Examples

1. In order to quadruple the resistance of a uniform wire, a part of its length was uniformly stretched till the final length of the entire wire was 1.5 times the original length, the part of the wire was fraction equal to



   (a) 1/8 (b) 1/6
   (c) 1/10 (d) 1/4

Sol. (a) Let l be the original length of wire and x be its length stretched uniformly such that final length is $1.5\,l$

Then $4R = \rho\dfrac{(l-x)}{A} + \rho\dfrac{(0.5l+x)}{A'}$

where $A' = \dfrac{x}{(0.5l+x)}A$

$\therefore\ 4\rho\dfrac{l}{A} = \rho\dfrac{l-x}{A} + \rho\dfrac{(0.5l+x)^2}{xA}$

or $4l = l - x + \dfrac{1}{4}\dfrac{l^2}{x} + \dfrac{x^2}{x} + \dfrac{lx}{x}$ or $x = l/8$

2. The n rows each containing m cells in series are joined in parallel. Maximum current is taken from this combination across an external resistance of 3Ω resistance. If the total number of cells used are 24 and internal resistance of each cell is $0.5\ \Omega$ then
   (a) $m = 8, n = 3$ (b) $m = 6, n = 4$
   (c) $m = 12, n = 2$ (d) $m = 2, n = 12$

Sol. (c) Total cells = $m \times n = 24$ (i)

For maximum current in the circuit $R = \dfrac{mr}{n}$

$\Rightarrow\ 3 = \dfrac{m}{n} \times (0.5) \Rightarrow m = 6n$ (ii)

On solving Eqs. (i) and (ii), we get $m = 12$, $n = 2$

3. In the circuit element given here, if the potential at point B, $V_B = 0$, then the potentials of A and D are given as

   (a) $V_A = -1.5V$, $V_D = +2V$
   (b) $V_A = +1.5V$, $V_A = +1.5V, V_D = +2V$
   (c) $V_A = +1.5V$, $V_D = +0.5V$
   (d) $V_A = +1.5V, V_D = -0.5V$

Sol. (d) Potential difference between A and

$V_A - V_B = 1 \times 1.5$

$\Rightarrow\ V_A - 0 = 1.5V \Rightarrow V_A = 1.5V$

Potential difference between B and C

$V_B - V_C = 1 \times 25 = 25V$

$\Rightarrow\ 0 - V_C = 25V \Rightarrow V_C = -2.5V$

Potential difference between C and D

$V_C - V_D = -2V \Rightarrow -2.5 - V_D = -2 \Rightarrow V_D = -0.5V$

4. When a resistance of 2 ohm is connected across the terminals of a cell, the current is 0.5 A. When the resistance is increased to 5 ohm, the current is 0.25 A. The e.m.f. of the cell is

(a) 1.0 V (b) 1.5 V
(c) 2.0 V (d) 2.5 V

Sol. (b) Since $i = \left(\dfrac{E}{R+r}\right)$ we get

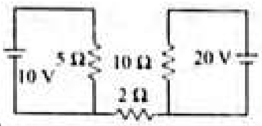
$$0.5 = \frac{E}{2+r} \quad \text{(i)}$$

$$0.25 = \frac{E}{5+r} \quad \text{(ii)}$$

Dividing (i) by (ii), we get $2 = \dfrac{5+r}{2+r} \Rightarrow r = 1\ \Omega$

$$\therefore \quad 0.5 = \frac{E}{2+1} \Rightarrow E = 1.5\ \text{V}$$

5. Find out the value of current through 2 Ω resistance for the given circuit.

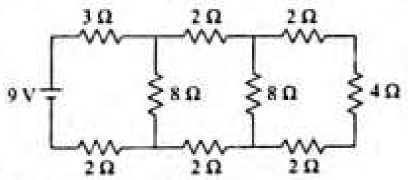


(a) 5 A (b) 2 A
(c) Zero (d) 4 A

Sol. (c) Since the current coming out from the positive terminal is equal to the current entering the negative terminal, therefore, current in the respective loop will remain confined in the loop itself.

$\therefore$ Current through 2 Ω resistor = 0

6. In the circuit shown in the figure, the current through

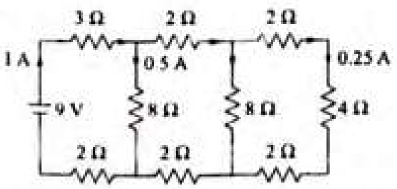


(a) The 3 Ω resistor is 0.50 A
(b) The 3 Ω resistor is 0.25 A
(c) The 4 Ω resistor is 0.50 A
(d) The 4 Ω resistor is 0.25 A

Sol. (d) Equivalent resistance of the circuit $R = 9\ \Omega$

$\therefore$ Main current $i = \dfrac{V}{R} = \dfrac{9}{9} = 1$ A

After proper distribution, the current through 4 Ω resistance is 0.25 A.



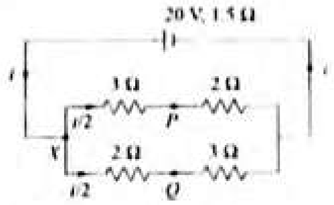
7. If in the circuit shown below, the internal resistance of the battery is 1.5 Ω and V_P and V_Q are the potentials at P and Q respectively, what is the potential difference between the points P and Q

(a) zero (b) 4 volt ($V_P > V_Q$)
(c) 4 volt ($V_Q > V_P$) (d) 2.5 volt ($V_Q > V_P$)

Sol. (d)

$$R_{eq} = \frac{5}{2}\ \Omega$$

$$i = \frac{20}{\frac{5}{2} + 1.5} = 5\ \text{A}$$



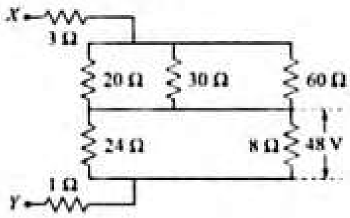
Potential difference between X and P,

$$V_x - V_P = \left(\frac{5}{2}\right) \times 3 = 7.5\ \text{V} \quad \text{(i)}$$

$$V_x - V_Q = \frac{5}{2} \times 2 = 5\ \text{V} \quad \text{(ii)}$$

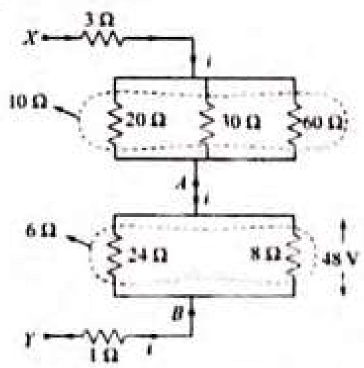
On solving (i) and (ii) $V_P - V_Q = -2.5$ volt; $V_Q > V_P$.

8. The potential difference across 8 ohm resistance is 48 volt as shown in the figure. The value of potential difference across X and Y points will be



(a) 160 volt (b) 128 volt
(c) 80 volt (d) 62 volt

Sol. (a) The given circuit can be redrawn as follows



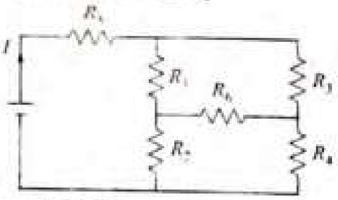
Resistance between A and $B = \dfrac{24 \times 8}{32} = 6\ \Omega$ Current between A and B = Current between X and $Y = i = \dfrac{48}{6} = 8$ A

Resistance between X and Y

$= (3 + 10 + 6 + 1) = 20\ \Omega$

$\Rightarrow$ Potential difference between X and $Y = 8 \times 20 = 160$ V

9. In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy



(a) $R_1R_2R_5 = R_3R_4R_5$

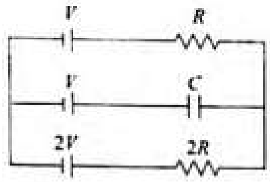
(b) $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$

(c) $R_1R_4 = R_2R_3$

(d) $R_1R_3 = R_2R_4 = R_5R_6$

Sol. (c) As I is independent of R_6 no current flows through R_6 this requires that the junction of R_1 and R_2 is at the same potential as the junction of R_3 and R_4. This must satisfy the condition $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ as in the Wheatstone bridge.

10. In the given circuit, with steady current, the potential drop across the capacitor must be



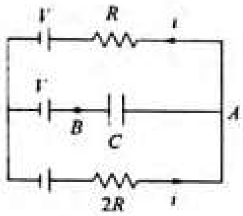
(a) V (b) $V/2$

(c) $V/3$ (d) $2V/3$

Sol. (c) Moving anticlockwise from

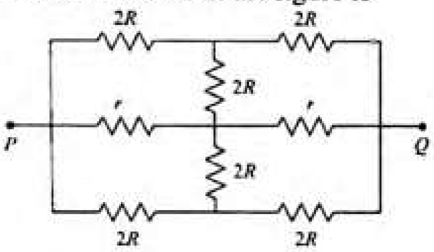
$$-iR - V + 2V - 2iR = 0$$

$$\text{or } 3iR = V \text{ or } i = \frac{V}{3R}$$



$$V_A - V_B = iR + V - V = iR = \frac{V}{3}$$

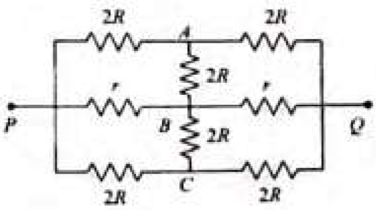
11. The effective resistance between points P and Q of the electrical circuit shown in the figure is



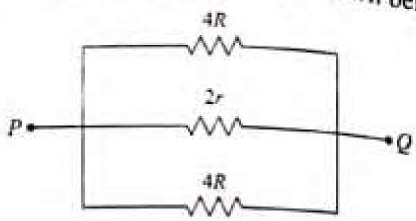
(a) $2Rr/(R + r)$ (b) $8R(R + r)(3R + r)$

(c) $2r + 4R$ (d) $5R/2 + 2r$

Sol. (a) In a circuit, any circuit element placed between points at the same potential can be removed, without affecting the rest of the circuit. Here, by symmetry, points A, B and C are at same potential, for any potential difference between P and Q.

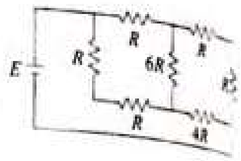


The circuit can therefore be reduced as shown below



Effective resistance $R_{eq} = \frac{2Rr}{R+r}$

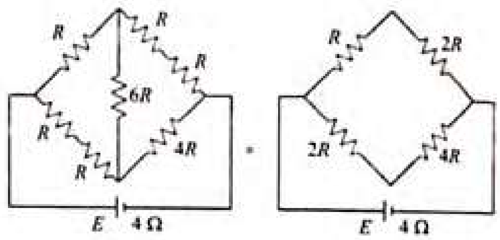
12. A battery of internal resistance $4\,\Omega$ is connected to the network of resistances as shown. In order to give the maximum power to the network, the value of R (in Ω) should be



(a) 4/9 (b) 8/9

(c) 2 (d) 18

Sol. (c) The equivalent network is

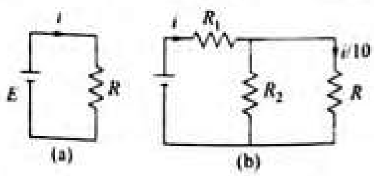


Clearly, the network of resistances is a balanced Wheatstone bridge. So R_{AB} is given by

$$\frac{1}{R_{AB}} = \frac{1}{3R} + \frac{1}{6R} = \frac{2+1}{6R} = \frac{1}{2R} \Rightarrow R_{AB} = 2R$$

For maximum power transfer $2R = 4\,\Omega \Rightarrow R = \frac{4}{2} = 2\Omega$

13. Consider the circuits shown in the figure. Both the circuits are taking same current from battery but current through R in the second circuit is $\frac{1}{10}$ th of current through R in the first circuit. If R is $11\,\Omega$, the value of R_1



(a) $9.9\,\Omega$ (b) $11\,\Omega$

(c) $8.8\,\Omega$ (d) $7.7\,\Omega$

Sol. (a) In Figure (b) current through $R_2 = i - \frac{i}{10} = \frac{9i}{10}$

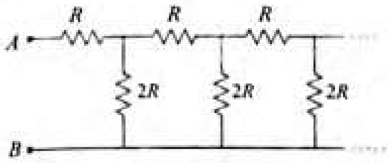
Potential difference across R_2 = Potential difference across R

$$\Rightarrow R_2 \times \frac{9}{10}i = R \times \frac{i}{10} \text{ i.e } R_2 = \frac{R}{9} = \frac{11}{9}\Omega$$

$$R_{eq} = \frac{R_2 \times R}{(R_2 + R)} = \frac{\frac{11}{9} \times \frac{11}{1}}{\frac{11}{9} + \frac{11}{1}} = \frac{11}{10}\,\Omega$$

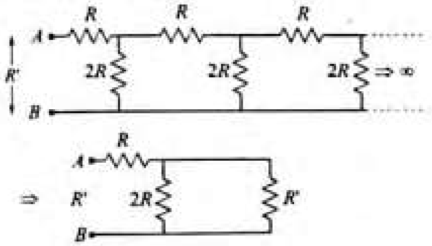
Total circuit resistance $= \frac{11}{10} + R_1 = R = 11 \Rightarrow R_1 = 9.9\,\Omega$

14. An infinite ladder network is arranged with resistances R and $2R$ as shown. The effective resistance between terminals A and B is



(a) ∞ (b) R
(c) $2R$ (d) $3R$

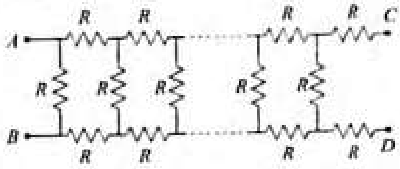
Sol. (c) Let equivalent resistance between A and B is R', so given circuit can be reduced as follows



$$R' = R + \frac{2R \times R'}{(2R + R')} \Rightarrow R'^2 - RR' - 2R^2 = 0$$

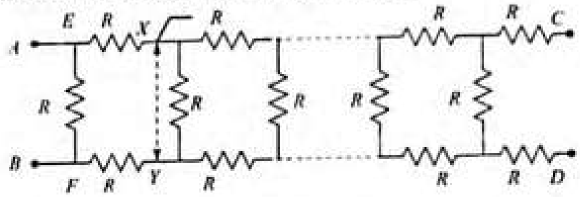
On solving the equation we get $R = 2R$.

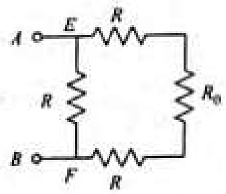
15. In the figure, the value of resistors to be connected between C and D so that the resistance of the entire circuit between A and B does not change with the number of elementary sets used is



(a) R (b) $R(\sqrt{3}-1)$
(c) $3R$ (d) $R(\sqrt{3}+1)$

Sol. (b) Cut the series from XY and let the resistance towards right of XY be R_0 whose value should be such that when connected across AB does not change the entire resistance. The combination is reduced to as shown below.





The resistance across EF, $= R_{EF} = (R_0 + 2R)$

Thus $R_{AB} = \frac{(R_0 + 2R)R}{R_0 + 2R + R} = \frac{R_0 R + 2R^2}{R_0 + 3R} = R_0$

$\Rightarrow \quad R_0{}^2 + 2RR_0 - 2R^2 + 0 \Rightarrow R(\sqrt{3}-1)$

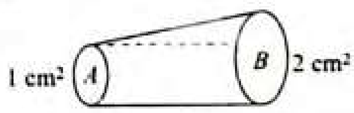
EXERCISES

Resistivity and Drift Velocity

1. The length of the resistance wire is increased by 10%. What is the corresponding change in the resistance of wire?
(a) 10% (b) 25%
(c) 21% (d) 9%

2. Two wires each of radius of cross section r but of different materials are connected together end to end (i.e. in series). If the densities of charge carriers in the two wires are in the ratio 1:4, the drift velocity of electrons in the two wires will be in the ratio
(a) 1 : 2 (b) 2 : 1
(c) 4 : 1 (d) 1 : 4

3. A solid conductor has a cross-section area 1 cm^2 and 2 cm^2 as shown in the figure. A current of 20 A entering at A. Then:

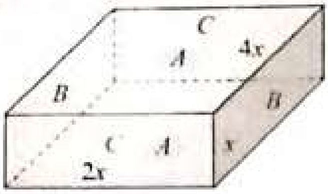


(a) current density at A = current density at B
(b) current density at A > current density at B
(c) current density at A < current density at B
(d) none of the above

4. A current I flows through a uniform wire of diameter d when the mean electron drift velocity is v. The same current will flow through a wire of diameter $d/2$ made of the same material if the mean drift velocity of the electron is

(a) $v/4$ (b) $v/2$
(c) $2v$ (d) $4v$

5. The following figure shows a rectangular block with dimensions x, $2x$ and $4x$. Electrical contacts can be made to the block between opposite pairs of faces (for example, between the faces labelled A-A, B-B and C-C). Between which two faces would the maximum electrical resistance be obtained (A-A: Top and bottom faces, B-B: Left and right faces, C-C: Front and rear faces)



(a) A-A (b) B-B
(c) C-C (d) Same for all three pairs

6. Two resistances R_1 and R_2 are made of different materials. The temperature coefficient of the material of R_1 is α and of the material of R_2 is $-\beta$. The resistance of the series combination of R_1 and R_2 will not change with temperature, if R_1/R_2 equals

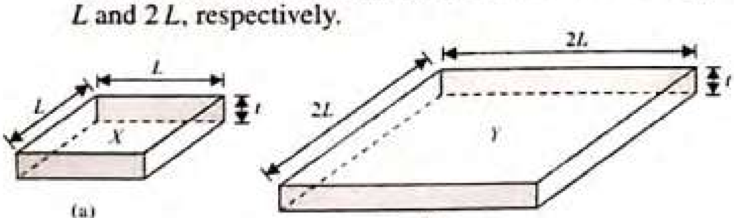
(a) $\dfrac{\alpha}{\beta}$ (b) $\dfrac{\alpha+\beta}{\alpha-\beta}$

(c) $\dfrac{\alpha^2+\beta^2}{\alpha\beta}$ (d) $\dfrac{\beta}{\alpha}$

7. A wire of cross-section area A, length L_1, resistivity ρ_1 and temperature coefficient of resistivity α_1 is connected in series to a second wire of length L_2, resistivity ρ_2, temperature coefficient of resistivity α_2 and the same area A, so that wires carry same current. Total resistance R is independent of temperature for small temperature change if (Thermal expansion effect is negligible)

(a) $\alpha_1 = -\alpha_2$ (b) $\rho_1 L_1 \alpha_1 + \rho_2 L_2 \alpha_2 = 0$
(c) $L_1\alpha_1 + L_2\alpha_2 = 0$ (d) None

8. Figure shows two squares, X and Y, cut from a sheet of metal of uniform thickness t. X and Y have sides of length L and $2L$, respectively.

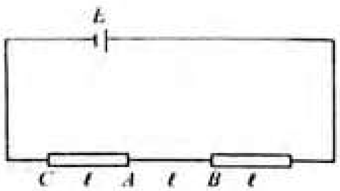


(a) (b)

The resistances R_x and R_y of the squares are measured between the opposite faces shaded in figure. What is the value of R_x/R_y?

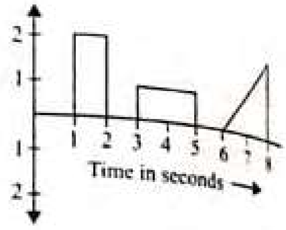
(a) 1/4 (b) 1/2
(c) 1 (d) 2

9. A cell of emf E volt with no internal resistance is connected to a wire whose cross section changes. The wire has three sections of equal length. The middle section has a radius a, whereas the radius of the outer two sections is $2a$. The ratio of the potential difference across section AB to the potential difference across section CA is



(a) 5 (b) 4
(c) 1/2 (d) 1/4

10. The plot represents the flow of current through a wire at three different times. The ratio of charges flowing through the wire at different times is (see figure)



(a) 2 : 1 : 2 (b) 1 : 3 : 3
(c) 1 : 1 : 1 (d) 2 : 3 : 4

11. Resistance of a resistor at temperature t°C is $R_t = R_0(1+\alpha t+\beta t^2)$, where R_0 is the resistance at 0°C. The temperature coefficient of resistance at temperature t°C is

(a) $\dfrac{(1+\alpha t+\beta t^2)}{\alpha+2\beta t}$ (b) $(\alpha+2\beta t)$

(c) $\dfrac{\alpha+2\beta t}{(1+\alpha t+\beta t^2)}$ (d) $\dfrac{\alpha+2\beta t}{2(1+\alpha t+\beta t^2)}$

12. The masses of the three wires of copper are in the ratio 1 : 3 : 5. And their lengths are in the ratio 5 : 3 : 1. The ratio of their electrical resistance is

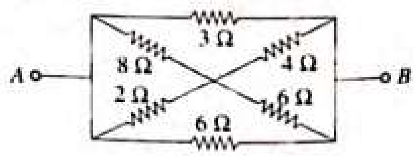
(a) 1 : 3 : 5 (b) 5 : 3 : 1
(c) 1 : 15 : 125 (d) 125 : 15 : 1

13. The temperature coefficient of resistance of conductor varies as $\alpha(T) = 3T^2 + 2T$. If R_0 is resistance at $T = 0$ and R is resistance at T, then

(a) $R = R_0(6T+2)$ (b) $R = 2R_0(3+2T)$
(c) $R = R_0(1+T^2+T^3)$ (d) $R = R_0(1-T+T^2+T^3)$

Combination of Resistance and Cells

14. The equivalent resistance between A and B in the network in figure is



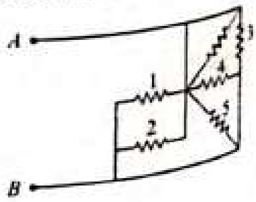
(a) $\dfrac{4}{3}\ \Omega$ (b) $\dfrac{3}{2}\ \Omega$

(c) 3 Ω (d) 2 Ω

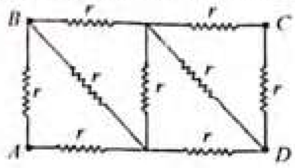
15. The circuit shown has resistors of equal resistance R. Find the equivalent resistance between A and B, when the key is closed.

(a) $\dfrac{11R}{12}$ (b) $\dfrac{13R}{12}$

(c) $\dfrac{R}{5}$ (d) $\dfrac{15R}{12}$



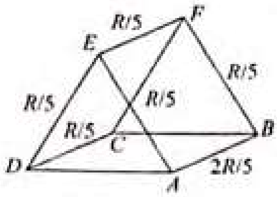
16. For the circuit shown in figure, the equivalent resistance between A and C is



(a) $\frac{12}{11}r$ (b) $\frac{13}{11}r$

(c) $\frac{14}{11}r$ (d) $\frac{15}{11}r$

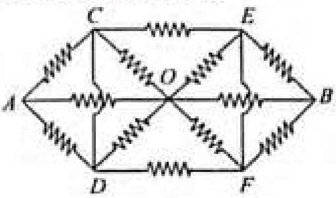
17. The current enters at A and comes out at D. Some of the resistances are shown. What should be resistance of wire CB so that it draws double of the current that enters the wire BF.



(a) $\left(\frac{9}{10}\right)R$ (b) $\left(\frac{8}{9}\right)R$

(c) $\left(\frac{7}{9}\right)R$ (d) $\left(\frac{3}{20}\right)R$

18. Find the equivalent resistance between A and B. Each resistor has same resistance R.



(a) $\frac{8}{5}R$ (b) $\frac{6}{5}R$

(c) $\frac{7}{5}R$ (d) $\frac{4}{5}R$

19. The emf of a cell is ε and its internal resistance is r. Its terminals are connected to a resistance R. The potential difference between the terminals is 1.6 V for $R = 4\ \Omega$, and 1.8 V for $R = 9\ \Omega$. Then,

(a) $\varepsilon = 1$ V, $r = 1\ \Omega$ (b) $\varepsilon = 2$ V, $r = 1\ \Omega$

(c) $\varepsilon = 2$ V, $r = 2\ \Omega$ (d) $\varepsilon = 2.5$ V, $r = 0.5\ \Omega$

20. N identical cells are connected to form a battery. When the terminals of the battery are joined directly (short-circuited), current I flows in the circuit. To obtain the maximum value of I,

(a) all the cells should be joined in series

(b) all the cells should be joined in parallel

(c) two rows of $N/2$ cells each should be joined in parallel

(d) $\sqrt{N}$ rows of $\sqrt{N}$ cells each should be joined in parallel, given that $\sqrt{N}$ is an integer

21. n identical cells, each of emf ε and internal resistance r, are joined in series to form a closed circuit. The potential difference across any one cell is

(a) zero (b) ε

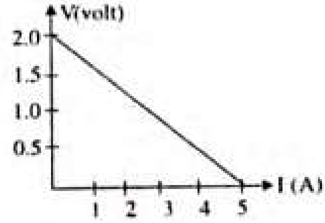
(c) $\frac{\varepsilon}{n}$ (d) $\frac{n-1}{n}\varepsilon$

22. n identical cells, each of emf ε and internal resistance r, are joined in series to form a closed circuit. One cell A is joined with reversed polarity. The potential difference across each cell, except A, is

(a) $\frac{2\varepsilon}{n}$ (b) $\frac{n-1}{n}\varepsilon$

(c) $\frac{n-2}{n}\varepsilon$ (d) $\frac{2n}{n-2}\varepsilon$

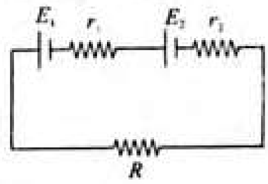
23. For a cell, a graph is plotted between the potential difference V across the terminals of the cell and the current I drawn from the cell (see figure). The emf and the internal resistance of the cell are E and r, respectively. Then



(a) $E = 2$ V, $r = 0.5\ \Omega$ (b) $E = 2$ V, $r = 0.4\ \Omega$

(c) $E > 2$ V, $r = 0.5\ \Omega$ (d) $E > 2$ V, $r = 0.4\ \Omega$

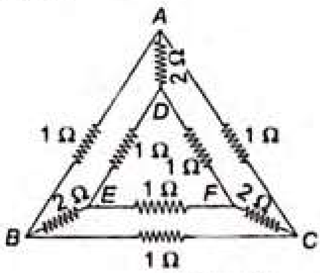
24. Under what condition, the current passing through the resistance R can be increased by short-circuiting the battery of emf E_2. The internal resistances of the two batteries are r_1 and r_2, respectively.



(a) $E_2 r_1 > E_1(R + r_2)$ (b) $E_1 r_2 > E_2(R + r_1)$

(c) $E_2 r_2 > E_1(R + r_2)$ (d) $E_1 r_1 > E_2(R + r_1)$

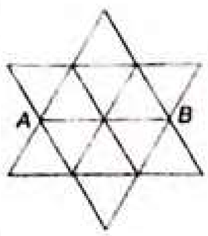
25. A network of nine conductors connects six points A, B, C, D, E and F as shown in figure. The figure denotes resistances in ohms. Find the equivalents resistance between A and D.



(a) $4\ \Omega$ (b) $1\ \Omega$

(c) $5\ \Omega$ (d) $6\ \Omega$

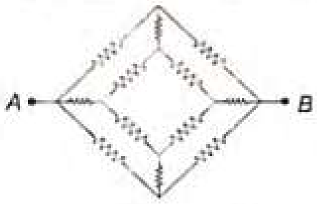
26. Find the equivalent resistance of the circuit between points A and B shown in figure is (each branch is of resistance = $1\ \Omega$)



(a) $\frac{22}{25}\Omega$ (b) $\frac{12}{25}\Omega$

(c) $\frac{22}{35}\Omega$ (d) $\frac{12}{35}\Omega$

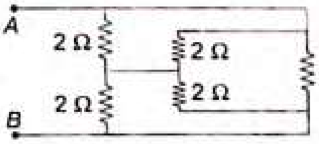
27. The figure shows a network of resistor each heaving value $12\,\Omega$. Find the equivalent resistance between points A and B.



(a) $9\,\Omega$ (b) $\frac{12}{5}\,\Omega$

(c) $8\,\Omega$ (d) $\frac{11}{3}\,\Omega$

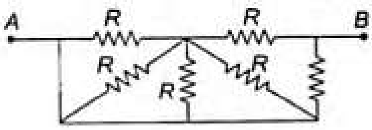
28. Find the equivalent resistance across AB.



(a) $1\,\Omega$ (b) $2\,\Omega$

(c) $3\,\Omega$ (d) $4\,\Omega$

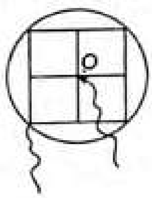
29. The equivalent resistance between the points A and B is



(a) $\frac{5R}{9}$ (b) $\frac{2R}{3}$

(c) R (d) None of these

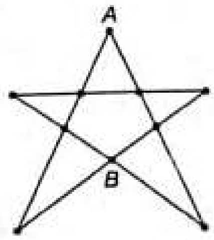
30. A network of twelve resistances each of resistance R form a square of squares as shown in the figure. The outer square is fitted in a metal ring of negligible resistance. Find the resistance between centre of the square and the ring.



(a) $R/4$ (b) $R/2$

(c) $3R/8$ (d) $R/12$

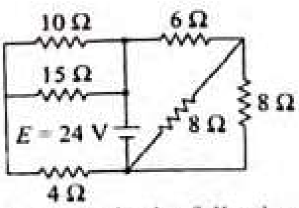
31. The resistance of all the wires between any two adjacent dots is R. The equivalent resistance between A and B as shown in figure is



(a) $\frac{7}{3}R$ (b) $\frac{7}{6}R$

(c) $\frac{14}{8}R$ (d) None of these

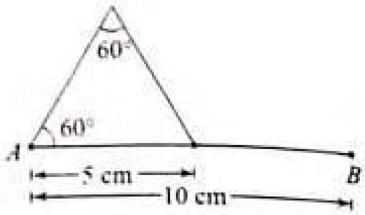
32. Find the equivalent resistance across the terminals of source of e.m.f. 24 V for the circuit shown in figure



(a) $15\,\Omega$ (b) $10\,\Omega$

(c) $5\,\Omega$ (d) $4\,\Omega$

33. A wire has resistance of $24\,\Omega$ is bent in the following shape. The effective resistance between A and B is

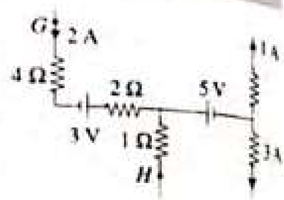


(a) $24\,\Omega$ (b) $10\,\Omega$

(c) $\frac{16}{3}\,\Omega$ (d) None of these

Kirchhoff's Law and Simple Circuits

34. In the part of a circuit shown in figure, the potential difference $(V_G - V_H)$ between points G and H will be



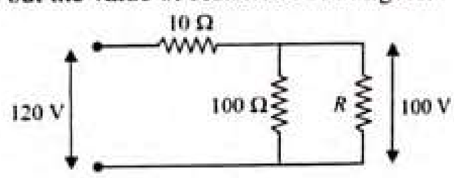
(a) 0 V

(b) 15 V

(c) 7 V

(d) 3 V

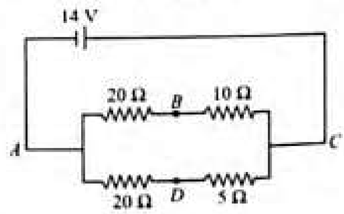
35. Find out the value of resistance R in figure.



(a) $100\,\Omega$ (b) $200\,\Omega$

(c) $50\,\Omega$ (d) $150\,\Omega$

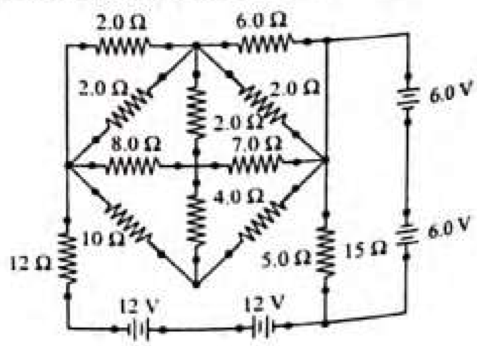
36. What resistor should be connected in parallel with the $20\,\Omega$ resistor in branch ADC in the circuit shown in figure so that potential difference between B and D may be zero?



(a) $20\,\Omega$ (b) $10\,\Omega$

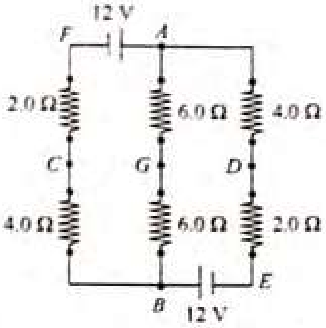
(c) $5\,\Omega$ (d) $5\,\Omega$

37. The current through the $8\,\Omega$ resistor (shown in figure) is

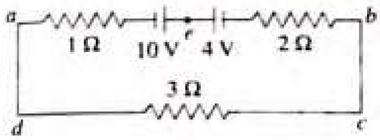


(a) 4 A (b) 2 A
(c) zero (d) 2.5 A

38. In the network shown in figure, the potential difference across A and B is
(a) 6 V
(b) zero
(c) 2 V
(d) 4 V

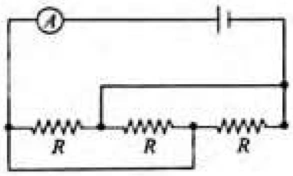


39. In the circuit shown in figure, the magnitudes and the direction of the flow of current, respectively, would be



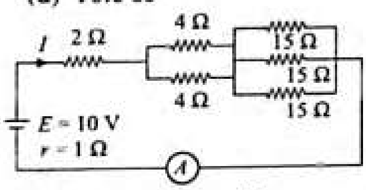
(a) 7/3 A from a to b via e (b) 7/3 A from b to a via e
(c) 1 A from b to a via e (d) 1 A from a to b via e

40. Figure represents a load consisting of three identical resistances connected to an electric energy source of emf 12 V and internal resistance 0.6 Ω. The ammeter reads 2 A. The magnitude of each resistance is

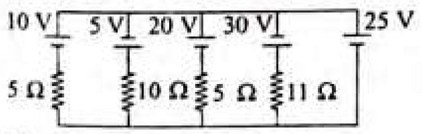


(a) 3.6 Ω (b) 7.2 Ω
(c) 16.2 Ω (d) 10.8 Ω

41. In the circuit shown in figure, the current I has a value equal to
(a) 1 A (b) 2 A
(c) 4 A (d) 3.5 A

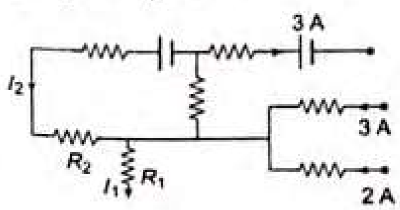


42. In the circuit shown, current through 25 V cell is



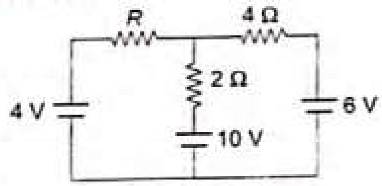
(a) 7.2 A (b) 10 A
(c) 12 A (d) 14.2 A

43. In the given circuit if I_1 and I_2 be the current in resistance R_1 and R_2 respectively then:



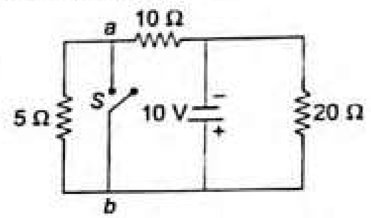
(a) $I_1 = 3$ A, $I_2 = 2$ A
(b) $I_1 = 0$, $I_2 = 2$ A
(c) $I_1 = 2$ A, $I_2 = 2$ A
(d) $I_1 = 2$ A and I_2 can't be determined with given data

44. For what value of R in circuit, current through 4 Ω resistance is zero?



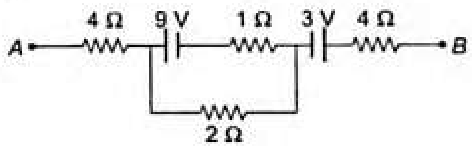
(a) 4 Ω (b) 1 Ω
(c) 3 Ω (d) 5 Ω

45. In the circuit shown below, the current that flows from a to b when the switch S is closed is



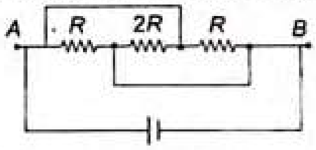
(a) −1.5 A (b) +1.5 A
(c) +1.0 A (d) −1.0 A

46. In the circuit shown in figure potential difference between point A and B is 16 V. Find the current passing through 2 Ω resistance.



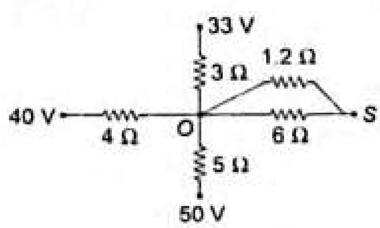
(a) 1.5 A (b) 3.5 A
(c) 1.0 A (d) 2.5 A

47. In the figure shown the current flowing through 2 R is



(a) from left to right (b) from right to left
(c) no current (d) none of these

48. In the given network, the potential at point O is (Given $V_O - V_s = 18$ V)



(a) 9 V (b) 780/47 V
(c) 680/43 V (d) none of these

Resistance Capacitor Circuits

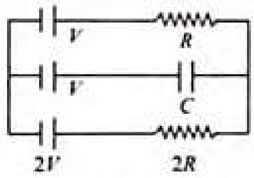
49. A capacitor is charged to a certain potential and then allowed to discharge through a resistance R. The ratio of charge on the capacitor to current in the circuit

(a) changes with time
(b) does not change with time and it is equal to time constant of circuit
(c) does not change with time, but not equal to time constant of circuit
(d) may or may not change depending upon the charge given to the capacitor

50. Two capacitors C_1 and C_2 ($C_1 > C_2$) are charged separately to same potential. Now they are allowed to discharge through similar resistors. Initial rate of discharging will be

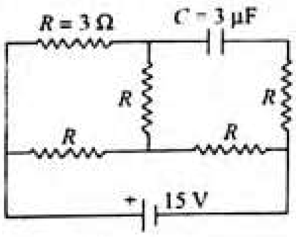
(a) more for C_1 (b) more for C_2
(c) same for both (d) cannot say

51. In the given circuit of figure, with steady current, the potential drop across the capacitor must be



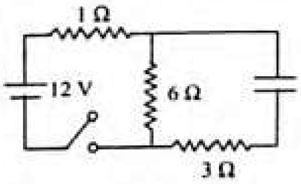
(a) V (b) $V/2$
(c) $V/3$ (d) $2V/3$

52. In the circuit shown in figure, the cell is ideal with emf 15 V. Each resistance is of 3 Ω. The potential difference across the capacitor in steady state is



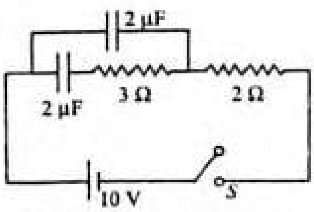
(a) 0 (b) 9 V
(c) 12 V (d) 15 V

53. When the switch is closed, then the initial current through 1 Ω resistor is (see figure)



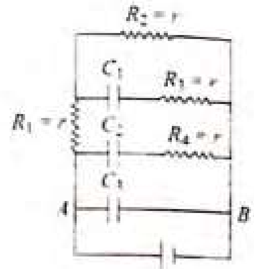
(a) 12 A (b) 4 A
(c) $\frac{10}{7}$ A (d) 3 A

54. Current through the battery at the instance when the switch S is closed is (see figure)



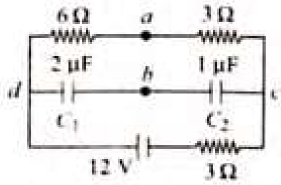
(a) zero (b) 2 A
(c) 4 A (d) 5 A

55. The equivalent resistance between points A and B in figure at steady state will be



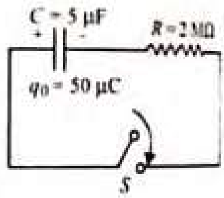
(a) $2r$ (b) $\frac{3}{5}r$
(c) $\frac{5}{3}r$ (d) none of these

56. What is the charge stored on each capacitor C_1 and C_2 in the circuit shown in figure?



(a) 6 μC, 6 μC (b) 6 μC, 3 μC
(c) 3 μC, 6 μC (d) 3 μC, 3 μC

57. For the arrangement shown in figure, the switch is closed at $t = 0$. The time after which the current becomes 2.5 μA is given by (take ln2 = 0.69)

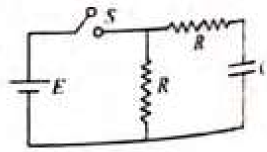


(a) 10 s (b) 5 s
(c) 7 s (d) 0.693 s

58. A capacitor discharges through a resistance. The stored energy U_0 in one capacitive time constant falls to

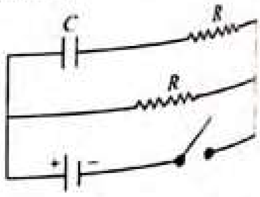
(a) U_0/e^2 (b) eU_0
(c) U_0/e (d) none of these

59. A capacitor C is connected to the two equal resistances as shown in figure. What is the ratio of the time constant during charging and discharging of the capacitance?



(a) 1:1 (b) 2 : 1
(c) 1 : 2 (d) 4 : 1

60. In the circuit shown in figure, when the switch is closed, the capacitor charges with a time constant

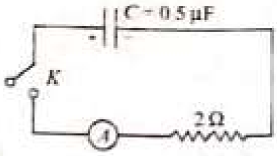


(a) RC (b) $2\,RC$
(c) $\frac{1}{2}RC$ (d) RC log2

61. In the question 60, if the switch is opened after the capacitor has been charged, it will discharge with a time constant

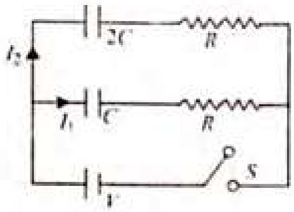
(a) RC (b) $2\,RC$
(c) $\frac{1}{2}RC$ (d) RC ln2

62. A charged capacitor is allowed to discharge through a resistor by closing the key at the instant $t = 0$ (see figure). At the instant $t = (\ln 4)$ μs, the reading of the ammeter falls half the initial value. The resistance of the ammeter is equal to



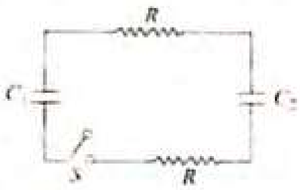
(a) 1 MΩ (b) 1 Ω
(c) 2 Ω (d) 2 MΩ

63. In the circuit shown in figure, switch S is closed at time $t = 0$. Let I_1 and I_2 be the currents at any finite time t, then the ratio I_1/I_2



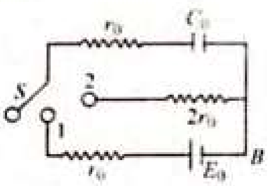
(a) is constant
(b) increases with time
(c) decreases with time
(d) first increases, then decreases

64. In the circuit shown in figure, $C_1 = 2C_2$. Initially, capacitor C_1 is charged to a potential of V. The current in the circuit just after the switch S is closed is



(a) 0 (b) $2V/R$
(c) ∞ (d) $V/2R$

65. In the circuit given in figure switch S is at position 1 for long time. Find the total heat generated in resistor of resistance $(2r_0)$, when the switch S is shifted from position 1 to position 2.



(a) $\frac{C_0E_0^2}{2}$ (b) $C_0E_0^2$

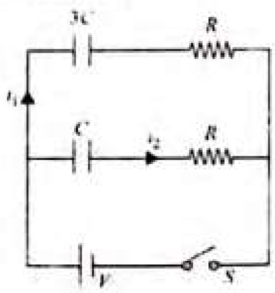
(c) $\frac{C_0E_0^2}{3}$ (d) none

66. A capacitor of capacitance C has charge Q. It is connected to an identical capacitor through a resistance. The heat produced in the resistance is

(a) $\frac{Q^2}{2C}$ (b) $\frac{Q^2}{4C}$

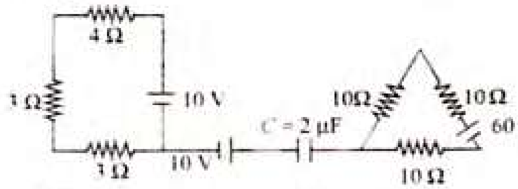
(c) $\frac{Q^2}{8C}$ (d) dependent on the value of the resistance

67. In the circuit shown, switch S is closed at $t = 0$. Let i_1 and i_2 be the currents at any finite time t, then the ratio i_1/i_2



(a) is constant
(b) increases with time
(c) decreases with time
(d) first increases and then decreases

68. In the circuit shown in figure, find the maximum energy stored on the capacitor. Initially, the capacitor was uncharged.



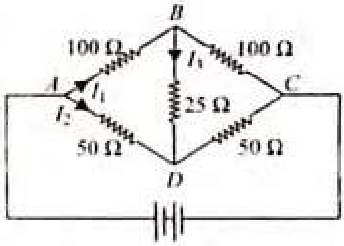
(a) 150 μC (b) 100 μC
(c) 50 μC (d) zero

Problems Based On Mixed Concepts

69. A wire of length L and three identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raised by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross section but of length $2L$. The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is

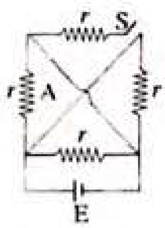
(a) 4 (b) 6
(c) 8 (d) 9

70. Figure shows a Wheatstone bridge circuit. Which of the following correctly shows the currents I_1, I_2, and I_3 in the decreasing order of magnitude?



(a) I_1, I_2, I_3 (b) I_2, I_3, I_1
(c) I_2, I_1, I_3 (d) I_3, I_2, I_1

71. In figure, after closing switch S, what is the change in current flowing through A? The battery is ideal.

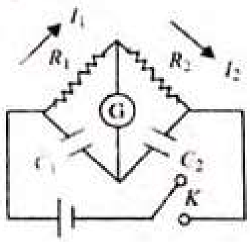


(a) no change
(b) decreases
(c) increases
(d) cannot say

72. In the above question, what would have been the change in current in A if battery were having some internal resistance.

(a) no change (b) decreases
(c) increases (d) cannot say

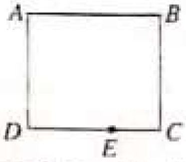
73. In the circuit in figure, if no current flows through the galvanometer when the key k is closed, the bridge is balanced. The balancing condition for bridge is



(a) $\frac{C_1}{C_2} = \frac{R_1}{R_2}$ (b) $\frac{C_1}{C_2} = \frac{R_2}{R_1}$

(c) $\frac{C_1^2}{C_2^2} = \frac{R_1^2}{R_2^2}$ (d) $\frac{G^2}{C_2^2} = \frac{R_2}{R_1}$

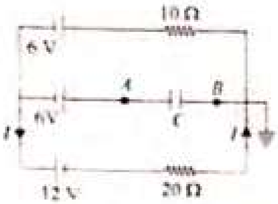
74. $ABCD$ is a square (see figure) where each side is a uniform wire of resistance 1 Ω. A point E lies on CD such that if a uniform wire of resistance 1 Ω is connected across AE and constant potential difference is applied across A and C, then B and E are equipotential.



(a) $\frac{CE}{ED}=1$ (b) $\frac{CE}{ED}=\frac{1}{\sqrt{2}}$

(c) $\frac{CE}{ED}=\frac{1}{\sqrt{2}}$ (d) $\frac{CE}{ED}=\sqrt{2}$

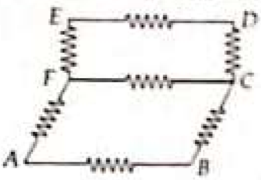
75. In the given circuit, with steady current, the potential of point A must be



(a) 1 V (b) 3 V

(c) 4 V (d) 2 V

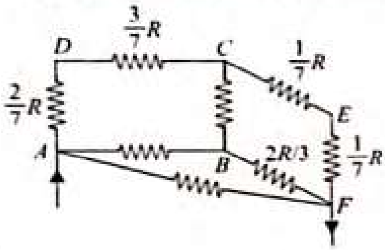
76. If the potential difference between A and D is V, what will be potential difference between F and C. Each of the resistance is R.



(a) $V/3$ (b) $V/4$

(c) $V/6$ (d) $V/8$

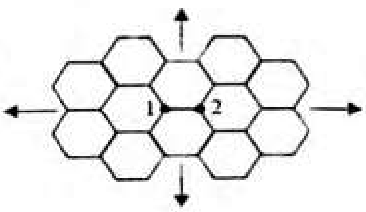
77. The current enters at A and leaves at F. The values of some resistances are shown. What should be the value of resistance AB so that no current will flow through CB?



(a) $\left(\frac{5}{7}\right)R$ (b) $\frac{R}{3}$

(c) $\left(\frac{5}{3}\right)R$ (d) $\left(\frac{5}{9}\right)R$

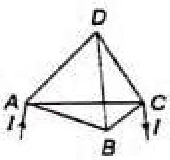
78. The given infinite grid consists of hexagonal cells of six resistors each of resistance R. Then $R_{12}=$



(a) $\frac{R}{3}$ (b) $\frac{2R}{3}$

(c) $\frac{4R}{3}$ (d) $\frac{3R}{4}$

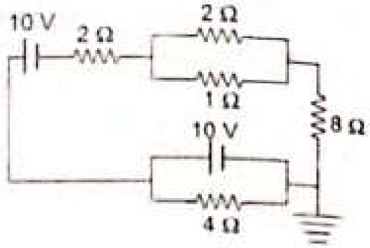
79. A wire frame in the form of a tetrahedron $ABCD$ is connected to a battery. The resistance of all the edges of the tetrahedron is equal. Indicate the edge of the frame whose removal does not affect I at all.



(a) AB (b) BC

(c) AC (d) BD

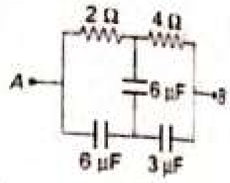
80. In the circuit shown the currents in 4 Ω and 8 Ω resistances are



(a) 2.5 A, 0 A (b) 5 A, 0 A

(c) 2.5 A, $\frac{40}{3}$ A (d) 5 A, $\frac{80}{3}$ A

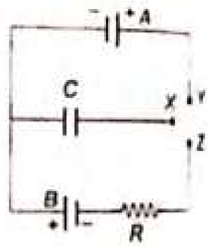
81. What is the equivalent capacitance between A and B in the circuit shown?



(a) 6 μF (b) 1.5 μF

(c) zero (d) 2 μF

82. In the circuit shown the cells are ideal and of equal emfs, the capacitance of the capacitor is C and the resistance of the resistor is R. X is first joined to Y and then to Z. After a long time, the total heat produced in the resistor will be



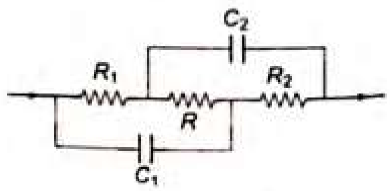
(a) equal to the energy finally stored in the capacitor

(b) half of the energy finally stored in the capacitor

(c) twice the energy finally stored in the capacitor

(d) 4 times the energy finally stored in the capacitor

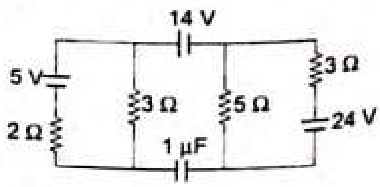
83. Which of the following conditions is correct in steady state?



(a) $R_1C_1=R_2C_2$ (b) $\frac{R_1}{C_1}=\frac{R_2}{C_2}$

(c) $R_1R_2=C_1C_2$ (d) all of these

84. The energy stored in the capacitor in the steady state is



(a) 338 μJ (b) 196 μJ

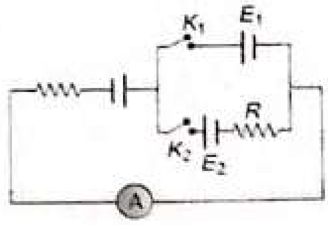
(c) 98 μJ (d) 8 μJ

85. In order to determine the e.m.f. of a storage battery it was connected in series with a standard cell (both are adding) in a certain circuit and a current I_1 was obtained. When polarity of the standard cell is reversed, a current I_2 was obtained in the same direction as that of I_1. What is the e.m.f. ε_1 of the storage battery? The e.m.f. of the standard cell is ε_2.

(a) $\varepsilon_1 = \dfrac{I_1 + I_2}{I_1 - I_2}\varepsilon_2$ (b) $\varepsilon_1 = \dfrac{I_1 + I_2}{I_2 - I_1}\varepsilon_2$

(c) $\varepsilon_1 = \dfrac{I_1 - I_2}{I_1 + I_2}\varepsilon_2$ (d) $\varepsilon_1 = \dfrac{I_2 - I_1}{I_1 + I_2}\varepsilon_2$

86. In the given arrangement, the reading of ammeter is same in each case when either K_1 or K_2 is closed. The reading of the ammeter is



(a) $\dfrac{E_1 - E_2}{R}$

(b) $\dfrac{E_1 + E_2}{R}$

(c) Data given is not sufficient

(d) None of above

ARCHIVES

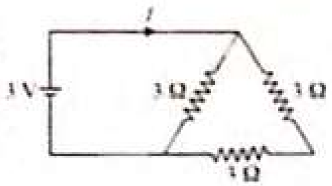
1. By increasing the temperature, the specific resistance of a conductor and a semiconductor

(a) increases for both (b) decreases for both

(c) increases, decreases (d) decreases, increases

(AIEEE 2002)

2. A 3 V battery with negligible internal resistance is connected in a circuit as shown in the figure. The current, I, in the circuit will be



(a) $\frac{1}{3}$ A (b) 1 A

(c) 1.5 A (d) 2 A **(AIEEE 2003)**

3. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter, the change in the resistance of the wire will be

(a) 300% (b) 200%

(c) 100% (d) 50% **(AIEEE 2003)**

4. The total current supplied to the circuit by the battery is

(a) 1 A

(b) 2 A

(c) 4 A

(d) 6 A **(AIEEE 2003)**

5. The resistance of the series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If $S = nP$, then the minimum possible value of n is

(a) 4 (b) 3

(c) 2 (d) 1 **(AIEEE 2004)**

6. An electric current is passed through a circuit containing two wires of the same material connected in parallel. If the lengths and radii of the wires are in the ratio of 4/3 and 2/3, then the ratio of the currents passing through the wires will be

(a) 3 (b) $\frac{1}{3}$

(c) $\frac{8}{9}$ (d) 2 **(AIEEE 2004)**

7. Two sources of equal emf are connected to an external resistance R. The internal resistance of the two sources are R_1 and R_2 $(R_2 > R_1)$. If the potential difference across the source having internal resistance is zero, then

(a) $R = \dfrac{R_1R_2}{(R_2 - R_1)}$ (b) $R = \dfrac{R_1R_2}{(R_2 + R_1)}$

(c) $R = R_2 - R_1$ (d) $R = R_2 \times \dfrac{(R_1 + R_2)}{(R_2 - R_1)}$

(AIEEE 2005)

8. Kirchhoff's first law $(\Sigma i = O)$ and second law $(\Sigma iR = \Sigma E)$, where symbols have their usual meanings, are, respectively, based on

(a) conservation of momentum, conservation of energy.

(b) conservation of charge, conservation of energy.

(c) conservation of charge, conservation of momentum.

(d) conservation of energy, conservation of charge.

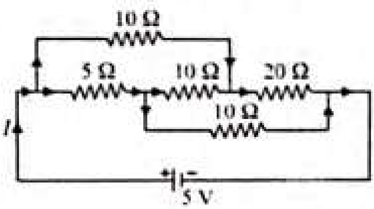
(AIEEE 2006)

9. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance is 0.005 Ω/°C, its resistance will become 200 Ω at the temperature of

(a) 500°C (b) 200°C

(c) 300°C (d) 400°C **(AIEEE 2006)**

10. The current I drawn from a 5 V source is



(a) 0.67 A (b) 0.17 A

(c) 0.33 A (d) 0.5 A **(AIEEE 2006)**

11. A material B has twice the specific resistance of A. A circular wire made of B has twice the diameter of a wire made of A. Then for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be

(a) $\frac{1}{4}$ (b) 2

(c) 1 (d) $\frac{1}{2}$ **(AIEEE 2006)**

12. The resistance of a wire is 5 Ω at 50°C and 6 Ω at 100°C. The resistance of the wire at 0°C will be

(a) 2 Ω (b) 1 Ω

(c) 4 Ω (d) 3 Ω **(AIEEE 2007)**

13. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 be the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 be the time taken for the charge to reduce to one-fourth of its initial value. Then the ratio t_1/t_2 will be

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 2 **(AIEEE 2010)**

14. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(a) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$ (b) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$

(c) $\alpha_1 + \alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}$ (d) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

(AIEEE 2010)

15. A resistor 'R' and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10}2.5 = 0.4$)

(a) 1.3×10^4 Ω (b) 1.7×10^5 Ω

(c) 2.7×10^6 Ω (d) 3.3×10^7 Ω

(AIEEE 2011)

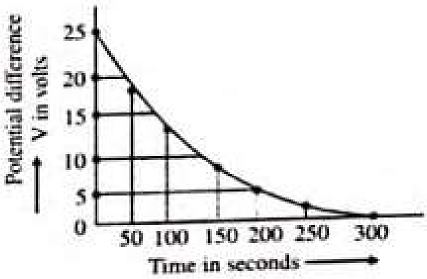
16. If a wire is stretched to make it 0.1% longer, its resistance will

(a) increase by 0.05% (b) increase by 0.2%

(c) decrease by 0.2% (d) decrease by 0.05%

(AIEEE 2011)

17. The figure shows an experimental plot discharging of a capacitor in an RC circuit. The time constant τ of this circuit lies between:

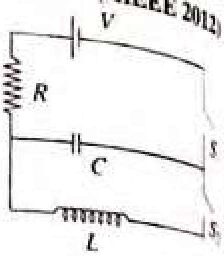


(a) 150 sec and 200 sec (b) 0 and 50 sec

(c) 50 sec and 100 sec (d) 100 sec and 150 sec

(AIEEE 2012)

18. In an LCR circuit as shown below, both switches are open initially. Now switch S_1 is closed and S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct?



(a) At $t = \tau$, $q = CV/2$

(b) At $t = 2\tau$, $q = CV(1 - e^{-2})$

(c) At $t = \tau/2$, $q = CV(1 - e^{-1})$

(d) Work done by the battery is half of the energy dissipated in the resistor. **(JEE Main 2013)**

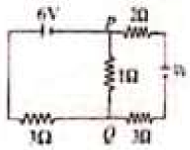
19. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to

(a) 1.6×10^{-8} Ω m (b) 1.6×10^{-7} Ω m

(c) 1.6×10^{-6} Ω m (d) 1.6×10^{-5} Ω m

(JEE Main 2015)

20. In the circuit shown, the current in the 1 Ω resistor is:



(a) 1.3 A, from P to Q

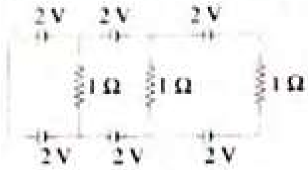
(b) 0 A

(c) 0.13 A, from Q to P

(d) 0.13 A, from P to Q

(JEE Main 2015)

21. In the given circuit the current in each resistance is

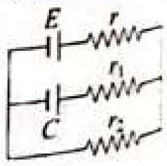


(a) 0.5 A (b) 0 A

(c) 1 A (d) 0.25 A

(JEE Main 2017)

22. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be



(a) $CE\frac{r_2}{(r + r_2)}$ (b) $CE\frac{r_1}{(r_1 + r)}$

(c) CE (d) $CE\frac{r_1}{(r_2 + r)}$

(JEE Main 2017)

23. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between

(a) 11.7 V and 11.8 V
(b) 11.6 V and 11.7 V
(c) 11.5 V and 11.6 V
(d) 11.4 V and 11.5 V

(JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	2. (c)	3. (b)	4. (d)	5. (c)	6. (d)	7. (b)	8. (c)	9. (b)	10. (c)
11. (c)	12. (d)	13. (c)	14. (a)	15. (c)	16. (d)	17. (d)	18. (d)	19. (b)	20. (b)
21. (a)	22. (a)	23. (b)	24. (b)	25. (b)	26. (c)	27. (a)	28. (a)	29. (a)	30. (c)
31. (b)	32. (c)	33. (b)	34. (c)	35. (a)	36. (a)	37. (c)	38. (b)	39. (d)	40. (c)
41. (a)	42. (c)	43. (d)	44. (b)	45. (d)	46. (b)	47. (b)	48. (b)	49. (b)	50. (c)
51. (c)	52. (c)	53. (b)	54. (d)	55. (a)	56. (a)	57. (c)	58. (a)	59. (c)	60. (a)
61. (b)	62. (c)	63. (c)	64. (d)	65. (c)	66. (b)	67. (b)	68. (d)	69. (b)	70. (c)
71. (a)	72. (b)	73. (b)	74. (d)	75. (d)	76. (b)	77. (c)	78. (b)	79. (d)	80. (a)
81. (d)	82. (d)	83. (d)	84. (a)	85. (a)	86. (c)				

Archives

1. (c)	2. (c)	3. (a)	4. (c)	5. (a)	6. (b)	7. (c)	8. (b)	9. (d)	10. (d)
11. (b)	12. (c)	13. (c)	14. (d)	15. (c)	16. (b)	17. (d)	18. (b)	19. (d)	20. (c)
21. (b)	22. (a)	23. (c)							

Chapter **20**

Heating Effect of Current and Electrical Measuring Instruments

GALVANOMETER

A galvanometer is a device that measures the magnitude and direction of current in a circuit. It is a very sensitive device, which can be used to measure only small currents, say, in µA. If we pass a current through a galvanometer more than the specified amount, it may get damaged. The maximum current that can be passed through a galvanometer is known as the full-scale deflection current (say, I_G). The resistance of a galvanometer is very small.

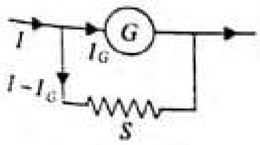
In a galvanometer, there is a needle attached to a coil, which is subject to a magnetic field. Attached to the coil is a spring similar to the hairspring on the balance wheel of a watch.

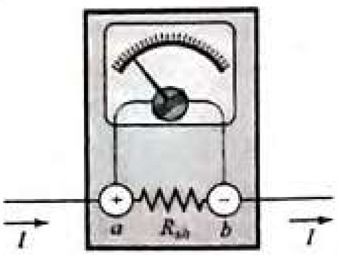
In equilibrium position, with no current in the coil, the needle is at zero. When there is current in the coil, the magnetic field exerts a torque on it, which is proportional to the current. This torque is responsible for the rotation of the coil and hence the needle deflects. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement.

AMMETER

A galvanometer cannot be used directly to measure big currents. To measure large currents, we have to carry out some modifications in the galvanometer. What we get after modifications is known as an *ammeter*.

Suppose we want to make an ammeter that can measure current up to I. Such an ammeter is said to have a range of $0 - I$. For this we have to connect a small resistance (known as *shunt*) in parallel with the galvanometer as shown in figure





The value of S is so selected that only I_G current passes through the galvanometer and the remaining $I - I_G$ through the shunt. Let the resistance of the galvanometer be G. Since G and S are in parallel, the potential difference across them should be same, i.e.,

$$I_G G = S(I - I_G) \text{ or } S = \frac{GI_G}{I - I_G}$$

The above equation gives the value of S to be connected in parallel with the galvanometer to convert it into an ammeter of range $0 - I$.

NOTES:

- **Resistance of ammeter:** G and S are in parallel, so their equivalent resistance is given by

$$R_A = \frac{SG}{S+G}$$

Since S is a very small resistance (it has to be small if we want most of the current to pass through it). R_A is even smaller than S. In general, the resistance of an ammeter is very small. For an ideal ammeter, resistance is zero.

- An ammeter is used in series to measure the current.
- The reading of an ammeter is generally lesser than the actual current in the circuit. It is because when we connect an ammeter in the circuit to measure the current, the ammeter introduces its own resistance in the circuit, which results in an increase in the resistance of the circuit and decrease in the current.
- The lower the value of shunt resistance, the higher the range of ammeter.
- **How an ammeter reads the current:** The current through the galvanometer is responsible for the deflection of the needle of the galvanometer. We have to use this fraction of current (I_G) in measuring the actual current I.

As the potential difference across G is the same as that across S, from the equation $I_G G = (I - I_G)S$, we get

$$I_G = \left(\frac{S}{G+S}\right) I \text{ or } \boxed{I_G \propto I}$$

Hence, I_G is proportional to I or the deflection of the needle is proportional to the current I. If the value of current I is changed, then the deflection of the needle also changes. The scale can be graduated to read the value of I directly.

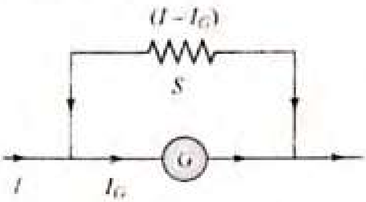
ILLUSTRATION 20.1 The deflection in a moving coil galvanometer falls from 50 divisions to 10 divisions when a shunt of 12 Ω is applied. What is the resistance of the galvanometer? Assume the main current to remain same.

Solution. In case of a galvanometer, $I \propto \theta$.

Given

$$\frac{I_G}{I} = \frac{10}{50} = \frac{1}{5}$$

i.e., $I_G = \frac{1}{5}I$



Now as in case of a shunted galvanometer as shown in figure,

$$(I - I_G)\,S = I_G G$$

or $$\left(I - \frac{1}{5}I\right) \times 12 = \frac{1}{5}IG$$

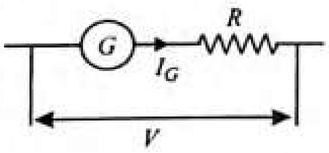
or $$G = 4 \times 12 = 48\ \Omega$$

VOLTMETER

A voltmeter an instrument used to find the potential difference across any two points in a circuit. A galvanometer can directly measure small potential differences only. To measure high potential differences, we have to do some modifications in the galvanometer. What we get after modifications is known as *voltmeter*.

Conversion of a Galvanometer into Voltmeter

Suppose we want to make a voltmeter that can measure the potential difference up to V; the range of the voltmeter is 0–V. For this, a suitable high resistance is connected in series with the galvanometer such that when a potential difference of V is applied, only a current I_G passes through the galvanometer as shown in figure. We can write



$$V = I_G(G + R) \text{ or } R = \frac{V}{I_G} - G$$

The above equation gives the value of R in terms of V.

NOTES:

- **Resistance of voltmeter:** $R_V = G + R$. Generally, R_V is very high, and for an ideal voltmeter, R_V is infinite.
- A voltmeter is used in parallel to measure potential difference.
- The higher the value of R, the higher the range of voltmeter.
- **How a voltmeter reads the potential difference:**
- Let V be the potential difference across a resistor to be measured. We have the relation,

$$I_G = \frac{V}{R+G} \Rightarrow \boxed{I_G \propto V}$$

- We know that the deflection of the needle is proportional to the current I_G and hence to V. The scale can be graduated to read the potential difference directly.

ILLUSTRATION 20.2 A galvanometer has a resistance of 30 Ω and a current of 2 mA is needed for a full-scale deflection. What is the resistance and how is it to be connected to convert the galvanometer (i) into an ammeter of 0.3 A range and (ii) into a voltmeter of 0.2 V range?

Solution. Given $G = 30\ \Omega$ and $I_G = 2$ mA

(i) To convert the galvanometer into an ammeter of range 0.3 A.

$$(I - I_G)\,S = I_G\,G \text{ or } (0.3 - 0.002)\,S = 0.002 \times 30$$

or $$S = \frac{0.002 \times 30}{0.298} = 0.2013\ \Omega$$

Hence a resistance of small value called shunt resistance ($S = 0.2013\ \Omega$) should be connected parallel with the galvanometer.

(ii) To convert the galvanometer into a voltmeter of range 0.2 V.

$$V = I_G(R + G) \text{ or } 0.2 = 2 \times 10^{-3}(30 + R) \text{ or } R = 70\ \Omega$$

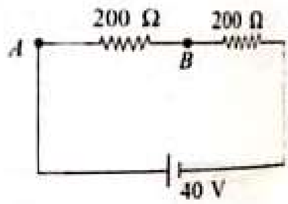
A resistance $R = 70\ \Omega$ should be connected in series with the galvanometer.

ILLUSTRATION 20.3

(i) In figure, find the potential difference between the points A and B.

(ii) Now we wish to measure this potential difference by using a voltmeter of resistance 2 kΩ. Find the reading of the voltmeter and percentage error.

(iii) Solve part (ii) if the voltmeter were of resistance 20 kΩ.



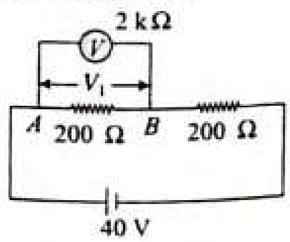
What conclusion do you draw from the results you get in the above parts?

Solution.

(i) As both the resistances are same, 40 V will be divided equally among both the resistances. Hence, the potential difference across A and B is 20 V.

(ii) Equivalent resistance of 200 Ω and 2 kΩ is

$$R_1 = \frac{200 \times 2000}{200 + 2000} = \frac{2000}{11}\ \Omega$$



Reading of voltmeter = potential difference across AB

$$= V_1 = 40 \times \left[\frac{\frac{2000}{11}}{\left(\frac{2000}{11}+200\right)}\right] = 19.05 \text{ V}$$

$$\text{Percentage error} = \frac{20-19.05}{20}\times 100 = 4.75\%$$

(iii) In this case, equivalent resistance of 200 Ω and 20 kΩ.

$$R_1 = \frac{200\times 20{,}000}{200+20{,}000} = \frac{20{,}000}{101}\ \Omega$$

Reading of voltmeter

= potential difference across AB

$$= V_2 = 40\left[\frac{\frac{20{,}000}{101}}{\frac{20{,}000}{101}+200}\right] = 19.90 \text{ V}$$

$$\text{Percentage error} = \frac{20-19.9}{20}\times 100 = 0.5\%$$

In case (iii), percentage error is less than that in case (ii). It means the more the resistance of the voltmeter, the more accurate the reading.

POTENTIOMETER

When we measure the current or potential difference using an ammeter or voltmeter, we do not get exact values, because when we connect the ammeter or voltmeter in the circuit, it disturbs the original circuit. But potentiometer is an instrument that can measure current or potential difference accurately. Basically, a potentiometer does not draw any current from the original circuit and hence does not disturb the original circuit. It is based on the principle that if a constant current is passed through a wire of uniform cross section, the potential difference across any segment of the wire is proportional to its length. Let a current I pass through a wire of uniform cross-sectional area A as shown below.

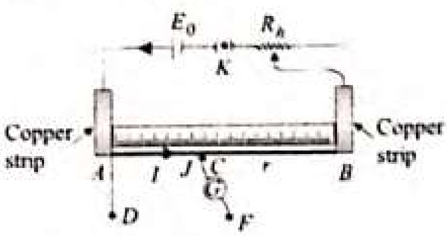
Consider a segment of length l of the wire. Resistance of this segment is given by $R = \rho l/A$. Potential difference across this segment is given by

$$V = IR = I\rho l/A \text{ or } V = kl \text{ or } k = V/l$$

where $k = I\rho/A$ is the potential difference per unit length known as *potential gradient*.

Construction of Potentiometer

A potentiometer consists of a wire AB of uniform cross section, generally 1–10 m long, fixed on a wooden board. Let the resistance of AB be r.



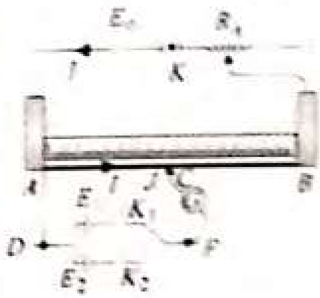
The ends A and B are connected to a battery E_0 (known as driving battery), a switch K, and a rheostat R_h. On closing the switch, current I is established in the wire AB, i.e.,

$$I = \frac{E_0}{R_h + r}$$

Current I can be adjusted by varying the value of R_h. A jockey J can slide freely on AB. The jockey touches the wire at C. A scale is fitted on the wooden board parallel to AB so that length AC can be read. Between points D and F, we can insert any cell, resistance, or any other device of which we need the current or voltage information. After inserting such a device, the jockey is slid on the wire, and the location of C is so selected that finally there is no current through the galvanometer. In this condition, the applied potential difference across D and F will be equal to the potential difference across the segment AC.

Uses of Potentiometer

Comparison of emf of two cells: Consider two cells of emfs E_1 and E_2 are connected across points D and F along with switches K_1 and K_2 as shown in figure. Initially, both the switches are open.



First close switch K_1 and move the jockey on the wire AB until the galvanometer shows no deflection. When $AC = \ell_1$, deflection in the galvanometer becomes zero. Then from the principle of potentiometer, we get

$$E_1 = k\ell_1 \qquad \text{(i)}$$

where k is the potential gradient of AB. We can say that E_1 is balanced at length ℓ_1.

Similarly, on opening switch K_1 and closing K_2, let E_2 be balanced at length ℓ_2, then

$$E_2 = k\ell_2 \qquad \text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{E_2}{E_1} = \frac{\ell_2}{\ell_1}$$

Thus we can compare the emfs of the cells.

Determination of internal resistance of a cell: Suppose we have a cell of emf E whose internal resistance r is to be measured.

Connect the cell in the circuit of the potentiometer as shown in figure. First keep switch S open and slide the jockey so that there is no deflection. Let the balancing length be ℓ_1. Then

$$E = k\ell_1 \qquad \text{(i)}$$

Now close switch S and again find the balancing length. Let the balancing length be ℓ_2. A separate current, say, I_1 is established in the lower circuit. I_1 is given by

$$I_1 = \frac{E}{R + r}$$

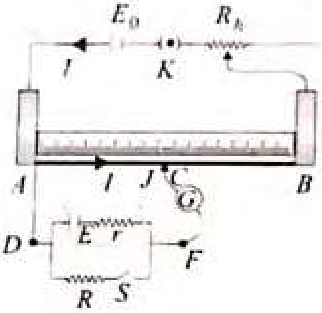
Let the terminal potential difference across the cell be V, then

$$V = k\ell_2 \text{ or } I_1 R = k\ell_2 \text{ or } \frac{E}{R+r}R = k\ell_2 \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{R+r}{R} = \frac{\ell_1}{\ell_2} \text{ or } r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R$$

Thus, we can measure the internal resistance of a cell.

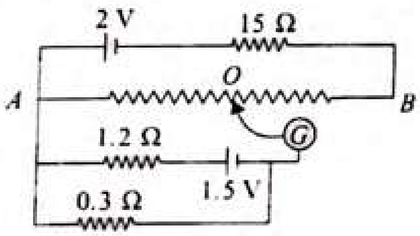


NOTE: We can also write

$$r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right)R = \left(\frac{k\ell_1 - k\ell_2}{k\ell_2}\right)R \text{ or } r = \left(\frac{E-V}{V}\right)R$$

Sensitivity of potentiometer: The sensitivity of a potentiometer means the smallest potential difference that can be measured with its help. It can be increased by decreasing the potential gradient. The same can be achieved by increasing the length of the potentiometer wire and by reducing the current in the potentiometer wire circuit with the help of a rheostat if the potentiometer wire is of fixed length.

ILLUSTRATION 20.4 In Figure, AB is a 1 m long uniform wire of 10 Ω resistance. Other data are shown in the figure. Calculate (i) potential gradient along AB and (ii) length AO when the galvanometer shows no deflection.



Solution.

i. Current in wire AB is $I = 2/(15 + 10) = 2/25$ A

Potential difference across AB is

$V = IR = 2/25 \times 10 = 0.8$ V

Potential gradient along AB is

$k = V/l = 0.8/1 = 0.8 \text{ Vm}^{-1}$

ii. Current through 0.3 Ω is $\dfrac{1.5}{1.2 + 0.3} = 1$ A

Potential difference across 0.3 Ω is $1 \times 0.3 = 0.3$ V

Let l_1 be the length AO, then $0.3 = 0.8 \times l_1$

or $l_1 = 0.3/0.8 = 0.375$ m

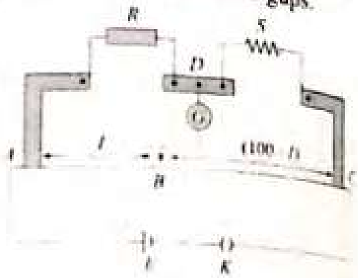
METER BRIDGE OR SLIDE WIRE BRIDGE

A slide wire bridge is a particular application of Wheatstone bridge and is used for (a) measuring an unknown resistance and (b) comparing two unknown resistances. A slide wire bridge works on the principle of Wheatstone bridge.

Construction

A slide wire bridge consists of a uniform wire AC, usually of eureka or manganin of 1 m length. It is stretched on a wooden board between two copper strips. A meter scale is fitted on the board parallel to the length of the wire. Another copper strip is fitted on the wooden board in order to provide two gaps.

In one of the gaps (say left gap), a resistance box R is connected, while in other gap (right gap), an unknown resistance S is connected. A cell E and a key K are connected across the ends A and C as shown in figure.



Checking of Connections

Close the key K and put the jockey at the end A of the wire and see the direction of deflection in the galvanometer. Now remove the jockey from A and put it at the end B of the wire and note the direction of deflection in the galvanometer. If the direction of deflection reverses, the connections are correct.

Working

Close the key K and take out some suitable low resistance R from the resistance box. Now move the jockey gently on wire AC till the galvanometer shows no deflection. Let this point be B on the wire. Let $AB = l$, then $BC = (100 - l)$.

Let the resistance of the wire between A and B be P and the resistance of the wire between B and C be Q. If r is the resistance of the wire of unit length, then

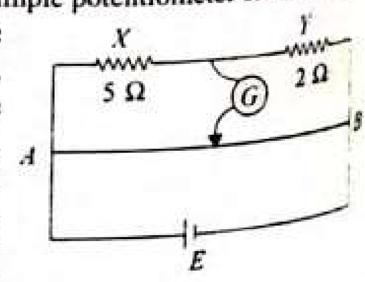
$P = lr$ and $Q = (100 - l)\,r$

According to the principle of Wheatstone bridge,

$$\frac{P}{Q} = \frac{R}{S} \text{ or } \frac{\ell r}{(100-\ell)r} = \frac{R}{S} \text{ or } S = \left(\frac{100-\ell}{\ell}\right)R$$

If λ and R are known, S can be calculated.

ILLUSTRATION 20.5 In the simple potentiometer circuit, where the length AB of the potentiometer wire is 1 m, the resistors X and Y have values of 5 Ω and 2 Ω, respectively. When X is shunted by a wire, the balance point is found to be 0.625 m from A. What is the resistance of the shunt?



Solution. Let R be the resistance of the shunted wire. The effective resistance of R and 5 Ω in parallel is $5R/(5+R)$.

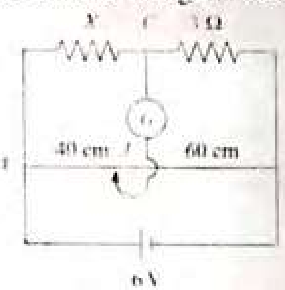
At balance point,

$$\frac{5R/(5+R)}{2}=\frac{0.625}{1-0.625}=\frac{0.625}{0.0375}=\frac{5}{3}$$

On solving, we get $R = 10\ \Omega$.

Concept Application Exercise 20.1

1. In the circuit shown in figure, a meter bridge is in its balance state. The meter bridge wire has a resistance of $1\ \Omega\ \text{cm}^{-1}$. Calculate the value of the unknown resistance X and the current drawn from the battery of negligible internal resistance.



2. A galvanometer with a coil of resistance $12.0\ \Omega$ shows full-scale deflection for a current of 2.5 mA. How will you convert the meter into
(a) an ammeter of range 0 to 7.5 A?
(b) a voltmeter of range 0 to 1.0 V?
3. What shunt resistance is required to make a 1.00 mA, $20.0\ \Omega$ meter into an ammeter with a range of 0 to 50.0 mA?
4. How can we make a galvanometer with $R_G = 20.0\ \Omega$ and $I_c = 0.00100$ A into a voltmeter with a maximum range of 10.0 V?
5. In an experiment with a potentiometer, the null point is obtained at a distance of 60 cm along the wire from the common terminal with a Leclanche cell. When a shunt resistance of $1\ \Omega$ is connected across the cell, the null point shifts to a distance of 30 cm from the common terminal. What is the internal resistance of the cell?
6. In the experiment of calibration of voltmeter, a standard cell of emf 1.1 V is balanced against 440 cm of potentiometer wire. The potential difference across the ends of a resistance is found to balance against 220 cm of the wire. The corresponding reading of the voltmeter is 0.5 V. Find the error in the reading of voltmeter.
7. It is required to measure the resistance of a circuit operating at 120 V. There is only one galvanometer of current sensitivity 10^{-6} A per division. How should the galvanometer be connected in the circuit to operate an ohmmeter? What minimum resistance can be measured with such a galvanometer if its full-scale has 40 divisions?

HEATING EFFECTS OF CURRENT

When an electric current is passed through a conductor, it becomes hot and its temperature starts rising. This is known as *heating effect of current* or *Joule's heating effect*. Here electric energy converts into heat energy. Various appliances such as geyser, iron, heater, fuse wire, etc. work on this basis.

Cause of Heating

When current is passed through a conductor, the electrons start drifting toward the positive end. They gain additional kinetic energy (KE) apart from thermal KE. These electrons suffer collisions with atoms/ions more violently and transfer their KE to atoms/ions. It increases the amplitude of vibrations of ions/atoms. Thus, the average KE of vibrations of atoms/ions increases, which shows up in the form of increased temperature. Here, the electric energy supplied by the source of emf is converted into heat.

Heat Produced by an Electric Current

Suppose a current I is flowing in a resistor of resistance R (figure). The amount of charge passed through the resistor in time t is $q = It$. Decrease in the potential of this charge is given by $V = IR$, and decrease in the potential energy of the charge is given by $qV = I^2Rt$.

This decrease in the energy will appear in the form of heat energy. So the electric energy produced in a resistor of resistance R in time t in which a current I is flowing is given by $H = I^2RT$, which is Joule's law of heating.

Thus, Joule's law of heating states that the amount of heat produced in a conductor is directly proportional to (1) the square of the current, (2) the resistance of the conductor, and (3) time. Other forms of H are as follows:

$$H = I^2Rt = \frac{V^2}{R} = VIt$$

Joule's heating effect is irreversible. The resistor will become hot (and not cool down), irrespective of the direction of current. As $H \propto I^2$, heating effect of current is common to both dc and ac. This is why instruments and appliances such as filament bulb, heater, geyser, press, toaster, etc. work on both dc and ac.

Electric Power Produced in the Circuit

The electric power generated in the circuit is the energy produced in the resistor per unit time. Thus,

$$P = \frac{H}{t} = I^2R = \frac{V^2}{R} = VI$$

Units of Electric Energy and Electric Power

Electric energy can be expressed in units such as J, cal, kWh, etc.

1 cal = 4.18 J = 4.2 J

Relation between kWh and J:

$1\ \text{kWh} = 1000\ \text{W} \times \text{h} = 1000\ \text{W} \times 3600\ \text{s} = 3.6 \times 10^6\ \text{Ws} = 3.6 \times 10^6\ \text{J}$

1 kWh is the energy consumed by an appliance of power 1 kW when it runs for 1 h.

Commercial unit: 1 kWh is one unit of electricity. To calculate the number of units, we can use the following relation:

$$\text{Number of units} = \frac{\text{watt} \times \text{hour}}{1000}$$

The energy dissipated in kWh can be calculated using the following relation:

$$E = \frac{V\,(\text{in volt}) \times I\,(\text{in ampere}) \times t\,(\text{in hour})}{1000}$$

Electric power can be expressed in units such as W, kW, MW, hp; they share the following relationships: 1 kW = 10^3 W, 1 MW = 10^6 W, 1 hp = 746 W.

Important Points

- If the resistances are connected in series, then using $P = I^2R$, the power developed will be higher in the resistor of higher value as current will be same in all resistors.
- If the resistances are connected in parallel, then using $P = V^2/R$, the power developed will be higher in the resistor of lower value as potential will be same across all resistors.

ILLUSTRATION 20.6 Two wires of same mass, having ratio of lengths 1:2, density 1:3, and resistivity 2:1, are connected one by one to the same voltage supply. The rate of heat dissipation in the first wire is found to be 10 W. Find the rate of heat dissipation in the second wire.

Solution. Given

$$\frac{\ell_1}{\ell_2} = \frac{1}{2}, \frac{d_1}{d_2} = \frac{1}{3}, \frac{\rho_1}{\rho_2} = \frac{2}{1}$$

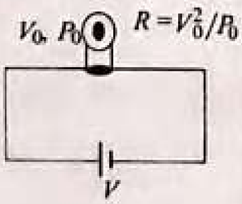
$$m = A_1\ell_1 d_1 = A_2\ell_2 d_2$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{V^2/R_2}{V^2/R_1} = \frac{R_1}{R_2} = \frac{\rho_1\ell_1A_2}{\rho_2\ell_2A_1} = \frac{\rho_1\ell_1^2d_1}{\rho_2\ell_2^2d_2} = \frac{2}{1}\times\left(\frac{1}{2}\right)^2\times\frac{1}{3} = \frac{1}{6}$$

or $\quad P_2 = \frac{P_1}{6} = \frac{10}{6} = \frac{5}{3}$ W

Important Points

Resistance of a bulb (or other appliances): Let a bulb be designed to operate on a voltage V_0 and its power indicated on it be P_0 (see figure). The resistance of the bulb is given by $R = V_0^2/P_0$.



Now let a potential difference of V is applied across this bulb, then power consumed is given by

$$P = \frac{V^2}{R} = \left(\frac{V}{V_0}\right)^2 P_0$$

If $V = V_0$, then $P = P_0$. The above formula is very convenient to calculate the power consumption when the applied voltage is different from the specified one.

An electric appliance consumes the specified power P_0 only if it runs at the specified voltage V_0. If the applied voltage V_A is greater than the specified voltage, the appliance may get damaged as in this situation, $I = V_A/R$ will exceed its current capacity $I_C = V_0/R$. Further, if an appliance is made to run at a voltage lower than the specified, then true power consumption will be less than the specified value.

ILLUSTRATION 20.7 A 500 W heating unit is designed to operate from a 200 V line. By what percentage will its heat output drop if the line voltage drops to 160 V? Find the heat produced by it in 10 min.

Solution. Actual power consumed is

$$P = \left(\frac{V}{V_0}\right)^2 P_0 = \left(\frac{160}{200}\right)^2 500 = 320 \text{ W}.$$

Heat output drop is 500 − 320 = 180 W.

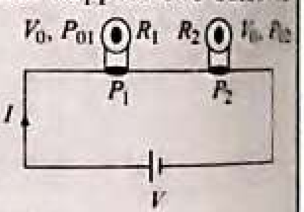
Percentage heat drop is $\frac{180}{500}\times 100 = 36\%$

Thus, heat produced in 10 min (600 s) is given by

$$H = 320 \times 600 = 192\,000 \text{ J} = 192 \text{ kJ}$$

Important Points

Two bulbs connected in series: Suppose two bulbs of same voltage rating V_0 and power ratings P_{01} and P_{02} are connected in series. Here $P_{01} > P_{02}$. Suppose potential V is applied across them as shown in figure.



Resistances of the bulbs are given by

$$R_1 = \frac{V_0^2}{P_{01}}, R_2 = \frac{V_0^2}{P_{02}}$$

Suppose powers produced in them are P_1 and P_2, respectively. Then $P_1 = I^2R_1$ and $P_2 = I^2R_2$. Now

$$P_{01} > P_{02} \Rightarrow R_1 < R_2 \Rightarrow P_1 < P_2$$

It means the bulb having more power rating will consume less power when connected in series. So the total power produced is

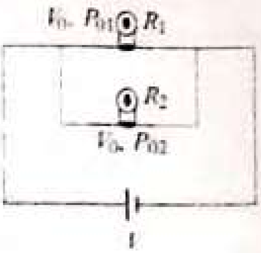
$$P = P_1 + P_2 = \frac{V^2}{R_1 + R_2} = \frac{V^2}{\frac{V_0^2}{P_{01}} + \frac{V_0^2}{P_{02}}} = \left(\frac{V}{V_0}\right)^2\left(\frac{P_{01}P_{02}}{P_{01}+P_{02}}\right)$$

If $V = V_0$, then

$$P = \frac{P_{01}P_{02}}{P_{01}+P_{02}} \text{ or } \frac{1}{P} = \frac{1}{P_{01}} + \frac{1}{P_{02}}$$

Note: If any one bulb in a series gets fused, then others will not glow.

Two bulbs connected in parallel: Suppose two bulbs of same voltage rating V_0 and power ratings P_{01} and P_{02} are connected in parallel. Here $P_{01} > P_{02}$. Suppose potential V is applied across them as shown in figure.



Resistances of the bulbs are

$$R_1 = \frac{V_0^2}{P_{01}}, R_2 = \frac{V_0^2}{P_{02}}$$

Let powers produced in them be P_1 and P_2, respectively.

Then $\quad P_1 = V^2/R_1$ and $P_2 = V^2/R_2$

$$P_{01} > P_{02} \Rightarrow R_1 < R_2 \Rightarrow P_1 > P_2$$

It means that the bulb having more power rating will consume more power when connected in parallel. So the total power produced is

$$P = P_1 + P_2 = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \frac{V^2}{V_0^2}P_{01} + \frac{V^2}{V_0^2}P_{02}$$

$$= \left(\frac{V^2}{V_0^2}\right)(P_{01} + P_{02})$$

If $V = V_0$, then $P = P_{01} + P_{02}$

Note: If one of the bulbs in a parallel connection gets fused, then others will continue to glow.

ILLUSTRATION 20.8 Two bulbs are marked 220 V–100 W and 220 V–50 W.

i. Which bulb will produce more illumination if they are connected in parallel to a 220 V supply?
ii. Which bulb will produce more illumination if they are connected in series to a 220 V supply?
iii. Also find the total power consumed by both the bulbs in each of the two parts above.

Solution.

i. The first bulb will produce more power. In parallel, the more the power rating, the more the power produced.
ii. The second bulb will produce more power. In series, the more the power rating, the lesser the power produced.
iii. In the first part,

$$P = P_{01} + P_{02} = 100 + 50 = 150 \text{ W}$$

In second part,

$$P = \frac{P_{01}P_{02}}{P_{01} + P_{02}} = \frac{100 \times 50}{150} = \frac{100}{3} \text{ W}$$

ILLUSTRATION 20.9 An electric tea kettle has two heating coils. When one of the coils is switched on, boiling begins in 6 min. When the other coil is switched on, boiling begins in 8 min. In what time will the boiling begin if both coils are switched on simultaneously (i) in series and (ii) in parallel.

Solution. Let the power of first coil be P_1 and that of the second coil is P_2. Let H be the amount of heat required to boil water. Then $H = P_1t_1 = P_2t_2$, where $t_1 = 6$ min and $t_2 = 8$ min.

i. When the coils are connected in series,

$$P = \frac{P_1P_2}{P_1 + P_2}$$

$$t = \frac{H}{P} = H\left[\frac{1}{P_1} + \frac{1}{P_2}\right] = H\left[\frac{t_1}{H} + \frac{t_2}{H}\right]$$

$$= t_1 + t_2 = 6 + 8 = 14 \text{ min}$$

ii. When the coils are connected in parallel.

$$P = P_1 + P_2$$

$$t = \frac{H}{P} = \frac{H}{P_1 + P_2} = \frac{H}{\frac{H}{t_1} + \frac{H}{t_2}}$$

$$= \frac{t_1t_2}{t_1 + t_2} = \frac{6 \times 8}{6 + 8} = 3.43 \text{ min}$$

Important Points

- Let a resistance R under a potential difference V dissipate power, then

$$P = \frac{V^2}{R}$$

So if the resistance is changed from R to R/n keeping V same, the power consumed will be

$$P' = \frac{V^2}{R/n} = n\frac{V^2}{R} = nP$$

That is, if for a given voltage, resistance is changed from R to R/n, power consumed changes from P to nP.

- If n equal resistances are connected in series with a voltage source, then power dissipated will be

$$P_S = \frac{V^2}{nR} \qquad [\text{as } R_S = nR]$$

And if the same resistances are connected in parallel with the same voltage source,

$$P_P = \frac{V^2}{R/n} = \frac{nV^2}{R} \qquad [\text{as } R_P = R/n]$$

$$\therefore \quad \frac{P_P}{P_S} = n^2 \quad \text{or} \quad P_P = n^2P_S$$

That is, power consumed by n equal resistors in parallel is n^2 times that of the power consumed in series, if V remains same.

Maximum Power Transfer Theorem

Suppose we want to find for what value of external resistance the maximum power will be drawn from a battery. For this, in the shown network (figure), let the power developed in resistance R be

$$P = I^2 R = \frac{E^2}{(R+r)^2} R \qquad \left(\text{as } I = \frac{E}{R+r}\right)$$

Now, for $dP/dR = 0$ (since P will be maximum if $dP/dR = 0$)

$$E^2 \frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4} = 0$$

or $(r + R) = 2R$ or $r = R$

It means the power output is maximum, when the external resistance equals the internal resistance i.e., when $R = r$.

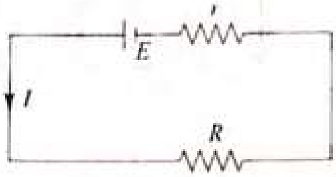


ILLUSTRATION 20.10 How will you connect (series and parallel) 24 cells each of internal resistance 1 Ω to get maximum power output across a load of 10 Ω?

Solution. Suppose there are m rows and each row has n cells. The total number of cells is $mn = 24$. Here

$$I = \frac{nE}{R + (nr/m)}$$

For maximum power

$$\frac{nr}{m} = R \text{ or } n = 10m \text{ or } 10m^2 = 24 \text{ or } m = \sqrt{2.4} = 1.55$$

i. If $m = 1$, $n = 24$, $I = 24E/34$, then

$$P_1 = \left(\frac{24E}{34}\right)^2 \times 10 = 4.98\,E^2$$

ii. If $m = 2$, $n = 12$, $I = 12E/16$, then

$$P_2 = \left(\frac{12E}{16}\right)^2 \times 10 = 5.625\,E^2$$

So we have two rows ($m = 2$) each containing 12 cells ($n = 12$) in series.

SOME APPLICATIONS

Fusing of bulb when it is switched on: Usually filament bulbs get fused when they are switched on. This is because with the rise in temperature, the resistance of the bulb increases and becomes constant in steady state. So the power consumed by the bulb (V^2/R) initially is more than that in steady state and hence the bulb glows more brightly in the beginning and may get fused.

ILLUSTRATION 20.11 Two wires made of tinned copper having identical cross section ($=10^{-6}$ m^2) and lengths 10 and 15 cm are to be used as fuses. Show that the fuses will melt at the same value of current in each case.

Solution. The temperature of the wire rises to a certain steady temperature when the heat produced per second by the current just becomes equal to the rate of loss of heat from its surface. Heat produced per second by the current is

$$I^2 R = I^2 \frac{\rho l}{\pi r^2}$$

where l is the length, r is the radius, and ρ is the specific resistance. Let H be the heat lost per second per unit surface area of the wire. If we neglect the loss of heat from the end faces of the wire, then heat lost per second by the wire is $H \times$ surface area of wire $= H \times 2\pi r l$

At steady state temperature,

$$H \times 2\pi r l = \frac{I^2 \rho l}{\pi r^2}$$

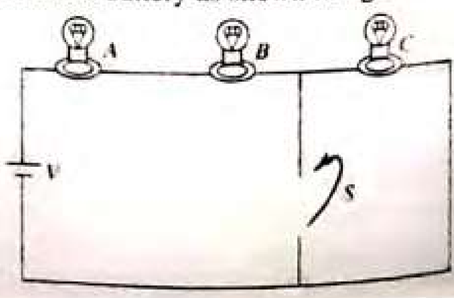
or $$H = \frac{I^2 \rho}{2\pi^2 r^3} \quad \text{(i)}$$

From Eq. (i) we note that the rate of loss of heat (H), which, in turn, depends on the temperature of the wire, is independent of the length of the wire. Hence, the fuses of two wires of the same values of r and ρ but of different lengths will melt for the same value of current in each case.

CONCEPT APPLICATION EXERCISE 20.2

1. A heater joined in series with a 50 W bulb is connected to the mains. If the 50 W bulb is replaced by a 100 W bulb, then will the heater now give more heat, less heat, or same heat? Why?
2. Each of the three resistors in figure has a resistance of 2 Ω and can dissipate a maximum of 18 W without becoming excessively heated. Find the maximum power the circuit can dissipate.

3. An electric bulb rated 220 V and 60 W is connected in series with another electric bulb rated 220 V and 40 W. The combination is connected across a source of emf 220 V. Which bulb will glow more?
4. We have a 30 W, 6 V bulb, which we want to glow by a supply of 120 V. What can be done for this?
5. Two heater coils made of the same material are connected in parallel across the mains: the length and the diameter of one coil is double that of the other. Which of them will produce more heat?
6. A series circuit consists of three identical lamps connected to a battery as shown in figure.



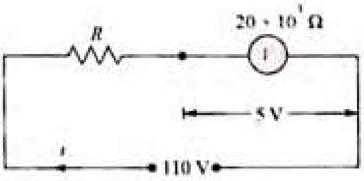
When the switch S is closed, what happens
(a) to the intensities of lamps A and B,
(b) to the intensity of lamp C,
(c) to the current in the circuit, and
(d) to the voltage drop across the three lamps?
Does the power dissipated in the circuit increase, decrease, or remain the same?

SOLVED EXAMPLES

1. A 100 V voltmeter of internal resistance 20 kΩ in series with a high resistance R is connected to a 110 V line. The voltmeter reads 5 V, the value of R is

(a) 210 kW (b) 315 kW
(c) 420 kW (d) 440 kW

Sol. (c)



Here $i = \dfrac{110}{20\times10^3+R}$

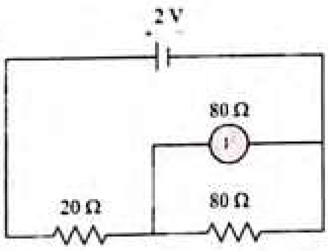
$\because \quad V = iR$

$\Rightarrow \quad 5 = \left(\dfrac{110}{20\times10^3+R}\right)\times20\times10^3$

$\Rightarrow \quad 10^5+5R = 22\times10^5 \;\Rightarrow\; R = 21\times\dfrac{10^5}{5} = 420\,\text{k}\Omega$

2. In the adjoining circuit, the e.m.f. of the cell is 2 volt and the internal resistance is negligible. The resistance of the voltmeter is 80 ohm. The reading of the voltmeter will be

(a) 0.80 volt (b) 1.60volt
(c) 1.33 volt (d) 2.00 volt



Sol. (c) Total resistance of the circuit $= \dfrac{80}{2}+20 = 60\ \Omega$

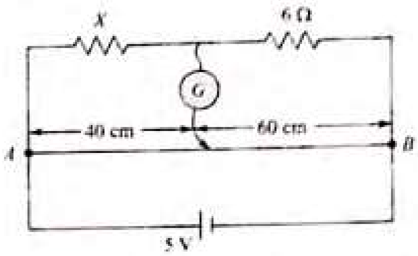
$\Rightarrow$ Main current $i = \dfrac{2}{60} = \dfrac{1}{30}$ A

Combination of voltmeter and 80 Ω resistance is connected in series with 20 Ω, so current through 20 Ω and this combination will be same $= \dfrac{1}{30}$ A.

Since the resistance of voltmeter is also 80 Ω, so this current is equally distributed in 80 Ω resistance and voltmeter (i.e. $\dfrac{1}{60}$ A through each)

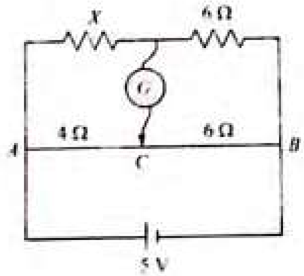
P.D. across 80 Ω resistance $= \dfrac{1}{60}\times80 = 1.33$ V

3. In the circuit shown, a meter bridge is in its balanced state. The meter bridge wire has a resistance 0.1 ohm/cm. The value of unknown resistance X and the current drawn from the battery of negligible resistance is



(a) 6 Ω, 5 A (b) 10 Ω, 0.1 A
(c) 4 Ω, 1.0 A (d) 12 Ω, 0.5 A

Sol. (c)



Resistance of the part AC

$R_{AC} = 0.1\times40 = 4\ \Omega$

and $R_{CB} = 0.1\times60 = 6\ \Omega$

In balanced condition

$\dfrac{X}{6} = \dfrac{4}{6} \Rightarrow X = 4\Omega$

Equivalent resistance $R_{eq} = 5\ \Omega$

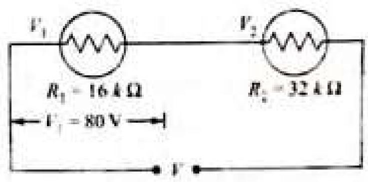
so current drawn from battery

$i = \dfrac{5}{5} = 1$ A.

4. Voltmeters V_1 and V_2 are connected in series across a D.C. line. V_1 reads 80 volts and has a per volt resistance of 200 ohms. V_2 has a total resistance of 32 kilo ohms. The line voltage is

(a) 120 volts (b) 160 volts
(c) 220 volts (d) 240 volts

Sol. (d)



$R_1 = 80\times200$
$= 16000\ \Omega$
$= 16\ \text{k}\Omega$

Current flowing through V_1 = Current flowing through

$V_2 = = \dfrac{18}{16\times10^3} = 5\times10^{-3}$ A.

So, potential differences across V_2 is

$V_2 = 5\times10^{-3}\times32\times10^3 = 160$ volt

Hence, line voltage $V = V_1+V_2 = 80+160 = 240$ V.

5. A voltmeter has a range 0-V with a series resistance R. With a series resistance $2R$, the range is 0-V'. The correct relation between V and V' is

(a) $V' = 2V$ (b) $V' > 2V$
(c) $V' >> 2V$ (d) $V' < 2V$

Sol. (d) For conversion of galvanometer (of resistances) into voltmeter, a resistance R is connected in series.

$\therefore \quad i_g = \dfrac{V_1}{R+G}$ and $i_g = \dfrac{V_2}{2R+G}$

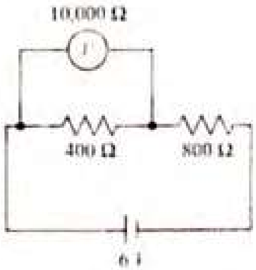
$$\Rightarrow \quad \frac{V_1}{R+G} = \frac{V_2}{2R+G} \Rightarrow \frac{V_2}{V_1} = \frac{2R+G}{R+G} = \frac{2(R+G)-G}{(R+G)}$$

$$2 - \frac{G}{(R+G)} \Rightarrow V_2 = 2V_1 - \frac{V_1 G}{(R+G)} \Rightarrow V_2 < 2V_1$$

6. Two resistances of 400 Ω and 800 Ω are connected in series with 6 volt battery of negligible internal resistance. A voltmeter of resistance 10,000 Ω is used to measure the potential difference across 400 Ω. The error in the measurement of potential difference in volts approximately is

(a) 0.01 (b) 0.02
(c) 0.03 (d) 0.05

Sol. (d) Before connecting voltmeter potential difference across 400 Ω resistance is



$$V_i = \frac{400}{(400+800)} \times 6 = 2\text{ V}$$

After connecting voltmeter equivalent resistance between A and B

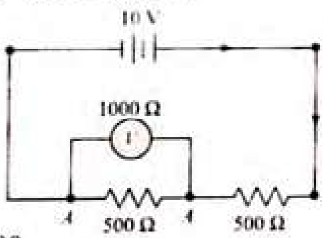
$$= \frac{400 \times 10{,}000}{(400+10{,}000)} = 384.6\Omega$$

Hence, potential difference measured by voltmeter

$$V_f = \frac{384.6}{(384.6+800)} \times 6 = 1.95\text{ V}$$

Error in measurement = $V_i - V_f = 2 - 1.95 = 0.05$ V.

7. What is the reading of voltmeter in the following figure

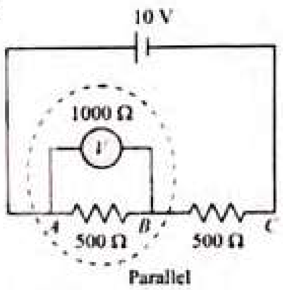


(a) 3 V (b) 2 V
(c) 5 V (d) 4 V

Sol. (d) Resistance between A and B $\dfrac{1000 \times 500}{(1500)} = \dfrac{1000}{3}\,\Omega$

So, equivalent resistance of the circuit

$$R_{eq} = 500 + \frac{1000}{3} = \frac{2500}{3}\,\Omega$$



Therefore, current drawn from the cell

$$i = \frac{10}{(2500/3)} = \frac{3}{250}\text{ A}$$

Reading of voltmeter i.e.

potential difference across $AB = \dfrac{3}{250} \times \dfrac{1000}{3} = 4$ V

8. Two resistances are connected in two gaps of a metre bridge. The balance point is 20 cm from the zero end. A resistance of 15 ohms is connected in series with the smaller of the two. The null point shifts to 40 cm. The value of the smaller resistance in ohms is

(a) 3 (b) 6 (c) 9 (d) 12

Sol. (c) Let S be larger and R be smaller resistance connected in two gaps of meter bridge.

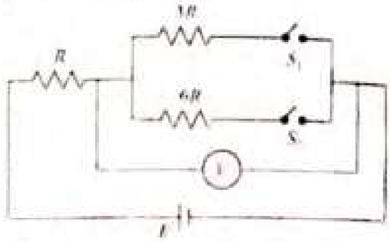
$$S = \left(\frac{100-l}{l}\right)R = \frac{100-20}{20}R = 4R$$

When 15Ω resistance is added to resistance R, then

$$S = \left(\frac{100-40}{40}\right)(R+15) = \frac{6}{4}(R+15)$$

From Eqs. (i) and (ii), $R = 9\,\Omega$

9. In the circuit shown in figure reading of voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed and reading of voltmeter is V_3 when both S_1 and S_2 are closed. Then



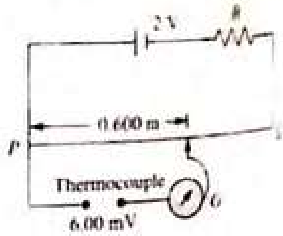
(a) $V_3 > V_2 > V_1$ (b) $V_2 > V_1 > V_3$
(c) $V_3 > V_1 > V_2$ (d) $V_1 > V_2 > V_3$

Sol. (b) In series : Potential difference ∝ R

When only S_1 is closed $V_1 = 3/4\ E = 0.75E$; when only S_2 is closed $V_2 = 6/7\ E = 0.86E$, and when both S_1 and S_2 are closed combined resistance of $6R$ and $3R$ is $2R$

$$V_3 = \left(\frac{2}{3}\right)E = 0.67E \Rightarrow V_2 > V_1 > V_3$$

10. Figure shows a simple potentiometer circuit for measuring a small e.m.f. produced by a thermocouple. The meter wire PQ has a resistance 5 Ω and the driver cell has an e.m.f. of 2 V. If a balance point is obtained 0.600 m along PQ when measuring an e.m.f. of 6.00 mV, what is the value of resistance R

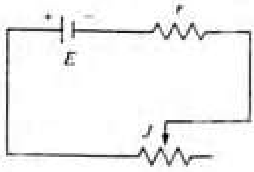


(a) 995 Ω (b) 1995 Ω
(c) 2995 Ω (d) None of these

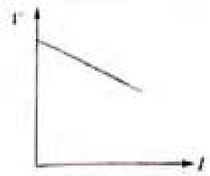
Sol. (a) The voltage per unit light of the metre wire PQ is $\left(\dfrac{6.00\text{ mV}}{0.600\text{ m}}\right)$ i.e. 10 mV/m. Hence potential difference across the metre wire is 10 mV/m × 1m = 10 mV. The current drawn from the driver cell is $i = \dfrac{10\text{ mV}}{5\,\Omega} = 2$ mA. The resistance

$$R = \frac{(2\text{ V} - 10\text{ mV})}{2\text{ mA}} = \frac{1990\text{ mV}}{2\text{ mA}} = 995\,\Omega$$

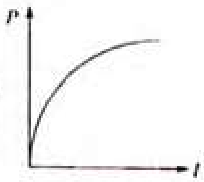
11. Battery shown in figure has e.m.f. E and internal resistance r. Current in the circuit can be varied by sliding the contact J. If at any instant current flowing through the circuit is I, potential difference between terminals of the cell is V, thermal power generated in the cell is equal to η fraction of total electrical power generated in it.; then which of the following graph is correct



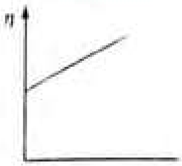
(a)



(b)



(c)



(d) Both (a) and (b) are correct

Sol. (d) Terminal voltage $V = E - Ir$. Hence the graph between V and i will be a straight line having negative slope and positive intercept.

Thermal power generated in the external circuit

$P = EI - I^2r.$

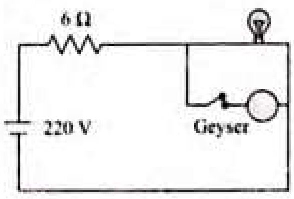
Hence graph between P and I will be a parabola passing through origin.

Also at an instant, thermal power generated in the cell $= I^2r$ and total electrical power generated in the cell $= Ei$. Hence the fraction $\eta = \frac{I^2r}{EI} = \left(\frac{r}{E}\right)I$; so $\eta \propto$ I. It means graph between η and I will be a straight line passing through origin.

12. The wiring of a house has resistance 6 Ω. A 100 W bulb is glowing. If a geyser of 1000 W is switched on, the change in potential drop across the bulb is nearly

(a) Nil (b) 23 V

(c) 32 V (d) 12 V



Sol. (b) $R_{Bulb} = \frac{220^2}{100} = 484\ \Omega$,

$R_{Geyser} = \frac{220^2}{1000} = 48.4\ \Omega$

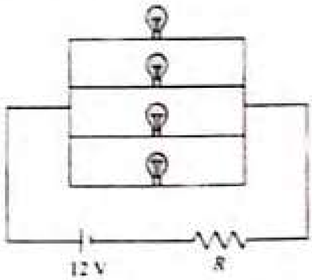
(i) When only bulb is ON, $V_{Bulb} = \frac{220 \times 484}{490} = 217.4$ V

(ii) When geyser is also switched ON, equivalent resistance of bulb and geyser is $R = \frac{484 \times 48.4}{484 + 48.4} = 44\Omega$

Voltage across the bulb $V_{Bulb} = \frac{220 \times 44}{50} = 193.6$ V

Hence the potential drop is $217.4 - 193.6 = 23.8$ V

13. Four identical electrical lamps are labelled 1.5 V, 0.5 A which describes the condition necessary for them to operate at normal brightness. A 12 V battery of negligible internal resistance is connected to lamps as shown, then



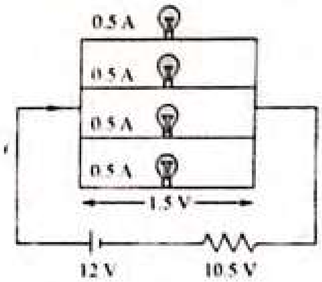
(a) The value of R for normal brightness of each lamp is (3/4) Ω

(b) The value of R for normal brightness of each lamp is (21/4) Ω

(c) Total power dissipated in circuit when all lamps are normally bright is 24 W

(d) Power dissipated in R is 21 W when all lamps are normally bright

Sol. (b) For normal brightness of each bulb see following circuit. Current through each bulb = 0.5 A



So main current $i = 2$ A

Also, voltage across the combination = 1.5 V

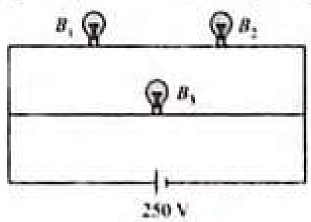
So voltage across the resistance = 10.5 V

Hence for resistance $V = iR \Rightarrow 10.5 = 2 \times R \Rightarrow R = \frac{21}{4}\Omega$

15. A 100 W bulb B_1, and two 60 W bulbs B_2 and B_3, are connected to a 250 V source, as shown in the figure. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3, respectively. Then

(a) $W_1 > W_2 = W_3$ (b) $W_1 > W_2 > W_3$

(c) $W_1 < W_2 = W_3$ (d) $W_1 < W_2 < W_3$



Sol. (d) $P = \frac{V^2}{R}$, so $R = \frac{V^2}{P}$

$$\Rightarrow \quad R_1 = \frac{V^2}{100} \text{ and } R_2 = R_3 = \frac{V^2}{60}$$

Now $W_1 = \frac{(250)^2}{(R_1 + R_2)^2} . R_1$, $W_2 = \frac{(250)^2}{(R_1 + R_2)^2} . R_2$

and $\quad W_3 = \frac{(250)^2}{R_3}$

$W_1 : W_2 : W_3 = 15 : 25 : 64$

or $\quad W_1 < W_2 < W_3$

15. The resistance of a heater coil is 110 ohm. A resistance R is connected in parallel with it and the combination is joined in series with a resistance of 11 ohm to a 220 volt main line. The heater operates with a power of 110 watt. The value of R in ohm is

(a) 12.22

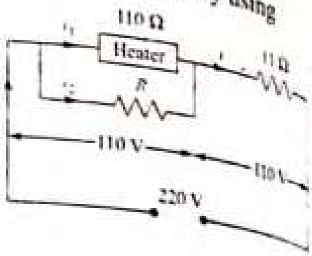
(b) 24.42

(c) Negative

(d) That the given values are not correct

Sol. **(a)** Power consumed by heater is 110 W so by using

$$P = \frac{V^2}{R}$$



$$110 = \frac{V^2}{110} \Rightarrow V = 110 \text{ V}$$

Also from figure

$$i_1 = \frac{110}{110} = 1 \text{ A}$$

and $\quad i = \frac{110}{11} = 10 \text{ A}$

So $i_2 = 10 - 1 = 9$ A

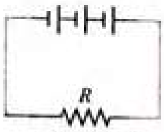
Applying Ohms law for resistance R, $V = iR$

$\Rightarrow \quad 110 = 9 \times R \Rightarrow R = 12.22\ \Omega$

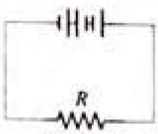
EXERCISES

Heating Effect of Current

1. Three identical cells, each having an emf 1.5 V and a constant internal resistance 2.0 Ω, are connected in series with a 4.0 Ω resistor R, first as in circuit (i), and second as in circuit (ii). Then



Circuit (i)



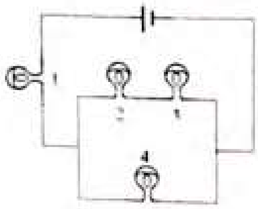
Circuit (ii)

$$\frac{\text{Power in } R \text{ in circuit (i)}}{\text{Power in } R \text{ in circuit (ii)}} =$$

(a) 9.0 (b) 7.2

(c) 1.8 (d) 3.0

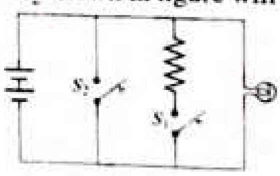
2. All bulbs in figure are identical. Which bulb lights more brightly?



(a) 1 (b) 2

(c) 3 (d) 4

3. Which of the two switches S_1 and S_2 shown in figure will produce short-circuiting?



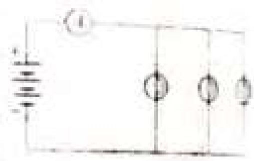
(a) S_1

(b) S_2

(c) Both S_1 and S_2

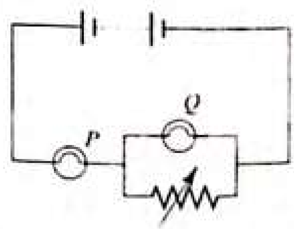
(d) Neither S_1 nor S_2

4. Three similar light bulbs are connected to a constant voltage dc supply as shown in figure. Each bulb operates at normal brightness and the ammeter (of negligible resistance) registers a steady current. The filament of one of the bulbs breaks. What happens to the ammeter reading and to the brightness of the remaining bulbs?



	Ammeter reading	*Bulb brightness*
(a)	increases	increases
(b)	increases	unchanged
(c)	unchanged	unchanged
(d)	decreases	unchanged

5. The circuit shown in figure contains a battery, a rheostat and two identical lamps. What will happen to the brightness of the lamps if the resistance of the rheostat is increased?



	Lamp P	*Lamp Q*
(a)	Less bright	Brighter
(b)	Less brighter	Less brighter
(c)	Brighter	Less brighter
(d)	No change	Brighter

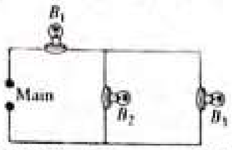
6. Two identical heaters rated 220 V, 1000 W are placed in series with each other across 220 V line; then the combined power is

(a) 1000 W (b) 2000 W

(c) 500 W (d) 4000 W

7. Two electric bulbs A and B are rated 60 and 100 W, respectively. If they are connected in parallel to the same source, then
   (a) both the bulbs draw the same current
   (b) bulb A draws more current than bulb B
   (c) bulb B draws more current than bulb A
   (d) currents drawn in the bulbs are in the ratio of their resistances

8. Three bulbs B_1, B_2, and B_3 are connected to the mains as shown in figure. How will the brightness of bulb B_1 be affected if B_2 or B_3 are disconnected from the circuit?
   (a) Bulb B_1 becomes brighter.
   (b) Bulb B_1 becomes dimmer.
   (c) No change occurs in the brightness.
   (d) Bulb B_1 becomes brighter if bulb B_2 is disconnected and dimmer if bulb B_3 is disconnected.



9. A cell of internal resistances r is connected to a load of resistance R. Energy is dissipated in the load, but some thermal energy is also wasted in the cell. The efficiency of such an arrangement is found from the expression

$$\frac{\text{Energy dissipated in the load}}{\text{Energy dissipated in the complete circuit}}$$

Which of the following gives the efficiency in this case?

(a) $\frac{r}{R}$ (b) $\frac{R}{r}$

(c) $\frac{r}{R+r}$ (d) $\frac{R}{R+r}$

10. Two identical batteries each of emf $E = 2$ V and internal resistance $r = 1\ \Omega$ are available to produce heat in an external circuit. What is the maximum rate of production of heat that can be obtained in the external circuit?
   (a) 1 W (b) 2 W
   (c) 4 W (d) 8 W

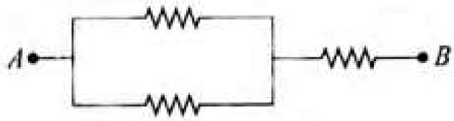
11. Two similar headlight lamps are connected in parallel to each other. Together, they consume 48 W from a 6 V battery. What is the resistance of each filament?
   (a) 6 Ω (b) 4 Ω
   (c) 3.0 Ω (d) 1.5 Ω

12. Two electric bulbs, rated for the same voltage, have powers of 200 W and 100 W, respectively. If their resistances are r_1 and r_2, respectively, then
   (a) $r_1 = 2r_2$ (b) $r_2 = 2r_1$
   (c) $r_2 = 4r_1$ (d) $r_1 = 4r_2$

13. If the current in an electric bulb decreases by 0.5%, the power in the bulb decreases by approximately
   (a) 1% (b) 2%
   (c) 0.5% (d) 0.25%

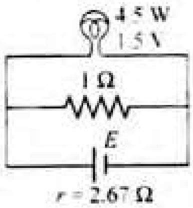
14. An electric bulb rated for 500 W at 100 V is used in a circuit having a 200 V supply. The resistance R that must be put in series with the bulb, so that the bulb draws 500 W, is
   (a) 18 Ω (b) 20 Ω
   (c) 40 Ω (d) 700 Ω

15. Three 10 Ω, 2 W resistors are connected as in figure. The maximum possible voltage between points A and B without exceeding the power dissipation limits of any of the resistors is



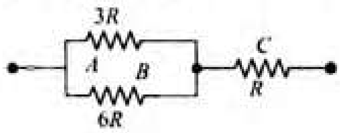
   (a) $5\sqrt{3}$ V (b) $3\sqrt{5}$ V
   (c) 15 V (d) 5/3 V

16. A torch bulb rated 4.5 W, 1.5 V is connected as shown in Figure. The emf of the cell needed to make the bulb glow at full intensity is
   (a) 4.5 V (b) 1.5 V
   (c) 2.67 V (d) 13.5 V

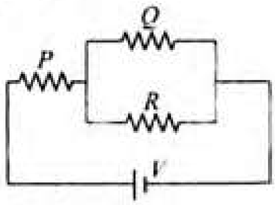


17. The main supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already giving light. What is the decrease in voltage across the bulb when a 240 W heater is switched on?
   (a) no change (b) 10 V
   (c) 20 V (d) more than 10 V

18. Figure shows a network of three resistances. When some potential difference is applied across the network, thermal powers dissipated by A, B, and C are in the ratio
   (a) 2 : 3 : 4 (b) 2 : 4 : 3
   (c) 4 : 2 : 3 (d) 3 : 2 : 4

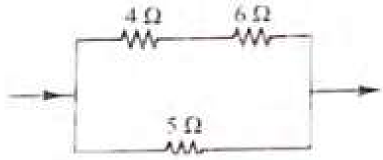


19. Resistors P, Q, and R in the circuit have equal resistances. If the battery is supplying a total power of 12 W, what is the power dissipated as heat in resistor R?
   (a) 2 W (b) 6 W
   (c) 3 W (d) 8 W



20. Three bulbs of 40 W, 60 W and 100 W are connected in series with a 240 V source.
   (a) The potential difference will be maximum across the 40 W bulb.
   (b) The current will be maximum in 100 W bulb.
   (c) The resistance of the 40 W bulb is minimum.
   (d) The current through the 60 W bulb will be 0.1 A.

21. In the circuit shown in figure, the heat produced in the 5 Ω resistor due to the current flowing through it is 10 cal s^{-1}. The heat generated in the 4 Ω resistor is



(a) 1 cals
(b) 2 cals
(c) 3 cals
(d) 4 cals

Ammeter and Voltmeter

22. A voltmeter has a resistance of G ohm and range V volt. The value of resistance used in series to convert it into voltmeter of range nV volt is
(a) nG
(b) $(n-1)G$
(c) G/n
(d) $G/(n-1)$

23. A galvanometer has a resistance of 3663 Ω. A shunt S is connected across it such that 1/34 of the total current passes through the galvanometer. The value of the shunt is
(a) 3663 Ω
(b) 111 Ω
(c) 107.7 Ω
(d) 3555.3 Ω

24. In Q.23, the combined resistance of the shunt and the galvanometer is
(a) 3665 Ω
(b) 111 Ω
(c) 107.7 Ω
(d) 3555.3 Ω

25. In Q.23, the external resistance that must be connected in series with the main circuit so that the total current in the main circuit remains unaltered even when the galvanometer is shunted is
(a) 3663 Ω
(b) 111 Ω
(c) 107.7 Ω
(d) 3555.3 Ω

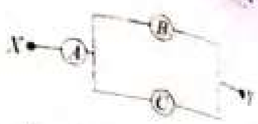
26. An ammeter is obtained by shunting a 30 Ω galvanometer with a 30 Ω resistance. What additional shunt should be connected across it to double the range?
(a) 15 Ω
(b) 10 Ω
(c) 5 Ω
(d) none of these

27. If a shunt 1/10 of the coil resistance is applied to a moving coil galvanometer, its sensitivity becomes
(a) 10 fold
(b) 11 fold
(c) 1/10 fold
(d) 1/11 fold

28. The resistance of a galvanometer is 10 Ω. It gives full-scale deflection when 1 mA current is passed. The resistance connected in series for converting it into a voltmeter of 2.5 V will be
(a) 24.9 Ω
(b) 249 Ω
(c) 2490 Ω
(d) 24900 Ω

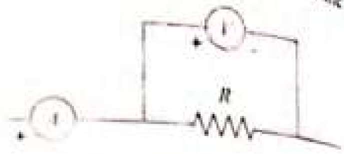
29. A milliammeter of range 10 mA has a coil of resistance 1 Ω. To use it as an ammeter of range 1A, the required shunt must have a resistance of
(a) $\frac{1}{101}$ Ω
(b) $\frac{1}{100}$ Ω
(c) $\frac{1}{99}$ Ω
(d) $\frac{1}{9}$ Ω

30. A, B, and C are voltmeters of resistance R, $1.5R$, and $3R$, respectively. When some potential difference is applied between X and Y, the voltmeter readings are V_A, V_B, and V_C, respectively. Then



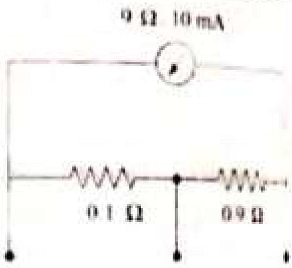
(a) $V_A = V_B = V_C$
(b) $V_A \neq V_B = V_C$
(c) $V_A = V_B \neq V_C$
(d) $V_B \neq V_A = V_C$

31. A candidate connects a moving coil voltmeter V, a moving coil ammeter A, and a resistor R as shown in figure. If the voltmeter reads 20 V and the ammeter reads 4 A, R is



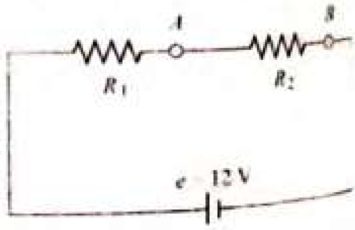
(a) equal to 5 Ω
(b) greater than 5 Ω
(c) less than 5 Ω
(d) greater or less than 5 Ω depending upon its material

32. A milliammeter of range 10 mA and resistance 9 Ω is joined in a circuit as shown in figure. The meter gives full-scale deflection for current I when A and B are used as its terminals. If current enters at A and leaves at B (C is left isolated), the value of I is



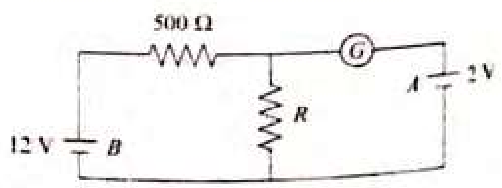
(a) 100 mA
(b) 900 mA
(c) 1 A
(d) 1.1 A

33. When an ammeter of negligible internal resistance is inserted in series with circuit, it reads 1 A. When a voltmeter of very large resistance is connected across R_1 it reads 3 V. But when the points A and B are short-circuited by a conducting wire, then the voltmeter measures 10.5 V across the battery. The internal resistance of the battery is equal to



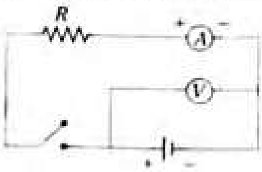
(a) 3/7 Ω
(b) 5 Ω
(c) 3 Ω
(d) none of these

34. In the circuit shown in figure, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be



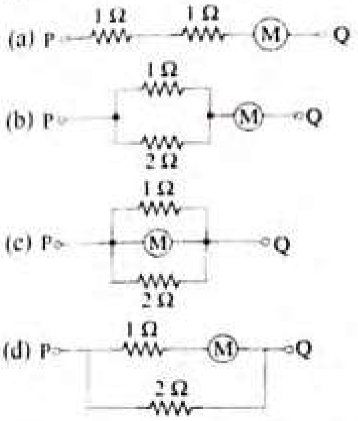
(a) 1000 Ω
(b) 500 Ω
(c) 100 Ω
(d) 200 Ω

35. In the circuit shown in figure, an ideal ammeter and an ideal voltmeter are used. When the key is open, the voltmeter reads 1.53 V. When the key is closed, the ammeter reads 1.0 A and the voltmeter reads 1.03 V. The resistance R is



(a) 0.5 Ω (b) 1.03 Ω
(c) 1.53 Ω (d) 0.53 Ω

36. In which of the following arrangements of resistors does the meter M, which has a resistance of 2 Ω, give the largest reading when the same potential difference is applied between points P and Q?



37. Two moving coil galvanometers 1 and 2 are with identical field magnets and suspension torque constants, but with coil of different number of turns N_1 and N_2, area per turn A_1 and A_2, and resistance R_1 and R_2. When they are connected in series in the same circuit, they show deflections θ_1 and θ_2. Then θ_1/θ_2 is

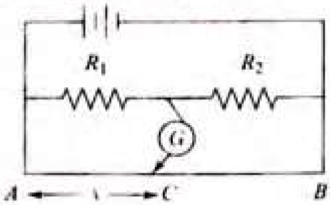
(a) $A_1 N_1 / A_2 N_2$ (b) $A_1 N_2 / A_2 N_1$
(c) $A_1 R_2 N_1 / A_2 R_2 N_2$ (d) $A_1 R_1 N_1 / A_2 R_2 N_2$

Meter Bridge and Potentiometer

38. If in an experiment of Wheatstone bridge, the positions of cells and galvanometer are interchanged, then the balance points will

(a) change
(b) remain unchanged
(c) depend on the internal resistance of the cell and resistance of the galvanometer
(d) none of these

39. In the shown arrangement of a meter bridge, if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?

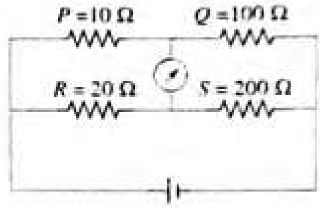


(a) x
(b) $x/4$
(c) $4x$
(d) $2x$

40. In a meter bridge experiment, the null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y?

(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm

41. Figure shows a balanced Wheatstone net. Now, it is disturbed by changing P to 11 Ω. Which of the following steps will not bring the bridge to balance again?



(a) increasing R by 2 Ω
(b) increasing S by 20 Ω
(c) increasing Q by 10 Ω
(d) making product $RQ = 2200\ (\Omega)^2$

42. The length of a wire of a potentiometer is 100 cm, and the emf of its standard cell is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at $l = 30$ cm from the positive end, the emf of the battery is

(a) $\dfrac{30E}{100}$ (b) $\dfrac{30E}{100.5}$

(c) $\dfrac{30E}{(100-0.5)}$ (d) $\dfrac{30\,(E-0.5i)}{100}$

where i is the current in the potentiometer wire.

43. In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is

(a) 2 Ω (b) 4 Ω
(c) 0.5 (d) 1 Ω

44. Sensitivity of a potentiometer can be increased by

(a) increasing the emf of the cell
(b) increasing the length of the potentiometer
(c) decreasing the length of the potentiometer wire
(d) none of the above

45. In an experiment to measure the internal resistance of a cell by a potentiometer, it is found that the balance point is at a length of 2 m when the cell is shunted by a 5 Ω resistance and is at a length of 3 m when the cell is shunted by a 10 Ω resistance, the internal resistance of the cell is then

(a) 1.5 Ω (b) 10 Ω
(c) 15 Ω (d) 1 Ω

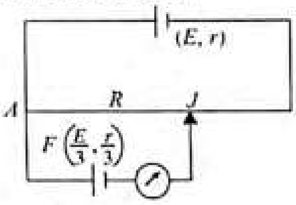
46. The emf of the driver cell of a potentiometer is 2 V, and its internal resistance is negligible. The length of the potentiometer wire is 100 cm, and resistance is 5 Ω. How much resistance is to be connected in series with the potentiometer wire to have a potential gradient of 0.05 mVcm^{-1}?

(a) 1990 Ω (b) 2000 Ω
(c) 1995 Ω (d) none of these

47. In the above question, if the balancing length for a cell of emf E is 60 cm, the value of E will be
(a) 3 mV (b) 5 mV
(c) 6 mV (d) 2000 mV

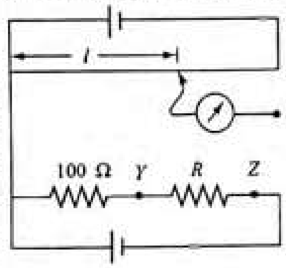
48. A potentiometer arrangement is shown in figure. The driver cell has emf E and internal resistance r.

The resistance of potentiometer wire AB is R. F is the cell of emf $E/3$ and internal resistance $r/3$. Balance point (J) can be obtained for all finite values of



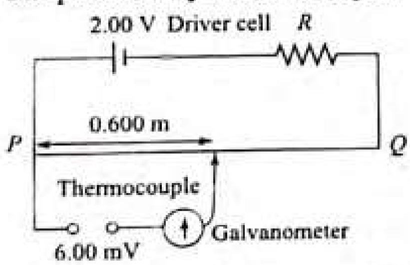
(a) $R > r/2$ (b) $R < r/2$
(c) $R > r/3$ (d) $R < r/3$

49. Figure shows a circuit that may be used to compare the resistance R of an unknown resistor with a 100 Ω standard. The distances l from one end of the potentiometer slider wire to the balance point are 400 mm and 588 mm when X is connected to Y and Z, respectively. The length of the slide wire is 1.00 m. What is the value of resistance R?



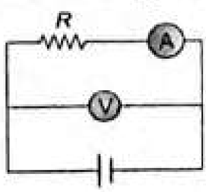
(a) 32 Ω (b) 47 Ω
(c) 68 Ω (d) 147 Ω

50. Figure shows a simple potentiometer circuit for measuring a small emf produced by a thermocouple.



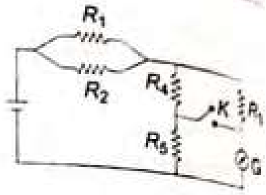
The meter wire PQ has a resistance of 5 Ω, and the driver cell has an emf of 2.00 V. If a balance point is obtained 0.600 m along PQ when measuring an emf of 6.00 mV, what is the value of resistance R?
(a) 95 Ω (b) 995 Ω
(c) 195 Ω (d) 1995 Ω

51. In the given circuit, the ratio of voltmeter reading to the ammeter reading is R'. If resistance of ammeter is R_A and that of voltmeter R_V, then



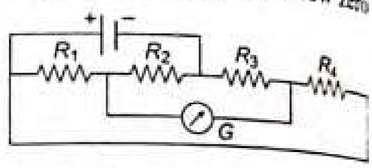
(a) $R = R' + R_A$
(b) $R = R' - R_A$
(c) $R_V = R + R_A$
(d) $R_V = R - R_A$

52. Whether the switch K is open or closed, the reading of galvanometer is the same. If I denotes the current then:



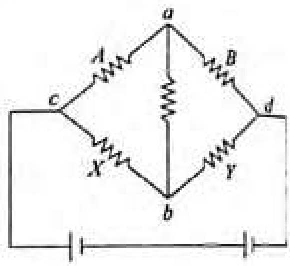
(a) $IR_4 = I_G$ (b) $IR_5 = I_G$
(c) $IR_3 = I_G$ (d) $IR_4 = IR_3$

53. In the given circuit, the galvanometer G will show zero deflection if



(a) $R_1R_2 = R_3R_4$
(b) $R_1R_3 = R_2R_4$
(c) $R_1R_4 = R_2R_3$
(d) galvanometer can't have zero current for any relation between the resistances

54. In the Wheatstone bridge (shown in the figure) $X = Y$ and $A > B$. The direction of the current between ab will be



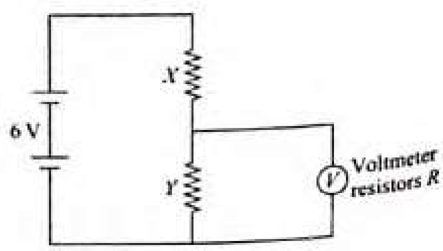
(a) From a to b
(b) From b to a
(c) From b to a through c
(d) From a to b through c

55. The figure shows a circuit diagram of a Wheatstone Bridge to measure the resistance G of the galvanometer. The relation $\frac{P}{Q} = \frac{R}{G}$ will be satisfied only when

(a) the galvanometer shows a deflection when switch S is closed
(b) the galvanometer shows a deflection when switch S is open
(c) the galvanometer shows no change in deflection whether S is open or closed
(d) the galvanometer shows no deflection

Problems Based on Mixed Concepts

56. In the circuit shown in figure, resistors X and Y, each with resistance R, are connected to a 6 V battery of negligible internal resistance. A voltmeter, also of resistance R, is connected across Y.

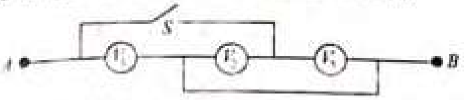


What is the reading of the voltmeter?

(a) zero (b) between zero and 3 V

(c) 3 V (d) between 3 V and 6 V

57. Three voltmeters are connected as shown.



A potential difference has been applied between A and B. On closing the switch S, readings of voltmeters?

(a) V_1 increases

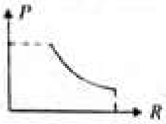
(b) V_1 decreases

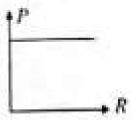
(c) V_2 and V_3 both increase

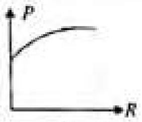
(d) One of V_2 and V_3 increases and other decreases.

58. A constant potential difference is applied across a resistance. Consider variation of resistance with temperature. Which graph represents best the variation of power produced in resistance versus resistance?

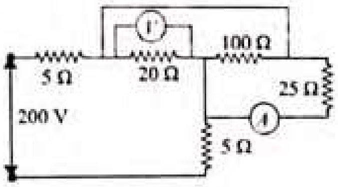
(a) 

(b) 

(c) 

(d) 

59. In figure, the voltmeter and ammeter shown are ideal. Then voltmeter and ammeter readings, respectively, are



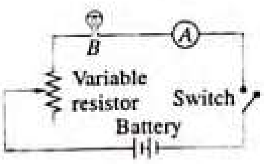
(a) 125 V, 3 A

(b) 100 V, 4 A

(c) 120 V, 4 A

(d) 120 V, 3 A

60. In the circuit in figure, bulb B does not glow although ammeter A indicates that the current is flowing. Why does the bulb not glow?



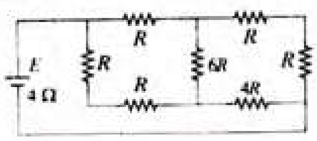
(a) The bulb is fused.

(b) There is a break in the circuit between bulb and ammeter.

(c) The variable resistor has too large resistance.

(d) There is a break in the circuit between the bulb and the variable resistance.

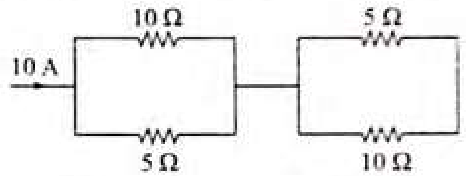
61. A battery of internal resistance 4 Ω is connected to the network of resistances as shown in figure. In order that the maximum power can be delivered to the network, the value of R in Ω should be



(a) 4/9 (b) 2

(c) 8/3 (d) 18

62. Four resistances carrying a current shown in figure are immersed in a box containing ice at 0°C. How much ice must be put in the box every 10 min to keep the average quantity of ice in the box constant? Latent heat of ice is 80 calg^{-1}



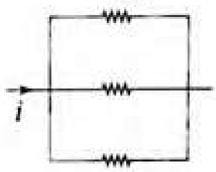
(a) 1.190 kg (b) 3.20 kg

(c) 4.2 kg (d) 0.25 kg

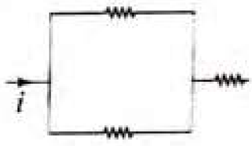
63. The three resistances of equal values are arranged in different combinations shown below. Arrange them in increasing order of power dissipation.

(I)

(II)



(III)



(IV)

(a) III < II < IV < I (b) II < III < IV < I

(c) I < IV < III < II (d) I < III < II < IV

64. A 25 W, 220 V bulb and a 100 W, 220 V bulb are connected in series across a 220 V line; which electric bulb will glow more brightly?

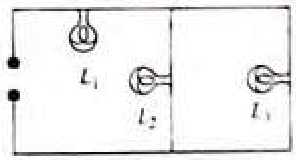
(a) 25 W bulb

(b) 100 W bulb

(c) Both will have equal incandescence

(d) Neither will give light

65. Figure shows three similar lamps L_1, L_2, and L_3 connected across a power supply. If the lamp L_3 fuses, how will the light emitted by L_1 and L_2 change?



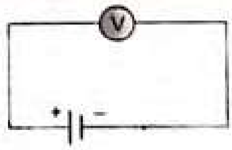
(a) no change

(b) brilliance of L_1 decreases and that of L_2 increases

(c) brilliance of both L_1 and L_2 increases

(d) brilliance of both L_1 and L_2 decreases

66. A voltmeter with resistance 2500 Ω indicates a voltage of 125 V in the circuit shown in figure. What is the series resistance to be connected to voltmeter so that it indicates 100 V? Consider ideal battery.

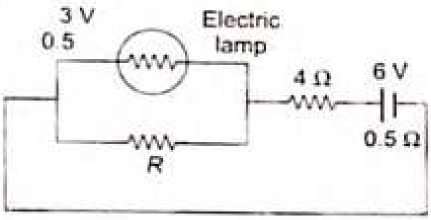


(a) 625 Ω (b) 120 Ω

(c) 550 Ω (d) Data are insufficient

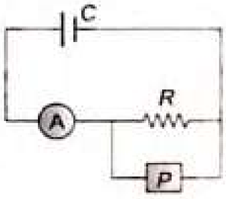
67. A resistance R is to be measured using a meter bridge, student chooses the standard resistance S to be 100 Ω. He finds the null point at $l_1 = 2.9$ cm. He is told to attempt to improve the accuracy. Which of the following is a useful way?
(a) He should measure l_1 more accurately
(b) He should change S to 1000 Ω and repeat the experiment
(c) He should change S to 3 Ω and repeat the experiment
(d) He should give up hope of a more accurate measurement with a meter bridge

68. What should be the value of resistance R in the circuit shown in figure so that the electric bulb consumes the rated power?



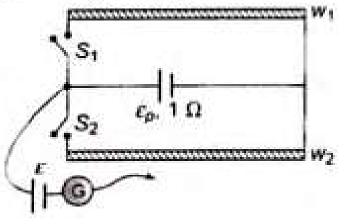
(a) 4.5 Ω (b) 6 Ω
(c) 12 Ω (d) None of these

69. An ammeter A of finite resistance, and a resistor R are joined in series to an ideal cell C. A potentiometer P is joined in parallel to R. The ammeter reading is I_0 and the potentiometer reading is V_0. P is now replaced by a voltmeter of finite resistance. The ammeter reading now is I and the voltmeter reading is V.



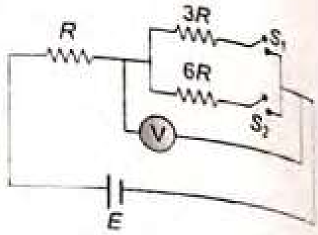
(a) $I > I_0$, $V < V_0$ (b) $I > I_0$, $V > V_0$
(c) $I = I_0$, $V < V_0$ (d) $I < I_0$, $V > V_0$

70. Two identical potentiometer wires w_1 and w_2 of equal length l, connected to a battery of emf ε_P and internal resistance 1 Ω through two switches s_1 and s_2. A battery of emf ε is balanced on these potentiometer wires one by one. If potentiometer wire w1 is of resistance 2 Ω and balancing length is $l/2$ on it, when only s_1 is closed and s_2 is open.



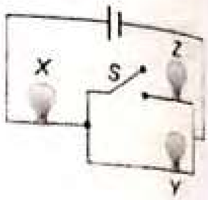
On closing s_2 and opening s_1 the balancing length on w_2 is found to be, then find the resistance of potentiometer wire w_2.
(a) 0.5 Ω (b) 1 Ω
(c) 0.25 Ω (d) 0.75 Ω

71. In the circuit shown, reading of voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed and reading of voltmeter is V_3 when both S_1 and S_2 are closed. Then:



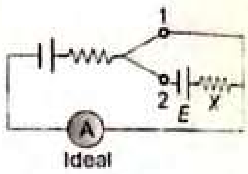
(a) $V_3 > V_2 > V_1$ (b) $V_2 > V_1 > V_3$
(c) $V_3 > V_1 > V_2$ (d) $V_1 > V_2 > V_3$

72. If X, Y and Z in figure are identical lamps, which of the following changes to the brightness of the lamps occur when switch S is closed?



(a) X stays the same, Y decreases
(b) X increases, Y decreases
(c) X increases, Y stays the same
(d) X decreases, Y increases

73. In the circuit shown the variable resistance X is to be adjusted such that the ideal ammeter reads the same in both the positions of the key, when connected independently to 1 and then to 2. The reading of the ammeter is 2A. If $E = 10$ V, then x is

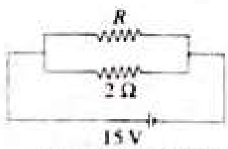


(a) 5 Ω (b) 20 Ω
(c) 50 Ω (d) cannot be determined

74. A battery of internal resistance 2 Ω is connected to a variable resistor whose value can vary from 4 Ω to 10 Ω. The resistance is initially set at 4 Ω. If the resistance is now increased then
(a) power consumed by it will decrease
(b) power consumed by it will increase
(c) power consumed by it may increase or may decrease
(d) power consumed will first increase then decrease

ARCHIVES

1. If in the circuit, the power dissipation is 150 W, then R is equal to



(a) 2 Ω (b) 6 Ω
(c) 5 Ω (d) 4 Ω

(AIEEE 2002)

2. A wire when connected to a 220 V main supply has power dissipation P_1. Now the wire is cut into two equal pieces which are connected in parallel to the same supply. The power dissipation in this case is P_2. Then $P_2 : P_1$ is
(a) 1 (b) 4
(c) 2 (d) 3

(AIEEE 2002)

3. The length of a wire of a potentiometer is 100 cm, and the emf of its standard cell is E volt. It is employed to measure the emf of a battery whose internal resistance is

0.5 Ω. If the balance point is obtained at $l = 30$ cm from the positive end, the emf of the battery is

(a) $\frac{30\,E}{100}$ (b) $\frac{30\,E}{100.5}$

(c) $\frac{30\,E}{(100-0.5)}$ (d) $\frac{30(E-0.5i)}{100}$ **(AIEEE 2003)**

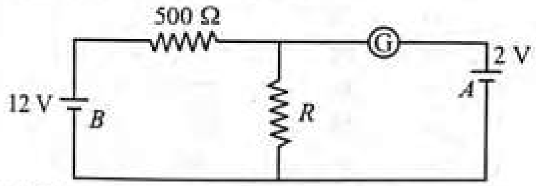
where i is the current in the potentiometer wire.

4. A 220 V, 1000 W bulb is connected across a 110 V main supply. The power consumed will be
(a) 1000 W (b) 750 W
(c) 500 W (d) 250 W **(AIEEE 2003)**

5. In a meter bridge experiment, the null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y?
(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm **(AIEEE 2004)**

6. The time taken by a 836 W heater to heat 1 L of water from 10°C to 40°C is
(a) 50 s (b) 100 s
(c) 150 s (d) 200 s **(AIEEE 2004)**

7. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be



(a) 1000 Ω (b) 500 Ω
(c) 100 Ω (d) 200 Ω **(AIEEE 2005)**

8. An energy source will supply a constant current into the load if its internal resistance is
(a) non-zero but less than the resistance of the load
(b) zero
(c) very large as compared to the load resistance
(d) equal to the resistance of the load **(AIEEE 2005)**

9. In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is
(a) 2 Ω (b) 4 Ω
(c) 0.5 Ω (d) 1 Ω **(AIEEE 2005)**

10. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be
(a) halved (b) one-fourth
(c) four times (d) doubled **(AIEEE 2005)**

11. The resistance of a hot tungsten filament is about 10 times the cold resistance. What will be the resistance of a 100 W, 200 V lamp when not in use?
(a) 200 Ω (b) 400 Ω
(c) 20 Ω (d) 40 Ω **(AIEEE 2005)**

12. In a Wheatstone's bridge, three resistances P, Q, and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be

(a) $\frac{P}{Q} = \frac{R(S_1+S_2)}{2S_1S_2}$ (b) $\frac{P}{Q} = \frac{R}{S_1+S_2}$

(c) $\frac{P}{Q} = \frac{2R}{S_1+S_2}$ (d) $\frac{P}{Q} = \frac{R\,(S_1+S_2)}{S_1S_2}$

(AIEEE 2006)

13. An electric bulb is rated 220 V – 100 W. The power consumed by it when operated on 110 V will be
(a) 25 W (b) 50 W
(c) 75 W (d) 40 W **(AIEEE 2006)**

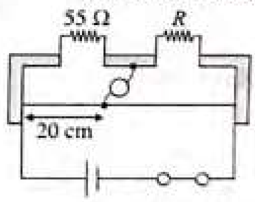
14. **Statement 1**

In a meter bridge experiment, the null point for an unknown resistance is measured. Now the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

Statement 2

The resistance of a metal increases with the increase in temperature.
(a) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1.
(b) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1.
(c) Statement 1 is true, statement 2 is false.
(d) Statement 1 is false, statement 2 is true.
(AIEEE 2008)

15. Shown in the figure is a meter bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is



(a) 110 Ω (b) 55 Ω
(c) 13.75 Ω (d) 220 Ω

(AIEEE 2008)

16. Two electric bulbs marked 25 W–220 V and 100 W–220 V are connected in series to a 440 V supply. Which of the bulbs will fuse?
(a) Both (b) 100 W
(c) 25 W (d) Neither **(AIEEE 2012)**

17. The supply voltage to a room is 120 V. The resistance of the lead wires is 6 Ω. A 60-W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240-W heater is switched on in parallel to the bulb?
(a) 2.9 V (b) 13.3 V
(c) 10.04 V (d) 0 V **(JEE Main 2013)**

18. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be

(a) 12 A (b) 14 A
(c) 8 A (d) 10 A **(JEE Main 2014)**

19. Which of the following statements is false?

(a) A rheostat can be used as a potential divider.
(b) Kirchhoff's second law represents energy conservation.
(c) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.
(d) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed. **(JEE Main 2017)**

20. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full-scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into voltmeter of range 0–10 V is

(a) $2.535 \times 10^3\ \Omega$ (b) $4.005 \times 10^3\ \Omega$
(c) $1.985 \times 10^3\ \Omega$ (d) $2.045 \times 10^3\ \Omega$

(JEE Main 2017)

21. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combinations is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances?

(a) 910 Ω (b) 990 Ω
(c) 505 Ω (d) 550 Ω

(JEE Main 2018)

22. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(a) 2.5 Ω (b) 1 Ω
(c) 1.5 Ω (d) 2 Ω

(JEE Main 2018)

ANSWER KEY

Exercises

1. (a)	**2.** (a)	**3.** (b)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (a)	**9.** (d)	**10.** (b)
11. (d)	**12.** (b)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (d)	**17.** (d)	**18.** (c)	**19.** (a)	**20.** (a)
21. (b)	**22.** (b)	**23.** (b)	**24.** (c)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (c)	**30.** (a)
31. (b)	**32.** (c)	**33.** (a)	**34.** (c)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (a)	**40.** (a)
41. (b)	**42.** (a)	**43.** (a)	**44.** (b)	**45.** (b)	**46.** (c)	**47.** (a)	**48.** (a)	**49.** (b)	**50.** (b)
51. (b)	**52.** (c)	**53.** (b)	**54.** (b)	**55.** (c)	**56.** (b)	**57.** (c)	**58.** (b)	**59.** (b)	**60.** (c)
61. (b)	**62.** (a)	**63.** (a)	**64.** (a)	**65.** (b)	**66.** (a)	**67.** (c)	**68.** (b)	**69.** (a)	**70.** (b)
71. (b)	**72.** (b)	**73.** (a)	**74.** (a)						

Archives

1. (b)	**2.** (b)	**3.** (a)	**4.** (d)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (d)
11. (d)	**12.** (d)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (c)
21. (d)	**22.** (c)								

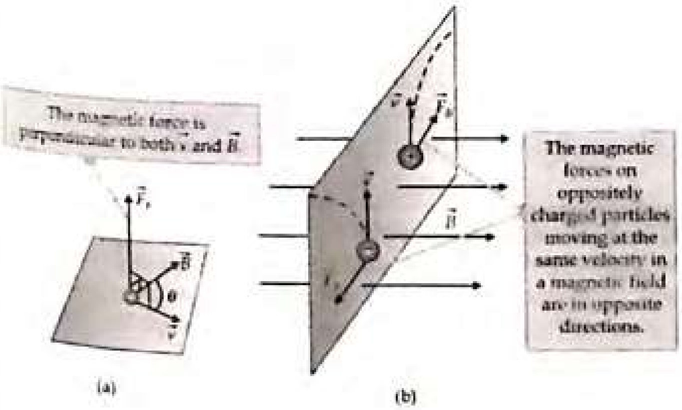
Chapter 21

Source and Effects of Magnetic Field

FORCE ON A MOVING CHARGE IN MAGNETIC FIELD

We can define a magnetic field $\vec{B}$ at some point in space in terms of the magnetic force $\vec{F}_B$ the field exerts on a charged particle moving with a velocity $\vec{v}$, which we call the test object. For the time being, let's assume no electric or gravitational fields are present at the location of the test object. Experiments on various charged particles moving in a magnetic field give the following results:

- The magnitude F_B of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; i.e. $\vec{F}_B$ perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$ [Figure (a)]. The magnetic force exerted on a positive charge is in the direction opposite to the direction of the magnetic force exerted on a negative charge moving in the same direction [Figure (b)].
- The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin\theta$, where θ is the angle the particle's velocity vector makes with the direction of $\vec{B}$.



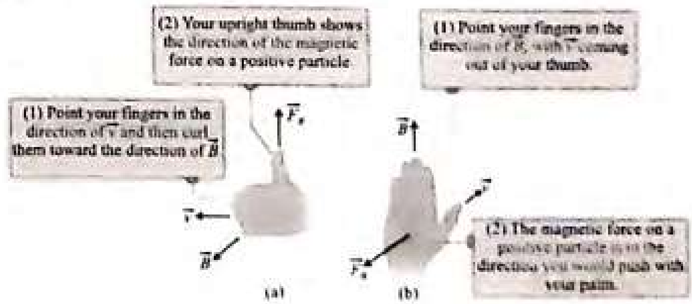
We can summarize these observations by writing the magnetic force in the form

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

which by definition of the cross product is perpendicular to both $\vec{v}$ and $\vec{B}$. We can regard this equation as an operational definition of the magnetic field at some point in space. That is, the magnetic field is defined in terms of the force acting on a moving charged particle.

Right Hand Rules for Determining the Direction of the Magnetic Force Acting on a Moving Charged Particle

Figure reviews two right-hand rules for determining the direction of the cross product $\vec{v} \times \vec{B}$ and determining the direction of $\vec{F}_B$. The rule in Figure (a) depends on our right-hand rule for the cross product in figure. Point the four fingers of your right hand along the direction of $\vec{v}$ with the palm facing $\vec{B}$ and curl them toward $\vec{B}$. Your extended thumb, which is at a right angle to your fingers, points in the direction of $\vec{v} \times \vec{B}$. Because $\vec{F}_B = q\vec{v} \times \vec{B}$, $\vec{F}_B$ is in the direction of your thumb if q is positive and is opposite the direction of your thumb if q is negative.



An alternative rule is shown in Figure (b). Here the thumb points in the direction of $\vec{v}$ and the extended fingers in the direction of $\vec{B}$. Now, the force $\vec{F}_B$ on a positive charge extends outward from the palm. The advantage of this rule is that the force on the charge is in the direction you would push on something with your hand: outward from your palm. The force on a negative charge is in the opposite direction. You can use either of these two right-hand rules.

The magnitude of the magnetic force on a charged particle is $F_B = |q| vB \sin\theta$, where u is the smaller angle between $\vec{v}$ and $\vec{B}$.

From this expression, we see that F_B is zero when $\vec{v}$ is parallel or antiparallel to $\vec{B}$ ($\theta = 0$ or $180°$) and maximum when $\vec{v}$ is perpendicular to $\vec{B}$ ($\theta = 90°$).

There are important differences between electric and magnetic forces on charged particles:

- The electric force is always parallel or antiparallel to the direction of the electric field, whereas the magnetic force is perpendicular to the magnetic field.
- The electric force acts on a charged particle independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion and the force is proportional to the velocity.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a constant magnetic field does no work when a charged particle is displaced.

LORENTZ FORCE

The combination of electric and magnetic forces is known as Lorentz forces. Let a charge particle moves in a magnetic field where both electric field ($\vec{E}$) and magnetic field ($\vec{B}$) are present. Then net force acting on the charge particle is given by

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

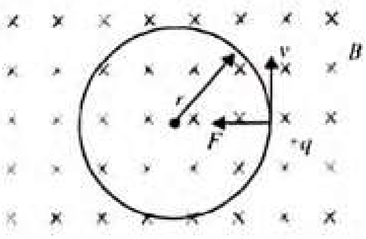
ILLUSTRATION 21.1 A charge $q = -4\,\mu C$ has an instantaneous velocity $\vec{v} = (2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6$ ms^{-1} in a uniform magnetic field $\vec{B} = (2\hat{i} + 5\hat{j} - 3\hat{k}) \times 10^{-2}$ T. What is the force on the charge?

Solution. $\vec{F} = q\vec{v} \times \vec{B}$

$$= (-4 \times 10^{-6})[(2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6]$$
$$\times (2\hat{i} + 5\hat{j} - 3\hat{k}) \times 10^{-2}]$$
$$= -(-16\hat{i} + 32\hat{j} + 64\hat{k}) \times 10^{-2} \text{ N}$$
$$= -16(\hat{i} + 2\hat{j} + 4\hat{k}) \times 10^{-2} \text{ N}$$

MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

1. Consider a charged particle of mass m projected in a uniform magnetic field $\vec{B}$ with an initial velocity vector $\vec{v}$ perpendicular to the field (figure). B points away from the reader.



A force will act on the particle as shown which is perpendicular to the velocity at any instant. And alsomagnitude of the velocity will remain same, only direction will change. Finally the particle will move in a circular path. The magnetic force provides the necessary centripetal force. So we can write

$$\frac{mv^2}{r} = qvB \quad \text{or} \quad r = \frac{mv}{qB}$$

The above equation gives the radius of the circular path.

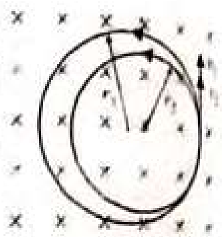
The angular speed ω of the particle is given by $\omega = \dfrac{v}{r} = \dfrac{qB}{m}$

The time period T of the motion is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Frequencey [or the number of revolutions completed in unit time] is given $f = \dfrac{1}{T} = \dfrac{qB}{2\pi m}$

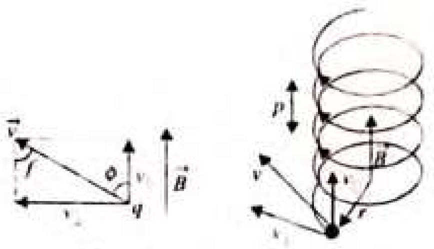
We see that time period is independent of velocity given to the charge particle. If two identical charge particle are given different speeds from a point in the same direction, then they will return to the initial point simultaneously.



2. **If particle is projected at some angle ϕ with magnetic field:** With perpendicular component of velocity it moves in a circular path of radius $r = \dfrac{mv_\perp}{qB} = \dfrac{mv\sin\phi}{qB}$

And, with parallel component of velocity it moves along the field lines. The resulting path is known as *helical*. The linear distance travelled (along the field line) in one revolution (time period) is called pitch (p) (figure).

$$\text{Pitch: } p = v_{\parallel}T = \frac{2\pi m v_{\parallel}}{qB} = \frac{2\pi m v\cos\phi}{qB}$$



(a) A charged particle moves in a uniform magnetic field $\vec{B}$, its velocity $\vec{v}$ making an angle ϕ with the field direction.
(b) The particle follows a helical path of radius r and pitch p.

ILLUSTRATION 21.2 A stream of protons and deuterons in a vacuum chamber enters a uniform magnetic field. Both protons and deuterons have been subjected to same accelerating potential, hence the kinetic energies of the particles are the same. If the ion-stream is perpendicular to the magnetic field and the protons move in a circular path of radius 15 cm, find the radius of the path traversed by the deuterons. Given that mass of deuteron is twice that of a proton.

Solution. The radius r of the path is given by

$$\frac{mv^2}{r} = qvB \quad \text{or} \quad r = \frac{mv}{qB}$$

For proton, $r_p = \dfrac{m_p v_p}{q_p B}$

For deuteron, $r_d = \dfrac{m_d v_d}{q_d B}$

$$\therefore \quad \frac{r_d}{r_p} = \frac{m_d v_d}{m_p v_p} \times \frac{q_p}{q_d} \qquad \text{(i)}$$

Given that $\frac{1}{2} m_p v_p^2 = \frac{1}{2} m_d v_d^2$ also $q_p = q_d$

$$\text{Now } \frac{v_d}{v_p} = \left(\frac{m_p}{m_d}\right)^{1/2} \qquad \text{(ii)}$$

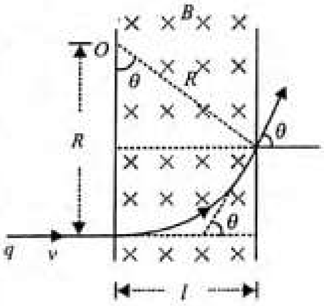
Subsituting the value of v_d/v_p from Eq. (ii) in Eq. (i) we get

$$\frac{r_d}{r_p} = \frac{m_d}{m_p}\left(\frac{m_p}{m_d}\right)^{1/2} = \left(\frac{m_d}{m_p}\right)^{1/2} = \sqrt{2} \qquad (\because \ m_d = 2m_p)$$

or $\quad r_d = \sqrt{2}.r_p = 1.414 \times 0.15 = 0.212$ m

ILLUSTRATION 21.3 An α-particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion, if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 tesla. (Given: mass of α-particle 6.4×10^{-27} kg).

Solution. The situation is shown in figure.



When a charged particle with charge q is accelerated through a potential difference V volt, then

$$\frac{1}{2}mv^2 = qV \qquad \text{(i)}$$

$$\text{or} \quad v = \sqrt{\left(\frac{2qV}{m}\right)} \qquad \text{(ii)}$$

α-particle in magnetic field moves in a circle of radius R which is given by

$$R = \frac{mv}{qB} \quad \text{or} \quad R = \frac{1}{B}\sqrt{\left(\frac{2mV}{q}\right)} \qquad \text{(iii)}$$

The change in direction of α-particle (θ) from figure is given by

$$\sin\theta = \frac{l}{R} = lB\sqrt{\left(\frac{q}{2mV}\right)}$$

Here $l = 0.1$ m, $B = 0.1$ tesla, $V = 10^4$ volt

$q = 2e = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ C

and $\quad m = 6.4 \times 10^{-27}$ kg

$$\therefore \quad \sin\theta = 0.1 \times 0.1 \times \sqrt{\left(\frac{3.2 \times 10^{-19}}{2 \times 6.4 \times 10^{-27} \times 10^4}\right)} = \frac{1}{2}$$

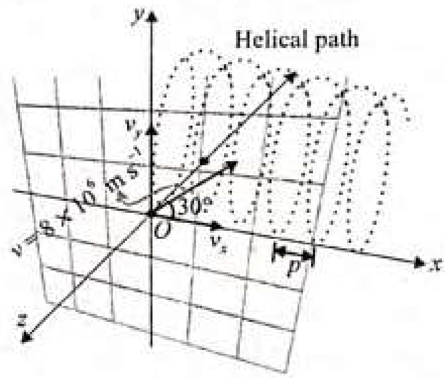
or $\quad \theta = 30°$

NOTE: In the above illustration,

1. If d were slightly less than (mv/qB), then the particle leaves the magnetic field just before it is able to turn back. Hence the angle of deviation is 90°.
2. If d were slightly greater than (mv/qB), then the particle turns around and completes a semicircle. When it leaves the magnetic field its angle of deviation is 180°.

ILLUSTRATION 21.4 A proton (charge 1.6×10^{-19} C, mass = 1.67×10^{-27} kg) is shot with a speed 8×10^6 m s^{-1} at an angle of 30° with the X-axis. A uniform magnetic field $B = 0.30$ T exists along the X-axis. Show that path of the proton is a helix. Find the radius and pitch of the helix.

Solution. The situation is shown in figure



$$v_x = v\cos 30° = (8 \times 10^6)(0.866) = 6.93 \times 10^6 \text{ m s}^{-1}$$

$$v_y = v\sin 30° = (8 \times 10^6)(0.5) = 4.0 \times 10^6 \text{ m s}^{-1}$$

Since the velocity has both components, parallel and transverse to magnetic field, so the resulting path will be helix. Due to combined action of v_x, v_y and B, the proton moves in a helical path.

The radius of helix is

$$r = \frac{mv_y}{qB} = \frac{(1.67 \times 10^{-27})(4 \times 10^6)}{(1.6 \times 10^{-19}) \times 0.3} = 0.139 \text{ m}$$

Time taken to complete one circle:

$$T = \frac{2\pi m}{qB} = \frac{2\pi 1.6 \times 10^{-27}}{1.6 \times 10^{-19} \times 0.3} = 2.16 \times 10^{-7} \text{ s}$$

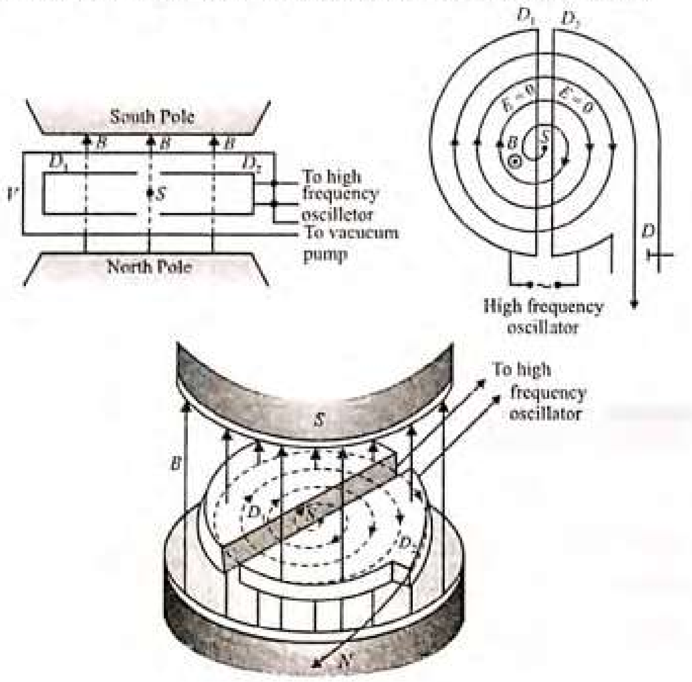
The pitch of the helix

= distance travelled by the proton along x-axis in time T.

$= v_x \times T = (6.93 \times 10^6) \times (2.19 \times 10^{-7}) = 1.515$ m

Cyclotron

It is a device which is used to accelerate charged particles.



Construction

It consists of two hollow flat semicircular metal boxes D_1 and D_2 called the "dees" on account of their shape like the letter D. The two dees are separated by narrow parallel gap. A high frequency oscillator, which provides an alternating potential of the order of 10^4 to 10^5 volt at a frequency of 10^7 cycles is connected between the two dees. The oscillator establishes an alternating electric field in the air gap, i.e., the electric field is once directed toward D_1 and then toward D_2. Thus D_1 and D_2 become alternately positive and negative at the same rate as the frequency of the oscillator. A source S is placed at the center of the dees which supplies the positive ions to be accelerated. These dees are mounted inside a vacuum chamber. The chamber is mounted horizontally between the pole pieces NS of a huge electromagnet capable of producing a vertical field of about 1.6 T.

Working

The positive ions emitted from the source will be acceleraed in the gap towards the dees which is negative at that time. Let it be D_2.Since there is no electric field inside the dees, the positive ions move with constant speed along circles of constant radius under the influence of magnetic field which is perpendicular to the dees. If by the time the ions emerge from D_2, the polarity of the applied potential is reversed (i.e., the dee D_1 now becomes negative), the positive ions will again face the negative dee and thus will be again accelerated by the field in the gap. Since their velocity is increased, they will now move through D_1 along circular arc of greater radius as shown in figure. Here the time of passage to complete the semi circle in the dee remains the same as in D_2. If the time of travel in D_1 is equal to half the time period of the oscillator voltage, the positive ions after coming from D_1 will find the reversed field and hence they are accelerated again in the gap D_1D_2. In this way, the positive ions move faster and faster in ever-expanding circles until they reach the outer edge of the dees where they are deflected by deflector plate and strike the target. Here it should be remembered that the time required for the positive ions to make one complete turn within dees is the same for all speeds and is equal to the time period of the oscillator.

Theory

When a particle of mass m and charge q moves with a velocity v in the magnetic field of flux density B, then the radius r of the circular path is given by

$$r = \frac{mv}{qB} \quad \text{(i)}$$

As the speed v increases every time the particle passes through the gap, so radius r also increases.

For resonance between the applied ac voltage and the moving particle, the time taken by the particle to travel the circular arc within any of the dees must be equal to half the time period of the applied oscillator voltage. Or the frequency of oscillator voltage should be same as the frequency of revolution of charge particle. So the frequency of oscillator is given by

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

The value of q/m being fixed for an ion hence the value of f is adjusted corresponding to B or vice-versa.

Energy of a Particle Accelerated by a Cyclotron

The ion will have maximum energy when it will travel at the boundary of dee. If the outside radius of dee is R then according to Eq. (i), the maximum velocity v_m of the ion may be written as

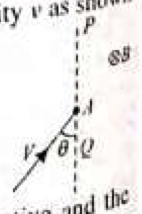
$$v_m = \frac{RqB}{m} \quad \text{(ii)}$$

and so the maximum kinetic energy of the ion will be given by

$$K_{max} = \frac{1}{2}mv_m^2 = \frac{R^2q^2B^2}{2m} \quad \text{(iii)}$$

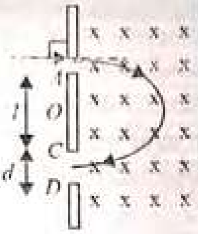
Concept Application Exercise 21.1

1. A positive charge particle of charge q and mass m enters into a uniform magnetic field with velocity v as shown in figure. There is no magnetic field to the left of PQ.
   Find (a) time spent, (b) distance travelled in the magnetic field, (c) impulse of magnetic force.



2. Repeat Question 1, if the charge is negative and the angle made by the boundary with the velocity is $\theta = \pi/6$.

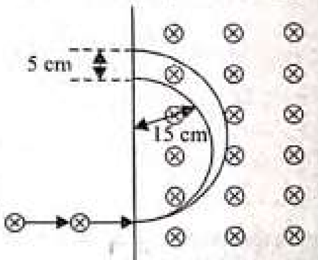
3. A beam of equally charged particles after being accelerated through a voltage V enters into a magnetic field B as shown in figure. It is found that all the particles hit the plate between C and D. Find the ratio between the masses of the heaviest and lightest particles of the beam.



4. A proton and an alpha particle are projected in a magnetic field which exists in the width of region d. Compare the angles of deviation suffered by the proton and the alpha particle if before entering the magnetic field both the particles
   (a) have the same momentum,
   (b) have the same kinetic energy, and
   (c) are accelerated through the same potential difference.

   Take $m_\alpha = 4m_p$, $q_\alpha = 2q_p$.

5. A beam of singly ionized atoms of carbon (each charge $+e$) all have the same speed enters a mass spectrometer, as shown in figure. The ions strike the photographic plate in two different locations 5 cm apart. The $^{12}C_6$ isotope traces a path of smaller radius, 15.0 cm. What is the atomic mass number of other isotope?



6. When a proton has a velocity $\vec{v} = (2\hat{i} + 3\hat{j}) \times 10^6\ \text{ms}^{-1}$, it experiences a force $\vec{F} = -(1.28 \times 10^{-13}\,\hat{k})$ N. When its velocity is along the z-axis, it experiences a force along the x-axis. What is the magnetic field?

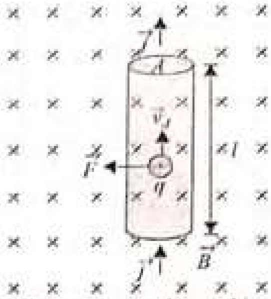
7. Protons having a kinetic energy of 5 MeV are moving in the positive x-direction and enter a magnetic field $B = 0.0500\,\hat{k}$ T directed out of the plane of the page and extending from $x = 0$ to $x = 1$ m as shown in figure.

   (a) Calculate the y-component of the protons' momentum as they leave the magnetic field.
   (b) Find the angle ϕ between the initial velocity vector of the proton beam and the velocity vector after the beam emerges from the field. Ignore relativistic effects and note that 1 eV $= 1.60 \times 10^{-19}$ J.

FORCE ON A CURRENT-CARRYING WIRE

Magnetic Force Acting on a Current-Carrying Conductor

Figure shows a straight segment of a conducting wire, with length l and cross-sectional area A; the current is from bottom to top. The wire is in a uniform magnetic field $\vec{B}$, perpendicular to the plane of the diagram and directed into the plane. Let us assume first that the moving charges are positive.



Forces on a moving positive charge in a current carrying conductor.

The drift velocity $\vec{v}_d$ is upward, perpendicular to $\vec{B}$. The average force on each charge is $\vec{F} = q\vec{v}_d \times \vec{B}$, directed to the left as shown in the figure; since $\vec{v}_d$ and $\vec{B}$ are perpendicular, the magnitude of the force is $F = qv_d B$.

We can derive an expression for the total force on all the moving charges in a length l of a conductor with cross-sectional area A. The number of charges per unit volume is n; a segment of conductor with length l has volume Al and contains a number of charges equal to nAl. The total force $\vec{F}$ on all the moving charges in this segment has magnitude

$$F = (nAl)\,(qv_dB) = (nqv_dA)(lB) \tag{i}$$

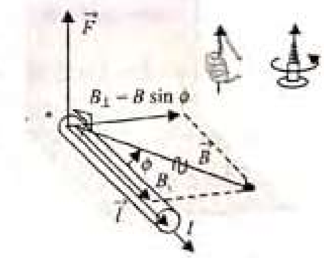
The current density is $J = nqv_d$. The product JA is the total current I, so we can rewrite Eq. (i) as,

$$F = IlB \tag{ii}$$

If the field $\vec{B}$ is not perpendicular to the wire but makes an angle ϕ with it, we handle the situation the same way we did for a single charge. Only the component of $\vec{B}$ perpendicular to the wire (and to the drift velocities of the charges) exerts a force; this component is $B_\perp = B \sin\phi$. The magnetic force on the wire segment is then

$$F = IlB_\perp = IlB \sin\phi \tag{iii}$$

The force is always perpendicular to both the conductor and the field, with the direction determined by the same right hand rule we used for a moving positive charge (as shown in figure). Hence, this force can be expressed as a vector product, just like the force on a single moving charge. We represent the segment of wire with a vector $\vec{l}$ along the wire in the direction of the current; then the force $\vec{F}$ on this segment is



A straight wire segment of length $\vec{l}$ carries a current I in the direction of $\vec{l}$. The magnetic force on this segment is perpendicular to both $\vec{l}$ and the magnetic field $\vec{B}$.

$$\vec{F} = I\vec{l} \times \vec{B} \text{ (magnetic force on a straight wire segment)} \tag{iv}$$

If the conductor is not straight, we can divide it into infinitesimal segments $d\vec{l}$. The force $d\vec{F}$ on each segment is

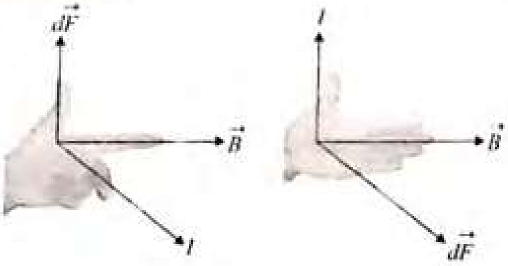
$d\vec{F} = I\vec{l} \times \vec{B}$ (magnetic force on an infinitesimal wire section)

Integrating this, we get total force $\vec{F} = \int I d\vec{l} \times \vec{B}$ (v)

Direction of Force on a Current-Carrying Wire in Magnetic Field

Left Hand Rule

If the thumb and first two fingers of the left hand are held each at right angles to the other, with the first finger pointing in the direction of the field and the second finger in the direction of the current, then the thumb predicts the direction of the thrust or force (see figure).



Right Hand Palm Rule

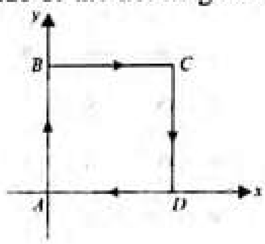
Stretch the fingers and thumb of right hand at right angles to each other. If the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current I, the normal to palm will point in the direction of force (or motion).

Regarding the force on a current carrying conductor in a magnetic field, it is worth mentioning that:

As the force $BI\,dL\sin\theta$ is not a function of position r, the magnetic force on a current element is non-central [a central force is of the form $F = Kf(r)\,\vec{n}_r$]

The force $d\vec{F}$ is always perpendicular to both $\vec{B}$ and $I\,d\vec{L}$ though $\vec{B}$ and $I\,d\vec{L}$ may or may not be perpendicular to each other.

Illustration 21.5 A square loop of edge l and carrying a current i, is placed with its edges parallel to the XY axes. There is present a non-uniform magnetic field in the region $\vec{B} = B_o\left(1+\frac{x}{l}\right)(-\hat{k})$. Find the magnitude of the net magnetic force experienced by the loop.



Solution. The magnetic field in the region

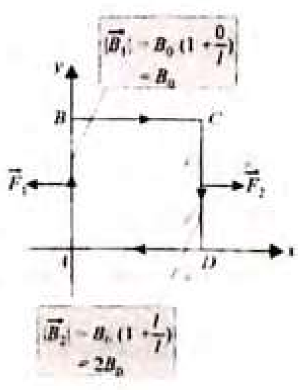
$$B = B_0\left(1+\frac{x}{l}\right)\vec{k}$$

Force on AB,

$$F_1 = iB_0\left(1+\frac{0}{l}\right)l = iB_0 l$$

Force on CD,

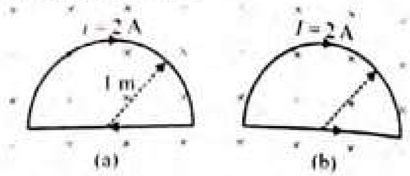
$$F_2 = iB_0\left(1+\frac{l}{l}\right)l = iB_0\,2l$$



The forces due to wires BC and AD will be equal and opposite. Hence, the force on BC and AD will cancel.

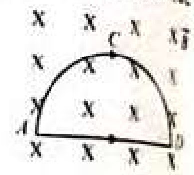
Net force, $F_2 - F_1 = iB_0 l$

Illustration 21.6 In figure, a semicircular wire loop is placed in uniform magnetic field $B = 1.0$ T. The plane of the loop is perpendicular to the magnetic field. Current $i = 2$ A flows in the loop in the direction shown. Find the magnitude of the magnetic force in both the cases (a) and (b). The rad-ius of the loop is 1 m.



Solution. Refer Figure (a): It forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero.

Refer Figure (b): In this case although it forms a closed loop, but current does not complete the loop. Hence, net force is not zero.



$$\vec{F}_{ACD} = \vec{F}_{AD}$$

$$\therefore \quad \vec{F}_{loop} = \vec{F}_{ACD} + \vec{F}_{AD} = 2\vec{F}_{AD}$$

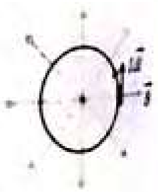
$$\therefore \quad |\vec{F}_{loop}| = 2|\vec{F}_{AD}|$$

$$= 2ilB\sin\theta\ (l = 2r = 2.0\text{ m})$$

$$= (2)(2)(2)(1)\sin 90^\circ = 8\text{ N}$$

Illustration 21.7 A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The center of the loop coincides with the center of the field. The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire.

Solution. Let us select a current element on circular loop Idl. The magnetic field in radial direction will be perpendicular to current element vector. From left hand rule or right palm rule, we can find the direction of force $d\vec{F}$ on the element which comes to be perpendicular into the plane of the paper.

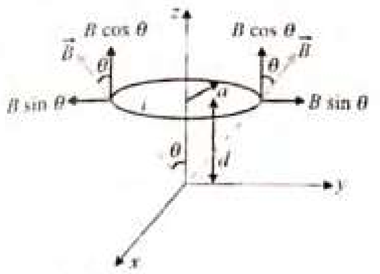


Required force $F = \int dF = \int idlB\sin 90^\circ = \int IdlB$

$$= 2\pi\,aiB \text{ perpendicular to the plane.}$$

Illustration 21.8 A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0\vec{e}_r$ where $\vec{e}_r$ denotes the unit vector along the radial direction. A circular loop of radius a carrying a current i, is placed with its plane parallel to the X-Y plane and the center at $(0,0,d)$. Find the magnitude of the magnetic force acting on the loop.

Solution. Magnetic force due to $B\cos\theta$ will be cancelled out. There will be no force due to magnetic field due to $B\cos\theta$ in downward direction.



Required magnetic force

$$F = \int dF = \int_0^{2\pi a} idlB \sin\theta = \frac{2\pi i a^2 B}{\sqrt{a^2 + d^2}}$$

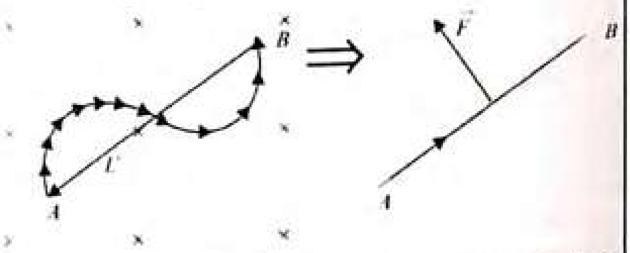
Important Points

- In case of current carrying conductor in a magnetic field, if the field is uniform, i.e., $\vec{B}$ = constant,

$$\vec{F} = \int I\,d\vec{L} \times \vec{B} = I\left[\int d\vec{L}\right] \times \vec{B}$$

For a conductor, $\int d\vec{L}$ represents the vector sum of all the length elements from initial to final point, which in accordance with the law of vector addition is equal to the length vector $\vec{L}$ joining initial to final point. So, a current carrying conductor of any arbitrary shape in a uniform field experiences a force

$$\vec{F} = i\left[\int d\vec{L}\right] \times \vec{B} = I\vec{L} \times \vec{B}$$



where $\vec{L}$ is the length vector joining initial and final points of the conductor as shown in figure.

- If the current carrying conductor in the form of a closed loop of any arbitrary shape is placed in a uniform field,

$$\vec{F} = \oint I d\vec{L} \times \vec{B} = I\left[\oint d\vec{L}\right] \times \vec{B}.$$

For a closed loop, the vector sum of $d\vec{L}$ is always zero.

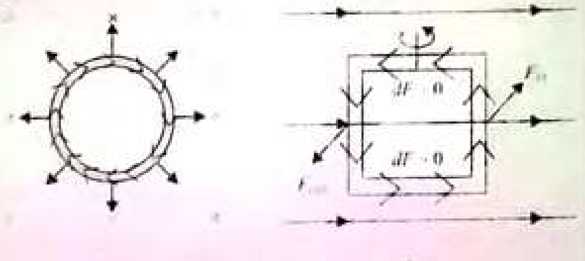
So, $\vec{F} = 0$ $\left[\text{as } \oint d\vec{L} = 0\right]$

i.e., the net magnetic force on a current loop in a uniform magnetic field is always zero as shown in figure.

- A current carrying loop in a uniform magnetic field

$\vec{F} = I(\oint d\vec{L}) \times \vec{B} = 0$

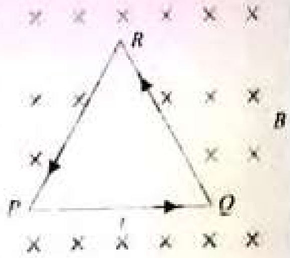
Here, it must be kept in mind that in this situation different parts of the loop may experience elemental force due to which the loop may be under tension or may experience a torque as shown in figure.



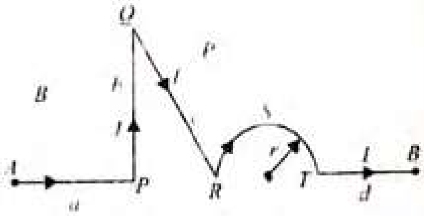
$\vec{F} = 0$, $\tau = 0$ — Current loop in a uniform field — $\vec{F} = 0$, $\tau \neq 0$

Concept Application Exercise 21.2

1. A wire is bent in the form of an equilateral triangle PQR of side 10 cm and carries a current of 5.0 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

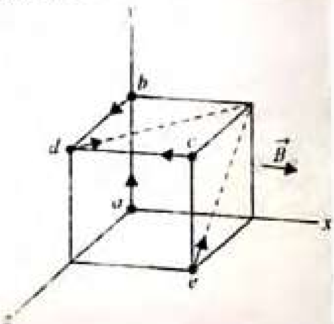


2. Calculate the force on a current carrying wire in a uniform magnetic field as shown in figure.



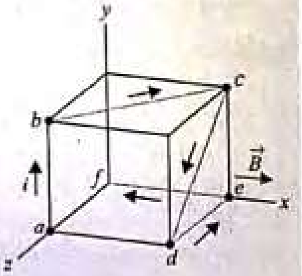
3. The horizontal component of the earth's magnetic field at a certain place is 3×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is
   (a) east to west, (b) south to north?

4. Each of the lettered points at the corners of the cube as shown in figure represents a positive charge q moving with a velocity of magnitude v in the direction indicated. The region in the figure is in a uniform magnetic field $\vec{B}$, parallel to the x-axis and directed toward right. Copy the figure, find the magnitude and



direction of the force on each charge and show the force in your diagram.

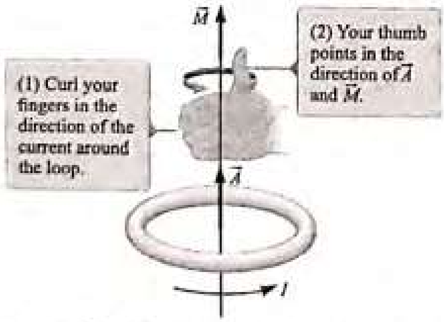
5. The cube as shown in figure, 75.0 cm on a side, is placed in a uniform magnetic field of 0.860 T parallel to the x-axis. The wire $abcdef$ carries a current of 6.58 A in the direction indicated.
   (a) Determine the magnitude and direction of the force acting on the segment ab.
   (b) Determine the magnitude and direction of the force acting on the segment bc.
   (c) Determine the magnitude and direction of the force acting on the segment cd.
   (d) Determine the magnitude and direction of the force acting on the segment de.
   (e) Determine the magnitude and direction of the force acting on the segment ef.
   (f) What are the magnitude and direction of the total force on the wire ?



MAGNETIC DIPOLE AND DIPOLE MOMENT

Magnetic dipole is the magnetic equivalent of electric dipole. The magnetic field pattern produced by a small current loop is similar to a bar magnet. Therefore, it also acts like a magnetic dipole. The magnetic moment of a flat current loop is defined as the product of the current I and the area A enclosed by it, i.e., $\vec{M} = I\vec{A}$.

The direction of the magnetic moment coincides with the direction of the area vector (which is the direction of the magnetic field).

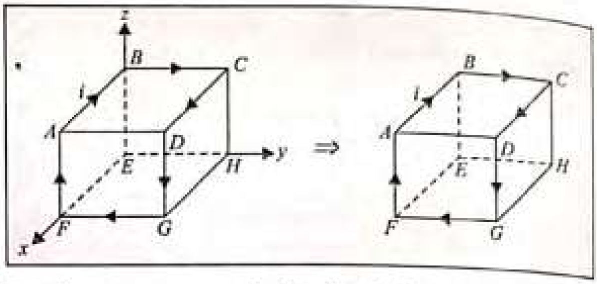


If the loop contains N number of turns, the magnetic moment is given by $M = NIA$

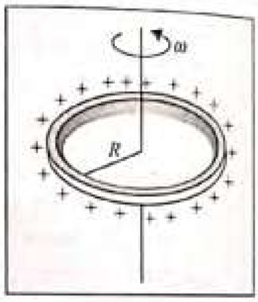
- Sometimes a current carrying loop does not lie in a single plane. But by assuming two equal and opposite currents in one branch (which obviously makes no change in the given circuit) two (or more) closed loops are completed in different planes. Now, the net magnetic moment of the given loop is the vector sum of individual loops. For example, in figure, six sides of a cube of side l carry a current i in the directions shown. By assuming two equal and opposite currents in wire AD, two loops in two different planes (xy and yz) are completed.

$$\vec{M}_{ABCDA} = -il^2\hat{k} \quad \text{and} \quad \vec{M}_{ADGFA} = -il^2\hat{i}$$

$$\Rightarrow \quad \vec{M}_{\text{net}} = -il^2(\hat{i} + \hat{k})$$



- Sometimes a non-conducting body is related with some angular speed. In this case, the ratio of magnetic moment and angular momentum is constant which is equal to $q/2m$, where q is the charge and m is the mass of the body. For example, in case of a ring of mass m, radius R, and charge q distributed on its circumference,



Angular momentum,

$$L = I\omega = (mR^2)(\omega) \qquad \text{(i)}$$

Magnetic moment,

$$M = iA = (qf)(\pi R^2) \qquad \text{(ii)}$$

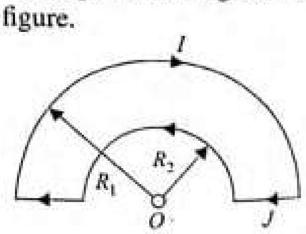
Here, f = frequency = $\dfrac{\omega}{2\pi}$

$$\therefore \quad M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\,\frac{\omega R^2}{2}$$

From Eqs. (i) and (ii), $\dfrac{M}{L} = \dfrac{q}{2m}$

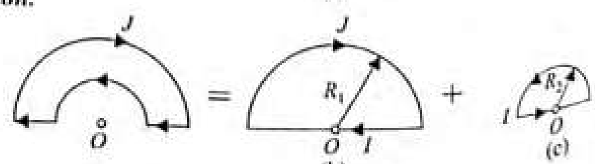
Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example for a disk or a sphere.

ILLUSTRATION 21.9 Compute the magnetic dipole moment of the loop shown in figure.



(a)

Solution.



(b) (c)

The given loop may be considered as the superposition of the two loops, as shown in the figure.

The resultant dipole moment is $M = \dfrac{\pi R_1^2 I}{2} - \dfrac{\pi R_2^2 I}{2}$

or $\quad M = \dfrac{\pi I}{2}\left(R_1^2 - R_2^2\right)$ (inward)

ILLUSTRATION 21.10 A circular loop of wire of radius R is bent about its diameter along two mutually perpendicular planes as shown in figure. If the loop carries a current I, then determine its magnetic moment.

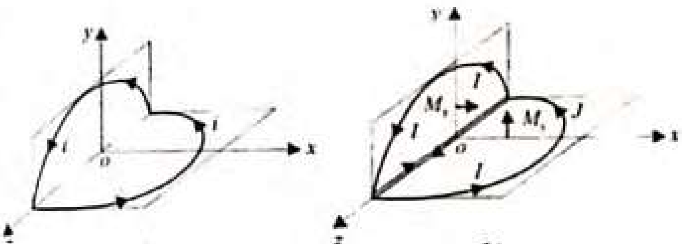
Solution. The given loop may be obtained by the superposition of two semicircular loops as shown in the figure.

The magnetic moment of the semicircle in the y-z plane is along the x-axis and that in the x-z plane is along the y-axis.

$$M_x = \frac{\pi R^2 I}{2}, \quad M_y = \frac{\pi R^2 I}{2}$$

The total magnetic moment is $\vec{M} = M_x\hat{i} + M_y\hat{j}$

or $\quad M = \dfrac{\pi R^2 I}{2}\left(\hat{i} + \hat{j}\right)$



(a) (b)

ILLUSTRATION 21.11 A non-conducting disk of mass M and radius R has a surface charge density σ and rotates with an angular velocity ω about its axis. Show that magnetic dipole moment and angular momentum are related as $\vec{\mu} = \left(\dfrac{Q}{2M}\right)\vec{L}$.

Solution. The charge is distributed on the surface of the disk. We consider a differential ring of radius r and thickness dr.

The charge on the element is $dq = \sigma dA = \sigma(2\pi r dr)$

The magnetic moment of the ring $d\mu = (dI)A = (dI)\pi r^2$

The current in the differential ring

$$dI = (dq)f = (\sigma dA)\,\frac{\omega}{2\pi} = (\sigma 2\pi r dr)\,\frac{\omega}{2\pi} = \sigma\omega r dr$$

The magnetic moment of the differential ring,

$$d\mu = (\sigma\omega r dr)\,\pi r^2 = \pi\sigma\omega r^3 dr$$

$$\mu = \int d\mu = \int_0^R \pi\sigma\omega r^3 dr = \frac{1}{4}\pi\sigma\omega R^4$$

The magnetic moment vector $\vec{\mu}$ is parallel to $\vec{\omega}$ if charge is positive.

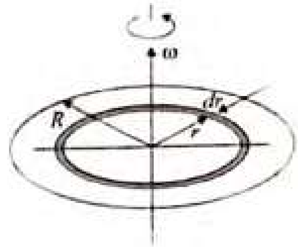
$$\vec{\mu} = \frac{1}{4}\pi\sigma R^4\vec{\omega}$$

In terms of total charge $Q = \sigma\pi R^2$, the magnetic moment is

$$\vec{\mu} = \frac{1}{4}QR^2\vec{\omega}$$

The angular momentum of disk is $\vec{L} = \left(\dfrac{1}{2}MR^2\right)\vec{\omega}$ and

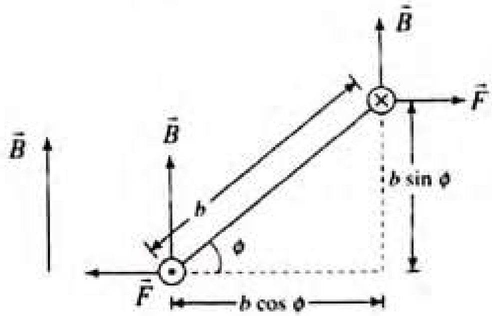
$\vec{\mu} = \left(\dfrac{Q}{2M}\right)\vec{L}$.



This is a general result for any rigid body of any arbitrary shape having mass M and charge Q.

TORQUE ON A CURRENT-CARRYING PLANAR LOOP IN A UNIFORM MAGNETIC FIELD

Figure shows a rectangular loop of wire with side lengths a and b. A line perpendicular to the plane of the loop (i.e., a normal to the plane) makes an angle ϕ with the direction of the magnetic field $\vec{B}$, and the loop carries a current I.



The magnitude of the net torque is

$$\tau = F(b)\sin\phi = (IBa)\,(b\sin\phi) \qquad \text{(i)}$$

The area A of the loop is equal to ab, so we can rewrite Eq. (i) as

$\tau = IBA\sin\phi$

(magnitude of torque on a current loop) (ii)

where ϕ is the angle between the normal to the loop (the direction of the vector area $\vec{A}$) and $\vec{B}$.

The product IA is called the magnetic dipole moment or magnetic moment of the loop

$$M = IA \qquad \text{(iii)}$$

We can express this interaction in terms of the torque vector $\vec{\tau}$, which we used for electric–dipole interactions. The

magnitude of $\vec{\tau}$ is equal to the magnitude of $\vec{M} \times \vec{B}$. So, we have

$$\vec{\tau} = \vec{M} \times \vec{B} \text{ (vector torque on a current loop)} \quad \text{(iv)}$$

ILLUSTRATION 21.12 A coil in the shape of an equilateral triangle of side 0.02 m is suspended from a vertex such that it is hanging in a vertical plane between the pole pieces of a permanent magnet producing a horizontal magnetic field of 5×10^{-2} T. Find the couple acting on the coil when a current of 0.1 A is passed through it and the magnetic field is parallel to its plane.

Solution. The couple acting on a closed loop is given by $\tau = NiAB \sin\theta$, where, N = number of loops, A = area of loop, B = magnetic field and θ = angle between magnetic field and normal to the surface of the loop.

Here $N = 1$, $i = 0.1$ A, $B = 5 \times 10^{-2}$ T, $\theta = 90°$ and

$$A = \frac{1}{2}(\text{Base}\times\text{height}) = \frac{1}{2}(0.02 \times 0.01732) = 1.732 \times 10^{-4}\,\text{m}^2$$

$$\therefore \quad \tau = 1 \times 0.1 \times (1.732 \times 10^{-4}) \times (5 \times 10^{-2}) \times 1$$

$$= 8.66 \times 10^{-7}\text{ Nm}$$

ILLUSTRATION 21.13 A circular coil of wire 8 cm in diameter has 12 turns and carries a current of 5 A. The coil is in a field where the magnetic induction is 0.6 T.

(a) What is the maximum torque on the coil?

(b) In what position would the torque be half as great as in (a)?

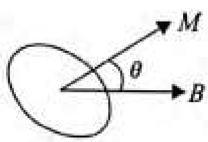
Solution.

(a) The dipole moment of the coil: $M = NiA$

Here $N = 12$ turns, $i = 5$ A and $A = \pi r^2 = \pi(4 \times 10^{-2})^2$

$$\therefore \quad M = 12 \times 5 \times \pi \times (4 \times 10^{-2})^2$$

$$= 0.3\text{ Am}^2$$



Maximum torque $= MB = 0.3 \times 0.60$

$= 0.18$ Nm

(b) If θ be the angle between the axis of the coil and the field

torque $= MB \sin\theta$

According to the problem, torque $= MB/2$

So $\dfrac{MB}{2} = MB\sin\theta \Rightarrow \sin\theta = \dfrac{1}{2}$ or $\theta = 30°$

Thus, the normal to the coil is at 30° to the field.

Energy of Dipole

When a magnetic dipole changes orientation in a magnetic field, the field does work on it. In an infinitesimal angular displacement $d\phi$, the work dW is given by $\tau\, d\phi$, and there is a corresponding change in potential energy.

Energy needed to rotate the loop through an angle $d\theta$ is

$$dU = \tau d\theta$$

$$\Rightarrow \quad \Delta U = \int_{\theta_1}^{\theta_2} \tau\, d\theta = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$$

$$\Rightarrow \quad \Delta U = MB(\cos\theta_1 - \cos\theta_2)$$

If we choose θ_1 such that at $\theta_1 = 90°$, $U_1 = 0$

$$U = -\vec{M}\cdot\vec{B}$$

This is the energy stored in the loop.

Important Points

Forces on the sides of a current carrying loop in a uniform magnetic field.

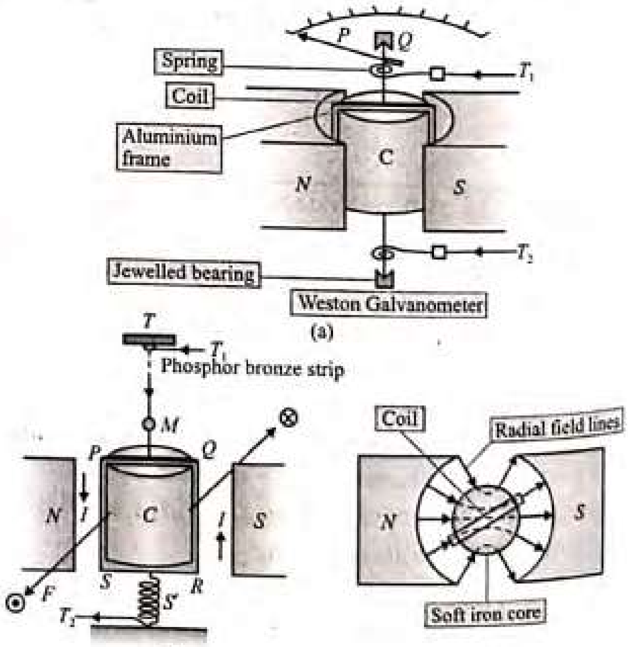
(a) The resultant force is zero; the net torque has magnitude $\tau = IAB \sin\phi$.

(b) The torque is maximum when the normal to the loop is perpendicular to $\vec{B}$.

(c) When the normal to the loop is parallel to $\vec{B}$, the torque is zero and the equilibrium is stable. If the normal is antiparallel to $\vec{B}$, the torque is also zero but the equilibrium is unstable.

MOVING COIL GALVANOMETER

This is a device which is used for detection and measurement of small electric current. The principle of a moving coil galvanometer is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.



(a) (b) (c)

Theory

Let the coil be suspended freely in the magnetic field.

Suppose, N = number of turns in the coil

A = area of the coil

B = magnetic field induction of radial magnetic field in which the coil is suspended.

Here, the magnetic field is radial, i.e., the plane of the coil always remains parallel to the direction of magnetic field, and hence the torque acting on the coil

$$\tau = NiAB \quad \text{(i)}$$

Due to this torque, the coil rotates. As a result, the suspension wire gets twisted. Now a restoring torque is developed in the suspension wire. The coil will rotate till the deflecting torque acting on the coil due to flow of current through it is balanced by the restoring torque developed in the suspension wire due to twisting. Let C be the restoring couple per unit twist in the suspension wire and θ be the angle through which the coil has turned. The couple for this twist θ is $C\theta$.

In equilibrium, deflecting couple = restoring couple

$$NiAB = C\theta$$

or $\quad i = C\theta/(NAB)$

or $\quad i = K\theta \quad$ (where $C/NAB = K$) $\quad$ (ii)

K is a constant for galvanometer and is known as *galvanometer constant*.

Hence, $i \propto \theta$

Therefore, the deflection produced in the galvanometer is directly proportional to the current flowing through it.

Current Sensitivity of the Galvanometer

The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer when a unit current is passed through it.

We know that, $NiAB = C\theta$

$\therefore$ Current sensitivity, $i_s = \dfrac{\theta}{i} = \dfrac{NAB}{C}$

The unit of current sensitivity is radian per ampere or deflection per ampere.

Voltage Sensitivity of the Galvanometer

The voltage sensitivity of the galvanometer is defined as the deflection produced in the galvanometer when a unit voltage is applied across the terminals of the galvanometer.

$\therefore$ Voltage sensitivity, $V_s = \dfrac{\theta}{V}$

If R be the resistance of the galvanometer and a current i is passed through it, then

$$V = iR$$

Voltage sensitivity, $V_s = \dfrac{\theta}{iR} = \dfrac{NAB}{CR}$

The unit of voltage sensitivity is radian per volt or deflection per volt.

Condition for Sensitive Galvanometer

A galvanometer is said to be more sensitive if it shows a large deflection evern for a small value of current.

We know that $\theta = \dfrac{NAB}{C} i$

For a given value of i, θ will be large if

(1) N is large, (2) A is large, (3) B is large, and (4) C is small.

The value of C for quartz and phosphor-bronze is very small so the suspension wire of quartz or phosphor-bronze is used. N and A cannot be increased beyond a certain limit, because otherwise size of the galvanometer increases. B can be increased using strong magnets.

ILLUSTRATION 21.14 A rectangular coil of area 5.0×10^{-4} m^2 and 60 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal field of 90 G (radial here means the field liners are in the plane of the coil for any rotation). What is the torsional constant of the hair spring connected to the coil if a current of 2.0 mA produces an angular deflection of 18°?

Solution. Torsional constant is given by: $C = \dfrac{INBA}{\theta}$

$$\Rightarrow \quad C = \frac{2 \times 10^{-3} \times 60 \times 90 \times 10^{-4} \times 5 \times 10^{-4}}{18^\circ}$$

$$= 3 \times 10^{-8} \text{ Nm/degree}$$

ILLUSTRATION 21.15 The coil of a galvanometer is 0.02×0.8 m^2. It consists of 200 turns of the wire and is in a magnetic field of 0.20 T. The restoring torque constant of suspension fibre is 10^{-5} Nm/degree. Assuming magnetic field to be radial

(a) What is the maximum current that can be measured by this galvanometer if scale can accommodate 45° deflection?

(b) What is the smallest current that can be detected if minimum observed deflection is 0.1°?

Solution. We know that $I = \dfrac{C}{NBA}\theta$

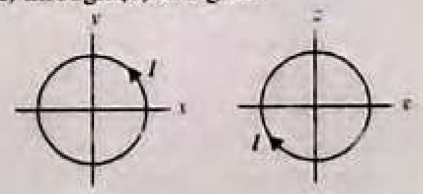
(a) $$I = \frac{10^{-5} \times 45^\circ}{200 \times 0.20 \times 0.02 \times 0.8} = 7.03 \times 10^{-4} \text{ A}$$

(b) $$I_{min} = \frac{C}{NBA}\theta_{min}$$

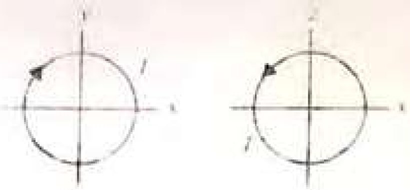
$$= \frac{10^{-5} \times 0.1^\circ}{200 \times 0.20 \times 0.02 \times 0.8} = 1.56 \times 10^{-6} \text{ A}$$

CONCEPT APPLICATION EXERCISE 21.3

1. A circular coil with area A and N turns is free to rotate about a diameter that coincides with the x-axis. Current I is circulating in the coil. There is a uniform magnetic field $\vec{B}$ in the positive y direction. Calculate the magnitude and direction of the torque $\vec{\tau}$ and the value of the potential energy U, when the coil is oriented as shown in parts (a) through (d) of figure.



(a) (b)



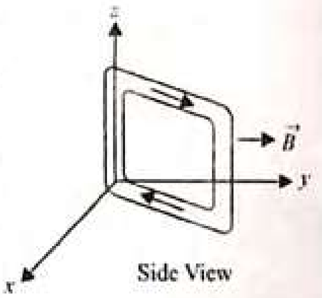
(c) (d)

2. The square loop in figure has sides of length 20 cm. It has 5 turns and carries a current of 2 A. The normal to the loop is at 37° to a uniform field $\vec{B} = 0.5\,\hat{j}$ T.

(a) Find the magnetic moment of the loop.

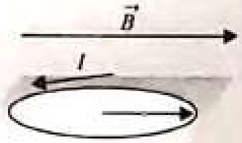
(b) Find the torque on the loop.

(c) Find the work needed to rotate the loop from its position of minimum energy to the given orientation.

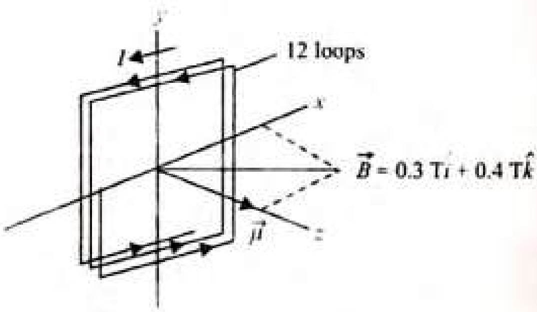


Side View

3. A circular wire loop of radius R, mass m carrying current I lies on a rough surface (as shown in figure). There is a horizontal magnetic field $\vec{B}$. How large can the current I be before one edge of the loop will lift off the surface?



4. A square 12-turn coil with sides of length 40 cm carries a current of 3 A. It lies in the x–y plane as shown in figure in a uniform magnetic field.

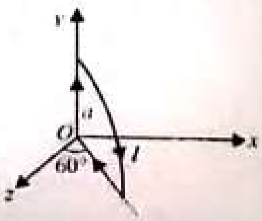


(a) Find the magnetic moment of the coil.

(b) Find the torque exerted on the coil.

(c) Find the potential energy of the coil.

5. Figure shows one quarter of a simple circular loop of wire that carries a current of 14 A. Its radius is $a = 5$ cm. A uniform magnetic field, B = 300 G, is directed in the $+x$ direction. Find the torque on the entire loop and the direction in which it will rotate.

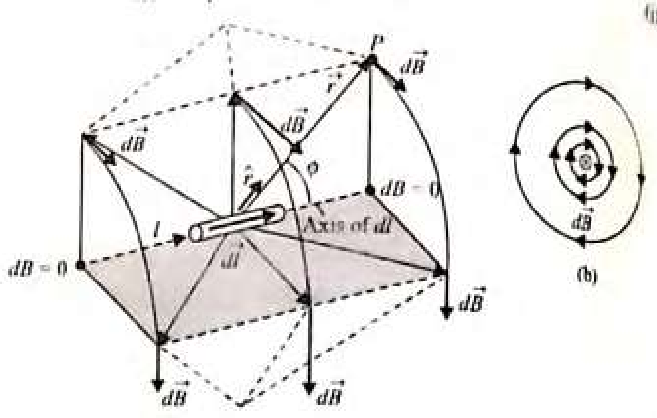


6. A galvanometer coil is replaced by another coil of diameter one-fourth of the original diameter and the total number of turns as ten times the original number. What will be the new deflection if the same current is passed through it? Old deflection is θ.

MAGNETIC FIELD OF A CURRENT ELEMENT

The magnetic field caused by a short segment $d\vec{l}$ of a current carrying conductor, as shown in Figure (a).

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\phi}{r^2} \quad \text{(i)}$$



(a) (b)

In vector form, using the unit vector $\hat{r}$ as in "magnetic field of a moving charge," we have

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2} \quad \text{(ii)}$$

(magnetic field of a current element)

Equations (i) and (ii) are called the law of Biot and Savart. We can use this law to find the total magnetic field $\vec{B}$ at any point in space due to the current in a complete circuit. To do this, we integrate equation (ii) over all segments $d\vec{l}$ that carry current symbolically.

$$\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}\times\hat{r}}{r^2} \quad \text{(iii)}$$

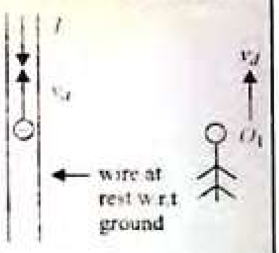
As Figure (a) shows, the field vectors $d\vec{B}$ and the magnetic field lines of a current element are exactly like those set up by a positive charge dQ moving in the direction of the drift velocity $\vec{v}_d$. The field lines are circles in planes perpendicular to $d\vec{l}$ and centered on the line of $d\vec{l}$. Their directions are given by the same right hand rule that we introduced for point charges

Important Points

Frame dependence of $\vec{B}$:

(a) The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving at some velocity $\vec{v}_1$ with respect to observer O_2 and at velocity $\vec{v}_2$ with respect to observer O_3. Then, $\vec{B}$ due to that charge w.r.t. O_1 will be zero and w.r.t. O_2 and O_3 it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different).

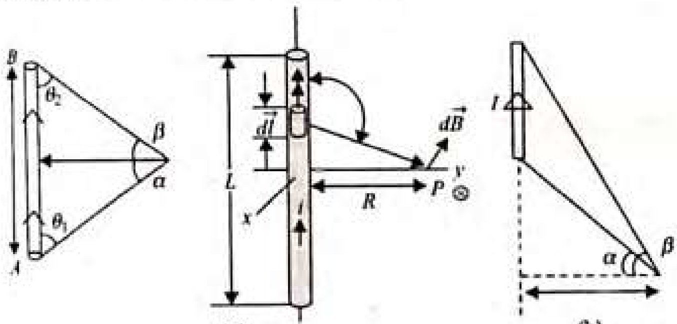
(b) In a current carrying wire electrons move in the opposite direction to that of the current and +ve ions (of the metal) are static w.r.t. the wire. Now, if some observer (O_1) moves with velocity v_d in the direction of motion of the electrons, then electrons will have zero velocity and +ve ions will have velocity v_d in the downward direction w.r.t. O_1. The density (n) of +ve ions is same as the density of free electrons and their charges are of the same magnitudes.



So, w.r.t. O_1 electrons will produce zero magnetic field but +ve ions will produce the magnetic field.

MAGNETIC FIELD DUE TO CURRENT IN A STRAIGHT WIRE

Magnetic field due to a wire segment carrying current i at P, when the wire segment subtends angles α and β as shown (figure), can be determined as follows.



(a) (b)

dB at P, due to dl is: $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\theta}{r^2}$

Now, $l = R\cot(\pi - \theta)$, $dl = R\,\text{cosec}^2\,\theta\, d\theta$

$r = R\,\text{cosec}\,\theta$

$$\Rightarrow \quad dB = \frac{\mu_0}{4\pi}\frac{I(\sin\theta)(R\,\text{cosec}^2\,\theta\, d\theta)}{R^2\,\text{cosec}^2\,\theta}$$

$$B = \frac{\mu_0 I}{4\pi R}\int_{\theta_1}^{180-\theta_2}\sin\theta\, d\theta = \frac{\mu_0 I}{4\pi R}[\cos\theta_1 + \cos\theta_2]$$

$$= \frac{\mu_0 I}{4\pi R}[\cos\theta_1 + \cos\theta_2] = \frac{\mu_0 I}{4\pi R}[\sin\alpha + \sin\beta].$$

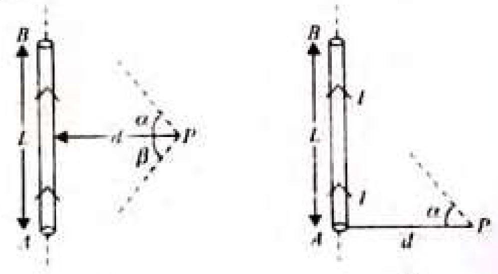
In the vector form, $\vec{B} = \dfrac{\mu I}{4\pi R}[\sin\alpha + \sin\beta](-\hat{k})$ [Figure (a)]

If the point under consideration where the magnetic field is to be calculated is not in front of the wire as shown in Figure (b), the magnetic field is given by $\vec{B} = \dfrac{\mu I}{4\pi R}[\sin\beta - \sin\alpha](-\hat{k})$ [Figure (b)].

Important Points

Thus, it is clear that in the case of a current carrying straight wire:

- For points along the length of the wire (but not on it), the field is always zero.
- The field is always perpendicular to the plane containing the wire and the point. So, in a plane perpendicular to the wire and containing the point, the lines of force are concentric circles encircling the wire as shown in figure.



(a) (b)

- If the wire is of infinite length and the point P is not near its ends as shown in Figure (a), $\alpha = \beta = (\pi/2)$; then

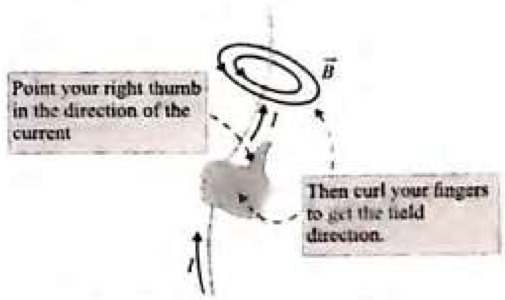
$$B = \frac{\mu_0}{4\pi}\frac{I}{d}[1+1] \quad \text{i.e.,} \quad B = \frac{\mu_0}{4\pi}\frac{2I}{d}$$

- If the point is near one end of an infinitely long wire as shown in Figure (b), $\alpha = (\pi/2)$ and $\beta = 0$.

So, $B = \dfrac{\mu_0 I}{4\pi d}[1+0]$ i.e., $B = \dfrac{\mu_0}{4\pi}\dfrac{I}{d}$

Right Hand Thumb Rule

Using this rule, the direction of magnetic field because of a current carrying wire may be obtained in the following ways: According to this rule, if we grasp the conductor in the palm of our right hand so that the thumb points in the direction of the flow of current, then the direction in which the fingers curl gives the direction of magnetic lines of force.



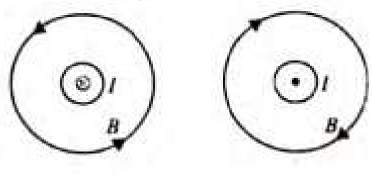
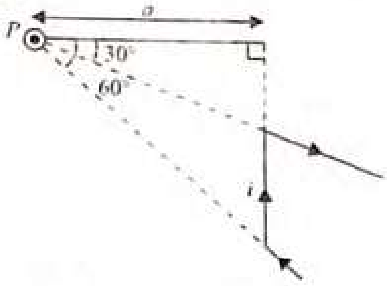


ILLUSTRATION 21.16 Find the magnitude and direction of magnetic field at point P due to the current carrying wire as shown in figure.

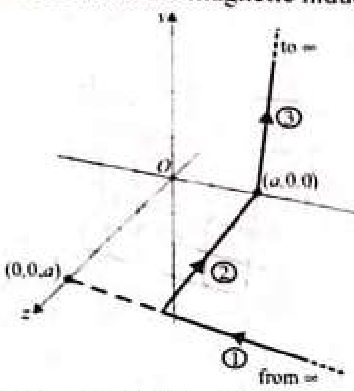


Solution. $B = \dfrac{\mu_0 I}{4\pi R}[\sin\theta_1 + \sin\theta_2]$

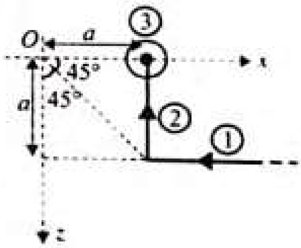
Here $\theta_1 = -30°$, $\theta_2 = 60°$. Putting these values, we get

$$B = \frac{\mu_0 I}{4\pi R}\left[-1/2 + \sqrt{3}/2\right] = \frac{\mu_0 I}{8\pi R}(\sqrt{3} - 1)$$

ILLUSTRATION 21.17 An infinite current carrying conductor is bent into three segments (1), (2) and (3) as shown in figure. If it carries a current i, find the magnetic induction at the origin.



Solution. Net magnetic field at O will be the vector sum of magnetic fields due to wires (1), (2) and (3).



Magnetic field due to wire (1),

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{i}{a}[\sin 90° - \sin 45°](-\hat{j})$$

$$= \frac{\mu_0}{4\pi}\frac{i}{a}\left[1 - \frac{1}{\sqrt{2}}\right](-\hat{j})$$

Magnetic field due to wire (2),

$$\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{i}{a}[\sin 45° + \sin 0°](\hat{j})$$

$$= \frac{\mu_0}{4\pi}\frac{i}{a}\left[\frac{1}{\sqrt{2}}\right](\hat{j})$$

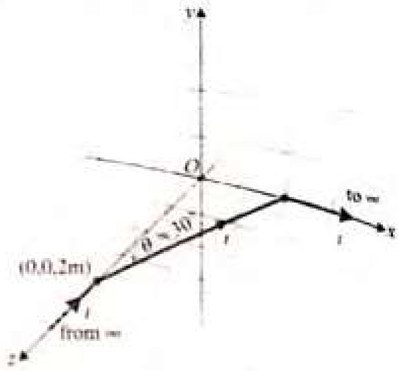
Magnetic field due to wire (3),

$$\vec{B}_3 = \frac{\mu_0}{4\pi}\frac{i}{a}(\hat{k})$$

Hence, net magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

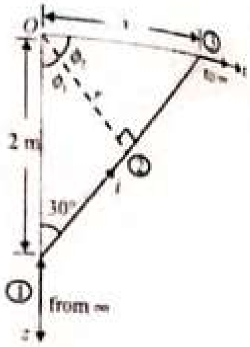
$$\Rightarrow \quad \vec{B} = \frac{\mu_0}{4\pi}\frac{i}{a}\left[(\sqrt{2} - 1)\hat{j} + \hat{k}\right]$$

ILLUSTRATION 21.18 Find the field at the origin O due to the current $i = 2$ A.



Solution. Magnetic field due to wires (1) and (3) will be zero.

Magnetic field due to wire (2),



$$B_2 = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin\phi_1 + \sin\phi_2]$$

Here, $r = 2\sin 30° = 1$ m.

$\phi_1 = 60°$, $\phi_2 = 30°$

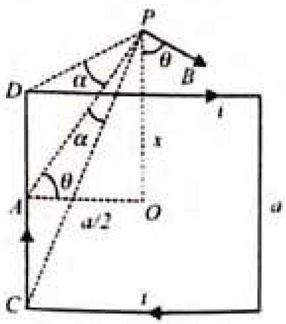
$$\vec{B}_2 = \frac{10^{-7} \times 2}{1}[\sin 60° + \sin 30°]$$

$$= (\sqrt{3} + 1) \times 10^{-7}\ \text{T}$$

ILLUSTRATION 21.19 A square loop of wire, edge length a, carries a current i. Compute the magnitude of the magnetic field produced at a point on the axis of the loop at a distance x from the center.

Solution. $AP = \sqrt{x^2 + \dfrac{a^2}{4}} = \dfrac{\sqrt{4x^2 + a^2}}{2}$

$$CP = \sqrt{AC^2 + AP^2} = \sqrt{x^2 + \frac{a^2}{2}} = \frac{\sqrt{4x^2 + 2a^2}}{2}$$



$$\sin\alpha = \frac{AC}{CP} = \frac{a}{\sqrt{4x^2 + 2a^2}}$$

Now $B = \dfrac{\mu_0 i}{4\pi(AP)} 2\sin\alpha = \dfrac{\mu_0 i a}{\pi\sqrt{4x^2 + a^2}\sqrt{4x^2 + 2a^2}}$

When the fields of all the four sides are considered, the horizontal components add to zero. So, the total field is given by

$$B_R = 4B\cos\theta = \frac{4\mu_0 i a^2}{\pi(4x^2+a^2)\sqrt{4x^2+2a^2}}$$

For $x = 0$, the expression reduces to $B_R = \dfrac{4\mu_0 i a^2}{\pi a^2\sqrt{2a^2}} = \dfrac{2\sqrt{2}\mu_0 i}{\pi a}$

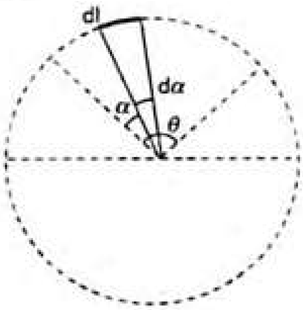
MAGNETIC FIELD AT THE CENTER OF A CURRENT-CARRYING CIRCULAR ARC

Figure shows a circular loop of radius r carrying a current I. Application of Biot and Savart law to a current element of length dl at angular position α with angular element $d\alpha$.

The magnetic field due to the element dl, $d\vec{B} = \dfrac{\mu_0 I(d\vec{l}\times\vec{r})}{4\pi r^3}$

Here, $\Rightarrow \quad dB = \dfrac{\mu_0 I}{4\pi}\dfrac{|dl|\,r\sin 90^\circ}{r^3}$ (as $d\vec{l} \perp \vec{R}$)

$\Rightarrow \quad dB = \dfrac{\mu_0 I}{4\pi r}(d\alpha)$ (as $dl = r\alpha$)



(a) (b) (c)

(a) A wire in the shape of a circular arc with center C carries current i.
(b) For any element of wire along the arc, the angle between the directions of $d\vec{s}$ and $\hat{r}$ is 90°.
(c) Determining the direction of the magnetic field at the center C due to the wire; the field is out of the page, in the direction of the fingertips, as indicated by the coloured dot at C.

$$B = \int dB = \frac{\mu_0 I}{4\pi r}\int_0^\theta d\alpha \Rightarrow B_{arc} = \frac{\mu_0 I}{4\pi r}\theta$$

where θ is the angle in radians.

Therefore, B at the center of a circular loop of radius R is

$$B = \frac{\mu_0 I}{4\pi r}(2\pi) = \frac{\mu_0 I}{2R}$$

ILLUSTRATION 21.20 Shown in figure is a conductor carrying current I. Find the magnetic field intensity at point O.

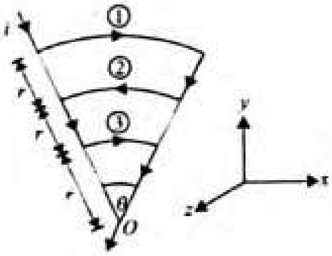
Solution. The magnetic field at the center of an arc is equal to

$$B = \frac{\mu_0 I}{4\pi r}\theta$$

Magnetic field due to arc (1), $B_1 = \dfrac{\mu_0 I\theta}{4\pi\times 3r}(-\hat{k})$

Magnetic field due to arc (2), $B_2 = \dfrac{\mu_0 I\theta}{4\pi\times 2r}(\hat{k})$

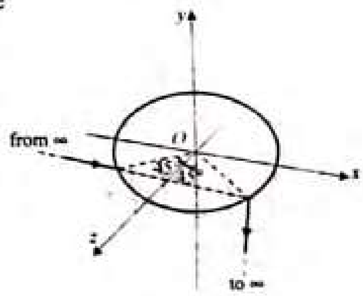
Magnetic field due to arc (3), $B_3 = \dfrac{\mu_0 I\theta}{4\pi r}(-\hat{k})$



Net magnetic field, $B = B_1 + B_2 + B_3$

Hence, net $B = \dfrac{\mu_0 I}{4\pi}\left[-\dfrac{1}{r}+\dfrac{1}{2r}-\dfrac{1}{3r}\right]\theta(\hat{k}) = -\dfrac{5\mu_0 I\theta}{24\pi r}\hat{k}$

ILLUSTRATION 21.21 What is the magnitude of magnetic field at the center O of loop of radius $\sqrt{2}$ m made of uniform wire when a current of 1 amp enters in the loop and taken out of it by a long wire as shown in the figure.

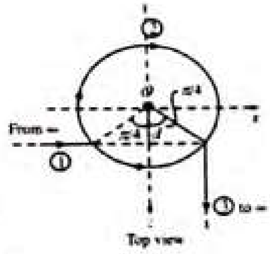


Solution. $\vec{B}_1$ = Magnetic field due to left wire

$$B_1 = \frac{\mu_0 I}{4\pi d}\left(\sin\frac{\pi}{2}-\sin\frac{\pi}{4}\right)(\hat{j})$$

$\vec{B}_2$ = Magnetic field due to right wire

$$= \frac{\mu_0 I}{4\pi d}\left(\sin\frac{\pi}{2}-\sin\frac{\pi}{4}\right)(-\hat{j})$$



In circular wire, $I \propto \dfrac{1}{R} \Rightarrow \dfrac{I_1}{I_2} = \dfrac{\theta}{2\pi-\theta}$

$\therefore \quad I_1 = \left(\dfrac{\theta}{2\pi-\theta}\right)I_2$

$\therefore \quad \vec{B}_3$ = Magnetic field due to circular wire

$$= \frac{\mu_0 I_1 \theta}{4\pi r} - \frac{\mu_0 I_2 (2\pi - \theta)}{4\pi r}$$

$$= \frac{\mu_0 I_2 \theta}{4\pi r} - \frac{\mu_0 \theta (2\pi - \theta)}{4\pi r (2\pi - \theta)} I_2 = 0$$

$$\therefore \quad \vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = 0$$

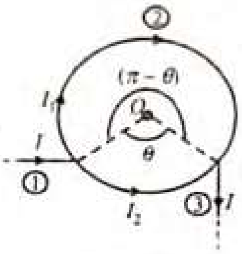
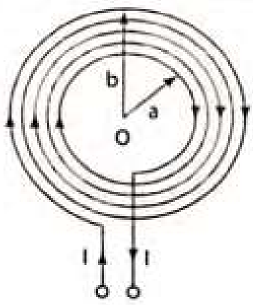


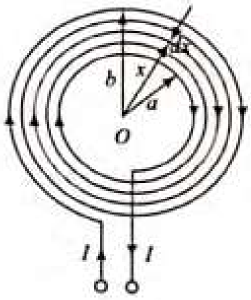
ILLUSTRATION 21.22 A thin insulated wire forms a plane spiral of $N = 100$ turns carrying a current $i = 8$ mA. The inner and outer radii are equal to $a = 5$ cm and $b = 10$ cm. Find the magnetic induction at the center of the spiral.



Solution. Let n be the number of turns per unit length along the radii of the spiral. Consider a ring of radii x and $x + dx$.

Number of turns in the ring $= n\, dx$

$\therefore$ Magnetic field at the center due to this ring $= \dfrac{\mu_0 (n\, dx) i}{2x}$



$\therefore$ B (total field)

$$= \int_a^b \frac{\mu_0 n i\, dx}{2x} = \frac{1}{2} \mu_0 n i \ln \frac{b}{a}$$

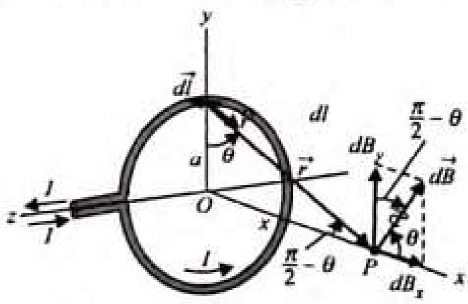
But $\quad n = \dfrac{N}{b-a}$

$$B = \frac{\mu_0 N i}{2(b-a)} \ln \frac{b}{a} = \frac{4\pi \times 10^{-7} \times 100 \times 8 \times 10^{-3}}{2(10-5) \times 10^{-2}} \ln \frac{10}{5}$$

$$= 6.9 \times 10^{-6}\ \text{T} = 6.9\ \mu\text{T}$$

MAGNETIC FIELD OF A CIRCULAR CURRENT LOOP

In Figure, the current in the segment $d\vec{l}$ causes the field $d\vec{B}$, which lies in the x-y plane. The currents in other $d\vec{l}$'s cause $d\vec{B}$'s with direction components perpendicular to the x-axis; these components add to zero. The x-components of the $d\vec{B}$'s combine to give the total field $\vec{B}$ at point P.



We can use the law of Biot and Savart to find the magnetic field at point P on the axis of the loop, at a distance x from the center. As Figure shows, $d\vec{l}$ and $\hat{r}$ are perpendicular and the direction of field $d\vec{B}$ caused by this particular element $d\vec{l}$ lies in the x-y plane. Since $r^2 = x^2 + a^2$, the magnitude dB of the field due to element $d\vec{l}$ is

$$dB = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \quad \text{(i)}$$

The components of the vector $d\vec{B}$ are

$$dB_x = dB \cos\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{a}{(x^2 + a^2)^{1/2}} \quad \text{(ii)}$$

$$dB_y = dB \sin\theta = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)} \frac{x}{(x^2 + a^2)^{1/2}} \quad \text{(iii)}$$

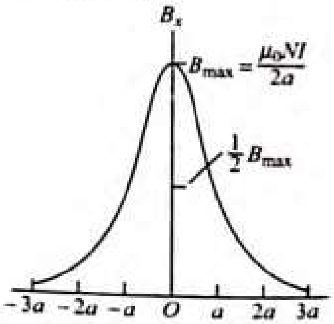
To obtain the x-component of the total field $\vec{B}$, we integrate Eq. (ii), including all the $d\vec{l}$'s around the loop. Everything in this expression except dl is constant and can be taken outside the integral. So, we have

$$B_x = \int \frac{\mu_0 I}{4\pi} \frac{a\, dl}{(x^2 + a^2)^{3/2}} = \frac{\mu_0 I a}{4\pi (x^2 + a^2)^{3/2}} \int dl$$

The integral of dl is just the circumference of the circle, i.e., $\int dl = 2\pi a$. So, we finally get

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \quad \text{(on the axis of a circular loop)} \quad \text{(iv)}$$

Now suppose that instead of the single loop in figure, we have a coil consisting of N loops, all with the same radius. The loops are closely spaced so that the plane of each loop is essentially the same distance x from the field point P. Each loop contributes equally to the field, and the total field is N times the field of a single loop:



Graph of the magnetic field along the axis of a circular coil with N turns. When x is much larger than a, the field magnitude decreases approximately as $1/x^3$.

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}} \quad \text{(on the axis of } N \text{ circular loops)} \quad \text{(v)}$$

If $x >> a$, then $B = \dfrac{\mu_0 I a^2}{2x^3} = \dfrac{\mu_0}{2\pi} \dfrac{I \pi a^2}{x^3} = \dfrac{\mu_0}{4\pi} \dfrac{I.2\pi a^2}{x^3}$

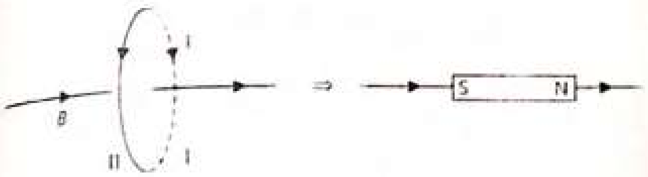
But $\pi a^2 = A =$ Area of cross section of the coil.

Thus, $B = \dfrac{\mu_0}{4\pi} \dfrac{2IA}{x^3} = \dfrac{\mu_0 2M}{4\pi x^3}$ or, $B = \dfrac{\mu_0}{4\pi} \dfrac{2M}{x^3}$

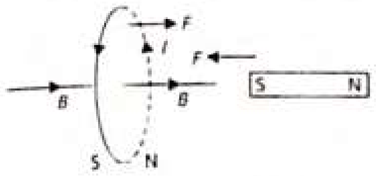
where $\vec{M} = I\vec{A}$ = Magnetic dipole moment of the loop. The direction of $\vec{M}$ is same as the direction of the normal to the area of the loop.

Important Points

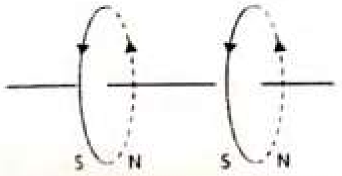
A loop as a magnet: The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.



The side I (the side from which $\vec{B}$ energes out) of the loop acts as "north pole" and side II (the side in which $\vec{B}$ enters) acts as the "south pole". It can be verified by studying force on one loop due to a magnet or a loop.



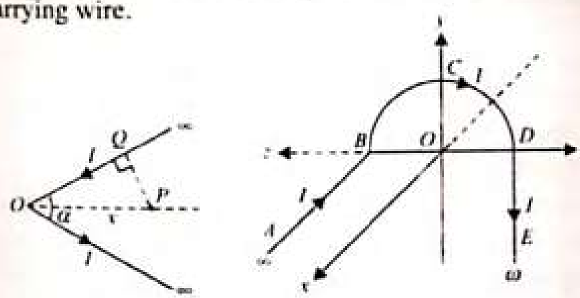
The loop and the magnet attract each other.



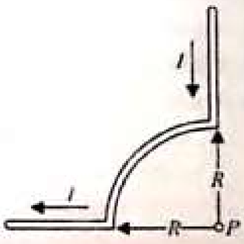
The two loops attract each other.

Concept Application Exercise 21.4

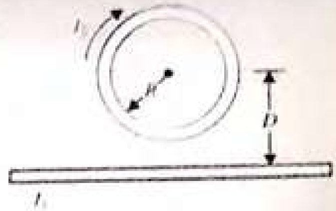
1. Find magnetic field at O by the system of current carrying wire.



2. The wire shown in figure, carries current I in the direction shown. The wire consists of a very long, straight section, a quarter-circle with radius R, and another long, straight section. What are magnitude and direction of net magnetic field at the center of curvature of quarter-circle section (point P)?



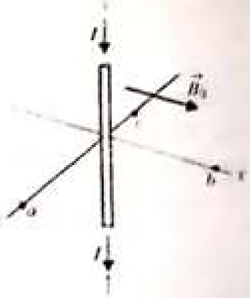
3. A circular loop has radius R and carries current I_2 in a clockwise direction (as shown in figure). The center of the loop is at a distance D above a long, straight wire. What are the magnitude and direction of the current I_1 in the wire if the magnetic field at the center of the loop is zero?



4. A wire is bent into the shape shown in Figure (a), and the magnetic field is measured at P_1 when the current in the wire is I. The same wire is then formed into the shape shown in Figure (b), and the magnetic field is measured at point P_2 when the current is again I. If the total length of wire is the same in each case, what is the ratio of B_1/B_2?

(a) (b)

5. A long, straight wire lies along the y-axis and carries a current $I = 8$ A in the $-y$-direction (as shown in figure). In addition to the magnetic field due to the current in the wire, a uniform magnetic field $\vec{B}_0$ with magnitude 1.50×10^{-6} T is in the $+x$ direction. What is the total field (magnitude and direction) at following points in the x-z plane?

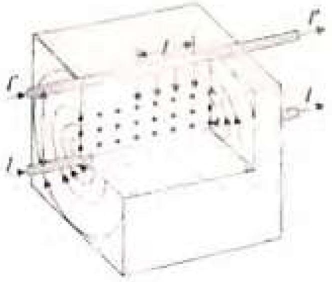


(a) $x = 0, z = 1.00$ m
(b) $x = 1.00$ m, $z = 0$
(c) $x = 0, z = -0.25$ m

6. Calculate the magnitude of the magnetic field at point P as shown in figure, in terms of R, I_1, and I_2.

FORCE BETWEEN TWO INFINITE PARALLEL CURRENT-CARRYING WIRES

Let two infinite parallel wires carrying currents I and I' be separated by a distance r (figure).



(a)

If we take an arbitrary point on the second wire, $B_{21} = \dfrac{\mu_0 I}{2\pi r}$

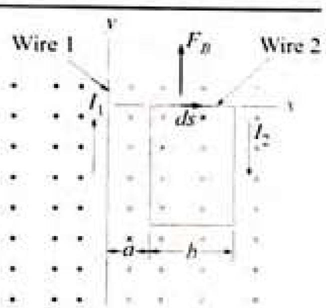
and $\vec{F}_{21} = I'(\vec{l} \times \vec{B}_{21}) \Rightarrow F_{21} = \dfrac{\mu_0 I I'}{2\pi r} L$

$\Rightarrow \dfrac{F_{21}}{L} = \dfrac{\mu_0 I I'}{2\pi r}$ this is force per unit length

By symmetry, $\dfrac{F_{12}}{l_1} = \dfrac{\mu_0 I I'}{2\pi r}$ (Force per unit length on the first wire due to second wire)

NOTE: Force per unit length $= \dfrac{\mu_0 I I'}{2\pi r}$. We note that the wires carrying current in the same direction attract. The wires carrying current in the opposite direction will repel each other.

ILLUSTRATION 21.23 Wire 1 in figure is oriented along the y-axis and carries a steady current I_1. A rectangular loop located to the right of the wire and in the x-y plane carries a current I_1. Find the magnetic force exerted by wire (1) on the top wire of length b in the loop, labeled "wire (2)" in the figure.



Solution. Consider the force exerted by wire 1 on a small segment ds of wire (2). This force is given by $d\vec{F}_B = I d\vec{s} \times \vec{B}$, where $I = I_2$ and $\vec{B}$ is the magnetic field created by the current in wire (1) at the position of $d\vec{s}$. The field at a distance x from wire (1) is

$$B = \frac{\mu_0 I_1}{2\pi x}(-\hat{k})$$

where the unit vector $-\hat{k}$ is used to indicate that the field due to the current in wire (1) at the position of $d\vec{s}$ points into the page. Because wire (2) is along the x-axis, $d\vec{s} = dx\hat{i}$, we find that

$$dF_B = \frac{\mu_0 I_1 I_2}{2\pi x}[\hat{i} \times (-\hat{k})]dx = \frac{\mu_0 I_1 I_2}{2\pi}\frac{dx}{x}\hat{j}$$

Integrating over the limits $x = a$ to $x = a + b$ gives

$$F_B = \frac{\mu_0 I_1 I_2}{2\pi}[\ln x]_a^{a+b}\hat{j} = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(1 + \frac{b}{a}\right)\hat{j}$$

The force on wire (2) points in the positive y-direction, as indicated by the unit vector $\hat{j}$ and as shown in the figure.

AMPERE'S LAW

Similar to Gauss law of electrostatics, this law provides a shortcut methods of finding magnetic field in cases of symmetry. According to this law, the line integral of magnetic field over the closed path ($\oint \vec{B}\cdot d\vec{l}$) is equal to μ_0 times the net current crossing the area enclosed by that path.

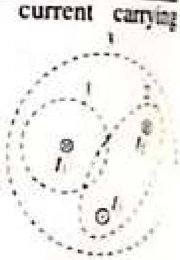
$$\oint_{\text{closed loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

This law is called Ampere's law and the closed loop on which it is applied is called the Amperian loop. The integral on the left hand side is called the *magnetic circulation*. Thus, Ampere's law can be stated as: *the magnetic circulation (C) around a closed loop is μ_0 times the net electric current enclosed by the loop.*

NOTE: While applying the above law, the following right hand convention is to be used

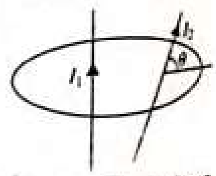
- Current into the plane of the paper is negative.
- Current out of the plane of the paper is positive.
- Circulation taken in the counterclockwise direction is positive.
- Circulation taken in clockwise direction is negative.

ILLUSTRATION 21.24 Figure shows three current carrying conductors and three imaginary loops. Calculate the current enclosed by each of the loops.



Solution. The current enclosed by the first loop is $-I_1$. The current enclosed by the second loop is $I_3 - I_2$ and the current enclosed by the third loop is $I_3 - I_1 - I_2$.

ILLUSTRATION 21.25 Figure shows two current carrying wires piercing the plane of an imaginary loop. One of the wires is normal to the plane of the loop while the other is at an angle. Calculate the net current enclosed.



Solution. The current enclosed is simply $I_1 + I_2$. There is no need to take any component of I_2 perpendicular to the plane of the loop. All that matters is just the magnitude of the net charge per unit time that crosses the plane of the loop.

Just as in the case of Gauss's law, the selection of the Amperian loop is critical for easy application of the law. The general rules while selecting the loop are:

- Estimate the direction of the magnetic field around the current carrying conductor.
- The loop should include the current carrying element whose magnetic field is to be calculated.

- The magnetic field should be constant over the whole loop or should be easy to calculate on the different portions of the loop.
- The dot product $\oint \vec{B}\cdot d\vec{l}$ should be easy to calculate.

NOTE:

- If B is everywhere tangent to the integration path and has the same magnitude B at every point on the path, then its line integral is equal to B multiplied by the circumference of the path.
- If B is everywhere perpendicular to the path, for all or some portion of the path, that portion of the path makes no contribution to the line integral.
- In the integral $\oint \vec{B}\cdot d\vec{l}$, $\vec{B}$ is always the total magnetic field at each point on the path. In general, this field is caused partly by currents enclosed by the path and partly by currents outside. Even when no current is enclosed by the path, the field at points on the path need not be zero. In that case, however, $\oint \vec{B}\cdot d\vec{l}$ is always zero.
- Some judgment is required in choosing an integration path. Two useful guiding principles are that the point or points at which the field is to be determined must lie on the path, and that the path must have enough symmetry so that the integral can be evaluated.

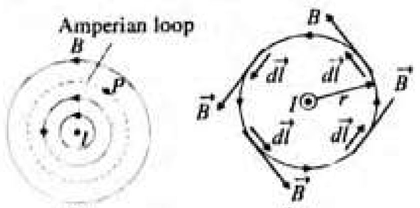
Field of a Long, Straight, Current-Carrying Conductor

Selection of Ampere Loop

We know that the magnetic field lines around a current carrying conductor are concentric circles in a plane perpendicular to the conductor. A natural choice for an Amperian loop is a circle concentric with the conductor as shown in figure. For this loop,

$\vec{B}$ is constant at all points due to symmetry.

$\vec{B}$ is always in the same direction as $d\vec{l}$.



(a) (b)

We exploit the cylindrical symmetry of the situation by taking circle as our integration path with radius r centered on the conductor and in a plane perpendicular to it. At each point, $\vec{B}$ is tangent to this circle. With our choice of integration path, Ampere's law becomes

$$\oint \vec{B}\cdot d\vec{l} = \oint \vec{B}_{\parallel}\, d\vec{l} = B(2\pi r) = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

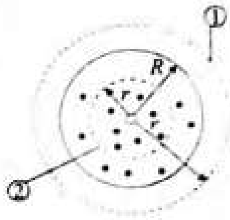
Ampere's law determines the direction of $\vec{B}$ as well as its magnitude. Since we go around the integration path in the counterclockwise direction, the positive direction for current is out of the plane. This is same as the actual current direction in the figure, so I is positive and the integral $\oint \vec{B}\ d\vec{l}$ is also positive.

Since the $d\vec{l}$'s run counterclockwise, the direction of $\vec{B}$ must be counterclockwise.

Magnetic Field B Outside and Inside a Cylindrical Wire

A steady current I flows along an infinitely long straight wire with circular cross section of radius R.

It can be concluded from the symmetry that the field lines of B are circles with their centers at the wire axis.



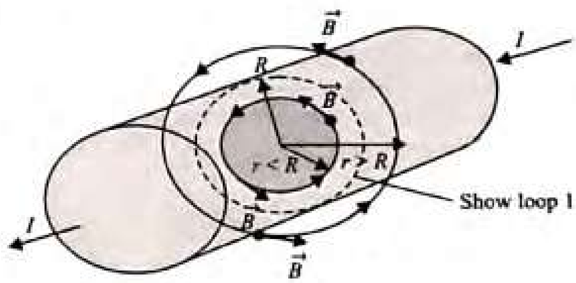
B Outside the Wire ($r \geq R$)

Consider a circular loop of radius r ($\geq R$) as shown by loop 1 in figure.

Here, the integral $\oint \vec{B}\cdot d\vec{l} = B\oint dl = B(2\pi r)$

Applying Ampere's law,

$$B(2\pi r) = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$



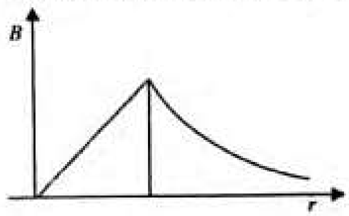
B Inside the Wire ($r \leq R$)

Consider the loop 2 as shown in figure.

Using Ampere's law,

$$B(2\pi r) = \mu_0\left[\frac{I}{\pi R^2}(\pi r^2)\right] \quad \text{or} \quad B = \frac{\mu_0 I r}{2\pi R^2}$$

The variation of B with r is shown in the figure.

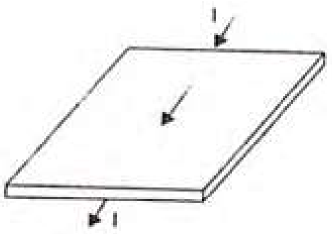


Magnetic Field of an Infinite Sheet of Given Linear Current Density

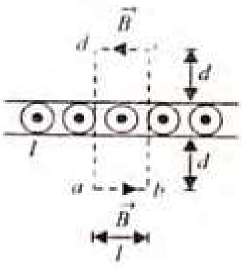
Selection of Amperian Loop for a Sheet of Current

Once again, the choice of Amperian loop will depend on the predicted nature and shape of the magnetic lines of force for

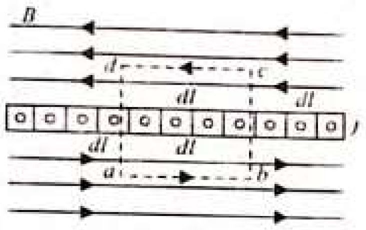
the present configuration. The infinite sheet of current can be treated as a bundle of infinite wires. The fields due to these wires will be concentric circles. The resultant magnetic field due to all these individual fields will be orientated parallel to the plate as shown in Figure (c).



(a)



(b)



(c)

Application of Ampere's Law

Figure (c) shows an infinite sheet of current with linear current density j (A m^{-1}). Due to symmetry, the field lines pattern above and below the sheet is uniform.

Consider a square loop of side l as shown in Figure (c).

According to Ampere's law,

$$\int_a^b \vec{B}\cdot\vec{dl}+\int_b^c \vec{B}\cdot\vec{dl}+\int_c^d \vec{B}\cdot\vec{dl}+\int_d^a \vec{B}\cdot\vec{dl}=\mu_0 I$$

Since $\vec{B}\perp\vec{dl}$ in the path $b\to c$ and $d\to a$, therefore,

$$\int_b^c \vec{B}\cdot\vec{dl}=0;\ \int_d^a \vec{B}\cdot\vec{dl}=0$$

Also, $\vec{B}\parallel\vec{dl}$ in the path $a\to b$ and $c\to d$, thus

$$\int_a^b B\cdot dl+\int_c^d B\cdot dl=2Bl$$

The current enclosed by the loop is $I=jl$

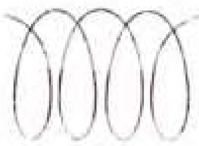
Therefore, according to Ampere's law, $2Bl=\mu_0(jl)$

or, $\quad B=\dfrac{\mu_0 j}{2}$

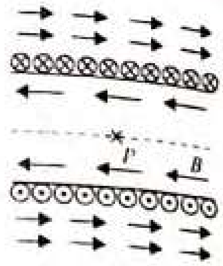
Field of a Long Solenoid

A solenoid consists of a helical winding of wire on a cylinder, usually circular in cross section. There can be hundreds or thousands of closely spaced turns, each of which can be regarded as a circular loop. There may be several layers of windings.

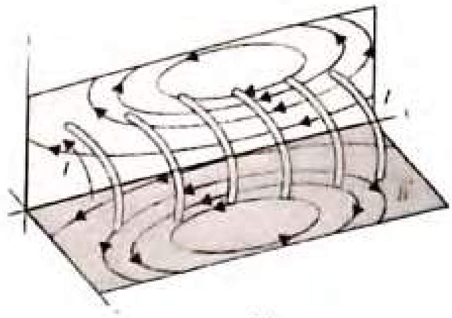
The field lines near the center of the solenoid are approximately parallel, indicating a nearly uniform $\vec{B}$; outside the solenoid, the field lines are spread apart, and the magnetic field is weak. If the solenoid is long in comparison with its cross-sectional diameter and the coils are tightly wound, the internal field near the midpoint of the solenoid's length is very nearly uniform over the cross section and parallel to the axis, and the external field near the midpoint is very small.



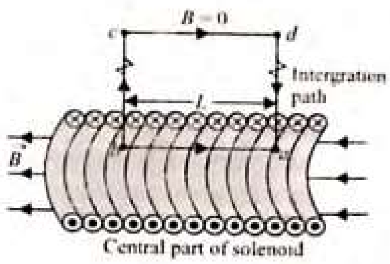
(a)



(b)



(c)



(d)

Application of Ampere's Law

Along sides bc and da, $B_{\parallel}=0$ because $\vec{B}$ is perpendicular to these sides; along side cd, $B_{\parallel}=0$ because $\vec{B}=0$.

The integral $\oint \vec{B}\cdot\vec{dl}$ around the entire closed path, therefore, reduces to BL.

The number of turns in length L is nL. Each of these turns passes once through the rectangle $abcd$ and carries a current I, where I is the current in the windings. The total current enclosed by the rectangle is then $I_{\text{encl}}=nLI$. Ampere's law then gives the magnitude

$$BL=\mu_0 nLI \tag{i}$$

$$B=\mu_0 nI \quad \text{(infinite solenoid)}$$

For points along the axis, the field is strongest at the center of the solenoid and drops off near the ends. For a solenoid that

is very long in comparison to its diameter, the field at each end is exactly half as strong as the field at the center.

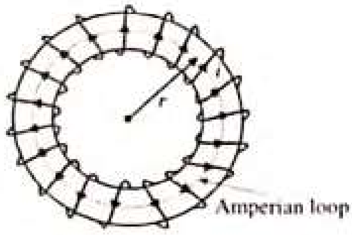
Toroid

If a solenoid is bent in a circular shape and the ends are joined, we get a toroid. Let N be the total number of turns in a toroid of radius r. Then number of turns per unit length: $n = N/2\pi r$

Apply Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 Ni$

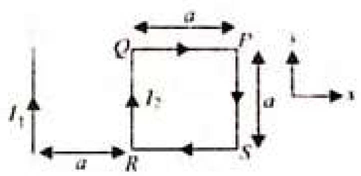
$$B = \frac{\mu_0 N i}{2\pi r} = \mu_0 ni$$

Magnetic field is almost constant inside the toroid and is always tangential to the circular closed path at any point.

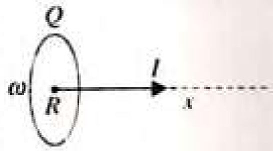


Concept Application Exercise 21.5

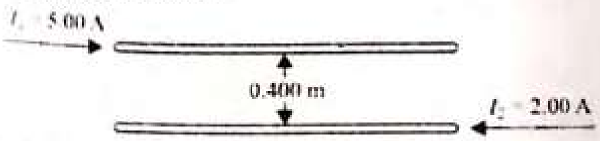
1. Find the magnetic force on loop $PQRS$ due to the wire.



2. A charge Q is uniformly distributed over a ring which is rotating with constant angular velocity ω about an axis passing through its center and perpendicular to the plane. A wire which carries a current I is lying perpendicular to the plane of the ring along its axis having one end at its center. Find resultant magnetic force on the wire by the ring.



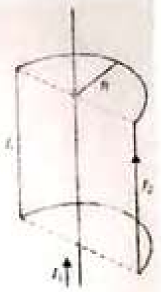
3. Two long, parallel wires are separated by a distance of 0.400 m (as shown in figure. The currents I_1 and I_2 have the directions shown.



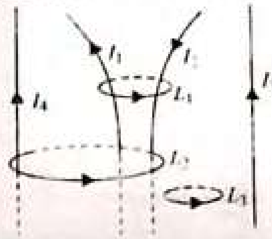
(a) Calculate the magnitude of the force exerted by each wire on a 1.20 m length of the other. Is the force attractive or repulsive?

(b) Each current is doubled, so that I_1 becomes 10.0 A and I_2 becomes 4.00 A. Now, what is the magnitude of the force that each wire exerts on a 1.20 m length of the other?

4. An infinitely long straight wire carrying a current I_1 is partially surrounded by a loop as shown in figure. The loop has a length L, radius R, and carries a current I_2. The axis of the loop coincides with the wire. Calculate the force exerted on the loop.

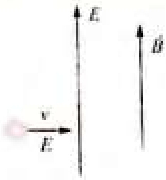


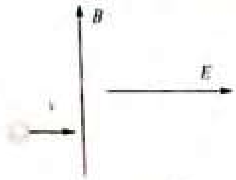
5. Find the values of $\oint \vec{B} \cdot d\vec{l}$ for the loops L_1, L_2, and L_3 in figure (The sense of $d\vec{l}$ is mentioned in the figure.)

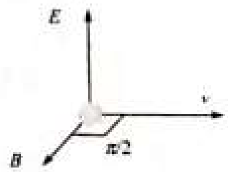


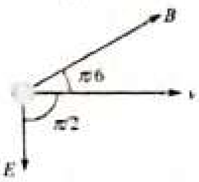
Solved Examples

1. A uniform magnetic field B and a uniform electric field E act in a common region. An electron is entering this region of space. The correct arrangement for it to escape undeviated is

(a) 

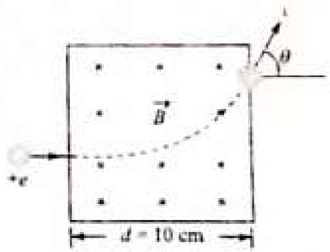
(b) 

(c) 

(d) 

Sol. (c) For undeviated motion $|\vec{F_e}| = |\vec{F_m}|$, which happened when $\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular to each other.

2. A proton accelerated by a potential difference 500 kV moves though a transverse magnetic field of 0.51 T as shown in figure. The angle θ through which the proton deviates from the initial direction of its motion is

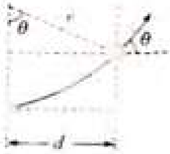


(a) 15°
(b) 30°
(c) 45°
(d) 60°

Sol. (b) According to following figure

$$\sin\theta = \frac{d}{r}$$

also $r = \frac{\sqrt{2mk}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

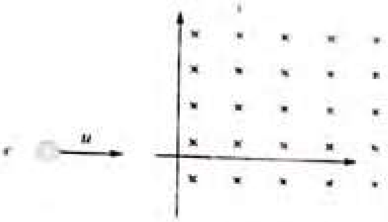


$\therefore \sin\theta = Bd\sqrt{\frac{q}{2mV}}$

$= 0.51 \times 0.1\sqrt{\frac{1.6\times10^{-19}}{2\times1.67\times10^{-27}\times500\times10^{3}}} = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ$

3. An electron moving with a speed u along the positive x-axis at $y = 0$ enters a region of uniform magnetic field $\vec{B} = -B_0\hat{k}$ which exists to the right of y-axis. The electron exits from the region after some time with the speed v at co-ordinate y, then



(a) $v > u, y < 0$ (b) $v = u, y > 0$
(c) $v > u, y > 0$ (d) $v = u, y < 0$

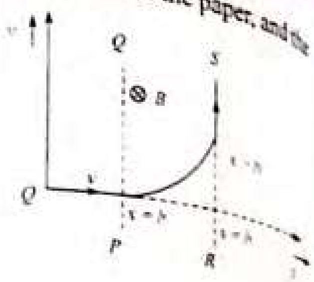
Sol. (d) The energy of a charged particle moving in magnetic field remains constant because the magnetic field does not do any work. Therefore kinetic energy is constant i.e. $u = v$.

4. The force on electron will act along negative y-axis initially. The electron will undergo circular motion in clockwise direction and emerge out the field. So $y < 0$. An ionized gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the $+x$ direction and a magnetic field along the $+z$ direction, then
   (a) Positive ions deflect towards $+y$ direction and negative ions towards $-y$ direction
   (b) All ions deflect towards $+y$ direction
   (c) All ions deflect towards $-y$ direction
   (d) Positive ions deflect towards $-y$ direction and negative ions towards $+y$ direction

Sol. (c) As the electric field is switched on, positive ion will start to move along positive x-direction and negative ion along negative x-direction. Current associated with motion of both types of ions is along positive x-direction. According to Fleming's left hand rule force on both types of ions will be along negative y-direction.

5. A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z direction, extending from $x = a$ to $x = b$. The minimum value of v required so that the particle can just enter the region $x > b$ is
   (a) $qb\,B/m$ (b) $q(b-a)B/m$
   (c) $qa\,B/m$ (d) $q(b+a)B/2m$

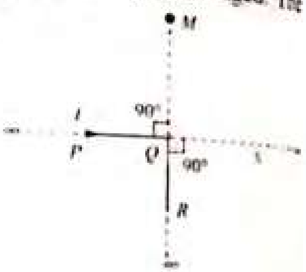
Sol. (b) In the figure, the z-axis points out of the paper, and the magnetic field is directed into the paper, existing in the region between PQ and RS. The particle moves in a circular path of radius r in the magnetic field. It can just enter the region $x > b$ for $r \geq (b-a)$



Now, $r = \frac{mv}{qB} \geq (b-a)$

or $v \geq \frac{q(b-a)B}{m} \Rightarrow v_{min} = \frac{q(b-a)B}{m}$

6. An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now another infinitely long straight conductor QS is connected at Q so that the current is $I/2$ in QR as well as in QS. *The current in PQ* remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by



(a) $\frac{1}{2}$

(b) 1

(c) $\frac{2}{3}$

(d) 2

Sol. (c) Magnetic field at any point lying on the current carrying straight conductor is zero.

Here H_1 = Magnetic field at M due to current in PQ

H_2 = Magnetic field at M due to QR

+ magnetic field at M due to QS

+ magnetic field at M due to PQ

$$= 0 + \frac{H_1}{2} + H_1 = \frac{3}{2}H_1 \Rightarrow \frac{H_1}{H_2} = \frac{2}{3}$$

7. A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the centre is
   (a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$
   (c) $\frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$ (d) $\frac{\mu_0 I^N}{2(b-a)}\ln\frac{b}{a}$

Sol. (c) Number of turns per unit width $= \frac{N}{b-a}$

Consider an elemental ring of radius x and with thickness dx

Number of turns in the ring $= dN = \frac{Ndx}{b-a}$

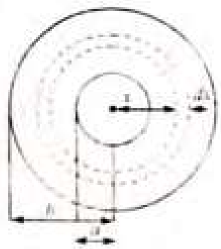
Magnetic field at the centre due to the ring element

$$dB = \frac{\mu_0 (dN) i}{2x} = \frac{\mu_0 i}{2} \frac{N dx}{(b-a)} \frac{1}{x}$$

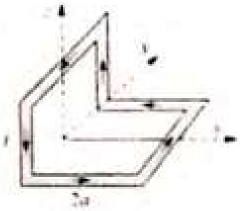
Field at the centre

$$= \int dB = \frac{\mu_0 N i}{2(b-a)} \int_a^b \frac{dx}{x}$$

$$= \frac{\mu_0 N i}{2(b-a)} \ln \frac{b}{a}$$



8. A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight sections of the loop is of length $2a$. The magnetic field due to this loop at the point $P(a, 0, a)$ points in the direction



(a) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$

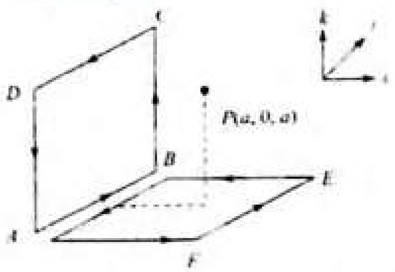
(b) $\frac{1}{\sqrt{3}}(-\hat{j}+\hat{k}+\hat{i})$

(c) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

(d) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

Sol. (d) The magnetic field at $P(a, 0, a)$ due to the loop is equal to the vector sum of the magnetic fields produced by loops $ABCDA$ and $AFEBA$ as shown in the figure.

Magnetic field due to loop $ABCDA$ will be along $\hat{i}$ and due to loop $AFEBA$, along $\hat{k}$. Magnitude of magnetic field due to both the loops will be equal.



Therefore, direction of resultant magnetic field at P will be $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$.

9. A long straight wire along the z-axis carries a current I in the negative z direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the $z=0$ plane is

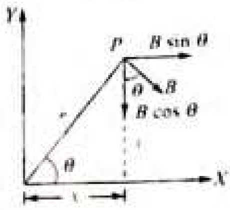
(a) $\frac{\mu_0 I(y\hat{i}-x\hat{j})}{2\pi(x^2+y^2)}$

(b) $\frac{\mu_0 I(x\hat{i}+y\hat{j})}{2\pi(x^2+y^2)}$

(c) $\frac{\mu_0 I(x\hat{j}-y\hat{i})}{2\pi(x^2+y^2)}$

(d) $\frac{\mu_0 I(x\hat{i}-y\hat{j})}{2\pi(x^2+y^2)}$

Sol. (a) Magnetic field at P is $\vec{B}$, perpendicular to OP in the direction shown in figure.



So, $\vec{B} = B\sin\theta\hat{i} - B\cos\theta\hat{j}$

Here $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

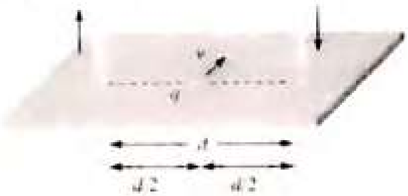
$$\vec{B} = \frac{\mu_0 I}{2\pi}\frac{1}{r^2}(y\hat{i}-x\hat{j}) = \frac{\mu_0 I(y\hat{i}-x\hat{j})}{2\pi(x^2+y^2)} \quad (\text{as } r^2 = x^2+y^2)$$

10. Two very long, straight and parallel wires carry steady currents I and I respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity v is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

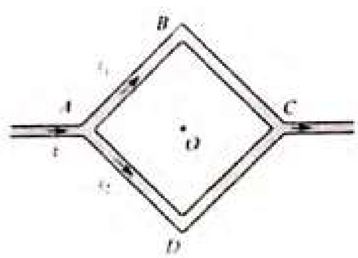
(a) $\frac{\mu_0 Iqv}{2\pi d}$ (b) $\frac{\mu_0 Iqv}{\pi d}$

(c) $\frac{2\mu_0 Iqv}{\pi d}$ (d) 0

Sol. (d) According to gives information following figure can be drawn, which shows that direction of magnetic field is along the direction of motion of charge so net force on it is zero.



11. Figure shows a square loop $ABCD$ with edge length a. The resistance of the wire ABC is r and that of ADC is $2r$. The value of magnetic field at the centre of the loop assuming uniform wire is



(a) $\frac{\sqrt{2}\mu_0 i}{3\pi a} \odot$

(b) $\frac{\sqrt{2}\mu_0 i}{3\pi a} \otimes$

(c) $\frac{\sqrt{2}\mu_0 i}{\pi a} \odot$

(d) $\frac{\sqrt{2}\mu_0 i}{\pi a} \otimes$

Sol. (b) According to question resistance of wire ADC is twice that of wire ABC. Hence current flows through ADC is half that of ABC i.e. $\frac{i_2}{i_1} = \frac{1}{2}$. Also $i_1 + i_2 = i \Rightarrow i_1 = \frac{2i}{3}$ and $i_2 = \frac{i}{3}$

Magnetic field at centre O due to wire AB and BC (parts 1 and 2) $B_1 = B_2 = \frac{\mu_0}{4\pi}\frac{2i_1 \sin 45^\circ}{a/2} \otimes = \frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}\,i_1}{a} \otimes$ and magnetic field at centre O due to wires AD and DC (i.e. parts 3 and 4)

$$B_3 = B_4 = \frac{\mu_0}{4\pi}\frac{2\sqrt{2}\,i_2}{a}$$

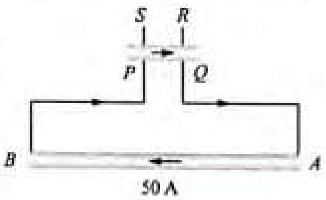
Also $i_1 = 2i_2$. So $(B_1 = B_2) > (B_3 = B_4)$
Hence net magnetic field at centre O

$$B_{net} = (B_1 + B_2) - (B_3 + B_4)$$

$$= 2\times\frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}\times\left(\frac{2}{3}i\right)}{a} - \frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}\left(\frac{i}{3}\right)\times 2}{a}$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{4\sqrt{2}\,i}{3a}(2-1)\otimes = \frac{\sqrt{2}\,\mu_0 i}{3\pi a}\otimes$$

12. A long wire AB is placed on a table. Another wire PQ of mass 1.0 g and length 50 cm is set to slide on two rails PS and QR. A current of 50 A is passed through the wires. At what distance above AB, will the wire PQ be in equilibrium



(a) 25 mm (b) 50 mm
(c) 75 mm (d) 100 mm

Sol. (a) Suppose in equilibrium wire PQ lies at a distance r above the wire AB

Hence in equilibrium $mg = Bil \Rightarrow mg = \frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right)\times il$

$$\Rightarrow \quad 10^{-3}\times 10 = 10^{-7}\times\frac{2\times(50)^2}{r} = 0.5 \Rightarrow r = 25 \text{ mm}$$

13. A steady current i flows in a small square loop of wire of side L in a horizontal plane. The loop is now folded about its middle such that half of it lies in a vertical plane. Let $\overline{\mu_1}$ and $\overline{\mu_2}$ respectively denote the magnetic moments due to the current loop before and after folding. Then

(a) $\overline{\mu_2} = 0$

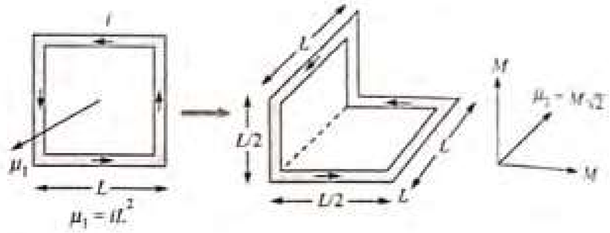
(b) $\overline{\mu_1}$ and $\overline{\mu_2}$ are in the same direction

(c) $\frac{|\overline{\mu_1}|}{|\overline{\mu_2}|} = \sqrt{2}$

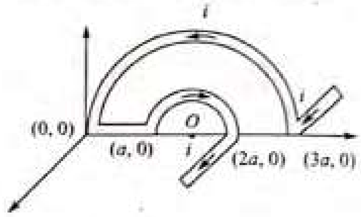
(d) $\frac{|\overline{\mu_1}|}{|\overline{\mu_2}|} = \left(\frac{1}{\sqrt{2}}\right)$

Sol. (c) Initial magnetic moment $= \mu_1 = iL^2$
After folding the loop, M = magnetic moment due to each part $= i\left(\frac{L}{2}\right)\times L = \frac{iL^2}{2} = \frac{\mu_1}{2}$

$$\Rightarrow \quad \mu_2 = M\sqrt{2} = \frac{\mu_1}{2}\times\sqrt{2} = \frac{\mu_1}{\sqrt{2}} \quad\text{or}\quad \frac{\mu_1}{\mu_2} = \sqrt{2}$$



14. In the given figure net magnetic field at O will be

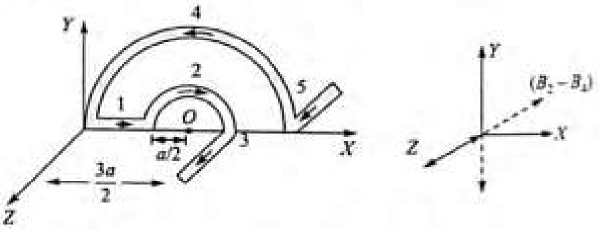


(a) $\frac{2\mu_0 i}{3\pi a}\sqrt{4-\pi^2}$ (b) $\frac{\mu_0 i}{3\pi a}\sqrt{4+\pi^2}$

(c) $\frac{2\mu_0 i}{3\pi a^2}\sqrt{4+\pi^2}$ (d) $\frac{2\mu_0 i}{3\pi a}\sqrt{(4-\pi^2)}$

Sol. (b) Magnetic field at 0 due to

Part (1) : $B_1 = 0$



Part (2): $B_2 = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{(a/2)}\otimes$ (along –Z-axis)

Part (3): $B_3 = \frac{\mu_0}{4\pi}\cdot\frac{i}{(a/2)}(\downarrow)$ (along – Y-axis)

Part (4): $B_4 = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{(3a/2)}\odot$ (along +Z-axis)

Part (5): $B_5 = \frac{\mu_0}{4\pi}\cdot\frac{i}{(3a/2)}(\downarrow)$ (along – Y-axis)

$$B_2 - B_4 = \frac{\mu_0}{4\pi}\cdot\frac{\pi i}{a}\left(2-\frac{2}{3}\right) = \frac{\mu_0 i}{3a}\otimes \text{(along – Z-axis)}$$

$$B_3 + B_5 = \frac{\mu_0}{4\pi}\cdot\frac{1}{a}\left(2+\frac{2}{3}\right) = \frac{8\mu_0 i}{12\pi a}(\downarrow) \text{ (along – Y-axis)}$$

Hence net magnetic field

$$B_{net} = \sqrt{(B_2-B_4)^2 + (B_3+B_5)^2} = \frac{\mu_0 i}{3\pi a}\sqrt{\pi^2+4}$$

15. In the following figure a wire bent in the form of a regular polygon of n sides is inscribed in a circle of radius a. Net magnetic field at centre will be

(a) $\frac{\mu_0 i}{2\pi a}\tan\frac{\pi}{n}$ (b) $\frac{\mu_0 n i}{2\pi a}\tan\frac{\pi}{n}$

(c) $\frac{2}{\pi}\frac{ni}{a}\mu_0\tan\frac{\pi}{n}$ (d) $\frac{ni}{2a}\mu_0\tan\frac{\pi}{n}$

***Sol.* (b)** Magnetic field at the centre due to one side

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2i\sin\theta}{r}$$

where $r = a\cos\theta$

So $B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2i\sin\theta}{a\cos\theta} = \frac{\mu_0 i}{2\pi a}\tan\theta$

Hence net magnetic field

$$B_{net} = n\times\frac{\mu_0 i}{2\pi a}\tan\frac{\pi}{n}$$

EXERCISES

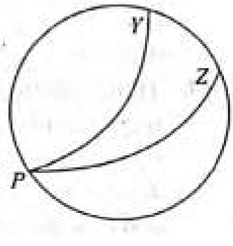
Motion of Charged Particles In Magnetic Field

1. A charged particle moves along a circle under the action of possible constant electric and magnetic fields. Which of the following are possible?

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$

(c) $E \neq 0, B = 0$ (d) $E \neq 0, B \neq 0$

2. Two particles Y and Z emitted by a radioactive source at P made tracks in a chamber as illustrated in the figure. A magnetic field acts downward into the paper. Careful measurements showed that both tracks were circular, the radius of Y track being half that of the Z track. Which one of the following statements is certainly true?



(a) Both particles Y and Z carried a positive charge

(b) The mass of particle Z was one-half that of particle Y

(c) The mass of particle Z was twice that of particle Y

(d) The charge of particle Z was twice that of particle Y

3. A proton and an α-particle enter a uniform magnetic field moving with the same speed. If the proton takes 25 μs to make 5 revolutions, then the periodic time for the α-particle would be

(a) 50 μs (b) 25 μs

(c) 10 μs (d) 5 μs

4. An electron is launched with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. The angle θ between $\vec{v}$ and $\vec{B}$ lies between 0 and $\pi/2$. Its velocity vector $\vec{v}$ returns to its initial value in a time interval of

(a) $\frac{2\pi m}{eB}$

(b) $\frac{2\times 2\pi m}{eB}$

(c) $\frac{\pi m}{eB}$

(d) depends upon angle between $\vec{v}$ and $\vec{B}$

5. A charged particle is whirled in a horizontal circle on a frictionless table by attaching it to a string fixed at one end. If a magnetic field is switched on in the vertical direction, the tension in the string

(a) will increase (b) will decrease

(c) remains same (d) may increase or decrease

6. A charged particle moves with velocity $\vec{v} = a\hat{i} + d\hat{j}$ in a magnetic field $\vec{B} = A\hat{i} + D\hat{j}$. The force acting on the particle has magnitude F. Then,

(a) $F = 0$, if $aD = dA$.

(b) $F = 0$, if $aD = -dA$.

(c) $F = 0$, if $aA = -dD$.

(d) $F \propto (a^2 + b^2)^{1/2} \times (A^2 + D^2)^{1/2}$

7. A particle with a specific charge s is fired with a speed v toward a wall at a distance d, perpendicular to the wall. What minimum magnetic field must exist in this region for the particle not to hit the wall?

(a) v/sd (b) $2v/sd$

(c) $v/2sd$ (d) $v/4sd$

8. A charged particle begins to move from the origin in a region which has a uniform magnetic field in the x-direction and a uniform electric field in the y-direction. Its speed is v when it reaches the point (x, y, z). Then, v will depend

(a) only on x

(b) only on y

(c) on both x and y, but not z

(d) on x, y and z

9. A charged particle enters a uniform magnetic field with velocity vector at an angle of 45° with the magnetic field. The pitch of the helical path followed by the particle is p. The radius of the helix will be

(a) $\frac{p}{\sqrt{2}\pi}$ (b) $\sqrt{2}p$

(c) $\frac{p}{2\pi}$ (d) $\frac{\sqrt{2}p}{\pi}$

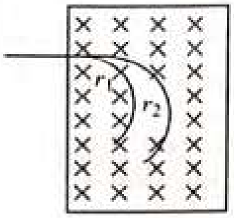
10. A particle of specific charge $q/m = \pi$ C kg^{-1} is projected from the origin toward positive x-axis with a velocity of

$10\ m\ s^{-1}$ in a uniform magnetic field $\vec{B} = -2\hat{k}$ T. The velocity $\vec{v}$ of particle after time $t = 1/12$ s will be (in m s^{-1})

(a) $5[\hat{i} + \sqrt{3}\,\hat{j}]$ (b) $5[\sqrt{3}\,\hat{i} + \hat{j}]$

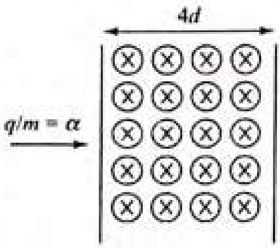
(c) $5[\sqrt{3}\,\hat{i} - \hat{j}]$ (d) $5[\hat{i} + \hat{j}]$

11. A beam of mixture of α particles and protons are accelerated through same potential difference before entering into the magnetic field of strength B. If $r_1 = 5$ cm, then r_2 is



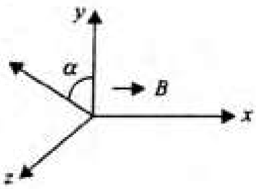
(a) 5 cm
(b) $5\sqrt{2}$ cm
(c) $10\sqrt{2}$ cm
(d) 20 cm

12. If a charged particle of charge to mass ratio $(q/m) = \alpha$ enters in a magnetic field of strength B at a speed $v = (2\alpha d)(B)$, then



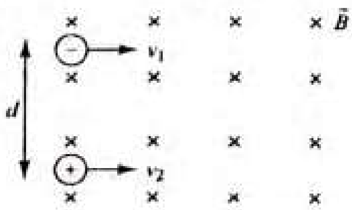
(a) angle subtended by the path of charged particle in magnetic field at the center of circular path is 2π
(b) the charge will move on a circular path and then will come out from magnetic field at some distance from the point of insertion
(c) the time for which particle will be in the magnetic field is $\dfrac{2\pi}{\alpha B}$
(d) angle subtended by the path of charged particle in magnetic field at the center of circular path is $\pi/2$

13. In a region of space, a uniform magnetic field B exists in the x-direction. An electron is fired from the origin with its initial velocity u making an angle α with the y direction in the y-z plane. In the subsequent motion of the electron,



(a) y-coordinate of the electron will never be negative
(b) z-coordinate of the electron will never be negative
(c) x-coordinate of the electron will never be negative
(d) trajectory of the electron would be helical

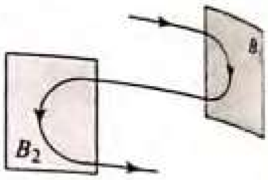
14. Two identical particles having the same mass m and charges $+q$ and $-q$ separated by a distance d enter a uniform magnetic field B directed perpendicular to paper inwards with speeds v_1 and v_2 as shown in figure. The particles will not collide if



(a) $d > \dfrac{m}{Bq}(v_1 + v_2)$ (b) $d < \dfrac{m}{Bq}(v_1 + v_2)$

(c) $d > \dfrac{2m}{Bq}(v_1 + v_2)$ (d) $v_1 = v_2$

15. Following figure shows the path of an electron that passes through two regions containing uniform magnetic fields of magnitudes B_1 and B_2. Its path in each region is a half circle, choose the correct option



(a) B_1 is into the page and it is stronger than B_2
(b) B_1 is into the page and it is weaker than B_2
(c) B_1 is out of the page and it is weaker than B_2
(d) B_1 is out of the page and it is stronger than B_2

16. An electron moves through a uniform magnetic field given by $\vec{B} = B_x\hat{i} + (3.0B_x)\hat{j}$. At a particular instant, the electron has velocity $\vec{v} = (2.0\hat{i} + 4.0\hat{j})$ m/s and the magnetic force acting on it is $(6.4 \times 10^{-19}\,\text{N})\hat{k}$. The value of B_x (in T) will be

(a) −3.0 (b) 3.0
(c) 2.0 (d) −2.0

17. Three identical charge particles A, B and C are projected perpendicular to the uniform magnetic field with velocities v_1, v_2 and v_3 ($v_1 < v_2 < v_3$), respectively such that T_1, T_2 and T_3 are their respective time period of revolution and r_1, r_2 and r_3 are respective radii of circular path described. Then:

(a) $\dfrac{r_1}{T_1} > \dfrac{r_2}{T_2} > \dfrac{r_3}{T_3}$ (b) $T_1 < T_2 < T_3$

(c) $\dfrac{r_1}{T_1} < \dfrac{r_2}{T_2} < \dfrac{r_3}{T_3}$ (d) $r_1 = r_2 = r_3$

18. An electric field acts along positive x-axis. A charged particle of charge q and mass m is released from origin and moves with velocity $\vec{v} = v_0\,\hat{j}$ under the action of electric field and magnetic field, $\vec{B} = B_0\hat{i}$. The velocity of particle becomes $2v_0$ after time $\dfrac{\sqrt{3}\,mv_0}{\sqrt{2}\,qE_0}$. Find the electric field.

(a) $\dfrac{\sqrt{2}}{\sqrt{3}}E_0\hat{i}$ (b) $\dfrac{\sqrt{3}}{\sqrt{2}}E_0\hat{i}$

(c) $\sqrt{3}\,E_0\hat{i}$ (d) $\sqrt{2}\,E_0\hat{i}$

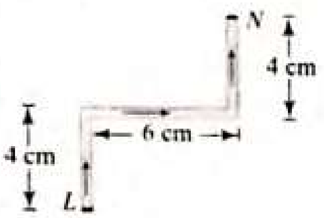
19. A magnetic field $\vec{B} = -B_0\hat{i}$ exists within a sphere of radius $R = v_0T\sqrt{3}$ where T is the time period of one revolution of a charged particle starting its motion from origin and moving with a velocity $\vec{v}_0 = \dfrac{v_0}{2}\sqrt{3}\,\hat{i} - \dfrac{v_0}{2}\hat{j}$. Find the number of turns that the particle will take to come out of the magnetic field.

(a) 5 (b) 1
(c) 4 (d) 2

20. A particle of specific charge $q/m = \pi$ C/kg is projected from the origin towards positive x-axis with a velocity of 10 m/s in a uniform magnetic field $\vec{B} = -2\hat{k}$ T. The velocity $\vec{v}$ of particle after time $t = \frac{1}{12}$ s will be (in m/s)
(a) $5[\hat{i} + \sqrt{3}\,\hat{j}]$ (b) $5(\sqrt{3}\,\hat{i} + \hat{j})$
(c) $5[\sqrt{3}\,\hat{i} - \hat{j}]$ (d) $5[\hat{i} + \hat{j}]$

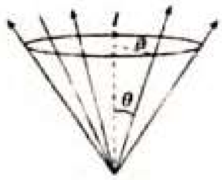
Force on a Conductor and Torque on a Current Carrying Loop In Magnetic Field

21. A current-carrying wire LN is bent in the form as shown below. If wire carries a current of 10 A and it is placed in a magnetic field of 5 T which acts perpendicular to the paper outwards then it will experience a force



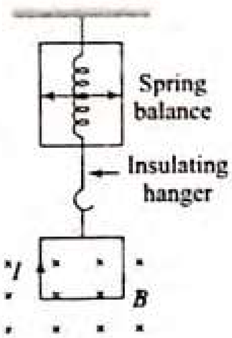
(a) Zero (b) 5 N
(c) 30 N (d) 20 N

22. A circular current loop of radius a is placed in a radial field B as shown. The net force acting on the loop is



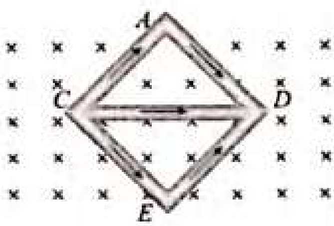
(a) zero (b) $2\pi BaI\cos\theta$
(c) $2\pi aIB\sin\theta$ (d) None

23. A square loop of side a hangs from an insulating hanger of spring balance. The magnetic field of strength B occurs only at the lower edge. It carries a current I. Find the change in the reading of the spring balance if the direction of current is reversed.



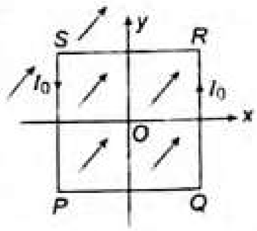
(a) IaB
(b) $2IaB$
(c) $\frac{IaB}{2}$
(d) $\frac{3}{2}IaB$

24. Same current $i = 2$ A is flowing in a wire frame as shown in the figure. The frame is a combination of two equilateral triangles ACD and CDE of side 1 m. It is placed in uniform magnetic field $B = 4$ T acting perpendicular to the plane of frame. The magnitude of magnetic force acting on the frame is



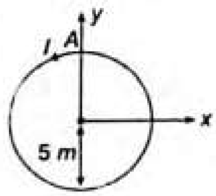
(a) 24 N (b) Zero
(c) 16 N (d) 8 N

25. A uniform, constant magnetic field is directed at an angle of 45° to the x-axis in the xy-plane, $PQRS$ is a rigid square wire frame carrying a steady current I_0, with its centre at the origin O. At time $t = 0$, the frame is at rest in the position shown in the figure, with its sides parallel to the x and y-axes. Each side of the frame is of mass M and length L.



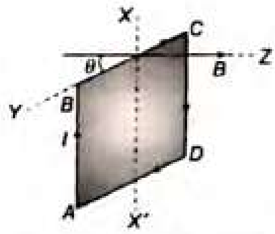
The torque $\vec{\tau}$ about O acting on the frame due to the magnetic field will be
(a) $\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}(-\hat{i} + \hat{j})$ (b) $\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}(\hat{i} - \hat{j})$
(c) $\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}(\hat{i} + \hat{j})$ (d) $\vec{\tau} = \frac{BI_0L^2}{\sqrt{2}}(-\hat{i} - \hat{j})$

26. A ring of radius 5 m is lying in the x-y plane and is carrying current of 1 A in anti-clockwise sense. If a uniform magnetic field $\vec{B} = 3\hat{i} + 4\hat{j}$ is switched on, then the co-ordinates of point about which the loop will lift up is



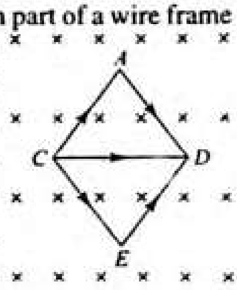
(a) (3, 4) (b) (4, 3)
(c) (3, 0) (d) (0, 3)

27. A square loop $ABCD$, carrying current I, is placed in a uniform magnetic field B, as shown. The loop can rotate about the axis XX'. The plane of the loop makes an angle θ ($\theta < 90°$) with the direction of B. Through what angle will the loop rotate by itself before the torque on it becomes zero?



(a) θ (b) $90° - \theta$
(c) $90° + \theta$ (d) $180° - \theta$

28. Let current $i = 2$A be flowing in each part of a wire frame as shown in figure. The frame is a combination of two equilateral triangles ACD and CDE of side 1 m. It is placed in uniform magnetic field $B = 4$ T acting perpendicular to the plane of frame. The magnitude of magnetic force acting on the frame is



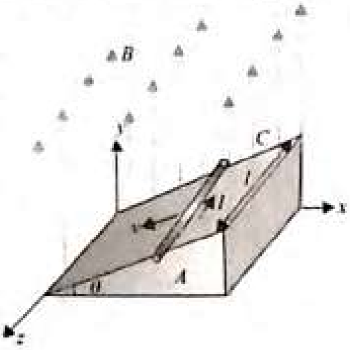
(a) 24 N (b) zero
(c) 16 N (d) 8 N

29. A loop of flexible conducting wire of length l lies in magnetic field B which is normal to the plane of loop. A current I is passed through the loop. The tension developed in the wire to open up is

(a) $\frac{\pi}{2} BIl$ (b) $\frac{BIl}{2}$

(c) $\frac{BIl}{2\pi}$ (d) BIl

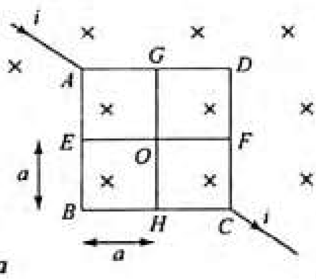
30. A conducting rod of length l and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v. A current i is flowing in the conductor in a direction perpendicular to paper inwards. A vertically upward magnetic field $\vec{B}$ exists in space. Then, magnitude of magnetic field $\vec{B}$ is



(a) $\frac{mg}{il}\sin\theta$ (b) $\frac{mg}{il}\tan\theta$

(c) $\frac{mg\cos\theta}{il}$ (d) $\frac{mg}{il\sin\theta}$

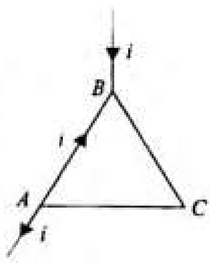
31. In figure, there is a uniform conducting structure in which each small square has side a. The structure is kept in a uniform magnetic field B. Then the magnetic force on the structure will be



(a) $2\sqrt{2}\, iBa$ (b) $\sqrt{2}\, iBa$

(c) $2iBa$ (d) iBa

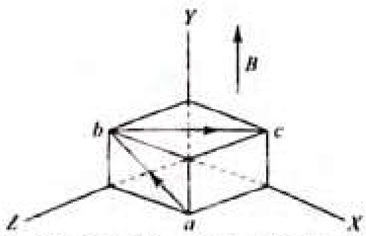
32. Figure shows an equilateral triangle ABC of side l carrying currents as shown, and placed in a uniform magnetic field B perpendicular to the plane of triangle. The magnitude of magnetic force on the triangle is



(a) ilB (b) $2ilB$

(c) $3\ ilB$ (d) zero

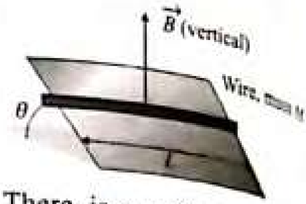
33. Two straight segments of wire ab and bc each carrying current I, are placed as shown in figure. The cube edge is 50 cm and magnetic field is uniform along Y-axis having magnitude 0.4 T. If $I = 3$ A, the force experienced by wire abc in the presence of magnetic field is



(a) $0.6\hat{i}$ (b) $1.2(\hat{i}+\hat{k})$

(c) $0.6\left(\sqrt{2}\hat{i}+\hat{j}-\sqrt{2}\hat{k}\right)$ (d) $0.6\left(\sqrt{2}\hat{i}-\hat{k}\right)$

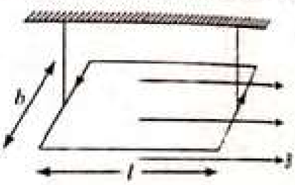
34. A straight piece of conducting wire with mass M and length L is placed on a frictionless incline tilted at an angle θ from the horizontal (as shown in figure). There is a uniform, vertical magnetic field at all points (produced by an arrangement of magnets not shown in figure). To keep the wire from sliding down the incline, a voltage source is attached to the ends of the wire. When just the right amount of current flows through the wire, the wire remains at rest. Determine the magnitude and direction of the current in the wire that will cause the wire to remain at rest.



(a) $\frac{Mg\tan\theta}{2LB}$ to the left (b) $\frac{Mg\tan\theta}{LB}$ to the right

(c) $\frac{Mg\tan\theta}{LB}$ to the left (d) $\frac{3Mg\tan\theta}{2LB}$ to the right

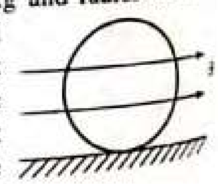
35. A uniform conducting rectangular loop of sides l, b and mass m carrying current i is hanging horizontally with the help of two vertical strings. There exists a uniform horizontal magnetic field B which is parallel to the longer side of loop. The value of tension which is least is



(a) $\frac{mg - Bb}{2}$ (b) $\frac{mg + Bb}{2}$

(c) $\frac{mg - 2iBb}{2}$ (d) $\frac{mg + 2Bb}{2}$

36. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current of $i = 4$ A. A horizontal magnetic field $B = 10$ T is switched on at time $t = 0$ as shown in figure 20.140. The initial angular acceleration of the ring will be



(a) 40π rad s^{-2} (b) 20π rad s^{-2}

(c) 5π rad s^{-2} (d) 15π rad s^{-2}

37. A moving coil galvanometer is based on the

(a) heating effect of current

(b) magnetic effect of current

(c) chemical effect of current

(d) Peltier effect of current

38. In a moving coil galvanometer, we use a radial magnetic field so that the galvanometer scale is

(a) logarithmic (b) exponential

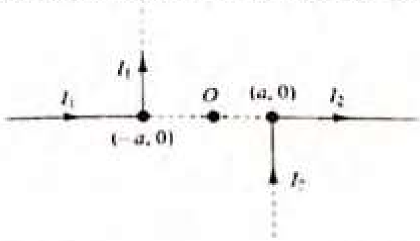
(c) linear (d) none of the above

39. The current that must flow through the coil of a galvanometer so as to produce a deflection of one division on its scale is called
(a) charge sensitivity of the galvanometer
(b) current sensitivity of the galvanometer
(c) micro-volt sensitivity
(d) none of the above

40. In a moving coil galvanometer, the deflection of the coil θ is related to the electric current i by the relation
(a) $i \propto \tan\theta$ (b) $i \propto \theta$
(c) $i \propto \theta^2$ (d) $i \propto \sqrt{\theta}$

Magnetic Field Due to Current Carrying Wire and Wire Loop

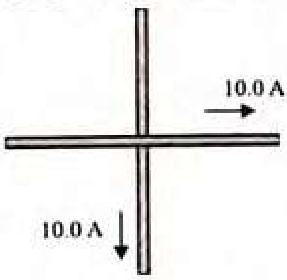
41. Currents I_1 and I_2 flow in the wires shown in figure. The field is zero at distance x to the right of O. Then



(a) $x = \left(\dfrac{I_1}{I_2}\right)a$ (b) $x = \left(\dfrac{I_2}{I_1}\right)a$

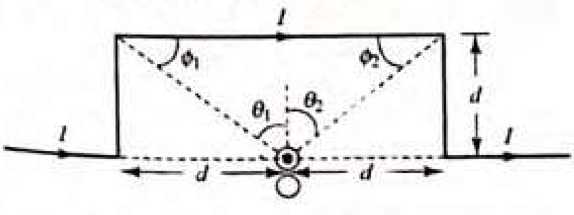
(c) $x = \left(\dfrac{I_1 - I_2}{I_1 + I_2}\right)a$ (d) $x = \left(\dfrac{I_1 + I_2}{I_1 - I_2}\right)a$

42. Two very long, straight wires carrying, currents as shown in figure. Find all locations where the net magnetic field is zero.



(a) $y = \sqrt{2}\,x$
(b) $y = x$
(c) $y = -x$
(d) $y = -(x/2)$

43. The magnetic field at O due to current in the infinite wire forming a loop as shown in Figure is



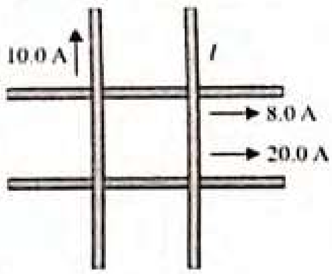
(a) $\dfrac{\mu_0 I}{2\pi d}(\cos\phi_1 + \cos\phi_2)$ (b) $\dfrac{\mu_0}{4\pi}\dfrac{2I}{d}(\tan\theta_1 + \tan\theta_2)$

(c) $\dfrac{\mu_0}{4\pi}\dfrac{I}{d}(\sin\phi_1 + \sin\phi_2)$ (d) $\dfrac{\mu_0}{4\pi}\dfrac{I}{d}(\cos\theta_1 + \sin\theta_2)$

44. Four parallel conductors, carrying equal currents, pass vertically through the four corners of a square $WXYZ$. In two conductors, the current is flowing into the page, and in the other two out of the page. In what directions must the currents flow to produce a resultant magnetic field in the direction shown at O, the center of the square ?

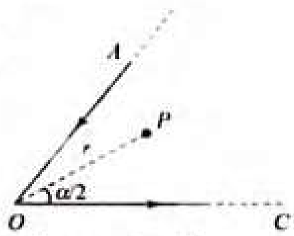
	Into the page	Out of the page
(a)	W and Y	X and Z
(b)	X and Z	W and Y
(c)	W and Z	X and Y
(d)	W and X	Y and Z

45. Four very long, current carrying wires in the same plane intersect to form a square 40.0 cm on each side, as shown in figure. Find the magnitude and direction of the current I so that the magnetic field at the center of the square is zero.



(a) 4.0 A toward the bottom of the page.
(b) 2.0 A toward the bottom of the page.
(c) 2.5 A toward the bottom of the page.
(d) 3.6 A toward the top of the page.

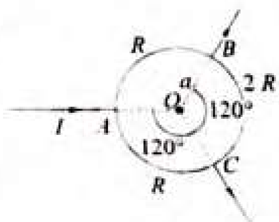
46. Two wires AO and OC carry equal currents i as shown in figure. One end of both the wires extends to infinity. Angle AOC is α. The magnitude of magnetic field at point P on the bisector of these two wires at a distance r from point O is



(a) $\dfrac{\mu_0}{2\pi}\dfrac{i}{r}\cot\left(\dfrac{\alpha}{2}\right)$ (b) $\dfrac{\mu_0}{4\pi}\dfrac{i}{r}\cot\left(\dfrac{\alpha}{2}\right)$

(c) $\dfrac{\mu_0}{2\pi}\dfrac{i}{r}\dfrac{\left(1+\cos\dfrac{\alpha}{2}\right)}{\sin\left(\dfrac{\alpha}{2}\right)}$ (d) $\dfrac{\mu_0}{4\pi}\dfrac{i}{r}\left(\dfrac{\alpha}{2}\right)$

47. The resistances of three parts of a circular loop are as shown in figure. The magnetic field at the center O is (current enters at A and leaves at B and C as shown)



(a) $\dfrac{\mu_0 I}{6a}$ (b) $\dfrac{\mu_0 I}{3a}$

(c) $\frac{2}{3}\frac{\mu_0 I}{a}$ (d) zero

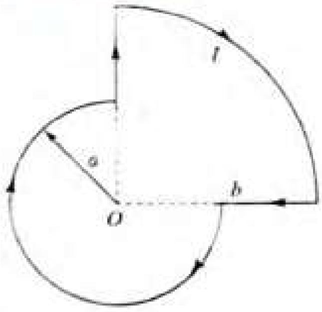
48. The magnetic induction at center O (figure) is

(a) $\frac{\mu_0 I}{2a}-\frac{\mu_0 I}{2b}\otimes$

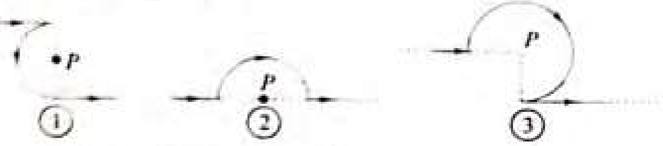
(b) $\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}\otimes$

(c) $\frac{3\mu_0 I}{8a}-\frac{\mu_0 I}{8b}\otimes$

(d) $\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}\odot$



49. Figure shows three cases: in all cases the circular part has radius r and straight ones are infinitely long. For the same current the ratio of field B at center P in the three cases $B_1 : B_2 : B_3$ is



(a) $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}-\frac{1}{2}\right)$

(b) $\left(-\frac{\pi}{2}+1\right):\left(\frac{\pi}{2}+1\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

(c) $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}\right)$

(d) $\left(-\frac{\pi}{2}-1\right):\left(\frac{\pi}{2}-\frac{1}{4}\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

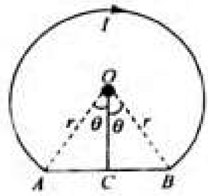
50. A wire is bent in the form of a circular arc with a straight portion AB. Magnetic induction at O when current I flowing in the wire, is

(a) $\frac{\mu_0}{2r}(\pi-\theta+\tan\theta)$

(b) $\frac{\mu_0 I}{2\pi r}(\pi+\theta-\tan\theta)$

(c) $\frac{\mu_0 I}{2\pi r}(\pi-\theta+\tan\theta)$

(d) $\frac{\mu_0 I}{2\pi r}(-\tan\theta+\pi-\theta)$.



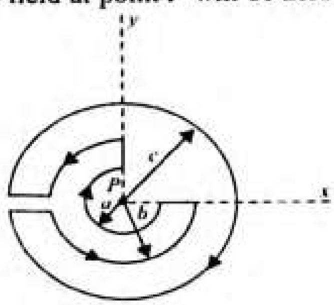
51. For $c = 2a$ if, the magnetic field at point P will be zero when

(a) $a=b$

(b) $a=\frac{3}{5}b$

(c) $a=\frac{5}{3}b$

(d) $a=\frac{1}{3}b$



52. A square conducting loop of side length L carries a current I. The magnetic field at the center of the loop is

(a) independent of L

(b) proportional to L

(c) inversely proportional to L

(d) linearly proportional to L

53. A circular current carrying coil has a radius R. The distance from the center of the coil on the axis where the magnetic induction will be $(1/8)^{\text{th}}$ of its value at the center of the coil, is

(a) $R/\sqrt{3}$ (b) $R\sqrt{3}$

(c) $2R\sqrt{3}$ (d) $(2\sqrt{3})R$

54. A circular loop is kept in that vertical plane which contains the north-south direction. It carries a current that is toward north at the topmost point. Let A be a point on the axis of the circle to the east of it and B a point on this axis to the west of it. The magnetic field due to the loop

(a) is toward east at A and toward west at B

(b) is toward west at A and toward east at B

(c) is toward east at both A and B

(d) is toward west at both A and B

55. Figure shows a small loop carrying a current I. The curved portion is an arc of a circle of radius R and the straight portion is a chord to the same circle subtending an angle θ. The magnetic induction at center O is

(a) zero

(b) always inward irrespective of the value of θ

(c) inward as long as θ is less than π

(d) always outward irrespective of the value of θ

56. Two concentric coils, each of radius equal to 2π cm, are placed at right angles to each other. Currents of 3 A and 4 A, respectively, are flowing through the two coils. The magnetic induction, in Wb m^{-2}, at the center of the coils will be [$\mu_0 = 4\pi\times10^{-7}$ Wb(Am^{-1})]

(a) 5×10^{-5} (b) 7×10^{-5}

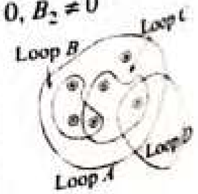
(c) 12×10^{-5} (d) 10^{-5}

57. A coaxial cable consists of a thin inner conductor fixed along the axis of a hollow outer conductor. The two conductors carry equal currents in opposite directions. Let B_1 and B_2 be the magnetic fields in the region between the conductors and outside the conductor, respectively. Then

(a) $B_1\neq0, B_2\neq0$ (b) $B_1=B_2=0$

(c) $B_1\neq0, B_2=0$ (d) $B_1=0, B_2\neq0$

58. Consider six wires coming into or out of the page, all with the same current. Rank the line integral of the magnetic field (from most positive to most negative) taken counterclockwise around each loop shown.



(a) $B > C > D > A$ (b) $B > C = D > A$
(c) $B > A > C = D$ (d) $C > B = D > A$

59. A very long straight con ducting wire, lying along the z-axis, carries a current of 2A. The integral $\oint \vec{B} \cdot d\vec{l}$ is computed along the straight line PQ, where P has the coordinates (2 cm, 0, 0) and Q has the coordinates (2cm, 2cm, 0). The integral has the magnitude (in SI units)

(a) $\frac{\pi}{2} \times 10^{-7}$ (b) $8\pi \times 10^{-7}$
(c) $2\pi \times 10^{-7}$ (d) $\pi \times 10^{-7}$

60. A current of $1/(4\pi)$ ampere is flowing in a long straight conductor. The line integral of magnetic induction around a closed path enclosing the current carrying conductor is

(a) 10^{-7} Wb m^{-1} (b) $4\pi \times 10^{-7}$ Wb m^{-1}
(c) $16\pi^2 \times 10^{-7}$ Wb m^{-1} (d) zero

Problems Based on Mixed Concepts

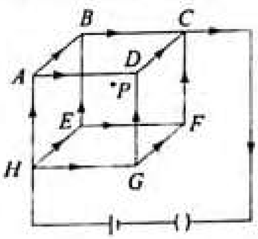
61. A point charge is moving in clockwise direction in a circle with constant speed. Consider the magnetic field produced by the charge at a fixed point P (not at the center of circle) on the axis of the circle. Then,

(a) it is constant in magnitude only
(b) it is constant in direction only
(c) it is constant both in direction and magnitude
(d) it is constant neither in magnitude nor in direction

62. An infinitely long current carrying wire carries current i. A charge of mass m and charge q is projected with speed v parallel to the direction of current at a distance r from it. Then, the radius of curvature at the point of projection is

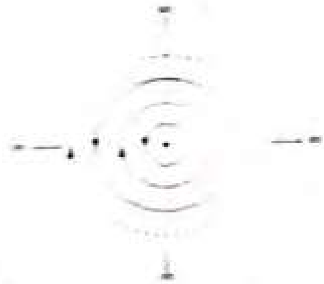
(a) $\frac{2rmv}{q\mu_0 i}$ (b) $\frac{2\pi rmv}{q\mu_0 i}$
(c) r (d) cannot be determined

63. A steady current is set up in a cubic network composed of wires of equal resistance and length d as shown in figure. What is the magnetic field at center P due to the cubic network?



(a) $\frac{\mu_0}{4\pi} \frac{2I}{d}$ (b) $\frac{\mu_0}{4\pi} \frac{3I}{\sqrt{2}d}$
(c) zero (d) $\frac{\mu_0}{4\pi} \frac{\theta \pi I}{d}$

64. In figure, infinite conducting rings each having current i in the direction shown are placed concentrically in the same plane as shown in the figure. The radii of rings are $r, 2r, 2^2r, 2^3r, \ldots, \infty$. The magnetic field at the center of rings will be

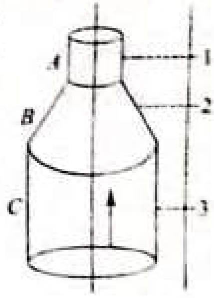


(a) zero
(b) $\frac{\mu_0 i}{r}$
(c) $\frac{\mu_0 i}{2r}$
(d) $\frac{\mu_0 i}{3r}$

65. From a cylinder of radius R, a cylinder of radius $R/2$ is removed, as shown in figure. Current flowing in the remaining cylinder is I. Then, magnetic field strength is

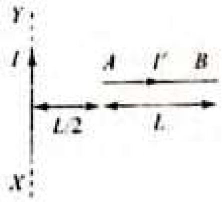
(a) zero at point A
(b) zero at point B
(c) $\frac{\mu_0 I}{2\pi R}$ at point A
(d) $\frac{\mu_0 I}{3\pi R}$ at point B

66. A long, straight, hollow conductor (tube) carrying a current has two sections A and C of unequal cross sections joined by a conical section B. 1, 2, and 3 are points on a line parallel to the axis of the conductor. The magnetic fields at 1, 2 and 3 have magnitudes B_1, B_2 and B_3. Then,



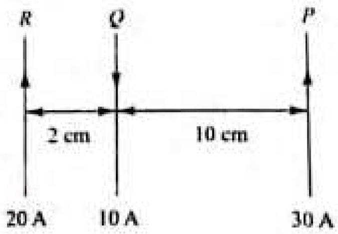
(a) $B_1 = B_2 = B_3$
(b) $B_1 = B_2 \neq B_3$
(c) $B_1 < B_2 < B_3$
(d) B_2 cannot be found unless the dimensions of the section B are known

67. A conductor AB of length L carrying a current I', is placed perpendicular to a long straight conductor XY carrying a current I as shown in figure. The force on AB has magnitude



(a) $\frac{\mu_0 I'I}{2\pi} \log 2$
(b) $\frac{\mu_0 I'I}{2\pi} \log 3$
(c) $\frac{3\mu_0 I'I}{2\pi} \log \frac{3}{2}$
(d) $\frac{2\mu_0 I'I}{3\pi}$

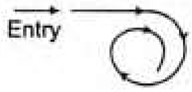
68. Three long, straight and parallel wires are arranged as shown in figure. The force experienced by 10 cm length of wire Q is



(a) 1.4×10^{-4} N toward the right

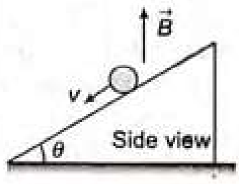
(b) 1.4×10^{-4} N toward the left
(c) 2.6×10^{-4} N toward the right
(d) 2.6×10^{-4} N toward the left

69. A charged particle enters a uniform magnetic field perpendicular to its initial direction travelling in air. The path of the particle is seen to follow the path in figure. Which of statements 1–3 is/are correct?
(1) The magnetic field strength may have been increased while the particle was travelling in air
(2) The particle lost energy by ionising the air
(3) The particle lost charge by ionising the air



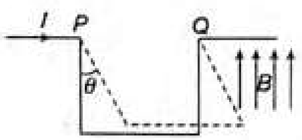
(a) 1, 2, 3 are correct (b) 1, 2 only are correct
(c) 2, 3 only are correct (d) 1 only

70. A conducting rod of length l and mass m is moving down a smooth inclined plane of inclination θ with constant velocity v. A current i is flowing in the conductor in a direction perpendicular to paper inwards. A vertically upward magnetic field $\vec{B}$ exists in space. Then the magnitude of magnetic field $\vec{B}$ is



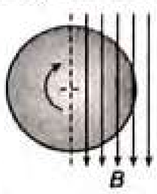
(a) $\frac{mg}{il}\sin\theta$ (b) $\frac{mg}{il}\tan\theta$
(c) $\frac{mg\cos}{il}\theta$ (d) $\frac{mg}{il\sin\theta}$

71. As shown in the figure, a three-sided frame is pivoted at P and Q and hangs vertically. Its sides are of same length and have a linear density of $\sqrt{3}$ kg/m. A current of $10\sqrt{3}$ A is sent through the frame, which is in a uniform magnetic field of 2 T directed upwards as shown. Then angle through which the frame will be deflected in equilibrium is (Take $g = 10$ m/s^2)



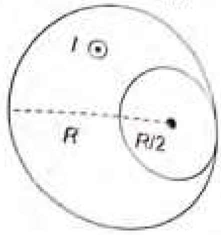
(a) 30° (b) 45°
(c) 60° (d) 90°

72. A thin non-conducting disc of radius R is rotating clockwise (see figure) with an angular velocity ω about its central axis, which is perpendicular to its plane. Both its surfaces carry +ve charges of uniform surface density. Half the disc is in a region of a uniform, unidirectional magnetic field B parallel to the plane of the disc, as shown. Then,



(a) The net torque on the disc is zero
(b) The net torque vector on the disc is directed leftwards
(c) The net torque vector on the disc is directed rightwards
(d) The net torque vector on the disc is parallel to B

73. The cross-section of an infinite cylinder is shown below. A current I uniformly distributed over its cross-section is flowing along its length. Now a cylinder of radius $R/2$ is checked out of this. Then

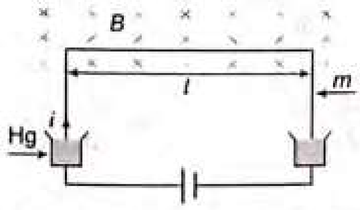


(a) magnetic fields at the centers of both cylinders is same both in magnitude and direction.
(b) magnetic fields at the centers of both cylinders is same in magnitude but opposite in direction.
(c) magnetic fields at the centers of both cylinders is same in direction but unequal in magnitude.
(d) magnetic field at the centre of both cylinders is opposite in direction and unequal in magnitude.

74. A non-conducting disc of radius 2 m is rotated about its axis of symmetry perpendicular to the plane of disc with a uniform angular speed ω. The disc carries a charge whose density is given as $\sigma = A - Br$, where A and B are positive constants and r is distance from the centre of disc. The value of A/B for which magnetic field at the centre of disc will be zero is
(a) 1 (b) 2
(c) $\frac{2}{3}$ (d) $\frac{4}{3}$

75. A charged particle is moving in a circular path in a magnetic field. A resistive force starts acting on the particle whose direction is oppositely directed to its motion and magnitude is directly proportional to its velocity. Now the particle starts moving in a spiral path; then
(a) Angular velocity of the particle decreases continuously.
(b) Angular momentum of the particle remains constant.
(c) Magnetic field is lying only perpendicular to the plane of coil.
(d) Net force acting on the particle remains constant.

76. A U-shaped wire of mass m and length l is immersed with its two ends in mercury (see figure). The wire is in a homogeneous field of magnetic induction B. If a charge, that is, a current pulse $q = \int i\,dt$, is sent through the wire, the wire will jump up.



If the wire reaches height h, calculate the magnitude of the charge or current pulse, assuming that the time of the current pulse is very small in comparison with the time of flight. Make use of the fact that impulse of force equals $\int F\,dt$, which equals mv. Evaluate q for

$B = 0.1$ Wb/m^2, $m = 10$ gm, $l = 20$ cm and $h = 3$ m.

(a) $\sqrt{24}$ C (b) $\sqrt{48}$ C

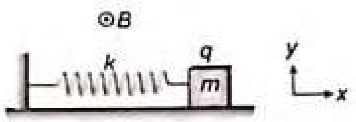
(c) $\sqrt{15}$ C (d) None of these

77. A block of mass m and charge q is released on a long smooth inclined plane. Magnetic field B is constant, uniform, horizontal and parallel to surface as shown. Find the time from start when block loses contact with the surface

(a) $\dfrac{m\cos\theta}{qB}$ (b) $\dfrac{m\,\text{cosec}\,\theta}{qB}$

(c) $\dfrac{m\cot\theta}{qB}$ (d) None

78. A spring of spring constant K is fixed at one end has a small block of mass m and charge q is attached at the other end. The block rests over a smooth horizontal surface. A uniform and constant magnetic field B exists normal to the plane of paper as shown in figure. An electric field $\vec{E} = E_0\,\hat{i}$ (E_0 is a positive constant) is switched on at $t = 0$ sec. The block moves on horizontal surface without ever lifting off the surface. Then the normal reaction acting on the block is



(a) maximum at extreme position and minimum at mean position.

(b) maximum at mean position and minimum at extreme position.

(c) is uniform throughout the motion.

(d) is both maximum and minimum at mean position.

79. A proton moves in the positive z-direction after being accelerated from rest through a potential difference V. The proton then passes through a region with a uniform electric field E in the positive x-direction and a uniform magnetic field B in the positive y-direction, but the proton's trajectory is not affected. If the experiment were repeated using a potential difference of 2 V, the proton would then be

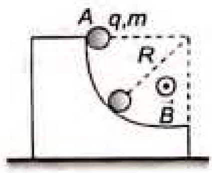
(a) deflected in positive x-direction

(b) deflected in negative x-direction

(c) deflected in positive y-direction

(d) deflected in negative y-direction

80. In the figure, a charged small sphere of mass m and charge q starts sliding from rest on a vertical fixed circular smooth track of radius R from position A as shown in the figure. There exists a uniform magnetic field B. Find the maximum force exerted by track on the sphere during its motion.



(a) $N_{max} = 2mg + qB\sqrt{2gR}$

(b) $N_{max} = 3mg + qB\sqrt{2gR}$

(c) $N_{max} = 3mg + 2qB\sqrt{2gR}$

(d) $N_{max} = 3mg + qB\sqrt{gR}$

ARCHIVES

1. If a current is passed through a spring, the spring will

(a) expand (b) compress

(c) remain same (d) none of these

(AIEEE 2002)

2. If an electron and a proton having same moment enter perpendicular to a magnetic field, then

(a) the curved path of electron and proton will be same (ignoring the sense of revolution).

(b) they will move undeflected.

(c) the curved path of electron is more curved than that of proton.

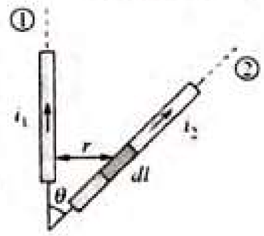
(d) the path of proton is more curved. **(AIEEE 2002)**

3. If current i flows in a circular coil A of radius R, and current $2i$ flows in another coil B of radius $2R$, then the ratio of the magnetic fields, B_A and B_B, produced by them will be

(a) 1 (b) 2

(c) 1/2 (d) 4 **(AIEEE 2002)**

4. Wires 1 and 2 carrying currents i_1 and i_2, respectively, are inclined at an angle θ to each other. What is the force on a small element dl of wire 2 at a distance r from wire 1 (as shown in the figure) due to the magnetic field of wire 1?



(a) $\dfrac{\mu_0}{2\pi r} i_1 i_2\, dl\tan\theta$ (b) $\dfrac{\mu_0}{2\pi r} i_1 i_2\, dl\sin\theta$

(c) $\dfrac{\mu_0}{2\pi r} i_1 i_2\, dl\cos\theta$ (d) $\dfrac{\mu_0}{4\pi r} i_1 i_2\, dl\cos\theta$

(AIEEE 2002)

5. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its
   (a) speed (b) mass
   (c) charge (d) magnetic induction
   (AIEEE 2002)

6. A particle of mass M and charge Q moving with a velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is
   (a) $BQv\,2\pi R$ (b) $\left(\frac{Mv^2}{R}\right)2\pi R$
   (c) zero (d) $BQ\,2\pi R$ **(AIEEE 2003)**

7. A particle of charge -16×10^{-18} C moving with a velocity 10 m/s along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electron field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is
   (a) 10^{-3} Wb/m^2 (b) 10^3 Wb/m^2
   (c) 10^8 Wb/m^2 (d) 10^{16} Wb/m^2 **(AIEEE 2003)**

8. A current i ampere flows along an infinitely long straight thin-walled tube. Then the magnetic induction at any point inside the tube is
   (a) infinite (b) zero
   (c) $\frac{\mu_0}{4\pi}\frac{2i}{r}$ tesla (d) $\frac{2i}{r}$ tesla
   (AIEEE 2004)

9. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be
   (a) nB (b) n^2B
   (c) $2nB$ (d) $2n^2B$ **(AIEEE 2004)**

10. The magnetic field due to a current-carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop?
    (a) 250 μT (b) 100 μT
    (c) 125 μT (d) 75 μT **(AIEEE 2004)**

11. Two long conductors, separated by a distance d, carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is
    (a) $-2F$ (b) $F/3$
    (c) $-2F/3$ (d) $-F/3$ **(AIEEE 2004)**

12. Two concentric coils, each of radius 2π cm, are placed at right angles to each other. The currents flowing in each coil are 3 A and 4 A, respectively. The magnetic induction (in Wb/m^2) at the centre of the coils ($\mu_0=4\pi\times10^{-7}$ Wb/A m^2) is
    (a) 7×10^{-5} (b) 5×10^{-5}
    (c) 10^{-6} (d) 12×10^{-5} **(AIEEE 2005)**

13. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity, then
    (a) it will turn towards the left of the direction of motion
    (b) it will turn towards the right of the direction of motion
    (c) its velocity will increase
    (d) its velocity will decrease. **(AIEEE 2005)**

14. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is
    (a) $\frac{2\pi qB}{m}$ (b) $\frac{2\pi m}{qB}$
    (c) $\frac{2\pi mq}{B}$ (d) $\frac{2\pi q^2B}{m}$ **(AIEEE 2005)**

15. Two thin, long parallel wires, separated by a distance d, carry a current of i ampere in the same direction. They will
    (a) repel each other with a force of $(\mu_0 i^2)/(2\pi d^2)$.
    (b) attract each other with a force of $(\mu_0 i^2)/(2\pi d^2)$.
    (c) repel each other with a force of $(\mu_0 i^2)/(2\pi d)$.
    (d) attract each other with a force of $(\mu_0 i^2)/(2\pi d)$.
    (AIEEE 2005)

16. A long solenoid has 200 turns per centimetre and carries a current i. The magnetic field at its centre is 6.28×10^{-2} Wb/m^2. Another long solenoid has 100 turns per centimetre and it carries a current $i/3$. The value of the magnetic field at its centre is
    (a) 1.05×10^{-1} Wb/m^2 (b) 1.05×10^{-4} Wb/m^2
    (c) 1.05×10^{-2} Wb/m^2 (d) 1.05×10^{-5} Wb/m^2
    (AIEEE 2006)

17. A charged particle moves through a magnetic field perpendicular to its direction. Then
    (a) both momentum and kinetic energy of the particle are not constant.
    (b) both momentum and kinetic energy of the particle are constant.
    (c) kinetic energy changes but the momentum remains constant.
    (d) the momentum changes but kinetic energy remains constant.
    (AIEEE 2007)

18. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

(a) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)^{1/2}$ (b) $\frac{\mu_0}{2\pi d}(I_1+I_2)$

(c) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)$ (d) $\frac{\mu_0}{2\pi d}\left(\frac{I_1+I_2}{d}\right)^{1/2}$

(AIEEE 2007)

19. A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $a/2$ and $2a$ is

(a) 4 (b) 1

(c) 1/2 (d) 1/4 **(AIEEE 2007)**

20. A current I flows along the length of an infinitely long, straight, thin-walled pipe. Then

(a) the magnetic field is different at different points inside the pipe.

(b) the magnetic field at any point inside the pipe is zero.

(c) the magnetic field at all points inside the pipe is the same, but not zero.

(d) the magnetic field is zero only on the axis of the pipe.

(AIEEE 2007)

21. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

(a) $\vec{v}=\frac{\vec{B}\times\vec{E}}{E^2}$ (b) $\vec{v}=\frac{\vec{E}\times\vec{B}}{B^2}$

(c) $\vec{v}=\frac{\vec{B}\times\vec{E}}{B^2}$ (d) $\vec{v}=\frac{\vec{E}\times\vec{B}}{E^2}$

(AIEEE 2007)

22. A horizontal overhead power line is at a height 4 m from the ground and carries a current 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0=4\pi\times10^{-7}$ Tm/A)

(a) 5×10^{-6} T southward

(b) 2.5×10^{-7} T northward

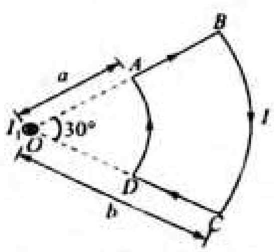
(c) 2.5×10^{-7} southward

(d) 5×10^{-6} northward

(AIEEE 2008)

Statement for Problems 23 and 24:

A current loop $ABCD$ is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius b) and DA (radius a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. The angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper, is kept at the origin.



(AIEEE 2009)

23. The magnitude of the magnetic field (B) due to loop $ABCD$ at the origin (O) is

(a) zero

(b) $\frac{\mu_0 I(b-a)}{24ab}$

(c) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$

(d) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

24. Due to the presence of the current I_1 at the origin,

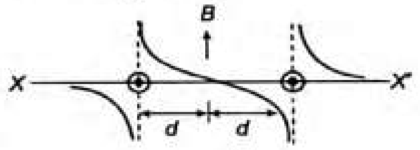
(a) the forces on AB and DC are zero.

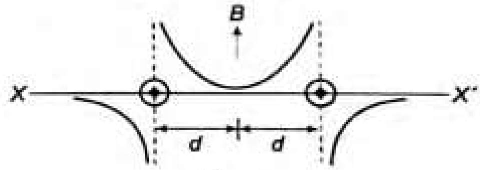
(b) the forces on AD and BC are zero.

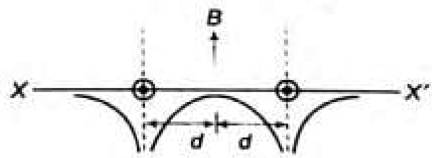
(c) the magnitude of the net force on the loop is given by $\frac{\mu_0 II}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

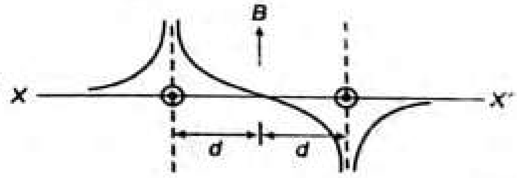
(d) the magnitude of the net force on the loop is given by $\frac{\mu_0 II_1}{24ab}(b-a)$

25. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field along the line XX' is given by

(a) 

(b) 

(c) 

(d) 

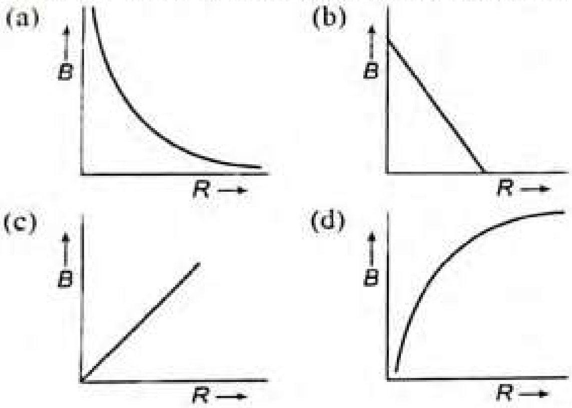
(AIEEE 2010)

26. A current I flows in an infinitely long wire with cross-section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is

(a) $\frac{\mu_0 I}{\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi^2 R}$

(c) $\frac{\mu_0 I}{2\pi R}$ (d) $\frac{\mu_0 I}{4\pi R}$ **(AIEEE 2011)**

27. A charge Q is uniformly distributed over the surface of non-condcting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc, then the variation of the magnetic induction at the centre of the disc will be represented by which of the following figures.



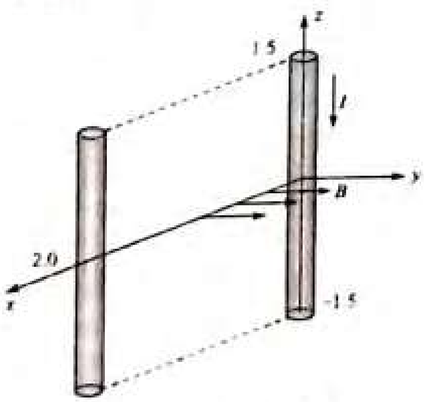
(AIEEE 2012)

28. Proton, deuteron and alpha particle of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are, respectively, r_p, r_d and r_α. Which one of the following relation is correct?

(a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p < r_d$

(c) $r_\alpha > r_d > r_p$ (d) $r_\alpha = r_d > r_p$ **(AIEEE 2012)**

29. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0\times10^{-4} e^{-0.2x}\,\hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis



(a) 14.85 W (b) 29.7 W

(c) 1.57 W (d) 2.97 W **(JEE Main 2014)**

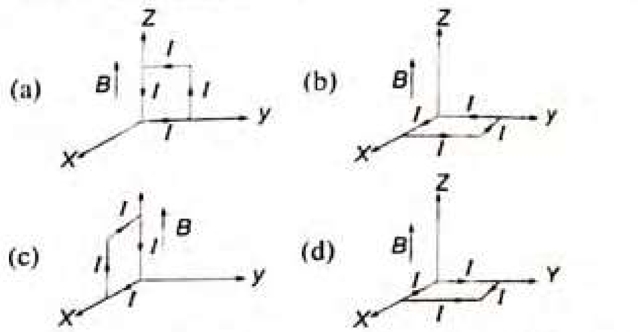
30. Two long current-carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then the value of I is (g = gravitational acceleration)

(a) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (b) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

(c) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$ (d) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

(JEE Main 2015)

31. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below:



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

(a) (a) and (b), respectively

(b) (a) and (c), respectively

(c) (b) and (d), respectively

(d) (b) and (c), respectively **(JEE Main 2015)**

32. Two identical wires A and B, each of length 'l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$

(c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$ **(JEE Main 2016)**

33. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(a) $r_e < r_\alpha < r_p$ (b) $r_e > r_p = r_\alpha$

(c) $r_e < r_p = r_\alpha$ (d) $r_e < r_p < r_\alpha$

(JEE Main 2018)

34. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio B_1/B_2 is

(a) $\frac{1}{\sqrt{2}}$ (b) 2

(c) $\sqrt{3}$ (d) $\sqrt{2}$ **(JEE Main 2018)**

ANSWER KEY

Exercises

1. (b)	2. (a)	3. (c)	4. (a)	5. (d)	6. (a)	7. (a)	8. (b)	9. (c)	10. (b)
11. (b)	12. (b)	13. (c)	14. (c)	15. (a)	16. (d)	17. (c)	18. (d)	19. (d)	20. (b)
21. (b)	22. (c)	23. (b)	24. (a)	25. (a)	26. (a)	27. (c)	28. (a)	29. (c)	30. (b)
31. (a)	32. (a)	33. (a)	34. (c)	35. (c)	36. (a)	37. (b)	38. (c)	39. (b)	40. (b)
41. (c)	42. (c)	43. (a)	44. (d)	45. (b)	46. (c)	47. (d)	48. (b)	49. (a)	50. (c)
51. (c)	52. (c)	53. (b)	54. (d)	55. (c)	56. (a)	57. (c)	58. (c)	59. (a)	60. (a)
61. (a)	62. (b)	63. (c)	64. (d)	65. (d)	66. (a)	67. (b)	68. (a)	69. (b)	70. (b)
71. (b)	72. (b)	73. (a)	74. (a)	75. (c)	76. (c)	77. (c)	78. (d)	79. (b)	80. (b)

Archives

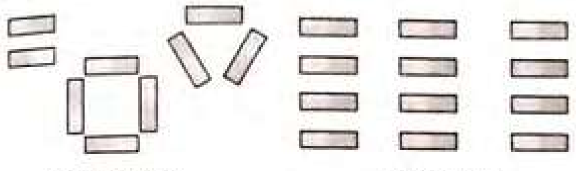
1. (b)	2. (a)	3. (a)	4. (c)	5. (a)	6. (c)	7. (b)	8. (b)	9. (b)	10. (a)
11. (c)	12. (b)	13. (d)	14. (b)	15. (d)	16. (c)	17. (d)	18. (a)	19. (a)	20. (b)
21. (b)	22. (a)	23. (b)	24. (b)	25. (a)	26. (a)	27. (a)	28. (b)	29. (d)	30. (b)
31. (c)	32. (d)	33. (c)	34. (d)						

Chapter 22

Magnetism and Matter

MAGNETISM

According to the molecular theory of magnetism, every molecule of a substance is a complete magnet in itself.



(Unmagnetised) (Magnetised)

However, in a *magnetised* substance the molecular magnets are randomly oriented to give zero net magnetic moment. On magnetising, the molecular magnets are realigned in a specific direction leading to a net magnetic moment.

BAR MAGNET

It consists of two equal and opposite magnetic poles separated by a small distance. Poles are not exactly at the ends. The shortest distance between two poles is called effective length (L_e) and is less than its geometric length (L_g).

Important Points of Bar Magnet

Directive properties: When a magnet suspended freely it stays in the earth's N-S direction (in magnetic meridian).

Monopole concept: If a magnet is broken into a number of pieces, each piece becomes a magnet. This in turn implies that monopoles do not exist (i.e., ultimate individual unit of magnetism in any magnet is called dipole).

Pole strength (m): The strength of a magnetic pole to attract magnetic materials towards itself is known as pole strength.

- It is a scalar quantity.
- Pole strength of N and S poles of a magnet is conventionally represented by $+m$ and $-m$ respectively.
- Its SI unit is amp × m or N/Tesla and dimensions are $[LA]$.
- Pole strength of the magnet depends on the nature of material of magnet and area of cross section. It does not depend upon length.

Magnetic moment or magnetic dipole moment ($\vec{M}$): It represents the strength of magnet.

Mathematically it is defined as the product of the strength of either pole and effective length.

i.e. $\vec{M} = m(2\vec{l})$

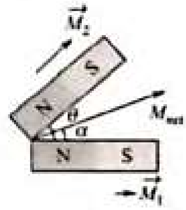
- It is a vector quantity directed from south to north.
- Its SI unit is A m^2 or N-m/tesla and dimensions are $[AL^2]$.

Combination of Bar Magnet

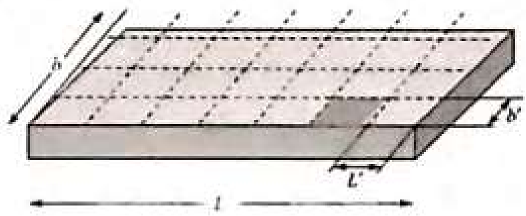
$$M_{\text{net}} = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$$

$$\tan\alpha = \frac{M_2\sin\theta}{M_1 + M_2\cos\theta}$$

$$M_{\text{net}} = 2M$$



Cutting of a bar magnet: Suppose we have a rectangular bar magnet having length, breadth and mass are L, b and w respectively, if it is cut in n equal parts along the length as well as perpendicular to the length simultaneously as shown in the figure then



Length of each part $L' = \frac{L}{\sqrt{n}}$, breadth of each part $b' = \frac{b}{\sqrt{n}}$,

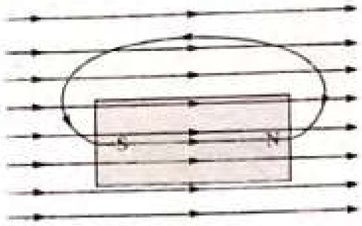
Mass of each part $w' = \frac{w}{n}$, pole strength of each part $m' = \frac{m}{\sqrt{n}}$,

Magnetic moment of each part $M' = m'L' = \frac{m}{\sqrt{n}} \times \frac{L}{\sqrt{n}} = \frac{M}{n}$

Magnetic Flux (ϕ) and Flux Density (B)

(i) The number of magnetic lines of force passing normally through a surface is defined as magnetic flux (ϕ). Its SI unit is weber (Wb) and CGS unit is maxwell. Remember 1 Wb = 10^8 maxwell.

(ii) When a piece of a magnetic substance is placed in an external magnetic field the substance becomes magnetised. The number of magnetic lines of induction inside a magnetised substance crossing unit area normal to their direction is called magnetic induction or magnetic flux density ($\vec{B}$). It is a vector quantity.



Its SI unit is Tesla which is equal to

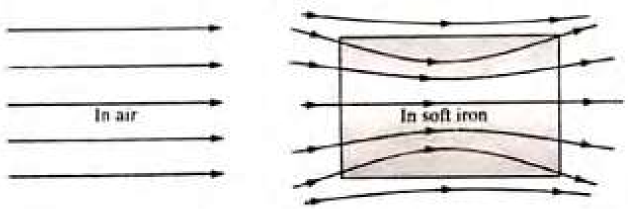
$$\frac{\text{Wb}}{\text{m}^2} = \frac{\text{N}}{\text{amp} \times \text{m}} = \frac{\text{J}}{\text{amp} \times \text{m}^2} = \frac{\text{volt} \times \text{sec}}{\text{m}^2}$$ and CGS unit is Gauss. Remember 1 tesla = 10^4 gauss.

(iii) Magnetic flux density can also be defined in terms of force experienced by a unit north pole placed in that field i.e. $B = F/m_0$.

Magnetic Permeability

It is the degree or extent to which magnetic lines of force can enter a substance and is denoted by μ.

The characteristic of a medium which allows magnetic flux to pass through it is called its permeability, e.g. permeability of soft iron is 1000 times greater than that of air.



Also $\mu = \mu_0\mu_r$; where μ_0 = absolute permeability of air or free space = $4\pi \times 10^{-7}$ tesla × m/A, and

μ_r = Relative permeability of the medium

$$= \frac{B}{B_0} = \frac{\text{flux density in material}}{\text{flux density in vacuum}}$$

Intensity of magnetising field ($\vec{H}$): It is the degree or extent to which a magnetic field can magnetise a substance. Also $H = B/\mu$. Its SI unit is

$$\text{A/m} = \frac{\text{N}}{\text{m}^2 \times \text{tesla}} = \frac{\text{N}}{\text{Wb}} = \frac{\text{J}}{\text{m}^3 \times \text{tesla}} = \frac{\text{J}}{\text{m} \times \text{Wb}}$$

Its CGS unit is oersted. Also 1 oersted = 80 A/m

Intensity of magnetisation (I): It is the degree to which a substance is magnetised when placed in a magnetic field.

It can also be defined as the pole strength per unit cross-sectional area of the substance or the induced dipole moment per unit volume. Hence $I = \frac{m}{A} = \frac{M}{V}$

It is a vector quantity. Its SI unit is A/m.

Magnetic susceptibility (χ_m): It is the property of the substance which shows how easily a substance can be magnetised. It can also be defined as the ratio of intensity of magnetisation (I) in a substance to the magnetic intensity (H) applied to the substance, i.e. $\chi_m = I/H$. It is a scalar quantity with no units and dimensions.

Relation between permeability and susceptibility: Total magnetic flux density B in a material is the sum of magnetic flux density in vacuum B_0 produced by magnetising force and magnetic flux density due to magnetisation of material B_m, i.e.

$$B = B_0 + B_m$$

$$\Rightarrow \quad B = \mu_0 H + \mu_0 I = \mu_0 (H + I) = \mu_0 H(1 + \chi_m).$$

Also $\mu_r = (1 + \chi_m)$

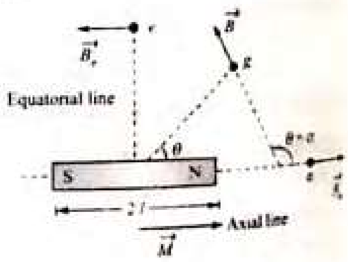
Force and Field

Coulombs law in magnetism: The force between two magnetic poles of strength m_1 and m_2 lying at a distance r is given by $F = k.\frac{m_1 m_2}{r^2}$. In SI units $k = \frac{\mu_0}{4\pi} = 10^{-7}$ Wb/Amp × m.

In CGS, units $k = 1$.

MAGNETIC FIELD

(i) Magnetic field due to an imaginary magnetic pole (pole strength m): It is given by $B = \frac{F}{m_0}$, also $B = \frac{\mu_0}{4\pi}.\frac{m}{d^2}$



(ii) Magnetic field due to a bar magnet: At a distance r from the centre of magnet

(a) On axial position $B_a = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$; If $l \ll r$ then $B_a = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$

(b) On equatorial position: $B_e = \frac{\mu_0}{4\pi}\frac{M}{(r^2 + l^2)^{3/2}}$; If $l \ll r$ then $B_e = \frac{\mu_0}{4\pi}\frac{M}{r^3}$

(c) General position: In general position for a short bar magnet $B_g = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{(3\cos^2\theta + 1)}$

Bar magnet in magnetic field: When a bar magnet is left free in an uniform magnetic field, it aligns it self in the directional field.

- Torque: $\tau = MB\sin\theta \Rightarrow \vec{\tau} = \vec{M}\times\vec{B}$
- Work: $W = MB(1 - \cos\theta)$
- Potential energy: $U = MB\cos\theta = -\vec{M}.\vec{B}$; ($\theta$= Angle made by the dipole with the field)

ILLUSTRATION 22.1 A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Solution. The pole strength is m = 120 CGS units = 12 A-m. Magnetic length is $2l$ = 10 cm or l = 0.05 m. Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-l^2)^2} = \frac{\mu_0}{4\pi}\frac{4mld}{(d^2-l^2)^2}$$

$$= \left(10^{-7}\frac{\text{T}-\text{m}}{\text{A}}\right)\frac{4\times(12\text{A}-\text{m})\times(0.05\text{m})\times(0.2\text{m})}{[(0.2\text{m})^2-(0.05\text{m})^2]^2}$$

$$= 3.4\times10^{-5}\text{ T}$$

ILLUSTRATION 22.2 Find the magnetic field due to a dipole of magnetic moment 1.2 A-m^2 at a point 1 m away from it in a direction making an angle of 60° with the dipole-axis.

Solution. The magnitude of the field is $B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$

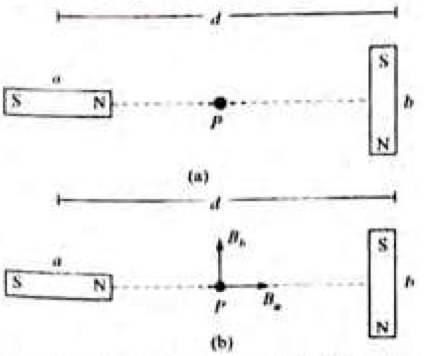
$$= \left(10^{-7}\frac{\text{T}-\text{m}}{\text{A}}\right)\frac{1.2\text{A}-\text{m}^2}{1\text{ m}^3}\sqrt{1+3\cos^2 60°}$$

$$= 1.6\times10^{-7}\text{ T}$$

The direction of the field makes an angle α with the radial line

where $\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$

ILLUSTRATION 22.3 Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



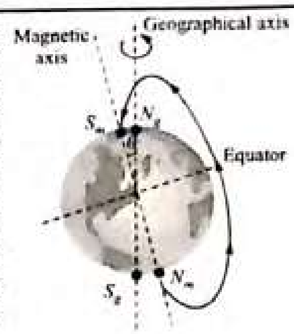
Solution. The point P is in end-on position for the dipole a and in broadside-on position for the dipole b. The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is $B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore,

$$B = \sqrt{B_a^2+B_b^2} = \frac{\mu_0 M}{4\pi(d/2)^3}\sqrt{1^2+2^2}\ \frac{2\sqrt{5}\mu_0 M}{\pi d^2}$$

The direction of this field makes an angle α with B_a such that $\tan\alpha = B_b/B_a = 1/2$.

EARTH'S MAGNETIC FIELD (TERRESTRIAL MAGNETISM)

As per the most established theory it is due to the rotation of the earth where by the various charged ions present in the molten state in the core of the earth rotate and constitute a current.



The magnetic field of earth is similar to one which would be obtained if a huge magnet is assumed to be buried deep inside the earth at its centre.

The axis of rotation of earth is called geographic axis and the points where it cuts the surface of earth are called geographical poles (N_g, S_g). The circle on the earth's surface perpendicular to the geographical axis is called equator.

A vertical plane passing through the geographical axis is called geographical meridian.

The axis of the huge magnet assumed to be lying inside the earth is called magnetic axis of the earth. The points where the magnetic axis cuts the surface of earth are called magnetic poles. The circle on the earth's surface perpendicular to the magnetic axis is called magnetic equator.

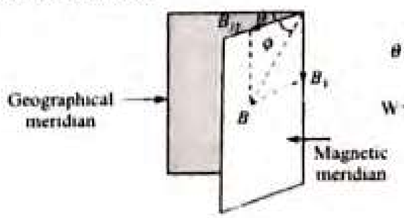
Magnetic axis and geographical axis do not coincide but they make an angle of 17.5° with each other.

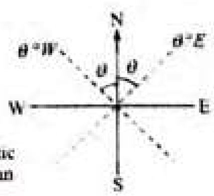
Magnetic equator divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called northern hemisphere while the other, the southern hemisphere. The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place it varies with time too.

Direction of earth's magnetic field is from S (geographical south) to N (Geographical north).

Elements of Earth's Magnetic Field

The magnitude and direction of the magnetic field of the earth at a place are completely given by certain quantities known as magnetic elements.





Magnetic declination (θ): It is the angle between geographic and the magnetic meridian planes.

Declination at a place is expressed at $\theta°$E or $\theta°$W depending upon whether the north pole of the compass needle lies to the east or to the west of the geographical axis.

Angle of inclination or dip (ϕ): It is the angle between the direction of intensity of total magnetic field of earth and a horizontal line in the magnetic meridian.

Horizontal component of earth's magnetic field (B_H): Earth's magnetic field is horizontal only at the magnetic equator. At any other place, the total intensity can be resolved into horizontal component (B_H) and vertical component (B_V).

Also $B_H = B\cos\phi$ (i)

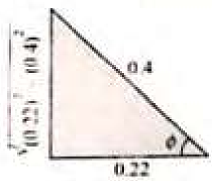
and $B_V = B\sin\phi$ (ii)

By squaring and adding Eqs. (i) and (ii), $B = \sqrt{B_H^2 + B_V^2}$

Dividing Eq. (ii) by Eq. (i) $\tan\phi = \dfrac{B_V}{B_H}$

- At equator $\theta = 0 \Rightarrow B_H = B, B_V = 0$ while at poles $\phi = 90°$ $\Rightarrow B_H = 0, B_V = B$

ILLUSTRATION 22.4 The horizontal component of the earth's magnetic field is 0.22 Gauss and total magnetic field is 0.4 Gauss. Find the angle of dip.



Solution. By using $B_H = B\cos\phi$

$$\Rightarrow \quad \cos\phi = \frac{B_H}{B} = \frac{0.22}{0.4}$$

$$\Rightarrow \quad \tan\phi = \frac{\sqrt{(0.4)^2-(0.22)^2}}{0.22}$$

$$\Rightarrow \quad \phi = \tan^{-1}(1.518)$$

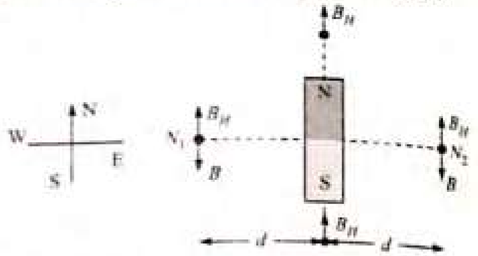
MAGNETIC MAPS AND NEUTRAL POINTS

Magnetic maps (i.e. declination, dip and horizontal component) over the earth vary in magnitude from place to place. It is found that many places have the same value of magnetic elements. The lines are drawn joining all place on the earth having same value of a magnetic elements. These lines form magnetic maps.

- **Isogonic lines:** These are the lines on the magnetic map joining the places of equal declination.
- **Agonic line:** The line which passes through places having zero declination is called agonic line.
- **Isoclinic lines:** These are the lines joining the points of equal dip or inclination.
- **Aclinic line:** The line joining places of zero dip is called aclinic line (or magnetic equator)
- **Isodynamic lines:** The lines joining the points or places having the same value of horizontal component of earth's magnetic field are called isodynamic lines.

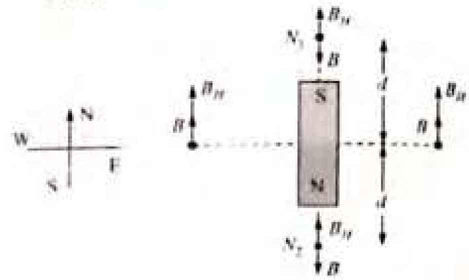
Neutral points: At the neutral point, magnetic field due to the bar magnet is just equal and opposite to the horizontal component of earth's magnetic field.

(i) Magnet is placed horizontally in a horizontal plane: N-pole of magnet is facing N-pole of earth



Two neutral points N_1 and N_2 are obtained on equatorial line of bar magnet as shown and at neutral points $B = B_H$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{M}{d^3} = B_H$$

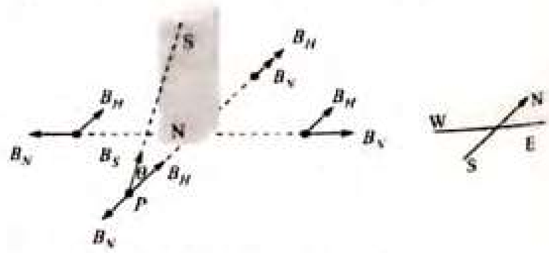


N-pole of magnet is facing N-pole of earth.

Two neutral points N_1 and N_2 are obtained on axial line of B or magnet and at neutral points

$$B = B_H \text{ i.e. } \frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3} = B_H$$

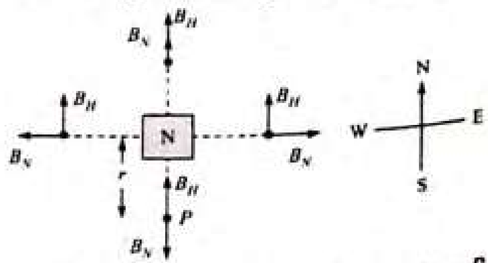
(ii) Magnet is placed vertically in a horizontal plane: N-pole of magnet is the horizontal plane



B_N = Magnetic field due to N-pole

B_S = Magnetic field due to S-pole

M = Pole strength of each pole of the magnet

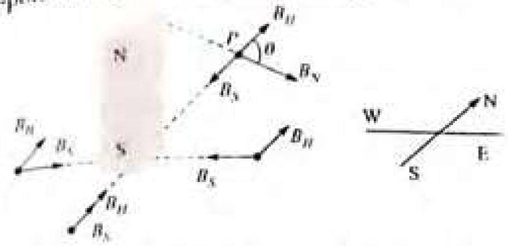


At neutral point P : $B_N - B_S\cos\theta = B_H$ ($B_S < B_N$)

If effect of S-pole is neglected: As seen from top, only one neutral point is obtained and at neutral point $B_N = B_H$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{m}{r^2} = B_H$$

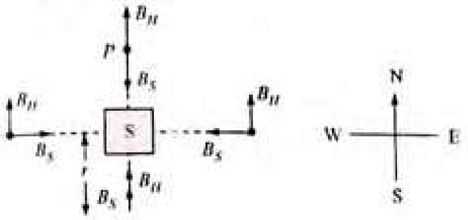
S-pole of magnet is the horizontal plane



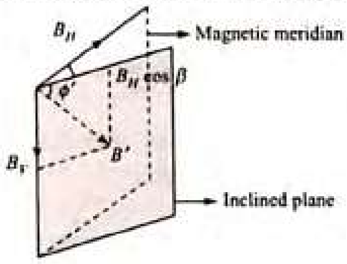
At neutral point P : $B_S - B_N \cos\theta = B_H$ $(B_S < B_N)$

If effect of N-pole is neglected: As seen from top, only one neutral point is obtained and at neutral point $B_S = B_H$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{m}{r^2} = B_H$$



- **Apparent dip:** In a vertical plane inclined at an angle β to the magnetic meridian, vertical component of earth's magnetic field remains unchanged while in the new inclined plane horizontal component



$B'_H = B_H \cos\phi'$

ϕ' = Apparent angle of dip

and $\tan\phi' = \dfrac{B_V}{B_H'} = \dfrac{B_V}{B_H \cos\beta} \Rightarrow \tan\phi' = \dfrac{\tan\phi}{\cos\beta}$

- If at any place the angle of dip is θ and magnetic latitude is λ then $\tan\theta = 2\tan\lambda$.
- At the poles and equator of earth the values of total intensity are 0.66 and 0.33 Oersted respectively.

ILLUSTRATION 22.5 If a magnet is suspended at an angle 30° to the magnetic meridian, it makes an angle of 45° with the horizontal. Find the real dip.

Solution. Let the real dip be ϕ, then $\tan\phi = \dfrac{B_V}{B_H}$

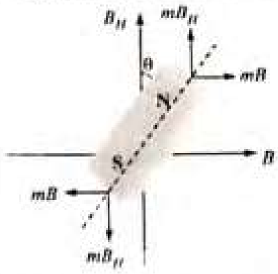
For apparent dip,

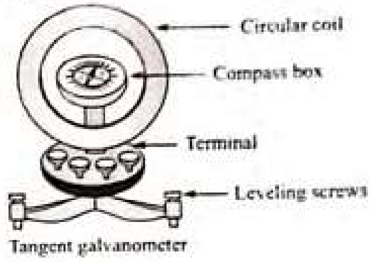
$$\tan\phi' = \frac{B_V}{B_H\cos\beta} = \frac{B_V}{B_H\cos 30^\circ} = \frac{2B_V}{\sqrt{3}B_H}$$

$$\text{or } \tan 45^\circ = \frac{2}{\sqrt{3}}\cdot\tan\phi \text{ or } \phi = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

TANGENT LAW AND ITS APPLICATION

When a small magnet is suspended in two uniform magnetic fields B and B_H which are at right angles to each other, the magnet comes to rest at an angle θ with respect to B_H such that $B = B_H \tan\theta$. This is called tangent law.





Tangent Galvanometer

It is an instrument which can detect/measure very small electric currents. It is also called as moving magnet galvanometer.

It consists of three circular coils of insulated copper wire wound on a vertical circular frame made of nonmagnetic material as ebonite or wood. A small magnetic compass needle is pivoted at the centre of the vertical circular frame.

This needle rotates freely in a horizontal plane inside a box made of nonmagnetic material. When the coil of the tangent galvanometer is kept in magnetic meridian and current passes through any of the coil then the needle at the centre gets deflected and comes to an equilibrium position under the action of two perpendicular field: one due to horizontal component of earth and the other due to field set up by the coil due to current (B).

In equilibrium $B = B_H \tan\theta$, where $B = \dfrac{\mu_0 ni}{2r}$; n = number of turns, r = radius of coil, i = the current to be measured, θ = angle made by needle from the direction of B_H in equilibrium.

Hence, $\dfrac{\mu_0 Ni}{2r} = B_H \tan\theta \Rightarrow i = k\tan\theta$, where $k = \dfrac{2rB_H}{\mu_0 N}$ is called reduction factor.

- Principle of moving coil galvanometer is $i \propto \tan\theta$. Since $i \propto \tan\theta$, so its scale is not uniform.
- When $\theta = 45^\circ$, reduction factor equals to current flows through coil.
- Sensitivity of this galvanometer is maximum at $\theta = 45^\circ$.
- This instrument is also called moving magnet type galvanometer.

MAGNETIC INSTRUMENTS

Magnetic instruments are used to find out the magnetic moment of a bar magnet, find out the horizontal component of earth's magnetic field, compare the magnetic moments of two bar magnets.

Deflection Magnetometer

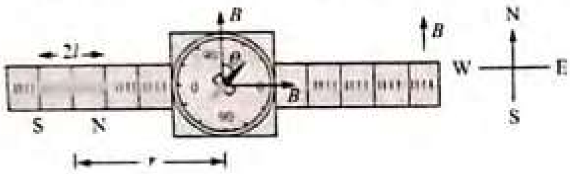
Its working is based on the principle of tangent law. It consists of a small compass needle, pivoted at the centre of a circular

box. The box is kept in a wooden frame having two meter scale fitted on its two arms. Reading of a scale at any point directly gives the distance of that point from the centre of compass needle.

Different Positions of Deflection Magnetometer

Deflection magnetometer can be used according to two following positions.

Tan A position: Arms of magnetometer are placed along E-W direction such that magnetic needle is acted upon by only horizontal component of earth's magnetic field (B_H) as shown.



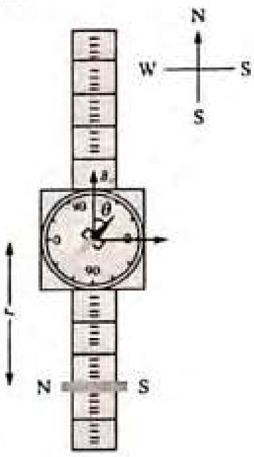
If a bar magnet is placed on one arm with its length parallel to arm, so magnetic needle comes under the influence of two mutual perpendicular magnetic fields (i) B_H and (ii) axial magnetic field of experimental bar magnet.

In equilibrium, $B = B_H \tan\theta$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\frac{2M}{r^3} = B_H \tan\theta$$

(M = Magnetic moment of experimental bar magnet)

Tan B position: Arms of magnetometer are placed along N-S direction such that magnetic needle aligns itself in the direction of earth's magnetic field (i.e. B_H) as shown.

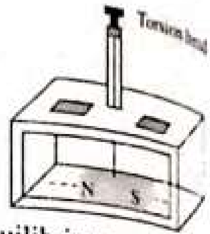


If a bar magnet is placed on one arm with its length perpendicular to arm, so magnetic needle comes under the influence of two mutual perpendicular magnetic fields (i) B_H and (ii) equatorial magnetic field of experimental bar magnet.

In equilibrium $B = B_H \Rightarrow \quad \frac{\mu_0}{4\pi}\cdot\frac{M}{r^3} = B_H \tan\theta$

- Deflection magnetometer is also used to compare the magnetic moments either by deflection method or by null deflection method. Deflection method : $\frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}$, Null deflection method: $\frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3$, where d_1 and d_2 are the positions of two bar magnet placed simultaneously on each arm.

Vibration Magnetometer



Vibration magnetometer is used for comparison of magnetic moments and magnetic fields. This device works on the principle, that whenever a freely suspended magnet in a uniform magnetic field, is disturbed from its equilibrium position, it starts vibrating about the mean position.

Time period of oscillation of experimental bar magnet (magnetic moment M) in earth's magnetic field (B_H) is given by the formula, $T = 2\pi\sqrt{\frac{I}{MB_H}}$, where I = moment of inertia of short bar magnet $= \frac{wL^2}{12}$ (w = mass of bar magnet).

Use of Vibration Magnetometer

Determination of magnetic moment of a magnet:

- The experimental (given) magnet is put into vibration magnetometer and its time period T is determined. Now

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \Rightarrow M = \frac{4\pi^2 I}{B_H.T^2}$$

- Comparison of horizontal components of earth's magnetic field at two places

$T = 2\pi\sqrt{\frac{I}{MB_H}}$; since I and M of the magnet are constants,

$$\text{so} \quad T^2 \propto \frac{1}{B_H} \Rightarrow \frac{(B_H)_1}{(B_H)_2} = \frac{T_2^2}{T_1^2}$$

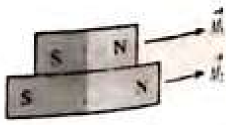
- Comparison of magnetic moment of two magnets of same size and mass

$T = 2\pi\sqrt{\frac{I}{M.B_H}}$; here I and B_H are constants.

$$\text{so} \quad M \propto \frac{1}{T^2} \Rightarrow \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

- Comparison of magnetic moments of two magnets of unequal sizes and masses (by sum and difference method): In this method, both the magnets vibrate simultaneously in the following positions.

Sum position: Two magnets are placed such that their magnetic moments are additive.



Net magnetic moment $M_s = M_1 + M_2$

Net moment of inertia $I_s = I_1 + I_2$

Time period of oscillation of this pair in earth's magnetic field (B_H)

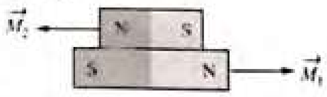
$$T_s = 2\pi\sqrt{\frac{I_s}{M_s B_H}} = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B_H}} \quad \text{(i)}$$

Frequency $\nu_s = \frac{1}{2\pi}\sqrt{\frac{M_s(B_H)}{I_s}}$

Difference position: Magnetic moments are subtractive.

Net magnetic moment $M_d = M_1 + M_2$

Net moment of inertia $I_d = I_1 + I_2$



and $T_d = 2\pi\sqrt{\dfrac{I_d}{M_d B_H}}$

$$= 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B_H}} \quad \text{(ii)}$$

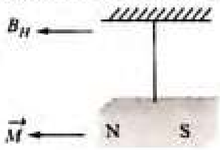
and $v_d = \dfrac{1}{2\pi}\sqrt{\dfrac{(M_1 + M_2)B_H}{(I_1 + I_2)}}$

From Eqs. (i) and (ii): we get $\dfrac{T_s}{T_d} = \sqrt{\dfrac{M_1 - M_2}{M_1 + M_2}}$

$$\Rightarrow \quad \frac{M_1}{M_2} = \frac{T_d^2 + T_s^2}{T_d^2 - T_s^2} = \frac{v_s^2 + v_d^2}{v_s^2 - v_d^2}$$

- To find the ratio of magnetic field: Suppose it is required to find the ratio B/B_H, where B is the field created by magnet and B_H is the horizontal component of earth's magnetic field.

To determine B/B_H a primary (main) magnet is made to first oscillate in earth's magnetic field (B_H) alone and its time period of oscillation (T) is noted.



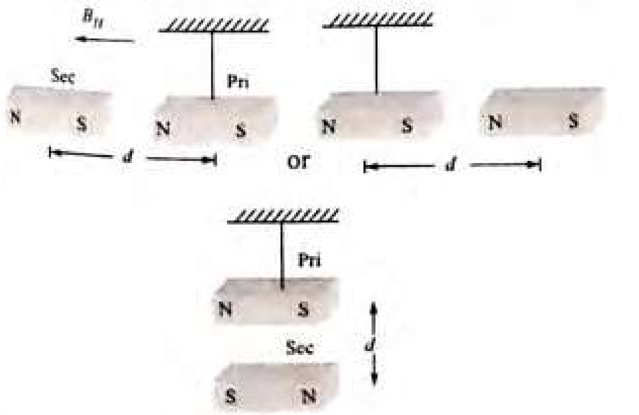
$$T = 2\pi\sqrt{\frac{I}{M B_H}} \text{ and frequency } v = \frac{1}{2\pi}\sqrt{\frac{M B_H}{I}}$$

Now a secondary magnet placed near the primary magnet so primary magnet oscillates in a new field which is the resultant of B and B_H and now time period, is noted again.

There are two important possibilities for placing secondary magnet.

Possibility-1

New field increases so time period of oscillation of primary magnet decreases.



Now time period $T' = 2\pi\sqrt{\dfrac{I}{M(B + B_H)}}$ or new frequency

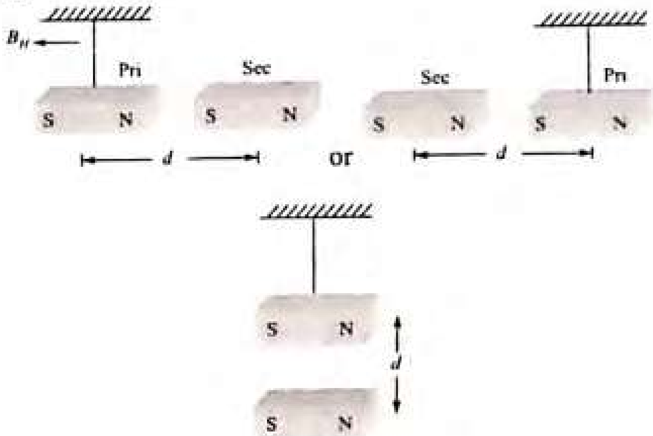
$$v' = \frac{1}{2\pi}\sqrt{\frac{M(M + B_H)}{I}}$$

Also $\left(\dfrac{v'}{v}\right)^2 = \sqrt{\dfrac{B + B_H}{B_H}} \Rightarrow \left(\dfrac{v'}{v}\right)^2 = \dfrac{B}{B_1} + 1$

$$\Rightarrow \quad \frac{B}{B_H} = \left(\frac{v'}{v}\right)^2 - 1$$

Possibility-2

Net field decreases, so time period of oscillation of primary magnet increases.



$$T' = 2\pi\sqrt{\frac{I}{M(B_H - B)}} \quad (B_H > B)$$

and $v' = \dfrac{1}{2\pi}\sqrt{\dfrac{M(B_H - B)}{I}}$

Also $\left(\dfrac{v'}{v}\right)^2 = \sqrt{\dfrac{B_H - B}{B_H}} \Rightarrow \left(\dfrac{v'}{v}\right)^2 = 1 - \left(\dfrac{B}{B_1}\right)$

$$\Rightarrow \quad \frac{B}{B_H} = 1 - \left(\frac{v'}{v}\right)^2$$

ILLUSTRATION 22.6 A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25μT.

(a) Find the magnetic moment of the magnet.

(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Solution.

(a) The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2) =$$

$$\frac{100 \times 10^{-3}}{12}[(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2] \text{ kg-m}^2$$

$$= \frac{25}{6} \times 10^{-5} \text{ kg -m}^2$$

We have, $T = 2\pi\sqrt{\dfrac{I}{MB}}$

or,

$$M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25 \times 10^{-5}\ \text{kg/m}^2}{6 \times (25 \times 10^{-6} T) \times \frac{\pi^2}{4} s^2} = 27\ \text{A-m}^2$$

(b) In this case, the moment of inertia becomes

$I' = \dfrac{m'}{12}(L^2 + b^2)$, where $b' = 0.5$ cm

The time period would be $T' = \sqrt{\dfrac{I'}{MB}}$ (i)

Dividing by Eqs. (i),

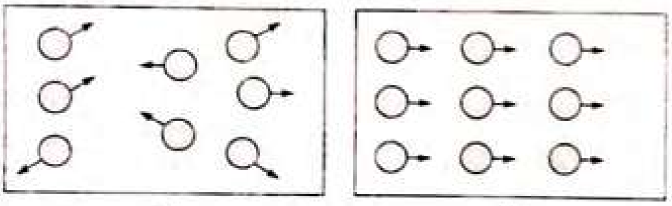
$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2 + b'^2)}}{\sqrt{\frac{m'}{12}(L^2 + b^2)}} = 0.992$$

or, $T' = \dfrac{0.992 \times \pi}{2}\ \text{s} = 0.496\ \pi\ \text{s}$

MAGNETIC MATERIALS

Types of Magnetic Materials

On the basis of mutual interactions or behaviour of various materials in an external magnetic field, the materials are divided in three main categories.



(a) When no field is applied

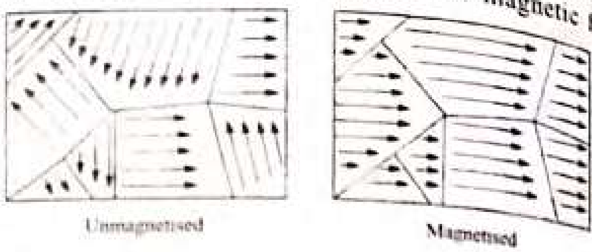
(b) On application of field (B)

(i) **Diamagnetic materials:** Diamagnetism is the intrinsic property of every material and it is generated due to mutual interaction between the applied magnetic field and orbital motion of electrons.

(ii) **Paramagnetic materials**: In these substances the inner orbits of atoms are incomplete. The electron spins are uncoupled, consequently on applying a magnetic field the magnetic moment generated due to spin motion align in the direction of magnetic field and induces magnetic moment in its direction due to which the material gets feebly magnetised. In these materials the electron number is odd.

(iii) **Ferromagnetic materials:** In some materials, the permanent atomic magnetic moments have strong tendency to align themselves even without any external field. These materials are called ferromagnetic materials.

In every unmagnetised ferromagnetic material, the atoms form domains inside the material. The atoms in any domain have magnetic moments in the same direction giving a net large magnetic moment to the domain. Different domains, however, have different directions of magnetic moment and hence the materials remain unmagnetised. On applying an external magnetic field, these domains rotate and align in the direction of magnetic field.



Unmagnetised Magnetised

Curie Law

The magnetic susceptibility of paramagnetic substances is inversely proportional to its absolute temperature i.e. $\chi \propto 1/T$ in $\chi = C/T$, where C = Curie constant, T = absolute temperature.

On increasing temperature, the magnetic susceptibility of paramagnetic materials decreases and vice versa.

The magnetic susceptibility of ferromagnetic substances does not change according to Curie law.

(i) **Curie temperature (T_c):** The temperature above which a ferromagnetic material behaves like a paramagnetic material is defined as Curie temperature (T_c).

or

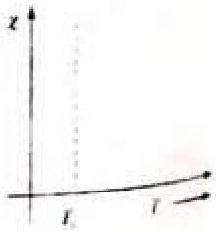
The minimum temperature at which a ferromagnetic substance is converted into paramagnetic substance is defined as Curie temperature.

For various ferromagnetic materials its values are different, e.g. for Ni, $T_{C_{Ni}} = 358°C$

for Fe, $T_{C_{Fe}} = 770°C$, for Co, $T_{C_{Co}} = 1120°C$

At this temperature the ferromagnetism of the substances suddenly vanishes.

(ii) **Curie–Weiss law:** At temperatures above Curie temperature the magnetic susceptibility of ferromagnetic materials is inversely proportional to $(T - T_c)$ i.e.



$$\chi \propto \frac{1}{T - T_c} \Rightarrow \chi = \frac{C}{(T - T_c)}$$

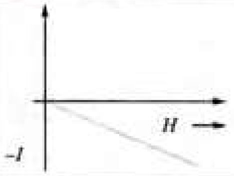
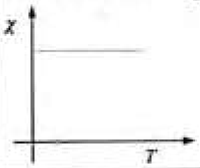
here T_c = Curie temperature

χ–T curve is shown (for Curie–Weiss law)

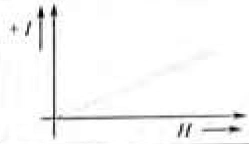
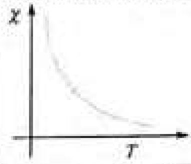
Comparative Study of Magnetic Materials

Diamagnetic Substances

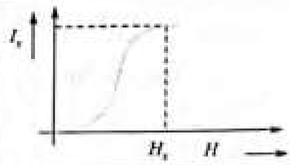
Property	Description
Cause of magnetism	Orbital motion of electrons
Explanation of magnetism	On the basis of orbital motion of electrons
Behaviour in a non-uniform magnetic field	These are repelled in an external magnetic field i.e. have a tendency to move from high to low field region

State of magnetisation	These are weekly magnetised in a direction opposite to that of applied magnetic field
Magnetic susceptibility χ	Low and negative $\lvert\chi\rvert \approx 1$
Dependence of χ on temperature	Does not depend on temperature (except Bi at low temperature)
Dependence of χ on H	Does not depend (independent)
Relative permeability (μ_r)	$\mu_r < 1$
Intensity of magnetisation (I)	I is in a direction opposite to that of H and its value is very low
I–H curves	
Magnetic moment (M)	The value of M is very low (≈ 0 and is in a direction opposite to H)
Transition of materials (at Curie temperature)	These do not change 
The property of magnetism	Diamagnetism is found in those materials the atoms of which have even number of electrons
Examples	Cu, Ag, Au, Zn, Bi, Sb, NaCl, H_2O air and diamond etc.

Paramagnetic Substances

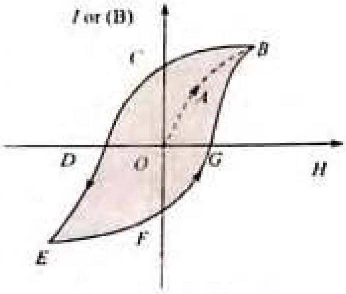
Property	Description
Cause of magnetism	Spin motion of electrons
Explanation of magnetism	On the basis of spin and orbital motion of electrons
Behaviour in a non-uniform magnetic field	These are feebly attracted in an external magnetic field i.e., have a tendency to move from low to high field region
State of magnetisation	These get weekly magnetised in the direction of applied magnetic field
Magnetic susceptibility χ	Low but positive $\chi \approx 1$
Dependence of χ on temperature	Inversely proportional to temperature $\chi \propto \frac{1}{T}$ or $\chi = \frac{C}{T}$. This is called Curie law, where C = Curie constant
Dependence of χ on H	Does not depend (independent)
Relative permeability (μ_r)	$\mu_r > 1$
Intensity of magnetisation (I)	I is in the direction of H but value is low
I–H curves	
Magnetic moment (M)	The value of M is very low and is in the direction of H
Transition of materials (at Curie temperature)	On cooling, these get converted to ferromagnetic materials at Curie temperature 
The property of magnetism	Paramagnetism is found in those materials the atoms of which have majority of electron spins in the same direction
Examples	Al, Mn, Pt, Na, $CuCl_2$, O_2 and crown glass

Ferromagnetic Substances

Property	Description
Cause of magnetism	Formation of domains
Explanation of magnetism	On the basis of domains formed
Behaviour in a non-uniform magnetic field	These are strongly attracted in an external magnetic field i.e. they easily move from low to high field region
State of magnetisation	These get strongly magnetised in the direction of applied magnetic field
Magnetic susceptibility χ	Positive and high $\chi \approx 10^2$
Dependence of χ on temperature	$\chi \propto \frac{1}{T - T_c}$ or $\chi = \frac{C}{T - T_c}$. This is called Curie–Weiss law. T_c = Curie temperature
Dependence of χ on H	Does not depend (independent)
Relative permeability (μ_r)	$\mu_r \gg 1$ $\mu_r \approx 10^2$
Intensity of magnetisation (I)	I is in the direction of H and value is very high.
I–H curves	
Magnetic moment (M)	The value of M is very high and is in the direction of H

Transition of materials (at Curie temperature)	These get converted into paramagnetic materials above Curie temperature 
The property of magnetism	Ferro-magnetism is found in those materials which when placed in an external magnetic field are strongly magnetised
Examples	Fe, Co, Ni, Cd, Fe_3O_4 etc.

Hysteresis



For ferromagnetic materials, by removing external magnetic field i.e. $H = 0$. The magnetic moment of some domains remain aligned in the applied direction of previous magnetising field which results into a residual magnetism.

The lack of retracibility as shown in figure is called hysteresis and the curve is known as hysteresis loop.

(i) When magnetising field (H) is increased from O, the intensity of magnetisation I increases and becomes maximum. This maximum value is called the saturation value.

(ii) When H is reduced, I reduces but is not zero when $H = 0$. The remainder value OC of magnetisation when $H = 0$ is called the residual magnetism or retentivity.

The property by virtue of which the magnetism (I) remains in a material even on the removal of magnetising field is called retentivity or residual magnetism.

(iii) When magnetic field H is reversed, the magnetisation decreases and for a particular value of H, denoted by H_c, it becomes zero i.e., $H_c = OD$ when $I = 0$. This value of H is called the coercivity.

(iv) So, the process of demagnetising a material completely by applying magnetising field in a negative direction is defined coercivity. Coercivity assesses the softness or hardness of a magnetic material. Coercivity signifies magnetic hardness or softness of substance :

Magnetic hard substance (steel) $\rightarrow$ High coercvity

Magnetic soft substance (soft iron) $\rightarrow$ Low coercivity

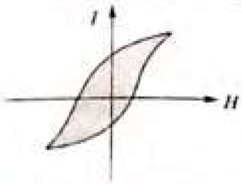
(v) When field H is further increased in reverse direction, the intensity of magnetisation attains saturation value in reverse direction (i.e. point E)

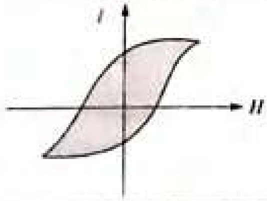
(vi) When H is decreased to zero and changed direction in steps, we get the part $EFGB$.

Thus complete cycle of magnetisation and demagnetisation is represented by $BCDEFGB$.

- The energy loss (or hysteresis energy loss) in magnetising and demagnetising a specimen is proportional to the area of hysteresis loop.

(vii) Comparison between soft iron and steel:

Soft iron

The area of hysteresis loop is less (low energy loss)
Less relativity and coercive force
Magnetic permeability is high
Magnetic susceptibility (χ) is high
Intensity of magnetisation (I) is high
It magnetised and demagnetised easily
Used in dynamo, transformer, electromagnet tape recorder and tapes etc.

Steel

The area of hysteresis loop is large (high energy loss)
More retentivity and coercive force
Magnetic permeability is less
χ is low
I is low
Magnetisation and demagnetisation is complicated
Used for making permanent magnet.

- An iron cored coil and a bulb are connected in series with an ac generator. If an iron rod is introduced inside a coil, then the intensity of bulb will decrease, because some energy lost in magnetising the rod.
- Hysteresis energy loss = Area bound by the hysteresis loop = $VAnt$ Joule

where V = Volume of ferromagnetic sample, A = Area of $B-H$ loop P, n = Frequency of alternating magnetic field and t = Time.

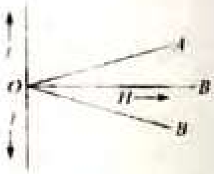
Concept Application Exercise 22.1

1. Figure shows the variation of intensity of magnetisation (I) versus the applied magnetic field intensity (H) for two magnetic materials A and B:

(a) Identify the materials A and B.
(b) For the material A, plot the variation of intensity of magnetisation versus temperature.

2. The earth field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8×10^{22} J T^{-1} located at its centre. Check the order or magnitude of this number in some way.
(a) Identify the materials A and B.

3. Figure shows the variation of intensity of magnetisation (I) versus the applied magnetic field intensity (H) for two magnetic materials A and B.



(a) Identify the materials A and B.
(b) Draw the variation of susceptibility with temperature for the material B.

4. Horizontal and vertical components of earth's magnetic field at a place are 0.22 tesla and 0.38 tesla respectively. Find the resultant intensity of earth's magnetic field.

5. Horizontal component of earth's magnetic field at a place is $\sqrt{3}$ times the vertical component. What is the value of angle of dip at this place?

6. The horizontal component of earth's magnetic field at a place is 0.3×10^{-4} T. If the angle of dip is 60°, what is the total earth's magnetic field?

7. The horizontal component of earth's magnetic field is 0.2 gauss and total magnetic field is 0.4 gauss. Find angle of dip.

SOLVED EXAMPLES

1. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque required to maintain the needle in this position will be

(a) $\sqrt{3}W$ (b) W
(c) $\frac{\sqrt{3}}{2}W$ (d) $2W$

Sol. (a) $W = MB(\cos\theta_1 - \cos\theta_2) = MB(\cos 0° - \cos 60°)$

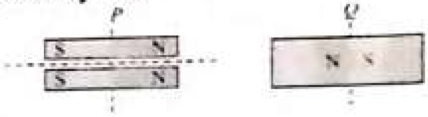
$$= MB\left(1 - \frac{1}{2}\right) = \frac{MB}{2}$$

and $\tau = MB\sin\theta = MB\sin 60° = MB\frac{\sqrt{3}}{2}$

$$\therefore \quad \tau = \left(\frac{MB}{2}\right)\sqrt{3} \Rightarrow \tau = \sqrt{3}W$$

2. Two similar bar magnets P and Q, each of magnetic moment M, are taken. If P is cut along its axial line and Q is cut along its equatorial line, all the four pieces obtained have
(a) equal pole strength
(b) magnetic moment $M/4$
(c) magnetic moment $M/2$
(d) magnetic moment M

Sol. (c) If pole strength, magnetic moment and length of each part are m', M' and L' respectively then



$$\Rightarrow \quad m' = \frac{m}{2}$$
$$\Rightarrow \quad m' = m$$
$$\Rightarrow \quad L' = L$$
$$\Rightarrow \quad L' = \frac{L}{2} \quad \Rightarrow \quad M' = \frac{M}{2} \quad \Rightarrow M' = \frac{M}{2}$$

3. A bar magnet is held perpendicular to a uniform magnetic field. If the couple acting on the magnet is to be halved by rotating it, then the angle by which it is to be rotated is
(a) 30° (b) 45°
(c) 60° (d) 90°

Sol. (c) $\tau = MB\sin\theta \Rightarrow \tau \propto \sin\theta$

$$\Rightarrow \quad \frac{\tau_1}{\tau_2} = \frac{\sin\theta_1}{\sin\theta_2} \Rightarrow \frac{\tau}{\tau/2} = \frac{\sin 90°}{\sin\theta_2}$$

$$\Rightarrow \quad \sin\theta_2 = \frac{1}{2} \Rightarrow \theta_2 = 30°$$

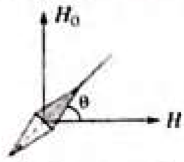
$\Rightarrow$ Angle of rotation $= 90° - 30 = 60°$

4. A current carrying coil is placed with its axis perpendicular to N-S direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop is H. If a magnet is suspended inside the loop, it makes angle θ with H. Then $\theta =$

(a) $\tan^{-1}\left(\frac{H_0}{H}\right)$ (b) $\tan^{-1}\left(\frac{H}{H_0}\right)$
(c) $\text{cosec}^{-1}\left(\frac{H}{H_0}\right)$ (d) $\cot^{-1}\left(\frac{H_0}{H}\right)$

Sol. (a) In given case H and H_0 are perpendicular to each other.

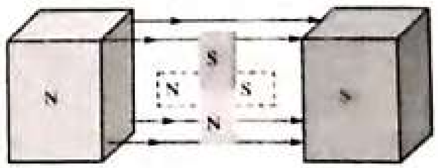
From figure $\tan\theta = \frac{H_0}{H}$



$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{H_0}{H}\right)$$

5. A small rod of bismuth is suspended freely between the poles of a strong electromagnet. It is found to arrange itself at right angles to the magnetic field. This observation establishes that bismuth is
(a) diamagnetic (b) paramagnetic
(c) ferri-magnetic (d) antiferro-magnetic

Sol. (a) A diamagnetic rod set itself perpen-dicular to the field if free to rotate between the poles of a magnet as in this situation the field is strongest near the poles.



6. Two identical magnetic dipoles of magnetic moments 1.0 A-m^2 each, placed at a separation of 2 m with their axis perpendicular to each other. The resultant magnetic field at a point midway between the dipoles is

(a) 5×10^{-7} T (b) $\sqrt{5} \times 10^{-7}$ T

(c) 10^{-7} T (d) None of these

Sol. (b) With respect to 1st magnet, P lies in end side-on position

$$\therefore \quad B_1 = \frac{\mu_0}{4\pi}\left(\frac{2M}{d^3}\right) \quad \text{(RHS)}$$

With respect to 2nd magnet, P lies in broad sideon position.

$$\therefore \quad B_2 = \frac{\mu_0}{4\pi}\left(\frac{M}{d^3}\right) \quad \text{(Upward)}$$

$$B_1 = 10^{-7} \times \frac{2\times 1}{1} = 2\times 10^{-7} T, B_2 = \frac{B_1}{2} = 10^{-7}\text{ T}$$

As B_1 and B_2 are mutually perpendicular, hence the resultant magnetic field

$$B_R = \sqrt{B_1^2 + B_2^2} = \sqrt{(2\times 10^{-7})^2 + (10^{-7})^2} = \sqrt{5}\times 10^{-7}\text{ T}$$

7. Two identical short bar magnets, each having magnetic moment M, are placed a distance of $2d$ apart with axes perpendicular to each other in a horizontal plane. The magnetic induction at a point midway between them is

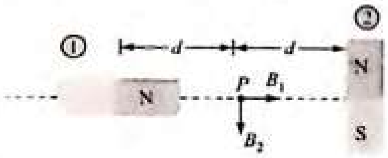
(a) $\frac{\mu_0}{4\pi}(\sqrt{2})\frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi}(\sqrt{3})\frac{M}{d^3}$

(c) $\left(\frac{2\mu_0}{\pi}\right)\frac{M}{d^3}$ (d) $\frac{\mu_0}{4\pi}(\sqrt{5})\frac{M}{d^3}$

Sol. (d) At point P net magnetic field $B_{net} = \sqrt{B_1^2 + B_2^2}$

where $B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3}$

and $B_2 = \frac{\mu_0}{4\pi}\cdot\frac{M}{d^3}$



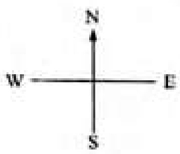
$$\Rightarrow \quad B_{net} = \frac{\mu_0}{4\pi}\cdot\frac{\sqrt{5}M}{d^3}$$

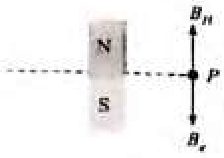
8. A short bar magnet with its north pole facing north forms a neutral point at P in the horizontal plane. If the magnet is rotated by 90° in the horizontal plane, the net magnetic induction at P is (Horizontal component of earth's magnetic field = B_H)

(a) 0 (b) $2B_H$

(c) $\frac{\sqrt{5}}{2}B_H$ (d) $\sqrt{5}B_H$

Sol. (d) Initially neutral point obtained on equatorial line and at neutral point $|B_H| = |B_e|$ where B_H = Horizontal component of earth's magnetic field, B_e = Magnetic field due to bar magnet on its equatorial line.



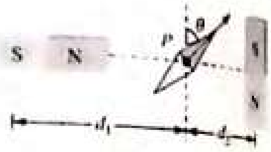


Finally, point P comes on axial line of the magnet and at P, net magnetic field

$$B = \sqrt{B_a^2 + B_H^2}$$

$$= \sqrt{(2B_e)^2 + (B_H)^2} = \sqrt{(2B_H)^2 + B_H^2} = \sqrt{5}\,B_H$$

9. Two magnets A and B are identical and these are arranged as shown in the figure. Their length is negligible in comparison to the separation between them. A magnetic needle is placed between the magnets at point P which gets deflected through an angle θ under the influence of magnets. The ratio of distance d_1 and d_2 will be



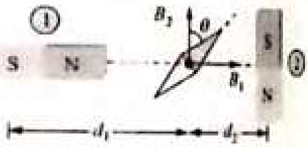
(a) $(2\tan\theta)^{1/3}$ (b) $(2\tan\theta)^{-1/3}$

(c) $(2\cot\theta)^{-1/3}$ (d) $(2\cot\theta)^{-1/3}$

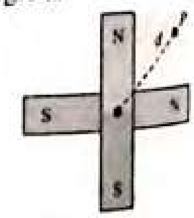
Sol. (c) In equilibrium $B_1 = B_2 \tan\theta$

$$\Rightarrow \quad \frac{\mu_0}{4\pi}\cdot\frac{2M}{d_1^3} = \frac{\mu_0}{4\pi}\cdot\frac{M}{d_2^3}\tan\theta$$

$$\Rightarrow \quad \frac{d_1}{d_2} = (2\cot\theta)^{1/3}$$



10. Two short magnets of equal dipole moments M are fastened perpendicularly at their centre (see figure). The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is



(a) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$

(b) $\frac{\mu_0}{4\pi}\frac{M\sqrt{2}}{d^3}$

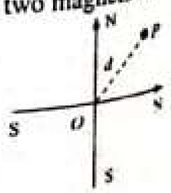
(c) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}M}{d^3}$

(d) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$

Sol. (c) Resultant magnetic moment of the two magnets is

$$M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

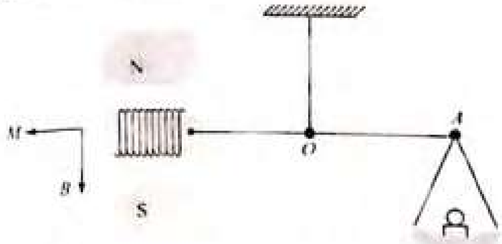
Imagine a short magnet lying along OP with magnetic moment equal to $M\sqrt{2}$. Thus point P lies on the axial line of the magnet.



$\therefore$ Magnitude of magnetic field at P is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\sqrt{2}M}{d^3}$$

11. A small coil C with $N = 200$ turns is mounted on one end of a balance beam and introduced between the poles of an electromagnet as shown in the figure. The cross-sectional area of coil is $A = 1.0\ cm^2$, length of arm OA of the balance beam is $l = 30$ cm. When there is no current in the coil the balance is in equilibrium. On passing a current $I = 22$ mA through the coil the equilibrium is restored by putting the additional counter weight of mass $\Delta m = 60\ mg$ on the balance pan. Find the magnetic induction at the spot where coil is located.



(a) 0.4 T (b) 0.3 T
(c) 0.2 T (d) 0.1 T

Sol. ***(a)*** On passing current through the coil, it acts as a magnetic dipole. Torque acting on magnetic dipole is counter balanced by the moment of additional weight about position O.

Torque acting on a magnetic dipole $\tau = MB\sin\theta = (NiA)B\sin 90^\circ = NiAB$.

Again τ = force × Lever arm = $\Delta mg \times l$

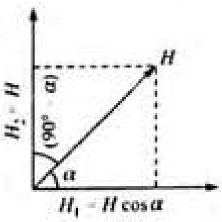
$\Rightarrow \quad NiAB = \Delta mgl$

$$\Rightarrow \quad B = \frac{\Delta mgl}{NiA} = \frac{60\times10^{-3}\times9.8\times30\times10^{-2}}{200\times22\times10^{-3}\times1\times10^{-4}} = 0.4\ T$$

12. If ϕ_1 and ϕ_2 be the angles of dip observed in two vertical planes at right angles to each other and ϕ be the true angle of dip, then

(a) $\cos^2\phi = \cos^2\phi_1 + \cos^2\phi_2$
(b) $\sec^2\phi = \sec^2\phi_1 + \sec^2\phi_2$
(c) $\tan^2\phi = \tan^2\phi_1 + \tan^2\phi_2$
(d) $\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$

Sol. ***(d)*** Let α be the angle which one of the planes make with the magnetic meridian the other plane makes an angle $(90^\circ - \alpha)$ with it. The components of H in these planes will be $H\cos\alpha$ and $H\sin\alpha$ respectively. If ϕ_1 and ϕ_2 are the apparent dips in these two planes, then



$$\tan\phi_1 = \frac{V}{H\cos\alpha} \text{ i.e. } \cos\alpha = \frac{V}{H\tan\phi_1} \qquad \text{(i)}$$

$$\tan\phi_2 = \frac{V}{H\sin\alpha} \text{ i.e. } \sin\alpha = \frac{V}{H\tan\phi_2} \qquad \text{(ii)}$$

Squaring and adding (i) and (ii), we get

$$\cos^2\alpha + \sin^2\alpha = \left(\frac{V}{H}\right)^2\left(\frac{1}{\tan^2\phi_1} + \frac{1}{\tan^2\phi_2}\right)$$

i.e. $\quad 1 = \dfrac{V^2}{H^2}(\cot^2\phi_1 + \cot^2\phi_2)$

or $\quad \dfrac{H^2}{V^2} = \cot^2\phi_1 + \cot^2\phi_2$ i.e.

$$\cot^2\phi = \cot^2\phi_1 + \cot^2\phi_2$$

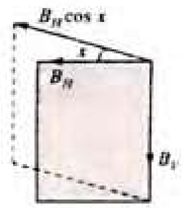
This is the required result.

13. A dip needle lies initially in the magnetic meridian when it shows an angle of dip θ at a place. The dip circle is rotated through an angle x in the horizontal plane and then it shows an angle of dip θ'. Then $\dfrac{\tan\theta'}{\tan\theta}$ is

(a) $\dfrac{1}{\cos x}$ (b) $\dfrac{1}{\sin x}$

(c) $\dfrac{1}{\tan x}$ (d) $\cos x$

Sol. ***(a)*** In first case $\tan\theta = \dfrac{B_V}{B_H}$ (i)



Second case $\tan\theta' = \dfrac{B_V}{B_H\cos x}$ (ii)

From Eqs. (i) and (ii), $\dfrac{\tan\theta'}{\tan\theta} = \dfrac{1}{\cos x}$

14. A dip circle is adjusted so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 40°. Now the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position the needle will dip by an angle

(a) 40° (b) 30°
(c) More than 40° (d) Less than 40°

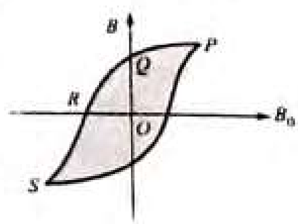
Sol. ***(c)*** $\tan\theta = \dfrac{B_V}{B_H}$ ... (i)

If apparent dip is θ' then

$$\tan\theta' = \frac{B'_V}{B'_H} = \frac{B_V}{B_H\cos 30^\circ} = \frac{B_V}{B_H\times\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \quad \tan\theta' = \left(\frac{2}{\sqrt{3}}\right)\tan\theta \Rightarrow \tan\theta' > \tan\theta \Rightarrow \theta' > \theta$$

15. The figure illustrates how B, the flux density inside a sample of unmagnetised ferromagnetic material varies with B_0, the magnetic flux density in which the sample is



kept. For the sample to be suitable for making a permanent magnet

(a) OQ should be large, OR should be small
(b) OQ and OR should both be large
(c) OQ should be small and OR should be large
(d) OQ and OR should both be small

Sol. *(b)* In the given figure OQ refers to retentivity while OR refers to coercivity, for permanents both retentivity and coercivity should be high.

EXERCISES

Magnetic Dipole, Magnet and Its Properties

1. The magnetic moment of atomic neon is [where μ_B = Bohr's magnetron]

(a) zero (b) $\frac{\mu_B}{2}$
(c) μ_B (d) $\frac{3\mu_B}{2}$

2. An electron is revolving in a circular orbit of radius r in a hydrogen atom. The angular momentum of the electron is l. The dipole moment associated with it is

(a) $(2e/m)l$ (b) $(e/2m)l$
(c) $(e/m)l$ (d) $(2m/e)l$

3. A steel wire of length l has a magnetic moment M. It is bent in L-shape at its mid-point. The new magnetic moment is

(a) M (b) $\frac{M}{\sqrt{2}}$
(c) $\frac{M}{2}$ (d) $2M$

4. A circular coil of 100 turns has an effective radius of 0.05 m and current of 0.1 A. How much work is done to turn it through 180° in a uniform field of 1.5 W/m^2 if the plane of the coil is initially perpendicular to the magnetic field?

(a) 0.5523 J (b) 0.3255 J
(c) 0.2355 J (d) 0.5235 J

5. Two like poles of strength m_1 and m_2 are far distance apart. The energy required to bring them r_0 distance apart is

(a) $\frac{\mu_0}{4\pi}\frac{m_1 m_2}{r_0}$ (b) $\frac{\mu_0}{8\pi}\frac{m_1 m_2}{r_0}$
(c) 0 (d) $\frac{\mu_0}{4\pi}\frac{m_1 m_2}{r_0^2}$

6. A magnet of pole strength m is divided into four equal parts so that the length and breadth of each part is half that of the original magnet. Then pole strength of each of the magnet is

(a) m (b) $m/4$
(c) $m/2$ (d) $4m$

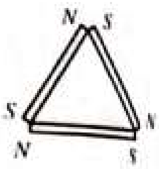
7. A magnetised wire of moment M is bent into an arc of a circle subtending an angle 60° at the centre, then the new magnetic moment is

(a) $(2M/\pi)$ (b) (M/π)
(c) $(3\sqrt{2}\,M/\pi)$ (d) $(3M/\pi)$

8. A thin bar magnet of length $2L$ is bent at the mid-point so that the angle between them is 60°. The new length of the magnet is

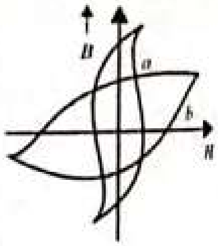
(a) $\sqrt{2}\,L$ (b) $\sqrt{3}\,L$
(c) $2L$ (d) L

9. Three identical bar magnets, each of magnetic moment M, are placed in the form of an equilateral triangle with north pole of one touching the south pole of the other (see figure). The net magnetic moment of the system is



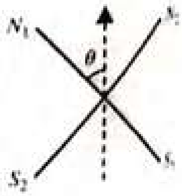
(a) zero (b) $3M$
(c) $3M/2$ (d) $M\sqrt{3}$

10. The B-H curves (a) and (b) shown in the figure are associated with



(a) a diamagnetic and a paramagnetic substance respectively
(b) a paramagnetic and a ferromagnetic substance respectively
(c) soft iron and steel, respectively
(d) steel and soft iron, respectively

11. Two magnets of equal mass are joined at right angles to each other. Magnet N_1S_1 has a magnetic moment $\sqrt{3}$ times that of N_2S_2. This arrangement is pivoted so that it is free to rotate in a horizontal plane. When in equilibrium, θ is



(a) 0° (b) 30°
(c) 45° (d) 60°

12. When a bar magnet is placed at 90° to a uniform magnetic field, it is acted upon by a couple which is maximum. For the couple to half the maximum value, the magnet should be inclined to the magnetic field at an angle of

(a) 45° (b) 30°
(c) 15° (d) 0°

13. The pole strength of a bar magnet is 48 Am and the distance between its poles is 25 cm. The moment of the couple by which it can be placed at an angle of 30° with the uniform magnetic field of flux density 0.15 N A^{-1} m^{-1} will be

(a) 12 N m (b) 18 N m

(c) 0.9 N m (d) None of the above

14. The distance of two points on the axis of a magnet from its centre are 10 cm. The ratio of magnetic fields at these points is 12.5:1. The length of the magnet is
(a) 5 cm (b) 25 cm
(c) 10 cm (d) 20 cm.

15. Two identical thin bar magnets each of length l and pole strength m are placed at right angles to each other with north pole of one touching south pole of other, the magnetic moment of the system is
(a) ml (b) $2ml$
(c) $\sqrt{2}\,ml$ (d) $ml/2$

16. The work done to turn a magnet by 60° from its equilibrium position is W in a uniform field. The torque required to hold it in that position will be
(a) $\frac{W}{2}$ (b) $\frac{W}{\sqrt{3}}$
(c) $\frac{\sqrt{3}}{2}W$ (d) $W\sqrt{3}$

17. A compass needle of magnetic moment 60 Am^2 pointing geographical north at a place where the horizontal component of earth field is 40 μ Wb m^{-2} experiences a torque of 1.2×10^{-3} Nm.
(a) 15° (b) 30°
(c) 45° (d) 60°

18. In the above question, if the coil is in the magnetic field of 1.2 T, the field being in the plane of coil, then the torque acting on it is
(a) 1.42 Nm (b) 2.84 Nm
(c) zero Nm (d) 0.71 Nm

19. The force between two short bar magnets with magnetic moments M_1 and M_2 whose centres are r metre apart is 0.8 N when their axes are in the same line. If the separation is increased to $2r$, then force between them is reduced to
(a) 4.0 N (b) 2.0 N
(c) 1.0 N (d) 0.5 N

20. A magnet is parallel to uniform magnetic field. If it is rotated by 60°, the work done is 0.8 J. How much work is done in moving it 30° further?
(a) 0.8 ergs (b) 0.4 J
(c) 8 J (d) 0.8×10^7 ergs

Earth Magnetism

21. The values of the apparent angles of dip in two planes at right angles to each other are 30° and 45°. Then the true value of the angle of dip at the place is
(a) $\tan^{-1} 1$ (b) $\tan^{-1} 2$
(c) $\cot^{-1} 2$ (d) $\cot^{-1} 4$

22. At a certain place the angle of dip is 30° and horizontal component of earth's field is 0.5 oersted, the earth's total magnetic field in oersted is
(a) $\sqrt{3}$ (b) 1
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

23. If the total magnetic field due to earth is 28 A/m, then the total magnetic induction due to earth is
(a) 28 T (b) 280 T
(c) 0.352 gauss (d) 0.352 T

24. The correct value of dip at a place is 45°. If the dip circle is rotated 45° out of the magnetic meridian then the tangent of the angle of the apparent dip is
(a) 1 (b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) $\frac{1}{2}$

25. The plane of dip circle is set in the geographic meridian and the apparent dip is θ_1. It is then set in a vertical plane perpendicular to the geographic meridian. Now, the apparent dip is θ_2. The angle of declination α at that place is
(a) $\tan\alpha = \sqrt{\tan\theta_1 \tan\theta_2}$
(b) $\tan\alpha = \sqrt{(\tan\theta_1)^2 + (\tan\theta_2)^2}$
(c) $\tan\alpha = \frac{\tan\theta_1}{\tan\theta_2}$
(d) $\tan\alpha = \frac{\tan\theta_2}{\tan\theta_1}$

26. In a vibration magnetometer, the time period of a bar magnet oscillating in horizontal component of earth's magnetic field is 2 s. When a magnet is brought near and parallel to it, the time period reduces to 1 s. The ratio H/F of the horizontal component H_a and the field F due to magnet will be
(a) 3 (b) 1/3
(c) $\sqrt{3}$ (d) $1/\sqrt{3}$

27. The magnetic needle of an oscillation magnetometer makes 0 oscillations per minute under the action of earth's magnetic field only. When a bar magnet is placed at some distance along the axis of the needle, it makes 14 oscillations per minute. If the bar magnet be turned so that its poles interchange the positions, the new frequency of the needle is
(a) 2 oscillations per minute
(b) 12 oscillations per minute
(c) 20 oscillations per minute
(d) 69.7 oscillations per minute

28. A magnetic needle of magnetic moment 60 Am^2 is directed towards the geographical north at a place. It experiences a torque of 1.2×10^{-3} Nm. If the earth's horizontal component of that place is 40 μ Wb m^{-2} then the angle of decimation at that plane will be
(a) 30° (b) 45°
(c) 60° (d) 90°

29. Two bar magnets of the same mass, same length and breadth but having magnetic moments M and $2M$ are joined together pole for pole and suspended by a string. The time period of assembly in a magnetic field of strength H is 3 seconds. If now the polarity of one of the magnets is reversed and the combination is again made to oscillate in the same field, the time of oscillation is

(a) $\sqrt{3}$ (b) $3\sqrt{3}$
(c) 3 (d) 6

30. A magnetic needle vibrates in a vertical plane parallel to the magnetic meridian about the horizontal axis. Its frequency is n. If the plane of oscillation is turned about a vertical axis by 90°, the frequency of oscillation will be

(a) n (b) zero
(c) $<n$ (d) $>n$

31. A magnet needle makes 4 vibrations per second at a place where $H = 3.5 \times 10^{-5}$ T. What is the value of H at place, where the same needle makes 3 vibrations per second?

(a) 1.98×10^{-5} T (b) 3.5×10^{-5} T
(c) 4.0×10^{-5} T (d) 3.96×10^{-5} T

32. Two bar magnets when placed with their similar poles together makes 20 vib/min. When one of them is reversed the number of vibrations become 15 vib/min. What is the ratio of their dipole moment?

(a) 7 : 45 (b) 25 : 7
(c) 1 : 4 (d) 4 : 1

33. A magnet oscillates in earth's field with a time period T. If its mass is quadrupled, then its new time period will be

(a) $4T$
(b) $2T$
(c) $T/2$
(d) unaffected but motion is not SHM

34. A magnet is cut into four equal parts by cutting it parallel to its length. What will be the time period of each part, if the time period of the original magnet in the same field is T_0?

(a) $\frac{T_0}{\sqrt{2}}$ (b) $\frac{T_0}{2}$

(c) $\frac{T_0}{4}$ (d) $4T_0$

35. A magnet is suspended in such a way that it oscillates in the horizontal plane. It makes 20 oscillations per minute at a plane where dip angle is 30° and 15 oscillations per minute at a place where dip angle is 60°. The ratio of earth's magnetic field at two places is

(a) $3\sqrt{3}:8$ (b) $16:9\sqrt{3}$
(c) 4 : 9 (d) $2\sqrt{2}:3$

36. Which of the following relation is correct in magnetism?

(a) $I^2 = V^2 + H^2$ (b) $I = V + H$
(c) $V = I^2 + H^2$ (d) $V^2 = I + H^2$

37. At a certain place, the horizontal component B_0 and the vertical component V_0 of the earth's magnetic field are equal in magnitude. The total intensity at the place will be

(a) B_0 (b) B_0^2
(c) $2B_0$ (d) $\sqrt{2}B_0$

38. A dip circle lies initially in the magnetic meridian. If it is now rotated through angle θ in the horizontal plane, then tangent of the angle of dip is changed in the ratio:

(a) $1 : \cos\theta$ (b) $\cos\theta : 1$
(c) $1 : \sin\theta$ (d) $\sin\theta : 1$

39. A current carrying coil is placed with its axis perpendicular to N-S direction. Let horizontal component of earth's magnetic field be H_0 and magnetic field inside the loop is H. If a magnet is suspended inside the loop, it makes angle θ with H. Then $\theta =$

(a) $\tan^{-1}\left(\frac{H_0}{H}\right)$ (b) $\tan^{-1}\left(\frac{H}{H_0}\right)$

(c) $\operatorname{cosec}^{-1}\left(\frac{H}{H_0}\right)$ (d) $\cot^{-1}\left(\frac{H_0}{H}\right)$

40. If a magnet is suspended at an angle 30° to the magnetic meridian, it makes an angle of 45° with the horizontal. The real dip is

(a) $\tan^{-1}(\sqrt{3}/2)$ (b) $\tan^{-1}(\sqrt{3})$
(c) $\tan^{-1}(\sqrt{3}/2)$ (d) $\tan^{-1}(2/\sqrt{3})$

Magnetic Materials

41. Among the following properties describing diamagnetism identify the property that is wrongly stated.

(a) Diamagnetic material do not have permanent magnetic moment
(b) Diamagnetism is explained in terms of electromagnetic induction
(c) Diamagnetic materials have a small positive susceptibility
(d) The magnetic moment of individual electrons neutralize each other

42. When a ferromagnetic material is heated to temperature above its Curie temperature, the material

(a) is permanently magnetized
(b) remains ferromagnetic
(c) behaves like a diamagnetic material
(d) behaves like a paramagnetic material

43. Which of the following is most suitable for the core of electromagnets?

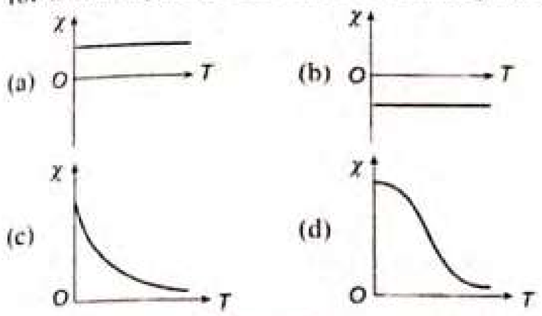
(a) Soft iron (b) Steel
(c) Copper-nickel alloy (d) Air

44. A ferromagnetic material is heated above its curie temperature. Which one is a correct statement?

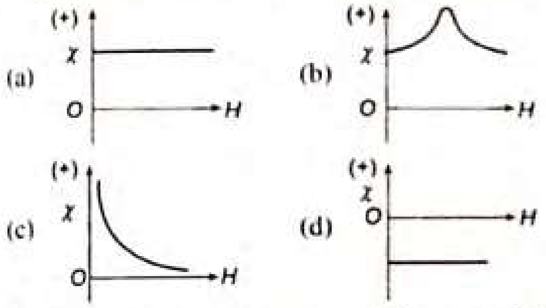
(a) Ferromagnetic domains are perfectly arranged
(b) Ferromagnetic domains becomes random
(c) Ferromagnetic domains are not influenced
(d) Ferromagnetic material changes itself into diamagnetic material

45. The material of permanent magnet has
(a) high retentively, low coercively
(b) low retentively, high coercively
(c) low retentively, low coercively
(d) high retentively, high coercively

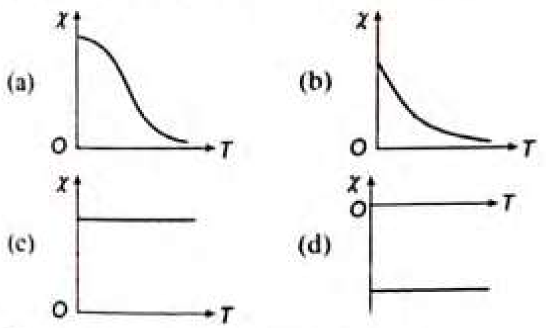
46. The variation of magnetic susceptibility (χ) with temperature for a diamagnetic substance is best represented by



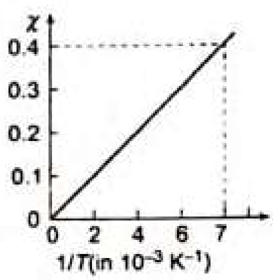
47. The variation of magnetic susceptibility (χ) with magnetising field for a paramagnetic substance is



48. The variation of magnetic susceptibility (χ) with absolute temperature T for a ferromagnetic material is



49. The χ–$1/T$ graph for an alloy of paramagnetic nature is shown in the figure. The curie constant is, then
(a) 57 K
(b) 2.8×10^{-3} K
(c) 570 K
(d) 17.5×10^{-3} K



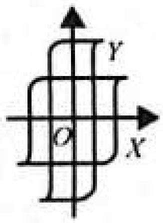
50. Which of the following is true?
(a) Diamagnetism is temperature dependent
(b) Paramagnetic is temperature dependent
(c) Paramagnetic is temperature independent
(d) None of these

51. If a magnetic substance is kept in a magnetic field, then which of the following is thrown out?
(a) Paramagnetic (b) Ferromagnetic
(c) Diamagnetic (d) Antiferromagnetic

52. A paramagnetic material is kept in a magnetic field. The field is increased till the magnetisation becomes constant. If the temperature is now decreased, the magnetisation
(a) will increase (b) decreases
(c) remain constant (d) may increase or decrease

53. The susceptibility of a ferromagnetic material is K at 27°C. At what temperature will its susceptibility be 0.5 K?
(a) 54°C (b) 327°C
(c) 600°C (d) 237°C

54. Two ferromagnetic materials X and Y have hysteresis curves of the shapes shown (see figure)



(a) X and Y are suitable for making a permanent magnet as well as an electromagnet.
(b) X is more suitable for making a permanent magnet while Y is more suitable for making an electromagnet.
(c) X is more suitable for making an electromagnet while Y is more suitable for making a permanent magnet.
(d) neither X nor Y is suitable for making either a permanent magnet or an electromagnet.

Problems Based on Mixed Concepts

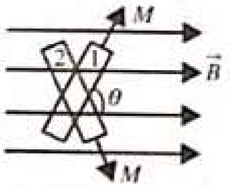
55. At a certain place, the angle of dip is 50° and the horizontal component of earth's field is 0.15 T. A beam of protons is moving vertically upwards with a speed of 3×10^5 m s^{-1}. The force on proton is [cos 50° = 0.6, sin 50° = 0.8]
(a) 4.39×10^{-15} N (b) 5.88×10^{-15} N
(c) 8.60×10^{-15} N (d) 7.20×10^{-15} N

56. The magnetic field intensity H at the centre of a long solenoid having n turns per unit length and carrying a current I, when no material is kept in it is
(a) $\mu_0 nI$ (b) nI
(c) $\frac{n}{I}$ (d) $\frac{\mu_0}{nI}$

57. A bar magnet is suspended horizontally by a torsionless wire in the magnetic meridian. In order to deflect the magnet through 30° from the magnetic meridian, the upper end of the wire has to be rotated by 180°. Now this magnet is replaced by another magnet. If it has to be deflected by 30° from the magnetic meridian, upper end of the wire has to be rotated by 270°. Ratio of the magnetic moments of two magnets is
(a) $\frac{8}{5}$ (b) $\frac{7}{6}$

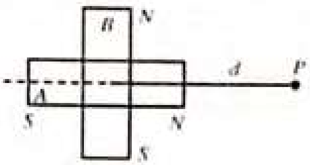
(c) $\frac{6}{7}$ (d) $\frac{5}{8}$

58. M and $M\sqrt{3}$ are the magnetic dipole moments of the two magnets which are joined to form a cross figure. The inclination of the system with the field, if their combination is suspended freely in a uniform external magnetic field B is



(a) $\theta = 30^\circ$ (b) $\theta = 45^\circ$
(c) $\theta = 60^\circ$ (d) $\theta = 15^\circ$

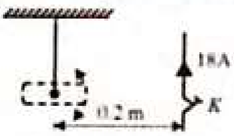
59. The magnetic induction at P, for the arrangement shown in the figure, when two similar short magnets of magnetic moment M are joined at the middle so that they are mutually perpendicular will be



(a) $\frac{\mu_0}{4\pi}\frac{M\sqrt{3}}{d^3}$ (b) $\frac{\mu_0}{4\pi}\frac{3M}{d^3}$

(c) $\frac{\mu_0 M\sqrt{5}}{4\pi d^3}$ (d) $\frac{\mu_0 2M}{4\pi d^3}$

60. Figure shows a short magnet executing small oscillations in a vibration magnetometer in earth's magnetic field having horizontal component 24 μT. The period of oscillation is 0.1 s. When the key K is closed, an upward current of 18 A is established as shown. The new time period is



(a) 0.1 s (b) 0.2 s
(c) 0.3 s (d) 0.4 s

61. A bar of mass M is suspended by two wires. Assume that a uniform magnetic field B is directed into the page. When the current through the bar is I, then the tension in each supporting wire is

(a) $\frac{Mg}{2}$ (b) $2BIL$

(c) $Mg - BIL$ (d) $\frac{Mg - BIL}{2}$

62. A coil in the shape of an equilateral triangle of side 0.02 m is suspended from its vertex such that it is hanging in a vertical plane between the pole pieces of permanent magnet producing a uniform field of 5×10^{-2} T. If a current of 0.1 A is passed through the coil, what is the couple acting?

(a) $5\sqrt{3} \times 10^{-7}$ N-m (b) $5\sqrt{3} \times 10^{-10}$ N-m

(c) $\frac{\sqrt{3}}{5} \times 10^{-7}$ N-m (d) None of these

63. A vibration magnetometer consists of two identical bar magnets placed one over the other such that they are mutually perpendicular and bisect each other. The time period of combination is 4 s. If one of the magnets is removed, find the period of other

(a) 5 s (b) 3.36 s
(c) 4.36 s (d) 5.36 s

64. A short bar magnet of magnetic moment 255 JT^{-1} is placed with its axis perpendicular to earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with earth's field. $H = 0.4 \times 10^{-4}$ T

(a) 5 m (b) 0.5 m
(c) 2.5 m (d) 0.25 m

65. A magnetic dipole is under the effect of two magnetic fields inclined at 75° to each other. One of the fields has a magnitude of 1.5×10^{-2} T. The magnets come to stable position at an angle of 30° with the direction of the above field. The magnitude of other field is

(a) $\frac{1.5}{2\sqrt{2}} \times 10^{-2}$ T (b) $\frac{1.5}{\sqrt{2}} \times 10^{-2}$ T

(c) $1.5 \times \sqrt{2} \times 10^{-2}$ T (d) 1.5×10^{-2} T

66. The dip at a place is δ. For measuring it, the axis of the dip needle is perpendicular to the magnetic meridian. If the axis of the dip needle makes angle θ with the magnetic meridian, the apparent dip will be given tan δ_1 which is equal to

(a) $\tan\delta \cos\theta$ (b) $\tan\delta \sec\theta$
(c) $\tan\delta \sin\theta$ (d) $\tan\delta \operatorname{cosec}\theta$

67. The dipole moment of each molecule of a paramagnetic gas is 1.5×10^{-23} A × m². The temperature of gas is 27°C and the number of molecules per unit volume in it is 2×10^{26} m^{-3}. The maximum possible intensity of magnetisation in the gas will be

(a) 3×10^{3} A/m (b) 4×10^{-3} A/m
(c) 5×10^{5} A/m (d) 6×10^{-4} A/m

68. Each atom of an iron bar (5 cm × 1 cm × 1 cm) has a magnetic moment 1.8×10^{-23} A m², knowing that the density of iron is 7.78×10^{3} kg^{-3} m, atomic weight is 56 and Avogadro's number is 6.02×10^{23}, the magnetic moment of bar in the state of magnetic saturation will be

(a) 4.75 Am² (b) 5.74 Am²
(c) 7.54 Am² (d) 75.4 Am²

69. An iron rod of volume 10^{-4} m³ and relative permeability 1000 is placed inside a long solenoid wound with 5 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod is

(a) 10 Am² (b) 15 Am²
(c) 20 Am² (d) 25 Am²

70. A bar magnet has coercivity 4×10^3 Am^{-1}. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. The current that should be sent through the solenoid is

(a) 2 A (b) 4 A
(c) 6 A (d) 8 A

71. The magnetic moment produced in a substance of 1 g is 6×10^{-7} ampere-metre². If its density is 5 g/cm³, then the intensity of magnetisation in A/m will be

(a) 8.3×10^6 (b) 3.0

(c) 1.2×10^{-7} (d) 3×10^{-6}

72. The area of hysteresis loop of a material is equivalent to 250 joule. When 10 kg material is magnetised by an alternating field of 50 Hz then energy lost in one hour will be if the density of material is 7.5 g/cm^3

(a) 6×10^4 J (b) 6×10^4 erg

(c) 3×10^2 J (d) 3×10^2 erg

73. The magnet of vibration magnetometer is heated so as to reduce its magnetic moment by 36%. By doing this the periodic time of the magnetometer will

(a) increases by 36% (b) increases by 25%

(c) decreases by 25% (d) decreases by 64%

ARCHIVES

1. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio T'/T is

(a) $\frac{1}{4}$ (b) $\frac{1}{2\sqrt{2}}$

(c) $\frac{1}{2}$ (d) 2 **(AIEEE 2003)**

2. Curie temperature is the temperature above which

(a) a paramagnetic material becomes ferromagnetic

(b) a ferromagnetic material becomes paramagnetic

(c) a paramagnetic material becomes diamagnetic

(d) a ferromagnetic material becomes diamagnetic **(AIEEE 2003)**

3. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. The torque needed to maintain the needle in this position will be

(a) $2W$ (b) $\sqrt{3}W$

(c) W (d) $\frac{\sqrt{3}}{2}W$ **(AIEEE 2003)**

4. The magnetic lines of force inside a bar magnet

(a) are from the south pole to the north pole of the magnet

(b) are from the north pole to the south pole of the magnet

(c) do not exist

(d) depend upon the area of cross-section of the bar magnet **(AIEEE 2003)**

5. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in an aberration magnetometer is 2 s. The magnet is cut along its length into three equal parts, which are then placed on each other with their like poles together. The time period of this combination will be

(a) 2 s (b) 2/3 s

(c) $2\sqrt{3}$ s (d) $2/\sqrt{3}$ s **(AIEEE 2004)**

6. The materials suitable for making electromagnets should have

(a) high retentivity and high coercivity

(b) low retentivity and low coercivity

(c) high retentivity and low coercivity

(d) low retentivity and high coercivity. **(AIEEE 2004)**

7. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 V, the resistance (in Ω) needed to be connected in series with the coil will be

(a) 9995 (b) 99995

(c) 10^6 (d) 10^3 **(AIEEE 2005)**

8. A magnetic needle is kept in a non-uniform magnetic field. It experiences

(a) a force but not torque

(b) a force and a torque

(c) neither a force, nor a torque

(d) a torque but not force **(AIEEE 2005)**

9. In a region, steady and uniform electric and magnetic fields are present. These fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be

(a) an ellipse (b) a circle

(c) a helix (d) a straight line

(AIEEE 2006)

10. Needles N_1, N_2, and N_3 are made of ferromagnetic, paramagnetic, and diamagnetic substances, respectively. A magnet when brought close to them will

(a) attract N_1 strongly, but repel N_2 and N_3 weakly

(b) attracts all three to them

(c) attract N_1 strongly, N_2 weakly and repel N_3 weakly

(d) attract N_1 and N_2 strongly but repel N_3

(AIEEE 2006)

11. The dimensions of magnetic field in M, L, T and C (Coulomb) is giving as

(a) $[MT^{-1}C^{-1}]$ (b) $[MT^{-2}C^{-1}]$

(c) $[MLT^{-1}C^{-1}]$ (d) $[MT^2C^{-2}]$ **(AIEEE 2008)**

12. The relative permeability and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material?

(a) $\varepsilon_r = 0.5, \mu_r = 0.5$ (b) $\varepsilon_r = 1.5, \mu_r = 1.5$

(c) $\varepsilon_r = 0.5, \mu_r = 1.5$ (d) $\varepsilon_r = 1.5, \mu_r = 0.5$

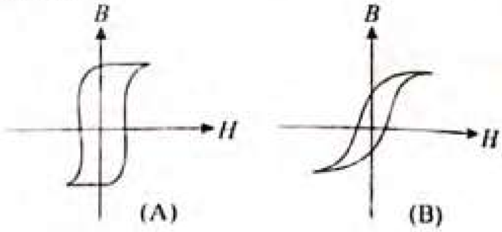
(AIEEE 2008)

13. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement I: Higher the range, greater is the resistance of ammeter.
Statement II: To increase the range of ammeter, additional shunt needs to be used across it.
(a) Statement I is true, statement II is true, statement II is not the correct explanation of statement I.
(b) Statement I is true but statement II is false.
(c) Statement I is false but statement II is true.
(d) Statement I is true but statement II is true, statement II is the correct explanation of statement I
(JEE Main 2013)

14. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2, respectively. They are placed on a horizontal table parallel to each other with their *N* poles pointing towards the south. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point *O* of the line joining their centres is close to (horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)
(a) 2.56×10^{-4} Wb/m^2 (b) 3.50×10^{-4} Wb/m^2
(c) 5.80×10^{-4} Wb/m^2 (d) 3.60×10^{-5} Wb/m^2
(JEE Main 2013)

15. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is:
(a) 3 A (b) 6 A
(c) 30 mA (d) 60 mA **(JEE Main 2014)**

16. Hysteresis loops for two magnetic materials A and B are given below:



(A) (B)

These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use
(a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers
(JEE Main 2016)

17. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is
(a) 6.98 s (b) 8.76 s
(c) 6.65 s (d) 8.89 s **(JEE Main 2017)**

ANSWER KEY

Exercises

1. (a)	2. (b)	3. (b)	4. (c)	5. (a)	6. (c)	7. (d)	8. (d)	9. (a)	10. (c)
11. (b)	12. (b)	13. (c)	14. (c)	15. (c)	16. (d)	17. (b)	18. (a)	19. (d)	20. (d)
21. (c)	22. (c)	23. (c)	24. (c)	25. (c)	26. (b)	27. (a)	28. (a)	29. (b)	30. (c)
31. (a)	32. (b)	33. (b)	34. (a)	35. (b)	36. (a)	37. (d)	38. (a)	39. (a)	40. (a)
41. (c)	42. (d)	43. (a)	44. (b)	45. (d)	46. (b)	47. (a)	48. (a)	49. (a)	50. (b)
51. (c)	52. (c)	53. (b)	54. (b)	55. (d)	56. (b)	57. (d)	58. (c)	59. (c)	60. (b)
61. (d)	62. (a)	63. (b)	64. (b)	65. (b)	66. (b)	67. (a)	68. (c)	69. (d)	70. (d)
71. (b)	72. (a)	73. (b)							

Archives

1. (c)	2. (b)	3. (b)	4. (a)	5. (b)	6. (c)	7. (a)	8. (b)	9. (d)	10. (c)
11. (a)	12. (d)	13. (c)	14. (a)	15. (a)	16. (d)	17. (c)			

Chapter 23

Electromagnetic Induction

MAGNETIC FLUX

Magnetic flux through any surface held in a magnetic field is measured as the total number of magnetic field lines crossing the surface. It is a scalar quantity.

Consider a plane area A placed in a magnetic field B as shown in figure. Area vector $\vec{A}$ makes an angle θ with the direction of magnetic field.

Now the magnetic flux passing through the area is defined as

$$\phi = \vec{B} \cdot \vec{A} \quad [\text{for uniform } \vec{B}]$$

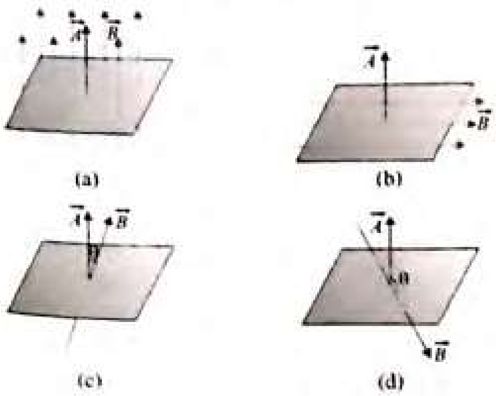
or $\quad \phi = BA\cos\theta = (B\cos\theta)\,A = B_{\perp} A$

where $B_{\perp} = B\cos\theta$ is the component of the magnetic field B perpendicular to the face of the area. Direction of area vector is normal to the face of the area.

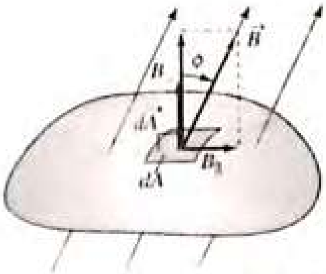
Special Cases

1. if $\theta = 0^\circ$, then $\phi = BA\cos 0^\circ \Rightarrow \phi = BA$ (maximum flux)
2. if $\theta = 90^\circ$, then $\phi = BA\cos 90^\circ \Rightarrow \phi = 0$
3. if $\theta < 90^\circ$, then $\cos\theta > 0 \Rightarrow \phi > 0$ (positive flux)
4. if $\theta > 90^\circ$, then $\cos\theta < 0 \Rightarrow \phi < 0$ (negative flux)

If the surface is not plane, we can divide any surface into elements of area dA (as shown in figure). For each element we determine $B_{\perp}$, the component of normal to the surface at the position of that element, as shown. From figure, $B_{\perp} = B\cos\phi$, where ϕ is the angle between the direction of B and a line perpendicular to the surface. In general, this component varies from point to point on the surface.



(a) (b) (c) (d)



We define the magnetic flux $d\Phi_B$ through the area element dA as

$$d\phi = B_{\perp}\, dA$$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\phi = \int B_{\perp}\, dA = \int B\cos\phi\, dA = \int \vec{B} \cdot d\vec{A}$$

(magnetic flux through a surface)

The unit of magnetic flux is tesla-meter^2 which is called weber (Wb) in honour of Wilhelm Weber. 1 Wb = 1 T m^2. Clearly, B can be measured in Wb m^{-2}. 1 Wb m^{-2} = 1 T. Sometimes it is referred to as *flux density*.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

On the basis of several experimental observations, Michael Faraday came to the following conclusions.

1. Whenever there is a relative motion between a magnet (source of magnetic field) and a closed conducting loop, electric current appears in the loop. It happens because of the change in magnetic flux associated with the loop.
2. Since emf causes current in a circuit, when loop and magnet are brought in relative motion current flows in the loop. This implies that an emf is set up in the loop. This emf is known as induced emf and its magnitude is directly proportional to the rate of change of magnetic flux through the loop with time.

Now, we come to Faradays's law of electromagnetic induction. In mathematical form, induced emf can be given by the expression $\varepsilon = -\dfrac{d\phi_B}{dt}$.

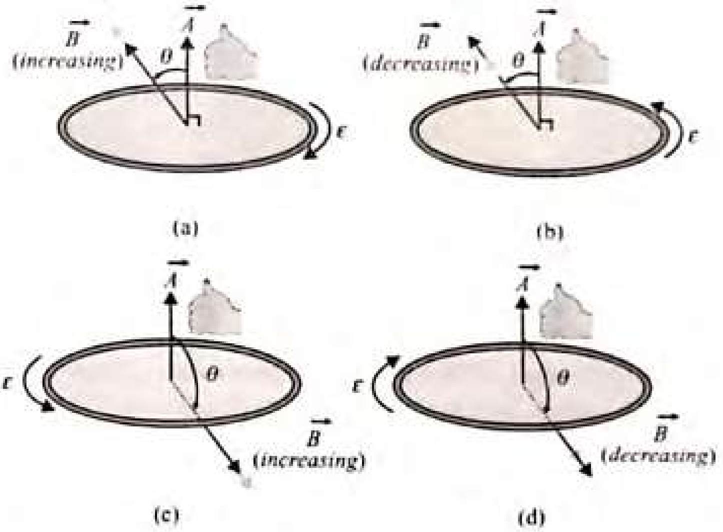
As such, Faraday's law in itself is complete to tell the magnitude and polarity of induced emf. But Lenz's rule is

commonly used to determine the polarity of induced emf or direction of induced current.

Direction of Induced emf

We can find the direction of an induced emf or current by using $\varepsilon = -d\Phi_B/dt$ together with some simple sign rules. Here is the procedure:

1. Define a positive direction for the area vector $\vec{A}$.
2. From the directions of $\vec{A}$ and the magnetic field $\vec{B}$, determine the sign of the magnetic flux Φ_B and its rate of change $d\Phi_B/dt$. Figure shows several examples.



For Figure (a):

1. $\vec{A}$ is upward so anticlock wise direction is positive.
2. $\phi = BA\cos\theta$

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\theta) \Rightarrow \varepsilon = -A\cos\theta\frac{dB}{dt} \qquad \text{(i)}$$

Here θ is acute, so cos θ is positive. B is increasing, so dB/dt is positive. Then we find that ε is negative. So ε is clockwise as shown.

For Figure (b): Here B is decreasing, so dB/dt is negative. Then from Eq. (i), ε is positive. So ε is anticlockwise as shown.

For Figure (c): Here θ is obtuse, so cos θ is negative. B is increasing, so dB/dt is positive. Then from Eq. (i), ε is positive. So ε is anticlockwise as shown.

For Figure (d): Here θ is obtuse, so cos θ is negative, B is decreasing so dB/dt is negative. Then from Eq. (i), ε is negative. So ε is clockwise as shown.

LENZ'S LAW

Direction of the Induced Current in a Circuit

Lenz's law states that "when the magnetic flux through a loop changes, a current is induced in the loop such that the magnetic field due to the induced current opposes the change in the magnetic flux through the loop".

The above rule can be systematically applied as follows to determine the direction of the induced currents.

- Identify the loop in which the induced current is to be determined.
- Determine the direction of the magnetic field in this loop (i.e., in or out of the loop).
- The direction of flux is the same as the direction of the magnetic field. Determine if the flux through the loop is increasing or decreasing (because of change in area or change in B).

Choose the appropriate current in the loop that will oppose the change in flux.

- If the flux is into the paper and increasing then the flux due to the induced current should be out of the paper.
- If the flux is into the paper and decreasing, the flux due to the induced current should be into the paper.
- If the flux is out of the paper and increasing, the flux due to the induced current should be into the paper.
- If the flux is out of the paper and decreasing, the flux due to the induced current should be out of the paper.

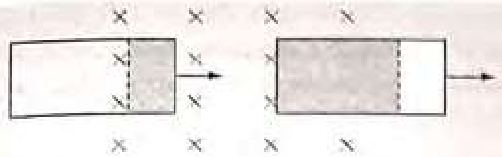
The above description is the physical interpretation of Lenz's law. We can determine the direction of the induced current mathematically by simply applying Lenz's law, $\xi_{ind} = -\frac{d\Phi_B}{dt}$ with the appropriate conventions.

The right hand sign convention used is as follows.

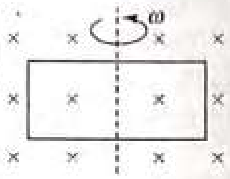
- For counterclockwise current, emf is positive.
- For clockwise current, emf is negative.
- Magnetic flux out of the paper is positive.
- Magnetic flux into the plane of the paper is negative.
- The rate of change of an increasing positive flux is positive.
- The rate of change of a decreasing positive flux is negative.
- The rate of change of an increasing negative flux is negative.
- The rate of change of a decreasing negative flux is positive.

NOTE: Note down the following points regarding Faraday's law:

1. As we have seen, induced emf is produced only when there is a change in magnetic flux passing through a loop. The flux passing through the loop is given by $\phi = BA\cos\theta$. Thus, flux can be changed in several ways:
   - The magnitude of $\vec{B}$ can change with time. In the problems if magnetic field is given as a function of time, it implies that the magnetic field is changing. Thus, $B = B(t)$.
   - The area enclosed by the loop can change with time. This can be done by pulling a loop inside (or outside) a magnetic field. By doing so, the area enclosed by the loop (hatched area) can be changed.



- The angle θ between $\vec{B}$ and the normal to the loop can change with time. This can be done by rotating a loop in a magnetic field.



- Any combination of the above can occur.

2. When the magnetic field passing through a loop is changed, an induced emf, and hence an induced current is produced in the circuit. If R is the resistance of the circuit, then induced current is given by

$$i=\frac{\varepsilon}{R}=\frac{1}{R}\left(\frac{-d\phi_B}{dt}\right)$$

Current starts flowing in the circuit, i.e., flow of charge takes place. Charge flown in the circuit in time dt will be given by

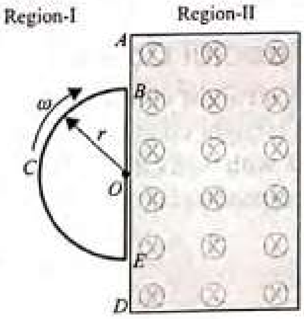
$$dq=i\,dt=\frac{1}{R}(-d\phi_B)$$

Thus, for a time interval Δt, we can write

$$\varepsilon=-\frac{\Delta\phi_B}{\Delta t},\; i=\frac{1}{R}\left(\frac{-\Delta\phi_B}{\Delta t}\right)\text{ and }\Delta q=i\Delta t=\frac{1}{R}(-\Delta\phi_B)$$

From these equations we can see that e and i are inversely proportional to Δt while Δq is independent of Δt. It depends on the magnitude of change in flux, not the time taken by it.

ILLUSTRATION 23.1 Space is divided by the line AD into two regions. Region I is field free and region II has a uniform magnetic field B directed into the plane of the paper. BCE is a semicircular conducting loop of radius r with center at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R.



(a) Obtain an expression for the magnitude of the induced current in the loop.

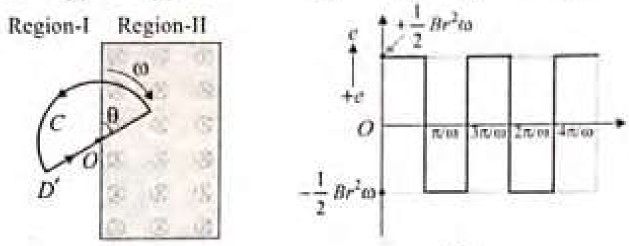
(b) Show the direction of the current when the loop is entering into region II.

(c) Plot a graph between the induced emf and the time of rotation for the two periods of rotation.

Solution.

(a) When the loop is rotated about an axis passing through center O and perpendicular to the plane of the paper, the angle between magnetic field vector $\vec{B}$ and area $\vec{A}$ is always 0°. When the loop is in region I, the magnetic flux linked with loop $= BA\cos 0 = 0$ the (since $B=0$ in region I). When the loop enters the magnetic field in region II, the magnetic flux linked with it is given by $\phi = BA$ where $A=\frac{1}{2}r^2\theta$. Therefore, emf induced

$$\varepsilon=-\frac{d\phi}{dt}=-\frac{d}{dt}(BA)=-B\frac{dA}{dt}=\frac{-Br^2}{2}\frac{d\theta}{dt}=-\frac{Br^2}{2}\omega$$



(a) (b)

As resistance of the loop is R, the current induced is given by

$$i=\frac{\varepsilon}{R}=\frac{1}{2}\frac{Br^2\omega}{R}$$

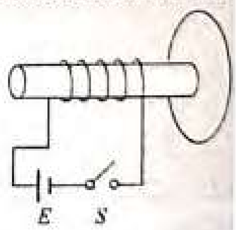
This is the required expression for current induced in the loop.

(b) According to Lenz's law, the direction of current induced is to oppose the change in magnetic flux. So, when entering into region II the field produced by the current induced must be upward. For this, the current in the loop must be anticlockwise as shown in Figure (a).

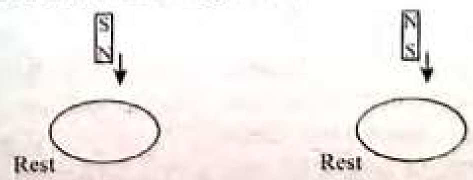
(c) When the loop enters the magnetic field, the magnetic flux linked with it increases and the emf $\varepsilon = 1/2\,Br^2\omega$ is induced in one direction. When the loop comes out of the field, the flux decreases and emf is induced in opposite sense. The graph for representing the emf induced versus time for two periods ($T = 2\pi/\omega$) is shown in Figure (b). Here we have taken anticlockwise direction as positive.

CONCEPT APPLICATION EXERCISE 23.1

1. A conducting ring is placed near a solenoid as shown in figure. Find the direction of the induced current in the ring.
   (a) At the instant the switch in the circuit containing the solenoid is closed.
   (b) After the switch has been closed for a long time.
   (c) At the instant the switch is opened.



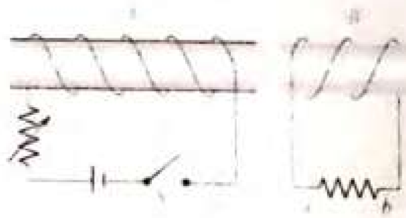
2. Identify the direction of induced current as seen from the above in the following cases.



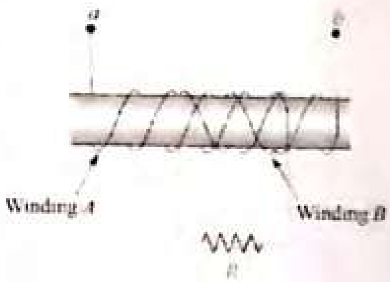
(a) (b)

(c) (d)

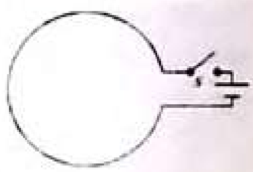
3. Using Lenz's law, determine the direction of the current in resistor ab in figure when
   (a) switch S is opened after having been closed for several minutes.
   (b) coil B is brought closer to coil A with the switch closed.
   (c) the resistance of R is decreased while the switch remains closed.



4. A cardboard tube is wrapped with two windings of insulated wire wound in opposite directions as shown in figure. Terminals a and b of winding A may be connected to a battery through a reversing switch. State whether the induced current in the resistor R is from left to right or from right to left in the following circumstances.
   (a) The current in winding A is from a to b and is increasing.
   (b) The current in winding A is from b to a and is decreasing.
   (c) The current in winding A is from b to a and is increasing.

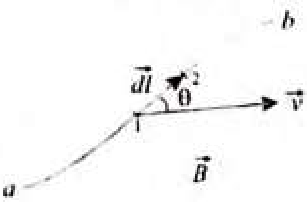


5. A small circular ring is inside a larger loop that is connected to a battery and a switch as shown in figure. Use Lenz's law to find the direction of the current induced in the small ring
   (a) just after switch S is closed;
   (b) after S has been closed for a long time;
   (c) just after S has been reopened after being closed for a long time.



MOTIONAL ELECTROMOTIVE FORCE

When a conductor moves in a magnetic field, then charges inside conductor experience magnetic force given by $\vec{F} = q\vec{v} \times \vec{B}$, where $\vec{v}$ is the velocity of conductor (here we assume that velocity of charges inside conductor is same as the velocity of conductor and random motion of charges inside the conductor is neglected). Due to this, force charges start moving in a particular direction in the conductor or we say that an emf is induced in the conductor. This emf is known as *motional* emf.



Consider a conductor of arbitrary shape as shown in figure moving in some magnetic field $\vec{B}$.

The different parts of the conductor may have different velocity. consider an element $\vec{dl}$ having velocity $\vec{v}$.

Force due to magnetic field on a charge in conductor.

$$\vec{F} = q\vec{v} \times \vec{B}$$

Work done by this force on charge q in passing through $\vec{dl}$

$$dW = \vec{F}\cdot\vec{dl} = q(\vec{v} \times \vec{B})\cdot\vec{dl}$$

So emf induced within $\vec{dl}$: $\quad d\varepsilon = \dfrac{dW}{q} = (\vec{v} \times \vec{B})\cdot\vec{dl}$

Integrate this to find net emf: $\quad \varepsilon = \int_a^b (\vec{v} \times \vec{B})\cdot\vec{dl}$

This emf is directed from a to b.

NOTE: We can also write $\varepsilon = \int_a^b \vec{B}\cdot(\vec{dl} \times \vec{v})$

As $\vec{dl} \times \vec{v}$ is the area swept per unit time by length $\vec{dl}$ and hence $\vec{B}\cdot(\vec{dl} \times \vec{v})$ is the flux of induction through the area. Therefore, the motional emf is equal to the flux of induction cut by the conductor per unit time.

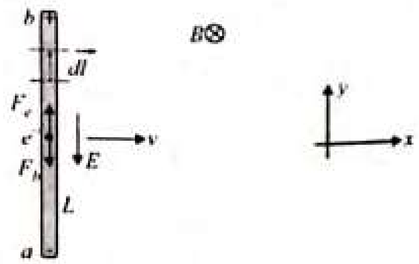
We can write the expression for induced emf in various forms:

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B})\cdot\vec{dl} = \int_a^b (\vec{B} \times \vec{dl})\cdot\vec{v} = \int_a^b (\vec{dl} \times \vec{v})\cdot\vec{B}$$

if any two out of $\vec{v}$, $\vec{B}$, and $\vec{dl}$ become parallel or antiparallel, ε will become zero.

Special Cases

A straight conductor moving in a magnetic field: Consider a straight conductor of lenght L is moving in a magnetic field B with velocity v. Here all three L, B, and v are mutually perpendicular.



We have $\vec{v} = v\hat{i}$, $\vec{B} = -B\hat{k}$, $\vec{dl} = dl\,\hat{j}$

Induced emf in the rod is given by

$$\varepsilon = \int_a^b (\vec{v} \times \vec{B})\cdot\vec{dl} = \int_a^b [v\hat{i} \times (-B\hat{k})]\cdot dl\,\hat{j}$$

$$= \int_a^b (Bv\hat{j})]\cdot dl\,\hat{j} = Bv\int_a^b dl = BvL \quad \Rightarrow \quad \varepsilon = BvL$$

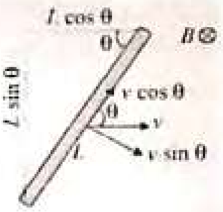
Here end b will be positive w.r.t. end a.

NOTE: If velocity is not perpendicular to length, then the velocity component perpendicular to the length of rod will be responsible for inducing emf.

So induced emf is given by: $\varepsilon = B(v \sin \theta)L$.

It can also be written as: $\varepsilon = Bv(L \sin \theta)$

where $L \sin \theta$ is the component of length perpendicular to v.



Rotating straight conductor: Consider a straight conductor AB of length L rotating about end A with angular velocity ω. Magnetic field B is perpendicular to the plane of rotation as shown in figure. We want to find emf induced in the conductor.

Take a very small element ab of length dx as shown.

Velocity of this element: $v = \omega x$

Small emf induced in this element: $d\varepsilon = (\vec{v} \times \vec{B}) \cdot \overrightarrow{dx}$

$\Rightarrow \qquad d\varepsilon = -vBdx$

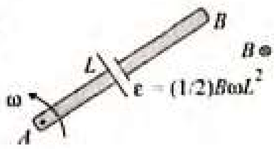
(because $\vec{v} \times \vec{B}$ will be opposite to $\overrightarrow{dx}$)

Negative sign indicates that a will be positive w.r.t. b.

Integrating: $\varepsilon = -\int_0^L Bv\,dx$

$$= -B\omega \int_0^L x\,dx = -\frac{1}{2}B\omega L^2$$

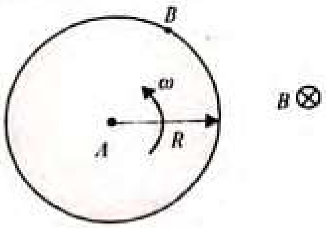
Negative sign indicates that end A will be at higher potential than B. Hence, magnitude of induced emf in the rod is $\varepsilon = \frac{1}{2}B\omega L^2$



with the polarities induced as shown in figure.

A rotating conducting disc: Consider a conducting disc of radius R rotating about its axis in a uniform magnetic field B. Magnetic field is perpendicular to the plane of disc.

Due to the rotation of the disc, electrons inside it experience magnetic force due to which emf is induced in the disc.



In the present situation electrons experience force away from the center. So center acquires positive potential w.r.t. circumference. Induced emf between center A and circumference B is

$$\varepsilon = \frac{1}{2}B\omega R^2$$

This expression is similar to the emf induced in a rotating rod with replacing L with R.

A conductor of arbitrary shape: Consider a conductor ab of arbitrary shape translating in a uniform magnetic field B.

Then induced emf in this conductor will be same as in a straight conductor connected between a and b, i.e., $\varepsilon = BvL \sin \beta$

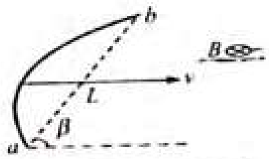
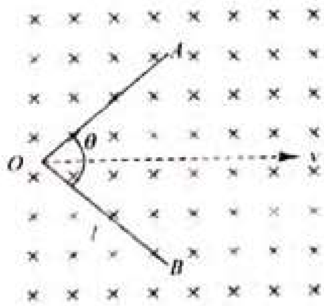


ILLUSTRATION 23.2 An angle $\angle AOB$ made of a conducting wire moves along its bisector through a magnetic field B as suggested by figure. Find the emf induced between the two free ends if the magnetic field is perpendicular to the plane at the angle.



Solution. The rod OA is equivalent to a battery of emf $vBl \sin \theta/2$. The positive charges of OA shift toward A due to the force. The positive terminal of the battery app-ears toward A. Similarly, the rod OB is equivalent to a battery of emf $vBl \sin \theta/2$ with the positive terminal toward O. The equivalent circuit is shown in figure. Clearly, the emf between points A and B is $2Blv \sin \theta/2$.

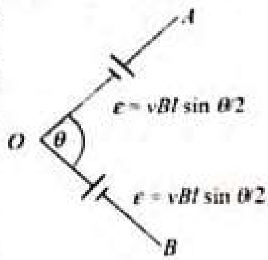
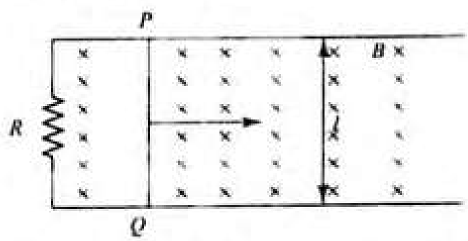


ILLUSTRATION 23.3 A conducting rod of length l slides at constant velocity v on two parallel conducting rails, placed in a uniform and constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail.



(a) Identify the cause which produces change in magnetic flux.

(b) Identify the direction of current in the loop.

(c) Determine the emf induced in the loop.

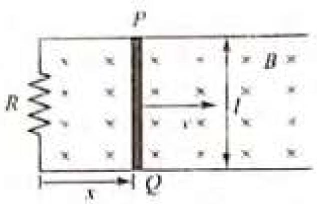
(d) Compute the electric power dissipated in the resistor.

(e) Calculate the mechanical power required to pull the rod at a constant velocity.

Solution.

(a) The change in area produces the change in magnetic flux.

(b) The direction of current in the loop is anticlockwise. As the rod moves toward right, the number of crosses in the loop increases with time and to oppose the increasing number of crosses in the loop, the current in the loop must be anticlock wise.



(c) According to Faraday's law,

$$|\varepsilon| = \frac{d\phi_B}{dt} = \frac{d}{dt}(Blx) = Bl\frac{dx}{dt} = Blv.$$

(d) The magnitude of current is $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$.

The electric power dissipated in the resistor is

$$P_{ele} = I^2R = \frac{B^2l^2v^2}{R}$$

(e) The mechanical power is $P_{mech} = F_{ext}v$.

The external force is equal and opposite to Ampere's force:

$$F_{ext} = -F_{Ampere}$$

Ampere's force is given by

$$F_{Ampere} = \int I d\vec{l} \times \vec{B} \text{ or } F_{Ampere} = BIl$$

Now, $P_{mech} = (BIl)v$.

Substituting the value of I, we get $P_{mech} = \frac{B^2l^2v^2}{R}$.

We see that mechanical power is same as electric power. It means mechanical power supplied to the rod is dissipated in the form of heat through resistor.

Motional emf when the Magnetic Field is Non-uniform

In some of the cases, motion of a conductor may be in a non-uniform magnetic field. Take the following steps while calculating motional emf.

Step 1: Determine the magnetic field at all points on the rod.

Step 2: Consider a small element at some distance from one end of the rod.

Step 3: Assuming B to be uniform over this element, calculate the potential difference across this element using the procedures outlined earlier.

Step 4: Integrate over the entrie rod to calculate the total induced emf.

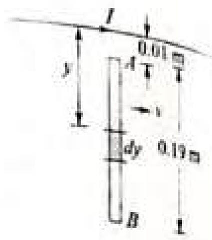
Let us learn to calculate the induced emf through the illustrations given below.

ILLUSTRATION 23.4 A copper rod of length 0.19 m is moving with uniform velocity 10 m s^{-1} parallel to a long straight wire carrying a current of 5.0 A. The rod is perpendicular to the wire with its ends at distances 0.01 and 0.2 m from it. Calculate the emf induced in the rod.

Solution. As shown in figure, consider an element of length dy at a distance y from the wire, then at this position of the element, the magnetic field due to the current-carrying wire PQ will be $B = \frac{\mu_0}{4\pi}\frac{2I}{y}$ into the plane of the paper.

So, the emf induced in the element

$$d\varepsilon = Bv\,dy = \frac{\mu_0}{4\pi}\frac{2I}{y}v\,dy$$



and hence the emf induced across the ends of the rod due to its motion in the field of the wire,

$$\varepsilon = \int_a^b d\varepsilon = \frac{\mu_0}{4\pi}2Iv\int_a^b \frac{dy}{y}, \text{ i.e., } \varepsilon = \frac{\mu_0}{4\pi}2Iv\log_e\left(\frac{b}{a}\right)$$

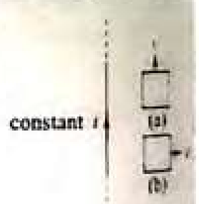
Substituting the given data with $b = (a + l)$,

$$\varepsilon = 10^{-7} \times 2 \times 5 \times 10 \log_e \frac{0.20}{0.01} = 10^{-5} \times \log_e 20$$

$$\simeq 30\ \mu\text{V}$$

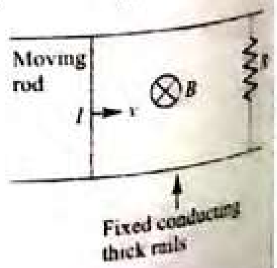
CONCEPT APPLICATION EXERCISE 23.2

1. Figure shows a long current-carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



2. A rod of length l is moving with velocity v in magnetic field B as seen in figure. Find the emf induced in all three cases.

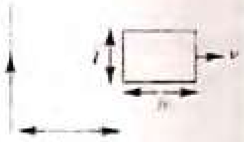
3. Figure shows a rod of length l and resistance r moving on two rails shorted by a resistance R. A uniform magnetic field B is present normal to the plane of rod and rails. Show the electrical equivalence of each branch.



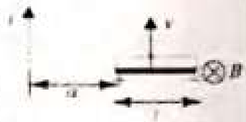
4. A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with

a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

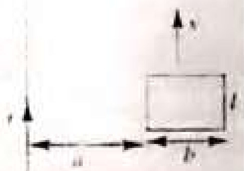
5. A rectangular loop, as shown in figure, moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.



6. A rod of length l is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the emf induced in the rod, if its nearest end is at a distance a from the wire.

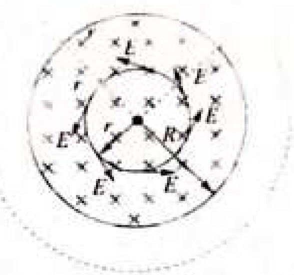


7. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the emf induced in the loop (figure) if its nearest end is at a distance a from the wire. Draw equivalent electrical diagram.



Time-Varying Magnetic Field

Consider a conducting loop of area A in a uniform but time-varying magnetic field. Rate of change of magnitude of magnetic field = dB/dt for the loop, flux linked with it = BA = ϕ (say) (take area vector directed along $\vec{B}$).



Hence, rate of change of flux ϕ is $A\dfrac{dB}{dt}$ and hence induced emf = $-A\dfrac{dB}{dt}$.

For non-zero values of dB/dt, there could be a definite current in the loop, whose direction can be obtained using Lenz's rule. For example, if $\dfrac{dB}{dt} > 0$, i.e., B is increasing with time, magnetic field produced by the induced current would oppose the existing magnetic field. Hence, the induced current would be anticlock-wise.

The current in the loop can be easily known if the resistance of the loop is known as $I = \varepsilon/R$.

In the case of motional emf, you learnt that the electric field caused due to drifting of electrons is responsible for the induced emf. Do we also have an electric field in the present case which is linked with the induced emf? The answer is partly "Yes" and partly "No."

Yes, as there is a definite field and the electric field that you know. The electric field that you learnt in electrostatics is conservative and the associated lines of force never form closed loops.

On the other hand, the field associated with the induced emf in case of time-varying magnetic field is non-conservative as, then only, we would have non-zero value for $\oint \vec{E}_n \cdot d\vec{l}$. Here $\vec{E}_n$ denotes the induced field caused by the time-varying magnetic field.

For the path described by the loop, $\varepsilon_{\text{induced}} = -\dfrac{d\phi}{dt} = \oint \vec{E}_n \cdot d\vec{l}$.

Consider a magnetic field where B (magnitude of the magnetic field) is a function of $r (r < R)$. The distance of the point from O is shown in figure. For the circular path shown in figure

For $r < R$, $E_n(2\pi r) = \pi r^2 \left|\dfrac{dB}{dt}\right|$

$$\Rightarrow \quad E_n = \frac{r}{2}\left|\frac{dB}{dt}\right|$$

For $r > R$, $E_n 2\pi r = \pi R^2 \left|\dfrac{dB}{dt}\right|$

$$\Rightarrow \quad E_n = \frac{R^2}{2r}\left|\frac{dB}{dt}\right|$$

Direction of $\vec{E}_n$ can be easily obtained as it would be responsible for the induced current when a conducting loop is placed on the given path. For example, in the present case, for $\dfrac{dB}{dt} > 0$, path is in anticlockwise direction. Variation of E with r is shown in figure.

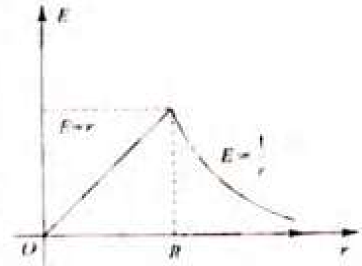


ILLUSTRATION 23.5 A thin non-conducting ring of mass m carrying a charge q can freely rotate about its axis. At the initial moment, the ring was at rest and no magnetic field was present. Then a uniform magnetic field was switched on, which was perpendicular to the plane of the ring and increased with time according to a certain law: $\dfrac{dB}{dt} = k$.

Find the angular velocity ω of the ring as a function of k.

Solution. $E = \dfrac{1}{2}R\dfrac{dB}{dt}$

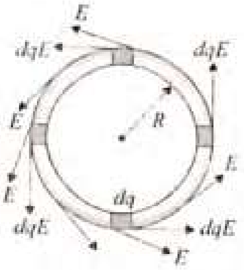
Electric force on charge dq is given by

$$dF = Edq = \frac{1}{2}R\left(\frac{dB}{dt}\right)dq$$

$$\Rightarrow \quad d\tau = RdF$$

$$\Rightarrow \quad d\tau = \frac{1}{2}R^2\left(\frac{dB}{dt}\right)dq$$

$$\Rightarrow \quad \tau = \frac{1}{2}R^2 kq$$



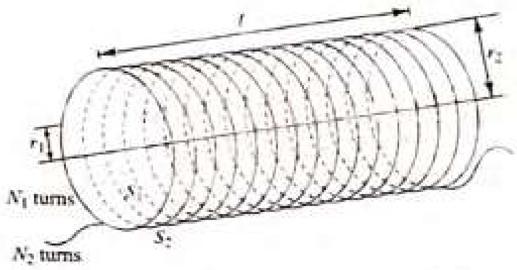
Now, $\tau = \frac{1}{2}R^2 kq$

$$\Rightarrow \quad I\alpha = \frac{1}{2}R^2 kq \quad \Rightarrow \quad mR^2\alpha = \frac{1}{2}R^2 kq$$

$$\Rightarrow \quad \alpha = \frac{kq}{2m} \quad \Rightarrow \quad \omega = \frac{kq}{2m}t$$

MUTUAL INDUCTANCE

Consider figure that shows two long coaxial solenoids each of length l. We denote the radius of the inner solenoid S_1 by r_1 and the number of turns per unit length by n_1. The corresponding quantities for the outer solenoid S_2 are r_2 and n_2, respectively.



We pass a time-varying current I_2 through S_2. This sets up a time-varying magnetic flux through S_1 which we designate by ϕ_1. Mutual inductance is the constant of proportionality given by

$$\phi_1 = M_{12}I_2 \qquad \text{(i)}$$

M_{12} is called the mutual inductance of circuit 1 with respect to current 2 and is sometimes also referred to as the coefficient of mutual induction.

From Faraday's law, the induced emf in S_1 is

$$\varepsilon_1 = -\frac{d\phi_1}{dt} = -M_{12}\frac{dI_2}{dt} \qquad \text{(ii)}$$

We now consider the reverse case. A time-varying current I_1 is passed through solenoid S_1 and the associated flux through S_2 is ϕ_2. $\phi_2 = M_{21}I_1$.

M_{21} is called the mutual inductance of circuit 2 with respect to current 1.

From Faraday's law, the induced emf in S_2 is

$$\varepsilon_2 = -\frac{d\phi_2}{dt} = -M_{21}\frac{dI_1}{dt}$$

It can be shown that $M_{21} = M_{12}$ (reciprocity theorem). Note that M is a purely geometrical quantity, depending only on the size, number of turns, relative position and relative orientation of the two coils. The SI unit of mutual inductance is called henry (H).

ILLUSTRATION 23.6 What is the mutual inductance of a system of coaxial cables carrying current in opposite directions as shown in figure. Their radii are a and b, respectively.

Solution. B between the space of the cables is $B = \mu_0 I/2\pi r$.

Ampere's law tells that B outside the cables is zero as the net current through the Amperian loop would be zero.

Taking an element of length l and thickness dr, flux through it is

$$d\phi = \frac{\mu_0 I}{2\pi r}l\,dr \quad \Rightarrow \quad \phi = \frac{\mu_0 Il}{2\pi}\int_a^b \frac{1}{r}dr = \frac{\mu_0 Il}{2\pi}\ln\frac{b}{a}$$

$$L = \frac{\phi}{I} = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right)$$

SELF-INDUCTANCE

When current is present in a circuit, it sets up a magnetic field that causes a magnetic flux through the same circuit; this flux changes when the current changes. Thus, any circuit that carries a varying current has an emf induced in it by the variation in its own magnetic field. Such an emf is called a self-induced emf. By Lenz's law, a self-induced emf always opposes the change in the current that causes the emf and so tends to make it more difficult for variations in the current to occur. For this reason, self-induced emfs can be of great importance whenever there is varying current.

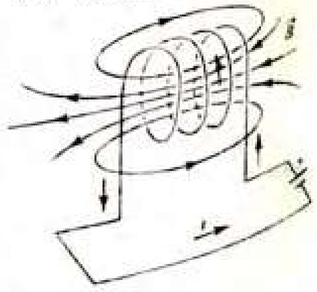
Self-induced emfs can occur in any circuit as there is always some magnetic flux through the closed loop of a current-carrying circuit. But the effect is greatly enhanced if the circuit includes a coil with N turns of wire (as shown in figure). As a result of current i, there is an average magnetic flux ϕ_B through each turn of the coil. In analogy to equation

$$M = \frac{N_2\phi_{B_2}}{i_1} = \frac{N_1\phi_{B_1}}{i_2} \quad \text{(mutual inductance)} \qquad \text{(i)}$$

We represent the self-inductance L of the circuit as

$$L = \frac{N\phi_B}{i} \quad \text{(self-inductance)} \qquad \text{(ii)}$$

To avoid any confusion with mutual inductance, self-inductance is simply called inductance. Comparing Eqs (i) and (ii), we see that the unit of self-inductance is the same as that of mutual inductance; the SI unit of self-inductance is henry (H).



If current i in the circuit changes, so does flux ϕ_B; on rearranging Eq. (ii) and taking the derivative with

respect to time, the rate of change is related by $N\dfrac{d\phi_B}{dt} = L\dfrac{di}{dt}$.

From Faraday's law for a coil with N turns, the self-induced emf is $\varepsilon = -\dfrac{N\,d\phi_B}{dt}$, so it follows that

$$\varepsilon = -L\frac{di}{dt} \text{ (self-induced emf)} \qquad \text{(iii)}$$

The minus sign in the equation is a reflection of Lenz's law; it says that self-induced emf in a circuit opposes any change in current in that circuit.

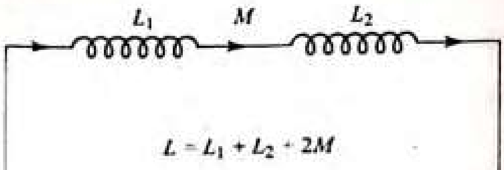
Equation (iii) also states that self-inductance of a circuit is the magnitude of the self-induced emf per unit rate of change of current. This relationship makes it possible to measure an unknown self-inductance in a relatively simple way. Change the current in the circuit at a known rate di/dt, measure the induced emf and take the ratio to determine L.

A circuit device that is designed to have a particular inductance is called an inductor. The usual circuit symbol for an inductor is ꝏꝏꝏ.

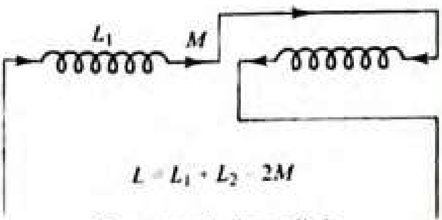
In the previous section, we have discussed the flux in one solenoid due to the current in the other. Consider the general case of current flowing simultaneously in two nearby coils. The flux linked with one coil will be due to the sum of two fluxes which exist independently. The law of superposition applies to magnetic fields. For example, Eq. (i) would generalize to $\phi_1 = L_{11}I_1 + M_{12}I_2$.

Therefore, using Faraday's law, $\varepsilon_1 = -L_1\dfrac{dI_1}{dt} - M_{12}\dfrac{dI_2}{dt}$

NOTE: In case of series grouping of two inductors, if mutual inductance is also taken into account, then $L = L_1 + L_2 \pm 2M$ as shown in figure.



Current in the two coils is in the same direction
(a)

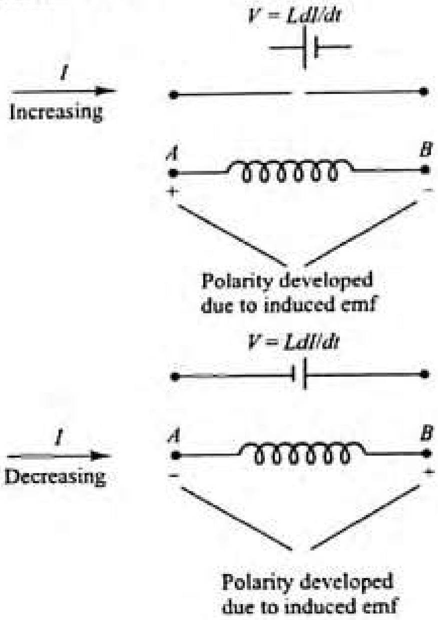


Current in the two coils is in opposite directions
(b)

APPLICATION OF KIRCHHOFF'S LAW

Suppose a current passes through an inductor from left to right and increases with time. From Lenz's law, the induced emf must oppose the change in the magnetic flux. The induced emf opposes the increasing flux by acting like a source of emf that opposes the external source of emf driving the current (as shown in figure). So we treat an inductor as a common circuit element labelled with polarity (+) and (−) marked according to Lenz's law.

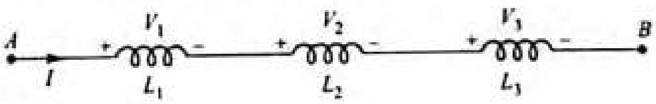
If the current is decreasing, then by Lenz's law the induced emf acts to help the decreasing flux. The inductor acts like a source of emf reinforcing the external emf driving the current. The induced emf acts to increase I (figure). Thus, we can consider an inductor as a battery whose polarity is decided according to Lenz's law.



SERIES AND PARALLEL COMBINATION OF INDUCTORS

Series Combination

Figure (a) shows a collection of inductors in series, all of them will have same current through them.



(a)

(b)

For each inductor, we have

$$V_1 = L_1\frac{dI}{dt},\ V_2 = L_2\frac{dI}{dt},\ V_3 = L_3\frac{dI}{dt}$$

We replace the series with a single equivalent inductance L_{eq} with the same potential difference V between the terminals A and B, as shown in Figure (b).

$$V = L_{eq}\frac{dI}{dt}$$

The potential difference V is the sum of potential differences.

$$V = V_1 + V_2 + V_3$$

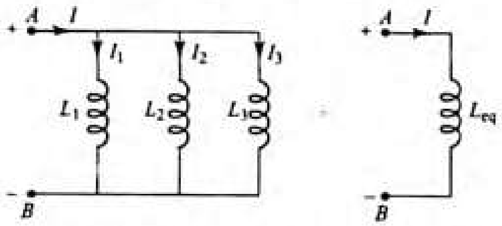
$$L_{eq}\frac{dI}{dt} = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} + L_3\frac{dI}{dt}$$

$$\Rightarrow \quad L_{eq} = L_1 + L_2 + L_3$$

Parallel Combination

When the circuit elements are in parallel, the potential difference across each circuit element is same. Thus, for each inductor, we have

$$V = L_1\frac{dI_1}{dt}, V = L_2\frac{dI_2}{dt}, V = L_3\frac{dI_3}{dt} \quad \text{(i)}$$



If we replace the combination with a single equivalent inductor L_{eq}, we get

$$V = L_{eq}\frac{dI}{dt} \quad \text{(ii)}$$

From Kirchhoff's current law (KCL), we get $I = I_1 + I_2 + I_3$. Differentiating this equation with respect to t, we get

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} + \frac{dI_3}{dt} \quad \text{(iii)}$$

On substituting derivatives from Eqs. (i) and (ii) in Eq. (iii), we have

$$\frac{V}{L_{eq}} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \text{ (inductors in parallel)}$$

ILLUSTRATION 23.7 The equivalent inductance of two inductors is 2.4 H when connected in parallel and 10 H when connected in series. What is the value of inductance of the individual inductors?

Solution. As inductances obey laws similar to the "grouping of resis-tances,"

$$L_1 + L_2 = 10 \text{ H and } \frac{L_1L_2}{L_1 + L_2} = 2.4 \text{ H}$$

Substituting the value of $(L_1 + L_2)$ from first expression into the second, $L_1L_2 = (2.4)(L_1 + L_2) = 2.4 \times 10 = 24$

so that $(L_1 - L_2)^2 = (L_1 + L_2)^2 - 4L_1L_2$, i.e., $L_1 - L_2 = 2$ H and as $L_1 + L_2 = 10$ H, so $L_1 = 6$ H and $L_2 = 4$ H

COMBINATION OF INDUCTORS WITH RESISTORS

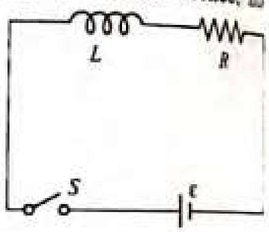
An *LR* circuit is analyzed in three states:

- Initial state, i.e., just after closing the switch or just after opening the switch.
- Transient state or instantaneous state, i.e., any time after closing or opening the switch.
- Steady state, i.e., a long time after closing or opening the switch. In this state, current in the inductor does not vary with time, i.e., $dI/dt = 0$.

Initial State

At $t = 0$, the current tends to increase very rapidly, therefore opposition produced by the inductor is infinite. Hence, no current flows through the circuit at the instant of closing the switch. The entire voltage is dropped across the inductor and no voltage is dropped across the resistor.

i.e., $V_L = \varepsilon$, $V_R = 0$ at $t = 0$



Steady State

At $t = \infty$, the current has risen to its maximum value, and the inductor does not produce any opposition. No voltage is dropped across the inductor, the entire voltage is dropped across the resistor.

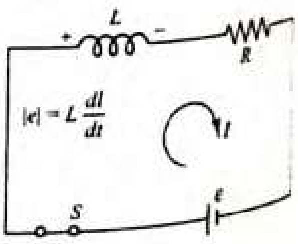
i.e., $V_L = 0$; $V_R = \varepsilon$ at $t = \infty$

Transient State

At any instant $(0 < t < \infty)$, both inductor and resistor share the total applied voltage.

Rise of Current

A series of combination of an inductor L and a resistor R is connected across a cell of emf ε through a switch S as shown in figure. When the switch is closed, current starts increasing in the inductor. This causes an induction of emf in the inductor. The induced emf opposes the growth of current in the circuit. Let at any time t, the current in the circuit be I.



From loop rule, we obtain

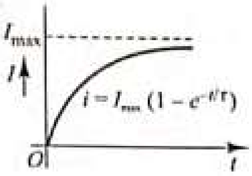
$$\varepsilon = L\frac{dI}{dt} + IR$$

$$\Rightarrow \quad \varepsilon - IR = L\frac{dI}{dt} \quad \Rightarrow \quad \frac{1}{L}\int_0^t dt = \int_0^I \frac{dI}{\varepsilon - IR}$$

$$\Rightarrow \quad \frac{t}{L} = \left|\frac{\ln[\varepsilon - IR]}{-R}\right|_0^I$$

$$\Rightarrow \quad -\frac{tR}{L} \ln\frac{\varepsilon - IR}{\varepsilon}$$

$$\Rightarrow \quad I = \frac{\varepsilon}{R}\left(1 - e^{-\frac{tR}{L}}\right) = I_{max}\left[1 - e^{-t/\tau}\right]$$



Current in the circuit increases with time

Here, I represents the instantaneous current in the circuit.

NOTE:

1. Final current in the circuit ε/R, which is independent of L.
2. After one time constant, current in the circuit = 63% of the final current (verify yourself).
3. More time constant in the circuit implies slower rate of change of current.
4. If there is any change in the circuit containing inductor, then there is no instantaneous effect on the flux of inductor, $L_1 i_1 = L_2 i_2$.

Decay of Current

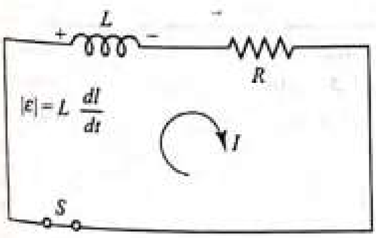
In this case, the source of emf is disconnected from the circuit (figure). Let initial current in the circuit be I_0.

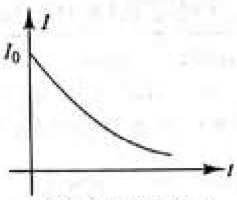
$$\Rightarrow \quad -\frac{LdI}{dt} - IR = 0$$

$$\Rightarrow \quad \int_{I_0}^{I}\frac{dI}{I} = -\frac{R}{L}\int_0^t dt$$

$$\Rightarrow \quad I = I_0 e^{-\frac{tR}{L}}$$

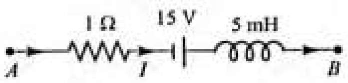
(L/R) is called time constant as its dimension is same as that of time.





Variation of electric current with time

ILLUSTRATION 23.8 The network shown in figure is a part of a complete circuit. What is the potential difference $V_B - V_A$, when current I is 5 A and is decreasing at a rate of 10^3 A s^{-1}?



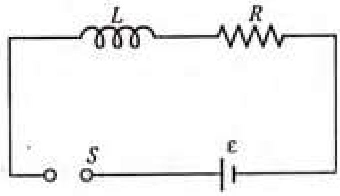
Solution. In accordance with the law of potential distribution, for the given network, $V_A - IR + E - L\frac{dI}{dt} = V_B$

And here I is decreasing (i.e., dI/dt is negative).

$$V_B - V_A = -5 \times 1 + 15 - 5 \times 10^{-3}(-10^3)$$

$$V_B - V_A = -5 + 15 + 5 = 15 \text{ V}$$

ILLUSTRATION 23.9 In an LR circuit as shown in figure, when the switch is closed, how much time will it take for the current to grow to a value n times the maximum value of current(where $n < 1$)?



Solution. We know that $i = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$

$$i = n\frac{\varepsilon}{R} \quad \text{(given)}$$

$$n\frac{\varepsilon}{R} = \frac{\varepsilon}{R}(1 - e^{-t/\tau})$$

or $\quad e^{-t/\tau} = 1 - n$

or $\quad t = \tau \ln\left(\frac{1}{1-n}\right)$

ENERGY STORED IN THE MAGNETIC FIELD OF AN INDUCTOR

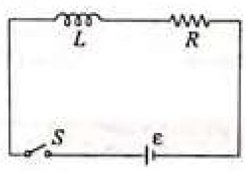
As $\varepsilon = IR + \frac{LdI}{dt}$, Hence $\varepsilon I = I^2R + LI\frac{dI}{dt}$

εI is the power supplied by the battery, I^2R is the electrical power dissipated in the resistance and $LI\frac{dI}{dt}$ is the rate of energy stored in the inductor.Therefore,

$\varepsilon Idt = I^2Rdt + LIdI$

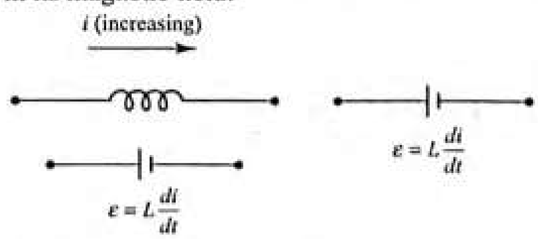
$\Rightarrow$ Energy stored in the inductor is

$$U_B = \int_0^I LIdI = \frac{1}{2}LI^2$$



Alternatively

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.



An increasing current in an inductor causes an emf between its terminals.

Work done per unit time is power $P = \dfrac{dW}{dt} = -\varepsilon i = -Li\dfrac{di}{dt}$

From $dW = dU$ or $\dfrac{dW}{dt} = -\dfrac{dU}{dt}$, we have

$$\frac{dU}{dt} = Li\frac{di}{dt} \quad \text{or} \quad dU = Li\,di$$

The total energy U supplied while the current increases from zero to a final value i is $U = L\int_0^i i\,di = \dfrac{1}{2}Li^2$

$$\therefore \quad U = \frac{1}{2}Li^2$$

The energy in an inductor is actually stored in the magnetic field within the coil. We can develop magnetic energy density u (energy stored per unit volume) analogous to those we obtained in electrostatics. We will concentrate on one simple case of an ideal long cylindrical solenoid. For a long solenoid, its magnetic field can be assumed completely within the solenoid. Energy U stored in the solenoid with current i is

$$U = \frac{1}{2}Li^2 = \frac{1}{2}(\mu_0 n^2 V)i^2$$

as $\quad L = \mu_0 n^2 V$

The energy per unit volume is $u = U/V$.

$$u = \frac{U}{V} = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\frac{(\mu_0 ni)^2}{\mu_0} = \frac{1}{2}\frac{B^2}{\mu_0}$$

as $\quad B = \mu_0\, \text{m}ni$

Thus, $u = \dfrac{1}{2}\dfrac{B^2}{\mu_0}$

This expression is similar to $u = \dfrac{1}{2}\varepsilon_0 E^2$ used in electrostatics. Although we have derived it for one special situation, it turns out to be correct for any magnetic field configuration.

ILLUSTRATION 23.10 Consider the RL circuit in figure. When the switch is closed in position 1 and opens in position 2, electrical work must be performed on the inductor and on the resistor. The energy stored in the inductor is for the magnetic field inside it which increases as I increases. In the resistor energy appears as heat.

(a) What is the ratio of P_L/P_R of the rate at which energy is stored in the inductor to the rate at which energy is dissipated in the resistor?

(b) Express the ratio P_L/P_R as a function of time.

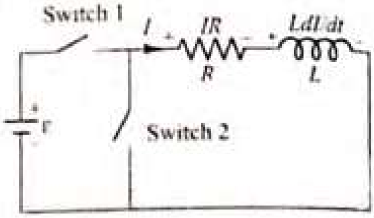
(c) If the time constant of circuit is t, what is the time at which $P_L = P_R$?

Solution.

(a) The energy stored in the inductor is $U_L = \dfrac{1}{2}LI^2$

$$\text{Power } P_L = \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) = LI\frac{dI}{dt} \qquad \text{(i)}$$

The power of a resistor $P_R = I^2R$ (ii)



$$\text{So the ratio is } \frac{P_L}{P_R} = \frac{LI\,dI/dt}{I^2R} = \frac{\tau(dI/dt)}{I} \qquad \text{(iii)}$$

(b) As $I = \dfrac{E}{R}\left[1 - e^{-(t/\tau)}\right]$ (iv)

$$\frac{dI}{dt} = \frac{d}{dt}\left[\frac{E}{R}\left(1 - e^{-(t/\tau)}\right)\right] = \frac{E}{R\tau}e^{-(t/\tau)} \qquad \text{(v)}$$

Inserting the value of dI/dt and I in Eq. (iii), we have

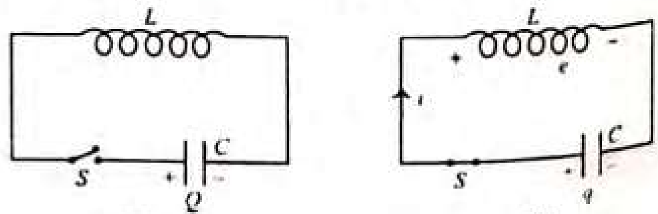
$$\frac{P_L}{P_R} = \tau\frac{\dfrac{E}{R\tau}e^{-t/\tau}}{\dfrac{E}{R}\left(1 - e^{-t/\tau}\right)} = \frac{e^{-(t/\tau)}}{1 - e^{-(t/\tau)}} = \frac{1}{e^{(t/\tau)} - 1} \qquad \text{(vi)}$$

When $t = 0$, this ratio tends to infinity, due to large initial value of dI/dt and small initial value of I. When t tends to infinity, after many time constants, the current tends to zero. As a result, P_L vanishes and the ratio goes to zero.

(c) From Eq. (vi), $P_L = P_R$ when $\dfrac{1}{e^{(t/\tau)} - 1} = 1$ which on simplification yields $e^{(t/\tau)} = 2$, $\dfrac{t}{\tau} = \ln 2 \Rightarrow t = 0.693\tau$

LC OSCILLATIONS

Consider a capacitor C charged to Q initially is connected to a pure inductor L through a switch as shown in figure. The circuit has no resistance. Let at $t = 0$, switch S be closed.



Due to the presence of inductor, current will not be developed instantly. It will take time to grow. We want to find expressions for current i in the circuit and charge q on the capacitor as a function of time t.

Let at any time $t = t$, $i = i$, and $q = q$.

Initial condition: at $t = 0$, $i = 0$, $q = Q$

We can write $i = -\dfrac{dq}{dt}$ (i)

Negative sign is because charge on the capacitor decreases with time.

Induced emf ε in the inductor with the polarities as shown, assuming that i increases with time, is

$$\varepsilon = L\frac{di}{dt} \qquad \text{(ii)}$$

Applying Kirchhoff's law in circuit: $\varepsilon = \frac{q}{C} \Rightarrow L\frac{di}{dt} = \frac{q}{C}$

Differentiating: $L\frac{d^2i}{dt^2} = \frac{1}{C}\frac{dq}{dt} \Rightarrow \frac{d^2i}{dt^2} = -\frac{i}{LC}$

$$\Rightarrow \frac{d^2i}{dt^2} = -\left(\frac{1}{\sqrt{LC}}\right)^2 i$$

General solution of this equation is

$$i = i_0 \sin(\omega t + \phi) \qquad \text{(iii)}$$

where $\omega = 1/\sqrt{LC}$; i_0 and ϕ are constants whose values are to be found using some given conditions.

We know that $i = 0$ at $t = 0$, putting this condition in Eq. (iii), we get $\phi = 0°$.

Equation (iii) reduces to $i = i_0 \sin \omega t$ (iv)

$$\Rightarrow -\frac{dq}{dt} = i_0 \sin \omega t$$

$$\Rightarrow \int dq = -\int i_0 \sin \omega t \, dt$$

$$\Rightarrow q = \frac{i_0}{\omega}\cos \omega t + C \qquad \text{(v)}$$

At $t = 0$, $q = Q$, putting in Eq. (v),

$$Q = \frac{i_0}{\omega}\cos 0° + C \Rightarrow C = Q - \frac{i_0}{\omega}$$

$$\Rightarrow q = Q - \frac{i_0}{\omega} + \frac{i_0}{\omega}\cos \omega t \qquad \text{(vi)}$$

Equations (iv) and (vi) give current and charge as a function of time where i_0 is the maximum current in the circuit which is still to be calcualted.

Energy considerations: As we have assumed that there is no resistance in the circuit, so there is no loss of energy in the circuit. Initially the whole energy is in the capacitor. As the time passes, charge on capacitor decreases and current increases. At a certain time charge on the capacitor will become zero and current in the circuit will be maximum which is i_0. At this time whole of the energy will be stored in the inductor. Apply conservation of energy:

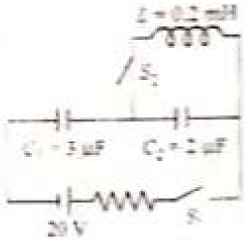
$$\frac{1}{2}Li_0^2 = \frac{Q^2}{2C} \Rightarrow i_0 = \frac{Q}{\sqrt{LC}} \Rightarrow i_0 = Q\omega$$

Put the value of i_0 in Eq. (vi), we get

$$q = Q\cos \omega t \qquad \text{(vii)}$$

Equation (vii) gives charge as a function of time.

ILLUSTRATION 23.11 The circuit shown in figure is in the steady state with switch S_1 closed. At $t = 0$, S_1 is opened and switch S_2 is closed.



(a) Derive an expression for the charge on capacitor C_2 as a function of time.

(b) Determine the first instant t, when the energy in the inductor becomes one-third of that in the capacitor.

Solution.

(a) In the steady state, C_1 and C_2 are in series arrangement, their equivalent is

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = 1.2\ \mu F$$

Charge on the capacitor C_2, $Q_0 = C_{eq}V = 1.2 \times 20 = 24\ \mu C$. When S_1 is opened, S_2 is closed. Capacitor C_2 starts discharging through the inductor and let at any time t, charge on the capacitor be Q. Then we know that

$$Q = Q_0 \cos \omega t$$

$$U_E + U_B = \frac{Q_0^2}{2C_2}$$

where $\omega = \frac{1}{\sqrt{LC_2}} = \frac{1}{\sqrt{0.2 \times 10^{-3} \times 2 \times 10^{-6}}} = 50{,}000\ \text{rad s}^{-1}$

At the time $t = t_1$, $U_B = \frac{1}{3}U_E$, but $U_B + U_E = \frac{Q_0^2}{2C_2}$

Hence, $U_E = \frac{3}{4}\left(\frac{1}{2}\frac{Q_0^2}{C_2}\right) \Rightarrow \frac{1}{2}\frac{Q^2}{C_2} = \frac{3}{4}\left(\frac{1}{2}\frac{Q_0^2}{C_2}\right)$

$$Q = \frac{\sqrt{3}}{2}Q_0 \quad \text{or} \quad Q_0 \cos \omega t_1 = \frac{\sqrt{3}}{2}Q_0$$

$$\omega t_1 = \frac{\pi}{6} \quad \text{or} \quad t_1 = \frac{\pi}{6\omega} = 1.05 \times 10^{-5} = 10.5\ \mu s$$

Hence, the current through battery is $I = \frac{E}{R}[1 - e^{-(R/L)t} + e^{-t/RC}]$

If the current has to reach its final value E/R instantaneously, the exponential terms must cancel out, i.e.,

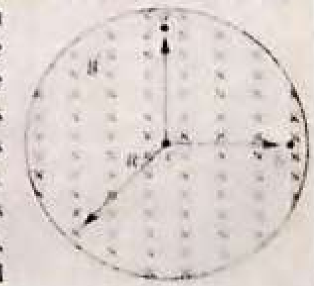
$e^{-t/\tau_L} = e^{-t/\tau_C}$, which is possible if $\tau_L = \tau_C$

$$L/R = RC, R = \sqrt{(L/C)}$$

For all $t > 0$, this is the desired result.

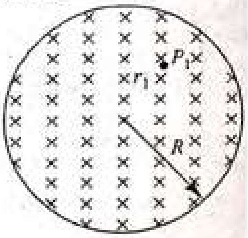
CONCEPT APPLICATION EXERCISE 23.3

1. The magnetic field at all points within a circular region of radius R is uniform in space and directed into the plane of the page in figure (the region could be a cross section inside the windings of a long, straight solenoid). If the magnetic field is increasing at a rate dB/dt, what are the magnitude and

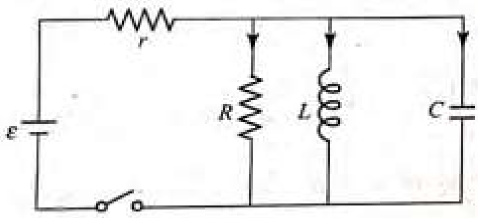


direction of the force on a stationary positive point charge q located at points a, b, and c? (Point a is a distance r above the center of the region, point b is at a distance r to the right of the center, and point c is at the center of the region.)

2. A magnetic field directed into the page changes with time according to $B = (0.0300t^2 + 1.40)$T, where t is in seconds. The field has a circular cross section of radius $R = 2.50$ cm. What are the magnitude and direction of the electric field at point P_1 when $t = 3.00$ s and $r_1 = 0.0200$ m?



3. Figure shows an LCR circuit. When the switch is closed, the currents through resistor R, inductor L, and capacitor C are I_1, I_2, and I_3, respectively. Determine the values of I_1, I_2, and I_3.

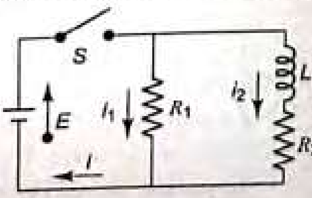


(a) at $t = 0$ (b) at $t = \infty$

4. It has been proposed to use large inductors as energy storage devices.
   (a) How much electrical energy is converted to light and thermal energy by a 200-Ω light bulb in one day?
   (b) If the amount of energy calculated in part (a) is stored in an inductor in which the current is 80.0 A, what is the inductance?

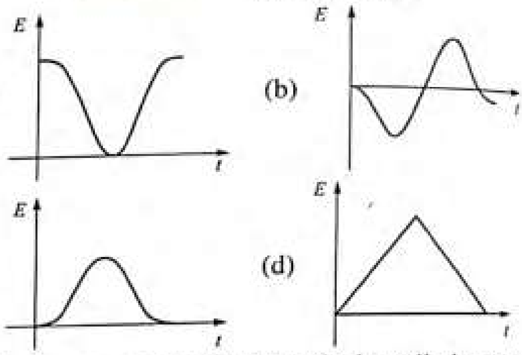
5. A 1 kΩ resistor is connected in series with a 10 mH inductor, a 30 V battery and an open switch. At time $t = 0$, the switch is suddenly closed.
   (a) What is the maximum current in this circuit and when does it occur?
   (b) What are the voltage drops across the inductor and across the resistor 20 μs after the switch is closed ?
   (c) On a single set of axes, sketch the voltage across the resistor and the voltage across the inductor as functions of time. Also, sketch a graph of the current in the circuit as a function of time.

6. In the circuit shown in figure, E = 10 V, R_1 = 5 Ω, R_2 = 10 Ω, and L = 5 H. For the two separate conditions, (i) switch S is just closed and (ii) switch S is closed for a long time, calculate
   (a) current i_1 through R_1,
   (b) current i_2 through R_2,
   (c) current i through the switchs,
   (d) the potential difference across R_2,
   (e) the potential difference across L,
   (f) di_2/dt.



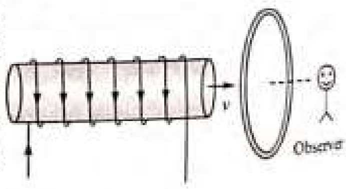
SOLVED EXAMPLES

1. The variation of induced emf (E) with time (t) in a coil if a short bar magnet is moved along its axis with a constant velocity is best represented as



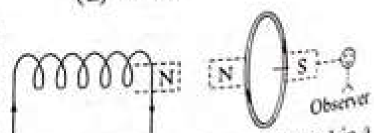
Sol. (b) As the magnet moves towards the coil, the magnetic flux increases (nonlinearly). Also there is a change in polarity of induced emf when the magnet passes on to the other side of the coil.

2. A current carrying solenoid is approaching a conducting loop as shown in the figure. The direction of induced current as observed by an observer on the other side of the loop will be

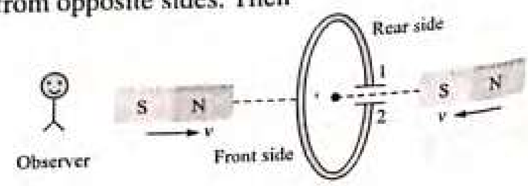


(a) anticlockwise (b) clockwise
(c) east (d) west

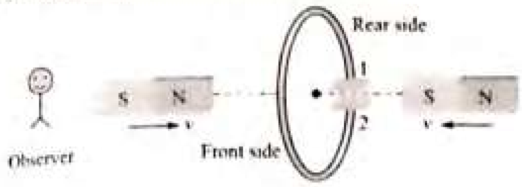
Sol. (b) The direction of current in the solenoid is anti-clockwise as seen by observer. On displacing it towards the loop a current in the loop will be induced in a direction so as to oppose the approach of solenoid. Therefore the direction of induced current as observed by the observer will be clockwise.



3. The north and south poles of two identical magnets approach a coil, containing a condenser, with equal speeds from opposite sides. Then

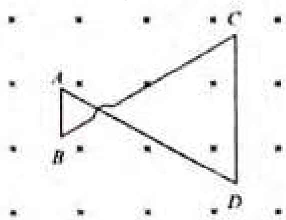


(a) plate 1 will be negative and plate 2 positive
(b) plate 1 will be positive and plate 2 negative
(c) both the plates will be positive
(d) both the plates will be negative



Sol. (b) By the movement of both the magnets, current will be anticlockwise, as seen from left side i.e. plate 1 will be positive and 2 will be negative.

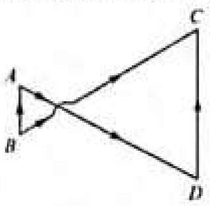
4. A conducting wire frame is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced current in wires AB and CD are



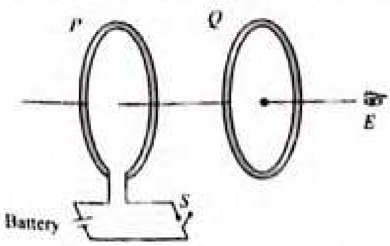
(a) B to A and D to C
(b) A to B and C to D
(c) A to B and D to C
(d) B to A and C to D

Sol. (a) Inward magnetic field (×) increasing. Therefore, induced current in both the loops should be anticlockwise. But as the area of loop on right side is more, induced emf in this will be more compared to the left side loop $\left(e=-\dfrac{d\varphi}{dt}=-A.\dfrac{dB}{dt}\right)$.

Therefore net current in the complete loop will be in a direction shown below. Hence only option (a) is correct.



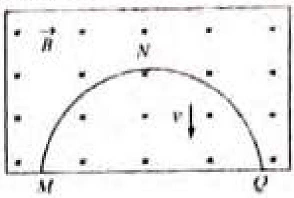
5. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q1} flows in Q. Then the directions of I_{Q1} and I_{Q1} (as seen by E) are



(a) respectively clockwise and anticlockwise
(b) both clockwise
(c) both anticlockwise
(d) respectively anticlockwise and clockwise

Sol. (d) When switch S is closed magnetic field lines passing through Q increases in the direction from right to left. So, according to Lenz's law induced current in Q i.e. I_{Q1} will flow in such a direction so that the magnetic field lines due to I_{Q1} passes from left to right through Q. This is possible when I_{Q1} flows in anticlockwise direction as seen by E. Opposite is the case when switch S is opened i.e. I_{Q2} will be clockwise as seen by

6. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction B. At the position MNQ, the speed of the ring is v and the potential difference developed across the ring is



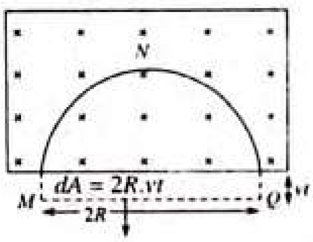
(a) zero
(b) $Bv\pi R^2/2$ and M is at higher potential
(c) πRBv and Q is at higher potential
(d) $2RBv$ and Q is at higher potential

Sol. (d) Rate of decrease of area of the semicircular ring

$$-\frac{dA}{dt}=(2R)\,v$$

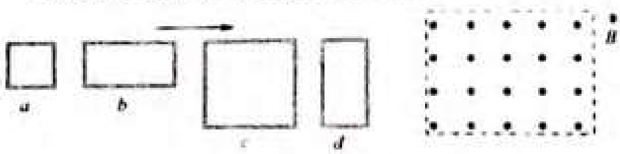
According to Faraday's law of induction induced emf

$$e=-\frac{d\varphi}{dt}=-B\frac{dA}{dt}=-B\,(2Rv)$$



The induced current in the ring must generate magnetic field in the upward direction. Thus Q is at higher potential

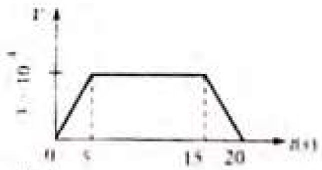
7. The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field $\vec{B}$ (directed out of the page) at the same constant velocity. Rank the four loops according to the maximum magnitude of the e.m.f. induced as they move through the field, greatest first

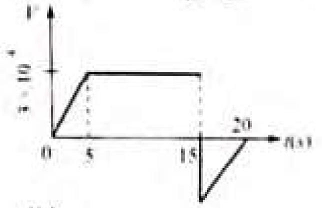


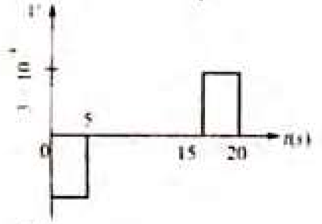
(a) $(\varepsilon_c=\varepsilon_d)<(\varepsilon_a=\varepsilon_b)$ (b) $(\varepsilon_c=\varepsilon_d)>(\varepsilon_a=\varepsilon_b)$
(c) $\varepsilon_c>\varepsilon_d>\varepsilon_b>\varepsilon_a$ (d) $\varepsilon_c<\varepsilon_d<\varepsilon_b<\varepsilon_a$

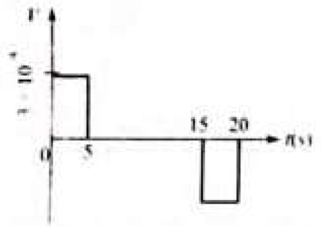
Sol. (b) Emf induces across the length of the wire which cuts the magnetic field. (Length of c = Length d) > (Length of $a = b$). So $(\varepsilon_c = \varepsilon_d) > (\varepsilon_a = \varepsilon_b)$

8. A square loop of side 5 cm enters a magnetic field with 1 cms^{-1}. The front edge enters the magnetic field at $t = 0$ then which graph best depicts emf

(a) 

(b) 

(c) 

(d) 

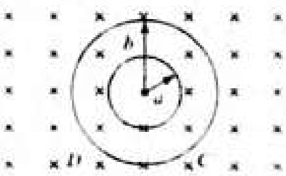
Sol. (c) When loop is entering in the field, magnetic flux (i.e. ×) linked with the loop increases so induced emf in it $\varepsilon = Bvl$

$= 0.6 \times 10^{-2} \times 5 \times 10^{-2} = 3 \times 10^{-4}$V (Negative).

When loop completely entered in the field (after 5 sec) flux linked with the loop remains constant so $\varepsilon = 0$.

After 15 sec, loop begins to exit out, linked magnetic flux decreases so induced emf $\varepsilon = 3 \times 10^{-4}$V (Positive).

9. Plane figures made of thin wires of resistance R = 50 milli ohm/metre are located in a uniform magnetic field perpendicular into the plane of the figures and which decrease at the rate dB/dt = 0.1 mT/s. Then currents in the inner and outer boundary are.



(The inner radius a = 10 cm and outer radius b = 20 cm)

(a) 10^{-4} A (Clockwise), 2×10^{-4} A (Clockwise)
(b) 10^{-4} A (Anticlockwise), 2×10^{-4} A (Clockwise)
(c) 2×10^{-4} A (clockwise), 10^{-4} A (Anticlockwise)
(d) 2×10^{-4} A (Anticlockwise), 10^{-4} A (Anticlockwise)

Sol. (a) Current in the inner coil $i = \dfrac{\varepsilon}{R} = \dfrac{A_1}{R_1}\dfrac{dB}{dt}$

Length of the inner coil = $2\pi a$

So its resistance $R_1 = 50 \times 10^{-3} \times 2\pi(a)$

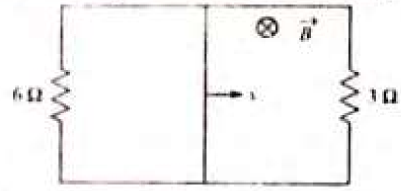
$$i_1 = \frac{\pi a^2}{50 \times 10^{-3} \times 2\pi(a)} \times 0.1 \times 10^{-3} = 10^{-4}\text{ A}$$

According to Lenz's law, direction of i_1 is clockwise.

Induced current in outer coil $i_2 = \dfrac{\varepsilon_2}{R_2} = \dfrac{A_2}{R_2}\dfrac{dB}{dt}$

$$\Rightarrow \quad i_2 = \frac{\pi b^2}{50 \times 10^{-3} \times (2\pi b)} \times 0.1 \times 10^{-3} = 2 \times 10^{-4} A\,(CW)$$

10. A rectangular loop with a sliding connector of length l = 1.0 m is situated in a uniform magnetic field B = 2T perpendicular to the plane of loop. Resistance of connector is $r = 2\ \Omega$. Two resistance of 6 Ω and 3 Ω are connected as shown in figure. The external force required to keep the connector moving with a constant velocity v = 2 m/s is



(a) 6 N
(b) 4 N
(c) 2 N
(d) 1 N

Sol. (c) Motional emf

$\varepsilon = Bvl \Rightarrow \varepsilon = 2 \times 2 \times 1 = 4$V

This acts as a cell of emf ε = 4V and internal resistance $r = 2\Omega$.
This simple circuit can be drawn as follows

Current through the connector $i = \dfrac{4}{2+2} = 1$ A

∴ Magnetic force on connector

$F_m = Bil = 2 \times 1 \times 1 = 2N$

(Towards left)

11. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($L > l$). The loop are coplanar and their centre coincide. The mutual inductance of the system is proportional to

(a) l/L
(b) l^2/L
(c) L/l
(d) L^2/l

Sol. (b) Magnetic field produced due to large loop

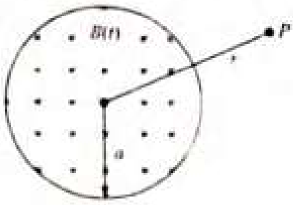
$$B = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}i}{L}$$

Flux linked with smaller loop

$$\varphi = B(l^2) = \frac{\mu_0}{4\pi}\frac{8\pi i l^2}{L}$$

$$\therefore\ \varphi = Mi \Rightarrow M = \frac{\varphi}{i} = \frac{\mu_0}{4\pi}\cdot\frac{8\sqrt{2}l^2}{L} \Rightarrow M \propto \frac{l^2}{L}$$

12. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region

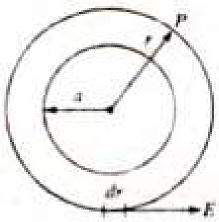


(a) is zero
(b) decreases as $1/r$
(c) increases as r
(d) decreases as $1/r^2$

Sol. (b) Construct a concentric circle of radius r. The induced electric field (E) at any point on the circle is equal to that at P. For this circle

$$\oint \vec{E}.d\vec{l} = \left|\frac{d\varphi}{dt}\right| = A\left|\frac{dB}{dt}\right|$$

or $E \times (2\pi r) = \pi a^2 . \left|\frac{dB}{dt}\right|$



$$\Rightarrow \quad E = \frac{a^2}{2r}\left|\frac{dB}{dt}\right| \Rightarrow E \propto \frac{1}{r}$$

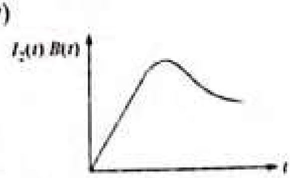
13. A coil of wire having finite inductance and resistance has a conducting ring placed coaxially within it. The coil is connected to a battery at time $t = 0$, so that a time-dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring, and $B(t)$ is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time ($t > 0$), the product $I_2(t)\,B(t)$

(a) Increases with time
(b) Decreases with time
(c) Does not vary with time
(d) Passes through a maximum

Sol. (d) Using k_1, k_2 etc, as different constants.

$I_1(t) = k_1[1 - e^{-t/\tau}]$, $B(t) = k_2 I_1(t)$

$$I_2(t) = k_3\frac{dB(t)}{dt} = k_4 e^{-t/\tau}$$

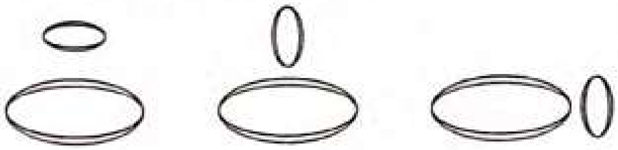


$\therefore\ I_2(t)\,B(t) = k_5[1 - e^{-t/\tau}][e^{-t/\tau}]$

This quantity is zero for $t = 0$ and $t = \infty$ and positive for other value of t. It must, therefore, pass through a maximum.

14. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be

(a) maximum in situation (a)
(b) maximum in situation (b)
(c) maximum in situation (c)
(d) the same in all situations



(a) (b) (c)

Sol. (a) The mutual inductance between two coils depends on their degree of flux linkage, i.e., the fraction of flux linked with one coil which is also linked to the other coil. Here, the two coils in arrangement (a) are placed with their planes parallel. This will allow maximum flux linkage.

15. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

(a) halved
(b) the same
(c) doubled
(d) quadrupled

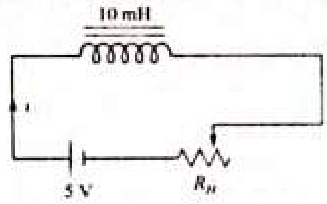
Sol. (b) Power $P = \frac{\varepsilon^2}{R}$ hence $\varepsilon = -\left(\frac{d\varphi}{dt}\right)$ where $\varphi = NBA$

$$\therefore \quad \varepsilon = -NA\left(\frac{dB}{dt}\right) \text{ Also } R \propto \frac{l}{r^2}$$

where R = resistance, r = radius, l = length

$$\therefore \quad P \propto \frac{N^2 r^2}{l} \Rightarrow \frac{P_1}{P_2} = 1$$

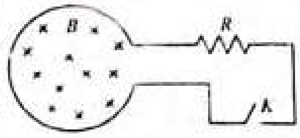
16. The resistance in the following circuit is increased at a particular instant. At this instant the value of resistance is 10 Ω. The current in the circuit will be now



(a) $i = 0.5\ A$
(b) $i > 0.5\ A$
(c) $i < 0.5\ A$
(d) $i = 0$

Sol. (b) If resistance is constant (10 Ω) then steady current in the circuit $i = 5/10 = 0.5$A. But resistance is increasing it means current through the circuit start decreasing. Hence inductance comes in picture which induces a current in the circuit in the same direction of main current. So $i > 0.5$ A.

17. Shown in the figure is a circular loop of radius r and resistance R. A variable magnetic field of induction $B = B_0 e^{-t}$ is established inside the coil. If the key (K) is closed, the electrical power developed right after closing the switch is equal to



(a) $\frac{B_0^2 \pi r^2}{R}$
(b) $\frac{B_0 10 r^3}{R}$

(c) $\dfrac{B_0^2\pi^2 r^4 R}{5}$ (d) $\dfrac{B_0^2\pi^2 r^4}{R}$

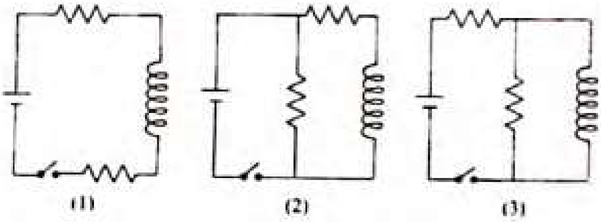
Sol. (d) $P = \dfrac{\varepsilon^2}{R}$, $\varepsilon = -\dfrac{d}{dt}(BA) = A\dfrac{d}{dt}(B_o e^{-t}) = AB_o e^{-t}$

$$\Rightarrow P = \frac{1}{R}(AB_o e^{-t})^2 = \frac{A^2 B_o^2 e^{-2t}}{R}$$

At the time of starting $t = 0$ so $P = \dfrac{A^2 B_o^2}{R}$

$$\Rightarrow P = \frac{(\pi r^2)^2 B_o^2}{R} = \frac{B_o^2 \pi^2 r^4}{R}$$

18. The figure shows three circuits with identical batteries, inductors, and resistors. Rank the circuits according to the current through the battery (i) just after the switch is closed and (ii) a long time later, greatest first



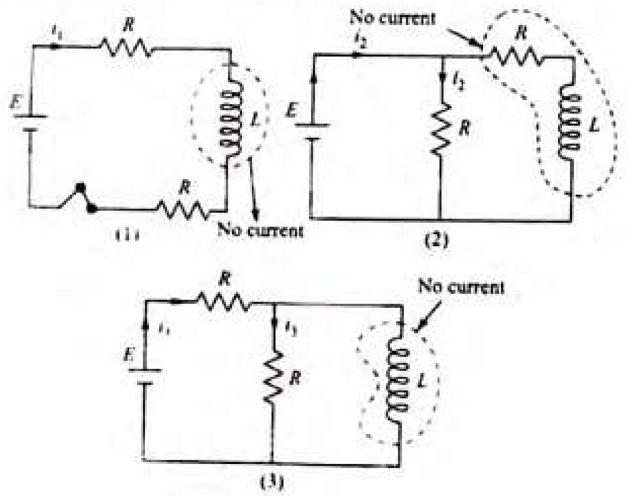
(a) (i) $i_2 > i_3 > i_1$ $(i_1 = 0)$ (ii) $i_2 > i_3 > i_1$

(b) (i) $i_2 < i_3 < i_1$ $(i_1 \neq 0)$ (ii) $i_2 > i_3 > i_1$

(c) (i) $i_2 > i_3 = i_1$ $(i_1 = 0)$ (ii) $i_2 > i_3 > i_1$

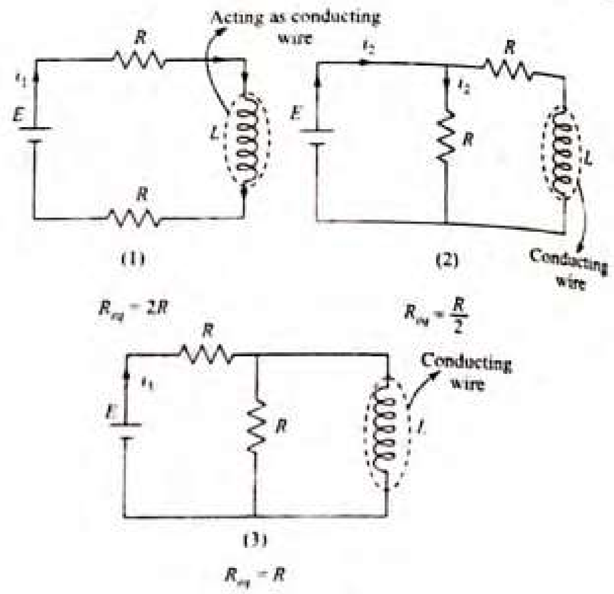
(d) (i) $i_2 = i_3 > i_1$ $(i_1 \neq 0)$ (ii) $i_2 > i_3 > i_1$

Sol. (a) Just after closing the switch.



$$i_1 = 0,\ i_2 = \frac{E}{R},\ i_3 = \frac{E}{2R} \text{ so } i_2 > i_3 > i_1\ (i_1 = 0)$$

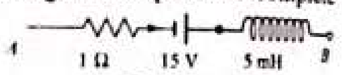
After a long time closing the switch



Here $i_1 = \dfrac{\varepsilon}{R}, i_2 = \dfrac{\varepsilon}{R/2}$ and $i_3 = \dfrac{\varepsilon}{2R}$

Hence $i_2 > i_3 > i_1$

19. The network shown in the figure is a part of a complete circuit. If at a certain instant the current i is 5 A and is decreasing at the rate of 10^3 A/s then $V_A - V_B$ is



(a) 5 V (b) 10 V (c) 15 V (d) 20 V

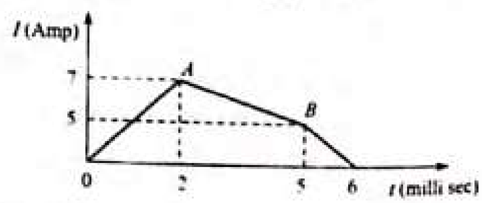
Sol. (c) By using Kirchoff's voltage law

$$V_A - i\,R + E - L\frac{di}{dt} = V_B \Rightarrow V_B - V_A = 15 \text{ volt}$$

20. The current through a 4.6 H inductor is shown in the following graph. The induced emf during the time interval t = 5 milli-sec to 6 milli-sec will be

(a) 10^3 V (b) -23×10^3 V

(c) 23×10^3 V (d) Zero



Sol. (c) Rate of decay of current between t = 5 ms to 6 ms

$$= \frac{di}{dt} = -(\text{Slope of the line } BC)$$

$$= -\left(\frac{5}{1 \times 10^{-3}}\right) = -5 \times 10^3 \text{ A/s}$$

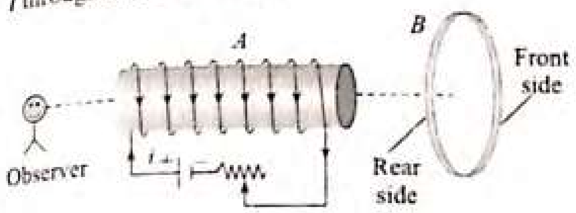
Hence induced emf

$$\varepsilon = -L\frac{di}{dt} = -4.6 \times (-5 \times 10^3) = 23 \times 10^3 \text{ V}$$

EXERCISES

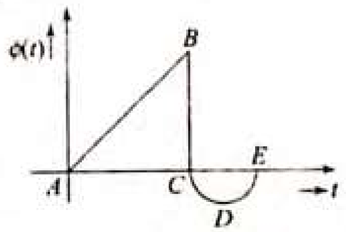
Faraday's Law and Lenz's Law

1. An aluminum ring B faces an electromagnet A. The current I through A can be altered



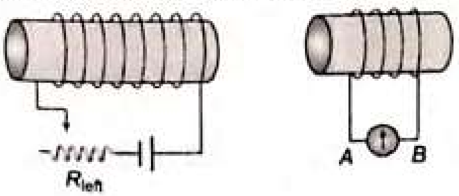
(a) whether I increases or decreases, B will not experience any force
(b) if I decrease, A will repel B
(c) if I increases, A will attract B
(d) if I increases, A will repel B

2. The graph shows the variation in magnetic flux $\phi(t)$ with time through a coil. Which of the statements given below is not correct?



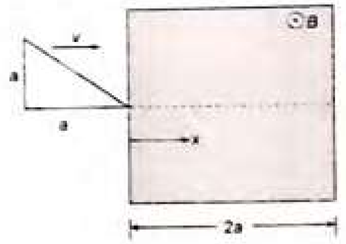
(a) There is a change in the direction as well as magnitude of the induced emf between B and D
(b) The magnitude of the induced emf is maximum between B and C
(c) There is a change in the direction as well as magnitude of induced emf between A and C
(d) The induced emf is zero at B

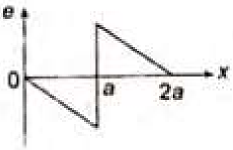
3. For the solenoids shown in the diagram (which are assumed to be close to each other), the resistance of the left-hand circuit is slowly increased. In which direction does the current flow through galvanometer in the right-hand circuit?

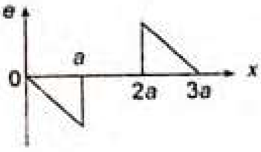


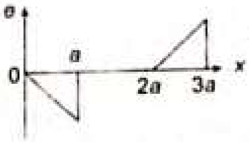
(a) B to A (b) A to B
(c) there is no current (d) cannot be determined

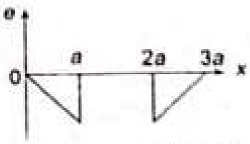
4. A right angled triangular loop (as shown in the figure) enters into uniform magnetic field (at right angle to the boundary of the field) directed into the paper. The loop moves always with constant speed. Draw the graph between induced emf e and the distance along the perpendicular to the boundary of the field (say x) along which loop moves.



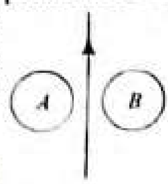
(a)


(b)


(c)


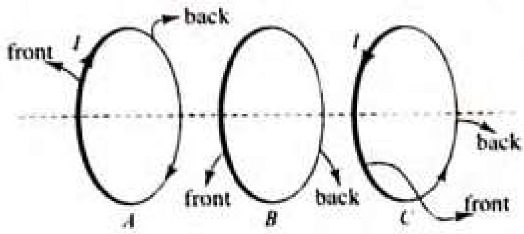
(d)


5. A and B are two metallic rings placed at opposite sides of an infinitely long straight conducting wire as shown in figure. If current in the wire is slowly decreased, the direction of the induced current will be



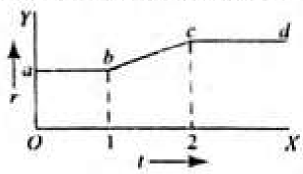
(a) clockwise in A and anticlockwise in B
(b) anticlockwise in A and clockwise in B
(c) clockwise in both A and B
(d) anticlockwise in both A and B

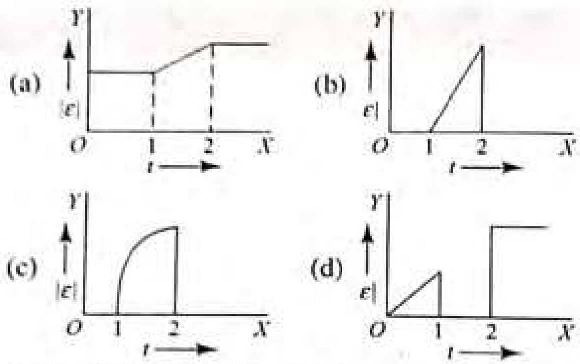
6. Three identical coils A, B, and C carrying currents are placed coaxially with their planes parallel to one another. A and C carry currents as shown in figure. B is kept fixed, while A and C both are moved toward B with the same speed. Initially, B is equally separated from A and C. The direction of the induced current in the coil B is



(a) same as that in coil A (b) same as that in coil B
(c) zero (d) none of these

7. A flexible wire bent in the form of a circle is placed in a uniform magnetic field perpendicularly to the plane of the coil. The radius of the coil changes as shown in figure. The graph of magnitude of induced emf in the coil is represented by



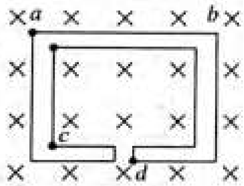


8. A thin circular ring of area A is perpendicular to uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

(a) $\dfrac{BR}{A}$ (b) $\dfrac{AB}{R}$

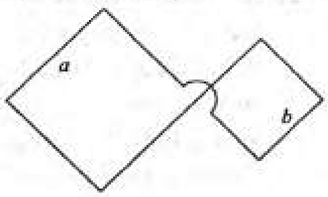
(c) ABR (d) B^2A/R^2

9. A wire is bent to form the double loop shown in figure. There is a uniform magnetic field directed into the plane of the loop. If the magnitude of this field is decreasing, the current will flow from



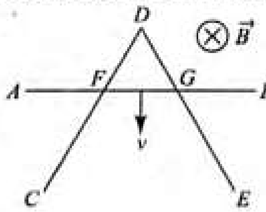
(a) a to b and c to d
(b) b to a and d to c
(c) a to b and d to c
(d) b to a and c to d

10. A plane loop, shaped as two squares of sides $a = 1$ m and $b = 0.4$ m is introduced into a uniform magnetic field $\perp$ to the plane of loop (figure). The magnetic field varies as $B = 10^{-3}\sin(100t)$ T. The amplitude of the current induced in the loop if its resistance per unit length is $r = 5\ \text{m}\Omega\ \text{m}^{-1}$ is



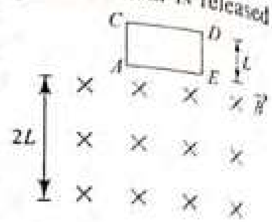
(a) 2 A (b) 3 A
(c) 4 A (d) 5 A

11. A long conducting wire AH is moved over a conducting triangular wire CDE with a constant velocity v in a uniform magnetic field $\vec{B}$ directed into the plane of the paper. Resistance per unit length of each wire is ρ. Then



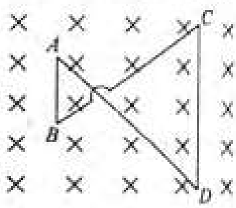
(a) a constant clockwise induced current will flow in the closed loop
(b) an increasing anticlockwise induced current will flow in the closed loop
(c) a decreasing anticlockwise induced current will flow in the closed loop
(d) a constant anticlockwise induced current will flow in the closed loop

12. A square coil $ACDE$ with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length $2L$ (figure). The acceleration of the coil is



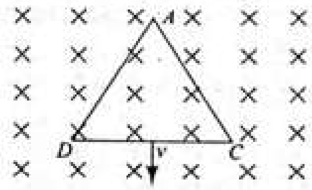
(a) less than g for all the time till the loop crosses the magnetic field completely
(b) less than g when it enters the field and greater than g when it comes out of the field
(c) g all the time
(d) less than g when it enters and comes out of the field but equal to g when it is within the field

13. A conducting wire frame is placed in a magnetic field which is directed into the plane of the paper (figure). The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are

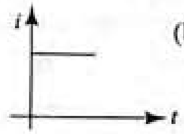
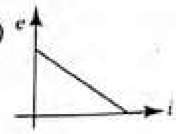
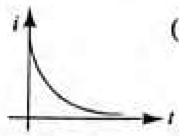
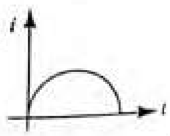


(a) B to A and D to C
(b) A to B and C to D
(c) A to B and D to C
(d) B to A and C to D

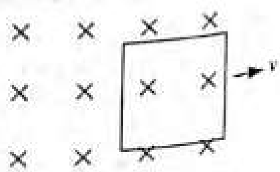
14. An equilateral triangular loop ADC having some resistance is pulled with a constant velocity v out of a uniform magnetic field directed into the paper (figure). At time $t = 0$, side DC of the loop is at edge of the magnetic field.



The induced current (i) versus time (t) graph will be as

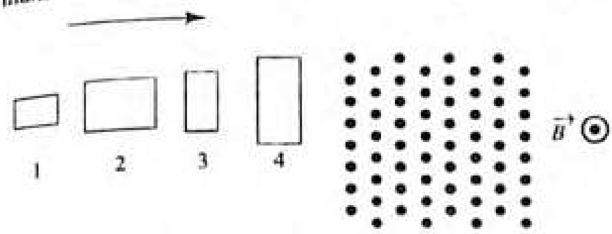
(a) 
(b) 
(c) 
(d) 

15. Figure shows a square loop of side 0.5 m and resistance 10 Ω. The magnetic field has a magnitude $B = 1.0$ T. The work done in pulling the loop out of the field slowly and uniformly in 2.0 s is



(a) 3.125×10^{-3} J (b) 6.25×10^{-4} J
(c) 1.25×10^{-2} J (d) 5.0×10^{-4} J

16. The four wire loops shown in figure have vertical edge lengths of either L, $2L$, or $3L$. They will move with the same speed into a region of uniform magnetic field $\vec{B}$ directed out of the page. Rank them according to the maximum magnitude of the induced emf greatest to least.



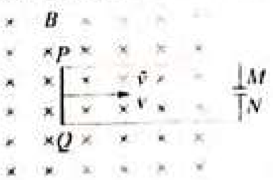
(a) 1 and 2 tie, then 3 and 4 tie
(b) 3 and 4 tie, then 1 and 2 tie
(c) 4, 2, 3, 1
(d) 4 then 2 and 3 tie, and then 1

17. A long solenoid having 200 turns per centimeter carries a current of 1.5 A. At the center of the solenoid is placed a coil of 100 turns of cross-sectional area 3.14×10^{-4} m^2 having its axis parallel to the field produced by the solenoid. When the direction of current in the solenoid is reversed within 0.05 s, the induced emf in the coil is
(a) 0.48 V (b) 0.048 V
(c) 0.0048 V (d) 48 V

18. A horizontal ring of radius $r = 1/2$ m is kept in a vertical constant magnetic field 1 T. The ring is collapsed from maximum area to zero area in 1 s. Then the emf induced in the ring is
(a) 1 V (b) $(\pi/4)$ V
(c) $(\pi/2)$ V (d) π V

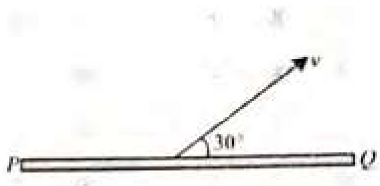
Motional EMF

19. A rod PQ is connected to the capacitor plates. The rod is placed in a magnetic field (B) directed downward perpendicular to the plane of the paper. If the rod is pulled out of magnetic field with velocity $\vec{v}$ as shown in figure.



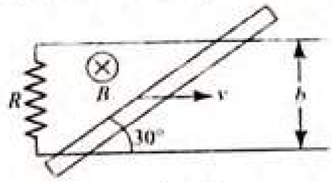
(a) Plate M will be positively charged.
(b) Plate N will be positively charged.
(c) Both plates will be similarly charged.
(d) No charge will be collected on plates.

20. A conducting rod PQ of length $l = 2$ m is moving at a speed of 2 m s^{-1} making an angle of 30° with its length. A uniform magnetic field $B = 2$ T exists in a direction perpendicular to the plane of motion. Then



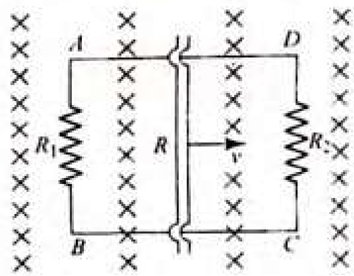
(a) $V_P - V_Q = 8$ V (b) $V_P - V_Q = 4$ V
(c) $V_Q - V_P = 8$ V (d) $V_Q - V_P = 4$ V

21. A wire is sliding as shown in figure. The angle between the acceleration and the velocity of the wire is



(a) 30° (b) 40°
(c) 120° (d) 90°

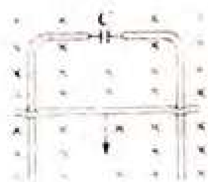
22. A rectangular loop with a sliding conductor of length l is located in a uniform magnetic field perpendicular to the plane of the loop (figure). The magnetic induction is B. The conductor has a resistance R. The sides AB and CD have resistances R_1 and R_2, respectively. Find the current through the conductor during its motion to the right with a constant velocity v.



(a) $\dfrac{Blv(R_1+R_2)}{R_1(R_1+R_2)}$ (b) $\dfrac{Bl^2v}{R_1+R_1R_2}$

(c) $\dfrac{Blv(R_1+R_2)}{R_1R_2+R(R_1+R_2)}$ (d) $\dfrac{Bl^2v}{R_1R_2+R(R_1+R_2)}$

23. A conductor of length l and mass m can slide without any friction along the two vertical conductors connected at the top through a capacitor (figure). A uniform magnetic field B is set up ⊥ to the plane of paper. The acceleration of the conductor



(a) is constant (b) increases
(c) decreases (d) cannot say

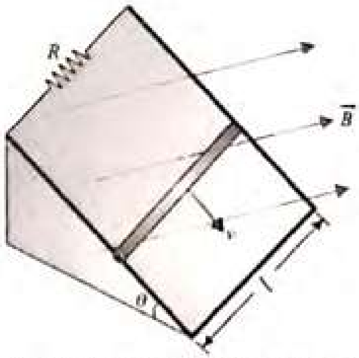
24. A conducting wire of mass m slides down two smooth conducting bars, set at an angle θ to the horizontal as shown in figure. The separation between the bars is l. The system is located in the magnetic field B, perpendicular to the plane of the sliding wire and bars. The constant velocity of the wire is

(a) $\dfrac{mgR\sin\theta}{B^2l^2}$

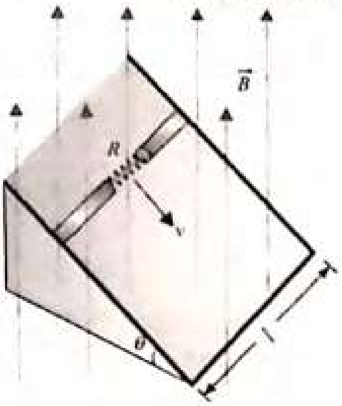
(b) $\dfrac{mgR\sin\theta}{Bl^3}$

(c) $\dfrac{mgR\theta}{B^2l^5}$

(d) $\dfrac{mgR\sin\theta}{Bl^4}$



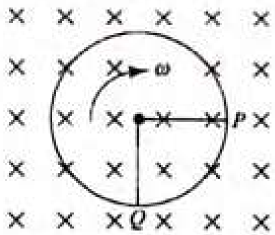
25. A wire of length l, mass m, and resistance R slides without any friction down the parallel conducting rails of negligible resistance (figure). The rails are connected to each other at the bottom by a resistanceless rail parallel to the wire so that the wire and the rails form a closed rectangular conducting loop. The plane of the rails makes an angle θ with the horizontal and a uniform vertical magnetic field of induction B exists throughout the region. Find the steady-state velocity of the wire.



(a) $\dfrac{mg}{R}\dfrac{\sin\theta}{B^2l^2\cos^2\theta}$ (b) $\dfrac{mg}{R}\dfrac{\sin^2\theta}{B^2l^2\cos^2\theta}$

(c) $\dfrac{mgR\sin\theta}{B^2l^2\cos^2\theta}$ (d) $mgR\dfrac{\sin^2\theta}{B^2l^2\cos\theta}$

26. A conducting ring of radius r is rolling without slipping with a constant angular velocity ω (figure). If the magnetic field strength is B and is directed into the page then the emf induced across PQ is

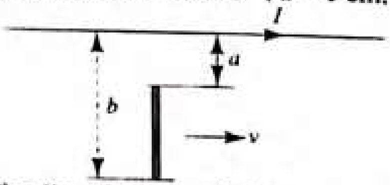


(a) $B\omega r^2$ (b) $\dfrac{B\omega r^2}{2}$

(c) $4B\omega r^2$ (d) $\dfrac{\pi^2r^2B\omega}{8}$

27. Figure shows a copper rod moving with velocity v parallel to a long straight wire carrying current = 100 A. Calculate the induced emf in the rod, where $v = 5\ \text{m s}^{-1}$, $a = 1$ cm, $b = 100$ cm.

(a) 0.23 mV
(b) 0.46 mV
(c) 0.16 mV
(d) 0.32 mV

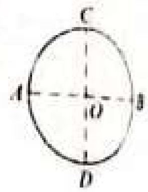


28. A semicircular wire of radius R is rotated with constant angular velocity about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced emf between the ends is

(a) $B\omega R^2/2$ (b) $2B\omega R^2$
(c) is variable (d) none of these

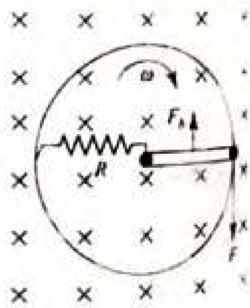
29. Two identical cycle wheels (geometrically) have different number of spokes connected from center to rim. One is having 20 spokes and the other having only 10 (the rim and the spokes are resistanceless). One resistance of value R is connected between center and rim. The current in R will be
(a) double in the first wheel than in the second wheel
(b) four times in the first wheel than in the second wheel
(c) will be double in the second wheel than that of the first wheel
(d) will be equal in both these wheels

30. A vertical conducting ring of radius R falls vertically with a speed V in a horizontal uniform magnetic field B which is perpendicular to the plane of the ring. Which of the following statements is correct?



(a) A and B are at the same potential
(b) C and D are at the same potential
(c) current flows in clockwise direction
(d) current flows in anticlockwise direction

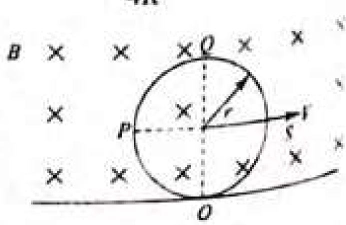
31. A metallic ring of radius r with a uniform metallic spoke of negligible mass and length r is rotated about its axis with angular velocity ω in a perpendicular uniform magnetic field B as shown in figure. The central end of the spoke is connected to the rim of the wheel through a resistor R as shown. The resistor does not rotate, its one end is always at the center of the ring and the other end is always in contact with the ring. A force F as shown is needed to maintain constant angular velocity of the wheel. F is equal to (the ring and the spoke has zero resistance)



(a) $\dfrac{B^2\omega r^2}{8R}$ (b) $\dfrac{B^2\omega r^2}{2R}$

(c) $\dfrac{B^2\omega r^3}{2R}$ (d) $\dfrac{B^2\omega r^3}{4R}$

32. A conducting ring of radius r and resistance R rolls on a horizontal surface with constant velocity v. The magnetic field B is uniform and



is normal to the plane of the loop. Choose the correct option.

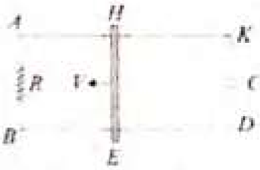
(a) The induced emf between O and Q is Brv.

(b) An induced current $I = 2Bvr/R$ flows in the clockwise direction.

(c) An induced current $I = 2Bvr/R$ flows in the anticlockwise direction.

(d) No current flows.

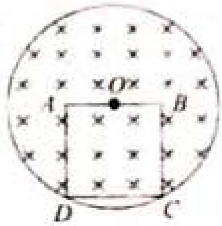
33. In the circuit shown in figure, a conducting wire HE is moved with a constant speed v toward left. The complete circuit is placed in a uniform magnetic field $\vec{B}$ perpendicular to the plane of the circuit inward. The current in $HKDE$ is



(a) clockwise (b) anticlock wise
(c) alternating (d) zero

Induced Electric Field, Self and Mutual Inductance

34. A square non-conducting loop, 20 cm. on a side is placed in a magnetic field. The centre of side AB coincides with the centre of magnetic field. The magnetic field is increasing at the rate of 2 T/s. The potential difference between B and C is



(a) 30 mV (b) zero
(c) 10 mV (d) 20 mV

35. Refer to above question, the potential difference between C and D is

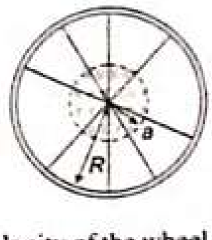
(a) 80 mV (b) zero
(c) 40 mV (d) 60 mV

36. A ring of mass m, radius r having charge q uniformly distributed over it and free to rotate about its own axis is placed in a region having a magnetic field B parallel to its axis. If the magnetic field is suddenly switched off, the angular velocity acquired by the ring is

(a) $\dfrac{qB}{m}$ (b) $\dfrac{2qB}{m}$

(c) $\dfrac{qB}{2m}$ (d) none of these

37. A line charge l per unit length is pasted uniformly onto the rim of a wheel of mass m and radius R. The wheel has light non-conducting spokes and is free to rotate about a vertical axis as shown in figure. A uniform magnetic field B exists as shown in figure. What is the angular velocity of the wheel when the field is suddenly switched off?



(a) $\dfrac{2\pi\lambda a^2 B}{mR}$ (b) $\dfrac{\pi\lambda a^2 B}{mR}$

(c) $\dfrac{3\pi\lambda a^2 B}{mR}$ (d) $\dfrac{\pi\lambda a^2 B}{2mR}$

38. A thin non-conducting ring of mass m carrying a charge Q can freely rotate about its axis. Initially, the ring is rest and no magnetic field is present. Then a uniform field of magnetic induction was switched on, which was perpendicular to the plane of the ring and increased with time as a given function $B(t)$. The angular velocity $\omega(t)$ of the ring as a function of the field $B(t)$ will be given by

(a) $\omega(t) = \dfrac{qB(t)}{m}$ (b) $\omega(t) = \dfrac{qB(t)}{2m}$

(c) $\omega(t) = \dfrac{qB(t)}{2\pi m}$ (d) $\omega = 0$

39. An emf of 96.0 mV is induced in the windings of a coil when the current in a nearby coil is increasing at the rate of 1.20 A/s. The mutual inductance of the two coils is

(a) 40 mH (b) 20 mH
(c) 10 mH (d) 80 mH

40. Two single-turn circular loops of wire have radii R and r, with $R >> r$. The loops lie in the same plane and are concentric. The mutual inductance of the pair is (approximately)

(a) $\dfrac{\mu_0 \pi r^2}{R}$ (b) $\dfrac{2\mu_0 \pi r^2}{R}$

(c) $\dfrac{\mu_0 \pi r^2}{2R}$ (d) $\dfrac{3\mu_0 \pi r^2}{2R}$

41. A magnetic flux of 5×10^{-4} Wb is associated with every 10 turns of a 500 turns coil. The electric current flowing through the wire is 5 A. What is the self-inductance of the coil?

(a) 0.5 H (b) 5×10^{-3} H
(c) 5.0 H (d) 5×10^{-2} H

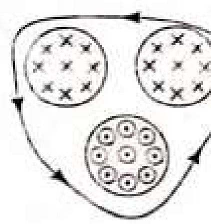
42. A wire of fixed length is wound in such a way that it forms a solenoid of length 'l' and radius 'r'. Its self-inductance is found to be L. Now if same wire is wound in such a way that it forms a solenoid of length $l/2$ and radius $r/2$, then the self-inductance will be

(a) $2L$ (b) L
(c) $4L$ (d) $8L$

43. A rectangular loop of sides a and b is placed in xy plane. A very long wire is also placed in xy plane such that side of length a of the loop is parallel to the wire. The distance between the wire and the nearest edge of the loop is d. The mutual inductance of this system is proportional to

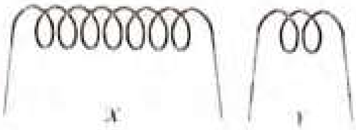
(a) a (b) b
(c) $1/d$ (d) current in wire

44. Figure shows three regions of magnetic field each of area A, and in each region, magnitude of magnetic field decreases at a constant rate α. If $\vec{E}$ is the induced electric field, then the value of the line integral $\oint \vec{E} \cdot d\vec{r}$ along the given loop is equal to



(a) αA (b) $-\alpha A$ (c) $3\alpha A$ (d) $3\alpha A$

45. A mutual inductor consists of two coils X and Y as shown in figure in which one-quarter of the magnetic flux produced by X links with Y, giving a mutual inductance M. What will be the mutual inductance when Y is used as the primary?

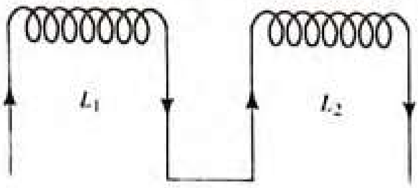


(a) $M/4$ (b) $M/2$
(c) M (d) $2M$

46. A small coil of radius r is placed at the center of a large coil of radius R, where $R >> r$. The two coils are coplanar. The mutual inductance between the coils is proportional to

(a) r/R (b) r^2/R
(c) r^2/R^2 (d) r/R^2

47. A circuit contains two inductors of self-inductance L_1 and L_2 in series (figure). If M is the mutual inductance, then the effective inductance of the circuit shown will be

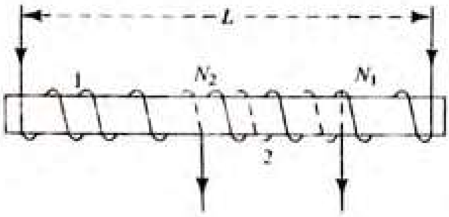


(a) $L_1 + L_2$ (b) $L_1 + L_2 - 2M$
(c) $L_1 + L_2 + M$ (d) $L_1 + L_2 + 2M$

48. The coefficient of mutual inductance of two circuits A and B is 3 mH and their respective resistances are 10 and 4 Ω. How much current should change in 0.02 s in circuit A, so that the induced current in B should be 0.0060 A?

(a) 0.24 A (b) 1.6 A
(c) 0.18 A (d) 0.16 A

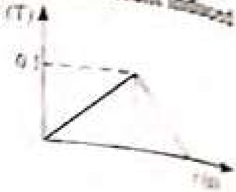
49. A long solenoid of length L, cross section A having N_1 turns has wound about its center a small coil of N_2 turns as shown in figure. The mutual inductance of two circuits is



(a) $\dfrac{\mu_0 A(N_1/N_2)}{L}$ (b) $\dfrac{\mu_0 A(N_1 N_2)}{L}$

(c) $\mu_0 A N_1 N_2 L$ (d) $\dfrac{\mu_0 A N_1^2 N_2}{L}$

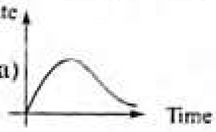
50. A closed loop of cross-sectional area 10^{-2} m^2 which has inductance $L = 10$ mH and negligible resistance is placed in a time-varying magnetic field. Figure shows the variation of B with time for the interval 4 s. The field is perpendicular to the plane of the loop (given at $t = 0$, $B = 0$, $I = 0$). The value of the maximum current induced in the loop is



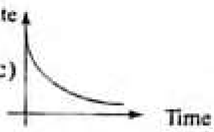
(a) 0.1 mA
(b) 10 mA
(c) 100 mA
(d) Data insufficient

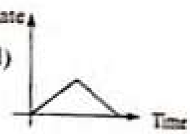
L.R and L.C Circuits

51. In an LR circuit connected to a battery, the rate at which energy is stored in the inductor is plotted against time during the growth of current in the circuit. Which of the following best represents the resulting curve?

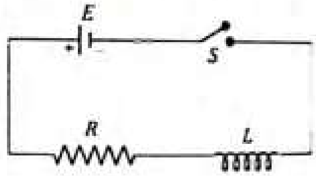
(a) 

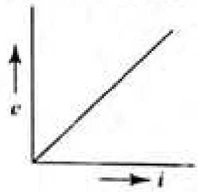
(b) 

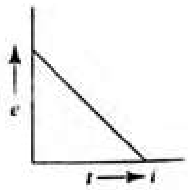
(c) 

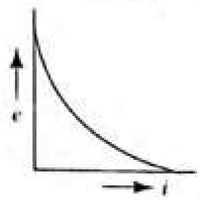
(d) 

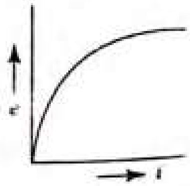
52. Switch S of the circuit shown in figure is closed at $t = 0$. If e denotes the induced emf in L and i the current flowing through the circuit at time t, then which of the following graphs correctly represents the variation of e with i ?



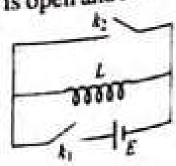
(a) 

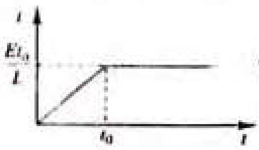
(b) 

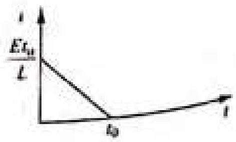
(c) 

(d) 

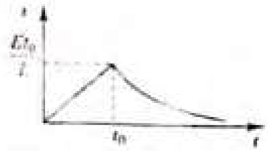
53. In the circuit shown in figure, switch k_2 is open and switch k_1 is closed at $t = 0$. At time $t = t_0$, switch k_1 is opened and switch k_2 is simultaneously closed. The variation of inductor current with time is



(a) 

(b) 

(c) 

(d) 

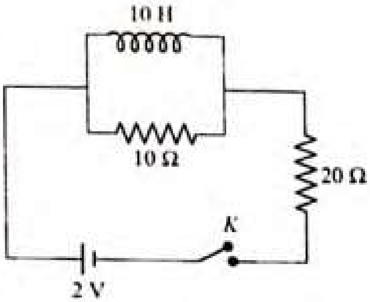
(d) $\frac{2}{3}$ A

54. A simple LR circuit is connected to a battery at time $t = 0$. The energy stored in the inductor reaches half its maximum value at time

(a) $\frac{R}{L}\ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$ (b) $\frac{L}{R}\ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

(c) $\frac{L}{R}\ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$ (d) $\frac{R}{L}\ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

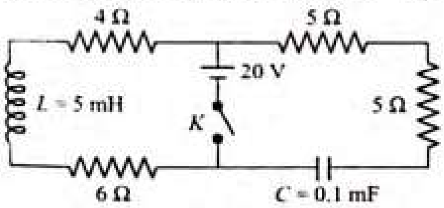
55. Two resistors of 10 Ω and 20 Ω and an ideal inductor of 10 H are connected to a 2 V battery as shown in figure. Key K is inserted at time $t = 0$. The initial ($t = 0$) and final ($t \to \infty$) currents through the battery are



(a) $\frac{1}{15}$ A, $\frac{1}{10}$ A (b) $\frac{1}{10}$ A, $\frac{1}{15}$ A

(c) $\frac{2}{15}$ A, $\frac{1}{10}$ A (d) $\frac{1}{15}$ A, $\frac{2}{25}$ A

56. In the circuit shown in figure, key K is closed at $t = 0$, the current through the key at the instant $t = 10^{-3}\ln 2$ s is



(a) 2 A (b) 3.5 A
(c) 2.5 A (d) 0

57. The total heat produced in resistor R in an RL circuit when the current in the inductor decreases from I_0 to 0 is

(a) LI_0^2 (b) $\frac{1}{2}LI_0^2$

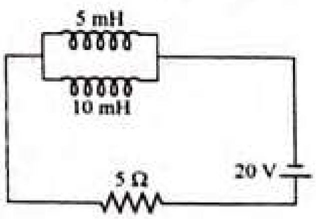
(c) $\frac{3}{2}LI_0^2$ (d) $\frac{1}{3}LI_0^2$

58. In the given circuit (figure), current through the 5 mH inductor in steady state is

(a) $\frac{2}{3}$ A

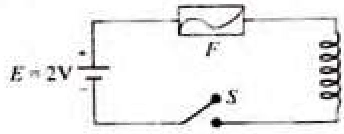
(b) $\frac{8}{3}$ A

(c) $\frac{1}{3}$ A



59. In the circuit shown (figure), the cell is ideal. The coil has an inductance of 4 H and zero resistance. F is a fuse of zero resistance and will blow when the current through it reaches 5 A. The switch is closed at $t = 0$. The fuse will blow

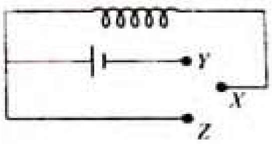
(a) almost at once
(b) after 2 s
(c) after 5 s
(d) after 10 s



60. In the circuit shown (figure), the coil has inductance and resistance. When X is joined to Y, the time constant is τ during the growth of current.

When the steady state is reached, heat is produced in the coil at a rate P. X is now joined to Z. After joining X and Z:

(a) The total heat produced in the coil is $P\tau$
(b) The total heat produced in the coil is 1/2 $P\tau$
(c) The total heat produced in the coil is $2P\tau$
(d) The data given are not sufficient to reach a conclusion



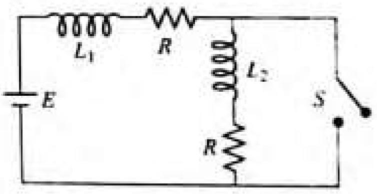
61. Switch S shown in figure is closed for $t < 0$ and is opened at $t = 0$. When currents through L_1 and L_2 are equal, their common value is

(a) $\frac{E}{R}$

(b) $\frac{E(L_2 + L_1)}{RL_1}$

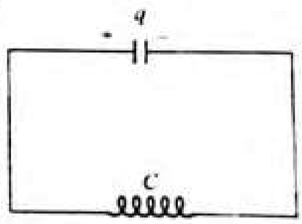
(c) $\frac{EL_1}{R(L_1 + L_2)}$

(d) $\frac{E}{R}\frac{(L_1 + L_2)}{L_2}$

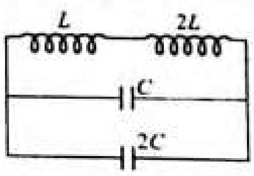


62. In an LC circuit shown in figure, $C = 1$ F, $L = 4$ H. At time $t = 0$, charge in the capacitor is 4 C and it is decreasing at the rate of $\sqrt{5}$ C s^{-1}. Choose the correct statement.

(a) Maximum charge in the capacitor can be 6 C
(b) Maximum charge in the capacitor can be 8 C
(c) Charge in the capacitor will be maximum after time 3 $\sin^{-1}$ (2/3) s
(d) None of these



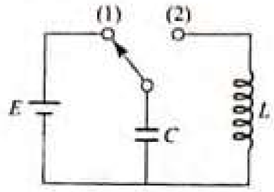
63. The frequency of oscillation of current in the inductance is



(a) $\dfrac{1}{3\sqrt{LC}}$ (b) $\dfrac{1}{6\pi\sqrt{LC}}$

(c) $\dfrac{1}{\sqrt{LC}}$ (d) $\dfrac{1}{2\pi\sqrt{LC}}$

64. In the following electrical network at $t < 0$ (figure), key is placed on (1) till the capacitor got fully charged. Key is placed on (2) at $t = 0$. Time when the energy in both the capacitor and the inductor will be same for the first time is



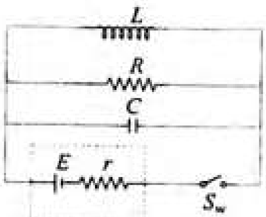
(a) $\dfrac{\pi}{4}\sqrt{LC}$

(b) $\dfrac{3\pi}{4}\sqrt{LC}$

(c) $\dfrac{\pi}{3}\sqrt{LC}$

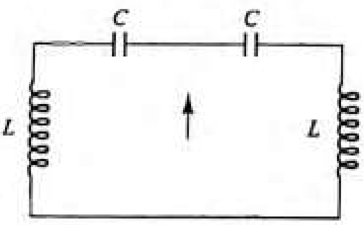
(d) $\dfrac{2\pi}{3}\sqrt{LC}$

65. A pure inductor L, a capacitor C and a resistance R are connected across a battery of emf E and internal resistance r as shown in figure. Switch S_w is closed at $t = 0$, select the correct alternative(s).



(a) Current through resistance R is zero all the time
(b) Current through resistance R is zero at $t = 0$ and $t \to \infty$
(c) Maximum charge stored in the capacitor is CE
(d) Maximum energy stored in the inductor is equal to the maximum energy stored in the capacitor

66. The natural frequency of the circuit shown in figure is



(a) $\dfrac{1}{\sqrt{LC}}$

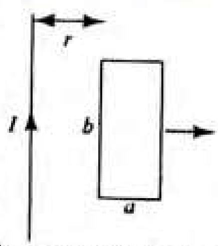
(b) $\dfrac{1}{\sqrt{2LC}}$

(c) $\dfrac{2}{\sqrt{LC}}$

(d) none of these

Problems Based on Mixed Concepts

67. A rectangular loop of wire with dimensions shown in figure is coplanar with a long wire carrying current I. The distance bet-ween the wire and the left side of the loop is r. The loop is pulled to the right as indicated. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop when the loop is pulled?

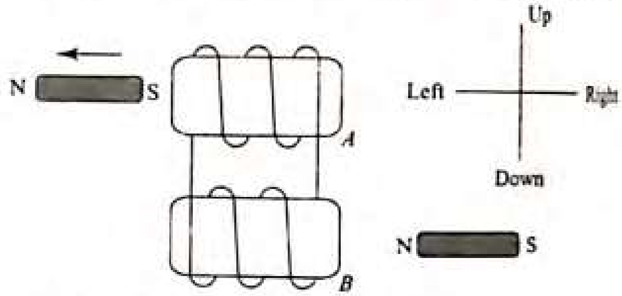


Induced current	Force on left side	Force on right side
(a) Counterclockwise	To the left	To the left
(b) Counterclockwise	To the right	To the left
(c) Clockwise	To the right	To the left
(d) Clockwise	To the left	To the right

68. In Figures (a) and (b), two air-cored solenoids P and Q have been shown. They are placed near each other. In Figure (a), when I_P, the current in P, changes at the rate of 5 A s^{-1}, an emf of 2 mV is induced in Q. The current in P is then switched off, and the current changing at 2 A s^{-1} is fed through Q as shown in the figure. What emf will be induced in P?

(a) 8×10^{-4} V (b) 2×10^{-3} V
(c) 5×10^{-3} V (d) 8×10^{-2} V

69. A bar magnet was pulled away from a hollow coil A as shown in figure. As the south pole came out of the coil, the bar magnet next to hollow coil B experienced a magnetic force



(a) to the right (b) to the left
(c) upward (d) equal to zero

70. The capacitor of an oscillatory circuit of frequency 10000 Hz is enclosed in a container. When the container is evacuated, the frequency changes by 50 Hz, the dielectric constant of the gas is
(a) 1.1 (b) 1.01
(c) 1.001 (d) 1.0001

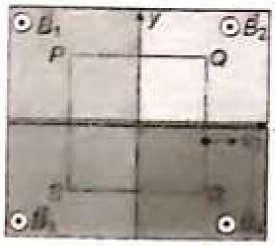
71. A simple pendulum with bob of mass m and conducting wire of length L swings under gravity through an angle 2θ. The earth's magnetic field component in the direction perpendicular to swing is B. Maximum potential difference induced across the pendulum is

(a) $2BL\sin\left(\dfrac{\theta}{2}\right)(gL)^{1/2}$ (b) $BL\sin\left(\dfrac{\theta}{2}\right)(gL)$

(c) $BL\sin\left(\dfrac{\theta}{2}\right)(gL)^{3/2}$ (d) $BL\sin\left(\dfrac{\theta}{2}\right)(gL)^2$

72. Four different uniform and constant magnetic fields $\vec{B}_1 = B_0\,\hat{k}$, $\vec{B}_2 = 2B_0\,\hat{k}$, $\vec{B}_3 = 3B_0\,\hat{k}$ and $\vec{B}_4 = 4B_0\,\hat{k}$ exist in first, second, third and fourth quadrant respectively of x-y

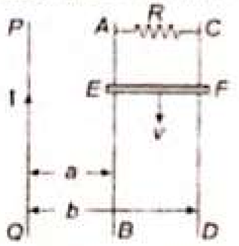
plane as shown in figure. A square wireframe $PQRS$ of one side l and total resistance R lying in x–y plane is moving with velocity $\vec{v} = v_0\hat{i}$ such that its centre lies on x-axis and side PQ is parallel to x-axis. B_0 and v_0 are positive constants. Then the magnitude of current induced in the square loop at the instant its centre lies of origin is



(a) 0 (b) $\frac{Blv_0}{R}$

(c) $\frac{Blv_0}{2R}$ (d) $\frac{2Blv_0}{R}$

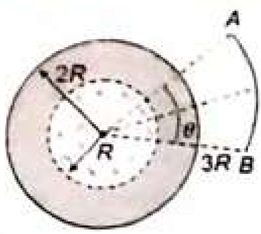
73. PQ is an infinite current carrying conductor. AB and CD are smooth conducting rods on which a conductor EF moves with constant velocity v as shown. The force needed to maintain constant speed of EF is



(a) $\frac{1}{vR}\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$ (b) $\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2 \frac{1}{vR}$

(c) $\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2 \frac{v}{R}$ (d) $\frac{v}{R}\left[\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$

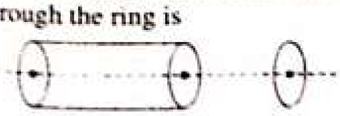
74. In the given figure two concentric cylindrical region in which time varying magnetic field is present as shown. From the center to radius R magnetic field is perpendicular into the plane varying as $dB/dt = 2k_0$ and in a region from R to $2R$ magnetic field is perpendicular out of the plane varying as $dB/dt = 4k_0$. Find the induced emf across an arc AB of radius $3R$.



(a) $6R^2k\theta$ (b) $5R^2k\theta$

(c) $7R^2k\theta$ (d) none of these

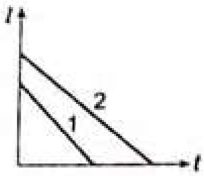
75. The diagram shows a solenoid carrying time varying current $I = I_0 t$. On the axis of the solenoid, a ring has been placed. The mutual inductance of the ring and the solenoid is M and the self-inductance of the ring is L. If the resistance of the ring is R then maximum current which can flow through the ring is



(a) $\frac{(2M + L)I_0}{R}$ (b) $\frac{MI_0}{R}$

(c) $\frac{(2M - L)I_0}{R}$ (d) $\frac{(M + L)I_0}{R}$

76. Two identical inductance carry currents that vary with time according to linear laws (as shown in figure). In which of two inductance is the self-induction emf greater?



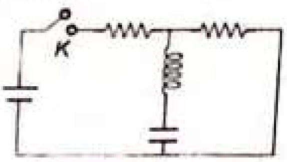
(a) 1
(b) 2
(c) same
(d) data are insufficient to decide

77. An inductor coil stores 32 J of magnetic field energy and dissipates energy as heat at the rate of 320 W when a current of 4 amp is passed through it. Find the time constant of the circuit when it is formed across an ideal battery.

(a) $\tau = 0.2$ sec (b) $\tau = 0.32$ sec

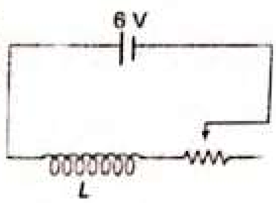
(c) $\tau = 0.5$ sec (d) $\tau = 1$ sec

78. In the circuit shown in figure if the switch K is pressed, then



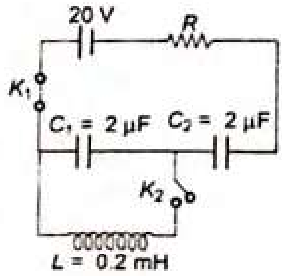
(a) initial current in the circuit will be greater than steady state current
(b) initial current in the circuit will be equal to steady state current
(c) initial current in the circuit will be less than steady state current
(d) initial current in the circuit will be zero

79. In the circuit shown, sliding contact is moving with uniform velocity towards right. Its value at some instance is 12 Ω. The current in the circuit at this instant of time will be



(a) 0.5 A
(b) more than 0.5 A
(c) less than 0.5 A
(d) may be less or more than 0.5 A depending on the value of L

80. A circuit containing capacitors C_1 and C_2 as shown in the figure are in steady state with key K_1 closed. At the instant $t = 0$, if K_1 is opened and K_2 is closed then the maximum current in the circuit will be

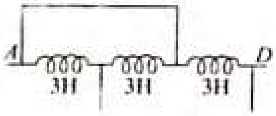


(a) 1 A (b) $\frac{1}{2}$ A

(c) 2 A (d) None

ARCHIVES

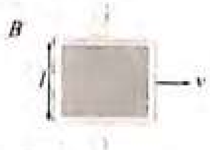
1. The inductance between A and D is



(a) 3.66 H (b) 9 H
(c) 0.66 H (d) 1 H

(AIEEE 2002)

2. A conducting square loop of side L and resistance R moves in its place with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane at the loop, exists everywhere with half the loop outside the field, as shown in the figure. The induced emf is



(a) zero (b) RvB
(c) $\frac{vBL}{R}$ (d) vBL **(AIEEE 2002)**

3. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
(a) the currents in the two coils.
(b) the rates at which currents are changing in the two coils.
(c) the relative position and orientation of the two coils
(d) the material of the wire of the coils **(AIEEE 2003)**

4. When the current changes from +2 A to −2 A in 0.05 s, an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is
(a) 0.1 H (b) 0.2 H
(c) 0.4 H (d) 0.8 H **(AIEEE 2003)**

5. In an oscillating LC circuit, the maximum charge in the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is
(a) Q (b) $\frac{Q}{2}$
(c) $\frac{Q}{\sqrt{3}}$ (d) $\frac{Q}{\sqrt{2}}$ **(AIEEE 2003)**

6. A coil having n turns and resistance R ohm is connected with a galvanometer of resistance $4R$ ohm. This combination is moved in time t second from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is
(a) $-\frac{W_2 - W_1}{5Rnt}$ (b) $-\frac{n(W_2 - W_1)}{5Rt}$
(c) $-\frac{W_2 - W_1}{Rnt}$ (d) $-\frac{n(W_2 - W_1)}{Rt}$

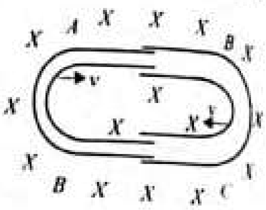
(AIEEE 2004)

7. In a uniform magnetic field of induction B, a wire in the form of a semi-circle of radius r rotates about the diameter of the circle with an angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, the mean power generated per period of rotation is
(a) $\frac{B\pi r^2 \omega}{2R}$ (b) $\frac{(B\pi r^2 \omega)^2}{8R}$
(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega H_2)^2}{8R}$

(AIEEE 2004)

8. A metal conductor of length 1 m rotates vertically about one of its ends at an angular velocity 5 radians per second. If the horizontal component of the earth's magnetic field is 0.2×10^{-4} T, then the emf developed between the two ends of the conductor is
(a) 5 μV (b) 50 μV
(c) 5 mV (d) 50 mV **(AIEEE 2004)**

9. A conducting U-tube can slide inside another as shown in the figure maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l, and v, where l is the width of each tube, will be

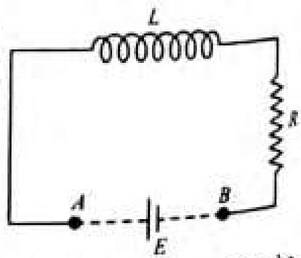


(a) $2Blv$ (b) zero
(c) $-Blv$ (d) Blv **(AIEEE 2005)**

10. The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of
(a) 2 μF (b) 1 μF
(c) 8 μF (d) 4 μF **(AIEEE 2005)**

11. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in
(a) 0.3 s (b) 0.15 s
(c) 0.1 s (d) 0.05 s **(AIEEE 2005)**

12. An inductor (L = 100 mH), a resistor (R = 100 Ω), and a battery (E = 100 V) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 mm after the short circuit is



(a) 0.1 A (b) 1 A
(c) $\frac{1}{e}$ A (d) e A **(AIEEE 2006)**

13. The flux linked with a coil at any instant t is given by $\phi = 10t^2 - 50t + 250$. The induced emf at t = 3 s is
(a) 10 V (b) 190 V
(c) −190 V (d) −10 V **(AIEEE 2006)**

14. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. Two seconds after the connection is made, the current flowing (in A) in the circuit is

(a) e (b) e^{-1}

(c) $1 - e^{-1}$ (d) $1 - e$ **(AIEEE 2007)**

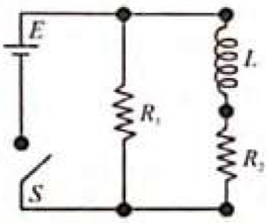
15. Two coaxial solenoids are made by winding a thin insulated wire over a pipe of cross-sectional area $A = 10\ \text{cm}^2$ and length 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}$ T m/A)

(a) $4.78\pi \times 10^{-5}$ H (b) $2.4\pi \times 10^{-4}$ H

(c) $2.4\pi \times 10^{-5}$ H (d) $4.8\pi \times 10^{-4}$ H

(AIEEE 2007)

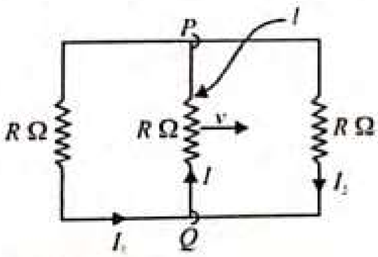
16. An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2 ohm and R_2 = 2 Ω are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t = 0. The potential drop across L as a function of time is



(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V

(c) $6(1 - e^{-t/0.2})$ V (d) $12e^{-5t}$ V **(AIEEE 2009)**

17. A rectangular loop has a sliding connector PQ of length l and resistance R ohm and it is moving with a speed v as shown. The set up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2, and I are



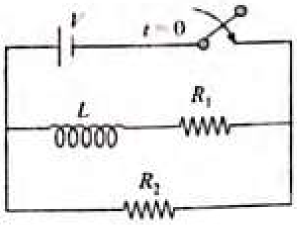
(a) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$

(d) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ **(AIEEE 2010)**

18. In the circuit shown below, the key K is closed at t = 0. The current through the battery is



(a) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(d) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$ **(AIEEE 2010)**

19. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is

(a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$

(c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$ **(AIEEE 2011)**

20. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1}m^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is

(a) 1 mV (b) 0.75 mV

(c) 0.50 mV (d) 0.15 mV **(AIEEE 2011)**

21. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to

(a) developement of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

(AIEEE 2012)

22. A metallic rod of length 'l' is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is

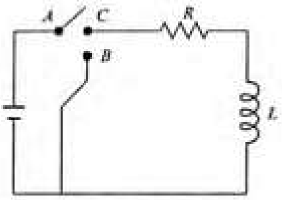
(a) $\frac{3B\omega l^2}{2}$ (b) $\frac{4B\omega l^2}{2}$

(c) $\frac{5B\omega l^2}{2}$ (d) $\frac{2B\omega l^2}{2}$ **(JEE Main 2013)**

23. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

(a) 6×10^{-11} weber
(b) 3.3×10^{-11} weber
(c) 6.6×10^{-9} weber
(d) 9.1×10^{-11} weber **(JEE Main 2013)**

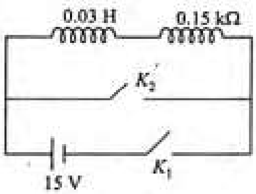
24. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to:



(a) -1 (b) $\frac{1-e}{e}$

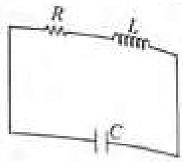
(c) $\frac{e}{1-e}$ (d) 1 **(JEE Main 2014)**

25. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15 V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ ms, the current in the circuit will be ($e^5 \cong 150$)



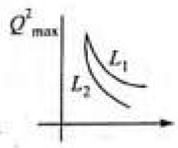
(a) 100 mA (b) 67 mA
(c) 6.7 mA (d) 0.67 mA
(JEE Main 2015)

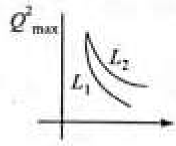
26. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below:

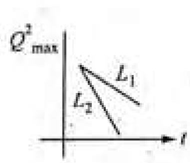


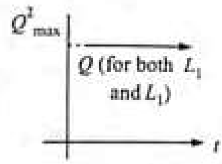
If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly?

(Plots are schematic and not drawn to scale)

(a)


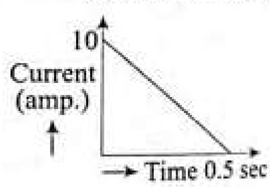
(b)


(c)


(d)


(JEE Main 2015)

27. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



(a) 250 Wb (b) 275 Wb
(c) 200 Wb (d) 225 Wb
(JEE Main 2017)

ANSWER KEY

Exercises

1. (d)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (b)
11. (d)	**12.** (d)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (b)
21. (c)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (d)	**30.** (b)
31. (d)	**32.** (d)	**33.** (d)	**34.** (d)	**35.** (c)	**36.** (c)	**37.** (b)	**38.** (b)	**39.** (d)	**40.** (c)
41. (b)	**42.** (a)	**43.** (a)	**44.** (b)	**45.** (c)	**46.** (b)	**47.** (d)	**48.** (d)	**49.** (b)	**50.** (c)
51. (a)	**52.** (b)	**53.** (a)	**54.** (c)	**55.** (a)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (d)	**60.** (b)
61. (c)	**62.** (a)	**63.** (b)	**64.** (a)	**65.** (b)	**66.** (a)	**67.** (d)	**68.** (a)	**69.** (a)	**70.** (b)
71. (a)	**72.** (a)	**73.** (a)	**74.** (b)	**75.** (b)	**76.** (a)	**77.** (a)	**78.** (b)	**79.** (b)	**80.** (a)

Archives

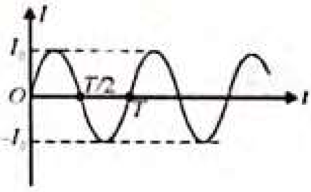
1. (d)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (d)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (b)
11. (c)	**12.** (c)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (d)
21. (d)	**22.** (c)	**23.** (d)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (a)			

Chapter **24**

Alternating Current

INTRODUCTION

We have studied about the dc circuits. But most of the power generated and used in the world is in the form of alternating current (ac). An alternating current is the current whose value changes with time in both magnitude and direction. We will study about a special kind of alternating current which varies sinusoidally. It is represented by $I = I_0 \sin(\omega t + \phi)$



where $I \rightarrow$ instantaneous current

$I_0 \rightarrow$ peak value or maximum value of ac

$\omega t + \phi \rightarrow$ phase at any time t

$\phi \rightarrow$ initial phase or phase constant

$\omega \rightarrow$ angular frequency

$T \rightarrow$ time period

(where $T = 2\pi/\omega$)

$f \rightarrow$ frequency (where $f = 1/T$)

The graph of I vs. t is shown for $\phi = 0$

Time period, T: It is the time taken to complete one complete cycle or one oscillation (one cycle consists of variation of current from 0 to maximum, maximum to 0, 0 to negative maximm, and negative maximum to 0).

Frequency, f: It is the number of cycles or oscillations completed per unit time.

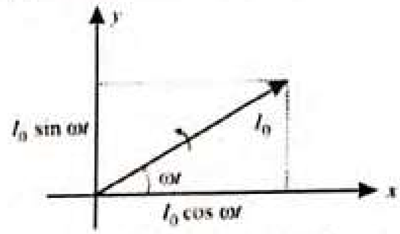
NOTE:

1. The equation of current can also be written in cosine form as $I = I_0 \cos(\omega t + \phi')$
2. To produce alternating current, emf should be alternating. An alternating emf can be represented by $E = E_0 = \sin(\omega t + \phi_1)$
3. An alternating emf is produced by a dynamo or an electronic oscillator.
4. The frequency of ac in India is 50 Hz, i.e., f = 50 Hz, so $\omega = 2\pi f = 100\pi$ rad s^{-1}.
5. The ac can be converted into dc with the help of a rectifier while dc into ac with the help of an inverter.
6. It cannot produce chemical effects of current such as electroplating or electrolysis as due to large ions, they cannot follow the frequency of ac.
7. It can be stepped up or stepped down with the help of transformer (while dc cannot be).

PHASOR DIAGRAMS

Generally, currents and voltages in ac circuits are represented in the form of phasors or anticlockwise rotating vectors. The length of arrow represents the peak value of the quantity and its projection on x- or y-axis gives its instantaneous value.

For example, let $I = I_0 \sin \omega t$, then it will be represented as shown in the figure Length of arrow is I_0 which represents the peak value of I. Its projection on y-axis is $I_0 \sin \omega t$ which represents the instantaneous value. ωt is the phase angle which increases with time.



NOTE: If the equation of current were in cosine form as $I = I_0 \cos \omega t$, then projection on x-axis will represent the instantneous value.

MEAN OR AVERAGE VALUE OF CURRENT

Mean or Average value of any current in a given time is defined as that constant value of current which will send the same amount of charge in a given circuit as sent by actual current in the same time interval.

$$I_{av}(t_2 - t_1) = \int_{t_1}^{t_2} I(t)\,dt \quad \Rightarrow \quad I_{av} = \frac{\int_{t_1}^{t_2} I(t)\,dt}{t_2 - t_1}$$

Average value of $I = I_0 \sin \omega t$

1. Over first half cycle: for this $t_1 = 0$, $t_2 = T/2 = \pi/\omega$

$$I_{av}\left(\frac{T}{2} - 0\right) = \int_0^{T/2} I_0 \sin \omega t\, dt = \left[\frac{-I_0 \cos \omega t}{\omega}\right]_0^{\pi/\omega}$$

$$\Rightarrow \quad I_{av}\left(\frac{T}{2}\right) = \frac{2I_0}{\omega} = \frac{2I_0 T}{2\pi}$$

$$\Rightarrow \quad I_{av} = \frac{2I_0}{\pi} = 0.637\, I_0$$

2. Over full cycle: for this $t_1 = 0$, $t_2 = T = 2\pi/\omega$

$$I_{av}(T - 0) = \int_0^{T} I_0 \sin \omega t\, dt = \left[\frac{-I_0 \cos \omega t}{\omega}\right]_0^{2\pi/\omega}$$

$$\Rightarrow \quad I_{av} = 0$$

So average value of current for the full cycle is zero. We can also think in this way:

Average value during positive half cycle: $0.637\, I_0$

Average value during negative half cycle: $-0.637\, I_0$

So over full cycle, $I_{av} = 0.637\, I_0 - 0.637\, I_0 = 0$

NOTE:

1. Average value of emf can also be found in the same way as we find for current. For this we can write the formula:

$$E_{av} = \frac{\int_{t_1}^{t_2} E(t)\,dt}{t_2 - t_1}$$

2. Average value of alternating current or emf over a long period of time will be zero, because for this time interval $t_2 - t_1$ will be large whereas quantity in the numerator will be finite.

Root Mean Square (rms) Value of Current

The root mean square value of any current is defined as that value of steady current (constant), which would generate the same amount of heat in a given resistance in a given time as generated by actual current passing through the same resistance for the same given time.

The rms value is also known as *effective value* or *virtual value* of current. It is denoted by by I_v.

$$I_v^2 R(t_2 - t_1) = \int_{t_1}^{t_2} [I(t)]^2 R dt \quad \Rightarrow \quad I_v = \sqrt{\frac{\int_{t_1}^{t_2} [I(t)]^2\, dt}{t_2 - t_1}}$$

We square the instantaneous current I, take the average (mean) value of I^2, and finally take the square root of that average. This procedure defines the root mean square current, denoted as I_{rms} or I_v. Even when I is negative, I^2 is always positive, so I_v is never zero (unless I is zero at every instant).

rms value of $I = I_0 \sin \omega t$:

1. Over first half cycle: for this $t_1 = 0$, $t_2 = T/2 = \pi/\omega$

$$I_v^2\left(\frac{T}{2} - 0\right) = \int_0^{T/2} I_0^2 \sin^2 \omega t\, dt = I_0^2 \int_0^{T/2} \left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

$$\Rightarrow \quad I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

2. Over full cycle: for this $t_1 = 0$, $t_2 = T = 2\pi/\omega$

$$I_v^2 (T - 0) = \int_0^{T} I_0^2 \sin^2 \omega t\, dt = I_0^2 \int_0^{T} \left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

$$\Rightarrow \quad I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

The rms value of ac over half cycle or full cycle is same. This is also valid over a long period of time.

NOTE:

1. Similarly, root mean square value of an emf.

$$E_v = \frac{E_0}{\sqrt{2}} = 0.707\, E_0$$

2. Ordinary dc ammeters and voltmeters cannot measure alternating current or alternating emf. They will show zero reading. For this purpose, we use ac ammeters and voltmeters. They are known as hot wire instruments. They measure virtual values of current or voltage. For example, if we measure $I = 10 \sin \omega t$ A from ac ammeter, it will show $I_v = \frac{10}{\sqrt{2}}$ A.

3. The symbol for an ac source is shown below.

ILLUSTRATION 24.1 The plate on the back of a personal computer says that it draws 2.7 A from a 120-V, 60-Hz line. For this computer, what is (a) the average of the square of the current, (b) the current amplitude, (c) the average current for a positive half cycle, and (d) the average current for a full cycle?

Solution.

(a) The current given is the rms value: $I_{rms} = 2.7$ A. The average of the square of the current is nothing but $(I_{rms})^2$. So, $(I_{rms})^2 = (2.7\text{ A})^2 = 7.3\text{ A}^2$

(b) The current amplitude I_0 is

$$I_0 = \sqrt{2}\, I_{rms} = \sqrt{2}\,(2.7\text{ A}) = 3.8\text{ A}$$

(c) For a positive half cycle, $I_{av} = \frac{2I_0}{\pi} = \frac{2 \times 3.8}{\pi} = 2.42$ A

(d) For a full cycle, $I_{av} = 0$

ILLUSTRATION 24.2 A voltage, $E = 60 \sin(314t)$, is applied across a resistor of 20 Ω. What will be the reading of I_{rms}

(a) in an ac ammeter?

(b) in an ordinary moving coil ammeter in series with the resistor?

Solution. Given that $E = 60 \sin(314t)$

(a) An ac ammeter will read the rms value

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{60}{\sqrt{2}} = 42.4\text{ V}$$

$$I_{rms} = \frac{E_{rms}}{R} = \frac{42.4}{20} = 2.12\text{ A}$$

Therefore, ac ammeter will read 2.12 A.

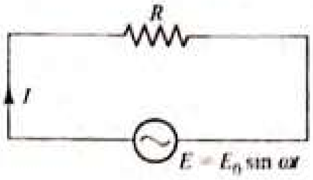
(b) An ordinary moving coil ammeter will read the average value of alternating current. Since the average value of ac is zero, this meter will give zero reading.

AC CIRCUITS

Basic ac circuit elements are resistors, inductors, and capacitors. We will discuss the behaviour of each of them when connected in ac circuits.

AC Circuits Containing Resistor Only

Consider resistor R is connected across emf $E = E_0 \sin \omega t$ as shown in the figure.

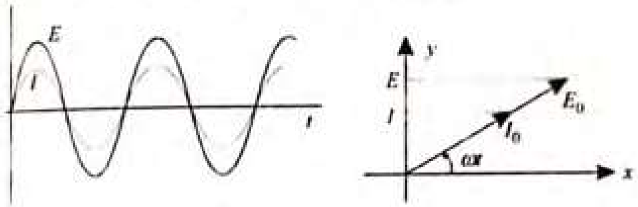


Due to this emf, current will flow in the resistance which will vary in both magnitude and direction. At any time, current is given by

$$I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t$$

$$= I_0 \sin \omega t \quad \left[\text{where } I_0 = \frac{E_0}{R} \rightarrow \text{Current amplitude}\right]$$

Variation of E and I is shown in Figure (a) and their phasor diagram representation is shown in Figure (b).



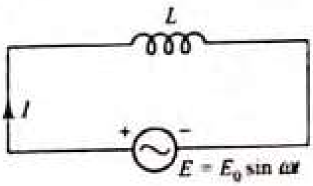
(a) (b)

From both the diagrams we see that phase of current and emf are same.

Same phase means both are varying in the same manner. Both are becoming zero at the same time, attaining their positive maximum, and negative maximum values at the same time, etc. The current and voltage phasors rotate together; they are parallel at each instant. Their projections on the vertical axis represent their instantaneous values.

AC Circuits Containing Inductor Only

Consider a pure inductor (having no resistance) of inductance L is connected as shown in the figure.



Induced emf in the inductor (with the polarities as shown in the figure, assuming current increases with time): $E = L\dfrac{dI}{dt}$

As there is only inductor in the circuit, induced emf should be equal to applied emf.

$$\Rightarrow \quad \frac{LdI}{dt} = E_0 \sin \omega t$$

$$\Rightarrow \quad \int dI = \int \frac{E_0}{L} \sin \omega t \, dt$$

$$\Rightarrow \quad I = -\frac{E_0}{\omega L} \cos \omega t + C$$

where C is constant of integration. To determine the value of C, let us take the average of current for one time period.

$$I_{av} = \frac{\int_0^T I\,dt}{T} = \frac{1}{T}\int_0^T \left(-\frac{E_0}{\omega L}\cos \omega t + C\right)dt$$

$$= \frac{1}{T}\left[\frac{-E_0}{\omega^2 L}\sin \omega t + Ct\right]_0^T = \frac{1}{T}[Ct] = C$$

But for any kind of alternating current $I_{av} = 0$ over one cycle Hence, $C = 0$. So, finally, we get

$$I = -\frac{E_0}{\omega L}\cos \omega t$$

$$= \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

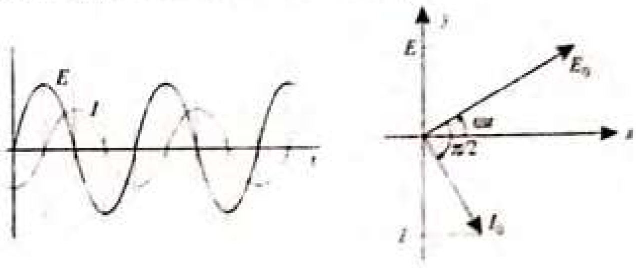
$$= \frac{E_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $X_L = \omega L$ is the resistance offered by inductor. It is also known as inductive reactance. Its unit is Ω.

$$I_0 = \frac{E_0}{X_L} \rightarrow \text{Current amplitude}$$

Variation of E and I is shown in Figure (a) and their phasor diagram respresentation is shown in Figure (b).



(a) (b)

From Figure (a), voltage and current are "out of step" or out of phase by a quarter cycle. Voltage peak occurs a quarter cycle earlier than the current peak. Thus, we say as that the voltage leads the current by 90°.

Also from Figure (b), we see that the phase of emf is ωt and that of current is $\omega t - \pi/2$. So, phase of current is $\pi/2$ less than emf. Hence, current lags behind emf by phase $\pi/2$, or emf leads current by phase $\pi/2$.

A phase difference of $\pi/2$ is equivalent to time difference of $T/4$. We see that current attains its zero or maximum (positive or negative) values after time $T/4$ of emf has attained the same type of value.

NOTE:

1. For dc circuit $\omega = 0 \Rightarrow X_L = 0$. So inductor offers zero resistance to a dc circuit.
2. X_L depends upon the frequency of source.

Inductive Reactance

Inductive reactance X_L is a description of the self-induced emf that opposes any change in the current through the inductor. Inductive reactance and self-induced emf increase with more rapid variation in current (that is, increasing angular frequency, ω) and increasing inductance L.

If an oscillating voltage of a given amplitude E_0 is applied across the inductor terminals, the resulting current will have a smaller amplitude I for larger values of X_L. since X_L is proportional to frequency, a high-frequency voltage applied to the inductor gives only a small current, while a lower-frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio- interference filters, to block high frequencies while permitting lower frequencies or dc to pass through. A circuit device that uses an inductor for this purpose is called a *low-pass filter*.

ILLUSTRATION 24.3 Suppose you want the current amplitude in a pure inductor in a radio receiver to be 250 μA when the voltage amplitude is 3.60 V at a frequency of 1.60 MHz (corresponding to the upper is 3.60 V AM broadcast band). What inductive reactance is needed? What inductance is required?

Solution. We are given the current amplitude, I_0, and the voltage amplitude, E_0. The inductive reactance

$$X_L = \frac{E_0}{I_0} = \frac{3.60}{250 \times 10^{-6}} = 1.44 \times 10^4\ \Omega = 14.4\ \text{k}\Omega$$

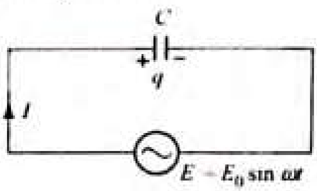
$$L = \frac{X_L}{2\pi f} = \frac{1.44 \times 10^4\ \Omega}{2\pi(1.60 \times 10^6\ \text{Hz})} = 1.43 \times 10^{-3}\ \text{H} = 1.43\ \text{mH}$$

AC Circuits Containing Capacitor Only

Consider a capacitor of capacitance C is connected as shown in the figure. Charge on the capacitor at any time is

$$q = CE = CE_0 \sin \omega t$$

Current in the circuit at any time is



$$I = \frac{dq}{dt} = CE_0 \omega \cos \omega t$$

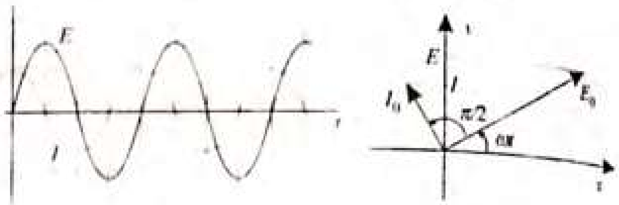
$$\Rightarrow \quad I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $I_0 = \dfrac{E_0}{X_C}$ and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

X_C represents effective resistance offered by the capacitor. It is known as capacitive reactance. More the frequency, lesser is the reactance. Further, if $f = 0$, $X_C = \infty$. So for dc circuit, capacitor offers infinite resistance or it blocks dc. Unit of X_C is Ω.

Variation of E and I is shown in Figure (a) and their phasor diagram respresentation is shown in Figure (b).



(a) (b)

The capacitor voltage and the current are out of phase by a quarter cycle. The peak of voltage occurs quarter cycle after the corresponding current peak. And we say that the voltage phasor is behind the current phasor by a quarter cycle or 90°.

Capacitive Reactance

The capacitive reactance of a capacitor is inversely proportional to both the capacitance C and the angular frequency ω. Greater the capacitance and higher the frequency, smaller is the capacitive reactance, X_C. Capacitors tend to pass high-frequency current and block low-frequency currents and dc, just the opposite of inductors. A device that preferentially passes signals of high frequency is called a *high-pass filter*.

NOTE: In case of resistor, resistance is offered due to the obstruction to the passage of current. In inductor, it is induced emf. In capacitor, it is potential difference developed across the capacitor.

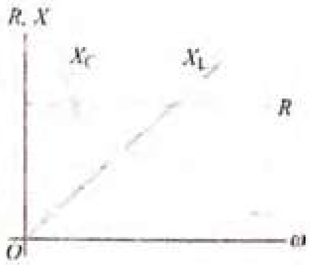
The whole of the above discussion can be generalized as:
Let $E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$. Here we have assumed that current leads emf by an angle, ϕ. ϕ is also known as phase factor. Here $I_0 = E_0/Z$, where Z is known as impedance of the circuit. Its unit is Ω. We can also write $Z = \dfrac{E_0}{I_0} = \dfrac{E_v}{I_v}$

Values of ϕ and Z for different types of circuits is given in the table below:

Type of circuit	Phase factor	Impedance
Purely resistive circuit	$\phi = 0°$	$Z = R$
Purely inductive circuit	$\phi = -\pi/2$	$Z = X_L = \omega L$
Purely capacitive circuit	$\phi = \pi/2$	$Z = X_C = \dfrac{1}{\omega C}$

The reciprocal of reactance is called susceptance and that of impedance is called admittance.

Figure shows how the resistance of a resistor and the reactances of an inductor and a capacitor vary with angular frequency, ω. Resistance R is independent of frequency, while the reactances X_L and X_C are not. If $\omega = 0$, corresponding to a dc circuit, there is no current through a capacitor because $X_C \to \infty$, and there is no inductive effect because $X_L = 0$. In the limit $\omega \to \infty$, X_L also approaches infinity, and the current through an inductor becomes vanishingly small; recall that the self-induced emf opposes rapid changes in the current. In this same limit, X_C and the voltage across a capacitor both approach zero; the current changes direction so rapidly that no charge can build up on either plate.



AC Circuit Containing Resistor, Inductor, and Capacitor in Series (Series *LCR* Circuit)

Figure shows a circuit in which a resistor of resistance R, an inductor of inductance L, and a capacitor of capacitance C are connected in series across an alternating source of emf

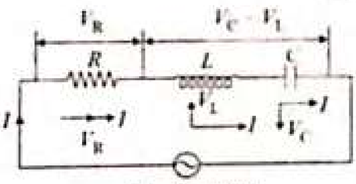
$$E = E_0 \sin \omega t \tag{i}$$

We want to determine the instantaneous current I in the circuit and its phase relationship to the applied voltage.

Let us assume that current I is flowing in the circuit as

$$I = I_0 \sin(\omega t + \phi) \tag{ii}$$

where I_0 is peak current and ϕ is the phase by which current leads emf.



V_R, V_L, and V_C are the instantaneous voltages across R, L and C, respectively. We can write the following relations:

$$V_R = V_{R0} \sin(\omega t + \phi) \tag{iii}$$

$$V_L = V_{L0} \sin\left(\omega t + \phi + \frac{\pi}{2}\right)$$

$$= V_{L0} \cos(\omega t + \phi) \tag{iv}$$

$$V_C = V_{C0} \sin\left(\omega t + \phi - \frac{\pi}{2}\right)$$

$$= -V_{C0} \cos(\omega t + \phi) \tag{v}$$

The above equations are written keeping in mind the phase relationship between current and voltage across each of the element resistor, inductor, and capacitor.

Equation (iii) shows that current and voltage across R are in phase (having same phase $\omega t + \phi$).

Equation (iv) shows that voltage across inductor leads the current by $\pi/2$.

Equation (v) shows that voltage across capacitor lags current by $\pi/2$.

V_{R0}, V_{L0}, and V_{C0} are the peak values of voltages across R, L, and C, respectively. We can write

$$\begin{bmatrix} V_{R0} = I_0 R \\ V_{L0} = I_0 X_L \\ V_{C0} = I_0 X_C \end{bmatrix} \tag{vi}$$

At any instant:

$$E = V_R + V_L + V_C$$

$$\Rightarrow \quad E_0 \sin \omega t = V_{R0} \sin(\omega t + \phi) + (V_{L0} - V_{C0}) \cos(\omega t + \phi) \tag{vii}$$

Equation (vii) is a general equation and it is true at any instant of time. Replacing ωt by $\omega t + \pi/2$ in Eq. (vii), we get

$$E_0 \cos \omega t = V_{R0} \cos(\omega t + \phi) - (V_{L0} - V_{C0}) \sin(\omega t + \phi) \tag{viii}$$

Squaring and adding (vii) and (viii),

$$E_0^2 = V_{R0}^2 + (V_{L0} - V_{C0})^2 \tag{ix}$$

$$= I_0^2\left[R^2 + (X_L - X_C)^2\right] \qquad \text{[using (vi)]}$$

$$\Rightarrow \quad \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{x}$$

Equation (x) gives impedance of the circuit.

Dividing Eq. (vii) by Eq. (viii) and simplifying, we get

$$\tan \phi = \frac{V_{C0} - V_{L0}}{V_{R0}} \tag{xi}$$

$$= \frac{I_0 X_C - I_0 X_L}{I_0 R}$$

$$= \frac{X_C - X_L}{R} \tag{xii}$$

Using Eq. (xii), we can find ϕ, the phase difference between current and voltage. It is to be noted that $-\pi/2 \le \phi \le \pi/2$.

If $X_C < X_L$, then ϕ is negative

If $X_C > X_L$, then ϕ is positive

If $X_C = X_L$, then $\phi = 0$

If $R = 0$, then $\phi = \frac{\pi}{2}$ (if $X_C > X_L$) or $\phi = -\frac{\pi}{2}$ (if $X_L > X_C$)

NOTE: Equation (ix) can also be written in terms of virtual voltages.

$$E_v^2 = V_{Rv}^2 + (V_{Lv} - V_{Cv})^2$$

because $E_0 = \sqrt{2}\, E_v$, $V_{R0} = \sqrt{2}\, V_{Rv}$, $V_{L0} = \sqrt{2}\, V_{Lv}$

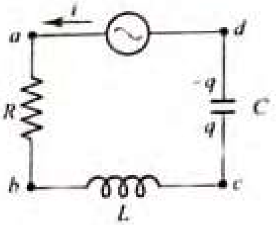
and $V_{C0} = \sqrt{2}\, V_{Cv}$

Similarly, Eq. (xi) can also be written in terms of virtual voltages:

$$\tan \phi = \frac{V_{Cv} - V_{Lv}}{V_{Rv}}$$

We can also write $V_{RV} = I_{VR}$
$V_{LV} = I_V X_L$
$V_{CV} = I_V X_C$

ILLUSTRATION 24.4 In the series circuit of the figure, suppose $R = 300\ \Omega$, $L = 60$ mH, $C = 0.50\ \mu$F, source amplitude is $E_0 = 50$ V and $\omega = 10{,}000$ rad s^{-1}. Find the reactances X_L and X_C, the impedance Z, the current amplitude I_0, the phase angle ϕ, and the voltage amplitude across each circuit element.



Solution. The inductive and capacitive reactances are

$$X_L = \omega L = (10{,}000\ \text{rad/s})(60\ \text{mH}) = 600\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(10{,}000\ \text{rad s}^{-1})(0.50 \times 10^{-6}\ \text{F})} = 200\ \Omega$$

The impedance Z of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(300\,\Omega)^2 + (600\,\Omega - 200\,\Omega)^2} = 500\ \Omega$$

With source voltage amplitude $E_0 = 50$ V, the current amplitude is

$$I_0 = \frac{E_0}{Z} = \frac{50\ \text{V}}{500\,\Omega} = 0.10\ \text{A}$$

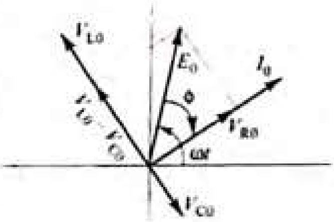
The phase angle ϕ is

$$\phi = \tan^{-1}\frac{X_C - X_L}{R} = \tan^{-1}\left(-\frac{400\,\Omega}{300\,\Omega}\right) = -53°$$

The voltage amplitudes V_{R0}, V_{L0}, and V_{C0} across the resistor, inductor, and capacitor, respectively, are

$$V_{R0} = I_0 R = (0.10\ \text{A})(300\ \Omega) = 30\ \text{V}$$
$$V_{L0} = I_0 X_L = (0.10\ \text{A})(600\ \Omega) = 60\ \text{V}$$
$$V_{C0} = I_0 X_C = (0.10\ \text{A})(200\ \Omega) = 20\ \text{V}$$

Note that $X_L > X_C$ and hence the voltage amplitude across the inductor is greater than that across the capacitor and ϕ is negative. The value $\phi = -53°$ means that the voltage leads the current by 53°; this is like the situation shown in the figure.



Note that the source voltage amplitude $V = 50$ V is not equal to the sum of the voltage amplitude across the separate circuit elements (that is, 50 V π 30 V + 60 V + 0 V).

POWER IN SERIES *LCR* CIRCUITS

Alternating currents play a central role in the system for distributing, converting, and using electrical energy, so it is important to look at power relationships in ac circuits.

The power in an electric circuit is the rate at which electrical enegy is consumed in the circuit. Let an emf $E = E_0 \sin \omega t$ be applied to a series LCR circuit. The curent in the circuit is $I = I_0 \sin(\omega t + \phi)$, where ϕ is the phase difference between current and voltage. The instantaneous power is given by

$$P = EI = E_0 I_0 \sin \omega t \sin(\omega t + \phi)$$

If the average power consumed in the cycle from time t_1 to t_2 is P_{av}, then

$$P_{av}(t_2 - t_1) = \int_{t_1}^{t_2} P\,dt$$

Average power over 1 cycle, i.e., $t_1 = 0$ to $t_2 = T$ is given by

$$P_{av}T = \int_0^T E_0 I_0 \sin \omega t \sin(\omega t + \phi)\,dt$$

$$\Rightarrow \quad P_{av}T = E_0 I_0 \int_0^T \left(\sin^2 \omega t \cos\phi + \frac{\sin 2\omega t}{2}\sin\phi\right)dt$$

$$\left[\begin{array}{ll} \text{remember} & \int_0^T \sin^2 \omega t\,dt = \frac{T}{2} \\ \text{and} & \int_0^T \sin 2\omega t\,dt = 0 \end{array}\right]$$

$$\Rightarrow \quad P_{av}T = \frac{E_0 I_0}{2}\cos\phi\,T$$

$$\Rightarrow \quad P_{av} = E_v I_v \cos\phi$$
$$= P_{ap}\cos\phi,$$

where $P_{ap} = E_v I_v \rightarrow$ apparent power or virtual power

and $\cos\phi = \dfrac{P_{av}}{P_{ap}} \rightarrow$ power factor

also $P_{av} = E_v I_v \cos\phi = (I_v Z) I_v \dfrac{R}{Z} = I_v^2 R$

For pure resistive circuit: $\phi = 0$ and $Z = R$

$$P_{av} = E_v I_v = (I_v Z) I_v = I_v^2 Z = I_v^2 R$$

For pure inductive circuit: $\phi = -\dfrac{\pi}{2}$

So $P_{av} = 0$

Therefore, the average power over a complete cycle of ac through an ideal inductor is zero. Actually, whatever energy is needed in building up current in inductance is returned back during the decay of current.

For pure capacitive circuit: $\phi = \dfrac{\pi}{2}$

$$P_{av} = 0$$

In this case too, the average power is zero. Actually, whatever energy is needed in building up the voltage across capacitor is returned to the source during discharging of capacitor.

So average power consumed in pure inductive or pure capacitive circuit is zero. So there is no loss of energy in the

inductor or capacitor. But in a resistance, loss of energy occurs. It cannot store the energy like inductor or capacitor.

The current through pure L or C, which dissipates no power is called *idle current* or *wattless current*. L and C are most suitable for controlling the current in ac circuits.

Wattless Component of Current

$I_v \sin\phi$ is known as wattless component of current.

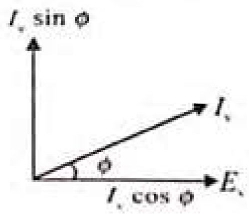


ILLUSTRATION 24.5 The potential difference E and current I flowing through the ac circuit is given by $E = 5\cos(\omega t - \pi/6)$ V and $I = 10 \sin \omega t$ A. Find the average power dissipated in the circuit.

Solution. $E = 5\cos[\omega t - (\pi/6)]$,

$I = 10\sin\omega t = 10\cos[\omega t - (\pi/2)]$

Phase difference between E and I is given by:

$$\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

Average power dissipated:

$$P = \frac{E_0 I_0}{2}\cos\phi = \frac{5\times 10}{2}\times\frac{1}{2} = 12.5 \text{ W}$$

RESONANCE IN SERIES *LCR* CIRCUIT

When the current in the series LCR circuit is maximum possible, then the circuit is said to be in resonance. We know that in a series LCR circuit, amplitude of current is given by

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

where $X_L = \omega L$ and $X_C = 1/\omega C$

If we vary the angular frequency ω of the source, X_L and X_C both vary and I_0 also varies. I_0 is maximum when Z is minimum. Z is minimum when $X_L = X_C$. Let at $\omega = \omega_0$ when $X_L = X_C$

$$\Rightarrow \quad \omega_0 L = \frac{1}{\omega_0 C} \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

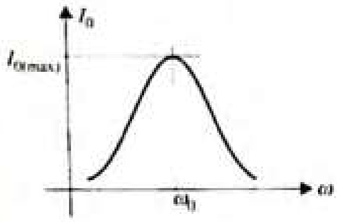
$$\Rightarrow \quad 2\pi f_0 = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

So at $\omega = \omega_0$, maximum possible current flows in circuit and the circuit is said to be in resonance. ω_0 is known as resonance frequency. At resonance frequency, impedance is minimum which is given by $Z_{min} = R$ and at resonance, the value of I_0 is maximum which is given by

$$I_{0(max)} = \frac{E_0}{R}$$

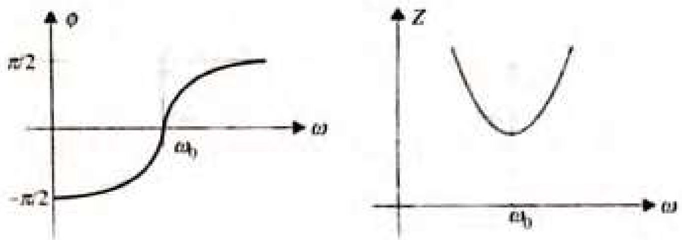
The variation of I_0 with ω is as shown in the figure.

A series resonance circuit is used in tuning circuits. The tuning circuits of radio and TV sets, etc. are series LCR circuits.



The instantaneous voltages across L and C always differ in phase by 180°; they have opposite sign at each instant. At the resonance frequency, $X_L = X_C$ and the voltages $V_L = V_C$. Then the instantaneous voltages across L and C add to zero at each instant, and the total voltage across the L–C combination is exactly zero. The voltage across the resistor is then equal to the source voltage. So at the resonance frequency, the circuit behaves as if the inductor and the capacitor were not there at all!

This variation of ϕ and Z with angular frequency is shown in the figure.



NOTE: At resonance power, factor is maximum. It is given by

$$\cos\phi = \frac{Z}{R} = \frac{R}{R} = 1 \qquad (\because Z = R \text{ at resonance})$$

Thus, power factor is unity at resonance. Maximum power is dissipated during resonance.

Q-Factor (or Quality Factor) of Resonance

The quality factor Q is a parameter which is used to describe the sharpness of resonance curve. The quality factor Q of a series resonance circuit is defined as the ratio of voltage drop across inductor (or capacitor) at resonance to the applied voltage. Thus,

$$Q = \frac{\text{voltage across } L \text{ (or } C\text{) at resonance}}{\text{applied voltage}} \qquad \text{(i)}$$

$$= \frac{I_v \omega_0 L}{I_v R} = \omega_0\left(\frac{L}{R}\right)$$

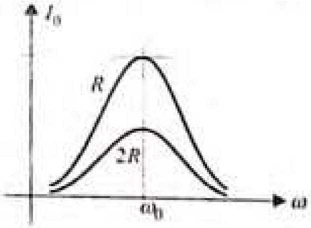
$$= \frac{1}{\sqrt{LC}}\times\frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \text{(ii)}$$

The rapidity with which the current falls from its resonant value $I_{0(max)} = E_0/R$ with change in applied frequency is known as sharpness of resonance. More quickly the current falls from

its maximum value with change in frequency, more is the sharpness of resonance.

If Q-factor is high, the current will respond to very narrow range of frequencies and the resonance will be sharper. Usual value of Q is 10 to 100 but it can go up to 200. Q-factor is a dimensionless quantity. Equation (ii) shows that when R increases, Q-factor of the circuit decreases.

In the figure, two curves are plotted, one when the circuit resistance is low, i.e., R, and the other when the circuit resistance is high, i.e., $2R$. We observe the following points:



- The peak of the curve depends on the resistance of the circuit. When R is low, the peak is high and vice-versa. The peak is related to sharpness of resonance. More is height of peak, more sharp is the resonance.
- It is clear from the figure that the width of resonance curve increases on increasing the value of R. Therefore, the resonance curve becomes flatter.
- The series resonant circuit is sometimes called the *acceptor circuit*. The reason is that impedance of the circuit is minimum at resonance and, due to this fact, it readily accepts that current out of the many currents whose frequency is equal to its resonant frequency.

  Lesser the band width, sharper is the resonance.

ILLUSTRATION 24.6 A series LCR circuit with $L = 0.12$ H, $C = 480$ nF, and $R = 23\ \Omega$ is connected to a 230-V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum? Find this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency?

(d) What is the Q-factor of the circuit?

Solution.

(a) Current amplitude is maximum at resonance and source frequency at resonance is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.12\times 480\times 10^{-9}}}$$

$$= 663.14 \text{ Hz}$$

and $\quad \omega_0 = 2\pi f_0 = 4166.66 \text{ rad s}^{-1}$

The maximum current amplitude is given by

$$I_{0(\max)} = \frac{E_0}{R} = \frac{230\sqrt{2}}{23} = 14.14 \text{ A}$$

(b) Average power absorbed by the circuit is maximum at resonance for which frequency is calculted above as 663.14 Hz. Maximum power absorbed is given by

$$P_{\max} = \frac{E_v^2}{R} = \frac{(230)^2}{23} = 2300 \text{ W}$$

(c) $P = E_v I_v \cos\phi = E_v I_v \dfrac{R}{Z} = \dfrac{E_v^2}{Z^2} R$ and $P_{\max} = \dfrac{E_v^2}{R}$

Given $P = P_{\max}/2$

$$\Rightarrow \quad \frac{E_v^2 R}{Z^2} = \frac{1}{2}\frac{E_v^2}{R} \quad \Rightarrow \quad Z^2 = 2R^2$$

$$\Rightarrow \quad R^2 + (X_C - X_L)^2 = 2R^2$$

$$\Rightarrow \quad X_C - X_L = \pm R$$

Solve to get

$$\omega_1 = \frac{-RC + \sqrt{R^2C^2 + 4LC}}{2LC} = 4071.9 \text{ rad s}^{-1}$$

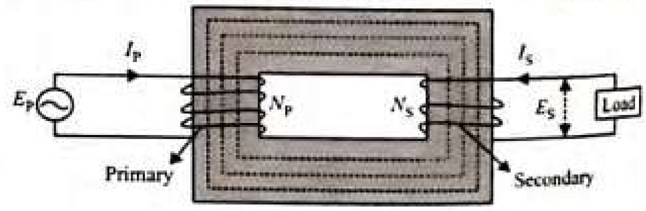
and $\quad \omega_2 = \dfrac{RC + \sqrt{R^2C^2 + 4LC}}{2LC} = 4263.6 \text{ rad s}^{-1}$

(d) Q-factor is given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{23}\sqrt{\frac{0.12}{480\times 10^{-9}}} = 21.74$$

TRANSFORMERS

A transformer is an ac device which transfers electric power from one circuit to another. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. Here it is worth mentioning that while transferring the power, frequency is not altered. Figure shows a basic transformer.



Theory and working: Let N_P and N_S be the number of turns in the primary and secondary coils, respectively. Here we assume that the entire magnetic flux of the primary coil is linked to the secondary coil. This results in the same magnetic flux per turn linked with each of the two coils. Let ϕ be the flux linked with each turn of either coil at any instant. Then

emf induced in primary coil, $E_P = -\dfrac{N_P d\phi}{dt}$

emf induced in secondary coil, $E_S = -\dfrac{N_S d\phi}{dt}$

If an alternating emf E_P is applied across that primary coil, then Kirchhoff's loop law applied to primary circuit gives

$$E_P - N_P\frac{d\phi}{dt} = 0$$

or $$E_P = N_P \frac{d\phi}{dt} \qquad \text{(i)}$$

Here, we neglect the resistance of primary circuit.

If E_S represents the alternating emf in secondary coil, then

$$E_S = -N_S \frac{d\phi}{dt} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{E_S}{E_P} = -\frac{N_S}{N_P} \quad \text{or} \quad E_S = -\frac{N_S}{N_P} E_P$$

The negative sign shows that E_S is 180° out of phase with E_P. Ignoring negative sign,

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} \qquad \text{(iii)}$$

Let I_P and I_S be the currents at any instant in primary and secondary coils, respectively. Considering the transformer to be ideal (no loss of energy by any means),

$$E_P I_P = E_S I_S \Rightarrow \frac{E_S}{E_P} = \frac{I_P}{I_S} \qquad \text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

or $$I_S = I_P\left(\frac{N_P}{N_S}\right) = \frac{I_P}{K} \qquad \text{(v)}$$

where $K = (N_S/N_P)$ = transformer ratio.

When $K > 1$, then $N_S > N_P$. The transformer is known as *step up transformer*. So, in a step up transformer, the number of turns in secondary coil is more than the number of turns in primary coil. This transformer converts a low voltage at high current to a high voltage at low current.

When $K > 1$, then $N_S < N_P$. The transformer is known as *step-down transformer*. So, in a step down transformer, the number of turns in primary coil is more than the number of turns in secondary coil. This transformer converts high voltage at low current to a low voltage at high current.

When a resistance R is introduced in the secondary circuit, then a current I_S flows in it. The current is in phase with the voltage. The current is given by $I_S = E_S/R$

where $I_S = I_P\left(\frac{N_P}{N_S}\right)$ and $E_S = E_P\left(\frac{N_S}{N_P}\right)$

Putting these values, we get

$$I_P\left(\frac{N_P}{N_S}\right) = E_P \times \left(\frac{N_S}{N_P}\right) \times \frac{1}{R}$$

or $$I_P = \frac{E_P}{(N_P/N_S)^2 R} = \frac{E_P}{R_{eq}}$$

where $R_{eq} = \left(\frac{N_P}{N_s}\right)^2 R$

This current I_P is the same as if we connect a resistance $R_{eq} = (N_P/N_S)^2 R$ across the primary. This effect is called *impedance transformation*. Impedance matching is an important function of the transformer.

Efficiency of a transformer: There are some losses of energy due to primary resistance, hysteresis in the core, eddy currents in the core, etc., in an ordinary tranformer. The efficiency of a transformer is defined as the ratio of output power to the input power

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{E_S I_S}{E_P I_P}$$

For an indeal transformer, $\eta = 1$ (100 %). Due to transformer, losses, the efficiency is less than one (less than 100 %).

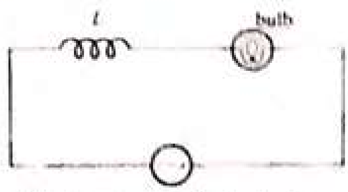
It is important to mention that a transformer is an ac device and it cannot work on dc.

SKIN EFFECT

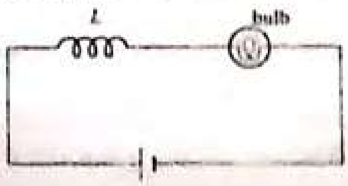
When a steady current flows through a cylindrical conductor, it is distributed uniformly over the whole cross-section of the conductor. But when an alternating current of high frequency flows through the conductor, the curent density is not uniform throughout the cross-sectional area. The current density is more near the surface than inside the conductor. So the high frequency alternating current is confined to the surface layer. The phenomenon is called *skin effect*. Since the high frequency alternating current does not pass through the entire cross-section, hence the effective resistance of the conductor for ac is much higher, than dc. So in the transmission of ac, a bundle of wire is preferred than a thick wire to reduce the resistance caused by skin effect.

CONCEPT APPLICATION EXERCISE 24.1

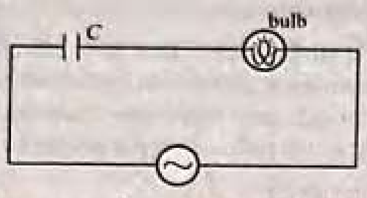
1. Can we use 15 Hz ac for lighting purposes?
2. A soft iron rod is inserted in the solenoid. After this what will be the effect on the brightness of the bulb?
3. In the above question, what will be the effect on the brightness of bulb if frequency of the source ω is increased?



4. An iron rod is inserted in the inductor, what will be the effect on the brightness of the bulb?



5. In the circuit shown in the figure, what will be the effect on the brightness of bulb if frequency of the source ω is increased?



6. In the above question, what will be the effect on the brightness of bulb if capacitance C is reduced?

SOLVED EXAMPLES

1. When 100 volts dc is supplied across a solenoid, a current of 1.0 ampere flows in it. When 100 volt ac is applied across the same coil, the current drops to 0.5 ampere. If the frequency of ac source is 50 Hz, then the impedance and inductance of the solenoid are
   (a) 200 Ω and 0.55 henry
   (b) 100 Ω and 0.86 henry
   (c) 200 Ω and 1.0 henry
   (d) 100 Ω and 0.93 henry

Sol. (a) For dc, $R = \frac{V}{i} = \frac{100}{1} = 100\ \Omega$

For ac, $Z = \frac{V}{i} = \frac{100}{0.5} = 200\ \Omega$

$\because \quad Z = \sqrt{R^2 + (\omega L)^2}$

$\Rightarrow \quad 200 = \sqrt{(100)^2 + 4\pi^2(50)^2 L^2}$

$\therefore \quad L = 0.55$ H

2. A bulb and a capacitor are in series with an ac source. On increasing frequency how will glow of the bulb change
   (a) The glow decreases
   (b) The glow increases
   (c) The glow remain the same
   (d) The bulb quenches

Sol. (b) This is because, when frequency ν is increased, the capacitive reactance $X_C = \frac{1}{2\pi\nu C}$ decreases and hence the current through the bulb increases

3. $\frac{2.5}{\pi}\mu$F capacitor and 3000-ohm resistance are joined in series to an ac source of 200 volt and 50 sec^{-1} frequency. The power factor of the circuit and the power dissipated in it will respectively
   (a) 0.6, 0.06 W (b) 0.06, 0.6 W
   (c) 0.6, 4.8 W (d) 4.8, 0.6 W

Sol. (c) $Z = \sqrt{R^2 + \left(\frac{1}{2\pi\nu C}\right)^2}$

$$= \sqrt{(3000)^2 + \frac{1}{\left(2\pi \times 50 \times \frac{2.5}{\pi} \times 10^{-6}\right)^2}}$$

$$\Rightarrow \quad Z = \sqrt{(3000)^2 + (4000)^2} = 5 \times 10^3\ \Omega$$

So power factor $\cos\varphi = \frac{R}{Z} = \frac{3000}{5 \times 10^3} = 0.6$ and power

$$P = V_{rms} i_{rms} \cos\varphi = \frac{V_{rms}^2 \cos\varphi}{Z} \Rightarrow P = \frac{(200)^2 \times 0.6}{5 \times 10^3} = 4.8\ \text{W}$$

4. The self-inductance of a choke coil is 10 mH. When it is connected with a 10 V dc source, then the loss of power is 20 watt. When it is connected with 10 volt ac source loss of power is 10 watt. The frequency of ac source will be
   (a) 50 Hz (b) 60 Hz
   (c) 80 Hz (d) 100 Hz

Sol. (c) With dc : $P = \frac{V^2}{R} \Rightarrow R = \frac{(10)^2}{20} = 5\Omega$;

With ac : $P = \frac{V_{rms}^2 R}{Z^2} \Rightarrow Z^2 = \frac{(10)^2 \times 5}{10} = 50\Omega^2$

Also $Z^2 = R^2 + 4\pi^2\nu^2 L^2$

$\Rightarrow \quad 50 = (5)^2 + 4(3.14)^2\nu^2(10 \times 10^{-3})^2 \Rightarrow \nu = 80$ Hz

5. In an LCR circuit $R = 100$ ohm. When capacitance C is removed, the current lags behind the voltage by $\pi/3$. When inductance L is removed, the current leads the voltage by $\pi/3$. The impedance of the circuit is
   (a) 50 ohm (b) 100 ohm
   (c) 200 ohm (d) 400 ohm

Sol. (b) When C is removed circuit becomes RL circuit hence

$$\tan\frac{\pi}{3} = \frac{X_L}{R} \qquad \text{(i)}$$

When L is removed circuit becomes RC circuit hence

$$\tan\frac{\pi}{3} = \frac{X_C}{R} \qquad \text{(ii)}$$

From Eqs. (i) and (ii) we obtain $X_L = X_C$. This is the condition of resonance and in resonance $Z = R = 100\ \Omega$

6. A group of electric lamps having a total power rating of 1000 watt is supplied by an ac voltage $E = 200\sin(310t + 60°)$. Then the rms value o the circuit current is
   (a) 10 A (b) $10\sqrt{2}$ A
   (c) 20 A (d) $20\sqrt{2}$ A

Sol. (b) $P = \frac{1}{2} V_0 i_0 \cos\varphi$

$\Rightarrow \quad 1000 = \frac{1}{2} \times 200 \times i_0 \cos 60°$

$\Rightarrow \quad i_0 = 20$ A

$\Rightarrow \quad i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 10\sqrt{2}$ A

7. In an LR-circuit, the inductive reactance is equal to the resistance R of the circuit. An e.m.f. $E = E_0\cos(\omega t)$ applied to the circuit. The power consumed in the circuit is

(a) $\frac{E_0^2}{R}$ (b) $\frac{E_0^2}{2R}$

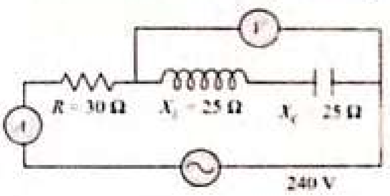
(c) $\frac{E_0^2}{4R}$ (d) $\frac{E_0^2}{8R}$

Sol. (c) $P = E_{rms} i_{rms} \cos\varphi = \frac{E_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \times \frac{R}{Z}$

$\Rightarrow \quad \frac{E_0}{\sqrt{2}} \times \frac{E_0}{Z\sqrt{2}} \times \frac{R}{Z} \Rightarrow P = \frac{E_0^2 R}{2Z^2}$

Given $X_L = R$ so, $Z = \sqrt{2}R \Rightarrow P = \frac{E_0^2}{4R}$

8. In the circuit shown in figure neglecting source resistance the voltmeter and ammeter reading will respectively, will be



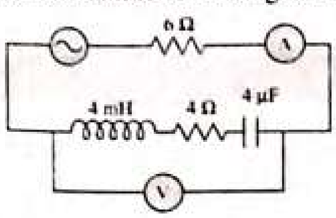
(a) 0 V, 3 A (b) 150 V, 3 A
(c) 150 V, 6 A (d) 0 V, 8 A

Sol. (d) The voltage V_L and V_C are equal and opposite so voltmeter reading will be zero.

Also $R = 3\ \Omega, X_L = X_C = 25\ \Omega$

So $\quad i = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{R} = \frac{240}{30} = 8$ A

9. In the circuit shown in the figure, the ac source gives a voltage $V = 20\cos(2000t)$ Neglecting source resistance, the voltmeter and ammeter reading will be



(a) 0 V, 0.47 A (b) 1.68 V, 0.47 A
(c) 0 V, 1.4 A (d) 5.6 V, 1.4 A

Sol. (d) $Z = \sqrt{(R)^2 + (X_L - X_C)^2}$;

$R = 10\ \Omega$, $X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\ \Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\Omega$ i.e. $Z = 10\Omega$

Maximum current $i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2$ A

Hence $i_{rms} = \frac{2}{\sqrt{2}} = 1.4$ A

and $V_{rms} = 4 \times 1.41 = 5.64$ V

10. An ac source of angular frequency ω is fed across a resistor r and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in then circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency ω

(a) $\sqrt{\frac{3}{5}}$ (b) $\sqrt{\frac{2}{5}}$

(c) $\sqrt{\frac{1}{5}}$ (d) $\sqrt{\frac{4}{5}}$

Sol. (a) At angular frequency ω, the current in RC circuit is given by

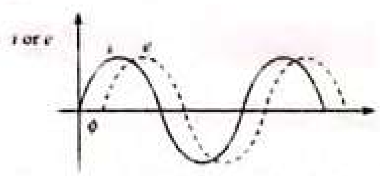
$$i_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \quad \text{(i)}$$

Also $$\frac{i_{rms}}{2} = \frac{V_{rms}}{\sqrt{R^2 + \left(\frac{1}{\frac{\omega}{3}C}\right)^2}} = \frac{V_{rms}}{\sqrt{R^2 + \frac{9}{\omega^2 C^2}}} \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$3R^2 = \frac{5}{\omega^2 C^2} \Rightarrow \frac{\frac{1}{\omega C}}{R} = \sqrt{\frac{3}{5}} \Rightarrow \frac{X_C}{R} = \sqrt{\frac{3}{5}}$$

11. When an ac source of e.m.f. $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\pi/4$, as shown in the diagram. If the circuit consists possibly only of RC or LC in series, find the relationship between the two elements



(a) $R = 1$ kΩ, $C = 10$ μF (b) $R = 1$ kΩ, $C = 1$ μF
(c) $R = 1$ kΩ, $L = 10$ μH (d) $R = 1$ kΩ, $L = 1$ H

Sol. (a) As the current i leads the voltage by $\pi/4$, it is an RC circuit, hence $\tan\varphi = X_C/R \Rightarrow \tan \pi/4 = 1/\omega CR$

$\Rightarrow \quad \omega CR = 1$ as $\omega = 100$ rad/sec

$$\Rightarrow \quad CR = \frac{1}{100}\sec^{-1}$$

From all the given options only option (a) is correct.

12. Same current is flowing in two alternating circuits. The first circuit contains only inductance and the other contains only a capacitor. If the frequency of the e.m.f. of ac is increased, the effect on the value of the current will be

(a) Increases in the first circuit and decreases in the other
(b) Increases in both the circuits
(c) Decreases in both the circuits
(d) Decreases in the first circuit and increases in the other

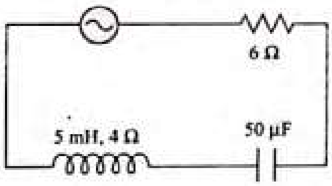
Sol. (d) For the first circuit $i = \frac{V}{Z} = \frac{V}{\sqrt{R^2+\omega^2L^2}}$

$\therefore$ Increase in ω will cause a decrease in i.

For the second circuit $i = \frac{V}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}$

$\therefore$ Increase in ω will cause an increase in i.

13. In the circuit shown below, the ac source has voltage $V = 20\cos(\omega t)$ volts with $\omega = 2000$ rad/sec. The amplitude of the current will be nearest to



(a) 2 A (b) 3.3 A
(c) $2/\sqrt{5}$ A (d) $\sqrt{5}$ A

Sol. (a) $R = 6 + 4 = 10\Omega$

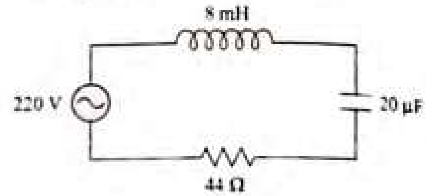
$$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2000\times 50\times 10^{-6}} = 10\Omega$$

$$\therefore \quad Z = \sqrt{R^2+(X_L-X_C)^2} = 10\Omega$$

Amplitude of current $= i_0 = \frac{V_0}{Z} = \frac{20}{10} = 2$ A

14. For the series LCR circuit shown in the figure, what is the resonance frequency and the amplitude of the current at the resonating frequency



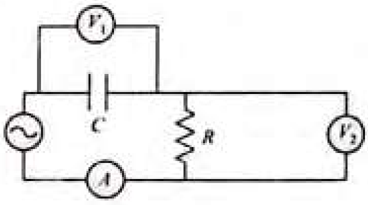
(a) 2500 rad-s^{-1} and $5\sqrt{2}$ A
(b) 2500 rad-s^{-1} and 5 A
(c) 2500 rad-s^{-1} and $\frac{5}{\sqrt{2}}$ A
(d) 25 rad-s^{-1} and $5\sqrt{2}$ A

Sol. (b) Resonance frequency

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{8\times10^{-3}\times 20\times 10^{-6}}} = 2500 \text{ rad/sec}$$

Resonance current $= \frac{V}{R} = \frac{220}{44} = 5$ A

15. The diagram shows a capacitor C and a resistor R connected in series to an ac source. V_1 and V_2 are voltmeters and A is an ammeter



Consider now the following statements

I. Readings in A and V_2 are always in phase
II. Reading in V_1 is ahead in phase with reading in V_2
III. Readings in A and V_1 are always in phase which of these statements are/is correct

(a) I only (b) II only
(c) I and II only (d) II and III only

Sol. (b) In RC series circuit voltage across the capacitor leads the voltage across the resistance by $\pi/2$

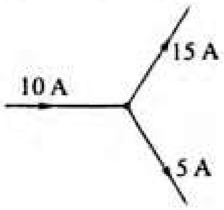
EXERCISES

Properties of Alternating Current

1. If $i = t^2$; $0 < t < T$ then r.m.s. value of current is

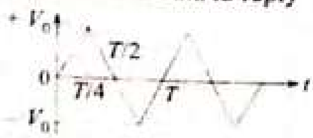
(a) $\frac{T^2}{\sqrt{2}}$ (b) $\frac{T^2}{2}$
(c) $\frac{T^2}{\sqrt{5}}$ (d) None of these

2. Is the following circuit correctly drawn?



(a) Yes (b) No
(c) Cannot be predicted (d) Insufficient data to reply

3. The voltage time (V–t) graph for triangular wave having peak value, V_0 is as shown in figure. The average value of voltage V in time interval from $t = 0$ to T is



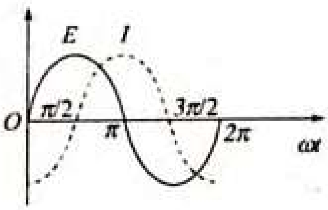
(a) 0 (b) $\frac{V_0}{2}$
(c) $\frac{V_0}{\sqrt{2}}$ (d) none of these

4. In the above question the average value of voltage (V) in one time period will be
(a) $\frac{V_0}{\sqrt{3}}$ (b) $\frac{V_0}{2}$
(c) $\frac{V_0}{\sqrt{2}}$ (d) zero

5. Voltage and current in an ac circuit are given by $V = 5\sin\left(100\pi t - \frac{\pi}{6}\right)$ and $I = 4\sin\left(100\pi t + \frac{\pi}{6}\right)$
(a) Voltage leads the current by 30°
(b) Current leads the voltage by 30°
(c) Current leads the voltage by 60°
(d) Voltage leads the current by 60°

6. An A.C. is given by equation $I = I_1 \cos \omega t + I_2 \sin \omega t$. The r.m.s. value of current is given by
(a) $\frac{I_1 + I_2}{2}$ (b) $\frac{(I_1 + I_2)^2}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}}\sqrt{I_1^2 + I_2^2}$ (d) $\frac{I_1^2 + I_2^2}{2}$

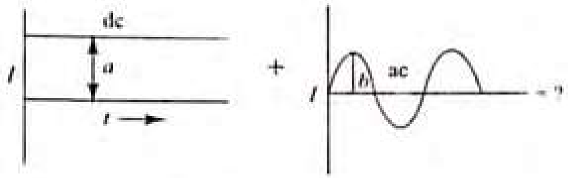
7. The variation of the instantaneous current (I) and the instantaneous emf (E) in a circuit is as shown in figure Which of the following statements is correct?



(a) The voltage lags behind the current by $\pi/2$
(b) The voltage leads the current by $\pi/2$
(c) The voltage and the current are in phase
(d) The voltage leads the current by π

8. A direct current of 5 A is superimposed on an alternating current $I = 10 \sin \omega t$ flowing through a wire. The effective value of the resulting current will be
(a) (15/2) A (b) $5\sqrt{3}$ A
(c) $5\sqrt{5}$ A (d) 15 A

9. If a direct current of value a ampere is superimposed on an alternative current $I = b \sin \omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit?



(a) $\left[a^2 - \frac{1}{2}b^2\right]^{1/2}$ (b) $[a^2 + b^2]^{1/2}$
(c) $\left[\frac{a^2}{2} + b^2\right]^{1/2}$ (d) $\left[a^2 + \frac{1}{2}b^2\right]^{1/2}$

10. Two alternating voltage generators produce emfs of the same amplitude E_0 but with a phase difference of $\pi/3$. The resultant emf is
(a) $E_0 \sin[\omega t + (\pi/3)]$ (b) $E_0 \sin[\omega t + (\pi/6)]$
(c) $\sqrt{3}\, E_0 \sin[\omega t + (\pi/6)]$ (d) $\sqrt{3}\, E_0 \sin[\omega t + (\pi/2)]$

11. A sinusoidal alternating current of peak value I_0 passes through a heater of resistance R. What is the mean power output of the heater?
(a) $I_0^2 R$ (b) $\frac{I_0^2 R}{2}$
(c) $2I_0^2 R$ (d) $\sqrt{2}\, I_0^2 R$

12. An ac voltage is represented by $E = 220\sqrt{2}\cos(50\pi)t$ How many times will the current become zero in 1 s?
(a) 50 times (b) 100 times
(c) 30 times (d) 25 times

13. The rms value of an ac of 50 Hz is 10 A. The time taken by an alternating current in reaching from zero to maximum value and the peak value will be
(a) 2×10^{-2} s and 14.14 A
(b) 1×10^{-2} s and 7.07 A
(c) 5×10^{-3} s and 7.07 A
(d) 5×10^{-3} s and 14.14 A

14. The peak value of an alternating emf E given by $E = E_0 \cos\omega t$ is 10 V and frequency is 50 Hz. At time $t = (1/600)$ s, the instantaneous value of emf is
(a) 10 V (b) $5\sqrt{3}$ V
(c) 5 V (d) 1 V

15. Using an ac voltmeter, the potential difference in the electrical line in a house is read to be 234 V. If the line frequency is known to be 50 cycles per second, the equation for the line voltage is
(a) $V = 165 \sin(100\,\pi t)$ (b) $V = 331 \sin(100\,\pi t)$
(c) $V = 220 \sin(100\,\pi t)$ (d) $V = 440 \sin(100\,\pi t)$

16. Current in an ac circuit is given by $I = 3 \sin \omega t + 4 \cos \omega t$, then
(a) rms value of current is 5 A
(b) mean value of this current in any one half period will be $6/\pi$
(c) if voltage applied is $V = V_m \sin \omega t$, then the circuit may be containing resistance and capacitance only

(d) if voltage applied is $V = V_m \sin \omega t$, the circuit may contain resistance and inductance only

Different A-C Circuits

17. A resistance of 20 Ω is connected to a source of an alternating potential $V = 220 \sin(100\ \pi t)$. The time taken by the current to change from the peak value to rms value, is
(a) 0.2 s (b) 0.25 s
(c) 2.5×10^{-3} s (d) 2.5×10^{-3} s

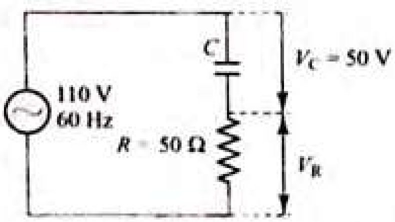
18. An inductive coil has resistance of 100 Ω. When an ac signal of frequency 1000 Hz is fed to the coil, the applied voltage leads the current by 45°. What is the inductance of the coil?
(a) 2 mH (b) 3.3 mH
(c) 16 mH (d) $\sqrt{5}$ mH

19. In an ac circuit, the potential differences across an inductance and resistance joined in series are, respectively, 16 V and 20 V. The total potential difference across the circuit is
(a) 20 V (b) 25.6 V
(c) 31.9 V (d) 53.5 V

20. A 50 W, 100 V lamp is to be connected to an ac mains of 200 V, 50 Hz. What capacitor is essential to be put in series with the lamp?
(a) $\frac{25}{\sqrt{2}}$ μF (b) $\frac{50}{\pi\sqrt{3}}$ μF
(c) $\frac{50}{\sqrt{2}}$ μF (d) $\frac{100}{\pi\sqrt{3}}$ μF

21. A capacitor of 10 μF and an inductor of 1 H are joined in series. An ac of 50 Hz is applied to this combination. What is the impedance of the combination?
(a) $\frac{5(\pi^2-5)}{\pi}$ Ω (b) $\frac{10(10-\pi^2)}{\pi}$ Ω
(c) $\frac{10(\pi^2-5)}{\pi}$ Ω (d) $\frac{5(10-\pi^2)}{\pi}$ Ω

22. In the circuit given in the figure, $V_C = 50$ V and $R = 50\ \Omega$. The values of C and V_R are

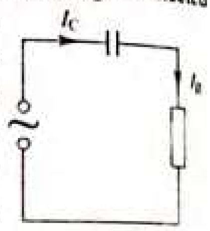


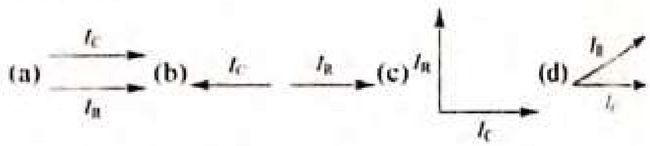
(a) 3.3 mF, 60 V (b) 104 μF, 98 V
(c) 52 μF, 98 V (d) 2 μF, 60 V

23. A 220-V, 50 Hz ac generator is connected to an inductor and a 50 Ω resistance in series. The current in the circuit is 1.0 A. What is the PD across inductor?
(a) 102.2 V (b) 186.4 V
(c) 214 V (d) 170 V

24. An inductor and a resistor are connected in series with an ac source. In this circuit.
(a) the current and the PD across the resistance lead the PD across the inductance
(b) the current and the PD across the resistance lag behind the PD across the inductance by an angle $\pi/2$
(c) the current and the PD across the resistance lag behind the PD across the inductance by an angle π
(d) the PD across the resistance lags behind the PD across the inductance by an angle $\pi/2$ but the current in resistance leads the PD across the inductance by $\pi/2$

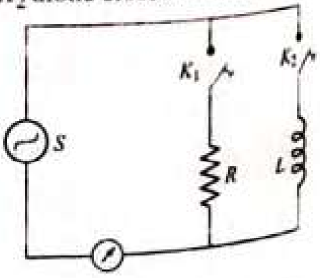
25. Figure shows a source of alter-nating voltage connected to a cap-acitor and a resistor. Which of the following phasor diagrams correctly describes the phase relationship between I_C, the current between the source and the capacitor, and I_R, the current in the resistor?





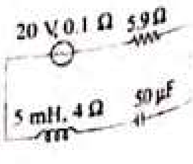
26. When 100 V dc is applied across a solenoid, a current of 1.0 A flows in it. When 100 V ac is applied across the same coil, the current drops to 0.5 A. If the frequency of the ac source is 50 Hz, the impedance and inductance of the solenoid are
(a) 200 Ω and 0.55 H (b) 100 Ω and 0.86 H
(c) 200 Ω and 1.0 H (d) 100 Ω and 0.93 H

27. In the circuit shown in the figure, R is a pure resistor, L is an inductor of negligible resistance (as compared to R), S is a 100 V, 50 Hz ac source of negligible resistance. With either key K_1 alone or K_2 alone closed, the current is I_0. If the source is changed to 100 V, 100 Hz the current with K_1 alone closed and with K_2 alone closed will be, respectively,



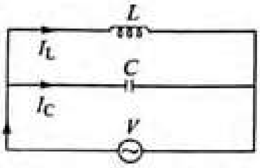
(a) $I_0, \frac{I_0}{2}$
(b) $I_0, 2I_0$
(c) $2I_0, I_0$
(d) $2I_0, \frac{I_0}{2}$

28. In the circuit of the figure, the source frequency is $\omega = 2000$ rad s^{-1}. The current in the circuit will be



(a) 2 A (b) 3.3 A
(c) $2/\sqrt{5}$ A (d) $\sqrt{5}$ A

29. For the circuit shown in the figure, current in inductance is 0.8 A while that in capacitance is 0.6 A. What is the current drawn from the source?

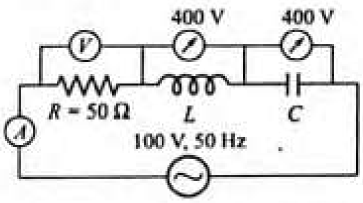


(a) 0.1 A (b) 0.3 A
(c) 0.6 A (d) 0.2 A

30. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited, the voltage across the inductance will be
(a) 10 V (b) $10/\sqrt{2}$ V
(c) (10/3)V (d) 20 V

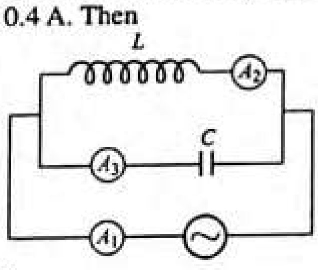
31. An LCR circuit contains resistance of 100 Ω and a supply of 200 V at 300 rad angular frequency. If only capacitance is taken out from the circuit and the rest of the circuit is joined, current lags behind the voltage by 60°. If, on the other hand, only inductor is taken out, the current leads by 60° with the applied voltage. The current flowing in the circuit is
(a) 1 A (b) 1.5 A
(c) 2 A (d) 2.5 A

32. In the series LCR circuit (see figure), the voltmeter and ammeter readings are:



(a) $V = 100$ V, $I = 2$ A (b) $V = 100$ V, $I = 5$ A
(c) $V = 1000$ V, $I = 2$ A (d) $V = 300$ V, $I = 1$ A

33. For the circuit shown in the figure, the ammeter A_2 reads 1.6 A and ammeter A_3 reads 0.4 A. Then

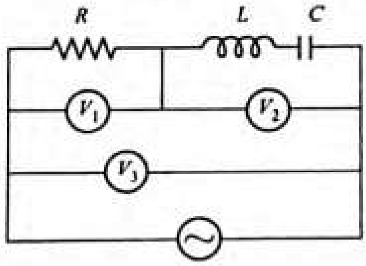


(a) $\omega = \dfrac{4}{\sqrt{LC}}$
(b) $f = \dfrac{2\pi}{\sqrt{LC}}$
(c) the ammeter A_1 reads 1.2 A
(d) the ammeter A_1 reads 2 A

34. The value of current in two series LCR circuits at resonance is same. Then
(a) both circuits must be having same value of capacitance and inductance
(b) in both circuits ratio of L and C will be same
(c) for both the circuits X_L/X_C must be same at that frequency
(d) both circuits must have same impedance at all frequencies

35. In series LCR circuit voltage drop across resistance is 8 V, across inductor is 6 V and across capacitor is 12 V. Then
(a) voltage of the source will be leading current in the circuit
(b) voltage drop across each element will be less than the applied voltage
(c) power factor of circuit will be 4/3
(d) none of these

36. Which voltmeter will give zero reading at resonance?



(a) V_1 (b) V_2
(c) V_3 (d) None

Power in A-C Circuits

37. A bulb is connected first with dc and then ac of same voltage then it will shine brightly with
(a) AC
(b) DC
(c) Brightness will be in ratio 1/1.4
(d) Equally with both

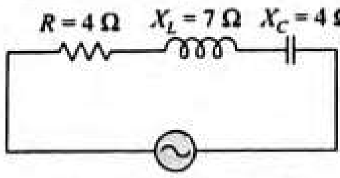
38. A sinusoidal ac current flows through a resistor of resistance R. If the peak current is I_p, then the power dissipated is
(a) $I_p^2 R\cos\theta$ (b) $\dfrac{1}{2}I_p^2 R$
(c) $\dfrac{4}{\pi}I_p^2 R$ (d) $\dfrac{1}{\pi}I_p^2 R$

39. What will be the phase difference between virtual voltage and virtual current, when the current in the circuit is wattless?
(a) 90° (b) 45°
(c) 180° (d) 60°

40. In an ac circuit, V and I are given by

$V = 100\sin(100\,t)$ volts, $I = 100\sin\left(100t + \dfrac{\pi}{3}\right)$ mA. The power dissipated in circuit is
(a) 10^4 watt (b) 10 watt
(c) 2.5 watt (d) 5 watt

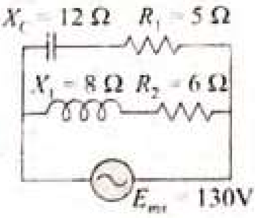
41. For the series L-C-R circuit as shown in figure, the heat developed in 80 s and amplitude of wattless current i.



$V = 25\sin(100\pi t + \pi/2)$

(a) 4000 J, 3 A (b) 8000 J, 3 A
(c) 4000 J, 4 A (d) 8000 J, 5 A

42. What is the amount of power delivered by the ac source in the circuit shown (in watts).



(a) 500 watt (b) 1014 watt
(c) 1514 watt (d) 2013 watt

43. Power factor is one for
(a) pure resistor
(b) pure inductor
(c) pure capacitor
(d) either an inductor or a capacitor

44. An rms voltage of 110 V is applied across a series circuit having a resistance 11 Ω and an impedance 22 Ω. The power consumed is
(a) 275 W (b) 366 W
(c) 550 W (d) 1100 W

45. A resistor and an inductor are connected to an ac supply of 120 V and 50 Hz. The current in the circuit is 3 A. If the power consumed in the circuit is 108 W, then the resistance in the circuit is
(a) 12 Ω (b) 40 Ω
(c) $\sqrt{(52 \times 28)}\ \Omega$ (d) 360 Ω

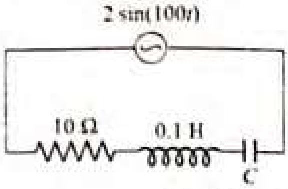
46. A resistor and a capacitor are connected to an ac supply of 200 V, 50 Hz in series. The current in the circuit is 2 A. If the power consumed in the circuit is 100 W then the resistance in the circuit is
(a) 100 Ω (b) 25 Ω
(c) $\sqrt{125 \times 75}\ \Omega$ (d) 400 Ω

47. In the above question, the capacitive reactance in the circuit is
(a) 100 Ω (b) 25 Ω
(c) $\sqrt{125 \times 75}\ \Omega$ (d) 400 Ω

48. In the above question, the capacitance in the circuit is
(a) $\dfrac{100}{100\pi}$ F (b) $\dfrac{25}{100\pi}$ F
(c) $\dfrac{\sqrt{125 \times 75}}{100\pi}$ F (d) $\dfrac{1}{100\pi\sqrt{125 \times 75}}$ F

49. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When an alternating emf of 220 V at 50 cps is applied to it, then the wattless component of the current in the circuit is (take $0.7\pi = 2.2$]
(a) 5 A (b) 0.5 A
(c) 0.7 A (d) 7 A

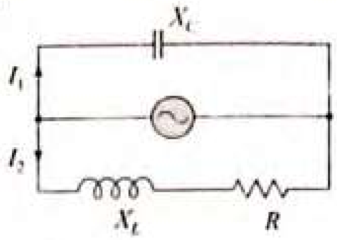
50. The power factor of the circuit in the figure is $1/\sqrt{2}$. The capacitance of the circuit is equal to



(a) 400 μF (b) 300 μF
(c) 500 μF (d) 200 μF

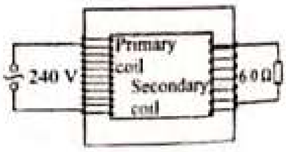
Problems Based on Mixed Concepts

51. In the shown AC circuit phase different between currents I_1 and I_2 is



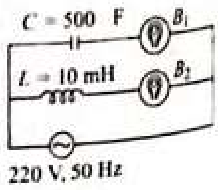
(a) $\dfrac{\pi}{2} - \tan^{-1}\dfrac{X_L}{R}$ (b) $\tan^{-1}\dfrac{X_L - X_C}{R}$
(c) $\dfrac{\pi}{2} + \tan^{-1}\dfrac{X_L}{R}$ (d) $\tan^{-1}\dfrac{X_L - X_C}{R} + \dfrac{\pi}{2}$

52. Figure shows an iron-cored transformer assumed to be 100% efficient. The ratio of the secondary turns to the primary turns is 1:20.



A 240 V ac supply is connected to the primary coil and a 6 Ω resistor is connected to the secondary coil. What is the current in the primary coil?
(a) 0.10 A (b) 0.14 A
(c) 2 A (d) 40 A

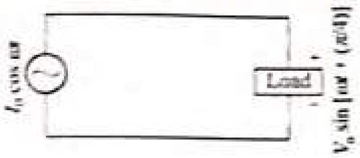
53. In the circuit shown in the figure, if both the bulbs B_1 and B_2 are identical,



(a) their brightness will be the same
(b) B_2 will be brighter than B_1
(c) B_1 will be brighter than B_2
(d) only B_2 will glow because the capacitor has infinite impedance

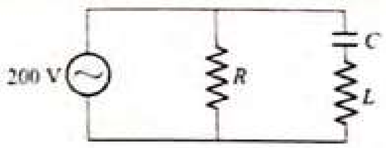
54. An ideal choke takes a current of 10 A when connected to an ac supply of 125 V and 50 Hz. A pure resistor under the same conditions takes a current of 12.5 A. If the two are connected to an ac supply of 100 V and 40 Hz, then the current in series combination of above resistor and inductor is
(a) $10/\sqrt{2}$ A (b) 12.5 A
(c) 20 A (d) 10 A

55. A current source sends a current $I = I_0 \cos(\omega t)$. When connected across an unknown load, it gives a voltage output of $V = V_0 \sin[\omega t - (\pi/4)]$ across that load. Then the voltage across the current source may be brought in phase with the current through it by



(a) connecting an inductor in series with the load
(b) connecting a capacitor in series with the load
(c) connecting an inductor in parallel with the load
(d) connecting a capacitor in parallel with the load

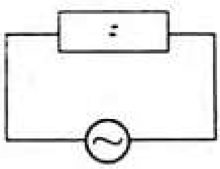
56. In the circuit shown in the figure, $X_C = 100\ \Omega$, $X_L = 200\ \Omega$ and $R = 100\ \Omega$. The effective current through the source is



(a) 2 A (b) $2\sqrt{2}$ A
(c) 0.5 A (d) $\sqrt{0.4}$ A

57. For an *LCR* series circuit with an ac source of angular frequency ω,
(a) circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$
(b) circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$
(c) power factor of circuit will be unity if capacitive reactance equals inductive reactance
(d) current will be leading voltage if $\omega > \frac{1}{\sqrt{LC}}$

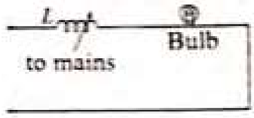
58. In a black box of unknown elements (*L* or *R* or any other combination), an ac voltage $E = E_0 \sin(\omega t + \phi)$ is applied and current in the circuit was found to be $I = I_0 \sin[\omega t + \phi + (\pi/4)]$. Then the unknown elements in the box may be



(a) only capacitor
(b) inductor and resistor both
(c) either capacitor, resistor, and inductor or only capacitor and resistor
(d) only resistor

59. In an ideal transformer, the voltage and the current in the primary coil are 200 V and 2 A, respectively. If the voltage in the secondary coil is 2000 V, then the value of current in the secondary coil will be
(a) 0.2 A (b) 2 A
(c) 10 A (d) 20 A

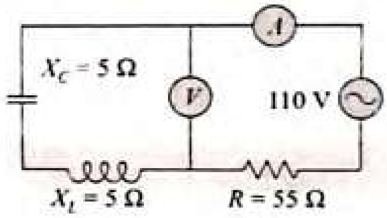
60. A typical light dimmer used to dim the stage lights in a theatre consists of a variable induction for *L* (where inductance is adjustable between zero and L_{max}) connected in series with a light bulb *B* as shown. The mains electrical supply is 220 V at 50 Hz, the light bulb is rated at 220 V, 1100 W. What L_{max} is required if the rate of energy dissipation in the light bulb is to be varied by a factor of 5 from its upper limit of 1100 W?



(a) 0.69 H (b) 0.28 H
(c) 0.38 H (d) 0.56 H

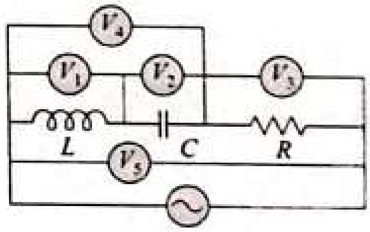
61. A telephone wire of length 200 km has a capacitance of 0.014 μF per km. If it carries an ac of frequency 5 kHz, what should be the value of an inductor required to be connected in series so that the impedance of the circuit is minimum
(a) 0.35 mH (b) 35 mH
(c) 3.5 mH (d) Zero

62. The reading of ammeter in the circuit shown will be



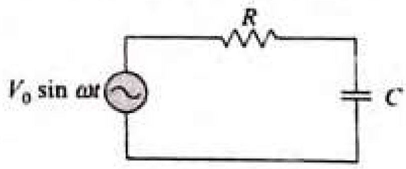
(a) 2 A (b) 2.4 A
(c) Zero (d) 1.7 A

63. In the adjoining ac circuit the voltmeter whose reading will be zero at resonance is



(a) V_1 (b) V_2
(c) V_3 (d) V_4

64. An ac voltage source $V = V_0 \sin \omega t$ is connected across resistance *R* and capacitance *C* as shown in figure. It is given that $R = 1/\omega C$. The peak current is I_0. If the angular frequency of the voltage source is changed to $\omega/\sqrt{3}$ then the new peak current in the circuit is



(a) $\frac{I_0}{2}$ (b) $\frac{I_0}{\sqrt{2}}$

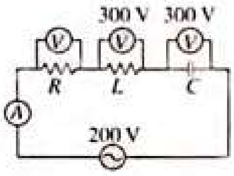
(c) $\frac{I_0}{\sqrt{3}}$ (d) $\frac{I_0}{3}$

65. A 300 Ω resistor is connected in series with a parallel-plate capacitor across the terminals of a 50.0 Hz ac generator. When the gap between the plates is empty, its capacitance is 70/22 μF. The ratio of the rms current in the circuit when the capacitor is empty to that when ruby mica of dielectric constant $k = 5.0$ is inserted between the plates, is equal to

(a) 0.1 (b) 0.3

(c) 0.6 (d) 2.9

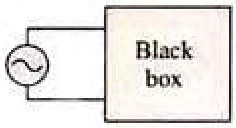
66. In the series circuit shown in the figure the voltmeter reading will be (all the meters are ideal):



(a) 300 V (b) 200 V

(c) 100 V (d) 600 V

67. A combination of elements is enclosed in a black box and the voltage and currents are measured across this black box the expression for applied voltage, $V_S = 200\sqrt{2}\sin\omega t$ and the current flowing in is $i = 2\sqrt{2}\sin(\omega t + \pi/4)$ where $\omega = 100\pi$ rad/sec. Then the wrong statement is:



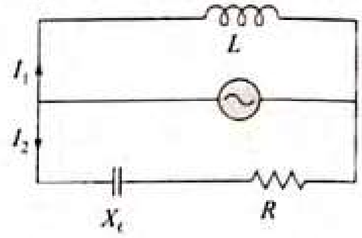
(a) There must be a capacitor in the black box

(b) Power factor of circuit = 0.707

(c) There must be a resistor in the box

(d) There must be an inductor in the box

68. For the given circuit as summing inductor and source to be ideal, the phase difference between currents I_1 and I_2 is



(a) $\tan^{-1}\left(\frac{X_C}{R}\right) - \frac{\pi}{2}$ (b) $\tan^{-1}\left(\frac{X_C}{R}\right)$

(c) $\tan^{-1}\left(\frac{X_C}{R}\right) + \frac{\pi}{2}$ (d) $\frac{\pi}{2}$

ARCHIVES

1. The power factor of an AC circuit having resistance R and inductance L connected in series and an angular velocity ω is

(a) $\frac{R}{\omega L}$ (b) $\frac{R}{(R^2 + \omega^2 L^2)^{1/2}}$

(c) $\frac{\omega L}{R}$ (d) $\frac{R}{(R^2 - \omega^2 L^2)^{1/2}}$

(AIEEE 2002)

2. In a transformer, the number of turns in the primary are 140 and that in the secondary are 280. If the current in primary is 4 A, then that in the secondary is

(a) 4 A (b) 2 A

(c) 6 A (d) 10 A **(AIEEE 2002)**

3. An inductor of inductance L and resistor of resistance R are joined in series and connected by a source of frequency ω. Power dissipated in the circuit is

(a) $\frac{(R^2 + \omega^2 L^2)}{V}$ (b) $\frac{V^2 R}{(R^2 + \omega^2 L^2)}$

(c) $\frac{V}{(R^2 + \omega^2 L^2)}$ (d) $\frac{\sqrt{R^2 + \omega^2 L^2}}{V^2}$

(AIEEE 2002)

4. The core of any transformer is laminated to

(a) increase the secondary voltage

(b) reduce the energy loss due to eddy currents

(c) make it light weight

(d) make it robust and strong **(AIEEE 2003)**

5. In an LCR circuit, the capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be changed from L to

(a) $4L$ (b) $2L$

(c) $L/2$ (d) $L/4$ **(AIEEE 2004)**

6. Alternating current cannot be measured by a DC ammeter because

(a) AC cannot pass through DC ammeter

(b) AC changes direction

(c) the average value of current for complete cycle is zero

(d) DC ammeter will get damaged **(AIEEE 2004)**

7. In an LCR series AC circuit, the voltage across each of the components, L, C, and R, is 50 V. The voltage across the LC combination will be

(a) 50 V (b) $50\sqrt{2}$ V

(c) 100 V (d) 0 V (zero) **(AIEEE 2004)**

8. In an LCR circuit capacitance is changed from C to $2C$. For the resonant frequency to remain unchanged, the inductance should be change from L to

(a) $4L$ (b) $2L$

(c) $L/2$ (d) $L/4$ **(AIEEE 2004)**

9. The phase difference between an alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit?
(a) L alone (b) L, C
(c) R, L (d) C alone **(AIEEE 2005)**

10. A circuit has a resistance of 12 Ω and an impedance of 15 Ω. The power factor of the circuit will be
(a) 0.125 (b) 1.25
(c) 0.4 (d) 0.8 **(AIEEE 2005)**

11. In a series resonant LCR circuit, the voltage across R is 100 V and $R = 1$ kΩ with $C = 2$ μF. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is
(a) 250 V (b) 4×10^{-3} V
(c) 2.5×10^{-2} V (d) 40 V **(AIEEE 2006)**

12. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of the emf generated in the coil is
(a) $N.A.B.R.$ (b) $N.A.B.\ \omega$
(c) $N.A.B.R.\ \omega$ (d) $N.A.B$ **(AIEEE 2006)**

13. In an AC circuit, the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin(\omega t - \pi/2)$. The power consumption in the circuit is given by
(a) $P = 0$ (b) $P = \dfrac{E_0 I_0}{2}$
(c) $P = \sqrt{2}\, E_0 I_0$ (d) $P = \dfrac{E_0 I_0}{\sqrt{2}}$ **(AIEEE 2007)**

14. In a series LCR circuit, $R = 200\ \Omega$ and the voltage and the frequency of the main supply are 220 V and 50 Hz, respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the LCR circuit is
(a) 305 W (b) 210 W
(c) Zero W (d) 242 W **(AIEEE 2010)**

15. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to
(a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H **(JEE Main 2016)**

16. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \dfrac{1}{\sqrt{LC}}$. The quality factor, Q is given by
(a) $\dfrac{CR}{\omega_0}$ (b) $\dfrac{\omega_0 L}{R}$
(c) $\dfrac{\omega_0 R}{L}$ (d) $\dfrac{R}{(\omega_0 C)}$
(JEE Main 2018)

17. In an ac circuit, the instantaneous emf and current are given by $e = 100 \sin 30\, t$
$$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$$
In one cycle of alternate current, the average power consumed by the circuit and the wattles current are, respectively
(a) 50, 0 (b) 50, 10
(c) $\dfrac{1000}{\sqrt{2}}, 10$ (d) $\dfrac{50}{\sqrt{2}}, 0$
(JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	**2.** (a)	**3.** (a)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (b)	**9.** (d)	**10.** (c)
11. (b)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (b)
21. (b)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (a)	**26.** (a)	**27.** (a)	**28.** (a)	**29.** (d)	**30.** (b)
31. (c)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (d)	**36.** (b)	**37.** (d)	**38.** (b)	**39.** (a)	**40.** (c)
41. (a)	**42.** (c)	**43.** (a)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (d)	**49.** (b)	**50.** (c)
51. (c)	**52.** (a)	**53.** (b)	**54.** (a)	**55.** (a)	**56.** (b)	**57.** (c)	**58.** (c)	**59.** (a)	**60.** (b)
61. (a)	**62.** (c)	**63.** (d)	**64.** (b)	**65.** (b)	**66.** (b)	**67.** (d)	**68.** (c)		

Archives

1. (b)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (c)	**6.** (d)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (d)
11. (a)	**12.** (b)	**13.** (a)	**14.** (d)	**15.** (d)	**16.** (b)	**17.** (c)			

Chapter 25

Electromagnetic Waves

INTRODUCTION

In 1865, Maxwell discovered the displacement current. This brought together the phenomenon of electricity and magnetism into a coherent and unified theory. After this discovery, Maxwell predicted the transmission of energy by electromagnetic waves, which could travel with the speed of light. This led him to conclude that the light itself is an electromagnetic wave.

According to him, an accelerated charge produces a sinusoidal time-varying magnetic field, which in turn produces a sinusoidal time-varying electric field. The two fields so produced are mutually perpendicular time-varying electric and magnetic fields. They constitute electromagnetic waves which can propagate through empty space.

The velocity of electromagnetic waves in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where μ_0 (= 1.257×10^{-6} T mA^{-1}) and ε_0 (= 8.854×10^{-12} C^2 N^{-1} m^{-2}) are, respectively, the absolute permeability and the absolute permittivity of free space. The velocity of electromagnetic waves in free space (vacuum) is equal to the velocity of light in vacuum, i.e., 3×10^8 m s^{-1}.

ILLUSTRATION 25.1 What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Å, and radiowaves of wavelength 500 m?

Solution. Speed is the physical quantity which is the same in vacuum for given X-ray, red light and radiowave.

Value of speed of electromagnetic waves in vacuum is $c = 3 \times 10^8$ m s^{-1}.

ILLUSTRATION 25.2 A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

Solution. The electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ are in the xy plane.

Using $c = \lambda v$, we get $v = \dfrac{c}{\lambda}$ and $\lambda = \dfrac{3 \times 10^8}{30 \times 60} = 10$ m

MAXWELL'S DISPLACEMENT CURRENT

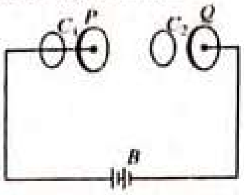
According to Ampere's circuital law, the magnetic field B is related to steady current I as

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \text{(i)}$$

where I is the current travelling the surface bounded by closed path C.

In 1864, Maxwell showed that relation (i) is logically inconsistent. He accounted for this inconsistency as follows:

Consider a parallel plate capacitor having plates P and Q being charged with battery B.



During charging, a current I flows through the connecting wires which changes with time. This current will produce magnetic field around the wires which can be detected using a magnetic compass needle. Consider two loops c_1 and c_2 parallel to the plates P and Q of the capacitor. c_1 is enclosing only the connecting wire attached to the plate P of the capacitor and c_2 lies in the region between the two plates of capacitor. For the loop c_1, a current I is flowing through it, hence Ampere's circuital law for loop c_1 gives

$$\oint_{c_1} \vec{B} \cdot d\vec{l} = \mu_0 I \qquad \text{(ii)}$$

Since the loop c_2 lies in the region between the plates of the capacitor, no current flows in this region. Hence Ampere's circuital law for loop c_2 gives

$$\oint_{c_2} \vec{B} \cdot d\vec{l} = 0 \qquad \text{(iii)}$$

The relations (ii) and (iii) continue to be true even if two loops c_1 and c_2 are infinitesimally close to the plate P of the capacitor. On the other hand, as the loops c_1 and c_2 are infinitesimally close, it is expected that

$$\oint_{c_1} \vec{B} \cdot d\vec{l} = \oint_{c_2} \vec{B} \cdot d\vec{l} \qquad \text{(iv)}$$

Thus, relation (iv) is in contradiction with relations (ii) and (iii). This led Maxwell to point out that Ampere's circuital law as given by (i) is logically inconsistent.

Idea of Displacement Current. Maxwell predicted that not only a current flowing in a conductor produces magnetic field but also a time-varying electric field (i.e., changing electric

field) in a vacuum/free space (or in a dielectric) produces a magnetic field. It means a changing electric field gives rise to a current which flows through a region so long as the electric field is changing there. Maxwell also predicted that this current produces the same magnetic field as a conduction current can produce. This current is known as 'displacement current'.

Thus, displacement current is that current which comes into play in the region in which the electric field and hence the electric flux is changing with time.

Maxwell defined this displacement current in space where electric field is changing with time as

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt} \qquad \text{(v)}$$

where ϕ_E is the electric flux.

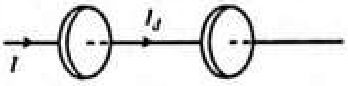
Maxwell also found that conduction current (I) and displacement current (I_D) together have the property of continuity, although, individually, they may not be continuous.

This idea led Maxwell to modify Ampere's circuital law in order to make the same logically consistent. He states Ampere circuital law to the form, $\oint_c \vec{B}\cdot d\vec{l} = \mu_0(I + I_D)$

$$= \mu_0\left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

It is now called as Ampere Maxwell law.

ILLUSTRATION 25.3 The following figure shows capacitor made of two circular plates. The capacitor is being charged by an external source which supply constant current equal to 0.15 A. Obtain displacement charged by constant current across plates. Given $\frac{dV}{dt} = 1.87\times10^9\ \text{V s}^{-1}$.



Solution. Displacement current $I_d = \frac{\varepsilon_0\, d\phi_C}{dt}$

$$\text{But } \phi_E = EA = \frac{d}{\varepsilon_0 A}A = \frac{q}{\varepsilon_0}$$

$$\therefore \qquad I_d = \varepsilon_0 \frac{d}{dt}\frac{q}{\varepsilon_0} = \frac{\varepsilon_0}{\varepsilon_0}\frac{dq}{dt} = \frac{dq}{dt} = I$$

Here $I = 0.15$ A $\qquad \therefore I_d = 0.15$ A

MAXWELL EQUATIONS

The four Maxwell's equations and Lorentz force law together constitute the foundations of classical electromagnetism. The Maxwell's equations are:

1[st] equation: $\oiint \vec{E}\cdot d\vec{S} = \frac{q}{\varepsilon_0}$

2[nd] equation: $\oiint \vec{B}\cdot d\vec{S} = 0$

3[rd] equation: $\oiint \vec{B}\cdot d\vec{l} = \frac{-d}{dt}\iint \vec{B}\cdot d\vec{S}$

4[th] equation: $\oiint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d}{dt}\cdot\iint \vec{B}\cdot d\vec{S} + \mu_0 \cdot I$

The Lorentz equation is

$\vec{F} = q\left(\vec{E}\times\vec{v}\times\vec{B}\right)$

$E_z = Ez_0 \sin(\omega t - ky)$

$B_x = Bx_0 \sin(\omega t - ky)$, where $\frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Since $\omega = 2\pi f$, where f is the frequency and $k = 2\pi/\lambda$, where λ is the wavelength.

Therefore, $\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$

But $f\lambda$ gives the velocity of the wave. Hence $f\lambda = c = \omega k$.
Hence, the velocity of the waves is given by

$$c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\,\varepsilon_0}}$$

ILLUSTRATION 25.4 Suppose that the electric field amplitude of an electromagnetic wave is E_0 = 120 N/C and that its frequency is ν = 50.0 MHz. (a) Determine B_0, ω, k, and λ. (b) Find expressions for $\vec{E}$ and $\vec{B}$.

Solution.

(a) (i) Using $\frac{E_{max}}{B_{max}} = c$, we get

$$B_{max} = \frac{E}{c} = \frac{120}{3\times10^8} = 40\times10^{-7}\ \text{T}$$

i.e., $B_{max} = 400$ nT

(ii) $\omega = 2\pi\nu = 2\times\pi\times50\times10^6$
$= 3.14\times10^8$ rad s^{-1}

(iii) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{6} = \frac{2\times3.14}{6} = 1.05$ rad s^{-1}

(iv) $c = \lambda\nu$ or $\lambda = \frac{c}{\nu} = \frac{3\times10^8}{50\times10^6} = 6$ m

(b) $\vec{E} = E_{max}\sin(kx - \omega t)\hat{j}$

$= 120\sin(1.05x - 3.14\times10^8\times t)\hat{j}$

$\vec{B} = B_{max}\times\sin(kx - \omega t)\hat{k}$

$= 400\sin(1.05x - 3.14\times10^8\times t)\hat{k}$

ILLUSTRATION 25.5 Suppose that the electric field part of a electromagnetic wave in vacuum is

$$\vec{E} = (3.1\ \text{N/C})\cos\left((1.8\ \text{rad/m})y + (5.4\times10^6\ \text{rad/s})t\right)$$

(a) What is the direction of propagation?
(b) What is the wavelength λ?
(c) What is the frequency ν?
(d) What is the amplitude of the magnetic field part of the wave?

(e) Write an expression for the magnetic field part of the wave.

Solution. Here $\vec{E} = 3.1\cos\{1.8y + (5.4\times10^6)t\}\hat{i}$

Comparing it with standard equation

$$\vec{E} = E_0\cos(kx + \omega t)d$$

(a) $-\hat{j}$ direction

(b) Using $k = \dfrac{2\pi}{\lambda}$, we get $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{1.8} = 3.5\text{ m}$

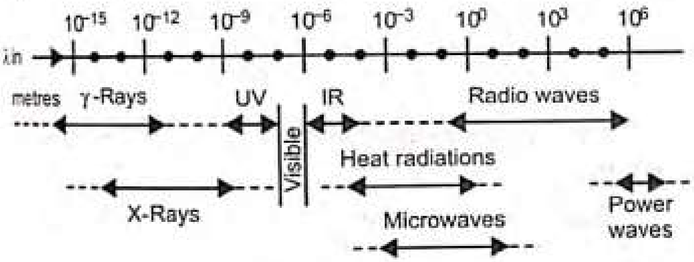
(c) Frequency, $\nu = \dfrac{\omega}{2\pi} = \dfrac{5.4\times10^6}{2\pi} = 0.86\text{ MHz}$

(d) $c = \dfrac{E}{B}$ or $B = \dfrac{E}{c} = \dfrac{3.1}{3\times10^8} = 10\text{ nT}$

(e) $\vec{B} = 10\cos(1.8y + 5.4\times10^6)\hat{k}$

ELECTROMAGNETIC WAVE SPECTRUM

Broadly, the various regions of the electromagnetic wave spectrum may be assigned the wavelength ranges as follows:



ENERGY DENSITY AND INTENSITY

The electric and magnetic fields in a plane electromagnetic wave are given by

$$E = E_0\sin\omega(t - x/c)$$

and $$B = B_0\sin\omega(t - x/c)$$

In any small volume dV, the energy of the electric field is

$$U_E = \frac{1}{2}\varepsilon_0 E^2\,dV$$

and the energy of the magnetic field is $U_B = \dfrac{1}{2\mu_0}B^2\,dV$

Thus, the total energy is $U = \dfrac{1}{2}\varepsilon_0 E^2\,dV + \dfrac{1}{2\mu_0}B^2\,dV$

The energy density is $u = \dfrac{1}{2}\varepsilon_0 E^2 + \dfrac{1}{2\mu Z_0}B^2$

$$u = \frac{1}{2}\varepsilon_0 E_0^2\sin^2\omega(t - x/c) + \frac{1}{2\mu_0}B_0^2\sin^2\omega(t - x/c)$$

If we take the average over a long time, the $\sin^2$ terms have an average value of 1/2

Thus, $u_{av} = \dfrac{1}{4}\varepsilon_0 E_0^2 + \dfrac{1}{4\mu_0}B_0^2$

Now, $E_0 = cB_0$ and $\mu_0\varepsilon_0 = \dfrac{1}{c^2}$ so that, $\mu_0 = \dfrac{1}{\varepsilon_0 c^2}$ and

$$B_0 = \frac{E_0}{c}\quad \frac{1}{4\mu_0}B_0^2 = \frac{\varepsilon_0 c^2}{4}\left(\frac{E_0}{c}\right)^2 = \frac{1}{4}\varepsilon_0 E_0^2$$

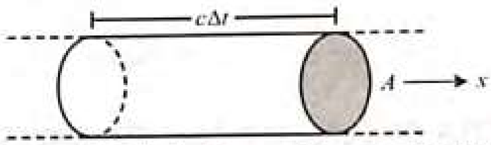
Thus, the electric energy density is equal to the magnetic density in average.

or, $$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{2}\varepsilon_0 E_0^2$$

Also $$u_{av} = \frac{1}{4\mu_0}B_0^2 + \frac{1}{4\mu_0}B_0^2 = \frac{1}{2\mu_0}B_0^2$$

INTENSITY

The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.



Consider a cylindrical volume with area of cross section A and length $c\,\Delta t$ along the x-axis. The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained is

$$U = u_{av}(c\,\Delta t)A$$

The intensity is $I = \dfrac{U}{A\Delta t} = u_{av}c$ in terms of maximum electric field, $I = \dfrac{1}{2}\varepsilon_0 E_0^2 c$

MOMENTUM

The electromagnetic wave also carries linear momentum with it. The linear momentum carried by the portion of wave having energy U is given by $p = U/c$.

Thus, if the wave incident on a material surface is completely absorbed, it delivers energy U and momentum $p = U/c$ to the surface. If the wave is totally reflected, the momentum delivered is $2U/c$ because the momentum of the wave changes from p to $-p$. It follows that electromagnetic waves incident on a surface exert a force on the surface.

ATMOSPHERE

The atmosphere of earth extends upon about 300 km. The composition of atmosphere is as given below.

Troposphere

Troposphere extends up to a height of about 12 km. This layer consists of water droplets, vapour, dust particles etc. Density of air decreases as we move from bottom to top of this layer. This layer is responsible for all the weather phenomena affecting our environment.

Stratosphere

This layer extends from about 10 km to about 50 km. In the upper part of the atmosphere, we have a layer of ozone. The density of air at the top of the stratosphere is about 10^{-3} times the density at the surface of earth.

Mesosphere

The mesosphere extends from about 50 km to about 80 km.

Ionosphere

The atmosphere above the mesosphere is called ionosphere. This layer is composed partly of electrons and positive ions. The rest of the atmosphere is composed mostly of neutral molecules.

The atmosphere is transparent to visible radiation and we can see the sun and the stars through it clearly. However, most infrared radiation is not able to pass through, as it is absorbed by the atmosphere. Low lying clouds in the atmosphere also prevent infrared radiation from passing through. The ozone layer blocks the passage of ultraviolet radiation from the sun.

PROPERTIES OF ELECTROMAGNETIC WAVES

(a) In electromagnetic waves the electric field vector $\vec{E}$, magnetic field vector $\vec{B}$ and propagation vector $\vec{k}$ are mutually perpendicular such that $\vec{E}$, $\vec{B}$, and $\vec{k}$ form a right angle system. Hence electromagnetic waves are transverse in nature.

(b) Electromagnetic waves travel with speed of light. In vacuum their speed is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

In isotropic medium, their speed is

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu.\mu_0\varepsilon.\varepsilon_0}}$$

$$= \frac{1}{\sqrt{\mu_r\varepsilon_r}} \cdot \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{c}{n}$$

where $n = \sqrt{\mu_r\, \varepsilon_r}$ is the refractive index of the medium,
μ_r = relative permeability of medium, and
ε_r = relative permittivity of medium or electric dielectric constant.

(c) The Poynting vector $\vec{S} = \vec{E} \times \vec{H}$ represents the direction of energy flow per unit area per second along the direction of wave propagation. Its unit is Wm^{-2}.

(d) The medium offers hindrance to the propagation of wave. Such a hindrance is called Wave Impedance (Z) and its value in a medium is given by

$$Z = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$(\because\ \mu = \mu_r\,\mu_0 \text{ and } \varepsilon : \varepsilon_0\,\varepsilon_r\ )$

where μ_r and ε_r are relative permeability and relative permittivity of the medium.

For vacuum or free space

$$Z = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.6\ \Omega$$

Also in free space

$$\frac{E}{B} = \frac{E}{\mu_0 H} = \frac{1}{\mu_0}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\frac{E}{B} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

$$E = Bc$$

(e) In free space

The electrostatic energy density is equal to magnetostatic energy density.

Average electric energy density = Average magnetic energy density

$$= \frac{1}{4}\varepsilon_0\, E_0^2 = \frac{1}{4}\mu_0\, H_0^2 = \frac{B_0^2}{4\mu_0}$$

Total average energy density $= \frac{1}{2}\varepsilon_0\, E_0^2$ (in free space)

(f) In a medium

Electric energy density $= \frac{1}{2}\varepsilon E^2$

Magnetic energy density $= \frac{B^2}{2\mu}$

Total energy density $= \frac{1}{2}\varepsilon E^2 + \frac{B^2}{2\mu}$

(g) The electric and magnetic fields satisfy the following wave equations, which can be obtained from Maxwell's third and fourth equations.

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

(h) Electromagnetic waves carry momentum and hence can exert pressure (P) on surfaces, which is known as radiation pressure. For an electromagnetic wave with Poynting vector $\vec{S}$, incident upon a perfectly absorbing surface.

$$P = \frac{S}{c}$$

and if incident upon a perfectly reflecting surface, then

$$P = \frac{2S}{c}$$

(i) The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive

x-direction can also be written as

$$E = E_0 \sin (kx - \omega t)$$

$$B = B_0 \sin (kx - \omega t)$$

where ω is the angular frequency of the wave and k is the angular wave number or propagation constant which are given by

$$\omega = 2\pi f \text{ and } k = \frac{2\pi}{\lambda}$$

(j) The intensity of a sinusoidal plane electromagnetic wave is defined as the average value of Polynting vector taken over one cycle.

ILLUSTRATION 25.6 About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation

(a) At a distance of 1 m from the bulb?

(b) At a distance of 10 m?

Assume that the radiation is emitted isotropicailly and neglect reflection.

Solution.

(a) Visible power = 5 W

$\therefore$ Average intensity of radiation at 1 m

$$= \frac{\text{Power}}{4\pi r^2} = \frac{5}{4 \times \pi \times 1} = 0.4 \text{ W m}^{-2}$$

(b) Average intensity of radiation at 10 m

$$= \frac{\text{Power}}{4\pi r^2} = \frac{5}{4 \times \pi \times 10^2} = 0.004 \text{ W m}^{-2}$$

ILLUSTRATION 25.7 Given below are some famous numbers associated with electromagnetic radiation in different contexts in physics. State the part of the em spectrum of which each belongs.

(a) 21 cm (wavelength emitted by atomic hydrogen in interstellar space)

(b) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen, known as Lamb shift)

(c) 2.7 K (temperature associated with the isotropic radiation filling all space-thought to be a relic of the 'big-bang' origin of the universe)

(d) 5890 Å–58963 Å (double lines of sodium)

(e) 14.4 keV (energy of a particular transition n ^{57}Fe nucleus associated with a famous high resolution spectroscopic method of sodium)

Solution.

(a) Wavelength is of the order of 10^{-2} m, i.e., short radio wave.

(b) Frequency is of the order of 10^{-2} Hz, i.e,. short radio wave.

(c) $\lambda_m T = 0.29$ or

$$\lambda_m = \frac{0.29}{T} = \frac{0.29}{2.7} = 0.09 \text{ cm} = 0.0009 \text{ m}$$

Wavelength is of the order of 10^{-4} m, i.e., microwave.

(d) Wavelengths are of the order of 10^{-7} m, i.e., visible radiations.

(e) Energy $= \frac{hc}{\lambda e}$ or $\lambda = \frac{hc}{\text{Energy} \times e}$

$$\lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{14.4 \times 10^3 \times 1.6 \times 10^{-19}}$$

i.e., $\lambda = 0.86 \times 10^{-10} = 8.6 \times 10^{-11}$ m

i.e., X-rays.

SOLVED EXAMPLES

1. In an apparatus, the electric field was found to oscillate with an amplitude of 18 V/m. The magnitude of the oscillating magnetic field will be

(a) 4×10^{-6}T (b) 6×10^{-8}T

(c) 9×10^{-9}T (d) 11×10^{-11}T

Sol. (b) $c = \frac{E}{B} \Rightarrow B = \frac{E}{c} = \frac{18}{3 \times 10^8} = 6 \times 10^{-8}\text{T}$.

2. According to Maxwell's hypothesis, a changing electric field gives rise to

(a) an e.m.f. (b) electric current

(c) magnetic field (d) pressure radiant

Sol. (c) According to the Maxwell's EM theory, the EM waves propagation contains electric and magnetic field vibration in mutually perpendicular direction. Thus the changing of electric field give rise to magnetic field.

3. In an electromagnetic wave, the electric and magnetising fields are 100 Vm^{-1} and 0.265 Am^{-1}. The maximum energy flow is

(a) 26.5 W/m^2 (b) 36.5 W/m^2

(c) 46.7 W/m^2 (d) 765 W/m^2

Sol. (a) Here E_0 = 100 V/m, B_0 = 0.265 A/m.

$\therefore$ Maximum rate of energy flow $S = E_0 \times B_0$

$$= 100 \times .265 = 26.5 \frac{\text{W}}{\text{m}^2}$$

4. The 21 cm radio wave emitted by hydrogen in interstellar space is due to the interaction called the hyperfine interaction is atomic hydrogen. the energy of the emitted wave is nearly

(a) 10^{-17} joule (b) 1 joule

(c) 7×10^{-3} joule (d) 10^{-24} joule

Sol. (d) $E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{21 \times 10^{-2}} = 0.94 \times 10^{-24} = 10^{-24}$ J

5. The oscillating electric and magnetic vectors of an electromagnetic wave are oriented along

(a) The same direction but differ in phase by 90°

(b) The same direction and are in phase

(c) Mutually perpendicular directions and are in phase

(d) Mutually perpendicular directions and differ in phase by 90°

Sol. (c) $\vec{E}$ and $\vec{B}$ are mutually perpendicular to each other and are in phase i.e. they become zero and minimum at the same place and at the same time.

6. In which one of the following regions of the electromagnetic spectrum will the vibrational motion of molecules give rise to absorption
   (a) ultraviolet (b) microwaves
   (c) infrared (d) radio waves

Sol. (b) Molecular spectra due to vibrational motion lie in the microwave region of EM-spectrum. Due to Kirchhoff's law in spectroscopy the same will be absorbed.

7. An electromagnetic wave travels along z-axis. Which of the following pairs of space and time varying fields would generate such a wave
   (a) E_x, B_y (b) E_y, B_x
   (c) E_z, B_x (d) E_y, B_z

Sol. (a) E_x and B_y would generate a plane EM wave travelling in z-direction. $\vec{E}$, $\vec{B}$ and $\hat{k}$ form a right handed system $\hat{k}$ is along z-axis. As $\hat{i} \times \hat{j} = \hat{k}$

$\Rightarrow E_x \hat{i} \times B_y \hat{j} = C\hat{k}$ i.e. E is along x-axis and B is along y-axis.

8. A signal emitted by an antenna from a certain point can be received at another point of the surface in the form of
   (a) sky wave (b) ground wave
   (c) sea wave (d) both (a) and (b)

Sol. (d) Ground wave and sky wave both are amplitude modulated wave and the amplitude modulated signal is transmitted by a transmitting antenna and received by the receiving antenna at a distance place.

9. The electromagnetic waves do not transport
   (a) energy (b) charge
   (c) momentum (d) information

Sol. (b) EM waves transport energy, momentum and information but not charge. EM waves are uncharged

10. A plane electromagnetic wave is incident on a material surface. If the wave delivers momentum p and energy E, then
   (a) $p = 0, E = 0$ (b) $p \neq 0, E \neq 0$
   (c) $p \neq 0, E = 0$ (d) $p = 0, E \neq 0$

Sol. (b) EM waves carry momentum and hence can exert pressure on surfaces. They also transfer energy to the surface so $p \neq 0$ and $E \neq 0$.

11. An electromagnetic wave, going through vacuum is described by $E = E_0 \sin(kx - \omega t)$. Which of the following is independent of wavelength
   (a) k (b) ω
   (c) k/ω (d) $k\omega$

Sol. (c) The angular wave number $k = \dfrac{2\pi}{\lambda}$; where λ is the wave length. The angular frequency is $\omega = 2\pi v$.

The ratio $\dfrac{k}{\omega} = \dfrac{2\pi/\lambda}{2\pi v} = \dfrac{1}{v\pi} = \dfrac{1}{c}$ = constant

12. An electromagnetic wave going through vacuum is described by $E = E_0 \sin(kx - \omega t)$; $B = B_0 \sin(kx - \omega t)$. Which of the following equation is true
   (a) $E_0 k = B_0 \omega$ (b) $E_0 \omega = B_0 k$
   (c) $E_0 B_0 = \omega k$ (d) None of these

Sol. (a) $\dfrac{E_0}{B_0} = C$. also $k = \dfrac{2\pi}{\lambda}$ and $\omega = 2\pi v$

These relation gives $E_0 K = B_0 \omega$

13. A radio receiver antenna that is 2 m long is oriented along the direction of the electromagnetic wave and receives a signal of intensity 5×10^{-16} W/m^2. The maximum instantaneous potential difference across the two ends of the antenna is
   (a) 1.23 μV (b) 1.23 mV
   (c) 1.23 V (d) 12.3 mV

Sol. (a) $I = \dfrac{1}{2}\varepsilon_0 C E_0^2$

$$\Rightarrow E_0 = \sqrt{\frac{2I}{\varepsilon_0 C}} = \sqrt{\frac{2 \times 5 \times 10^{-16}}{8.85}} = 0.61 \times 10^{-6} \frac{\text{V}}{\text{m}}$$

Also $E_0 = \dfrac{V_0}{d}$

$\Rightarrow V_0 = E_0 d = 0.61 \times 10^{-6} \times 2 = 1.23\ \mu V$

14. Electromagnetic waves travel in a medium which has relative permeability 1.3 and relative permittivity 2.14. Then the speed of the electromagnetic wave in the medium will be
   (a) 13.6×10^6 m/s (b) 1.8×10^2 m/s
   (c) 3.6×10^8 m/s (d) 1.8×10^8 m/s

Sol. (d) $v = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}} = \dfrac{3 \times 10^8}{\sqrt{1.3 \times 2.14}} = 1.8 \times 10^8$ m/s

15. If c is the speed of electromagnetic waves in vacuum, its speed in a medium of dielectric constant K and relative permeability μ_r is
   (a) $v = \dfrac{1}{\sqrt{\mu_r K}}$ (b) $v = c\sqrt{\mu_r K}$
   (c) $v = \dfrac{c}{\sqrt{\mu_r K}}$ (d) $v = \dfrac{K}{\sqrt{\mu_r C}}$

Sol. (c) Speed of light of vacuum $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ and in another medium $v = \dfrac{1}{\sqrt{\mu \varepsilon}}$

$$\therefore \frac{c}{v} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} = \sqrt{\mu_r K} \Rightarrow v = \frac{c}{\sqrt{\mu_r K}}$$

EXERCISES

Problems Based on Basic Theory

1. The intensity of sunlight (in W/m^2) at the solar surface will be
   (a) 5.6×10^6 (b) 5.6×10^7
   (c) 4.2×10^6 (d) 4.2×10^7
2. Which of the following has zero average value in a plane electromagnetic wave?
   (a) Electric field (b) Magnetic potential
   (c) Electric energy (d) Magnetic energy
3. The wave impedance of free space is
   (a) 0 Ω (b) 376.6 Ω
   (c) 1883 Ω (d) 3776 Ω
4. The frequency 1057 MHz of radiation arising from two close energy levels in hydrogen belongs to
   (a) radio waves (b) infrared waves
   (c) micro waves (d) γ-rays
5. The wave associated with 2.7 K belongs to
   (a) radio waves (b) micro waves
   (c) ultraviolet ways (d) infrared waves
6. The percentage power of X-ray increases with the increases in its
   (a) velocity (b) intensity
   (c) frequency (d) wavelength
7. The shortest wavelength of X-rays remitted from an X-ray tube depends upon
   (a) nature of the gas in the tube
   (b) voltage applied to the tube
   (c) current in the tube
   (d) nature of target of the tube
8. X-rays are not used for radar purposes, because they are not
   (a) reflected by target
   (b) partly absorbed by target
   (c) electromagnetic waves
   (d) completely absorbed by target
9. The energy of X-rays photon is 3.3×10^{-16} J. Its frequency is
   (a) 2×10^{19} Hz (b) 5×10^{18} Hz
   (c) 5×10^{17} Hz (d) 5×10^{16} Hz
10. In X-ray tube, the accelerating potential applied at the anode is V volt. The minimum wavelength of the emitted X-ray will be
    (a) eV/h (b) h/eV
    (c) eV/ch (d) hc/eV
11. The area of television telecast is made twice, the height of antenna will be changed as
    (a) halved (b) doubled
    (c) quardupled (d) kept unchanged
12. If a source is transmitting electromagnetic wave of frequency 8.2×10^6 Hz, then the wavelength of the electromagnetic waves transmitted from the source will be
    (a) 36.6 m (b) 40.5 m
    (c) 42.3 m (d) 50.9 m
13. A plane electromagnetic wave
    $$E_z = 100 \cos(6 \times 10^8 t + 4x) \text{ V/m}$$
    propagates in a medium of dielectric constant
    (a) 1.5 (b) 2.0
    (c) 2.4 (d) 4.0

Maxwell Equations and Properties of Electromagnetic Waves

14. If μ_0 be the permeability and K_0 the dielectric constant of a medium, its refractive index is given by
    (a) $\dfrac{1}{\sqrt{\mu_0 K_0}}$ (b) $\dfrac{1}{\mu_0 K_0}$
    (c) $\sqrt{\mu_0 K_0}$ (d) $\mu_0 K_0$
15. If ε_0 and μ_0 represent the permittivity and permeability of vacuum and ε and μ represent the permittivity and permeability of medium, the refractive index of the medium is given by
    (a) $\sqrt{\dfrac{\varepsilon_0\mu_0}{\varepsilon\mu}}$ (b) $\sqrt{\dfrac{\varepsilon\mu}{\varepsilon_0\mu_0}}$
    (c) $\sqrt{\dfrac{\varepsilon}{\mu_0\varepsilon_0}}$ (d) $\sqrt{\dfrac{\mu_0\varepsilon_0}{\varepsilon}}$
16. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.5×10^{10} Hz and amplitude 480 V/m. The amplitude of the oscillating magnetic field will be
    (a) 1.52×10^{-8} Wb/m^2 (b) 1.52×10^{-7} Wb/m^2
    (c) 1.6×10^{-6} Wb/m^2 (d) 1.6×10^{-7} Wb/m^2
17. The magnetic field between the plates of radius 12 cm separated by a distance of 4 mm of a parallel plate capacitor of capacitance 100 pF along the axis of plates having conduction current of 0.15 A is
    (a) zero (b) 1.5 T
    (c) 15 T (d) 0.15 T
18. A flood light is covered with a fitter that transmits red light. The electric field of the emerging beam is represented by a sinusoidal plane wave
    $$E_x = 36 \sin (1.20 \times 10^7 z - 3.6 \times 10^{15} t) \text{ V/m}$$
    The average intensity of beam in W/m^2 will be
    (a) 6.88 (b) 3.44
    (c) 1.72 (d) 0.86
19. The average energy-density of electromagnetic wave given by $E = (50 \text{ N/C}) \sin(\omega t - kx)$ will be nearly
    (a) 10^{-8} J/m^3 (b) 10^{-7} J/m^3
    (c) 10^{-6} J/m^3 (d) 10^{-5} J/m^3
20. A larger parallel plate capacitor, whose plates have an area of 1 m^2 are separated from each other by 1 mm. If the plates has dielectric constant 10, then the displacement current at this instant is

(a) 25 μA (b) 11 μA
(c) 2.2 μA (d) 1.1 μA

21. A parallel plate capacitor with plate area A and separation between the plates d, is charged by a constant current i. Consider a plane surface of area $A/2$ parallel to the plates and drawn simultaneously between the plates. The displacement current through this area is
(a) i (b) $i/2$
(c) $i/4$ (d) $i/8$

22. The sum delivers 10^4 W/m^2 of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions (10×10) m^2 will be
(a) 10^4 W (b) 10^5 W
(c) 10^6 W (d) 10^7 W

23. The average value of electric energy density in an electromagnetic wave is (E_0 is peak value)
(a) $\frac{1}{2}\varepsilon_0 E_0^2$ (b) $\frac{E_0^2}{2\varepsilon_0}$
(c) $\varepsilon_0 E_0^2$ (d) $\frac{1}{4}\varepsilon_0 E_0^2$

24. The sun delivers 10^3 W/m^2 of electromagnetic flux to the earth's surface. The total power that is incident on a roof of dimensions 8 m × 20 m will be
(a) 2.56×10^4 W (b) 6.4×10^5 W
(c) 4.0×10^5 W (d) 1.6×10^5 W

25. In Q. 24, the radiation force on the roof will be
(a) 8.53×10^{-5} N (b) 2.3×10^{-3} N
(c) 1.33×10^{-3} N (d) 5.33×10^{-4} N

26. A plane electromagnetic wave of wave intensity 6 W/m^2 strikes a small mirror of area 39 cm^2, held perpendicular to the approaching wave. The momentum transferred in kg ms^{-1} by the wave to the mirror each second will be
(a) 1.2×10^{-10} (b) 2.4×10^{-9}
(c) 3.6×10^{-8} (d) 4.8×10^{-7}

Problems Based on Mixed Concepts

27. The wave of wavelength 5900 Å emitted by any atom or molecule must have some finite total length which is known as coherence length. For sodium light, this length is 2.4 cm. The number of oscillations in this length will be
(a) 4.068×10^8 (b) 4.068×10^4
(c) 4.068×10^6 (d) 4.068×10^5

28. A radiowave has a maximum electric field intensity of 10^{-4} V m^{-1} on arrival at a receiving antenna. The maximum magnetic flux density of such a wave is
(a) zero (b) 3×10^4 T
(c) 5.8×10^{-9} T (d) 3.3×10^{-13} T

29. The transmitting antenna of a radio-station is mounted vertically. At a point 10 km due north of the transmitter the peak electric field is 10^{-3} V/m. The amplitude of the radiated magnetic field is
(a) 3.33×10^{-10} T (b) 3.33×10^{-12} T
(c) 10^{-3} T (d) 3×10^5 T

30. A circular ring of radius r is placed in a homogeneous magnetic field perpendicular to the plane of the ring. The field B changes with time according to the equation $B = Kt$, where K is a constant and t is the time. The electric field in the ring is
(a) $\frac{Kr}{4}$ (b) $\frac{Kr}{3}$
(c) $\frac{Kr}{2}$ (d) $\frac{K}{2r}$

31. A plane em wave of wave intensity of 10 W/m^2 strikes a small mirror of area 20 cm^2, held perpendicular to the approaching wave. The radiation force on the mirror will be
(a) 6.6×10^{-11} N (b) 1.33×10^{-11} N
(c) 1.33×10^{-10} N (d) 6.6×10^{-10} N

32. A TV tower has a height of 100 m. How much population is covered by the TV broadcast if the average population density around the tower is 1000 km^{-2}? (radius of the earth $= 6.37 \times 10^6$ m)
(a) 4 lakh (b) 4 billion
(c) 40000 (d) 40 lakh

33. In a region of free space the electric field at some instant of time is $\vec{E} = (80\hat{i} + 32\hat{j} - 64\hat{k})$ V/m and the magnetic field is $\vec{B} = (0.2\hat{i} + 0.08\hat{j} - 0.29\hat{k})\mu$T. The pointing vector for these fields is
(a) $-11.52\hat{i} + 28.8\hat{j}$ (b) $-28.8\hat{i} + 11.52\hat{i}$
(c) $28.8\hat{i} - 11.52\hat{j}$ (d) $11.52\hat{i} - 28.8\hat{j}$

34. A plane electromagnetic wave propagating in the x-direction has wavelength of 60 mm. The electric field is in the y-direction and its maximum magnitude is 33 V/m^{-1}. The equation for the electric field as function of x and t is
(a) $11 \sin \pi(t - x/c)$ (b) $33 \sin \pi \times 10^{11}(t - x/c)$
(c) $33 \sin \pi(t - x/c)$ (d) $11 \sin \pi \times 10^{11}(t - x/c)$

35. In an electromagnetic wave, the electric field oscillated sinusoidally with amplitude 45 Vm^{-1}, the rms value of oscillating magnetic field will be
(a) 1.6×10^{-8} T (b) 16×10^{-9} T
(c) 144×10^{-8} T (d) 11.3×10^{-8} T

36. The magnetic field in the plane electromagnetic wave is given by
$B_z = 2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ tesla.
The expression for electric field will be
(a) $E_z = 30\sqrt{2}\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ V/m
(b) $E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ V/m
(c) $E_y = 30\sqrt{2}\sin(0.5 \times 10^{11} x + 0.5 \times 10^3 t)$ V/m
(d) $E_y = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ V/m

ARCHIVES

1. Electromagnetic waves are transverse in nature as is evident by
   (a) polarisation (b) interference
   (c) reflection (d) diffraction
   (AIEEE 2002)

2. Which of the following are not electromagnetic waves?
   (a) cosmic rays (b) gamma rays
   (c) β-rays (d) X-rays **(AIEEE 2002)**

3. The dimensions of $1/\mu_0\varepsilon_0$, where symbols have their usual meanings, are
   (a) $[L^{-1}T]$ (b) $[L^{-2}T^2]$
   (c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$ **(AIEEE 2003)**

4. Which of the following radiations has the least wavelength?
   (a) γ-rays (b) β-rays
   (c) α-rays (d) X-rays **(AIEEE 2003)**

5. An electromagnetic wave of ν = 3 MHz passes from vacuum into dielectric medium with $\varepsilon = 4.0\,\varepsilon_0$. Then
   (a) the wavelength is doubled and the frequency becomes half.
   (b) the wavelength is doubled and the frequency remains same.
   (c) the wavelength and frequency both remain unchanged.
   (d) the wavelength is halved but the frequency remains same. **(AIEEE 2004)**

6. The rms value of the electric field of the light coming from sun is 720 N/C. The average energy density of the emf is
   (a) 3.3×10^{-3} J/m^3 (b) 4.58×10^{-6} J/m^3
   (c) 6.37×10^{-9} J/m^3 (d) 81.35×10^{-12} J/m^3
   (AIEEE 2006)

7. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then
   (a) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$
   (b) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$
   (c) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$
   (d) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$ **(AIEEE 2012)**

8. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is
   (a) 6 V/m (b) 9 V/m
   (c) 12 V/m (d) 3 V/m **(JEE Main 2013)**

9. Match List-I (Electromagnetic wave type) with List-II (Its association / application) and select the correct option from the choices given below the lists:

List–I	List–II
(A) Infrared waves	(i) To treat muscular strain
(B) Radio waves	(ii) For broadcasting
(C) X-rays	(iii) To detect fracture of bones
(D) Ultraviolet rays	(iv) Absorbed by the ozone layer of the atmosphere

	(A)	(B)	(C)	(D)
(a)	(iii)	(ii)	(i)	(iv)
(b)	(i)	(ii)	(iii)	(iv)
(c)	(iv)	(iii)	(ii)	(i)
(d)	(i)	(ii)	(iv)	(iii)

(JEE Main 2014)

10. During the propagation of electromagnetic waves in a medium:
    (a) Electric energy density is equal to the magnetic energy density.
    (b) Both electric and magnetic energy densities are zero.
    (c) Electric energy density is double of the magnetic energy density.
    (d) Electric energy density is half of the magnetic energy density. **(JEE Main 2014)**

11. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is
    (a) 1.73 V/m (b) 2.45 V/m
    (c) 5.48 V/m (d) 7.75 V/m
    (JEE Main 2015)

12. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (Speed of light $= 3 \times 10^8$ ms^{-1})
    (a) 17.3 GHz (b) 15.3 GHz
    (c) 10.1 GHz (d) 12.1 GHz
    (JEE Main 2017)

13. An EM wave from air enters a medium. The electric fields are
$$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right] \text{ in air}$$
$$\vec{E}_2 = E_{02}\,\hat{x}\cos[k(2z - ct)] \text{ in medium,}$$
    where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. if ε_{r1} and ε_{r2} refer to relative permittivities of air and medium respectively, which of the following options is correct ?
    (a) $\dfrac{\varepsilon_{r1}}{\varepsilon_{r2}} = \dfrac{1}{2}$ (b) $\dfrac{\varepsilon_{r1}}{\varepsilon_{r2}} = 4$
    (c) $\dfrac{\varepsilon_{r1}}{\varepsilon_{r2}} = 2$ (d) $\dfrac{\varepsilon_{r1}}{\varepsilon_{r2}} = \dfrac{1}{4}$
    (JEE Main 2018)

ANSWER KEY

Exercises

1. (b)	2. (a)	3. (c)	4. (a)	5. (b)	6. (c)	7. (b)	8. (a)	9. (a)	10. (d)
11. (b)	12. (a)	13. (b)	14. (c)	15. (b)	16. (c)	17. (a)	18. (c)	19. (a)	20. (c)
21. (b)	22. (c)	23. (d)	24. (d)	25. (d)	26. (a)	27. (b)	28. (d)	29. (b)	30. (c)
31. (c)	32. (d)	33. (d)	34. (b)	35. (d)	36. (d)				

Archives

1. (a)	2. (c)	3. (c)	4. (a)	5. (d)	6. (b)	7. (b)	8. (a)	9. (b)	10. (a)
11. (b)	12. (a)	13. (d)							

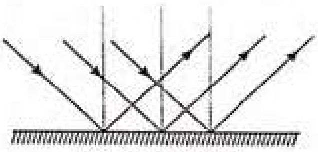
Chapter 26

Geometrical Optics

REFLECTION OF LIGHT

When a ray of light after incidenting on a boundary separating two media comes back into the same media, then this phenomenon is called reflection of light.

(a) Regular reflection: When the reflection takes place from a perfect plane surface, it is called *regular reflection*. In this case, the reflected light has large intensity in one direction and negligibly small intensity in other directions.

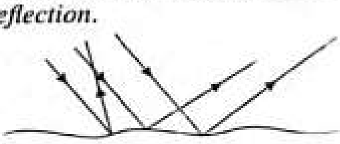


Regular reflection

(b) Diffused reflection: When the surface is rough, we do not get a regular behavior of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface and it is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called as *diffused reflection*.

For example, reflection from a wall, from a newspaper, etc.

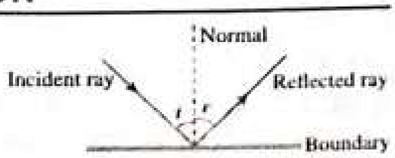
This is why you cannot see your face in newspaper and in the wall.



Diffused reflection

LAWS OF REFLECTION

(a) The incident ray, the reflected ray, and the normal at the point of incidence lie in the same plane. This plane is called the *plane of incidence (or plane of reflection)*.



(b) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e., $\angle i = \angle r$.

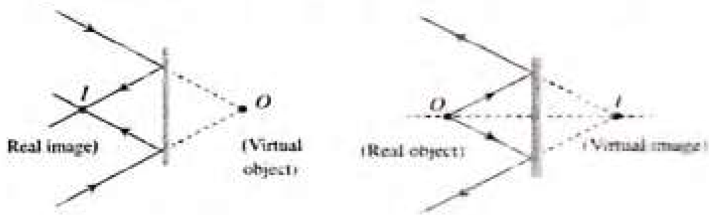
Object and Image

Object (O): Object is defined as point of intersection of incident rays.

(b) Image (I): Image is defined as point of intersection of reflected rays (in case of reflection) or refracted rays (in case of refraction).

Real and Virtual Images

If light rays, after reflection or refraction, actually meets at a point, then real image is formed; if they appear to meet, virtual image is formed.



Formation of Images by a Plane Mirror

When you look into a plane (flat) mirror, you see an image of yourself that has three properties:

1. The image is upright but laterally inverted.
2. The image is of the same size as you are.
3. The image is located as far behind the mirror as you are in front of it.

Figure (a) shows a light ray leaving the top of an object. This ray reflects from the mirror (angle of reflection equals angle of incidence) and enters the eye. To the eye, it appears that the ray originates from behind the mirror, somewhere back along the dashed line. Actually, rays going in all directions leave each point on the object but only a small bundle of such rays is intercepted by the eye. Part (b) of the figure shows a bundle of two rays leaving the top of the object. All the rays

that leave a given point on the object, no matter what angle θ they have when they strike the mirror, appear to originate from a corresponding point on the image behind the mirror [see the dashed lines in part (b)].

The image is inverted if the extended object lies perpendicular to the plane mirror.

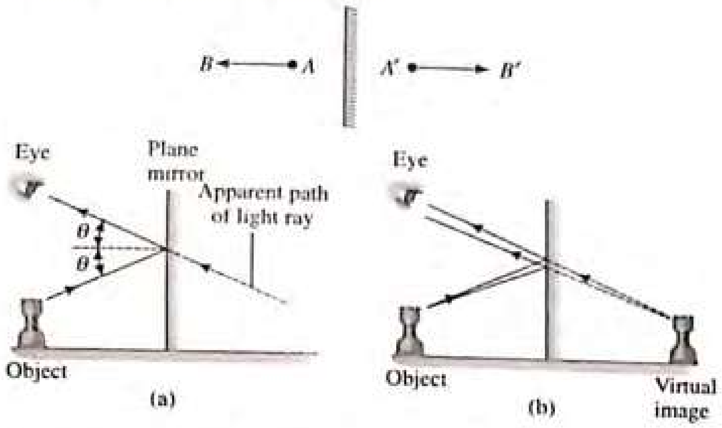
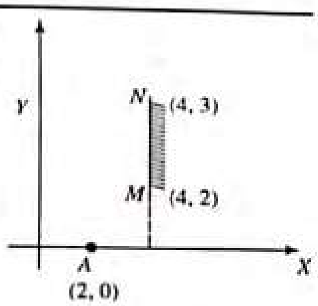
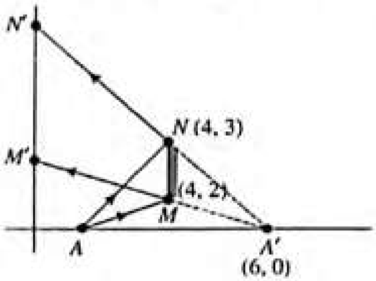


ILLUSTRATION 26.1 Find the region on Y axis in which reflected rays are present. Object is at $A(2, 0)$ and MN is a plane mirror, as shown.



Solution. For plane mirror, distance of object from mirror is equal to distance of image from the mirror, hence the image of point A, in the mirror is at A' (6, 0).

Join $A'M$ and extend to cut Y axis at M' (Ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join $A'N$ and extend to cut Y axis at N' (ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A').



From geometry, we can observe

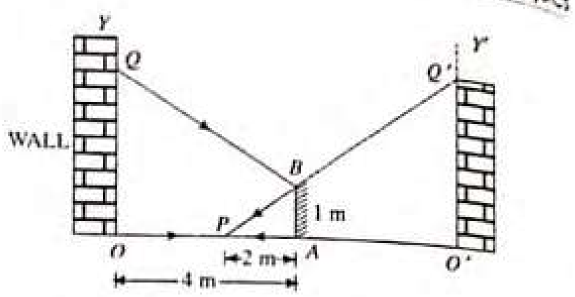
$M' \equiv (0, 6)$

$N' \equiv (0, 9)$

$M'N'$ is the region on Y axis in which reflected rays are present.

ILLUSTRATION 26.2 A plane mirror 1 m high hangs on a wall. A man stands at a distance 2 m away from the mirror. What is the height of the portion of the opposite wall in the room that can be seen by the man in the mirror, without changing the position of his head? The wall is 4 m from the mirror.

Solution. Figure shows the position of the head (P) of the man at a distance PA = 2 m from the plane mirror AB = 1 m. The vertical wall OY is at a distance OA = 4 m from the mirror.



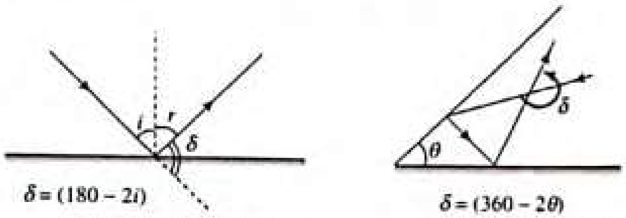
The image of the wall will be formed in the vertical plane $O'Y'$ through the point O', such that $O'A = OA = 4$ m. If the eye is assumed to be at the point P, then it can receive the rays of light from the section OQ of the wall after reflection from the mirror and likewise, its image $O'Q'$ (= OQ) will be formed in the mirror. Now, the triangles PAB and $PO'Q'$ are similar.

$$\therefore \quad \frac{O'Q'}{AB} = \frac{PO'}{PA}$$

$$\text{or} \quad O'Q' = \frac{PO'}{PA} \times AB = \frac{(2+4)}{2} \times 1 = 3 \text{ m}$$

Deviation (δ) of Ray by Plane Mirror

Deviation produced by a plane mirror and by two inclined plane mirrors.

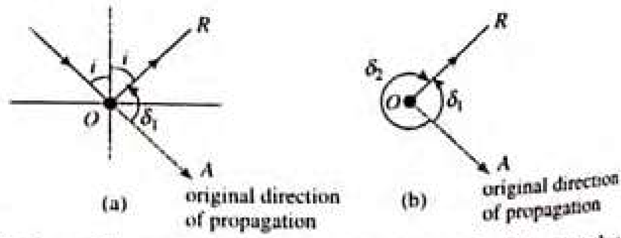


$\delta = (180 - 2i)$ $\quad$ $\delta = (360 - 2\theta)$

(A) Single Reflection $\quad$ (B) Double Reflection

ILLUSTRATION 26.3 Show that for a light ray incident at an angle 'i' on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\delta = \pi + 2i$.

Solution.



(a) $\quad$ (b)

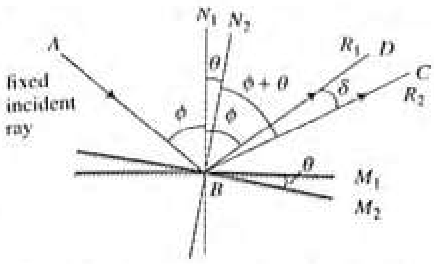
Sol. From Figure (b), it is clear that light ray bends either by δ_1 anticlockwise or by $\delta_2 (= 2\pi - \delta_1)$ clockwise.

From Figure (a), $\delta_1 = \pi - 2i$

$\therefore \quad \delta_2 = \pi + 2i$

ILLUSTRATION 26.4 For a fixed incident light ray, if the mirror is rotated through an angle θ (about an axis which lies in the

plane of mirror and perpendicular to the plane of incidence), show that the reflected ray turns through an angle 2θ in same sense.



Solution. In the figure, M_1, N_1 and R_1 indicate the initial position of mirror, initial normal, and initial direction of reflected light ray, respectively. M_2, N_2 and R_2 indicate the final position of mirror, final normal, and final direction of reflected light ray, respectively. From the figure, it is clear that $\angle ABC = (2\phi + \delta)$ $= (2\phi + 2\theta)$ or $\delta = 2\theta$.

ILLUSTRATION 26.5 Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in figure. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?

Solution. When a plane mirror rotates through certain angle, the reflected ray turns through twice the angle of rotation. Therefore, angle between the incident ray AO and the reflected ray,

$$\theta = 3.5° \times 2 = 7°$$

From the right-angled ΔOAB, we have

$$\tan 7° = \frac{AB}{AO}$$

or $$0.1228 = \frac{d}{1.5}$$

or $$d = 0.1228 \times 1.5$$
$$= 0.1842 \text{ m} = 18.42 \text{ cm}$$

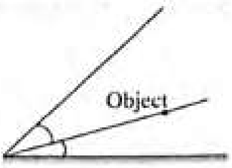
Images formed by two inclined plane mirrors: When two plane mirrors are inclined to each other at an angle θ, then number of images (n) formed of an object which is kept between them.

(i) $n = \left(\frac{360°}{\theta} - 1\right)$; If $\frac{360°}{\theta}$ = even integer

(ii) If $\frac{360°}{\theta}$ = odd integer, then there are two possibilities

(A) Object is placed symmetrically

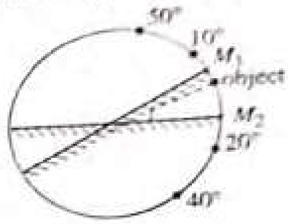
$n = \left(\frac{360}{\theta} - 1\right)$



(B) Object is placed asymmetrically

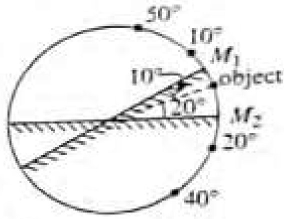
$n = \frac{360}{\theta}$

ILLUSTRATION 26.6 Two mirrors are inclined by an angle 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.



Solution. Figure is self-explanatory.

Number of Images



(i) Using direct formula: $\frac{360°}{30°}$

= 12 (even number)

$\therefore$ Number of images = 12 − 1

= 11

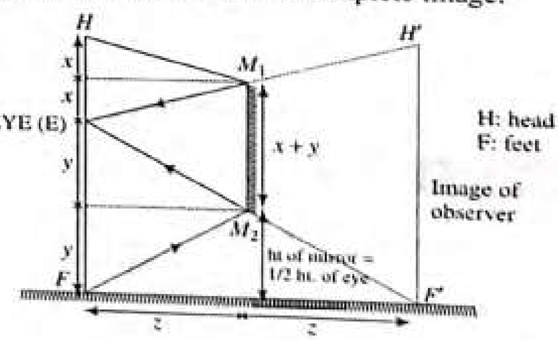
(ii) By counting: See the following table.

Image formed by Mirror M_1 (angles are measured from the mirror M_1.)		Image formed by Mirror M_2 (angles are measured from the mirror M_2.)
10°	+30°	20°
50°	+30°	40°
70°	+30°	80°
110°	+30°	100°
130°	+30°	140°
170°	+30°	160°
Stop because next angle will be more than 180°		Stop because next angle will be more than 180°

To check whether the final images made by the two mirrors coincide or not: add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide. Here last angles made by the mirrors + the angle between the mirrors = 160° + 170° + 30° = 360°. Therefore in this case the last images coincide. Therefore the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 − 1 (as the last images coincide) = 6 + 6 − 1 = 11.

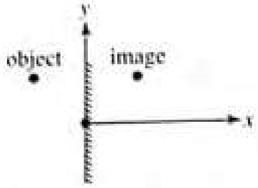
Important Points

- After reflection, velocity, wave length and frequency of light remain same but intensity decreases.
- There is a phase change of π if reflection takes place from denser medium.
- A man of height h requires a mirror of length at least equal to $h/2$, to see his own complete image.



Relation between Velocity of Object and Image

From mirror property:



$x_{\text{image, mirror}} = -x_{\text{object, mirror}}$,
$y_{\text{image, mirror}} = y_{\text{object, mirror}}$ and
$z_{\text{image, mirror}} = z_{\text{object, mirror}}$

Here $x_{\text{image, mirror}}$ means 'x' coordinate of image with respect to mirror. Similarly, others have meaning.

Differentiating w.r.t. time, we get

$$(v_{\text{image, mirror}})_x = -(v_{\text{object, mirror}})_x,$$
$$(v_{\text{image, mirror}})_y = -(v_{\text{object, mirror}})_y \text{ and}$$
$$(v_{\text{image, mirror}})_z = -(v_{\text{object, mirror}})_z$$

$\Rightarrow$ for x axis,

$$v_{\text{image, ground}} - v_{\text{mirror, ground}} = -(v_{\text{object, ground}} - v_{\text{mirror, ground}})$$

but for y axis and z axis,

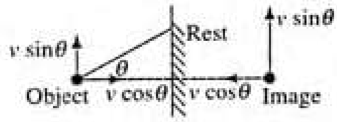
$$v_{\text{image, ground}} - v_{\text{mirror, ground}} = (v_{\text{object, ground}} - v_{\text{mirror, ground}})$$

or $\quad v_{\text{image, ground}} = v_{\text{object, ground}}$.

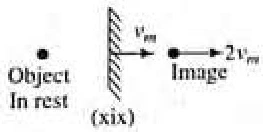
here: $v_{\text{image, ground}}$ = velocity of image with respect to ground.

Important Points

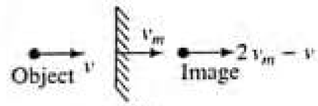
- When the object moves with speed u towards (or away from) the plane mirror, then image also moves towards (or away) with speed u. But relative speed of image *w.r.t.* object is $2u$.
- When mirror moves towards the stationary object with speed u, the image will move with speed $2u$ in same direction as that of mirror.



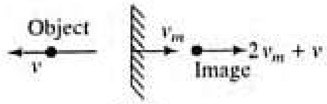
(xvii)



(xix)

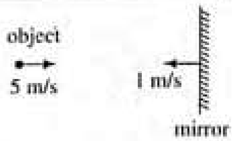


(xviii)



(xx)

ILLUSTRATION 26.7 An object moving with 5 m/s towards right while the mirror moving with 1 m/s towards the left as shown. Find the velocity of image.

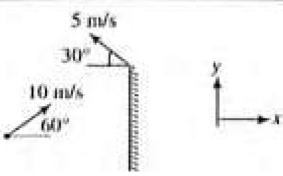


Solution. Let us take rightward direction as positive direction.

$$v_{\text{image, ground}} - v_{\text{mirror, ground}} = -(v_{\text{object, ground}} - v_{\text{mirror, ground}})$$
$$v_{\text{image, ground}} - (-1) = -[5 - (-1)]$$
$$v_i = -7 \text{ m/s}.$$

Hence image will move with velocity 7 m/s towards left.

ILLUSTRATION 26.8 A plane mirror and a point object are moving as shown in figure, find the velocity of image.



Solution. Let us consider x-direction first. Along x direction, applying

$$v_{\text{image}} - v_{\text{mirror}} = -(v_{\text{object}} - v_{\text{mirror}})$$
$$v_{\text{image}} - (-5\cos 30^\circ) = -(10\cos 60^\circ - (-5\cos 30^\circ))$$
$$v_{\text{image}} = -5(1+\sqrt{3}) \text{ m/s}$$

Similarly, along y direction we can write $v_{\text{object}} = v_{\text{image}}$

$$v_{\text{image}} = 10\sin 60^\circ = 5\sqrt{3} \text{ m/s}$$

Hence, velocity of the image

$$\vec{v}_{\text{image}} = -5(1+\sqrt{3})\hat{i} + 5\sqrt{3}\hat{j} \text{ m/s}$$

REFLECTION FROM A CURVED SURFACE

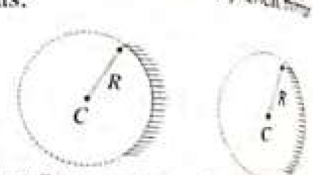
Spherical Mirrors

A spherical mirror is formed by polishing one surface of a part of sphere. A spherical mirror is a reflecting surface whose shape is a section of a sphserical surface (see figure).

Spherical mirror

Depending upon which part is shining, the spherical mirror is classified as:

Concave mirror: If the inside surface of the mirror is polished, it is a concave mirror [see Figure (a)].

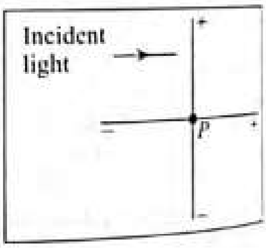


(a) Concave mirror (b) Convex mirror

Convex mirror: If the outside surface of the mirror is polished, it is a convex mirror [see Figure (b)].

SIGN CONVENTION: CARTESIAN CONVENTION

We will use following sign convention for problem solving in case of reflection as well as refraction.



1. All distances are measured from the pole.
2. Distances measured in the direction of incident rays are taken as positive
3. Distances measured in the direction opposite to that of the incident rays are taken as negative.
4. Distances above the principal axis are taken as positive.
5. Distances below the principal axis are taken as negative.

Angles measured from the normal in anticlockwise sense are positive, while that in clockwise sense are negative.

POSITION, SIZE AND NATURE OF IMAGE FORMED BY SPHERICAL MIRRORS

Consider figure where O is a point object and I is the corresponding image.

Here, $f = \dfrac{R}{2}$ and $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$

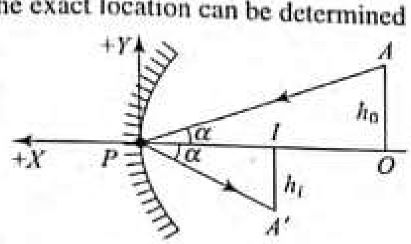
(mirror formula)

Image Formation in Convex Mirror

Magnification: The lateral magnification is defined as the ratio

$$m_v = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_0}$$

To compute the vertical magnification, consider the extended object OA shown in figure. The base of the object, O, will map on to a point I on the principal axis which can be determined from the equation $(1/u) + (1/v) = (1/f)$. The image of the top of the object, A, will map on to a point A' that will lie on the perpendicular through I. The exact location can be determined by drawing a ray from A passing through the pole and intercepting the line through I at A'.



A real image of an extended object in front of a concave mirror

Consider the triangles APO and $A'PI$ in the figure. As the two triangles are similar, we get

$$\tan\alpha = \frac{AO}{PO} = \frac{A'I}{PI} \quad \text{or} \quad \frac{A'I}{AO} = \frac{PI}{PO}$$

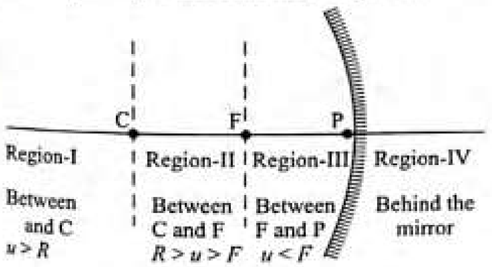
Applying the sign convention, we get, $u = -PO$

$$v = -PI \;\Rightarrow\; h_0 = +AO \;\Rightarrow\; h_i = -A'I$$

$$\text{Therefore, } -\frac{h_i}{h_0} = \frac{v}{u} \quad \text{or} \quad m_v = \frac{h_i}{h_0} = -\frac{v}{u}$$

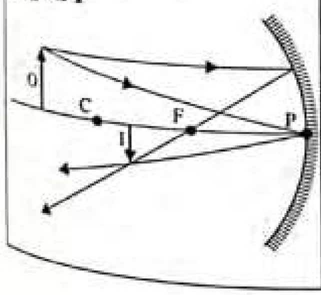
Nature of Image Formed by Concave Mirror

We can divide principal axis in three regions.



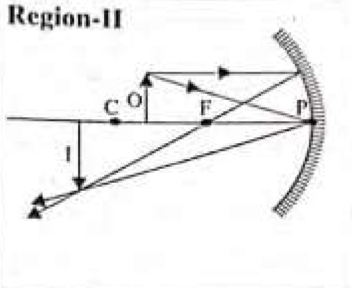
Now we analyze the corresponding image formed.

Region-I



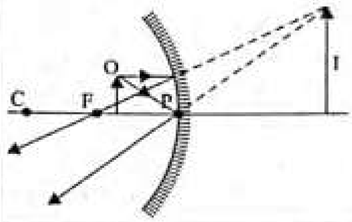
- Object placed between center of curvature and infinity. Image real, inverted, smaller (diminished)
- If object is moved away from center of curvature towards infinity, magnification goes on decreasing.

Region-II



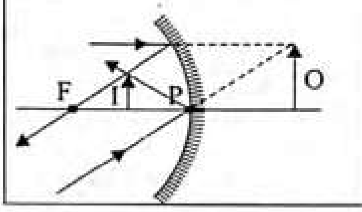
- Object placed between f and center of curvature. Image is real, inverted, magnified.
- If object is moved from focus towards center of curvature, magnification goes on decreasing at center of curvature it becomes unity.

Region-III



- Object placed between pole and focus. Image is virtual, erect, magnified.
- If object is moved towards pole magnification goes on decreasing at pole magnification m tends to unity.

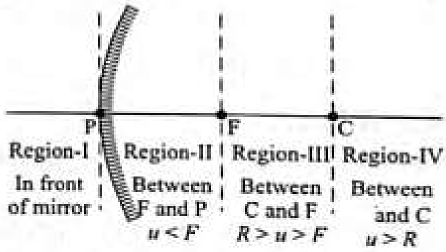
Region-IV



- Object is virtual point object. Image real, erect, smaller (diminished)
- As object moves away from pole, magnification goes on decreasing.

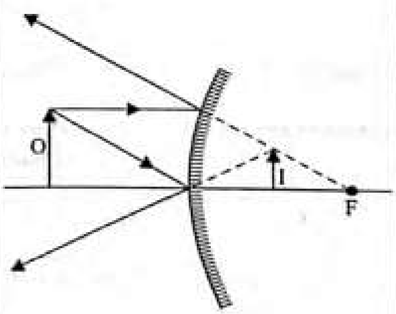
Nature of Image Formed by Convex Mirror

Figure shows convex mirror with four different object positions as shown in figure

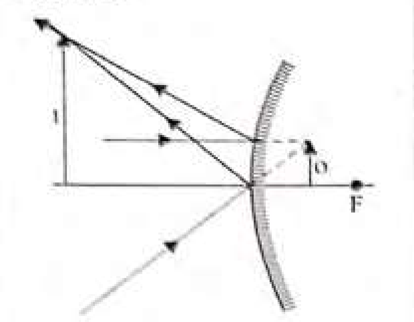
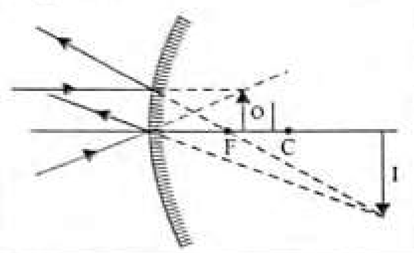
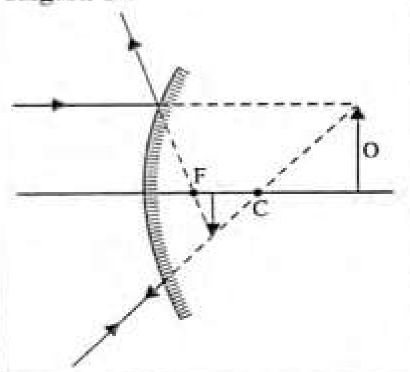


Now we analyze the corresponding image formed.

Region-I



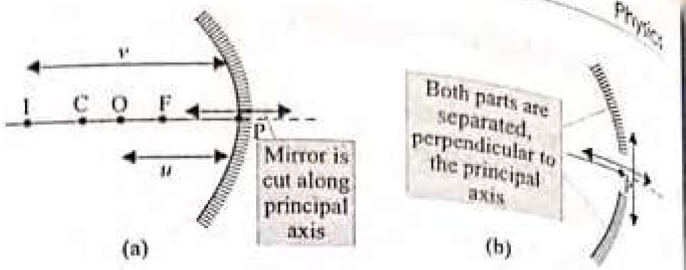
- Object placed in front of mirror. For all positions of object infront of mirror. Image is virtual, erect, smaller in size.
- As object moved towards pole, magnification increases and tends to unity at pole.

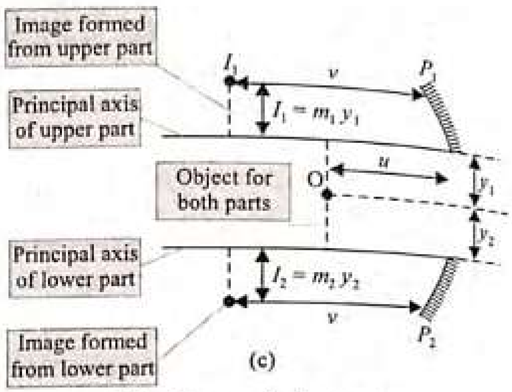
Region-II 	• Virtual object placed between pole and focus. Image formed is real, erect, enlarged. • As object is moved away from pole, magnification increases.
Region-III 	• Virtual object placed between F and C. Image formed virtual, inverted, enlarged. • As object is moved toward C magnification decreases.
Region-IV 	• Virtual object is placed beyond C. Image is virtual inverted smaller. • As object is moved away from C, image size further decrease magnification decreases.

Cutting of a Mirror

Mirror is cut along its principal axis and two parts are separated perpendicular to the principal axis.

1. For first part the object O will be below principal axis and for second part the object O will be above principal axis.
2. For first part image will form above its principal axis, and for second part the image will form below principal axis at a distance m_1y_1 and m_2y_2, respectively.





For image partition from principal axis,

$$m = -\frac{v}{u} = \frac{I_1}{O} \Rightarrow |I_1| = my_1$$

Thus, $s = my_1 + y_1 + m_r y_2 + y_2$; $s = (m + 1)(y_1 + y_2)$

If s is the separation distance between two images I_1 and I_2 of the same object O formed due to two halves of the mirror.

Mirror is cut along its principal axis and two parts are separated parallel to the principal axis.

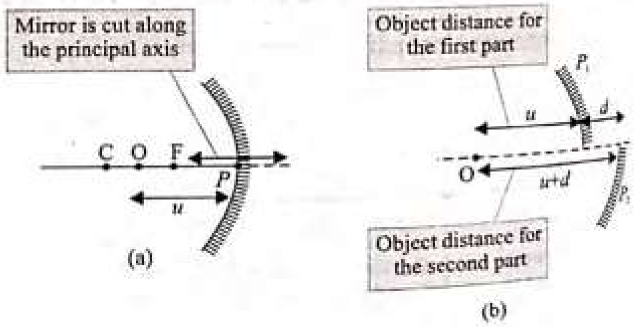
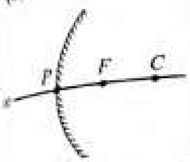
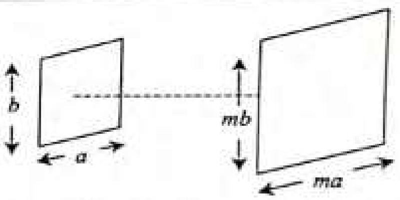


Table 26.1 Position, size and nature of image formed by the spherical mirror

Mirror	Location of the object	Location of the image	Magnification, size of the image	Nature	
				Real virtual	Erect inverted
(a) Concave 	At infinity, i.e., $u = \infty$	At focus, i.e., $v = f$	$m << 1$, diminished	Real	inverted
	Away from center of curvature ($u > 2f$)	Between f and $2f$, i.e., $f < v < 2f$	$m < 1$, diminished	Real	inverted
	At center of curvature $u = 2f$	At center of curvature, i.e., $v = 2f$	$m = 1$, same size as that of the object	Real	inverted

	Between center of curvature and focus; $f < u < 2f$	Away from the center of curvature $v > 2f$	$m > 1$, magnified	Real	inverted
	At focus, i.e., $u = f$	At infinity, i.e., $v = \infty$	$m = \infty$, magnified	Real	inverted
	Between pole and focus $u < f$	$v > u$	$m > 1$, magnified	Virtual	erect
(b) Convex 	At infinity, i.e., $u = \infty$	At focus, i.e., $v = f$	$m < 1$, diminished	Virtual	erect
	Anywhere between infinity and pole	Between pole and focus	$m < 1$, diminished	Virtual	erect

NOTE:
- In case of convex mirrors, as the object moves away from the mirror, the image becomes smaller and moves closer to the focus.
- For convex mirror maximum image distance is it's focal length.
- In concave mirror, minimum distance between a real object and its real image is zero (i.e., when $u = v = 2f$).

Linear magnification		Areal magnification
Transverse	**Longitudinal**	
When a object is placed perpendicular to the principle axis, then linear magnification is called lateral or transverse magnification. It is given by $m = \frac{I}{O} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$ (*Always use sign convention while solving the problems)	When object lies along the principal axis then its longitudinal magnification $m = \frac{I}{O} = \frac{-(v_2 - v_1)}{(u_2 - u_1)}$ If object is small; $m = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2$ Also length of image $= \left(\frac{v}{u}\right)^2 \times$ Length of object (L_0) $(L_i) = \left(\frac{f}{u-f}\right)^2 \cdot L_o$	 If a 2D-object is placed with its plane perpendicular to principal axis Its areal magnification $M_s = \frac{\text{Area of image}(A_i)}{\text{Area of object}(A_o)} = \frac{ma \times mb}{ab} = m^2$ $\Rightarrow m_s = m^2 = \frac{A_i}{A_o}$

NOTE:
- Do not put the sign of quantity which is to be determined.
- If a spherical mirror produces an image m times the size of the object (m = magnification) then u, v and f are given by the followings

$$u = \left(\frac{m-1}{m}\right)f, \quad v = -(m-1)f \quad \text{and} \quad f = \left(\frac{m}{(m-1)}\right)u \text{ (use sign convention)}$$

ILLUSTRATION 26.9 An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance of 5 cm, then the image becomes triple its size. Find the nature and radius of curvature of the mirror.

Solution. Since the image is formed on the screen, it is real. Real object and real image implies concave mirror.

Applying $m = \frac{f}{f-u}$ or $-2 = \frac{f}{f-u}$ (i)

After shifting, $-3 = \frac{f}{f-(u+5)}$ (ii)

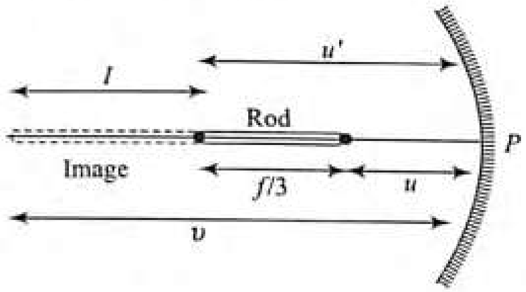
Here, care should be taken that distance of the object becomes $u + 5$ not $u - 5$: In a concave mirror the size of real image will increase only when the real object is brought closer to the mirror. In doing so, its x-coordinate will increase.

From (i) and (ii), we get

$f = -30$ cm or $R = 60$ cm

ILLUSTRATION 26.10 A thin rod of length $f/3$ is placed along the optical axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. Calculate the magnification.

Solution. As in question, image touches the rod, i.e., image and object coincides, hence one end of the rod should be at the center of curvature. It is also written that image is enlarged, it indicates that the orientation of rod should be toward focus then only we can get enlarged image along the principal axis. Let l be the length of the image.



Then, $m = \dfrac{l}{f/3} \Rightarrow l = \dfrac{mf}{3}$

Also, one end of the image coincides with the object, $u' = 2f$.

Now, $u' = u + \dfrac{f}{3} \Rightarrow u = 2f - \dfrac{f}{3} = \dfrac{5f}{3}$

$$v = -\left(u + \frac{f}{3} + \frac{mf}{3}\right).$$

Putting in mirror formula, we get

$$\frac{1}{u + f/3 + mf/3} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{3}{5f + f + mf} + \frac{3}{5f} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{m+6} = \frac{2}{15} \Rightarrow m = \frac{3}{2}$$

RELATION BETWEEN OBJECT AND IMAGE VELOCITY

Case (i). Differentiate equation $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ with respect to time.

$$\Rightarrow -\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow -\frac{1}{v^2}v_{im} - \frac{1}{u^2}v_{OM} = 0$$

$\dfrac{dv}{dt} = v_{im}$ = velocity of image w.r.t. mirror

$$\Rightarrow v_{im} = -\frac{v^2}{u^2}v_{OM};$$

$\dfrac{du}{dt} = v_{OM}$ = velocity of object w.r.t. mirror

$$v_{Im} = -m^2 v_{OM}$$

The negative sign shows that if u is decreasing, v will increase, i.e., if real object approaches the mirror, its real image will recede from the mirror.

In this case, $|m| = 1$, hence $\left|\dfrac{dv}{dt}\right| < \left|-\dfrac{du}{dt}\right|$

When the object is at center of curvature, $\left|\dfrac{dv}{dt}\right| = \left|-\dfrac{du}{dt}\right|$

Case (ii). Object moves between center of curvature and focus.

In this case $|m| = 1$, hence $\left|\dfrac{dv}{dt}\right| > \left|-\dfrac{du}{dt}\right|$.

Speed of image is more than speed of object.

Case (iii). Object moves between focus and pole of the mirror.

In this case image is virtual, hence $\dfrac{1}{(+v)} + \dfrac{1}{(-u)} = \dfrac{1}{(-f)}$

$$\Rightarrow \frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt}$$

If u is decreasing, v will also decrease, ie., if real object approaches mirror, image will also do so.

As $|m| > 1$, speed of image will be greater than speed of object.

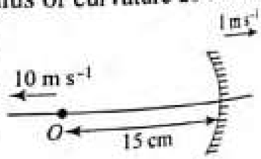
Size of image for a small size object placed along principal axis.

Differentiating $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$ w.r.t. t, we get

$$\frac{dv}{dt} = -m^2\frac{du}{dt} \Rightarrow dv = -m^2 du$$

NOTE: du/dt and dv/dt and are the velocities with respect to mirror not w.r.t. ground. When the mirror is at rest, then velocity of object or image w.r.t. mirror is same as velocity of object or mirror w.r.t. ground.

ILLUSTRATION 26.11 A mirror of radius of curvature 20 cm and an object which is placed at a distance of 15 cm are both moving with velocities 1 m s^{-1} and 10 m s^{-1} as shown in figure. Find the velocity of image at this situation.



Solution. Using $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{2}{R}$, we get

$$\frac{1}{v} - \frac{1}{15} = -\frac{1}{10} \Rightarrow v = -30\text{ cm}$$

Now, using $v_{im} = -\dfrac{v^2}{u^2}v_{OM}$

$$(v_i - v_m) = -\frac{v^2}{u^2}(v_o - v_m)$$

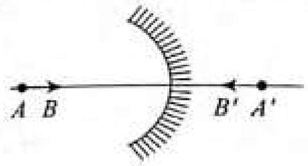
$$\Rightarrow v_i - (1) = -\frac{(-30)^2}{(-15)^2}[(-10) - (1)]$$

$$\Rightarrow v_i = 45\text{ cm s}^{-1}$$

So, the image will move with velocity 45 cm s^{-1}.

NOTE: Some important points about curved mirror:

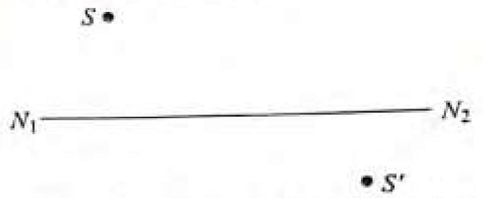
- Lateral magnification (or transverse magnification) denoted by m is defined as $m = h_2/h_1$ and is related as $m = v/u$. From the definition of m, the positive sign of m indicates erect image and the negative sign indicates inverted image.
  In case of successive reflections from mirrors, the overall lateral magnification is given by $m_1 \times m_2 \times m_3 \times \ldots$, where m_1, m_2, etc. are lateral magnifications produced by individual mirrors.
- On differentiating the mirror formula, we get $= dv/du = v^2/u^2$. Mathematically, du' implies a small change in the position of object and dv' implies corresponding small change in the position of image. If a small object lies along principal axis, du may indicate the size of object and dv the size of its image along principal axis. (Note that the focus should not lie in between the initial and final points of object.) In this case, dv/du is called longitudinal magnification. Negative sign indicates inversion of image irrespective of the nature of the image and the mirror.



- Velocity of image
  - Object moving perpendicular to the principal axis:
    We have $h_2/h_1 = v/u$ or $h_2 = (-v/u)h_1$
    If a point object moves perpendicular to the principal axis, x-coordinate of both the object and the image remains constant. On differentiating the above relation w.r.t. time, we get
    $$\frac{dh_2}{dt} = -\frac{v}{u}\frac{dh_1}{dt}$$
    Here, dh_1/dt denotes velocity of object perpendicular to the principal axis and dh_2/dt denotes velocity of image perpendicular to the principal axis.
  - Object moving along the principal axis: On differentiating the mirror formula with respect to time, we get $dv/dt = -v^2/u^2(du/dt)$, where dv/dt is the velocity of image along the principal axis and du/dt is the velocity of object along principal axis. Negative sign implies that the image, in case of mirror, always moves in the direction opposite to that of the object. This discussion is for velocity with respect to mirror and along the x-axis.
  - Object moving at an angle with the principal axis: Resolve the velocity of object along and perpendicular to the principal axis and find the velocities of image in these directions separately and then find the reusltant.
- Newton's formula: $XY = f^2$
  X and Y are the distances (along the principal axis) of the object and image, respectively, from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.
- Optical power of a mirror (in diopters): $P = -1/f$
  f = focal length with sign and is in meters.
- If object lying along the principal axis is not of very small size, the longitudinal magnification = $(v_2 - v_1)/(u_2 - u_1)$ (it will always be inverted)

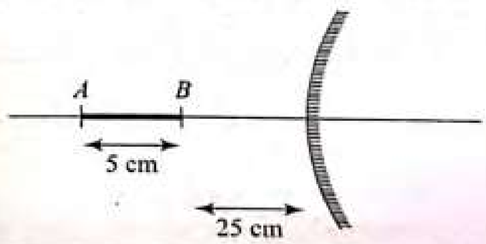
CONCEPT APPLICATION EXERCISE 26.1

1. Point S' is the image of a point source of light S in a spherical mirror whose optical axis is N_1N_2 (shown in figure). Find by construction the position of the center of the mirror and its focus.



2. The positions of optical axis N_1N_2 of a spherical mirror, the source and the image are known (as shown in figure). Find by construction the positions of the center of the mirror, its focus, and the pole for the cases
   (a) A– source, B– image;
   (b) B– source, A– image.

3. The image of a real object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the position of object and magnification.
4. When an object is placed at a distance of 25 cm from a mirror, the magnification is m_1. The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes m_2. If $m_1/m_2 = 4$, then calculate the focal length of the mirror.
5. A short linear object is placed at a distance u along the axis of a spherical mirror of focal length f.
   (a) Obtain an expression for the longitudinal magnification.
   (b) Also, obtain an expression for the ratio of the velocity of image (v) to the velocity of object (u).
6. A convex mirror of focal length 10 cm is shown in figure. A linear object AB = 5 cm is placed along the optical axis. Point B is at distance 25 cm from the pole of mirror. Calculate the size of the image of AB.



REFRACTION OF LIGHT

Deviation or bending of light rays from their original path while passing from one medium to another is called *refraction*.

It is due to change in speed of light as light passes from one medium to another medium. If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

When a light ray passes from one medium to another such that it undergoes a change in velocity, refraction takes place. Hence, wavelength of light changes, but frequency remains same.

Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum.

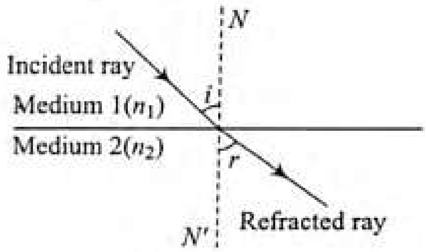
$$\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

More (less) refractive index implies less (more) speed of light in that medium, which therefore is called denser (rarer) medium.

Laws of Refraction

(a) The incident ray, the normal to any refracting surface at the point of incidence, and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

(b) $\frac{\sin i}{\sin r}$ = constant for any pair of media and for light of a given wavelength (figure). This is known as *Snell's law*.



Also, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

For applying in problems remember

$n_1 \sin i = n_2 \sin r$

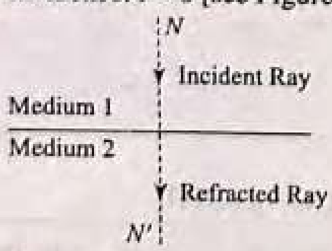
$\frac{n_2}{n_1} = {}_1n_2$ = refractive index of the second medium with respect to the first medium.

c = speed of light in air (or vacuum)

$= 3 \times 10^8 \text{ m s}^{-1}$

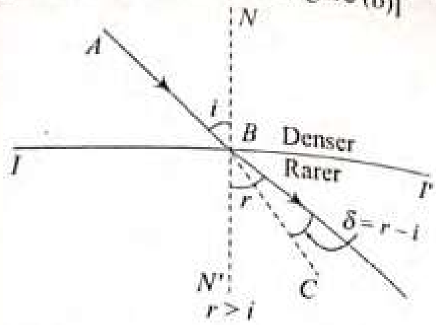
Special Cases

Case 1. Normal incidence: $i = 0$ [see Figure (a)]

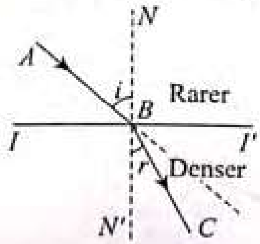


From Snell's law: $r = 0$

Case 2. When light moves from denser to rarer medium, it bends away from the normal [see Figure (b)]



Case 3. When light moves from rarer to denser medium, it bends towards the normal [see Figure (c)]



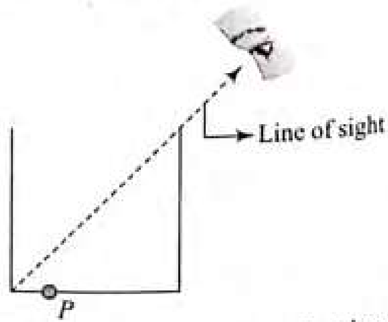
NOTE:

- Higher the value of R.I., denser (optically) is the medium.
- Frequency of light does not change during refraction.
- Refractive index of the medium relative to vacuum

$$= \sqrt{\mu_r \varepsilon_r}$$

$n_{\text{vacuum}} = 1$; $n_{\text{air}} \geq 1$; n_{water} (average value) = 4/3; n_{glass} (average value) = 3/2

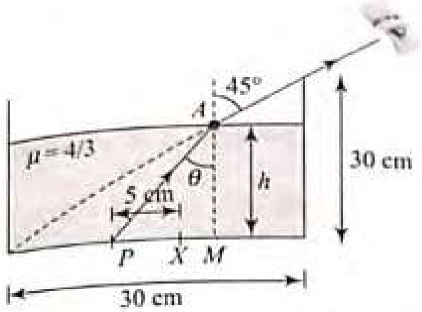
ILLUSTRATION 26.12 A cylindrical vessel, whose diameter and height both are equal to 30 cm, is placed on a horizontal surface and a small particle P is placed in it at a distance of 5.0 cm from the center. An eye is placed at a position such that the edge of the bottom is just visible. The particle P is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle P visible?



Solution. If we pour water in vessel, refraction will take place at air and water interface.

Applying Snell's law at A, we get $1 \cdot \sin 45° = \frac{4}{3} \sin \theta$

Here, θ is the angle which the incidence ray of light makes with the normal.



$$\Rightarrow \quad 1\left(\frac{1}{\sqrt{2}}\right) = \frac{4}{3}\sin\theta \quad \Rightarrow \quad \sin\theta = \frac{3}{4\sqrt{2}}$$

$$\tan\theta = \frac{3}{\sqrt{16 \times 2 - 9}} = \frac{3}{\sqrt{23}}$$

$\Rightarrow$ In ΔAPM,

$$\tan\theta = \frac{3}{\sqrt{23}} = \frac{PM}{AM} = \frac{5+x}{h} = \frac{5+h-15}{h}$$

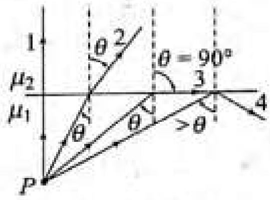
$$\frac{3}{\sqrt{23}} = \frac{h-10}{h} \quad \Rightarrow \quad 3h = h\sqrt{23} - 10\sqrt{23}$$

$$h = \left(\frac{10\sqrt{23}}{\sqrt{23}-3}\right) \text{cm}$$

Hence, water should be poured upto height $\frac{10\sqrt{23}}{\sqrt{23}-3}$ cm to make the particle 'P' visible.

CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Consider a ray of light that travels from a denser medium to rarer medium. As the angle of incidence increases in the denser medium the angle of refraction in the rarer medium increases (see figure). The angle of incidence for which the angle of refraction becomes 90° is called *critical angle*.

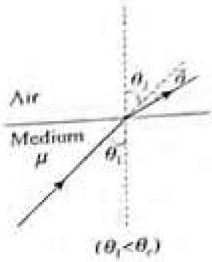


$$\frac{\sin c}{\sin 90°} = {}_2\mu_1 = \frac{\mu_1}{\mu_2} \quad \Rightarrow \quad \sin c = \frac{1}{{}_2\mu_1} = \frac{\mu_2}{\mu_1}$$

or $\quad \sin c = \dfrac{\text{R.I. of rarer medium}}{\text{R.I. of denser medium}}$

When the angle of incidence of a ray traveling from a denser medium to rarer medium is greater than the critical angle, no refraction occurs. The incident ray is totally reflected back into the same medium. Here, the laws of reflection hold good. Some light is also reflected before the critical angle is achieved, but not totally.

Deviation of ray due to refraction: The figure shows a light ray travelling from a denser to rarer medium at an angle θ_1 smaller than the critical angle θ_c. The deviation δ of the light ray is given by $\delta = \theta_2 - \theta_1$. Since $\mu_{\text{denser}} \sin\theta_1 = \mu_{\text{rarer}} \sin\theta_2$, therefore,



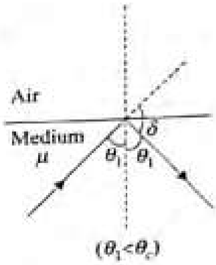
$$\sin\theta_2 - \frac{\mu_D}{\mu_R}\sin\theta_1 = \mu\sin\theta_1$$

$$\theta_2 = \sin^{-1}(\mu \sin\theta_1)$$

$$\delta = \sin^{-1}(\mu\sin\theta_1) - \theta_1$$

This is a non-linear increasing equation. The maximum value of δ occurs when $\theta_1 = \theta_c$, and is equal to

$$\delta_{\max} = \frac{\pi}{2} - \theta_c$$

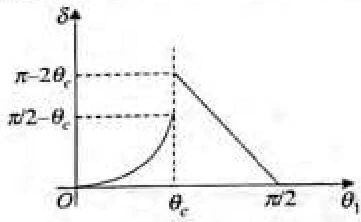


If the light is incident at angle $\theta_1 > \theta_c$, then the angle of deviation is given by, $\delta = \pi - 2\theta_1$

This is a linearly decreasing function. The maximum value of δ occurs when $\theta_1 = \theta_c$ and is equal to

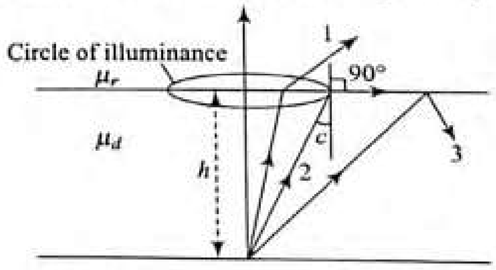
$$\delta_{\max} = \pi - 2\theta_c$$

The variation of δ with the angle of incidence θ_1 is plotted in figure.



Conditions of Total Internal Reflection

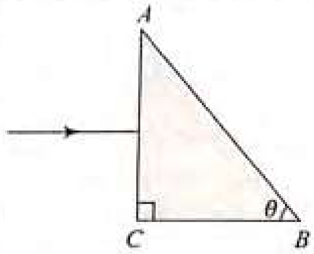
1. Light is incident on the interface from denser medium.
2. Angle of incidence should be greater than the critical angle ($i > c$). Figure shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d. Subscript r and d stand for rarer and denser media, respectively.



In figure, ray 1 strikes the surface at an angle less than critical angle c and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle greater than the critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called *circle of illuminance* (C.O.I). All light rays striking inside the circle of illuminance get refracted in the rarer medium. If an observer is in the rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in

the rarer medium and then the object cannot be seen from the rarer medium. Radius of C.O.I. can be easily found.

ILLUSTRATION 26.13 What should be the value of angle θ so that light entering normally through the surface AC of a prism ($n = 3/2$) does not cross the second refracting surface AB?



Solution. Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.

$$i = 90 - \theta$$

For the required condition:

$$i > C \Rightarrow \quad 90° - \theta > C$$

or $\quad \sin(90° - \theta) > \sin C$

or $\quad \cos\theta > \sin C = \dfrac{1}{3/2}$

$$= \frac{2}{3} \quad \text{or} \quad \theta < \cos^{-1}\frac{2}{3}$$

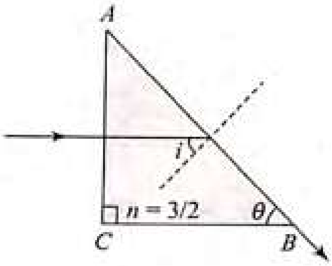
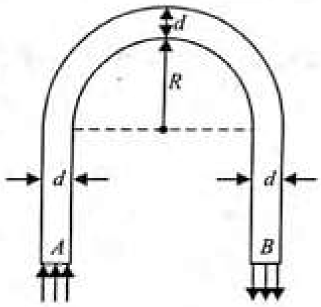


ILLUSTRATION 26.14 A glass rod having square cross-section is bent into the shape as shown in the figure. The radius of the inner semi-circle is R and width of the rod is d. Find the minimum value of d/R so that the light that enters at A will emerge at B. Refractive index of glass is $\mu = 1.5$.



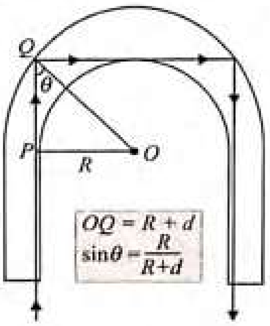
Solution. Consider the figure. If smallest angle of incidence θ is greater than critical angle then all light will emerge out at B.

$$\Rightarrow \quad \theta \geq \sin^{-1}\left(\frac{1}{\mu}\right) \quad \Rightarrow \quad \sin\theta \geq \frac{1}{\mu}$$

From figure, $\sin\theta = \dfrac{R}{R+d}$

$$\Rightarrow \quad \frac{R}{R+d} \geq \frac{1}{\mu} \quad \Rightarrow \quad \left(1 + \frac{d}{R}\right) \leq \mu$$

$$\Rightarrow \quad \frac{d}{R} \leq \mu - 1 \quad \Rightarrow \quad \left(\frac{d}{R}\right)_{max} = 0.5$$



APPARENT SHIFT OF AN OBJECT DUE TO REFRACTION

Due to bending of light at the interface of two different media, the image formed due to refraction appears at a place other than the object position. This image formation due to refraction creates illusion of shifting of the object position.

Locating the position of image formed becomes much simpler if we restrict ourselves to nearly normal incident rays.

Consider an object O in the medium (R.I = μ). After refraction, the ray at the interface bend. When the bent ray falls in our eye our eye perceives it along a straight line and it appears at I (see figure).

For nearly normal incident rays, θ_1 and θ_2 will be very small.

$$\tan\theta_1 = \sin\theta_1 = \frac{AB}{\text{object distance from the refracting surface}}$$

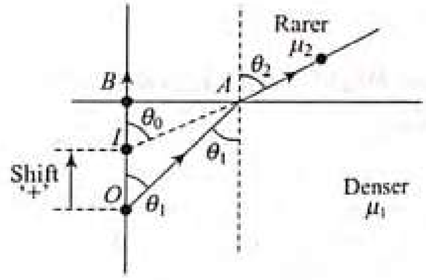
$$\tan\theta_2 = \sin\theta_2 = \frac{AB}{\text{image distance from the refracting surface}}$$

$$\Rightarrow \quad \frac{\text{Image distance from the refracting surface}}{\text{Object distance from the refracting surface}} = \frac{\sin\theta_1}{\sin\theta_2} = {}_1\mu_2 = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad \frac{\frac{AB}{OB}}{\frac{BA}{BI}} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{BI}{OB} = \frac{\text{Apprant depth}}{\text{Real depth}} = \frac{\mu_2}{\mu_1}$$

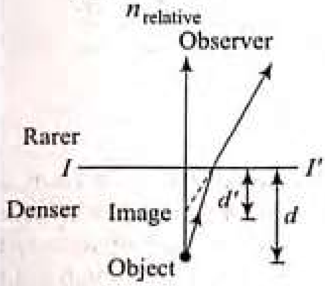
So, Shift = Real depth – Apparent depth = Real depth $\left(1 - \dfrac{\mu_2}{\mu_1}\right)$

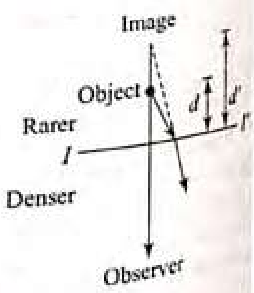


NOTE: Apparent depth and shift of submerged object

At near normal incidence (small angle of incidence i), apparent depth (d') is given by:

$$d' = \frac{d}{n_{relative}}$$





where

$$n_{relative} = \frac{n_i \text{ (R.I. of medium of incidence)}}{n_r \text{ (R.I. of medium of refraction)}}$$

d = distance of object from the interface = real depth

d' = distance of image from the interface
= apparent depth

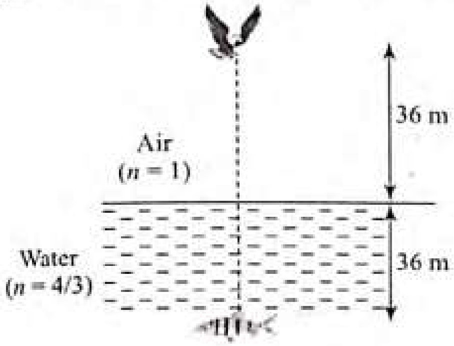
In general, we can write

$$\frac{n_i}{d} = \frac{n_r}{d'}$$

$$\text{Apparent shift} = d\left(1 - \frac{1}{n_{rel}}\right)$$

ILLUSTRATION 26.15 See figure and answer the following questions.

(a) Find apparent height of the bird.
(b) Find apparent depth of the fish.
(c) At what distance will the bird appear to the fish?
(d) At what distance will the fish appear to the bird?



Solution.

(a) Here bird is an object and fish is an observer. Hence, apparent height observed by the fish

$$d_B' = \frac{d}{n_{rel}} = \frac{d}{\left(\frac{n_{air}}{n_{water}}\right)} \Rightarrow d_B' = \frac{36}{\frac{1}{\left(\frac{4}{3}\right)}} = \frac{36}{3/4} = 48 \text{ m}$$

(b) Here the fish is an object and the bird is an observer. Hence, apparent height observed by the bird

$$d_F' = \frac{d}{n_{rel}} = \frac{d}{\left(\frac{n_{water}}{n_{air}}\right)} \Rightarrow d_F' = \frac{36}{4/3} = 27 \text{ m}$$

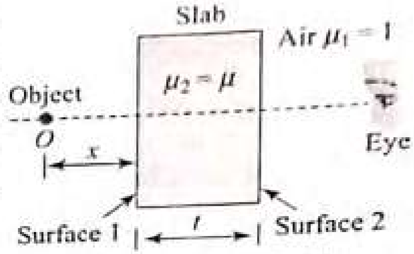
(c) For the fish, the bird will be observed at a distance d_B' from the fish: $d_B = 36 + 48 = 84$ m

(d) For the bird, the fish will be observed at a distance 'd_F' from the bird: $d_F = 27 + 36 = 63$ m

REFRACTION THROUGH A PARALLEL SLAB

A slab is formed when a medium is isolated from its surroundings by two plane surfaces parallel to each other. In this section, we will determine the position and nature of the image formed when a slab is placed in front of an object.

Consider an object O placed a distance x in front of a glass slab of thickness "t" and refractive index μ. The observer is on the other side of the slab. A ray of light from the object first refracts at surface (1) then refracts at surface (2) before reaching the observer (figure). Let us analyse the location of image as seen by observer taking one step at a time.

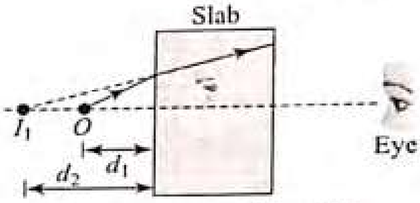


All objects viewed throgh a transparent glass slab

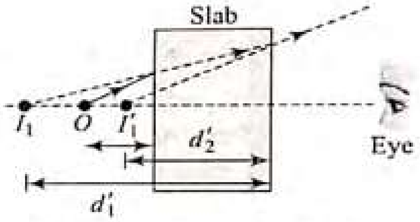
Refraction at surface 1:

Here, $\mu_1 = 1$, $\mu_2 = \mu$, d_1 = real depth, d_2 = apparent depth

$$\text{Apparent depth, } d_2 = \frac{d_{real}}{n_{relative}} = \frac{d_1}{\left(\frac{n_{incident}}{n_{refraction}}\right)} = \frac{d_1}{\left(\frac{\mu_1}{\mu_2}\right)}$$



(a) The apparent position of the object after refraction at the first surface.



(b) Final position of the object after refraction at both surfaces

$$d_2 = \frac{x}{(1/\mu)}$$

$$d_2 = \mu x \tag{i}$$

Thus, the first image is formed behind the object at the point I_1. I_1 now serves as the object for the second surface [see Figure (a)].

Refraction at surface 2:

Here, $n_{incident} = \mu_2 = \mu$

$n_{refraction} = \mu_1 = 1$

$$\text{Apparent depth, } d_2' = \frac{d_{real}}{n_{real}}$$

$$d_2' = \frac{d_1'}{\left(\frac{\mu}{1}\right)} \text{ but } d_1' = (d_2 + t)$$

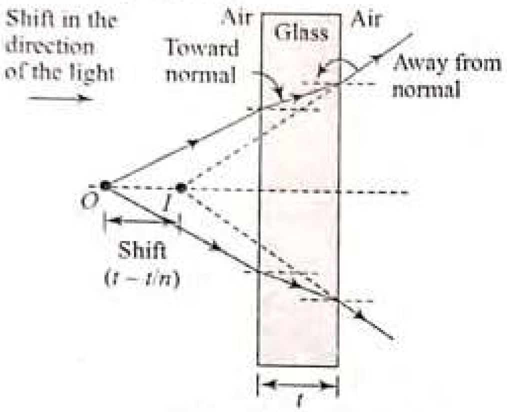
$$d_2' = \frac{(\mu x + t)}{\mu} \tag{ii}$$

Thus, the final image I_1' is at a distance $x + (t/\mu)$ behind the second interface. The original object was at a distance $x + t$ behind the second interface. Therefore, the image appears shifted by

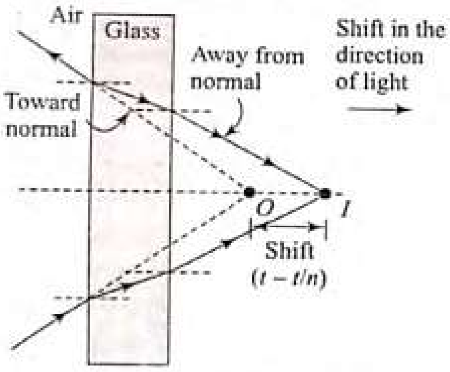
$$s = \left[x + \frac{t}{\mu}\right] - [(x+t)] = t\left[1 - \frac{1}{\mu}\right] \quad \text{(iii)}$$

We can say a slab appears to shift the object along the perpendicular to the slab by a distance $t[1-(1/\mu)]$ in the direction of the travelling ray [see Figure (b)].

Image formation by a slab:



(a) Image formed is virtual



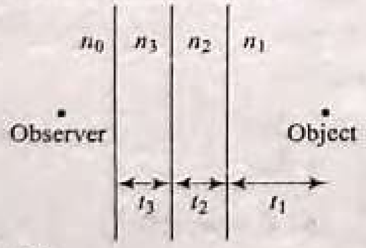
(b) Image formed is real

In both cases, shifting is in the direction of the traveling ray.

NOTE: Refraction through a composite slab (or refraction through a number of parallel media, as seen from a medium of R.I. n_0)

- Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1rel}} + \frac{t_2}{n_{2rel}} + \frac{t_3}{n_{3rel}} + \cdots + \frac{t_n}{n_{nrel}}$$

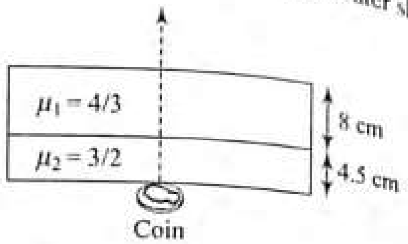


- Apparent shift

$$= t_1\left[1 - \frac{1}{n_{1rel}}\right] + t_2\left[1 - \frac{1}{n_{2rel}}\right] + \cdots + t_n\left[1 - \frac{n}{n_{nrel}}\right]$$

where t represents thickness and n represents the R.I. of the respective media, relative to the medium of observer (i.e., $n_{1rel} = n_1/n_0$, $n_{2rel} = n_2/n_0$, etc.)

ILLUSTRATION 26.16 In figure, determine the apparent shift in the position of the coin. Also, find the effective refractive index of the combination of the glass and water slab.



Solution. Total apparent shift is

$$s = t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right)$$

$$= 8\left(1 - \frac{1}{\frac{4}{3}}\right) + 4.5\left(1 - \frac{1}{\frac{3}{2}}\right) = 2 + 1.5 = 3.5 \text{ cm}$$

The apparent depth of the coin from the top is $t = (8 + 4.5) - 3.5 = 9$ cm and, the real depth of the coin is $t_1 + t_2 = 8 + 4.5 = 12.5$

Therefore, the effective refractive index is

$$\mu_{\text{eff}} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

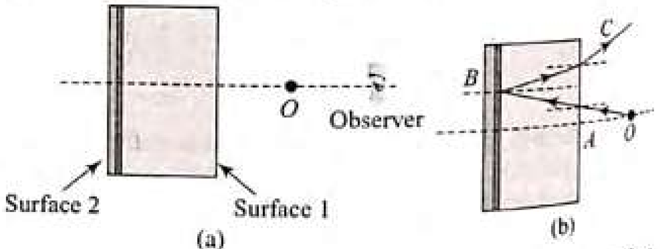
$$= \frac{t_1 + t_2}{t} = \frac{12.5}{9}$$

$$= 1.39$$

SLAB AND MIRROR COMBINED

Let us observe what happens if one surface of the slab is silvered.

Consider the silvered slab shown in Figure (a). An object is placed in front of a silvered glass slab.



Here, a ray of light from the object first refracts at surface 1. It is then reflected from surface 2 before refracting again at surface 1 and emerging [see Figure (b)]. So, we can consider a silvered slab as a combination of

1. A refracting surface,
2. A reflecting surface, and
3. A refracting surface again.

The above situation can be considered as a combination of a slab and a plane mirror placed together. Thus, a silvered slab is a combination of

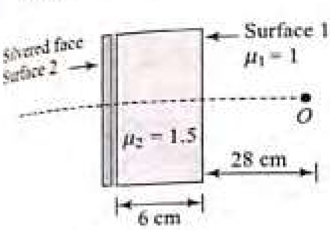
1. A glass slab,
2. A plane mirror, and
3. A glass slab again.

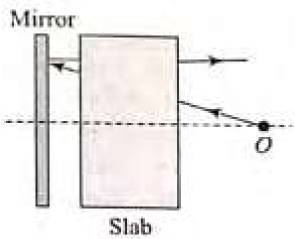
To learn the concept, we will discuss the situation through an illustration.

ILLUSTRATION 26.17 An object is placed in front of a slab ($\mu = 1.5$) of thickness 6 cm at a distance 28 cm from it. Other face of the slab is silvered. Find the position of final image.

Solution. Method of interface: A ray of light from the object O undergoes refraction, reflection and then refraction.

Refraction at surface 1:





Here, $\mu_1 = 1, \mu_2 = 1.5$

$d_1 = 28$ cm, $d_2 = ?$

Since $d_2 = \dfrac{d_1}{n_{\text{relative}}} = \dfrac{d_1}{(1/\mu)}$; $\quad \left[n_{\text{rel}} = \dfrac{n_{\text{incident}}}{n_{\text{refracted}}} = \dfrac{1}{\mu}\right]$

$\Rightarrow \quad d_2 = \mu d_1 = 1.5 \times 28 = 42$ cm

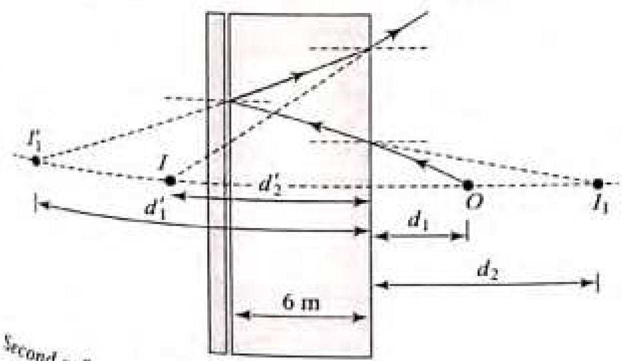
Therefore, $d_2 = 42$ cm from the first interface. The first image I_1 is formed 42 cm in front of the slab.

Reflection at surface 2:

The object for reflection at the second surface is the image from refraction at the first.

Therefore, object distance from the mirror is $= 42 + 6$
$= 48$ cm.

As a result of reflection, the image will be formed as far behind the mirror as the object is in front of it. Therefore, the second image I_1' is formed 48 cm behind the mirror.



Second refraction at surface 1:

Object distance from surface 1,

$d_1 = 48 + 6 = 54$ cm

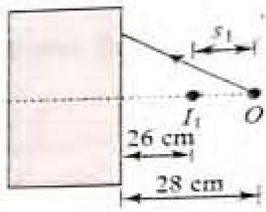
$$d_2' = \frac{d_1'}{n_{\text{relative}}} = \frac{d_1'}{\left(\dfrac{n_{\text{incident}}}{n_{\text{refraction}}}\right)} = \frac{54}{(1.5/1)} = 36 \text{ cm}$$

So, the final image is at a distance $d_2 = 36$ cm behind the first interface.

Hence, final image is formed 36 cm behind surface 1 or 30 cm behind surface 2.

Method 2: Shifting of object

A ray of light from the object first encounters a glass slab, then a mirror, and finally a glass slab again.



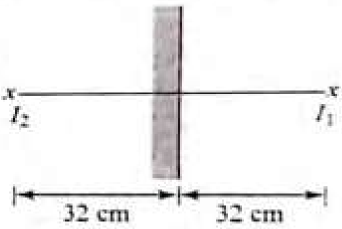
Glass slab: A slab simply shifts the object along the axis by a distance

$$s_1 = t\left(1 - \frac{1}{\mu}\right) = 2 \text{ cm}$$

Direction of shift of object is towards left. Therefore, the object appears to be at I_1 which is $28 - 2 = 26$ cm from the slab.

For mirror, the object for the mirror is the image I_1 formed after shift due to the slab.

Therefore, object distance from the mirror is $26 + 6 = 32$ cm. The image will now be formed 32 cm behind the mirror. Now, reflected rays are travelling from left to right.

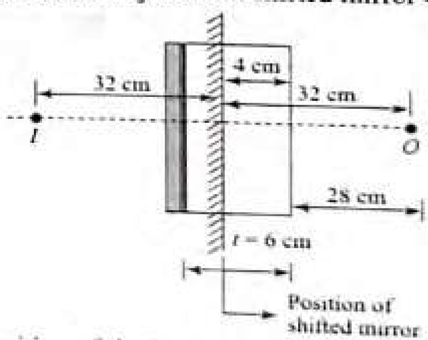


The ray now travels through the slab again but this time from right to left. Therefore, it is shifted again by a distance of 2 cm, but towards the right. Thus, final position of the image is $32 - 2 = 30$ cm behind the mirror.

Method 3: Shifting of mirror

By the principle of reversibility of light, we can say if light rays are coming from the mirror and passing through the slab, the mirror will shift 2 m towards right for observer in front of the slab.

The position of the object from shifted mirror = 32 cm.

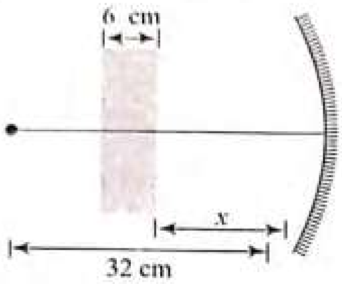


So, the position of the image formed by shifted mirror will be 32 cm behind it. Hence, the position of the image from surface 2 is 30 cm left to it and 36 cm left of surface 1.

Let us learn the combination of the slab and mirror through some more illustrations.

ILLUSTRATION 26.18 A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between the object and mirror.

Find the position of final image when the distance x shown in figure is, (a) 5 cm and (b) 20 cm.



Solution. The normal shift produced by a glass slab is,

$$S = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)(6) = 2 \text{ cm}$$

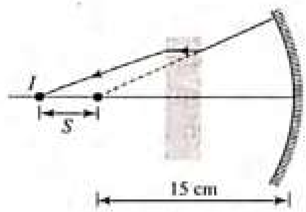
i.e., for the mirror the object is placed at a distanced $(32 - S)$ = 30 cm from it.

Applying mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

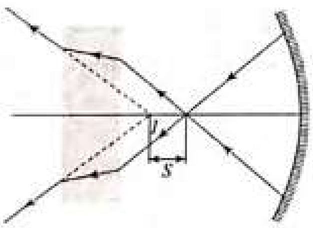
or $\quad \frac{1}{v} + \frac{1}{30} = -\frac{1}{10}$

or $\quad v = -15$ cm

(a) When $x = 5$ cm: The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance $S = 2$ cm. Hence, the final real image is formed at a distance $(15 + 2) = 17$ cm from the mirror.

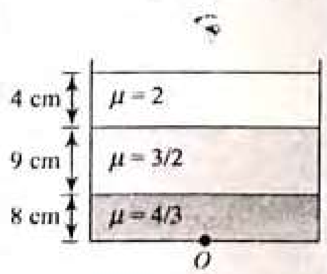


(b) When $x = 20$ cm: This time also the final image is at a distance 17 cm from the mirror but it is virtual as shown.

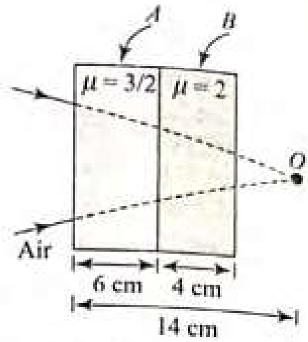


CONCEPT APPLICATION EXERCISE 26.2

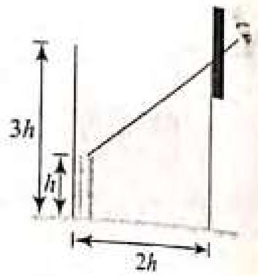
1. A tank contains three layers of immiscible liquids. The first layer is of water with refractive index 4/3 and thickness 8 cm. The second layer is of oil with refractive index 3/2 and thickness 9 cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.



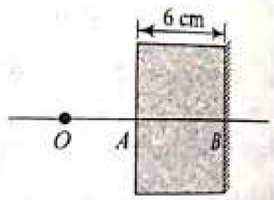
2. A convergent beam is incident on two slabs placed in contact as shown in figure. Where will the rays finally converge?



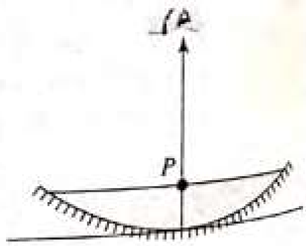
3. A observer can see through a pin hole, the top of a thin rod of height h, placed as shown in figure. The beaker's height is $3h$ and its radius is h. When the beaker is filled with a liquid upto a height $2h$, he can see the lower end of the rod. Find the refractive index of the liquid.



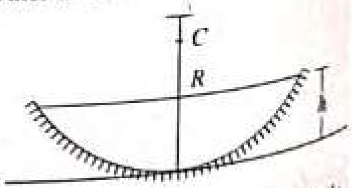
4. An object O is placed at 8 cm in front of a glass slab, whose one face is silvered as shown in figure. The thickness of the slab is 6 cm. If the image formed 10 cm behind the silvered face, find the refractive index of glass.



5. A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it up to a height of 5.00 cm. A small dust particles floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. The refractive index of water is 1.33.

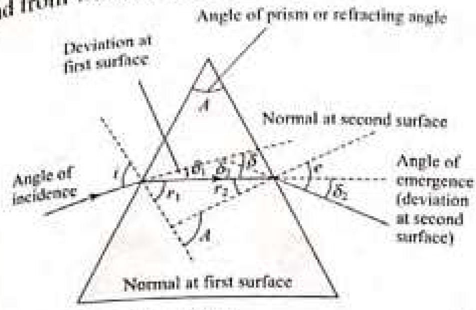


6. A concave mirror of radius R is kept on a horizontal table. Water (refractive index $= \mu$) is poured into it upto a height h. Where should an object be placed so that its image is formed on itself ?



PRISM

1. **Angle of prism or reflecting angle (A):**
The angle between the faces on which light is incident and from which it emerges (figure).



2. **Angle of deviation (δ):**
It is the angle between the emergent and the incident ray. It other words, it is the angle through which incident ray turns in passing through a prism.

$$\delta = (i - r_1) + (e - r_2) = i + e - (r_1 + r_2) = i + e - A$$

Condition of No Emergence

A ray of light incident on a prism of angle A and refractive index μ will not emerge out of a prism (whatever may be the angle of incidence) if $A > 2\theta_c$, where θ_c is the critical angle, i.e., $\mu > 1 / [\sin (A/2)]$ (see figure).

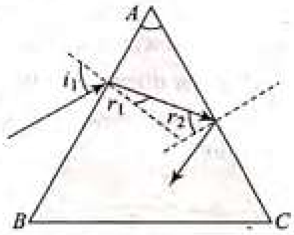
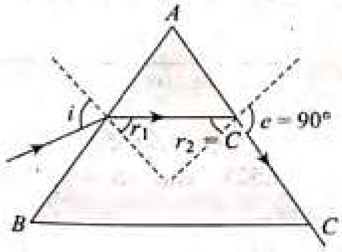


ILLUSTRATION 26.19 What should be the minimum value of refractive index of a prism, refracting angle A, so that there is no emergent ray irrespective of the angle of incidence?



Solution. If the ray just emerges from face AC,

$e = 90°$ and $r_2 = C$ (i)

From Snell's law at face AB, we have

$1 \sin i = n \sin r_1$ (ii)

$A = r_1 + r_2 = r_1 + C$ (iii)

From Eq. (ii), n is minimum when r_1 is maximum, i.e., $r_2 = C$.
In this case, $i = 90°$
From Eq. (iii),

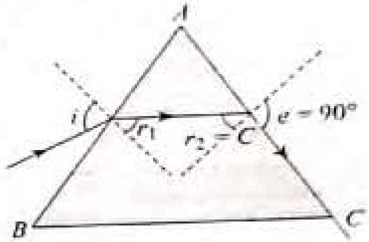
$$A = 2C \quad \text{or} \quad C = A/2$$

As $\sin C = \dfrac{1}{n} \Rightarrow \sin \dfrac{A}{2} = \dfrac{1}{n}$

$$n = \operatorname{cosec} \frac{A}{2}$$

Condition of Grazing Emergence

By the condition of grazing emergence, we mean the angle of incidence i at which the angle of emergence becomes 90° (see figure).



Consider a prism with refracting angle A and refractive index n.

The ray grazes face AC.

So, $e = 90°, r_2 = C$ (i)

and $A = r_1 + r_2 = r_1 + C$ (ii)

Also, $\sin C = 1/n$

From Snell's law at face AB, $1 \sin i = n \sin r_1$

$$\sin i = n \sin (A - C)$$
$$= n[\sin A \cos C - \cos A \sin C]$$
$$= n\left[\sin A\sqrt{1-\sin^2 C} - \cos A \sin C\right]$$
$$= \sqrt{n^2 - 1}\sin A - \cos A$$
$$i = \sin^{-1}\left[\sqrt{n^2-1}\sin A - \cos A\right]$$

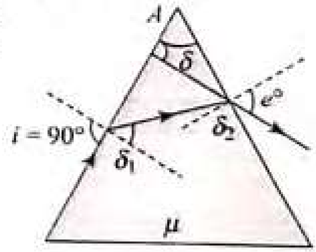
Condition of Maximum Deviation

Maximum deviation occurs when the angle of incidence is 90° (figure).

$$\delta_{max} = 90° + e - A$$

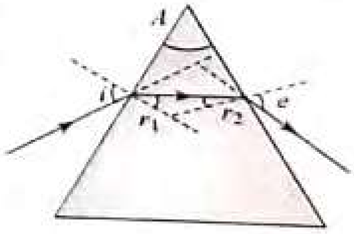
where

$$e = \sin^{-1}[\mu \sin (A - \theta_c)]$$



Condition of Minimum Deviation

The minimum deviation occurs when the angle of incidence is equal to the angle of emergence (figure).



i.e., $i = e$; $\delta_{min} = 2i - A$.

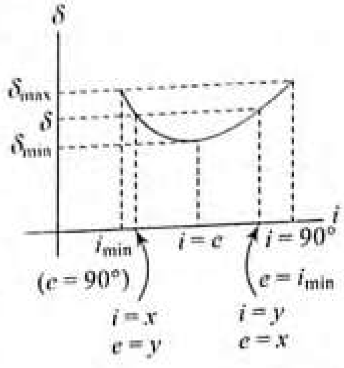
Using Snell's law, we get

$$\mu = \frac{\sin[(\delta_{min} + A)/2]}{\sin[A/2]}$$

Note that in the condition of minimum deviation the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

1. Variation of δ versus i (shown in figure).

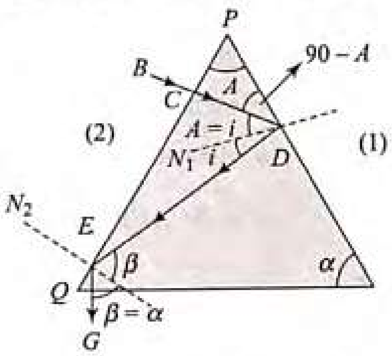
For each δ (except δ_{min}), there are two values of angle of incidence. If i and e are interchanged, then we get the same value of δ because of reversibility principle of light.



2. There is one and only one angle of incidence for which the angle of deviation is minimum.

ILLUSTRATION 26.20 The cross section of a glass prism has the form of an isosceles triangle. One of the refracting faces is silvered. A ray of light falling normally on the other refracting face, being reflected twice, emerges through the base of the prism perpendicular to it. Find the angles of the prism.

Solution The incident ray BC at normal incidence is reflected at silvered face, along DE and at E it again suffers reflection along EF. Since the ray emerges normally from the base, therefore the ray EF must fall normally on the base and emerges along EG.



We find $i = A$.

Also, $\beta = \alpha$.

Since $EN_2 \| CD$, $\beta = 2i$ (alternate angles)

$\therefore \quad \alpha = 2A \quad (\beta = \alpha, i = A)$ (i)

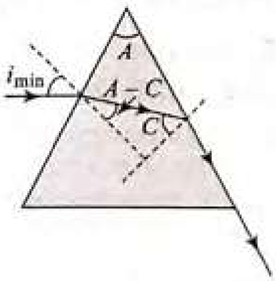
Also, $2\alpha + A = 180°$ ($\because$ Sum of angles of a triangle = 180°) (ii)

Solving Eqs. (i) and (ii), we get $A = 36°$, $\alpha = 72°$.

ILLUSTRATION 26.21 For a prism, $A = 60°$, $n = \sqrt{7/3}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface.

Also, find δ_{max}.

Solution. In minimum incidence case, the angles will be as shown in figure.



Applying Snell's law, we get

$$1 \times \sin i_{min} = \sqrt{\frac{7}{3}} \sin (A - C)$$

$$= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C)$$

$$= \sqrt{\frac{7}{3}} \left(\sin 60° \sqrt{1 - \frac{3}{7}} - \cos 60° \sqrt{\frac{3}{7}} \right) = \frac{1}{2}$$

$\therefore \quad i_{min} = 30°$

$\therefore \quad \delta_{max} = i_{min} + 90° - A = 30° + 90° - 60° = 60°$

THIN PRISMS

In thin prisms, the distance between the refracting surfaces is negligible and the angle of prism (A) is very small. Since $A = r_1 + r_2$, therefore if A is small then both r_1 and r_2 are also small, and the same is true for i_1 and i_2.

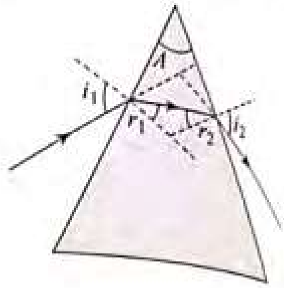
According to Snell's law,

$\sin i_1 = \mu \sin r_1$

or $\quad i_1 = \mu r_1$

$\sin i_2 = \mu \sin r_2$

or $\quad i_2 = \mu r_2$



Therefore, deviation,

$$\delta = (i_1 - r_1) + (i_2 - r_2)$$

$$\Rightarrow \quad \delta = (r_1 + r_2)(\mu - 1)$$

$$\Rightarrow \quad \delta = A(\mu - 1)$$

DISPERSION OF LIGHT

When a ray of light passes through a prism, it splits up into rays of constituent colors or wavelengths. This phenomenon is called dispersion of light.

This phenomenon arises due to the fact that refractive index varies with wavelength. It has been observed for a prism that μ decreases with the increase of wavelength, i.e., $\mu_{blue} > \mu_{red}$

Angular dispersion: $\theta = \delta_v - \delta_r$

Dispersive power: Ratio of angular dispersion to mean deviation.

$$\omega = \frac{\delta_v - \delta_r}{\delta}$$

where δ is deviation of mean ray (yellow)

As $\quad d_v = (\mu_v - 1)A$, $d_r = (\mu_r - 1)A$

$$\Rightarrow \quad \omega = \frac{\mu_v - \mu_r}{\mu_y - 1} \quad \text{where} \quad \mu_y = \frac{\mu_v - \mu_r}{2}$$

ILLUSTRATION 26.22 Calculate the dispersive power for crown glass from the given data

$\mu_v = 1.523$, and $\mu_r = 1.5145$.

Solution. Here, $\mu_v = 1.523$ and $\mu_r = 1.5145$

Mean refractive index,

$$\mu = \frac{1.523 + 1.5145}{2} = 1.51875$$

Dispersive power is given by,

$$\omega = \frac{\mu_v - \mu_r}{(\mu - 1)} = \frac{1.523 - 1.5145}{(1.51875 - 1)} = 0.1639$$

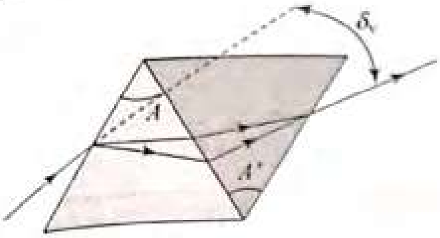
Deviation Without Dispersion

This means an achromatic combination of two prisms in which net or resultant dispersion is zero and deviation is produced.

For the two prisms,

$$(\mu_v - \mu_1)A + (\mu_v - \mu_r) A = 0$$

$$\Rightarrow \quad A' = \frac{(\mu_v - \mu_r) A}{\mu_v - \mu_r} \quad \text{and} \quad \omega\delta + \omega'\delta' = 0$$

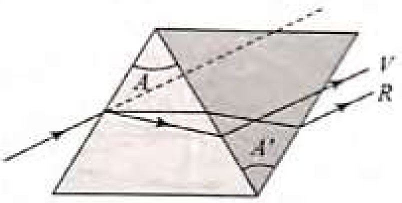


$$\delta = \delta_1\left[1 - \frac{\omega}{\omega'}\right]$$

where ω and ω' are the dispersive powers of the two prisms and δ and δ' their mean deviations.

Dispersion Without Deviation

A combination of two prisms in which deviation produced for the mean ray by the first prism is equal and opposite to that produced by the second prism is called a direct vision prism.



This combination produces dispersion without deviation.

For deviation to be zero, $(\delta + \delta') = 0$

$\Rightarrow \quad (\mu - 1)A + (\mu' - 1)A = 0$

$\Rightarrow \quad A' = \dfrac{(\mu - 1)A}{(\mu' - 1)}$

(–ve sign $\Rightarrow$ prism A' has to be kept inverted)

$\theta = \delta(\omega - \omega')$

ILLUSTRATION 26.23 A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow, and violet light are 1.514, 1.517, and 1.523, respectively, for crown glass and 1.613, 1.620, and 1.632 for flint glass.

Solution. The deviation produced by the crown prism is $\delta = (\mu - 1)A$ and by the flint prism is $\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$\delta_{net} = \delta - \delta' = (\mu - 1)A - (\mu - 1)A'. \quad \text{(i)}$$

(a) If the net deviation for the mean ray is zero,

$(\mu - 1)A = (\mu' - 1)A'$

or $\quad A' = \dfrac{(\mu - 1)}{(\mu' - 1)}A = \dfrac{1.517 - 1}{1.620 - 1} \times 5°$

(b) The angular dispersion produced by the crown prism is

$\delta_v - \delta_r = (\mu_v - \mu_r)A$

and that by the flint prism is $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is $(\delta_v - \delta_r)A - (\mu'_v - \mu'_r)A$

$= (1.523 - 1.514) \times 5° - (1.632 - 1.613) \times 4.2°$

$= -0.0348°.$

The angular dispersion has magnitude 0.0348°.

(a) If the net deviation for the mean ray is zero.

$(\mu - 1)A = (\mu' - 1)A'$

or $\quad A = \dfrac{(\mu' - 1)}{(\mu' - 1)}A = \dfrac{1.517 - 1}{1.620 - 1} \times 5°$

(b) The angular dispersion produced by the crown prism is

$\delta_v - \delta_r = (\mu_v - \mu_r)A$

and that by the flint prism is $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is

$(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A$

$= (1.523 - 1.514) \times 5° - (1.632 - 1.613) \times 4.2°$

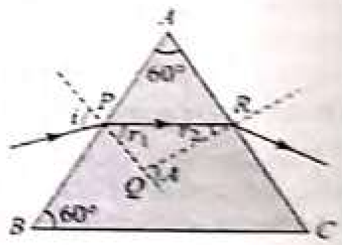
$= -0.0348°.$

The angular dispersion has magnitude 0.034°.

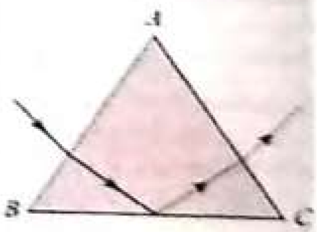
CONCEPT APPLICATION EXERCISE 26.3

1. A ray of light suffers minimum deviation through a prism of refractive index $\sqrt{2}$. What is the angle of prism if the angle of incidence is double the angle of refraction within the prism?

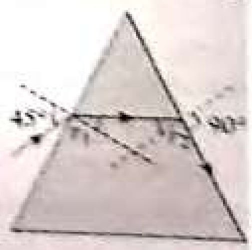
2. A ray of light undergoes a deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. What is the angle subtended by the ray inside the prism with the base of the prism?



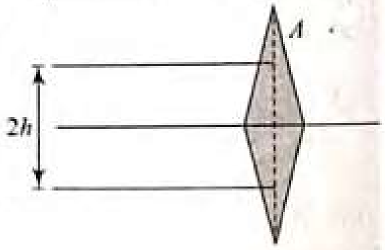
3. The path of a ray of light passing through an equilateral glass prism ABC is shown in figure. The ray of light is incident on face BC at the critical angle for just total internal reflection. The total angle of deviation after the refraction at face AC is 108°. Calculate the refractive index of the glass.



4. In an isosceles prism of angle 45°, it is found that when the angle of incidence is same as the prism angle, the emergent ray grazes the emergent surface. Find the refractive index of the material of the prism. For what angle of incidence, the angle of deviation will be minimum?

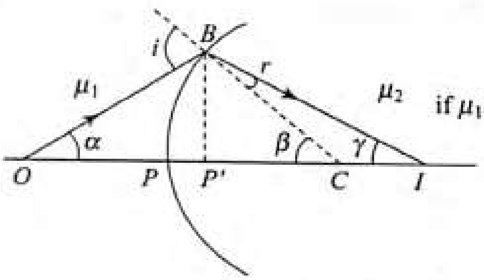


5. Refracting angle of a prism $A = 60°$ and its refractive index is $n = 3/2$. What is the angle of incidence i to get minimum deviation. Also, find the minimum deviation. Assume the surrounding medium to be air ($n = 1$).
6. Two identical thin isosceles prisms of refracting angle A and refractive index μ are placed with their bases touching each other and this system can collectively act as a crude converging lens. A parallel beam of light is incident on this system as shown in figure. Find the focal length of this so called converging lens.



REFRACTION AT SPHERICAL SURFACES

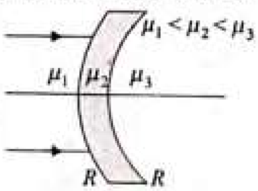
Consider the point object O placed in a medium with refractive index equal to μ_1.



Therefore, for a spherical surface, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

The symbols should be carefully remembered as: μ_2–refractive index of the medium into which light rays are entering; μ_1–refractive index of the medium from which light rays are coming. Care should also be taken while applying the sign convention to R.

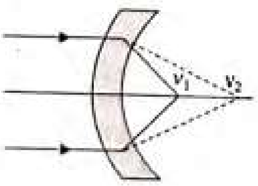
ILLUSTRATION 26.24 Find the focal length of the lens shown in figure. The radii of curvature of both the surfaces are equal to R.



Solution. For an object placed at infinity, the image after first refraction will be formed at v_1

$$\therefore \quad \frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{+R} \qquad \text{(i)}$$

The image after second refraction will be found at v_2



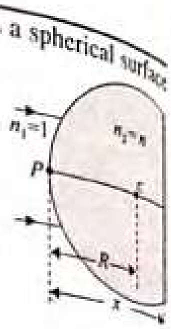
$$\therefore \quad \frac{\mu_3}{v_2} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{+R} \qquad \text{(ii)}$$

Adding (i) and (ii)

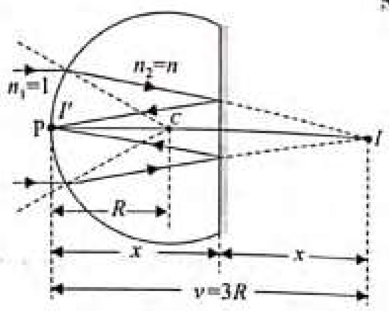
$$\frac{\mu_3}{v_2} = \frac{\mu_3 - \mu_1}{R} \quad \Rightarrow \quad v_2 = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

Therefore, focal length will be $\dfrac{\mu_3 R}{\mu_3 - \mu_1}$.

ILLUSTRATION 26.25 Shown in the figure is a spherical surface of radius of curvature R and R.I. $(=n)1.5$. Find the distance of the silvering of the plane surface (x) so as to form an image at the (P) pole due to a very far object.



Solution. Since light comes from very far object



$$u = \infty \Rightarrow \frac{n}{v} - \frac{1}{\infty} = \frac{n-1}{R} \Rightarrow v = \frac{nR}{n-1} = \frac{1.5R}{1.5-1} = 3R$$

Due to presence of the plane mirror, the image found at I behaves as a virtual object for the plane mirror & a real image I' is formed in front of the plane mirror, at the pole P.

$$\Rightarrow \quad x + x = v$$

$$\Rightarrow \quad x = \frac{v}{2} = \frac{3R}{2} \Rightarrow x = 1.5R$$

Lateral Magnification for Refracting Spherical Surface

Lateral magnification, $m = \dfrac{\text{Image height}}{\text{Object height}} = \dfrac{-(A'B')}{AB}$

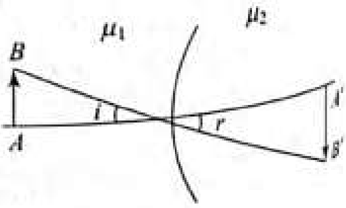
Now, $\dfrac{\sin i}{\sin r} = \dfrac{\mu_2}{\mu_1}$

For small angles of incidence and thus refraction,

$\sin i = \tan i$ and $\sin r = \tan r$

$\Rightarrow \quad \dfrac{\tan i}{\tan r} = \dfrac{\mu_2}{\mu_1}$; in triangles ABP and $A'B'P$, we have

$$\frac{AB / PA}{A'B' / P'A'} = \frac{\mu_2}{\mu_1}$$

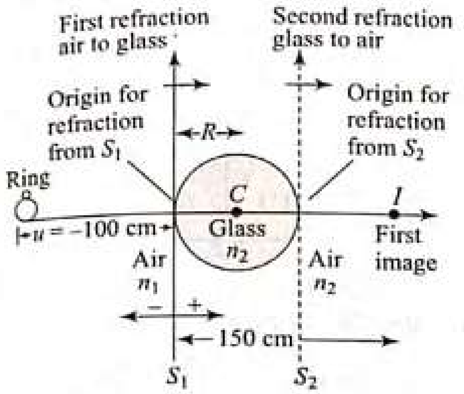


$$\Rightarrow \quad \frac{A'B'}{AB} = -\frac{\mu_1}{\mu_2}\frac{PA'}{PA}$$

$$= -\frac{\mu_1}{\mu_2}\frac{v}{(-u)} = \frac{\mu_1}{\mu_2}\frac{v}{u}$$

ILLUSTRATION 26.26 A ring of radius 1 cm is placed 1 m in front of a spherical glass ball of radius 25 cm with refractive index 1.50. Determine the position of the final image of the ring and its magnification.

Solution. Light rays from the object are refracted through the glass ball twice: first at surface S_1, from air to glass, and second, at surface S_2, from glass to air. We use paraxial approximation, so that single surface refraction equation can be used.



Refraction at first surface

$n_1 = 1$, $n_2 = 1.5$, $u = -100$ cm, $R = +25$ cm

The radius of curvature is positive because center of curvature is to the right.

Substituting these values in single surface refraction equation,

$$\frac{1.5}{v} - \frac{1}{(-100)} = \frac{(1.5-1)}{25}$$

On solving for v, we get $v = +150$ cm

The image is located 150 cm to the right of the first refracting surface. The magnification due to refraction at first surface,

$$\mu_1 = \frac{n_1 v}{n_2 u} = \frac{1(150)}{1.5(-100)} = -1$$

Refraction at second surface

For refraction at second surface, the origin of the cartesian coordinate system has to be shifted to the vertex of the second refracting surface.

The object distance for refraction at S_2 is

$u' = +(150 - 50) = 100$ cm.

This is the virtual object for S_2; the light rays converging to I_1 are refracted at S_2 before they can actually converge to form the image.

$n_1 = 1.5$, $n_2 = 1$, $u = 100$ cm, $R = -25$ cm

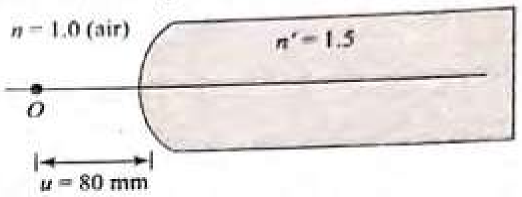
$$\frac{1}{v} - \frac{1.5}{(+100)} = \frac{(1-1.5)}{(-25)} \Rightarrow v = +200/7 \text{ cm}$$

$$\mu_2 = \frac{n_1 v}{n_2 u} = \frac{1.5(200/7)}{1(100)} = \frac{3}{7}$$

$$m = m_1 \times m_2 = -1 \times \frac{3}{7} = -\frac{3}{7}$$

CONCEPT APPLICATION EXERCISE 26.4

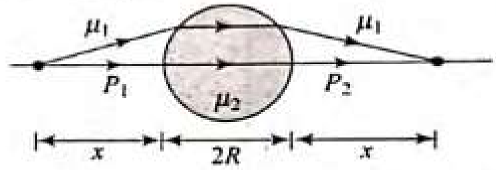
1. One end of a cylindrical glass rod shown in Figure is ground to a hemispherical surface of radius $R = 20$ mm.



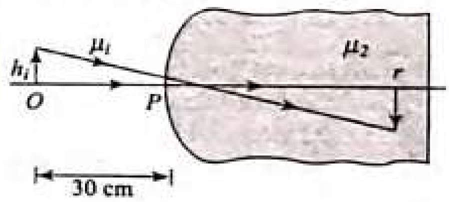
   (a) Find the image distance of a point object on the axis of the rod, 80 mm to the left of the vertex. The rod is in air.

   (b) Let the same rod be immersed in water of index 4/3, the other quantities having the same values as before. Find the image distance.

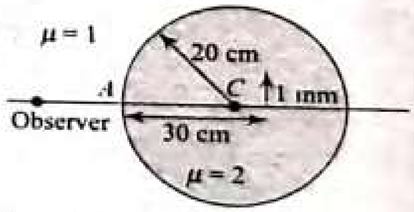
2. A sphere of radius R made of material of refractive index μ_2 is placed in a medium of refractive index μ_1. Where would an object be placed so that a real image is formed at equidistant from the sphere?



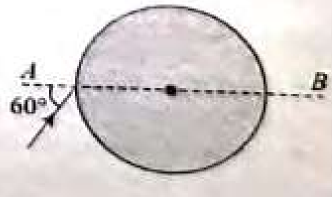
3. A small object of height 0.5 cm is placed in front of a convex surface of glass ($\mu = 1.5$) of radius of curvature 10 cm. Find the height of the image formed in glass.



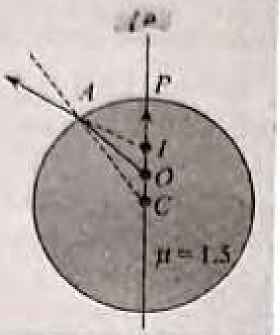
4. An object of height 1 mm is placed inside a sphere of refractive index $\mu = 2$ and radius of curvature 20 cm as shown in the figure. Find the position, size, and nature of image, for the situation shown in figure. Draw ray diagram.



5. A ray of light falls on a transparent sphere with center at C as shown in figure. The ray emerges from the sphere parallel to line AB. Find the refractive index of the sphere.

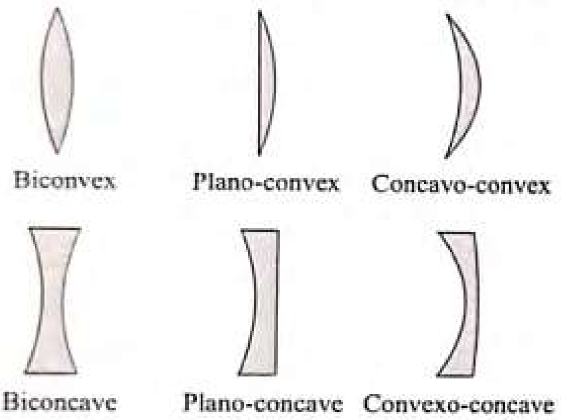


6. There is a small air bubble inside a glass sphere ($\mu = 1.5$) of radius 10 cm. The bubble is 4.0 cm below the surface and is viewed normally from the outside. Find the apparent depth of the bubble.



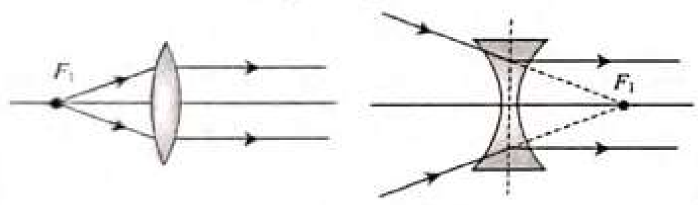
THIN LENS

A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends (see figures).

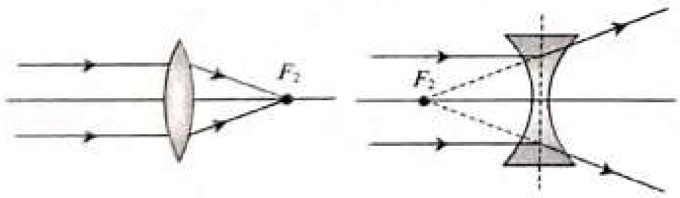


Principle focus

First principal focus

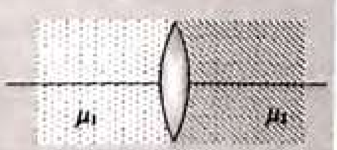


Second principal focus



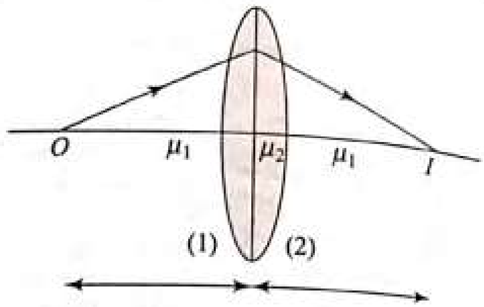
NOTE:

- Second principal focus is the principal focus of the lens.
- When medium on two sides of lens is same then $|F_1| = |F_2|$.



- If medium on two sides of lens are not same, then the ratio of two focal lengths $\dfrac{f_1}{f_2} = \dfrac{\mu_1}{\mu_2}$

Consider figure with the two refracting surfaces having radii of curvature equal to R_1 and R_2, respectively. The refractive indices of the surrounding medium and of the material of the lens are μ_1 and μ_2, respectively.



Now, using the result that we obtained for refraction at single spherical surface, we get

For the first surface, $\dfrac{\mu_2}{v_1} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R_1}$ (i)

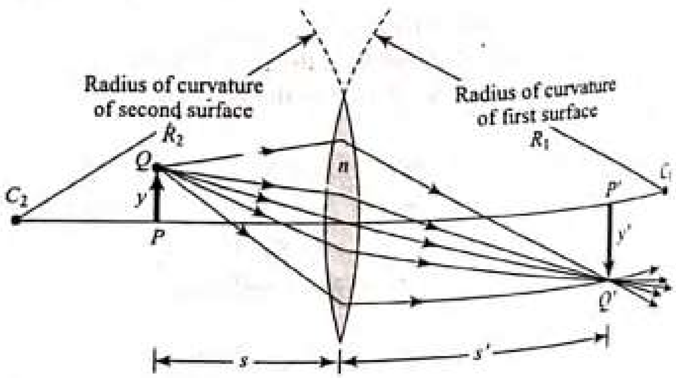
For the second surface, $\dfrac{\mu_1}{v} - \dfrac{\mu_2}{v_1} = \dfrac{\mu_1 - \mu_2}{R_2}$ (ii)

Adding Eqs. (i) and (ii), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\mu_1\left(\frac{1}{v} - \frac{1}{u}\right) = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\left(\frac{1}{v} - \frac{1}{u}\right) = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(iii)}$$



Lensmaker's Formula

In the above equation, if the object is at infinity and image is formed at the focus, then $u = \infty$ and $v = f$.

$$\Rightarrow \quad \frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(iv)}$$

Thin Lens Formula

Now comparing Eqs. (iii) and (iv), we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

For a spherical thin lens *having the same medium on both sides*:

$$\frac{1}{v}-\frac{1}{u}=(n_{rel}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{(v)}$$

where $n_{rel}=\dfrac{n_{lens}}{n_{medium}}$ and R_1 and R_2 are x-coordinates of the center of curvature of the first and second surfaces, respectively.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \text{(Lensmaker's Formula)} \quad \text{(vi)}$$

Lens has two focii:

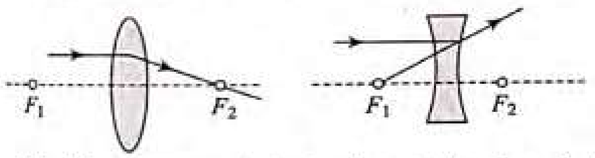
If $u=\infty$, then $\dfrac{1}{v}-\dfrac{1}{\infty}=\dfrac{1}{f} \Rightarrow v=f$

If incident rays are parallel to principal axis, then the refracted rays will cut the principal axis at f. It is called the second focus. *In case of converging lens, it is positive and in case of diverging lens it is negative.*

If $v=\infty$, that means

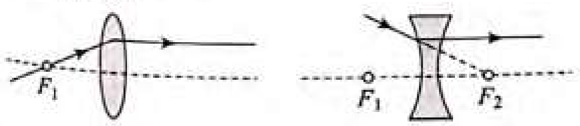
$$\frac{1}{\infty}-\frac{1}{u}=\frac{1}{f} \Rightarrow u=-f$$

1. *A ray initially parallel to the principal axis will pass (or appear to pass) through focus (figure).*



If incident rays cut principal axis at $-f$, then the refracted rays will become parallel to the principal axis. It is called the first focus. In case of converging lens, it is negative (f is positive) and in the case of diverging lens it is positive (f is negative).

2. *A ray which initially passes (or appears to pass) through focus will emerge from the lens parallel to the principal axis (figure).*



From the relation $\dfrac{1}{f}=(n_{rel}-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$, it can be seen that the second focal length depends on two factors.

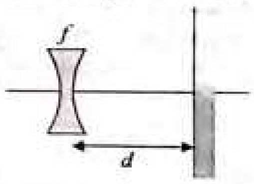
1. The factor $\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$ is
   (a) *Positive* for all types of *convex* lenses and
   (b) *Negative* for all types of *concave* lenses.
2. The factor $(n_{rel}-1)$ is
   (a) *Positive* when *surrounding medium is rarer than the medium of lens.*
   (b) *Negative* when *surrounding medium is denser than the medium of lens.*

So, a lens is *converging iff f is positive* which happens when *both the factors (a) and (b) are of same sign.*

And a lens is *diverging iff f is negative* which happens when the *factors (a) and (b) are of opposite signs.*

Equiconvex lens	**Plano convex lens**
$R_1=R$ and $R_2=-R$  $f=\dfrac{R}{2(\mu-1)}$ for $\mu=1.5, f=R$	$R_1=\infty, R_2=-R$  $f=\dfrac{R}{(\mu-1)}$ for $\mu=1.5, f=2R$
Equi concave lens	**Plano concave lens**
$R_1=-R, R_2=+R$  $f=-\dfrac{R}{2(\mu-1)}$ for $\mu=1.5, f=-R$	$R_1=\infty, R_2=R$  $f=\dfrac{R}{2(\mu-1)}$ for $\mu=1.5, f=-2R$

ILLUSTRATION 26.27 A diverging lens of focal length 10 cm is placed 10 cm in front of a plane mirror as shown in figure. Light from a very far away source falls on the lens. What is the distance of final image?

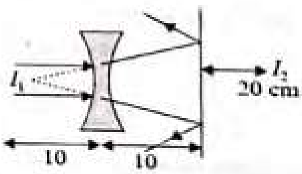


Solution $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

$$\frac{1}{v}-\frac{1}{-30}=+\frac{1}{-10}$$

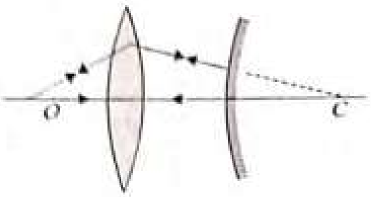
$$\Rightarrow \frac{1}{v}=\frac{-1}{10}-\frac{1}{30}=-\frac{4}{30}$$

$$\Rightarrow v=-7.5$$

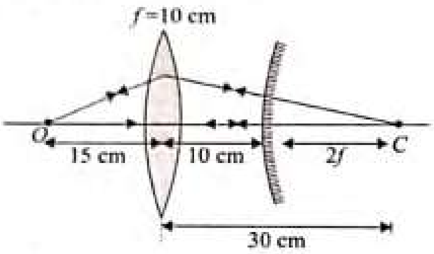


The final image distance is 2.5 cm in front of the mirror.

ILLUSTRATION 26.28 An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm on the other side of the lens, a convex mirror is placed at its focus such that the image formed by combination coincides with the object itself. Find the focal length of concave mirror.



Solution. For retracing of ray: ray must fall normally on mirror i.e., toward the center of curvature.



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f};\ u=-15$$

$$\frac{1}{v}+\frac{1}{15}=\frac{1}{10};\ f=10\text{ cm}$$

$v = 30$ cm

For mirror $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, $u = 2f$

$$\frac{1}{v}+\frac{1}{2f}=\frac{1}{f}\Rightarrow V=2f$$

Hence, from the ray diagram,

$2f = 30 - 10$

$f = 10$ cm

POWER OF A LENS

Power of a lens is defined as the reciprocal of focal length, where f is measured in meter.

$$P=\frac{1}{f}$$

The unit of power is diopter, $1\text{ D} = 1\text{ m}^{-1}$.

Sign convention: Focal length of a *converging* lens is taken as *positive* and that of a *diverging* lens is taken as *negative*.

In terms of power, the above expression may be written as $P = P_1 + P_2 + P_3 + \cdots + P_n$

ILLUSTRATION 26.29 A lens has a power of +5 diopters in air. What will be its power if completely immersed in water? (${}_a\mu_w = 4/3$ and ${}_a\mu_g = 3/2$)

Solution. Let f_a and f_w be the focal lengths of the lens in air and water, respectively, then

$$P_a=\frac{1}{f_a}\quad\text{or}\quad +5=\frac{1}{f_a}$$

$$f_a = 0.2\text{ m} = 20\text{ cm}$$

Now, $$\frac{1}{f_a}=({}_a\mu_g-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad \text{(i)}$$

and $$\frac{1}{f_w}=({}_w\mu_g-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \quad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f_w}{f_a}=\left[\frac{{}_a\mu_g-1}{{}_w\mu_g-1}\right]$$

Again, $${}_w\mu_g=\frac{{}_a\mu_g}{{}_a\mu_w}=\frac{3/2}{4/3}=\frac{9}{8}$$

$$\Rightarrow\quad \frac{f_w}{f_a}=\frac{(3/2)-1}{(9/8)-1}=\frac{(1/2)}{(1/8)}=4$$

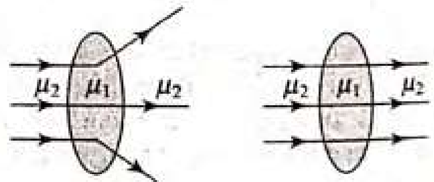
$$f_w = f_a\times 5 = 20\times 4 = 80\text{ cm} = 0.8\text{ m}$$

$$P_w=\frac{1}{f_w}=\frac{1}{0.8}=1.25\text{ diopter.}$$

CONCEPT APPLICATION EXERCISE 26.5

1. The layered lens shown in figure is made of two kinds of glasses. How many and what kind of images will be produced by this lens with a point source placed on the optical axis? Neglect the reflection of light at the boundaries between layers.

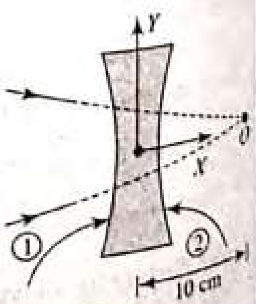
2. What is the relation between the refractive indices μ, μ_1 and μ_2 if the behavior of light rays is as shown in figure?



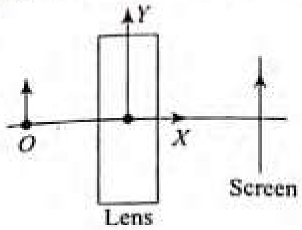
(a) (b)

3. A biconvex lens has radii of curvature 20 cm and 40 cm. The refractive index of the material of the lens is 1.5. An object is placed 40 cm in front of the lens. Calculate the position of the image.

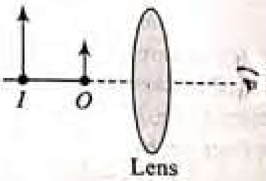
4. A converging bundle of rays is intercepted by a biconcave lens. The radii of curvature of both surfaces are 20 cm and the refractive index of the material of the lens is 1.5. If the rays originally converged to a point 10 cm in front of the lens, where will they now converge after passing through the lens?



5. A candle is placed 15 cm in front of a lens. If the image of the candle captured on a screen is magnified two times, calculate the focal length and nature of the lens.

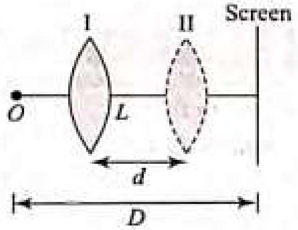


6. A man wishes to view an object through a convex lens of focal length 10 cm. The final image is to be erect and magnified 2 times. How far from the object must he hold the lens?



LENS DISPLACEMENT METHOD

Consider a convex lens L placed between an object O and a screen S. The distance between the object and the screen is D and the positions of the object and the screen are held fixed. The lens can be moved along the axis of the system and at a position I a sharp image will be formed on the screen. Interestingly, there is another position on the same axis where a sharp image will once again be obtained on the screen.



The position is marked as II in figure.

Why are there two and only two positions where a sharp image is formed?

In the figure, let the distance of position I from the object be x_1.

Then the distance of the screen from the lens is $D - x_1$. Therefore, $u = -x_1$ and $v = +(D - x_1)$.

Substituting in the lens equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{D - x_1} + \frac{1}{x_1} = \frac{1}{f} \quad \text{(i)}$$

At position II, let the distance of the lens from the screen be x_2. Then, the distance of the lens from the object is $D - x_2$. Therefore, $u = -x_2$ and $v = +(D - x_2)$.

Substituting in the lens equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{D - x_2} + \frac{1}{x_2} = \frac{1}{f} \quad \text{(ii)}$$

Comparing Eqs. (i) and (ii), we realize that there are only two solutions:

1. $x_1 = x_2$; or
2. $D - x_1 = x_2$ and $D - x_2 = x_1$

The first solution is trivial. Therefore, if the first position of the lens, for a sharp image, is x_1 from the object, the second position is at $D - x_1$ from the object.

Let the distance between the two positions I and II be d. From the diagram, it is clear that

$$D = x_1 + x_2 \quad \text{and} \quad d = x_2 - x_1 \quad \text{(iii)}$$

Solving the two equations in (iii), we obtain

$$x_1 = \frac{D - d}{2} \quad \text{and} \quad D - x_1 = \frac{D + d}{2} \quad \text{(iv)}$$

Substituting Eq. (iv) in Eq. (i), we get

$$\frac{1}{f} = \frac{2}{D - d} + \frac{2}{D + d}$$

or $$f = \frac{D^2 - d^2}{4D} \quad \text{(v)}$$

Also, $$d = \sqrt{D^2 - 4Df} = \sqrt{D(D - 4f)} \quad \text{(vi)}$$

We notice from Eq. (vi) that a solution for d is possible only when $D \geq 4f$.

When $D < 4f$, there is no position for which a sharp image can be formed.

When $D = 4f$, there is only one position where a sharp image is formed.

When $D > 4f$, there are two positions where a sharp image is formed.

The method is suitable for convex lenses only.

What can we say about the size of the object when we know the size of the images?

The magnifications in the first and second positions are

$$m_1 = \frac{h_{i,1}}{h_0} = \frac{D - x_1}{x_1} = \frac{D + d}{D - d}$$

$$m_2 = \frac{h_{i,2}}{h_0} = \frac{D - x_2}{x_2} = \frac{D - d}{D + d}$$

Thus, in the first position the image is magnified while in the second it is reduced. The product of the two magnifications is equal to unity, i.e.,

$$m_1 \times m_2 = \frac{h_{i,1} \times h_{i,2}}{h_0^2} = 1 \quad \text{or} \quad h_0 = \sqrt{h_{i,1} \times h_{i,2}} \quad \text{(vii)}$$

The displacement method is not suitable for concave lenses.

ILLUSTRATION 26.30 (a) A screen is kept at a distance of 1 m from the object. A converging lens between the object and the screen, when placed at any of the two positions which are 60 cm apart, forms a sharp image of the object on the screen. Find the focal length of the lens.

(b) In the two positions of the lens, lateral size of the image is 4 cm and 9 cm. Find the size of the object.

Solution. This problem is based on 'Displacement method' which is commonly used to determine the focal length of a converging lens.

(a) As discussed earlier,

$$f = \frac{D^2 - x^2}{4D}$$

where D is the distance between the object and the screen. x is the distance between the two positions of lens for which sharp images of the given object are obtained on the screen.

Here, $D = 1$ m $= 100$ cm

$x = 60$ cm

$$f = \frac{(100)^2 - (60)^2}{4 \times 100}$$

$f = 16$ cm

(b) Size of the object can be obtained from the relation

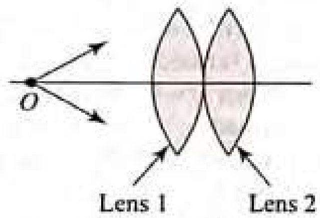
$$O = \sqrt{I_1 I_2}$$

I_1, I_2 = lateral size of image in the two positions of lens

$$O = \sqrt{4 \times 9} = 6 \text{ cm}$$

Lenses Placed Very Close to Each Other

Consider the arrangement shown in figure. Let us assume that f_1 and f_2 are the focal lengths of the individual lenses, respectively (note that the lenses themselves may be convex or concave and this is captured in the signs of f_1 and f_2). We are now interested in finding out the net focal length of the system and position of image for various positions of the object.



A system of two lense placed close together

Let v_1 be the position of the image formed by the first lens. The lens formula for the first lens gives

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_2} \quad \text{(i)}$$

The image of the first lens now serves as the object for the second lens. If the image after the second lens if formed at v, then for the second lens, we have

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \text{(ii)}$$

Adding Eqs. (i) and (ii), we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{(iii)}$$

We can, therefore, see that the system behaves as a single lens with equivalent focal length "f_e" given by

$$\frac{1}{f_e} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{(iv)}$$

And we can use the same lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_e} \quad \text{(v)}$$

What can we say about the magnification of the system?

Let m_1 be the magnification of the first lens and m_2 be the magnification of the second lens. Let the height of the object be h_0, the height of the first image be h_1, and the height of the final image be h_2. Therefore,

$$m_1 = \frac{h_1}{h_0} \quad \text{(vi)}$$

$$m_2 = \frac{h_2}{h_1} \quad \text{(vii)}$$

And the net magnification

$$m = \frac{h_2}{h_0} = \frac{h_2}{h_1} \times \frac{h_1}{h_0} = m_1 m_2 \quad \text{(viii)}$$

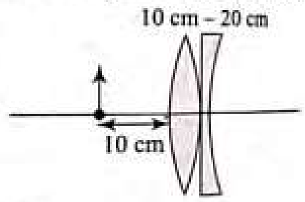
To summarize: we can say that if a set of thin lenses are placed next to each other such that the distance between the lenses is zero, then the net effective focal length of the system f_{eff} is given by

$$\frac{1}{f_{eff}} = \sum \frac{1}{f_i}$$

where f_i is the focal length of every lens and the net magnification is given by

$$m_{eff} = m_1 \times m_2 \times m_3 \times \cdots = \prod_{i=1}^{n} m_i \quad \text{(ix)}$$

ILLUSTRATION 26.31 Find the lateral magnification produced by the combination of lenses shown in figure.



Solution. If lenses are placed in contact, the focal length of single equivalent length can be given as

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$$

$\Rightarrow \quad f = +20$ cm

Now using lens formula considering system of lenses as a single lens, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\therefore \quad \frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{10} = \frac{-1}{20} \Rightarrow v = -20 \text{ cm}$$

Now using magnification formula, we get

$$m = \frac{-20}{-10} = 2$$

Lens with One Silvered Surface

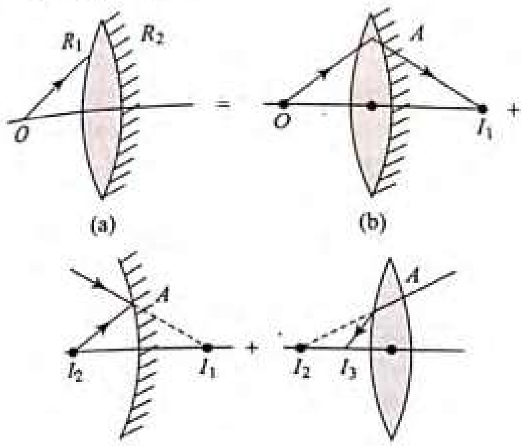
If the back surface of a lens is silvered and an object is placed in front of it then:

1. First, light will pass through the lens and it will form the image I_1.

2. Image I_1 will act as an object for silvered surface which acts as curved mirror and forms image I_2 of object I_1.
3. The light after reflection from silvered surface will again pass through the lens and lens will form final image I_3 of object I_2.

This is shown in figure. In such a situation, power of the silvered lens will be

$$P = P_L + P_M + P_L$$



(a) (b)

(c) (d)

With $P_L = \dfrac{1}{f_L}$, where $\dfrac{1}{f_L} = (\mu - 1)\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

And $P_M = -\dfrac{1}{f_M}$, where $f_M = \dfrac{R_2}{2}$

So, the system will behave as a curved mirror of focal length F given by $F = -1/P$.

Illustration 26.32 The radius of curvature of the convex face of a plano-convex lens is 12 cm and its refractive index is 1.5.

(a) Find the focal length of this lens. The plane surface of the lens is now silvered.

(b) At what distance from the lens will parallel rays incident on the convex face converge?

(c) Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens.

(d) Calculate the image distance when the object is placed as in (c).

Solution.

(a) As for a lens, by lensmaker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here $\mu = 1.5$; $R_1 = 12$ cm; and $R_2 = \infty$

So, $\dfrac{1}{f} = (1.5 - 1)\left[\dfrac{1}{12} - \dfrac{1}{-\infty}\right]$ i.e., $f = 24$ cm

i.e., the lens is convergent with focal length 24 cm.

(b) As light after passing through the lens will be incident on the mirror which will reflect it back through the lens again, so

$$P = P_L + P_M + P_L = 2P_L + P_M$$

But $P_L = \dfrac{1}{f_L} = \dfrac{1}{0.24}$

and $P_M = -\dfrac{1}{\infty} = 0$ $\left[\text{as } f_M = \dfrac{R}{2} = \infty\right]$

So, $P = 2 \times \dfrac{1}{0.24} + 0 = \dfrac{1}{0.12}$ D

The system is equivalent to a concave mirror of focal length F.

$$P = -\frac{1}{F}$$

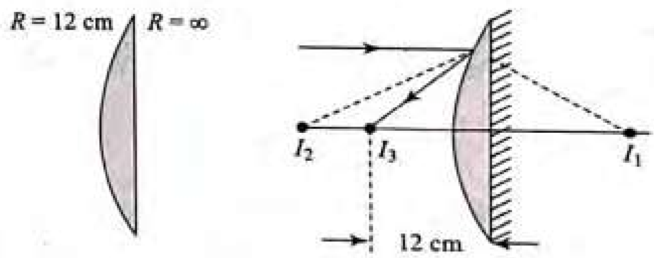
i.e., $F = -\dfrac{1}{P} = -0.12 \text{ m} = -12 \text{ cm}$

i.e., the system will behave as a concave mirror of focal length 12 cm. So, as for parallel incident rays $u = -\infty$, from mirror formula $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$, we have

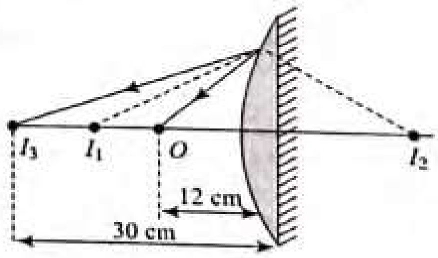
$$\frac{1}{v} + \frac{1}{-\infty} = \frac{1}{-12} \Rightarrow v = -12 \text{ cm}$$

i.e., parallel incident rays will focus at a distance of 12 cm in front of the lens as shown in Figures (b), (c) and (d) When object is at 20 cm in front of the given silvered lens, which behaves as a concave mirror of focal length 12 cm, from mirror formula $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$, we have

$$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-12} \Rightarrow v = -30 \text{ cm}$$



(a) (b)



(c)

i.e., the silvered lens will form an image at a distance of 30 cm in front of it as shown in Figure (c).

COMBINATION OF LENSES AND MIRRORS

When several lenses or mirrors are used coaxially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens (or mirror) acts as an object for the third, and so on. The total magnification in such situations will be given by

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \cdots, \quad \text{i.e.,} \quad m = m_1 \times m_2 \times \cdots$$

Here, is it worthy to note that

1. If two thin lenses of equal focal length but of opposite nature (i.e., one convergent and other divergent) are put in contact, the resultant focal length of the combination will be

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{-f} = 0, \quad \text{i.e.,} \quad F = \infty \text{ and } P = 0$$

i.e., the system will behave as a plane glass plate.

2. If two thin lenses of the same nature are put in contact, then as

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{F} > \frac{1}{f_1} \quad \text{and} \quad \frac{1}{F} > \frac{1}{f_2}$$

i.e., $F < f_1$ and $F < f_2$

i.e., the resultant focal length will be lesser than the smallest individual.

3. If two thin lenses of opposite nature with different focal lengths are put in contact, the resultant focal length will be of the same nature as that of the lens of the shorter focal length but its magnitude will be more than that of the shorter focal length.

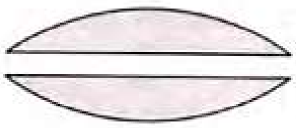
4. If a lens of focal length f is divided into two equal parts as shown in Figure (a) and each part has focal length f' then as

$$\frac{1}{f} = \frac{1}{f'} + \frac{1}{f'}, \quad \text{i.e.,} \quad f' = 2f$$

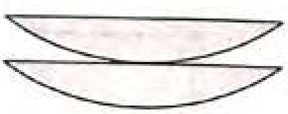
i.e., each part will have focal length $2f$.

Now, if these parts are put in contact as in Figure (b) or (c), the resultant focal length of the combination will be

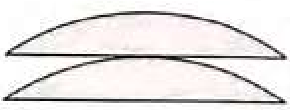
$$\frac{1}{F} = \frac{1}{2f} + \frac{1}{2f}, \quad \text{i.e.,} \quad F = f (= \text{initial value})$$



(a)



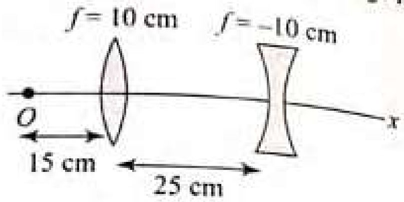
(b)



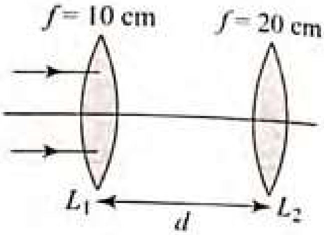
(c)

CONCEPT APPLICATION EXERCISE 26.6

1. In figure, find the position of final image formed.

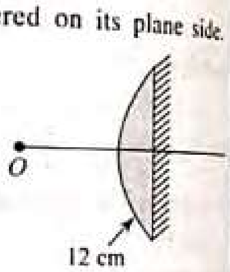


2. Figure shows two converging lenses. Incident rays are parallel to the principal axis. What should be the value of d so that final rays are also parallel?



Here, the diameter of ray beam becomes wider.

3. A plano-convex lens is silvered on its plane side. The radius of curvature of the other face is 12 cm and the refractive index of the material of the lens is 1.5. An object is placed 24 cm in front of the silvered lens. Where will the image be formed?



4. A convex lens of focal length 10 cm is placed 30 cm in front of a second convex lens also of the same focal length. A plane mirror is placed after the two lenses. Where should a point object be placed in front of the first lens so that it images on to itself?

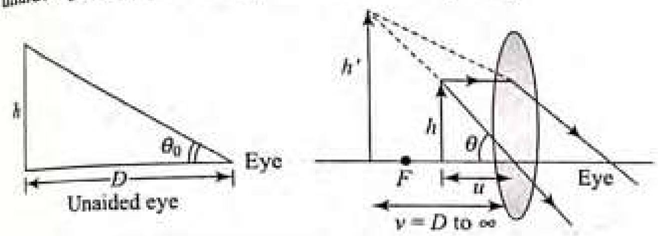
5. A concave mirror of focal length 30 cm is placed on the flat horizontal surface with its concave side up. Water with refractive index 1.33 is poured into the lens. Where should an object be placed if its image is to be captured on a screen with a magnification of 2?

6. The convex side of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature of 20 cm. The concave surface has a radius of curvature of 60 cm. What is the focal length of the lens? The convex side is silvered and placed on a horizontal surface. What is the effective focal length of the silvered lens? The concave part is filled with water with refractive index 1.33. What is the effective focal length of the combined glass and water lens? If the convex side is silvered, what is the new effective focal length of the silvered compound lens?

OPTICAL INSTRUMENTS

Simple Microscope or Magnifier or Magnifying Glass

A converging lens used for the above mentioned purpose is termed as magnifier or simple microscope. The magnifier forms a virtual image of the object and an eye looks at this virtual image, i.e., we can say the object is placed in between the focus and optical center of lens and the eye close to it on the other side. Since a normal eye can focus sharply on an object anywhere between the near point and infinity, the image can be seen equally clearly if it is formed anywhere within this range. Magnifying power or angular magnification of an optical instrument is defined as the ratio of visual angle with instrument to the maximum visual angle for clear vision when the eye is unaided [i.e., when the object is at the near point].



$$\gamma = \text{Magnifying power} = \frac{\tan\theta}{\tan\theta_0}$$

where θ and θ_0 are clearly shown in figure.

$$\tan\theta = \frac{h'}{v},$$

where $v = D$ to ∞ and D is the least distance of distinct vision.

$$\tan\theta_0 = \frac{h}{D}$$

$$\gamma = \frac{h'}{v} \times \frac{D}{h},$$

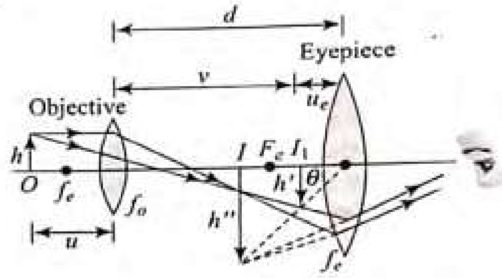
where $\frac{h'}{v} = \frac{h}{u}$, u being the object distance from the lens.

For $v = D$, $\gamma = \left[1 + \frac{D}{f}\right]$. In this situation, the magnifying power is maximum and eye is under maximum strain. For $v = \infty$, $\gamma = \frac{D}{f}$. In this situation, magnifying power is minimum and eye is least strained.

Compound Microscope

When an angular magnification higher than that attainable with a simple magnifier is desired, it is necessary to use a compound microscope, simply termed as a microscope.

It consists of two convergent lenses of short focal lengths and apertures arranged coaxially. One is termed as objective while the other is eyepiece or ocular; an object is placed near to objective while an eyepiece is arranged in such a way that it is facing the eye. The objective has smaller aperture and focal length as compared to the eyepiece. The separation between the objective and eyepiece can be varied. Construction and ray diagram outlining the basic principle of a microscope is shown in figure.



Object O is placed just beyond the focus of the objective, whose image I_1 is formed by the objective within the focus and optical center of eye piece.

Image I_1 is real, inverted, and enlarged image w.r.t. object O. The eyepiece then forms a virtual image of the object between D and ∞. This final image is inverted, virtual, and enlarged w.r.t. object O.

Angular magnification or magnifying power of the instrument is

$$\gamma = \frac{\tan\theta}{\tan\theta_0}$$

$$\tan\theta = \frac{h'}{u_e} \quad \text{and} \quad \tan\theta_0 = \frac{h}{D}$$

So, $$\gamma = \frac{h'}{h} \times \frac{D}{u_e} = m\gamma_e = -\frac{v}{u} \times \gamma_e$$

where m is the lateral magnification produced by the objective and γ_e is the angular magnification produced by the eyepiece.

1. When final image is formed at the least distance of distinct vision (D), the eye is most strained and
$$\gamma = -\frac{v}{u}\left[1 + \frac{D}{f_e}\right]$$
with $L = v + \frac{f_e D}{f_e + D}$ γ is maximum in this mode.
2. In general, as f_0 is small and object is close to the first objective focus, i.e., $u \simeq f_0$ and L; v as $u_e \ll v$, so for normal adjustment [i.e., when the final image is at for point] $\gamma \simeq \frac{L}{f_0} \times \frac{D}{f_e}$
3. In this case, γ is –ve. Hence, the final image is inverted.
4. In normal adjustment, if eyepiece and objective lenses are interchanged, the angular magnification remains unchanged.
5. Due to small apertures of lenses, spherical aberration is decreased to a great extent.
6. F_0 is taken smaller than f_e to increase the field of view and to increase brightness of the image.

7. With respect to microscope, the minimum distance between two lines at which they are just distinct is called limit of resolution (Δx) and the reciprocal of limit of resolution is called resolving power.

$$\text{Resolving power} = \frac{1}{\Delta x} \propto \frac{1}{\lambda}$$

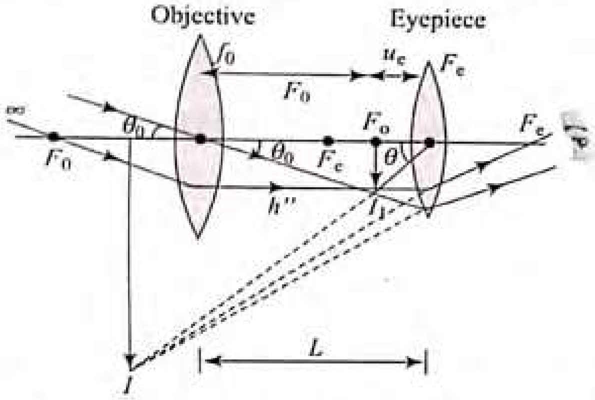
Telescopes

Refracting Telescope

The optical system of a refracting telescope is essentially the same as that of a compound microscope. In both instruments, the image formed by the objective is viewed through an ocular (eyepiece); the difference is that the telescope is used to examine large objects at large distances while microscope is used to examine small objects close at hands.

Astronomical Telescope

The working diagram of astronomical telescope is as shown in figure.

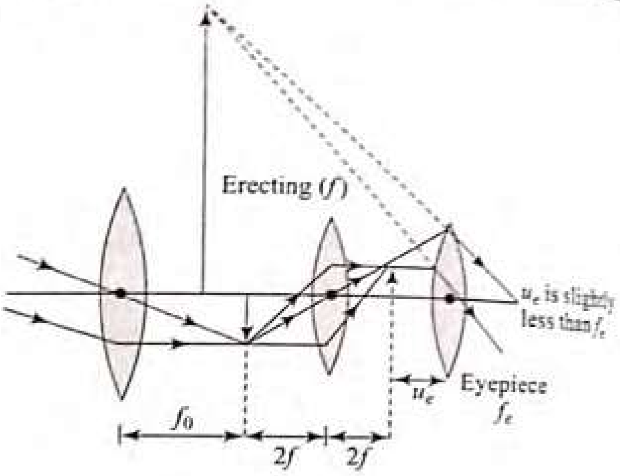


1. While seeing the object through telescope, the object is between ∞ and $2F$ of the objective which forms real, inverted, and diminished image within the focus and optical center of eyepiece which then forms its virtual, erect (w.r.t. intermediate image), and magnified image at a distance between D to ∞ from the eye. This means the final image is inverted (w.r.t. object) at a distance between D to ∞ from the eye.
2. In a telescope, the aperture and focal length of objective are greater than that of eyepiece.
3. Magnifying power, $\gamma = \dfrac{\tan\theta}{\tan\theta_0} = -\dfrac{h''/u_e}{h''/f_0} = -\dfrac{f_0}{u_e}$
4. In general case, the final image is formed at ∞ and hence $\gamma = -\dfrac{f_0}{f_e}$. In this case, the eye is least strained and magnification power is minimum with length of tube, $L = f_0 + f_e$.
5. In normal adjustment (i.e., final image is at ∞), to have large magnifying power, f_0 must be as large as practically possible while f_e has to be kept small.
6. Resolving power of a telescope depends on the aperture of objective and wavelength of light.

$$\text{RP} \propto \frac{\text{Aperture of objective}}{\text{Wavelength}}$$

Terrestrial Telescope

While an inverted image is not a disadvantage if the instrument is to be used for astronomical observation, it is desirable that a terrestrial telescope shall form an erect image. This can be accomplished by using an erecting lens in the astronomical telescope (figure).



Magnifying power for terrestrial telescope is,

$$\gamma_e = \frac{f_0}{f_e} \qquad \text{[for relaxed eye]}$$

Length of tube, $L_t = f_0 + 4f + f_e$

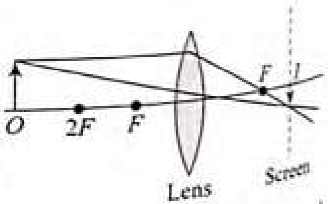
Reflecting Telescope

It is an astronomical telescope, in which an objective lens is replaced by a concave mirror.

The reflecting telescope is cheap, light, and portable as compared to the astronomical telescope.

Lens Camera

In lens camera, the image of the object formed by a converging lens is allowed to fall on a screen (figure). In camera, the aperture of lens and its separation from film/screen can be adjusted. Generally, the object is placed between F and $2F$ and hence the image forms between F an $2F$ which is real (hence can be taken on film), inverted, and diminished.



During photographing of the object, the first image is focused on the film by adjusting the separation between film and lens. After this, the film is exposed to light through a shutter for a given time. How much energy is incident on the film also depends on the aperture of the lens.

For proper exposure of a particular film, a definite amount of energy is incident on the film. Let the film has been exposed for time t and the intensity of light is I, then $I \times A \times t =$ constant, where A is light transmitting area of the lens.

$A \propto d^2$, where d is the aperture of lens.

So, $Id^2 I \simeq$ constant

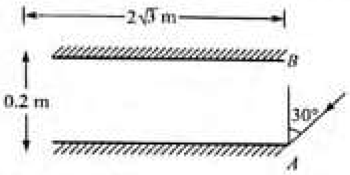
The ratio of focal length to aperture of lens is called f-number of the camera.

$$f\text{-number} = \frac{\text{Focal length}}{\text{Aperture}}$$

For a given camera, f-number gives us the idea about aperture.

SOLVED EXAMPLES

1. Two plane mirrors. A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle of 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is

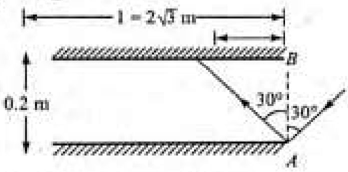


(a) 28 (b) 30
(c) 32 (d) 34

Sol. (b) From the following ray diagram

$$d = 0.2 \tan 30° = \frac{0.2}{\sqrt{3}}$$

$$\Rightarrow \frac{l}{d} = \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$$



Therefore, maximum number of reflections are 30.

2. A square of side 3 cm is placed at a distance of 25 cm from a concave mirror of focal length 10 cm. The centre of the square is at the axis of the mirror and the plane is normal to the axis. The area enclosed by the image of the square is

(a) 4 cm^2 (b) 6 cm^2
(c) 16 cm^2 (d) 36 cm^2

Sol. (a) $m = \frac{I}{O} = \frac{f}{u-f} = \frac{10}{25-10} = \frac{10}{15} = \frac{2}{3}$

$$m^2 = \frac{A_i}{A_o} \Rightarrow A_i = m^2 \times A_o = \left(\frac{2}{3}\right)^2 \times (3)^2 = 4\,\text{cm}^2$$

3. A concave mirror is placed at the bottom of an empty tank with face upwards and axis vertical. When sunlight falls normally on the mirror, it is focussed at distance of 32 cm from the mirror. If the tank filled with water $\left(\mu = \frac{4}{3}\right)$ up to a height of 20 cm, then the sunlight will now get focussed at

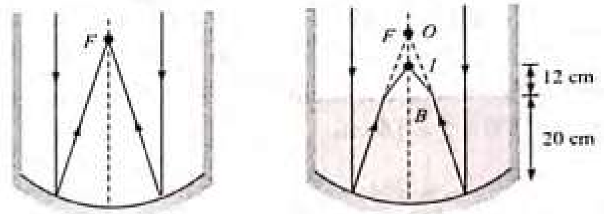
(a) 16 cm above water level
(b) 9 cm above water level
(c) 24 cm below water level
(d) 9 cm below water level

Sol. (b) Sun is at infinity i.e. $u = \infty$ so from mirror formula we have $\frac{1}{f} = \frac{1}{-32} + \frac{1}{(-\infty)} \Rightarrow f = -32\text{cm}$.

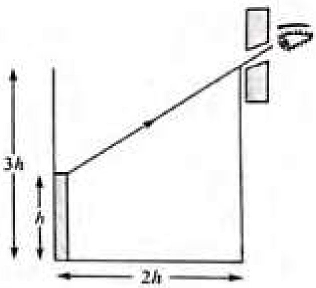
When water is filled in the tank upto a height of 20 cm, the image formed by the mirror will act as virtual object for water surface. Which will form it's image at I such that

$$\frac{\text{Actual height}}{\text{Apperant height}} = \frac{\mu_w}{\mu_a}, \text{ i.e., } \frac{BO}{BI} = \frac{4/3}{1}$$

$$\Rightarrow BI = BO \times \frac{3}{4} = 12 \times \frac{3}{4} = \text{cm}.$$



4. An observer can see through a pinhole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is $3h$ and its radius h. When the beaker is filled with a liquid up to a height $2h$, he can see the lower end of the rod. Then the refractive index of the liquid is

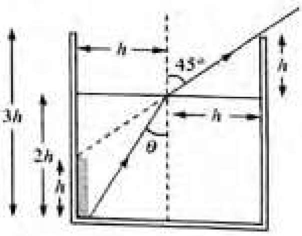


(a) 5/2 (b) $\sqrt{(5/2)}$
(c) $\sqrt{(3/2)}$ (d) 3/2

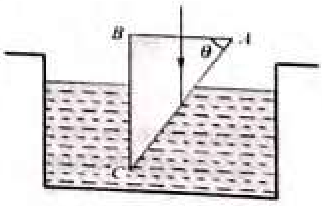
Sol. (b) The line of sight of the observer remains constant, making an angle of 45° with the normal.

$$\sin\theta = \frac{h}{\sqrt{h^2 + (2h)^2}} = \frac{1}{\sqrt{5}}$$

$$\mu = \frac{\sin 45°}{\sin\theta} = \frac{1/\sqrt{2}}{1/\sqrt{5}} = \sqrt{\left(\frac{5}{2}\right)}$$



5. A glass prism (μ = 1.5) is dipped in water (μ = 4/3) as shown in figure. A light ray is incident normally on the surface AB. It reaches the surface BC after totally reflected, if



(a) $\sin\theta \geq 8/9$
(b) $2/3 < \sin\theta < 8/9$

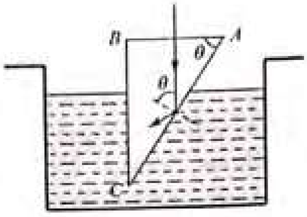
(c) $\sin\theta \le 2/3$
(d) It is not possible

Sol. (a) For TIR at AC, $\theta > C$

$\Rightarrow \sin\theta \ge \sin C$

$\Rightarrow \sin\theta \ge \dfrac{1}{{}_w\mu_g}$

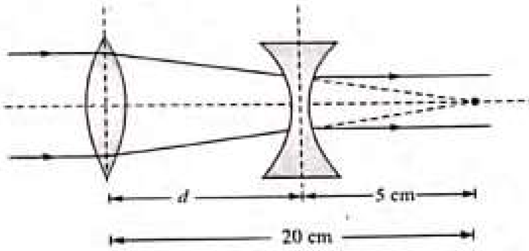
$\Rightarrow \sin\theta \ge \dfrac{\mu_w}{\mu_g} \Rightarrow \sin\theta \ge \dfrac{8}{9}$



6. A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with the distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then distance d in cm will be

(a) 25 (b) 15
(c) 30 (d) 50

Sol. (b) From figure it is clear that separation between lenses
$d = 20 - 5 = 15$ cm



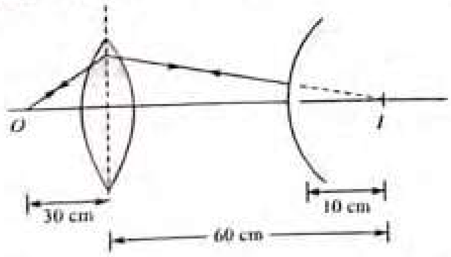
7. A luminous object is placed at a distance of 30 cm from the convex lens of focal length 20 cm. On the other side of the lens, at what distance from the lens a convex mirror of radius of curvature 10 cm be placed in order to have an upright image of the object coincident with it

(a) 12 cm (b) 30 cm
(c) 50 cm (d) 60 cm

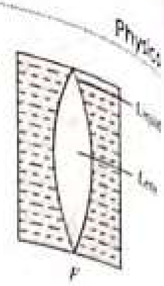
Sol. (c) For lens $u = 30$ cm, $f = 20$ cm, hence by using

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{+20} = \frac{1}{v} - \frac{1}{-30} \Rightarrow v = 60 \text{ cm}$$

The final image will coincide the object, if light ray falls normally on convex mirror as shown. From figure, it could be seen clearly that separation between lens and mirror is 60 – 10 = 50 cm.

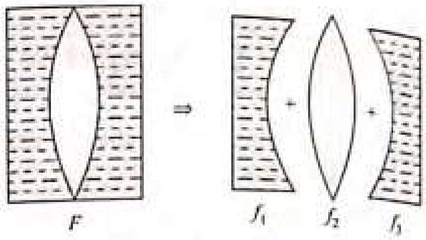


8. Shown in the figure here is a convergent lens placed inside a cell filled with a liquid. The lens has focal length + 20 cm when in air and its material has refractive index 1.50. If the liquid has refractive index 1.60, the focal length of the system is



(a) +80 cm
(b) –80 cm
(c) –24 cm
(d) –100 cm

Sol. (d) $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} + \dfrac{1}{f_3}$



$$\frac{1}{f_1} = (1.6-1)\left(\frac{1}{\infty} - \frac{1}{20}\right) = -\frac{0.6}{20} = -\frac{3}{100} \quad ...(i)$$

$$\frac{1}{f_2} = (1.5-1)\left(\frac{1}{20} - \frac{1}{-20}\right) = \frac{1}{20} \quad ...(ii)$$

$$\frac{1}{f_3} = (1.6-1)\left(\frac{1}{-20} - \frac{1}{\infty}\right) = -\frac{3}{100} \quad ...(iii)$$

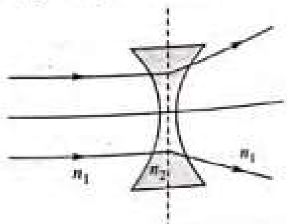
$$\Rightarrow \frac{1}{F} = -\frac{3}{100} + \frac{1}{20} - \frac{3}{100} \Rightarrow F = -100 \text{ cm}$$

9. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 and L_2 having refractive indices n_1 and n_2 respectively ($n_2 > n_1 > 1$). The lens will diverge a parallel beam of light if it is filled with

(a) Air and placed in air
(b) Air and immersed in L_1
(c) L_1 and immersed in L_2
(d) L_2 and immersed in L_1

Sol. (d) $\dfrac{1}{f} = \left(\dfrac{n_2}{n_1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

where n_2 and n_1 are the refractive indices of the material of the lens and of the surroundings respectively. For a double concave lens,



$\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ is always negative.

Hence, f is negative only when $n_2 > n_1$

10. The object distance u, the image distance v and the magnification m in a lens follow certain linear relations. These are

(a) $\frac{1}{u}$ versus $\frac{1}{v}$ (b) m versus u
(c) u versus v (d) m versus v

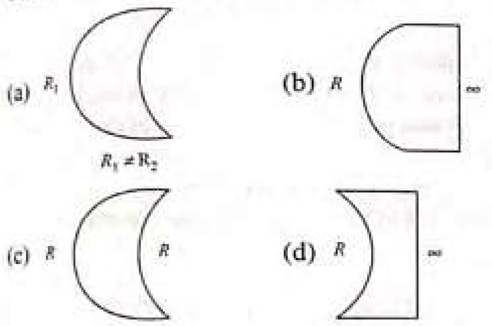
Sol. (a, d) For a lens $\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$ .....(i)

Also $m=\frac{f-v}{f}=1-\frac{v}{f} \Rightarrow m=\left(-\frac{1}{f}\right)v+1$ ....(ii)

On comparing equations (i) and (ii) with $y=mx+c$

It is clear that relationship between $\frac{1}{v}$ vs $\frac{1}{u}$ and m vs v is linear.

11. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams

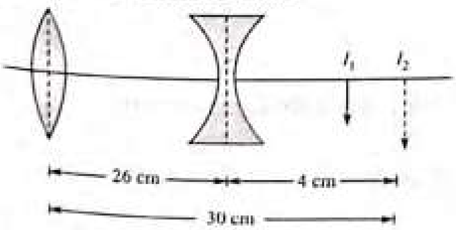


Sol. (c) The dispersion produced by a spherical surface depends on its radius of curvature. Hence, a lens will not exhibit dispersion only if it's two surfaces have equal radii, with one being convex and the other concave.

12. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image

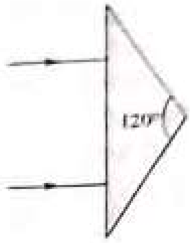
(a) 1.25 cm (b) 2.5 cm
(c) 1.05 cm (d) 2 cm

Sol. (b) Convex lens will form image I_1 at its focus which acts like a virtual object for concave lens.



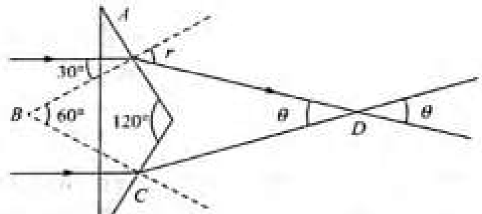
Hence, for concave lens, $u=+4$ cm, $f=20$ cm. So by lens formula $\frac{1}{-20}=\frac{1}{v}-\frac{1}{4} \Rightarrow v=5\text{cm}$, i.e., distance of final image (I_2) from concave lens $v=5$ cm by using $\frac{v}{u}=\frac{I}{O} \Rightarrow \frac{5}{4}=\frac{I}{2} \Rightarrow (I_2)=2.5\text{cm}$

13. An isosceles prism of angle 120° has a refractive index of 1.44. Two parallel monochromatic rays enter the prism parallel to each other in air as shown. The rays emerging from the opposite faces



(a) Are parallel to each other
(b) Are diverging
(c) Make an angle $2\sin^{-1}(0.72)$ with each other
(d) Make an angle $2\{\sin^{-1}(0.720-30°\}$ with each other

Sol. (d) At point A. $\frac{\sin 30°}{\sin r}=\frac{1}{1.44}$



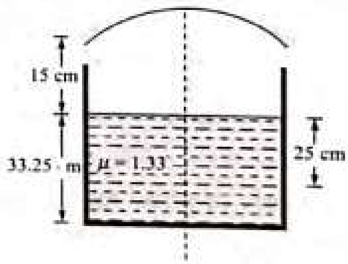
$\Rightarrow r=\sin^{-1}(0.72)$; also $\angle BAD = 180° - \angle r$

In rectangle $ABCD$, $\angle A+\angle B+\angle C+\angle D=360°$

$\Rightarrow (180°-r)+60°+(180°-r)+\theta=360°$

$\Rightarrow \theta=2[\sin^{-1}(0.72)-30°]$

14. A container is filled with water ($\mu=1.33$) up to a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. The focal length of the mirror is



(a) 10 (b) 15
(c) 20 (d) 25

Sol. (c) Distance of object from mirror

$$=15+\frac{33.25}{4}\times 3=39.93 \text{ cm}$$

Distance of image from mirror $=15+\frac{25}{4}\times 3=33.75$

For mirror, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

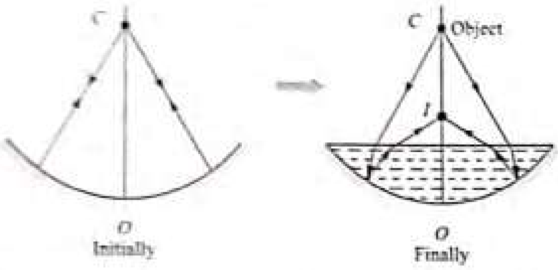
$$\Rightarrow \quad \frac{1}{-33.75}-\frac{1}{39.93}=\frac{1}{f} \Rightarrow f=-18.3 \text{ cm.}$$

15. A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let O be the pole of

the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be

(a) Real, and will remain at C
(b) Real, and located at a point between C and ∞
(c) Virtual and located at a point between C and O
(d) Real, and located at a point between C and O

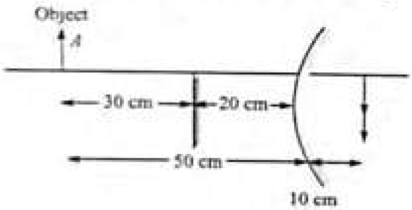
Sol. (d) From the following figures it is clear that real image (I) will be formed between C and O



16. An object is placed infront of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and plane mirror is 30 cm, it is found that there is no parallax between the images formed by two mirrors. Radius of curvature of mirror will be

(a) 12.5 cm (b) 25 cm
(c) $\frac{50}{3}$ cm (d) 18 cm

Sol. (b) Since there is no parallex, it means that both images (By plane mirror and convex mirror) coinciding each other.



According to property of plane mirror it will form image at a distance of 30 cm behind it. Hence for convex mirror $u = -50$ cm, $v = +10$ cm

By using $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{+10} + \frac{1}{-50} = \frac{4}{50}$

$\Rightarrow f = \frac{25}{2}$ cm $\Rightarrow R = 2f = 25$ cm

17. A cube of side 2 m is placed in front of a concave mirror focal length 1 m with its face P at a distance of 3 m and face Q at a distance of 5 m from the mirror. The distance between the images of face P and Q and height of images of P and Q are

(a) 1 m, 0.5 m, 0.25 m (b) 0.5 m, 1 m, 0.25 m
(c) 0.5 m, 0.25 m, 1 m (d) 0.25 m, 1 m, 0.5 m

Sol. (d) For surface P, $\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u} = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow v_1 = \frac{3}{2}$ m

For surface Q, $\frac{1}{v_2} = \frac{1}{f} - \frac{1}{u} = 1 - \frac{1}{5} = \frac{4}{5} \Rightarrow v_2 = \frac{5}{4}$ m

$\therefore \quad v_1 - v_2 = 0.25$ m

Magnification of $P = \frac{v_1}{u} = \frac{3/2}{3} = \frac{1}{2}$

$\therefore \quad$ Height of $P = \frac{1}{2} \times 2 = 1$ m

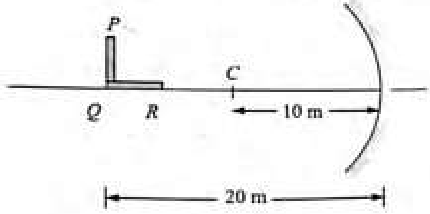
Magnification of $Q = \frac{v_2}{u} = \frac{5/4}{5} = \frac{1}{4}$

$\therefore \quad$ Height of $Q = \frac{1}{4} \times 2 = 0.5$ m

18. A small piece of wire bent in to an L shape with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror whose radius of curvature is 10 cm. If the bend is 20 cm from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is

(a) 1 : 2 (b) 3 : 1
(c) 1 : 3 (d) 2 : 1

Sol. (b) Focal length of mirror $f = \frac{R}{2} = \frac{10}{2} = 5$ cm



For part PQ : transverse magnification

length of image $L_1 = \left(\frac{f}{f-u}\right) \times L_0$

$= \left(\frac{-5}{-5-(-20)}\right) \times L_0 = \frac{-L_0}{3}$

For part QR : longitudinal magnification

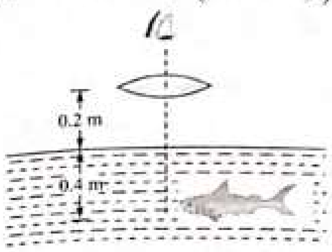
Length of image $L_2 = \left(\frac{f}{f-u}\right)^2 L_0$

$= \left(\frac{-5}{-5-(-20)}\right)^2 \times L_0 = \frac{L_0}{9}$

$\Rightarrow \quad \frac{L_1}{L_2} = \frac{3}{1}$

19. A small fish 0.4 m below the surface of a lake, is viewed through a simple converging lens of focal length 3 m. The

lens is kept at 0.2 m above the water surface such that fish lies on the optical axis of the lens. The image of the fish seen by observer will be at $\left(\mu_{water} = \frac{4}{3}\right)$



(a) A distance of 0.2 m from the water surface
(b) A distance of 0.6 m from the water surface
(c) A distance of 0.3 m from the water surface
(d) The same location of fish

Sol. (d) Apparent distance of fish from lens

$$u = 0.2 + \frac{h}{\mu}$$

$$= 0.2 + \frac{0.4}{4/3} = 0.5 \text{ m}$$

From $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \frac{1}{(+3)} = \frac{1}{v} - \frac{1}{(-0.5)} \quad v = -0.6 \text{ m}$$

The image of the fish is still where the fish is 0.4 m below the water surface.

20. In an astronomical telescope in normal adjustment, a straight black line of length L is drawn on the objective lens. The eyepiece forms a real image of this line. The length of this image is l. The magnification of the telescope is

(a) $\frac{L}{l}$ (b) $\frac{L}{l}+1$

(c) $\frac{L}{l}-1$ (d) $\frac{L+l}{L-l}$

Sol. (a) Here we treat the line on the objective as the object and the eyepiece as the lens.

Hence, $u = -(f_o + f_e)$ and $f = f_e$

Now $\frac{1}{v} - \frac{1}{-(f_o+f_e)} = \frac{1}{f_e}$

Solving we get $v = \frac{(f_o+f_e)f_e}{f_o}$

Magnification $= \left|\frac{v}{u}\right| = \frac{f_e}{f_o} = \frac{\text{Image size}}{\text{Object size}} = \frac{l}{L}$

$\therefore$ Magnification of telescope in normal adjustment

$$= \frac{f_o}{f_e} = \frac{L}{l}$$

EXERCISES

Reflection through Plain and Spherical Mirror

1. A convex mirror of focal length f forms an image which is $1/n$ times the object. The distance of the object from the mirror is

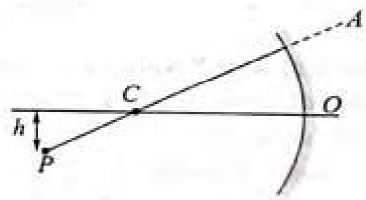
(a) $(n-1)f$ (b) $\left(\frac{n-1}{n}\right)f$

(c) $\left(\frac{n+1}{n}\right)f$ (d) $(n+1)f$

2. A point object is placed at a distance of 10 cm and its real image is formed at a distance of 20 cm from a concave mirror. If the object is moved by 0.1 cm towards the mirror, the image will shift by about

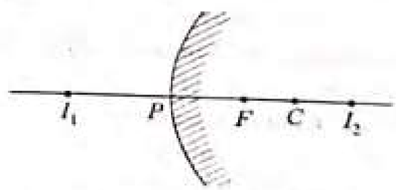
(a) 0.4 cm away from the mirror
(b) 0.4 cm towards the mirror
(c) 0.8 cm away from the mirror
(d) 0.8 cm towards the mirror

3. Consider a line through center of curvature. The object P is h distance below the optical axis. The image of point object P is H distance either above or below the optical axis. Then,



(a) $h = H$ (b) $h > H$
(c) $h < H$ (d) $h \le H$

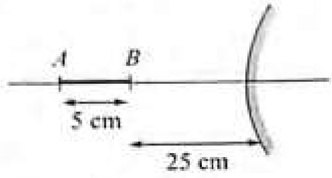
4. P, F and C are the pole, focus and centre of curvature of a convex mirror. There are two positions of images I_1 and I_2 as shown, for a virtual object placed at different positions. The positions of the virtual object corresponding to I_1 and I_2 respectively will be



(a) between F and C, between C and I_2
(b) between P and F, between F and C

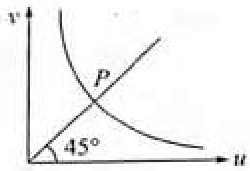
(c) between P and C, between, C and I_2
(d) between P and F, between C and I_2

5. A convex mirror of focal length 10 cm is shown in figure. A linear object AB = 5 cm is placed along the optical axis. Point B is at distance 25 cm from the pole of mirror. The size of image of AB is



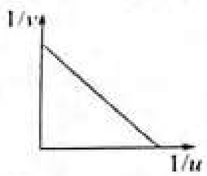
(a) 1/14 cm (b) 5/14 cm
(c) 3/14 cm (d) 4/14 cm

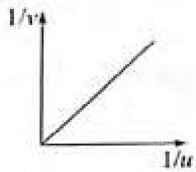
6. The graph shows variation of v with change in u for a mirror. Points plotted above the point P on the curve are for values of v

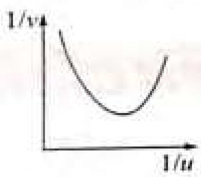


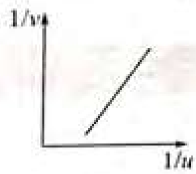
(a) Smaller than f
(b) Smaller than $2f$
(c) Larger than $2f$
(d) Larger than f

7. For a concave mirror, if real image is formed the graph between $1/u$ and $1/v$ is of the form

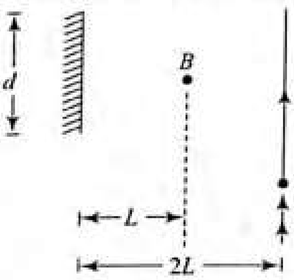
(a) 

(b) 

(c) 

(d) 

8. A point source of light B is placed at a distance L in front of the center of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown in figure. the greatest distance over which he can see the image of the light source in the mirror is



(a) $d/2$ (b) d
(c) $2d$ (d) $3d$

9. A convex mirror of radius of curvature 1.6 m has an object placed at a distance of 1 m from it. The image is formed at a distance of
(a) 8/13 m in front of the mirror
(b) 8/13 m behind the mirror
(c) 4/9 m in front of the mirror
(d) 4/9 m behind the mirror

10. In the above question, the magnification is
(a) 4/9 (b) −4/9
(c) 9/4 (d) 8/13

11. With a concave mirror, an object is placed at a distance x_1 from the principal focus, on the principal axis. The image is formed at a distance x_2 from the principal focus. The focal length of the mirror is
(a) x_1x_2 (b) $(x_1 + x_2)/2$
(c) $\sqrt{x_1/x_2}$ (d) $\sqrt{x_1x_2}$

12. A concave mirror is placed on a horizontal table with its axis directed vertically upward. Let O be the pole of the mirror C its center of curvature and F is the focus. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be
(a) real and will remain at C
(b) real and located at a point between C and ∞
(c) real and located at a point between C and O
(d) real and located at a point between C and F

13. A plane glass mirror of thickness 3 cm of material of $\mu = 3/2$ is silvered on the back surface. When a point object is placed 9 cm from the front surface of the mirror, then the position of the brightest image from the front surface is
(a) 9 cm (b) 11 cm
(c) 12 cm (d) 13 cm

14. An object is placed at a distance of 25 cm from the pole of a convex mirror and a plane mirror is set at a distance 5 cm from convex mirror so that the virtual images formed by the two mirrors do not have any parallax. The focal length of the convex mirror is
(a) 37.5 cm (b) −7.5 cm
(c) −37.5 cm (d) +7.5 cm

15. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm. If the object is moved with a speed of 9 cm s^{-1} the speed with which the image moves is
(a) 0.1 m s^{-1} (b) 1 m s^{-1}
(c) 3 m s^{-1} (d) 9 m s^{-1}

16. A convex mirror and a concave mirror of radius 10 cm each are placed 15 cm apart facing each other. An object is placed midway between them. If the reflection first takes place in the concave mirror and then in convex mirror, the position of the final image is
(a) on the pole of the convex mirror
(b) on the pole of the concave mirror
(c) at a distance of 10 cm from the convex mirror
(d) at a distance of 5 cm from the concave mirror

17. A small piece of wire bent into an L shape, with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror

whose radius of curvature is 10 cm. If the bend is 20 cm from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is
(a) 1 : 2 (b) 3 : 1
(c) 1 : 3 (d) 2 : 1

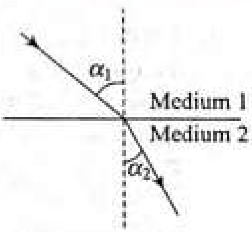
18. The image of an object placed on the principal axis of a concave mirror of focal length 12 cm is formed at a point which is 10 cm more distance from the mirror than the object. The magnification of the image is
(a) 8/3 (b) 2.5
(c) 2 (d) −1.5

19. A spherical mirror forms an image of magnification 3. The object distance, if focal length of mirror is 24 cm, may be
(a) 32 cm, 24 cm (b) 32 cm , 16 cm
(c) 32 cm only (d) 16 cm only

20. A candle is placed 20 cm from the surface of a convex mirror and a plane mirror is also placed so that the virtual images in the two mirrors coincide. If the plane mirror is 12 cm away from the object, what is the focal length of the convex mirror?
(a) 20 cm (b) 15 cm
(c) 10 cm (d) 5 cm

Refraction of Light at Plane and Curved Surface

21. A beam of light propagates through medium 1 and falls onto another medium 2, at an angle α_1 as shown in figure. After that, it propagates in medium 2 at an angle α_2 as shown. The light's wavelength in medium 1 is λ_1. What is the wavelength of light in medium 2?



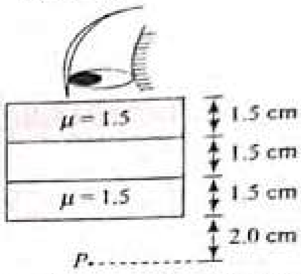
(a) $\dfrac{\sin\alpha_1}{\sin\alpha_2}\lambda_1$ (b) $\dfrac{\sin\alpha_2}{\sin\alpha_1}\lambda_1$
(c) $\dfrac{\cos\alpha_1}{\cos\alpha_2}\lambda_1$ (d) $\dfrac{\cos\alpha_2}{\cos\alpha_1}\lambda_1$

22. Light from a denser medium 1 passes to a rarer medium 2. When the angle of incidence is θ the partially reflected and refracted rays are mutually perpendicular. The critical angle will be
(a) $\sin^{-1}(\cot\theta)$ (b) $\sin^{-1}(\tan\theta)$
(c) $\sin^{-1}(\cos\theta)$ (d) $\sin^{-1}(\sec\theta)$

23. Light of wavelength 500 nm traveling with a speed of 2.0×10^8 m s^{-1} in a certain medium enters another medium of refractive index 5/4 times that of the first medium. What are the wavelength and speed in the second medium?

	Wavelength (nm)	speed (ms^{-1})
(a)	400	1.6×10^8
(b)	400	2.5×10^8
(c)	500	2.5×10^8
(d)	625	1.6×10^8

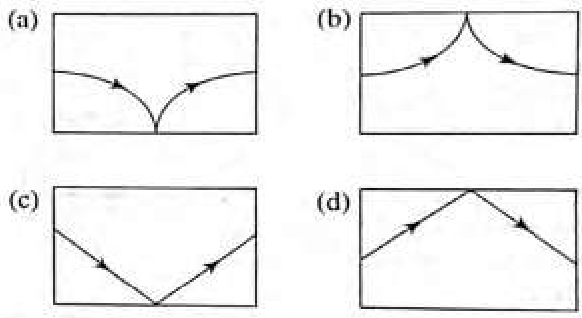
24. The image of point P when viewed from top of the slabs will be



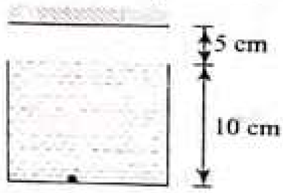
(a) 2.0 cm above P
(b) 1.5 cm above P
(c) 2.0 cm below P
(d) 1 cm above P

25. The critical angle for light going from medium X into medium Y is θ. The speed of light in medium X is v. The speed of light in medium Y is
(a) $v\cos\theta$ (b) $v/\cos\theta$
(c) $v\sin\theta$ (d) $v/\sin\theta$

26. A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. Which of the following represents the path of a ray of light inside the liquid?



27. Consider the situation shown in figure. Water ($\mu = 4/3$) is filled in a beaker upto a height of 10 cm. A plane mirror is fixed at a height of 5 cm from the surface of water. Distance of image from the mirror after reflection from it of an object O at the bottom of the beaker is



(a) 15 cm (b) 12.5 cm
(c) 7.5 cm (d) 10 cm

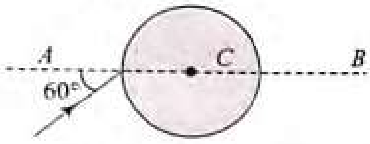
28. The velocity of light in a medium is half its velocity in air. If a ray of light emerges from such a medium into air, the angle of incidence, at which it will be totally internally reflected, is
(a) 15° (b) 30°
(c) 45° (d) 60°

29. A circular beam of light of diameter d = 2 cm falls on a plane surface of glass. The angle of incidence is 60° and refractive index of glass is $\mu = 3/2$. The diameter of the refracted beam is
(a) 4.00 cm (b) 3.0 cm
(c) 3.26 cm (d) 2.52 cm

30. Critical angle of glass is θ_1 and that of water is θ_2. The critical angle for water and glass surface would be ($\mu_g = 3/2$, $\mu_w = 4/3$)

(a) less than θ_2 (b) between θ_1 and θ_2
(c) greater than θ_2 (d) less than θ_1

31. A ray of light falls on a transparent sphere with centre at C as shown in figure.



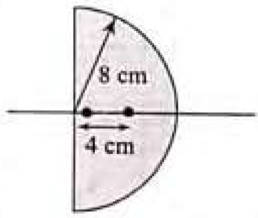
The ray emerges from the sphere parallel to line AB. The refractive index of the sphere is

(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 3/2 (d) 1/2

32. A ray of light is incident on a glass sphere of refractive index 3/2. What should be the angle of incidence so that the ray which enters the sphere does not come out of the sphere?

(a) $\tan^{-1}(2/3)$ (b) 60°
(c) 90° (d) 30°

33. A plastic hemisphere has a radius of curvature of 8 cm and an index of refraction of 1.6. On the axis halfway between the plane surface and the spherical one (4 cm from each) is a small object O.



The distance between the two images when viewed along the axis from the two sides of the hemisphere is approximately

(a) 1.0 cm (b) 1.5 cm
(c) 3.75 cm (d) 2.5 cm

34. Refraction takes place at a convex spherical boundary separating glass–air medium. For the image to be real, the object distance ($\mu_g = 3/2$) should be/is

(a) greater than three times the radius of curvature of the refracting surface
(b) greater than two times the radius of curvature of the refracting surface
(c) greater than the radius of curvature of the refracting surface
(d) independent of the radius of curvature of the refracting surface

35. The refractive index of a prism is 2. The prism can have a maximum refracting angle of

(a) 90° (b) 60°
(c) 45° (d) 30°

36. One of the refracting surfaces of a prism of angle 30° is silvered. A ray of light incident at angle of 60° retraces its path. The refractive index of the material of prism is

(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 3/2 (d) 2

37. Angle of minimum deviation is equal to the angle of prism A of an equilateral glass prism. The angle of incidence at which minimum deviation will be obtained is

(a) 60° (b) 30°
(c) 45° (d) $\sin^{-1}(2/3)$

38. What is the angle of incidence for an equilateral prism of refractive index $\sqrt{3}$ so that the ray is parallel to the base inside the prism?

(a) 30° (b) 45°
(c) 60° (d) Either 30° or 60°

39. A ray of light passes from glass, having a refractive index of 1.6, to air. The angle of incidence for which the angle of refraction is twice the angle of incidence is

(a) $\sin^{-1}\left(\frac{4}{5}\right)$ (b) $\sin^{-1}\left(\frac{3}{5}\right)$

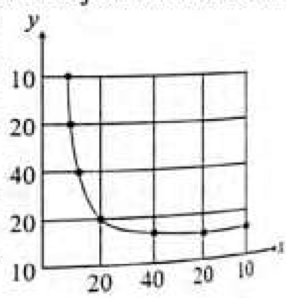
(c) $\sin^{-1}\left(\frac{5}{8}\right)$ (d) $\sin^{-1}\left(\frac{2}{5}\right)$

40. A clear transparent glass sphere (μ=1.5) of radius R is immersed in a liquid of refractive index 1.25. A parallel beam of light incident on it will converge to a point. The distance of this point from the center will be

(a) $-3R$ (b) $+3R$
(c) $-R$ (d) $+R$

Convex and Concave Lenses and Prism

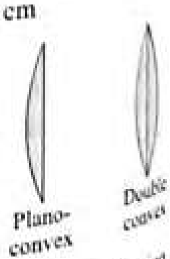
41. A lens forms a real image of an object. The distance from the object to the lens is x cm and that from the lens to the image is y cm. The graph (see figure) shows the variation of y with x.



It can be deduced that the lens is

(a) converging and of focal length 10 cm
(b) converging and of focal length 20 cm
(c) converging and of focal length 40 cm
(d) diverging and of focal length 20 cm

42. You are given two identical plano-convex lenses. When you place an object 20 cm to the left of a single plano-convex lens, the image appears 40 cm to the right of the lens. You then arrange the two plano-convex lenses back to back to form a double convex lens. If the object is 20 cm to the left of this new lens, what is the approximate location of the image?



(a) 10 cm to the right of the lens
(b) 20 cm to the right of the lens
(c) 80 cm to the right of the lens
(d) 80 cm to the left of the lens

43. A thin equiconvex lens ($\mu = 3/2$) of focal length 10 cm is cut and separated and a material of refractive index 3 is filled between them. What is the focal length of the combination?
(a) -10 cm (b) $-10/4$ cm
(c) $-10/3$ cm (d) None of these

44. Consider an equiconvex lens of radius of curvature R and focal length f. If $f > R$, the refractive index μ of the material of the lens
(a) is greater than zero but less than 1.5
(b) is greater than 1.5 but less than 2.0
(c) is greater than 1.0 but less than 1.5
(d) none of these

45. Two identical glass ($\mu_g = 3/2$) equiconvex lenses of focal length f are kept in contact. The space between the two lenses is filled with water ($\mu_w = 4/3$). The focal length of the combination is
(a) f (b) $\frac{f}{2}$
(c) $\frac{4f}{3}$ (d) $\frac{3f}{4}$

46. An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens, the image formed is virtual. If the sizes, of the images formed are equal, the focal length of the lens will be
(a) 15 cm (b) 17 cm
(c) 21 cm (d) 11 cm

47. Point object O is placed on the principal axis of a convex lens of focal length 20 cm at a distance of 40 cm to the left of it. The diameter of the lens is 10 cm. If the eye is placed 60 cm to the right of the lens at a distance h below the principal axis, then the maximum value of h to see the image will be
(a) 0 cm (b) 5 cm
(c) 2.5 cm (d) 10 cm

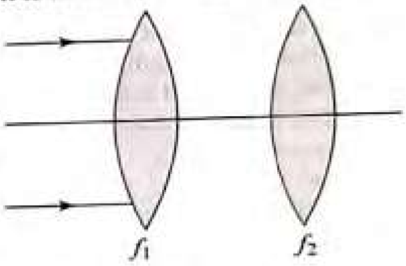
48. A convex lens of focal length 20 cm and a concave lens of focal length f are mounted coaxially 5 cm apart. Parallel beam of light incident on the convex lens emerges from the concave lens as a parallel beam. Then, f in cm is
(a) 35 (b) 25
(c) 20 (d) 15

49. An equiconvex lens is made from glass of refractive index 1.5. If the radius of each surface is changed from 5 cm to 6 cm, then the power
(a) remains unchanged (b) increases by 3.33 D
(c) decreases by 3.33 D (d) decreases by 5.5 D

50. An object is put at a distance of 5 cm from the first focus of a convex lens of focal length 10 cm. If a real image is formed, its distance from the lens will be
(a) 15 cm (b) 20 cm
(c) 25 cm (d) 30 cm

51. A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If the lenses are made of different material of refractive indices μ_1 and μ_2 and R is the radius of curvature of the curved surface of the lenses, then focal length of the combination is
(a) $\frac{R}{\mu_1 - \mu_2}$ (b) $\frac{2R}{\mu_2 - \mu_1}$
(c) $\frac{R}{2(\mu_1 - \mu_2)}$ (d) $\frac{R}{2 - (\mu_1 + \mu_2)}$

52. A parallel beam of light is incident on the system of two convex lenses of focal lengths f_1 = 20 cm and f_2 = 10 cm. What should be the distance between the two lenses so that rays after refraction from both the lenses pass undeviated?

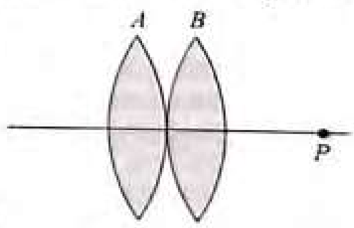


(a) 60 cm (b) 30 cm
(c) 90 cm (d) 40 cm

53. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness t and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. The thickness t is
(a) 10 cm (b) 5 cm
(c) 20 cm (d) 15 cm

54. A lens forms a virtual, diminished image of an object placed at 2 m from it. The size of image is half of the object. Which one of the following statements is correct regarding the nature and focal length of the lens?
(a) Concave, $|f| = 1$ m (b) Convex, $|f| = 1$ m
(c) Concave, $|f| = 2$ m (d) Convex, $|f| = 2$ m

55. Two convex lenses placed in contact form the image of a distant object at P. If lens B is moved to the right, the image will



(a) move to the left
(b) move to the right
(c) remain at P
(d) move either to the left or right, depending upon focal lengths of the lenses.

56. An object 15 cm high is placed 10 cm from the optical center of a thin lens. Its image is formed 25 cm from the optical center on the same side of the lens as the object. The height of the image is
(a) 2.5 cm (b) 0.2 cm
(c) 16.7 cm (d) 37.5 cm

57. A convex lens of power +6 dioptre is placed in contact with a concave lens of power –4 dioptre. What will be the nature and focal length of this combination?
    (a) Concave, 25 cm (b) Convex, 50 cm
    (c) Concave, 20 cm (d) Convex, 100 cm
58. A convex lens of focal length 1.0 m and a concave lens of focal length 0.25 m are 0.75 m apart. A parallel beam of light is incident on the convex lens. The beam emerging after refraction from both lenses is
    (a) parallel to the principal axis
    (b) convergent
    (c) divergent
    (d) none of the above
59. A convex lens A of focal length 20 cm and a concave lens G of focal length 5 cm are kept along the same axis with the distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then distance d in cm will be
    (a) 25 (b) 15
    (c) 30 (d) 50
60. A convex lens forms an image of an object placed 20 cm away from it at a distance of 20 cm on the other side of the lens. If the object is moves 5 cm toward the lens, the image will be
    (a) 5 cm toward the lens
    (b) 5 cm away from the lens
    (c) 10 cm toward the lens
    (d) 10 cm away from the lens
61. When light rays are incident on a prism at an angle of 45°, the minimum deviation is obtained. If refractive index of the material of prism is $\sqrt{2}$, then the angle of prism will be
    (a) 30° (b) 40°
    (c) 50° (d) 60°
62. A light ray is incident by grazing one of the face of a prism and after refraction ray does not emerge out, what should be the angle of prism while critical angle is C?
    (a) Equal to $2C$ (b) Less than $2C$
    (c) More than $2C$ (d) None of the above
63. When a ray of light is incident normally on one refracting surface of an equilateral prism (Refractive index of the material of the prism = 1.5?
    (a) Emerging ray is deviated by 30°
    (b) Emerging ray is deviated by 45°
    (c) Emerging ray just grazes the second refracting surface
    (d) The ray undergoes total internal reflection at the second refracting surface
64. When a glass prism of refracting angle 60° is immersed in a liquid its angle of minimum deviation is 30°. The critical angle of glass with respect to the liquid medium is
    (a) 42° (b) 45°
    (c) 50° (d) 52°
65. A prism of refractive index μ and angle A is placed in the minimum deviation position. If the angle of minimum deviation is A, then the value of A in terms of μ is
    (a) $\sin^{-1}\left(\frac{\mu}{2}\right)$ (b) $\sin^{-1}\sqrt{\frac{\mu-1}{2}}$
    (c) $2\cos^{-1}\left(\frac{\mu}{2}\right)$ (d) $\cos^{-1}\left(\frac{\mu}{2}\right)$

Optical Instruments

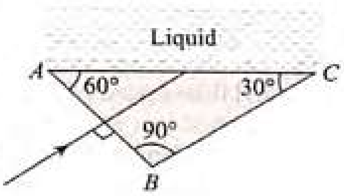
66. The focal lengths of the objective and the eye-piece of a compound microscope are 2.0 cm and 3.0 cm respectively. The distance between the objective and the eye-piece is 15.0 cm. The final image formed by the eye-piece is at infinity. The two lenses are thin. The distances in cm of the object and the image produced by the objective measured from the objective lens are respectively
    (a) 2.4 and 12.0 (b) 2.4 and 15.0
    (c) 2.3 and 12.0 (d) 2.3 and 3.0
67. The objective lens of a compound microscope produces magnification of 10. In order to get an overall magnification of 100 when image is formed at 25 cm from the eye, the focal length of the eye lens should be
    (a) 4 cm (b) 10 cm
    (c) $\frac{25}{9}$ cm (d) 9 cm
68. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eye piece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of the eye piece are
    (a) $f_o = 45$ cm and $f_e = -9$ cm
    (b) $f_o = 7.2$ cm and $f_e = 5$ cm
    (c) $f_o = 50$ cm and $f_e = 10$ cm
    (d) $f_o = 30$ cm and $f_e = 6$ cm
69. In Gallilean telescope, if the powers of an objective and eye lens are respectively +1.25 D and –20 D, then for relaxed vision, the length and magnification will be
    (a) 21.25 cm and 16 (b) 75 cm and 20
    (c) 75 cm and 16 (d) 8.5 cm and 21.25
70. The magnifying power of an astronomical telescope is 8 and the distance between the two lenses is 54 cm. The focal length of eye lens and objective lens will be respectively
    (a) 6 cm and 48 cm (b) 48 cm and 6 cm
    (c) 8 cm and 64 cm (d) 64 cm and 8 cm
71. A telescope of diameter 2 m uses light of wavelength 5000 Å for viewing stars. The minimum angular separation between two stars whose image is just resolved by this telescope is
    (a) 4×10^{-4} rad (b) 0.25×10^{-6} rad
    (c) 0.31×10^{-6} rad (d) 5.0×10^{-3} rad
72. Magnification of a compound microscope is 30. Focal length of eye-piece is 5 cm and the image is formed at a distance of distinct vision of 25 cm. The magnification of the objective lens is
    (a) 6 (b) 5
    (c) 7.5 (d) 10

73. A Galileo telescope has an objective of focal length 100 cm and magnifying power 50. The distance between the two lenses in normal adjustment will be
(a) 98 cm (b) 100 cm
(c) 150 cm (d) 200 cm

74. A compound microscope has an eye piece of focal length 10 cm and an objective of focal length 4 cm. Calculate the magnification, if an object is kept at a distance of 5 cm from the objective so that final image is formed at the least distance vision (20 cm)
(a) 12 (b) 11
(c) 10 (d) 13

Problems Based on Mixed Concepts

75. Light is incident normally on face AB of a prism as shown in Figure. A liquid of refractive index μ is placed on face AC of the prism. The prism is made of glass of refractive index 3/2.



The limit of μ for which total internal reflection takes place on face AC is
(a) $\mu > \frac{3}{4}$ (b) $\mu < \frac{3\sqrt{3}}{4}$
(c) $\mu > \sqrt{3}$ (d) $\mu < \frac{\sqrt{3}}{2}$

76. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i. The reflected and refracted rays make an angle of $\pi/2$ with each other. If the angles of reflection and refraction are r and r', then the critical angle will be
(a) $\tan^{-1}(\sin i)$ (b) $\sin^{-1}(\sin r)$
(c) $\sin^{-1}(\tan i)$ (d) $\sin^{-1}(\tan r)$

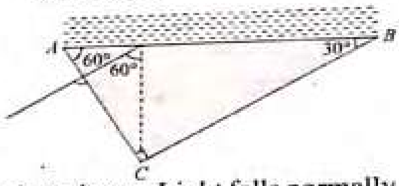
77. A ray of light traveling in glass ($\mu = 3/2$) is incident on a horizontal glass–air surface at the critical angle θ_C. If a thin layer of water ($\mu = 4/3$) is now poured on the glass–air surface, the angle at which the ray emerges into air at the water–air surface is
(a) 60° (b) 45°
(c) 90° (d) 180°

78. A ray of light enters a rectangular glass slab of refractive index $\sqrt{3}$ at an angle of incidence 60°. It travels a distance of 5 cm inside the slab and emerges out of the slab. The perpendicular distance between the incident and the emergent rays is
(a) $5\sqrt{3}$ cm (b) $\frac{5}{2}$ cm
(c) $5\sqrt{3/2}$ cm (d) 5 cm

79. A ray of monochromatic light is incident on the refracting face of a prism (angle 75°). It passes through the prism and is incident on the other face at the critical angle. If the refractive index of the prism is $\sqrt{2}$, then the angle of incidence on the first face of the prism is
(a) 15° (b) 30°
(c) 45° (d) 60°

80. An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at its focus such that the image formed by the combination coincides with the object itself. The focal length of the convex mirror is
(a) 20 cm (b) 10 cm
(c) 15 cm (d) 30 cm

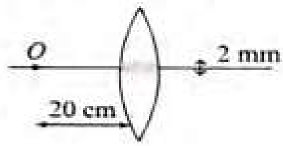
81. ACB is right-angled prism with other angles as 60° and 30°. Refractive index of the prism is 1.5. AB has thin layer of liquid on it as shown. Light falls normally on the face AC. For total internal reflections, maximum refractive index of the liquid is



(a) 1.4 (b) 1.3
(c) 1.2 (d) 1.6

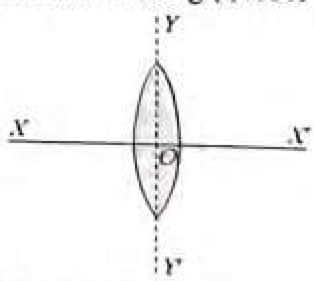
82. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices n_1 and n_2, respectively ($n_2 > n_1 > 1$). The lens will diverge parallel beam of light if it is filled with
(a) air and placed in air (b) air and immersed in L_1
(c) L_1 and immersed in L_2 (d) L_2 and immersed in L_1

83. A convex lens of focal length 10 cm is painted black at the middle portion as shown in figure. An object is placed at a distance of 20 cm from the lens. Then,



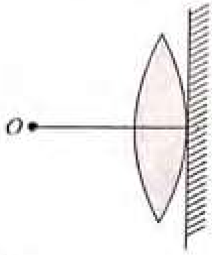
(a) only one image will be formed by the lens
(b) the distance between the two images formed by such a lens is 6 mm
(c) the distance between the images is 4 mm
(d) the distance between the images is 2 mm

84. An equiconvex lens is cut into two halves along (i) XOX' and (ii) YOY'' as shown in Figure. Let f, f', f'' be the focal length of the lens, of each half in case (i), and of each half in case (ii), respectively.



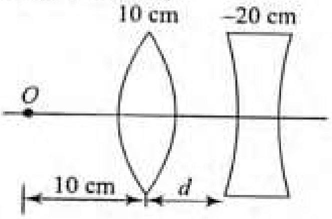
Choose the correct statement from the following:
(a) $f' = f$, $f'' = 2f$ (b) $f' = 2f$, $f'' = f$
(c) $f' = f$, $f'' = f$ (d) $f' = 2f'$, $f'' = 2f$

85. Behind a thin converging lens having both the surfaces of the same radius 10 cm, a plane mirror has been placed. The image of an object at a distance of 40 cm from the lens is formed at the same position. What is the refractive index of the lens?



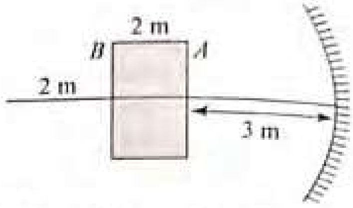
(a) 1.5 (b) 5/3
(c) 9/8 (d) None of these

86. What should be the value of distance d so that final image is formed on the object itself. (Focal lengths of the lenses are written on the lenses.)



(a) 10 cm (b) 20 cm
(c) 5 cm (d) None of these

87. A cube of side 2 m is placed in front of a concave mirror of focal length 1 m with its face A at a distance of 3 m and face B at a distance of 5 m form the mirror. The distance between the images of faces A and B and heights of images of A and B are, respectively,



(a) 1 m, 0.5 m, 0.25 m (b) 0.5 m, 1 m, 0.25 m
(c) 0.5 m, 0.25 m, 1 m (d) 0.25 m, 1 m, 0.5 m

88. A plano-convex lens when silvered on the plane side behaves like a concave mirror of focal length 60 cm. However, when silvered on the convex side, it behaves like a concave mirror of focal length 20 cm. Then, the refractive index of the lens is
(a) 3.0 (b) 1.5
(c) 1.0 (d) 2.0

89. Two thin lenses are placed 5 cm apart along the same axis and illuminated with a beam of light parallel to that axis. The first lens in the path of the beam is a converging lens of focal length 10 cm whereas the second is a diverging lens of focal length 5 cm. If the second lens is now moved toward the first, the emergent light
(a) remains parallel
(b) remains convergent
(c) remains divergent
(d) changes from parallel to divergent

ARCHIVES

1. An astronomical telescope has a large aperture to
(a) reduce spherical aberration.
(b) have high resolution.
(c) increases the span of observation.
(d) have low dispersion. **(AIEEE 2002)**

2. A light bulb is placed between two plane mirrors inclined at an angle of 60°. The number of images formed are
(a) 6 (b) 2
(c) 5 (d) 4 **(AIEEE 2002)**

3. The phenomenon utilised in an optical fibre is
(a) refraction (b) interference
(c) polarization (d) total internal reflection **(AIEEE 2002)**

4. Colour of the sky is blue due to
(a) scattering of light (b) total internal reflection
(c) total emission (d) none of the above **(AIEEE 2002)**

5. Wavelength of light used in an optical instrument are $\lambda_1 = 4000$ Å and $\lambda_2 = 5000$ Å, then ratio of their respective resolving power (corresponding to λ_1 and λ_2) is
(a) 16 : 25 (b) 9 : 1
(c) 4 : 5 (d) 5 : 4 **(AIEEE 2002)**

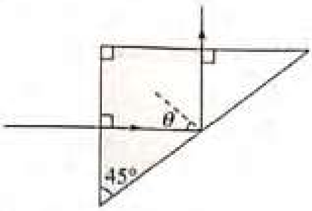
6. The image formed by the objective of a compound microscope is
(a) virtual and enlarged (b) virtual and diminished
(c) real and diminished (d) real and enlarged **(AIEEE 2003)**

7. The aperture of the objective lens of a telescope is made large so as to
(a) increase the magnifying power of the telescope
(b) increase the resolving power of the telescope
(c) make image aberration less
(d) focus on distant objects **(AIEEE 2003)**

8. Consider telecommunication through optical fibres. Which of the following statements is not true?
(a) Optical fibres may have homogeneous core with a suitable cladding
(b) Optical fibres can be of graded refractive index
(c) Optical fibres are subject to electromagnetic interference from outside
(d) Optical fibres have extremely low transmission loss **(AIEEE 2003)**

9. To get three images of a single object, one should have two plane mirrors at an angle of
(a) 30° (b) 60°
(c) 90° (d) 150° **(AIEEE 2003)**

10. A light ray is incident perpendicular to a face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, what can be concluded about the refractive index n?



(a) $n < \frac{1}{\sqrt{2}}$ (b) $n < \sqrt{2}$

(c) $n > \frac{1}{\sqrt{2}}$ (d) $n > \sqrt{2}$ **(AIEEE 2004)**

11. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now this lens is used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object?

(a) 20 cm (b) 30 cm

(c) 60 cm (d) 80 cm **(AIEEE 2004)**

12. The angle of incidence at which reflected light is totally polarised for reflection from air to glass (refractive index n) is

(a) $\sin^{-1}(n)$ (b) $\sin^{-1}(1/n)$

(c) $\tan^{-1}(1/n)$ (d) $\tan^{-1}(n)$ **(AIEEE 2004)**

13. A thin glass (refractive index 1.5) lens has optical power of –5 D in air. Its optical power in a liquid medium is

(a) 1 D (b) –25 D

(c) 25 D (d) –1 D **(AIEEE 2005)**

14. A fish looking up through water sees the outside world contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the surface, the radius of this circle (in cm) is

(a) $4\sqrt{5}$ (b) $36\sqrt{5}$

(c) $36\sqrt{7}$ (d) $\frac{36}{\sqrt{7}}$ **(AIEEE 2005)**

15. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which dots can be resolved by the eye? [Take wavelength of light = 500 nm]

(a) 6 m (b) 3 m

(c) 5 m (d) 1 m **(AIEEE 2005)**

16. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be the angles of minimum deviation for red and blue lights, respectively, in a prism of this glass. Then

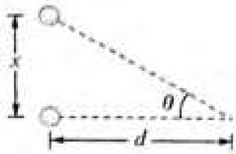
(a) D_1 can be less than or greater than D_2 depending upon the angle of prism.

(b) $D_1 > D_2$

(c) $D_1 = D_2$

(d) $D_1 < D_2$ **(AIEEE 2006)**

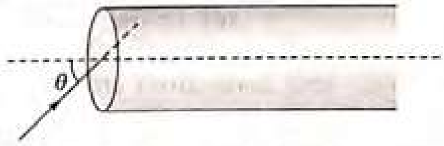
17. Two lenses of powers –15 D and +5 D are in contact with each other. The focal length of the combination is



(a) –10 cm (b) +20 cm

(c) +10 cm (d) –20 cm **(AIEEE 2007)**

18. A transparent solid cylindrical rod has a refractive index of $2/\sqrt{3}$. It is surrounded by air. A light ray is incident at the midpoint of one end of the rod as shown in the figure.



The incident angle θ for which the light ray grazes along the wall of the rod is

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **(AIEEE 2009)**

19. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is

(a) 1/10 m/s (b) 1/15 m/s

(c) 10 m/s (d) 15 m/s **(AIEEE 2011)**

20. Let the x–y plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ in incident on the plane of separation. The angle of refraction in medium 2 is

(a) 30° (b) 45°

(c) 60° (d) 75° **(AIEEE 2011)**

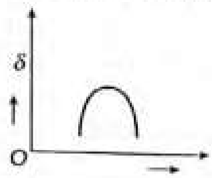
21. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50, is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film?

(a) 7.2 m (b) 2.4 m

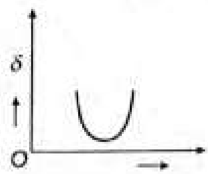
(c) 3.2 m (d) 5.6 m **(AIEEE 2012)**

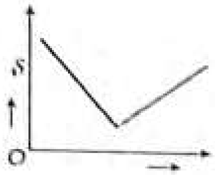
22. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by

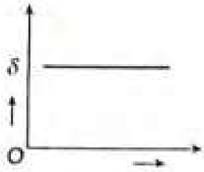
(a)



(b)



(c) 

(d) 

(JEE Main 2013)

23. Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is

(a) 20 cm (b) 30 cm
(c) 10 cm (d) 15 cm **(JEE Main 2013)**

24. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\sqrt{\frac{3}{2}}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is

(a) $f_2 > f$ and f_1 becomes negative
(b) both f_1 and f_2 become negative
(c) $f_1 = f_2 < f$
(d) $f_1 > f$ and f_2 becomes negative **(JEE Main 2014)**

25. A green light is incident from the water to the air–water interface at the critical angle (θ). Select the correct statement

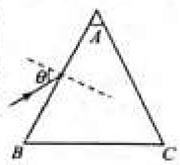
(a) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.

(b) The entire spectrum of visible light will come out of the water at various angles to the normal.

(c) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.

(d) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium. **(JEE Main 2014)**

26. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided:



(a) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(b) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(c) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin\left(\frac{1}{\mu}\right)\right)\right]$

27. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears

(a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer

(JEE Main 2016)

28. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index?

(a) 1.5 (b) 1.6
(c) 1.7 (d) 1.8

(JEE Main 2016)

29. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

(a) real and at a distance of 40 cm from the divergent lens
(b) real and at a distance of 6 cm from the convergent lens
(c) real and at a distance of 40 cm from convergent lens
(d) virtual and at a distance of 40 cm from convergent lens **(JEE Main 2017)**

ANSWER KEY

Exercises

1. (a)	2. (a)	3. (b)	4. (b)	5. (b)	6. (c)	7. (a)	8. (d)	9. (d)	10. (a)
11. (d)	12. (d)	13. (d)	14. (a)	15. (b)	16. (a)	17. (b)	18. (d)	19. (b)	20. (d)
21. (b)	22. (b)	23. (a)	24. (d)	25. (d)	26. (a)	27. (b)	28. (b)	29. (c)	30. (c)
31. (b)	32. (c)	33. (d)	34. (a)	35. (b)	36. (b)	37. (a)	38. (c)	39. (b)	40. (b)
41. (a)	42. (a)	43. (c)	44. (c)	45. (d)	46. (d)	47. (c)	48. (d)	49. (c)	50. (d)
51. (a)	52. (b)	53. (d)	54. (c)	55. (b)	56. (d)	57. (b)	58. (a)	59. (b)	60. (d)
61. (d)	62. (c)	63. (d)	64. (b)	65. (c)	66. (a)	67. (c)	68. (d)	69. (c)	70. (a)
71. (c)	72. (b)	73. (a)	74. (a)	75. (c)	76. (c)	77. (c)	78. (b)	79. (c)	80. (b)
81. (b)	82. (d)	83. (a)	84. (a)	85. (c)	86. (a)	87. (d)	88. (b)	89. (d)	

Archives

1. (b)	2. (c)	3. (d)	4. (a)	5. (d)	6. (d)	7. (b)	8. (c)	9. (c)	10. (d)
11. (a)	12. (d)	13. (None)	14. (d)	15. (c)	16. (d)	17. (a)	18. (d)	19. (b)	20. (b)
21. (d)	22. (b)	23. (b)	24. (d)	25. (d)	26. (a)	27. (c)	28. (a)	29. (c)	

Chapter 27

Wave Optics

HUYGENS' WAVE THEORY

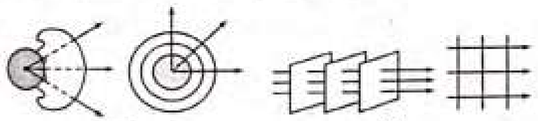
Huygens assumed that a body emits light in the form of waves. According to him, each point source of light is a center of disturbance from which waves spread in all directions.

The locus of all the particles of the medium vibrating in the same phase at a given instant is called the wavefront.

WAVEFRONTS

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outward from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront.

1. Rays are perpendicular to wavefronts.
2. The time taken by light to travel from one wavefront to another is the same along any ray.



(a) (b)

Wavefronts and the corresponding rays in two cases: (a) diverging spherical wave and (b) plane wave. The figure on the left shows a wave (e.g., light) in three dimensions. The figure on the right shows a wave in two dimensions (e.g., a water surface).

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (b).

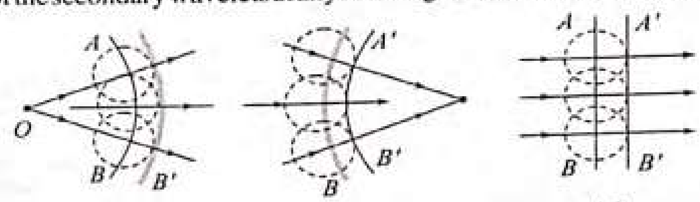
A linear source such as a slit illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wavefronts may be regarded as planar.

A wavefront is a surface joining the points of same phase. For a point source, the wavefronts are spherical which become almost plane at a very large distance. For example, wavefronts of sunlight are plane.

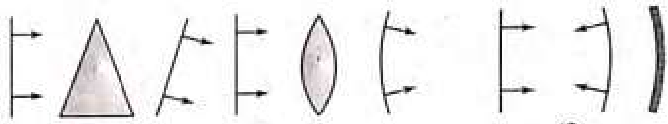
Important Points

- Every point on a wavefront is an independent source to produce secondary wavefronts.
- All points on a wavefront vibrate in same phase with same frequency.
- Wavefronts move with the velocity of wave in that medium.

Each point on a wavefront is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium. The forward envelope of the secondary wavelets at any instant gives the new wave-front.



(a) (b) (c)



(d) (e) (f)

CONDITIONS OF INTERFERENCE

We know that the superpositions of two mechanical waves can be constructive or destructive. In constructive interference, the amplitude of the resultant wave at a given position or time is greater than that of either individual wave.

If two light bulbs are placed side by side, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two light bulbs do not maintain a constant phase relationship with each other over time. Light waves from an ordinary source such as a light bulb undergo random phase changes in time intervals less than a nanosecond. Therefore, the conditions for constructive interference, destructive interference, or some intermediate state are maintained only for such short time intervals. Because the eye cannot follow such rapid changes, no interference effects are observed. Such light sources are said to be incoherent.

In order to observe interference in light waves, the following conditions must be met:

1. The sources must be coherent—that is, they must maintain a constant phase with respect to each other.
2. The source should be monochromatic—that is, of a single wavelength.

COHERENT SOURCES

Two sources are said to be coherent if they produce waves of same frequency with a constant phase difference. Unlike sound waves, two independent sources of light cannot be coherent. Since sound is a bulk property of matter, therefore, two independent sources of sound can be identical in all respects and can produce coherent waves. On the contrary, light is not a bulk property of matter, it is a property of each individual atom. As the individual atoms emit light randomly and independently, therefore, two independent sources of light cannot be coherent.

Coherent sources can be obtained by splitting a light beam from a source into two. This can be done in two ways:

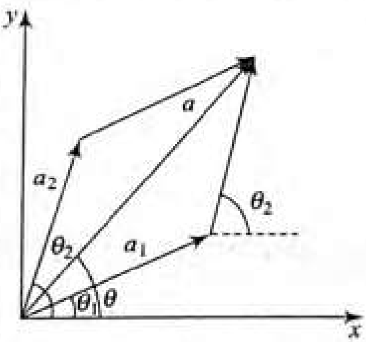
1. **Division of wavefront:** It is done in Young's double-slit experiment, Fresnel's biprism, Lloyd's mirror, etc.
2. **Division of amplitude:** It is usually done by partial reflection and transmission at a boundary, as it occurs in thin film interference, Newton's rings, etc.

INTERFERENCE

We have learned that in the superposition of waves there is interference of two sinusoidal waves. Let the two sinusoidal waves be

$$y_1 = a_1 \cos(\omega t + \theta_1) \qquad \text{(i)}$$

$$y_2 = a_2 \cos(\omega t + \theta_2) \qquad \text{(ii)}$$



According to the principle of superposition, the resultant displacement would be given by

$$y(t) = a\cos(\omega t + \theta) \qquad \text{(iii)}$$

$$a = \left[a_1^2 + a_2^2 + 2a_1a_2\cos\theta\right]^{1/2} \qquad \text{(iv)}$$

$$\therefore \quad \tan\theta = \frac{a_2 \sin\theta}{a_1 + a_2\cos\theta} \qquad \text{(v)}$$

From Eq. (iv), we find that if $\theta = 0, 2\pi, 4\pi, \ldots$, then $a = a_1 + a_2$.

When two displacements are in phase, then the resultant amplitude will be the sum of the two amplitudes; this is constructive interference.

Similarly, if $\theta = 0, 3\pi, 5\pi, \ldots$, then $a = a_1 - a_2$.

For a path difference Δx, the phase difference (in radians) is

$$\delta = \frac{2\pi}{\lambda}\ \text{(path difference)} = \frac{2\pi}{\lambda}(\Delta x)$$

The factor $\Delta x/\lambda$ is number of wavelengths corresponding to path difference. Each wavelength corresponds to a phase difference of π.

The intensity of a wave is proportional to square of amplitude, therefore, Eq. (iv) reduces to

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\delta$$

where $\delta = \frac{2\pi}{\lambda}(\Delta x)$.

As the maximum and minimum values of $\cos\delta$ are +1 and −1, respectively, the maximum and minimum values of I are given by

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad \text{and} \quad I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

The conditions for maximum intensity are

$$\delta = 2n\pi, \quad n = 0, 1, 2, \ldots \quad \text{or} \quad \Delta x = n\lambda$$

The conditions for minimum intensity are

$$\delta = (2n-1)\pi, \quad n = 0, 1, 2, 3, \ldots$$

or $\quad \Delta x = (n - 1/2)\lambda, \quad n = 0, 1, 2, 3, \ldots$

When $I_1 = I_2 = I_0$, then

$$I = 4I_0\cos^2 \delta/2$$

Clearly, if $d = \pm\pi, \pm 3\pi, \ldots$, the resultant intensity will be zero and we will have a minimum. When $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$, the intensity will be maximum ($I = 4I_0$).

NOTE: In interference, energy is neither created nor destroyed but it is conserved.

In case of interference, intensity at a point is given by

$$I = I_1 + I_2 + 2(\sqrt{I_1 I_2})\ \cos\delta$$

Now, as at a given point phase difference can have any value between 0 and 2π, the average value of intensity will be

$$I_{av} = \left[\frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta}\right] = \frac{1}{2\pi}\int_0^{2\pi}(I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\delta)d\delta$$

$$= I_1 + I_2\ [\text{as} \int_0^{2\pi}(\cos\delta)d\delta = 0]$$

As the average value of intensity is equal to the sum of individual intensities, in interference, energy is neither created nor destroyed but is redistributed and, hence, in interference, energy is conserved.

ILLUSTRATION 27.1 In Young's experiment, the interfering waves have amplitudes in the ratio 3:2. Find the ratio of (a) amplitudes and (b) intensities, between the bright and dark fringes.

Solution. Here, we have to obtain the ratio

$$\frac{A_{max}}{A_{min}} = \frac{A_1 + A_2}{A_1 - A_2}$$

and also the corresponding ratio of intensities

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

But $\quad \frac{A_1}{A_2} = \frac{3}{2} \quad$ (given)

By componendo and dividendo,

$$\frac{A_1+A_2}{A_1-A_2}=\frac{3+2}{3-2}=5$$

Hence,

$$\frac{A_{max}}{A_{min}}=5 \quad \text{and} \quad \frac{I_{max}}{I_{min}}=\frac{(A_1+A_2)^2}{(A_1-A_2)^2}=25$$

ILLUSTRATION 27.2 Determine the resultant of two waves given by $y_1 = 4\sin(200\pi t)$ and $y_2 = 3\sin(200\pi t + \pi/2)$.

Solution. In general, resultant of two waves given by $y_1 = A_1 \sin\omega t$ and $y_2 = A_2\sin(\omega t+\phi)$ can be obtained by the principle of superposition, i.e.,

$$y = y_1 + y_2 = A_1\sin\omega t + A_2\sin(\omega t+\phi)$$
$$= A\sin(\omega t+\alpha)$$

where

$$A = [A_1^2 + A_2^2 + 2A_1A_2\cos\phi]^{1/2}$$

$$\alpha = \tan^{-1}\left[\frac{A_2\sin\phi}{A_1+A_2\cos\phi}\right]$$

In the present case,

$$y_1 = 4\sin 200\pi t \quad \text{and} \quad y_2 = 3\sin\left(200\pi t+\frac{\pi}{2}\right)$$

Here, $A_1 = 4$, $A_2 = 3$, and $\phi = \pi/2$.

Resultant wave will be $y = A\sin(200\pi t+\alpha)$ with

$$A = [A_1^2 + A_2^2 + 2A_1A_2\cos\phi]^{1/2}$$

$\Rightarrow \quad A = [4^2 + 3^2 + 2(4)(3)\cos\pi/2]^{1/2}$

$\Rightarrow \quad A = 5$

$$\tan\alpha = \frac{A_2\sin\phi}{A_1+A_2\cos\phi}=\frac{3\sin\pi/2}{4+3\cos\pi/2}=\frac{3}{4}$$

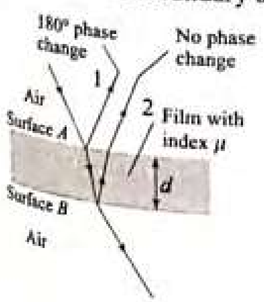
or $\quad \alpha = 37° \approx 0.2\pi$ rad

Hence, the resultant wave is $y = 5\sin(200\pi t + 0.2\pi)$.

THIN-FILM INTERFERENCE

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d, then waves reflected from the upper surface interfere with waves reflected from the lower surface.

Clearly, the waves reflected from the lower surface travel an extra optical path of $2\mu d$, where μ is refractive index of the film. a medium of lower index of refraction. The transmitted wave that crosses the boundary also undergoes no phase change.



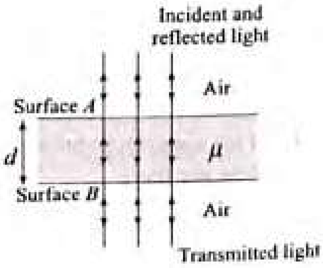


Figure shows that ray reflecting from a medium of higher refractive index undergoes a 180° phase change. The right side shows the analogy with a reflected pulse on a string.

Figure shows that a ray reflecting from a medium of lower refractive index undergoes no phase change. Consequently, condition for constructive and destructive interference in the reflected light is given by

$$2\mu d = \begin{cases} n\lambda & \text{(for destructive interference)} \\ \left(n+\frac{1}{2}\right)\lambda & \text{(for constructive interference)} \end{cases} \qquad \text{(i)}$$

where $n = 0, 1, 2, \ldots$ and λ = wavelength in free space. Interference will also occur in the transmitted light and here conditions of constructive and destructive interference will be the reverse of (1), i.e.,

$$2\mu d = \begin{cases} n\lambda & \text{(for constructive interference)} \\ \left(n+\frac{1}{2}\lambda\right) & \text{(for destructive interference)} \end{cases} \qquad \text{(ii)}$$

ILLUSTRATION 27.3 White light is incident normally on a glass plate of thickness 0.50×10^{-6} and index of refraction 1.50. Which wavelengths in the visible region (400 nm–700 nm) are strongly reflected by the plate?

Solution. The light of wavelength λ is strongly reflected if

$$2\mu d = \left(n+\frac{1}{2}\right)\lambda \qquad \text{(i)}$$

where n is a non-negative integer.

Here $\quad 2\mu d = 2 \times 1.50 \times 0.5 \times 10^{-6}\,\text{m} = 1.5 \times 10^{-6}\,\text{m}$.

Putting $\lambda = 400$ nm in Eq. (i), we get

$$1.5\times10^{-6}\,\text{m} = \left(n+\frac{1}{2}\right)(400\times10^{-9}\,\text{m})$$

or $\quad n = 3.25$

Putting $\lambda = 700$ nm in Eq. (i), we get

$$1.5\times10^{-6}\,\text{m} = \left(n+\frac{1}{2}\right)(700\times10^{-9}\,\text{m})$$

or $\quad n = 1.66$

Thus, within 400 nm to 700 nm, the integer n can take the values 2 and 3. Putting these values of n in (i), the wavelengths become

$$\lambda = \frac{4\mu d}{2n+1} = 600\text{ nm and } 429\text{ nm}$$

Thus, light of wavelengths 429 nm and 600 nm are strongly reflected.

CONCEPT APPLICATION EXERCISE 27.1

1. Two coherent sources of light of intensity ratio β produce interference pattern. Prove that in the interference pattern

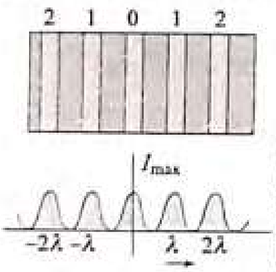
$$\frac{I_{max}-I_{min}}{I_{max}+I_{min}}=\frac{2\sqrt{\beta}}{1+\beta}$$

where I_{max} and I_{min} are maximum and minimum intensities in the resultant wave.

2. Find the maximum intensity in case of interference of n identical waves each of intensity I_0 if the interference is (a) coherent and (b) incoherent.
3. Explain why two flashlights held close together do not produce an interference pattern on a distant screen.
4. Refractive index of a thin soap film of a uniform thickness is 1.34. Find the smallest thickness of the film that gives an interference maximum in the reflected light when light of wavelength 5360 Å falls at normal incidence.
5. A thin transparent film of thickness 3000 Å and refractive index 1.5 is deposited on a sheet made of a metal. Assuming normal incidence of light and also that the film is a plane parallel one, what will be the color of a pot made from this sheet when observed in white light?
6. By an anodizing process, a transparent film of aluminium oxide of thickness t = 250 nm and index of refraction n_2=1.80isdepositedonasheetofpolishedaluminium.What is the color of utensils made from this sheet with observer in white light? Assume normal incidence of the light.

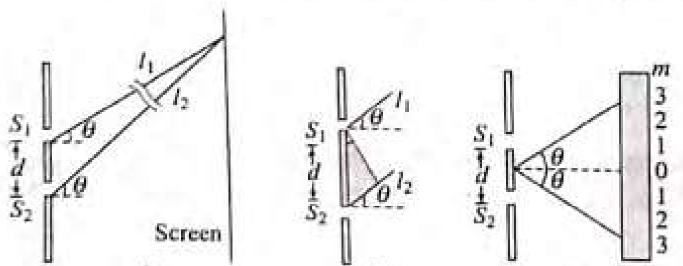
YOUNG'S DOUBLE-SLIT EXPERIMENT

The brightness of the fringes in Young's experiment varies, as the photograph in figure shows. Below the photograph is a graph to suggest the way in which the intensity varies for the fringe pattern. The central fringe has the greatest intensity. To either side of the center, the intensities of the other fringes decrease symmetrically in a way that depends on how small the slit widths are relative to the wavelength of the light.



Light intensity versus d sin θ for a double slit interference pattern when the screen is far from the two slits ($L >> d$)

The position of the fringes observed on the screen in Young's experiment can be calculated with the aid of figure. If the screen is located far away compared with the separation d of the slits, then the lines labeled l_1 and l_2 in part (a) are nearly parallel. Being nearly parallel, these lines make approximately equal angles θ with the horizontal. The distances l_1 and l_2 differ by an amount Δl, which is the length of the short side of the shaded triangle in part (b) of the figure. Since the triangle is a right triangle, it follows that $\Delta l = d \sin \theta$. Constructive interference occurs when the distances differ by an integer number m of wavelengths λ, or $\Delta l = d \sin \theta = m\lambda$. Therefore, the angle θ for the interference maxima can be determined from the following expression.



(a) (b) (c)

(a) Rays from slits S_1 and S_2, which make approxi-mately the same angle θ with the horizontal, strike a distant screen at the same spot. (b) The difference in the path lengths of the two rays is $\Delta l = d \sin \theta$. (c) The angle θ is the angle at which a bright fringe (m = 2, here) occurs on either side of the central bright fringe (m = 0).

Bright fringes of a double slit:

$$\sin \theta = m\frac{\lambda}{d}, m = 0, 1, 2, 3, \ldots \quad \text{(i)}$$

The value of m specifies the order of the fringe. Thus, $m = 2$ identifies the 'second-order' bright fringe. Part (c) of the figure stresses that the angle θ given by Eq. (i) locates bright fringes on either side of the midpoint between the slits. A similar line of reasoning leads to the conclusion that the dark fringes, which lie between the bright fringes, are located according to the following expression.

Dark fringes of a double slit:

$$\sin \theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}, m = 0, 1, 2, 3, \ldots \quad \text{(ii)}$$

Position of Bright and Dark Fringes in YDSE

Let us consider point P on the distant screen, at a distance D from the slits with $D \gg d$. The small arc of the circle from P is almost a straight line. From figure, path difference $\Delta x = d \sin \theta$.

The condition for constructive interference is

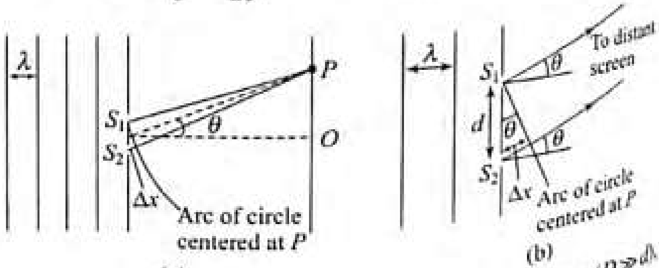
$$\Delta x = S_2P - S_1P = \pm n\lambda, \text{ where } n = 0, 1, 2, \ldots \quad \text{(iii)}$$

$$d \sin \theta = \pm n\lambda \quad \text{[Condition for maxima]}$$

The condition for destructive interference is

$$\Delta x = S_2P - S_1P = \pm\left(n - \frac{1}{2}\right)\lambda, \text{ where } n = 1, 2, \ldots$$

$$d \sin \theta = \pm\left(n - \frac{1}{2}\right)\lambda \quad \text{[Condition for minima] (iv)}$$



(a) (b)

1. If the separation between screen and slits is large ($D \gg d$), then we have

$$\sin \theta \approx \tan \theta = \theta = \frac{y}{D}$$

where y is the vertical distance from the center of the pattern.

Position of nth bright and dark fringes are, respectively,

$$\frac{y_n d}{D} = n\lambda \quad \text{or} \quad y_n = n\lambda\left(\frac{D}{d}\right)$$

and $$\frac{y_n d}{D} = \left(n - \frac{1}{2}\right)\lambda \quad \text{or} \quad y_n = \left(n - \frac{1}{2}\right)\lambda\frac{D}{d}$$

Each value of n corresponds to particular bright or dark fringe. The absolute value of n is called the order of interference.

2. The nth and $(n + 1)$th maxima are given by

$$y_n = \frac{n\lambda D}{d} \quad \text{or} \quad y_{n+1} = \frac{(n+1)\lambda D}{d}$$

Fringe Width

Separation between adjacent maxima is defined as fringe width β.

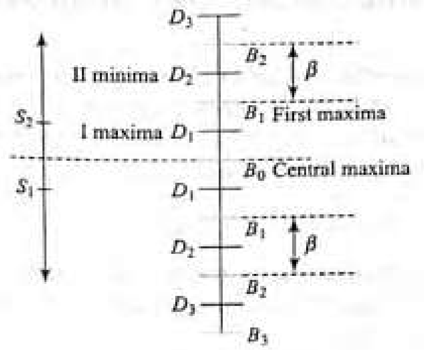
$$\therefore \quad \beta = y_{n-1} - y_n = \frac{\lambda D}{d}$$

This is also the expression for separation between adjacent minima. As long as d and θ are small, the separation between interference fringes is independent of n (order of fringe), i.e., the fringes are evenly spaced. As vertical distance y is related to θ by $\theta = y/D$, so $\Delta\theta = \Delta y/D = \beta/D = \lambda/d$ is referred to as angular fringe width.

NOTE: Fringe width is directly proportional to wavelength and inversely proportional to the distance between the two slits.

From the above equation, following points can be noted.

1. Fringe width is independent of n, i.e., all the interference fringes have same width in experiments where there is a division of wavefront of the incoming waves.



Fringe pattern in YDSE

2. Fringe width is directly proportional to the wavelength of light used, i.e., $\beta \propto \lambda$. So, fringes for red light are wider than those for blue light.
3. Fringe width is inversely proportional to the separation between the slits, i.e., $\beta \propto (1/d)$. Thus, with increase in separation between the sources, fringe width decreases.
4. With increase in distance between screen and plane of slits, fringe width β increases linearly with D. However, with increase in D, intensity of light sources and hence of interfering waves is adversely affected.
5. If the interference experiment is performed in a medium of refractive index μ (say water) instead of air, the wavelength of light will change from λ to (λ/μ) and so

$$\beta' = \frac{D}{d}\left[\frac{\lambda}{\mu}\right] = \frac{\beta}{\mu} \qquad (v)$$

That is, fringe width reduces and becomes $(1/\mu)$ times of its value in air.

If monochromatic light is replaced by white light, due to overlapping of patterns (each corresponding to a single wavelength with fringe width $\beta \propto \lambda$), central band, i.e., principal maxima will be white with red edges.

On either side of it, we shall get a few colored bands and then uniform illumination.

Following are the conditions for observing sustained interference with good contrast:

1. The initial phase difference between the interfering waves must remain constant otherwise the interference will not be sustained.
2. The frequencies and wavelength of the two waves should be equal. If not, the phase difference will not remain constant and so the interference will not be sustained.
3. The light must be monochromatic. This eliminates overlapping of pattern as each wavelength corresponds to one interference pattern.
4. The amplitudes of the interfering waves must be equal. This improves constant with $I_{max} = 4I_0$ and $I_{min} = 0$.
5. The sources must be close to each other, otherwise due to small fringe, width fringes may be so close to each other that the eye cannot resolve them, resulting in uniform illumination.
6. The sources must be narrow. A board source will be equal to a large number of narrow sources and each set of two sources will give its own pattern and overlapping of patterns will result in uniform illumination.

Maximum Order of Interference Fringes

The position of nth order maxima on the screen,

$y = \frac{n\ D}{\ }$, $n = 0, \pm1, \pm2$ [for interference maxima, but n cannot take infinitely large values, as that would violate the approximation (II)]

i.e., θ is small or $y \ll D \Rightarrow \frac{y}{D} = \frac{n\lambda}{d} \ll 1$

Hence, the above formula for interference maxima is applicable when $n \ll d/\lambda$.

When n becomes comparable to d/λ, path difference can no longer be given by dy/D. Hence, for maxima,

$$\Delta x = n\lambda \Rightarrow d\sin\theta = n\lambda \Rightarrow n = \frac{d\sin\theta}{\lambda}$$

Hence, the highest order of interference maxima,

$$n_{max} = \left[\frac{d}{\lambda}\right] \quad \text{(vi)}$$

where [ ] represents the greatest integer function.

Similarly, the highest order of interference minima,

$$n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] \quad \text{(vii)}$$

ILLUSTRATION 27.4 Monochromatic light of wavelength 5000 Å is used in YDSE, with slit width, $d = 1$ mm, distance between screen and slits, $D = 1$ m. If intensites at the two slits are $I_1 = 4I_0$ and $I_2 = I_0$, find:

(a) fringe width β;
(b) distance of 5th minima from the central maxima on the screen;
(c) intensity at $y = \frac{1}{3}$ mm;
(d) distance of the 1000th maxima; and
(e) distance of the 5000th maxima.

Solution.

(a) $\beta = \dfrac{\lambda D}{d} = \dfrac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5$ mm

(b) $y = (2n-1)\dfrac{\lambda D}{d}, n = 5 \Rightarrow y = 2.25$ mm

(c) At $y = \dfrac{1}{3}$ mm, $y \ll D \Rightarrow \Delta x = \dfrac{dy}{D}$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$$

Now, resultant intensity,

$$I = I_1 + I_2 + 2\cos\Delta\phi = 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi$$

$$= 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$$

(d) $\dfrac{d}{\lambda} = \dfrac{10^{-3}}{0.5 \times 10^{-6}} = 2000$

$n = 1000$ is not $\ll 2000$

Hence, now $\Delta x = d\sin\theta$ must be used.

$$\therefore \quad d\sin\theta = n\lambda = 1000\,\lambda \Rightarrow \sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2}$$
$$\Rightarrow \theta = 30^\circ$$

$$y = D\tan\theta = \frac{1}{\sqrt{3}}\text{ m}$$

(e) Highest order maxima, $n_{max} = \left[\dfrac{d}{\lambda}\right] = 2000$

Hence, $n = 5000$ is not possible.

Shape of Interference Fringes in YDSE

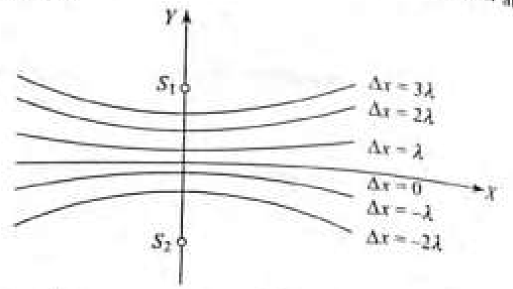
We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$$S_2P - S_1P = \Delta x = \text{constant} \quad \text{(i)}$$

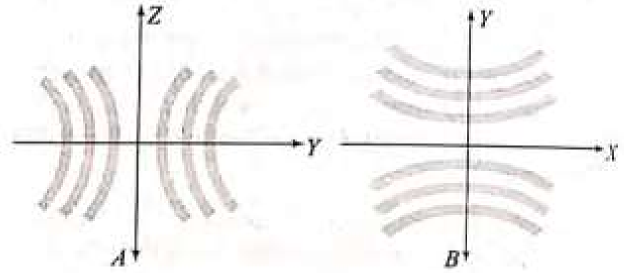
If $\Delta x = \pm\lambda/2$, the fringe represents 1st minima.
If $\Delta x = \pm 3\lambda/2$, it represents 2nd minima.
If $\Delta x = 0$, it represents central maxima.
If $\Delta x = \pm\lambda$, it represents 1st maxima, etc.

Equation (i) represents a hyperbola with its two foci at S_1 and S_2 (see figure).



The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

If the screen is perpendicular to the X-axis, i.e., in the YZ plane, as is generally the case, firinges are hyperbolic with a straight central section [see Figure (a)].



(a) (b)

the screen is in the XY plane, again fringes are hyperbolic [see Figure (b)].

Young's Double-Slit Experiment with White Light

The central maxima will be white because all wavelengths will constructively interference here. However, slightly below (or above) the position of central maxima fringes will be colored. For example, if P is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{violet}}{2} = 190 \text{ nm}$$

then completely destructive interference will occur for violet light. Hence, we will have a line devoid of violet color that will appear reddish. And if

$$S_2P - S_1P = \frac{\lambda_{red}}{2} = 350 \text{ nm}$$

then completely destructive interference for red light results and the line at this position will be violet. The colored fringes disappear at points far away from the central white fringe; for these points, there are so many wavelengths which interfere constructively that we obtain a uniform white illumination. For example, if $S_2P - S_1P = 3000$ nm, then constructive interference will occur for wavelengths $\lambda = 3000/n$ nm. In the visible region these wavelengths are 750 nm (red), 600 nm (yellow), 500 nm

(greenish-yellow), and 430 nm (violet). Clearly, such a light will appear white to the unaided eye.

ILLUSTRATION 27.5 White coherent light (4000–7000 Å) is sent through the slits of a YDSE. The separation between the slits is 0.5 mm and screen is 50 cm away from the slits. There is a hole in the screen at a point 1.0 mm away (along the width of the fringe) from the central line.

(a) Which wavelength(s) will be absent in the light coming from the hole?

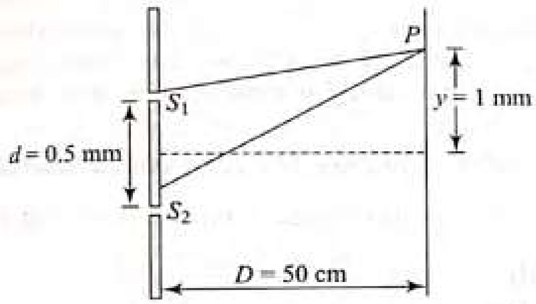
(b) Which wavelength(s) will have a strong intensity?

Solution.

(a) The absent wavelength will correspond to minima at this position order of minima corresponding to 4000 Å,

$$y_n = \frac{(2n-1)D\lambda}{2d} \Rightarrow (2n-1)$$

$$= \frac{y_n 2d}{D\lambda} \Rightarrow n = \frac{1}{2}\left[\frac{2d\,y_n}{D\lambda} + 1\right]$$



$$n_1 = \frac{1}{2}\left[\frac{0.5\times10^{-3}\times1\times10^{-3}\times2}{50\times10^{-2}\times4000\times10^{-10}} + 1\right]$$

$$= \frac{1}{2}\left[\frac{2\times10^4}{4000} + 1\right] = 3$$

Order of minima corresponding to 7000 Å,

$$n_2 = \frac{1}{2}\left[\frac{2\times10^4}{7000} + 1\right] = 1.9$$

Number of integers between 1.9 and 3.0 are 2 and 3.

Wavelength corresponding to $n = 2$ is

$$\lambda_{(2)} = \frac{y_n 2d}{(2n-1)D} = \frac{1\times10^{-3}\times2\times0.5\times10^{-3}}{(2\times2-1)\times50\times10^{-2}}$$

$$= \frac{10^{-4}}{(2\times2-1)\times50} = \frac{2\times10^{-6}}{3} = 0.6667\times10^{-6}\text{ m}$$

$$= 667\text{ nm}$$

Now, wavelength corresponding to $n = 3$ is

$$\lambda_{(3)} = \frac{2\times10^{-6}}{(2\times3-1)} = 400\text{ nm}$$

Hence the wavelengths 400 nm and 667 nm will be absent at the hole.

(b) The wavelengths corresponding to strong intensity will correspond to maxima at the position. Order of maxima corresponding to λ,

$$y_n = \frac{nD\lambda}{d} \Rightarrow n = \frac{y_n d}{D\lambda} = \frac{1\times10^{-3}\times0.5\times10^{-3}}{50\times10^{-2}\times\lambda}$$

$$n_{4000} = \frac{10^{-6}}{4000\times10^{-10}} = \frac{10000}{4000} = 2.5$$

and $\quad n_{7000} = \dfrac{10000}{7000} = 1.4$

Integer between 1.4 and 2.5 is 2.

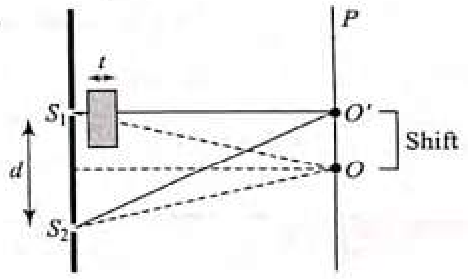
$$\therefore \quad \lambda_{(2)} = \frac{y_n d}{nD} = \frac{10^{-6}}{2} = 500\text{ nm}$$

Displacement of Fringes

When we introduce a thin transparent plate in front of one of the slits in YDSE, the fringe pattern shifts toward the side where the plate is present.

The path length S_1O' is

$$(S_1O')_{new} = (S_1O' - t)_{air} + t_{plate}$$
$$= (S_1O' - t)_{air} + (\mu t)_{air}$$
$$= (S_1O') - (t)_{air} + (\mu t)_{air}$$
$$= (S_1O') + (\mu t - t)$$
$$= (S_1O') + [t(\mu - 1)]$$



Path difference, $S_2O' - S_1O'_{new} = \Delta x$

$$S_2O' - (S_1O')_{new} = S_2O' - \{(S_1O') + [t\mu - 1]\}$$

$$= d\sin\theta - [t(\mu-1)] = \frac{yd}{D} - [t(\mu-1)]$$

$$\Delta x = d\sin\theta - [t(\mu - 1] = \frac{yd}{D} - [t(\mu-1)]$$

$$\Rightarrow \quad y = \frac{\Delta x D}{d} + \frac{D}{d}[t(\mu-1)]$$

If a transparent sheet of thickness t and refractive index μ is introduced in one of the paths of the interfering waves, then the path increases by $(\mu - 1)t$ and the whole fringe pattern shifts by

$$s = \frac{D}{d}(\mu - 1)t$$

NOTE: This fringe shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upward and if the glass slab is placed before the lower slit, the fringe pattern gets shifted downward.

ILLUSTRATION 27.6 A monochromatic light of $\lambda = 500$ Å is incident on two identical slits separated by a distance of 5×10^{-4}m. The interference pattern is seen on a screen placed at a distance of 1 m from the plane of slits. A thin glass plate of thickness 1.5×10^{-6} m and refractive index $\mu = 1.5$ is placed between one of the slits and the screen. Find the intensity at the center of the screen.

Solution. In case of interference,

$$I_R = I_1 + I_2 + 2(\sqrt{I_1 I_2}) \cos \phi$$

Now, as for identical slits $I_1 = I_2 = I$, so

$$I_R = 2(1 + \cos \phi) = 4I \cos^2 (\phi/2)$$

But for central maxima,

$$\phi = 0°, \text{ and here } I_R = I_0 \text{ (given)}$$

$$I_0 = 4I \cos(0°) = 4I$$

Hence, $I_R = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$ (i)

Now, when the glass plate is introduced, path difference between the waves at the position of central maxima will become

$$\Delta x = (\mu - 1)t = (1.5 - 1)1.5 \times 10^{-6} = 7.5 \times 10^{-7} \text{ m} \quad \text{(ii)}$$

$$\therefore \quad \phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{5 \times 10^{-7}} \times 7.5 \times 10^{-7} = 3\pi$$

So, intensity at central maxima will now be

$$I_R = I_0 \cos^2\left(\frac{3\pi}{2}\right) = I_0 \times 0 = 0$$

CONCEPT APPLICATION EXERCISE 27.2

1. In Young's double slit experiment with wavelength 5890 Å, there are 60 fringes in the field of vision. How many fringes will be observed in the same field of vision if wavelength used is 5460 Å?
2. In YDSE performed with wavelength $\lambda = 5890$ Å, the angular fringe width is 0.40°. What is the angular fringe width if the entire set-up is immersed in water?
3. In YDSE for wavelength $\lambda = 589$ nm, the interference fringes have angular separation of 3.50×10^{-3} rad. For what wavelength would the angular separation be 10.0% greater?
4. A beam of light consisting of wavelengths 6000 Å and 4500 Å is used in a YDSE with $D = 1$ m and $d = 1$ mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.
5. White light is used in a YDSE with $D = 1$ m and $d = 0.9$ mm. Light reaching the screen at position $y = 1$ mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.
6. Monochromatic light of wavelength of 600 nm is used in YDSE. One of the slits is covered by a transparent sheet of thickness 1.8×10^{-5} m made of a material of refractive index 1.6. How many fringes will shift due to the introduction of the sheet?

Fresnel's Biprism

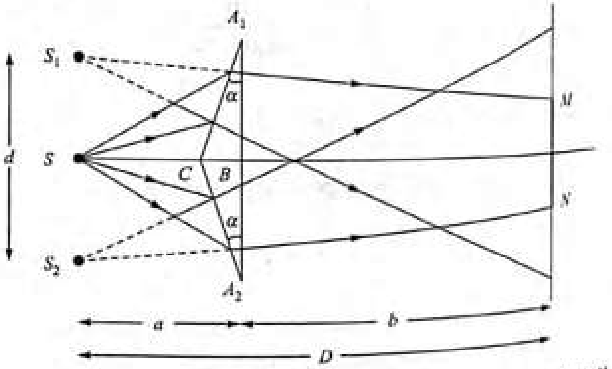
It is an optical device of producing interference of light. Fresnel's biprism is made by joining base to base two thin prism (A_1BC and A_2BC as shown in the figure) of very small angle or by grinding a thick glass plate. Acute angle of prism is about 1/2° and obtuse angle of prism is about 179°.

When a monochromatic light source is kept in front of biprism two coherent virtual source S_1 and S_2 are produced.

Interference fringes are found on the screen (in the MN region) placed behind the biprism interference fringes are formed in the limited region which can be observed with the help eye piece.

Fringe width is measured by a micrometer attached to the eye piece. Fringes are of equal width and its value is $\beta = \dfrac{\lambda D}{d}$

$$\Rightarrow \lambda = \frac{\beta d}{D}$$

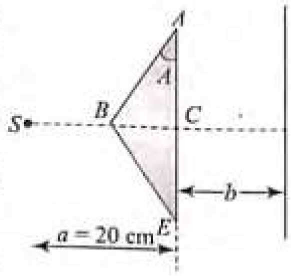


Let the separation between S_1 and S_2 be d and the distance of slits and the screen from the biprism be a and b respectively i.e., $D = (a + b)$. If angle of prism is α and refractive index is μ then $d = 2a(\mu - 1)\alpha$

$$\therefore \quad \lambda = \frac{\beta[2a(\mu-1)\alpha]}{(a+b)} \Rightarrow \beta = \frac{(a+b)\lambda}{2a(\mu-1)\alpha}$$

ILLUSTRATION 27.7 Two small angled transparent prisms (each of refracting angle $A = 1°$) are so placed that their bases coincide, so that common base is BC. This device is called Fresnel's biprism and is used to obtain coherent sources of a point source S illuminated by monochromatic light of

wavelength 6000 Å placed at a distance $a = 20$ cm. Calculate the separation between coherent sources. If a screen is placed at a distance $b = 80$ cm from the device, what is the fringe width of fringes obtained (Refractive index of material of each prism = 1.5).



Solution. The virtual images of slit S are formed at S_1 and S_2 by means of prism ABC and BCE, respectively. These images act as coherent sources. The deviation produced by small-angled prism is

$$\delta = (\mu - 1)A$$

Also $\tan \delta = \delta = \dfrac{SS_1}{AS_1} = \dfrac{d/2}{a}$

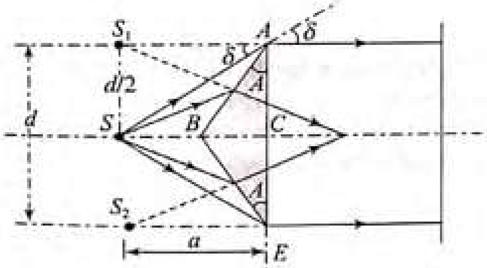
Therefore, separation between coherent sources is

$$d = 2a\delta = 2a(\mu - 1)A$$

Here, $a = 20$ cm $= 0.20$ m

$$A = 1° = \frac{\pi}{180} = \frac{3.14}{180} \text{ rad}$$

$$\therefore \quad d = 2 \times 0.20 \times (1.5 - 1)\frac{3.14}{180} \text{ m} = 3.84 \times 10^{-3} \text{ m}$$



Separation between coherent sources and screen is

$$D = a + b = 20 + 80 = 100 \text{ cm} = 1 \text{ m}$$

$$\lambda = 6000 \text{ Å} = 6000 \times 10^{-10} \text{ cm}$$

Therefore, fringe width

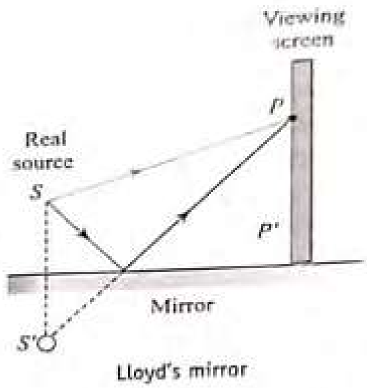
$$\beta = \frac{D\lambda}{d} = \frac{1 \times 6000 \times 10^{-10}}{3.48 \times 10^{-3}}$$

$$= 1.724 \times 10^{-4} \text{ m} = 0.1724 \text{ mm}$$

LLOYD'S MIRROR EXPERIMENT

Change of Phase Due to Reflection

Young's method of producing two coherent light sources involves illuminating a pair of slits with a single source. Another simple, although ingenious, arrangement for producing an interference pattern with a single light source is known as Lloyd's mirror. A light source is placed at point S, close to a mirror, as illustrated in figure. Waves can reach the viewing point P either by the direct path SP or by the path involving reflection from the mirror. The reflected ray can be treated as a ray originating at the source S' behind the mirror. Source S', which is the image of S, can be considered a virtual source.

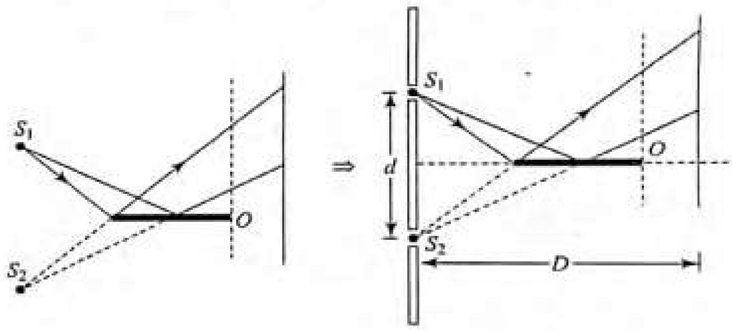


Lloyd's mirror

At points far from the source, we would expect an interference pattern due to waves from S and S', just as is observed for two real coherent sources. An interference pattern is indeed observed. However, the positions of the dark and bright fringes are reversed relative to the pattern of two real coherent sources (Young's experiment). This is because the coherent sources S and S' differ in phase by 180°. This 180° phase change is produced by reflection.

An interference pattern is produced on a screen at P as a result of the combination of the direct ray and the reflected ray.

The reflected ray undergoes a phase change of 180°.

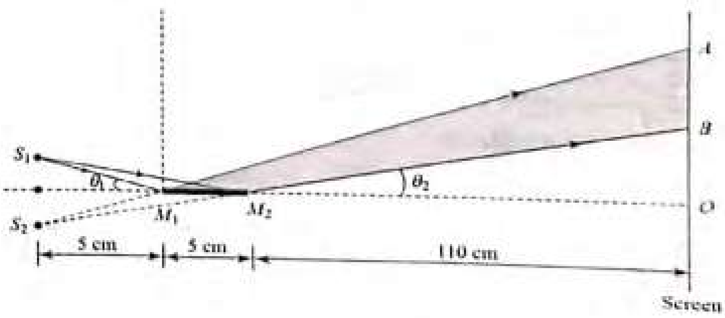


The direct beam does not suffer any phase change of π radian. Hence, at point P on the screen the conditions for minima and maxima are

$$S_2P - S_1P = n\lambda, \; n = 0, 1, 2, 3, \ldots \qquad \text{[minima]}$$

$$S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda \qquad \text{[maxima]}$$

ILLUSTRATION 27.8 A Lloyd's mirror of length 5 cm is illuminated with monochromatic light of wavelength λ



(= 6000 Å) from a narrow 1 mm slit in its plane and 5 cm plane from its near edge. Find the fringe width on a screen 120 cm from the slit and width of interference pattern on the screen.

Solution. In a plane mirror, image is formed behind it as the object is in front of it. So,

$$d = 2 \text{ mm} = 0.2 \text{ cm}$$

$$\lambda = 6000 \text{ Å} = 6000 \times 10^{-8} \text{ cm}$$

$$D = 120 \text{ cm}$$

Therefore, fringe width,

$$\beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-5} \times 120}{0.2} = 0.036 \text{ cm}$$

The width of fringe pattern is AB. From figure,

$$\tan \theta_1 = \frac{0.1}{5} \quad \text{and} \quad \tan \theta_2 = \frac{0.1}{10}$$

In right-angled triangles AM_1O and BM_2O,

$$\tan \theta_1 = \frac{0.1}{5} = \frac{OA}{M_1O} \Rightarrow OA = 115 \times \frac{0.1}{5} \text{ cm},$$

$$\tan\theta_2 = \frac{0.1}{10} = \frac{OB}{OM_2} \Rightarrow OB = 110 \times \frac{0.1}{10} \text{ cm}$$

Hence, width of fringe pattern is

$$OA - OB = \frac{115 \times 0.1}{5} - \frac{110 \times 0.1}{10} = 1.2 \text{ cm}$$

DIFFRACTION

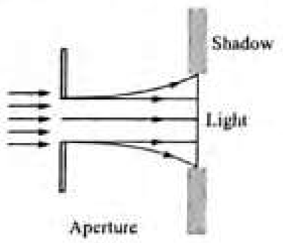
Diffraction is the penetration of light wave towards the geometrical shadow region around a sharp obstacle such as the edge of a slit, sharp aperture etc.

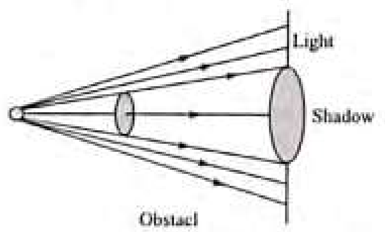
(a) In Fresnel class of diffraction, the source and/or screen are at a finite distance from the aperture. The wavefront is either spherical or cylindrical in shape

(b) In Fraunhoffer class of diffraction, the source and screen are at infinite distance from the diffracting aperture. The wavefront is plane.

Diffraction of Light

It is the phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength of light.

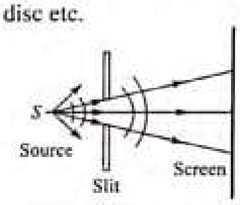
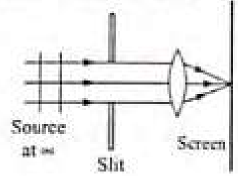




- Diffraction is the characteristic of all types of waves.
- Greater the wavelength of wave, higher will be it's degree of diffraction.
- Experimental study of diffraction was extended by Newton as well as Young. Most systematic study carried out by Huygens on the basis of wave theory.
- The minimum distance at which the observer should be from the obstacle to observe the diffraction of light of wavelength λ around the obstacle of size d is given by $x = \frac{d^2}{4\lambda}$.

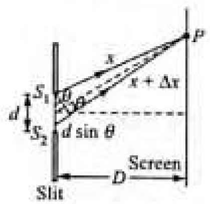
Types of Diffraction

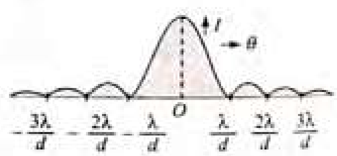
The diffraction phenomenon is divided into two types

Fresnel diffraction	Fraunhofer diffraction
(i) If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel type.	(i) In this case both source and screen are effectively at infinite distance from the diffracting device.
(ii) Common examples: Diffraction at a straight edge, narrow wire or small opaque disc etc. 	(ii) Common examples: Diffraction at single slit, double slit and diffraction grating. 

Diffraction of Light at a Single Slit

In case of diffraction at a single slit, we get a central bright band with alternate bright (maxima) and dark (minima) bands of decreasing intensity as shown





- Width of central maxima $\beta_0 = \frac{2\lambda D}{d}$; and angular width $= \frac{2\lambda}{d}$
- Minima occurs at a point on either side of the central maxima, such that the path difference between the waves from the two ends of the aperture is given by $\Delta = n\lambda$; where $n = 1, 2, 3...$

  i.e., $d \sin \theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$
- The secondary maxima occurs, where the path difference between the waves from the two ends of the aperture is given by $\Delta = (2n+1)\frac{\lambda}{2}$; where $n = 1, 2, 3...$

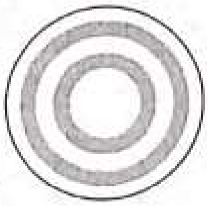
i.e., $d\sin\theta = (2n+1)\dfrac{\lambda}{2} \Rightarrow \sin\theta = \dfrac{(2n+1)\lambda}{2d}$

Comparison Between Interference and Diffraction

Interference	Diffraction
Results due to the superposition of waves from two coherent sources.	Results due to the superposition of wavelets from different parts of same wave front. (single coherent source)
All fringes are of same width $\beta = \dfrac{\lambda D}{d}$	All secondary fringes are of same width but the central maximum is of double the width $\beta_0 = 2\beta = 2\dfrac{\lambda D}{d}$
All fringes are of same intensity	Intensity decreases as the order of maximum increases.
Intensity of all minimum may be zero	Intensity of minima is not zero.
Positions of nth maxima and minima $x_{n(\text{Bright})} = \dfrac{n\lambda D}{d}$, $x_{n(\text{Dark})} = (2n-1)\dfrac{\lambda D}{d}$	Positions of nth secondary maxima and minima $x_{n(\text{Bright})} = (2n+1)\dfrac{\lambda D}{d}$, $x_{n(\text{Dark})} = \dfrac{n\lambda D}{d}$
Path difference for nth maxima $\Delta = n\lambda$	for nth secondary maxima $\Delta = (2n+1)\dfrac{\lambda}{2}$
Path difference for nth minima $\Delta = (2n-1)\lambda$	Path difference for nth minima $\Delta = n\lambda$

Diffraction and Optical Instruments

The objective lens of optical instrument like telescope or microscope etc. acts like a circular aperture. Due to diffraction of light at a circular aperture, a converging lens cannot form a point image of an object rather it produces a brighter disc known as Airy disc surrounded by alternate dark and bright concentric rings.



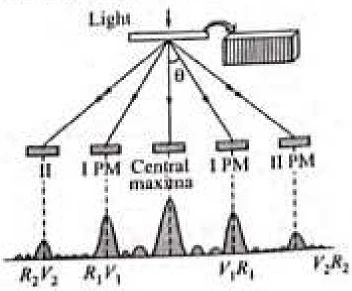
The angular half width of Airy disc $= \theta = \dfrac{1.22\lambda}{D}$ (where D = aperture of lens)

The lateral width of the image $= f\theta$ (where f = focal length of the lens)

Diffraction of light limits the ability of optical instruments to form clear images of objects when they are close to each other.

Diffraction Grating

Consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the diffraction of principle maxima (PM) is given by d

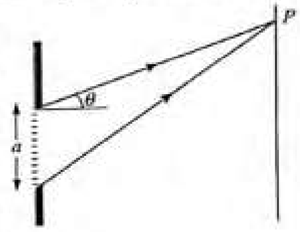


$\sin\theta = n\lambda$; where d = distance between two consecutive slits and is called grating element.

Single Slit Fraunhoffer Diffraction

A slit of width 'a' is divided into N parallel strips of width Δx. Each strip acts as a radiator of Huygen's wavelets and produces a characteristic wave disturbance at P, whose position on the screen for a particular arrangement of apparatus can be described by the angle θ.

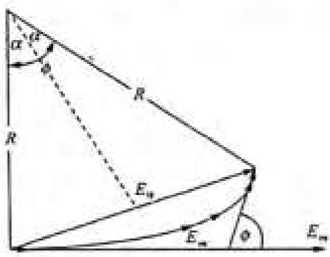
If the strips are narrow enough - which we assume - all points on a given strip have essentially the same optical path length to P, and therefore all the light from the strip will have the same phase when it arrives at P. The amplitudes ΔE_0 of the wave disturbances at P from the various strips may be taken as equal if θ is not too large.



The wave disturbances from adjacent strips have a constant phase difference $\Delta\phi$ between them at P, given by

$$\frac{\text{phase difference}}{2\pi} = \frac{\text{path difference}}{\lambda} \Rightarrow \Delta\phi = \frac{2\pi}{\lambda}(\Delta x\sin\theta)$$

Thus at P, N vectors with the same amplitude ΔE_0, the same frequency and the same phase difference $\Delta\phi$ between adjacent members combine to produce a resultant disturbance. In the limiting case ($N \to \infty$), $\Delta\phi \to 0$



From geometry, $E_\theta = 2R\sin\left(\dfrac{\phi}{2}\right)$, $\phi = \dfrac{E_m}{R}$

$$\Rightarrow \quad E_\theta = \frac{E_m}{\phi/2}.\sin\left(\frac{\phi}{2}\right)$$

or $E_\theta = E_m.\dfrac{\sin\alpha}{\alpha}$, where $\alpha = \dfrac{\phi}{2}$

As $\phi = \dfrac{2\pi}{\lambda}[a\sin\theta] \Rightarrow \alpha = \dfrac{\pi a\sin\theta}{\lambda}$

As $I \propto E^2 \Rightarrow I_\theta = I_m\left(\dfrac{\sin\alpha}{\alpha}\right)^2$

In brief, $E_\theta = E_m\left(\dfrac{\sin\alpha}{\alpha}\right)$; $I_\theta = I_m\left(\dfrac{\sin\alpha}{\alpha}\right)^2$; $\alpha = \dfrac{\phi}{2}\ \dfrac{\pi a\sin\theta}{\lambda}$

Single Slit Diffraction Formula

Minima occurs when, $\alpha = n\pi$, $n = 1, 2, 3...$

$$\Rightarrow \quad \frac{\pi a\sin\theta}{\lambda} = n\pi \quad \Rightarrow a\sin\theta = n\lambda$$

[Condition for minima]

If θ is small, $\theta_{\min} = \dfrac{n\lambda}{a}$.

ILLUSTRATION 27.10 A beam of light of wave length 600 nm from a distant source falls on a single slit 1 mm wide and the

resulting diffraction pattern is observed on a screen 2 m away. Find the distance between the first dark fringes on either side of the central bright.

Solution.

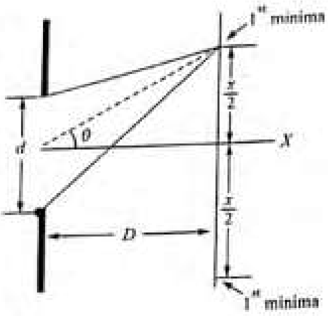
For first minimum putting $n = 1$ in the condition for minima given as

$$d \sin \theta = n\lambda$$

we obtain $d \sin \theta = \lambda$ ...(1)

where $\sin \theta \cong 0 \cong \dfrac{x/2}{D}$

$\Rightarrow \quad \sin \theta = \dfrac{x}{2D}$ ...(2)



Using (1) and (2) we obtain, $\dfrac{\lambda}{d} = \dfrac{x}{2D}$

$$x = \frac{2\lambda D}{d} = \frac{2(600 \times 10^{-9})(2)}{10^{-3}}$$

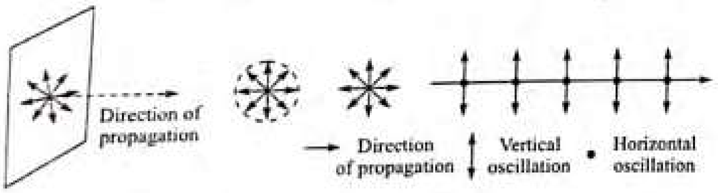
$$= 2.4 \times 10^{-3} \text{ m} = 2.4 \text{ mm}$$

POLARISATION OF LIGHT

Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. We generally prefer to describe light as electric field oscillations.

Unpolarised Light

The light having electric field oscillations in all directions in the plane perpendicular to the direction of propagation is called Unpolarised light. The oscillation may be resolved into horizontal and vertical component.



Polarised Light

The light having oscillations only in one plane is called Polarised or plane polarised light.

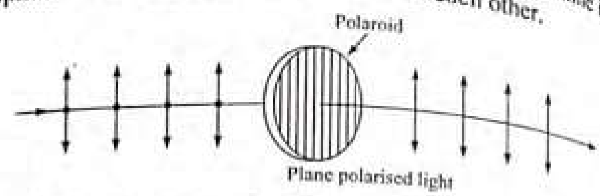
- The plane in which oscillation occurs in the polarised light is called plane of oscillation.
- The plane perpendicular to the plane of oscillation is called plane of polarisation.
- Light can be polarised by transmitting through certain crystals such as tourmaline or polaroids.

Polaroids

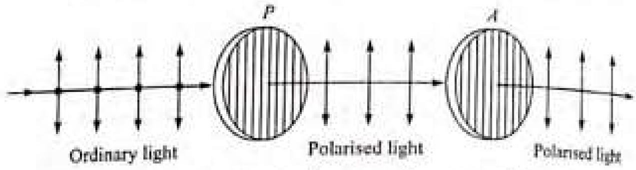
It is a device used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal.

or

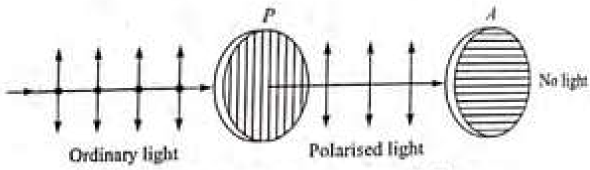
It is a thin film of ultramicroscopic crystals of quinine idosulphate with their optic axis parallel to each other.



- Polaroids allow the light oscillations parallel to the transmission axis pass through them.
- The crystal or polaroid on which unpolarised light is incident is called polariser. Crystal or polaroid on which polarised light is incident is called analyser.



Transmission axes of the polariser and analyser are parallel to each other, so whole of the polarised light passes through analyser

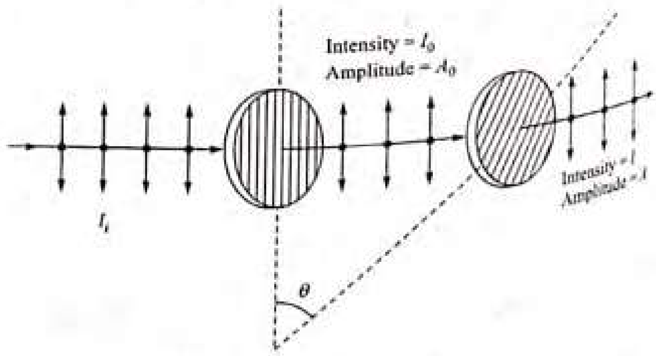


Transmission axis of the analyser is perpendicular to the polariser, hence no light passes through the analyser

When unpolarised light is incident on the polariser, the intensity of the transmitted polarised light is half the intensity of unpolarised light.

Malus law

This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.



(i) $I = I_0 \cos^2 \theta$ and $A^2 = A_0^2 \cos^2 \theta$

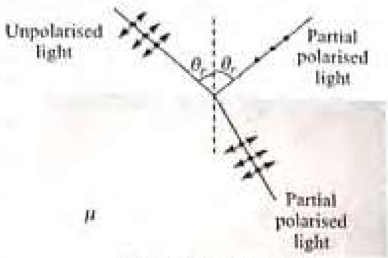
$\Rightarrow A = A_0 \cos \theta$

If $\theta = 0°$, $I = I_0$, $A = A_0$. If $\theta = 45°$, $I = \dfrac{I_0}{2}$, $A = \dfrac{A_0}{\sqrt{2}}$,

If $\theta = 90°$, $I = 0$, $A = 0$.

(ii) If I_i = Intensity of unpolarised light.

So $I_0 = \frac{I_i}{2}$ i.e. if an unpolarised light is converted into plane polarised light (say by passing it through a plaroid or a Nicol-prism), its intensity becomes half, and



Polarisation by reflection

$I = \frac{I_i}{2}\cos^2\theta$

Percentage of polarisation $= \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100$

Brewster's Law

Brewster discovered that when a beam of unpolarised light is reflected from a transparent medium (refractive index = μ), the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation θ_p).

Also $\mu = \tan\theta_p$

Brewster's law:

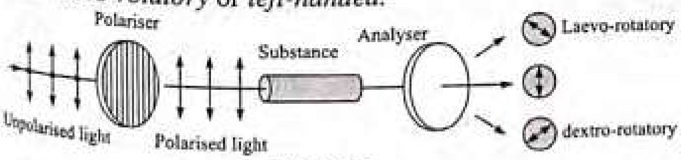
(i) For $i < \theta_P$ or $i > \theta_P$
Both reflected and refracted rays becomes partially polarised.

(ii) For glass, $\theta_p \cup 57°$ and for water, $\theta_p \cup 53°$.

Optical Activity and Specific Rotation

When plane polarised light passes through certain substances, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle. This phenomenon is called optical activity or optical rotation and the substances optically active.

If the optically active substance rotates the plane of polarisation clockwise (looking against the direction of light), it is said to be *dextro-rotatory* or *right-handed*. However, if the substance rotates the plane of polarisation anti-clockwise, it is called *laevo-rotatory* or *left-handed*.



Polarimeter

The optical activity of a substance is related to the asymmetry of the molecule or crystal as a whole, e.g., a solution of cane-sugar is dextro-rotatory due to asymmetrical molecular structure while crystals of quartz are dextro or laevo-rotatory due to structural asymmetry which vanishes when quartz is fused.

Optical activity of a substance is measured with help of polarimeter in terms of 'specific rotation' which is defined as the rotation produced by a solution of length 10 cm (1 dm) and of unit concentration (i.e., 1 g/cc) for a given wavelength of light at a given temperature, i.e., $[\alpha]^{\lambda}_{t°C} = \frac{\theta}{L \times C}$ where θ is the rotation in length L at concentration C.

Applications and Uses of Polarisation

- By determining the polarising angle and using Brewster's law, i.e. $\mu = \tan\theta_p$, refractive index of dark transparent substance can be determined.
- It is used to reduce glare.
- In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display (LCD).
- In CD player polarised laser beam acts as needle for producing sound from compact disc which is an encoded digital format.
- It has also been used in recording and reproducing three-dimensional pictures.
- Polarisation of scattered sunlight is used for navigation in solar-compass in polar regions.
- Polarised light is used in optical stress analysis known as 'photoelasticity'.
- Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of 'optical activity'.

Concept Application Exercise 27.3

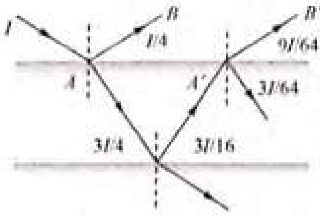
1. (i) Light of wavelength 5.4×10^{-5} cm passes through a slit 0.12 cm wide and form a diffraction pattern on a screen 2.7 m away. What is the width of the central maxima? (ii) If the apparatus is immersed in a liquid of refractive index 1.35, what would be the width of the central fringe?
2. Determine the angular separation between central maximum and first order maximum of the diffraction pattern due to a single silt of width 0.25 mm, when light of wavelength 55890 Å is incident on it normally.
3. A parallel beam of light of wavelength 600 nm is incident normally on a slit of width a. If the distance between the slit and the screen is 0.8 m and the distance of the second order maximum from the centre of the screen is 15 mm, calculate the width of the slit.
4. An unpolarised beam of light is incident on a group of four polarising sheets, which are arranged in such a way that the characteristic direction of each polarising sheet makes an angle of 30° with that o the preceeding sheet. What fraction of light is transmitted?
5. Refractive index of water is 1.33. Calculate the angle of polarisation for light reflected from the surface of a lake.

Solved Examples

1. A ray of light of intensity I is incident on a parallel glass-slab at a point A as shown in figure. It undergoes partial reflection and refraction. At each reflection 25% of incident energy is reflected. The rays AB and $A'B'$ undergo interference. The ratio I_{max}/I_{min} is

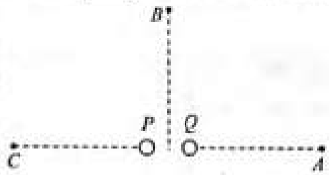
(a) 4 : 1 (b) 8 : 1
(c) 7 : 1 (d) 49 : 1

Sol. (d) From figure $I_1 = \dfrac{I}{4}$ and $I_2 = \dfrac{9I}{64} \Rightarrow \dfrac{I_2}{I_1} = \dfrac{9}{16}$



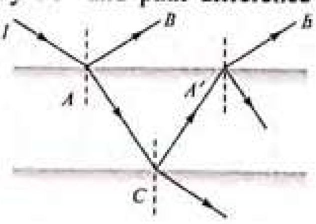
By using $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{\dfrac{I_2}{I_1}}+1}{\sqrt{\dfrac{I_2}{I_1}}-1}\right) = \left(\dfrac{\sqrt{\dfrac{9}{16}}+1}{\sqrt{\dfrac{9}{16}}-1}\right) = \dfrac{49}{1}$

2. Figure shows that P and Q are two equally intense coherent sources emitting radiations of wavelength 20 m. The separation PQ is 5.0 m and phase of P is ahead of the phase of Q by 90°. A, B and C are three distant points of observation equidistant from the mid-point of PQ. The intensity of radiations at A, B, C will bear the ratio



(a) 0 : 1 : 4 (b) 4 : 1 : 0
(c) 0 : 1 : 2 (d) 2 : 1 : 0

Sol. (d) Since P is ahead of Q by 90° and path difference between P and Q is $\lambda/4$. Therefore at A, phase difference is zero, so intensity is $4I$. At C it is zero and at B, the phase difference is 90°, so intensity is $2I$.



3. In Young's double slit experiment, the intensity on the screen at a point where path difference is λ is K. What will be the intensity at the point where path difference is $\lambda/4$?

(a) $\dfrac{K}{4}$ (b) $\dfrac{K}{2}$
(c) K (d) Zero

Sol. (b By using phase difference $\phi = \dfrac{2\pi}{\lambda}(\Delta)$

For path difference λ, phase difference $\phi_1 = 2\pi$ and for path difference $\lambda/4$, phase difference $\phi_2 = \pi/2$.

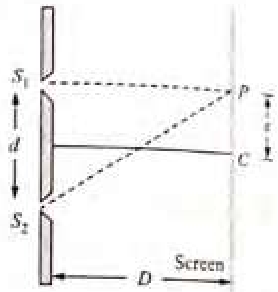
Also, $I = 4I_0 \cos^2\dfrac{\phi}{2} \Rightarrow \dfrac{I_1}{I_2} = \dfrac{\cos^2(\phi_1/2)}{\cos^2(\phi_2/2)}$

$$\Rightarrow \frac{K}{I_2} = \frac{\cos^2(2\pi/2)}{\cos^2\left(\dfrac{\pi/2}{2}\right)} = \frac{1}{1/2} \Rightarrow I_2 = \frac{K}{2}.$$

4. The maximum intensity in Young's double slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10\,d$

(a) $\dfrac{I_0}{2}$ (b) $\dfrac{3}{4}I_0$
(c) I_0 (d) $\dfrac{I_0}{4}$

Sol. (a) Suppose P is a point in front of one slit at which intensity is to be calculated from figure. It is clear that $x = \dfrac{d}{2}$. Path difference between the waves reaching at P,



$$\Delta = \frac{xd}{D} = \frac{\left(\dfrac{d}{2}\right)d}{10d}$$
$$= \frac{d}{20} = \frac{5\lambda}{20} = \frac{\lambda}{4}$$

Hence, corresponding phase difference $\phi = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{4} = \dfrac{\pi}{2}$

Resultant intensity at P

$$I = I_{max}\cos^2\frac{\phi}{2}$$
$$= I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

5. In a Young's double slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelength $\lambda_0 = 750$ nm and $\lambda = 900$ nm. The minimum distance from the common central bright fringe on a screen 2 m from the slits where a bright fringe from one interference pattern coincides with a bright fringe from the other is

(a) 1.5 mm (b) 3 mm
(c) 4.5 mm (d) 6 mm

Sol. (c) From the given data, note that the fringe width (β_1) for $\lambda_1 = 900$ nm is greater than fringe width (β_2) for $\lambda_2 = 750$ nm. This means that although the central maxima of the two coincide, first maxima for $\lambda_1 = 900$ nm will be further away from the first maxima for $\lambda_2 = 750$ nm, and so on. A stage may come when this mismatch equals β_2, then again maxima of $\lambda_1 = 900$ nm, will coincide with a maxima of $\lambda_2 = 750$ nm, let this correspond to n^{th} order fringe for λ_1. Then it will correspond to $(n + 1)^{th}$ order fringe for λ_2.

Therefore, $\dfrac{n\lambda_1 D}{d} = \dfrac{(n+1)\lambda_2 D}{d}$

$\Rightarrow$ $n \times 900 \times 10^{-9} = (n + 1)750 \times 10^{-9} \Rightarrow n = 5$

Minimum distance from central maxima

$$= \frac{n\lambda_1 D}{d} = \frac{5 \times 900 \times 10^{-9} \times 2}{2 \times 10^{-3}}$$
$$= 45 \times 10^{-4}\text{ m} = 4\text{ mm}$$

6. In a double slit arrangement, fringes are produced using light of wavelength 4800 Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7. By doing so, the central bright shifts to original fifth bright fringe from centre. Thickness of glass plate is

(a) 8 μm (b) 6 μm
(c) 4 μm (d) 10 μm

Sol. (a) Shift $\Delta x = \frac{\beta}{\lambda}(\mu - 1)t$

Shift due to one plate $\Delta x_1 = \frac{\beta}{\lambda}(\mu_1 - 1)t$

Shift due to another plate $\Delta x_2 = \frac{\beta}{\lambda}(\mu_2 - 1)t$

Net shift $\Delta x = \Delta x_2 - \Delta x_1 = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t$ ...(i)

Also it is given that $\Delta x = 5\beta$ ...(ii)

Hence $5\beta = \frac{\beta}{\lambda}(\mu_1 - \mu_2)t$

$$\Rightarrow t = \frac{5\lambda}{(\mu_2 - \mu_1)} = \frac{5 \times 4800 \times 10^{-10}}{(1.7 - 1.4)}$$

$$= 8 \times 10^{-6}\ \text{m} = 8\ \mu\text{m}.$$

7. In a YDSE bichromatic lights of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is

(a) 4 mm (b) 5.6 mm
(c) 14 mm (d) 28 mm

Sol. (d) Let nth minima of 400 nm coincides with mth minima of 560 nm then

$$(2n-1)400 = (2m-1)560 \Rightarrow \frac{2n-1}{2m-1} = \frac{7}{5} = \frac{14}{10} = \frac{21}{15}$$

i.e., 4th minima of 400 nm coincides with 3rd minima of 560 nm.

The location of this minima is

$$= \frac{7(1000)(400 \times 10^{-6})}{2 \times 0.1} = 14\,\text{mm}$$

Next, 11th minima of 400 nm will coincide with 8th minima of 560 nm.

Location of this minima is

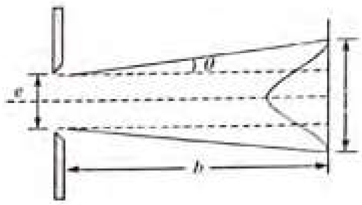
$$= \frac{21(1000)(400 \times 10^{-6})}{2 \times 0.1} = 42\,\text{mm}$$

∴ Required distance = 28 mm

8. In a single slit diffraction of light of wavelength λ by a slit of width e, the size of the central maximum on a screen at a distance b is

(a) $2b\lambda + e$ (b) $\frac{2b\lambda}{e}$
(c) $\frac{2b\lambda}{e} + e$ (d) $\frac{2b\lambda}{e} - e$

Sol. (c) The direction in which the first minima occurs is θ (say). Then $e \sin\theta = \lambda$ or $e\theta = \lambda$ or, $\theta = \frac{\lambda}{e}$ ($\because \theta = \sin\theta$ when θ is very small)



Width of the central maximum $= 2b\theta + e = 2b.\frac{\lambda}{e} + e$

9. In a single slit diffraction experiment first minimum for red light (660 nm) coincides with first maximum of some other wavelength λ'. The value of λ' is

(a) 4400 Å (b) 6600 Å
(c) 2000 Å (d) 3500 Å

Sol. (a) In a single slit diffraction experiment, position of minima is given by $d \sin\theta = n\lambda$

So for first minima of red $\sin\theta = 1 \times \left(\frac{\lambda_R}{d}\right)$

and as first maxima is midway between first and second minima, for wavelength λ', its position will be

$$d\sin\theta' = \frac{\lambda' + 2\lambda'}{2} \Rightarrow \sin\theta' = \frac{3\lambda'}{2d}$$

According to given condition $\sin\theta = \sin\theta'$

$$\Rightarrow \lambda' = \frac{2}{3}\lambda_R \text{ so } \lambda' = \frac{2}{3} \times 660 = 440\,\text{nm} = 4400\,\text{Å}$$

10. The ratio of intensities of consecutive maxima in the diffraction pattern due to a single slit is

(a) 1 : 4 : 9 (b) 1 : 2 : 3
(c) $1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2}$ (d) $1 : \frac{1}{\pi^2} : \frac{9}{\pi^2}$

Sol. (c) $I = I_0\left[\frac{\sin\alpha}{\alpha}\right]^2$, where $\alpha = \frac{\phi}{2}$

For n^{th} secondary maxima, $d\sin\theta = \left(\frac{2n+1}{2}\right)\lambda$

$$\Rightarrow \alpha = \frac{\phi}{2} = \frac{\pi}{\lambda}[d\sin\theta] = \left(\frac{2n+1}{2}\right)\pi$$

$$\therefore I = I_0\left[\frac{\sin\left(\frac{2n+1}{2}\right)\pi}{\left(\frac{2n+1}{2}\right)\pi}\right]^2 = \frac{I_0}{\left\{\frac{(2n+1)}{2}\pi\right\}^2}$$

So $I_0 : I_1 : I_2 = I_0 : \frac{4}{9\pi^2}I_0 : \frac{4}{25\pi^2}I_0$

$$= 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2}$$

11. White light may be considered to be a mixture of waves with λ ranging between 3900 Å and 7800 Å. An oil film of thickness 10000 Å is examined normally by reflected light. If $\mu = 1.4$, then the film appears bright for

(a) 4308 Å, 5091 Å, 6222 Å
(b) 4000 Å, 5091 Å, 5600 Å
(c) 4667 Å, 6222 Å, 7000 Å
(d) 4000 Å, 4667 Å, 5600 Å, 7000 Å

Sol. (a) The film appears bright when the path difference ($2\mu t \cos r$) is equal to odd multiple of $\frac{\lambda}{2}$.

i.e., $2\mu t\cos r = (2n-1)\lambda/2$ where $n = 1, 2, 3...$

$$\therefore \lambda = \frac{4\mu t \cos r}{(2n-1)}$$

$$= \frac{4 \times 1.4 \times 10000 \times 10^{-10} \times \cos 0}{(2n-1)} = \frac{56000}{(2n-1)} \text{Å}$$

$\therefore \lambda =$ 56000 Å, 18666 Å, 8000 Å, 6222 Å, 5091 Å, 4308 Å, 3733 Å.

The wavelength which are not within specified range are to be refracted.

12. The ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between two fringes from the centre is

(a) 2 (b) 1/2
(c) 4 (d) 16

Sol. (a) $I = 4I_0 \cos^2\frac{\phi}{2}$

At central position, $I_1 = 4I_0$ ...(i)

Since the phase difference between two successive fringes is 2π, the phase difference between two points separated by a distance equal to one quarter of the distance between the two successive fringe, $\delta = (2\pi)\left(\frac{1}{4}\right) = \frac{\pi}{2}$ radian

$$\Rightarrow I_2 = 4I_0\cos^2\left(\frac{\frac{\pi}{2}}{2}\right) = 2I_0 \qquad \text{...(ii)}$$

Using (i) and (ii), $\frac{I_1}{I_2} = \frac{4I_0}{2I_0} = 2$

13. Specific rotation of sugar solution is 0.01 SI units. 200 kgm^{-3} of impure sugar solution is taken in a polarimeter tube of length 0.25 m and an optical rotation of 0.4 rad is observed. The percentage of purity of sugar in the sample is

(a) 80% (b) 89%
(c) 11% (d) 20%

Sol. (a) Specific rotation

$$(\alpha) = \frac{\theta}{lc} \Rightarrow c = \frac{\theta}{\alpha l} = \frac{0.4}{0.01 \times 0.25} = 160\ \text{kg/m}^3$$

Now, percentage purity of sugar solution

$$= \frac{160}{200} \times 100 = 80\%$$

14. A 20 cm length of a certain solution causes right-handed rotation of 38°. A 30 cm length of another solution causes left-handed rotation of 24°. The optical rotation caused by 30 cm length of a mixture of the above solutions in the volume ratio 1 : 2 is

(a) Left-handed rotation of 14°
(b) Right-handed rotation of 14°
(c) Left-handed rotation of 3°
(d) Right-handed rotation of 3°

Sol. (d) As $\theta \propto l$

Volume ratio 1 : 2 in a tube of length 30 cm means 10 cm length of first solution and 20 cm length of second solution.

Rotation produced by 10 cm length of first solution

$$\theta_1 = \frac{38°}{20} \times 10 = 19°$$

Rotation produced by 20 cm length of second solution

$$\theta_2 = -\frac{24°}{30} \times 20 = -16°$$

$\therefore$ Total rotation produced = 19° − 16° = 3°

15. Unpolarized light of intensity 32 Wm^{-2} passes through three polarizers such that transmission axes of the first and second polarizer makes and angle 30° with each other and the transmission axis of the last polarizer is crossed with that of the first. The intensity of final emerging light will be

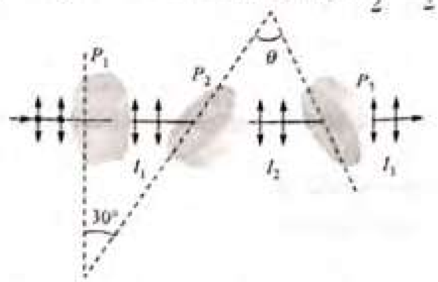
(a) 32 Wm^{-2} (b) 3 Wm^{-2}
(c) 8 Wm^{-2} (d) 4 Wm^{-2}

Sol. (b) Angle between P_1 and P_2 = 30° (given)

Angle between P_2 and $P_3 = \theta = 90° - 30° = 60°$

$I_0 = 32$ Wm^{-2}

The intensity of light transmitted by P_1 is $I_1 = \frac{I_0}{2} = \frac{32}{2} = 16$ Wm^{-2}



According to Malus law the intensity of light transmitted by P_2 is $I_2 = I_1\cos^2 30° = 16\left(\frac{\sqrt{3}}{2}\right)^2 = 12$ Wm^{-2}

Similarly intensity of light transmitted by P_3 is

$I_3 = I_2 \cos^2\theta = 12\cos^2 60^\circ = 12\left(\frac{1}{2}\right)^2 = 3\ \text{Wm}^{-2}$

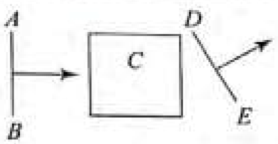
16. Yellow light is used in single slit diffraction experiment with slit width 0.6 mm. If yellow light is replaced by X-rays then the pattern will reveal
(a) that the central maxima is narrower
(b) no diffraction pattern
(c) more number of fringes
(d) less number of fringes

Sol. (b) Diffraction is obtained when the slit width is of the order of wavelength of EM waves (or light). Here wavelength of X-rays (1-100 Å) is very lesser than slit width (0.6 mm). Therefore, no diffraction pattern will be observed.

EXERCISES

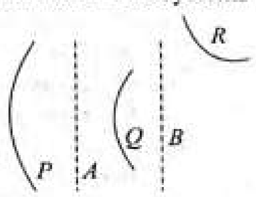
Huygens's Wave Principle and Interference of Light

1. A wave front AB passing through a system C emerges as DE. The system C could be



(a) a slit (b) a biprism
(c) a prism (d) a glass slab

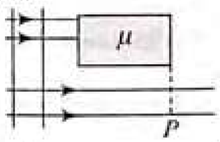
2. Figure shows wavefront P passing through two systems A and B, and emerging as Q and then as R. The systems A and B could, respectively, be

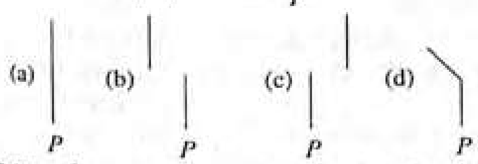


(a) a prism and a convergent lens
(b) a convergent lens and a prism
(c) a divergent lens and a prism
(d) a convergent lens and a divergent lens

3. Light waves travel in vacuum along the y-axis. Which of the following may represent the wavefront?
(a) x = constant (b) y = constant
(c) z = constant (d) $x + y + z$ = constant

4. A plane wavefront traveling in a straight line in vacuum encounters a medium of refractive index μ. At P, the shape of the wavefront is





5. The wavefront of a light beam is given by the equation $x + 2y + 3z = c$ (where c is arbitrary constant), then the angle made by the direction of light with the y-axis is
(a) $\cos^{-1}\frac{1}{\sqrt{14}}$ (b) $\sin^{-1}\frac{2}{\sqrt{14}}$
(c) $\cos^{-1}\frac{2}{\sqrt{14}}$ (d) $\sin^{-1}\frac{3}{\sqrt{14}}$

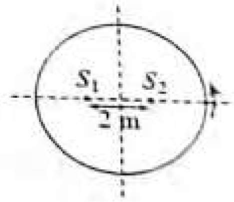
6. Sources 1 and 2 emit lights of different wavelengths whereas sources 3 and 4 emit lights of different intensities. The coherence
(a) can be obtained by using sources 1 and 2
(b) can be obtained by using sources 3 and 4
(c) cannot be obtained by any of these sources
(d) since contrast suffers when sources 3 and 4 are used so coherence cannot be obtained by using sources 3 and 4

7. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first ray travels a path of length L_1 through a medium of refractive index μ_1. The second ray travels a path of length L_2 through a medium of refractive index μ_2. The two waves are then combined to observe interference effects. The phase difference between the two, when they interfere, is
(a) $\frac{2\pi}{\lambda}(L_1 - L_2)$ (b) $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$
(c) $\frac{2\pi}{\lambda}(\mu_2 L_1 - \mu_1 L_2)$ (d) $\frac{2\pi}{\lambda}\left[\frac{L_1}{\mu_1} - \frac{L_2}{\mu_2}\right]$

8. Microwaves from a transmitter are directed toward a plane reflector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima, the detector travels a distance of 0.14 m. What is the frequency of transmitter?
(a) 1.5×10^{10} Hz (b) 3.0×10^{10} Hz
(c) 1.5×10^{9} Hz (d) 3.0×10^{9} Hz

9. Two beams of light having intensities I and $4I$ interfere to produce a fringe pattern on a screen. The phase between the beams is $\pi/2$ at point A and π at point B. Then, the difference between the resultant intensities at A and B is
(a) $2I$ (b) $4I$
(c) $5I$ (d) $7I$

10. Two point sources separated by 2.0 m are radiating in phase with λ = 0.50 m. A detector moves in a circular path around the two sources in a plane containing them. How many maxima are detected?



(a) 16 (b) 20
(c) 24 (d) 32

11. Two coherent sources of intensities, I_1 and I_2 produce an interference pattern. The maximum intensity in the interference pattern will be
(a) $I_1 + I_2$ (b) $I_1^2 + I_2^2$

(c) $(I_1 + I_2)^2$ (d) $(\sqrt{I_1} + \sqrt{I_2})^2$

12. Following transverse waves,

$y_1 = 2\sin(100\pi t - 5.3x)$,

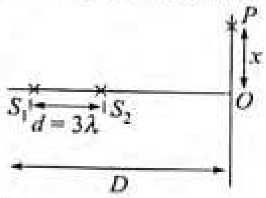
$y_2 = 2\sqrt{2}\sin\left(100 - 5.3x + \dfrac{\pi}{4}\right)$

and $y_3 = \sin(100\pi t - 5.3x)$

superpose in a homogenous medium. Find the resultant amplitude at $x = 0$.

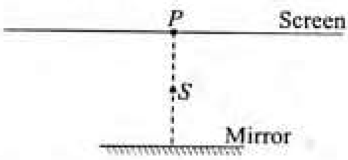
(a) 1 unit (b) 4 unit
(c) 5 unit (d) 2 unit

13. Two coherent narrow slits emitting light of wavelength λ in the same phase are placed parallel to each other at a small separation of 3λ. The light is collected on a screen S which is placed at a distance D ($>>\lambda$) from the slits. The smallest distance x such that the P is a maxima.



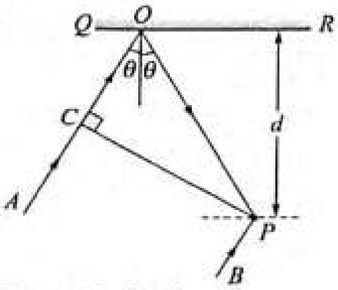
(a) $\sqrt{3}D$ (b) $\sqrt{8}D$
(c) $\sqrt{5}D$ (d) $\sqrt{5}\dfrac{D}{2}$

14. Due to interference between direct and reflected light from mirror, maxima is formed at point P. By what minimum distance mirror is shifted downward to find minima at point P. (Assume that, wavelength of light is 600 mm)



(a) 100 nm (b) 200 nm
(c) 300 nm (d) 400 nm

15. In the adjacent diagram, CP represents a wavefront and AO and BP, the corresponding two rays. Find the condition on θ for constructive interference at P between the ray BP and reflected ray OP



(a) $\cos\theta = 3\lambda/2d$ (b) $\cos\theta = \lambda/4d$
(c) $\sec\theta - \cos\theta = \lambda/d$ (d) $\sec\theta - \cos\theta = 4\lambda/d$

Young's Double Slit Experiment

16. In a YDSE with identical slits, the intensity of the central bright fringe is I_0. If one of the slits is covered, the intensity at the same point is

(a) $2I_0$ (b) I_0
(c) $I_0/2$ (d) $I_0/4$

17. The maximum intensity in Young's double-slit experiment is I_0. Distance between the slits is $d = 5\lambda$, where λ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10d$?

(a) $\dfrac{I_0}{2}$ (b) $\dfrac{3}{4}I_0$
(c) I_0 (d) $\dfrac{I_0}{4}$

18. If one of the two slits of Young's double-slit experiment is painted so that it transmits half the light intensity as the second slit, then

(a) the fringe system will altogether disappear
(b) the bright fringes will become brighter and the dark fringes will become darker
(c) both dark and bright fringes will become darker
(d) dark fringes will become brighter and bright fringes darker

19. In a double-slit experiment, instead of taking slits of equal width, one slit is made twice as wide as the other. Then in the interference pattern

(a) the intensities of both the maxima and the minima increase
(b) the intensity of the maxima increases and the minima has zero intensity
(c) the intensity of the maxima decreases and that of the minima increases
(d) the intensity of the maxima decreases and the minima has zero intensity

20. In Young's double-slit experiment, the slits are illuminated by monochromatic light. The entire set-up is immersed in pure water. Which of the following act *cannot* restore the original fringe width?

(a) Bringing the slits close together.
(b) Moving the screen away from the slit plane.
(c) Replacing the incident light by that of longer wavelength.
(d) Introducing a thin transparent slab in front of one of the slits.

21. In Young's double-slit experiment $d/D = 10^{-4}$ (d = distance between slits, D = distance of screen from the slits). At point P on the screen, resulting intensity is equal to the intensity due to the individual slit I_0. Then, the distance of point P from the central maximum is ($\lambda = 6000$ Å)

(a) 2 mm (b) 1 mm
(c) 0.5 mm (d) 4 mm

22. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude A and of wavelength λ. In another experiment with the same setup, the two slits are sources of equal amplitude A and wavelength λ, but are incoherent. The ratio of intensity of light at the mid-point of the screen in the first case to that in the second case is

(a) 1:1 (b) 1:2
(c) 2:1 (d) 4:1

23. In Young's double-slit experiment, the y-coordinates of central maxima and 10th maxima are 2 cm and 5 cm,

respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5, the corresponding y-co-ordinates will be

(a) 2 cm, 7.5 cm (b) 3 cm, 6 cm
(c) 2 cm, 4 cm (d) 4/3 cm, 103 cm

24. A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double-slit experiment. What should be the minimum thickness t which will make the intensity at the center of the fringe pattern zero?

(a) $(\mu - 1)\frac{\lambda}{2}$ (b) $(\mu - 1)\lambda$

(c) $\frac{\lambda}{2(\mu - 1)}$ (d) $\frac{\lambda}{(\mu - 1)}$

25. In Young's double-slit experiment, how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe ($\lambda = 2000$ Å)?

(a) 12 (b) 7
(c) 18 (d) 4

26. Young's double-slit experiment is made in a liquid. The 10th bright fringe in liquid lies where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately

(a) 1.8 (b) 1.54
(c) 1.67 (d) 1.2

27. In Young's double-slit experiment, 30 fringes are obtained in the field of view of the observing telescope, when the wavelength of light used is 4000 Å. If we use monochromatic light of wavelength 6000 Å, the number of fringes obtained in the same field of view is

(a) 30 (b) 45
(c) 20 (d) none of these

28. In Young's double-slit experiment, the separation between two coherent sources S_1 and S_2 is d and the distance between the source and screen is D. In the interference pattern, it is found that exactly in front of one slit, there occurs a minimum. Then the possible wavelengths used in the experiment are

(a) $\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D}$ (b) $\lambda = \frac{d^2}{D}, \frac{d^2}{5D}, \frac{d^2}{9D}$

(c) $\lambda = \frac{d^2}{D}, \frac{d^2}{2D}, \frac{d^2}{3D}$ (d) $\lambda = \frac{d^2}{3D}, \frac{d^2}{7D}, \frac{d^2}{11D}$

29. In a two-slit experiment with white light, a white fringe is observed on a screen kept behind the slits. When the screen in moved away by 0.05 m, this white fringe

(a) does not move at all
(b) gets displaced from its earlier position
(c) becomes colored
(d) disappears

30. In a double-slit experiment, the slits are separated by a distance d and the screen is at a distance D from the slits. If a maximum is formed just opposite to each slit, then what is the order of the fringe so formed?

(a) $\frac{d^2}{2\lambda D}$ (b) $\frac{2d^2}{\lambda D}$

(c) $\frac{d^2}{\lambda D}$ (d) $\frac{d^2}{4\lambda D}$

31. In Young's double-slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen is given by

(a) 12 (b) 18
(c) 24 (d) 30

32. In the ideal double-slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is

(a) 2λ (b) $2\lambda/3$
(c) $\lambda/3$ (d) λ

33. A double-slit arrangement produces interference fringes for sodium light (λ = 589 nm) that have an angular separation of 3.50×0^{-3} rad. For what wavelength would the angular separation be 10% greater?

(a) 527 nm (b) 648 nm
(c) 722 nm (d) 449 nm

34. In Young's double-slit experiment, the intensity of light at a point on the screen, where the path difference is λ, is I. The intensity of light at a point where the path difference becomes $\lambda/3$ is

(a) $\frac{I}{4}$ (b) $\frac{I}{3}$

(c) $\frac{I}{2}$ (d) I

35. A double-slit experiment is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6830 Å. Then the fringe width is

(a) 6.3×10^{-4} m (b) 8.3×10^{-4} m
(c) 6.3×10^{-2} m (d) 6.3×10^{-5} m

36. In Young's double-slit experiment, the slit separation is 0.5 mm and the screen is 0.5 m away from the slit. For a monochromatic light of wavelength 500 nm, the distance of 3rd maxima from the 2nd minima on the other side of central maxima is

(a) 2.75 mm (b) 2.5 mm
(c) 22.5 mm (d) 2.25 mm

37. In Young's double-slit experiment, the angular width of a fringe formed on a distant screen is 1°. The wavelength of light used is 6000 Å. What is the spacing between the slits?

(a) 344 mm (b) 0.1344 mm
(c) 0.0344 mm (d) 0.034 mm

38. In a double-slit experiment, the distance between the slits is d. The screen is at a distance D from the slits. If a bright fringe is formed opposite to a slit on the screen, the order of the fringe is

(a) $\frac{d^2}{2\lambda D}$ (b) $\frac{d}{2\lambda D}$

(c) $\frac{d^2}{4\lambda D}$ (d) 0

39. In YDSE, find the thickness of a glass slab ($\mu = 1.5$) which should be placed before the upper slit S_1 so that the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å.

(a) 5×10^{-6} m (b) 3×10^{-6} m
(c) 10×10^{-6} m (d) 5×10^{-5} m

Diffraction and Polarization

40. A slit of width a is illuminated by white light. For red light (λ = 6500 Å), the first minima is obtained at $\theta = 30°$. Then the value of a will be

(a) 3250 Å (b) 6.5×10^{-4} mm
(c) 1.24 microns (d) 2.6×10^{-4} cm

41. What will be the angle of diffracting for the first minimum due to Fraunhofer diffraction with sources of light of wave length 550 nm and slit of width 0.55 mm?

(a) 0.001 rad (b) 0.01 rad
(c) 1 rad (d) 0.1 rad

42. Angular width (β) of central maximum of a diffraction pattern on a single slit does not depend upon

(a) distance between slit and screen
(b) wavelength of light used
(c) width of the slit
(d) frequency of light used

43. A plane wavefront ($\lambda = 6 \times 10^{-7}$ m) falls on a slit 0.4 mm wide. A convex lens of focal length 0.8 m placed behind the slit focusses the light on a screen. What is the linear diameter of second maximum?

(a) 6 mm (b) 12 mm
(c) 3 mm (d) 9 mm

44. What will be the angular width of central maxima in Fraunhofer diffraction when light of wavelength 6000 Å is used and slit width is 12×10^{-5} cm.

(a) 2 rad (b) 3 rad
(c) 1 rad (d) 8 rad

45. Direction of the first secondary maximum in the Fraunhofer diffraction pattern at a single slit is given by (a is the width of the slit)

(a) $a\sin\theta = \frac{\lambda}{2}$ (b) $a\cos\theta = \frac{3\lambda}{2}$

(c) $a\sin\theta = \lambda$ (d) $a\sin\theta = \frac{3\lambda}{2}$

46. Angular width of central maxima in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. The wavelength of this light will be

(a) 6000 Å (b) 4200 Å
(c) 3000 Å (d) 1800 Å

47. In a Fresnel's diffraction arrangement, the screen is at a distance of 2 meter from a circular aperture. It is found that for light of wavelengths λ_1 and λ_2, the radius of 4th zone for λ_1 coincides with the radius of 5th zone for λ_2. Then the ratio $\lambda_1 : \lambda_2$ is

(a) $\sqrt{4/5}$ (b) $\sqrt{5/4}$
(c) 5/4 (d) 4/5

48. If in single slit diffraction pattern, first minima for red light (600 nm) coincides with first maxima of some other wavelength λ, then λ would be

(a) 400 nm (b) 440 nm
(c) 0.3 nm (d) 900 nm

49. Out of the following statements which is not correct?

(a) When unpolarised light passes through a Nicol's prism, the emergent light is elliptically polarised
(b) Nicol's prism works on the principle of double refraction and total internal reflection
(c) Nicol's prism can be used to produce and analyse polarised light
(d) Calcite and Quartz are both doubly refracting crystals

50. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is

(a) Zero (b) I_0

(c) $\frac{1}{2}I_0$ (c) $\frac{1}{4}I_0$

51. Two Nicols are oriented with their principal planes making an angle of 60°. The percentage of incident unpolarized light which passes through the system is

(a) 50% (b) 100%
(c) 12.5% (d) 37.5%

52. Unpolarized light falls on two polarizing sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the final transmitted light is one-third the maximum intensity of the first transmitted beam

(a) 75° (b) 55°
(c) 35° (d) 15°

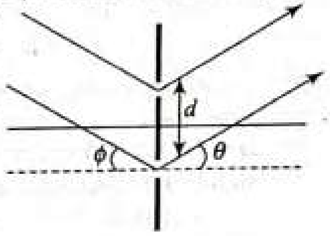
53. Two polaroids are placed in the path of unpolarized beam of intensity I_0 such that no light is emitted from the second polaroid. If a third polaroid whose polarization axis makes an angle θ with the polarization axis of first polaroid, is placed between these polaroids then the intensity of light emerging from the last polaroid will be
(a) $\left(\frac{I_0}{8}\right)\sin^2 2\theta$ (b) $\left(\frac{I_0}{4}\right)\sin^2 2\theta$
(c) $\left(\frac{I_0}{24}\right)\cos^4 2\theta$ (d) $I_0 \cos^4 \theta$

54. Specific rotation of sugar solution is 0.01 SI units. 200 kgm^{-3} of impure sugar solution is taken in a polarimeter tube of length 0.25 m and an optical rotation of 0.4 rad is observed. The percentage of purity of sugar is the sample is
(a) 80% (b) 89%
(c) 11% (d) 20%

55. A beam of natural light falls on a system of 6 polaroids, which are arranged in succession such that each polaroid is turned through 30° with respect to the preceding one. The percentage of incident intensity that passes through the system will be
(a) 100% (b) 50%
(c) 30% (d) 12%

Problems Based on Mixed Concepts

56. Light is incident at an angle ϕ with the normal to a plane containing two slits of separation d. Select the expression that correctly describes the positions of the interference maxima in terms of the incoming angle ϕ and outgoing angle θ.



(a) $\sin\phi + \sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}$
(b) $d\sin\theta = m\lambda$
(c) $\sin\phi - \sin\theta = (m+1)\frac{\lambda}{d}$
(d) $\sin\phi + \sin\theta = m\frac{\lambda}{d}$

57. Blue light of wavelength 480 nm is most strongly reflected off a thin film of oil on a glass slab when viewed near normal incidence. Assuming that the index of refraction of the oil is 1.2 and that of the glass is 1.6, what is the minimum thickness of the oil film (other than zero)?
(a) 100 nm (b) 200 nm
(c) 300 nm (d) none

58. Two identical coherent sources are placed on a diameter of a circle of radius R at separation x ($\ll R$) symmetrical about the center of the circle. The sources emit identical wavelength λ each. The number of points on the circle of maximum intensity is ($x = 5\lambda$)
(a) 20 (b) 22
(c) 24 (d) 26

59. To produce a minimum reflection of wavelengths near the middle of visible spectrum (550 nm), how thick should a coating of MgF_2 ($\mu = 1.38$) be vacuum-coated on a glass surface?
(a) 10^{-7} m (b) 10^{-10} m
(c) 10^{-9} m (d) 10^{-8} m

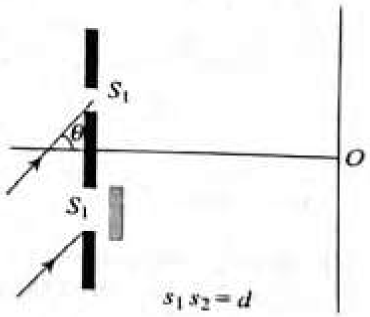
60. A thin film of refractive index 1.5 and thickness 4×10^{-5} cm is illuminated by light normal to the surface. What wavelength within the visible spectrum will be intensified in the reflected beam?
(a) 4800 Å (b) 5800 Å
(c) 6000 Å (d) 6800 Å

61. A plane wave of monochromatic light falls normally on a uniform thin film of oil which covers a glass plate. The wavelength of source can be varied continuously. Complete destructive interference is observed for $\lambda = 5000$ Å and $\lambda = 1000$ Å and for no other wavelength in between. If μ of oil is 1.3 and that of glass is 1.5, the thickness of the film will be
(a) 6.738×10^{-5} cm (b) 5.7×10^{-5} cm
(c) 4×10^{-5} cm (d) 2.8×10^{-5} cm

62. A light ray of frequency v and wavelength λ enters a liquid of refractive index 3/2. The ray travels in the liquid with
(a) frequency v and wavelength $\left(\frac{2}{3}\right)\lambda$
(b) frequency v and wavelength $\left(\frac{3}{2}\right)\lambda$
(c) frequency v and wavelength λ
(d) frequency $\left(\frac{3}{2}\right) v$ and wavelength λ

63. Light of wavelength $\lambda = 5890$ Å falls on a double-slit arrangement having separation $d = 0.2$ mm. A thin lens of focal length $f = 1$ m is placed near the slits. The linear separation of fringes on a screen placed in the focal plane of the lens is
(a) 3 mm (b) 4 mm
(c) 2 mm (d) 1 mm

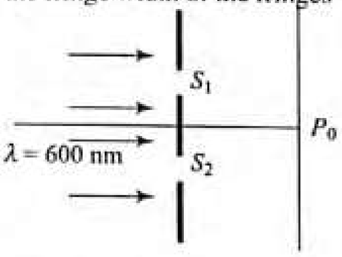
64. A monochromatic beam of light falls on YDSE apparatus at some angle (say θ) as shown in Figure. A thin sheet of glass is inserted in front of the lower slit s_2. The central bright fringe (path difference = 0) will be obtained



(a) at O
(b) above O
(c) below O

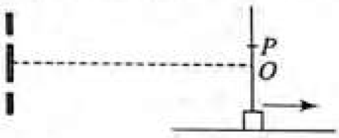
(d) anywhere depending on angle θ, thickness of plate t, and refractive index of glass μ

65. In a standard Young's double-slit experiment with coherent light of wavelength 600 nm, the fringe width of the fringes in the central region (near the central fringe, P_0) is observed to be 3 mm. An extremely thin glass plate is introduced in front of the first slit, and the fringes are observed to be displaced by 11 mm. Another thin plate is placed before the second slit and it is observed that the fringes are now displaced by an additional 12 mm. If the additional optical path lengths introduced are Δ_1 and Δ_2, then



(a) $11\Delta_1 = 12\Delta_2$ (b) $12\Delta_1 = 11\Delta_2$
(c) $11\Delta_1 > 12\Delta_2$ (d) none of the above

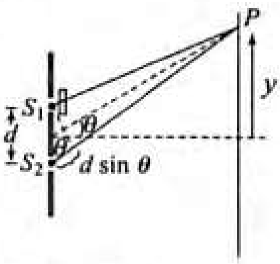
66. In Young's double-slit experiment, the first maxima is observed at a fixed point P on the screen. Now, the screen is continuously moved away from the plane of slits. The ratio of intensity at point P to the intensity at point O (center of the screen)



(a) remains cosntant
(b) keeps on decreasing
(c) first decreases and then increases
(d) first decreases and then becomes constant

67. A parallel beam of white light is incident on a thin film of air of uniform thickness. Wavelengths 7200 Å and 5400 Å are observed to be missing from the spectrum of reflected light viewed normally. The other wavelength in the visible region missing in the reflected spectrum is
(a) 6000 Å (b) 4320 Å
(c) 5500 Å (d) 6500 Å

68. In YDSE, light of wavelength $\lambda = 5000$ Å is used, which emerges in phase from two slits a distance $d = 3 \times 10^{-7}$ m apart. A transparent sheet of thickness $t = 1.5 \times 10^{-7}$ m, refractive index $n = 1.17$, is placed over one of the slits. Where does the central maxima of the interference now appear from the center of the screen? (Find the value of y?)



(a) $\dfrac{D(\mu-1)t}{2d}$ (b) $\dfrac{2D(\mu-1)t}{d}$
(c) $\dfrac{D(\mu+1)t}{d}$ (d) $\dfrac{D(\mu-1)t}{d}$

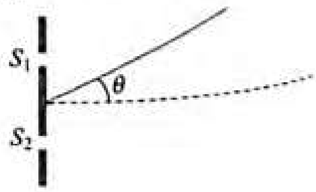
69. In Young's double-slit experiment, the wavelength of light was changed from 7000 Å to 3500 Å. While doubling the separation between the slits, which of the following is not true for this experiment?
(a) The width of fringes changes.
(b) The color of bright fringes changes.
(c) The separation between successive bright fringes changes.
(d) The separation between successive dark fringes remains unchanged.

70. Calculate the wavelength of light used in an interference experiment from the following data:
Fringe width = 0.03 cm. Distance between slits and eyepiece through which the interference pattern is observed is 1 m. Distance between the images of the virtual source when a convex lens of focal length 16 cm is used at a distance of 80 cm from the eyepiece is 0.8 cm.
(a) 6000 Å (b) 0.00006 Å
(c) 6000 cm (d) 0.00006 m

71. In Young's double-slit experiment using monochromatic light, the light pattern shifts by a certain distance on the screen when a mica sheet of refractive index μ and thickness t microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of light?
(a) $(1/2)t(\mu-1)$ (b) $t(\mu-1)$
(c) μt (d) $3\mu t$

72. In Young's double-slit experiment, the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slits. It is found that the ninth bright fringe is at a distance of 7.5 mm from the second dark fringe from the center of the fringe pattern. The wavelength of the light used is
(a) 5000 Å (b) $\dfrac{5000}{7}$ Å
(c) 2500 Å (d) $\dfrac{2500}{7}$ Å

73. Figure shows two coherent sources S_1 and S_2 emitting wavelength λ. The separation $S_1S_2 = 1.5\lambda$ and S_1 is ahead in phase by $\pi/2$ relative to S_2. Then the maxima occur in direction θ given by $\sin^{-1}$ of
(i) 0 (ii) 1/2
(iii) −1/6 (iv) −5/6



Correct options are
(a) (ii), (iii), and (iv)
(b) (i), (ii), and (iii)
(c) (i), (iii), and (iv)
(d) All the above

74. Light from a source emitting two wavelengths λ_1 and λ_2 is allowed to fall on Young's double-slit apparatus after filtering one of the wavelengths. The position of interference maxima is noted. When the filter is removed both the wavelengths are incident and it is found that maximum intensity is produced where the fourth maxima occured previously. If the other wavelength is filtered, at the same location the third maxima is found. What is the ratio of wavelengths?
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

ARCHIVES

1. The wavelengths of light used in an optical instrument are $\lambda_1 = 4000$ Å and $\lambda_2 = 5000$ Å, then the ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is
(a) 16 : 25 (b) 9 : 1
(c) 4 : 5 (d) 5 : 4 **(AIEEE 2002)**

2. The transverse nature of light is shown by
(a) interference of light (b) refraction of light
(c) polarisation of light (d) dispersion of light
(AIEEE 2002)

3. To demonstrate the phenomenon of interference, we require two sources which emit radiation
(a) of the same frequency and having a definite phase relationship
(b) of nearly the same frequency
(c) of the same frequency
(d) of different wavelengths **(AIEEE 2003)**

4. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment is
(a) infinite (b) five
(c) three (d) zero **(AIEEE 2004)**

5. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refraction index n) is
(a) $\sin^{-1}(n)$ (b) $\sin^{-1}\left(\frac{1}{n}\right)$
(c) $\tan^{-1}\left(\frac{1}{n}\right)$ (d) $\tan^{-1}(n)$ **(AIEEE 2004)**

6. In Young's double-slit experiment, a monochromatic source is used. The shape of the interference fringes formed on the screen is
(a) a parabola (b) a straight line
(c) a circle (d) a hyperbola
(AIEEE 2005)

7. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 m. Approximately, what is the maximum distance at which these dots can be resolved by the eye? (Take wavelength of light as 500 nm.)
(a) 3 m (b) 6 m
(c) 1 m (d) 5 m **(AIEEE 2005)**

8. If I_0 is the intensity of the principal maximum in the single-slit diffraction pattern, then what will be its intensity when the slit width is doubled?
(a) $\frac{I_0}{2}$ (b) I_0
(c) $4I_0$ (d) $2I_0$ **(AIEEE 2005)**

9. When an unpolarised light of intensity I_0 is incident on a polarising sheet, the intensity of the light which does not get transmitted is
(a) I_0 (b) zero
(c) $\frac{1}{4}I_0$ (d) $\frac{1}{2}I_0$ **(AIEEE 2005)**

10. In Young's double-slit experiment, the intensity at a point where the path difference is $\lambda/6$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, I/I_0 is equal to
(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) $\frac{1}{\sqrt{2}}$ **(AIEEE 2007)**

11. A mixture of lights, consisting of wavelength 590 nm and an unknown wavelength illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of the known light coincides with the fourth bright fringe of the unknown light. From this data, the wavelength of the unknown light is
(a) 393.4 nm (b) 885.0 nm
(c) 442.5 nm (d) 776.8 nm **(AIEEE 2009)**

Statement for Problems 12–14:

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

(AIEEE 2010)

12. As the beam enters the medium, it will
(a) diverge
(b) converge
(c) diverge near the axis and converge near the periphery
(d) travel as a cylindrical beam

13. The initial shape of the wave front of the beam is
(a) convex (b) concave

(c) convex near the axis and concave near the periphery
(d) planar

14. The speed of light in the medium is
(a) minimum on the axis of the beam
(b) the same everywhere in the beam
(c) directly proportional to the intensity I
(d) maximum on the axis of the beam

15. **Direction:** The question has a paragraph followed by two statements, Statement 1 and Statement 2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement 1: When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement 2: The centre of the interference pattern is dark.
(a) Statement 1 is true but statement 2 is false.
(b) Statement 1 is true, statement 2 is true, statement 2 is the correct explanation of statement 1.
(c) Statement 1 is true, statement 2 is true, statement 2 is not the correct explanation of statement 1.
(d) Statement 1 is false but statement 2 is true.
(AIEEE 2011)

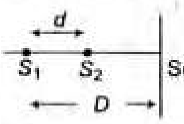
16. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by

(a) $\frac{I_m}{9}(4+5\cos\phi)$ (b) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$

(c) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$
(AIEEE 2012)

17. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is:
(a) $I_0/2$ (b) $I/4$
(c) $I_0/8$ (d) I **(JEE Main 2013)**

18. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be



(a) straight lines (b) semi-circles
(c) concentric circles (d) points **(JEE Main 2013)**

19. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 300 makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then I_A/I_B equals
(a) 1 (b) 1/3
(c) 3 (d) 3/2 **(JEE Main 2014)**

20. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygen's principle leads us to conclude that as it travels, the light beam
(a) becomes narrower
(b) goes horizontally without any deflection
(c) bends downwards
(d) bends upwards **(JEE Main 2015)**

21. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is
(a) 1 μm (b) 30 μm
(c) 100 μm (d) 300 μm **(JEE Main 2015)**

22. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when:

(a) $a=\frac{\lambda^2}{L}$ and $b_{min}=\left(\frac{2\lambda^2}{L}\right)$

(b) $a=\sqrt{\lambda L}$ and $b_{min}=\left(\frac{2\lambda^2}{L}\right)$

(c) $a=\sqrt{\lambda L}$ and $b_{min}=\sqrt{4\lambda L}$

(d) $a=\frac{\lambda^2}{L}$ and $b_{min}=\sqrt{4\lambda L}$ **(JEE Main 2016)**

23. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is
(a) 9.75 mm (b) 15.6 mm
(c) 1.56 mm (d) 7.8 mm
(JEE Main 2017)

24. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance ? (i.e. distance between the centres of each slit)
(a) 100 μm (b) 25μm
(c) 50 μm (d) 75 μm
(JEE Main 2018)

25. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be 1/2. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $I/8$. The angle between polarizer A and C is
(a) 60° (b) 0°
(c) 30° (d) 45°
(JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	2. (b)	3. (b)	4. (c)	5. (c)	6. (b)	7. (b)	8. (a)	9. (b)	10. (a)
11. (d)	12. (c)	13. (b)	14. (c)	15. (b)	16. (d)	17. (a)	18. (d)	19. (a)	20. (d)
21. (a)	22. (c)	23. (c)	24. (c)	25. (b)	26. (a)	27. (c)	28. (a)	29. (a)	30. (a)
31. (b)	32. (a)	33. (b)	34. (a)	35. (a)	36. (d)	37. (c)	38. (a)	39. (a)	40. (c)
41. (a)	42. (a)	43. (a)	44. (c)	45. (d)	46. (b)	47. (c)	48. (a)	49. (a)	50. (c)
51. (c)	52. (b)	53. (a)	54. (a)	55. (d)	56. (d)	57. (b)	58. (a)	59. (a)	60. (a)
61. (a)	62. (a)	63. (a)	64. (d)	65. (b)	66. (c)	67. (b)	68. (d)	69. (d)	70. (a)
71. (a)	72. (a)	73. (a)	74. (c)						

Archives

1. (d)	2. (c)	3. (a)	4. (b)	5. (d)	6. (d)	7. (d)	8. (c)	9. (d)	10. (c)
11. (c)	12. (b)	13. (d)	14. (a)	15. (b)	16. (d)	17. (b)	18. (c)	19. (b)	20. (d)
21. (b)	22. (c)	23. (d)	24. (b)	25. (d)					

Chapter 28

Dual Nature of Radiation and Matter

QUANTUM THEORY OF LIGHT

In 1900, Planck proposed that electromagnetic radiation (or light) is quantized and exists in elementary amounts (or quanta) that we now call *photons*. According to this proposal, the quantum of light wave of frequency f has the energy:

$E = hf$

Here h is Planck's constant and it has the value, $h = 6.63 \times 10^{-34}$ J s $= 4.14 \times 10^{-15}$ eV s

We can say that the light energy from any source is always an integral multiple of a smaller energy value called quantum of light. Hence, energy: $Q = NE$, where N (number of photons) $= 1, 2, 3, \ldots$

Here energy is quantized. $E = hf$ is the quantum of energy, it is a packet of energy called *photon*.

Also $E = hf = \dfrac{hc}{\lambda}$ and $hc = 12400$ eV

$\Rightarrow \quad E = \dfrac{12400}{\lambda\,(\text{in Å})}$ eV

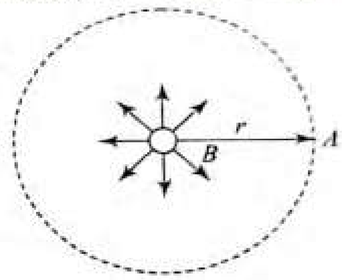
So, the least energy a light of frequency f can have is hf. The light cannot have energy like $1.5hf$ or πhf.

Properties of Photon

1. a source of radiation emits energy in the form of photons and these photons travel in straight line with the speed of light. Photon is not a material particle, it is a packet of energy.
2. Energy of a photon depends upon its frequency and it does not change with change in medium.
3. With change in medium, the speed and wavelength of the photon change but frequency does not change.
4. Photons are electrically neutral. They are not deflected by electric or magnetic fields.
5. Under suitable conditions, they can show diffraction.
6. a photon does not exist at rest. Its rest mass is zero. Equivalent mass of photon: $E = mc^2 = hf \Rightarrow m = \dfrac{hf}{c^2} = \dfrac{h}{c\lambda}$
7. Photons have momentum: $p = mc = \dfrac{h}{c\lambda}c = \dfrac{h}{\lambda}$
8. (Intensity of a light beam) $\propto$ (number of photons present in it).

Photon Counts Emitted by a Source Per Second

Consider a light bulb of power P watt as the source of light energy (see figure). If the wavelength of light emitted by the bulb is λ, then energy of each photon emitted by the bulb can be given as



$$E = h\nu = \frac{hc}{\lambda}$$

As the power of the source is P watt, we can say that the source is emitting light energy P joule per second in the form of photons. If it is 100% efficient, then the number of photons emitted per second by the source can be given as

$$n = \frac{\text{Power of the source}}{\text{Energy of photon}} = \frac{P}{E} = \frac{P\lambda}{hc}$$

These photons are assumed to be emitted uniformly in all directions. Here, we can consider that all the light energy emitted by the source is uniformly distributed in the spherical region with center at the source.

ILLUSTRATION 28.1 Calculate the number of photons emitted in 10 h by a 60 W sodium lamp ($\lambda = 5893$ Å).

Solution. The energy of the photon $= \dfrac{hc}{\lambda}$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{5893 \times 10^{-10}}$$

$$= 3.374 \times 10^{-19}\ \text{J}$$

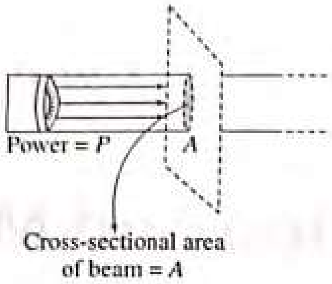
Therefore, the number of photons emitted by sodium lamp in 10 h

$$= \frac{60 \times 10 \times 3600}{3.374 \times 10^{-19}} = 6.40 \times 10^{24}$$

INTENSITY OF LIGHT DUE TO A LIGHT SOURCE

The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.

In figure, a source of light emits a uniform cylindrical light beam of cross-sectional area A. If the source emits a total power P in the beam, then the intensity of light beam is



$$I = \frac{P}{A} \text{ W m}^{-2}$$

As cross-sectional area of the beam is constant throughout, the beam intensity at every point remains constant.

Similarly, we can find the intensity of light due to a point isotropic source as shown in figure.

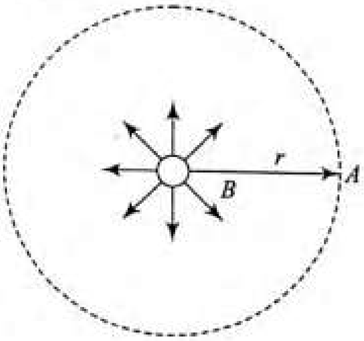


Figure shows a point source of light of power P watt emitting light in all directions uniformly. If we want to calculate intensity of light at a point A, at a distance r from the source, then it can be given as

$$I = \frac{P}{4\pi r^2} \text{ W m}^{-2} \qquad \text{(i)}$$

Here, it can be assumed that the whole power P is incident on the normal area of a hypothetical sphere of radius r passing through point A with center at the source as shown. Thus, energy crossing per unit area per second at point A can be given by Eq. (i).

Photon Flux

It is defined as the number of photons incident on a normal surface per second per unit area. Consider a light beam of intensity I W m^{-2} having wavelength λ incident on a surface. Then, number of photons per second per unit area in the beam can be given as

$$\text{Photon flux, } \phi = \frac{\text{Intensity}}{\text{Energy of a photon}} = \frac{I}{hc/\lambda} = \frac{I\lambda}{hc}$$

If we consider a point source of power P watt which emits light in all directions, then it produces photons per second at a rate

$$n = \frac{\text{Power of a source}}{\text{Energy of a photon}} = \frac{P}{(hc/\lambda)} = \frac{P\lambda}{hc}$$

All these photons are distributed in the three-dimensional spherical space around the source. If we find the photon flux at a distance r from the point source, it can be given as

$$\phi = \frac{\text{Number of photons per second}}{\text{Surface area of sphere of radius } r} = \frac{n}{4\pi r^2}$$

Illustration 28.2 A bulb lamp emits light of mean wavelength of 4500 Å. The lamp is rated at 150 W and 8% of the energy appears as emitted light. How many photons are emitted by the lamp per second?

Solution. The energy of photon $= \dfrac{hc}{\lambda}$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{4500 \times 10^{-10}}$$

According to the given problem, power of the lamp is 150 W. As 8% of the light energy is utilized for emission of light, hence the utilized energy for emission per second is given by

$$\text{Useful power} = 150 \times \frac{8}{100} = 12 \text{ W} = 12 \text{ J s}^{-1}$$

Therefore, number of photons emitted per second by the lamp

$$\frac{\text{Useful power}}{\text{Energy of photon}} = \frac{12 \times 4500 \times 10^{-10}}{(6.63 \times 10^{-34})(3 \times 10^{8})}$$

$$= 27.17 \times 10^{18}$$

Photon Density in a Light Beam

Photon density is defined as the number of photons per unit volume.

A light source emits photons, which move away from the source with speed of light. If a light source of power P watt is producing a uniform cylindrical light beam of cross-sectional area S (see figure), then the intensity of light beam is

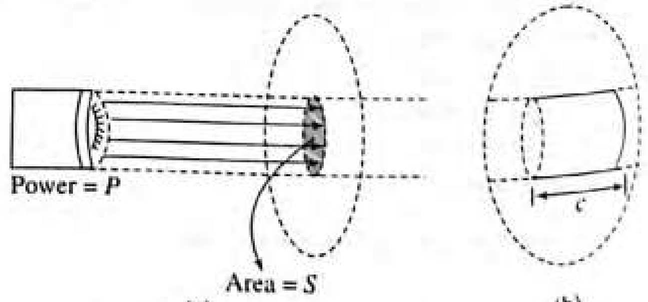
$$I = \text{Power crossing per unit area} = \frac{P}{S}$$

The photon flux, i.e., number of photons crossing per unit area per second at the cross-sectional area S of light beam, is

$$\phi = \frac{I\lambda}{hc} \quad [\text{where } \lambda \text{ is wavelength of light}]$$

In duration of 1s, these photons will cover a distance c. The volume of the region crossed by photons in one second $= S(c \times 1) = Sc$.

Thus, the total number of photons crossing cross-sectional area S is given as



(a) (b)

$$N = \frac{I\lambda}{hc} \times S$$

Thus, photon density in the light beam can be given as

$$\rho_{ph} = \frac{N}{Sc} = \frac{I\lambda}{hc^2} = \frac{\phi}{c} \text{ photons m}^{-2}$$

As the beam is uniform and cylindrical, the photon density throughout the beam remains constant and at any point in space, photon density can be given as

$$\rho_{ph} = \frac{\phi}{c} = \frac{\text{Photon flux}}{\text{Speed of light}}$$

Similarly, for a point isotropic source of light, we can say that as the emitted photons move away from the source, the distance between photons increases and the photon density decreases. If we wish to find photon density at a distance r from a point source of light of power P watt, then we first find the photon flux at a distance r from the source which is given as

$$\phi = \frac{P\lambda/hc}{4\pi r^2}$$

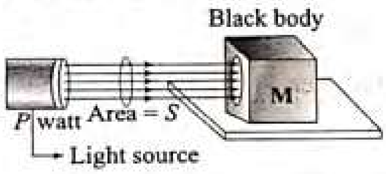
Thus, at a distance r from the source photon density can be given as

$$\rho_{ph} = \frac{\phi}{c} = \frac{P\lambda}{4\pi r^2 hc^2}$$

Force Exerted by a Light Beam on a Surface

Figure shows a black body of mass M placed on a smooth surface on which a light beam of cross-sectional area S is incident. The beam is produced by a torch of power P watt. If λ is the wavelength of light produced by the torch, then the number of photons emitted per second is

$$N = \frac{P\lambda}{hc}$$



We know that momentum in each photon is

$$p = \frac{h}{\lambda}$$

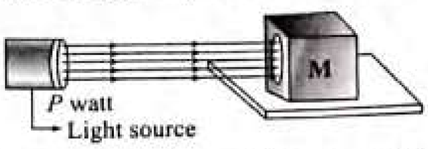
As all the photons incident on the black body will be absorbed by it, here the total momentum absorbed by the body per second or force exerted on the body is

$$F = \frac{P\lambda}{hc} \times \frac{h}{\lambda} = \frac{P}{c}$$

In the above case, if the surface of body is perfectly reflecting like a mirror, then the force exerted on the body will become

$$F = \frac{2P}{c}$$

ILLUSTRATION 28.3 A source of light of power P is shown in figure. Find the force on the block placed in the path of the light rays. The surface of body on which the light beam is incident is having a reflection coefficient $a_r = 0.7$ and absorption coefficient $a_a = 0.3$.



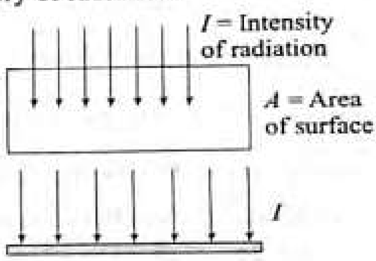
Solution. In this case, 70% of the incident photons are reflected back and 30% are absorbed by the body. Thus, the photon which is absorbed will impart a momentum h/λ to the body and the photon which is reflected will impart the change in momentum $2h/\lambda$ to the body. Thus, the net force acting on body can be given as

$$F = \frac{0.7P\lambda}{hc} \times \frac{2h}{\lambda} + \frac{0.3P\lambda}{hc} \times \frac{h}{\lambda} = \frac{1.7P}{c}$$

RADIATION PRESSURE/FORCE

The force (pressure) experienced by surface exposed to radiation is known as radiation force (pressure). This force can be calculated using photon theory of radiation.

Let us consider a surface of area A exposed to radiation of intensity I as shown in figure. The radiation is falling normally on the surface. Assume absorption and reflection coefficients of surface are a and r, respectively. Assume no trans-mission.



Energy received by the surface per second: $E = IA$

Number of photons received by surface per second:

$$N = \frac{E}{\text{Energy of one photon}} = \frac{E}{hc/\lambda} = \frac{IA\lambda}{hc}$$

We know that force is the rate of change of momentum,

i.e., $\vec{F} = \dfrac{d\vec{p}}{dt}$

Let us consider the following cases:

Case I: The whole amount of radiation falling on the surface is absorbed by the surface.

For this, absorption coefficient $a = 1$, reflection coefficient $r = 0$.

Initial momentum of one photon $= \dfrac{h}{\lambda}$ (downward)

Final momentum of the photon = 0

Change in momentum of one photon $= \dfrac{h}{\lambda}$ (upward)

Energy incident per unit time = IA

Number of photons incident per unit time:

$$N = \frac{IA}{hf} = \frac{IA\lambda}{hc}$$

Therefore, the total change in momentum per unit time

$$= N\frac{h}{\lambda} = \frac{IA\lambda}{hc} \times \frac{h}{\lambda} = \frac{IA}{c} \quad \text{(upward)}$$

Force on photons = Total change in momentum per unit time

$$= \frac{IA}{c} \quad \text{(upward)}$$

Therefore, from the third law, force on plate due to photons:

$$F = \frac{IA}{c} \quad \text{(downward)}$$

Therefore, pressure $= \frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$

Case II: The whole amount of radiation falling on the surface is reflected by the surface.

For this, reflection coefficient $r = 1$, absorption coefficient $a = 0$.

Initial momentum of one photon $= \frac{h}{\lambda}$ (downward)

Final momentum of the photon $= \frac{h}{\lambda}$ (upward)

Change in momentum of one photon $= \frac{h}{\lambda} + \frac{h}{\lambda} = \frac{2h}{\lambda}$ (upward)

Energy incident per unit time $= IA$

Number of photons incident per unit time: $N = \frac{IA\lambda}{hc}$

Therefore, the total change in momentum per unit time

$$N\frac{2h}{\lambda} = \frac{IA\lambda}{hc} \cdot \frac{2h}{\lambda} = \frac{2IA}{c}$$

Force on photons = Total change in momentum per unit time

$$= \frac{2IA}{c} \quad \text{(upward)}$$

Therefore, force on plate due to photons:

$$F = \frac{2IA}{c} \quad \text{(downward)}$$

Pressure $P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$

ILLUSTRATION 28.4 A plate of mass 10 g is in equilibrium in air due to the force exerted by a light beam on the plate. Calculate power of the beam. Assume that the plate is perfectly absorbing.

Solution. For equilibrium, the force exerted by the light beam should balance the weight of plate.

$$F_{photon} = mg$$

$$\left(F_{photon} = \frac{IA}{c} = \frac{P}{c}, \text{ where power } P = IA\right)$$

$\Rightarrow \quad \frac{P}{c} = 10 \times 10^{-3} \times 10$

$\Rightarrow \quad P = 3 \times 10^7$ W

ILLUSTRATION 28.5 A radiation of wavelength 200 nm is propagating in the form of a parallel surface. The intensity of the beam is 5 mW and its cross-sectional area is 1.0 mm^2. Find the pressure exerted by radiation on the metallic surface if the radiation is completely reflected.

Solution. Energy of one photon $E = hc/\lambda$

If power of source is P, the number of photons incident on the metallic surface is

$$\frac{P}{E} = \frac{\lambda P}{hc}$$

Momentum of incident photons $= h/\lambda$

Change in momentum due to reflection $= 2h/\lambda$

The total momentum imparted to the surface per second is

$$\frac{2h}{\lambda}\frac{\lambda P}{hc} = \frac{2P}{c}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \left(\frac{2P}{c}\right)\Big/A = \frac{2P}{cA} = \frac{2 \times 5 \times 10^{-3}}{3 \times 10^8 \times 10^{-6}}$$

$$= 3.33 \times 10^{-5}\ \text{N m}^{-2}$$

MATTER WAVES (DE BROGLIE WAVES)

de Broglie suggested that a moving body behaves in certain ways as though it has a wave nature. His conjecture of dual nature of matter was based on the following two points: (1) dual nature of radiation, and (2) nature loves symmetry.

de Broglie deduced the connection between particle and wave properties from the Einstein–Planck expression for the energy of an electromagnetic wave and the classical result for the momentum of such a wave. The two expressions are

$$E = hf \quad \text{(i)}$$

and $\quad p = \frac{E}{c} \quad$ (ii)

From Eqs. (i) and (ii), we get

$$\lambda = \frac{h}{p} \quad \text{(iii)}$$

de Broglie said that Eq. (iii) is a completely general one that is applied to material particle as well as photons. The waves associated with moving particle are called *matter waves or de Broglie waves.* The wavelength associated with a moving particle is known as *de Broglie wavelength* and it is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{(iv)}$$

where p = momentum of the particle, m = mass of the particle, and v = velocity of the particle.

Also, KE $= \frac{p^2}{2m} \quad \Rightarrow \quad \lambda = \frac{h}{\sqrt{2m\text{KE}}}$

de Broglie wavelength associated with charged particles

1. For electrons ($m_e = 9.1 \times 10^{-31}$ kg):

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}\ \text{m} = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

2. For protons ($m_p = 1.67 \times 10^{-27}$ kg):

$$\lambda = \frac{0.286}{\sqrt{V}} \text{Å}$$

3. For deuterons ($m_d = 2 \times 1.67 \times 10^{-27}$ kg):

$$\lambda = \frac{0.202}{\sqrt{V}} \text{Å}$$

4. For α-particles ($m_\alpha = 4 \times 1.67 \times 10^{-27}$ kg):

$$\lambda = \frac{0.101}{\sqrt{V}} \text{Å}$$

de Broglie wavelength associated with uncharged particles

1. For neutrons ($m_n = 1.67 \times 10^{-27}$ kg):

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27}\, E}}$$

2. For thermal neutrons at ordinary temperatures:

$$E = kT$$

$$\lambda = \frac{h}{\sqrt{2mkT}} = \frac{30.835}{\sqrt{T}} \text{Å}$$

3. For gas molecules:

$$\lambda = \frac{h}{m \times c_{rms}}$$

4. For gas molecules at T K:

$$E = \frac{3}{2}kT \quad \Rightarrow \quad \lambda = \frac{h}{\sqrt{3mkT}}$$

Properties of Matter Waves

1. Matter waves are different from electromagnetic waves. This is an obvious fact as matter waves are related to moving particles and independent of the fact whether particle is neutral or charged.
2. Matter waves travel with speed more than that of light.

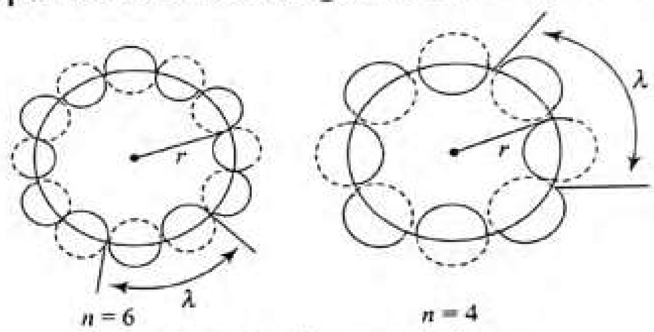
$$v_{\text{matter waves}} = f\lambda = \frac{E}{h} \times \frac{h}{P} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

As velocity of particle can never exceed speed of light, hence $v_{\text{matter waves}} > c$. As a matter of fact, matter waves travel with the body in a group. Group velocity is same as velocity of body, whereas the individual waves constituting the group may theoritically move with speed greater than that of light.
3. In ordinary situation, de Broglie wavelength is very small and wave nature of matter can be ignored. To appreciate this point, let us calculate de Broglie wavelength of 46 g of golf ball moving with a speed of 30 m s^{-1}.

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(0.046)(30)} = 4.8 \times 10^{-34} \text{ m}$$

The wavelength of golf ball is so small compared with its dimensions that we would not expect to detect any wave aspects in its behavior using ordinary instrument.
4. The wave and particle aspects of moving bodies can never be observed at the same time, i.e., the two nature are mutually exclusive. Which nature dominates in a given situation will be directed by how its de Broglie wavelength compares with its dimensions and dimensions of whatever it interacts with.
5. The square of the amplitude of de Broglie wave at any point is proportional to the probability of finding the particle at that point.

Application of de Broglie Wave Hypothesis



Standing de Broglie waves of electrons around the circumference of Bohr's orbit

Electron microscope: de Broglie idea of wave associated with a moving material particle had led to the development of a microscope using a beam of fast moving electrons. The microscope is highly suitable for large magnification in the study of atomic structure.

Quantization of orbit: The concept of matter waves was used to justify quantization of angular momentum as proposed in Bohr's atomic model. If we consider standing electron wave, then to maintain a standing wave over the circumference of a circular orbit, the wavelength must be an integral fraction of that circumference.

i.e., $2\pi r = n\lambda = \frac{nh}{P} = \frac{nh}{mv}$ or $mvr = \frac{nh}{2\pi}$

ILLUSTRATION 28.6 An electron and a photon have the same de Broglie wavelength. Which one of these has higher kinetic energy?

Solution. Let λ be the de Broglie wavelength of the electron and the photon. If m and v are the mass and the velocity of the electron, then de Broglie wavelength of the electron

$$\lambda = \frac{h}{mv}$$

The photon has zero rest mass. Therefore, energy of the photon is totally kinetic in nature. Since the wavelength of the photon is same as that of the electron, the kinetic energy of the photon having wavelength λ,

$$E_1 = \frac{hc}{\lambda} = \frac{hc}{h/mv}$$

or $\quad E_1 = mvc \qquad$ (i)

Now, the kinetic energy of the electron,

$$E_2 = \frac{1}{2}mv^2 = mv \times \frac{v}{2} \qquad \text{(ii)}$$

Since $c > v/2$ (as $c > v$), from the results (i) and (ii), it follows that

$$E_1 > E_2$$

i.e., kinetic energy of the photon is greater than that of the electron. As it moves with the speed c, it is faster than electron.

ILLUSTRATION 28.7 An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Solution. Here, $V = 50$ kV. Therefore, energy of electrons,

$$E = 50 \text{ keV} = 50 \times 10^3 \times 1.6 \times 10^{-19} = 8.0 \times 10^{-15} \text{ J}$$

Now,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Taking $m = 9.1 \times 10^{-31}$ kg, we have

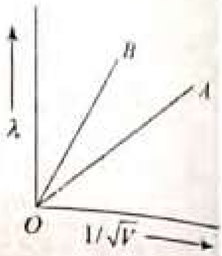
$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 8.0 \times 10^{-15}}}$$

$$= \frac{6.62 \times 10^{-34}}{1.207 \times 10^{-22}} = 5.485 \times 10^{-12} \text{ m}$$

The resolving power of a microscope is inversely proportional to the wavelength of the radiation used. Since wavelength of the yellow light is 5990 Å, i.e., 5.99×10^{-7} m, power of electron microscope is 10^5 times as large as that of the optical microscope.

CONCEPT APPLICATION EXERCISE 28.1

1. Find the de Broglie wavelength of 2 MeV proton. Mass of proton $= 1.64 \times 10^{-27}$ kg, $h = 6.625 \times 10^{-34}$ J s.
2. Find the de Broglie wavelength of a neutron at 127°C. Given that Boltzmann's constant $k = 1.38 \times 10^{-23}$ J molecule^{-1} K^{-1}. Planck's constant $= 6.625 \times 10^{-34}$ J s, mass of neutron $= 1.66 \times 10^{-27}$ kg.
3. The intensity of direct sunlight before it passes through the earth's atmosphere is 1.4 kWm^{-2}. If it is completely absorbed, find the corresponding radiation pressure.
4. An electron is accelerated by a potential difference of 25 V. Find the de Broglie wavelength associated with it.
5. Why are de Broglie waves associated with a moving football not apparent to us?
6. The de Broglie wavelength of a particle of kinetic energy K is λ. What would be the wavelength of the particle, if its kinetic energy were $K/4$?
7. The two lines A and B in figure show the plot of de Broglie wavelength (λ) as a function of $1/\sqrt{V}$ (V is the accelerating potential) for two particles having the same charge. Which of the two represents the particle of heavier mass?



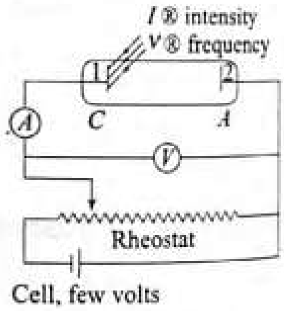
PHOTOELECTRIC EFFECT

When electromagnetic radiations of suitable wavelength (or frequency) are incident on a metallic surface then electrons are emitted, this phenomenon is called *photoelectric effect*. The electrons so emitted are known as *photoelectrons*. And the current so obtained is known as *photoelectric current*.

Alkali metals Li, Na, K, Cs, etc., show photoelectric emission with visible light. Zn, Cd, Mg, etc., show photoelectric emission with ultraviolet light.

Study of Photoelectric Effect

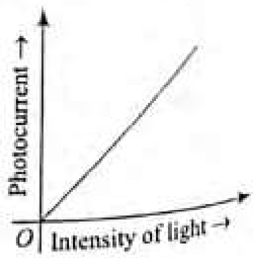
Figure shows the experimental setup to study the photoelectric effect.



Here plate 1 is called emitter or cathode and plate 2 is called collector or anode. A suitable potential difference $V = V_A - V_C$ can be applied between them. On cathode, an electromagnetic radiation of intensity I and frequency f is allowed to fall upon. Due to this, electrons are ejected out from plate 1 and they travel to plate 2 as it is at positive potential V w.r.t. plate 1. Thus, a current is established in the circuit known as photoelectric current (PEC).

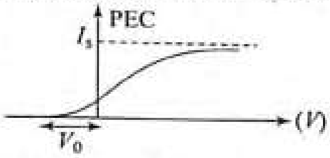
Observations (by Einstein)

1. If in this experiment the frequency f and potential difference V are kept constant and intensity I is varied (or more number of photons are allowed to fall per unit time), a graph between intensity of light and photoelectric current (PEC) is found to be a straight line as shown in figure.



The photoelectric current is directly proportional to the intensity of incident radiation.

2. A graph between photoelectric current and potential difference V is found as shown in figure. Here, I and f are kept constant. If we increase V, then PEC also increases, but after increasing V up to some value there is no increase in PEC. This value of PEC is known as saturation current (I_s). At this value, all the photoelectrons emitted from cathode reach the anode and there is no possibility of increasing the current by increasing V. Hence in case of saturation current, the rate of emission of photoelectrons is the rate of flow of photoelectrons from cathode to anode.



V_0 is known as stopping potential or cut-off potential. It is the minimum negative potential given to plate A w.r.t. C at which PEC just becomes zero.

Einstein's Photoelectric Equation

According to Einstein, photon energy (hf) falling on a metal is utilized for two purposes:

1. Partly for getting the electron free from the atom and away from the metal surface. This energy is known as the photoelectric work function of the metal and is represented by W_0.
2. The balance of the photon energy is used up in giving the electron a kinetic energy KE.

So, we can write:

$$hf = W_0 + \text{KE}$$

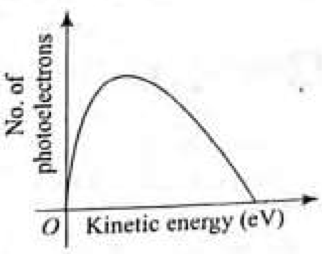
$$\Rightarrow \quad \text{KE} = hf - W_0$$

The above equation says that when a single photon carrying an energy hf enters into the surface, there it is absorbed by a single electron. Part of this energy W_0 (called the work function of the emitting surface) is used in causing the electron to escape from the metal surface. The excess energy ($hf - W_0$) becomes the electron's kinetic energy. If the electron does not lose energy by internal collisions as it escapes from the metal, it will still have this much kinetic energy after it emerges. Thus, ($hf - W_0$) represents the maximum kinetic energy that a photoelectron can have outside the surface.

So, we can write:

$$\text{KE}_{max} = hf - W_0$$

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from 0 to KE_{max}. The energy distribution of photoelectrons is as shown in figure.



In case the photon energy (say for $f = f_0$) is just sufficient to liberate the electron only, then kinetic energy of the liberated electron will be zero. Then, $hf_0 = W_0$.

Here, f_0 is known as *threshold frequency*. The corresponding wavelength is known as *threshold wavelength*. If the frequency of incident light is less than f_0, no photoelectric emission takes place.

Maximum kinetic energy of photoelectrons is

$$\text{KE}_{max} = hf - hf_0 = h(f - f_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$= 124000\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)\text{eV} \qquad \text{(i)}$$

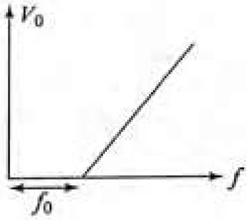
For a given frequency, the stopping potential is related to the maximum KE of photoelectrons which are just stopped from reaching the anode A. For this,

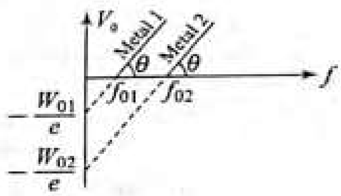
$$eV_0 = \text{KE}_{max} = \frac{1}{2}mv_{max}^2 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$eV_0 = h(f - f_0)$$

$$\Rightarrow \quad V_0 = \frac{h}{e}f - \frac{hf_0}{e} = \frac{h}{e}f - \frac{W_0}{e}$$

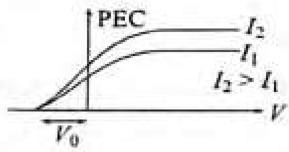




Stopping potential varies linearly with frequency. Slope of the graph (h/e) is same for all metals.

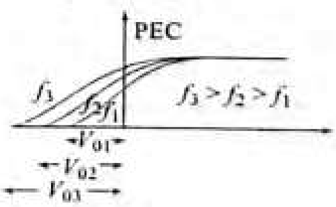
For $f \le f_0$, stopping potential is zero.

3. If intensity is increased (keeping the frequency constant), then saturation current is increased by the same factor by which intensity increases. Stopping potential is same, so maximum value of kinetic energy is not affected.



So, maximum KE of photoelectrons emitted is independent of intensity of falling radiation.

4. If light of different frequencies (keeping the intensity same) is used, then plots are obtained which are shown in figure. It is clear from the graph, as f increases, the magnitude of stopping potential increases. It means the maximum value of kinetic energy increases.



So, maximum KE of photoelectrons emitted depends on the frequency of incident light.

Laws of Photoelectric Effect

1. The number of photoelectrons emitted per second is proportional to the intensity of the incident light.
2. There is a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.
3. Above the threshold frequency, the maximum velocity with which electrons emerge is dependent solely on the frequency and not on the intensity of the incident light.
4. The emission of photoelectrons is an instantaneous process. It means time difference between incidence of light and emission of photoelectrons is very small, may be even less than 10^{-9} s.

Failure of Classical Wave Theory of Light to Explain the Laws of Photoelectric Effect

The intensity problem: If intensity of light falling on metal is increased, then, according to wave theory of light, oscillating electric field vector $\vec{E}$ of the light wave increases in amplitude. The force applied to the electron in the metal will be $e\vec{E}$ due to the falling radiation and this force will increase on increasing the intensity of light. This suggests that the kinetic energy of the emitted photoelectrons should also increase due to larger force applied in emitting them out as the light beam is made more intense. However, observations show that maximum kinetic energy is independent of the light intensity.

The frequency problem: According to the wave theory, the photoelectric effect should occur for any frequency of the light, provided that the light is intense enough to supply the energy needed to eject the photoelectrons. However, observations show that the photoelectric effect does not occur if the frequency of falling radiation is less than the threshold frequency, no matter how intense the light beam is.

The time delay problem: In the classical theory, the light energy is uniformly distributed over the wave front. So, when the light falls on metal, the energy of the incident light will not entirely go to a particular electron in the metal but will be distributed uniformly to other electrons also. So, the electron will take some time to accumulate enough energy to escape from the metal surface. Hence, there should be a measurable time lag between the impinging of the light on the surface and the ejection of the photoelectron. However, no detectable time lag has ever been measured.

Quantum theory solves these problems in providing the correct interpretation of photoelectric effect as follows:

1. Let a light of intensity I fall on area A. Let N be the number of photons falling per unit time, then

$$IA = Nhf \quad \Rightarrow \quad N = \frac{IA}{hf}$$

If we double the light intensity, we double the number of photons and thus double the photoelectric current; we do not change the energy of the individual photon. Hence, kinetic energy of emitted electrons does not change on increasing the intensity.

2. The second objection (the frequency problem) is met if KE_{max} equals zero. In this case we have $hf_0 = W$, which asserts that the photon has just enough energy to eject the photoelectrons and no extra to appear as kinetic energy. If f is reduced below f_0, hf will be smaller than W and the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photoelectrons.
3. The third objection (the time delay problem) follows from the photon theory because the required energy is supplied in a concentrated bundle or in the form of energy packets. It is not spread uniformly over the beam cross section as in the wave theory. Basically, the emission of photoelectron is a single step process in which energy from photon is supplied to electron at once and the electron is ejected immediately.

ILLUSTRATION 28.8 What is the energy (in eV) of a photon of wavelength 12400 Å?

Solution. Using Planck's formula, we have $E = hf = \frac{hc}{\lambda}$ where $h = 6.63 \times 10^{-34}$ J s; $c = 3 \times 10^8$ m s^{-1}; $\lambda = 12400 \times 10^{-10}$ m

$$\therefore \quad E = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{12400 \times 10^{-10}} = 1 \text{ eV}$$

NOTE: In general, a photon of wavelength λ (in Å) will have energy E (in eV) as given by $E = 12400/\lambda$.

ILLUSTRATION 28.9 Will photoelectrons be emitted from a copper surface, of work function 4.4 eV, when illuminated by a visible light?

Solution. The threshold wavelength λ_0 corresponding to work function W is given by

$$\lambda_0 = \frac{hc}{W} \quad \text{or} \quad \lambda_0 = \frac{12400}{4.4} = 2820 \text{ Å}$$

Since λ_0 does not lie in the visible range (4000 Å to 7500 Å), therefore it cannot eject photoelectrons from copper.

ILLUSTRATION 28.10 The stopping potential for photoelectrons emitted from a surface illuminated by light wavelength of 5893 Å is 0.36 V. Calculate the maximum kinetic energy of photoelectrons, the work function of the surface, and the threshold frequency.

Solution. We know that

$$KE_{max} = hf - \phi = \left(\frac{hc}{\lambda}\right) - \phi$$

or $$\phi = \frac{hc}{\lambda} - KE_{max}$$

Also, $KE_{max} = eV_s = 0.36$ eV

$$\Rightarrow \quad \phi = \frac{(6.62\times10^{-34})\times(3\times10^{8})}{5893\times10^{-10}} - 0.36\times1.6\times10^{-19}$$

$= 1.746\text{ eV}$

The threshold frequency is given by

$$f_0 = \frac{\phi}{h} = \frac{2.794\times10^{-19}}{6.62\times10^{-34}} = 4.22\times10^{14}\text{ Hz}$$

Concept Application Exercise 28.2

1. One milliwatt of light of wavelength 4560 Å is incident on a cesium surface. Calculate the photoelectric current liberated assuming a quantum efficiency of 0.5%. Given Planck's constant $h = 6.62\times10^{-34}$ J-s and velocity of light $c = 3\times10^{8}\ \text{ms}^{-1}$.
2. Light quanta with energy 4.9 eV eject photoelectrons from metal with work function 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.
3. The maximum KE of photoelectrons emitted from a certain metallic surface is 30 eV when monochromatic radiation of wavelength λ falls on it. When the same surface is illuminated with light of wavelength 2λ, the minimum KE of photoelectrons is found to be 10 eV. (a) Calculate the wavelength λ and (b) determine the maximum wavelength of incident radiation for which photoelectric emission is possible.
4. A horizontal photoelectric plate whose work function is 1.9 eV is moved vertically downward at a constant speed v in a room full of radiation of wavelength 250 nm and above. What should be the minimum value of v so that the vertically upward component of velocity of ejected photoelectrons is non-positive?
5. A beam of 1.5 mW of 400 nm light is directed at a photoelectric cell. If 0.1% of the incident photons produce electrons, find the current in the photocell. Assume all the photoelectrons to reach the opposite plate.
6. A uniform monochromatic beam of light of wavelength 365×10^{-9} m and intensity $10^{-8}\ \text{Wm}^{-2}$ falls on a surface having absorption coefficient 0.8 and work function 1.6 eV. Determine the rate of number of electrons emitted per m^2, power absorbed per m^2 and the maximum kinetic energy of emitted photoelectrons.

Solved Examples

1. When a point source of monochromatic light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are 0.6 volt and 18 mA respectively. If the same source is placed 0.6 m away from the photoelectric cell, then
   (a) The stopping potential will be 0.2 V
   (b) The stopping potential will be 0.6 V
   (c) The saturation current will be 6 mA
   (d) The saturation current will be 18 mA

Sol. (b) Cut-off voltage is independent of intensity and hence, remains the same. Since distance becomes 3 times, intensity (I) becomes $\frac{I}{9}$. Hence, photocurrent also decreases by this factor i.e., becomes $\frac{18}{9} = 2$ mA.

2. Ultraviolet light of wavelength 300 nm and intensity 1.0 watt/m^2 falls on the surface of a photosensitive material. If 1% of the incident photons produce photoelectrons, then the number of photoelectrons emitted from an area of 1.0 cm^2 of the surface is nearly
   (a) 9.61×10^{14} per sec (b) 4.12×10^{13} per sec
   (c) 1.51×10^{12} per sec (d) 2.13×10^{11} per sec

Sol. (c) Intensity of light,

$$I = \frac{Power}{Area} = \frac{nhc}{A\lambda} \Rightarrow \text{Number of photon, } n = \frac{IA\lambda}{hc}$$

$$\therefore \quad \text{Number of photo electron} = \frac{1}{100}\times\frac{IA\lambda}{hc}$$

$$= \frac{1}{100}\times\frac{1\times10^{-4}\times300\times10^{-9}}{6.6\times10^{-34}\times3\times10^{8}} = 1.5\times10^{12}$$

3. Light from a hydrogen discharge tube is incident on the cathode of a photoelectric cell. The work function of the cathode surface is 4.2 eV. In order to reduce the photocurrent to zero the voltage of the anode relative to the cathode must be made
   (a) -4.2 V (b) -9.4 V
   (c) -17.8 V (d) $+9.4$ V

Sol. (b) $E = W_0 + eV_0$

For hydrogen atom, $E = +13.6$ eV

$\therefore \quad +13.6 = 4.2 + eV_0$

$$\Rightarrow \quad V_0 = \frac{(13.6-4.2)eV}{e} = 9.4\text{ V}$$

Potential at anode $= -9.4$ V

4. Work function of lithium and copper are respectively 2.3 eV and 4.0 eV. Which metal will be useful for the photoelectric cell working with visible light? ($h = 6.6\times10^{-34}$ J-s, $c = 3\times10^{8}$ m/s)
   (a) Lithium (b) Copper
   (c) Both (d) None of these

Sol. (a) From $\lambda_0 = \frac{12375}{W_0}$

The maximum wavelength of light required for the photoelectron emission, $(\lambda_0)_{Li} = \frac{12375}{2.3} = 5380$ Å

Similarly $(\lambda_0)_{Cu} = \frac{12375}{4} = 3094$ Å

The wavelength 3094 Å does not lie in the visible region, but it is in the ultraviolet region. Hence to work with visible light, lithium metal will be used for photoelectric cell.

5. The ratio of de Broglie wavelength of a α-particle to that of a proton being subjected to the same magnetic field so that the radii of their paths are equal to each other assuming the field induction vector $\vec{B}$ is perpendicular to the velocity vectors of the α-particle and the proton is

(a) 1 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 2

Sol. (c) When a charged particle (charge q, mass m) enters perpendicularly in a magnetic field (B) than, radius of the path

$r = \frac{mv}{qB} \Rightarrow mv = qBr$.

Also de Broglie wavelength, $\lambda = \frac{h}{mv}$

$$\Rightarrow \lambda = \frac{h}{qBr} \Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \frac{q_p r_p}{q_\alpha r_\alpha} = \frac{1}{2}$$

6. K_α wavelength emitted by an atom of atomic number $Z = 11$ is λ. The atomic number for an atom that emits K_α radiation with wavelength 4λ is

(a) $Z = 6$ (b) $Z = 4$

(c) $Z = 11$ (d) $Z = 44$

Sol. (a) $\sqrt{f_1} = \sqrt{\frac{v}{\lambda_1}} = a(11-1)$ and $\sqrt{f_2} = \sqrt{\frac{v}{\lambda_2}} = a(Z-1)$

By dividing, $\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{10}{Z-1} \Rightarrow \sqrt{\frac{4}{1}} = \frac{10}{Z-1} \Rightarrow Z = 6$

7. The potential energy of a particle of mass m is given by

$$U(x) = \begin{cases} E_0; & 0 \le x \le 1 \\ 0; & x > 1 \end{cases}$$

λ_1 and λ_2 are the de Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, the ratio $\frac{\lambda_1}{\lambda_2}$ will be

(a) 2 (b) 1

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

Sol. (c) K.E. = $2E_0 - E_0 = E_0$ (for $0 \le x \le 1$) $\Rightarrow \lambda_1 = \frac{h}{\sqrt{2mE_0}}$

K.E. = $2E_0$ (for $x > 1$) $\Rightarrow \lambda_2 = \frac{h}{\sqrt{4mE_0}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{2}$.

8. The minimum intensity of light to be detected by human eye is 10^{-10} W/m². The number of photons of wavelength 5.6×10^{-7} m entering the eye, with pupil area 10^{-6} m², per second for vision will be nearly

(a) 100 (b) 200

(c) 300 (d) 400

Sol. (c) By using $I = \frac{P}{A}$; where P is radiation power, we get

$$P = I \times A \Rightarrow \frac{nhc}{t\lambda} = IA \Rightarrow \frac{n}{t} = \frac{IA\lambda}{hc}$$

Hence number of photons entering the eye per sec

$$\left(\frac{n}{t}\right) = \frac{10^{-10} \times 10^{-6} \times 5.6 \times 10^{-7}}{6.6 \times 10^{-34} \times 3 \times 10^8} = 300.$$

9. In a photocell, bichromatic rays of light of wavelength 2475 Å and 6000 Å are incident on cathode whose work function is 4.8 eV. If a uniform magnetic field of 3×10^{-5} tesla exists parallel to the plate, the radius of the path describe by the photoelectron will be (mass of electron $= 9 \times 10^{-31}$ kg)

(a) 1 cm (b) 5 cm

(c) 10 cm (d) 25 cm

Sol. (b) Energy of photons corresponding to light of wave length $\lambda_1 = 2475$ Å is $E_1 = \frac{12375}{2475} = 5$ eV

and that corresponding to $\lambda_2 = 6000$ Å is

$$E_2 = \frac{12375}{6000} = 2.06 \text{ eV}$$

As $E_2 < W_0$ and $E_1 > W_0$,

photoelectric emission is possible with λ_1 only. Maximum kinetic energy of emitted photoelectrons, $K = E - W_0 = 5 - 4.8 = 0.2$ eV.

Photo electrons experience magnetic force and move along a circular path of radius,

$$r = \frac{\sqrt{2mk}}{QB} = \frac{\sqrt{2 \times 9 \times 10^{-31} \times 0.2 \times 1.6 \times 10^{-19}}}{1.6 \times 10^{-19} \times 3 \times 10^{-5}}$$

$= 0.05$ m $= 5$ cm.

10. Two metallic plates A and B, each of area 5×10^{-4} m² are placed parallel to each other at a separation of 1 cm. Plate B carries a positive charge of 33.7 pC. A monochromatic beam of light, with photons of energy 5 eV each, starts falling on plate A at $t = 0$, so that 10^{16} photons fall on it per square meter per second. Assume that one photoelectron is emitted for every 10^6 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2 eV. Electric field between the plates at the end of 10 seconds is

(a) 2×10^3 N/C (b) 10^3 N/C

(c) 5×10^3 N/C (d) Zero

Sol. (a) Number of photoelectrons emitted up to $t = 10$ sec are

$$n = \frac{(\text{Number of photons per unit area per unit time}) \times (\text{Area}) \times (\text{Time})}{10^6}$$

$$= \frac{1}{10^6}[(10)^{16} \times (5 \times 10^{-4}) \times (10)] = 5 \times 10^7$$

At time $t = 10$ s

Charge on plate A; $q_A = +ne = 5 \times 10^7 \times 1.6 \times 10^{-19}$
$= 8 \times 10^{-12}$ C $= 8$ pC

and charge on plate B; $q_B = 33.7 - 8 = 25.7$ pC

Electric field between the plates

$$E = \frac{(q_B - q_A)}{2\varepsilon_0 A} = \frac{(25.7 - 8) \times 10^{-12}}{2 \times 8.85 \times 10^{-12} \times 5 \times 10^{-4}} = 2 \times 10^3 \frac{\text{N}}{\text{C}}.$$

11. The eye can detect 5×10^4 photons per square metre per sec of green light ($\lambda = 5000$ Å) while the ear can detect 10^{-13}(W/m^2). The factor by which the eye is more sensitive as a power detector than the ear is close to

(a) 5 (b) 10
(c) 10^6 (d) 15

Sol. (a) $E = \frac{12375}{5000} = 2.475 \text{ eV} \approx 4 \times 10^{-19}$ J

So the minimum intensity to which the eye can respond

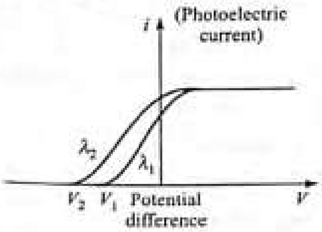
I_{Eye} = (Photon flux) × (Energy of a photon)

$\Rightarrow I_{Eye} = (5 \times 10^4) \times (4 \times 10^{-19}) \simeq 2 \times 10^{-14}$ (W/m^2)

Now as lesser the intensity required by a detector for detection, more sensitive it will be

$\frac{S_{Eye}}{S_{Ear}} = \frac{I_{Ear}}{I_{Eye}} = \frac{10^{-13}}{2 \times 10^{-14}} = 5$ i.e., as intensity (power) detector, the eye is five times more sensitive than ear.

12. In the following diagram if $V_2 > V_1$ then



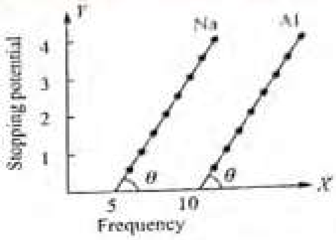
(a) $\lambda_1 = \sqrt{\lambda_2}$ (b) $\lambda_1 < \lambda_2$
(c) $\lambda_1 = \lambda_2$ (d) $\lambda_1 > \lambda_2$

Sol. (d) $V_0 = \left(\frac{h}{e}\right)\nu - \left(\frac{W_0}{e}\right)$. From the graph $V_2 > V_1$.

$$\Rightarrow \frac{h\nu_2}{e} - \frac{W_0}{e} > \frac{h\nu_1}{e} - \frac{W_0}{e} \Rightarrow \nu_2 > \nu_1$$

$\Rightarrow \lambda_1 > \lambda_2$ (as $\lambda \propto \frac{1}{\nu}$)

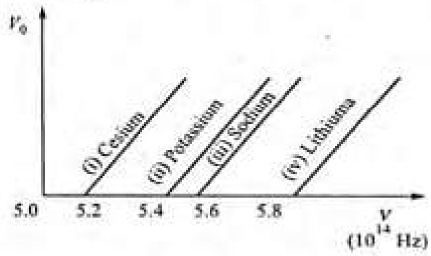
13. From the figure describing photoelectric effect, we may infer correctly that



(a) Na and Al both have the same threshold frequency.
(b) Maximum kinetic energy for both the metals depend linearly on the frequency.
(c) The stopping potentials are different for Na and Al for the same change in frequency.
(d) Al is a better photosensitive material than Na.

Sol. (b) Stopping potential equals to maximum kinetic energy. Since stopping potential is varying linearly with the frequency, max. KE for both the metals also vary linearly with frequency.

14. The figure shows different graphs between stopping potential (V_0) and frequency (ν) for photosensitive surface of cesium, potassium, sodium and lithium. The plots are parallel. Correct ranking of the targets according to their work function greatest first will be



(a) (i) > (ii) > (iii) > (iv) (b) (i) > (iii) > (ii) > (iv)
(c) (iv) > (iii) > (ii) > (i) (d) (i) = (iii) > (ii) = (iv)

Sol. (c) The graph between V_0 and ν cut the ν-axis at ν_0.

For the given graphs $(\nu_0)_{(iv)} > (\nu_0)_{(iii)} > (\nu_0)_{(ii)} > (\nu_0)_{(i)}$

$\therefore (W_0)_{(iv)} > (W_0)_{(iii)} > (W_0)_{(ii)} > (W_0)_{(i)}$

15. If 10000 V is applied across an X-ray tube, what will be the ratio of de Broglie wavelength of the incident electrons to the shortest wavelength of X-ray produced ($\frac{e}{m}$ for electron is 1.8×10^{11} C kg^{-1})

(a) 1 (b) 0.1
(c) 0.2 (d) 0.3

Sol. (b) For the incident electron, $\frac{1}{2}mv^2 = eV$ or $P^2 = 2meV$

$\therefore$ de Broglie wavelength, $\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$

Shortest X-ray wavelength $\lambda_2 = \frac{hc}{eV}$

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{1}{c}\sqrt{\left(\frac{V}{2}\right)\left(\frac{e}{m}\right)} = \frac{\sqrt{\frac{10^4}{2} \times 1.8 \times 10^{11}}}{3 \times 10^8} = 0.1$$

EXERCISES

Photon Momentum and Energy, De-Broglie Hypothesis

1. Out of a photon and an electron, the equation $E = pc$, is valid for
   (a) both (b) neither
   (c) photon only (d) electron only

2. How many photons are emitted per second by a 5 mW laser source operating at 632.8 nm?
   (a) 1.6×10^{16} (b) 1.6×10^{13}
   (c) 1.6×10^{10} (d) 1.6×10^{3}

3. If 5% of the energy supplied to a bulb is irradiated as visible light, how many quanta are emitted per second by a 100 W lamp? Assume wavelength of visible light as 5.6×10^{-5} cm.
   (a) 1.4×10^{19} (b) 3×10^{3}
   (c) 1.4×10^{-19} (d) 3×10^{4}

4. A modern 200 W sodium street lamp emits yellow light of wavelength 0.6 μm. Assuming it to be 25% efficient in converting electrical energy to light, the number of photons of yellow light it emits per second is
   (a) 62×10^{20} (b) 3×10^{19}
   (c) 1.5×10^{20} (d) 6×10^{18}

5. How many photons of a radiation of wavelength $\lambda = 5 \times 10^{-7}$ m must fall per second on a blackened plate in order to produce a force of 6.62×10^{-5} N?
   (a) 3×10^{19} (b) 5×10^{22}
   (c) 2×10^{22} (d) 1.67×10^{18}

6. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de Broglie wavelength associated with an electron to that associated with proton is
   (a) 1 (b) m_p/m_e
   (c) m_e/m_p (d) $\sqrt{m_p/m_e}$

7. A material particle with a rest mass m_0 is moving with a velocity of light c. Then, the wavelength of the de Broglie wave associated with it is
   (a) (h/m_0c) (b) zero
   (c) ∞ (d) (m_0c/h)

8. If λ_0 stands for mid-wavelength in the visible region, the de Broglie wavelength for 100 V electrons is nearest to
   (a) $\lambda_0/5$ (b) $\lambda_0/50$
   (c) $\lambda_0/500$ (d) $\lambda_0/5000$

9. An electron is accelerated through a potential difference of V volt. It has a wavelength λ associated with it. Through what potential difference an electron must be accelerated so that its de Broglie wavelength is the same as that of a proton? Take mass of proton to be 1837 times larger than the mass of electron.
   (a) V volt (b) $1837V$ volt
   (c) $V/1837$ volt (d) $\sqrt{1837}\,V$ volt

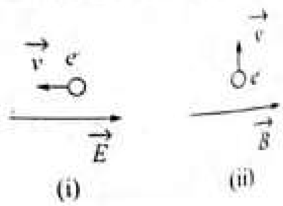
10. The potential difference applied to an X-ray tube is V. The ratio of the de Broglie wavelength of electron to the minimum wavelength of X-ray is directly proportional to
   (a) V (b) $\sqrt{V}$
   (c) $V^{3/2}$ (d) $V^{7/2}$

11. What is the de Broglie wavelength of the wave associated with an electron that has been accelerated through a potential difference of 50.0 V?
   (a) 2.7×10^{-10} (b) 1.74×10^{-10}
   (c) 3.6×10^{-9} (d) 4.9×10^{-11}

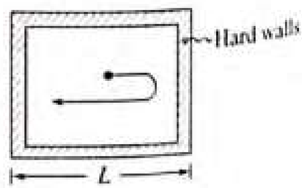
12. Find the ratio of de Broglie wavelength of a proton and an α-particle which have been accelerated through same potential difference.
   (a) $2\sqrt{2} : 1$ (b) 3:2
   (c) $3\sqrt{2} : 1$ (d) 2:1

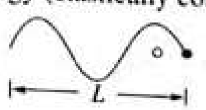
13. Two electrons are moving with same speed v. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after some time de Broglie wavelengths of two are λ_1 and λ_2, respectively. Now,
   (a) $\lambda_1 = \lambda_2$
   (b) $\lambda_1 > \lambda_2$
   (c) $\lambda_1 < \lambda_2$
   (d) λ_1 can be greater than or less than λ_2

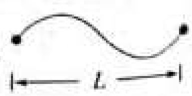
14. An electron is moving through a field. It is moving (i) opposite an electric field (ii) perpendicular to a magnetic field as shown. For each situation the de-Broglie wave length of electron
   (a) increasing, increasing
   (b) increasing, decreasing
   (c) decreasing, same
   (d) same, same



15. Which of the following is not a possible de-Broglie's wavelength of a particle, which moves inside a cubical box of side length L, without losing any energy (elastically colliding with walls of cube)?



   (a) 

   (b) 

(c) 

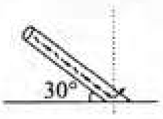
(d) All of these

16. Radiation pressure on any surface
(a) is dependent on wavelength of the light used
(b) is dependent on nature of surface and intensity of light used
(c) is dependent on frequency and nature of surface
(d) depends on the nature of source from which light is coming and on nature of surface on which it is falling

17. If the momentum of an electron is changed by Δp, then the de-Broglie wavelength associated with it changes by 0.50%. The initial momentum of the electron will be
(a) $\frac{\Delta p}{200}$ (b) $\frac{\Delta p}{199}$
(c) $199\Delta p$ (d) $400\ \Delta p$

18. A photon of wavelength 4400 Å is passing through vacuum. The effective mass and momentum of the photon are respectively
(a) 5×10^{-36} kg, 1.5×10^{-27} kg-m/s
(b) 5×10^{-35} kg, 1.5×10^{-26} kg-m/s
(c) Zero, 1.5×10^{-26} kg-m/s
(d) 5×10^{-36} kg, 1.67×10^{-43} kg-m/s

19. The number of photons ($\lambda = 6630$ Å) that strike per second on a totally reflecting screen (as shown in figure), so that a force of 1N is exerted on the screen, is approximately



(a) 10^{23} (b) 10^{27}
(c) 10^{25} (d) 10^{26}

20. There are n_1 photons of frequency γ_1 in a beam of light. In an equally energetic beam, there are n_2 photons of frequency γ_2. Then the correct relation is
(a) $\frac{n_1}{n_2}=1$ (b) $\frac{n_1}{n_2}=\frac{\gamma_1}{\gamma_2}$
(c) $\frac{n_1}{n_2}=\frac{\gamma_2}{\gamma_1}$ (d) $\frac{n_1}{n_2}=\frac{\gamma_1^2}{\gamma_2^2}$

21. Electrons are accelerated in television tubes through potential differences of about 10 kV. The highest frequency of the electromagnetic waves emitted when these electrons strike the screen of the tube is
(a) 2.4×10^{18} Hz (b) 3.6×10^{18} Hz
(c) 2.2×10^{17} Hz (d) 3.2×10^{16} Hz

Photoelectric Effect

22. In a photoelectric effect, electrons are emitted
(a) with a maximum velocity proportional to the frequency of the incident radiation
(b) at a rate that is independent of the intensity of the incident radiation
(c) only if the frequency of the incident radiation is above a certain threshold value
(d) only if the temperature of the emitter is high

23. With respect to electromagnetic theory of light, the photoelectric effect is best explained by statement
(a) Light waves carry energy and when light is incident on the metallic surface, the energy absorbed by the metal may somehow concentrate on individual electrons and reappear as their kinetic energy when ejected
(b) Particles of light (photons) collide with the metal and the electrons take this energy and may eject
(c) When light waves fall on a metallic surface, the stability of atoms is disturbed and the electrons come out to make the system stable
(d) None of the above

24. If the intensity of radiation incident on a photocell be increased four times, then the number of photoelectrons and the energy of photoelectrons emitted respectively become
(a) four times, doubled
(b) doubled, remains unchanged
(c) remains unchanged, doubled
(d) four times, remains unchanged

25. The work function of a metal is W and λ is the wavelength of the incident radiation. There is no emission of photoelectrons when
(a) $\lambda > hc/W$ (b) $\lambda = hc/W$
(c) $\lambda < hc/W$ (d) $\lambda \leq hc/W$

26. If a surface has work function of 3.00 eV, the longest wavelength of light which will cause the emission of electrons is
(a) 4.8×10^{-7} m (b) 5.99×10^{-7} m
(c) 4.13×10^{-7} m (d) 6.84×10^{-7} m

27. The work function for sodium surface is 2.0 eV and that for aluminium surface is 4.2 eV. The two metals are illuminated with appropriate radiations so as to cause photoemission. Then
(a) the threshold frequency for sodium will be less than that for aluminium
(b) the threshold frequency of sodium will be more than that of aluminium
(c) both sodium and aluminium will have the same threshold frequency
(d) none of the above

28. A metal surface is illuminated by a light of given intensity and frequency to cause photoemission. If the intensity of illumination is reduced to one-fourth of its original value, then the maximum KE of emitted photoelectrons will become
(a) $(1/16)^{th}$ of original value
(b) unchanged
(c) twice the original value
(d) four times the original value

29. The frequency of incident light falling on a photosensitive metal plate is doubled, the KE of the emitted photoelectrons is
(a) double the earlier value
(b) unchanged
(c) more than doubled
(d) less than doubled

30. Lights of two different frequencies whose photons have energies 1 and 2.5 eV, respectively, successively illuminate a metal whose work function is 0.5 eV. The ratio of the maximum speeds of the emitted electrons will be
(a) 1:5 (b) 1:4
(c) 1:2 (d) 1:1

31. A proton when accelerated through a potential difference of V volt has a wavelength λ associated with it. An α-particle in order to have the same λ must be accelerated through a potential difference of
(a) V volt (b) $4V$ volt
(c) $2V$ volt (d) $(V/8)$ volt

32. Given that a photon of light of wavelength 10,000 Å has an energy equal to 1.23 eV. When light of wavelength 5000 Å and intensity I_0 falls on a photoelectric cell, the saturation current is 0.40×10^{-6} A and the stopping potential is 1.36 V; then the work function is
(a) 0.43 eV (b) 1.10 eV
(c) 1.36 eV (d) 2.47 eV

33. In Q. 32, if the intensity of light is made $4I_0$, then the stopping potential will become
(a) 1.36×1 V (b) 1.36×2 V
(c) 1.36×3 V (d) 1.36×4 V

34. In Q. 32, if the intensity of light is made $4I_0$, then the saturation current will become
(a) 0.40×1 μA (b) 0.40×2 μA
(c) 0.40×4 μA (d) 0.40×8 μA

35. In Q. 32, if the cathode and the anode are kept at the same potential, the emitted electrons have
(a) the same KE equal to 1.36 eV
(b) the average KE equal to (1.36/2) eV
(c) the maximum KE equal to 1.36 eV
(d) the minimum KE equal to 1.36 eV

36. In Q. 32, if the wavelength is changed to 4000 Å, then stopping potential will become
(a) 1.36 V (b) 3.40 V
(c) 1.60 V (d) 1.97 V

37. The work function of a metallic surface is 5.01 eV. The photoelectrons are emitted when light of wavelength 2000 Å falls on it. The potential difference applied to stop the fastest photoelectrons is [$h = 4.14 \times 10^{-15}$ eVs]
(a) 1.2 V (b) 2.24 V
(c) 3.6 V (d) 4.8 V

38. If a photocell is illuminated with a radiation of 1240 Å, then stopping potential is found to be 8 V. The work function of the emitter and the threshold wavelength are
(a) 1 eV, 5200 Å (b) 2 eV, 6200 Å
(c) 3 eV, 7200 Å (d) 4 eV, 4200 Å

39. Silver has a work function of 4.7 eV. When ultraviolet light of wavelength 100 mm is incident upon it, a potential of 7.7 V is required to stop the photoelectrons from reaching the collector plate. How much potential will be required to stop the photoelectrons when light of wavelength 200 mm is incident upon silver?
(a) 1.5 V (b) 3.85 V
(c) 2.35 V (d) 15.4 V

40. When a centimeter thick surface is illuminated with light of wavelength λ, the stopping potential is V. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is $V/3$. Threshold wavelength for the metallic surface is
(a) $4\lambda/3$ (b) 4λ
(c) 6λ (d) $8\lambda/3$

41. Light of wavelength λ strikes a photoelectric surface and electrons are ejected with kinetic energy K. If K is to be increased to exactly twice its original value, the wavelength must be changed to λ' such that
(a) $\lambda' < \lambda/2$ (b) $\lambda' > \lambda/2$
(c) $\lambda > \lambda' > \lambda/2$ (d) $\lambda' = \lambda/2$

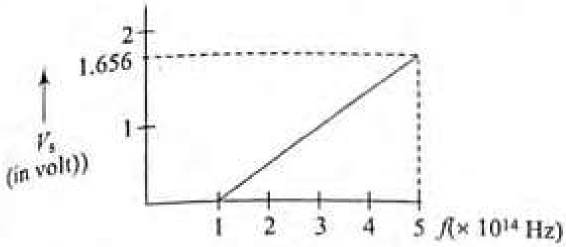
42. The KE of the photoelectrons is E when the incident wavelength is $\lambda/2$. The KE becomes $2E$ when the incident wavelength is $\lambda/3$. The work function of the metal is
(a) hc/λ (b) $2hc/\lambda$
(c) $3hc/\lambda$ (d) $hc/3\lambda$

43. The threshold frequency for certain metal is v_0. When light of frequency $2v_0$ is incident on it, the maximum velocity of photoelectrons is 4×10^6 m s^{-1}. If the frequency of incident radiation is increased to $5v_0$, then the maximum velocity of photoelectrons will be
(a) $4/5 \times 10^6$ m s^{-1} (b) 2×10^6 m s^{-1}
(c) 8×10^6 m s^{-1} (d) 2×10^7 m s^{-1}

44. Work function of nickel is 5.01 eV. When ultraviolet radiation of wavelength 200 Å is incident on it, electons are emitted. What will be the maximum velocity of emitted electrons?
(a) 3×10^8 m s^{-1} (b) 6.46×10^5 m s^{-1}
(c) 10.36×10^5 m s^{-1} (d) 8.54×10^6 m s^{-1}

45. The kinetic energy of most energetic electrons emitted from a metallic surface is doubled when the wavelength λ of the incident radiation is changed from 400 nm to 310 nm. The work function of the metal is
(a) 0.9 eV (b) 1.7 eV
(c) 2.2 eV (d) 3.1 eV

46. Figure shows the plot of the stopping potential versus the frequency of the light used in an experiment on photoelectric effect. The ratio h/e is



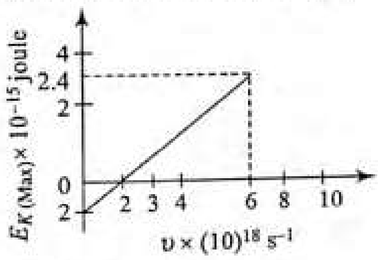
(a) 10^{-15} V s (b) 2×10^{-15} V s
(c) 3×10^{-15} V s (d) 4.14×10^{-15} V s

47. In Q. 46, the work function is
(a) 0.212 eV (b) 0.313 eV
(c) 0.414 eV (d) 0.515 eV

48. The maximum velocity of electrons emitted from a metal surface is v. What would be the maximum velocity if the frequency of incident light is increased by a factor of 4?
(a) $2v$ (b) $> 2v$
(c) $< 2v$ (d) between $2v$ and $4v$.

49. In a photocell, with excitation wavelength λ, the faster electron has speed v. If the excitation wavelength is changed to $3\lambda/4$, the speed of the fastest electron will be
(a) $v(3/4)^{1/2}$ (b) $v(4/3)^{1/2}$
(c) less than $v(4/3)^{1/2}$ (d) greater than $v(4/3)^{1/2}$

50. In the experiment on photoelectric effect, the graph between $E_{K(\max)}$ and v is found to be a straight line as shown in figure. The threshold frequency and Planck's constant according to this graph are



(a) 3.33×10^{18} s^{-1}, 6×10^{-34} J-s
(b) 6×10^{18} s^{-1}, 6×10^{-34} J-s
(c) 2.66×10^{18} s^{-1}, 4×10^{-34} J-s
(d) 4×10^{18} s^{-1}, 3×10^{-34} J-s

Problems Based on Mixed Concepts

51. When a certain metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is $3V_0$ and when the same surface is illuminated with light of wavelength 2λ, the stopping potential is V_0. The threshold wavelength of this surface for photoelectric effect is
(a) 6λ (b) $4\lambda/3$
(c) 4λ (d) 8λ

52. Threshold frequency for a certain metal is v_0. When light of frequency $2v_0$ is incident on it, the maximum velocity of photoelectrons is 4×10^8 cm s^{-1}. If frequency of incident radiation is increased to $5v_0$, then the maximum velocity of photoelectrons, in cm s^{-1}, will be
(a) $(4/5) \times 10^8$ (b) 2×10^8
(c) 8×10^8 (d) 20×10^8

53. Light of wavelength 0.6 μm from a sodium lamp falls on a photocell and causes the emission of photoelectrons for which the stopping potential is 0.5 V. With light of wavelength 0.4 μm from a mercury vapor lamp, the stopping potential is 1.5 V. Then, the work function [in electron volts] of the photocell surface is
(a) 0.75 eV (b) 1.5 eV
(c) 3 eV (d) 2.5 eV

54. Ultraviolet light of wavelength 300 nm and intensity 1.0 W m^{-2} falls on the surface of a photosensitive material. If one per cent of the incident photons produce photoelectrons, then the number of photoelectrons emitted per second from an area of 1.0 cm^2 of the surface is nearly
(a) 9.61×10^{14} s^{-1} (b) 4.12×10^{13} s^{-1}
(c) 1.51×10^{12} s^{-1} (d) 2.13×10^{11} s^{-1}

55. Monochromatic light incident on a metal surface emits electrons with kinetic energies from zero to 2.6 eV. What is the least energy of the incident photon if the tightly bound electron needs 4.2 eV to remove?
(a) 1.6 eV (b) From 1.6 eV to 6.8 eV
(c) 6.8 eV (d) More than 6.8 eV

56. A cesium photocell, with a steady potential difference of 60 V across it, is illuminated by a small bright light placed 1 m away. When the same light is placed 2 m away, the electrons crossing the photocell
(a) each carry one-quarter of their previous momentum
(b) each carry one-quarter of their pervious energy
(c) are one-quarter as numerous
(d) are half as numerous

57. An image of the sun is formed by a lens, of the focal length of 30 cm, on the metal surface of a photoelectric cell and a photoelectric current I is produced. The lens forming the image is then replaced by another of the same diameter but of focal length 15 cm. The photoelectric current in this case is
(a) $\frac{I}{2}$ (b) I
(c) $2I$ (d) $4I$

58. A homogeneous ball (mass = m) of ideal black material at rest is illuminated with a radiation having a set of photons (wavelength = λ), each with the same momentum and the same energy. The rate at which photons fall on the ball is n. The linear acceleration of the ball is
(a) $m\lambda/nh$ (b) $nh/m\lambda$
(c) $nh/(2\pi)(m\lambda)$ (d) $2pm\lambda/nh$

59. The eye can detect 5×10^4 photons (m^2 s)$^{-1}$ of green light ($\lambda = 5000$ Å), while ear can detect 10^{-13} W m^{-2}. As a power detector, which is more sensitive and by what factor?

(a) Eye is more sensitive and by a factor of 5.00
(b) Ear is more sensitive by a factor of 5.00
(c) Both are equally sensitive
(d) Eye is more sensitive by a factor of 10^{-1}

60. A photon of wavelength 0.1 Å is emitted by a helium atom as a consequence of the emission of photon. The KE gained by helium atom is
(a) 0.05 eV (b) 1.05 eV
(c) 2.05 eV (d) 3.05 eV

61. A monochromatic source of light is placed at a large distance d from a metal surface. Photoelectrons are ejected at rate n, the kinetic energy being E. If the source is brought nearer to distance $d/2$, the rate and kinetic energy per photoelectron become nearly
(a) $2n$ and $2E$ (b) $4n$ and $4E$
(c) $4n$ and E (d) n and $4E$

62. An α-particle and a proton are fired through the same magnetic field which is perpendicular to their velocity vectors. The α-particle and the proton move such that radius of curvature of their paths is same. Find the ratio of their de Broglie wavelengths.
(a) 2:3 (b) 3:4
(c) 5:7 (d) 1:2

63. All electrons ejected from a surface by incident light of wavelength 200 nm can be stopped before traveling 1 m in the direction of a uniform electric field of 4 NC^{-1}. The work function of the surface is
(a) 4 eV (b) 6.2 eV
(c) 2 eV (d) 2.2 eV

64. If the short wavelength limit of the continous spectrum coming out of a Coolidge tube is 10 Å, then the de Broglie wavelength of the electrons reaching the target metal in the Coolidge tube is approximately
(a) 0.3 Å (b) 3 Å
(c) 30 Å (d) 10 Å

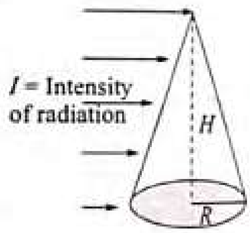
65. A 60 W bulb is placed at a distance of 4 m from you. The bulb is emitting light of wavelength 600 nm uniformly in all directions. In 0.1 s, how many photons enter your eye if the pupil of the eye is having a diameter of 2 mm?
[Take hc = 1240 eV-nm]
(a) 2.84×10^{12} (b) 2.84×10^{11}
(c) 9.37×10^{11} (d) 6.48×10^{11}

66. A photosensitive material is at 9 m to the left of the origin and the source of light is at 7 m to the right of the origin along x-axis. The photosensitive material and the source of light start from rest and move, respectively, with $8\hat{i}$ m s^{-1} and $4\hat{i}$ m s^{-1}. The ratio of intensities at $t = 0$ to $t = 3$ s as received by the photosensitive material is
(a) 16 : 1 (b) 1 : 16
(c) 2 : 7 (d) 7 : 2

67. A sodium metal piece is illuminated with light of wavelength 0.3 μm. The work function of sodium is 2.46 eV. For this situation, mark out the correct statement(s).
(a) The maximum kinetic energy of the ejected photoelectrons is 1.68 eV
(b) The cut-off wavelength for sodium is 505 nm
(c) The minimum photon energy of incident light for photoelectric effect to take place is 2.46 eV
(d) All of the above

68. The kinetic energy of a particle is equal to the energy of a photon. The particle moves at 5% of the speed of light. The ratio of the photon wavelength to the de Broglie wavelength of the particle is
[No need to use relativistic formula for the particle.]
(a) 40 (b) 4
(c) 2 (d) 80

69. The radiation force experienced by body exposed to radiation of intensity I, assuming surface of body to be perfectly absorbing is:



(a) $\dfrac{\pi R^2 I}{c}$ (b) $\dfrac{\pi R^2 I}{2c}$
(c) $\dfrac{IRH}{2c}$ (d) $\dfrac{IRH}{c}$

70. A silver ball of radius 4.8 cm is suspended by a thread in the vacuum chamber. UV light of wavelength 200 nm is incident on the ball for some times during which a total energy of 1×10^{-7} J falls on the surface. Assuming on an average one out of 10^3 photons incident is able to eject electron. The potential on sphere will be
(a) 1 V (b) 2 V
(c) 3 V (d) Zero

71. The potential energy of a particle of mass m is given by

$$U(x) = \begin{cases} E_0; & 0 \le x \le 1 \\ 0; & x > 1 \end{cases}$$

λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, the ratio $\dfrac{\lambda_1}{\lambda_2}$ will be
(a) 2 (b) 1
(c) $\sqrt{2}$ (d) $\dfrac{1}{\sqrt{2}}$

72. In a photo-emissive cell, with exciting wavelength λ, the maximum kinetic energy of the electron is K. If the exciting wavelength is changed to $3\lambda/4$, the kinetic energy of the fastest emitted electron will be
(a) $\frac{3K}{4}$ (b) $\frac{4K}{3}$
(c) less than $\frac{4K}{3}$ (d) more than $\frac{4K}{3}$

73. As energy associated with changes with its wavelength, often the reciprocal of the wavelength $1/\lambda$ is used to describe energy associated with that wavelength. Then, mark the correct equivalence.
(a) $1 \text{ eV} = 5092.6 \text{ cm}^{-1}$ and $1 \text{ cm}^{-1} = 1.2398 \times 10^{-4}$ eV
(b) $1 \text{ eV} = 1239.8 \text{ cm}^{-1}$ and $1 \text{ cm}^{-1} = 8068 \times 10^{-7}$ eV
(c) $1 \text{ eV} = 8068.8 \text{ cm}^{-1}$ and $1 \text{ cm}^{-1} = 6.65 \times 10^{-19}$ eV
(d) $1 \text{ eV} = 8065 \times 10^{-19} \text{ cm}^{-1}$ and $1 \text{ cm}^{-1} = 1.2398 \times 10^{-4}$ eV

74. Let a light beam of total intensity 1 μW cm^{-2} falls on a clean iron sample piece with J work function 4.5 eV and of 1.0 cm^2 area. Assume that iron sample reflects 96% of light and that only 3% of the absorbed energy lies in the violet region which can cause photoemission (of wavelength 250 nm). Number of electrons emitted will be
(a) 1.5×10^{10} electrons per second
(b) 3.0×10^{9} electrons per second
(c) 3.0×10^{10} electrons per second
(d) 1.5×10^{9} electrons per second

75. An electron beam accelerated from rest through a potential difference of 5000 V in vacuum is allowed to impinge on a surface normally. The incident current is mA and if the electrons come to rest on striking the surface the force on it is
(a) 1.1924×10^{-8} N (b) 2.1×10^{-8} N
(c) 1.6×10^{-8} N (d) 1.6×10^{-8} N

ARCHIVES

1. Sodium and copper have work functions 2.3 eV and 4.5 eV, respectively. Then the ratio of the wavelengths is nearest to
(a) 1 : 2 (b) 4 : 1
(c) 2 : 1 (d) 1 : 4 **(AIEEE 2002)**

2. The formation of covalent bonds in compounds exhibits
(a) wave nature of electron.
(b) particle nature of electron.
(c) both wave and particle natures of electron.
(d) none of these **(AIEEE 2002)**

3. Two identical photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are, respectively, v_1 and v_2, then
(a) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$
(b) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$
(c) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$
(d) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 + f_2)$ **(AIEEE 2003)**

4. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photoelectrons from a metal versus the frequency of the incident radiation gives a straight line whose slope
(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depends both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation **(AIEEE 2004)**

5. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately
(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm **(AIEEE 2004)**

6. A charged oil drop is suspended in a uniform field of 3×10^4 V/m so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge as 9.9×10^{-15} kg and $g = 10$ m/s^2)
(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C
(c) 1.6×10^{-18} C (d) 408×10^{18} C
(AIEEE 2004)

7. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed 1/2 m away, the number of electrons emitted by the photocathode would
(a) increase by a factor of 2
(b) decrease by a factor of 2
(c) increase by a factor of 4
(d) decrease by a factor of 4 **(AIEEE 2005)**

8. If the kinetic energy of a free electron doubles, its de Broglie wavelength changes by the factor
(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$
(c) 2 (d) $\frac{1}{2}$ **(AIEEE 2005)**

9. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead which will reduce the intensity to $I/2$ will be
(a) 18 mm (b) 12 mm
(c) 6 mm (d) 9 mm **(AIEEE 2005)**

10. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface 5V. The incident radiation lies in
(a) X-ray region (b) ultra-violet region
(c) infra-red region (d) visible region
(AIEEE 2006)

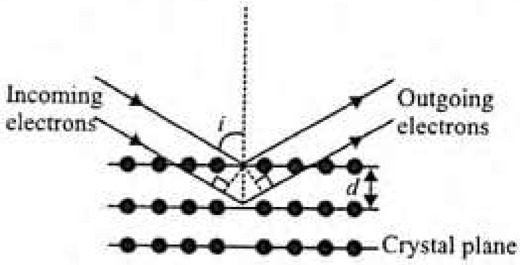
11. The time by a photoelectron to come out after the photon strikes is approximately
(a) 10^{-1} s (b) 10^{-4} s
(c) 10^{-10} s (d) 10^{-16} s **(AIEEE 2006)**

12. A photon of frequency ν has momentum associated with it equal to (c is the velocity of photon)
(a) $h\nu c$ (b) $\frac{h\nu}{c^2}$
(c) $\frac{h\nu}{c}$ (d) $\frac{\nu}{c}$ **(AIEEE 2007)**

Statement for Problems 13–15:

These questions are based on the following paragraph.

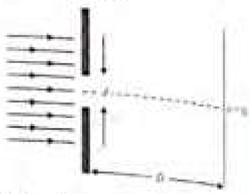
The wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see the figure). **(AIEEE 2008)**

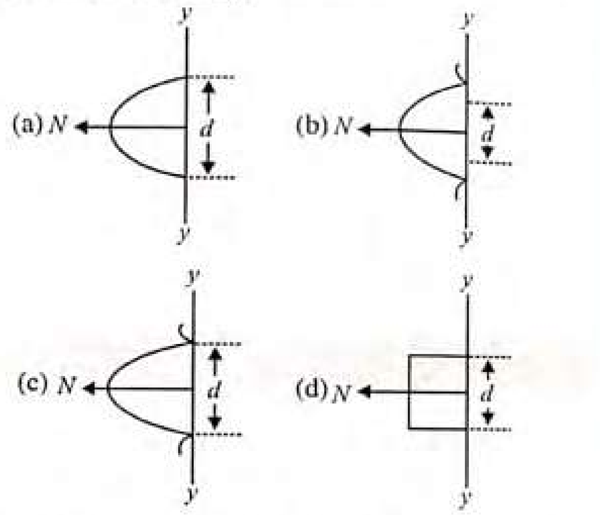


13. Electrons accelerated by potential V are diffracted from a crystal. If d = 1 Å and i = 30°, V should be about ($h = 6.6 \times 10^{-34}$ Js, $me = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)
(a) 500 V (b) 1000 V
(c) 2000 V (d) 50 V

14. If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see the figure), the de Broglie wavelenght λ_{dB} of electrons can be calculated by the relationship (n is an integer)
(a) $2d \sin i = n\lambda_{dB}$ (b) $d \cos i = n\lambda_{dB}$
(c) $d \sin i = n\lambda_{dB}$ (d) $2d \cos i = n\lambda_{dB}$

15. In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de Broglie wavelengths. They are detected on a screen at a distance D from the slit (see the figure). Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ($y = 0$ corresponds to the middle of the slit)?





16. The surface of a metal is illuminated with a light of wavelength 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is (hc = 1240 eV nm)
(a) 3.09 eV (b) 1.41 eV
(c) 151 eV (d) 1.68 eV **(AIEEE 2009)**

17. **Statement 1:** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement 2: Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.
(a) Statement 1 is true, statement 2 is true; statement 2 is the correct explanation for statement 1.
(b) Statement 1 is true, statement 2 is true; statement 2 is not the correct explanation for statement 1.
(c) Statement 1 is false but statement 2 is true.
(d) Statement 1 is true but statement 2 is false.
(AIEEE 2010)

18. If a source of power 4 kW produces 10^{20} photons per second, the radiation belongs to a part of the spectrum called
(a) X-rays (b) ultraviolet rays
(c) microwaves (d) γ-rays **(AIEEE 2010)**

19. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = (a/x^{12}) - (b/x^6)$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$

(c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$ **(AIEEE 2010)**

20. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements

Statement 1: A metallic surface is irradiated by a monochromatic light of frequency $f > f_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement 2: The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(a) Statement 1 is true but statement 2 is false.
(b) Statement 1 is true, statement 2 is true, statement 2 is the correct explanation of statement 1.
(c) Statement 1 is true, statement 2 is true, statement 2 is not the correct explanation of statement 1.
(d) Statement 1 is false but statement 2 is true.

(AIEEE 2011)

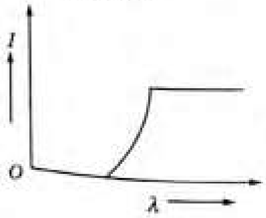
21. **Statement 1:** Davisson-Germer experiment established the wave nature of electrons.

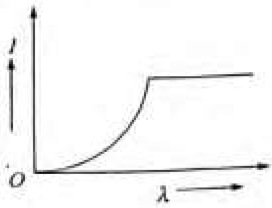
Statement 2: If electrons have wave nature, they can interfere and show diffraction.

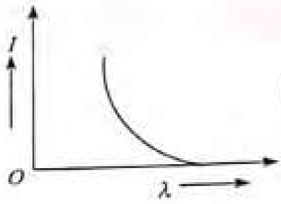
(a) Statement 1 is false but statement 2 is true.
(b) Statement 1 is true but statement 2 is false
(c) Statement 1 is true, statement 2 is true, statement 2 is the correct explanation for statement 1
(d) Statement 1 is true, statement 2 is true, statement 2 is not the correct explanation of statement 1

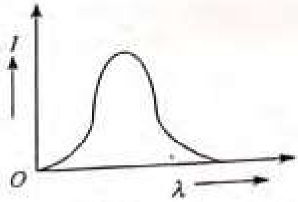
(AIEEE 2012)

22. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows:

(a)


(b)


(c)


(d)


(JEE Main 2013)

23. Match List I (Fundamental Experiment) with List II (its conclusion) and select the correct option from the choices given below the list:

	List I		List II
(p)	Franck–Hertz experiment	(i)	Particle nature of light
(q)	Photo electric experiment	(ii)	Discrete energy levels of atom
(r)	Davison–Germer experiment	(iii)	Wave nature of electron
		(iv)	Structure of atom

(a) (p) - (i), (q) - (iv), (r) - (iii)
(b) (p) - (ii), (q) - (iv), (r) - (iii)
(c) (p) - (ii), (q) - (i), (r) - (iii)
(d) (p) - (iv), (q) - (iii), (r) - (ii) **(JEE Main 2015)**

24. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be:

(a) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (b) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(c) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (d) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

(JEE Main 2016)

25. A particle A of mass m and initial velocity v collides with a particle B of mass $m/2$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is

(a) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

(c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (d) $\frac{\lambda_A}{\lambda_B} = 2$

(JEE Main 2017)

ANSWER KEY

Exercises

1. (c)	2. (a)	3. (a)	4. (c)	5. (b)	6. (d)	7. (b)	8. (d)	9. (c)	10. (b)
11. (b)	12. (a)	13. (d)	14. (c)	15. (d)	16. (b)	17. (c)	18. (a)	19. (b)	20. (c)
21. (a)	22. (c)	23. (a)	24. (d)	25. (a)	26. (c)	27. (a)	28. (b)	29. (c)	30. (c)
31. (d)	32. (b)	33. (a)	34. (c)	35. (c)	36. (d)	37. (a)	38. (b)	39. (a)	40. (b)
41. (c)	42. (a)	43. (c)	44. (b)	45. (c)	46. (d)	47. (c)	48. (b)	49. (d)	50. (a)
51. (c)	52. (c)	53. (b)	54. (c)	55. (c)	56. (c)	57. (b)	58. (b)	59. (a)	60. (c)
61. (c)	62. (d)	63. (d)	64. (a)	65. (b)	66. (b)	67. (d)	68. (a)	69. (d)	70. (c)
71. (c)	72. (d)	73. (c)	74. (d)	75. (a)					

Archives

1. (c)	2. (c)	3. (b)	4. (d)	5. (c)	6. (a)	7. (c)	8. (b)	9. (b)	10. (b)
11. (c)	12. (c)	13. (d)	14. (d)	15. (b)	16. (b)	17. (d)	18. (a)	19. (c)	20. (d)
21. (c)	22. (c)	23. (c)	24. (a)	25. (d)					

Chapter 29

Atomic Structure

BOHR MODEL OF HYDROGEN ATOM

In 1913, Bohr presented a model that led to equations such as Balmer's for predicting the specific wavelength that the hydrogen atom radiates. Bohr's theory begins with Rutherford's picture of an atom as a nucleus surrounded by electrons moving in circular orbits, the necessary centripetal force to electrons is provided by the electrostatic force of attraction between nucleus and electrons. In analyzing this picture, Bohr made a number of assumptions in order to combine the new quantum ideas of Planck and Einstein with the traditional description of a particle in uniform circular motion.

1. Adopting Planck's idea of quantized energy levels, Bohr hypothesized that in a hydrogen atom there can be only certain values of the total energy (electron kinetic energy plus potential energy) for the electrons. These allowed energy levels correspond to different orbits for the electron as it moves around the nucleus, the larger orbits being associated with larger total energies. Bohr assumed that an electron in one of these orbits does not radiate electromagnetic wave. For this reason, the orbits are called stationary orbits or stationary states. Bohr recognized that radiationless orbits violated the laws of physics. But the assumption of such orbits was necessary because the traditional laws indicated that an electron radiates electromagnetic waves as it accelerates around a circular path, and the loss of the energy carried by the waves would lead to the collapse of the orbit.
2. To incorporate Einstein's photon concept, Bohr theorized that a photon is emitted only when the electron changes orbits from a larger one with higher energy to a smaller one with lower energy. But how do electrons get into the higher-energy orbits? They get there by picking up energy when atoms collide, which happens more often when a gas is heated, or by acquiring energy when a high voltage is applied to a gas.

   When an electron in an initial orbit with a large energy E_i changes to a final orbit with a smaller energy E_f, the emitted photon has an energy of $E_i - E_f$, consistent with the law of conservation of energy. But according to Einstein, the energy of a photon is hf, where f is its frequency and h is Planck's constant and since the frequency of an electromagnetic wave is related to the wavelength by $\lambda = c/f$, Bohr could use this equation to determine the wavelengths radiated by a hydrogen atom. First, however, he had to derive expressions for the energies E_i and E_f.
3. Bohr found that the magnitude of the electron's angular momentum is quantized, and this magnitude for the electron must be an integral multiple of $h/2\pi$. The magnitude of the angular momentum is $L = mvr$ for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \qquad (n = 1, 2, 3....)$$

Radius of Orbit

We have

$$\frac{mv^2}{r} = \frac{1}{4\pi e_o}\frac{(Ze)(e)}{r^2} \qquad \text{(i)}$$

and $$mvr = \frac{nh}{2\pi} \qquad \text{(ii)}$$

From Eq. (ii), $v = \dfrac{nh}{2\pi mr}$

Substituting the value of v in Eq. (i), we get

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z} \quad \text{or} \quad r_n = (0.53)\frac{n^2}{Z}\,\text{Å}$$

So, for H-like atoms, $r_n \propto \dfrac{n^2}{Z}$.

Stationary orbits are not equally spaced.

Velocity of Electron in nth Orbit

Since $v = \dfrac{nh}{2\pi mr}$ and $r = \dfrac{n^2h^2\varepsilon_0}{\pi me^2 Z}$

Put the value of r to get

$$v = \frac{Ze^2}{2\varepsilon_0 nh} \quad \text{or} \quad v = \left(\frac{e^2}{2\varepsilon_0 ch}\right)\left(\frac{cZ}{n}\right)$$

or $$v = \alpha\left(\frac{cZ}{n}\right)$$

where $\alpha = e^2/2h\varepsilon_0 c$ is Summerfield's fine structure constant (a pure number) whose value is 1/137. Therefore,

$$v = \frac{1}{137}\left(\frac{cZ}{n}\right)$$

i.e., velocity of electron in Bohr's first orbit of hydrogen ($Z = 1$) is $c/137$, in second orbit is $c/274$, and so on.

Orbital Frequency of Electron

$$f = \frac{v}{2\pi r}$$

Put the value of v and r to get: $f = \dfrac{mZ^2e^4}{4\varepsilon_0^2 n^3 h^3} \Rightarrow f \propto \dfrac{Z^2}{n^3}$

ILLUSTRATION 29.1 The electron in a hydrogen atom makes a transition $n_1 \to n_2$, where n_1 and n_2 are the principal quantum numbers of the two energy states. Assume Bohr's model to be valid. The time period of the electron in the initial state is eight times that in the final state. What are the possible values of n_1 and n_2?

Solution. Here, $Z = 1$. We can derive the expression for time period of electron in nth orbit as

$$T_n = \frac{1}{f_n} \propto n^3$$

Hence $\dfrac{T_1}{T_2} = \dfrac{n_1^3}{n_2^3}$. As $T_1 = 8T_2$, therefore we get

$$8 = \left(\frac{n_1}{n_2}\right)^3 \Rightarrow n_1 = 2n_2$$

Thus, the possible values of n_1 and n_2 are $n_1 = 2$, $n_2 = 1$; $n_1 = 4$, $n_2 = 2$; $n_1 = 6$, $n_2 = 3$; and so on.

ILLUSTRATION 29.2 What is the angular momentum of an electron in Bohr's hydrogen atom whose energy is -3.4 eV?

Solution. First, we need to identify the quantum number of the energy level.

As $E = -\dfrac{13.6}{n^2}$, we have

$$-3.4 = -\frac{13.6}{n^2} \quad \text{or} \quad n = 2$$

The angular momentum quantization gives, $L = mvr = \dfrac{nh}{2\pi}$

Substituting $n = 2$, we get $\quad L = \dfrac{2h}{2\pi} = \dfrac{h}{\pi}$

ENERGY OF ELECTRON IN NTH ORBIT

Kinetic energy: Since we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \quad \text{or} \quad \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

or $\quad \text{KE} = \dfrac{Ze^2}{8\pi\varepsilon_0 r}$

Potential energy: $U = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(e)}{r} \quad$ or $\quad U = -\dfrac{Ze^2}{4\pi\varepsilon_0 r}$

Total energy: $E = \text{KE} + \text{PE} = \dfrac{Ze^2}{8\pi\varepsilon_0 r} - \dfrac{Ze^2}{4\pi\varepsilon_0 r}$

or $\quad E = -\dfrac{Ze^2}{8\pi\varepsilon_0 r}$

So we conclude that

$$\text{Total energy} = -\text{KE} = \frac{3}{4}(\text{PE})$$

Using $r = \dfrac{n^2h^2\varepsilon_0}{\pi me^2 Z}$, we get

$$E = -\left(\frac{me^4}{8h^2\varepsilon_0^2}\right)\frac{Z^2}{n^2} = -(13.6)\frac{Z^2}{n^2}\ \text{eV}$$

Also $E = -\left(\dfrac{me^4}{8\varepsilon_0^2 ch^3}\right)ch\dfrac{Z^2}{n^2} = -(Rch)\dfrac{Z^2}{n^2}$

where R = Rydberg's constant $= \dfrac{me^4}{8\varepsilon_0^2 ch^3} = 1.097 \times 10^7\ \text{m}^{-1}$

Substituting values of m, e, ε_0, and h with $n = 1$, we get the least energy of the atom in first orbit, which is -13.6 eV.

Rch = Rydberg's energy $= 2.17 \times 10^{-18}$ J ≈ 13.6 eV is the electron energy in first orbit of H atom.

Hence, $E_1 = -13.6$ eV $\quad$ (i)

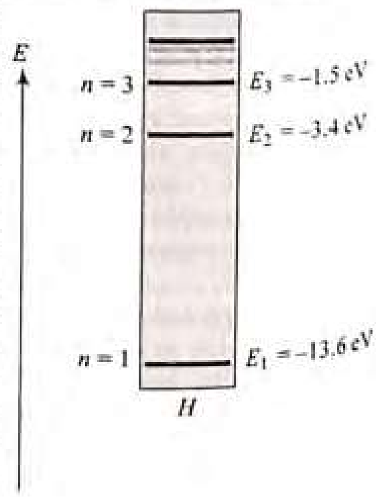
and $\quad E_n = \dfrac{E_1}{n^2} = -\dfrac{13.6}{n^2}$ eV $\quad$ (ii)

Substituting $n = 2, 3, 4, \ldots$, etc., we get energies of atom in different orbits.

The significance of negative sign in Eq. (ii) is that the electron is bound to the nucleus by attractive forces and to separate the electron from the nucleus energy must be supplied to it. Giving different values to n, we can calculate the orbital energy or binding energy of the electron in different orbitals.

$E_1 = -13.6$ eV	where	$n = 1$ (K-shell)
$E_2 = -3.4$ eV		$n = 2$ (L-shell)
$E_3 = -1.5$ eV		$n = 3$ (M-shell)
$\vdots$		
$E_\infty = 0$ eV		$n = \infty$ (Limiting case)

Now, we shall consider the energy level diagram. This diagram can be drawn in terms of various electron orbits to the scale of their radii but it is customary to draw horizontal lines to the energy scale as shown in figure.



The diagram is known as energy level diagram. The lowest energy level ($n = 1$) corresponds to normal unexcited state of hydrogen. This state is also called as ground state. In energy level diagram, the lower energies (more negative) are at the bottom

while higher energies (less negative) are at the top. By such a consideration, the various electron jumps between allowed orbits will be vertical arrows between different energy levels. The energy of radiated photon is greater when the length of arrow is greater.

FREQUENCY OF EMITTED RADIATION

If electron makes a transition (jumps) from final state n_f to the initial state n_i, then frequency of emitted radiation f is given by

$$hf = E_f - E_i \quad \text{or} \quad f = \frac{E_f - E_i}{h} = -Z^2 Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

If λ is the wavelength of emitted radiation, then

$$f = \frac{c}{\lambda} = -Z^2 Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

So reciprocal of wavelength or wave number ($\bar{v}$) is given by

$$\bar{v} = \frac{1}{\lambda} = -Z^2 R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

This relation holds for radiations emitted by hydrogen-like atoms, i.e.,

$H(Z = 1)$, $He^+(Z = 2)$, $Li^{++}(Z = 3)$, and $Be^{+++}(Z = 4)$

If the electron makes a transition from $n = 1$ to the higher states, then absorption of radiation takes place.

ILLUSTRATION 29.3 Suppose potential energy between electron and proton at separation r is given by $U = k \log r$, where k is a constant. For such a hypothetical hydrogen atom, calculate the radius of nth Bohr's orbit and its energy levels.

Solution. For a conservative force field, $F = -\dfrac{dU}{dr} = -\dfrac{k}{r}$

This force $F = -k/r$ provides the centripetal force for the circular motion of electron.

So, $\dfrac{mv^2}{r} = \dfrac{k}{r} \Rightarrow m^2v^2 = mk \Rightarrow mv = \sqrt{mk}$ (i)

Applying Bohr's quantization rule, $mvr = \dfrac{nh}{2\pi}$ (ii)

From Eqs. (i) and (ii), we get $r = \dfrac{nh}{2\pi\sqrt{mk}}$

From Eq. (i), KE of electron $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}k$

Total energy of electron = KE + PE $= \dfrac{1}{2}k + k\log r$

$$= \frac{k}{2}\left[1 + \log\frac{n^2h^2}{4\pi^2 mk}\right]$$

HYDROGEN-LIKE ATOMS

We can extend the Bohr model to other one-electron atoms, such as singly ionized helium (He^+), doubly ionized lithium (Li^{2+}), and so on. Such atoms are called *hydrogen-like atoms*.

The Bohr model of hydrogen can be extended to hydrogen-like atoms, i.e., one-electron atoms, the nuclear charge is $+Ze$, where Z is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace e^2 everywhere by Ze^2. Thus, the equations for r_n, v_n and E_n are altered as under:

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2} = a_0\frac{n^2}{Z} \quad \text{or} \quad r_n \propto \frac{n^2}{Z} \qquad \text{(i)}$$

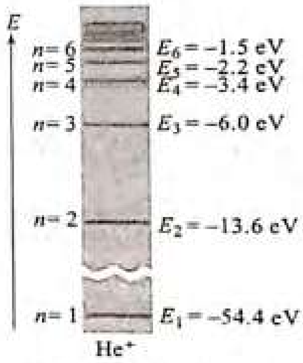
where $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{Ze^2}{2\varepsilon_0 nh} = \frac{Z}{n}v_1 \quad \text{or} \quad v_n \propto \frac{Z}{n} \qquad \text{(ii)}$$

where $v_1 = 2.19 \times 10^6$ m s^{-1} (speed of electron in first orbit of H)

$$E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{Z^2}{n^2}E_1 \quad \text{or} \quad E_n \propto \frac{Z^2}{n^2} \qquad \text{(iii)}$$

where $E_1 = -13.60$ eV (energy of atom in first orbit of H)



Energy level diagram of He

Important Points

For single-electron system

Ground state: Lowest energy state of any atom or ion is called ground state of the atom.

Ground state energy of H atom = −13.6 eV

Ground state energy of He^+ ion = −54.4 eV

Ground state energy of Li^{++} ion = −122.4 eV

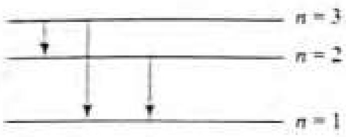
ILLUSTRATION 29.4 A gas of hydrogen-like atoms can absorb radiations of 698 eV. Consequently, the atoms emit radiations of only three different wavelengths. All the wavelengths are equal to or smaller than that of the absorbed photon.

(a) Determine the initial state of the gas atoms.
(b) Identify the gas atoms.
(c) Find the minimum wavelength of the emitted radiation.
(d) Find the ionization energy and the respective wavelength for the gas atoms.

Solution.

(a) Since three radiations are emitted, therefore, the final excited state of the gas is $n = 3$.

The initial state of the gas atoms is $n = 2$ as all the wavelengths are smaller and the energy will be higher.



(b) $13.6Z^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = 68$

or $13.6Z^2\left[\frac{5}{36}\right] = 68 \Rightarrow Z = 6$

(c) The minimum wavelength corresponds to the transition $n = 3$ to $n = 1$

$$\frac{1}{\lambda_{\min}} = RZ^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right]$$

or $$\lambda_{\min} = \frac{9}{8RZ^2} = \frac{9}{8(1.097\times10^7)(6)^2} = 28.5\ \text{Å}$$

(d) The ionization energy of the gas atoms is

$$E = 13.6Z^2 = (13.6)(6)^2 = 489.6\ \text{eV}$$

$$\lambda = \frac{1}{RZ^2} = \frac{1}{(1.097\times10^7)(6)^2} = 25.32\ \text{Å}$$

IONIZATION ENERGY (IE) AND IONIZATION POTENTIAL (IP)

Ionization energy (IE): Total energy zero of a hydrogen atom corresponds to infinite separation between electron and nucleus. Total positive energy implies that the atom is ionized and electron is in unbound (isolated) state moving with certain kinetic energy. Minimum energy required to move an electron from ground state to $n = \infty$ is called ionization energy of the atom or ion.

$$E_{\text{ionization}} = E_{\infty} - E_n = -E_n = \frac{13.6Z^2}{n^2}\ \text{eV}$$

Ionization energy of H atom = 13.6 eV
Ionization energy of He^+ ion = 54.4 eV
Ionization energy of Li^{++} ion = 122.4 eV

Ionization potential (IP): Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionization energy of the atom is called ionization potential of the atom.

$$V_{\text{ionization}} = \frac{E_n}{e} = \frac{13.6Z^2}{n^2}\ \text{V}$$

IP of H atom = 13.6 V
IP of He^+ ion = 54.4 V
IP of Li^{++} ion = 122.4 V

EXCITATION ENERGY AND EXCITATION POTENTIAL

Excitation: The process of absorption of energy by an electron so as to raise it from a lower energy level to some higher energy level is called *excitation*.

Excited state: The states of an atom other than the ground state are called its excited states.

$n = 2$, first excited state
$n = 3$, second excited state
$n = 4$, third excited state
$n = n_0 + 1$, n_0th excited state

Excitation energy: Energy required to move an electron from ground state of the atom to any other excited state of the atom is called excitation energy of that state.

$$E_{\text{excitation}} = E_{\text{higher}} - E_{\text{lower}}$$

Energy in ground state of H atom = –13.6 eV
Energy in first excited state of H atom = –3.4 eV
The first excitation energy = – 3.4 – (– 13.6) = 10.2 eV.

Excitation potential: Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.

$$V_{\text{excitation}} = \frac{E_{\text{excitation}}}{e}$$

First excitation energy = 10.2 eV, so First excitation potential = 10.2 V.

$n = 1$,	$E_1 = -13.6$ eV.	This is the ground state energy.
$n = 2$,	$E_2 = -3.4$ eV.	This is the first excited level.
$n = 3$,	$E_3 = -1.51$ eV.	This is the second excited level.
⋮	⋮	⋮
$n = \infty$,	$E_\infty = 0$.	The atom is said to be ionized.

Binding Energy or Separation Energy

1. Energy liberated when constituents of a system are brought from infinity to assemble the system. The binding energy is negative of ionization energy.
   $$E_{\text{binding}} = E_n$$
2. Energy required to move an electron from any state to $n = \infty$ is called binding energy of that state or energy released during formation of an H-like atom/ion from $n = \infty$ to some particular state n is called binding energy of that state. Binding energy of ground state of H atom is 13.6 eV.

ATOMIC EXCITATION

An atom can be excited to an energy level above its ground state by following two ways: (1) by photon absorption, and (2) by collision.

When an atom absorbs photon, it becomes excited and returns to its ground state in an average of 10^{-8} s by emitting one or more photons. An atom will not absorb photon of any arbitrary energy. For a photon to be absorbed by an atom, it must have energy equal to difference of energy of ground state and any excited state. For example, when H atom absorbs a photon of wavelength 121.7 nm (10.2 eV), it will bring up H atom from $n = 1$ state to $n = 2$ state. This explains the origin of absorption spectra. On subsequent de-excitation, a photon of wavelength 121.7 nm is emitted when H atom drops from $n = 2$ to $n = 1$ state.

The second type of mechanism for excitation of atoms is by collision. For atomic excitation, collision has to be inelastic and kinetic energy lost in the process is utilized for atomic excitation. Numerical problems based on atomic excitation by collision are worked out using the following principles:

1. Conservation of linear momentum (COLM)
2. Conservation of energy (COE)

LIMITATIONS OF BOHR'S ATOMIC MODEL

1. It is applicable for hydrogen-like atoms, i.e., one electron systems only. For example, He^+, Li^{++}, etc.
2. Bohr's atomic model could not explain hyperfine structure of spectral lines. As the technology of spectroscopy advanced, wavelengths could be measured with greater accuracy and deviations were observed even in the case of hydrogen spectral lines. On examination, a spectral line was found to consist of very closely spaced lines, which is known as hyperfine structure of spectral line. For example, at least seven components having slightly different wavelengths are revealed in what was previously known as 656.3 nm line.
3. Bohr could not explain why electrons are permitted to disobey law of electrodynamics while revolving in stable orbits.
4. The model could not account for splitting of spectral line in magnetic and electric fields.

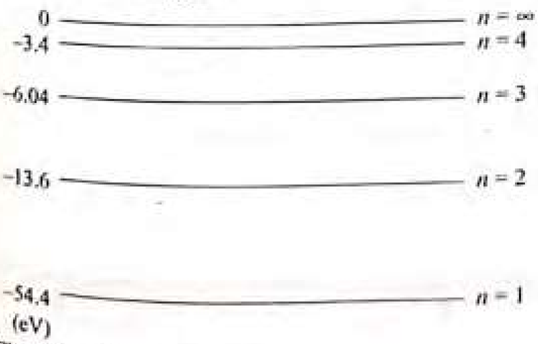
ILLUSTRATION 29.5 Find the ratio of ionization energy of Bohr's hydrogen atom and doubly ionized lithium ion (Li^{2+}).

Solution. The energy of ground state of Bohr's hydrogen-like atom is $E_n = -(13.6)Z^2$.

The ionization energy is equal in magnitude to energy of ground state; so $E_{\text{ionization}} = (13.6)Z^2$

$$\frac{(E_{\text{ionization}})_H}{(E_{\text{ionization}})_{Li^{2+}}} = \frac{(Z_H)^2}{(Z_{Li^{2+}})^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

ILLUSTRATION 29.6 The energy level diagram for a hydrogen-like atom is shown in figure.



(a) Find the value of Z.
(b) If initially the atom is in the ground state, then
 i. determine its first excitation potential, and
 ii. determine its ionization potential.
(c) Can it absorb a photon of 42 eV?
(d) Can it absorb a photon of 56 eV ?
(e) Calculate the radius of its first Bohr orbit.
(f) Calculate the kinetic energy and potential energy of an electron in the first orbit.

Solution.

(a) We know that $E_n = -13.6\dfrac{Z^2}{n^2}$ eV

Here, for $n = 1$, $E_1 = -54.4$ eV, therefore,

$$-54.4 = -13.6\,\frac{Z^2}{1^2}$$

$$\therefore \quad Z = \sqrt{\frac{54.4}{13.6}} = 2$$

(b) **i.** The first excitation energy is required to excite the electron from $n = 1$ to $n = 2$. Thus,

$$\Delta E_{12} = E_2 - E_1 = -13.6 - (54.4) = 40.8 \text{ eV}$$

Therefore, the first excitation potential is 40.8 V.

ii. The ionization energy is required to eject the electron from $n = 1$ to $n = \infty$. Thus,

$$\Delta E_{1\infty} = E_\infty - E_1 = 0 - (-54.4) = 54.4 \text{ eV}$$

Therefore, the ionization potential is 54.4 V.

(c) No. By absorbing a photon of 42 eV, the energy of electron will become

$$E = -54.4 + 42 = -12.4 \text{ eV}$$

This energy level exists between $n = 1$ and $n = 2$ shells which is not possible. Hence, an electron in the first orbit cannot absorb it.

(d) Yes. By absorbing a photon of 56 eV, the electron comes out of the atom where the energy is not quantized. Hence, a photon of 56 eV can be absorbed.

(e) We know that

$$r_n = 0.53\frac{n^2}{Z}\ (\text{Å})$$

Here, $n = 1$; $Z = 2$, therefore, $r = 0.53\dfrac{(1)^2}{2} = 0.265$ (Å)

(f) Since $K = -E$ and $U = 2E$, therefore,

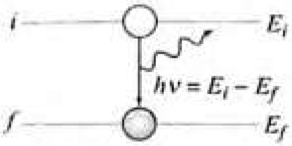
$K = 54.4 \text{ eV} = 8.7 \times 10^{18}$ J and

$U = -108.8 \text{ eV} = 1.74 \times 10^{17}$ J

WAVELENGTH OF PHOTON EMITTED IN DE-EXCITATION

According to Bohr, when an atom makes a transition from a higher energy level to a lower level, it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $h\nu = hc/\lambda$, then conservation of energy gives,

$$h\nu = \frac{hc}{\lambda} = E_i - E_f \qquad \text{(energy of emitted photon)} \quad \text{(i)}$$



By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is in red and is called H_α, the next line, in the blue-green is called H_β, and so on.

In 1885, Johann Balmer, a Swiss teacher found a formula that gives the wavelengths of these lines. This is now called the Balmer series. Balmer's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad \text{(ii)}$$

Here, $n = 3, 4, 5, \ldots$, etc.,

R = Rydberg constant = $1.097 \times 10^7 \text{ m}^{-1}$; and

λ is the wavelength of light/photon emitted during transition.

For $n = 3$, we obtain the wavelength of H_α line.

Similarly, for $n = 4$, we obtain the wavelength of H_β line. For $n = \infty$, the smallest wavelength (= 3646 Å) of this series is obtained. Using the relation $E = hc/\lambda$, we can find the photon energies corresponding to the wavelengths of the Balmar series.

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

This formula suggests that,

$$E_n = \frac{Rhc}{n^2}, n = 1, 2, 3, \ldots \qquad \text{(iii)}$$

The wavelengths corresponding to other spectral series (Lymen, Paschen, etc.) can be represented by formula similar to Balmer's formula.

Lymen series: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), n = 2, 3, 4, \ldots$

Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), n = 4, 5, 6, \ldots$

Brackett series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), n = 5, 6, 7, \ldots$

Pfund series: $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), n = 6, 7, 8$

The Lymen series is in the ultraviolet, and the Paschen, Brackett, and Pfund series are in the infrared region.

ILLUSTRATION 29.7 A hydrogen atom is in the third excited state. It makes a transition to a different state and a photon is either absorbed or emitted. Determine the quantum number n_f of the final state and the energy of the photon if it is

(a) emitted with the shortest possible wavelength,

(b) emitted with the longest possible wavelength, and

(c) absorbed with the longest possible wavelength.

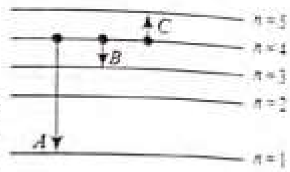
Solution.

(a) When a photon is emitted, it carries energy with it, so the final energy of the atom is less than its initial energy. So, the final quantum number is less than the initial when a photon is emitted.

An atom gains energy when a photon is absorbed. So, the final quantum number is greater than the initial when a photon is absorbed.

The largest possible energy arises when the electron jumps from the initial state ($n = 4$) to the ground state ($n_1 = 1$) as shown by transition A in figure. Therefore, the quantum number of final state is $n = 1$. The energy of the n^{th} state is given by

$$E_n = E_1\left(\frac{Z^2}{n^2}\right)$$



The energy of the photon emitted corresponding to transition from excited state quantum number n_u to lower energy state n_l can be calculated from

$$\frac{hc}{\lambda} = E_u - E_l = (13.6)(1)^2\left[\frac{1}{n_l^2} - \frac{1}{n_u^2}\right]$$

So, the energy of the photon with minimum wavelength is

$$\Delta E = E_4 - E_1$$
$$= (13.6)(1)^2\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 12.75 \text{ eV}$$

(b) The longest possible wavelength photon corresponds to minimum energy. This happens when the electron jumps from the initial state ($n_u = 4$) to the next lower state ($n_l = 3$) as shown by transition B in the figure. Therefore, the quantum number of the final state is $n = 3$. The energy of the photon is

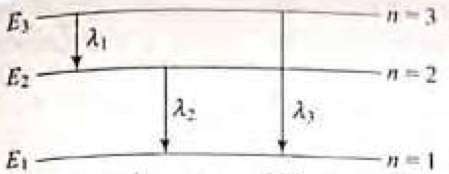
$$\Delta E = E_4 - E_3 = (13.6)(1)^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 0.661 \text{ eV}$$

(c) The absorbed photon has the longest possible wavelength when its energy is the smallest, i.e., the electron jumps from the initial state ($n_l = 4$) to the next higher state ($n_u = 5$). This is shown by transition C in the figure. Therefore, the quantum number of the final state is $n = 5$. The energy of the photon is

$$\Delta E = E_5 - E_4 = (13.6)(1)^2\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 0.306 \text{ eV}$$

CONCEPT APPLICATION EXERCISE 29.1

1. Three energy levels of an atom are shown in figure. The wavelength corresponding to three possible transitions are λ_1, λ_2, and λ_3. The value of λ_3 in terms of λ_1 and λ_2 is given by

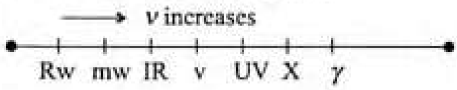


2. A hydrogen atom in a state of binding energy 0.85 eV makes a transition to a state of excitation energy of 10.2 eV.
   (a) What is the initial state of the hydrogen atom?
   (b) What is the final state of the hydrogen atom?
   (c) What is the wavelength of the photon emitted?
3. Calculate (a) the wavelength and (b) the frequency of the H_β line of the Balmer series for hydrogen.
4. Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?
5. Using the known values for hydrogen atom, calculate:
   (a) radius of the third orbit for Li^{2+}.
   (b) speed of electron in the fourth orbit for He^+.
6. Find the kinetic energy, potential energy, and total energy in the first and second orbits of hydrogen atom if potential energy in the first orbit is taken to be zero.

X-RAYS

X-rays are electromagnetic radiations of very short wavelength (0.1 Å to 100 Å) and high energy which are emitted when fast moving electrons or cathode rays strike a target of high atomic mass. These rays are invisible to eye. They are electromagnetic waves and have speed $c = 3 \times 10^8$ m s^{-1} in vacuum.

Its photons have energy around 1000 times more than the visible light.

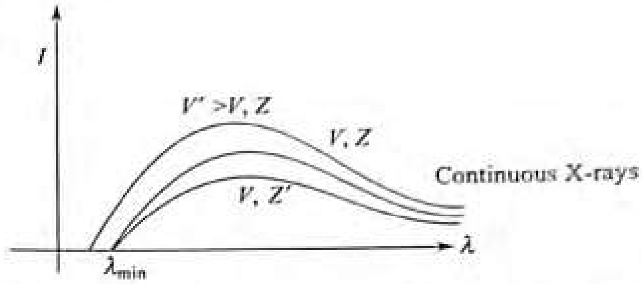


When fast-moving electrons having energy of the order of several keV strike the metallic target, then X-rays are produced.

Properties of X-Rays

1. These are highly penetrating rays and can pass through several materials which are opaque to ordinary light.
2. They ionize the gas through which they pass. While passing through a gas, they knock out electrons from several of the neutral atoms, leaving these atoms with +ve charge.
3. They cause fluorescence in several materials. A plate coated with barium platinocyanide, ZnS (zinc sulphide), etc. becomes luminous when exposed to X-rays.
4. They affect photographic plates especially designed for the purpose.
5. They are not deflected by electric and magnetic fields, showing that they are not charged particles.
6. They show all the properties of the waves except refraction. They show diffraction patterns when passed through a crystal which behaves like a grating.

Variation of intensity of X-rays with λ is plotted as shown in figure



The minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy (eV) of the striking electrons. Thus,

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV} = \frac{12400}{V_{(in\ volts)}}\ \text{Å}.$$

We see that cut-off wavelength λ_{min} depends only on accelerating voltage applied between the target and filament. It does not depend upon material of the target, it is same for two different metals (Z and Z').

ILLUSTRATION 29.8 An X-ray tube operates at 20 kV. Find the maximum speed of the electrons striking the anode, given the charge of electron is 1.6×10^{-19} coulomb and mass of electron is 9×10^{-31} kg.

Solution. When an electron of charge e is accelerated through a potential difference V, it acquires energy eV. If m be the mass of the electron and v_{max} the maximum speed of electron, then

$$\frac{1}{2}mv^2_{max} = eV \text{ or } v_{max} = \sqrt{\left(\frac{2eV}{m}\right)}$$

Substituting the given values, we get

$$v_{max} = \sqrt{\left(\frac{2 \times (1.6 \times 10^{-19}) \times 20,000}{9 \times 10^{-31}}\right)}$$

$$= 8.4 \times 10^7 \text{ m s}^{-1}.$$

ILLUSTRATION 29.9 The voltage applied to an X-ray tube being increased $\eta = 1.5$ times, the short wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda = 26$ pm. Find the initial voltage applied to the tube.

Solution. $\lambda_{min} = \dfrac{ch}{eV}$

Here, $\lambda_1 = \dfrac{ch}{eV_1}$ and $\lambda_2 = \dfrac{ch}{eV_2}$

or $$\frac{ch}{e}\left[\frac{1}{V_2} - \frac{1}{V_1}\right] = \lambda_2 - \lambda_1 = \Delta\lambda = 26 \times 10^{-12}$$

or $\dfrac{ch}{e}\left[\dfrac{2}{3V_1}-\dfrac{1}{V_1}\right]=26\times10^{-12}$ $(\because\ V_2=1.5\,V_1)$

Solving for V_1, we get $V_1 = 16000$ V or 16 kV

X-RAY SPECTRA AND ORIGIN OF X-RAYS

Experimental observation and studies of spectra of X-rays reveal that X-rays are of two types and so are their respective spectra: characteristic X-rays and continuous X-rays.

Characteristic X-Rays

These X-rays are called characteristic X-rays because they are characteristic of the element used as target anode. Characteristic X-rays have a line spectral distribution unlike the continuous X-rays. The wavelength spectrum of the X-frequencies corresponding to these lines are the characteristic of the material or the target, i.e., anode material.

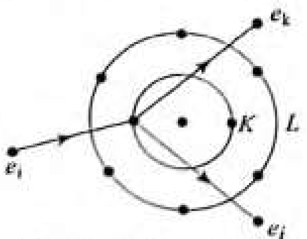
The spectrum of this group consists of several radiations with specific sharp wavelengths and frequency similar to the spectrum (line) of atoms like hydrogen. The wavelengths of this group show characteristic discrete radiations emitted by the atoms of the target material. The characteristic X-rays spectra help us to identify the element of target material.

Origin of Characteristic X-Rays

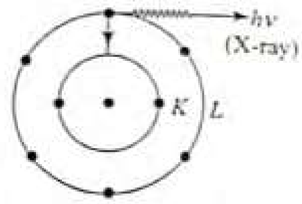
1. When the atoms of the target material are bombarded with high energy electrons (or hard X-rays), which possess enough energy to penetrate into the atom, they knockout the electron of inner shell (say K shell, $n = 1$). When an electron is missing in the K shell, an electron from next upper shell makes a quantum jump to fill the vacancy in the K shell. In the transition process, the electron radiates energy whose frequency lies in the X-ray region. The frequency of emitted radiation (i.e., of photon) is given by

$$v = RZ_e^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

where R is constant and Z_e is effective atomic number. Generally, Z_e is taken to be equal to $Z - s$, where Z is proton number or atomic number of the element and s is called the screening constant. Due to the presence of other electrons, the charge of the nucleus as seen by the electron will be different in different shells.

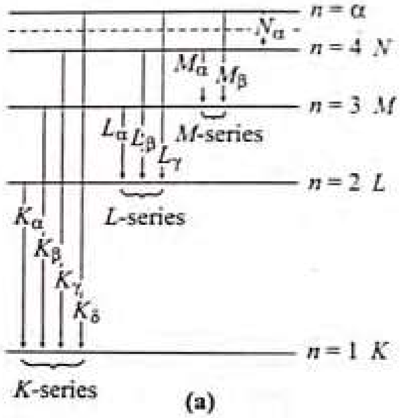


(a) Knocking out e' of K shell by incident electron e_i

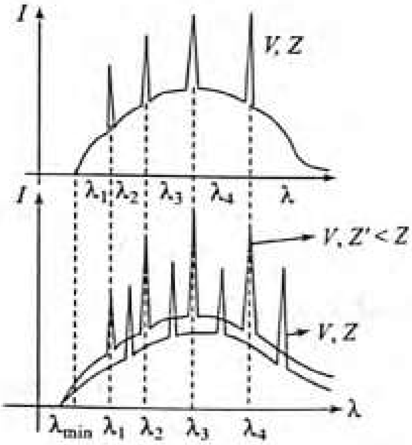


(b) Emission of X-ray photon (K_α series)

2. If cavity is created in L shell, then according to the transition of electrons from high orbits there will be L_α, L_β, ... lines, and this is called L-series. There may also be M-series as shown in Figure (a). Figure (b) shows the wavelength spectrum for the characteristic X-rays when cavity is created in K-shell.



(a)



(b)

The sharp peaks obtained in graph are known as characteristic X-rays because they are characteristic of the target material. The characteristic wavelengths of the material having atomic number Z are called *characteristic X-rays* and the spectrum obtained is called *characteristic spectrum*. If a target material of atomic number Z' is used, then peaks are shifted.

ILLUSTRATION 29.10 The K_α X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. What is the energy difference between K and L levels in this atom?

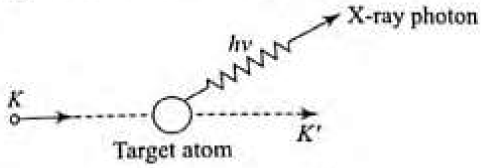
Solution. The origin of K_α X-ray can be understood on the basis of the Bohr model of atom. If there is only one electron in the K shell, and fill up this vacancy. An electron from higher shell will get de-excited. In this process of de-excitation of electron, a photon is emitted during this transition which is the X-ray.

$$\Delta E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{0.021 \times 10^{-9}}$$

$$= 9.46 \times 10^{-15}\ \text{J} = 5.9 \times 10^4\ \text{eV} = 59\ \text{keV}$$

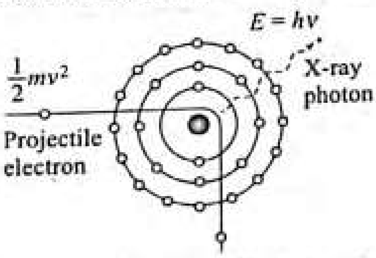
Continuous X-Rays

In addition to characteristic X-rays, tubes emit a continuous spectrum also. The characteristic line spectra is superimposed on a continuous X-rays spectra of varying intensities. The wavelengths of the continuous X-rays spectra are independent of the material. One important feature of continuous X-rays is that they end abruptly at a certain lower wavelength for a given voltage.



Origin of Continuous X-Rays

The mechanism for the production of continuous X-rays can be explained on the basis of figure. It shows an atom of the anode material of high atomic weight with its electron configuration. In the Coolidge tube, an electron is projected toward the anode with an accelerating voltage V. So, the kinetic energy of the projectile electron will be eV. As shown in the figure, when the projectile electron enters into the extremely high electric field of the nucleus of the atom of the anode material, it experiences strong electric force toward the nucleus of the atom and due to this strong attraction the velocity of this electron, when it emerge from the atom, will be highly reduced and negligible compared with the initial velocity of the projectile electron. This electron, in the influence of the highly positive nucleus, experiences a very high acceleration and according to the classical theory every accelerated charge particle emits the electromagnetic radiations, so this electron will also emit electromagnetic radiations. These electromagnetic radiations are called X-rays.



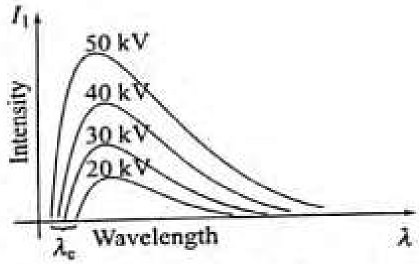
The electrons which pass through the atom very close to the nucleus, will be more accelerated and the energy corresponding to the electrons will be more as compared to those electrons which pass through the atom at relatively large distance from the nucleus. The maximum energy of X-ray photon will be corresponding to that electron which loses almost all of its energy during passing through the atom. The photon corresponding to this electron will have the shortest wavelength among all the photons radiated by other electrons. If this shortest wavelength is λ_c, then we have

$$\Delta E = \frac{1}{2}mv^2 = eV = \frac{hc}{\lambda_c}$$

or $$\lambda_c = \frac{hc}{eV} = \frac{12431}{V}\ \text{Å} \qquad \text{(iii)}$$

This is the minimum wavelength of X-rays emitted from an X-ray tube. Thus, from Eq. (iii), we can see that the maximum energy or minimum wavelength of X-rays emitted depends only on the potential difference applied across the discharge tube.

Thus, we can obtain X-rays in any range λ_c to ∞ by applying an appropriate voltage across the discharge tube, which will fix λ_c and other photons emitted from the tube will have wavelengths more than λ_c and ranging up to ∞. That is why these X-rays are called continuous X-rays as shown in figure.



As we can see in the graph, the intensity of emitted X-rays will be maximum (maximum number of photons) for a particular value of wavelength at a particular accelerating voltage across the discharge tube. At a particular voltage, the intensity of X-rays can be varied by changing the current in the circuit because the intensity of X-rays (number of photons) is proportional to the number of electrons attacking the anode. The broad continuous spectrum beyond the peak intensity is referred as "Bremsstrahlung."

ILLUSTRATION 29.11 When 0.50 Å X-rays strike a material, the photoelectrons from the K shell are observed to move in a circle of radius 23 mm in a magnetic field of 2×10^{-2} tesla acting perpendicular to direction of emission of photoelectrons. What is the binding energy of K-shell electrons?

Solution. The velocity of the photoelectrons is found by the relation:

$$evB = m\frac{v^2}{R} \quad \text{or} \quad v = \frac{e}{m}BR$$

The kinetic energy of the photoelectrons is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$

$$= \frac{1}{2}\frac{(1.6 \times 10^{-19})^2(2 \times 10^{-2})^2(23 \times 10^{-3})^2}{(9.1 \times 10^{-31})}$$

$$= 2.97 \times 10^{-15}\ \text{J}$$

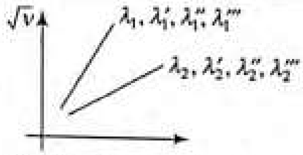
$$= (2.97 \times 10^{-15})\frac{1}{1.6 \times 10^{-19}} = 18.36\ \text{keV}$$

The energy of the incident photon is $E_v = \frac{hc}{\lambda} = \frac{12.4}{0.50} = 24.8$ keV

The binding energy is the difference between these two values:

$$BE = E_v - K = 24.8 - 18.6 = 6.2 \text{ keV}$$

MOSELEY'S LAW



Moseley measured the frequencies of characteristic X-rays for a large number of elements and plotted the square root of frequency against position number in periodic table. He discovered that the plot is very close to a straight line not passing through origin. The relation of straight line is expressed as $\sqrt{\nu} = a(Z - b)$, where a and b are constants. This relation is called *Moseley's law*. It helps to determine the atomic number Z of an atom. Here, b is the screening constant.

Moseley's observations can be mathematically expressed as $\sqrt{\nu} = a(Z - b)$, where a and b are positive constants for one type of X-rays and for all elements (independent of Z).

Moseley's law can be derived on the basis of Bohr's theory of atom.

Frequency of X-rays is given by

$$\sqrt{\nu} = \sqrt{cR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}\ (Z - b)$$

(For multielectron system)

b is known as the screening constant or shielding constant, and $(Z - b)$ is the effective nuclear charge.

For K_a line: $n_1 = 1$, $n_2 = 2$, therefore

$$\sqrt{\nu} = \sqrt{\frac{3Rc}{4}}\ (Z - b) = a(Z - b)$$

Here $a = \sqrt{\frac{3Rc}{4}}$ [$b = 1$ for K_α lines]

ILLUSTRATION 29.12 An X-ray tube with a copper target is found to be emitting lines other than those due to copper. The K_α line of copper is known to have a wavelength 1.5405 Å and the other two K_α lines observed have wavelengths 0.7090 Å and 1.6578 Å. Identify the impurities.

Solution. According to Moseley's equation for K_α radiation,

$$\frac{1}{\lambda} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] (\lambda = \text{wavelength corresponding to Cu})$$

Let λ_1 and λ_2 be the two other unknown wavelengths, then

$$\frac{\lambda_1}{\lambda} = \frac{(Z-1)^2}{(Z_1-1)^2} = \frac{0.7092}{1.5405}$$

For copper, $Z = 29$, therefore,

$$(Z_1 - 1) = 28\sqrt{\frac{1.5405}{0.7092}} = 41$$

or $Z_1 = 42$ (Molybdenum)

Similarly, $\frac{\lambda_2}{\lambda} = \frac{(28)^2}{(Z_2-1)^2} = \frac{1.6578}{1.5405}$

$$(Z_2 - 1) = 28\sqrt{\left(\frac{1.5405}{1.6578}\right)}$$

or $(Z_2 - 1) = 27$ or $Z_2 = 28$ (Nickel)

So, the impurities are molybdenum and nickel.

CONCEPT APPLICATION EXERCISE 29.2

1. An X-ray tube operates at 20 kV. Find the maximum speed of the electrons striking the anticathode, given the charge of electron = 1.6×10^{-19} coulomb and mass of electron = 9×10^{-31} kg.
2. (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation? (b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?
3. The wavelength of characteristic X-ray K_α line emitted from Zinc ($Z = 30$) is 1.415Å. Find the wavelength of the K_α line emitted from molybdenum ($Z = 42$).
4. If the short series limit of the Balmer series for hydrogen is 3644 Å, find the atomic number of the element which gives X-ray wavelengths down to 1 Å. Identify the element.
5. A material whose K-absorption edge is 0.2 Å is irradiated by X-rays of wavelength 0.15 Å. Find the maximum energy of the photoelectrons that are emitted from the K shell.
6. Calculate the wavelength of the emitted characteristic X-ray from a tungsten ($Z = 74$) target when an electron drops from an M shell to a vacancy in the shell.

SOLVED EXAMPLES

1. An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is
   (a) 79.0 (b) 51.8
   (c) 49.2 (d) 38.2

Sol. (a) After the removal of first electron remaining atom will be hydrogen like atom.

So energy required to remove second electron from the atom $E = 13.6 \times \frac{2^2}{1} = 54.4$ eV

$\therefore$ Total energy required = 24.6 + 54.4 = 79 eV

2. A double charged lithium atom is equivalent to hydrogen whose atomic number is 3. The wavelength of required radiation for emitting electron from first to third Bohr

orbit in Li^{++} will be (Ionisation energy of hydrogen atom is 13.6 eV)

(a) 182.51 Å (b) 177.17 Å
(c) 142.25 Å (d) 113.74 Å

Sol. (d) $E_n = -13.6\dfrac{Z^2}{n^2}$ eV. Required energy for said transition,

$$\Delta E = E_3 - E_1 = 13.6\, Z^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right]$$

$$\Rightarrow \Delta E = 13.6\times 3^2\left[\frac{8}{9}\right] = 108.8 \text{ eV}$$

$$\Delta E = 108.8 \times 1.6 \times 10^{-19} \text{ J}$$

Now $\Delta E = \dfrac{hc}{\lambda} = 108.8\times 1.6\times 10^{-19}$

$$\Rightarrow \lambda = \frac{6.6\times 10^{-34}\times 3\times 10^8}{108.8\times 1.6\times 10^{-19}} = 0.11374\times 10^{-7}\text{ m} = 113.74 \text{ Å}$$

3. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Boh'r atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (given in terms of the Rydberg constant R for the hydrogen atom) equal to

(a) $9/(5R)$ (b) $36/(5R)$
(c) $18/(5R)$ (d) $4/R$

Sol. (c) In hydrogen atom $E_n = -\dfrac{Rhc}{n^2}$

Also $E_n \propto m$, where m is the mass of the electron. Here the electron has been replaced by a particle whose mass is double of an electron. Therefore, for this hypothetical atom energy in n^{th} orbit will be given by $E_n = -\dfrac{2Rhc}{n^2}$

The longest wavelength λ_{max} (or minimum energy) photon will correspond to the transition of particle from $n = 3$ to $n = 2$

$$\Rightarrow \frac{hc}{\lambda_{max}} = E_3 - E_2 = Rhc\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

This gives $\lambda_{max} = \dfrac{18}{5R}$

4. The transition from the state $n = 4$ to $n = 3$ in a hydrogen-like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition

(a) $2 \to 1$ (b) $3 \to 2$
(c) $4 \to 2$ (d) $5 \to 4$

Sol. (d) As the transition $n = 4$ and $n = 3$, results in UV radiation and infrared radiation involves smaller amounts of energy UV. So we require a transition involving initial values of n greater than 4 e.g. $5 \to 4$.

5. The electric potential between a proton and an electron is given by $V = V_0 \ln\dfrac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write the variation of r_n with n, n being the principal quantum number

(a) $r_n \propto n$ (b) $r_n \propto 1/n$
(c) $r_n \propto n^2$ (d) $r_n \propto 1/n^2$

Sol. (a) Potential energy $U = eV = eV_0 \ln\dfrac{r}{r_0}$

$\therefore$ Force $F = -\left|\dfrac{dU}{dr}\right| = \dfrac{eV_0}{rr_0}$.

Therefore the force will provide the necessary centripetal force. Hence, $\dfrac{mv^2}{r} = \dfrac{eV_0}{rr_0} \Rightarrow v = \sqrt{\dfrac{eV_0}{mr_0}}$ ...(i)

and $mvr = \dfrac{nh}{2\pi}$ ...(ii)

From equations (i) and (ii), $m\sqrt{\dfrac{eV_0}{mr_0}}\cdot r = \dfrac{nh}{2\pi}$ or $r = \dfrac{nh}{2\pi}\sqrt{\dfrac{r_0}{meV_0}}$

or $r \propto n$

6. A hydrogen like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. The value of n will be

(a) 1 (b) 2
(c) 3 (d) 4

Sol. (b) Let ground state energy (in eV) be E_1

Then from the given condition

$$E_{2n} - E_1 = 204 \text{ eV or } \frac{E_1}{4n^2} - E_1 = 204 \text{ eV}$$

$$\Rightarrow \quad E_1\left(\frac{1}{4n^2}-1\right) = 204 \text{ eV} \quad \text{(i)}$$

and $E_{2n} - E_n = 40.8$ eV

$$\Rightarrow \quad \frac{E_1}{4n^2} - \frac{E_1}{n^2} = E_1\left(-\frac{3}{4n^2}\right) = 40.8 \text{ eV} \quad \text{(ii)}$$

From equations (i) and (ii), $\dfrac{1-\dfrac{1}{4n^2}}{\dfrac{3}{4n^2}} = 5 \Rightarrow n = 2$

7. The wavelength K_α X-rays produced by an X-ray tube is 0.76 Å. The atomic number of anticathode material is

(a) 82 (b) 41
(c) 20 (d) 10

Sol. (b) For K_α X-ray line

$$\frac{1}{\lambda_\alpha} = R(Z-1)^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = \frac{3R}{4}(Z-1)^2$$

On putting the given values

$$\frac{1}{0.76\times 10^{-10}} = \frac{3}{4}\times 1.09\times 10^7 (Z-1)^2$$

$$\Rightarrow \quad (Z-1)^2 = 1600 \Rightarrow Z - 1 = 40 \Rightarrow Z = 41$$

8. Consider a hydrogen like atom whose energy in n^{th} exicited state is given by $E_n = -\frac{13.6Z^2}{n^2}$ when this excited atom makes a transition from excited state to ground state, most energetic photons have energy $E_{max} = 52.224$ eV and least energetic photons have energy $E_{min} = 1.224$ eV. The atomic number of atom is

(a) 2 (b) 5
(c) 4 (d) None of these

Sol. (a) Maximum energy is liberated for transition $E_n \to 1$ and minimum energy for $E_n \to E_{n-1}$

Hence $\frac{E_1}{n^2} - E_1 = 52.224$ eV (i)

and $\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224$ eV (ii)

Solving equations (i) and (ii) we get

$E_1 = -54.4$ eV and $n = 5$

Now $E_1 = -\frac{13.6Z^2}{1^2} = -54.4$ eV. Hence Z = 2

9. If the series limit of Lymen series for Hydrogen atom is equal to the series limit of Balmer series for a hydrogen like atom, then atomic number of this hydrogen-like atom will be

(a) 1 (b) 2
(c) 3 (d) 4

Sol. (b) By using $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For hydrogen atom, $\frac{1}{(\lambda_{min})_H} = R\left[\frac{1}{1^2} - \frac{1}{\infty}\right] = R$

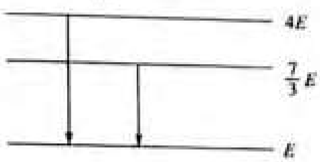
$\Rightarrow (\lambda_{min})_H = \frac{1}{R}$ (i)

For hydrogen like atom $\left(\frac{1}{\lambda_{min}}\right)_{atom} = RZ^2\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$

$\Rightarrow (\lambda_{min})_{atom} = \frac{4}{RZ^2}$ (ii)

From equations (i) and (ii), $\frac{1}{R} = \frac{4}{RZ^2} \Rightarrow Z = 2.$

10. The following diagram indicates the energy levels of a certain atom when the system moves from 4*E* level to *E*. A photon of wavelength λ_1 is emitted. The wavelength of photon produced during it's transition from $\frac{7}{3}E$ level to *E* is λ_2. The ratio $\frac{\lambda_1}{\lambda_2}$ will be



(a) $\frac{9}{4}$ (b) $\frac{4}{9}$
(c) $\frac{3}{2}$ (d) $\frac{7}{3}$

Sol. (b) Transition from 4*E* to *E*

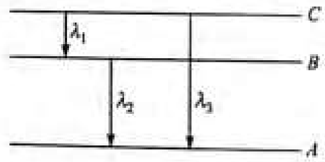
$(4E - E) = \frac{hc}{\lambda_1} \Rightarrow \lambda_1 = \frac{hc}{3E}$ (i)

Transition from $\frac{7}{3}E$ to *E*

$\left(\frac{7}{3}E - E\right) = \frac{hc}{\lambda_2} \Rightarrow \lambda_2 = \frac{3hc}{4E}$ (ii)

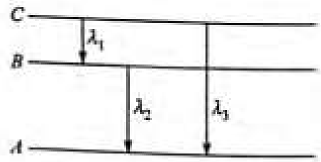
From equations (i) and (ii): $\frac{\lambda_1}{\lambda_2} = \frac{4}{9}$

11. Energy levels A, B, C of a certain atom corresponding to increasing values of energy i.e. $E_A < E_B < E_C$. If $\lambda_1, \lambda_2, \lambda_3$ are the wavelengths of radiations corresponding to the transitions *C* to *B*, *B* to *A* and *C* to *A*, respectively, which of the following statements is correct



(a) $\lambda_3 = \lambda_1 + \lambda_2$ (b) $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$
(c) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (d) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

Sol. (b) Let the energy in *A*, *B* and *C* state be E_A, E_B and E_C, then from the figure



$(E_C - E_B) + (E_B - E_A) = (E_C - E_A)$ or $\frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} = \frac{hc}{\lambda_3}$

$\Rightarrow \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$

12. In hydrogen atom, if the difference in the energy of the electron in $n = 2$ and $n = 3$ orbits is *E*, the ionization energy of hydrogen atom is

(a) 13.2 *E* (b) 7.2 *E*
(c) 5.6 *E* (d) 3.2 *E*

Sol. (b) Energy, $E = K\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$ (K = constant)

$n_1 = 2$ and $n_2 = 3$. So $E = K\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = K\left[\frac{5}{36}\right]$

For removing an electron $n_1 = 1$ to $n_2 = \infty$

Energy $E_1 = K[1] = \frac{36}{5}E = 7.2E$

$\therefore$ Ionization energy = $7.2E$

13. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true
(a) Its kinetic energy increases and its potential and total energies decrease
(b) Its kinetic energy decreases, potential energy increases and its total energy remains the same
(c) Its kinetic and total energies decrease and its potential energy increases
(d) Its kinetic, potential and total energies decreases

Sol. (a) For hydrogen and hydrogen like atoms

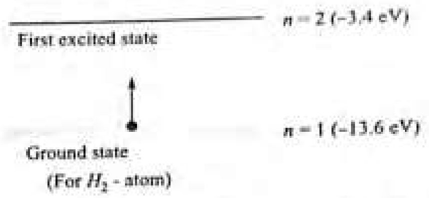
$$E_n = -13.6\frac{z^2}{n^2}\text{ eV}$$

$$U_n = 2E_n = -27.2\frac{z^2}{n^2}\text{ eV} \quad\text{and}\quad K_n = |E_n| = 13.6\frac{z^2}{n^2}\text{ eV}$$

From these three relations, we can see that as n decreases, K_n will increase but E_n and U_n will decrease.

14. The energy required to excite an electron from the ground state of hydrogen atom to the first excited state, is
(a) 1.602×10^{-14}J (b) 1.69×10^{-16} J
(c) 1.632×10^{-18} J (d) 1.656×10^{-20} J

Sol. (c) Energy to excite the e^- from $n = 1$ to $n = 2$



$$E = -3.4 - (-13.6) = 10.2\text{eV} = 10.2 \times 1.6 \times 10^{-19}$$
$$= 1.6382 \times 10^{-18}\text{ J}$$

15. The first line in the Lyman series has wavelength λ. The wavelength of the first line in Balmer series is
(a) $\frac{2}{9}\lambda$ (b) $\frac{9}{2}\lambda$
(c) $\frac{5}{27}\lambda$ (d) $\frac{27}{5}\lambda$

Sol. (d) For first line in Lyman series, $\lambda_{L_1} = \frac{4}{3R}$ (i)

For first line in Balmer series, $\lambda_{B_1} = \frac{36}{5R}$ (ii)

From equations (i) and (ii),

$$\frac{\lambda_{B_1}}{\lambda_{L_1}} = \frac{27}{5} \Rightarrow \lambda_{B_1} = \frac{27}{5}\lambda_{L_1} \quad\text{or}\quad \lambda_{B_1} = \frac{27}{5}\lambda$$

EXERCISES

Bohr's Hydrogen Atom Model

1. The minimum energy to ionize an atom is the energy required to
(a) add one electron to the atom
(b) excite the atom from its ground state to its first excited state
(c) remove one outermost electron from the atom
(d) remove one innermost electron from the atom

2. Which of the following is true when Bohr gave his model for hydrogen atom?
(a) It was not known that hydrogen lines could be explained as differences of terms like R/n^2 with R being a constant and n an integer.
(b) It was not known that positive charge is concentrated in a nucleus of small size.
(c) It was not known that radiant energy occurred in energy bundles defined by $h\nu$ with h being a constant and ν a frequency.
(d) Bohr knew terms like R/n^2 and in the process of choosing allowed the orbits to fit them, he got "angular momentum = $n_i/2\pi$ as a deduction.

3. The force acting on the electron in a hydrogen atom depends on the principal quantum number as
(a) $F \propto n^2$ (b) $F \propto \frac{1}{n^2}$
(c) $F \propto n^4$ (d) $F \propto \frac{1}{n^4}$

4. Magnetic moment due to the motion of the electron in nth energy of hydrogen atom is proportional to
(a) n (b) n^0
(c) n^5 (d) n^3

5. Angular momentum (L) and radius (r) of a hydrogen atom are related as
(a) Lr = constant (b) Lr^2 = constant
(c) Lr^4 = constant (d) none of these

6. The angular momentum of an electron in an orbit is quantized because it is a necessary condition for the compatibility with
(a) wave nature of electron
(b) particle nature of electron
(c) Pauli's exclusion behavior
(d) none of these

7. The maximum angular speed of the electron of a hydrogen atom in a stationary orbit is
(a) 6.2×10^{15} rad s^{-1} (b) 4.1×10^{16} rad s^{-1}
(c) 24×10^{10} rad s^{-1} (d) 9.2×10^{6} rad s^{-1}

8. When an electron jumps from n_1th orbit to n_2th orbit, the energy radiated is given by
(a) $h\nu = E_1/E_2$ (b) $h\nu = E_2/E_1$
(c) $h\nu = E_1 - E_2$ (d) $h\nu = E_2 - E_1$.

9. In Bohr's model of hydrogen atom, let PE represent potential energy and TE the total energy. In going to a higher orbit,
(a) PE increases, TE decreases
(b) PE decreases, TE increases
(c) PE increases, TE increases
(d) PE decreases, TE decreases

10. If the electron in an hydrogen atom jumps from an orbit with level $n_i = 3$ to an orbit with level $n_f = 2$, the emitted radiation has a wavelength given by
(a) $\lambda = \frac{R}{6}$ (b) $\lambda = \frac{36}{5R}$
(c) $\lambda = \frac{6}{R}$ (d) $\lambda = \frac{5R}{36}$

11. The energy change is greatest for a hydrogen atom when its state changes from
(a) $n = 2$ to $n = 1$ (b) $n = 3$ to $n = 2$
(c) $n = 4$ to $n = 3$ (d) $n = 5$ to $n = 4$.

12. The electron in a hydrogen atom jumps from the ground state to the higher energy state where its velocity is reduced to one-third of its initial value. If the radius of the orbit in the ground state is r, the radius of new orbit will be
(a) $3r$ (b) $9r$
(c) $\frac{r}{3}$ (d) $\frac{r}{9}$

13. The energy of an electron in an excited hydrogen atom is -3.4 eV. Then, according to Bohr's Theory, the angular momentum of this electron, in J s, is
(a) 2.11×10^{-34} (b) 3×10^{-34}
(c) 1.055×10^{-34} (d) 0.5×10^{-34}

14. The ionization potential of H atoms is 13.6 V. The energy difference between $n = 2$ and $n = 3$ levels is nearest to
(a) 1.9 eV (b) 2.3 eV
(c) 3.4 eV (d) 4.5 eV

15. If 13.6 eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is
(a) 10.2 eV (b) 0 eV
(c) 3.4 eV (d) 6.8 eV

16. The potential energy of an electron in the fifth orbit of hydrogen atom is
(a) 0.54 eV (b) -0.54 eV
(c) 1.08 eV (d) -1.08 eV

17. If the radius of an orbit is r and the velocity of electron in it is v, then the frequency of electron in the orbit will be
(a) $2\pi r v$ (b) $\frac{2\pi}{vr}$
(c) $\frac{vr}{2\pi}$ (d) $\frac{v}{2\pi r}$

18. As the quantum number increases, the difference of energy between conservative energy levels:
(a) decreases
(b) increases
(c) first decreases and then increases
(d) remains the same

19. The ratio of the speed of the electron in the first Bohr orbit of hydrogen and the speed of light is equal to (where e, h, and c have their usual meanings in cgs system)
(a) $2\pi hc/e^2$ (b) $er^2h/2\pi c$
(c) $e^2c/2\pi h$ (d) $2\pi e^2/hc$

20. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would have been
(a) 32 (b) 60
(c) 64 (d) 4

21. The orbital velocity of an electron in the ground state is v. If the electron is excited to energy state -0.54 eV, its orbital velocity will be
(a) v (b) $\frac{v}{3}$
(c) $\frac{v}{5}$ (b) $\frac{v}{7}$

22. In a hydrogen atom, the transition takes place from $n = 3$ to $n = 2$. If Rydberg's constant is 1.09×10^{7} m^{-1}, the wavelength of the line emitted is
(a) 6606 Å (b) 4861 Å
(c) 4340 Å (d) 4101 Å

23. Let the potential energy of a hydrogen atom in the ground state be zero. Then, its energy in the first excited state will be
(a) 10.2 eV (b) 13.6 eV
(c) 23.8 eV (d) 27.2 eV

24. In a hydrogen atom, the electron is in nth excited state. It comes down to the first excited state by emitting 10 different wavelengths. The value of n is
(a) 6 (b) 7
(c) 8 (d) 9

Hydrogen Like Atom and Atomic Spectrum

25. In which of the following systems will the radius of the first orbit ($n = 1$) be minimum?
(a) Doubly ionized lithium
(b) Singly ionized helium
(c) Deuterium atom
(d) Hydrogen atom

26. The radius of the Bohr orbit in the ground state of hydrogen atom is 0.5 Å. The radius of the orbit of the electron in the third excited state of He^+ will be
(a) 8 Å (b) 4 Å
(c) 0.5 Å (d) 0.25 Å

27. Which of the following parameters are the same for all hydrogen-like atoms and ions in their ground states?
(a) Radius of the orbit
(b) Speed of the electron
(c) Energy of the atom
(d) Orbital angular momentum of the electron

28. The ratio of maximum to minimum possible radiation energy in Bohr's hypothetical hydrogen atom is equal to
(a) 2 (b) 4
(c) 4/3 (d) 3/2

29. A hydrogen-like atom emits radiations of frequency 2.7×10^{15} Hz when it makes a transition from $n = 2$ to $n = 1$. The frequency emitted in a transition from $n = 3$ to $n = 1$ will be
(a) 1.8×10^{15} Hz (b) 3.2×10^{15} Hz
(c) 4.7×10^{5} Hz (d) 6.9×10^{15} Hz

30. The ionization energy of the ionized sodium atom Na^{10+} is
(a) 13.6 eV (b) 13.6×11 eV
(c) (13.6/11) eV (d) $13.6 \times (11^2)$ eV

31. Magnetic field at the center (at nucleus) of the hydrogen-like atoms (atomic number = z) due to the motion of electron in nth orbit is proportional to
(a) $\frac{n^3}{z^5}$ (b) $\frac{n^4}{z}$
(c) $\frac{z^2}{n^3}$ (a) $\frac{z^3}{n^5}$

32. The ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) is
(a) 1 (b) 8
(c) 4 (d) 16

33. Hydrogen atoms are excited from ground state to the state of principal quantum number 4. Then, the number of spectral lines observed will be
(a) 3 (b) 6
(c) 5 (d) 2

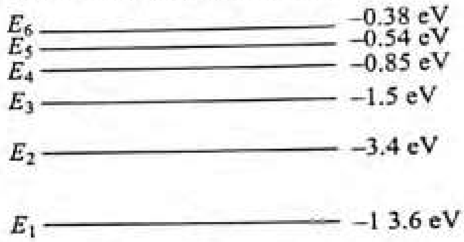
34. The wavelength of the first line of Lyman series in hydrogen atom is 1216. The wavelength of the first line of Lyman series for 10 times ionized sodium atom will be added
(a) 0.1 Å (b) 1000 Å
(c) 100 Å (d) 10 Å

35. Let ν_1 be the frequency of series limit of Lyman series, ν_2 the frequency of the first line of Lyman series, and ν_3 the frequency of series limit of Balmer series. Then which of the following is correct?
(a) $\nu_1 - \nu_2 = \nu_3$ (b) $\nu_2 - \nu_1 = \nu_3$
(c) $\nu_3 = \frac{1}{2}(\nu_1 + \nu_2)$ (d) $\nu_2 + \nu_1 = \nu_3$

36. A sample of hydrogen is bombarded by electrons. Through what potential difference should the electrons be accelerated so that third line of Lyman series be emitted?
(a) 2.55 V (b) 10.2 V
(c) 12.09 V (d) 12.75 V

37. If R is the Rydberg constant for hydrogen, then the wave number of the first line in the Lyman series is
(a) $\frac{R}{2}$ (b) $2R$
(c) $\frac{R}{4}$ (d) $\frac{3R}{4}$

38. In figure, E_1 to E_6 represent some of the energy levels of an electron in the hydrogen atom.



Which one of the following transitions produces a photon of wavelength in the ultraviolet region of the electromagnetic spectrum?
(a) $E_2 - E_1$ (b) $E_3 - E_2$
(c) $E_4 - E_3$ (d) $E_6 - E_4$

39. An atom emits a spectral line of wavelength λ when an electron makes a transition between levels of energy E_1 and E_2. Which expression correctly relates λ, E_1, and E_2?
(a) $\lambda = \frac{hc}{E_1 + E_2}$ (b) $\lambda = \frac{2hc}{E_1 + E_2}$
(c) $\lambda = \frac{2hc}{E_1 - E_2}$ (d) $\lambda = \frac{hc}{E_1 - E_2}$

40. If the series limit wavelength of the Lyman series for hydrogen atom is 912 Å, then the series limit wavelength for the Balmer series for the hydrogen atom is
(a) 912 Å/2 (b) 912 Å
(c) 912×2 Å (d) 912×4 Å

41. A stationary hydrogen atom of mass M emits a photon corresponding to the first line of Lyman series. If R is the Rydberg's constant, the velocity that the atom acquires is
(a) $\frac{3}{4}\frac{Rh}{M}$ (b) $\frac{Rh}{4M}$
(c) $\frac{Rh}{2M}$ (d) $\frac{Rh}{M}$

42. An electron jumps from the fourth orbit to the second orbit of hydrogen atom. Given: the Rydberg's constant

$R = 10^5\ cm^{-1}$. The frequency, in Hz, of the emitted radiation will be

(a) $\frac{3}{16} \times 10^5$ (b) $\frac{3}{6} \times 10^{15}$

(c) $\frac{9}{16} \times 10^5$ (d) $\frac{9}{16} \times 10^{15}$

43. The shortest wavelength of the Brackett series of a hydrogen-like atom (atomic number = z) is the same as the shortest wavelength of the Balmer series of hydrogen atom. The value of z is

(a) 2 (b) 3

(c) 4 (d) 6

44. If λ_1 and λ_2 are the wavelengths of the first members of the Lyman and Paschen series, respectively, then $\lambda_1:\lambda_2$ is

(a) 1:3 (b) 1:30

(c) 7:50 (d) 7:108

45. In hydrogen and hydrogen-like atoms, the ratio of $E_{4n} - E_{2n}$ and $E_{2n} - E_n$ varies with atomic number z and principal quantum number n as

(a) $\frac{z^2}{n^2}$ (b) $\frac{z^4}{n^4}$

(c) $\frac{z}{n}$ (d) none of these

X-rays

46. If energy of K-shell electron is – 40000 eV and If 60000 V potential is applied at Coolidge tube then which of the following X-ray will get form?

(a) Continuous

(b) White X-rays

(c) Continuous and all series of characteristic

(d) None of these

47. X-rays are produced by accelerating electrons by voltage V and let they strike a metal of atomic number Z. The highest frequency of X-rays produced is proportional to

(a) V (b) Z

(c) $(Z-1)$ (d) $(Z-1)^2$

48. Let λ_α, λ_β and λ_α' denote the wavelengths of the X-rays of the K_α, K_β and L_α lines in the characteristic X-rays for a metal

(a) $\lambda_\alpha > \lambda_\alpha' > \lambda_\beta$ (b) $\lambda_\alpha' > \lambda_\beta > \lambda_\alpha$

(c) $\frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha} + \frac{1}{\lambda_\alpha'}$ (d) $\frac{1}{\lambda_\alpha} + \frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha'}$

49. The energy ratio of two K_α photons obtained in X-ray from two metal targets of atomic numbers Z_1 and Z_2 is

(a) $\frac{Z_1}{Z_2}$ (b) $\left(\frac{Z_1}{Z_2}\right)^2$

(c) $\left(\frac{Z_1-1}{Z_2-1}\right)^2$ (d) $\sqrt{\frac{(Z_1-1)}{(Z_2-1)}}$

50. If the minimum wavelength obtained in an X-ray tube is 2.5×10^{-10} m, the operating potential of the tube will be

(a) 2 kV (b) 3 kV

(c) 4 kV (d) 5 kV

51. The wavelength K_α of X-rays produced by the X-ray tube is 0.76 Å. The atomic number of the node material of the tube is

(a) 30 (b) 40

(c) 50 (d) 60

52. The shortest wavelength produced in an X-ray tube operating at 0.5 million volt is

(a) dependent on the target element

(b) about 2.5×10^{-12} m

(c) double of the shortest wavelength produced at 1 million volt

(d) dependent only on the target material

53. If the K_α radiation of Mo ($Z = 42$) has a wavelength of 0.71 Å find the wavelength of the corresponding radiation of Cu ($Z = 29$)

(a) 1 Å (b) 2 Å

(c) 1.52 Å (d) 1.25 Å

54. A beam of electron accelerated by a large potential difference V is made to strike a metal target to produce X-rays. For which of the following values of V, will the resulting X-rays have the lowest minimum wavelength?

(a) 10 kV (b) 20 kV

(c) 30 kV (d) 40 kV

55. In X-ray tube, when the accelerating voltage V is halved, the difference between the wavelength of K_α line and minimum wavelength of continuous X-ray spectrum

(a) remains constant

(b) becomes more than two times

(c) becomes half

(d) becomes less than two times

56. X-rays emitted from a copper target and a molybdenum target are found to contain a line of wavelength 22.85 nm attributed to the K_α line of an impurity element. The K_α lines of copper ($Z = 29$) and molybdenum ($Z = 42$) have wavelengths 15.42 nm and 7.12 nm, respectively. The atomic number of the impurity element is

(a) 22 (b) 23

(c) 24 (d) 25

Problems Based on Mixed Concepts

57. In interpreting Rutherford's experiments on the scattering of alpha particles by thin foils, one must examine what the known factors were, and what the experiment concluded. Which of the following are true in this context?

(a) The number of electrons in the target atoms (i.e., Z) was settled by these experiments.
(b) The validity of Coulomb's law for distances as small as 10^{-13} was known before these experiments.
(c) The experiments settled that size of the nucleus could not be larger than a certain value.
(d) The experiments also settled that size of the nucleus could not be smaller than a certain value.

58. When the hydrogen atom emits a photon in going from $n = 5$ to $n = 1$ state, its recoil speed is nearly
(a) 10^{-4} m s^{-1} (b) 2×10^{-2} m s^{-1}
(c) 4 m s^{-1} (d) 8×10^2 m s^{-1}

59. The frequency of emission line for any transition in positronium atom (consisting of a positron and an electron) is x times the frequency for the corresponding line in the case of H atom, where x is
(a) $\sqrt{2}$ (b) $1/2\sqrt{2}$
(c) $2\sqrt{2}$ (d) 1/2

60. If wavelength of photon emitted due to transition of an electron from the third orbit to the first orbit in a hydrogen atom is λ, then the wavelength of photon emitted due to transition of electron from the fourth orbit to the second orbit will be
(a) $\frac{128}{27}\lambda$ (b) $\frac{25}{9}\lambda$
(c) $\frac{36}{7}\lambda$ (d) $\frac{125}{11}\lambda$

61. Given: mass number of gold = 197, density of gold = 19.7 g cm^{-3}, Avogadro's number = 6×10^{23}. The radius of the gold atom is approximately:
(a) 1.5×10^{-8} m (b) 1.5×10^{-9} m
(c) 1.5×10^{-10} m (d) 1.5×10^{-12} m

62. Transitions between three energy levels in a particular atom give rise to three spectral lines of wavelengths, in increasing magnitudes, λ_1, λ_2, and λ_3. Which one of the following equations correctly relates λ_1, λ_2, and λ_3?
(a) $\lambda_1 = \lambda_2 - \lambda_3$ (b) $\lambda_1 = \lambda_3 - \lambda_2$
(c) $\frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$ (d) $\frac{1}{\lambda_1} = \frac{1}{\lambda_3} - \frac{1}{\lambda_2}$

63. Suppose two deuterons must get as close as 10^{-14} m in order for the nuclear force to overcome the repulsive electrostatic force. The height of the electrostatic barrier is nearest to
(a) 0.14 MeV (b) 2.3 MeV
(c) 1.8×10 MeV (d) 0.56 MeV

64. An electron in H atom makes a transition from $n = 3$ to $n = 1$. The recoil momentum of H atom will be
(a) 6.45×10^{-27} N s (b) 6.8×10^{-27} N s
(c) 6.45×10^{-24} N s (d) 6.8×10^{-24} N s

65. The electron in a hydrogen atom makes a transition from $n = n_1$ to $n = n_2$ state. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are
(a) $n_1 = 4, n_2 = 2$ (b) $n_1 = 8, n_2 = 2$
(c) $n_1 = 8, n_2 = 3$ (d) $n_1 = 6, n_2 = 2$

66. An electron revolving in an orbit of radius 0.5 Å in a hydrogen atom executes 10^{16} revolutions per second. The magnetic moment of electron due to its orbital motion will be
(a) 1.256×10^{-23} A m^2 (b) 653×10^{-26} A m^2
(c) 10^{-3} A m^2 (d) 256×10^{-26} A m^2

67. The total energy of an electron in the ground state of hydrogen atom is −13.6 eV. The potential energy of an electron in the ground state of Li^{2+} ion will be
(a) 122.4 eV (b) −122.4 eV
(c) 244.8 eV (d) −244.8 eV

68. The wavelength of radiation required to excite the electron from the first orbit to the third orbit in a doubly ionized lithium atom will be
(a) 134.25 Å (c) 125.5 Å
(c) 113.7 Å (d) 110 Å

69. If potential energy between a proton and an electron is given by $|U| = ke^2/2R^3$, where e is the charge of electron and R is the radius of atom, then radius of Bohr's orbit is given by (h = Planck's constant, k = constant)
(a) $\frac{ke^2m}{h^2}$ (b) $\frac{6\pi^2}{n^2}\frac{ke^2m}{h^2}$
(c) $\frac{2\pi}{n}\frac{ke^2m}{h^2}$ (d) $\frac{4\pi^2ke^2m}{n^2h^2}$

70. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of
(a) 10^{-15} cm (b) 10^{-13} cm
(c) 10^{-12} cm (d) 10^{-19} cm

71. An electron with kinetic energy E eV collides with a hydrogen atom in the ground state. The collision is observed to be elastic for
(a) $0 < E < \infty$ (b) $0 < E < 10.2$ eV
(c) $0 < E < 13.6$ eV (d) $0 < E < 3.4$ eV

72. When photons of wavelength λ_1 are incident on an isolated sphere suspended by an insulated thread, the corresponding stopping potential is found to be V. When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice the above value. If light of wavelength λ_3 is used, calculate the stopping potential for this case.
(a) $\frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{1}{\lambda_1}\right]$ (b) $\frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{2\lambda_2} - \frac{3}{2\lambda_1}\right]$
(c) $\frac{hc}{e}\left[\frac{1}{\lambda_3} + \frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$ (d) $\frac{hc}{e}\left[\frac{1}{\lambda_3} - \frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$

73. The minimum kinetic energy required for ionization of a hydrogen atom is E_1 in case electron is collided with hydrogen atom. It is E_2 if the hydrogen ion is collided and E_3 when helium ion is collided. Then,
(a) $E_1 = E_2 = E_3$ (b) $E_1 > E_2 > E_3$
(c) $E_1 < E_2 < E_3$ (d) $E_1 > E_3 > E_2$

74. A hydrogen atom is in an excited state of principle quantum number n. It emits a photon of wavelength λ when it returns to the ground state. The value of n is
(a) $\sqrt{\lambda R(\lambda R - 1)}$ (b) $\sqrt{\frac{\lambda(R-1)}{\lambda R}}$
(c) $\sqrt{\frac{\lambda R}{\lambda R - 1}}$ (d) $\sqrt{\lambda(R-1)}$

75. A neutron moving with a speed v makes a head-on collision with a hydrogen atom in ground state kept at rest. The minimum kinetic energy of the neutron for which inelastic collision will take place is (assume that mass of proton is nearly equal to the mass of neutron)
(a) 10.2 eV (b) 20.4 eV
(c) 12.1 eV (d) 16.8 eV

76. According to Bohr's theory of hydrogen atom, the product of the binding energy of the electron in the nth orbit and its radius in the nth orbit
(a) is proportional to n^2
(b) is inversely proportional to n^3
(c) has a constant value of 10.2 eV-Å
(d) has a constant value of 7.2 eV-Å

77. An electron and a photon have same wavelength. If p is the momentum of electron and E the energy of photon, the magnitude of p/E in SI unit is
(a) 3.0×10^8 (b) 3.33×10^{-9}
(c) 9.1×10^{-31} (d) 6.64×10^{-34}

78. An electron is in an excited state in a hydrogen like atom. It has a total energy of –3.4 eV. The kinetic energy of electron is E and its de Broglie wavelength is λ
(a) $E = 6.8$ eV; $\lambda = 6.6 \times 10^{-10}$ m
(b) $E = 3.4$ eV; $\lambda = 6.6 \times 10^{-10}$ m
(c) $E = 3.4$ eV; $\lambda = 6.6 \times 10^{-11}$ m
(d) $E = 6.8$ eV; $\lambda = 6.6 \times 10^{-11}$ m

79. The circumference of the second Bohr orbit of electron in hydrogen atom is 600 nm. The potential difference that must be applied between the plates so that the electrons have the de Broglie wavelength corresponding in this circumference is
(a) 10^{-5} V (b) $\frac{5}{3} \times 10^{-5}$ V
(c) 5×10^{-5} V (d) 3×10^{-5} V

80. An electron in a Bohr orbit of hydrogen atom with the quantum number n_2 has an angular momentum 4.2176×10^{-34} kg m^2 s^{-1}. If the electron drops from this level to the next lower level, the wavelength of this line is
(a) 18 nm (b) 187.6 pm
(c) 1876 Å (d) 1.876×10^4 Å

ARCHIVES

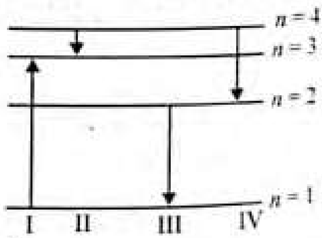
1. If 13.6 eV energy is required to ionise a hydrogen atom, then the required energy to remove an electron from $n = 2$ is
(a) 10.2 eV (b) 0 eV
(c) 3.4 eV (d) 6.8 eV **(AIEEE 2002)**

2. In the hydrogen atom, when electron jumps from second to first orbit, then energy emitted is
(a) –13.6 eV (b) –27.2 eV
(c) –6.8 eV (d) None of these
(AIEEE 2002)

3. The wavelengths involved in the spectrum of deuterium ($^2_1$D) are slightly different from that of hydrogen spectrum because
(a) the attraction between the electron and the nucleus is different in the two cases.
(b) the size of the two nuclei is different.
(c) the nuclear forces are different in the two cases.
(d) the masses of the two nuclei are different.
(AIEEE 2003)

4. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is
(a) 122.4 eV (b) 30.6 eV
(c) 13.6 eV (d) 3.4 eV **(AIEEE 2003)**

5. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



(a) II (b) I
(c) IV (d) III **(AIEEE 2005)**

6. Which of the following transitions in hydrogen atoms emits photons of the highest frequency?
(a) $n = 6$ to $n = 2$ (b) $n = 2$ to $n = 1$
(c) $n = 1$ to $n = 2$ (d) $n = 2$ to $n = 6$
(AIEEE 2007)

7. Suppose an electron is attracted towards the origin by a force k/r where k is a constant and r is the distance of the electron from the origin. By applying the Bohr model to this system, the radius of the nth orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Which of the following is true?

(a) $T_n \propto \frac{1}{n}, r_n \propto n$

(b) $T_n \propto \frac{1}{n}, r_n \propto n^2$

(c) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$

(d) T_n independent of n, $r_n \propto n$ **(AIEEE 2008)**

8. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from

(a) $2 \to 1$ (b) $3 \to 2$

(c) $4 \to 2$ (d) $5 \to 4$ **(AIEEE 2009)**

9. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is

(a) 12.1 eV (b) 36.3 eV

(c) 108.8 eV (d) 122.4 eV **(AIEEE 2011)**

10. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be

(a) 2 (b) 3

(c) 5 (d) 6 **(AIEEE 2012)**

11. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by

(a) $\frac{n^2h^2}{2(m_1+m_2)r^2}$ (b) $\frac{2n^2h^2}{(m_1+m_2)r^2}$

(c) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ (d) $\frac{(m_1+m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$

(AIEEE 2012)

12. In a hydrogen-like atom an electron makes transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to

(a) $\frac{1}{n^2}$ (b) $\frac{1}{n^{3/2}}$

(c) $\frac{1}{n^3}$ (d) $\frac{1}{n}$ **(JEE Main 2015)**

13. The radiation corresponding to 3×2 transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

(a) 0.8 eV (b) 1.6 eV

(c) 1.8 eV (d) 1.1 eV **(JEE Main 2015)**

14. As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion

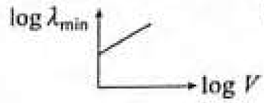
(a) its kinetic energy increases but potential energy and total energy decrease

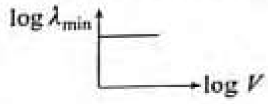
(b) kinetic energy, potential energy and total energy decrease

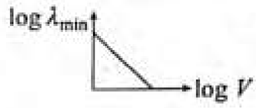
(c) kinetic energy decreases, potential energy increases but total energy remains same

(d) kinetic energy and total energy decrease but potential energy increases **(JEE Main 2015)**

15. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log \lambda_{min}$ with $\log V$ is correctly represented in

(a) 

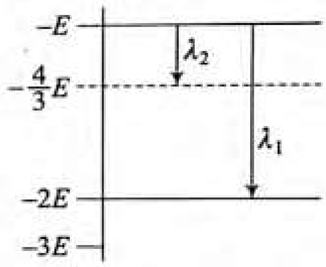
(b) 

(c) 

(d) 

(JEE Main 2017)

16. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by



(a) $r = \frac{3}{4}$ (b) $r = \frac{1}{3}$

(c) $r = \frac{4}{3}$ (d) $r = \frac{2}{3}$ **(JEE Main 2017)**

17. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(a) $\Lambda_n^2 \approx \lambda$ (b) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

(c) $\Lambda_n \approx A + B\lambda_n$ (d) $\Lambda_n^2 \approx A + B\lambda_n^2$

(JEE Main 2018)

18. If the series limit frequency of the Lyman series is v_L, then the series limit frequency of the Pfund series is:

(a) $\frac{v_L}{25}$ (b) $25\ v_L$

(c) $16\ v_L$ (d) $\frac{v_L}{16}$

(JEE Main 2018)

ANSWER KEY

Exercises

1. (c)	2. (d)	3. (d)	4. (a)	5. (d)	6. (a)	7. (b)	8. (c)	9. (c)	10. (b)
11. (a)	12. (b)	13. (a)	14. (a)	15. (c)	16. (d)	17. (d)	18. (a)	19. (d)	20. (b)
21. (c)	22. (a)	23. (c)	24. (a)	25. (a)	26. (b)	27. (d)	28. (c)	29. (b)	30. (d)
31. (d)	32. (b)	33. (b)	34. (d)	35. (a)	36. (d)	37. (d)	38. (a)	39. (d)	40. (d)
41. (a)	42. (d)	43. (a)	44. (d)	45. (d)	46. (c)	47. (d)	48. (c)	49. (c)	50. (d)
51. (b)	52. (c)	53. (b)	54. (d)	55. (d)	56. (c)	57. (c)	58. (c)	59. (d)	60. (a)
61. (c)	62. (c)	63. (a)	64. (a)	65. (a)	66. (a)	67. (d)	68. (c)	69. (b)	70. (c)
71. (b)	72. (b)	73. (c)	74. (c)	75. (b)	76. (d)	77. (b)	78. (b)	79. (b)	80. (d)

Archives

1. (c)	2. (d)	3. (d)	4. (b)	5. (d)	6. (b)	7. (d)	8. (d)	9. (c)	10. (d)
11. (c)	12. (c)	13. (d)	14. (a)	15. (c)	16. (b)	17. (b)	18. (a)		

Chapter 30

Nuclear Physics

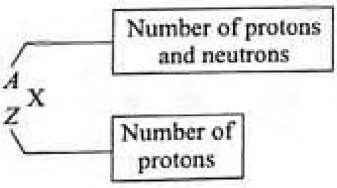
NUCLEAR STRUCTURE

The nucleus of an atom consists of neutrons and protons, collectively referred to as nucleons. The neutron, discovered in 1932 by the English physicist James Chadwick (1891–1974), carries no electrical charge and has a mass slightly larger than that of a proton.

The number of protons in the nucleus is different in different elements and is given by the atomic number Z. In an electrically neutral atom, the number of nuclear protons equals the number of electrons in orbit around the nucleus. The number of neutrons in the nucleus is N. The total number of protons and neutrons is referred to as the atomic mass number A because the total nuclear mass is approximately equal to A times the mass of a single nucleon:

$$\underset{\text{[Number of proton and neutrons (atomic mass number or nucleon number)]}}{A} = \underset{\text{[Number of protons (atomic number)]}}{A} + \underset{\text{[Number of neutrons]}}{N} \quad \text{(i)}$$

Sometimes A is also called the nucleon number. A short-hand notation is often used to specify Z and A along with the chemical symbol for the element. For instance, the nuclei of all naturally occurring aluminum atoms have $A = 27$, and the atomic number for aluminum is $Z = 13$. In short-hand notation, then, the aluminium nucleus is specified as $^{27}_{13}Al$. The number of neutrons in an aluminium nucleus is $N = A - Z = 14$. In general, for an element whose chemical symbol is X, the symbol for the nucleus is shown in figure.



For a proton the symbol is ^1_1H, since the proton is the nucleus of a hydrogen atom. A neutron is denoted by ^1_0n. In the case of an electron we use $^{0}_{-1}e$, where $A = 0$ because an electron is not composed of protons or neutrons and $Z = -1$ because the electron has a negative charge.

ILLUSTRATION 30.1 How many electrons, protons, and neutrons are there in 12 g of $_6C^{12}$ and in 14 g of $_6C^{14}$?

Solution. Mass number (or atomic weight) of $_6C^{12}$ is 12. Therefore, the number of atoms in 12 g of $_6C^{12}$ is

Avogadro number $= 6 \times 10^{23}$

The number of electrons in 12 g of $_6C^{12}$ is

$6 \times 6 \times 10^{23} = 36 \times 10^{23}$

The number of protons in 12 g of $_6C^{12}$ is 36×10^{23}.

The number of neutrons in 12 g of $_6C^{12}$ is

$(A - Z) \times 6 \times 10^{23} = (12 - 6) \times 6 \times 10^{23} = 36 \times 10^{23}$

Similarly, number of electrons in 14 g of $_6C^{14}$ is 36×10^{23}.

Number of protons in 14 g of $_6C^{14}$ is 36×10^{23}.

Number of neutrons in 14 g of $_6C^{14}$ $= (A - Z) \times 6 \times 10^{23}$
$= (14 - 6) \times 6 \times 10^{23}$
$= 48 \times 10^{23}$

ATOMIC MASS NUMBER

It is the nearest integer value of mass represented in amu (atomic mass unit).

Atomic masses refer to the masses of neutral atoms. Thus, an atomic mass always includes the mass of its Z electrons. The atomic mass of an atom is measured relative to the mass of an atom of the neutral carbon-12 isotope (the nucleus plus six electrons). Atomic masses are measured in atomic mass units, which are denoted by the symbol u. Atomic mass units are defined in terms of the mass of the isotope ^{12}C, whose atomic mass is defined to be exactly 12 u.

$$1 \text{ amu} = \frac{1}{12} \times [\text{mass of one atom of } _6C^{12} \text{ atom at rest and in ground state}]$$
$$= 1.6603 \times 10^{-27} \text{ kg}$$

Table 30.1 Properties of particles in the atom

Particle	Electric charge (C)	Mass: Kilogram (kg)	Mass: Atomic mass units (u)
Electron	-1.60×10^{-19}	9.109390×10^{-31}	5.485799×10^{-4}
Proton	$+1.60 \times 10^{-19}$	1.672623×10^{-27}	1.007276
Neutron	0	1.674929×10^{-27}	1.008665

SOME DEFINITIONS

Isotopes: The nuclei having the same number of protons but different number of neutrons are called isotopes.

Isotopes of carbon: $^{6}_{11}C$, $^{12}_{6}C$, $^{13}_{6}C$, $^{14}_{6}C$

Isotopes of hydrogen: ^1_1H, ^2_1H, ^3_1H

The isotopes ^1H is called ordinary hydrogen or simply hydrogen; ^2H is called deuterium. Deuterium, sometimes known as heavy hydrogen, can combine with oxygen to form heavy water (D_2O). The third isotope of hydrogen, ^3H, is called tritium, which is unstable.

Isotones: Nuclei with the same neutron number N but different atomic number Z are called isotones.

$$^{14}_{6}C,\ ^{15}_{7}N,\ ^{16}_{8}C,\ ^{17}_{9}F$$

Isobars: The nuclei with the same mass number but different atomic number are called isobars.

$$^{16}_{6}C,\ ^{16}_{7}N,\ ^{16}_{8}O,\ ^{16}_{9}F$$

SIZE OF NUCLEI

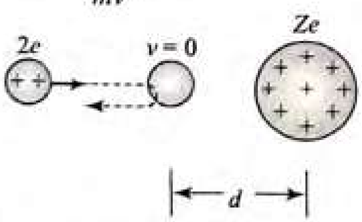
The size and structure of nuclei were first investigated in the scattering experiments of Rutherford. Using the principle of conservation of energy, Rutherford found an expression for how close an alpha particle moving directly toward the nucleus can come to the nucleus before being turned around by Coulomb repulsion.

In such a head-on collision, the kinetic energy of the incoming alpha particle must be converted completely to electrical potential energy when the particle stops at the point of closest approach and turns around (see figure). If we equate the initial kinetic energy of the alpha particle to the maximum electrical potential energy of the system (alpha particle + target nucleus), we have

$$\frac{1}{2}mv^2 = k_e\frac{q_1q_2}{r}k_e\frac{(2e)(Ze)}{d}$$

where d is the distance of closest approach. Solving for d, we get

$$d = \frac{4k_eZ_e^2}{mv^2}$$



An alpha particle on a head-on collision with a nucleus of charge Ze. Because of the Coulomb repulsion between charges of the same sign, the alpha particle approaches to a distance d from the target nucleus called the distance of closest approach

From this expression, Rutherford found that alpha particles approached to within 3.2×10^{-14} m of a nucleus when the foil was made of gold. Thus, the radius of the gold nucleus must be less than this value. For silver atoms, the distance of closest approach was 2×10^{-14} m. From these results, Rutherford concluded that the positive charge in an atom is concentrated in a small sphere, which he called the nucleus, whose radius is no greater than about 10^{-14} m. Because such small lengths are common in nuclear physics, a convenient unit of length is the femtometer (fm), sometimes called the Fermi, defined as

$$1 \text{ fm} = 10^{-15} \text{ m}$$

Since the time of Rutherford's scattering experiments, a multitude of other experiments have shown that most nuclei are approximately spherical and have an average radius given by

$$r = r_0A^{1/3} \quad \text{(i)}$$

where A is the total number of nucleons and r_0 is a constant equal to 1.2×10^{-15} m. Because the volume of a sphere is proportional to the cube of its radius, it follows from Eq. (i) that the volume of a nucleus (assumed to be spherical) is directly proportional to A, the total number of nucleons. This suggests that all nuclei have nearly the same density. Nucleons combine to form a nucleus as though they were tightly packed spheres (see figure).

A nucleus can be modelled as a cluster of tightly packed spheres, each of which is a nucleon

ILLUSTRATION 30.2 The most common kind of iron nucleus has a mass number of 56. Find the radius, approximate mass, and approximate density of the nucleus.

Solution. We use two key ideas. The radius and mass of a nucleus depend on the mass number A and density is mass divided by volume.

We use Eq. (i) to determine the radius of the nucleus. The mass of the nucleus in atomic mass units is approximately equal to the mass number. The radius is

$$R = R_0A^{1/3} = (1.2 \times 10^{-15} \text{ m})(56)^{1/3}$$
$$= 4.6 \times 10^{-15} \text{ m} = 4.6 \text{ fm}$$

Since $A = 56$, the mass of the nucleus is approximately 56 u, or $m \approx (56)(1.66 \times 10^{-27} \text{ kg}) = 9.3 \times 10^{-26}$ kg. The volume is

$$v = \frac{4}{3}\pi R^3 = \frac{4}{3} \times 3.14 \times (4.6 \times 10^{-15})^3$$
$$\approx 4.1 \times 10^{-13} \text{ m}^3$$

And the density ρ is approximately

$$\rho = \frac{m}{V} \approx \frac{9.3 \times 10^{-26} \text{ kg}}{4.1 \times 10^{-43} \text{ m}^3} = 2.3 \times 10^{17} \text{ kg m}^{-3}$$

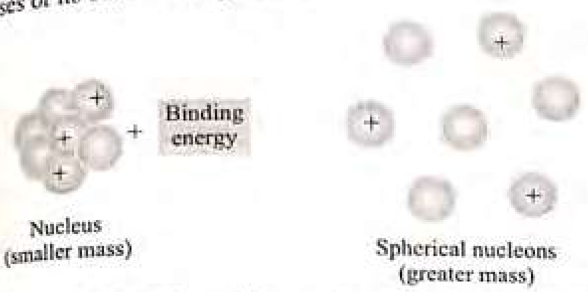
The density of solid iron is about 700 kg m^{-3}, so we see that the iron nucleus is more than 10^{13} times as dense as the bulk material. Densities of this magnitude are also found in neutron stars, which are similar to gigantic nuclei made almost entirely of neutrons. A 1 cm cube of material with this density would have a mass of 2.3×10^{11} kg or 230 million metric tons.

NUCLEAR BINDING ENERGY

We know that in a stable nucleus, because of strong nuclear force of attraction, the nucleons are held tightly together in a

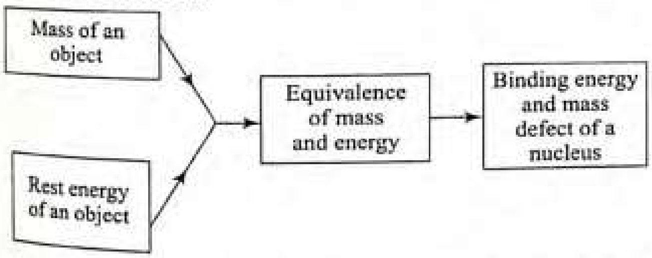
small volume. As the system is stable, we can relatively say that the total potential energy of the system is negative and to separate all the nucleons from each other, some energy must be supplied to break the nucleus. The more stable the nucleus is, the greater is the energy needed to break it apart. This required energy is called binding energy of the nucleus.

The mass of each element atom is less than the sum of masses of its constituent particles.



If we think from where this energy comes, we can simply say by Einstein's mass–energy relationship, some amount of mass from independent nucleons is converted into energy and released when nucleons bind with each other to form a stable nucleus. This is the energy that hold the nucleons together in a nucleus, we call this binding energy of nucleus. So, when this energy is supplied to a nucleus, it splits into its constituent particles.

The binding energy used to disassemble the nucleus appears as extra mass of the separated nucleons. In other words, the sum of the individual masses of the separated protons and neutrons is greater by an amount Δm than the mass of the stable nucleus. The difference in mass Δm is known as the mass defect of the nucleus.



For the reaction given in Eq. (ii), if we calculate the difference in masses of nucleons and that of nucleon X, then it is given as

$$\Delta m = Zm_p + (A - Z)m_n - M_X \qquad \text{(iii)}$$

This difference in masses of independent nucleons and mass of nucleus is called mass defect of the nuclear reaction. Using mass defect Δm, we can find the energy released in a nuclear reaction, e.g., the binding energy of the above nucleus X can be given as

$$\Delta E_{BE} = \Delta mc^2 \qquad \text{(iv)}$$

In a similar way, we can find the binding energy for any nucleus in nature for a known composition.

Mass Defect

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$$M_{\text{expected}} = Zm_p + (A - Z)m_n$$
$$M_{\text{observed}} = M_{\text{atom}} - Zm_e$$

It is found that

$$M_{\text{observed}} < M_{\text{expected}}$$

Hence, mass defect is defined as

$$\text{Mass defect} = M_{\text{expected}} - M_{\text{observed}}$$

$$\Rightarrow \quad \Delta m = [Zm_p + (A - Z)m_n] - [M_{\text{atom}} - Zm_e]$$

MASS–ENERGY EQUIVALENCE

We have discussed in previous section that using mass defect of a nuclear reaction, we can find the energy released in the nuclear process. We know generally nuclear masses are given in atomic mass unit where

$$1 \text{ amu} = 1.656 \times 10^{-27} \text{ kg}$$

If in a reaction 1 amu mass is converted into energy, then using Einstein's mass–energy relationship the amount of energy released is

$$\Delta E' = \Delta mc^2 = (1.656 \times 10^{-27})(3 \times 10^8)^2 \text{ J}$$
$$= \frac{(1.656 \times 10^{-27})(3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV}$$
$$= 931.5 \times 10^6 \text{ eV} = 931.5 \text{ MeV}$$

Thus, we can say that 1 amu mass is equivalent to 931.5 MeV energy.

ILLUSTRATION 30.3 The most abundant isotope of helium has a ${}_2^4$H nucleus whose mass is 6.6447×10^{-27} kg. For this nucleus, find (a) the mass defect and (b) the binding energy.

Given: Mass of the electron: $m_e = 5.485799 \times 10^{-4}$ u, mass of the proton: $m_p = 1.007276$ u and mass of the neutron: $m_n = 1.008\,665$ u.

Solution. The symbol ${}_2^4$He indicates that the helium nucleus contains $Z = 2$ protons and $N = 4 - 2 = 2$ neutrons. To obtain the mass defect Δm, we first determine the sum of the individual masses of the separated protons and neutrons. Then, we subtract from this sum the mass of the nucleus.

(a) We find that the sum of the individual masses of the nucleons is

$$\underbrace{2(1.6726 \times 10^{-27}\text{ kg})}_{\text{Two protons}} + \underbrace{2(1.6749 \times 10^{-27}\text{ kg})}_{\text{Two neutrons}}$$
$$= 6.6950 \times 10^{-27} \text{ kg}$$

This value is greater than the mass of the intact He nucleus, and the mass defect is

$$\Delta m = 6.6950 \times 10^{-27} \text{ kg} - 6.6447 \times 10^{-27} \text{ kg}$$
$$= 0.0503 \times 10^{-27} \text{ kg}$$

(b) According to Eq. (iv), the binding energy is given by

$$\Delta E_{BE} = (\Delta m)c^2 = (0.0503 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m s}^{-1})^2$$
$$= 4.53 \times 10^{-12} \text{ J}$$

Usually, binding energies are expressed in energy units of electron volt instead of joule (1 eV = 1.60×10^{-19} J).

$$\therefore \quad \text{Binding energy} = (4.53 \times 10^{-12}\ \text{J})\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right)$$

$$= 2.83 \times 10^{7}\ \text{eV} = 28.3\ \text{MeV}$$

In this result, one million electron volt is denoted by the unit MeV. The value of 28.3 MeV is more than two million times greater than the energy required to remove an orbital electron from an atom.

BINDING ENERGY PER NUCLEON

If we wish to see how nuclear binding energy varies from nucleus to nucleus, it is necessary to compare the binding energy per nucleon basis. The binding energy per nucleon for a given element can be given as binding energy divided by the nucleon number A as

$$(\text{BE})_N = \frac{\Delta mc^2}{A} \qquad \text{(v)}$$

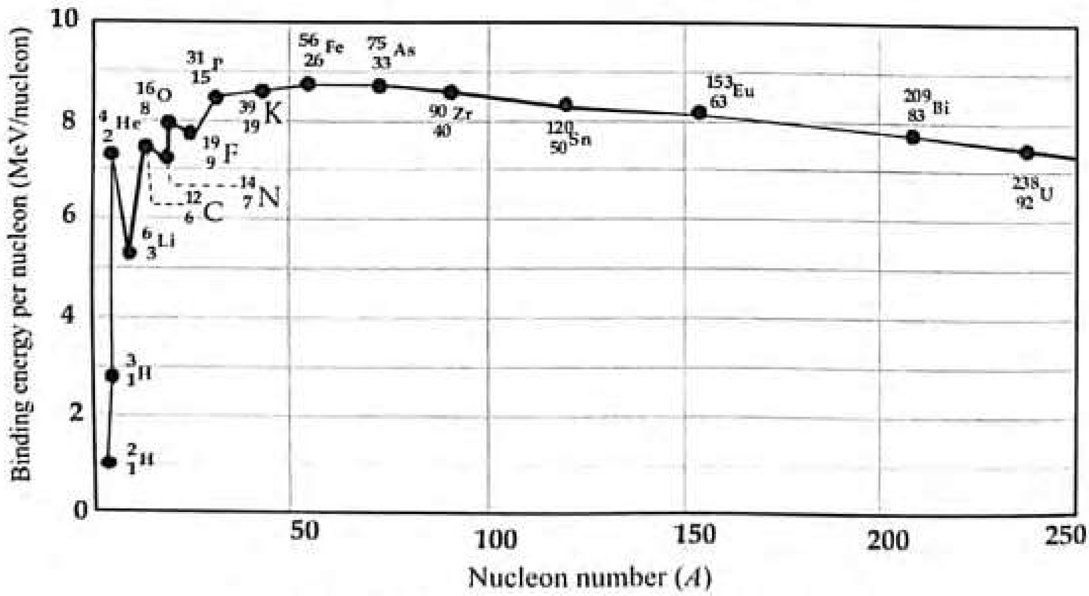
This value $(\text{BE})_N$ in Eq. (v) gives the criterion of stability among different elements. We can define binding energy per nucleon theoretically as the amount of energy needed to remove a nucleon from the nucleus of an element. For example, let us consider two elements X and Y with mass numbers A_X and A_Y ($A_X > A_Y$) and binding energies ΔE_X and ΔE_Y such that $\Delta E_X > \Delta E_Y$. Here, one can say that as for element X biding energy ΔE_X is more as compared to that for element Y, nucleus X is more stable than nucleus Y. But if we find the $(\text{BE})_N$ values for both elements, it is given as

$$(\text{BE}_N)_X = \frac{\Delta E_X}{A_X};\ (\text{BE}_N)_Y = \frac{\Delta E_Y}{A_Y}$$

These values are such that $(\text{BE}_N)_X < (\text{BE}_N)_Y$ which implies that to remove a nucleon from element X requires less energy than from element Y. This implies that for the nucleus of Y it is difficult to remove one nucleon from its nucleus, thus structure of Y is more stable than X.

Variation of Binding Energy Per Nucleon with Mass Number

The binding energy per nucleon is a characteristic property of elements. The graph in figure shows the variation of binding energy per nucleon with mass number for all the elements of periodic table. In the graph, we can see that the binding energy per nucleon increases rapidly for nuclei with small masses and reaches a maximum of approximately 8.8 MeV/nucleon for iron $\left({}^{56}_{26}\text{Fe}\right)$. That is, nuclei with mass numbers greater or less than 60 are not as strongly bound as those near the middle of the periodic table. As we shall see later, this fact allows energy to be released in fission and fusion reactions. The curve is slowly varying for $A > 40$, which suggests that the nuclear force saturates. In other words, a particular nucleon can interact with only a limited number of other nucleons, which can be viewed as the "nearest neighbors" in the close-packed structure illustrated in figure.



A plot of binding energy per nucleon versus the nucleon number A.

For greater nucleon numbers, the binding energy per nucleon decreases gradually. Later, the binding energy per nucleon decreases enough so there is insufficient binding energy to hold the nucleon together in the nucleus. It is observed that nuclei with $A > 209$ are unstable and hence radioactive.

In the figure, in the beginning there are some fluctuations in the graph. We can see that binding energies per nucleon for ^{4}He, ^{12}C, and ^{16}O are relatively higher as compared to their neighboring elements or these elements have nuclei which are relatively more stable than their neighbors. This is because

of the existence of nuclear energy levels in the nucleus. Each nuclear energy level can contain two neutrons of opposite spins or two protons of opposite spins. Energy levels in nuclei are filled in sequence, just as energy levels in atoms, to achieve configurations of minimum energy and therefore maximum stability. Similar to the case of atomic orbitals, here also the configuration of opposite spins in nucleons with pairs in nuclear energy levels are more stable. This concept can be used to explain the reason of more stability of ^4He, ^{12}C, and ^{16}O compared to their neighboring elements.

NOTE: If binding energy per nucleon is more for a nucleus, then it is more stable. For example, if

$$\left(\frac{BE_1}{A_1}\right) > \left(\frac{BE_2}{A_2}\right)$$

then nucleus 1 would be more stable.

Important observations from the graph

1. The maximum binding energy per nucleon is 8.8 MeV and it is for ^{56}Fe. Thus, iron nucleus is the most stable and the most tightly bound.
2. The fact that (BE/A) varies by less than 10 percent above $A = 10$ suggests that each nucleon in the nucleus interacts only with the neighbors, independently of the total number of nucleons present in the nucleus.
3. A small decrease after $A = 56$ is due to the destabilizing effect of the long-range repulsive Coulomb force.
4. Not only the Coulomb force reduces the BE, it also shifts the neutron–proton ratio in heavy nuclei toward the neutron excess, by an amount that will increase with A.

ILLUSTRATION 30.4 Calculate the binding energy for nucleon of $_6^{12}C$ nucleus, if mass of proton m_p = 1.0078 u, mass of neutron m_n = 1.0087 u, mass of C_{12}, m_C = 12.0000 u; and 1 u = 931.4 MeV.

Solution. In the nucleus of carbon, there are 6 protons and 6 neutrons.

Now,

$$6m_p = 6(1.0078) = 6.0468 \text{ u}$$
$$6m_n = 6(1.0087) = 6.05224 \text{ u}$$
$$\text{Total mass} = 6m_p + 6m_n = 12.0990 \text{ u}$$
$$m_C = 12.0000 \text{ u}$$
$$\text{Mass defect, } \Delta m = 0.0990 \text{ u}$$
$$BE = \Delta m \times 931.4 = (0.0990)\,931.4 = 92.2 \text{ MeV}$$

Q VALUES

We have just examined some nuclear reactions for which mass numbers and atomic numbers must be balanced in the equations. We shall now consider the energy involved in these reactions, because energy is another important quantity that must be conserved.

Let us illustrate this procedure by analyzing the following nuclear reaction:

$$_1^2H + {}_7^{14}N \rightarrow {}_6^{12}C + {}_2^4He \qquad (i)$$

The total mass on the left side of the equation is the sum of the mass of $_1^2H$ (2.014102 u) and the mass of $_7^{14}N$ (14.003074 u), which equals 16.017176 u. Similarly, the mass on the right side of the equation is the sum of the mass of $_6^{12}C$ (12.000000 u) plus the mass of $_2^4He$ (4.002602 u), for a total of 16.002602 u. Thus, the total mass before the reaction is greater than the total mass after the reaction. The mass difference in this reaction is greater than the total mass after the reaction. The mass difference in this reaction is equal to 16.017176 u – 16.002602 u = 0.014574 u. This "lost" mass is converted to the kinetic energy of the nuclei present after the reaction. In energy units, 0.0145774 u is equivalent to 13.576 MeV of kinetic energy carried away by the carbon and helium nuclei.

The energy required to balance the equation is called the Q value of the reaction. In Eq. (i), the Q value is 13.576 MeV. Nuclear reactions in which there is a release of energy—that is, positive Q values—are said to be exothermic reactions.

The energy balance sheet is not complete; however, we must also consider the kinetic energy of the incident particle before the collision. As an example, let us assume that the deuteron in Eq. (i) has a kinetic energy of 5 MeV. Adding this to our Q value, we find that the carbon and helium nuclei have a total kinetic energy of 18.576 MeV following the reaction. Now, consider the reaction

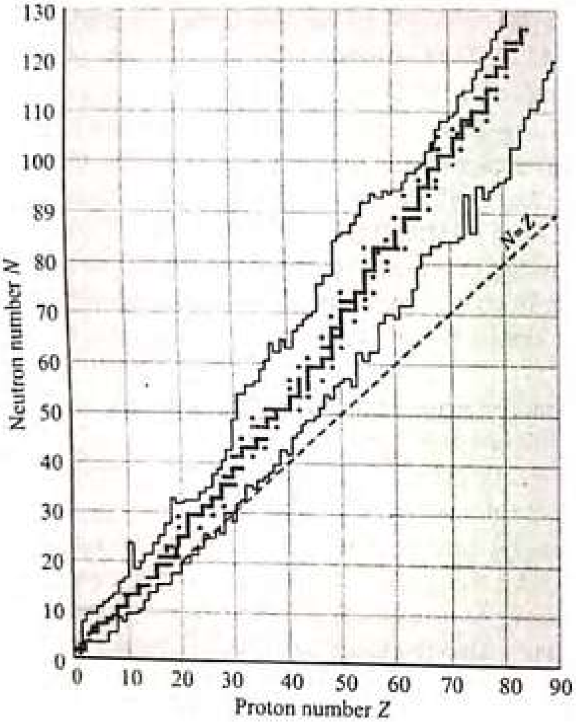
$$_2^4He + {}_7^{17}N \rightarrow {}_8^{17}O + {}_1^1H \qquad (ii)$$

Before the reaction, the total mass is the sum of the masses of the alpha particle and the nitrogen nucleus: 4.002602 u + 14.003074 u = 18.005676 u. After the reaction, the total mass is the sum of the masses of the oxygen nucleus and the proton: 16.999133 u + 1.07825 u = 18.006958 u. In this case, the total mass deficit is 0.001282 u, equivalent to an energy deficit of 1.194 MeV. This deficit is expressed by the negative Q value of the reaction, –1.194 MeV. Reactions with negative Q values are called endothermic reactions. Such reactions will not take place unless the incoming particle has at least enough kinetic energy to overcome the energy deficit.

NUCLEAR STABILITY

There are about 260 stable nuclei; hundreds of others have been observed but are unstable. A plot of N versus Z for a number of stable nuclei is given in figure. Note that light nuclei are most stable if they contain equal number of protons and neutrons—that is, if $N = Z$ but heavy nuclei are more stable if $N > Z$. This can be partially understood by recognizing that, as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart. As a result, more neutrons are needed to keep the nucleus stable, because

neutrons experience only the attractive nuclear forces. In effect, the additional neutrons "dilute" the nuclear charge density. Eventually, when $Z = 83$, the repulsive forces between protons cannot be compensated by the addition of more neutrons. Elements that contain more than 83 protons do not have stable nuclei, but decay or disintegrate into other particles in various amounts of time. The masses of several stable particles are given in Table 30.1.



Concept Application Exercise 30.1

1. How many electrons, protons, and neutrons are there in a nucleus of atomic number 11 and mass number 24?
2. Calculate the average binding energy per nucleon of $^{93}_{41}Nb$ having mass 92.906 u.
3. Protons and neutrons exist together in an extremely small space within the nucleus. How is this possible when protons repel each other?
4. Following data are available about three nuclei P, Q, and R. Arrange them in decreasing order of stability.

	P	Q	R
Atomic mass number (A)	10	5	6
Binding energy (MeV)	100	60	66

5. A nucelus has binding energy of 100 MeV. It further releases 10 MeV energy. Find the new binding energy of the nucleus.
6. A nuclear reaction is given as
$$A + B \to C + D$$
Binding energies of A, B, C, and D are given as B_1, B_2, B_3 and B_4. Find the energy released in the reaction.

RADIOACTIVITY

We have discussed that inside a nucleus electrostatic attraction is counterbalanced by short-range strong nuclear forces and the nucleus becomes stable. Despite the forces being balanced, many nuclides are unstable because of nuclear size or the limited range of nuclear forces. Due to slight imbalance in small sized nuclides, these nuclides spontaneously disintegrate into other nuclides. This phenomenon of spontaneous disintegration is called radioactivity. Further, in this chapter, we will discuss the aspects of radioactivity by which unstable nuclides disintegrate to achieve stability.

When an unstable or radioactive nucleus disintegrates spontaneously, certain kinds of particles and/or high-energy photons are released. These particles and photons are collectively called "rays". Three kinds of rays are produced by naturally occurring radioactivity: α-rays, β-rays, and γ-rays. They are named according to the first three letters of the Greek alphabet, alpha (α), beta (β), and gamma (γ), to indicate the extent of their ability to penetrate matter. α-rays are the least penetrating, being blocked by a thin ($\approx$0.01 mm) sheet of lead, while β-rays penetrate lead to a much greater distance ($\approx$0.1 mm). γ-rays are the most penetrating and can pass through an appreciable thickness ($\approx$100 mm) of lead.

α-Decay

When a nucleus disintegrates and produces α-rays, it is said to undergo α-decay. Experimental evidence shows that α-rays consist of positively charged particles, each one being the ^4_2He nucleus of helium. Thus, an α-particle has a charge of +2e and a nucleon number of $A = 4$. Since the grouping of 2 protons and 2 neutrons in a ^4_2He nucleus is particularly stable, as we have seen in connection with figure, it is not surprising that an α-particle can be ejected as a unit from a more massive unstable nucleus.

α-Decay:

$$_ZX^A \to {}_{Z-2}Y^{A-4} + {}_2He^4 + Q$$

Q value: It is defined as the energy released during the decay process.

Q value = rest mass energy of reactants – rest mass energy of products.

This energy is available in the form of increase in KE of the products. Let us consider the following:

$$M_X = \text{mass of atom } {}_ZX^A$$
$$M_Y = \text{mass of atom } {}_{Z-2}Y^{A-4}$$
$$M_{He} = \text{mass of atom } {}_2He^4$$
$$Q \text{ value} = [(M_X - Zm_e) - \{(M_Y - (Z-2)\,m_e) + (M_{He} - 2m_e)\}]c^2$$
$$= [M_X - M_Y - M_{He}]c^2$$

Considering actual number of electrons in α-decay

$${}_Z X^A \rightarrow Y + {}_2He^4 + Q$$

α-particle

Z-electrons Z-electrons (it has change +2e)

$$Q \text{ value} = [M_X - (M_Y + 2m_e) - (M_{He} - 2m_e)]c^2$$
$$= [M_X - M_Y - M_{He}]c^2$$

β-Decay

The β-rays are deflected by the magnetic field in a direction opposite to that of the positively charged α-rays. Consequently, these β-rays, which are the most common kind, consist of negatively charged particles or β-particles. Experiment shows that β-particles are electrons. When a radioactive nucleus undergoes β-decay, the daughter nucleus has the same number of nucleons as the parent nucleus, but the atomic number is changed by 1.

β^- Decay

$${}^{A}_{Z}P \underset{\text{Parent nucleus}}{} \rightarrow {}^{A}_{Z+1}D \underset{\text{Daughter nucleus}}{} \rightarrow {}^{0}_{-1}e \underset{\beta^- \text{ particle (electron)}}{}$$

The electron emitted in β^- decay does not actually exist within the parent nucleus and is not one of the orbital electrons. Instead, the electron is created when a neutron decays into a proton and an electron; when this occurs, the proton number of the parent nucleus increases from Z to Z + 1 and the nucleon number remains unchanged. The electron is usually fast-moving and escapes from the atom, leaving behind a positively charged atom.

β^+ Decay

A second kind of β-decay sometimes occurs. In this process, the particle emitted by the nucleus is a positron rather than an electron. A positron, also called a β^+ particle, has the same mass as an electron but carries a charge of $+e$ instead of $-e$. The disintegration process for β^+ decay is

$${}^{A}_{Z}P \underset{\text{Parent nucleus}}{} \rightarrow {}^{A}_{Z-1}P \underset{\text{Daughter nucleus}}{} \rightarrow {}^{0}_{1}e \underset{\beta^+ \text{ particle (positron)}}{}$$

The emitted positron does not exist within the nucleus but, rather, is created when a nuclear proton is transformed into a neutron. In the process, the proton number of the parent nucleus decreases from Z to Z – 1, and the nucleon number remains the same. As with β^- decay, the laws of conservation of charge and nucleon number are obeyed, and there is a transmutation of one element into another.

In the following illustration, we will discuss how energy is released during β^- decay, just as it is during α-decay, and that the conservation of mass/energy applies.

ILLUSTRATION 30.5 The atomic mass of thorium ${}^{234}_{90}Th$ is 234.04359 u, while that of protactinium ${}^{234}_{91}Pa$ is 234.04330 u. Find the energy released when β^- decay changes ${}^{234}_{90}Th$ into ${}^{234}_{91}Pa$.

Solution. To find the energy released, we follow the usual procedure of determining how much the mass has decreased because of the decay and then calculating the equivalent energy. The decay and the masses are shown below:

$$\underset{234.04359u}{{}^{234}_{90}Th} \rightarrow \underbrace{{}^{234}_{91}Pa + e^{0}_{-1}}_{234.04330u}$$

When the ${}^{234}_{90}Th$ nucleus of a thorium atom is converted into a ${}^{234}_{91}Pa$ nucleus, the number of orbital electrons remains the same, so the resulting protactinium atom is missing one orbital electron. However, the given mass includes all 91 electrons of a neutral protactinium atom. In effect, then, the value of 234.04330 u for ${}^{234}_{91}Pa$ already includes the mass of the β^- particle. The mass decrease that accompanies the β^- decay is 234.04359 u – 234.04330 u = 0.00029 u. The equivalent energy (1 u = 931.5 MeV) is 0.27 MeV. This is the maximum kinetic energy that the emitted electron can have.

Neutrino

When a β-particle is emitted by a radioactive nucleus, energy is simultaneously released. Experimenta lly, however, it is found that most β-particles do not have enough kinetic energy to account for all the energy released. If a β-particle carries away only part of the energy, where does the remainder go? In 1930, Pauli proposed that a third particle must be present to carry away the "missing" energy and to conserve momentum. Enrico Fermi later developed a complete theory of beta decay and named this particle the neutrino ("little neutral one") because it had to be electrically neutral and have little or no mass. Although it eluded detection for many years, the neutrino (ν) was finally detected experimentally in 1956. For instance, the β-decay of thorium ${}^{234}_{90}Th$ is more correctly written as

$${}^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e + \overline{\nu}$$

The bar above ν is included because the neutrino emitted in this particular decay process is an antimatter neutrino or antineutrino. A normal neutrino (ν without the bar) is emitted when β^+ decay occurs.

The neutrino has zero electrical charge and is extremely difficult to detect because it interacts very weakly with matter. The neutrino has the following properties:

1. Zero electric charge.
2. A mass much smaller than that of the electron, but probably not zero. (Recent experiments suggest that the neutrino definitely has mass, but the value is uncertain—perhaps less than 1 eV/c^2.)
3. A spin of $\frac{1}{2}$.
4. Very weak interaction with matter, making it quite difficult to detect.

Thus, with the introduction of the neutrino, we are now able to represent the beta decay process of equation in its correct form:

$${}^{14}_{6}C = {}^{14}_{7}N + e^- + \overline{\nu} \qquad \text{(i)}$$

where the bar in the symbol $\overline{\nu}$ indicates and antineutrino. To explain what an antineutrino is, let us first consider the following decay:

$$^{12}_{7}N = ^{12}_{6}C + e^{+} + \nu \quad \text{(ii)}$$

Here, we see that when ^{12}N decays into ^{12}C, a particle is produced that is identical to the electron except that it has a positive charge of +e. This particles is called a positron. Because it is like the electron in all respects except charge, the positron is said to be the antiparticle to the electron. We shall discuss antiparticles further in the chapter. For now, suffice it to say that in beta decay, an electron and an antinueutrino are emitted or a positron and a neutrino are emitted.

γ-Decay

Very often a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second decay to a lower energy state, perhaps to the ground state, by emitting one or more high-energy photons. The process is very similar to the emission of light by an atom. An atom emits radiation to release some extra energy when an electron "jumps" from a state of higher energy to a state of lower energy. Likewise, the nucleus uses essentially the same method to release any extra energy it may have following a decay or some other nuclear event. In neutrons in the nucleus s they move from a higher energy level to a lower level. The photons emitted in such a de-excitation process are called gamma rays, which have very high energy relative to the energy of visible light.

A nucleus may reach an excited state as the result of a violent collision with another particle. However, it is more common for a nucleus to be in an excited state as a result of alpha or beta decay. The following sequence of events represents a typical situation in which gamma decay occurs:

$$^{12}_{5}B = ^{12}_{6}C^{*} + e^{-} + \bar{\nu} \quad \text{(iii)}$$

$$^{12}_{6}C^{*} = ^{12}_{6}C + \gamma \quad \text{(iv)}$$

Equation (iii) represents beta decay in which ^{12}B decays to $^{12}C^{*}$ where the asterisk indicates that the carbon nucleus is left in an excited state following the decay. The excited carbon nucleus then decays to the ground state by emitting a gamma ray, as indicated by Eq. (iv). Note that gamma emission does not result in any change in either Z or A.

ILLUSTRATION 30.6 What is the wavelength of the 0.186 MeV γ-ray photon emitted by radium $^{226}_{88}Ra$?

Solution. The photon energy is the difference between two nuclear energy levels. Equation $E_i - E_f = hf$ gives the relation between the energy level separation ΔE and the frequency f of the photon as $\Delta E = hf$. Since $f\lambda = c$, the wavelength of the photon is $\lambda = hc/\Delta E$.

First, we must convert the photon energy into joule:

$$\Delta E = (0.186 \times 10^{6}\,\text{eV})\left(\frac{1.60 \times 10^{-19}}{1\,\text{eV}}\right)$$
$$= 2.98 \times 10^{-14}\,\text{J}$$

The wavelength of the photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}\,\text{J.s})(3.00 \times 10^{8}\,\text{m s}^{-1})}{2.98 \times 10^{-14}\,\text{J}}$$
$$= 6.67 \times 10^{-12}\,\text{m}$$

Measurement of Radioactivity

Radioactivity of an element is measured in terms of "activity". The activity of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms disintegrate. If N are the number of nuclei present in a radioactive sample at an instant, then activity of this sample is given as

$$A_c = -\frac{dN}{dt} \quad \text{(i)}$$

Here, dN/dt is negative as with time the number of elements always decreases due to disintegration: due to negative sign, A_c is always taken positive. The activity of a substance is measured in terms of "dps" or disintegrations per second. The SI unit of activity is named after Becquerel, defined as

1 becquerel = 1 Bq = 1 disintegration/second

Generally, activities of radioactive samples in nature are very high. That is why becquerel being a very small unit, in normal practice, more often MBq or GBq is used. For the same traditional units, Curie (Ci) and Rutherford (Ru) are also used. These are defined as

$1\ \text{Ci} = 3.7 \times 10^{10}$ disintegrations s^{-1}

$1\ \text{Ru} = 10^{6}$ disintegrations s^{-1}

Curie was originally defined as roughly the activity of 1 g of $^{226}_{88}Ra$. Similarly, it was observed that 1 kg of ordinary potassium has an activity of about 1 mCi (10^{-3} Ci) because in ordinary potassium small proportion of radioisotope $^{40}_{19}K$ is also present.

Fundamental Laws of Radioactivity

On the basis of experiments performed by Rutherford and Soddy, some conclusions were made for behavior radioactive elements and the properties of radioactivity. These conclusions are summarized as fundamental laws of radioactivity. These are discussed as follows.

1. Radioactivity is purely a nuclear process. It is not concerned in any manner with the electrons orbiting the nucleus.
2. As radioactivity is a nuclear process, it is independent from any chemical property of the element. As we have discussed that radioactive property of an element is only the process concerned with nucleus of the element, it does not affect the electronic configuration of the element. If this element takes part in a chemical reaction, the product formed will also have the radioactive property in the same fraction by which the radioactive atom is present in the molecule of product.
3. As radioactivity is a random process, its study is only possible by laws of probability. In a group of several radioactive atoms, which one will disintegrate first is just a matter of chance.
4. As radioactivity is a random process, the disintegration density throughout the volume of a radioactive element remains constant. If an element X decays to a daughter nuclide Y, then in a given volume of the element, all portions of volume will have same ratio of number of atoms of Y to that of X. Thus, homogenity is maintained.

Thus, due to randomness, the amount of disintegrations per unit volume per second (called disintegration density) remains approximately constant in the whole volume of the substance.

Radioactive Decay Law

This law relates the activity of a substance with the number of active or undecayed atoms present in a group of radioactive atoms at an instant of time. This law is stated as follows.

"The activity of a radioactive element at any instant is directly proportional to the number of undecayed active atoms (parent atoms) present at that instant."

Let us consider that at $t = 0$, there are N_0 parent atoms in a substance and after a time t, N atoms are left undecayed. This implies that in the duration from $t = 0$ to $t = t$, $N_0 - N$ atoms are decayed to their daughter element. If in further time from $t = t$ to $t = t + dt$, dN more atoms will decay then at time $t = t$, we can say that the activity of the element is given as

$$A_c = \left|\frac{dN}{dt}\right| \quad \text{(i)}$$

Now, according to radioactive decay law, we have

$$\left|\frac{dN}{dt}\right| \propto N \quad \text{or} \quad A_c = \left|\frac{dN}{dt}\right| = \lambda N \quad \text{(ii)}$$

Here, λ is the proportionality constant, called decay constant for the decay process. The value of decay constant differs for different elements. From Eq. (ii), we can see that if λ is high the element will have high value of activity and if λ is less, the activity will be relativity less. Thus, we can say that the decay constant for a radioactive element gives a relative criteria of its stability as well as rate of reaction. If the value of λ for an element is more, it is more active or relatively less stable and if for an element λ is less, it is more stable. From Eq. (ii), we can also write

$$\lambda = \frac{\left|\frac{dN}{dt}\right|}{N} \quad \text{(iii)}$$

Thus, decay constant of a process can be given as "activity per atom" (as given in Eq. (iii)). This shows that for a given radioactive element the activity per atom always remains constant whereas we have already discussed that with time the overall activity of a substance decreases with time as the number of parent elements continuously decreases with time. Now, from Eq. (iii), we can write

$$\frac{dN}{dt} = -\lambda N \quad \text{(iv)}$$

Here, negative sign shows that $\left|\frac{dN}{dt}\right|$, the rate at which the activity elements are disintegrating, is negative or number of active elements are decreasing with time. Now, we have from Eq. (iv),

$$\frac{dN}{N} = -\lambda dt$$

Decay constant is different for different radioactive substances, but it does not depend on the amount of substance and time. SI unit of λ is s^{-1}.

If $\lambda_1 > \lambda_2$, then the first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant λ

$$\begin{array}{lccc} & A & \xrightarrow{\lambda} & B \\ t=0 & N_0 & & 0 \end{array}$$

where N_0 = number of active nuclei of A at $t = 0$.

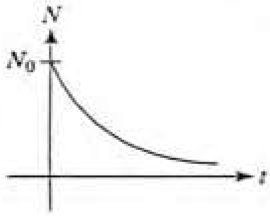
$$\begin{array}{lccc} t=t & N & & N' \end{array}$$

where N = Number of active nuclei of A at $t = t$.

Integrating the above expression within time limits from $t = 0$ to $t = t$, we have

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda\, dt \quad \text{or} \quad [\ln N]_{N_0}^{N} = -\lambda t$$

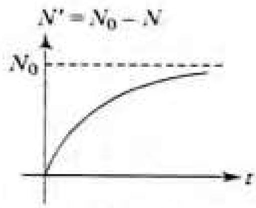
$$\text{or} \quad \ln\left(\frac{N}{N_0}\right) = -\lambda t \quad \text{or} \quad N = N_0 e^{-\lambda t} \quad \text{(v)}$$



Equation (v) gives the number of active parent atoms N present at time t in the mixture. This equation is called radioactive decay equation. From Eq. (v), we can have

$$\lambda N = \lambda N_0 e^{-\lambda t}$$

$$\text{or} \quad A_c = A_{c_0} e^{-\lambda t} \quad \text{(vi)}$$



Here, $A_{c_0} = \lambda N_0$ is the initial activity of the substance at $t = 0$. Equation (vi) is another form of radioactive decay equation. This equation can be used to find the activity of a radioactive substance at any instant of time.

Number of nuclei decayed (i.e., the number of nuclei of B formed),

$$N' = N_0 - N = N_0 - N_0 e^{-\lambda t}$$

$$\text{or} \quad N' = N_0(1 - e^{-\lambda t})$$

Activity

The activity of a radioactive sample is the number of disintegrations per second that occur. Each time a disintegration occurs, the number N of radioactive nuclei decreases. As a result, the activity can be obtained by dividing ΔN (the change in the number of nuclei) by Δt (the time interval during which the change take place); the average activity over the time interval Δt is the magnitude of $\Delta N/\Delta t$. Since the decay of any individual nucleus is completely random, the number of disintegrations per second that occur in a sample is proportional to the number of radioactive nuclei present, so that

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

where λ is a proportionality constant referred to as the decay constant. The minus sign is present in this equation because each disintegration decreases the number N of nuclei originally present.

The SI unit for activity is the becquerel (Bq); one becquerel equals one disintegration per second. Activity is also measured in terms of a unit called the curie (Ci), in honour of Marie Curie (1867–1934) and Pierre Curie (1859–1906), the discoverers of radium and polonium. Historically, the curie was chosen as a unit because it is roughly the activity of one gram of pure radium. In terms of becquerels,

$$1\ \text{Ci} = 3.70 \times 10^{10}\ \text{Bq}$$

The activity of the radium put into the dial of a watch to make it glow in the dark is about 4×10^4 Bq, and the activity used in radiation therapy for cancer treatment is approximately a billion times greater, or 4×10^{13} Bq.

Half-Life Time

In the previous section, we have discussed that during decay of a radioactive sample, the amount of radionuclide fall off exponentially with time. Every radioactive sample has a characteristic half-life. Half-life time is defined as the time duration in which half of the total number of nuclei will decay or be left undecayed. Say, for example, at any instant if we look into the quantity of a sample of radioactive element, it is observed that after every 3 hour some half-lives are only a millionth of a second for highly active elements and some less active elements have half-life in billions of years.

In some nuclear power plants, a major problem is disposal of the radioactive wastes since some of the nuclide present in waste have long half-lives.

For a radioactive element in a sample if at $t = 0$, N_0 nuclei are present of active parent element and during observation after $t = T$, $N_0/2$ are left, then this duration T can be taken as half-life of this element. From radioactive decay equation, we have

$$N = N_0 e^{-\lambda t} \qquad \text{(i)}$$

Here, at $t = T$, $N = N_0/2$. Thus, we have in Eq. (i)

$$\frac{N_0}{2} = N_0 e^{-\lambda t} \quad \text{or} \quad \ln\left(\frac{1}{2}\right) = -\lambda T \quad \text{or} \quad \ln(2) = \lambda T$$

or $$T = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} \qquad \text{(ii)}$$

Number of nuclei present after n half-lives, i.e., after a time $t = n\, t_{1/2}$,

$$N = N_0 e^{-\lambda t} = N_0 e^{-\lambda n t_{1/2}} = N_0\, e^{-\lambda n \frac{\ln 2}{\lambda}}$$

$$= N_0 e^{\ln 2^{(-n)}} = N_0\,(2)^{-n} = N_0 (1/2)^n = \frac{N_0}{2^n}$$

$$\left[n = \frac{t}{t_{1/2}}\text{. It may be a fraction, need not to be an integer.}\right]$$

or $$N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0\left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0\left(\frac{1}{2}\right)^3 \cdots \xrightarrow{n} N_0\left(\frac{1}{2}\right)^n$$

ILLUSTRATION 30.7 A radioactive sample has 6.0×10^{18} active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

Solution. In one half-life, the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$6.0 \times 10^{18} \times \left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right) = 1.5 \times 10^{18}$$

ILLUSTRATION 30.8 The half-life of a radioactive nuclide is 20 h. What fraction of original activity will remain after 40 h?

Solution. 40 h means 2 half-lives. Thus,

$$A = \frac{A_0}{2^2} = \frac{A_0}{4} \quad \text{or} \quad \frac{A}{A_0} = \frac{1}{4}$$

So, one-fourth of the original activity will remain after 40 h.

Specific activity: The activity per unit mass is called specific activity.

Average Life

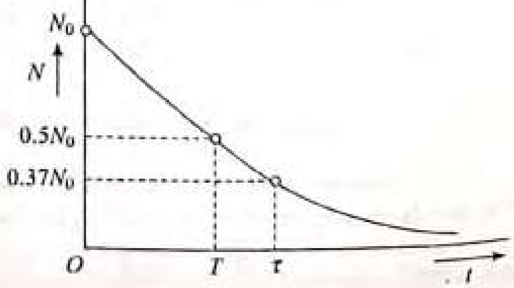
$$T_{av} = \frac{\text{Sum of ages of all the nuclei}}{N_0} = \frac{\int_0^\infty N_0 e^{-\lambda t}\, dt}{N_0} = \frac{1}{\lambda}$$

Important Points

- Time constant or average life is more than the half-life, i.e.,

  $$\tau = T_{av} > T$$

- The fractional radioactive decay in one mean life (or time constant) is more than that in half life, as shown in figure.



- The number of undecayed nuclei present after n mean life is

$$N = (0.37)^n N_0 = \left(\frac{1}{e}\right)^n N_0$$

- If a nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2, then the effective decay constant of the nuclide is

$$\lambda = \lambda_1 + \lambda_2 ;\ N = N_0 e^{-(\lambda_1 + \lambda_2)t}$$

ILLUSTRATION 30.9 A 50.0 g sample of carbon is taken from the pelvis bone of a skeleton and is found to have a ^{14}C decay rate of 200.0 decays min^{-1}. It is known that carbon from a living organism has a decay rate of decays/min^{-1} g^{-1} and that ^{14}C has a half-life of 5730 year = 3.01×10^9 min. Find the age of the skeleton.

Solution. Let us start with the equation $N = N_0 e^{-\lambda t}$ and multiply both sides by λ to get $\lambda N = \lambda N_0 e^{-\lambda t}$ which is equivalent to

$$R = R_0 e^{-\lambda t} \quad \text{or} \quad \frac{R}{R_0} = e^{-\lambda t}$$

where R is the present activity and R_0 was the activity when the skeleton was a part of a living organism. We can solve for the time by taking the natural log of both sides of this equation.

$$\ln\left(\frac{R}{R_0}\right) = \ln(e^{-\lambda t}) = -\lambda t$$

$$t = -\frac{\ln\left(\frac{R}{R_0}\right)}{\lambda}$$

Because we are given the decay rate and mass of the sample, we can find R_0 as

$$R_0 = \left(15.0 \frac{\text{decays}}{\text{min g}}\right)(50.0\ \text{g}) = 750 \frac{\text{decays}}{\text{min}}$$

The decay constant is found as

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.01 \times 10^9\ \text{min}} = 2.30 \times 10^{-10}\ \text{min}^{-1}$$

NUCLEAR FISSION

Nuclear fission occurs when a heavy nucleus, such as ^{235}U, splits, or fissions, into two smaller nuclei. In such a reaction, the total mass of the products is less than the original mass of the heavy nucleus.

Nuclear fission was first observed in 1939 by Otto Hahn and Fritz Strassman, following some basic studies by Fermi. After bombarding uranium ($Z = 92$) with neutrons, Hahn and Strassman discovered among the reaction products two medium-mass elements, barium and lanthanum. Shortly thereafter, Lisa Meithner and Otto Frisch explained what had happened. The uranium nucleus had split into two nearly equal fragments after absorbing a neutron. Such an occurrence was of considerable interest to physicists attempting to understand the nucleus. But it was to have even more far-reaching consequences. Measurements showed that about 200 MeV of energy is released in each fission event, and this fact was to affect the course of human history.

The fission of ^{235}U by slow (low-energy) neutrons can be represented by the equation

$$_0^1 + {}_{92}^{235}\text{U} \rightarrow {}_{92}^{236}\text{U}^* \rightarrow X + Y + \text{neutrons} \qquad \text{(i)}$$

where ^{236}U* is an intermediate state that lasts only for about 10^{-12} s before splitting into X and Y. The resulting nuclei, X and Y, are called fission fragments. Many combinations of X and Y satisfy the requirements of conservation of energy and charge. In the fission of uranium, there are about 90 different daughter nuclei that can be formed. The process also results in the production of several (typically two or three) neutrons per fission event. On the average, 2.47 neutrons are released per event.

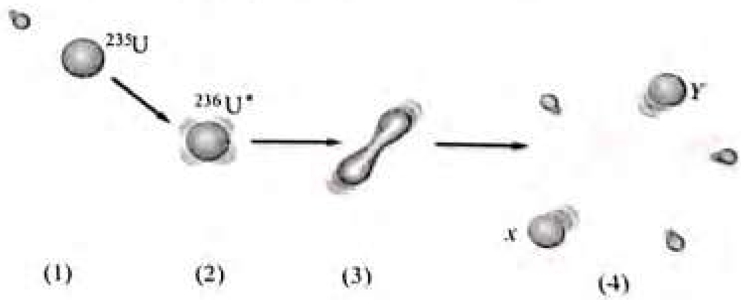
A typical reaction of this type is

$$_0^1 n + {}_{92}^{235}\text{U} \rightarrow {}_{56}^{141}\text{Ba} + {}_{36}^{92}\text{Kr} + 3\,{}_0^1 n \qquad \text{(ii)}$$

The fission fragments, barium and krypton, and the released neutrons have a great deal of kinetic energy following the fission event.

The breakup of the uranium nucleus can be equated to the case of a drop of water when excess energy is added to it. All of the atoms in the drop have energy, but not enough to break up the drop. However, if enough energy is added to set the drop vibrating, it will undergo elongation and compression until the amplitude of vibration becomes large enough to cause the drop to break apart. In the uranium nucleus, a similar process occurs (figure). The sequence of events is as follows:

1. ^{235}U nucleus captures a thermal (slow-moving) neutron.
2. This capture results in the formation of ^{236}U*, and the excess energy of this nucleus causes it to undergo violent oscillations.
3. The ^{236}U* nucleus becomes highly elongated, and the force of repulsion between protons in the two halves of the dumbbell shape tends to increase the distortion.
4. The nucleus splits into two fragments, emitting several neutrons in the process.



Let us estimate the disintegration energy, Q, released in a typical fission process. From figure, we see that the binding energy per nucleon is about 7.2 MeV for heavy nuclei (those having a mass number of approximately 240) and about 8.2 MeV for nuclei of intermediate mass. This means that the nucleons in the fission fragments are more tightly bound and, therefore, have less mass than the nucleons in the original heavy nucleus. This decrease in mass per nucleon appears as released energy when fission occurs. The amount of energy released is (8.2–7.2) MeV per nucleon. Assuming a total of 240 nucleons, we find that the energy released per fission event is

$$Q = (240\,\text{nucleons})(8.2\,\text{MeV/nucleon} - 7.2\,\text{MeV/nucleon}) = 240\text{ MeV}$$

This is a very large amount of energy relative to the amount released in chemical processes.

In nuclear fission, heavy nuclei of A above 200 break up into two or more fragments of comparable masses. The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use ${}_{92}U^{236}$ as the fission material. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy ≈ 0.04 eV, also called thermal neutrons). A ${}_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming ${}_{92}U^{236}$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$${}_{92}U^{236} \rightarrow {}_{53}I^{137} + {}_{39}Y^{97} + 2\,{}_{0}n^{1}$$

$${}_{92}U^{236} \rightarrow {}_{53}I^{140} + {}_{36}Y^{94} + 2\,{}_{0}n^{1}$$

and a number of other combinations.

1. On an average, 2.5 neutrons are emitted in each fission event.
2. Mass lost per reaction ≈ 0.2 amu
3. In nuclear fission, the total BE increases and excess energy is released.
4. In each fission event, about 200 MeV of energy is released, a large part of which appears in the form of kinetic energies of the two fragments. Neutrons take away about 5 MeV. For example,

$${}^{235}_{92}U + {}_{0}n^{1} \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}_{0}n^{1} + \text{energy}$$

$$Q\text{ value} = [(M_U - 92m_e + m_n) - \{(M_{Ba} - 56\,m_e) + (M_{Kr} - 36m_e) + 3m_n\}]c^2$$
$$= [(M_U + m_n) - (M_{Ba} + M_{Kr} + 3m_n)]c^2$$

5. A very important and interesting feature of neutron-induced fission is the chain reaction. For working of nuclear reactor, refer your text book.

ILLUSTRATION 30.10 Calculate the total energy released if 1.0 kg of ^{235}U undergoes fission, taking the disintegration energy per event to be Q = 208 MeV (a more accurate value than the estimate given previously).

Solution. We need to know the number of nuclei in 1.00 kg of uranium. Because A = 235, the number of nuclei is

$$N = \left(\frac{6.02 \times 10^{23}\text{ nuclei mol}^{-1}}{235\text{ g mol}^{-1}}\right)(1.00 \times 10^3\text{ g})$$
$$= 2.56 \times 10^{24}\text{ nuclei}$$

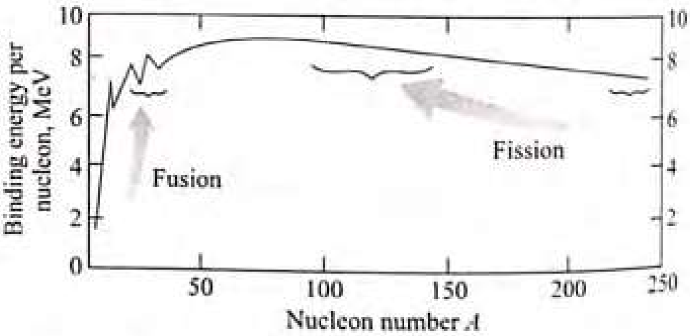
Hence, the disintegration energy is

$$E = nQ = (2.56 \times 10^{24}\text{ nuclei})\left(208\,\frac{\text{MeV}}{\text{nucleus}}\right)$$
$$= 5.32 \times 10^{26}\text{ MeV}$$

Because 1 MeV is equivalent to 4.45×10^{-20} kWh, $E = 2.3.7 \times 10^7$ kWh. This is enough energy to keep a 100 W light bulb burning for about 30000 years.

NUCLEAR FUSION

Figure shows that the binding energy for lighter nuclei (those having a mass number lower than 20) is much smaller than the binding energy for heavier nuclei. This suggests a possible process that is the reverse of fission. When two light nuclei combine to form a heavier nucleus, the process is called nuclear fussion. Because the mass of the final nucleus is less than the masses of the original nuclei, there is a loss of mass accompanied by a release of energy. Although fusion power plants have not yet been developed, a worldwide effort is under way to harness the energy from fusion reactions in the laboratory. Later, we shall discuss the possibilities and advantages of this process for generating electric power.



Fusion in the Sun

All stars generate their energy through fusion process. About 90% of the stars, including the Sun, fuse hydrogen, whereas some older stars fuse helium or other heavier elements. Stars are born in regions of space containing vast clouds of dust and gas. Recent mathematical models of these clouds indicate that star formation is triggered by shock waves passing through a cloud. These shock waves are similar to sonic booms and are produced by events such as the explosion of a nearby star, called a supernova explosion. The shock wave compresses certain regions of the cloud, causing these regions to collapse under their own gravity. As the gas falls inward toward the center, the atoms gain speed, which causes the temperature

of the gas to rise. Two conditions must be met before fusion reactions in the star can sustain its energy needs.

1. The temperature must be high enough (about 10^7 K for hydrogen) to allow the kinetic energy of the positively charged hydrogen nuclei to overcome their mutual Coulomb repulsion as they collide.
2. The density of nuclei must be high enough to ensure a high rate of collision.

When fusion reactions occur at the core of a star, the energy liberated eventually becomes sufficient to prevent further collapse of the star under its own gravity. The star then continues to live out the remainder of its life under a balance between the inward force of gravity pulling it toward collapse and the outward force due to thermal effects and radiation pressure.

The proton–proton cycle is a series of three nuclear reactions that are believed to be the stages in the liberation of energy in the Sun and other stars rich in hydrogen. An overall view of the proton–proton cycle is that four protons combine to form an alpha particle and two positrons, with the release of 25 MeV of energy in the process.

The specific steps in the proton–proton cycle are

$$ {}^1_1\text{H} + {}^1_1\text{H} \to {}^2_1\text{H} + e^+ + \nu $$

$$ {}^1_1\text{H} + {}^2_1\text{H} \to {}^3_2\text{He} + \gamma $$

This second reaction is followed by either hydrogen–helium fusion or helium–helium fusion:

$$ {}^1_1\text{H} + {}^3_2\text{He} \to {}^4_2\text{He} + e^+ + \nu $$

or

$$ {}^3_2\text{He} + {}^3_2\text{He} \to {}^4_2\text{He} + {}^1_1\text{H} + {}^1_1\text{H} $$

The energy liberated is carried primarily by gamma rays, positrons, and neutrinos, as can be seen from the reactions. The gamma rays are soon absorbed by the dense gas, thus, raising its temperature. The positrons combine with electrons to produce gamma rays, which in turn are also absorbed by the gas within a few centimeters. The neutrinos, however, almost never interact with matter; hence, they escape from the star, carrying about 2% of the generated energy with them. These energy-liberating fusion reactions are called thermonuclear fusion reactions. The hydrogen (fusion) bomb, first exploded in 1952, is an example of an uncontrolled thermonuclear fusion reaction.

NOTE:

- Some unstable light nuclei of A below 20, fuse together, the BE per nucleon increases and hence the excess energy is released. The easiest thermonuclear reaction that can be handled on earth is the fusion of two deuterons (D–D reaction) or fusion of a deuteron with a triton (D–T reaction).

$$ {}_1\text{H}^2 + {}_1\text{H}^2 \to {}_2\text{He}^3 + n + 3.3\ \text{MeV (D–D)} $$

$$ Q\text{-value} = [2(M_\text{D} - m_e) - \{(M_{\text{He-3}} - 2m_e) + m_n\}]c^2 $$

$$ = [2M_\text{D} - (M_{\text{He-3}} + m_n)]c^2 $$

$$ {}_1\text{H}^2 + {}_1\text{H}^2 \to {}_1\text{H}^3 + {}_1\text{H}^1 + 4.0\ \text{MeV (D–D)} $$

$$ Q\text{-value} = [2(M_\text{D} - m_e) - \{(M_\text{T} - m_e) + (M_\text{H} - m_e)\}]c^2 $$

$$ = [2M_\text{D} - (M_\text{T} + M_\text{H})]c^2 $$

$$ {}_1\text{H}^2 + {}_1\text{H}^3 \to {}_2\text{He}^4 + n + 17.6\ \text{MeV (D–T)} $$

$$ Q\text{-value} = [\{(M_\text{D} - m_e) + (M_\text{T} - m_e)\} - \{(M_{\text{He-4}} - 2m_e) + m_n\}]c^2 $$

$$ = [(M_\text{D} + M_\text{T}) - (M_{\text{He-4}} + m_n)]c^2 $$

In case of fission and fusion, $\Delta m = \Delta m_{atom} = \Delta m_{nuclear}$

- These reactions take place at ultra-high temperature ($\approx 10^7 - 10^9$). At high pressure, it can take place at low temperature also. For these reactions to take place, nuclei should be brought up to 1 fermi distance which requires very high kinetic energy.
- Energy released in fusion exceeds the energy liberated in the fission of heavy nuclei.

Fusion Reactors

The enormous amount of energy released in fusion reactions suggests the possibility of harnessing this energy for useful purposes on earth. A great deal of effort is focused on developing a sustained and controllable thermonuclear reactor—a fusion power reactor. Controlled fusion is often called the ultimate energy source because of the vast availability of its fuel source: water. An additional advantage of fusion reactors is that comparatively few radioactive by products are formed. As noted in Eq. (iii), the end product of the fusion of hydrogen nuclei is safe, nonradioactive helium. Unfortunately, a thermonuclear reactor that can deliver a net power output over a reasonable time interval is not yet a reality, and many difficulties must be solved before a successful device is constructed.

We have seen that the Sun's energy is based, in part, on a set of reactions in which ordinary hydrogen is converted to helium. Unfortunately, the proton–proton interaction is not suitable for use in a fusion reactor because the event requires very high pressures and densities. The process works in the Sun only because of the extremely high density of protons in the Sun's interior. In fact, even at the densities and temperature that exist at the center of the Sun, the average proton takes 14 billion years to react!

The fusion reactions that appear most promising in the construction of a fusion power reactor involve deuterium and tritium, which are isotopes of hydrogen. These reactions are

$$ {}^2_1\text{H} + {}^2_1\text{H} \to {}^3_2\text{He} + {}^1_0n, \qquad Q = 3.27\ \text{MeV} $$

$$ {}^2_1\text{H} + {}^2_1\text{H} \to {}^3_1\text{H} + {}^1_1\text{H}, \qquad Q = 4.03\ \text{MeV} $$

$$ {}^2_1\text{H} + {}^3_1\text{H} \to {}^4_2\text{He} + {}^1_0n, \qquad Q = 17.59\ \text{MeV} $$

where the Q values refer to the amount of energy released per reaction. As noted earlier, deuterium is available in almost unlimited quantities form our lakes and oceans and is very inexpensive to extract. Tritium, however, is radioactive ($T_{1/2}$ = 12.3 yr) and undergoes beta decay to ^{3}He. For this reason, tritium does not occur naturally to any great extent and must be artificially produced.

One of the major problems in obtaining energy from nuclear fusion is the fact that the Coulomb repulsion force

between two charged nuclei must be overcome before they can fuse. The fundamental challenge is to give the two nuclei enough kinetic energy to overcome this repulsive force. This can be accomplished by heating the fuel to extremely high temperatures (about 10^8 K, far greater than the interior temperature of the Sun). As you might expect, such high temperatures are not easy to obtain in a laboratory or a power plant. At these high temperatures, the atoms are ionized and the system consists of a collection of electrons and nuclei, commonly referred to as a plasma.

ILLUSTRATION 30.11 On disintegration of one atom of ^{235}U, the amount of energy obtained is 200 MeV. The power obtained in a reactor is 1000 kilowatt. How many atoms are disintegrated per second in the reactor? What is the decay in mass per hour?

Solution. Power produced in the reactor is

$$P = 1000 \text{ kW}$$
$$= 1000 \times 10^3 \text{ W}$$
$$= 10^6 \text{ J s}^{-1}$$
$$= \frac{10^6}{1.6 \times 10^{-19}} \text{eV s}^{-1}$$
$$= 6.25 \times 10^{18} \text{ MeV s}^{-1}$$

As in each disintegration 200 MeV energy is released, number of atoms disintegrated per second are

$$N = \frac{6.25 \times 10^{18}}{200} = 3.125 \times 10^{16} \text{ s}^{-1}$$

Energy released per second is 10^6 J.
Energy released per hour $= 10^6 \times 60 \times 60$ J
Thus, mass decay per hour can be given as

$$\Delta m = \frac{\Delta E}{c^2} \qquad \text{(Einstein's mass–energy formula)}$$
$$= \frac{10^6 \times 60 \times 60 \text{ J}}{(3 \times 10^8 \text{ m s}^{-1})^2}$$
$$= 4 \times 10^{-8} \text{ kg} = 4 \times 10^{-5} \text{ g}$$

ILLUSTRATION 30.12 What is the power output of a $_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel, and if each fission gives 185 MeV of usable energy?

Solution. 235 amu of uranium gives 185 MeV energy. Therefore, the energy given by 1 amu of $_{92}U^{235}$ is

$$\frac{185}{2.35} \text{MeV} = \frac{185}{235} \times 1.6 \times 10^{-13} \text{ J}$$

But 1 amu $= 1.66 \times 10^{-27}$ kg. Therefore, energy released by 1.66×10^{-27} kg of $_{92}U^{235}$ is $\frac{185 \times 1.6 \times 10^{-13}}{235}$ J

Hence, energy released by 2 kg of $_{92}U^{235}$,

$$W = \frac{185 \times 1.6 \times 10^{-13} \times 2}{235 \times 1.66 \times 10^{-27}} = 1.517 \times 10^{14} \text{ J}$$

Therefore, power output of reactor $= \frac{W}{t} = \frac{1.517 \times 10^{14} \text{ J}}{(30 \text{ days})}$

$$= \frac{1.517 \times 10^{14} \text{ J}}{30 \times 24 \times 60 \times 60 \text{ s}} = 5.85 \times 10^7 \text{ W}$$

CONCEPT APPLICATION EXERCISE 30.2

1. A radioactive sample has mass m, decay constant λ, and molecular weight M. If the Avogadro number is N_A, then
   (a) find the initial number of nuclei present;
   (b) find the number of decayed nuclei after a time t;
   (c) find the activity of the sample after a time t;
2. Calculate the time taken to decay 100 percent of a radioactive sample in terms of
   (a) half-life T and
   (b) mean-life T_{av}
3. The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 ($t_2 > t_1$). Obtain an expression for its mean-life.
4. An α-particle strikes an aluminium atom $^{27}_{13}Al$. As a result, an unknown nucleus $^{A}_{Z}X$ and a neutron are formed.
   $$^{2}_{4}He + ^{27}_{13}Al \rightarrow ^{A}_{Z}X + ^{1}_{0}n$$
   Identify the nucleus produced, indicating its atomic number Z (the number of protons) and its mass number A (the number of nucleons).
5. A neutron is observed to strike an $^{16}_{8}O$ nucleus and a deuteron is given off. What is the nucleus that result?
6. A $^{238}_{92}U$ undergoes alpha decay. What is the resulting daughter nucleus?

SOLVED EXAMPLES

1. At any instant the ratio of the amount of radioactive substances is 2 : 1. If their half lives be respectively 12 and 16 hours, then after two days, what will be the ratio of the substances
   (a) 1 : 1 (b) 2 : 1
   (c) 1 : 2 (d) 1 : 4

Sol. (a) Number of half lives in two days four substance 1 and 2 respectively are $n_1 = \frac{2 \times 24}{12} = 4$ and $n_2 = \frac{2 \times 24}{1.6} = 3$

By using $N = N_0 \left(\frac{1}{2}\right)^n$

$$\Rightarrow \frac{N_1}{N_2} = \frac{(N_0)_1}{(N_0)_2} \times \frac{\left(\frac{1}{2}\right)^{n_1}}{\left(\frac{1}{2}\right)^{n_2}} = \frac{2}{1} \times \frac{\left(\frac{1}{2}\right)^4}{\left(\frac{1}{2}\right)^3} = \frac{1}{1}$$

2. A radioactive sample has half-life of 5 years. Probability of decay in 10 years will be
(a) 100% (b) 75%
(c) 50% (d) 25%

Sol. (b) Number of half lives $n = \frac{10}{5} = 2$, now $\frac{N}{N_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Fraction decayed $= 1 - \frac{N}{N_0} = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow$ In percentage $= \frac{3}{4} \times 100 = 75\%$

3. During mean life of a radioactive element, the fraction that disintegrates is
(a) e (b) $\frac{1}{e}$
(c) $\frac{e-1}{e}$ (d) $\frac{e}{e-1}$

Sol. (c) By using $N = N_0 e^{-\lambda t}$ and average life time $t = \frac{1}{\lambda}$

So $N = N_0 e^{-\lambda \times 1/\lambda} = N_0 e^{-1} \Rightarrow \frac{N}{N_0} = e^{-1} = \frac{1}{e}$

Now disintegrated fraction $= 1 - \frac{N}{N_0} = 1 - \frac{1}{e} = \frac{e-1}{e}$

4. The half-life of a sample of a radioactive substance is 1 hour. If 8×10^{10} atoms are present at $t = 0$, then the number of atoms decayed in the duration $t = 2$ hour to $t = 4$ hour will be
(a) 2×10^{10} (b) 1.5×10^{10}
(c) Zero (d) Infinity

Sol. (b) $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$

No. of atoms at $t = 2$ hr, $N_1 = 8 \times 10^{10} \left(\frac{1}{2}\right)^{\frac{2}{1}} = 2 \times 10^{10}$

No. of atoms at $t = 4$ hr, $N_2 = 8 \times 10^{10} \left(\frac{1}{2}\right)^{\frac{4}{1}} = \frac{1}{2} \times 10^{10}$

$\therefore$ No. of atoms decayed in given duration

$= \left(2 - \frac{1}{2}\right) \times 10^{10} = 1.5 \times 10^{10}$

5. Activity of radioactive element decreased to one third of original activity R_0 in 9 years. After further 9 years, its activity will be
(a) R_0 (b) $\frac{2}{3} R_0$
(c) $R_0/9$ (d) $R_0/6$

Sol. (c) Activity $R = R_0 e^{-\lambda t}$

$$\frac{R_0}{3} = R_0 e^{-\lambda \times 9} \Rightarrow e^{-9\lambda} = \frac{1}{3} \quad \text{...(i)}$$

After further 9 years $R' = R e^{-\lambda t} = \frac{R_0}{3} \times e^{-\lambda \times 9}$ ...(ii)

From equation (i) and (ii) $R' = \frac{R_0}{9}$.

6. A star initially has 10^{40} deuterons. It produces energy via the processes

${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + p$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$

The masses of the nuclei are as follows :

$M(H^2) = 2.014$ amu; $M(p) = 1.007$ amu;

$M(n) = 1.008$ amu; $M(He^4) = 4.001$ amu

If the average power radiated by the star is 10^{16}W, the deuteron supply of the star is exhausted in a time of the order of
(a) 10^6 s (b) 10^8 s
(c) 10^{12} s (d) 10^{16} s

Sol. (c) Mass defect $= 3 \times 2.014 - 4.001 - 1.007 - 1.008$

$= 0.026$ amu $= 0.026 \times 931 \times 10^6 \times 1.6 \times 10^{-19}$ J

$= 3.82 \times 10^{-12}$ J

Power of star $= 10^{16}$W

Number of deuterons used $= \frac{10^{16}}{\Delta M} = 0.26 \times 10^{28}$

Deuteron supply exhausts in $\frac{10^{40}}{0.26 \times 10^{28}} = 10^{12}$ s.

7. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time
(a) $1/10\lambda$ (b) $1/11\lambda$
(c) $11/10\lambda$ (d) $1/9\lambda$

Sol. (d) $N_1 = N_0 e^{-10\lambda t}$ and $N_2 = N_0 e^{-\lambda t}$

$$\Rightarrow \frac{N_1}{N_2} = \frac{1}{e} = e^{-1} = e^{(-10\lambda + \lambda)t} = e^{-9\lambda t} \Rightarrow t = \frac{1}{9\lambda}.$$

8. Half life of a radio-active substance is 20 minutes. The time between 20% and 80% decay will be
(a) 20 minutes (b) 40 minutes
(c) 30 minutes (d) 25 minutes

Sol. (b) Here $T_{1/2} = 20$ minutes; we know $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$

For 20% decay $\frac{N}{N_0} = \frac{80}{100} = \left(\frac{1}{2}\right)^{t_1/20}$ ... (i)

For 80% decay $\frac{N}{N_0} = \frac{20}{100} = \left(\frac{1}{2}\right)^{t_2/20}$ ... (ii)

Dividing (ii) by (i)

$\frac{1}{4}=\left(\frac{1}{2}\right)^{\frac{(t_2-t_1)}{20}}$: On solving we get $t_2-t_1=40$ min.

9. After 280 days, the activity of a radioactive sample is 6000 dps. The activity reduces to 3000 dps after another 140 days. The initial activity of the sample in dps is

(a) 6000 (b) 9000
(c) 3000 (d) 24000

Sol. (d) Here the activity of the radioactive sample reduces to half in 140 days. Therefore, the half life of the sample is 140 days. 280 days is it's two half lives. So before two half lives it's activity was (2^2 × present activity).

$\therefore$ Initial activity $= 2^2 \times 6000 = 24000$ dps

10. A radioactive sample of U^{238} decay to Pb through a process for which half life is 4.5×10^9 years. The ratio of number of nuclei of Pb to U^{238} after a time of 1.5×10^9 years (given $2^{1/3} = 1.26$)

(a) 0.12 (b) 0.26
(c) 1.2 (d) 0.37

Sol. (b) Here $\frac{N}{N_0}=\left(\frac{1}{2}\right)^n=\left(\frac{1}{2}\right)^{1/3}$

where n = Number of half lives $=\frac{1}{3}$

$$\Rightarrow \frac{N}{N_0}=\frac{1}{1.26} \Rightarrow \frac{N_U}{N_{Pb}+N_U}=\frac{1}{1.26}$$

$$\Rightarrow N_{Pb}=0.26N_U \Rightarrow \frac{N_{Pb}}{N_U}=0.26$$

11. The ratio of radii of nuclei ${}_{13}Al^{27}$ and ${}_{52}X^A$ is 3 : 5. The number of neutrons in the nuclei of X will be

(a) 52 (b) 73
(c) 125 (d) 13

Sol. (b) $r \propto A^{1/3} \Rightarrow \frac{r_1}{r_2}=\left(\frac{A_1}{A_2}\right)^{1/3}$

$$\Rightarrow \frac{3}{5}=\left(\frac{27}{A}\right)^{1/3} \Rightarrow \frac{27}{125}=\frac{27}{A} \Rightarrow A=125$$

Number of nuclei in atom X = $A - 52 = 125 - 52 = 73$.

12. A nucleus ${}_ZX^A$ emits 9α-particles and 5β particle. The ratio of total protons and neutrons in the final nucleus is

(a) $\frac{Z-13}{(A-Z-23)}$ (b) $\frac{(Z-18)}{(A-36)}$
(c) $\frac{(Z-13)}{(A-36)}$ (d) $\frac{(Z-13)}{(A-Z-13)}$

Sol. (a) ${}_ZX^A \xrightarrow{9\alpha} {}_{Z-18}X^{A-36} \xrightarrow{5\beta} {}_{Z-13}X^{A-36}$

Number of protons, $P = (Z = 13)$

Number of neutrons, $N = (A-36)-(Z-13) = (A-Z-23)$

$$\therefore \frac{P}{N}=\frac{(Z-13)}{(A-Z-23)}$$

13. Which sample contains greater number of nuclei: a 5.00-μCi sample of ${}^{240}Pu$ (half-life 6560y) or a 4.45-μCi sample of ${}^{243}Am$ (half-life 7370 y)

(a) ${}^{240}Pu$ (b) ${}^{243}Am$
(c) Equal in both (d) None of these

Sol. (c) The activity $\left(-\frac{dN}{dt}\right)=\lambda N \Rightarrow N=\left(-\frac{dN}{dt}\right)\left(\frac{T_{1/2}}{\log_e 2}\right)$

Taking the ratio of this expression for ${}^{240}Pu$ to this same expression for ${}^{243}Am$

$$\frac{N_{Pu}}{N_{Am}}=\frac{\left(-\frac{dN_{Pu}}{dt}\right)(T_{1/2})_{Pu}}{\left(-\frac{dN_{Am}}{dt}\right)(T_{1/2})_{Am}}=\frac{(5\mu Ci)\times(6560y)}{(4.45\mu Ci)\times(7370y)}=1$$

i.e. the two samples contains equal number of nuclei.

14. The fission of ${}^{235}U$ can be triggered by the absorption of a slow neutrons by a nucleus. Similarly a slow proton can also be used. This statement is

(a) Correct
(b) Wrong
(c) Information is insufficient
(d) None of these

Sol. (b) Because the neutron has no electric charge, it experience no electric repulsion from a U^{235} nucleus. Hence a slow moving neutron can approach and enter a U^{235} nucleus, thereby providing the excitation needed to trigger fission. By contrast a slow moving proton feels a strong repulsion from a U^{235} nucleus. It never gets close to the nucleus, so it cannot trigger fission.

15. The radioactivity of a given sample of whisky due to tritium (half life 12.3 years) was found to be only 3% of that measured in a recently purchased bottle marked "7 years old". The sample must have been prepared about

(a) 220 years back (b) 300 years back
(c) 400 years back (d) 70 years back

Sol. (d) After one half life period, the activity of Tritium becomes 50%.

After 2 half life period 25%

After 3 half life period 12.5%

After 4 half life period 6.25%

After 5 half life period 3.12% × 3%

It is 5 × 12.5 years + 7 years, i.e., approximately 70 years only.

EXERCISES

Nuclear Reaction, Mass Energy and Binding Energy

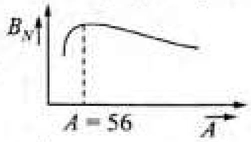
1. In the nuclear reaction ${}_{92}U^{238} \rightarrow {}_Z Th^A + {}_2He^4$, the values of A and Z are
(a) $A = 234, Z = 94$ (b) $A = 234, Z = 90$
(c) $A = 238, Z = 94$ (d) $A = 238, Z = 90$

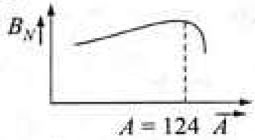
2. If m, m_n and m_p are masses of ${}_ZX^A$ nucleus, neutron and proton respectively
(a) $m = (A - Z) m_n + Zm_p$ (b) $m < (A - Z) m_n + Zm_p$
(c) $m > (A - Z) m_n + Zm_p$ (d) $m = (A - Z) m_p + Zm_n$.

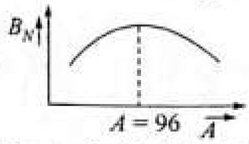
3. If M is the atomic mass and A is the mass number, packing fraction is given by
(a) $\frac{A}{M-A}$ (b) $\frac{A-M}{A}$
(c) $\frac{M}{M-A}$ (d) $\frac{M-A}{A}$

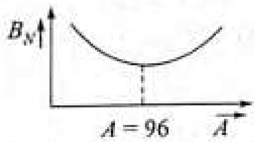
4. Binding energy of a nucleus is
(a) energy given to its nucleus during its formation
(b) total mass of nucleus converted to energy units
(c) loss of energy from the nucleus during its formation
(d) total K.E. and P.E. of the nucleons in the nucleus

5. The dependence of binding energy per nucleon, B_N on the mass number, A, is represented by

(a) 

(b) 

(c) 

(d) 

6. The energy in MeV is released due to transformation of 1 kg mass completely into energy ($c = 3 \times 10^8$ m/s)
(a) 7.625×10 MeV (b) 10.5×10^{29} MeV
(c) 2.8×10^{-28} MeV (d) 5.625×10^{29} MeV

7. The binding energies per nucleon of deuteron (${}_1H^2$) and helium atom (${}_2He^4$) are 1.1 MeV and 7 MeV. If two deuteron atoms react to form a single helium atom, then the energy released is
(a) 13.9 MeV (b) 26.9 MeV
(c) 23.9 MeV (d) 19.2 MeV

8. Mark out the incorrect statement.
(a) A free neutron can transform itself into photon.
(b) A free proton can transform itself into neutron.
(c) In beta minus decay, the electron originates from nucleus.
(d) All of the above.

9. Which of the following statements is incorrect for nuclear forces?
(a) These are strongest in magnitude.
(b) They are charge dependent.
(c) They are effective only for short ranges.
(d) They result from interaction of every nucleon with the nearest limited number of nucleons.

10. If the Q value of an endothermic reaction is 11.32 MeV, then the minimum energy of the reactant nuclei to carry out the reaction is (in laboratory frame of reference)
(a) 11.32 MeV
(b) Slightly less than 11.32 MeV
(c) Slightly greater than 11.32 MeV
(d) Data is insufficient

11. The binding energies of nuclei X and Y are E_1 and E_2, respectively. Two atoms of X fuse to give one atom of Y and an energy Q is released. Then,
(a) $Q = 2E_1 - E_2$ (b) $Q = E_2 - 2E_1$
(c) $Q < 2E_1 - E_2$ (d) $Q > E_2 - 2E_1$

12. Binding energy per nucleon of ${}_1H^2$ and ${}_2He^4$ are 1.1 MeV and 7.0 MeV, respectively. Energy released in the process ${}_1H^2 + {}_1H^2 = {}_2He^4$ is
(a) 20.8 MeV (b) 16.6 MeV
(c) 25.2 MeV (d) 23.6 MeV

13. ${}_{92}U^{238}$ absorbs a neutron. The product emits an electron. This product further emits an electron. The result is
(a) ${}_{94}Pu^{239}$ (b) ${}_{90}Pu^{239}$
(c) ${}_{93}Pu^{237}$ (d) ${}_{94}Pu^{237}$

14. Neutron decay in the free space is given as follows: ${}_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + [\ ]$
Then, the parenthesis represents
(a) photon (b) graviton
(c) neutrino (d) antineutrino

15. A nucleus ${}^A_Z X$ emits an α-particle. The resultant nucleus emits a β^+ particle. The respective atomic and mass numbers of the final nucleus will be
(a) $Z - 3, A - 4$ (b) $Z - 1, A - 4$
(c) $Z - 2, A - 4$ (d) $Z, A - 2$

16. Calculate the binding energy of a deuteron atom, which consists of a proton and a neutron, given that the atomic mass of the deuteron is 2.014102 u.
(a) 0.002388 MeV (b) 2.014102 MeV
(c) 2.16490 MeV (d) 2.224 MeV

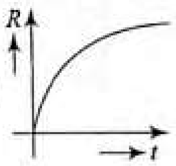
17. The rest mass of a deuteron is equivalent to an energy of 1876 MeV, that of a proton to 939 MeV, and that of a neutron to 940 MeV.
A deuteron may disintegrate to a proton and a neutron if it
(a) emits an X-ray photon of energy 2 MeV
(b) captures an X-ray photon of energy 2 MeV
(c) emits an X-ray photon of energy 3 MeV
(d) captures an X-ray photon of energy 3 MeV

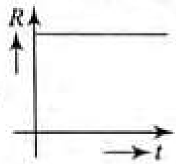
18. A helium atom, a hydrogen atom and a neutron have masses of 4.003 u, 1.008 u and 1.009 u (unified atomic mass units), respectively. Assuming that hydrogen atoms and neutrons can fuse to form helium, what is the binding energy of a helium nucleus?
(a) 2.01 u (b) 3.031 u
(c) 1.017 u (d) 0.031 u

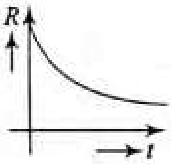
Radioactivity

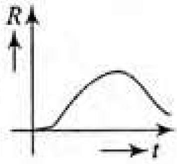
19. In which of the following processes, the number of protons in the nucleus increase?
(a) α-decay (b) β^--decay
(c) β^+-decay (d) k-capture

20. A radioactive nucleus X decays to a stable nucleus Y. Then, time graph of rate of formation of Y against time t will be:

(a) 

(b) 

(c) 

(d) 

21. In the disintegration series

$$^{238}_{92}U \xrightarrow{\alpha} X \xrightarrow{\beta^-} {}^{A}_{Z}Y$$

the values of Z and A, respectively, will be
(a) 92, 326 (b) 88, 230
(c) 90, 234 (d) 91, 234

22. In the case of thorium ($A = 232$ and $Z = 90$), we obtain an isotope of lead ($A = 208$ and $Z = 82$) after some radioactive disintegrations. The number of α- and β-particles emitted are, respectively,
(a) 6, 3 (b) 6, 4
(c) 5, 5 (d) 4, 6

23. The fraction of a radioactive material which remains active after time t is 9/16. The fraction which remains active after time $t/2$ will be
(a) $\frac{4}{5}$ (b) $\frac{7}{8}$
(c) $\frac{3}{5}$ (d) $\frac{3}{4}$

24. A radioactive nucleus undergoes a series of decays according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72, respectively, then what are these number for A_4?
(a) 172 and 69 (b) 174 and 70
(c) 176 and 69 (d) 176 and 70

25. Certain radioactive substance reduces to 25% of its value in 16 days. Its half-life is
(a) 32 days (b) 8 days
(c) 64 days (d) 28 day

26. The half-life of a radioactive decay is x times its mean life. The value of x is
(a) 0.3010 (b) 0.6930
(c) 0.6020 (d) $\frac{1}{0.6930}$

27. The activity of a radioactive element decreases to one-third of the original activity A_0 in a period of 9 years. After a further lapse of 9 years, its activity will be
(a) A_0 (b) $\frac{2}{3}A_0$
(c) $\frac{A_0}{9}$ (d) $\frac{A_0}{6}$

28. A certain radioactive element has half-life of 4 days. The fraction of material that decays in 2 days is
(a) 1/2 (b) $1/\sqrt{2}$
(c) $\sqrt{2}$ (d) $(\sqrt{2}-1)/\sqrt{2}$

29. The activity of a radioactive sample is 1.6 curie, and its half-life is 2.5 days. Its activity after 10 days will be
(a) 0.8 curie (b) 0.4 curie
(c) 0.1 curie (d) 0.16 curie

30. A newly prepared radioactive nuclide has a decay constant λ of 10^{-6} s^{-1}. What is the approximate half-life of the nuclide?
(a) 1 hour (b) 1 day
(c) 1 week (d) 1 month

31. The half-life of a certain radioactive isotope is 32 h. What fraction of a sample would remain after 16 h?
(a) 0.25 (b) 0.71
(c) 0.29 (d) 0.75

32. The mean life time of a radionuclide, if its activity decreases by 4% for every 1 h, would be (product is non-radioactive, i.e., stable)
(a) 25 h (b) 1.042 h
(c) 2 h (d) 30 h

33. The probability of survival of a radioactive nucleus for one mean life is
(a) $\frac{1}{e}$ (b) $1-\frac{1}{e}$
(c) $\frac{\ln 2}{e}$ (d) $1-\frac{\ln 2}{e}$

34. Two radioactive materials X_1 and X_2 have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, the ratio of the number of nuclei of X_1 to that of X_2 will be $1/e$ after a time
(a) $\frac{1}{10\lambda}$ (b) $\frac{1}{11\lambda}$
(c) $\frac{11}{10\lambda}$ (d) $\frac{1}{9\lambda}$

35. A radioactive substance is being consumed at a constant rate of 1 s^{-1}. After what time will the number of radioactive nuclei become 100. Initially, there were 200 nuclei present.
(a) 1 s (b) $\frac{1}{\ln(2)}$ s
(c) ln(2) s (d) 2 s

36. The ratio of molecular mass of two radioactive substances is 3/2 and the ratio of their decay constants is 4/3. Then, the ratio of their initial activity per mole will be
(a) 2 (b) $\frac{8}{9}$
(c) $\frac{4}{3}$ (d) $\frac{9}{8}$

37. N_1 atoms of a radioactive element emit N_2 beta particles per second. The decay constant of the element is (in s^{-1})
(a) $\frac{N_1}{N_2}$ (b) $\frac{N_2}{N_1}$
(c) $N_1 \ln(2)$ (d) $N_2 \ln(2)$

38. A sample of a radioactive element has a mass of 10 g at an instant $t = 0$. The approximate mass of this element in the sample after two mean lives is
(a) 1.35 g (b) 2.50 g
(c) 3.70 g (d) 6.30 g

39. After an interval of one day, 1/16th initial amount of a radioactive material remains in a sample. Then, its half-life is
(a) 6 h (b) 12 h
(c) 1.5 h (d) 3 h

40. The half-life of At is 100 μs. The time taken for the radioactivity of a sample of At to decay to 1/16th of its initial value is
(a) 400 μs (b) 6.3 μs
(c) 40 μs (d) 300 μs

41. Atomic mass number of an element is 232 and its atomic number is 90. The end product of this radioactive element is an isotope of lead (atomic mass 208 and atomic number 82). The number of α-and β-particles emitted are
(a) $\alpha = 3, \beta = 3$ (b) $\alpha = 6, \beta = 4$
(c) $\alpha = 6, \beta = 0$ (d) $\alpha = 4, \beta = 6$

42. The rate of decay of a radioactive element at any instant is 10^3 disintegrations s^{-1}. If the half-life of the elements is 1 s, then the rate of decay after 1 s will be
(a) 500 s^{-1} (b) 1000 s^{-1}
(c) 250 s^{-1} (d) 2000 s^{-1}

43. The percentage of quantity of a radioactive material that remains after 5 half-lives will be
(a) 31% (b) 3.125%
(c) 0.3% (d) 1%

44. If 10% of a radioactive substance decays in every 5 year, then the percentage of the substance that will have decayed in 20 years will be
(a) 40% (b) 50%
(c) 65.6% (d) 34.4%

45. The half-life period of RaB($_{82}Pb^{214}$) is 26.8 min. The mass of one curie of RaB is
(a) 3.71×10^{10} g (b) 3.71×10^{-10} g
(c) 8.61×10^{10} g (d) 3.064×10^{-8} g

Nuclear Fission, Fusion and Nuclear Energy

46. Assuming that about 200 MeV of energy is released per fission of $_{92}U^{235}$ nuclei, then the mass of U^{235} consumed per day in a fission reactor of power 1 megawatt will be approximately
(a) 10^{-2} g (b) 1 g
(c) 100 g (d) 10, 000 g

47. The fission of a heavy nucleus gives, in general, two smaller nuclei, two or three neutrons, some β-particles, and some γ-radiation. It is always true that the nuclei produced
(a) have a total rest-mass that is greater than that of the original nucleus
(b) have large kinetic energies that carry off the greater part of the energy released
(c) travel in exactly opposite directions
(d) have neutron-to-proton ratios that are too low for stability

48. 1.00 kg of ^{235}U undergoes fission process. If energy released per event is 200 MeV, then the total energy released is
(a) 5.12×10^{24} MeV (b) 6.02×10^{23} MeV
(c) 5.12×10^{26} MeV (d) 6.02×10^{26} MeV

49. U-235 can decay by many ways, let us here consider only two ways, A and B. In decay of U-235 by means of A, the energy released per fission is 210 MeV while in B it is 186 MeV. Then, the uranium 235 sample is more likely to decay by
(a) scheme A
(b) scheme B
(c) equally likely for both schemes
(d) it depends on half-life of schemes A and B

50. A star initially has 10^{40} deuterons. It produces energy via the processes $^2_1H + ^2_1H \rightarrow ^3_1He + p$ and $^2_1H + ^3_1H \rightarrow ^4_2He + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of
[Given: $M(^2H) = 2.014$ u, $M(n) = 1.008$ u, $M(p) = 1.008$ u, and $M(^4He) = 4.001$ u]
(a) 10^6 s (b) 10^8 s
(c) 10^{12} s (d) 10^{16} s

51. What is the power output of $_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy? Avogadro's number = 6.02×10^{26} per kilomole.

(a) 45 megawatt (b) 58.46 megawatt
(c) 72 megawatt (d) 92 megawatt

52. Stationary nucleus ^{238}U decays by a emission generating a total kinetic energy T:

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}\alpha$$

What is the kinetic energy of the α-particle?

(a) Slightly less than $T/2$ (b) $T/2$
(c) Slightly less than T (d) Slightly greater than T

53. In an α-decay, the kinetic energy of α-particle is 48 MeV and Q value of the reaction is 50 MeV. The mass number of the mother nucleus is (assume that daughter nucleus is in ground state)

(a) 96 (b) 100
(c) 104 (d) none of these

54. A heavy nucleus having mass number 200 gets disintegrated into two small fragments of mass numbers 80 and 120. If binding energy per nucleon for parent atom is 6.5 MeV and for daughter nuclei is 7 MeV and 8 MeV, respectively, then the energy released in the decay will be

(a) 200 MeV (b) –220 MeV
(c) 220 MeV (d) 180 MeV

55. If the energy released in the fission of one nucleus is 200 MeV. Then the number of nuclei required per second in a power plant of 16 kW will be

(a) 0.5×10^{14} (b) 0.5×10^{12}
(c) 5×10^{12} (d) 5×10^{14}

56. To generate a power of 3.2 mega watt, the number of fissions of U^{235} per minute is

(Energy released per fission = 200 MeV, 1eV = 1.6×10^{-19} J)

(a) 6×10^{18} (b) 6×10^{17}
(c) 10^{17} (d) 6×10^{16}

57. The sun radiates energy in all directions. The average radiations received on the earth surface from the sun is 1.4 kilowatt/m^2. The average earth- sun distance is 1.5×10^{11} metres. The mass lost by the sun per day is

(1 day = 86400 seconds)

(a) 4.4×10^9 kg (b) 7.6×10^{14} kg
(c) 3.8×10^{12} kg (d) 3.8×10^{14} kg

58. An atomic power nuclear reactor can deliver 300 MW. The energy released due to fission of each nucleus of uranium atom U^{238} is 170 MeV. The number of uranium atoms fissioned per hour will be

(a) 30×10^{25} (b) 4×10^{22}
(c) 10×10^{20} (d) 5×10^{15}

59. Assuming that about 20 MeV of energy is released per fusion reaction

$_1H^2 + {}_1H^2 \rightarrow {}_0n^1 + {}_2He^4$

then the mass of $_1H^2$ consumed per day in a fusion reactor of power 1 megawatt will approximately be

(a) 0.001 g (b) 0.1 g
(c) 10.0 g (d) 1000 g

Problems Based on Mixed Concepts

60. If mass of U^{235} = 235.12142 amu, mass of U^{236} = 236.1205 amu and mass of neutron = 1.008665 amu, then the energy required to remove one neutron from the nucleus U^{236} is nearly about

(a) 75 MeV (b) 6.5 MeV
(c) 1 eV (d) zero

61. A certain radioactive material can undergo three different types of decay, each with a different decay constant λ, 2λ, and 3λ. Then, the effective decay constant λ_{eff} is

(a) 6λ (b) 4λ
(c) 2λ (d) 3λ

62. A sample of radioactive material decays simultaneously by two processes A and B with half-lives $\frac{1}{2}$ and $\frac{1}{4}$ h, respectively. For the first half hour it decays with the process A, next one hour with the process B, and for further half an hour with both A and B. If, originally, there were N_0 nuclei, find the number of nuclei after 2 h of such decay.

(a) $\dfrac{N_0}{(2)^8}$ (b) $\dfrac{N_0}{(2)^4}$
(c) $\dfrac{N_0}{(2)^6}$ (d) $\dfrac{N_0}{(2)^5}$

63. An element X decays, first by positron emission and then two α-particles are emitted in successive radioactive decay. If the product nucleus has a mass number 229 and atomic number 89, the mass number and atomic number of element X are

(a) 237, 93 (b) 237, 94
(c) 221, 84 (d) 237, 92

64. $A \xrightarrow{\lambda} B \xrightarrow{2\lambda} C$

	A	B	C
$T = 0$	N_0	0	0
T	N_1	N_2	N_3

The ratio of N_1 to N_2 when N_2 is maximum is

(a) at no time this is possible
(b) 2
(c) 1/2
(d) $\dfrac{\ln 2}{2}$

65. On an average, a neutron loses half of its energy per collision with a quasi-free proton. To reduce a 2 MeV neutron to a thermal neutron having energy 0.04 eV, the number of collisions required is nearly

(a) 50 (b) 52
(c) 26 (d) 15

66. Atomic masses of two isobars $^{64}_{29}Cu$ and $^{64}_{30}Zn$ are 63.9298 u and 63.9292 u, respectively. It can be concluded from this data that

(a) both the isobars are stable
(b) ^{64}Zn is radioactive, decaying to ^{64}Cu through β-decay

(c) ^{64}Cu is radioactive, decaying to ^{64}Zn through β-decay
(d) ^{64}Cu is radioactive, decaying to ^{64}Zn through γ-decay

67. If a nucleus such as ^{226}Ra that is initially at rest undergoes α-decay, then which of the following statements is true?
(a) The alpha particle has more kinetic energy than the daughter nucleus.
(b) The alpha particle has less kinetic energy than the daughter nucleus.
(c) The alpha particle and daughter nucleus both have same kinetic energy
(d) We cannot say anything about kinetic energy of alpha particle and daughter nucleus.

68. $^{40}_{19}$K isotope of potassium has a half-life of 1.4×10^9 yr and decays to form stable argon, $^{40}_{18}$Ar. A sample of rock has been taken which contains both potassium and argon in the ratio 1:7, i.e.,

$$\frac{\text{Number of potassium-40 atoms}}{\text{Number of argon-40 atoms}} = \frac{1}{7}$$

Assuming that when the rock was formed no argon-40 was present in the sample and none has escaped subsequently, determine the age of the rock.
(a) 4.2×10^9 years (b) 9.8×10^9 years
(c) 1.4×10^9 years (d) 10×10^9 years

69. Consider one of fission reactions of ^{235}U by thermal neutrons $^{235}_{92}\text{U} + n \rightarrow ^{94}_{38}\text{Sr} + ^{140}_{54}\text{Xe} + 2n$. The fission fragments are however unstable and they undergo successive β-decay until $^{94}_{38}$Sr becomes $^{94}_{40}$Zr and $^{140}_{54}$Xe becomes $^{140}_{58}$Ce. The energy released in this process is
[Given: $m(^{235}\text{U}) = 235.439$ u, m (n) = 1.00866 u, m (^{94}Zr) = 93.9064 u, $m(^{140}\text{Ce}) = 139.9055$ u, 1 u = 931 MeV]
(a) 156 MeV (b) 208 MeV
(c) 456 MeV (d) cannot be computed

70. What is the age of an ancient wooden piece if it is known that the specific activity of C^{14} nuclide in it amounts to 3/5 of that in fresh trees? Given: the half of C nuclide is 5570 years and $\log_e(5/3) = 0.5$.
(a) 1000 years (b) 2000 years
(c) 3000 years (d) 4000 years

71. The initial activity of a certain radioactive isotope was measured as 16000 counts min^{-1}. Given that the only activity measured was due to this isotope and that its activity after 12 h was 2100 counts min^{-1}, its half-life, in hours, is nearest to [Given $\log_e(7.2) = 2$]
(a) 9.0 (b) 6.0
(c) 4.0 (d) 3.0

72. The minimum frequency of a γ-ray that causes a deuteron to disintegrate into a proton and a neutron is ($m_d = 2.0141$ amu, $m_p = 1.0078$ amu, $m_n = 1.0087$ amu.)
(a) 2.7×10^{20} Hz (b) 5.4×10^{20} Hz
(c) 10.8×10^{20} Hz (d) 21.6×10^{20} Hz

73. Samples of two radioactive nuclides, X and Y, each have equal activity A at time $t = 0$. X has a half-life of 24 years and Y a half-life of 16 years. The samples are mixed together. What will be the total activity of the mixture at $t = 48$ years?
(a) $\frac{1}{2}A_0$ (b) $\frac{1}{4}A_0$
(c) $\frac{3}{16}A_0$ (d) $\frac{3}{8}A_0$

74. ^{238}U decays with a half-life of 4.5×10^9 years, the decay series eventually ending at ^{206}Pb, which is stable. A rock sample analysis shows that the ratio of the number of atoms of ^{206}Pb to ^{238}U is 0.0058. Assuming that all the ^{206}Pb is produced by the decay of ^{238}U and that all other half-lives on the chain are negligible, the age of the rock sample is ($\ln 1.0058 = 5.78 \times 10^{-3}$)
(a) 38×10^8 years (b) 38×10^6 years
(c) 19×10^8 years (d) 19×10^6 years

75. The compound unstable nucleus $^{236}_{92}$U often decays in accordance with the following reaction $^{236}_{92}\text{U} \rightarrow ^{140}_{54}\text{Xe} + ^{94}_{38}\text{Sr} +$ other particles
In the nuclear reaction presented above, the "other particle" might be
(a) an alpha particle, which consists of two protons and two neutrons
(b) two protons
(c) one proton and one neutron
(d) two neutrons

76. Why is a $^4_2$He nucleus more stable than a $^4_3$Li nucleus?
(a) The strong nuclear force is larger when the neutron to proton ratio is higher.
(b) The laws of nuclear physics forbid a nucleus from containing more protons than neutrons.
(c) Forces other than the strong nuclear force make the lithium nucleus less stable.
(d) None of the above.

77. Consider the reaction,

$$^2_1\text{H} + ^2_1\text{H} \longrightarrow ^3_2\text{He} + ^1_0n$$

$m(^2_1\text{H}) = 2.014082$ u, $m(^3_2\text{He}) = 3.016029$ u, $m(^1_0n) = 1.008665$ u
Then, mark the correct option.
(a) Threshold energy of 3.23 MeV is required to initiate the reaction
(b) Reaction occurs when total KE of reactants exceeds 3.23 MeV
(c) Reaction occurs such that final total KE is 3.23 MeV lesser than total initial KE
(d) No, threshold energy is required for the reaction

78. Three radioactive sources A, B and C of same activity 25 m Ci are dumped in a pond as waste. Source A emits 1.0 MeV γ-rays, source B emits 2.0 MeV γ-rays, and source C emits 2.0 MeV α-rays. Order of relative danger of A, B and C is

(a) $A > B > C$ (b) $B > A > C$
(c) $C > B > A$ (d) $A = B = C$

ARCHIVES

1. If N_0 is the original mass of a substance of half-life period $t_{1/2}$ = 5 years, then the amount of substance left after 15 years is

(a) $\frac{N_0}{8}$ (b) $\frac{N_0}{16}$
(c) $\frac{N_0}{2}$ (d) $\frac{N_0}{4}$ **(AIEEE 2002)**

2. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit
(i) electrons (ii) protons
(iii) He^{2+} (iv) neutrons
The emission at the instant can be
(a) i, ii, iii (b) i, ii, iii, iv
(c) iv (d) ii, iii **(AIEEE 2002)**

3. Beta rays emitted by a radioactive material are
(a) electromagnetic radiation
(b) the electrons orbiting around the nucleus
(c) charged particles emitted by nucleus
(d) neutral particles **(AIEEE 2002)**

4. Which of the following cannot be emitted by a radioactive substance enduring its decay?
(a) electrons (b) protons
(c) neutrinos (d) helium nuclei **(AIEEE 2003)**

5. A radioactive sample at any instant has disintegration rate 5000 disintegrations per minute. After 5 min, the rate is 1250 integrations per minute. Then the decay constant (per minute) is
(a) 0.8 ln 2 (b) 0.4 ln 2
(c) 0.2 ln 2 (d) 0.1 ln 2 **(AIEEE 2003)**

6. A nucleus with $Z = 92$ emits the following in a sequence:
$\alpha, \beta^-, \beta^{-1}, \alpha, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^{-1}, \alpha, \beta^+, \beta^+, \alpha$
The Z of the resultant nucleus is
(a) 74 (b) 76
(c) 78 (d) 82 **(AIEEE 2003)**

7. Which of the following atoms has the lowest ionisation potential?
(a) ${}^{16}_{8}O$ (b) ${}^{14}_{7}N$
(c) ${}^{133}_{55}Cs$ (d) ${}^{40}_{15}Ar$ **(AIEEE 2003)**

8. In the nuclear fusion reaction ${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{5}_{2}He + n$, given that the repulsive potential energy between the two nuclei is -7.7×10^{-14} J. The temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant k is 1.38×10^{-23} J/K]
(a) 10^9 K (b) 10^7 K
(c) 10^5 K (d) 10^3 K **(AIEEE 2003)**

9. If the binding energy of the electron in hydrogen atoms is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is
(a) 122.4 eV (b) 30.6 eV
(c) 13.6 eV (d) 3.4 eV **(AIEEE 2003)**

10. A nucleus is disintegrated into two nuclear parts having velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be
(a) $2^{1/3} : 1$ (b) $1 : 3^{1/2}$
(c) $3^{1/2} : 1$ (d) $1 : 2^{1/3}$ **(AIEEE 2004)**

11. The binding energy per nucleon of deuteron (${}^{2}_{1}H$) and helium nucleus (${}^{4}_{2}He$) is 1.1 MeV and 7 MeV, respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is
(a) 13.9 MeV (b) 29.9 MeV
(c) 23.6 MeV (d) 19.2 MeV **(AIEEE 2004)**

12. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of
(a) 1 Å (b) 10^{-19} cm
(c) 10^{-12} cm (d) 10^{-15} cm **(AIEEE 2004)**

13. Starting with a sample of pure ${}^{66}Cu$, 7/8 of it decays into Zn in 15 min. The corresponding half-life is
(a) $7\frac{1}{2}$ min (b) 5 min
(c) 15 min (d) 10 min **(AIEEE 2005)**

14. If the radius of a ${}^{27}_{13}Al$ nucleus is estimated to be 3.6 fm, then the radius of a ${}^{125}_{52}Te$ nucleus is nearly
(a) 5 fm (b) 4 fm
(c) 8 fm (d) 6 fm **(AIEEE 2005)**

15. A nuclear transformation is denoted by $X\ (n, \alpha)\ {}^{7}_{3}Li$. Which of the following is the nucleus of element X?
(a) ${}^{11}_{4}Be$ (b) ${}^{9}_{5}B$
(c) ${}^{10}_{5}B$ (d) ${}^{12}C_6$ **(AIEEE 2005)**

16. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to 1/8. The thickness of lead which will reduce the intensity to 1/2 will be
(a) 12 mm (b) 18 mm
(c) 9 mm (d) 6 mm **(AIEEE 2005)**

17. In the nuclear reaction: $X(n, \alpha)_3Li^7$ the term X will be 3

(a) $_5B^{10}$ (b) $_5B^9$
(c) $_5B^{11}$ (d) $_2He^4$ **(AIEEE 2005)**

18. Starting with a sample of pure ^{66}Cu, $\frac{7}{8}$ of it decays into Zn in 15 min. The corresponding half-life is

(a) 5 min (b) $7\frac{1}{2}$ min
(c) 10 min (d) 15 min **(AIEEE 2005)**

19. If the binding energies per nucleon in ^7_3Li and ^4_2He nuclei are 5.60 MeV and 7.06 MeV, respectively, then in the reaction

$p + ^7_3Li \rightarrow 2\,^4_2He$

the energy of proton must be

(a) 1.46 MeV (b) 39.2 MeV
(c) 28.24 MeV (d) 17.22 MeV **(AIEEE 2006)**

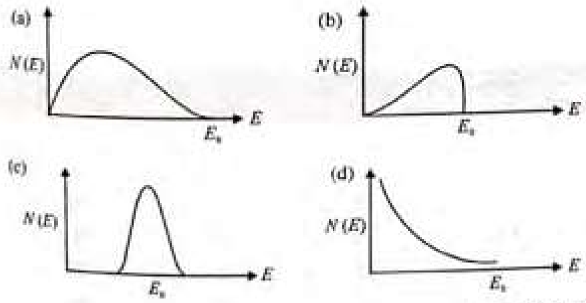
20. When $_3Li^7$ nuclei are bombarded by protons, and the resultant nuclei are $_4Be^8$, the emitted particles will be

(a) gamma photons (b) neutrons
(c) alpha particles (d) beta particles **(AIEEE 2006)**

21. An alpha nucleus of energy $(1/2)\,mv^2$ bombards a heavy nuclei target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to

(a) $\frac{1}{v^4}$ (b) $\frac{1}{Ze}$
(c) v^2 (d) $\frac{1}{m}$ **(AIEEE 2006)**

22. The energy spectrum of β particles [number $N(E)$ as a function of β energy E] emitted from a radioactive source is



(AIEEE 2006)

23. The half-life period of a radioactive element X is the same as the mean life of another radioactive element Y. Initially they have the same number of atoms. Then

(a) Y decays faster than X
(b) X and Y decay equally initially
(c) X and Y decay at the same rate always
(d) Y will decay faster than X **(AIEEE 2007)**

24. If M_O is the mass of an oxygen isotope $_8O_{17}$, M_P and M_N are the masses of a proton and a neutron, respectively, then the nuclear binding energy of the isotope is

(a) $(M_O - 8\,M_P - 9\,M_N)c^2$
(b) M_Oc^2
(c) $(M_O - 17\,M_N)c^2$
(d) $(M_O - 17\,M_P)c^2$ **(AIEEE 2007)**

25. In gamma ray emission from a nucleus,

(a) there is no change in the proton number and the neutron number
(b) only the neutron number changes
(c) only the proton number changes
(d) both the neutron number and the proton number changes **(AIEEE 2007)**

26. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electron charge on the moon}}{\text{electronic charge on the earth}}$ to be

(a) $\frac{g_M}{g_E}$ (b) 1
(c) 0 (d) $\frac{g_E}{g_M}$ **(AIEEE 2007)**

Direction for Question 27:

The question contains Statement 1 and Statement 2. Of the four choices given, choose the one that best describes the two statements.

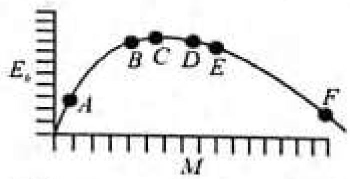
27. **Statement 1:** Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

Statement 2: For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

(a) Statement 1 is true but statement 2 is false.
(b) Statement 1 is false but statement 2 is true.
(c) Statement 1 is true, statement 2 is true; statement 2 is the correct explanation for statement 1.
(d) Statement 1 is true, statement 2 is true; statement 2 is not the correct explanation for statement 1.

(AIEEE 2008)

28. The figure given below is a plot of binding energy per nucleon, E_b, against the nuclear mass M; A, B, C, D, E, and F correspond to different nuclei. Consider four reactions:



(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$
(iii) $D + E \rightarrow F + \varepsilon$ (iv) $F \rightarrow D + E + \varepsilon$

Here ε is the energy released. In which reaction ε is positive?

(a) (i) and (iv) (b) (i) and (iii)
(c) (ii) and (iv) (d) (ii) and (iii) **(AIEEE 2009)**

Direction for Questions 29 and 30:

These questions are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal masses $M/2$ each. The speed of light is c. **(AIEEE 2010)**

29. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then
(a) $E_2 = 2E_1$ (b) $E_1 > E_2$
(c) $E_2 > E_1$ (d) $E_1 = 2E_2$

30. The speed of daughter nuclei is
(a) $c\dfrac{\Delta m}{M+\Delta m}$ (b) $c\sqrt{\dfrac{2\Delta m}{M}}$
(c) $c\sqrt{\dfrac{\Delta m}{M}}$ (d) $c\sqrt{\dfrac{m}{\Delta M}}$

31. A radioactive nucleus (initial mass number A and atomic number Z) emits three α-particles and two positrons. The ratio of the number of neutrons to that of protons in the final nucleus will be
(a) $\dfrac{A-Z-8}{Z-4}$ (b) $\dfrac{A-Z-4}{Z-8}$
(c) $\dfrac{A-Z-12}{Z-4}$ (d) $\dfrac{A-Z-4}{Z-2}$ **(AIEEE 2010)**

32. The half-life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when 2/3 of it has decayed and time t_1 when 1/3 of it had decayed is
(a) 7 min (b) 14 min
(c) 20 min (d) 28 min **(AIEEE 2011)**

33. Assume that a neutron breaks into a proton and an electron. The energy released during this process is: (mass of neutron = 1.6725×10^{-27} kg, mass of proton = 1.6725×10^{-27} kg, mass of electron = 9×10^{-31} kg)
(a) 0.73 MeV (b) 7.10 MeV
(c) 6.30 MeV (d) 5.4 MeV **(AIEEE 2012)**

34. Hydrogen $(_1H^1)$, Deuterium $(_1H^2)$, singly ionised Helium $(_2He^4)^+$ and doubly ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct?
(a) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (b) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$
(c) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = \lambda_4$
(JEE Main 2014)

35. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be
(a) 1 : 16 (b) 4 : 1
(c) 1 : 4 (d) 5 : 4 **(JEE Main 2016)**

36. A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At some time t, the ratio of the number of B to that of A is 0.3. Then, t is given by
(a) $t = T\log(1.3)$ (b) $t = \dfrac{T}{\log(1.3)}$
(c) $t = \dfrac{T}{2}\dfrac{\log 2}{\log 1.3}$ (d) $t = T\dfrac{\log 1.3}{\log 2}$
(JEE Main 2017)

ANSWER KEY

Exercises

1. (b)	2. (b)	3. (d)	4. (c)	5. (a)	6. (d)	7. (c)	8. (a)	9. (b)	10. (c)
11. (b)	12. (d)	13. (a)	14. (d)	15. (a)	16. (d)	17. (d)	18. (d)	19. (b)	20. (c)
21. (d)	22. (b)	23. (d)	24. (a)	25. (b)	26. (b)	27. (c)	28. (d)	29. (c)	30. (c)
31. (b)	32. (a)	33. (a)	34. (d)	35. (c)	36. (c)	37. (b)	38. (a)	39. (a)	40. (a)
41. (b)	42. (a)	43. (b)	44. (d)	45. (d)	46. (b)	47. (b)	48. (c)	49. (a)	50. (c)
51. (b)	52. (c)	53. (b)	54. (c)	55. (d)	56. (a)	57. (d)	58. (b)	59. (b)	60. (b)
61. (a)	62. (a)	63. (b)	64. (b)	65. (c)	66. (c)	67. (a)	68. (a)	69. (b)	70. (d)
71. (c)	72. (b)	73. (d)	74. (b)	75. (d)	76. (c)	77. (d)	78. (c)		

Archives

1. (a)	2. (a)	3. (c)	4. (b)	5. (b)	6. (c)	7. (c)	8. (a)	9. (b)	10. (d)
11. (c)	12. (c)	13. (b)	14. (d)	15. (c)	16. (a)	17. (a)	18. (a)	19. (d)	20. (a)
21. (d)	22. (a)	23. (d)	24. (a)	25. (a)	26. (b)	27. (a)	28. (a)	29. (c)	30. (b)
31. (b)	32. (c)	33. (None)	34. (a)	35. (d)	36. (d)				

Chapter 31

Electronic Devices

ENERGY BANDS

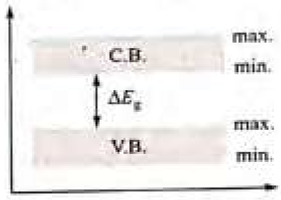
In an isolated atom, the valence electrons can exist only in one of the allowed orbitals each of a sharply defined energy called energy levels. But when two atoms are brought nearer to each other, there are alterations in energy levels and they spread in the form of bands. Energy bands are of the following types: valence band and conduction band.

Valence band: The energy band formed by a series of energy levels containing valence electrons is known as valence band. At 0 K, the electrons fill the energy levels in valence band starting from lowest one.

- This band is always fulfill by electron.
- This is the band of maximum energy.
- Electrons are not capable of gaining energy from external electric field.
- No flow of current due to such electrons.
- The highest energy level which can be occupied by an electron in valence band at 0 K is called fermi level.

Conduction band: The higher energy level band is called the conduction band.

- It is also called empty band of minimum energy.
- This band is partially filled by the electrons.
- In this band, the electrons can gain energy from external electric field.
- The electrons in the conduction band are called free electrons. They are able to move anywhere within the volume of the solid.
- Current flows due to such electrons.



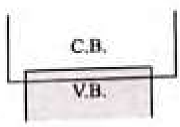
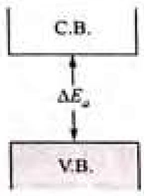
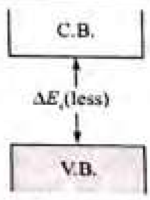
FORBIDDEN ENERGY GAP (ΔE_G)

Energy gap between conduction band and valence band $\Delta E_g = (\text{C.B.})_{min} - (\text{V.B.})_{max}$

- No free electron present in forbidden energy gap.
- Width of forbidden energy gap upon the nature of substance.
- As temperature increases (↑), forbidden energy gap decreases (↓) very slightly.

TYPES OF SOLIDS

On the basis of band structure of crystals, solids are divided in three categories: conductors, insulators and semiconductors.

S.No.	Properties	Conductors	Insulators	Semiconductors
(1)	Electrical conductivity	10^2 to 10^8 Ω/m	10^{-8} Ω/m	10^{-5} to 10^0 Ω/m
(2)	Resistivity	10^{-2} to 10^{-8} Ω-m (negligible)	10^8 Ω-m	10^5 to 10^0 Ω-m
(3)	Band structure			
(4)	Energy gap	Zero or very small	Very large; for diamond, it is 6 eV	For Ge, $E_g = 0.7$ eV; for Si, $E_g = 1.1$ eV
(5)	Current carries	Free electrons	—	Free electrons and holes
(6)	Condition of V.B. and C.B. at ordinary temperature	V.B. and C.B. are completely filled or C.B. is somewhat empty	V.B. – completely filled C.B. – completely unfilled	V.B. – somewhat empty C.B. – somewhat filled
(7)	Temperature co-efficient of resistance (α)	Positive	Zero	Negative

(8)	Effect of temperature on conductivity	Decreases	—	Increases
(9)	Effect of temperature on resistance	Increases	—	Decreases
(11)	Examples	Cu, Ag, Au, Na, Pt, Hg, etc.	Wood, plastic, mica, diamond, glass, etc.	Ge, Si, Ga, As etc.
(12)	Electron density	$10^{29}/m^3$	—	Ge ~ $10^{19}/m^3$ Si ~ $10^{16}/m^3$

SEMICONDUCTORS

Holes in semiconductors: At absolute zero temperature (0 K) conduction band of semiconductor is completely empty and the semiconductor behaves as an insulator. When temperature increases the valence electrons acquires thermal energy to jump to the conduction band (due to the braking of covalent bond). If they jumps to C.B. they leaves behind the deficiency of electrons in the valence band. This deficiency of electron is known as *hole* or cotter. A hole is considered as a seat of positive charge, having magnitude of charge equal to that of an electron.

Holes acts as virtual charge, although there is no physical charge on it.

Effective mass of hole is more than electron.

Mobility of hole is less than electron.

TYPES OF SEMICONDUCTORS

Intrinsic Semiconductor

A pure semiconductor is called intrinsic semiconductor. It has thermally generated current carriers.

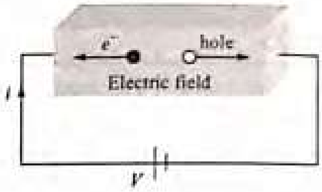
- They have four electrons in the outermost orbit of atom and atoms are held together by covalent bond
- Free electrons and holes both are charge carriers and n_e (in C.B.) = n_h (in V.B.)
- The drift velocity of electrons (v_e) is greater than that of holes (v_h)
- For them, fermi energy level lies at the centre of the C.B. and V.B.
- In pure semiconductor, impurity must be less than 1 in 10^8 parts of semiconductor.
- In intrinsic semiconductor,

  $n_e^{(o)} = n_h^{(o)} = n_i = AT^{3/2}e^{-\Delta E_f/2KT}$, where $n_e^{(o)}$ is equal to electron density in conduction band, $n_h^{(o)}$ is the hole density in V.B., and n_i is the density of intrinsic carriers.
- Because of less number of charge carriers at room temperature, intrinsic semiconductors have low conductivity so they have no practical use.

Net Current and Conductivity

When some potential difference is applied across a piece of intrinsic semiconductor, current flows in it due to both electron and holes, i.e., $i = i_e + i_h \Rightarrow i = eA[n_e v_e + n_h v_h]$

Hence, conductivity of semiconductor, $\sigma = e[n_e\mu_e + n_h\mu_h]$



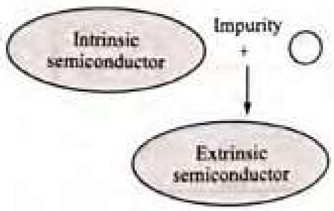
where v_e = drift velocity of electron, v_h = drift velocity of holes, E = applied electric field, $\mu_e = v_e/E$ = mobility of e^- and $\mu_h = v_h/E$ = mobility of holes.

- $(n_i)_{Ge} = 2.4 \times 10^{19}/m^3$ and $(n_i)_{Si} = 1.5 \times 10^{16}/m^3$
- At room temperature $\sigma_{Ge} > \sigma_{Si}$
- $\mu_e > \mu_h$
- Conductivity of semiconductor increases with temperature because number density of charge carriers increases.
- In a doped semiconductor, the number density of electrons and holes is not equal. But it can be established that $n_e n_h = n_i^2$, where n_e, n_h are the number density of electrons and holes, respectively, and n_i is the number density of intrinsic curries (i.e. electrons or holes) in a pure semiconductor. This product is independent of donor and acceptor impurity doping.

Extrinsic Semiconductor

(i) It is also called impure semiconductor.

(ii) The process of adding impurity is called doping.



(iii) The number of atoms of impurity element is about 1 in 10^8 atoms of the semiconductor.

(iv) $n_e \neq n_h$

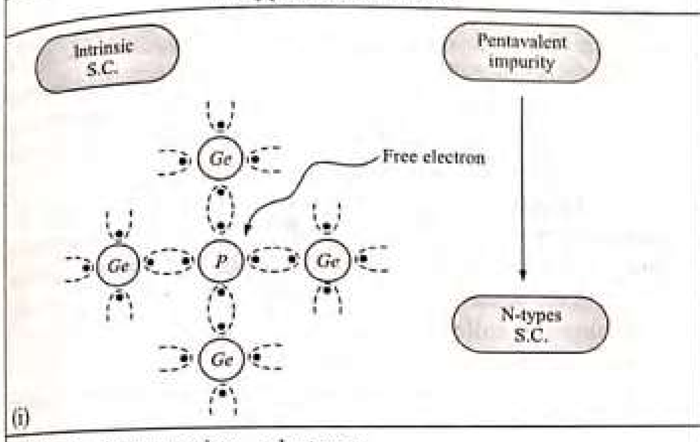
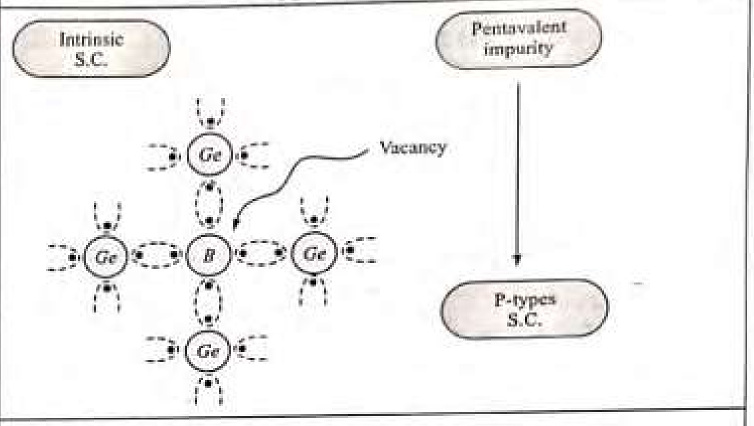
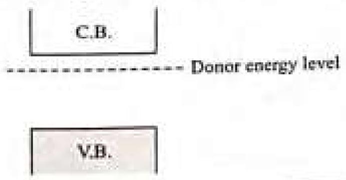
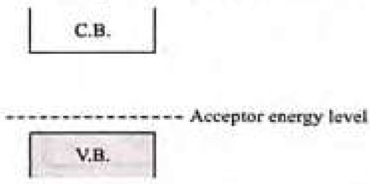
(v) In these fermi level shifts towards valence or conduction energy bands.

(vi) Impurities are of two types:

Pentavalent impurity	Trivalent impurity
The elements whose atom has five valance impurities e.g. As, P, Sb etc. These are also called donor impurities. These impurities are also called donor impurities because they donates extra free electron.	The elements whose each atom has three valance electrons are called trivalent impurities e.g. In, Ga, Al, B, etc. These impurities are also called acceptor impurities as they accept electron.

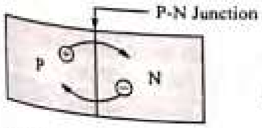
(vii) Their conductivity is high and they are practically used.

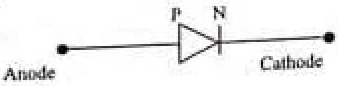
Types of Extrinsic Semiconductor

N-type semiconductor	*P*-type semiconductor
 (i)	
(ii) Major charge carriers – electrons Minor charge carriers – holes	Major charge carriers – holes Minor charge carriers – electrons
(iii) $n_e >> n_h$; $i_e >> i_h$	$n_h >> n_e$; $i_h >> i_e$
(iv) Conductivity $\sigma \approx n_e \mu_e e$	Conductivity $\sigma \approx n_h \mu_h e$
(iv) *N*-type semiconductor is electrically neutral (not negatively charged)	*P*-type semiconductor is also electrically neutral (not positively charged)
(v) Impurity is called donor impurity because one impurity atom generate one e^-.	Impurity is called acceptor impurity.
(vi) Donor energy level lies just below the conduction band. 	Acceptor energy level lies just above the valence band. 

P-N Junction Diode

When a *P*-type semiconductor is suitably joined to an *N*-type semiconductor, then resulting arrangement is called *P-N* junction or *P-N* junction diode

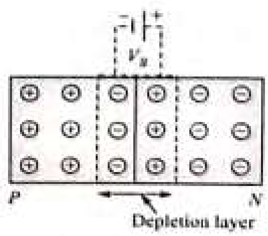




Depletion Region

On account of difference in concentration of charge carrier in the two sections of *P-N* junction, the electrons from *N*-region diffuse through the junction into *P*-region and the hole from *P* region diffuse into *N*-region.

Due to diffusion, neutrality of both *N* and *P*-type semiconductor is disturbed, a layer of negative charged ions appear near the junction in the *P*-crystal and a layer of positive ions appears near the junction in *N*-crystal. This layer is called depletion layer



- The thickness of depletion layer is 1 micron = 10^{-6} m.

- Width of depletion layer $\propto \dfrac{1}{\text{Dopping}}$
- Depletion is directly proportional to temperature.
- The *P-N* junction diode is equivalent to capacitor in which the depletion layer acts as a dielectric.

Potential Barrier

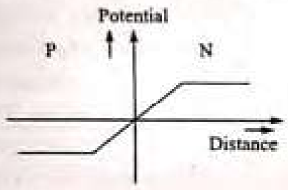
The potential difference created across the *P-N* junction due to the diffusion of electron and holes is called potential barrier.

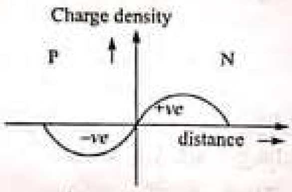
For Ge $V_B = 0.3V$ and for Si, $V_B = 0.7$ V

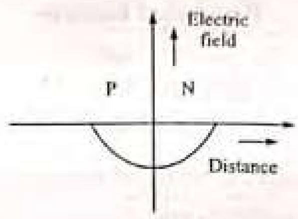
On the average the potential barrier in *P-N* junction is ~0.5 V and the width of depletion region ~10^{-6}.

So the barrier electric field, $E = \dfrac{V}{d} = \dfrac{0.5}{10^{-6}} = 5 \times 10^5$ V/m

Some Important Graphs







Diffusion and Drift Current

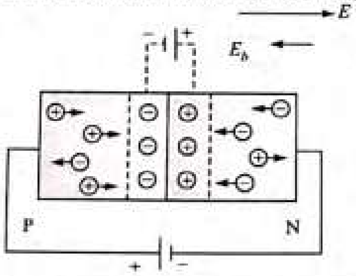
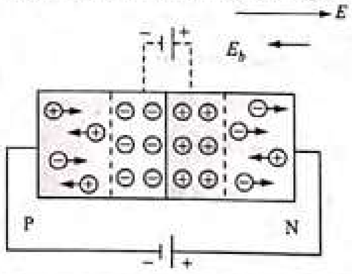
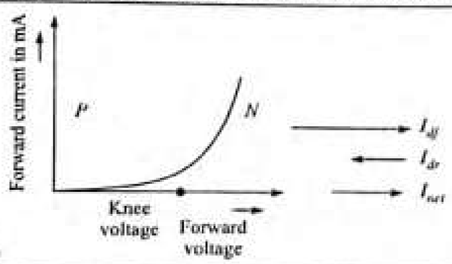
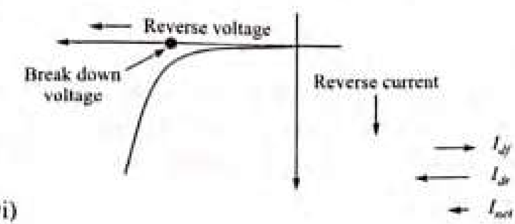
Because of concentration difference holes/electron try to diffuse from their side to other side. Only these holes/electrons crosses the junction, having high kinetic energy. This diffusion results is an electric current from the *P*-side to the *N*-side known as diffusion current (i_{df})

As electron hole pair (because of thermal collisions) are continuously created in the depletion region. These is a regular flow of electrons towards the *N*-side and of holes towards the *P*-side. This makes a current from the *N*-side to the *P*-side. This current is called the drift current (i_{dr}).

- In steady state $i_{df} = i_{dr}$ so $i_{net} = 0$
- When no external source is connected, diode is called unbiased.

Biasing

Means the way of connecting emf source to *P-N* junction diode.

Forward biasing	Reverse biasing
(i) Positive terminal of the battery is connected to the P-crystal and negative terminal of the battery is connected to N-crystal 	(i) Positive terminal of the battery is connected to the N-crystal and negative terminal of the battery is connected to P-crystal 
(ii) Width of depletion layer decreases	(ii) Width of depletion layer increases
(iii) $R_{Forward}$ ∪ 10Ω–25Ω	(iii) $R_{Reverse} \gg 10^5 \Omega$
(iv) Forward bias opposes the potential barrier and for $V > V_B$ a forward current is set up across the junction.	(iv) Reverse bias supports the potential barrier and no current flows across the junction due to the diffusion of the majority carriers. (A very small reverse currents may exist in the circuit due to the drifting of minority carriers across the junction)
(v) Cut-in (knee) voltage: The voltage at which the current starts to increase. For Ge, it is 0.3 V and for Si, it is 0.7 V.	(v) Break down voltage: Reverse voltage at which break down of semiconductor occurs. For Ge, it is 25 V and for Si, it is 35 V.
(vi) 	(vi) 

Reverse Breakdown and Special Purpose Diodes

Zener breakdown

When reverse bias is increased the electric field at the junction also increases. At some stage the electric field becomes so high that it breaks the covalent bonds creating electron, hole pairs. Thus a large number of carriers are generated. This causes a large current to flow. This mechanism is known as Zener breakdown.

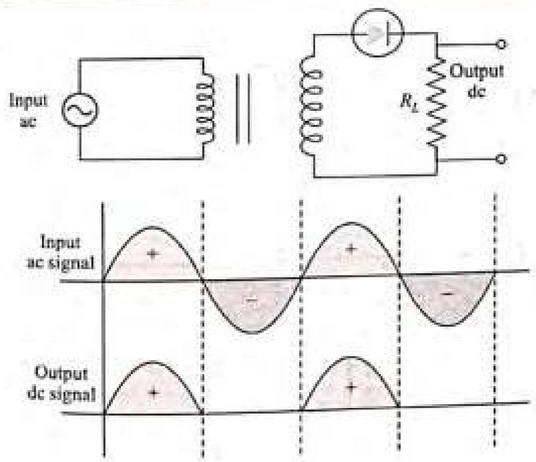
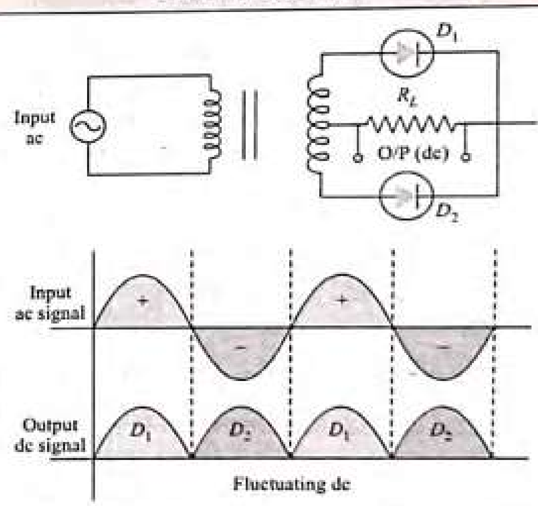
Avalanche breakdown

At high reverse voltage, due to high electric field, the miniority charge carriers, while crossing the junction acquires very high velocities. These by collision breaks down the covalent bonds, generating more carriers. A chain reaction is established, giving rise to high current. This mechanism is called avalanche breakdown.

Special purpose diodes

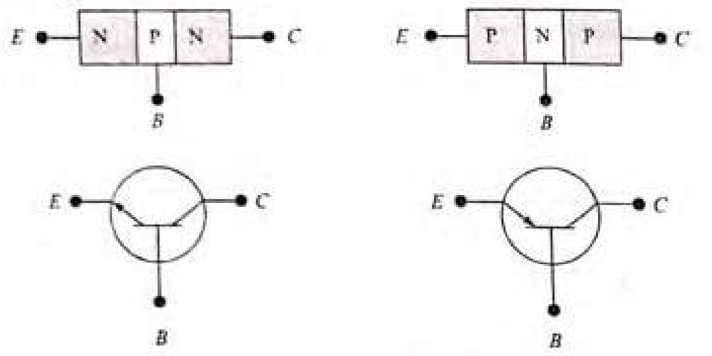
Zener diode	Light emitting diode (LED)	Photo diode	Solar cells
 It is a highly doped p-n junction which is not damaged by high reverse current. The breakdown voltage is made very sharp. In the forward bias, the zener diode acts as ordinary diode. It can be used as voltage regulator	 Specially designed diodes, which give out light radiations when forward biases. LEDS are made of *GaAsp*, *Gap*, etc.	 In these diodes electron and hole pairs are created by junction photoelectric effect. That is the covalent bonds are broken by the EM radiations absorbed by the electron in the V.B. These are used for detecting light signals.	It is based on the photovoltaic effect. One of the semiconductor region is made so thin that the light incident on it reaches the p-n junction and gets absorbed. It converts solar energy into electrical energy.

P-N Junction Diode as a Rectifier

Half wave rectifier	Full wave rectifier
	
During positive half cycle Diode: forward biased Output signal: obtained **During negative half cycle** Diode: reverse biased Output signal: not obtained	**During positive half cycle** Diode : D_1 forward biased D_2 reverse biased Output signal: obtained due to D_1 only **During negative half cycle** Diode : D_1 reverse biased D_2 forward biased Output signal: obtained due to D_2 only Note : Fluctuating dc → $\boxed{\text{Filter}}$ → Constant dc

TRANSISTOR

A junction transistor is formed by sandwiching a thin layer of *P*-type semiconductor between two *N*-type semiconductors or by sandwiching a thin layer of *n*-type semiconductor between two *P*-type semiconductor.



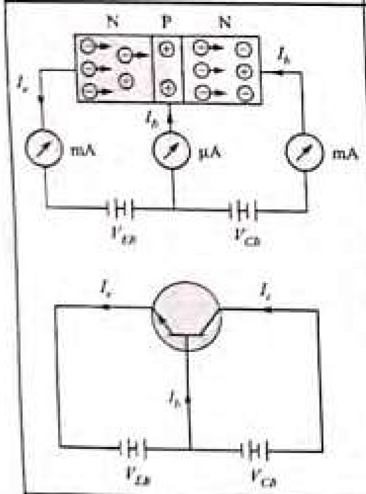
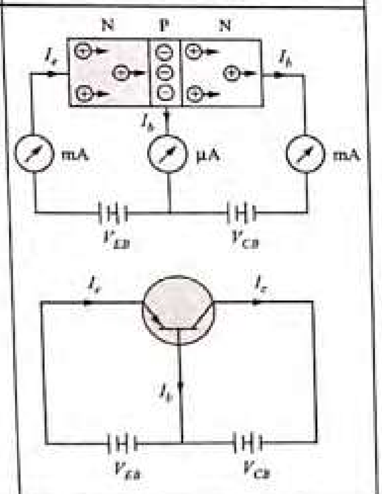
E – Emitter (emits majority charge carriers)

C – Collects majority charge carriers

B – Base (provide proper interaction between *E* and *C*)

- In normal operation, base-emitter is forward biased and collector base junction is reverse biased.

Working of transistor: In both transistors, emitter–base junction is forward biased and collector–base junction is reverse biased.

NPN–transistor	*PNP*–transistor
	
5% emitter electron combine with the holes in the base region resulting in small base current. Remaining 95% electrons enter the collector region.	5% emitter holes combine with the electrons in the base region resulting in small base current. Remaining 95% holes enter the collector region.
$I_e > I_c$, and $I_e = I_b + I_c$	$I_e > I_c$, and $I_e = I_b + I_c$

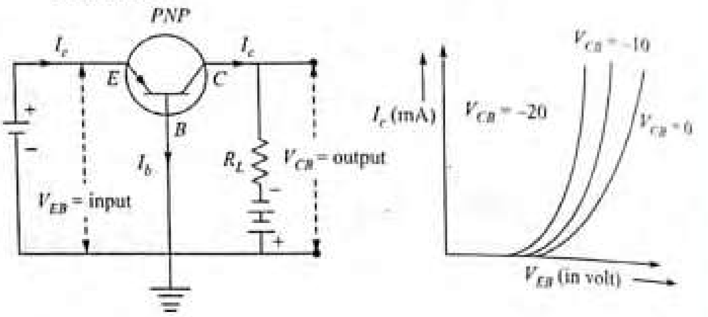
- In a transistor circuit the reverse bias is high as compared to the forward bias. So that it may exert a large attractive force on the charge carriers to enter the collector region.

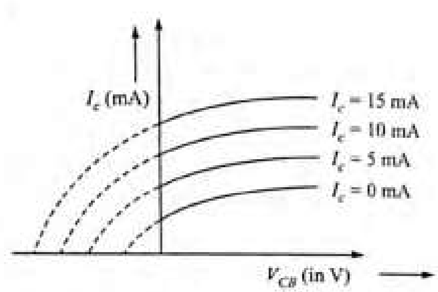
Characteristics of transistors : A transistor can be connected in a circuit in the following three different configurations.

(i) Common base (CB) (ii) Common emitter (CE)

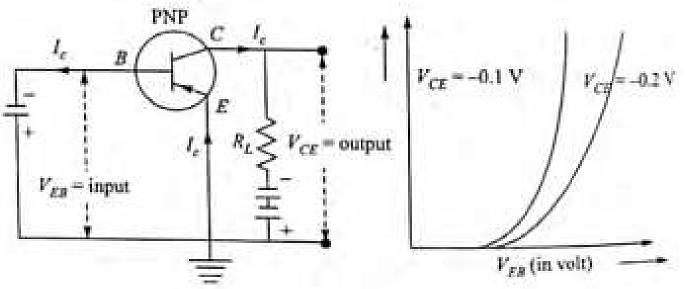
(iii) Common collector (CC)

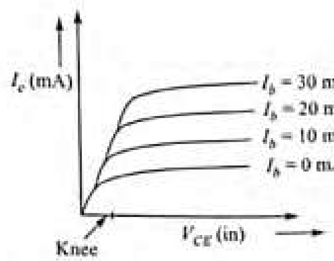
(i) **CB characteristics:** The graphs between voltages and currents when base of a transistor is common to input and output circuits are known as CB characteristic of a transistor.





(ii) **CE characteristics:** The graphs between voltages an d currents when emitter of a transistor is common to input and output circuits are known as CE characteristics of a transistor.

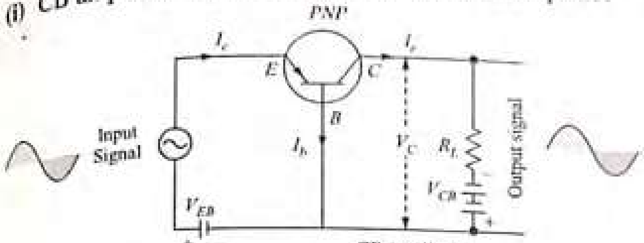




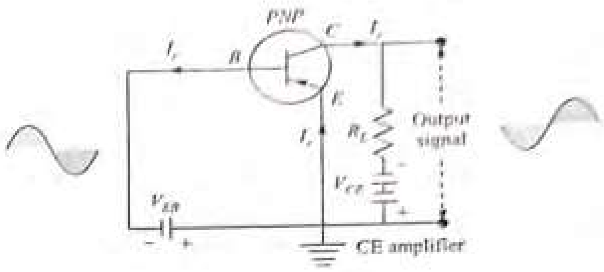
Transistor as an amplifier: A device which increases the amplitude of the input signal is called amplifier.

The transistor can be used as an amplifier in the following three configuration

(i) CB amplifier (ii) CE amplifier (iii) CC amplifier



CB amplifier



CE amplifier

Parameters of CE/CB amplifiers

Transistor as C.E. amplifier	Transistor as C.B. amplifier
(i) Current gain (α)	(i) Current gain (β)
(a) $\alpha_{ac} = \dfrac{\text{Small change in collector current } (\Delta i_c)}{\text{Small change in collector current } (\Delta i_e)}$; V_B (constant)	(a) $\beta_{ac} = \left(\dfrac{\Delta i_c}{\Delta i_b}\right)$ V_{CE} = constant
$\alpha_{dc}(\text{or}\alpha) = \dfrac{\text{Collector current}(i_c)}{\text{Emitter current }(i_e)}$ Value of α_{dc} lies between 0.95 to 0.99.	(b) $\beta_{dc} = \dfrac{i_c}{i_b}$ valve of β_{ac} lies between 15 and 20
(ii) Voltage gain $A_v = \dfrac{\text{Change in output voltage}(\Delta V_o)}{\text{Change in input voltage }(\Delta V_i)}$ $\Rightarrow A_v = \alpha_{ac} \times$ Resistance gain	(ii) Voltage gain $A_v = \dfrac{\Delta V_o}{\Delta V_i} = \beta_{ac} \times$ Resistance gain
(iii) Power gain $= \dfrac{\text{Change in output power}(\Delta P_o)}{\text{Change in input power}(\Delta P_c)}$ $\Rightarrow$ Power gain $= \alpha_{ac}^2 \times$ Resistance gain	(iii) Power gain $= \dfrac{\Delta P_o}{\Delta P_i} = \beta_{ac}^2 \times$ Resistance gain **Note: Trans conductance (gm) :** The ratio of the change in collector current to the change in emitter base voltage is called trans conductance. i.e. $g_m = \dfrac{\Delta i_c}{\Delta V_{EB}}$. Also $g_m = \dfrac{A_V}{R_L}$; R_L = Load resistance

Relation between α and β: $\beta = \dfrac{\alpha}{1-\alpha}$ or $\alpha = \dfrac{\beta}{1+\beta}$

Comparison between CB, CE and CC Amplifier

S. No.	Characteristic	Amplifier		
		CB	CE	CC
(i)	Input resistance (R_i)	≈ 50 to 200 Ω low	≈ 1 to 2 kΩ medium	≈ 150–800 kΩ high
(ii)	Output resistance (R_o)	≈ 1–2 MΩ high	≈ 50 kΩ medium	≈ 50 Ω low
(iii)	Current gain	0.8–0.9 low	20–200 high	20–200 high
(iv)	Voltage gain	Medium	High	Low
(v)	Power gain	Medium	High	Low
(vi)	Phase difference between input and output voltages	Zero	180°	Zero
(vii)	Used as amplifier for	Current	Power	Voltage

ILLUSTRATION 31.1 In half-wave rectification, what is the output frequency if the input frequency is 50 Hz? What is the output frequency of a full-wave rectifier for the same input frequency?

Solution. Since in half-wave rectification, only one ripple is obtained per cycle at the output, output frequency of the half-wave rectifier = input frequency = 50 Hz.

Further, in full-wave rectification, as two ripples are obtained per cycle at the output, output frequency of the full-wave rectifier = 2 × input frequency = 2 × 50 Hz = 100 Hz.

ILLUSTRATION 31.2 For a CE-transistor amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2V. Suppose the current amplification factor of the transistor is 100. Find the input signal voltage and base current, if the base resistance is 1 kΩ.

Solution. Here; $V_0 = 2$ V, $\beta = 100$; R_C (collector resistance) = 2 kΩ, R_B = 1 kΩ

As $\frac{V_0}{V_i} = \beta\left(\frac{R_C}{R_B}\right)$;

$$V_i = V_0\left(\frac{R_B}{R_C}\right)\left(\frac{1}{\beta}\right) = (2V)\left(\frac{1k\Omega}{2k\Omega}\right)\left(\frac{1}{100}\right) = 0.01\text{ V}$$

ILLUSTRATION 31.3 In a p-n junction diode, the current I can be expressed as, $I = I_0\left[\exp\left(\frac{eV}{k_BT} - 1\right)\right]$ where I_0 is called the reverse saturation current, V is the voltage across the diode and is positive for forward bias and negative for reverse bias, and I is the current through the diode, k_B is the Boltzmann constant (8.6×10^{-5} eV/K) and T is the absolute temperature. If for a diode, $I_0 = 5 \times 10^{-12}$ A and $T = 300$ K, then

(a) what will be the forward current at a forward voltage of 0.6 V?

(b) what will be the increase in the current if the voltage across the diode is increased to 0.7 V?

(c) what is the dynamic resistance?

(d) what will be the current if reverse bias voltage changes from 1 V to 2 V?

Solution. (a) As $\frac{eV}{k_BT} = \frac{e\times 0.6\text{ V0}}{(8.6\times10^{-5}\text{eV/K})300\text{K}} = 23.26;$

$$\exp\left(\frac{eV}{k_BT}\right) = (23.26) = 1.259\times10^{10}$$

Thus, $I = I_0[1.259 \times 10^{10} - 1]$

$= (5 \times 10^{-12}\text{ A})(1.259 \times 10^{10}) = 0.063$ A

(b) When $V = 0.7$ V, $\frac{eV}{k_BT} = 27.13$ and

$$\exp\left(\frac{eV}{k_BT}\right) = \exp(27.13) = 6.07\times10^{11}$$

Thus, $I = (5 \times 10^{-12})(6.07 \times 10^{11} - 1) = 3.035$ A,

$\Delta I = 3.035\text{ A} - 0.063\text{ A} = 2.972$ A

(c) Dynamic resistance, $r_{dc} = \frac{\Delta V}{\Delta I} = \frac{(0.7-0.6)\text{V}}{2.972\text{A}} = 0.336\Omega$

(d) The reverse current is given by, $I = I_0\left[\exp\left(\frac{-eV}{k_BT}\right) - 1\right]$

When $V = 1$ V, $\frac{eV}{k_BT} = 38.76$ and

$I = I_0[\exp(-38.76) - 1] = -I_0$

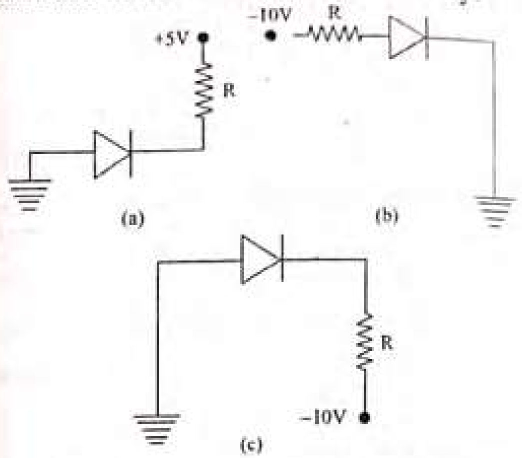
When $V = 2$V, $\frac{eV}{k_BT} = 77.52$ and

$I = I_0[\exp(-77.52) - 1] = -I_0$

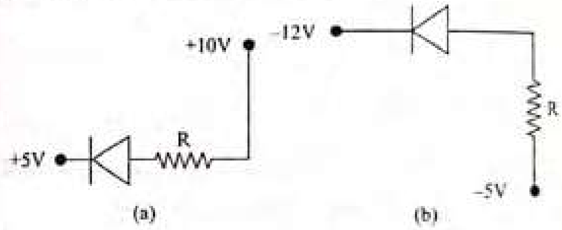
Thus, the reverse current remains almost constant (i.e., $I_0 = 5 \times 10^{-2}$ A) with change in voltage. This shows that in the reverse bias, the dynamic resistance is infinite.

CONCEPT APPLICATION EXERCISE 31.1

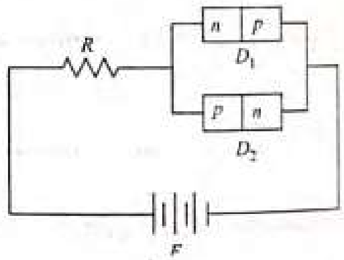
1. In the following circuits, which of the diodes is forward-biased and which is reverse-biased and why?



2. In the following circuits, which of the diodes is forward biased and which is reverse biased?



3. Figure below shows two p-n junctions diodes along with a resistance R and a d.c. battery E. Indicate the path and direction of flow of appreciable current in the circuit.

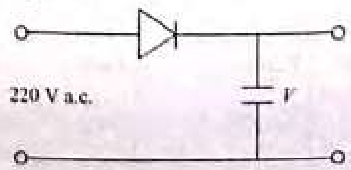


4. The following table provides the set of values of V and I, obtained for a given diode:

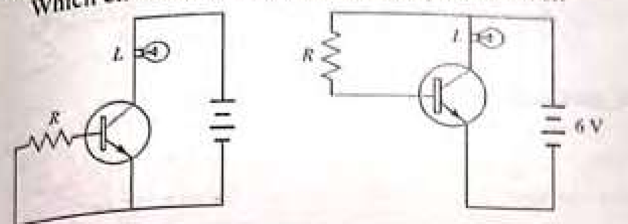
Forward biasing		Reverse biasing	
V	I	V	I
2 V	60 mA	0 V	0 μA
2.4 V	80 mA	−2 V	−0.25 μA

Assuming the characteristics to be nearly linear, over this range, calculate the forward and reverse bias resistance of the given diode.

5. A diode is connected to 220 V (r.m.s) a.c. in series with a capacitor as shown in figure. What is the voltage V across the capacitor?



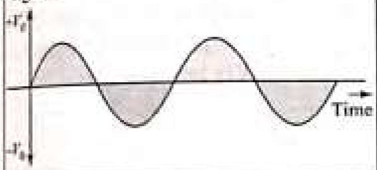
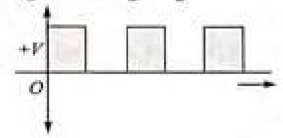
6. If only one of the circuits given below, the lamp L lights. Which circuit is it? Give reason for your answer.



DIGITAL ELECTRONICS

Voltage Signal and Binary System

Voltage signal

Analogue voltage signal	Digital voltage signal
The signal which represents the continuous variation of voltage with time is known as analogue voltage signal 	The signal which has only two values. i.e. either a constant high value of voltage or zero value is called digital voltage signal 

Binary system

- A number system which has only two digits i.e. 0 (low value) and 1 (high value) is known as binary system
- The electrical circuit which operates only in these two state, i.e., 1 (on or high) and 0 (i.e., off or low) are known as digital circuits.
- Different names for the two states of digital signals:

State Code	Name for the State							
1	On	Up	Closed	Excited	True	Pulse	High	Yes
0	Off	Down	Open	Unexcited	False	No pulse	Low	No

BOOLEAN ALGEBRA

In Boolean algebra only two states of variables (0 and 1) are allowed.

The variables (A, B, C, ...) of Boolean Algebra are subjected to three operations.

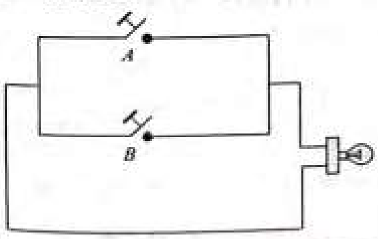
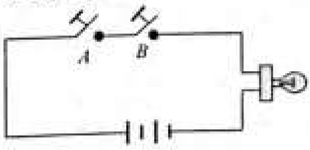
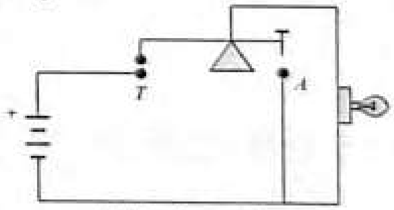
Basic Boolean Postulates and Laws

- Boolean Postulates : $0 + A = A$, $1 \cdot A = A$, $1 + A = 1$, $0 \cdot A = 0$, $A + \bar{A} = 1$
- Identity law: $A + A = A$, $A \cdot A = A$
- Negation law: $\bar{\bar{A}} = A$
- Commutative law: $A + B = B + A$, $A \cdot B = B \cdot A$
- Associative law: $(A + B) + C = A + (B + C)$, $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
- Distributive law: $A \cdot (B + C) = A \cdot B + A \cdot C$
- De Morgan's laws: $\overline{A+B} = \bar{A} \cdot \bar{B}$ and $\overline{A \cdot B} = \bar{A} + \bar{B}$ also $A + \bar{A}B = A + B$ and $A(\bar{A} + B) = AB$

LOGIC GATES AND TRUTH TABLE

Logic gate: The digital circuit that can be analysed with the help of Boolean algebra is called logic gate or logic circuit. A logic gate has two or more inputs but only one output.

There are primarily three logic gates namely the OR gate, the AND gate and the NOT gate.

	OR Operation	AND Operation	NOT Operation
(i)	Represented by (+) sign	Represented by (·) sign	Represented by bar over the variables
(ii)	Boolean expression $Y = A + B$ 	Boolean expression $Y = A \cdot B$ 	Boolean expression  $Y = \bar{A}$

Truth table: The operation of a logic gate or circuit can be represented in a table which contains all possible inputs and their corresponding outputs is called the truth table. To write the truth table, we use binary digits 1 and 0.

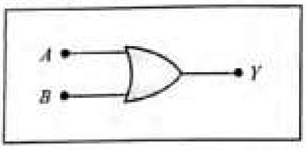
Different Logic Gates

OR Gate

(i) It has two inputs (A and B) and only one output (Y)

(ii) Boolean expression is $Y = A + B$

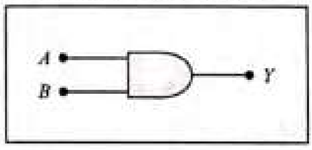
(iii) Truth table and logic symbol



A	B	$Y = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

AND Gate

(i) It has two inputs and one output **(ii)** Boolean expression is $Y = A \cdot B$ **(iii)** truth table and logic symbol



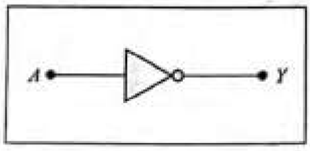
A	B	$Y = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

NOT Gate

(i) It has only one input and only one output

(ii) Boolean expression is $Y = \bar{A}$

(iii) Truth table and logic symbol :



A	$Y = \bar{A}$
0	1
1	0

COMBINATION OF LOGIC GATES

NAND Gate

From AND and NOT gate

Boolean expression and truth table : $Y = \overline{A \cdot B}$

A	B	$Y' = A \times B$	Y
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

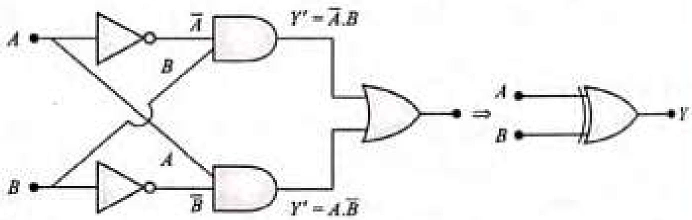
NOR Gate

Boolean expression and truth table : $Y = \overline{A + B}$

A	B	$Y' = A + B$	Y
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

XOR Gate

The logic gate which gives high output (i.e., 1) if either input A or input B but not both are high (i.e. 1) is called exclusive OR gate or the XOR gate. It may be noted that if both the inputs of the XOR gate are high, then the output is low (i.e., 0).

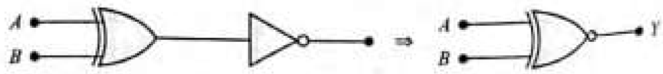


Boolean expression and truth table: $Y = A \oplus B = \bar{A}B + A\bar{B}$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive NOR (XNOR) Gate

XOR + NOT $\rightarrow$ XNOR



Boolean expression : $Y = A \odot B = \bar{A}\,\bar{B} + AB$

Logic Gates Using NAND Gate

The NAND gate is the building block of the digital electronics. All the logic gates like the OR, the AND and the NOT can be constructed from the NAND gates.

Construction of NOT Gate from NAND Gate

(i) When both the inputs (A and B) of the NAND gate are joined together then it works as the NOT gate.

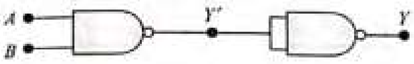
(ii) Truth table and logic symbol

Input	Output
$A = B$	Y
0	1
1	0

Construction of AND Gate from NAND Gate

(i) When the output of the NAND gate is given to the input of the NOT gate (made from the NAND gate), then the resultant logic gate works as the AND gate

(ii) Truth table and logic symbol

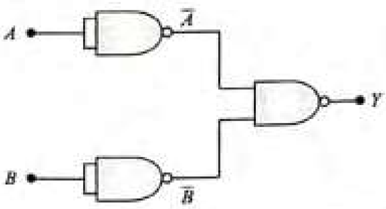


A	B	Y'	Y
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

Construction of OR Gate by NAND Gate

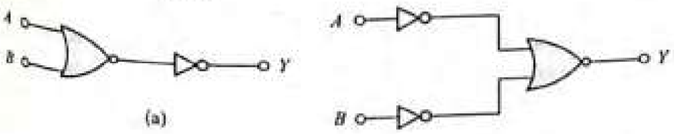
(i) When the outputs of two NOT gates (obtained from the NAND gate) is given to the inputs of the NAND gate, the resultant logic gate works as the OR gate

(ii) Truth table and logic symbol



A	B	$\bar{A}$	$\bar{B}$	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

ILLUSTRATION 31.4 You are given two circuits as shown in the figure below. Show that circuit (a) acts as OR gate while circuit (b) acts as AND gate.



(a) (b)

Solution. (a) The circuit consists of NOR gate followed by a NOT gate. The truth table is as follows:

A	B	A + B	$Y' = \overline{A+B}$	$Y = \overline{Y'}$
0	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	0	1

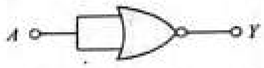
Since Y = A + B, the circuit acts as an OR gate.

(b) The circuit consists of two NOT gates followed by a NOR gate. The truth table is as follows.

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$	$Y = \overline{\bar{A}+\bar{B}}$
0	0	1	1	1	0
0	1	1	0	1	0
1	0	0	1	1	0
1	1	0	0	0	1

Since Y = A.B, the circuit acts as an AND gate.

ILLUSTRATION 31.5 Write the truth table for a NAND gate connected as given in figure.



Solution. The truth table of a NAND gate is as given below:

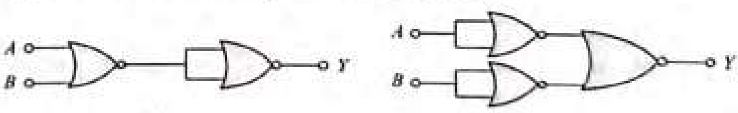
A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

When the two inputs of a NAND gates are joined as shown in figure, i.e., when A = B = 0 or A = B = 1, the above truth table of the NAND gate reduces to the one as given below:

A	B	Y
0	0	1
1	1	0

It is the truth table of a NOT gate. Therefore, the given circuit carries out the logic operation of a NOT gate.

ILLUSTRATION 31.6 You are given two circuits as shown in figure, which consist of NAND gates. Identify the logic operation carried out by the two circuits.



(a) (b)

Solution. Refer to Figure (a): Here, the output of NAND gate is connected to NOT gate (obtained from NAND gate). Let Y′ be the output of NAND gate and the final output of the combination of two gates be Y. The output of a NAND gate is 0 only when both the inputs are zero, while in NOT gate, the input gets inverted. Using these facts, the truth table for the given arrangement can be written as below:

A	B	Y	Y′
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

=

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

It is the truth table of AND gate. Therefore, the given circuit acts as AND gate.

Refer to Figure (b): Here, the outputs of two NOT gates (obtained from NAND gates) are connected to NAND gate. Let Y_1 and Y_2 be the outputs of the to NOT gates and the final output of the combination of three gates be Y. In a NOT gate, the input gets inverted, while the output of a NAND gate is 0 only when both the inputs are zero. Using these facts, the truth table for the given arrangement can be written as below:

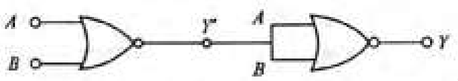
A	B	Y_1	Y_2	Y
0	0	1	1	0
1	0	0	1	1
0	1	1	0	1
1	1	0	0	1

=

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

It is the truth table of OR gate. Therefore, the given circuit acts as OR gate.

ILLUSTRATION 31.7 Write the truth table for circuit given in figure consisting of NOR gate and identify the logic operation (OR, AND, NOT) which this circuit is performing.

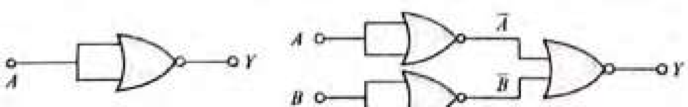


Solution. The first gate is a NOR gate. The second gate is also a NOR gate, with both the input terminals connected together. The truth table for the above circuit is as follows.

A	B	A + B	$Y = \overline{A+B}$	A = Y′	B = Y′	$Y = \overline{A+B}$
0	0	0	1	1	1	0
0	1	1	0	0	0	1
1	0	1	0	0	0	1
1	1	1	0	0	0	1

Since in this case $Y = \overline{A+B} = A + B$, the given circuit performs the function of an OR gate.

ILLUSTRATION 31.8 Write the truth table for the circuits given in figure consisting of NOR gates only. Identify the logic operations (OR, AND, NOT) performed by the two circuits.



(a) (b)

Solution. (a) The circuit of Figure (a) is a NOR gate with its input terminals connected together. The truth table for this circuit is as follows.

A	B = A	A + B	$Y = \overline{A+B}$
0	0	0	1
1	1	1	0

Since $Y = \overline{A+B} = \bar{A}$, a NOR gate with both its terminals connected together, performs the NOT operation.

(b) The inputs A and B are inverted by the two NOT gates [obtained from NOR gates as detailed in (a)].

These outputs $\bar{A}$ and $\bar{B}$ are fed to a NOR gate and the truth table is as follows:

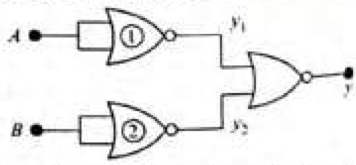
A	B	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$	$\overline{\bar{A}+\bar{B}}$
0	0	1	1	1	0
0	1	1	0	1	0
1	0	0	1	1	0
1	1	0	0	0	1

Since in the case $Y = \overline{\bar{A}+\bar{B}} = A \cdot B$, the circuit [Figure (b)] performs the function of an AND gate.

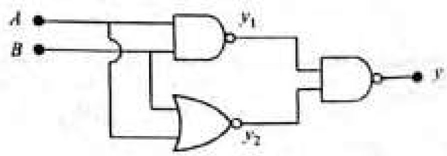
CONCEPT APPLICATION EXERCISE 31.2

1. Produce the truth table of the combination of two NAND gates arranged as shown in figure. Hence, show that it behaves as AND gate.

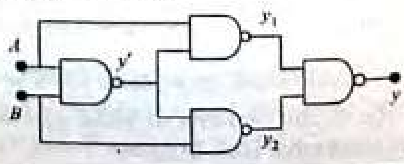
2. The inputs A and B are inverted by using two NOT gates and their outputs are fed to the NOR gate as shown in figure. Analyse the action of the gates 1 and 2 and identify the logic gate of the complete circuit so obtained. Give its symbol and the truth table.



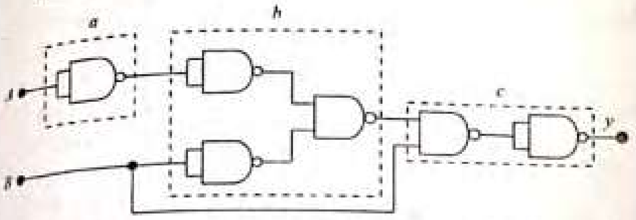
3. Produce the truth table of the combination of gate as shown in figure.



4. Produce the truth table of the combination of four NAND gates arranged as shown in figure. Hence, show that it behaves as XOR gate.



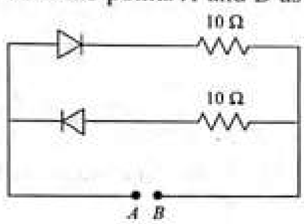
5. Identify which basic gate, OR, AND and NOT is represented by the circuits in the dotted line boxes, a, b and c shown in figure.



Give the truth table for the entire circuit for all possible values of A and B.

SOLVED EXAMPLES

1. A 2 V battery is connected across the points A and B as shown in the figure given below. Assuming that the resistance of each diode is zero in forward bias and infinity in reverse bias, the current supplied by the battery when its positive terminal is connected to A is

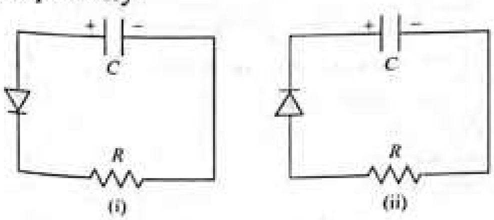


(a) 0.2 A (b) 0.4 A
(c) Zero (d) 0.1 A

Sol. (a) Since diode in upper branch is forward biased and in lower branch is reversed biased. So current through circuit $i = \frac{V}{R + r_d}$; here r_d = diode resistance in forward biasing = 0

$$\Rightarrow i = \frac{V}{R} = \frac{2}{10} = 0.2A.$$

2. Two identical capacitors A and B are charged to the same potential V and are connected in two circuits at $t = 0$, as shown in figure. The charge on the capacitors at time $t = CR$ are respectively



(a) VC, VC (b) $\frac{VC}{e}, VC$
(c) $\frac{VC}{e}, VC$ (d) $\frac{VC}{e}, \frac{VC}{e}$

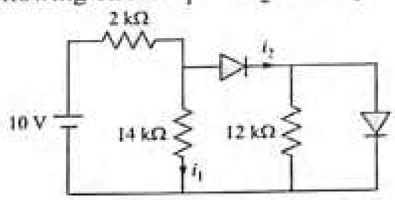
Sol. (b) Time $t = CR$ is known as time constant. It is time in which charge on the capacitor decreases to $\frac{1}{e}$ times of it's initial charge (steady state charge).

In Figure (i) PN junction diode is in forward bias, so current will flow the circuit i.e., charge on the capacitor decrease and in time t it becomes $Q = \frac{1}{e}(Q_0)$; where $Q_o = CV$

$$\Rightarrow Q = \frac{CV}{e}$$

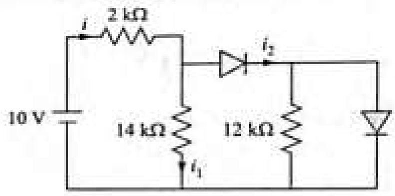
In Figure (ii) P-N junction diode is in reverse bias, so no current will flow through the circuit. Hence, change on capacitor will not decay and it remains same i.e. CV after time t.

3. In the following circuit I_1 and I_2 are respectively



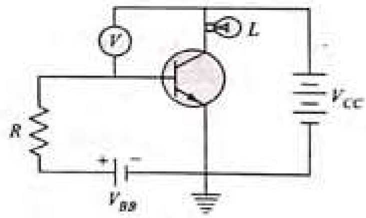
(a) 0, 0 (b) 5 mA, 5 mA
(c) 5 mA, 0 (d) 0, 5 mA

Sol. (d) Equivalent circuit can be redrawn as follows

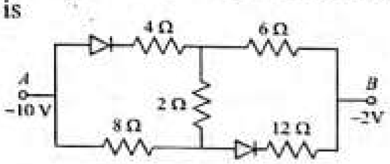


$$i = \frac{10}{2} = 5\text{ mA} = i_2$$

$$i_1 = 0$$

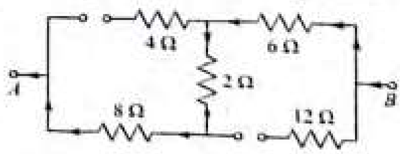


4. In the following circuit, the equivalent resistance between A and B is



(a) $\frac{20}{3}\Omega$ (b) $10\ \Omega$
(c) $16\ \Omega$ (d) $20\ \Omega$

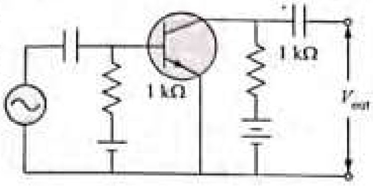
Sol. (c) According to the given figure A is at lower potential *w.r.t.* B. Hence both diodes are in reverse biasing, so equivalent, circuit can be redrawn as follows.



$\Rightarrow$ Equivalent resistance between A and B

$R = 8 + 2 + 6 = 16\ \Omega.$

5. In the following common emitter configuration an NPN transistor with current gain $\beta = 100$ is used. The output voltage of the amplifier will be



(a) 10 mV
(b) 0.1 V
(c) 1.0 V
(d) 10 V

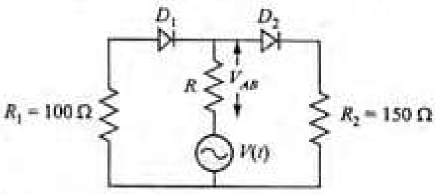
Sol. (c) Voltage gain $= \dfrac{\text{Output voltage}}{\text{Input voltage}}$

$\Rightarrow V_{out} = V_{in} \Rightarrow$ Voltage gain

$\Rightarrow V_{out} = V_{in} \Rightarrow$ Current gain $\Rightarrow$ Resistance gain

$= V_{in} \times \beta \times \dfrac{R_L}{R_{BE}} = 10^{-3} \times 100 \times \dfrac{10}{1} = 1\ \text{V}$

6. In the circuit given below, $V(t)$ is the sinusoidal voltage source, voltage drop $V_{AB}(t)$ across the resistance R

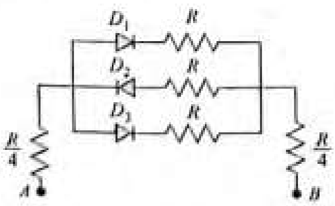


(a) is half wave rectified
(b) is full wave rectified
(c) has the same peak value in the positive and negative half cycles
(d) has different peak values during positive and negative half cycle

Sol. (d) In positive half cycle, one diode is in forward biasing and other is in reverse biasing while in negative half cycle their polarity reverses, and direction of current is opposite through R for positive and negative half cycles, so output is not rectified.

Since R_1 and R_2 are different hence the peaks during positive half and negative half of the input signal will be different.

7. In the following circuits, PN-junction diodes D_1, D_2 and D_3 are ideal for the following potential of A and B. The correct increasing order of resistance between A and B will be

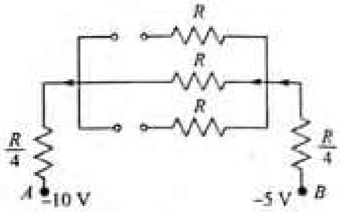


(i) -10 V, -5V
(ii) -5V, -10 V
(iii) -4V, -12V

(a) (i) < (ii) < (iii)
(b) (iii) < (ii) < (i)
(c) (ii) = (iii) < (i)
(d) (i) = (iii) < (ii)

Sol. (c) (i) $V_A = -10$ V and $V_B = -5$ V

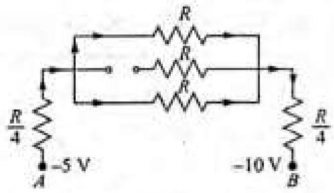
Diodes D_1 and D_3 are reveres biased and D_2 is forward biased.



$\Rightarrow R_{AB} = R + \dfrac{R}{4} + \dfrac{R}{4} = \dfrac{3}{2}R.$

(ii) When $V_A = -5$ V and $V_B = -10$ V

Diodes D_2 is reverse biased D_1 and D_3 are forward biased.

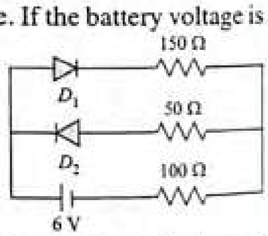


$\Rightarrow R_{AB} = \dfrac{R}{4} + \dfrac{R}{2} + \dfrac{R}{4} = R.$

(iii) In this case, equivalent resistance between A and B is also R.

Hence, (ii) = (iii) < (i).

8. The circuit shown in following figure contains two diode D_1 and D_2 each with a forward resistance of 50 ohms and with infinite backward resistance. If the battery voltage is 6 V, the current through the 100 ohms resistance (in amperes) is

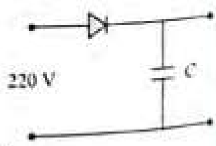


(a) Zero
(b) 0.02
(c) 0.03
(d) 0.036

Sol. (b) According to the given polarity, diode D_1 is forward biased while D_2 is reverse biased. Hence current will pass through D_1 only.

So current $i = \dfrac{6}{(150 + 50 + 100)} = 0.02$ A

9. A diode is connected to 220 V (rms) ac in series with a capacitor as shown in figure. The voltage across the capacitor is

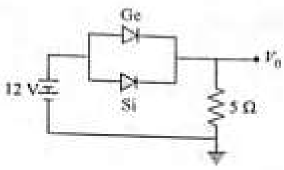


(a) 220 V
(b) 110 V
(c) 311.1 V
(d) $\dfrac{220}{\sqrt{2}}$ V

Sol. (d) The diode D will conduct for positive half cycle of $a.c.$ supply because this is forward biased. For negative half cycle of $a.c.$ supply, this is reverse biased and does not conduct. So out put would be half wave rectified and for half wave rectified out put

$$V_{rms} = \frac{V_0}{2} = \frac{200\sqrt{2}}{2} = \frac{200}{\sqrt{2}}$$

10. Ge and Si diodes conduct at 0.3 V and 0.7 V, respectively. In the following figure if Ge diode connection are reversed, the valve of V_0 changes by



(a) 0.2 V
(b) 0.4 V
(c) 0.6 V
(d) 0.8 V

Sol. (b) Consider the case when Ge and Si diodes are connected as show in the given figure.

Equivalent voltage drop across the combination Ge and Si diode = 0.3 V

$\Rightarrow$ Current $i = \dfrac{12-0.3}{5\text{k}\Omega} = 2.34\,\text{mA}$

$\therefore$ Out put voltage $V_0 = Ri = 5\ \text{k}\Omega \times 2.34\ \text{mA} = 11.7\ \text{V}$

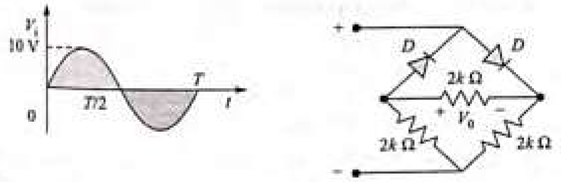
Now consider the case when diode connection are reversed. In this case voltage drop across the diode's combination = 0.7 V

$\Rightarrow$ Current $i = \dfrac{12-0.7}{5\text{k}\Omega} = 2.26\,\text{mA}$

$\therefore V_0 = iR = 2.26\ \text{mA} \times 5\text{k}\Omega = 11.3\ \text{V}$

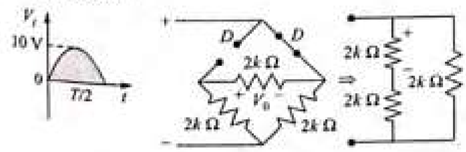
Hence charge in the value of $V_0 = 11.7 - 11.3 = 0.4$ V

11. In the circuit shown in figure, the maximum output voltage V_0 is



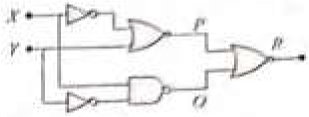
(a) 0 V
(b) 5 V
(c) 10 V
(d) $\dfrac{5}{\sqrt{2}}$ V

Sol. (b) For the positive half cycle of input the resulting network is shown below

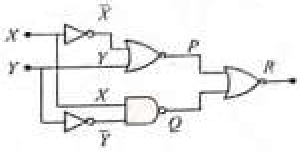


$$\Rightarrow (V_0)_{max} = \frac{1}{2}(V_i)_{max} = \frac{1}{2} \times 10 = 5\text{V}.$$

12. Figure gives a system of logic gates. From the study of truth table it can be found that to produce a high output (1) at R, we must have
(a) $X = 0, Y = 1$
(b) $X = 1, Y = 1$
(c) $X = 1, Y = 0$
(d) $X = 0, Y = 0$



Sol. (c)

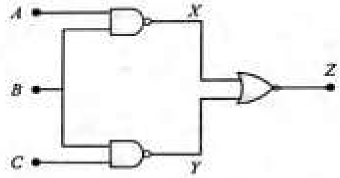


The truth table can be written as

X	Y	$\bar{X}$	$\bar{Y}$	$P = \overline{\bar{X}+Y}$	$Q = \overline{X.\bar{Y}}$	$R = \overline{P+Q}$
0	1	1	0	1	1	0
1	1	0	0	1	1	0
1	0	0	1	0	0	1
0	0	1	1	1	1	0

Hence $X = 1$, $Y = 0$ gives output $R = 1$

13. The shows two NAND gates followed by a NOR gate. The system is equivalent to the following logic gate



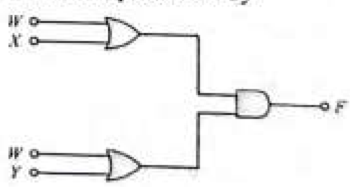
(a) OR
(b) AND
(c) NAND
(d) None of these

Sol. (b) The truth table of the circuit is given

A	B	C	$X = \overline{AB}$	$Y = \overline{BC}$	$Z = \overline{X+Y}$
0	0	0	1	1	0
1	0	0	1	1	0
0	0	1	1	1	0
1	0	1	1	1	0
0	1	0	1	1	0
1	1	0	0	1	0
0	1	1	1	0	0
1	1	1	0	0	1

Output Z of single three input gate is that of AND gate.

14. The diagram of a logic circuit is given below. The output F of the circuit is represented by



(a) $W(X+Y)$
(b) $W \cdot (X \cdot Y)$
(c) $W + (X \cdot Y)$
(d) $W + (X + Y)$

***Sol.* (c)** Output of upper OR gate = $W + X$

Output of lower OR gate = $W + Y$

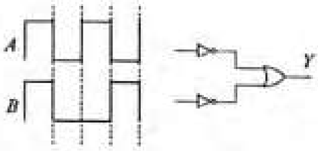
Net output $F = (W + X)(W + Y)$

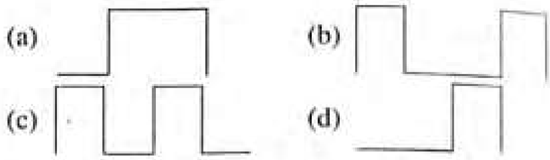
$= WW + WY + XW + XY$ (Since $WW = W$)

$= W(1 + Y) + XW + XY$ (Since $1 + Y = 1$)

$= W + XW + XY = W(1 + X) + XY = W + XY$

15. In a given circuit as shown the two input waveform A and B are applied simultaneously. The resultant waveform Y is





***Sol.* (a)** (1 = high, 0 = low)

Input to A is in the sequence, 1,0,1,0.

Input to B is in the sequence, 1, 0, 0, 1.

Sequence is inverted by NOT gate.

Thus inputs to OR gate becomes 0, 1, 0, 1 and output of OR gate becomes 0, 1, 1, 1

Since for OR gate $0 + 1 = 1$. Hence choice (a) is correct.

EXERCISES

Semiconductors and Diodes

1. Three semiconductors are arranged in the increasing order of their energy gap as follows. The correct arrangement is
(a) Tellurium, germanium, silicon
(b) Tellurium, silicon, germanium
(c) Silicon, germanium, tellurium
(d) Silicon, tellurium, germanium

2. Electrical conductivity of a semiconductor
(a) Decreases with the rise in its temperature
(b) Increases with the rise in its temperature
(c) Does not change with the rise in its temperature
(d) First increases and then decreases with the rise in its temperature

3. A doped semiconductor has impurity levels 30 meV below the conduction band. In a thermal collision, an amount kT of energy is given to the extra electron loosely bound to the impurity of ion. This electron just manages to be in the conduction band. The value of temperature T is
(a) 3 K (b) 34 K
(c) 34.8 K (d) 348 K

4. A pure semiconductor has equal electron and hole concentration of $10^{16}\ m^{-3}$. Doping by indium increases n_h to $4.5 \times 10^{22}\ m^{-3}$. What is n_e in the doped semiconductor?
(a) $10^6\ m^{-3}$ (b) $10^{22}\ m^{-3}$
(c) $\dfrac{10^{32}}{4.5\times10^{22}}\ m^{-3}$ (d) $4.5 \times 10^{22}\ m^{-3}$

5. A semiconductor device is connected in a serried circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, the current drops almost to zero. The device may be
(a) A p-type semiconductor
(b) An n-type semiconductor
(c) A p-n junction
(d) An intrinsic semiconductor

6. A p-n junction diode when forward-biased has a drop of 0.5 V. We want to see 1.5 V battery to forward-bias the diode. What resistance must be connected in series with the diode so that the maximum current does not exceed 1 mA?
(a) $10^{-3}\ \Omega$ (b) $10^{-6}\ \Omega$
(c) $10^3\ \Omega$ (d) $10^6\ \Omega$

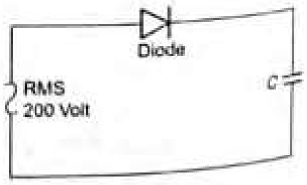
7. Barrier potential of a p-n junction diode does not depend on
(a) Diode design (b) Temperature
(c) Forward-biased (d) Doping density

8. Reverse bias applied to a junction diode
(a) lowers the potential barrier
(b) raises the potential barrier
(c) increases the majority carrier current
(d) increases the minority carrier current.

9. When on impurity is doped into an intrinsic semiconductor, the conductivity of the semiconductor
(a) Increases (b) Decreases
(c) Remains the same (d) Becomes zero

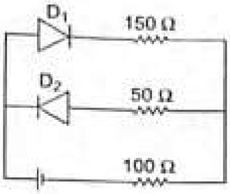
10. When a p-n junction diode is forward-biased, energy is released at the junction due to the recombination of electrons and holes. This energy is in
(a) Visible region (b) Infrared region
(c) UV region (d) X-ray region

11. A sinusoidal voltage of rms value 200 V is connected to the diode and capacitor C in the circuit shown so that half-wave rectification occurs. The final potential difference in volt, across C, is



(a) 500 (b) 200
(c) 283 (d) 141

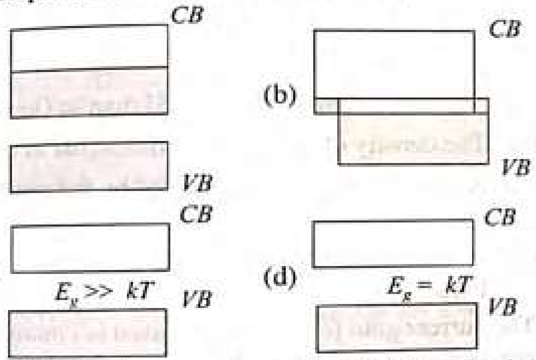
12. The circuit shown in the following figure contains two diodes, each with a forward resistance of 50 Ω and with infinite backward resistance. If the battery voltage is 6V, the current through 100 Ω resistance (in amperes) is:



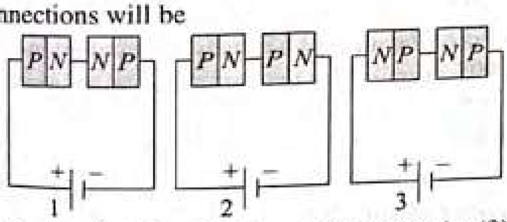
(a) Zero (b) 0.02
(c) 0.03 (d) 0.036

13. Let n_p and n_e be the number of holes and conduction electrons, respectively, in a semiconductor. Then
(a) $n_p > n_e$ in an intrinsic semiconductor
(b) $n_p = n_e$ in an extrinsic semiconductor
(c) $n_p = n_e$ in an intrinsic semiconductor
(d) $n_e > n_p$ in an intrinsic semiconductor

14. Which of the energy band diagrams shown in the figure corresponds to that of a semiconductor?



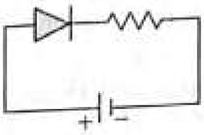
15. Two *PN* junctions can be connected in series by three different methods as shown in the figure. If the potential difference in the junctions is the same, then the correct connections will be

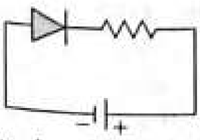


(a) in circuits (1) and (2) (b) in circuits (2) and (3)
(c) in circuits (1) and (3) (d) only in circuit (1)

16. Which one is in forward bias?

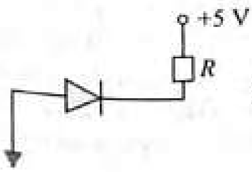
(a) 

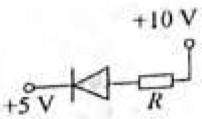
(b) 

(c) 

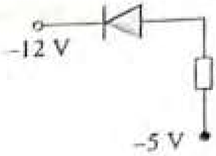
(d) None of these

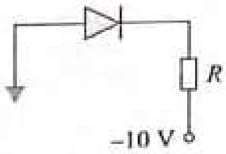
17. In the given figure, which of the diodes are forward biased?

(1) 

(2) 

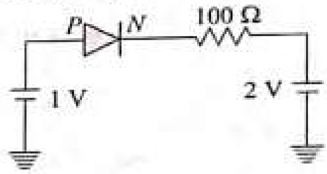
(3) 

(4) 

(5) 

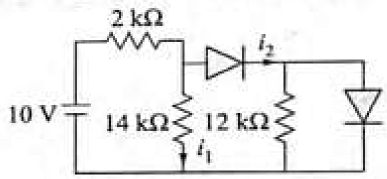
(a) 1, 2, 3 (b) 2, 4, 5
(c) 1, 3, 4 (d) 2, 3, 4

18. The current through an ideal *P-N* junction shown in the following circuit diagram will be



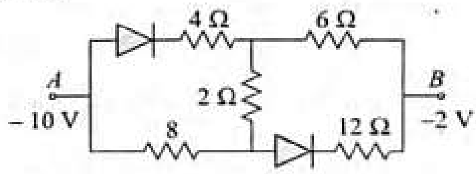
(a) zero (b) 1 mA
(c) 10 mA (d) 30 mA

19. In the following circuit, find I_1 and I_2.



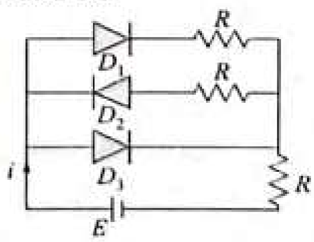
(a) 0, 0 (b) 5 mA, 5 mA
(c) 5 mA, 0 (d) 0, 5 mA

20. In the following circuit, the equivalent resistance between *A* and *B* is



(a) $\frac{20}{3}$ Ω (b) 10 Ω
(c) 16 Ω (d) 20 Ω

21. In the following circuit of *PN* junction diodes D_1, D_2 and D_3 are ideal, then *i* is



(a) E/R (b) $E/2R$
(c) $2E/3R$ (d) Zero

Junction Transistor

22. In *NPN* transistor, if doping in base region is increased, then collector current
    (a) increases (b) decreases
    (c) remain same (d) none of these

23. In an *NPN* transistor, the collector current is 24 mA. If 80% of electrons reach collector, its base current in mA is
    (a) 36 (b) 26
    (c) 16 (d) 6

24. A triode has a plate resistance of 10 kΩ and amplification factor 24. If the input signal voltage is 0.4 V (r.m.s.), and the load resistance is 10 kΩ, then, the output voltage (r.m.s.) is
    (a) 4.8 V (b) 9.6 V
    (c) 12.0 V (d) none of these

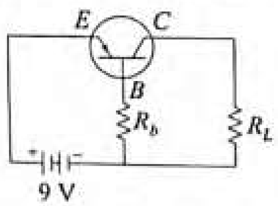
25. In a *PNP* transistor working as a common-base amplifier, current gain is 0.96 and emitter current is 7.2 mA. The base current is
    (a) 0.4 mA (b) 0.2 mA
    (c) 0.29 mA (d) 0.35 mA

26. For a transistor, the current amplification factor is 0.8. The transistor is connected in common emitter configuration. The change in the collector current when the base current changes by 6 mA is
    (a) 6 mA (b) 4.8 mA
    (c) 24 mA (d) 8 mA

27. In a common base amplifier circuit, calculate the change in base current if that in the emitter current is 2 mA and $\alpha = 0.98$.
    (a) 0.04 mA (b) 1.96 mA
    (c) 0.98 mA (d) 2 mA

28. In case of *NPN*-transistors, the collector current is always less than the emitter current because
    (a) collector side is reverse biased and emitter side is forward biased
    (b) after electrons are lost in the base and only remaining ones reach the collector
    (c) collector side is forward biased and emitter side is reverse biased
    (d) collector being reverse biased attracts less electrons

29. In a transistor circuit shown here the base current is 35 μA. The value of the resistor R_b is

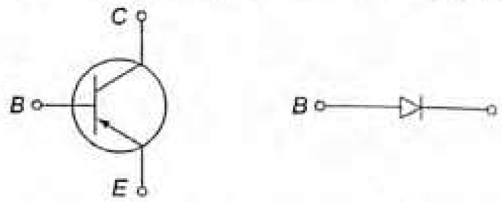


    (a) 123.5 kΩ
    (b) 257 kΩ
    (c) 380.05 kΩ
    (d) None of these

30. For a transistor, in a common emitter arrangement, the alternating current gain β is given by
    (a) $\beta = \left(\dfrac{\Delta I_C}{\Delta I_B}\right)_{V_C}$ (b) $\beta = \left(\dfrac{\Delta I_B}{\Delta I_C}\right)_{V_C}$
    (c) $\beta = \left(\dfrac{\Delta I_C}{\Delta I_E}\right)_{V_C}$ (d) $\beta = \left(\dfrac{\Delta I_E}{\Delta I_C}\right)_{V_C}$

31. The relation between α and β parameters of current gains for a transistor is given by
    (a) $\alpha = \dfrac{\beta}{1-\beta}$ (b) $\alpha = \dfrac{\beta}{1+\beta}$
    (c) $\alpha = \dfrac{1-\beta}{\beta}$ (d) $\alpha = \dfrac{1+\beta}{\beta}$

32. Which of the following statements is incorrect?



(a) (b)

    (a) The energy gap is larger in Si than in Ge
    (b) The density of Ge is over 2 times that of Si
    (c) A *npn* transistor is represented by the symbol shown in Figure (a).
    (d) A diode is represented by the symbol shown in Figure (b).

33. The current gain for a transistor used in common-emitter configuration is 98. If the load resistance is 1 MΩ and the internal resistance is 60 Ω, what is the voltage gain
    (a) 90 (b) 95
    (c) 100 (d) None of the above

34. The current gain α of a transistor in common-base mode is 0.995. Its gain β in the common-emitter mode is nearly
    (a) 9.5 (b) 1.005
    (c) 200 (d) 100

35. A transistor is used in a common-emitter mode in an amplifier circuit. When a voltage of 20 mV is added to the base-emitter voltage, the base current changes by 20 μA and the collector current changes by 2 mA. The load resistor is 5 kΩ. What is the volume of β ?
    (a) 10 (b) 100
    (c) 1000 (d) 10^6

36. In question 35, the input resistance is
    (a) 1 kΩ (b) 2 kΩ
    (c) 3 kΩ (d) 4 kΩ

37. In question 35, the transconductance is
    (a) 0.1 mho (b) 0.2 mho
    (c) 0.3 mho (d) 0.4 mho

38. In a silicon transistor, the base current is changed by 20 μA. This results in a change of 0.02 V in base to emitter voltage and a change of 2 mA in the collector current. The transistor is used as an amplifier with the load resistance of 5 kΩ. The input resistance of the transistor is

(a) 0.5 kΩ (b) 1 kΩ
(c) 1.5 kΩ (d) 2 kΩ

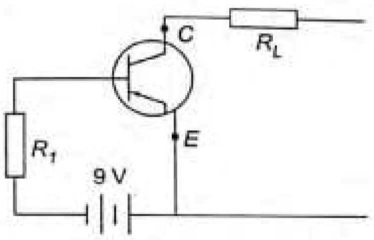
39. In question 38, the ac current gain is
(a) 0 (b) 1
(c) 0.1 (d) 10000

40. In question 38, the transconductance in Ω^{-1} is
(a) 0 (b) 0.2
(c) 0.1 (d) 0.05

41. In question 38, the voltage gain of the amplifier is
(a) 5 (b) 50
(c) 500 (d) 5000

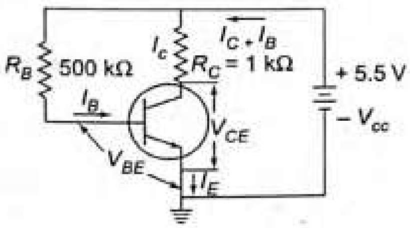
42. In question 38, the change in output across load is
(a) 5 V (b) 10 V
(c) 15 V (d) 16 V

43. In the circuit shown, the base current is 30 μA. The value of R_1 is



(a) 3 kΩ (b) 30 kΩ
(c) 300 kΩ (d) 3×10^4 kW

44. In the circuit shown in the figure, the base current I_B is 10 μA and the collector is 5.2 mA. The value of V_{BE} is

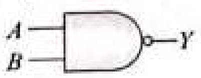


(a) 0.1 V (b) 0.3 V
(c) 0.5 V (d) 0.7 V

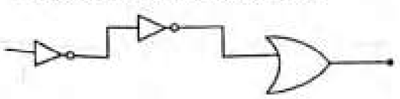
Digital Electronics

45. Symbol  represents
(a) NAND gate (b) NOR gate
(c) NOT gate (d) XNOR gate

46. Which gate is represented by this figure?
(a) NAND gate
(b) AND gate
(c) NOT gate
(d) OR gate



47. Identify the gate from the following:



(a) NOT gate (b) AND gate
(c) OR gate (d) None of these

48. The following truth table corresponds to the logic gate:

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

(a) NAND (b) AND
(c) XOR (d) OR

49. Which of the following gates corresponds to the truth table given below?

A	B	X
1	1	0
1	0	1
0	1	1
0	0	1

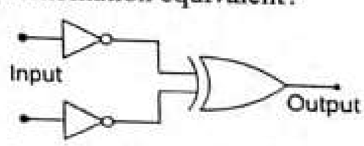
(a) XOR (b) OR
(c) NAND (d) NOR

50. How many NAND gate are used to form AND gate?
(a) 1 (b) 2
(c) 3 (d) 4

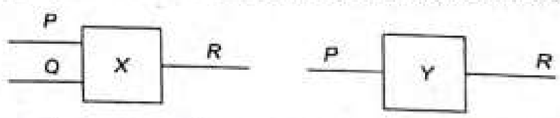
51. Sum of the two binary numbers $(1000010)_2$ and $(11011)_2$ is
(a) $(111101)_2$ (b) $(111111)_2$
(c) $(101111)_2$ (d) $(111001)_2$

52. Which of the following is not correct?
(a) $\overline{\overline{A}.\overline{B}} = A + B$ (b) $\overline{\overline{A}.\overline{B}} = A.B$
(c) $\overline{(\overline{A.B}).(\overline{A.B})} = A.B + A.B$ (d) $\bar{1} + \bar{1} = 1$

53. The figure shows a combination of logic gates.
To what single gate is this combination equivalent?
(a) EX-NOR
(b) NOR
(c) EX-OR
(d) OR



54. Logic gates X and Y have the truth tables shown below

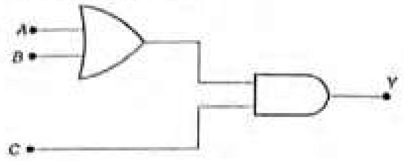


P	Q	R	P	R
0	0	0	0	1
1	0	0	1	0
0	1	0		
1	1	1		

When the output of X is connected to the input of Y, the resulting combination is equivalent to a single

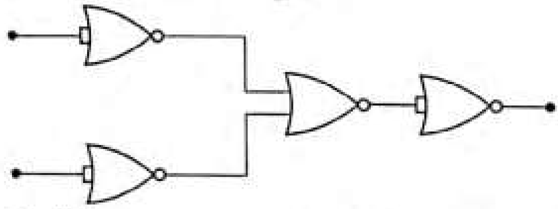
(a) NOT gate (b) OR gate
(c) NOR gate (d) NAND gate

55. In order to obtain an output $Y = 1$ from the circuit of the figure, the inputs must be



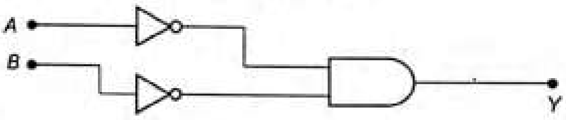
	A	B	C
(a)	0	1	0
(b)	1	0	0
(c)	1	0	1
(d)	1	1	0

56. Identify the gate in the figure.



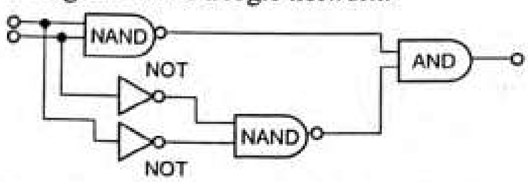
(a) AND (b) XOR
(c) NOR (d) NAND

57. What is the output in the figure?



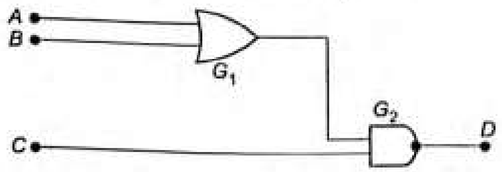
(a) $\overline{A.B}$ (b) $\overline{\bar{A}.\bar{B}}$
(c) $\overline{\overline{A.B}}$ (d) $\bar{A}.\bar{B}.$

58. The diagram shows a logic network.



Which single gate is equivalent to the network?
(a) EX-NOR (b) NOR
(c) EX-OR (d) OR

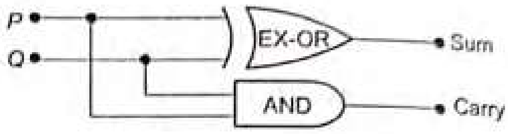
59. For the given combination of gates, if the logic states of inputs A, B, C are as follows $A = B = C = 0$ and $A = B = 1$, $C = 0$, then the logic states of output D are

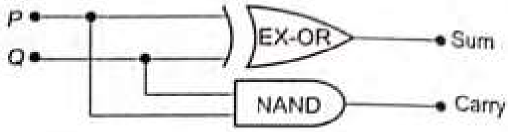


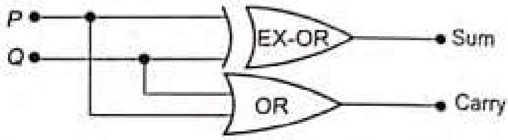
(a) 0, 0 (b) 0, 1
(c) 1, 0 (d) 1, 1

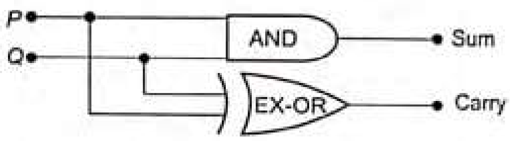
60. How many NAND gates are required to get a half adder.
(a) 3 (b) 5
(c) 9 (d) 7

61. Which circuit adds two bits P and Q to provide the correct range of outputs for a half-adder?

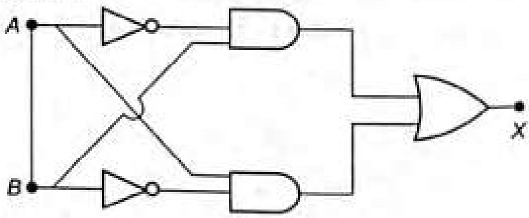
(a)


(b)


(c)


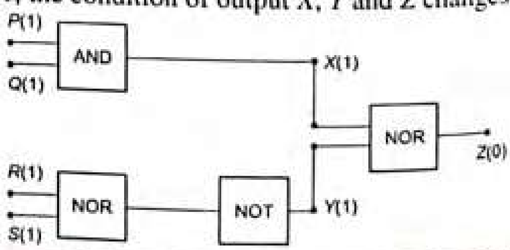
(d)


62. With reference to the figure which of the following is possible?



(a) $A = 0, B = 0, X = 1$ (b) $A = 0, B = 1, X = 0$
(c) $A = 0, B = 0, X = 0$ (d) $A = 1, B = 1, X = 0$

63. The circuit diagram (see the figure) shows a logic combination with the states outputs X, Y and Z given for inputs P, Q, R and S at state 1 (i.e., high). When inputs P and R change to state 0, (i.e., low) with inputs Q and S still at 1, the condition of output X, Y and Z changes to



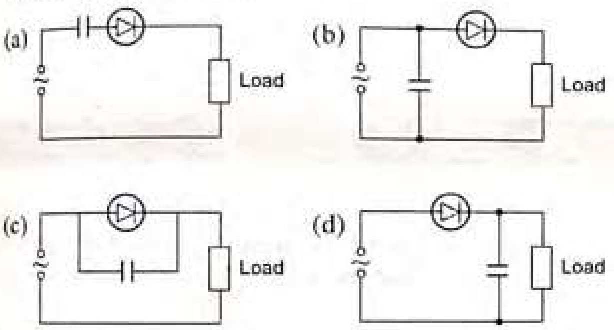
	(a)	(b)	(c)	(d)
X	1	1	0	0
Y	0	1	1	0
Z	0	1	0	1

Problems Based on Mixed Concepts

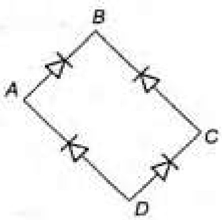
64. The process of negative feedback in electronics can involve returning a fraction of the output of an operational amplifier to the input. This may affect the gain and the bandwidth (the range of frequency over which the gain is constant). Which one of the following combinations of effects is correct?

	Effect on gain	Effect on bandwidth
(a)	Increased	Decreased
(b)	Decreased	Decreased
(c)	Increased	Unchanged
(d)	Decreased	Increased

65. A capacitor is to be used to provide smoothing for a half wave rectifier. In which of the following diagrams is the capacitor correctly connected?



66. In the figure, the input is across terminals A and C and the output is across B and D. Then the output is



(a) Zero
(b) Same as the input
(c) Full wave rectified
(d) Half-wave rectified.

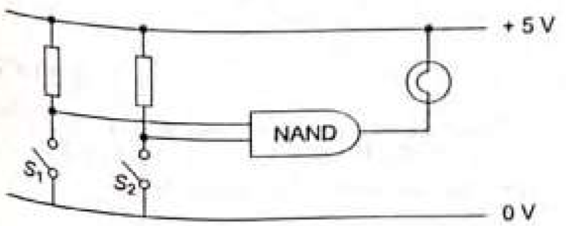
67. In a common-emitter amplifier, using output resistance of 5000 ohm and input resistance of 2000 ohm, if the peak value of input signal voltage is 10 mV and $\beta = 50$, then the peak value of output voltage is

(a) 5×10^{-6} V (b) 2.5×10^{-4} V
(c) 1.25 V (d) 125 V

68. In question 67, the power gain is

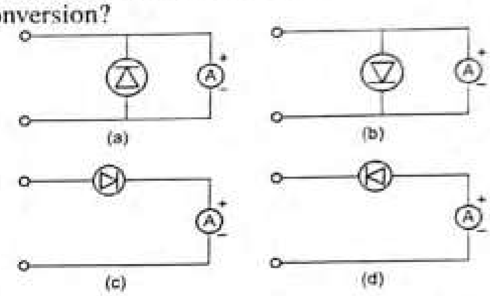
(a) 125×50 (b) $\frac{125}{90}$
(c) 1.25×50 (d) 2.5×10^4

69. The figure shows a circuit designed to control a lamp. For what positions of the switches S_1 and S_2 will the lamp be lit?

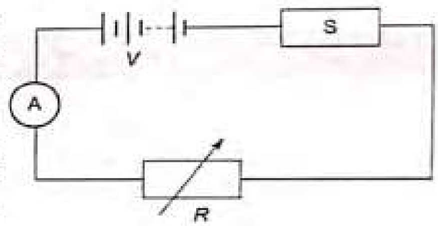


	S_1	S_2
(a)	Closed	Closed
(b)	Closed	Open
(c)	Open	Closed
(d)	Either open or closed	Either open or closed

70. A moving-coil ammeter is to be adapted to detect small alternating currents. Which of the following diagrams shows how a diode could be connected in order to make the conversion?



71. Figure shows a piece of semiconductor (pure one) S in series with a variable resistor R and a source of constant voltage V. S is heated and the current is kept constant by adjustment of R. Which of the following factors will decrease during this process?

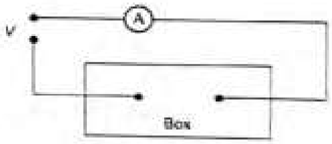


1. The drift velocity of the conduction electrons in S
2. The dc resistance of S
3. The number of conduction electrons in S

Which of the following is correct?

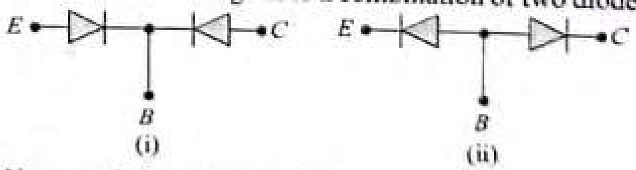
(a) Only 1 (b) 1 and 2
(c) 1, 2 and 3 (d) Only 3

72. A semiconductor diode and a resistor of constant resistance are connected in some way inside a box having two external terminals. When a potential difference V of 1 V is applied, $I = 25$ mA. If potential difference is reversed, $I = 50$ mA. Forward resistance and diode resistance are



(a) 40 Ω, 20 Ω (b) 40 Ω, 40 Ω
(c) 0 Ω, ∞ (d) 6 Ω, 12 Ω

73. A transistor is analogous to a combination of two diodes.

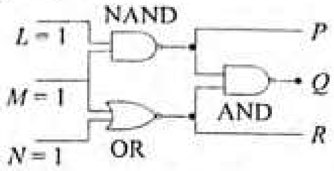


Now, mark the correct option.

(a) (i) is for p-n-p and (ii) is n-p-n transistor
(b) (i) is for p-n-p and (ii) for p-n-p transistor

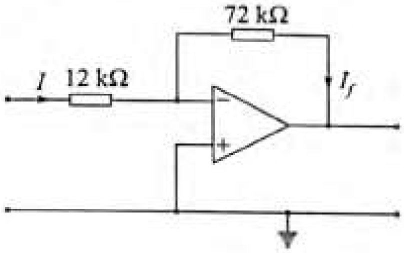
(c) Both (i) and (ii) are for *n-p-n* transistor
(d) Both (i) and (ii) are for *p-n-p* transistor

74. The diagram shows a logic network.
If the inputs L, M and N are all at logic 1, what are the logic states of P, Q and R?



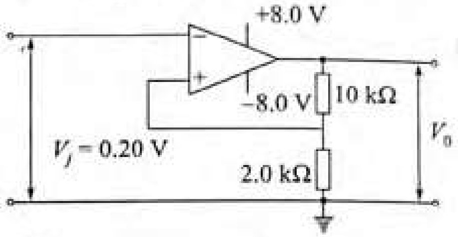
(a) –6 (b) I
(c) $-I/6$ (d) $6I$

75. In the circuit shown in the diagram, the operational amplifier may be assumed to be ideal. The current in the 12 kΩ resistor is I.



What is the current I_f in the 72 kΩ resistor ?
(a) –6 (b) I
(c) $-1/6\ I$ (d) $6I$

76. An input voltage V_i of 0.20 V is applied to an operational amplifier connected as shown in the diagram.
What is the output voltage V_0?



(a) 0.20 V (b) 1.2 V
(c) 0.80 V (d) 8.0 V

ARCHIVES

1. A piece of copper and another of germanium are cooled to 77 K. The resistance of
(a) each of them decreases.
(b) each of them increases.
(c) of Cu decreases and Ge increases.
(d) of Cu increases and Ge decreases. **(AIEEE 2004)**

2. The electrical conductivity of a semiconductor increases when electromagnetic radiations of wavelength shorter than 2480 nm is incident on it. The band gap (in eV) for the semiconductor is
(a) 0.7 eV (b) 0.5 eV
(c) 0.6 eV (d) 1.1 eV **(AIEEE 2005)**

3. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be
(a) 70.7 Hz (b) 100 Hz
(c) 25 Hz (d) 50 Hz **(AIEEE 2005)**

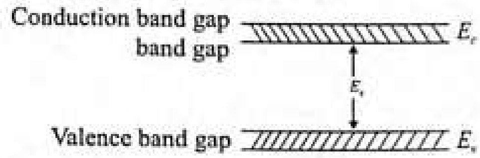
4. In a common base amplifier, the phase difference between the input signal voltage and output voltage is
(a) $\frac{\pi}{2}$ (b) 0
(c) π (d) $\frac{\pi}{4}$ **(AIEEE 2005)**

5. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by
(a) van der Waals binding (b) metallic binding
(c) ionic binding (d) covalent binding
(AIEEE 2006)

6. If the ratio of concentration of electrons to that of holes in a semiconductor is 7/5 and the ratio of currents is 7/4, what is the ratio of their drift velocities?
(a) $\frac{4}{7}$ (b) $\frac{5}{8}$
(c) $\frac{4}{5}$ (d) $\frac{5}{4}$ **(AIEEE 2006)**

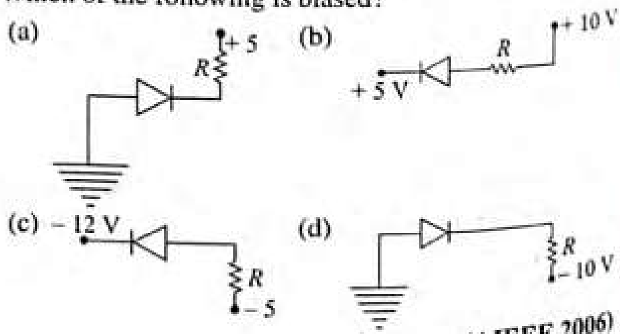
7. In the common base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be
(a) 48 (b) 49
(c) 50 (d) 51 **(AIEEE 2006)**

8. If the lattice constant of this semiconductor is decreased, then which of the following is correct?



(a) All E_c, E_g, and E_v decrease.
(b) All E_c, E_g, and E_v increase.
(c) E_c and E_v increase, but E_g decreases.
(d) E_c and E_v decrease, but E_g increases.
(AIEEE 2006)

9. Which of the following is biased?

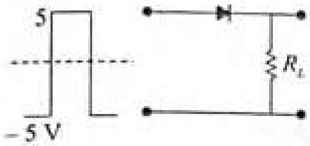


(AIEEE 2006)

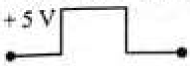
10. Carbon, silicon, and germanium have four valence electrons each. At room temperature, which one of the following statements is most appropriate?

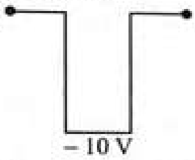
(a) The number of free conduction electrons is negligibly small in all the three.
(b) The number of free electrons for conduction is significant in all the three.
(c) The number of free electrons for conduction is significant only in Si and Ge but small in C.
(d) The number of free conduction electrons is significant in C but small in Si and Ge. **(AIEEE 2007)**

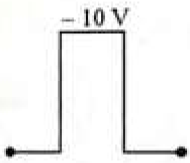
11. If in a *pn* junction diode, a square input signal of 10 V is applied as shown,

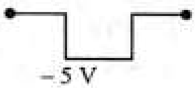


then the output signal across R_L will be

(a) 

(b) 

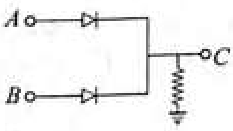
(c) 

(d) 

(AIEEE 2007)

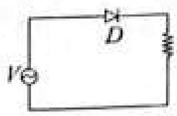
12. A working transistor with its three legs marked P, Q, and R is tested using a multimeter, no conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor?
(a) It is a *pnp* transistor with R as emitter.
(b) It is an *npn* transistor with R as collector.
(c) It is an *npn* transistor with R as base.
(d) It is *pnp* transistor with R as collector. **(AIEEE 2008)**

13. In the circuit shown, A and B represent two inputs and C represents the output. The circuit represents

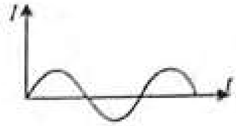


(a) NAND gate (b) OR gate
(c) NOR gate (d) AND gate **(AIEEE 2008)**

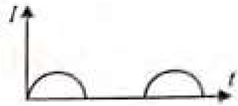
14. A *pn* junction (D) shown in the figure can act as a rectifier. An alternating current source (V) is connected in the circuit.

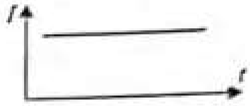


The current (I) in the resistor R can be shown by

(a) 

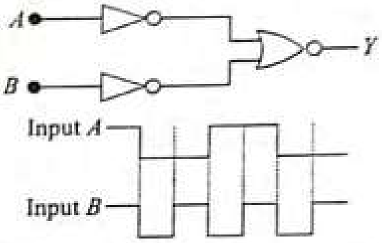
(b) 

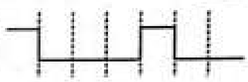
(c) 

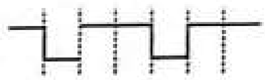
(d) 

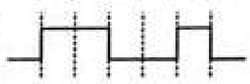
(AIEEE 2009)

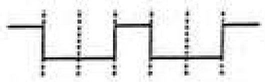
15. The logic circuit below has the input waveforms A and B as shown. Pick out the correct output waveform.



(a) 

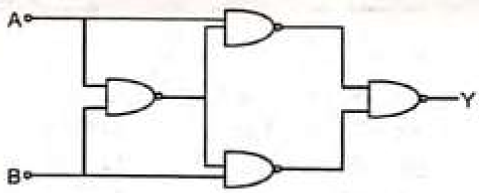
(b) 

(c) 

(d) 

(AIEEE 2009)

16. The truth table for system of four NAND gates as shown in figure is



(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

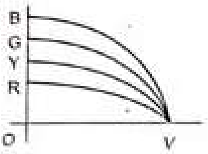
(AIEEE 2012)

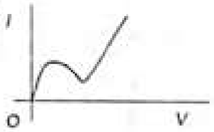
17. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kΩ. Find the maximum modulated frequency which could be detected by it.
(a) 10.62 kHz (b) 5.31 MHz

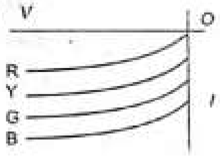
(c) 5.31 kHz (d) 10.62 MHz

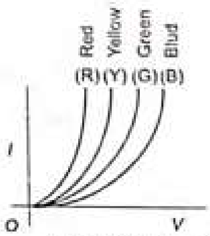
(JEE Main 2013)

18. The *I*–*V* characteristic of an LED is

(a) 

(b) 

(c) 

(d) 

(JEE Main 2013)

19. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring $\pm$ 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

(a) 0.5 mA (b) 0.05 mA
(c) 0.2 mA (d) 0.02 mA

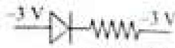
(JEE Main 2014)

20. The forward-biased diode connection is

(a) 

(b) 

(c) 

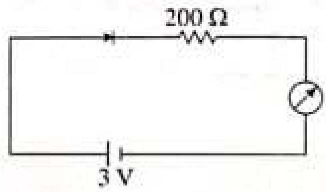
(d) 

(JEE Main 2014)

21. In a common emitter amplifier circuit using an *n-p-n* transistor, the phase difference between the input and the output voltages will be

(a) 135° (b) 180°
(c) 45° (d) 90° **(JEE Main 2017)**

22. The reading of the ammeter for a silicon diode in the given circuit is



(a) 13.5 mA (b) 0 (zero)
(c) 15 mA (d) 11.5 mA

(JEE Main 2018)

ANSWER KEY

Exercises

1. (a)	2. (b)	3. (d)	4. (c)	5. (c)	6. (c)	7. (a)	8. (b)	9. (a)	10. (b)
11. (c)	12. (b)	13. (c)	14. (d)	15. (b)	16. (b)	17. (b)	18. (a)	19. (d)	20. (c)
21. (a)	22. (b)	23. (d)	24. (a)	25. (c)	26. (c)	27. (a)	28. (b)	29. (b)	30. (a)
31. (b)	32. (c)	33. (d)	34. (c)	35. (b)	36. (a)	37. (a)	38. (b)	39. (a)	40. (c)
41. (b)	42. (c)	43. (c)	44. (c)	45. (b)	46. (a)	47. (d)	48. (d)	49. (c)	50. (b)
51. (a)	52. (d)	53. (c)	54. (d)	55. (c)	56. (d)	57. (d)	58. (c)	59. (b)	60. (d)
61. (a)	62. (d)	63. (c)	64. (d)	65. (d)	66. (c)	67. (c)	68. (a)	69. (d)	70. (c)
71. (b)	72. (b)	73. (a)	74. (b)	75. (b)	76. (b)				

Archives

1. (b)	2. (b)	3. (b)	4. (b)	5. (d)	6. (d)	7. (b)	8. (d)	9. (a)	10. (b)
11. (a)	12. (d)	13. (b)	14. (c)	15. (a)	16. (a)	17. (c)	18. (d)	19. (c)	20. (c)
21. (b)	22. (d)								

Chapter 32

Communication Systems

INTRODUCTION

The term communication refers to the transmitting, receiving, and processing of information by electronic means.

BASIC COMMUNICATION SYSTEM

A basic communication system consists of an information source, a transmitter, a link, and a receiver.

Information The idea/message that is to be conveyed is information. The message may be individual one or a set of messages. It may be a symbol, code, group of words, or any pre-decided unit.

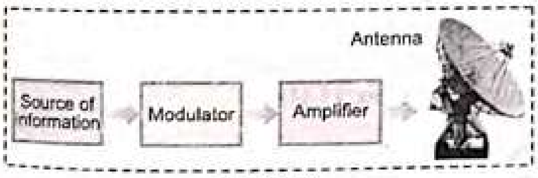
Transmitter In radio transmission, the transmitter consists of a transducer, modulator, amplifier, and transmitting antenna.

Transducer: Converts sound signals into electric signal.

Modulator: Mixing of audio electric signal with high frequency radio wave.

Amplifier: Boosting the power of modulated signal.

Antenna: Signal is radiated in the space with the aid of an antenna.



Transmitter

Communication channel The function of communication channel is to carry the modulated signal from transmitter to receiver. The communication channel is also called transmission medium or link.

The term channel refers to the frequency range allocated to a particular service or transmission.

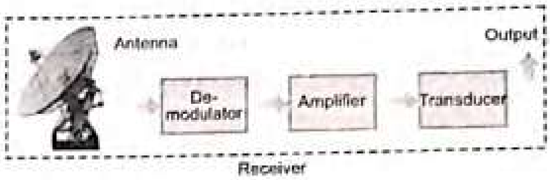
Receiver A receiver consists of the following components:

Pickup antenna: To pick the signal

Demodulator: To separate out the audio signal from the modulated signal

Amplifier: To boost up the weak audio signal

Transducer: To convert back audio signal in the form of electrical pulses into sound waves.



Receiver

TYPES OF COMMUNICATION SYSTEM

Communication systems can be classified according to the nature of information or mode of transmission or types of transmission channel or types of modulation.

Classification according to the nature of information source

1. Speech transmission
2. Picture transmission
3. Facsimile transmission (FAX): This involves exact reproduction of a document or picture which are static.

Classification according to the mode of transmission

1. *Analog communication*: The communication system which makes use of analog signals is called analog communication system.
2. *Digital communication*: In this system, digital signals are used.

Classification according to the transmission channel

1. Line communication
2. Space communication

Classification according to the type of modulation

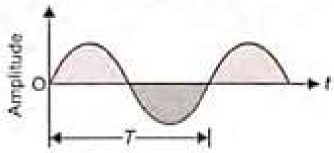
1. Amplitude modulation (AM)
2. Frequency modulation (FM)
3. Phase modulation (PM)
4. Pulse amplitude modulation (PAM)
5. Pulse time modulation (PTM)
6. Pulse code modulation (PCM)

ANALOG AND DIGITAL SIGNALS

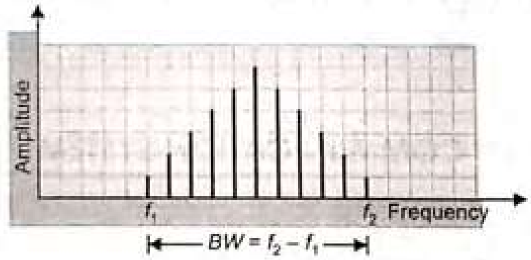
In communication system, a signal means a time, varying electrical signal containing informations.

Analog signals It is a continuous wave form which changes smoothly over time.

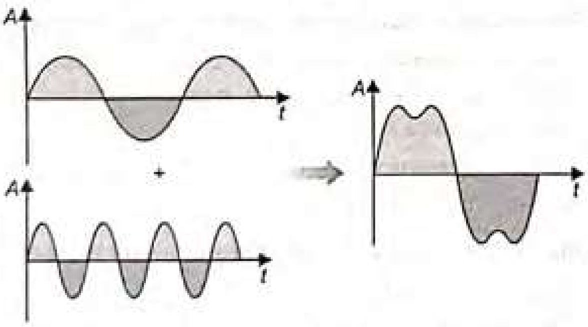
- Such signals can be easily generated from the source of information by using an appropriate transducer, e.g., pressure variations in the sound waves can be converted into corresponding current or voltage pulses with the help of a microphone.
- A simple analog signal is represented by a sine wave.



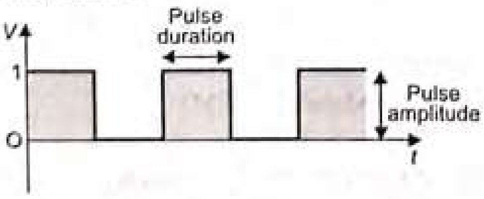
- The frequency of analog signals associated with speed or music varies over a range between 20 Hz to 20 kHz.



- The range over which the frequencies of a signal vary is called bandwidth.
- The term base band designates the band of frequencies representing the signal supplied by the source of information.
- A signal consists of two or more waves of different frequencies known as a complex analog signal.



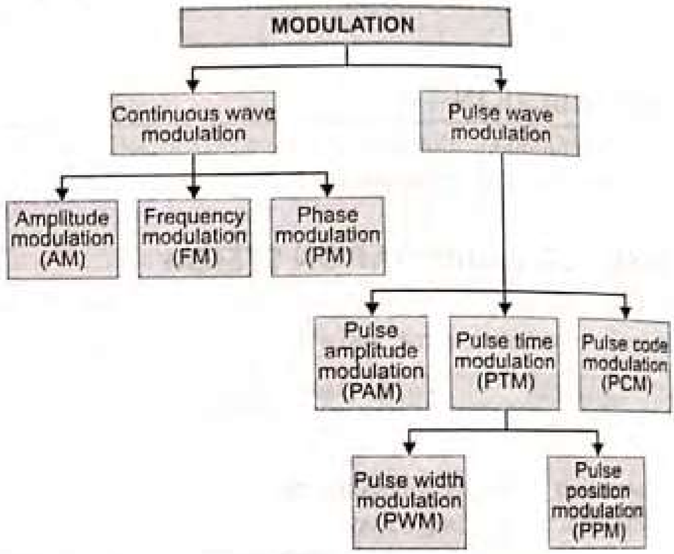
Digital signals A digital signal is a discontinuous function of time. It has only two voltage levels, i.e., either low (0) or high (1). Either of 0 and 1 is known as bit. A group of bit is called byte. A byte comprising 2 bits can give four code combinations, i.e., 00, 01, 10, and 11.



The number of code combination increases with number of bits in a byte is given by $N = 2^x$, where x = number of bits in a byte. The number of binary digits (bits) per second, which describe a digital signal is called its bit rate. Bit rate is expressed in bits per second (bps).

MODULATION

Digital and analog signals to be transmitted are usually of low frequency and hence cannot be transmitted as such. These signals require some carrier to be transported. These carriers are known as carrier waves or high-frequency signals. The process of placement of a low-frequency (LF) signal over the high-frequency (HF) signal is known as modulation various types of modulations are shown in figure.



Need for Modulation

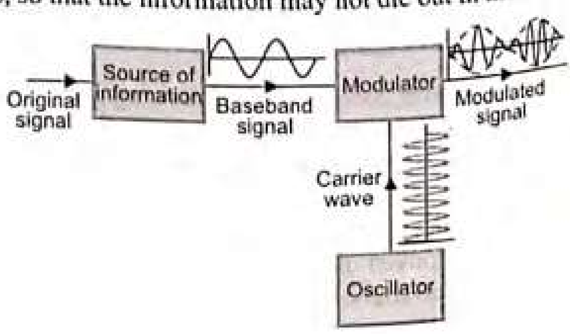
A sound wave (20 Hz to 20 kHz) cannot be transmitted directly from one place to another for the following reasons:

Height of Antenna

For efficient radiation and reception, the height of transmitting and receiving antennas should be comparable to a quarter of wavelength of the frequency used. For 15 kHz it is 5000 m (too large) and for 1 MHz it is 75 m. The energy radiated from an antenna is practically zero, when the frequency of the signal to be transmitted is below 15 Hz.

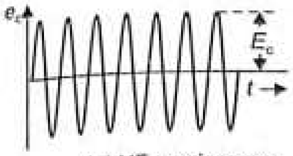
Detecting Signals

All audible signals are in the range of 20 Hz to 20 kHz. So the signals from all sources remain heavily mixed up in air. It will be very difficult to differentiate or detect the broadcast signal at the receiving station. Thus, modulation is necessary for a low-frequency signal especially when it is to be sent to a distant place, so that the information may not die out in the way itself.

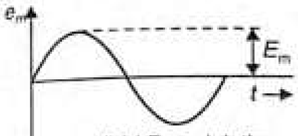


Amplitude Modulation

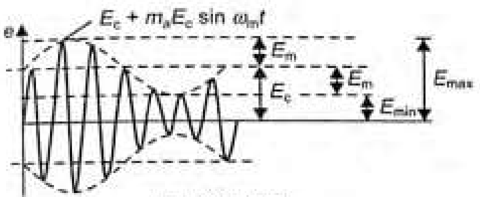
The process of changing the amplitude of a carrier wave in accordance with the amplitude of the audio frequency (AF) signal is known as amplitude modulation (AM). In AM frequency of the carrier wave remains unchanged. The amplitude of modulated wave is varied in accordance with the amplitude of modulating wave.



(a) HF carrier wave



(b) LF modulating



(c) AM wave

Modulation index The ratio of change of amplitude of the carrier wave to the amplitude of original carrier wave is called the modulation factor or degree of modulation or modulation index (m_a).

$$m_a = \frac{\text{Change in amplitude of carrier wave}}{\text{Amplitude of original carrier wave}} = \frac{kE_m}{E_c}$$

where k is a factor which determines the maximum change in the amplitude for a given amplitude E_m of the modulating signal. If $k = 1$, then

$$m_a = \frac{E_m}{E_c} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

If a carrier wave is modulated by several sine waves, the total modulated index m_t is given by

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \cdots}$$

Voltage equation for AM wave Suppose voltage equations for carrier wave and modulating wave are $e_c = E_c \cos \omega_c t$ and $e_m = E_m \sin \omega_m t = m_a E_c \sin \omega_m t$, where e_c is the instantaneous voltage of carrier wave, E_c amplitude of carrier wave, $\omega_c (=2\pi f_c)$ angular velocity at carrier frequency f_c, e_m instantaneous voltage of modulating, E_m amplitude of modulating wave, and $\omega_m (=2\pi f_m)$ angular velocity of modulating frequency f_m.

Voltage equation for AM wave is

$$\begin{aligned} e &= E \sin \omega_c t \\ &= (E_c + e_m) \sin \omega_c t \\ &= (E_c + E_m \sin \omega_m t) \sin \omega_c t \\ &= E_c \sin \omega_c t + \frac{m_a E_c}{2} \cos(\omega_c - \omega_m)t - \frac{m_a E_c}{2} \cos(\omega_c + \omega_m)t \end{aligned}$$

The above AM wave indicated that the AM wave is equivalent to the summation of three sinusoidal waves, one having amplitude E_c and the other two having amplitude $m_a E_c/2$.

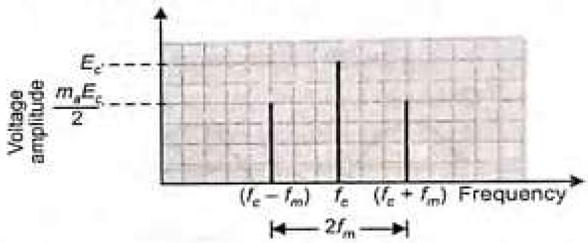
Side band frequencies: An AM wave contains three frequencies: f_c (called carrier frequency), $(f_c + f_m)$, and $(f_c - f_m)$ (latter two called side band frequencies).

$(f_c + f_m)$: Upper side band (USB) frequency

$(f_c - f_m)$: Lower side band (LSB) frequency

Side band frequencies are generally close to the carrier frequency.

Bandwidth in AM wave: The two side bands lie on either side of the carrier frequency at equal frequency interval f_m. So, bandwidth $= (f_c + f_m) - (f_c - f_m) = 2f_m$.



Power in AM waves Power dissipated in any circuit, $P = V^2_{rms}/R$. Hence,

Carrier power, $P_c = \frac{(E_c/\sqrt{2})^2}{R} = \frac{E_c^2}{2R}$

Total power of side bands,

$$P_{sb} = \frac{(m_a E_c/2\sqrt{2})^2}{R} + \frac{(m_a E_c/2\sqrt{2})}{R} = \frac{m_a^2 E_c^2}{4R}$$

Total power of AM wave,

$$P_t = P_c + P_{sb} = \frac{E_c^2}{2R}\left(1 + \frac{m_a^2}{2}\right)$$

$$\frac{P_t}{P_c} = \left(1 + \frac{m_a^2}{2}\right) \text{ and } \frac{P_{sb}}{P_t} = \frac{m_a^2/2}{\left(1 + \frac{m_a^2}{2}\right)}$$

Maximum power in AM (without distortion) will occur when $m_a = 1$, i.e., $P_t = 1.5P_c = 3P_{sb}$

If I_c = unmodulated current and I_t = total or modulated current, then $\frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} \Rightarrow \frac{I_t}{I_c} = \sqrt{\left(1 + \frac{m_a^2}{2}\right)}$

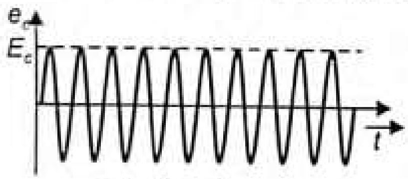
Limitations of Amplitude Modulation

1. Noisy reception
2. Low efficiency
3. Small operating range
4. Poor audio quality

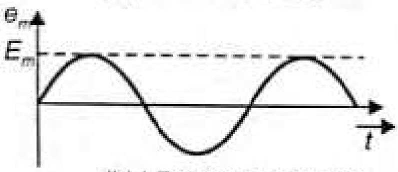
Frequency Modulation

The process of changing the frequency of a carrier wave in accordance with the audio frequency signal is known as frequency modulation.

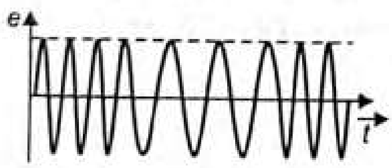
- Audio quality of AM transmission is poor. There is always a need to eliminate amplitude sensitive noise. This is possible if we eliminate amplitude variation. (i.e., a need to keep the amplitude of the carrier constant). This is precisely what we do in FM.
- In FM the overall amplitude of FM wave remains constant at all times.
- In FM, the total transmitted power remains constant.



(a) HF carrier wave



(b) LF modulating wave



(c) FM wave

Frequency deviation The maximum change in frequency from mean value (f_c) is known as frequency deviation. This is also the change or shift either above or below the frequency f_c and is called as frequency deviation.

$$\therefore \quad \delta = (f_{\max} - f_c) = f_c - f_{\min} = k_f \cdot \frac{E_m}{2\pi}$$

where k_f is the constant of proportionality. It determines the maximum variation in frequency of the modulated wave for a given modulating signal.

Carrier swing (CS) The total variation in frequency from the lowest to the highest is called the carrier swing.

i.e., $\quad CS = 2 \times \Delta f$

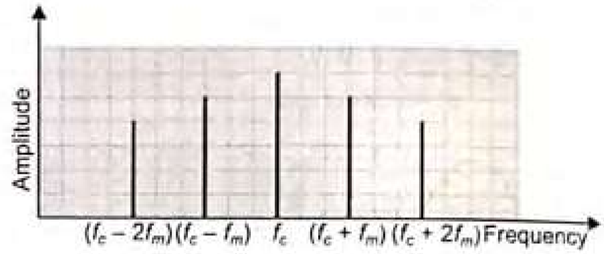
Frequency modulation index (m_f) The ratio of maximum frequency deviation to the modulating frequency is called modulation index.

$$m_f = \frac{\delta}{f_m} = \frac{f_{\max} - f_c}{f_m} = \frac{f_c - f_{\min}}{f_m} = \frac{k_f E_m}{f_m}$$

Frequency spectrum FM side band modulated signal consist of infinite number of side bands whose frequencies are

$$(f_c \pm f_m), (f_c \pm 2f_m), (f_c \pm 3f_m), \ldots$$

The number of side bands depends on the modulation index m_f.



In FM signal, the information (audio signal) is contained in the side bands. Since the side bands are separated from each other by the frequency of modulating signal f_m, so

$$\text{Bandwidth} = 2n \times f_m$$

where n = number of significant side band pairs

Deviation ratio The ratio of maximum permitted frequency deviation to the maximum permitted audio frequency is known as deviation ratio. Thus,

$$\text{Deviation ratio} = \frac{(\Delta f)_{\max}}{(f_m)_{\max}}$$

Percent Modulation

The ratio of actual frequency deviation to the maximum allowed frequency deviation is defined as percent modulation. Thus,

$$\text{Percent modulation, } m = \frac{(\Delta f)_{\text{actual}}}{(\Delta f)_{\max}}$$

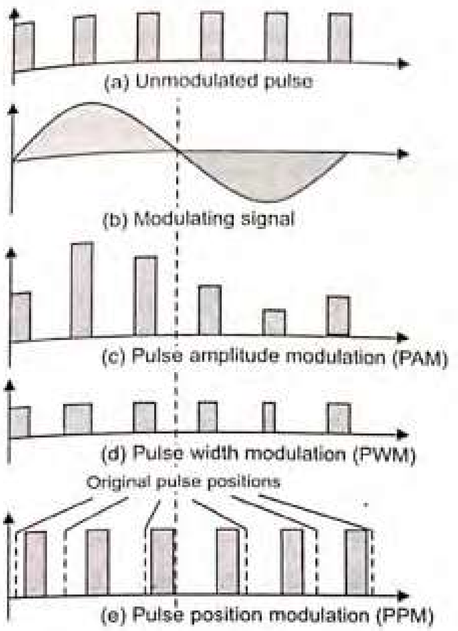
Pulse Modulation

In pulse modulation, the carrier wave is in the form of pulses. Mentioned below are the types of pulse modulation.

Pulse amplitude modulation (PAM) The amplitude of the pulse varies in accordance with the modulating signal.

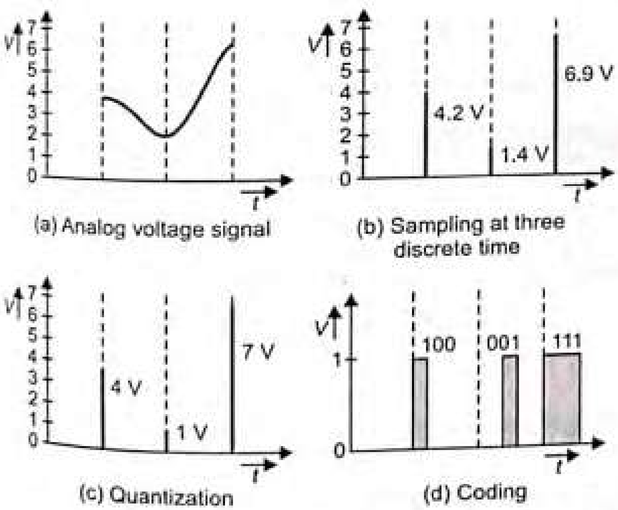
Pulse width modulation (PWM) The pulse duration varies in accordance with the modulating signal.

Pulse position modulation (PPM) In PPM, the position of the pulses of the carrier wave train is varied in accordance with the instantaneous value of the modulating signal.



Pulse Code Modulation

The pulse amplitude, pulse width, and pulse position modulations are not completely digital. A completely digital modulation is obtained by pulse code modulation (PCM). An analog signal is pulse code modulated by following three operations:



Sampling It is the process of generating pulses of zero width and of amplitude equal to the instantaneous amplitude of the analog signal. The number of samples taken per second is called sampling rate.

Quantization The process of dividing the maximum amplitude of the analog voltage signal into a fixed number of levels is called quantization. For example, amplitude 5 V of the analog voltage signal divides into six quantization levels, viz., 0, 1, 2, 3, 4, 5.

Pulses having amplitude between −0.5 V and 0.5 V are approximated (quantized) to a value of 0 V, amplitude between 0.5 V and 1.5 V are approximated to a value of 1 V, and so on.

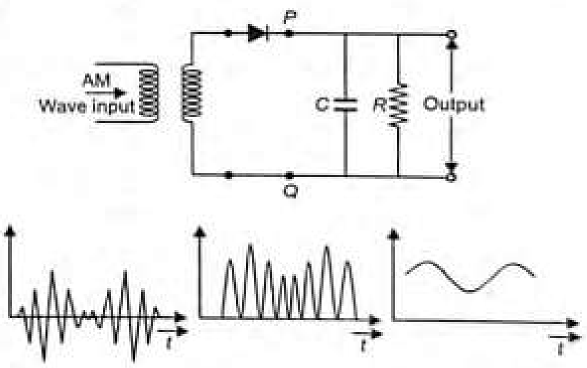
Coding The process of digitizing the quantized pulses according to some code is called coding. For example, consider that voltage amplitude of an analog signal varies between 0 and 7 V.

DEMODULATION

The process of extracting the audio signal from the modulated wave is known as demodulation or detection. The wireless signals consist of radio frequency (high frequency) carrier wave modulated by audio frequency (low frequency). The diaphragm of a telephone receiver or a loud speaker cannot vibrate with high frequency. So it is necessary to separate the audio frequencies from the radio frequency carrier wave.

Simple demodulator circuit A diode can be used to detect or demodulate an amplitude modulated (AM) wave. A diode basically acts as a rectifier, i.e., it reduces the modulated carrier wave into positive envelope only.

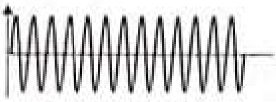
An AM wave input is shown in figure. It appears at the output of the diode across PQ as a rectified wave (since a diode conducts only in the positive half cycle). This rectified wave after passing through the RC network does not contain the radio frequency carrier component. Instead, it has only the envelope of the modulated wave.



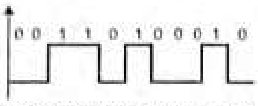
In the actual circuit the value of RC is chosen such that $1/f_c << RC$, where f_c is the frequency of carrier signal.

DATA TRANSMISSION AND RETRIEVAL

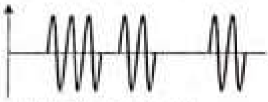
The term data is applied to a representation of facts, concepts, or instructions suitable for communication, interpretation, or processing by human beings or by automatic means. Data in most cases consist of pulse type of signals.



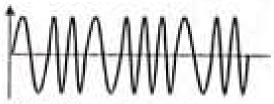
(a) Carrier wave



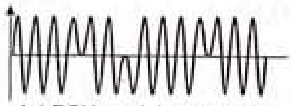
(b) Modulating source code



(c) ASK modulated wave



(d) FSK modulated wave



(e) PSK modulated wave

The PCM signal is a series of 1s and 0s. Following three modulation techniques are used to transmit a PCM signal:

Amplitude shift keying (ASK) Two different amplitudes of the carrier represent the two binary values of the PCM signal. This method is also known as on-off keying (OOK):

1: Presence of carrier of same constant amplitude.

0: Carrier of zero amplitude.

Frequency shift keying (FSK) The binary values of the PCM signal are represented by two frequencies.

1: Increase in frequency.

0: Frequency unaffected.

Phase shift keying (PSK) The phase of the carrier wave is changed in accordance with modulating data signal.

1: Phase changed by π.

0: Phase remains unchanged.

The analog signal is sampled by the sampler. The sampled pulses are then quantized. The encoder codes the quantized pulses according to the binary codes. After modulating the PCM signal (by ASK, FSK, or PSK method), the modulated signal is then transmitted into free space in the form of bits.

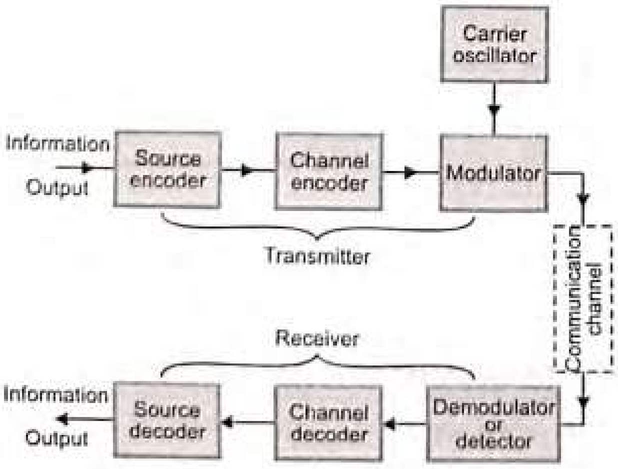


ILLUSTRATION 32.1 A TV transmitter has a range of 50 km. What is the height of the TV transmission tower? Radius of earth $R_e = 6.4 \times 10^6$ m.

Solution. Here $d = 50$ km $= 50 \times 10^3$ m

$\therefore$ Height of T. V. transmission tower

$$h = \frac{d^2}{2R_e} = \frac{(50\times10^3)^2}{2\times6.4\times10^6} = 195.3\,\text{m}$$

ILLUSTRATION 32.2 A TV tower has a height of 100 m. How much population is covered by the TV broadcast if the average population density around the tower is 1000 km^{-2} (radius of earth $= 6.4 \times 10^6$ m).

Solution. Here $h = 100$ m, $R_e = 6.4 \times 10^6$ m

Average population density

$$= 1000\text{ km}^{-2} = 1000 \times (1000\text{ m})^{-2} = 10^{-3}\text{ m}^2$$

The TV transmission range, $d = \sqrt{2hR_e}$

Therefore, area of TV transmission range

$$A = \pi d^2 = \pi\, 2h\, R_e = 2\pi hR_e$$

$$= 2 \times 3.14 \times 100 \times 6.4 \times 10^6\text{ m}^2 = 4.0 \times 10^9\text{ m}^2$$

Therefore, population covered by TV broadcast

$$= 4.0 \times 10^9 \times 10^{-3} = 4.0 \times 10^6$$

ILLUSTRATION 32.3 On a particular day, the maximum frequency reflected from the ionosphere is 10 MHz. On another day, it was found to increase to 11 MHz. Calculate the ratio of maximum electron density of the ionosphere on the two days. Point out a plausible explanation for this.

Solution. $f_c = 9\sqrt{N_{max}}$

$$N_{max} = \frac{f_c^2}{81} \propto f_c^2 \Rightarrow \frac{(N_2)_{max}}{(N_1)_{max}} = \left(\frac{11}{10}\right)^2 = \frac{121}{100} = 1.21$$

The variation may be due to atmospheric disturbances.

ILLUSTRATION 32.4 Frequencies higher than 10 MHz are found not to be reflected by the ionosphere on a particular day at a place. Calculate the maximum electron density of the ionosphere.

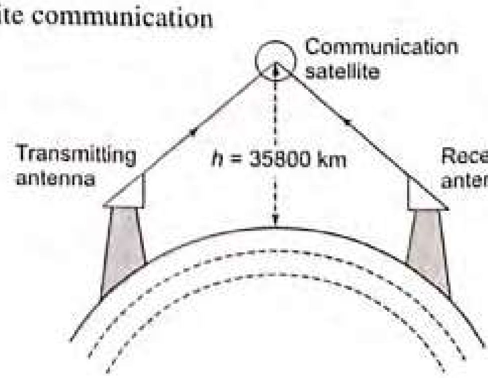
Solution. Given $f_c = 10$ MHz $= 10 \times 10^6$ Hz

If N_{max} is the maximum electron density, then

$$f_c = 9\sqrt{N_{max}}$$

$$N_{max} = \frac{f_c^2}{81} = \frac{(10\times10^6)^2}{81} = 1.2 \times 10^{12}\text{ m}^{-3}$$

Satellite communication



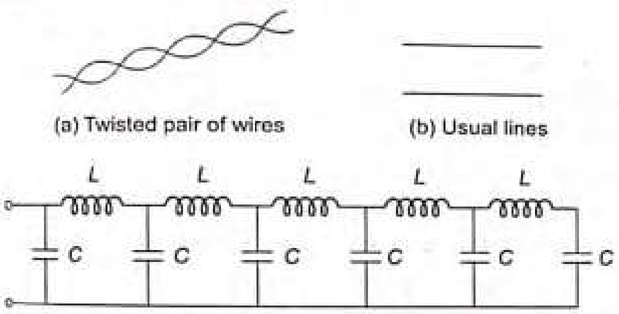
The height of communication satellite from earth's surface is 35,800 km. These communication satellites are required to cover the entire globe of earth.

Remote Sensing

Remote sensing is the process of finding information about an object at a distance without going near it. It is achieved by launching satellites in sun synchronous orbits.

Line Communication

The line communication is conventional and it is used to transmit electricity and telephone signals through a pair of wires. The most used transmission lines are (i) a pair of conducting wires and (ii) a coaxial cable. It consists of two concentric conductors separated by an isolated material. The equivalent circuit of a transmission line consists of a sequence of inductance L and capacitance C.



(a) Twisted pair of wires (b) Usual lines

(c) Equivalent circuit

Optical Communication

In optical communication, communication is through carrier optical signals. Optical signals cannot be transmitted through cables. The essentials of optical communication are

(i) **Light Source and Modulator:** Light sources are LED, diode lasers.

(ii) **Communication Channel:** The optical fibres of small diameters = 10^{-5} m based on total internal reflection have been discovered.

(iii) **Optical Signal Detectors:** These are photodiode, Avalanche photodiode.

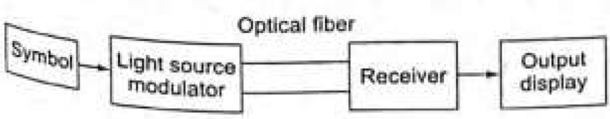
Advantages of optical communication are:

(i) Wide channel band width has large channel carrying capacity.

(ii) Low transmission losses

(iii) Signal security

The schematic optical communication is shown in the figure.



Laser

Laser is abbreviation of light amplification of stimulated emission of radiation. It is based on stimulated emission. A laser device consists of a resonant cavity having one totally and one partially reflecting face. The active substance has metastable states and most of the atoms are in excited metastable state so that population inversion is achieved. When a photon of proper frequency is incident on the active material, it stimulates emission of photons. Finally an intense, monochromatic, highly directional coherent laser beam is available.

SOLVED EXAMPLES

1. A sky wave with a frequency 55 MHz is incident on D-region of earth's atmosphere at 45°. The angle of refraction is (electron density for D-region is 400 electron/cm^3)

(a) 60° (b) 45°
(c) 30° (d) 15°

Sol. (b) $n_{eff} = n_0\sqrt{1-\left(\dfrac{80.5\,N}{\nu^2}\right)} = 1\sqrt{1-\dfrac{80.5\times(400\times10^6)}{(55\times10^6)^2}} \approx 1$

Also $n_{eff} = \dfrac{\sin i}{\sin r}$

$\Rightarrow \sin r = \sin i \Rightarrow r = i = 45°$

2. In a diode AM-detector, the output circuit consist of $R = 1$ kΩ and $C = 10$ pF. A carrier signal of 100 kHz is to be detected. Is it good?

(a) Yes
(b) No
(c) Information is not sufficient
(d) None of these

Sol. (b) For demodulation, $\dfrac{1}{f_c} << RC$

$$\frac{1}{f_c} = \frac{1}{100\times10^3} = 10^{-5}\,\text{s}$$

$$RC = 10^3 \times 10 \times 10^{-12}\,\text{s} = 10^{-8}\,\text{s}$$

We see that $1/f_c$ here is not less than RC as required by the above condition. Hence, this is not good.

3. A photodetector is made from a semiconductor $In_{0.53}Ga_{0.47}As$ with $E_g = 0.73$ eV. What is the maximum wavelength, which it can detect

(a) 1000 nm (b) 1703 nm
(c) 500 nm (d) 173 nm

Sol. (b) Limiting value of $h\nu$ is E_g, such that $h\nu = \dfrac{hc}{\lambda} = E_g$

or $\lambda = \dfrac{hc}{E_g} = \dfrac{6.63\times10^{-34}\,\text{J-s}\times3\times10^8\,\text{ms}^{-1}}{0.73\times1.6\times10^{-19}\,\text{J}} = 1703$ nm

4. A transmitter supplies 9 kW to the aerial when unmodulated. The power radiated when modulated to 40% is
(a) 5 kW (b) 9.72 kW
(c) 10 kW (d) 12 kW

Sol. (b) $P_t = P_c\left[1+\frac{m^2}{2}\right] = 9\left[1+\frac{(0.4)^2}{2}\right]$

$= 9\left[1+\frac{0.16}{2}\right]$ $(\because m = 40\% = 0.4)$

$= 9\,(1.08) = 9.72$ kW

5. The antenna current of an AM transmitter is 8 A when only carrier is sent but increases to 8.96 A when the carrier is sinusoidally modulated. The percentage modulation is
(a) 50% (b) 60%
(c) 65% (d) 71%

Sol. (d) We know that $\left(\frac{I_t}{I_c}\right)^2 = 1+\frac{m^2}{2}$

Here, $I_t = 8.96$ A and $I_c = 8$ A

$\therefore \left(\frac{8.96}{8}\right)^2 = 1+\frac{m^2}{2}$ or $1.254 = 1+\frac{m^2}{2}$

or $\frac{m^2}{2} = 0.254$

or $m^2 = 0.508$ or $m = 0.71 = 71\%$

6. The total power content of an AM wave is 1500 W. For 100% modulation, the power transmitted by the carrier is
(a) 500 W (b) 700 W
(c) 750 W (d) 1000 W

Sol. (d) $\frac{P_t}{P_c} = 1+\frac{m^2}{2}$ or $P_c = P_t\left[\frac{2}{2+m^2}\right]$

$\therefore P_c = 1500\left[\frac{2}{2+1}\right]$

$\because m = 100\% = 1 = 1000$ W

7. The total power content of an AM wave is 900 W. For 100% modulation, the power transmitted by each side band is
(a) 50 W (b) 100 W
(c) 150 W (d) 200 W

Sol. (c) $P_c = P_t\left[\frac{2}{2+m^2}\right] = 900\left[\frac{2}{2+1}\right] = 600$ W

Now, $P_{LSB} = \frac{m^2}{4}\times P_c = \frac{1}{4}\times 600 = 150$ W

8. The modulation index of an FM carrier having a carrier swing of 200 kHz and a modulating signal 10 kHz is
(a) 5 (b) 10
(c) 20 (d) 25

Sol. (b) CS $= 2\times\Delta f$ or Δf = CS/2

$\therefore \Delta f = \frac{200}{2} = 100\,\text{kHz}$

Now $m_f = \frac{\Delta f}{f_m} = \frac{100}{10} = 10$

9. A 500-Hz modulating voltage fed into an FM generator produces a frequency deviation of 2.25 kHz. If amplitude of the voltage is kept constant but frequency is raised to 6 kHz, then the new deviation will be
(a) 4.5 kHz (b) 54 kHz
(c) 27 kHz (d) 15 kHz

Sol. (b) $m_f = \frac{\delta}{f_m} = \frac{2250}{500} = 4.5$

$\therefore$ New deviation $= 2(m_f f_m) = 2\times 4.5\times 6 = 54$ kHz

10. The bit rate for a signal, which has a sampling rate of 8 kHz and where 16 quantisation levels have been used is
(a) 32000 bits/sec (b) 16000 bits/sec
(c) 64000 bits/sec (d) 72000 bits/sec

Sol. (a) If n is the number of bits per sample, then number of quantisation level $= 2^n$

Since the number of quantisation level is 16

$\Rightarrow 2^n = 16 \Rightarrow n = 4$

$\therefore$ Bit rate = Sampling rate × No. of bits per sample

$= 8000\times 4 = 32{,}000$ bits/sec.

11. An amplitude modulated wave is modulated to 50%. What is the saving in power if carrier as well as one of the side bands are suppressed?
(a) 70% (b) 65.4%
(c) 94.4% (d) 25.5%

Sol. (c) $P_{sb} = P_c\left(\frac{m_a}{2}\right)^2 = P_c\frac{(0.5)^2}{4} = 0.0625\,P_c$

Also $P = P_c\left(1+\frac{m_a^2}{2}\right) = P_c\left(1+\frac{(0.5)^2}{2}\right) = 1.125P_c$

$\therefore$ % saving $= \frac{(1.125P_c - 0.0625P_c)}{1.125P_c}\times 100 = 94.4\%$.

12. In a radio receiver, the short wave and medium wave stations are tuned by using the same capacitor but coils of different inductance L_s and L_m, respectively, then
(a) $L_s > L_m$ (b) $L_s < L_m$
(c) $L_s = L_m$ (d) None of these

Sol. (b) As $\nu = \frac{c}{\lambda} \Rightarrow \nu_m = \frac{c}{\lambda_m}$ and $\nu_s = \frac{c}{\lambda_s}$

$\because \lambda_m > \lambda_s \Rightarrow \nu_m < \nu_s$

Also $\nu_m = \frac{1}{2\pi\sqrt{L_m C}}$ and $\nu_s = \frac{1}{2\pi\sqrt{L_s C}}$

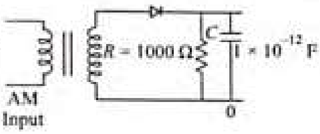
13. A ground receiver station is receiving a signal at 5 MHz and transmitted from a ground transmitter at a height of 300 m, located at a distance of 100 km from the receiver station. The signal is coming via. [Radius of earth $= 6.4 \times 10^6$ m. N_{max} of isosphere $= 10^{12}$ m^3]
(a) Space wave (b) Sky wave propagation
(c) Satellite transponder (d) All of these

Sol. (b) Maximum distance covered by space wave communication $\sqrt{2Rh} = 62$ km

Critical frequency $= f_c = 9(N_{max})1/2 = 9$ MHz

5 MHz $< f_c$, sky wave propagation (ionospheric propagation)

14. In the given detector circuit, the suitable value of carrier frequency is



(a) $<< 10^9$ Hz (b) $<< 10^5$ Hz
(c) $>> 10^9$ Hz (d) None of these

Sol. (a) Using $\dfrac{1}{f_{carrier}} << RC$

We get time constant, $RC = 1000 \times 10^{-12} = 10^{-9}$ s

Now $v = \dfrac{1}{T} = \dfrac{1}{10^{-9}} = 10^9$ Hz

Thus, the value of carrier frequency should be much less than 10^9 Hz, say 100 kHz.

$$\Rightarrow \frac{v_m}{v_s} = \sqrt{\frac{L_s}{L_m}} \Rightarrow L_s < L_m.$$

EXERCISES

1. In short wave communication waves, which of the following frequencies will be reflected back by the ionospheric layer, having electron density 10^{11} per m^3
(a) 2 MHz (b) 10 MHz
(c) 12 MHz (d) 18 MHz

2. Range of frequencies allotted for commercial FM radio broadcast is
(a) 88 to 108 MHz (b) 88 to 108 kHz
(c) 8 to 88 MHz (d) 88 to 108 GHz

3. The velocity factor of a transmission line x. If dielectric constant of the medium is 2.6, the value of x is
(a) 0.26 (b) 0.62
(c) 2.6 (d) 6.2

4. A step index fibre has a relative refractive index of 0.88%. What is the critical angle at the corecladding interface?
(a) 60° (b) 75°
(c) 45° (d) None of these

5. A laser beam of pulse power 10^{12} watt is focussed on an object are 10^{-4} cm^2. The energy flux in watt/cm^2 at the point of focus is
(a) 10^{20} (b) 10^{16}
(c) 10^8 (d) 10^4

6. The carrier frequency generated by a tank circuit containing 1 nF capacitor and 10 μH inductor is
(a) 1592 Hz (b) 1592 MHz
(c) 1592 kHz (d) 159.2 Hz

7. For television broadcasting, the frequency employed is normally
(a) 30–300 MHz (b) 30–300 GHz
(c) 30–300 kHz (d) 30–300 Hz

8. Maximum useable frequency (MUF) in F-region layer is x, when the critical frequency is 60 MHz and the angle of incidence is 70°. Then x is
(a) 150 MHz (b) 170 MHz
(c) 175 MHz (d) 190 MHz

9. Laser beams are used to measure long distances because
(a) They are monochromatic
(b) They are highly polarised
(c) They are coherent
(d) They have high degree of parallelism

10. An oscillator is producing FM waves of frequency 2 kHz with a variation of 10 kHz. What is the modulating index?
(a) 0.20 (b) 5.0
(c) 0.67 (d) 1.5

11. The maximum peak to peak voltage of an AM wire is 24 mV and the minimum peak to peak voltage is 8 mV. The modulation factor is
(a) 10% (b) 20%
(c) 25% (d) 50%

12. Sinusoidal carrier voltage of frequency 1.5 MHz and amplitude 50 V is amplitude modulated by sinusoidal voltage of frequency 10 kHz producing 50% modulation. The lower and upper side-band frequencies in kHz are
(a) 1490, 1510 (b) 1510, 1490
(c) $\dfrac{1}{1490}, \dfrac{1}{1510}$ (d) $\dfrac{1}{1510}, \dfrac{1}{1490}$

13. Which of the following is the disadvantage of FM over AM?
(a) Larger band width requirement
(b) Larger noise
(c) Higher modulation power
(d) Low efficiency

14. An AM wave has 1800 W of total power content. For 100% modulation, the carrier should have power content equal to
(a) 1000 watt (b) 1200 watt
(c) 1500 watt (d) 1600 watt

15. An antenna is a device
   (a) That converts electromagnetic energy into radio frequency signal
   (b) That converts radio frequency signal into electromagnetic energy
   (c) That converts guided electromagnetic waves into free space electromagnetic waves and vice-versa
   (d) None of these

16. In an FM system, a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is
   (a) 7.143   (b) 8
   (c) 0.71   (d) 350

17. The phenomenon by which light travels in an optical fibres is
   (a) Reflection
   (b) Refraction
   (c) Total internal reflection
   (d) Transmission

18. Advantage of optical fibre
   (a) High bandwidth and EM interference
   (b) Low bandwidth and EM interference
   (c) High band width, low transmission capacity and no EM interference
   (d) High bandwidth, high data transmission capacity and no EM interference

19. In frequency modulation
   (a) The amplitude of modulated wave varies as frequency of carrier wave
   (b) The frequency of modulated wave varies as amplitude of modulating wave
   (c) The amplitude of modulated wave varies as amplitude of carrier wave
   (d) The frequency of modulated wave varies as frequency of modulating wave

20. Audio signal cannot be transmitted because
   (a) The signal has more noise
   (b) The signal cannot be amplified for distance communication
   (c) The transmitting antenna length is very small to design
   (d) The transmitting antenna length is very large and impracticable

21. In which of the following remote sensing technique is not used?
   (a) Forest density   (b) Pollution
   (c) Wetland mapping   (d) Medical treatment

22. For sky wave propagation of a 10 MHz signal, what should be the minimum electron density in ionosphere?
   (a) $\sim 1.2 \times 10^{12}\ m^{-3}$ (b) $\sim 10^{6}\ m^{-3}$
   (c) $\sim 10^{14}\ m^{-3}$ (d) $\sim 10^{22}\ m^{-3}$

23. What should be the maximum acceptance angle at the aircore interface of an optical fibre if n_1 and n_2 are the refractive indices of the core and the cladding, respectively?
   (a) $\sin^{-1}(n_2/n_1)$ (b) $\sin^{-1}\sqrt{n_1^2 - n_2^2}$
   (c) $\left[\tan^{-1}\frac{n_2}{n_1}\right]$ (d) $\left[\tan^{-1}\frac{n_1}{n_2}\right]$

24. The audio signal used to modulate $60 \sin(2\pi \times 10^6 t)$ is $15 \sin 300\pi t$. The depth of modulation is
   (a) 50%   (b) 40%
   (c) 25%   (d) 15%

25. The impedance of coaxial cable, when its inductance is 0.40 μH and capacitance is 1×10^{-11} F, can be
   (a) $2 \times 10^2\ \Omega$ (b) $100\ \Omega$
   (c) $3 \times 10^3\ \Omega$ (d) $3 \times 10^{-2}\ \Omega$

26. An optical fibre communication system works on a wavelength of 1.3 μm. The number of subscribers it can feed if a channel requires 20 kHz are
   (a) 2.3×10^{10} (b) 1.15×10^{10}
   (c) 1×10^5 (d) None of these

27. In an FM system a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is
   (a) 7.143   (b) 8
   (c) 0.71   (d) 350

28. The electron density of E, F_1, F_2 layers of ionosphere is 2×10^{11}, 5×10^{11} and $8 \times 10^{11}\ m^{-3}$ respectively. What is the ratio of critical frequency for reflection of radiowaves
   (a) 2 : 4 : 3   (b) 4 : 3 : 2
   (c) 2 : 3 : 4   (d) 3 : 2 : 4

29. Mean optical power launched into an 8 km fibre is 120 μW and mean output power is 4 μW, then the overall attenuation is (Given log 30 = 1.477)
   (a) 14.77 dB   (b) 16.77 dB
   (c) 3.01 dB   (d) None of these

30. A antenna current of an AM broadcast transmitter modulated by 50% is 11 *A*. The carrier current is
   (a) 10.35 A   (b) 9.25 A
   (c) 10 A   (d) 5.5 A

31. A transmitter transmits a power of 10 kW when modulation is 50%. Power of carrier wave is
   (a) 5 kW   (b) 8.89 kW
   (c) 14 kW   (d) 5.7 kW

32. A telephone link operating at a central frequency of 10 GHz is established. If 1% of this is available then how many telephone channel can be simultaneously given when each telephone covering a band width of 5 kHz

(a) 2×10^4 (b) 2×10^6
(c) 5×10^4 (d) 5×10^6

33. In AM, the cent percent modulation is achieved when
(a) Carrier amplitude = signal amplitude
(b) Carrier amplitude ≠ signal amplitude
(c) Carrier frequency = signal frequency
(d) Carrier frequency ≠ signal frequency

ARCHIVES

1. Consider telecommunication through optical fibres. Which of the following statements is not true
(a) Optical fibres may have homogeneous core with a suitable cladding
(b) Optical fibres can be of graded refractive index
(c) Optical fibres are subject to electromagnetic interference from outside
(d) Optical fibres have extremely low transmission loss
(AIEEE 2003)

2. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.
Statement 1: Sky wave signals are used for long distance radio communication. These signals are in general, less table than ground wave signals.
Statement 2: The state of ionosphere varies from hour to hour, day to day and season to season.
(a) Statement 1 is true, Statement 2 is false.
(b) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1
(c) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1
(d) Statement 1 is false, Statement 2 is true
(AIEEE 2011)

3. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is
(a) 80 km (b) 16 km
(c) 40 km (d) 64 km **(AIEEE 2012)**

4. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?
(a) $\omega_m+\omega_c$ (b) $\omega_c-\omega_m$
(c) ω_m (d) ω_c **(JEE Main 2017)**

5. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10 % of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?
(a) 2×10^6 (b) 2×10^3
(c) 2×10^4 (d) 2×10^5
(JEE Main 2018)

ANSWER KEY

Exercises

1. (a)	2. (a)	3. (b)	4. (d)	5. (b)	6. (c)	7. (a)	8. (c)	9. (d)	10. (b)
11. (d)	12. (c)	13. (a)	14. (b)	15. (c)	16. (a)	17. (c)	18. (d)	19. (d)	20. (d)
21. (d)	22. (a)	23. (b)	24. (c)	25. (a)	26. (b)	27. (a)	28. (c)	29. (a)	30. (a)
31. (b)	32. (a)	33. (a)							

Archives

1. (c)	2. (d)	3. (a)	4. (c)	5. (d)

Chapter 33

Experimental Skills

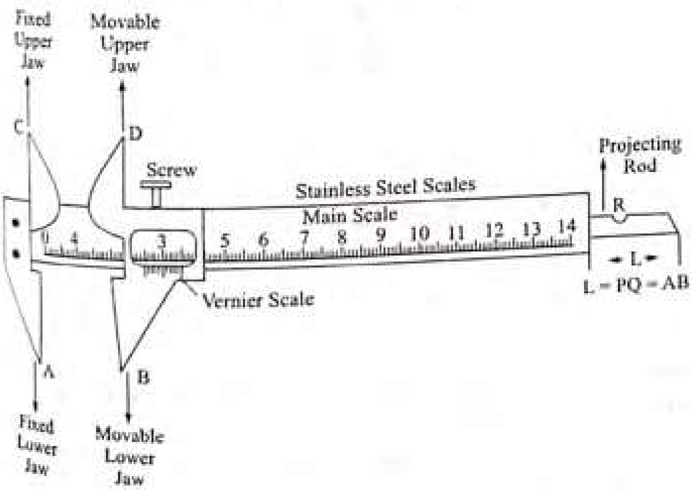
EXPERIMENT 1

Vernier Calipers: It is used to measure length, internal and external diameters and depth of a cylindrical vessel.

1. **Description of Vernier Calipers:** It consists of a main scale M graduated in cm and mm over which an auxiliary scale (or Vernier scale) V can slide along the length of main scale M, the divisions of the Vernier scale being either slightly longer or slightly smaller than the divisions of the main scale.

   The main scale has two fixed jaws A and C as shown while B and D are the jaws of Vernier scale, the position of Vernier scale is fixed with the help of screw S. In general, the Vernier scale has 10 divisions over a length of 9 mm.

   We can measure the external diameter of an object, internal diameter of a hollow object or depth of some vessel using Vernier calipers. The diameter (external) of an object can be determined by placing the object in between the jaws A and B while the internal diameter can be measured by inserting C and D in the object.



While performing the experiment using Vernier calipers, first of all, the jaws of Vernier scale, i.e., A and B touch each other while straight edges of C and D touch each other. If the instrument is free from zero error, then the zero of main scale coincides with the zero of Vernier scale.

After noticing whether the instrument is having zero error or not, hold the object whose external diameter is to be computed between jaws A and B as shown in the figure. Now, the most important task is to measure the diameter i.e., to take the readings of Vernier caliper.

To measure the depth of the vessel, a metallic strip E is connected to the back of M and Vernier scale. When jaws A and B touch each other, the edge of E touches the edge of M. When Vernier is separated from M, E moves outwards.

2. **Vernier constant:** The difference between one scale division and one Vernier division is called Vernier constant or the least count of the Vernier because it is the smallest length that can be measured accurately with its help.

   To find the Vernier constant (or least count),

   (i) find the magnitude of the smallest division of the main scale.

   (ii) count the total number of division on the vernier scale.

   (iii) slide the movable jaw that the zero mark (the first division) of the vernier scale coincides with any of the main scale divisions.

   (iv) find the number of scale divisions which coincide with the total number of Vernier divisions.

   In general, if n Vernier divisions are equal to $(n - 1)$ scale divisions, then

   $$\text{Vernier constant} = 1\text{S.D.} - \text{V.D.} = \left(1 - \frac{n-1}{n}\right)$$

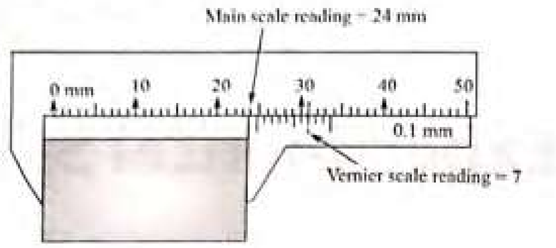
   $$\text{S.D.} = \frac{1}{n}\ \text{S.D.}$$

   If the mth vernier division coincides with any one of the scale division, then

   $$\text{Fraction to be added} = m \times \frac{1}{n} = \frac{m}{n}$$

   Hence, the value of L = Completer main scale reading before zero of the V.S. mark + $\frac{m}{n}$

3. How to read Vernier calipers



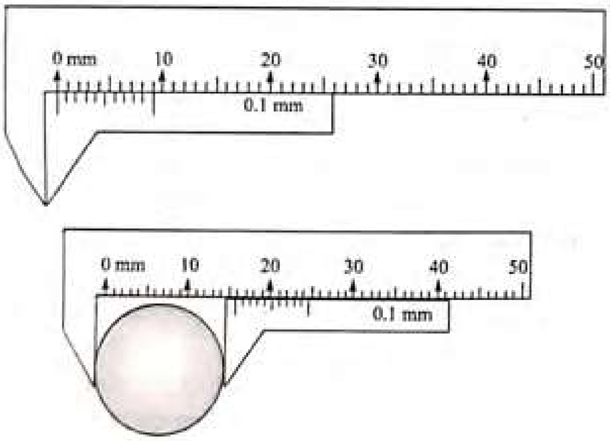
Thickness of the object = 24. ------
= 24 mm + 7 (0.1 mm)

Formula says
Thickness of the object = [Main Scale Reading] + [Vernier Scale Reading] [least count]

[Mark on vernier scale which exactly co-insides with some mark of main scale]

[1 main scale division] – [1 vernier scale division]
= 1 mm – 0.9 mm
= 0.1 mm

ILLUSTRATION 33.1 Read the Vernier scale.

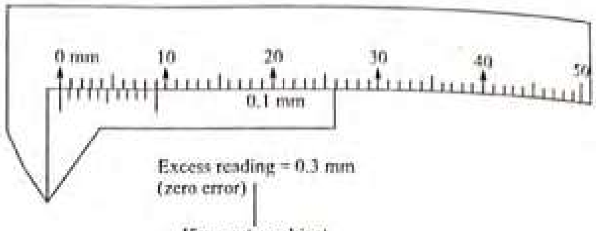


Solution. Thickness of the object = (Main scale reading) + (Vernier scale Reading) (Least count)

where least count = (Main scale division – Vernier scale division)
= 1 mm – 0.9 mm
= 0.1 mm

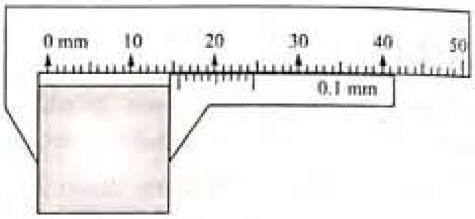
So thickness of the object = 15 mm + (6) (0.1mm) = 15.6 mm

Zero error: If there is no object between the jaws (i.e. jaws are in contact), the Vernier should give zero reading. But due to some extra material on jaws, even if there is no object between the jaws, it gives some excess reading. This excess reading is called *zero error*.



Excess reading = 0.3 mm
(zero error)

If we put an object between the jaws



It gives 13.8 mm reading

In which there is 0.3 mm excess reading which has to be removed (subtracted)

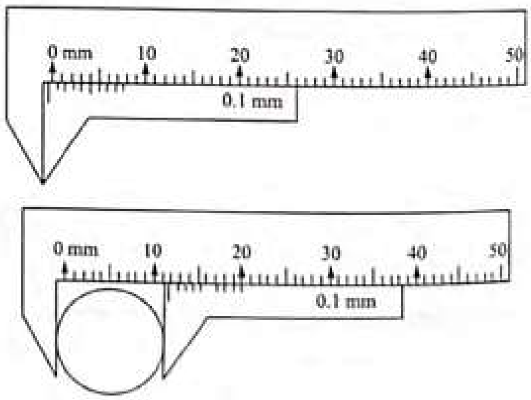
So, actual thickness = 13.8 – 0.3 = 13.5 mm

Observed reading — Excess reading (zero error)

So we can formulate it as

Actual reading = Observed reading – Excess reading (Zero error)

ILLUSTRATION 33.2 Find the thickness of the object using the defected vernier calipers.



Solution. Zero error = Main scale reading + (Vernier scale reading) (least count)

= –1 mm + 6 (0.1 mm)
= –0.4 mm

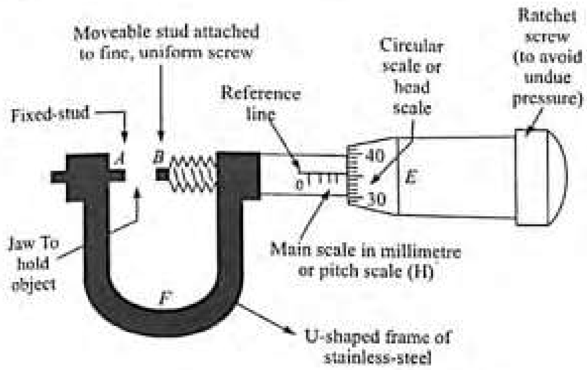
Observed reading = 11.8 mm

So actual thickness = 11.8 – (–0.4) = 12.2 mm

EXPERIMENT 2

Screw gauge: It is used to measure the thickness/diameter of thin sheet or wire.

1. It works on the principle of micrometer screw. H is the linear scale or pitch scale while E is the cap scale. A hollow cylindrical cap k is capable to rotate over h (hub or linear scale) when the screw is rotated. When zero of pitch scale coincides with zero of cap scale, then the instrument is free from zero error.

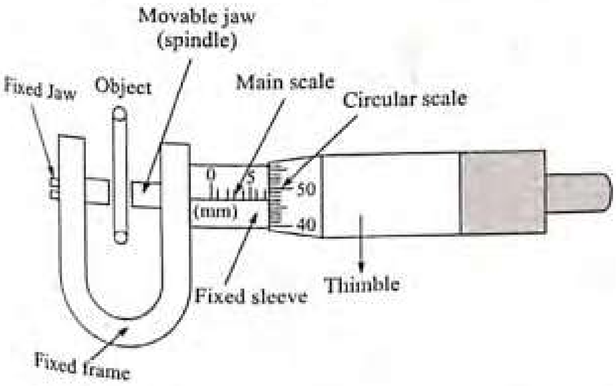


The wire whose diameter has to be measured is held between A and B. Reading of screw gauge is Measured diameter $= N + n \times$ Least count

Where N is the division of linear scale beyond which the edge of the cap lies, and n is the division of circular scale that lies over reference line. If some zero error is there, subtract it from the above reading.

When the zero of circular scale advances beyond the reference line, the zero error is negative and if it is left behind the reference line, the zero error is positive. In positive zero error, zero linear scale is not hidden from the circular scale, while in negative zero error, zero of the linear scale is hidden from circular scale.

2. **Description of screw gauge:**



The object to be measured is put between the jaws. The sleeve is the hollow part, fixed with the frame and main scale is printed on it.

The spindle and thimble are welded, and move together by means of a screw. The circular scale is printed on the thimble as shown. It generally consists of 100 divisions (sometime 50 divisions also)

The main scale has mm marks (Sometimes it also has 1/2 mm marks below mm marks.)

(Usually if pitch of the screw gauge is 1mm then there are 1mm marks on main scale and if pitch is 1/2 mm then there are 1/2 mm marks also)

This instrument can read upto 0.01 mm (10 μm) accuracy that is why it is called micrometer

3. **Least count of the screw gauge:** The pitch of he screw is defined as the distance through which the screw moves forward or backward parallel to its axis when one complete rotation is given to the circular cap. To find pitch:
   (a) rotate the circular scale h and coincide the zero mark with the reference line. Notice the reading on the pitch scale.
   (b) give four complete rotations to the circular scale and note the reading on the pitch scale again.
   (c) calculate the pitch of the screw by dividing the distance moved by the number of rotations.

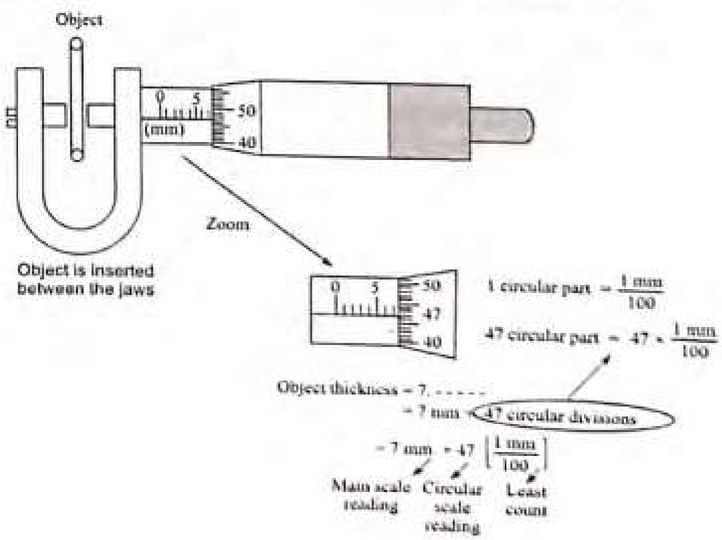
$$\text{Pitch of screw} = \frac{\text{Distance travelled on the pitch scale}}{\text{Number of rotations}}$$

NOTE: The pitch of a screw gauge is usually 0.5 mm or one millimeter.

To find the least count: The least count of the screw gauge is defined as the distance through which the screw moves backward or forward when the cap is rotated trough on the circular scale. Note the number of divisions on the circular scale H, then

$$\text{Least count} = \frac{\text{Pitch of the screw}}{\text{No. of rotations on the circular scale}}$$

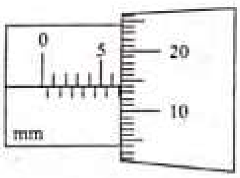
How to find thickness of an object by screw gauge!



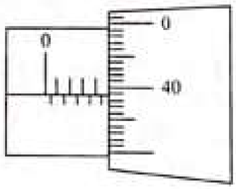
Thickness of object = Reading of screw gauge = [Main scale reading] + [Circular scale reading] × [Least count]

Least count = $\dfrac{\text{Pitch}}{\text{Numbers of divisions on circular sScale}}$

ILLUSTRATION 33.3 Read the screwgauge shown below:



Object thickness = 6.5 mm + 14 $\left[\dfrac{1/2\text{ mm}}{50}\right]$



Object thickness = 4.5 mm + 39 $\left[\dfrac{1/2\text{ mm}}{50}\right]$ = 4.89 mm

Solution.

- Main scale has 1/2 mm marks.
- Circular scale has 50 division.
- In complete rotation, the screw advances by 1/2 mm.

4. **Zero correction:** In some instruments when the jaws A and B are brought in contact without applying any undue pressure, the zero of the circular scale does not coincide with the reference line. In some instruments zero mark goes beyond the reference line, while in others it is left behind when two studs A and B are in contact with each other without undue pressure.

   To find zero correction, count the number of divisions on the circular scale that the zero mark has advanced beyond or is left behind the reference line. Multiply this number with the least count. This is the zero correction. If the zero of the circular scale has advanced beyond the reference line, the zero correction is positive, but if it is left behind the reference line, it is negative.

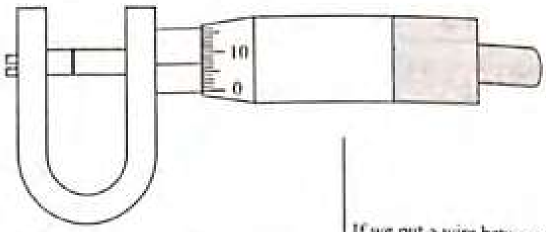
   Remember zero above, add and zero below, subtract.

   Zero correction is invert of zero error:

   Zero correction = –(Zero error)

   Actual reading = Observed reading – Zero error

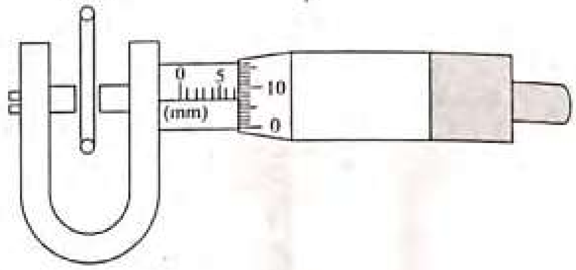
   = Observed reading + Zero correction



Excess reading = 0.07 mm (Zero error)

If we put a wire between jaws

Object



It gives 3.07 mm

In which there is 0.07 mm excess reading which has to be substructed

So actual thickness = 3.07 – 0.07 = 3.00 mm

(3.07: Observed reading; 0.07: Excess reading (zero error))

Actual reading = Observed reading – Excess reading (zero error)

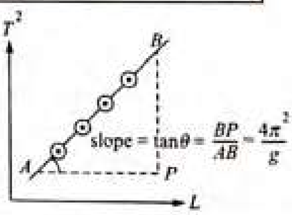
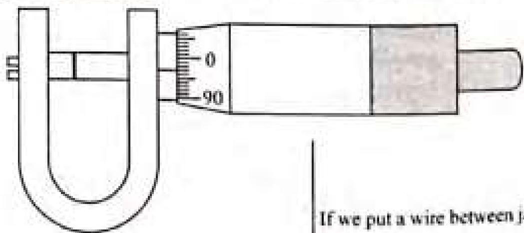
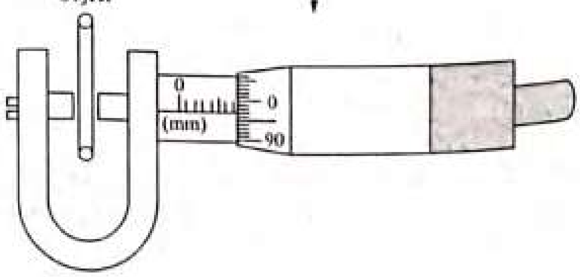


ILLUSTRATION 33.4 Find the thickness of the wire.



If we put a wire between jaws

Object



Solution. Excess reading (zero error) = –0.07 mm

It is giving 7.95 mm in which there is –0.07 mm excess reading, which has to be removed (subtracted).

So actual reading = 7.95 – (–0.07) = 8.02 mm

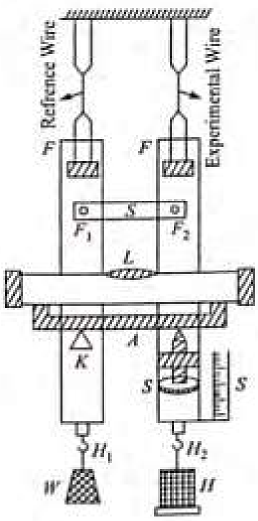
EXPERIMENT 3

Determination of Young's Modulus by Searle's Method

Searle's apparatus (static method): Figure shows a Searle's apparatus. It consists of two metal frames F_1 and F_2 hinged together so that they can have only vertical relative motion. A spirit level L is supported at one end on rigid cross bar frame whose other end rests on the tip of a micrometer screw S, which moves vertically through rigid cross bar.

If there is any relative motion between the two frames, the spirit level no longer remains horizontal and the bubble is displaced. To bring the bubble back to its original position, the screw has to be moved up and down. The distance through which the screw has to be moved gives the relative motion between the two frames.

The frames are suspended by two identical long wires of the steel from the same rigid horizontal support. The wire B is an experimental wire and the wire A acts as a reference wire. The two frames are provided with two hooks H_1 and H_2 at their lower ends. The hook H_1 carries a constant weight W to keep the wire taut. To the hook H_2, a hanger is attached over which slotted weights can be put to apply the stretching force.



Working procedure: First of all, measure the length of experimental wire from the point where it leaves the fixed support to be the point where it is fixed in the frame. Then measure the diameter of experimental wire using the screw gauge. Initially the system has to be held in such a way that the bubble in the SL is exactly at the centre (when there is no load on hanger), at this stage note down the micrometer reading. Now, load the experimental wire in steps of 0.5 kg or 1 kg by placing them on hanger upto a permissible value (half of the breaking force0 as a result of which wire gets elongated by small amount and SL gets disturbed from the horizontal setting. This increased in length is measured by turning the screw M upwards so at to restore the SL.

$$\Delta L = n \times \text{pitch} + f \times \text{least count}$$

Where $n \rightarrow$ the no. of turns of the micrometer screw.

$f \rightarrow$ the difference in the cap reading.

The load on he hanger is then reduced in same steps and SL is restored to its horizontal position. The mean of these two observations gives the true increase n length of the wire corresponding to the given value of load.

Observation table and calculations

Load on the Hanger	Spherometer Position		Mean,	Extension, Δl
	Loading (x)	Unloading (y)	$z = \dfrac{x+y}{2}$	

Prepare the Observation Table as shown, and then plot Δl versus load (on hanger) graph.

$$\tan\theta = \frac{\Delta l}{\text{load}} = \frac{\Delta l}{mg}$$

$$\text{From } y = \frac{mgL}{A \times \Delta l}$$

$$Y = \frac{L}{A} \times \frac{1}{\tan\theta}; \; L \text{ and } A \text{ has been already measured.}$$

Errors and Precautions

Both types of errors, systematic and random, can occur which can be reduced by careful experimentation and appropriate averaging process. We should take the following precautions:

(a) Both the wires should be identical.

(b) Both the wires should be supported from the same rigid support.

(c) Wire should be loaded and unloaded in the same steps.

(d) For every loading and unloading, allow the wire to become stabilized.

ILLUSTRATION 33.5 A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ±0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ±0.01 mm. Take $g = 9.8$ m/s^2 (exact). The Young's modulus obtained for the reading is

(a) $(2.0 \pm 0.3) \times 10^{11}$ N/m^2

(b) $(2.0 \pm 0.2) \times 10^{11}$ N/m^2

(c) $(2.0 \pm 0.1) \times 10^{11}$ N/m^2

(d) $(2.0 \pm 0.05) \times 10^{11}$ N/m^2

Solution. From $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{4F}{\pi d^2} \times \dfrac{l}{\Delta l}$

where $d \to$ diameter of wire
$\Delta l \to$ elongation in wire
$L \to$ length of wire
$F \to$ load supported by wire

$$\Rightarrow Y = \frac{4 \times 1.0 \times 9.8 \times 2}{\frac{22}{7} \times \left(0.4 \times 10^{-3}\right)^2 \times \left(0.8 \times 10^{-3}\right)}$$

$$= 2.0 \times 10^{11} \text{ N/m}^2$$

From $Y = \dfrac{4F}{\pi d^2} \times \dfrac{l}{\Delta l}$

$\Rightarrow$ Error (relative) in Y is given by,

$\dfrac{\delta Y}{Y} = -2\dfrac{\delta d}{d} - \dfrac{\delta(\Delta l)}{\Delta l}$ as F and l are constants.

As for random errors, we have to consider the worst case:

$$\frac{\delta Y}{Y} = \pm\left[2 \times \frac{0.01}{0.4} + \frac{0.05}{0.8}\right] = \frac{9}{80}$$

$$= \pm \frac{9}{80} \times Y = 0.225 \times 10^{11} = 0.2 \times 10^{11}$$

[By rounding off]

So, $Y = (2 \times 10^{11} \pm 0.2 \times 10^{11})$ N/m^2

ILLUSTRATION 33.6 In Searle's experiment to find Young's modulus, the diameter of wire is measured as $D = 0.05$ cm, length of wire is $L = 125$ cm, and when a weight, $m = 20.0$ kg is put, extension in wire was found to be 0.100 cm. Find maximum permissible error in young's modulus (Y).

Solution. $\dfrac{mg}{\pi d^2/4} = Y\left(\dfrac{x}{\ell}\right) \Rightarrow Y = \dfrac{mg\ell}{(\pi/4)\, d^2 x}$

$$\left(\frac{dY}{Y}\right)_{\max} = \frac{\Delta m}{m} + \frac{\Delta \ell}{\ell} + 2\frac{\Delta d}{d} + \frac{\Delta x}{x}$$

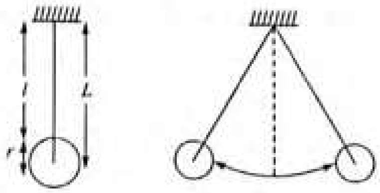
$m = 20.0$ kg $\Rightarrow$ $\Delta m = 0.1$ kg
$\lambda = 125$ m $\Rightarrow$ $\Delta\lambda = 1$ cm
$d = 0.050$ cm $\Rightarrow$ $\Delta d = 0.001$ cm
$x = 0.100$ cm $\Rightarrow$ $\Delta x = 0.001$ cm

$$\left(\frac{dY}{Y}\right)_{\max} = \left(\frac{0.1 \text{ kg}}{20.0 \text{ kg}} + \frac{1 \text{ cm}}{125 \text{ cm}} + \frac{0.001 \text{ cm}}{0.05 \text{ cm}} + \frac{0.001 \text{ cm}}{0.100 \text{ cm}}\right) \times 100\%$$

$$= 4.3\ \%$$

EXPERIMENT 4

Determining the value of g using a simple pendulum.



In this experiment, a small spherical bob is hanged with a cotton thread. This arrangement is called simple pendulum. The bob is displaced slightly and allowed to oscillate. To find time period, the time taken for 50 oscillations is noted using a stop watch.

Theoretically, $T = 2\pi\sqrt{\dfrac{L}{g}} \quad \Rightarrow \quad g = 4\pi^2 \dfrac{L}{T^2}$ (1)

where L = Equivalent length of pendulum = Length of thread (L) + Radius (r) of bob,

T = time period of the simple pendulum

$$= \frac{\text{Time taken for 50 oscillations}}{50}$$

So g can be easily determined by equation (1)

Graphical method to find g:

$$T^2 = \left(\frac{4\pi^2}{g}\right)L \qquad (2)$$

So, $T^2 \propto L$

- Find T for different values of L.
- Plot T^2 v/s L curve. From equation (2), it should be a straight line, with slope = $\left(\dfrac{4\pi^2}{g}\right)$.
- Find slope of T^2 v/s L graph and equate it to $\left(\dfrac{4\pi^2}{g}\right)$ and get g.

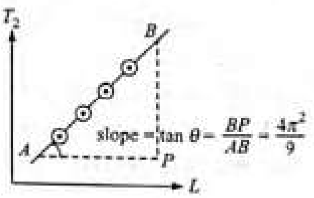
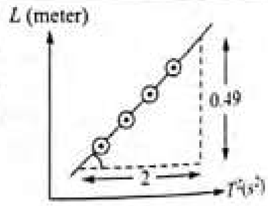


ILLUSTRATION 33.7 For different values of L, we get different values of T. The curve between L v/s T^2 is shown. Estimate g from this curve. (Take $\pi^2 = 10$)



Solution. $L = \left(\dfrac{g}{4\pi^2}\right)T^2$,

Slope = $\left(\dfrac{g}{4\pi^2}\right)$

The curve between L v/s T^2

Slope = $\dfrac{0.49}{2} = \dfrac{g}{4\pi^2} \Rightarrow g = 9.8$ m/s

ILLUSTRATION 33.8 In certain observation, we got $\lambda = 23.2$ cm, $r = 1.32$ cm, and time taken for 10 oscillation was 10.0 sec. Find maximum permissible error in (g).

Solution. $\lambda = 23.2 \Rightarrow \Delta\lambda = 0.1$ cm
$r = 1.32$ cm $\Rightarrow \Delta r = 0.01$ cm

$t = 10.0$ sec $\Rightarrow$ $\Delta t = 0.1$ sec

$$\left(\frac{dg}{g}\right)_{max} = \left(\frac{0.1\text{ cm} + 0.01\text{ cm}}{23.2\text{ cm} + 1.32\text{ cm}} + 2\frac{0.1\text{ sec}}{10.0\text{ sec}}\right) \times 100\% = 1.2\%$$

ILLUSTRATION 33.9 To find the value of 'g' using simple pendulum. $T = 2.00$ s; $l = 1.00$ m was measured. Estimate maximum permissible error in 'g'. Also find the value of 'g'. (use $\pi^2 = 10$)

Solution. $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2 \ell}{T^2}$

$$\left(\frac{dg}{g}\right)_{max} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T} = \left(\frac{0.01}{1.00} + 2\frac{0.01}{2.00}\right) \times 100\% = 2\%$$

Value of $g = \frac{4\pi^2 \ell}{T^2} = \frac{4 \times 10 \times 1.00}{(2.00)^2} = 10\text{ m/s}^2$

$$\left(\frac{dg}{g}\right)_{max} = \frac{2}{100} \text{ so } \frac{dg_{max}}{10.0} = \frac{2}{100}$$

So $(dg)_{max} = 0.2 =$ max error in g

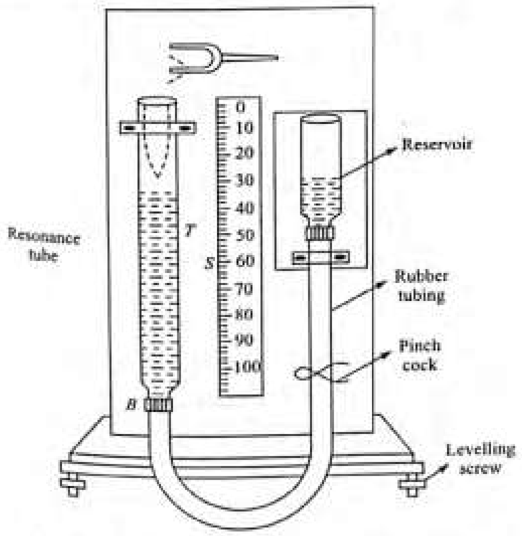
So $g = (10.0 \pm 0.2)\text{ m/s}^2$

EXPERIMENT 5

Find the speed of sound in air at room temperature using a resonance tube.

Speed of sound using resonance column method

- The resonance tube (A) is a long narrow tube which is connected to a reservoir (B) by a rubber tube. The resonance tube is attached to a wooden frame, to which a scale has been attached. Both tubes contain water. By raising or lowering the reservoir, we can decrease or increase the length of air column in A.



- A vibrating tuning fork of known frequency (f) is held at the mouth of A. When the length of air column in A is practically zero, you would hear no sound. Now lower the reservoir to increase the length of air column. As length of air column increases, intensity of sound increases – reaches a maximum and then dies away.

Repeat the procedure, adjust B for the length of air column corresponding to maximum intensity of sound and then measure the length l_1 of the air column. The loud sound is heard because the natural frequency of the air column of length l_1 is matches with the frequency of tuning fork and hence resonance occurs. This is the first position of resonance. After going for end corrections, $l_1 + e = \lambda/4$ [from theory of organ pipes] where λ is the wavelength of sound wave. When you lower B still further so that the length of the air column is increased, you will find another position of resonance. Measure the length l_2 of air column, this is the second position of resonance.

$$l_2 + e = \frac{3\lambda}{4}$$

$$\Rightarrow \frac{\lambda}{2} = l_2 - l_1$$

$v = f\lambda$ where v is velocity of sound in air.

$$\Rightarrow \quad v = f \times 2\,(l_2 - l_1)$$

Maximum permissible error in speed of sound due to error in f_0, ℓ_1, ℓ_2:

For resonance tube experiment:

$v = 2f_0(\ell_2 - \ell_1)$

$\ln v = \ln 2 + \ln f_0 + \ln(\ell_2 - \ell_1)$

Max. permissible error in speed of sound

$$= \left(\frac{dv}{v}\right)_{max} = \frac{\Delta f_0}{f_0} + \frac{\Delta \ell_2 + \Delta \ell_1}{(\ell_2 - \ell_1)}$$

ILLUSTRATION 33.10 If a tuning fork of (340 Hz ± 1%) is used in the resonance tube method, and the first and second resonance lengths are 24.0 cm and 74.0 cm, respectively, find max. permissible error in speed of sound.

Solution. $\ell_1 = 20.0\text{ cm} \rightarrow \Delta\ell_1 = 0.1\text{ cm}$

$\ell_2 = 74.0\text{ cm} \rightarrow \Delta\ell_2 = 0.1\text{ cm}$

$f_0 = (340\text{ Hz} \pm 1\%)$ $\quad \frac{\Delta f_0}{f_0} = 1\% = \frac{1}{100}$

$$\left(\frac{dv}{v}\right)_{max} = \frac{\Delta f_0}{f_0} + \frac{\Delta \ell_2 + \Delta \ell_1}{\ell_2 - \ell_1}$$

ILLUSTRATION 33.11 In resonance tube exp., we find $\ell_1 = 25.0$ cm and $\ell_2 = 75.0$ cm. If there is no error in frequency, what will be the max permissible error in speed of sound (take $f_0 = 325$ Hz)?

Solution. $v = 2f_0(\ell_2 - \ell_1)$

$(dv) = 2f_0(d\ell_2 - d\ell_1)$

$(dv)_{max} = \text{max of } [2f_0 \pm (d\ell_2 + d\ell_1)]$

$= 2f_0(d\ell_2 + d\ell_1)$

$\ell_1 = 25.0\text{ cm} \Rightarrow \Delta\ell_1 = 0.1\text{ cm}$

(place value of last number)

$\ell_1 = 75.0$ cm $\Rightarrow$ $\Delta\ell_2 = 0.1$ cm (place value of last number)

So max permissible error in speed of sound

$(dv)_{max} = 2(325\text{Hz})\ (0.1\text{ cm} + 0.1\text{ cm}) = 1.3$ m/s

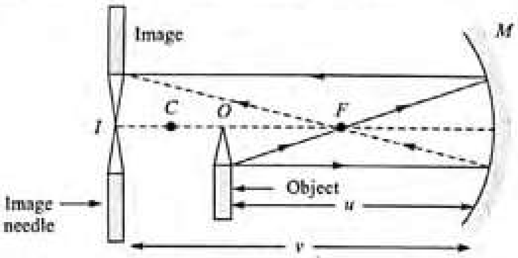
Value of $v = 2f_0(\ell_2 - \ell_1) = 2(325\text{Hz})(75.0\text{ cm} - 25.0\text{ cm}) = 325$ m/s

So $v = (325 \pm 1.3)$ m/s

EXPERIMENT 6

Object: To determine the focal length of a concave mirror.

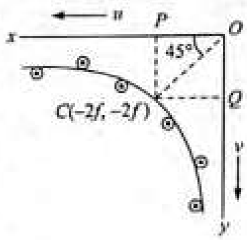
Material required: Optical bench, upright stands, optical needles, knitting needle and concave mirror.



Theory: When an object needle is placed at a distance u from the pole of the concave mirror by removing parallex between the image needle and inverted image of object needle, the position of image is located. Let the distance between pole of mirror and image needle be v. Then the focal length (f) of the mirror is given by $f = \dfrac{u \times v}{u + v}$

Graphical method:

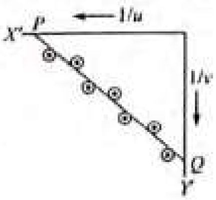
(1) **u–v graph:** To calculate the focal length f. A plot of graph between u (along negative x-axis) and v (along negative y-axis) is a rectangular hyperbola. Draw a line OC at an angle of 45° with x-axis. The point where it meets the hyperbola is $(-2f, -2f)$. Draw lines OP and OQ perpendicular to x and y axes.



$$f_1 = \frac{OP}{2}$$

and $$f_2 = \frac{OP}{2},\ f = \frac{f_1 + f_2}{2}$$

(2) **$\dfrac{1}{u}$ and $\dfrac{1}{v}$ graph:** A graph plotted between $1/u$ (along negative x-axis) $1/v$ (along negative y-axis) is a straight line which cuts the axes at points P and Q at an angle of 45° with each axes. The intercept $OP = \dfrac{1}{f_1}$ and $OQ = \dfrac{1}{f_2}$.



Then mean focal length, $f = \dfrac{f_1 + f_2}{2}$.

Procedure: Find focal length of concave mirror by focusing the image of a distant object on the wall. Take an optical needle as an object needle (O). The tip of the object needle and the pole of the mirror should be on the same line. Place the object needle at a distance 1.5 times the focal length of the mirror. Place another image needle behind the object needle and on the same side of the mirror. Remove the parallax between the inverted image of the object needle and the image needle. Adjust the position of the upright by moving it forward and backwards, so that the parallax is removed. In this position, image needle and the inverted image of image needle will move together when the eye is moved to the left or right. Adjust the height of the needle to remove the parallax tip to tip. Note the positions of the object, image and mirror uprights. Repeat the above set for five different positions of the object needle. Apply index correction for object and image needle separately. Then plot graphs between u-v and $1/u$ and $1/v$ and find focal length f of the mirror.

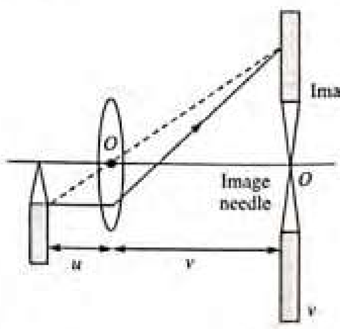
Precautions:

1. The uprights supporting the needles and the mirror should be rigid.
2. The tips of the needles and the pole of the mirror should be at the same horizontal level.
3. The parallax between the image needle and the image of the object needle should be removed tip to tip.
4. While removing the parallax, the eye should be positioned at a distance of at least 30 cm the image needle.

EXPERIMENT 7

To find the focal length of a convex lens.

Material required: An object bench, three uprights, a convex lens, a lens holder, two optical needle and a half metre scale, knitting needle.



Theory: When a ray of light, parallel to the principal axis is incident on convex lens, after refraction, it passes through the focus, the nature and sizer of the image depends on the position of the object. The focal length of convex lens is given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

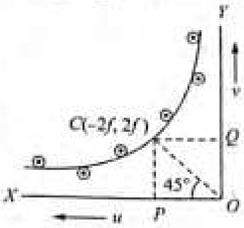
where v = distance of the image from the optical centre.

u = distance of the object from the optical centre.

f = focal length of the lens.

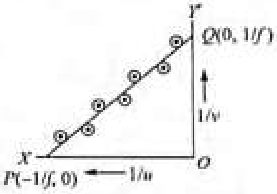
Graphical Method:

(1) **u–v graph:** Plot a graph between u (along negative x-axis) and v (along positive y-axis). It will rectangular hyperbola. Draw a line OC making an angle of 45° with x-axis. The point C where it meets the curve is $(-2f, 2f)$ half the value of either coordinate gives us the value of focal length f.



(2) $\frac{1}{u}$ **and** $\frac{1}{v}$ **graph:**

Plot a graph $1/u$ and $1/v$ by selecting a suitable scale. It will be a straight line cutting x-axis and the y-axis at an angle of 45° at points P and Q. The intercepts $OP = 1/f_1$ and $OQ = 1/f_2$ then mean of focal length, $f = \frac{f_1 + f_2}{2}$ cm.



Procedure: Find rough focal image of the convex lens by focusing the image of a distance object on the wall. Place the optical needle on one side of the lens at position between f and $2f$ and the image needle on the outer side. Remove the parallax between the image of the object pin and the image pin. (To remove the parallax, let the tips of the image pin and the image of the object pin coincide. The image in is now moved forward or backward so that the image of the object pin and the image pin move together when the eye is moved to left or right. Note the position of lens, object pin and image pin on the optical bench. Repeat the experiment with different position of object needle without disturbing the lens position. Find the index correction to be applied to u and v.

Precautions:

1. The uprights should be vertical.
2. The tips of the needles should as high as the optical centre of the lens.
3. Parallax should be removed tip to tip.
4. The image and object needles should not be interchanged for different sets of observations.
5. White removing the parallax, the eye should be placed at a distance of about 30 cm from the image needle.
6. Index corrections should be applied to both object and image distances.

ILLUSTRATION 33.12 In u–v method to find focus distance of a concave mirror, if object distance was found to be, 10.0 cm and, image distance was also found to be 10.0 cm, then find maximum permissible error in f due to error is u and v measurement.

Solution. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \Rightarrow \quad \frac{1}{(-10)}+\frac{1}{(-10)}=\frac{1}{f}$

$\Rightarrow \quad |f| = 5$ cm

$$(df)_{\max} = \left(\frac{\Delta u}{u^2}+\frac{\Delta v}{v^2}\right)\times f^2$$

$$= \left(\frac{0.1}{10^2}+\frac{0.1}{10^2}\right)\times 5^2 = 0.05 \text{ cm}$$

So, $f = (5 \pm 0.05)$ cm

SOLVED EXAMPLES

1. The main scale of a Vernier callipers reads 10 mm in 10 divisions. 10 divisions of Vernier scale coincide with 9 divisions of the main scale. When the two jaws of the callipers touch each other, the fifth division of the Vernier coincides with 9 main scale divisions and the zero of the Vernier is to the right of zero of main scale. When a cylinder is tightly placed between the two jaws, the zero of Vernier scale lies slightly to the left of 3.2 cm and the fourth Vernier division coincides with a main scale division. The diameter of the cylinder is

(a) 3.09 cm (b) 3.14 cm
(c) 3.04 cm (d) none of these

Sol. (a) Zero erro = 0.5 mm = 0.05 m

Deserved reading of cylinder = 3.1 cm + (4) (0.01 cm)
= 3.14 cm

Actual thickness of cylinder = (3.14) – (0.06)
= 3.09 cm

2. A student performs an experiment for determination of $g\left(=\frac{4\pi^2\ell}{T^2}\right)$, L = 1m, and he commits an error of ΔL. For T, he takes the time of n oscillations with the stop watch of least count Δt and he commits a human error of 0.1 sec. For which of the following data, the measurement of g will be most accurate?

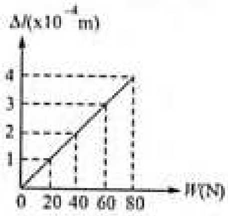
(a) $\Delta L = 0.5, \Delta t = 0.1, n = 20$
(b) $\Delta L = 0.5, \Delta t = 0.1, n = 50$
(c) $\Delta L = 0.5, \Delta t = 0.01, n = 20$
(d) $\Delta L = 0.1, \Delta t = 0.05, n = 50$

Sol. (d) Here $T = \frac{\text{Total time}}{\text{Total oscillation}} = \frac{t}{n}$. So $dT = \frac{dt}{n}$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

In option (d), error in L (ΔL) is minimum and number of repetition of measurement are maximum. So dT will be less. So, in this case, the error in g is minimum.

3. The adjacent graph shows the extra extension (Δx) of a wire length 1 m suspended from the top of a roof at one end with an extra load ΔW connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire.



(a) 2×10^{11} N/m^2
(b) 2×10^{-11} N/m^2
(c) 3×10^{13} N/m^3
(d) 2×10^{16} N/m^2

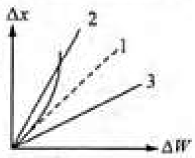
Sol. (a) $\Delta l = \left(\dfrac{l_0}{AY}\right)\Delta W$ (i)

Hence, slope of the line $\dfrac{\Delta l}{\Delta W} = \dfrac{(1.0\times10^{-4}-0)}{(20-0)}$

But from (i), $\dfrac{\Delta l}{\Delta W} = \dfrac{l_0}{AY}$

$Y = 20\times10^{10} = 2.0\times10^{11}$ N/m^2

4. In the experiment, the curve between Δx and ΔW is shown as dotted line (1). If we use an another wire of same material, but with double length and double radius, which of the curve is expected?



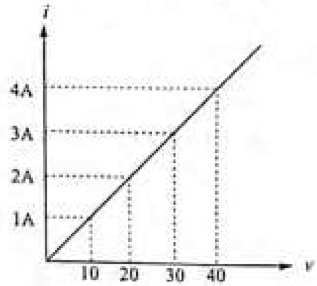
(a) 1 (b) 2
(c) 3 (d) 4

Sol. (c) Initially slope $= \dfrac{\Delta x}{\Delta W} = \dfrac{l_0}{(\pi r_0^2)\,(y_0)}$

In second case (slope)$' = \dfrac{(2l_0)}{\pi(2r_0)^2\,y_0} = \dfrac{1}{2}\dfrac{l_0}{(\pi r_0^2)y_0}$

So slope will be halved, Answer will be 3.

5. If emf of battery is 100 V, then what was the resistance of Rheostat adjusted at reading (2)? ($i = 2$ A, $V = 20$ V).



(a) 10 Ω (b) 20 Ω
(c) 30 Ω (d) 40 Ω

Sol. (d) From the curve slope $= \dfrac{i}{v} = \dfrac{1}{R} = \dfrac{1}{10}$

$R = 10\,\Omega$

For second reading, $i = \dfrac{\text{Emf}}{R+R_{rh}} \Rightarrow 2 = \dfrac{100}{10+R_{rh}}$

$\Rightarrow R_{rh} = 40\,\Omega$

6. In the experiment of Ohm's law. When potential difference 10.0 V is applied, current measured is 1.00 A. If length of wire of found to be 10.0 cm, and diameter of wire is 2.50 mm, then the maximum permissible error in resistivity will be

(a) 1.8% (b) 10.2%
(c) 3.8% (d) 5.75%

Sol. (c) $\left(\dfrac{d\rho}{\rho}\right)_{max} = 2\left(\dfrac{0.01}{2.50}\right)+\left(\dfrac{0.1}{10.0}\right)+\left(\dfrac{0.1}{10.0}\right)+\left(\dfrac{0.01}{1.00}\right)$

$= 3.8\%$

7. The %error in measurement of resistivity is mostly effected by %error in

(a) length measurement
(b) voltage measurement
(c) current measurement
(d) diameter measurement

Sol. (d) $\left(\dfrac{d\rho}{\rho}\right)_{max}$ is mostly effected by % error in diameter.

8. From some instruments, current measured is $i = 10.0$ A, potential difference measured is $V = 100.0$ V, length of wire is 31.4 cm, and diameter of wire is 2.00 mm (all in correct significant figure). The resistivity of wire(in correct significant figure) will be (use $\pi = 3.14$)

(a) 1.00×10^{-4} Ω-m (b) 1.0×10^{-4} Ω-m
(c) 1×10^{-4} Ω-m (d) 1.000×10^{-4} Ωm

Sol. (a) $\rho = \dfrac{\pi D^2}{4L}\dfrac{V}{I} = \dfrac{(3.14)\,(2.00\times10^{-3})^2}{4(0.314)}\left(\dfrac{100.0}{10.0}\right)$

and answer should be in three Significant figures. So $\rho = 1.00\times10^{-4}$ Ω -m

9. In the previous question, maximum permissible error is resistivity and resistance measurement will be (respectively)

(a) 2.14%, 1.5% (b) 1.5%, 2.45%
(c) 2.41%, 1.1% (d) None of these

Sol. (c) $\left(\dfrac{dR}{R}\right)_{max} = \dfrac{\Delta i}{i}+\dfrac{\Delta v}{v} = \dfrac{0.1}{10.0}+\dfrac{0.1}{100.0} = 1.1\%$

$\left(\dfrac{dR}{R}\right)_{max} = 2.41\%$

10. If resistance S in RB = 300Ω, then the balanced length is found to be 25.0 cm from end A. The diameter of unknown wire is 1mm and length of the unknown wire is 31.4 cm. The specific resistivity of the wire should be

(a) 2.5×10^{-4} Ω-m (b) 3.5×10^{-4} Ω-m
(c) 4.5×10^{-4} Ω-m (d) None of these

Sol. (a) $\dfrac{R}{300} = \dfrac{25}{75} \Rightarrow R = 100\,\Omega$

$\rho = \dfrac{R\pi d^2}{4L} = 2.5\times10^{-4}$ Ω-m

11. In the previous question, if R and S are interchanged, the balanced point is shifted by

(a) 30 cm (b) 40 cm
(c) 50 cm (d) None of these

Sol. *(c)* If R and S wave interchanged, $l = 75$ cm, $100 - l = 25$ cm
Balance point will be shifted by $75 - 25 = 50$ cm

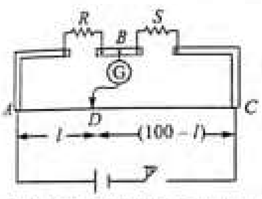
12. In a meter bridge, null point is at $l = 33.7$ cm, when the resistance S is shunted by 12 Ω resistance the null point is found to be shifted by a distance of 18.2 cm. The value of unknown resistance R should be
(a) 13.5 Ω (b) 6.85 Ω
(c) 3.42 Ω (d) None of these

Sol. *(b)* According to the condition of balance of Wheatstone bridge,

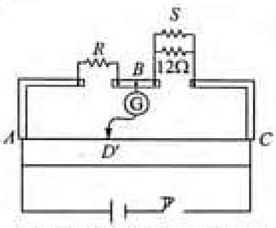
$$\frac{R}{S} = \frac{l}{(100 - l)}$$

Here $l = 33.7$ cm and $(100 - l) = 100 - 33.7 = 66.3$ cm

$$\Rightarrow \quad \frac{R}{S} = \frac{33.7}{66.3} \qquad \text{(i)}$$



Before applying shunt resistance



After applying shunt resistance

As resistance S' is due to a parallel combination of resistance S and a resistance of 12 Ω, hence

$$\frac{1}{S'} = \frac{1}{S} + \frac{1}{12} = \frac{12 + S}{12S} \text{ or } S' = \frac{12S}{12 + S}$$

Since S' is less than S, the ratio R/S' will be greater than R/S, hence the null point should shift towards the end C.

$$\Rightarrow \quad \frac{R}{S'} = \frac{33.7 + 18.2}{66.3 - 18.2} = \frac{51.9}{48.1} \text{ or } \frac{R(12 + S)}{12S} = \frac{51.9}{48.1}$$

$$\Rightarrow \quad \frac{R}{S} = \frac{12 \times 51.9}{(12 + S) \times 48.1} \qquad \text{(ii)}$$

After solving Eqs. (i) and (ii) we get
$S = 13.47$ Ω and $R = 6.85$ Ω

13. To find index error for u, when a knitting needle of length 20.0 cm is adjusted between pole and object needle, the separation between the indices of object needle and mirror was observed to be 20.2 cm. Index correction for u is
(a) −0.2 cm (b) 0.2 cm
(c) −0.1 cm (d) 0.1 cm

Sol. *(b)* Index error (excess reading)
= Observed reading − Actual reading
= (20.2 − 20.0) cm
= 0.2 cm

14. To find index error for v, when the same knitting needle is adjusted between the pole and the image needle, the separation between the indices of image needle and mirror was found to be 19.9 cm. The index error for v is
(a) 0.1 cm (b) −0.1 cm
(c) 0.2 cm (d) −0.2 cm

Sol. *(b)* $e = 19.9$ cm $- 20.0$ cm $= -0.1$ cm

15. In some observation, the observed object distance (separation between indices of object needle and mirror) is 30.2 cm, and the observed image distance is 19.9 cm. Using index correction from previous two equations, estimate the focus distance of the concave mirror.
(a) 12.0 cm (b) 11.0 cm
(c) 13.0 cm (d) 10.0 cm

Sol. *(a)* $u = 30.2 - 0.2$ (Excess reading)
$= 30.0$ cm.
$v = 19.9 - (-0.1)$ (Excess reading)
$= 20.0$ cm

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow f = 12.0 \text{ cm}$$

EXERCISES

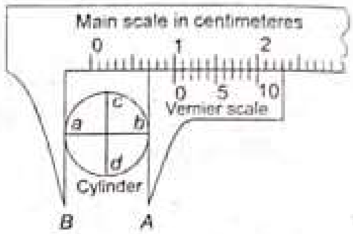
1. The vernier constant of a calliper is 0.01 cm. Taking all possible precautions, the most possible in the measurement with this vernier calliper will be
(a) 0.005 cm (b) 0.001 cm
(c) 0.01 cm (d) 0.02 cm

2. A vernier calliper has 20 divisions on the vernier scale which coincide with 19 divisions of the main scale. What is the vernier constant of the instrument if the main scale division is of 2 mm?
(a) 1 mm (b) 0.1 mm
(c) 0.2 mm (d) 0.01 mm

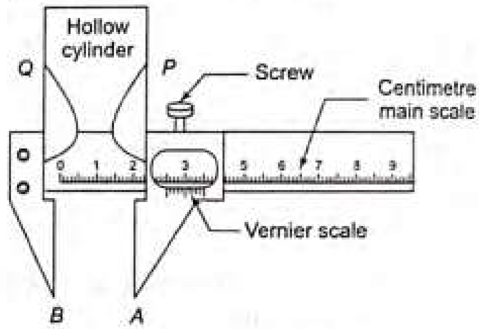
3. In vernier callipers N divisions of vernier scale coincide with $(N - 1)$ divisions of main scale in which length of a division is 1 mm. The least count of the instrument in centimetres is
(a) $\frac{1}{10N}$ (b) $N - 1$
(c) N (d) $\frac{1}{N} - 1$

4. The main scale of vernier callipers is divided into 0.5 mm and its least count is to be 0.005 cm. The number of vernier divisions is
(a) 10 (b) 20
(c) 30 (d) 40

5. The side of a cubical block when measured with a vernier calliper is 2.5 cm. If its vernier constant is 0.01 cm, then maximum possible error in the area of the side of the block is
   (a) $\pm 0.01\ cm^2$ (b) $\pm 0.02\ cm^2$
   (c) $\pm 0.05\ cm^2$ (d) $\pm 0.10\ cm^2$
6. The length of a wire is measured with a metre rod having least count 1 mm. Its external diameter is measured with vernier calliper having least count 0.1 mm. If the length and radius of the wire is 5 cm and 20 mm respectively, the percentage error in the calculated value of the volume of the wire is
   (a) 1% (b) 3%
   (c) 4% (d) 5%
7. The vernier constant of a vernier calliper is 0.01 cm. If the length of an object measured by this is 1 mm, the percentage accuracy in the measurement is
   (a) 1% (b) 10%
   (c) 50% (d) 100%
8. If a vernier calliper has least count 0.05 mm, the accuracy of its measurement is
   (a) 0.05 mm (b) Less than 0.05 mm
   (c) Greater than 0.05 mm (d) None of these
9. In a vernier calliper 10 smallest divisions of the vernier scale are equal to 9 small divisions on the main scale. If the smallest division on the main scale is 0.5 mm, then the vernier constant is
   (a) 0.5 mm (b) 0.1 mm
   (c) 0.05 mm (d) 0.005 mm
10. One centimetre on the main scale of a vernier calliper is divided into 10 equal parts. If 10 divisions of the vernier scale coincide with 8 divisions of the main scale, the least count of the calliper is
   (a) 0.005 cm (b) 0.05 cm
   (c) 0.02 cm (d) 0.01 cm
11. The depth of a vessel is measured by vernier callipers by the help of
   (a) The thin rod projecting at the back
   (b) The lower pair of jaws
   (c) The upper pair of jaws
   (d) None of these
12. What is zero error in vernier callipers?
   (a) If the zeros of vernier scale and main scale do not coincide when the two jaws are in close contact without any gap it has a zero error
   (b) When the zero of the vernier scale is not marked correctly
   (c) When the zero of main scale is not marked correctly
   (d) None of the above
13. What is the vernier constant of the vernier callipers?
   (a) The difference between one scale division and one vernier division
   (b) The value of one scale division
   (c) The value of one vernier division
   (d) $\dfrac{\text{One vernier division}}{\text{One scale division}}$
14. What is the diameter of the cylinder held with its cross section in the jaws of vernier callipers?



   (a) 1.06 cm (b) 1.16 cm
   (c) 1. 26 cm (d) None of these
15. What is the reading for the internal diameter of a hollow cylinder held between the upper pair of jaws of a vernier calliper? (Least count is 0.1 cm.)



   (a) 3.1 cm (b) 3.15 cm
   (c) 31.5 cm (d) 0.315 cm
16. The diameter of a wire is measured with a screw gauge having least count 0.01 mm. Out of the following which one correctly expresses the diameter?
   (a) 2.00 mm (b) 0.2 cm
   (c) 0.20 cm (d) 0.002 mm
17. The pitch of screw gauge is 0.5 mm. Its head scale contains 50 divisions. The least count of the instrument is
   (a) 0.01 mm (b) 0.1 mm
   (c) 0.25 mm (d) 0.02 mm
18. The screw gauge has least count of 0.005 mm and its head scale is divided into 100 equal divisions. What is the distance between two consecutive threads of its screw?
   (a) 0.5 mm (b) 0.05 mm
   (c) 0.01 mm (d) 0.1 mm
19. The least count of a screw gauge is 1/100 mm and the pitch of the screw is 1 mm. What is the maximum percentage error of the instrument?
   (a) 5% (b) 2%
   (c) 1% (d) 10%
20. The radius of a ball bearing measured by a screw gauge is 3.75 mm. The pitch of the screw is 1 mm and it has 100 divisions on its head scale. What is the percentage

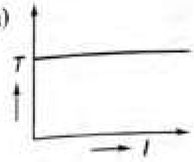
error in the volume of the ball bearing which can be assumed to be a perfect sphere?

(a) 2% (b) 1.5%

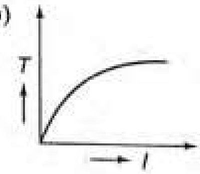
(c) 0.8% (d) 1%

21. The graph between time period T and length of simple pendulum L is depicted by

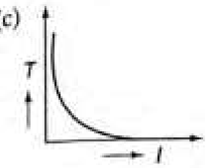
(a)



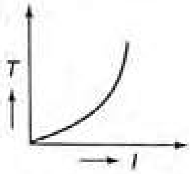
(b)



(c)



(d)



22. The bob of a simple pendulum is a hollow ball filled with water. A plugged hole near the bottom of oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

(a) First increase and then decrease to original value

(b) First decrease and then increase to original value

(c) Remain unchanged

(d) Increase towards a saturation value

23. Total energy of particle executing SHM is proportional to

(a) Frequency of oscillation

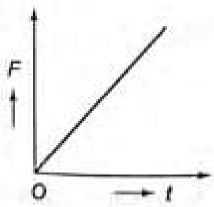
(b) Displacement from mean position

(c) Square of amplitude

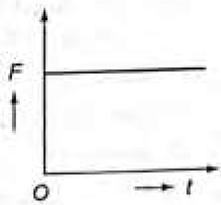
(d) Velocity in equilibrium

24. The graph between square of amplitude and time elapsed is depicted by

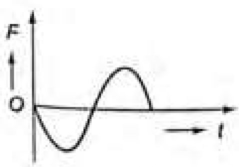
(a)



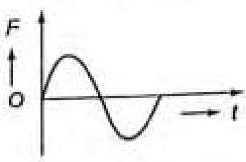
(b)



(c)



(d)

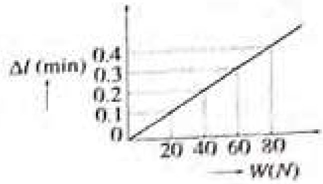


25. When the stress is increased beyond the elastic limit the length of the wire starts increasing without increasing the force. The point is called

(a) Triple point (b) Yield point

(c) Inverse point (d) Breaking point

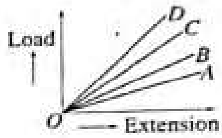
26. The following graph shows the variation of extension Δl of a wire of length 1 m suspended from the ceiling of room at one end a load W (in Newtons) connected to free end of wire. If cross sectional area of the wire is 10^{-6} m^2, then Young's modulus of the material of the wire is



(a) 2×10^{-11} N/m^2 (b) 2×10^{11} N/m^2

(c) 5×10^{-12} N/m^2 (d) 5×10^{12} N/m^2

27. The load versus extension graph for four wires of same material is shown in the figure below



The thinnest wire is represented by the line

(a) OA (b) OB

(c) OC (d) OD

28. In a resonance tube, we get

(a) Stationary longitudinal wave

(b) Stationary transverse wave

(c) Progressive longitudinal wave

(d) Progressive transverse wave

29. A tuning fork of frequency 500 Hz is sounded and a resonance is obtained at 17 cm and second at 52 cm. The velocity of sound is

(a) 650 m/s (b) 350 m/s

(c) 700 m/s (d) 190 m/s

30. In resonance tube apparatus, the first and second resonance is obtained at 15 cm and 48 cm, the end correction for this apparatus is equal to

(a) 6 cm (b) 3 cm

(c) 1.5 cm (d) None of these

31. The Wheatstone bridge is in the more balanced state when the ratio of arms P and Q is equal to

(a) Zero (b) Less than one

(c) One (d) More than one

32. The main function of the Wheatstone bridge is to

(a) Measure the resistance only

(b) Compare the resistance

(c) Measure current and the resistance

(d) Measure the current only

33. The metre bridge is most sensitive when

(a) Balance point is near 100 cm mark

(b) Balance point is near 50 cm mark

(c) Balance point is near 150 cm mark

(d) Balance point is near 75 cm mark

34. When the metre bridge is most sensitive

(a) $P = Q, R = X$ (b) $P = Q = R = X$

(c) $P = R, Q = X$ (d) $P \neq Q \neq R \neq X$

35. The material of a metre bridge is

(a) Constantan (b) Copper

(c) Silver (d) Platinum

36. The strips fitted on a metre bridge are
(a) Thin copper strips (b) Thick copper strips
(c) Thin constantan strips (d) Thick constantan strips

37. In making an Ohm's law circuit, which of the following connections is correct?
(a) Voltmeter in series and ammeter in parallel
(b) Voltmeter in parallel and ammeter in series
(c) Both voltmeter and ammeter in parallel
(d) Both voltmeter and ammeter in series

38. A rheostat is used in an electrical circuit
(a) To change the resistance of the circuit
(b) To change the potential difference
(c) To change the emf
(d) To change the current though a particular instrument

39. The material of the wire chosen for rheostat is
(a) Copper (b) Aluminium
(c) Constantan (d) Lead

40. Resistance of a conductor depends upon
(a) Applied potential difference across conductor and current passing through conductor
(b) Length and area of cross section of conductor
(c) Temperature of conductor
(d) Both (b) and (c)

41. The V–I graph for ohmic resistance is
(a) A straight line (b) A rectangular hyperbola
(c) A parabola (d) None of the above

42. Which of the following shows V–I straight line curve?
(a) NaCI solution (b) Junction diode
(c) Thyristor (d) Gold

43. The ohmic resistance can be measured by
(a) Ohm's law apparatus (b) Wheatstone Bridge
(c) AVO meter (d) All of the above

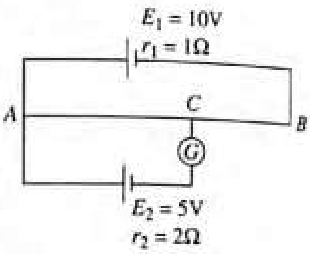
44. Rheostat is
(a) A variable resistance used for adjusting potential difference across conductor
(b) A constant resistance used for adjusting current in circuit
(c) A variable resistance used for adjusting current in circuit
(d) A constant high resistance used for adjusting potential difference across conductor

45. If the positions of ammeter and voltmeter are interchanged in Ohm's law circuit, then
(a) Both instruments will be damaged for flow of large current
(b) No instrument will be harmed for flow of very small current
(c) No effect on readings of both instruments
(d) Both show reading out of scale

46. The potential gradient of a potentiometer can be increased by which of the following operations?
(a) By decreasing the current through it
(b) By using a wire of material of low specific resistance
(c) By decreasing the area of cross section of the potentiometer wire.
(d) By increasing the area of cross section of the potentiometer wire.

47. A potentiometer wire has length l. A cell of emf E is balanced at a length $l/3$ from the positive end of the wire. If the length of the wire is increased by $l/2$, at what distance will the same cell give a balance point
(a) $\frac{5\ell}{3}$ (b) $\frac{\ell}{8}$
(c) $\frac{\ell}{2}$ (d) $\frac{4\ell}{3}$

48. If potentiometer wire of length l = 100 cm and resistance 9 Ω is joined to a cell of emf E_1 = 10 V and internal resistance r_1 = 1 Ω. Another cell of emf E_2 = 5 V and internal resistance r_2 = 2 Ω is connected as shown. The galvanometer G will show no deflection when the length AC is



(a) 55.55 cm (b) 53.65 cm
(c) 50 cm (d) 54.33 cm

49. Two cells of emf E_1 and E_2 are joined in series and the balancing length of the potentiometer wire is 625 cm. If the terminals of E_1 and E_2 are reversed, the balancing length obtained is 125 cm. Given $E_2 > E_1$, the ratio $E_1 : E_2$ will be
(a) 2 : 3 (b) 5 : 1
(c) 3 : 2 (d) 1 : 5

50. The potential gradient of a potentiometer is x_1. When the balancing length for E_1 cell is L, the cell E_2 is also balanced at this length L and its potential gradient is x_2. The ratio $x_1 : x_2$ will be
(a) $E_1^2 : E_2^2$ (b) $E_2^2 : E_1^2$
(c) $E_2 : E_1$ (d) $E_1 : E_2$

51. Potentiometer wire is made of a material of
(a) High specific resistance ρ and low temperature coefficient of resistance α
(b) High ρ and high α
(c) Low ρ and low α
(d) Low ρ and high α

52. The fall of potential per unit length of potentiometer wire depends upon
(a) Specific resistance (b) Current
(c) Area of cross section (d) All of the above

53. The sensitivity of the potentiometer is maximum for
(a) One wire (b) Two wires
(c) Four wires (d) Ten wires

54. The balance point will not be obtained on potentiometer wire
(a) If emf of cell used in circuit is greater than potential difference to be measured

(b) If emf of cell used in circuit is smaller than potential difference to be measured
(c) If large resistance with galvanometer is not used
(d) If area of cross section is not uniform

55. If the positions of an ideal voltmeter and the ammeter are interchanged in a DC circuit, then
(a) The voltmeter reads the emf of the source and the ammeter reading is zero
(b) The voltmeter reads the emf of the source and the ammeter shows maximum current.
(c) The voltmeter reading is zero and the ammeter reading is also zero
(d) Voltmeter reading is zero and the ammeter current is infinite

56. In an ammeter, 5% of the main current is passing through the galvanometer and if the resistance of the galvanometer is G, the resistance of shunt S will be
(a) $19G$ (b) $G/19$ (c) $20G$ (d) $G/20$

57. The sensitivity of a galvanometer depends on
(a) The magnetic field of cylindrical magnetic field used in the galvanometer
(b) Area of the coil
(c) Torsional constant of the spring
(d) All of the above

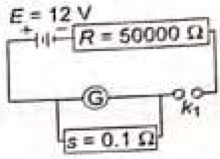
58. The resistance of a galvanometer by half-deflection method is given by
(a) $G=\dfrac{RS}{R-S}$ (b) $G=\dfrac{R-S}{RS}$
(c) $G=\dfrac{RS}{R+S}$ d. $G=\dfrac{R+S}{RS}$

59. The figure of merit (k) is related to E (emf of battery), R (high resistance used in circuit), G (galvanometer resistance) and θ (deflection in galvanometer) by the given formula
(a) $k=\dfrac{E}{(R+G)\theta}$ (b) $k=\dfrac{E\theta}{(R+G)}$
(c) $k=\dfrac{(R+G)\theta}{E}$ (d) $k=\dfrac{R+G}{E\theta}$

60. The deflection (θ) produced in the coil of galvanometer is
(a) Area of the coil
(b) Number of turns of coil
(c) Current passing through coil
(d) All of the above

61. The current passing through galvanometer is 50 A. The pointer of galvanometer shows reading at tenth division. The figure of merit is
(a) 6 A div^{-1} (b) 5 A div^{-1}
(c) 15 A div^{-1} (d) 500 A div^{-1}

62. In half-deflection method, a high resistance (R = 5000 Ω) is used in circuit. The shunt connected parallel to galvanometer is 0.1 Ω as shown in the figure. The resistance of galvanometer is given by

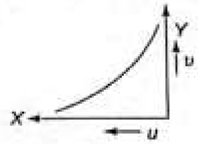


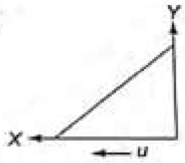
(a) 0.103020 Ω (b) 0.100002 Ω
(c) 0.200007 Ω (d) 0.111002 Ω

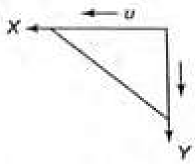
63. A concave mirror of focal length f produces an image n times the size of object. If the image is real, then distance of object from mirror is
(a) $(n-1)f$ (b) $[(n+1)/n]f$
(c) $[(n-1)/n]f$ (d) $(n+1)f$

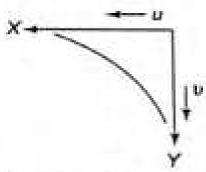
64. An object is placed in front of a concave mirror of focal length f. A virtual image is formed with a magnification of 2. To obtain a real image of same magnification, the object has to be moved by a distance
(a) f (b) $f/2$
(c) $3f/2$ (d) $2f/3$

65. The focal length of convex lens can be determined from the following graph

(a)


(b)


(c)


(d)


66. f_V and f_R are the focal lengths of a convex lens for violet and red light, respectively, and F_R and F_V are focal lengths of a concave lens for red and violet, respectively, then we must have
(a) $f_V<f_R$ and $F_V>F_R$ (b) $f_V<f_R$ and $F_V<F_R$
(c) $f_V>F_R$ and $f_V>f_R$ (d) $f_V<F_R$ and $f_V<f_R$

67. A convex lens of focal length F produces an image n times than of size of object. The distance of object from lens is
(a) nF (b) $\dfrac{F}{n}$
(c) $(n+1)F$ (d) $(n-1)F$

68. Which of the following is used to obtain real inverted and magnified image?
(a) Concave mirror (b) Convex mirror
(c) Concave lens (d) All of the above

69. The focal length of converging lens is F. An object is placed at a distance d from its first focal point. The ratio of size of real image to that of the object is

(a) $\frac{F}{d^2}$ (b) $\frac{d^2}{F}$ (c) $\frac{F}{d}$ (d) $\frac{d}{F}$

ARCHIVES

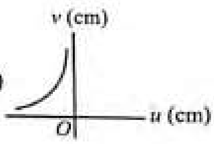
1. When temperature increases, the frequency of a tuning fork

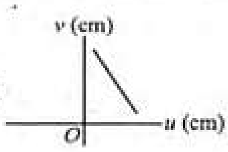
(a) increases
(b) decreases
(c) remains same
(d) increases or decreases depending on the material **(AIEEE 2002)**

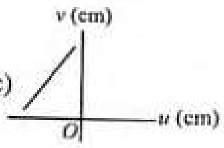
2. In a meter bridge experiment, the null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y?

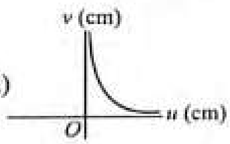
(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm **(AIEEE 2004)**

3. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like

(a)


(b)


(c)


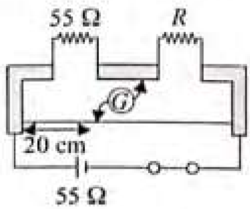
(d)


(AIEEE 2008)

4. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment, distances are measured by

(a) a metre scale provided on the microscope
(b) a screw gauge provided on the microscope
(c) a vernier scale provided on the microscope
(d) a standard laboratory scale **(AIEEE 2008)**

5. Shown in the figure is a meter bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is



(a) 110 Ω
(b) 55 Ω
(c) 13.75 Ω
(d) 220 Ω **(AIEEE 2008)**

6. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x centimetre for the second resonance. Then

(a) $54 > x > 36$ (b) $36 > x > 18$
(c) $18 > x$ (d) $x > 54$ **(AIEEE 2008)**

7. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of -0.03 mm. while measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

(a) 3.38 mm (b) 3.32 mm
(c) 3.73 mm (d) 3.67 mm **(AIEEE 2008)**

8. In an experiment, the angles are required to be measured using an instrument. Twenty-nine divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (0.5°), then the least count of the instrument is

(a) one minute (b) half minute
(c) one degree (d) half degree **(AIEEE 2009)**

9. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be

(a) $(2f, 2f)$ (b) $\left(\frac{f}{2}, \frac{f}{2}\right)$
(c) (f, f) (d) $(4f, 4f)$ **(AIEEE 2009)**

10. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading: 0 mm

Circular scale reading: 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is:

(a) 0.52 cm (b) 0.052 cm
(c) 0.026 cm (d) 0.005 cm **(AIEEE 2011)**

11. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is
(a) 6% (b) zero
(c) 1% (d) 3% **(AIEEE 2012)**

12. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading: 58.5 degree
Vernier scale reading: 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data:
(a) 58.59 degree (b) 58.77 degree
(c) 58.65 degree (d) 59 degree **(AIEEE 2012)**

13. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?
(a) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(b) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm
(c) A meter scale
(d) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm **(JEE Main 2014)**

14. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. The measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is
(a) 2% (b) 3%
(c) 1% (d) 5% **(JEE Main 2015)**

15. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be
(a) 92 ± 2 s (b) 92 ± 5.0 s
(c) 92 ± 1.8 s (d) 92 ± 3 s **(JEE Main 2016)**

16. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides the main scale line?
(a) 0.75 mm (b) 0.80 mm
(c) 0.70 mm (d) 0.50 mm
(JEE Main 2016)

ANSWER KEY

Exercises

1. (c)	2. (c)	3. (a)	4. (a)	5. (c)	6. (b)	7. (d)	8. (a)	9. (c)	10. (c)
11. (a)	12. (a)	13. (a)	14. (a)	15. (b)	16. (a)	17. (b)	18. (a)	19. (c)	20. (c)
21. (b)	22. (a)	23. (c)	24. (b)	25. (b)	26. (b)	27. (a)	28. (a)	29. (b)	30. (c)
31. (c)	32. (b)	33. (b)	34. (b)	35. (a)	36. (b)	37. (b)	38. (d)	39. (c)	40. (d)
41. (a)	42. (d)	43. (d)	44. (c)	45. (b)	46. (c)	47. (c)	48. (a)	49. (a)	50. (d)
51. (c)	52. (d)	53. (a)	54. (b)	55. (a)	56. (b)	57. (d)	58. (a)	59. (a)	60. (c)
61. (b)	62. (b)	63. (c)	64. (a)	65. (a)	66. (a)	67. (c)	68. (a)	69. (c)	

Archives

1. (b)	2. (a)	3. (a)	4. (c)	5. (d)	6. (d)	7. (a)	8. (a)	9. (a)	10. (b)
11. (a)	12. (c)	13. (d)	14. (b)	15. (a)	16. (b)				

Hints and Solutions

CHAPTER 1: UNITS, DIMENSIONS AND MEASUREMENT

Concept Application Exercise 1.1

1. (n) = Number of particle passing from unit area in unit time

$$= \frac{\text{Number of particles}}{A \times t} = \frac{[M^0L^0T^0]}{[L^2][T]} = [L^{-2}T^{-1}]$$

$[n_1] = [n_2]$ = Number of particles in unit volume = $[L^{-3}]$

Now from the given formula,

$$[D] = \frac{[n][x_2 - x_1]}{[n_2 - n_1]} = \frac{[L^{-2}T^{-1}][L]}{[L^{-3}]} = [L^2T^{-1}]$$

2. From the principle of dimensional homogeneity,

$[x^2] = [B]$

$\therefore \quad [B] = [L^2]$

As well as $[U] = \dfrac{[A][x^{1/2}]}{[x^2]+[B]} \Rightarrow [ML^2T^{-2}] = \dfrac{[A][L^{1/2}]}{[L^2]}$

$\therefore \quad [A] = [ML^{7/2}T^{-2}]$

Now $\quad [AB] = [ML^{7/2}T^{-2}] \times [L^2] = [ML^{11/2}T^{-2}]$

3. $[P] = [ML^2T^{-3}]$

Using the relation,

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^x\left[\frac{L_1}{L_2}\right]^y\left[\frac{T_1}{T_2}\right]^z$$

$$= 1\times10^6\left[\frac{1\text{ kg}}{10\text{ kg}}\right]^1\left[\frac{1\text{ m}}{1\text{ dm}}\right]^2\left[\frac{1\text{ s}}{1\text{min}}\right]^{-3}$$

[As 1 MW = 10^6 W]

$$= 10^6\left[\frac{1\text{ kg}}{10\text{ kg}}\right]\left[\frac{10\text{ dm}}{1\text{ dm}}\right]^2\left[\frac{1\text{ s}}{60\text{ s}}\right]^{-3} = 2.16\times10^{12}\text{ unit}$$

4. Unit of force $= \dfrac{\text{kg}\times\text{m}}{\text{s}^2}$

In new system $= \dfrac{1}{10}\text{ kg} \times \dfrac{100\text{ m}}{100\text{ s}\times100\text{ s}} = \dfrac{1}{1000}\ \dfrac{\text{kg}\times\text{m}}{\text{s}^2}$

5. $[E] = [ML^2T^{-2}]$

$= [100\text{ kg}] \times [1\text{ km}]^2 \times [100\text{ s}]^{-2}$

$= 100\text{ kg} \times 10^6\text{m}^2 \times 10^{-4}\text{ s}^{-2}$

$= 10^4\text{ kg} \times \text{m}^2\text{ s}^{-2} = 10^4\text{ J}$

6. Let $M = V^aF^bE^c$

Putting dimensions of each quantities on both sides

$$[M] = [LT^{-1}]^a\,[MLT^{-2}]^b\,[ML^2\,T^{-2}]^c$$

Equating powers of dimensions, we have

$$b + c = 1,\ a + b + 2c = 0,\text{ and } -a - 2b - 2c = 0$$

Solving these equations, $a = -2$, $b = 0$, and $c = 1$.

So $\quad M = [V^{-2}F^0E]$

Concept Application Exercise 1.2

1. Relative error in measurement of length is minimum. So this measurement is most accurate.
2. Area = $1.5 \times 1.203 = 1.8045\text{ cm}^2 = 1.8\text{ cm}^2$ (Upto correct number of significant figure).
3. Total surface area = $6 \times (5.402)^2 = 157.639\text{ cm}^2 = 157.6\text{ cm}^2$ (Up to correct number of significant figure)
   Total volume = $(5.402)^3 = 175.64\text{ cm}^3 = 175.6\text{ cm}^3$ (Up to correct number of significant figure).
4. 9.99 m + 0.0099 m = 9.999 m = 10.00 m (In proper significant figures).
5. $3.124 \times 4.576 = 14.295 = 14.3$ (Correct to three significant figures).
6. Value of current (3.23 A) has minimum significant figure (3). So the value of potential difference $V (= IR)$ has only three significant figures. Hence, its value is 35.0 V.

Concept Application Exercise 1.3

1. Probable error reduces to 1/5 as the number of observations is made 5 times.
2. Here $S = (13.8 \pm 0.2)$ cm; $t = (4.0 \pm 0.3)$ s

$$V = \frac{13.8}{4.0} = 3.45\text{ ms}^{-1}$$

$$\frac{\Delta V}{V} = \pm\left(\frac{\Delta S}{S} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{13.8} + \frac{0.3}{4.0}\right)$$

$$= \pm0.0895$$

$$\Delta V = \pm\,0.0895 \times 3.45 = \pm\,0.3$$

(rounded off to one place of decimal)

$$V = (3.45 \pm 0.3)\text{ ms}^{-1}$$

Percentage error $= \dfrac{\Delta V}{V} \times 100$

$$= 0.0895 \times 100 = 8.95\%$$

3. Volume $V = \frac{4}{3}\pi r^3 \Rightarrow \Delta V = \frac{4}{3}\pi 3r^2 \Delta r$

$$\frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100 = 3 \times 1\% = 3\%$$

4. In parallel, $R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{5.0 \times 10.0}{5.0 + 10.0} = \frac{50}{15} = 3.3\ \Omega$

Also $\frac{\Delta R_P}{R_P} \times 100 = \frac{\Delta R_1}{R_1} \times 100 + \frac{\Delta R_2}{R_2} \times 100$

$$+\frac{\Delta(R_1 + R_2)}{R_1 + R_2} \times 100$$

$$= \frac{0.2}{5.0} \times 100 + \frac{0.1}{10.0} \times 100 + \frac{0.3}{15} \times 100$$

$$= 7\%$$

$\therefore \quad R_P = 3.3\ \Omega \pm 7\%$

5. 35.0 V, the final result should contain three significant figures.

6. Length $l = 2.53 + 1.27 = 3.80$ cm, $\Delta l = 0.01 + 0.01 = 0.02$ (Most probable errors of both the rods are added)
Hence true value = (3.80 ± 0.02) cm

7. $P = \frac{F}{A} = \frac{F}{l^2}$; so maximum error in pressure (P)

$$\left(\frac{\Delta P}{P} \times 100\right)_{\max} = \frac{\Delta F}{F} \times 100 + 2\frac{\Delta l}{l} \times 100$$

$$= 4\% + 2 \times 2\% = 8\%$$

8. Quantity C has maximum power. So it brings maximum error in P.

Exercises

Basic Concept of Units

1. (c) Energy = force × distance, so if both are increased by 4 times, then energy will increase by 16 times.

2. (a) Physical quantity (p) = Numerical value (n) × Unit (u)
If physical quantity remains constant then
$n \propto 1/u \therefore n_1 u_1 = n_2 u_2$.

3. (c)

$$Y = \frac{F}{A} \cdot \frac{L}{\Delta L} = \frac{\text{dyne}}{\text{cm}^2} = \frac{10^{-5}\text{N}}{10^{-4}\text{m}^2} = 0.1\ \text{N/m}^2$$

4. (c) $n_2 = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^1\left(\frac{T}{T_2}\right)^{-2}$

$$= 100\left(\frac{\text{gm}}{\text{kg}}\right)^1\left(\frac{\text{cm}}{\text{m}}\right)^1\left(\frac{\text{sec}}{\text{min}}\right)^{-2}$$

$$= 100\left(\frac{\text{gm}}{10^3\text{gm}}\right)^1\left(\frac{\text{cm}}{10^2\text{cm}}\right)^1\left(\frac{\text{sec}}{60\,\text{sec}}\right)^{-2}$$

$$n_2 = \frac{3600}{10^3} = 3.6$$

5. (d) $P = nu \therefore n \propto \frac{1}{u}$

6. (c) $[x] = [bt^2] \Rightarrow [b] = [x/t^2] = \text{km/s}^2$

7. (d) ct^2 must have dimensions of L
$\Rightarrow$ c must have dimensions of L/T^2 i.e. LT^{-2}.

8. (b) Watt $= \frac{J}{\text{sec}} = \frac{\text{N-m}}{\text{sec}} = \text{kg}\frac{\text{m}^2}{\text{sec}^3}$

9. (b) $F = \frac{\text{Newton}}{\text{m}^2}$

$$\frac{hc}{d^2} = \text{joule-sec.}\left(\frac{\text{metre}}{\text{sec.}}\right)\frac{1}{(\text{metre})^2}$$

$$= \frac{\text{joule}}{\text{metre}} = \text{Newton}$$

$$\frac{hc}{d^4} = \left(\frac{hc}{d^2}\right)\frac{1}{d^2} = \frac{\text{Newton}}{\text{m}^2}, \text{ Hence } F = hc/d^4$$

$$\frac{hd}{c} = \frac{\text{Joule.sec}(\text{metre})^2}{\left(\frac{\text{metre}}{\text{sec.}}\right)} = \text{Newton-metre}^2\,\text{sec}^2$$

$$\frac{d^4}{hc} = \frac{1}{\left(\frac{hc}{d^4}\right)} = \frac{m^2}{\text{Newton}}$$

Dimensional Analysis

10. (d) As $\mu = \frac{\text{Velocity of light in vaccum}}{\text{Velocity of light in medium}}$, hence μ is dimensionless.
Thus, each term on the RHS of given equation should be dimensionless, i.e., B/λ^2 is dimensionless, i.e., B should have dimension of λ^2, i.e., cm^2, i.e., area.

11. (d) $[x] = [Ay] = [B] \Rightarrow [y] = [B/A]$
Also, $[x] \neq [A]$ and $[Cz]$ = dimensionless
$\Rightarrow [C] = [z^{-1}]$

12. (b) Dimension of L/R is same as that of time.

13. (c) $\frac{A}{B} = \frac{\text{Force}}{\text{Force}} = [M^0 L^0 T^0]$

$$Ct = \text{angle} \Rightarrow C = \frac{\text{Angle}}{\text{Time}} = \frac{1}{T} = T^{-1}$$

$$Dx = \text{angle} \Rightarrow D = \frac{\text{Angle}}{\text{Distance}} = \frac{1}{L} = L^{-1}$$

$$\therefore \frac{C}{D} = \frac{T^{-1}}{L^{-1}} = [M^0 L T^{-1}]$$

14. (c) $n = \frac{1}{2l}\sqrt{\frac{T}{m}} \Rightarrow n^2 = \frac{1}{4l^2}\frac{T}{m}$

$$m = \frac{T}{4l^2 n^2} = \left[\frac{MLT^{-2}}{L^2 \times T^{-2}}\right] = [ML^{-1}]$$

15. (b) $y = r\sin(\omega t - kx)$

Here ωt = angle $\Rightarrow \omega = \frac{1}{T} = T^{-1}$

Similarly, kx = angle $\Rightarrow k = \frac{1}{x} = L^{-1}$

$$\therefore \frac{\omega}{k} = \frac{T^{-1}}{L^{-1}} = LT^{-1}$$

Or simply ω/k represents wave velocity.

$$\frac{\omega}{k}=\frac{2\pi f}{2\pi/\lambda}=f\lambda=v, \text{ where } f \text{ is frequency.}$$

16. (d) We know that the speed of electromagnetic wave is

$$C=\frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \mu_0\varepsilon_0=\frac{1}{C^2}=[L^{-2}T^2]$$

17. (d) Momentum, $p=mv=MLT^{-1}=ML^{-3}L^4T^{-4}T^3$

$$=DV^4F^{-3}$$

18. (d) $AT^2=LT^{-2}\times T^2=[M^0LT^0]$

19. (d) $f=Cm^xk^y$. Writing dimensions on both sides,

$$[M^0L^0T^{-1}]=M^x[ML^0T^{-2}]^y=[M^{x+y}T^{-2y}]$$

Comparing dimensions on both sides, we have

$$0=x+y \text{ and } -1=-2y \Rightarrow y=\frac{1}{2},\ x=-\frac{1}{2}$$

Aliter. Remembering that the frequency of oscillation of loaded spring is

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}(k)^{1/2}m^{-1/2}$$

which gives $x=-\frac{1}{2}$ and $y=\frac{1}{2}$

20. (d) $\frac{C^2}{g}=\frac{L^2T^{-2}}{LT^{-2}}=[L]$

21. (d) $\frac{A}{B}=m$, $B=\frac{A}{m}=\frac{\text{Force}}{\text{Linear density}}=\frac{MLT^{-2}}{ML^{-1}}$

$$\therefore\ B=[M^0L^2T^{-2}]$$

$$\text{Latent heat}=\frac{\text{Heat energy}}{\text{Mass}}=\frac{ML^2T^{-2}}{M}=[M^0L^2T^{-2}]$$

Thus, B has same dimensions as that of latent heat.

22. (b) Here, b and $x^2=L^2$ have same dimensions.

Also, $a=\frac{L^2}{E\times t}=\frac{L^2}{(ML^2T^{-2})T}=M^{-1}T^1$

$$a\times b=[M^{-1}L^2T^1]$$

23. (a) $h=[ML^2T^{-1}]$, $G=[M^{-1}L^3T^{-2}]$, $C=[LT^{-1}]$

$$\therefore\ h^{1/2}G^{-1/2}C^{1/2}=M^{1/2}LT^{-1/2}\times M^{1/2}L^{-3/2}T^1\times L^{1/2}T^{-1/2}$$

$$=ML^0T^0=\text{Mass}$$

24. (b) $V=\frac{\pi}{8}\frac{Pr^4}{\eta l}=\frac{ML^{-1}T^{-2}L^4}{ML^{-1}T^{-1}L}=M^0L^3T^{-1}$

25. (a) Let $Y=[V^aA^bF^c]$

$$[ML^{-1}T^{-2}]=[LT^{-1}]^a[LT^{-2}]^b[MLT^{-2}]^c$$

$$ML^{-1}T^{-2}=M^cL^{a+b+c}T^{-a-2b-2c}$$

$\therefore\ c=1,\ a+b+c=-1,\ -a-2b-2c=-2$

On solving, we get $a=-4$, $b=2$, and $c=1$.

26. (a) Here $v=e^ah^b\mu^cG^d$. Taking the dimensions, $M^0LT^{-1}A^0$ $=[AT^1]^a[ML^2T^{-1}]^b[MLT^{-2}A^{-2}]^c[M^{-1}L^3T^{-2}]^d$

There will be four simultaneous equations by equating the dimensions of M, L, T, and A. These are $a-2c=0$, $a-b-2c-2d=-1$, $b+c-d=0$ and $2b+c+3d=1$.

Solving for a, b, c and d, we get

$$a=-2,\ b=1,\ c=-1,\ d=0$$

Thus, $v=e^{-2}h\mu^{-1}G^0$

27. (b) $\frac{M}{At}\propto P^xv^y$

$$\Rightarrow\ ML^{-2}T^{-1}=[ML^{-1}T^{-2}]^x[L^1T^{-1}]^y=M^xL^{-x+y}T^{-2x-y}$$

$$x=1,\ -x+y=-2,\text{ and } -2x-y=-1$$

From here, we get $y=-1$.

Thus, $x=-y$.

28. (b) Here x^2 has the dimensions of L^2, $B=[L^2]$

Also $ML^2T^{-2}=\frac{AL^{1/2}}{L^2}$ or $A=ML^{7/2}T^{-2}$

$$\therefore\ A\times B=ML^{11/2}T^{-2}$$

29. (c) By Coulomb's laws, $F=\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{r^2}$

$$\varepsilon_0=\frac{q_1q_2}{4\pi\times F\times r^2}$$

Taking dimensions,

$$\varepsilon_0=\frac{(AT)(AT)}{ML^1T^{-2}\times L^2}=[M^{-1}L^{-3}T^4A^2]$$

30. (d) $m\propto v^a\rho^bg^c$

$$ML^0T^0\propto(LT^{-1})^a(ML^{-3})^b(LT^{-2})^c$$

Comparing the powers of M, L, and T and solving, we get $b=1$, $c=-3$, $a=6\Rightarrow m\propto v^6$

Errors in Measurement

31. (c) Random error is reduced by making a large number of observations and taking mean of all the results.

32. (c) All the choices are equivalent but the answer must possess three significant digits as significant digits does not change on conversion from one system to another. So appropriate choice is (c).

33. (b) In a product, percentage errors are added up.

34. (c) 0.08076 has least number of significant figures, i.e., 4.

35. (c) $X=a+b \Rightarrow \Delta X=\Delta a+\Delta b$

Now $\frac{\Delta X}{X}\times 100=\frac{(\Delta a+\Delta b)}{a+b}\times 100$

$$=\left(\frac{\Delta a}{a+b}+\frac{\Delta b}{a+b}\right)\times 100$$

36. (c) $X=M^xL^{-y}T^{-z}$

$$\therefore\ \frac{\Delta X}{X}\times 100=x\left(\frac{\Delta M}{M}\times 100\right)+y\left(\frac{\Delta L}{L}\times 100\right)+z\left(\frac{\Delta T}{T}\times 100\right).$$

(errors are always added)

$$\therefore\ \frac{\Delta X}{X}\times 100=(ax+by+cz)\text{ percent}$$

37. (b) Percentage error in volume is

$$\frac{0.01}{15.12}\times 100+\frac{0.01}{10.15}\times 100+\frac{0.01}{5.28}\times 100=0.35\%$$

38. (d) Relative density $\rho_r=\frac{W_1}{W_1-W_2}=\frac{8.00}{8.00-6.00}=4.00$

$$\frac{\Delta\rho_r}{\rho_r}\times 100=\frac{\Delta W_1}{W_1}\times 100+\frac{\Delta(W_1-W_2)}{W_1-W_2}\times 100$$

$$=\frac{0.05}{8.00}\times 100+\frac{0.05+0.05}{2}\times 100=5.62\%$$

$\therefore\quad \rho_r=4.00\pm 5.62\%$

39. (c) Kinetic energy, $E=\frac{1}{2}mv^2$

$$\frac{\Delta E}{E}\times 100=\frac{\Delta m}{m}\times 100+2\frac{\Delta v}{v}\times 100$$

$$=2+2\times 3=8\%$$

40. (a) $X=\frac{a^{1/2}b^2}{c^3}$

$$\frac{\Delta X}{X}\times 100=\frac{1}{2}\frac{\Delta a}{a}\times 100+2\frac{\Delta b}{b}\times 100+3\frac{\Delta c}{c}\times 100$$

$$=\frac{1}{2}\times 1+2\times 3+3\times 2=12.5\%$$

41. (d) $R=\frac{V}{I}=\frac{8}{4}=2\,\Omega$

$$\frac{\Delta R}{R}\times 100=\frac{\Delta V}{V}\times 100+\frac{\Delta I}{I}\times 100$$

$$=\frac{0.5}{8}\times 100+\frac{0.2}{4}\times 100=11.25\%$$

$\Rightarrow\quad R=(2\pm 11.25\%)\,\Omega$

42. (d) Required percentage $=2\times\frac{0.02}{0.24}\times 100+\frac{1}{30}\times 100+\frac{0.01}{4.80}\times 100$

$$=16.7+3.3+0.2=20\%$$

43. (d) Maximum error in measuring mass = 0.001 g, because least count is 0.001 g. Similarly, maximum error in measuring volume is 0.01 cm^3.

$$\frac{\Delta\rho}{\rho}=\frac{\Delta M}{M}+\frac{\Delta V}{V}=\frac{0.001}{20.000}+\frac{0.01}{10.00}$$

$$=(5\times 10^{-5})+(1\times 10^{-3})=1.05\times 10^{-3}$$

$$\Delta\rho=(1.05\times 10^{-3})\times\rho$$

$$=1.05\times 10^{-3}\times\frac{20.000}{10.00}=0.002\ \text{g cm}^{-3}$$

44. (c) Given that $\frac{\Delta l}{l}\times 100=+1\%$

and $\frac{\Delta T}{T}\times 100=-3\%$

Percentage error in the measurement of g is

$$\left[\frac{4\pi^2 l}{T^2}\right]=100\times\frac{\Delta l}{l}-2\times\frac{\Delta T}{T}\times 100$$

$$=1\%-2[-3\%]=+7\%$$

45. (b) $T=2\pi\sqrt{\frac{L}{g}}$ or $T^2=4\pi^2\frac{L}{g}$

$$g=4\pi^2\frac{L}{T^2};\ \frac{\Delta g}{g}=\frac{\Delta L}{L}-2\frac{\Delta T}{T}$$

$$\frac{\Delta g}{g}\times 100=\frac{\Delta L}{L}\times 100-2\frac{\Delta T}{T}\times 100$$

Actual % error in $g=\frac{\Delta L}{L}\times 100-2\frac{\Delta T}{T}\times 100$

$$=+2\%-2\times 1\%=0\%$$

46. (d) Relative density $=\frac{W_a}{W_a-W_w}$, $\rho=\frac{W_a}{w}$

where ρ is relative density, W_a is weight in air, and w is loss in weight.

$$\frac{\Delta\rho}{\rho}=\frac{\Delta W_a}{W_a}-\frac{\Delta w}{w}$$

For maximum error, $\frac{\Delta\rho}{\rho}=\frac{\Delta W_a}{W_a}+\frac{\Delta w}{w}$

For maximum percentage error,

$$\frac{\Delta\rho}{\rho}\times 100=\frac{\Delta W_a}{W_a}\times 100+\frac{\Delta w}{w}\times 100$$

Given $\Delta W_a=0.1$ gf and $W_a=10.0$ gf

$w=10.0-5.0=5.0$ gf

$\Delta w=\Delta W_a+\Delta W_w=0.1+0.1=0.2$ gf

$$\frac{\Delta\rho}{\rho}\times 100=\left(\frac{0.1}{10.0}\right)\times 100+\left(\frac{0.2}{5.0}\right)\times 100$$

$$=1+4=5$$

47. (d) $X=M^{-1}L^3T^{-2}$

$$\frac{\Delta X}{X}=\frac{\Delta M}{M}+3\frac{\Delta L}{L}+2\frac{\Delta T}{T}$$

$$=2+3\times 3+2\times 4=19$$

48. (d) Required error is $2\times 2\%+1\%+1\%$, i.e., 6%.

Problems Based on Mixed Concepts

49. (d) Given $\lambda=km^pv^qh^k$. The dimensions of right-hand rule and left-hand side terms should be equal.

So $[M^0LT^0]=[M]^p[LT^{-1}]^q[ML^2T^{-1}]^r$

or $[M^0LT^0]=[M^{p+r}][L^{q+2r}][T^{-q-r}]$

Now, compare powers of M, L and T, we get

$p+r=0$ ...(i)

$q+2r=1$ ...(ii)

$-q-r=0$ ...(iii)

After solving $p=-1$, $q=-1$ and $r=1$ putting these values, we get

$\lambda=k\frac{h}{mv}$ which is the required relation.

50. (d) $T\propto p^ad^bE^c$

$$[T]=[ML^{-1}T^{-2}]^a[ML^{-3}]^b[ML^2T^{-2}]^c$$

Equating the exponent, $a+b+c=0$

$-a-3b+2c=0$ and $-a-2c=1$

By solving

$a=-\frac{5}{6}$

51. (a) $\left[\frac{\alpha Z}{K\theta}\right]=[M^0L^0T^0]$ & $[\alpha]=\left[\frac{K\theta}{Z}\right]$

But $[P]=\left[\frac{\alpha}{\beta}\right]$ So, $[\beta]=\left[\frac{\alpha}{P}\right]=\left[\frac{K\theta}{ZP}\right]$

$[\beta]=\left[\frac{ML^2T^{-2}}{LML^{-1}T^{-2}}\right]=[M^0L^2T^0]$

(Since dimensions of K' is of energy)

52. (c) It is clear that k should be dimensionless

$[1-k^2]=M^0\,L^0\,T^0$

Hence, $[k]=M^0\,L^0\,T^0$

Actually k is the coefficient of restitution.

53. (b) $F=kA^xv^y\rho^z$

$\Rightarrow\ MLT^{-2}=[L^2]^x[LT^{-1}]^y[ML^{-3}]^z$

$\Rightarrow\ z=1,\ 2x+y-3z=1,\ -y=-2\Rightarrow y=2$

$\Rightarrow\ x=1$

$F=kA^1v^2\rho^1=Av^2\rho,\ k=1$

54. (a) LHS$=[x_1+x_2]=[x_1]=[x_2]=[M^0LT^0]$

RHS $=\left[\frac{a_1a_2t^2}{2(a_1+a_2)}\right]$

$=\frac{[LT^{-2}][LT^{-2}][T^2]}{[LT^{-2}]}=[M^0LT^0]$

$\because$ LHS = RHS

According to homogeneity principle, equation is dimensionally correct.

Hence, option (a) is correct.

55. (a) $\therefore\ [E]=[ML^2T^{-2}]$

$[M]=[EL^{-2}T^2]$

$\because\ F=\frac{Gm_1m_2}{r_2}$

$\Rightarrow\ [G]=\frac{Fr^2}{m_1m_2}=[FL^2M^{-2}]$

$[FL^2][EL^{-2}T^{-2}]^{-2}=[FL^6\,E^{-2}]$

56. (b) $\rho=\frac{m}{\pi r^2L}$

$\frac{\Delta\rho}{\rho}\times 100=\left[\frac{\Delta m}{m}+2\frac{\Delta r}{r}+\frac{\Delta L}{L}\right]\times 100$

$=\left(0.01+2\frac{\Delta r}{r}+0.01\right)\times 100=4$ (given)

or $100\frac{\Delta r}{r}=1\quad\Rightarrow\Delta r=0.01r$

57. (b) $C=\frac{q}{V}=\frac{q^2}{W}$

$[X]\to[C]=[M^{-1}L^{-2}T^2Q^2]$

And $[B]=\left[\frac{F}{iL}\right]$

$[Z]\to[B]=\left[\frac{MLT^{-2}}{QT^{-1}L}\right]=[MT^{-1}Q^{-1}]$

Given, $Y=\frac{X}{3Z^2}$

$[Y]=\frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2}=[M^{-3}L^{-2}T^4Q^4]$

58. (c) $X=\left[\frac{l}{(A-l)}\right]R$

$\frac{\Delta X}{X}=\frac{\Delta l}{l}-\frac{\Delta l}{(A-l)}=0$

$l=A-l$

or $A=2l=140$ cm

59. (a) $V=\frac{S}{t}$

$\Delta V=\frac{1}{t}\cdot\frac{\partial S}{\partial S}\cdot\Delta S+\frac{S}{t^2}\Delta t=\left(\frac{\Delta S}{t}+\frac{S\Delta t}{t^2}\right)$

So $\frac{\Delta S}{t}+\frac{S\Delta t}{t^2}=\frac{\Delta S}{\Delta t}\cdot\frac{\Delta t}{t}+\frac{(\Delta S)\cdot S(\Delta t)^2}{(\Delta S)\cdot t^2(\Delta t)}$

or $\frac{\Delta S}{\Delta t}\left[\frac{\Delta t}{t}+\frac{S(\Delta t)^2}{(\Delta S)t^2}\right]=\pm\frac{\Delta S}{\Delta t}$ (given)

So, $\frac{\Delta t}{t}+\frac{S(\Delta t)^2}{(\Delta S)t^2}=\pm 1$.

60. (c) Logarithm has no dimensions.

$\therefore\ 1+\frac{\alpha l}{ma}$ = dimensionless

$\therefore\ \frac{\alpha l}{ma}=[M^0\,L^0\,T^0]$

$\Rightarrow\ \alpha=\frac{ma}{l}=\frac{[MLT^{-2}]}{[L]}=[MT^{-2}]$

$\because$ Dimensions of ϕ = Dimensions of $\frac{ma}{\alpha}$

$=\frac{[MLT^{-2}]}{[MT^{-2}]}=[L]=[M^0LT^0]$

61. (b) Logarithm has no dimensions.

$[xb]=[M^0L^0T^0]\ \Rightarrow\ [b]=[L^{-1}]$

Hence, option (b) is correct.

62. (d) In Bohr's model, $\frac{1}{\lambda}=\frac{me^4}{\varepsilon_0^2h^3c}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

where λ = wavelength, n_1 and n_2 are principal quantum numbers.

$\therefore\ \left[\frac{me^4}{\varepsilon_0^2h^3c}\right]=[L^{-1}]=[M^0L^{-1}T^0]$

63. (c) The quantity $\frac{t}{a}-1$ is dimensionless i.e., $[a]=[t]$

$\therefore \quad [\sqrt{2at-t^2}]=[t]$

or $\quad \left[\frac{dt}{\sqrt{2at-t^2}}\right]=\left[\frac{t}{t}\right]=[M^0L^0T^0]$

i.e. a^x should also be dimensionless or $x=0$.

64. (c) Quantity C has maximum power. So it brings maximum error in P.

65. (d) $[n]$ = Number of particles crossing a unit area in unit time $=[L^{-2}T^{-1}]$

$[n_2]=[n_1]$ = number of particles per unit volume $=[L^{-3}]$

$[x_2]=[x_1]$ = positions

$\therefore \quad D=\frac{[n][x_2-x_1]}{[n_2-n_1]}=\frac{[L^{-2}T^{-1}]\times[L]}{[L^{-3}]}=[L^2T^{-1}]$

66. (a) Dimension of $\alpha t=[M^0L^0T^0] \quad \therefore \quad [\alpha]=[T^{-1}]$

Again $\left[\frac{v_0}{\alpha}\right]=[L]$ so $[v_0]=[LT^{-1}]$

ARCHIVES

1. (a) Both torque and work have the same dimensional formula $[ML^2T^{-2}]$.

2. (b) $\frac{\text{Energy}}{\text{Volume}}=\frac{ML^2T^{-2}}{L^3}=[ML^{-1}T^{-2}]$ = Pressure

3. (c) $\omega=\frac{d\theta}{d}=[T^{-1}]$ and frequency $[n]=[T^{-1}]$

4. (c) [Momentum] $=[MLT^{-1}]$

[Planck's constant] $=[ML^2T^{-1}]$.

Clearly, momentum and Planck's constant do not have same dimensions.

5. (c) $C=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ or $\mu_0\varepsilon_0=\frac{1}{C^2}=\frac{1}{[LT^{-1}]^2}=[L^{-2}T^2]$

6. (c) $F=6\pi\eta rv$

$\therefore \quad [\eta]=\frac{[F]}{[r][V]}=\frac{[MLT^{-2}]}{[L][LT^{-1}]}=[ML^{-1}T^{-1}]$

7. (b) [Work, torque] $=[MLT^{-2}][L]=[ML^2T^{-2}]$

[Angular momentum, Planck's constant] $=[M][LT^{-1}][L]=[ML^2T^{-1}]$

[Impulse, momentum] $=[M][LT^{-1}]=[MLT^{-1}]$

[Moment of inertia] $=[ML^2]$

[Moment of force] $=[MLT^{-2}][L]=[ML^2T^{-2}]$

Clearly, the dimensions of moment of inertia are different from the dimensions of moment of force.

8. (d) Energy $=\frac{1}{2}[LI^2]=\frac{1}{2}L\left[\frac{Q}{t}\right]^2$

or $\quad [L]=\frac{[\text{Energy}]}{\left[\frac{Q}{t}\right]}=\frac{[ML^2T^{-2}T^2]}{[Q^2]}=\left[\frac{ML^2}{Q^2}\right]$

9. (d) The unit rad (radiation absorbed dose) is an old unit. It is a measure of the radiation dose (as energy per unit mass) actually absorbed by a specific object such as a patient's hand or chest.

10. (c) $F=qvB$ or $[B]=[F/qv]=[MT^{-1}C^{-1}]$

11. (d) Diameter = M.S.R. + C.S.R × L.C + Z.E.

$=3+35\times(0.5/50)+0.03=3.38$ mm

12. (c) Resistance of a given wire, $R=\frac{V}{I}$

$\therefore \quad \frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta I}{I}$

The percentage error in R is:

$$\frac{\Delta R}{R}\times100=\frac{\Delta V}{V}\times100+\frac{\Delta I}{I}\times100=3\%+3\%=6\%$$

13. (a) According to Coulomb's law,

$$F=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$\therefore \quad \epsilon_0=\frac{1}{4\pi}\frac{q_1q_2}{4\pi Fr^2}$

$$[\epsilon_0]=\frac{[AT][AT]}{[MLT^{-2}][L]^2}=[M^{-1}L^{-3}T^4A^2]$$

14. (a) Measured time period of 100 oscillations are 90 sec, 91 sec, 95 sec and 92 sec.

Mean value of time $=t_m=\frac{90+91+95+92}{4}=92$ sec

Absolute error in measurement

$|\Delta t_1|=|t_m-t_1|=2$ sec

$|\Delta t_2|=|t_m-t_2|=1$ sec

$|\Delta t_3|=|t_m-t_3|=3$ sec

$|\Delta t_4|=|t_m-t_4|=0$ sec

Mean absolute error $\Delta t_{mean}=\frac{2+1+3+0}{4}=1.5$ sec

But the least count of the measuring clock is 1 sec, so it cannot measure up to 0.5 second, so we have to round it off. So mean error will be 2 second.

Hence mean time $(92\pm2 \text{ sec})$.

15. (b) Least count $LC=\frac{\text{pitch}}{\text{no. of div. on circular scale}}$

$\Rightarrow LC=\frac{0.5\text{mm}}{50}=0.01$ mm

When jaws are closed, the zero error will be

= main scale reading + (circular scale reading) (Least count)

$=-0.5$ mm $+(45)(0.01)$

Hence zero error $e=-0.05$ mm.

When the sheet is placed between the jaws:

measured thickness $=0.5$ mm $+(25)(0.01)=0.75$ mm

Hence actual thickness or true reading = observed reading + zero error

$=0.75$ mm $-(-0.05)=0.80$ mm

16. (d) $Density(\rho)=\frac{\text{Mass}}{\text{Volume}}=\frac{M}{L^3}$

$$\frac{\Delta\rho}{\rho}\%=\frac{\Delta M}{M}\%+3.\frac{\Delta L}{L}\%=1.5+3(1)=4.5\%$$

CHAPTER 2: VECTORS

Concept Application Exercise 2.1

1. Net movement along x-direction,

$$S_x = (6-4)\cos 45^\circ\ \hat{i} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}\ \text{km}$$

Net movement along y-direction,

$$S_y = (6+4)\sin 45^\circ\ \hat{j} = 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2}\ \text{km}$$

Net movement from starting point,

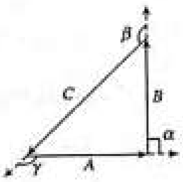
$$|\vec{s}| = \sqrt{{s_x}^2 + {s_y}^2} = \sqrt{(\sqrt{2})^2 + (5\sqrt{2})^2} = \sqrt{52}\ \text{km}$$

Angle which makes with the east direction,

$$\tan\theta = \frac{Y\text{-component}}{X\text{-component}} = \frac{5\sqrt{2}}{\sqrt{2}} \quad \therefore \quad \theta = \tan^{-1}(5)$$

2. For 17 N, both the vectors, should be parallel, i.e., angle between them should be zero.
For 7 N, both the vectors should be antiparallel, i.e., angle between them should be 180°.
For 13 N, both the vectors should be perpendicular to each other, i.e., angle between them should be 90°.

3. From polygon law, three vectors having summation zero should form a closed polygon (triangle). Since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude, i.e., the triangle should be right-angled triangle.
Angle between A and B, $\alpha = 90^\circ$
Angle between B and C, $\beta = 135^\circ$
Angle between A and C, $\gamma = 135^\circ$



4. $\vec{A} + \vec{B} = 4\hat{i} - 3\hat{j} + 6\hat{i} + 8\hat{j} = 10\hat{i} + 5\hat{j}$

$$|\vec{A} + \vec{B}| = \sqrt{(10)^2 + (5)^2} = 5\sqrt{5}$$

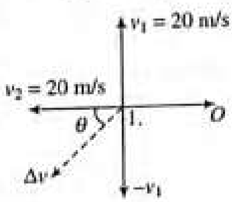
$$\tan\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

5. From figure, $\vec{v}_1 = 20\,\hat{j}$ and $\vec{v}_2 = -20\hat{i}$.

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = -20(\hat{i} + \hat{j})$$

$$|\Delta\vec{v}| = 20\sqrt{2}$$

and direction $\theta = \tan^{-1}(1) = 45^\circ$, i.e., S–W.



6. Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors. Then the sum is

$$\vec{n}_s = \hat{n}_1 + \hat{n}_2 \quad \text{or} \quad n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta$$
$$= 1 + 1 + 2\cos\theta$$

Since it is given that n_s is also a unit vector, therefore

$$1 = 1 + 1 + 2\cos\theta$$

or $\quad \cos\theta = -\frac{1}{2}$ or $\theta = 120^\circ$

Now the difference vector is $n_d = n_1 - n_2$

or $\quad n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1 + 1 - 2\cos(120^\circ)$

$\therefore \quad n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$

7. Let P be the smaller force and Q be the greater force. Then

$$P + Q = 18 \qquad \text{(i)}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12 \qquad \text{(ii)}$$

$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90 = \infty$$

$\therefore \quad P + Q\cos\theta = 0 \qquad \text{(iii)}$

By solving (i), (ii), and (iii), we will get $P = 5$ and $Q = 13$.

8. $x = 0$ means y-axis $\Rightarrow \vec{F}_1 = \hat{j}$

$y = 0$ means x-axis $\Rightarrow \vec{F}_2 = 2\hat{i}$

So resultant, $\vec{F} = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + \hat{j}$

9. $\vec{A} - 2\vec{B} + 3\vec{C} = (2\hat{i} + \hat{j}) - 2(3\hat{j} - \hat{k}) + 3(6\hat{i} - 2\hat{k})$

$$= 2\hat{i} + \hat{j} - 6\hat{j} + 2\hat{k} + 18\hat{i} - 6\hat{k} = 20\hat{i} - 5\hat{j} - 4\hat{k}$$

Concept Application Exercise 2.2

1. $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|}$

$$= \frac{a_1b_1 + a_2b_2 + a_3b_3}{|\vec{A}|.|\vec{B}|} = \frac{2\times4 + 4\times2 - 4\times4}{|\vec{A}|.|\vec{B}|} = 0$$

$\therefore \quad \theta = \cos^{-1}(0^\circ) \Rightarrow \theta = 90^\circ$

2. Let $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} + \lambda\hat{k}$

$\vec{A}$ and $\vec{B}$ are parallel to each other $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$,

i.e., $\quad \dfrac{2}{-4} = \dfrac{3}{-6} = \dfrac{-1}{\lambda} \Rightarrow 1 = 2$

3. If $\vec{A}$ and $\vec{B}$ are perpendicular to each other, then

$$\vec{A}.\vec{B} = 0 \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$$

So, $\quad 2(-4) + 3(-6) + (-1)(\lambda) = 0 \Rightarrow \lambda = -26$

4. $|\vec{A}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$

$$|\vec{B}| = \sqrt{(-1)^2 + 3^2 + 4^2} = \sqrt{1 + 9 + 16} = \sqrt{26}$$

$$\vec{A}.\vec{B} = 2(-1) + 3\times3 + (-1)(4) = 3$$

The projection of $\vec{A}$ on $\vec{B} = \dfrac{\vec{A}.\vec{B}}{|\vec{B}|} = \dfrac{3}{\sqrt{26}}$

5. $W = \vec{F}.\vec{S} = FS\cos\theta = 50\times10\times\cos60^\circ = 50\times10\times\frac{1}{2} = 250$ J.

6. $S = \vec{r_2} - \vec{r_1}$; $W = \vec{F}.\vec{S} = (4\hat{i} + \hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} + 15\hat{k})$
$= (4 \times 11 + 1 \times 11 + 3 \times 15) = 100$ J

7. $(\vec{A} + \vec{B})$ is perpendicular to $(\vec{A} - \vec{B})$.

Thus $(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$

or $A^2 + \vec{B}.\vec{A} - \vec{A}.\vec{B} - B^2 = 0$

Because of commutative property of dot product $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

$\therefore$ $A^2 - B^2 = 0$ or $A = B$

Thus, the ratio of magnitudes $A/B = 1$.

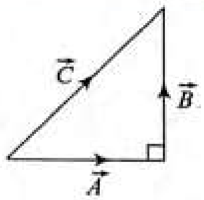
Exercises

Vector Addition

1. (a)

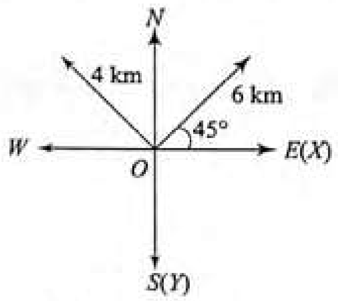
$$C = \sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2} = 5$$

$\therefore$ Angle between $\vec{A}$ is $\frac{\pi}{2}$



2. (a) If the angle between all forces which are equal and lying in one plane are equal, then resultant force will be zero.

3. (c)



Net movement along x-direction

$$S_x = (6 - 4) \cos 45° \; \hat{i} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \text{ km}$$

Net movement along y-direction

$$S_y = (6 + 4) \sin 45° \; \hat{j} = 10 \times \frac{1}{\sqrt{2}} = 5\sqrt{2} \text{ km}$$

Net movement from starting point

$$|\vec{S}| = \sqrt{S_x^2 + S_y^2} = \sqrt{(\sqrt{2})^2 + (5\sqrt{2})^2} = \sqrt{52} \text{ km}$$

Angle which makes with the east direction

$$\tan\theta = \frac{y\text{-component}}{x\text{-component}} = \frac{5\sqrt{2}}{\sqrt{2}} \quad \therefore \theta = \tan^{-1}(5)$$

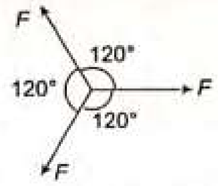
4. (b) $R_{net} = R + \sqrt{R^2 + R^2} = R + \sqrt{2}R = R(\sqrt{2} + 1)$

5. (d) We can observe $\vec{A} + \vec{B} + \vec{E} = 0$

$\Rightarrow$ $\vec{E} = -(\vec{A} + \vec{B})$

6. (d) $\vec{P} + \vec{Q} = P\hat{P} + Q\hat{Q}$

7. (b)



In N forces of equal magnitude works on a single point and their resultant is zero then angle between any two forces is given

$$\theta = \frac{360}{N} = \frac{360}{3} = 120°$$

8. (c) $R_x = 50 = R\cos 60° \Rightarrow R = 100\text{N}$

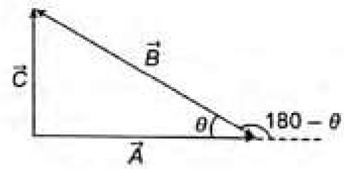
$$R_y = R\sin 60° = \frac{100\sqrt{3}}{2} = 50\sqrt{3} \text{ N}$$

$= 86.6\text{N} = 87\text{N}$

9. (a) In each of the options (b), (c) and (d), there is one force which is greater than sum of other two forces. hence resultant force cannot be zero for these.

10. (c) $A = C$

$$\tan\theta = \frac{C}{A} = 1$$



$\Rightarrow$ $\theta = 45° = \frac{\pi}{4}$

Angle between $\vec{A}$ and $\vec{B}$ is

$$180 - \theta = 180° - 45° = 135° = \frac{3\pi}{4} \text{rad}$$

Resultant of Vectors

11. (a) Resultant $\vec{R} = \vec{P} + \vec{Q} + \vec{P} - \vec{Q} = 2\vec{P}$

The angle between $\vec{P}$ and $2\vec{P}$ is zero.

12. (b) Minimum resultant $= |A - B| = |6 - 8| = 2$ kg
Maximum resultant $= A + B = 6 + 8 = 14$ kg
Only 9 kg lies between them.

13. (a) For 17 N both the vector should be parallel i.e. angle between them should be zero.
For 7 N both the vectors should be antiparallel i.e. angle between them should be 180°.
For 13 N both the vectors should be perpendicular to each other i.e. angle between them should be 90°.

14. (b) $\vec{C} + \vec{A} = \vec{B}$.

The value of C lies between $A - B$ and $A + B$

$\therefore$ $|\vec{C}| < |\vec{A}|$ or $|\vec{C}| < |\vec{B}|$

15. (a) $R_1^2 = (2P)^2 = P^2 + Q^2 + 2PQ\cos\theta$

$(P)^2 = P^2 + Q^2 - 2PQ\cos\theta$

Adding them

$$5P^2 = 2P^2 + 2Q^2$$

or $\quad 3P^2 = 2Q^2$

$$\frac{P}{Q} = \frac{\sqrt{2}}{\sqrt{3}}$$

16. (b) $(3Q)^2 = P^2 + Q^2 + 2PQ\cos\theta \quad$ ...(i)

$Q^2 = P^2 + Q^2 + 2PQ\cos\theta(180-\theta) \quad$ ...(ii)

$9Q^2 = P^2 + Q^2 + 2PQ\cos\theta$

$Q^2 = P^2 + Q^2 - 2PQ\cos\theta$

Adding $10Q^2 = 2P^2 + 2Q^2$

or $\quad 8Q^2 = 2P^2$

or $\quad \dfrac{P^2}{Q^2} = \dfrac{8}{2} = \dfrac{4}{1} \quad$ or $\quad \dfrac{P}{Q} = 2$

17. (b) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

By substituting, $A = F$, $B = F$ and $R = F$ we get

$\cos\theta = \dfrac{1}{2} \qquad \therefore \theta = 120°$

18. (c) $\vec{R}_1 = \vec{A} + \vec{B}$, $\vec{R}_2 = \vec{A} - \vec{B}$

$$R_1^2 + R_2^2 = \left(\sqrt{A^2+B^2}\right)^2 + \left(\sqrt{A^2+B^2}\right)^2 = 2(A^2+B^2)$$

19. (a) $A + B = 16$ (given) $\quad$ ...(i)

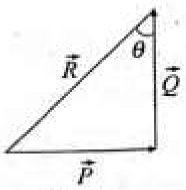
$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \tan 90°$$

$\therefore \quad A + B\cos\theta = 0 \Rightarrow \cos\theta = \dfrac{-A}{B} \quad$ ...(ii)

$8 = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad$ ...(iii)

By solving eqs (i), (ii) and (iii) we get $A = 6$ N, $B = 10$ N

20. (c)



$|\vec{P}| = 5, |\vec{Q}| = 12$ and $|\vec{R}| = 13$

$\cos\theta = \dfrac{Q}{R} = \dfrac{12}{13}$

$\therefore \quad \theta = \cos^{-1}\left(\dfrac{12}{13}\right)$

Expressing Vectors in Unit Vector Notation

21. (a) $\vec{P} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j} \quad \therefore \quad |\vec{P}| = \sqrt{\left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2} = 1$

$\therefore$ It is a unit vector.

22. (c) $\hat{R} = \dfrac{\vec{R}}{|R|} = \dfrac{\hat{i}+\hat{j}}{\sqrt{1^2+1^2}} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}$

23. (c) $\vec{R} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\therefore$ Length in XY plane $= \sqrt{R_x^2 + R_y^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$

24. (b) $\vec{A} = \hat{i} + \hat{j} \Rightarrow |A| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\cos\alpha = \dfrac{A_x}{|A|} = \dfrac{1}{\sqrt{2}} = \cos 45° \qquad \therefore \quad \alpha = 45°$

25. (b) Magnitude of unit vector = 1

$\Rightarrow \quad \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$

By solving we get $c = \sqrt{0.11}$

26. (c) $\vec{F} = F\cos 45°\hat{i} + F\sin 45°(-\hat{j})$

$\Rightarrow \quad \vec{F} = 50\sqrt{2}\,\hat{i} - 50\sqrt{2}\,\hat{j}$ N

27. (a) Let $\vec{C}$ is required vector, then $\vec{C} + \vec{A} + \vec{B} = -\hat{j}$

$\Rightarrow \quad \vec{C} + 2\hat{i} + 5\hat{j} - \hat{k} + 3\hat{i} - 4\hat{j} - \hat{k} = -\hat{j}$

$\Rightarrow \quad \vec{C} = -5\hat{i} - 2\hat{j} + 2\hat{k}$

28. (c) Displacement vector $\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

$= (3-2)\hat{i} + (4-3)\hat{j} + (5-5)\hat{k} = \hat{i} + \hat{j}$

29. (b) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$= (-4\hat{i} + 5\hat{i} - 3\hat{i} + 2\hat{i}) + (-5\hat{j} + 8\hat{j} + 4\hat{j} - 3\hat{j})$

$+ (5\hat{k} + 6\hat{k} - 7\hat{k} - 2\hat{k}) = 4\hat{j} + 2\hat{k}$

$\therefore$ The particle will move in y–z plane.

30. (a) Resultant of vectors $\vec{A}$ and $\vec{B}$

$\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$

$\vec{R} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$$

Dot Product

31. (d) $\cos\theta = \dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{1}{\sqrt{2}\sqrt{2}} = \dfrac{1}{2} \qquad \therefore \quad \theta = 60°$

32. (b) $\cos\theta = \dfrac{\vec{F}_1 . \vec{F}_2}{|F_1||F_2|}$

$$= \frac{(5\hat{i} + 10\hat{j} - 20\hat{k}).(10\hat{i} - 5\hat{j} - 15\hat{k})}{\sqrt{25+100+400}\sqrt{100+25+225}} = \frac{50 - 50 + 300}{\sqrt{525}\sqrt{350}}$$

$\Rightarrow \quad \cos\theta = \dfrac{1}{\sqrt{2}} \qquad \therefore \quad \theta = 45°$

33. (c) Dot product of two perpendicular vector will be zero.

34. (c) $\vec{R} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$

Comparing the given vector with $R = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$

$R_x = 1, R_y = 1, R_z = \sqrt{2}$ and $|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = 2$

$$\cos\alpha = \frac{R_x}{R} = \frac{1}{2} \Rightarrow \alpha = 60^\circ$$

$$\cos\beta = \frac{R_y}{R} = \frac{1}{2} \Rightarrow \beta = 60^\circ$$

$$\cos\gamma = \frac{R_z}{R} = \frac{1}{\sqrt{2}} \Rightarrow \gamma = 45^\circ$$

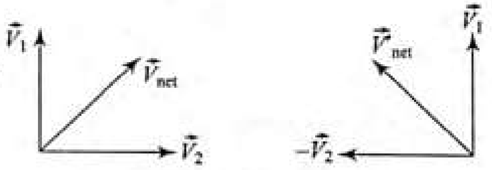
35. (c) $\sin^2\alpha + \sin^2\beta + \sin\gamma$

$= 1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma$

$= 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 3 - 1 = 2$

36. (c) Force F lie in the x-y plane so a vector along z-axis will be perpendicular to F.

37. (c)



According to problem $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$

$\Rightarrow \quad |\vec{V}_{net}| = |\vec{V}'_{net}|$

So V_1 and V_2 will be mutually perpendicular.

38. (a) $\cos\theta = \dfrac{\vec{P}\cdot\vec{Q}}{PQ} = 1 \quad \therefore \quad \theta = 0^\circ$

39. (c) $\vec{A}\cdot\vec{B} = AB\cos\theta$

In the problem $\vec{A}\cdot\vec{B} = -AB$, i.e., $\cos\theta = -1 \quad \therefore \quad \theta = 180^\circ$

i.e. $\vec{A}$ and $\vec{B}$ acts in the opposite direction.

40. (a) $\dfrac{\vec{A}\cdot\vec{B}}{|\vec{i}+\vec{j}|} = \dfrac{(2\hat{i}+3\hat{j})(\hat{i}+\hat{j})}{\sqrt{2}} = \dfrac{2+3}{\sqrt{2}} = \dfrac{5}{\sqrt{2}}$

Cross Product

41. (a) Let the components of $\vec{A}$ makes angles α, β and γ with x, y and z axis respectively then $\alpha = \beta = \gamma$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$\Rightarrow \quad 3\cos^2\alpha = 1 \Rightarrow \cos\alpha = \dfrac{1}{\sqrt{3}}$

$\therefore \quad A_x = A_y = A_z = A\cos\alpha = \dfrac{A}{\sqrt{3}}$

42. (b) $\vec{A}\times\vec{B} = 0 \quad \therefore \sin\theta = 0 \quad \therefore \theta = 0^\circ$

Two vectors will be parallel to each other.

43. (b) Vector $(\vec{P}+\vec{Q})$ lies in a plane and vector $(\vec{P}\times\vec{Q})$ is perpendicular to this plane i.e. the angle between given vectors is $\dfrac{\pi}{2}$.

44. (c) $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{C} = \vec{A}\times\vec{B} = (2\hat{i} + 2\hat{j} - \hat{k})\times(6\hat{i} - 3\hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 6 & -3 & 2 \end{vmatrix} = \hat{i} - 10\hat{j} - 18\hat{k}$$

Unit vector perpendicular to both $\vec{A}$ and $\vec{B}$

$$= \frac{\hat{i} - 10\hat{j} - 18\hat{k}}{\sqrt{1^2 + 10^2 + 18^2}} = \frac{\hat{i} - 10\hat{j} - 18\hat{k}}{5\sqrt{17}}$$

45. (a) $\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{AB} = \dfrac{15}{5\times 6} = \dfrac{1}{2} \Rightarrow \theta = 30^\circ$

46. (c) If $\vec{A}\cdot\vec{B} = 0$, $\vec{A}$ is perpendicular to $\vec{B}$.

If $\vec{A}\cdot\vec{C} = 0$, $\vec{A}$ is perpendicular to $\vec{C}$.

So $\vec{A}$ is perpendicular to both $\vec{B}$ and $\vec{C}$.

Also $\vec{B}\times\vec{C}$ is perpendicular to both $\vec{B}$ and $\vec{C}$. Hence, $\vec{A}$ is parallel to $\vec{B}\times\vec{C}$.

47. (c) $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

For parallel vectors $\theta = 0^\circ$ or 180°, $\sin\theta = 0$

$\therefore \quad \vec{A}\times\vec{B} = \hat{0}$

48. (c) Vector perpendicular to A and B, $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$\therefore$ Unit vector perpendicular to A and B

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}|\times|\vec{B}|\sin\theta}$$

49. (a) $\vec{A}\times\vec{B}$ is a vector perpendicular to plane $\vec{A}+\vec{B}$ and hence perpendicular to $\vec{A}+\vec{B}$.

50. (d) $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B}) = \vec{A}\times\vec{A} - \vec{A}\times\vec{B} + \vec{B}\times\vec{A} - \vec{B}\times\vec{B}$

$= 0 - \vec{A}\times\vec{B} + \vec{B}\times\vec{A} - 0 = \vec{B}\times\vec{A} + \vec{B}\times\vec{A} = 2(\vec{B}\times\vec{A})$

Problems Based on Mixed Concepts

51. (a) $\vec{P}_1 = mv\sin\theta\,\hat{i} - mv\cos\theta\,\hat{j}$

and $\vec{P}_2 = mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}$

So change in momentum

$\overline{\Delta P} = \vec{P}_2 - \vec{P}_1 = 2mv\cos\theta\,\hat{j}$, $|\Delta\vec{P}| = 2mv\cos\theta$

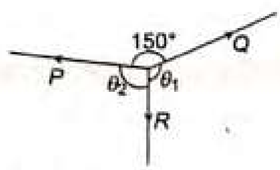
52. (d) $\vec{a}.\vec{b} = 0$ i.e., $\vec{a}$ and $\vec{b}$ will be perpendicular to each other

$\vec{a}.\vec{c} = 0$ i.e., $\vec{a}$ and $\vec{c}$ will be perpendicular to each other

$\vec{b}\times\vec{c}$ will be a vector perpendicular to both $\vec{b}$ and $\vec{c}$

So $\vec{a}$ is parallel to $\vec{b}\times\vec{c}$.

53. (c)



$$\frac{P}{\sin\theta_1} = \frac{Q}{\sin\theta_2} = \frac{R}{\sin 150^\circ}$$

$\Rightarrow \dfrac{1.93}{\sin\theta_1} = \dfrac{R}{\sin 150°}$

$\Rightarrow R = \dfrac{1.93 \times \sin 150°}{\sin\theta_1} = \dfrac{1.93 \times 0.5}{0.9659} = 1$

54. (a) $\tan\alpha = \dfrac{2p\sin\theta}{p + 2p\cos\theta}$ here $\theta = 90°$

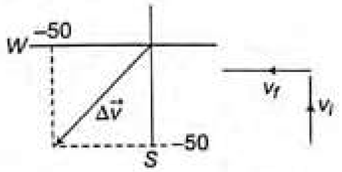
$\Rightarrow \infty = \dfrac{2p\sin\theta}{p + 2p\cos\theta} \Rightarrow p + 2p\cos\theta = 0$

$\Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{2\pi}{3}$

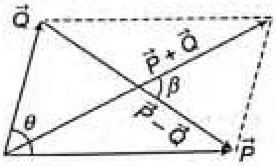
55. (b) $\vec{v}_i = 50\hat{j} \Rightarrow \vec{v}_f = -50\hat{i}$

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = -50\hat{i} - 50\hat{j}$

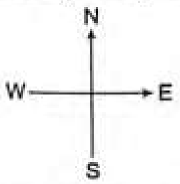
$|\Delta\vec{v}| = 50\sqrt{2}$ km/h, SW

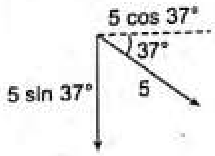


56. (c) β is the angle between $\vec{P} + \vec{Q}$ and $\vec{P} - \vec{Q}$. β can have any value from 0° to 180° depending upon magnitudes of $\vec{P}$ and $\vec{Q}$ and angle θ between them.



57. (a) Given resultant displacement is 6 km due east Which may be expressed as $6\hat{i}$.





Given displacements are

(i) 2 km due east or $2\hat{i}$

(ii) 5 km 37° south of east its components

along x-axis $= 5\cos 37° = 5 \times \dfrac{4}{5} = 4\hat{i}$

along y-axis $= 5\sin 37° = 5 \times \dfrac{3}{5} = -3\hat{j}$

(iii) unknown displacement let it be $a\hat{i} + b\hat{j}$

$\therefore\ 2\hat{i} + 4\hat{i} - 3\hat{j} + a\hat{i} + b\hat{j} = 6\hat{i}$

$\therefore\ 6\hat{i} + a\hat{i} - 3\hat{j} + b\hat{j} = 6\hat{i}$

comparing the two sides

$a = 0$ and $b = 3$

$\therefore$ Unknown displacement $= 3\hat{j}$

58. (b) Given speed $v = 50$ m/s in the direction $\vec{a} = 7\hat{i} - 24\hat{j}$

$\vec{v} = v\hat{a}$; $\hat{a}$ is the unit vector in the direction $7\hat{i} - 24\hat{j}$

$\hat{a} = \dfrac{(7\hat{i} - 24\hat{j})}{\sqrt{(24)^2 + (7)^2}} = \dfrac{(7\hat{i} - 24\hat{j})}{25}$

$\vec{v} = v\hat{a} = 50.\dfrac{(7\hat{i} - 24\hat{j})}{25} = 2(7\hat{i} - 24\hat{j})$ m/s

Initially the particle is at $\vec{r}_0 = (3\hat{i} - 7\hat{j})$ m

Position of the particle after 3 sec, $\vec{r} = \vec{r}_0 + \vec{v}t$

$\Rightarrow \vec{r} = (3\hat{i} - 7\hat{j}) + 3 \times 2(7\hat{i} - 24\hat{j}) = (45\hat{i} - 151\hat{j})$ m

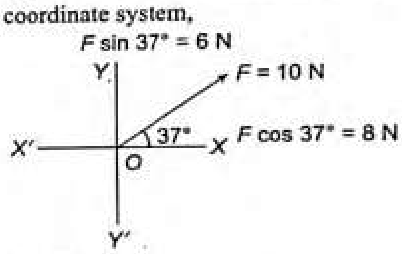
59. (b) Substituting $\vec{r} = (A\cos\omega t\,\hat{i} + A\sin\omega t\,\hat{j})$ in $\vec{v} = \dfrac{d\vec{r}}{dt}$ we have

$\vec{v} = A\dfrac{d}{dt}(\cos\omega t)\hat{i} + A\dfrac{d}{dt}(\sin\omega t)\hat{j}$

$= -A\omega\sin\omega t\hat{i} + A\omega\cos\omega t\,\hat{j}$

At $t = 0$, $v = -A\omega\sin 0\hat{i} + A\omega\cos 0\hat{j} = A\omega\hat{j}$

60. (c) In new coordinate system,

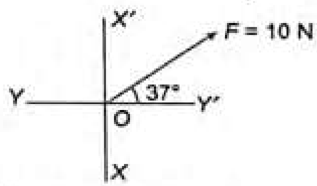


Initial coordinate system

$F\sin 37° = 6$ N

$F\cos 37° = 8$ N

x-component $F_x = F\sin 37° = 10 \times \dfrac{3}{5} = 6$N



New coordinate system

y-component $F_y = -F\cos 37° = -10 \times \dfrac{4}{5} = -8$N

Archives

1. (b) $\Delta v = 2v\sin\left(\dfrac{\theta}{2}\right) = 2 \times 5 \times \sin 45° = \dfrac{10}{\sqrt{2}}$

$\therefore\ a = \dfrac{\Delta v}{\Delta t} = \dfrac{10/\sqrt{2}}{10} = \dfrac{1}{\sqrt{2}}$ m/s^2

2. (c) For motion of the particle from (0, 0) to $(a, 0)$

$\vec{F} = -K(0\hat{i} + a\hat{j}) \Rightarrow \vec{F} = -Ka\hat{j}$

Displacement $\vec{r} = (a\hat{i} + 0\hat{j}) - (0\hat{i} + 0\hat{j}) = a\hat{i}$

So work done from (0, 0) to $(a, 0)$ is given by

$$W = \vec{F}.\vec{r} = -Ka\hat{j}.a\hat{i} = 0$$

For motion $(a, 0)$ to (a, a)

$\vec{F} = -K(a\hat{i} + a\hat{j})$ and displacement

$\vec{r} = (a\hat{i} + a\hat{j}) - (a\hat{i} + 0\hat{j}) = a\hat{j}$

So work done from $(a, 0)$ to (a, a) $W = \vec{F}.\vec{r}$

$= -K(a\hat{i} + a\hat{j}).a\hat{j} = -Ka^2 1$

So total work done $= -Ka^2$

3. (c) Let P be the smaller force and Q be the greater force then according to problem:

$$P + Q = 18 \quad ...(i)$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = 12 \quad ...(ii)$$

$$\tan\varphi = \frac{Q\sin\theta}{P + Q\cos\theta} = \tan 90 = \infty$$

$\therefore$ $P + Q\cos\theta = 0$ ...(iii)

By solving (i), (ii) and (iii) we will get $P = 5$, and $Q = 13$

4. (c) We know that $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$ because the angle between these two is always 90°.

But if the angle between $\vec{A}$ and $\vec{B}$ is 0 or π.

Then $\vec{A} \times \vec{B} = \vec{B} \times \vec{A} = 0$.

5. (b) Given $u = 3\hat{i} + 4\hat{j}, a = 0.4\hat{i} + 0.3\hat{j}$

$v = u + at$

$= 3\hat{i} + 4\hat{j} + (0.4\hat{i} + 0.3\hat{j})10$

$= 3\hat{i} + 4\hat{j} + 4\hat{i} + 3\hat{j} = 7\hat{i} + 7\hat{j}$

So, speed is equal to magnitude of velocity

$= \sqrt{7^2 + 7^2} = 7\sqrt{2}$ units

6. (c) $\vec{L} = m(\vec{r} \times \vec{v})$

$$\vec{L} = m\left[v_0\cos\theta t\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j}\right] \times \left[v_0\cos\theta\hat{i} + (v_0\sin\theta - gt)\hat{j}\right]$$

$$= mv_0\cos\theta t\left[-\frac{1}{2}\right]\hat{k} = -\frac{1}{2}mgv_0 t^2\cos\theta\hat{k}$$

7. (c) For a particle in uniform circular motion, $\vec{a} = \frac{v^2}{R}$ towards centre of circle

$$\vec{a} = \frac{v^2}{R}(-\cos\theta\hat{i} - \sin\theta\hat{j})$$

$$\vec{a} = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$$

8. (d) $\vec{v} = Ky\hat{i} + Kx\hat{j}$

$$\frac{dx}{dt} = Ky, \frac{dy}{dt} = Kx$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{Kx}{Ky}$$

$y = dy = x\,dx$

$y^2 = x^2 + c$

9. (a) $x = t$

$y = 2t - 5t^2$

Equation of trajectory is $y = 2x - 5x^2$

CHAPTER 3: MOTION IN ONE DIMENSION

Concept Application Exercise 3.1

1. $10 = u + \frac{a}{2}(2\times 2-1)$ and $25 = u + \frac{a}{2}(2\times 5-1)$

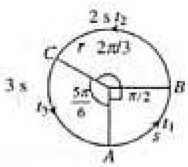
Solving, we get $u = \frac{5}{2}\,\text{ms}^{-1}$, $a = 5\,\text{m s}^{-2}$

$$D_7 = \frac{5}{2} + \frac{5}{2}[2\times 7-1] = 35\text{m}$$

2. $25 = u + \frac{a}{2}(2\times 5-1)$ and $33 = u + \frac{a}{2}(2\times 7-1)$

Solving, we get $u = 7\,\text{m s}^{-1}$, $a = 4\,\text{m s}^{-2}$

3. $t_1 = \frac{AB}{s} = \frac{\frac{\pi}{2}r}{s}$, $t_2 = \frac{BC}{2s} = \frac{\frac{2\pi}{3}r}{2s}$,

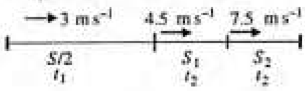


$$t_3 = \frac{CA}{3s} = \frac{\frac{5\pi}{6}r}{3s}$$

Average speed $= \frac{2\pi r}{t_1+t_2+t_3} = 1.8$ s

4. $t_1 = \frac{S/2}{3} = \frac{S}{6}$, $\quad t_2 = \frac{S_1}{4.5} = \frac{S_2}{7.5}$

$S_1 + S_2 = S/2$



$\Rightarrow \quad 4.5\,t_2 + 7.5\,t_2 = S/2$

$\Rightarrow \quad t_2 = \frac{S}{24}$

Average speed $= \frac{S}{t_1+2t_2} = 4\,\text{m s}^{-1}$

5. $\frac{D_4}{D_5} = \frac{(g/2)(2\times 4-1)}{(g/2)(2\times 5-1)} = \frac{7}{9}$

6. Both will pass with same speed, because free fall motion is independent of mass.

7. Average velocity on each lap will be zero, because displacement is zero.

Average speed $= \frac{2\pi r}{t} = \frac{2\pi\times 200}{62.8} = 20\,\text{m s}^{-1}$

8. $v_0 = 0 + \alpha t_1 \quad\Rightarrow\quad t_1 = v_0/\alpha,$

$0 = v_0 - \beta t_2 \quad\Rightarrow\quad t_2 = v_0/\beta$

$\Rightarrow \quad \frac{t_1}{t_2} = \frac{\beta}{\alpha}$

Concept Application Exercise 3.2

1. (a) $s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad -200 = 0\times t + \frac{1}{2}(-10)\,t^2 \quad\Rightarrow\quad t = \sqrt{40}$ s

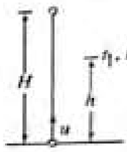
(b) $-200 = 10t - \frac{1}{2}10t^2$

$\Rightarrow \quad t^2 - 2t - 40 = 0 \quad\Rightarrow\quad t = 1 + \sqrt{41}$ s

(c) $-200 = -10t - \frac{1}{2}10t^2$

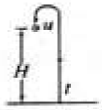
$\Rightarrow \quad t^2 + 2t - 40 = 0 \quad\Rightarrow\quad t = -1 + \sqrt{41}$ s

2. $H = \frac{u^2}{2g}$, $u = \frac{1}{2}g(t_1+t_2)$, $h = \frac{1}{2}gt_1t_2$



Given $\frac{t_1}{t_2} = \frac{1}{3}$; solving, we get $h = \frac{3}{4}H$

3. $u = \sqrt{2\times\frac{g}{8}H} \quad\Rightarrow\quad u = \frac{\sqrt{gH}}{2}$



$$-H = ut - \frac{1}{2}gt^2$$

$\Rightarrow \quad gt^2 - 2ut - 2H = 0$

$\quad gt^2 - \sqrt{gH}\,t - 2H = 0$

$\Rightarrow \quad gt^2 - 2\sqrt{gH}\,t + \sqrt{gH}\,t - 2H = 0$

$\quad \sqrt{g}\,t[\sqrt{g}\,t - 2\sqrt{H}] + \sqrt{H}\,[\sqrt{g}\,t - 2\sqrt{H}] = 0$

$\Rightarrow \quad (\sqrt{g}\,t + \sqrt{H})(\sqrt{g}\,t - 2\sqrt{H}) = 0 \quad\Rightarrow\quad t = 2\sqrt{\frac{H}{g}}$

4. Velocity after 50 m fall, $u_1 = \sqrt{2gh} = \sqrt{100}\,g$

$3^2 = u_1^2 + 2\times(-2)h \quad\Rightarrow\quad 9 = 100g - 4h$

$h = \frac{100\times 9.8 - 9}{4} = 242.75 \approx 243$ m

Net height = 243 + 50 = 293 m

5. $h = \frac{1}{2}gn^2, \frac{h}{2} = \frac{1}{2}g(n-1)^2$, where n is the total time taken.

Solving, we get

$$n = \frac{\sqrt{2}}{\sqrt{2}-1} \quad\Rightarrow\quad n = \left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right) = 2+\sqrt{2}\text{ s}$$

6. Every ball will travel a distance of 5 m in the last second of going up to its highest point, provided its velocity is greater than 10 m s^{-1} initially (it should at least travel a maximum height of 5 m).

7. The time needed for the flower to fall is

$$t = \sqrt{\frac{2\Delta h}{g}} = \sqrt{\frac{2(46.0\text{ m} - 1.80\text{ m})}{(9.80\text{ m s}^{-2})}} = 3.00\text{ s}$$

and so the professor should be at a distance

$$v_y t = (1.20\text{ m s}^{-1})(3.00\text{ s}) = 3.60\text{ m}$$

Concept Application Exercise 3.3

1. Distance to be covered = 1000 + 200 = 1200 m

Velocity = 72 km h^{-1} = 20 m s^{-1}

$$t = \frac{\text{Distance}}{\text{Velocity}} = \frac{1200}{20} = 60\text{ s}$$

2. Required separation = $(v_2 - v_1)t$

$$= (72-36)\frac{20}{60} = 12 \text{ km}$$

3. $$t = \frac{\text{Total length}}{\text{Relative velocity}} = \frac{110+90}{(36+54)\times(5/18)} = 8 \text{ s}$$

4. (a) When she walks in the same direction, relative velocity of woman w.r.t. ground:

$v_1 = 1 + 1.5 = 2.5 \text{ m s}^{-1}$,

$$\text{Time taken} = \frac{s}{v_1} = \frac{35}{2.5} = 14 \text{ s}$$

(b) When she walks in the opposite direction, relative velocity of woman w.r.t. ground:

$v_2 = 1.5 - 1 = 0.5 \text{ m s}^{-1}$,

$$\text{Time taken} = \frac{s}{v_2} = \frac{35}{0.5} = 70 \text{ s}$$

5. Velocity of passenger train in m s^{-1},

$$V_P = 72 \times \frac{5}{18} = 29 \text{ ms}^{-1}$$

Let velocity of goods train be V_G and its length be x meter. Velocity of passenger train to goods train.

$$\vec{V}_{PG} = \vec{V}_P - \vec{V}_G = \vec{V}_P + (-\vec{V}_G)$$

$$|\vec{V}_{PG}| = |\vec{V}_P| - |\vec{V}_G| = (20 - V_G)$$

In 25 s, the distance travelled by passenger train relative to goods train, $S_{PG} = x$. So,

$$(20 - V_G) = 20\,x \quad \text{(i)}$$

In next 30 s, the passenger train completely overtakes the goods train, which implies $S_{PG} = 240$ m.

$$(20 - V_G)\,30 = 240 \quad \text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$V_G = 12 \text{ m s}^{-1}$ and $x = 200$ m

Concept Application Exercise 3.4

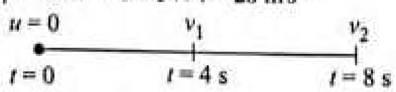
1. Let us first find the change in velocity. Change in velocity is the area under the acceleration–time graph. For first 20 s,

$$\text{Area} = \Delta v = \frac{1}{2}\times 10\times 20 + 10\times 20 = 300 \text{ m s}^{-1}$$

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t} = \frac{300}{20} = 15 \text{ m s}^{-2}$$

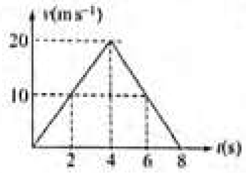
2. From 0 to 4 s, at $t = 0$, $a = 5 \text{ m s}^{-2}$

$v_1 = u + at = 0 + 5\times 4 = 20 \text{ m s}^{-1}$



From 4 to 8 s, $a = -5 \text{ m s}^{-2}$

$v_2 = v_1 + at \;\Rightarrow\; v_2 = 20 - 5\times 4 = 0 \text{ m s}^{-1}$



3. Slope for C is greatest, so C will have greater velocity and slope for A is least, so A will have least velocity.

4. (a) At point IV, velocity is zero because slope is zero at this point.
(b) At point I, slope is positive and constant.
(c) At point V, slope is negative and constant.
(d) At point II, slope suddenly increases.
(e) At point III, slope is positive but decreasing.

5. (a) The slope of position time graph gives the velocity

Slope of line A = velocity of car A

$$= \frac{14-6}{1.6-0} = \frac{8}{1.6} = 5 \text{ km h}^{-1}$$

Slope of line B = velocity of car B

$$= \frac{14-6}{1.4-0} = 10 \text{ km h}^{-1}$$

Slope of line = velocity of car C

$$= \frac{14-2}{1-0} = 12 \text{ km h}^{-1}$$

Here car C has highest speed and car A has lowest speed.

(b) As clear from graph, the position of all three cars are not same at any time. Hence, the three cars never at the same point on the road.

(c) From graph, it is clear car B is approximately 6 km from the origin.

(d) From graph, car A travels between the time it passed cars B and $C = 1.2 - 0.6 = 0.6$ h

(e) Relative velocity of car C with respect to car A,

$$\vec{v}_{C,A} = \vec{v}_C - \vec{v}_A = 12 - 5 = 7 \text{ km h}^{-1}$$

(f) Relative velocity of car B with respect to car C

$$\vec{v}_{B,C} = \vec{v}_B - \vec{v}_C = 10 - 12 = -2 \text{ km h}^{-1}$$

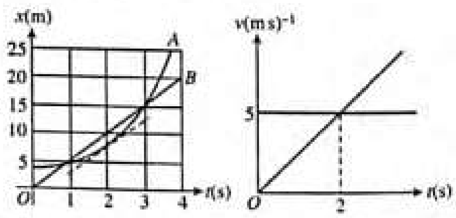
6. (a) The two cars have the same position at times when their x-t graphs cross. The figure shows this occurs at $t = 1$ s and $t = 3$ s.

(b) A and B have the same velocity when slope of the tangent on parabola is equal to the slope of the straight line. From position-time graph, it is clear it occurs at $t = 2$ s.

Car B travels with constant velocity.

$$v_B = \frac{20-0}{4-0} = 5 \text{ m s}^{-1}$$

(c)



(d) Car A passes car B when x_A moves above x_B in the x-t graph. This happens at $t = 3$ s.

(e) Car B passes car A when x_B moves above x_A in the x-t graph. This happens at $t = 1$ s.

Exercises

General Kinematics and Motion with Constant Acceleration

1. (c) To have distance equal to magnitude of displacement, the particle has to move in the same direction. The velocity may or may not be constant.

2. (d) $s = kt^{1/2} \Rightarrow a = \dfrac{d^2s}{dt^2} = -\dfrac{1}{4}kt^{-3/2}$

As t increases, retardation decreases.

3. (c) $x = at^2 - bt^3$

Velocity $= \dfrac{dx}{dt} = 2at - 3bt^2$

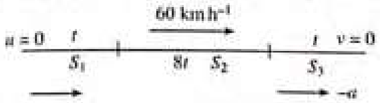
and acceleration $= \dfrac{d^2x}{dt^2} = 2a - 6bt$

Acceleration will be zero if

$2a - 6bt = 0 \Rightarrow t = \dfrac{2a}{6b} = \dfrac{a}{3b}$

4. (c) $S_1 = \dfrac{1}{2}at^2 = \dfrac{1}{2}(at)t = \dfrac{60t}{2} = 30t$

$S_2 = 60 \times 8t = 480t$, $S_3 = S_1 = 30t$



$v_{av} = \dfrac{S_1 + S_2 + S_3}{t + 8t + t} = 54\ \text{km h}^{-1}$

5. (a) $30 = u + a \times 2$, $60 = u + a \times 4$

Solve to get $u = 0$.

6. (d) $2ax = (50)^2 - (10)^2$ and $2(-a)(-x) = v^2 - (50)^2$

This gives $v^2 - (50)^2 = (10)^2$ i.e. $v = 70\ \text{m s}^{-1}$

7. (d) $v^2 - u^2 = 2as$, $v = 0$

$s \propto u^2$

When the initial velocity is made n times, the distance over which it can be stopped becomes n^2 times.

8. (a) $t = \alpha x^2 + \beta x$

Differentiating: $1 = 2\alpha \dfrac{dx}{dt}.x + \beta \dfrac{dx}{dt}$

$v = \dfrac{dx}{dt} = \dfrac{1}{\beta + 2\alpha x}$; $\dfrac{dv}{dt} = \dfrac{-2\alpha v}{(\beta + 2\alpha x)^2} = -2\alpha v^3$

9. (a) $t = \sqrt{x} + 3$.

Differentiating with respect to t, we get

$1 = \dfrac{1}{2\sqrt{x}}\dfrac{dx}{dt} + 0$ or $\dfrac{dx}{dt} = 2\sqrt{x}$

When velocity is zero, $2\sqrt{x} = 0$ or $x = 0$.

10. (a) Since the last five steps covering 5 m land the drunkard fell into the pit, the displacement prior to this is $(11 - 5)$ m = 6 m. Time taken for first eight steps (displacement in first eight steps = 5 − 3 = 2 m) = 8 s. Then time taken to cover first 6 m of journey $= \dfrac{6}{2} \times 8 = 24$ s

Time taken to cover last 5 m = 5 s

Total time = 24 + 5 = 29 s

11. (b) $200 = u \times 2 - (1/2)\,a(2)^2$

or $u - a = 100$ (i)

$200 + 220 = u(2 + 4) - (1/2)\,(2 + 4)^2 a$

or $u - 3a = 70$ (ii)

Solving Eqs (i) and (ii), we get $a = 15\ \text{cm s}^{-2}$ and $u = 115\ \text{cm s}^{-1}$.

Further, $v = u - at = 115 - 15 \times 7 = 10\ \text{cm s}^{-1}$.

12. (c) $\dfrac{v_{av}}{v_{max}} = \dfrac{\text{Total displacement}}{2\begin{pmatrix}\text{Total displacement}\\ \text{during acceleration}\\ \text{and retardation}\end{pmatrix} + \begin{pmatrix}\text{Displacement}\\ \text{during uniform}\\ \text{velocity}\end{pmatrix}}$

$\dfrac{v_{av}}{v_{max}} = \dfrac{8S}{2(S + 5S) + 2S} = \dfrac{8}{14} = \dfrac{4}{7}$

13. (c) If police is able to catch the dacoit after time t, then

$vt = x + \dfrac{1}{2}\alpha t^2$. This gives $\dfrac{\alpha}{2}t^2 - vt + x = 0$

or $t = \dfrac{v \pm \sqrt{v^2 - 2\alpha x}}{\alpha}$

For t to be real, $v^2 \geq 2ax$

14. (b) Distance covered $= S = v_{av} \times$ time

For first second: $S_1 = 5 \times 1 = 5$ m

For second second: $S_2 = 10 \times 1 = 10$ m

For third second: $S_3 = 15 \times 1 = 15$ m

Total distance travelled

$S = S_1 + S_2 + S_3 = 5 + 10 + 15 = 30$ m

15. (b) Given $v_{av} = \dfrac{v + u}{2} = 0.34$ and $v - u = 0.18$

Solving these two equations, we get

$u = 0.25\ \text{m s}^{-1}$, $v = 0.43\ \text{m s}^{-1}$. Given $s = 3.06$ m

Now use $v^2 - u^2 = 2as$ to find $a = 0.02\ \text{m s}^{-2}$.

16. (b) Suppose u be the initial velocity.

Velocity after time t_1: $v_{11} = u + at_1$

Velocity after time $t_1 + t_2$: $v_{22} = u + a(t_1 + t_2)$

Velocity after time $t_1 + t_2 + t_3$:

$v_{33} = u + a(t_1 + t_2 + t_3)$

Now $v_1 = \dfrac{u + v_{11}}{2} = \dfrac{u + u + at_1}{2} = u + \dfrac{1}{2}at_1$

$v_2 = \dfrac{v_{11} + v_{22}}{2} = u + at_1 + \dfrac{1}{2}at_2$

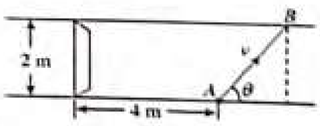
$v_3 = \dfrac{v_{22} + v_{33}}{2} = u + at_1 + at_2 + \dfrac{1}{2}at_3$

So $v_1 - v_2 = -\dfrac{1}{2}a(t_1 + t_2)$

$v_2 - v_3 = -\dfrac{1}{2}a(t_2 + t_3)$

$(v_1 - v_2):(v_2 - v_3) = (t_1 + t_2):(t_2 + t_3)$

17. (c) Let the man start crossing the road at an angle θ with the roadside. For safe crossing, the condition is that the man must cross the road by the time the truck describes the distance $(4 + 2\cot\theta)$



So, $\dfrac{4 + 2\cot\theta}{8} = \dfrac{2/\sin\theta}{v}$ or $v = \dfrac{8}{2\sin\theta + \cos\theta}$

For minimum v, $\dfrac{dv}{d\theta} = 0$

or $\dfrac{-8(2\cos\theta - \sin\theta)}{(2\sin\theta + \cos\theta)^2} = 0$ or $2\cos\theta - \sin\theta = 0$

or $\tan\theta = 2$, so $\sin\theta = \dfrac{2}{\sqrt{5}}, \cos\theta = \dfrac{1}{\sqrt{5}}$

$$v_{\min} = \frac{8}{2\left(\frac{2}{\sqrt{5}}\right) + \frac{1}{\sqrt{5}}} = \frac{8}{\sqrt{5}} = 3.57 \text{ ms}^{-1}$$

Problems Based on Motion under Gravity

18. (c) We have $h = \dfrac{1}{2}gT^2$

In $T/3$ second, distance fallen $= \dfrac{1}{2}g\left(\dfrac{T}{3}\right)^2 = \dfrac{h}{9}$

So position of the ball from the ground is

$$h - \frac{h}{9} = \frac{8h}{9} \text{ m}$$

19. (d) Distance covered by the object in first 2 s

$$h_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

Similarly, distance covered by the object in next 2 s will also be 20 m, hence the required height $= H - 20 - 20 = H - 40$ m

20. (c) $h = \dfrac{1}{2}gt^2$ and $h - 20 = \dfrac{1}{2}g(t-1)^2$

Solving them, we get $t = 2.5$ s and $h = 31.25$ m.

21. (a) Time of ascent = 1 s $\Rightarrow \dfrac{u}{g} = 1 \Rightarrow u = 10 \text{ ms}^{-1}$

Maximum height attained $= \dfrac{u^2}{2g} = \dfrac{10^2}{2 \times 10} = 5$ m

22. (c) Maximum height attained $\propto u^2$

23. (b) Given $7x = \dfrac{g}{2}(2n-1)$ and $x = \dfrac{1}{2}g(1)^2$

Solving these two equations, we get $n = 4$ s.

24. (b) The required ratio is 1 : 3 : 5 : ... so on

25. (c) Here $h = \dfrac{1}{2} \times 10 \times (5)^2 = 125\text{m}$

In 3 s it falls through: $h_1 = \dfrac{1}{2} \times 10 \times (3)^2 = 45\text{m}$

Rest 80 m is covered in 4 s. Hence, total time taken is
3 s + 4 s = 7 s

26. (b) When a body slides on an inclined plane, component of weight along the plane produces an acceleration.

$$a = \frac{mg\sin\theta}{m} = g\sin\theta = \text{Constant}$$

If s is the length of the inclined plane, then

$$s = 0 + \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta \times t^2$$

$$\frac{s'}{s} = \frac{t'^2}{t^2} \text{ or } \frac{s}{s'} = \frac{t^2}{t'^2}$$

Given $t = 4$ s and $s' = \dfrac{s}{4}$

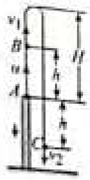
$$t' = t\sqrt{\frac{s'}{s}} = 4\sqrt{\frac{s}{4s}} = \frac{4}{2} = 2 \text{ s}$$

27. (a) Bomb B_1 will have less velocity upwards on dropping, so it will reach ground first.

28. (c) $H = \dfrac{u^2}{2g}$; given $v_2 = 2v_1$ (i)

A to B: $v_1^2 = u^2 - 2gh$ (ii)

A to C: $v_2^2 = u^2 - 2g(-h)$ (iii)



Solving (i), (ii) and (iii), we get the value of u^2 as $10gh/3$ and then we get the value of H by using

$$H = \frac{u^2}{2g} \text{ (see figure) or } H = \frac{5h}{3}$$

29. (b) Time taken by the same ball to return to the hands of the juggler is $\dfrac{2u}{g} = \dfrac{2 \times 20}{10} = 4$ s. So he is throwing the balls after 1 s each. Let at some instant he throws ball number 4. Before 1 s of throwing it, he throws ball 3. So the height of ball 3 is

$$h_3 = 20 \times 1 - \frac{1}{2}10(1)^2 = 15 \text{ m}$$

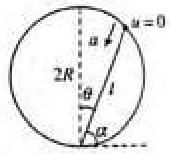
Before 2 s, he throws ball 2. So the height of ball 2 is

$$h_2 = 20 \times 2 - \frac{1}{2}10(2)^2 = 20 \text{ m}$$

Before 3 s, he throws ball 1. So the height of ball 1 is

$$h_1 = 20 \times 3 - \frac{1}{2}10(3)^2 = 15 \text{ m}$$

30. (d) $a = g\sin\alpha = g\sin(90° - \theta)$ (see figure)
$= g\cos\theta$
$l = 2R\cos\theta$



Now using $s = ut + \dfrac{1}{2}at^2$, we get

$$l = 0t + \frac{1}{2}g\cos\theta t^2$$

$$\Rightarrow 2R\cos\theta = \frac{1}{2}g\cos\theta t^2 \Rightarrow t = 2\sqrt{\frac{R}{g}}$$

This is independent of θ.

31. (c) Suppose the body be projected vertically upwards from A with a speed u_0.

Using equation $s = ut + \left(\dfrac{1}{2}\right)at^2$, we get

For first case: $-h = u_0t_1 - \left(\dfrac{1}{2}\right)gt_1^2$ (i)

For second case: $-h = -u_0t_2 - \left(\dfrac{1}{2}\right)gt_2^2$ (ii)

From (i) and (ii) we get

$$h = \frac{1}{2}gt_1t_2 \quad \text{(iii)}$$

For third case: $u = 0, t = ?$

$$-h = 0 \times t - \left(\frac{1}{2}\right)gt^2 \text{ or } h = \left(\frac{1}{2}\right)gt^2 \quad \text{(iv)}$$

Combining Eq. (iii) and Eq. (iv), we get

$$\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \text{ or } t = \sqrt{t_1t_2}$$

32. (a) Suppose h be the height of each storey. Then

$$25h = 0 + \frac{1}{2} \times 10 \times t^2 = \frac{1}{2} \times 10 \times 5^2 \text{ or } h = 5 \text{ m}$$

In first second, let the stone passes through n storey. So

$$n \times 5 = \frac{1}{2} \times 10 \times (1)^2 \quad \text{or} \quad n = 1$$

33. (b) Suppose v be the velocity attained by the body after time t_1. Then $v = u - gt_1$ (i)

Let the body reach the same point at time t_2. Now velocity will be downwards with same magnitude v. Then

$-v = u - gt_2$ (ii)

(i) – (ii) $\Rightarrow 2v = g(t_2 - t_1)$

or $\quad t_2 - t_1 = \frac{2v}{g} = \frac{2}{g}(u - gt_1) = 2\left(\frac{u}{g} - t_1\right)$

34. (a) The velocity v acquired by the parachutist after 10 s:

$v = u + gt = 0 + 10 \times 10 = 100\ \text{m s}^{-1}$

Then, $s_1 = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2} \times 10 \times 10^2 = 500\,\text{m}$

The distance travelled by the parachutist under retardation is

$s_2 = 2495 - 500 = 1995$ m

Let v_g be the velocity on reaching the ground. Then

$v_g^2 - v^2 = 2as_2$

or $\quad v_g^2 - (100)^2 = 2 \times (-2.5) \times 1995 \quad \text{or} \quad v_g = 5\ \text{m s}^{-1}$

35. (d) By the time fifth water drop starts falling, the first water drop reaches the ground.

$u = 0,\ h = \frac{1}{2}gt^2 \Rightarrow 5 = \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 1\,\text{s}$

Hence, the interval of falling of each water drop is

$\frac{1\,\text{s}}{4} = 0.25\,\text{s}$

When the fifth drop starts its journey towards ground, the third drop travels in air for

$0.25 + 0.25 = 0.5$

Therefore, height (distance) covered by third drop in air is

$h_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.5)^2 = 5 \times 0.25 = 1.25$ m

The third water drop will be at a height of

$5 - 1.25 = 3.75$ m

Relative Velocity in One Dimension

36. (d) Relative velocity of policeman w.r.t. the thief is $10 - 9 = 1\ \text{m s}^{-1}$. Since the relative separation between them is 100 m, the time taken will be = relative separation/relative velocity =100/1 = 100 s.

37. (a) Relative velocity of overtaking = 40 – 30 = 10 m s^{-1}. Total relative distance covered with this relative velocity during overtaking = 100 + 200 = 300 m

So time taken = 300/10 = 30 s

38. (d) Here relative velocity of the train w.r.t. other train is $V - v$. Hence, $0 - (V - v)^2 = 2ax$

or $\quad a = -\frac{(V-v)^2}{2x}$ Minimum retardation $= \frac{(V-v)^2}{2x}$

39. (d) $\vec{v}_A = 60\hat{i}, \vec{v}_B = 40\hat{i}$

$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = 40\hat{i} - 60\hat{i} = -20\hat{i}$

So direction is opposite to that of trains.

40. (b) Using $v_r^2 = u_r^2 + 2a_r s, u_r = 0$

$a_r = g - (-a) = (g + a)$

$v_r^2 = 0 + 2(g + a) \cdot h$

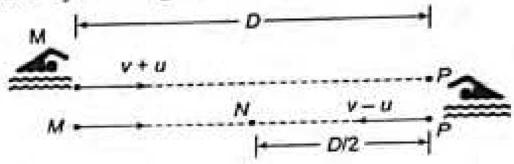
$v = \sqrt{2(g+a)h} = \sqrt{2 \times 12 \times 1.5} = 6$ m/s

41. (c) Time to meet the cars: $t = \frac{d}{v_1 + v_2}$

Distance travelled by bird in this time

$s = v_3 t = \frac{v_3 d}{v_1 + v_2} = \frac{10 \times 2000}{(20 + 30)} = 400$ m

42. (b) Time taken by man to go from M to P and then P to N = time taken by float to go from M to N.



$\frac{D}{v+u} + \frac{D}{2(v-u)} = \frac{D}{2u}$

Simplify to get $\frac{v}{u} = 3.$

43. (b) Let speed of elevator is v_e

$t_1 = \frac{L}{v_e} \Rightarrow 1\,\text{min} = \frac{L}{v_e}$

Let speed of person relative to elevator is v_p, then

$t_2 = \frac{L}{v_p} \Rightarrow 3\,\text{min} = \frac{L}{v_p}$

when the escalator is moving:

$t_3 = \frac{L}{v_e + v_p} = \frac{L}{\frac{L}{1\text{min}} + \frac{L}{3\text{min}}} = \frac{3}{4}\text{min} = 45\,\text{s}$

44. (d) Let the velocity of the scooter be v ms^{-1}. Then

$(v - 10)100 = 1000$ or $v = 20$ ms^{-1}

Understanding Motion Through Graphs

45. (a) Area from 0 to 10 s = $\frac{1}{2}[10 + 4]5 = 35$ m

Area from 10 to 12 s = $\frac{1}{2} \times 2 \times (-2.5) = -2.5$ m

Distance travelled = 35 + 2.5 = 37.5 m

46. (c) Maximum height will be attained at 110 s. Because after 110 s, velocity becomes negative and rocket will start coming down. Area from 0 to 110 s is

$\frac{1}{2} \times 110 \times 1000 = 55{,}000$ m = 55 km

47. (d) Displacement in 12 s = Area under v–t graph

$= \frac{1}{2} \times (12 + 5)4 = 34$ m

$V_{av} = \frac{\text{Displacement}}{\text{Time}} = \frac{34}{12} = \frac{17}{6}\ \text{m s}^{-1}$

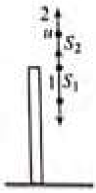
Hence, (a) is incorrect; (b) is incorrect because during first 3 s, velocity increases from 0 to 4 m s^{-1} option; (c) is incorrect, because in part AB velocity is constant.

48. (a) Maximum acceleration will be from 30 to 40 s, because slope in this interval is maximum.

$$a=\frac{v_2-v_1}{t_2-t_1}=\frac{60-20}{40-30}=4\text{ m s}^{-2}$$

49. (d) At $t=0$, velocity is positive and maximum. As the particle goes up, velocity decreases and becomes zero at the highest point. When the particle starts coming down, velocity increases in the negative direction.

50. (a) At time t, let the displacement of first stone be



$S_1=\frac{1}{2}gt^2$ and that of the second stone be

$S_2=ut-\frac{1}{2}gt^2$

Distance between two stones at time t:

$S=S_1+S_2=u \Rightarrow S=ut$

So the graph should be a straight line passing through origin as shown in option (a).

51. (b) Let the particle be thrown up with initial velocity u.

Displacement (s) at any time t is $S=ut-\frac{1}{2}gt^2$.

The graph should be parabolic downwards as shown in option (b).

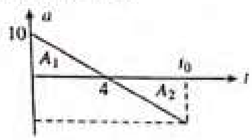
52. (a) From 0 to t_1, acceleration is increasing linearly with time; hence, v–t graph should be parabolic upwards.

From t_1 to t_2, acceleration is dereasing linearly with time; hence, the $v-t$ graph should be parabolic downwards.

53. (a) At $t=0$, slope of the x–t graph is zero; hence, velocity is zero at $t=0$. As time increases, slope increases in negative direction; hence, velocity increases in negative direction. At point (1), slope changes suddenly from negative to positive value; hence, velocity changes suddenly from negative to positive and then velocity starts decreasing and becomes zero at (2). Option (a) represents all these clearly.

54. (d) Before the second ball is dropped, the first ball would have travelled some distance say $S_0=\frac{1}{2}gt_0^2$. After dropping the second ball, the relative acceleration of both balls becomes zero. So distances between them increase linearly. After some time, the first ball will collide with the ground and the distance between them will start decreasing and the magnitude of relative velocity will be increasing for this time. Option (d) represents all these clearly.

55. (c) Particle will acquire the initial velocity when areas A_1 and A_2 are equal. For this, $t_0=8$ s.



56. (a) For 0 to 5 s, acceleration is positive, for 5 to 15 s acceleration is negative, for 15 to 20 s acceleration is positive.

Problems Based on Mixed Concepts

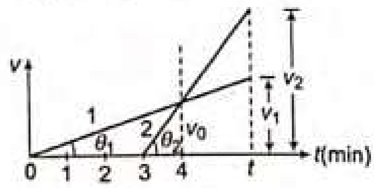
57. (b) First find relation between v and x

$$\frac{1}{v}=\frac{3x}{16}-\frac{1}{4}\Rightarrow\frac{dt}{dx}=\frac{3x}{16}-\frac{1}{4}\Rightarrow\int_{t_1}^{t_2}dt=\int_{4}^{12}\left(\frac{3x}{16}-\frac{1}{4}\right)dx$$

Solve to get $t_2-t_1=10$ s

58. (b) $\tan\theta_1=\frac{v_0}{4}$, $\tan\theta_2=\frac{v_0}{1}$

$\Rightarrow\ 4\tan\theta_1=\tan\theta_2=v_0$



Let they meet at time t, then their displacement should be same. It means area under v–t graph should be same.

$S_1=S_2$

$\Rightarrow\ \frac{1}{2}v_1t=\frac{1}{2}v_2(t-3)$

$\Rightarrow\ \frac{1}{2}[(\tan\theta_1)t]t=\frac{1}{2}[(\tan\theta_2)(t-3)](t-3)$

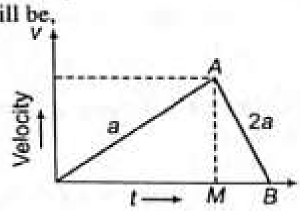
$\Rightarrow\ (\tan\theta_1)t^2=4(\tan\theta_1)(t-3)^2$

$\Rightarrow\ t=2(t-3)\Rightarrow t=6\text{ min}$

59. (d) Acceleration is positive in all three graphs, so velocity increases in all three.

60. (b)

Let OAB be the velocity–time graph of the lift. The ordinate at A (i.e., AM) represents maximum velocity. Total distance travelled will be,



Area of the $\Delta OAB=\frac{1}{2}\times OB\times AM$

$AM=v$, $OM=t_1$, $t_1+t_2=OB=t$, $MB=t_2$

$\Delta OAB=\frac{1}{2}\times tv=h$

or $\quad vt=2h$ ...(i)

Now $\frac{v}{t_1}=a$ or $t_1=\frac{v}{a}$ ...(ii)

and $\frac{v}{t_2}=2a$ or $t_2=\frac{v}{2a}$ ...(iii)

Adding (ii) and (iii)

$$t=t_1+t_2=\frac{v}{a}+\frac{v}{2a}=\frac{3v}{2a}=\frac{3}{2a}\times\frac{2h}{t}$$

or $\quad at^2=3h\Rightarrow h=\frac{at^2}{3}$

61. (b) Case-I: Actual time spent in decelerating

$=\frac{v-u}{a}=\frac{0-20}{-1.0}=20$ s

Distance travelled in decelerating

$=20\times20-\frac{1}{2}\times1\times20^2=400-200=200$ m

Time that train will have taken if it had travelled uniformly with 20 m/s for 200 m $= \frac{200}{20} = 10$ s

Extra time spent in decelerating = 20 – 10 = 10 s

Case-II: Actual time spent in accelerating

$$= \frac{v-u}{a} = \frac{20-0}{0.5} = 40 \text{ s}$$

Distance travelled in these 40 s

$$= 0 \times 40 + \frac{1}{2} \times 0.5 \times 40^2 = 400 \text{m}$$

Time that the train will have taken if travelled uniformly with 20 m/s

$$= \frac{400}{20} = 20 \text{ s}$$

Extra time lost due to acceleration = 40 – 20 = 20 s

Total extra time = 10 s + 2 min + 30 s = 2 min 30 s

62. (b) With respect to lift initial speed $= v_0$

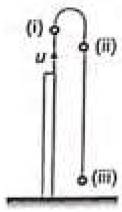
Acceleration $= -2g$

Displacement = 0 $\therefore \quad S = ut + \frac{1}{2}at^2$

$$0 = v_0 T' + \frac{1}{2} \times 2g \times T'^2$$

$$\therefore \quad T' = \frac{v_0}{g} = \frac{1}{2} \times \frac{2v_0}{g} = \frac{1}{2}T$$

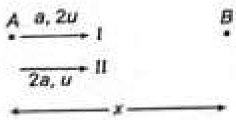
63. (c)



64. (d) Let both will meet at point B

$$x = 2ut + \frac{1}{2}at^2 \Rightarrow x = ut + \frac{1}{2}(2a)t^2$$

So $\quad 2ut + \frac{1}{2}at^2 = ut + at^2$



$$ut = \frac{1}{2}at^2 \Rightarrow t = \frac{2u}{a}$$

So $\quad x = 2u\left(\frac{2u}{a}\right) + \frac{1}{2}a\left(\frac{2u}{a}\right)^2 = \frac{6u^2}{a}$

65. (a) At $t = 0$

$\frac{dx}{dt} = 0$ for particles 1, 2 and 3 and $\left|\frac{d^2x}{dt^2}\right| > 0$ for > 0

and $\frac{dx}{dt} = -3.4$ m/s for particle 4 and $\frac{d^2x}{dt^2}$ is negative for $t > 0$

Therefore for $t > 0$; $\left|\frac{dx}{dt}\right|$ is increasing in all.

66. (b) Let velocity of bodies be v_1 and v_2.

in first case

$$u_1 = v_1 = v_2 \quad \text{.... (i)}$$

in second case

$$u_2 = v_1 - v_2 \quad \text{... (ii)}$$

$$\therefore \quad v_1 = \frac{u_1 + u_2}{2} \quad \text{and} \quad v_2 = \frac{u_1 - u_2}{2}$$

Here $u_1 = \frac{16}{10}$ m/s and $u_2 = \frac{3}{5}$

After solving we have

$v_1 = 1.1$ m/s and $v_2 = 0.5$ m/s.

67. (c) For P, in t sec.

$$x_1 = \frac{1}{2}X t^2 = \frac{Xt^2}{2} \Rightarrow v_1 = Xt$$

$$x_2 = (Xt)t + \frac{1}{2}2Xt^2 \Rightarrow x_2 = 2Xt^2$$

$$x_p = x_1 + x_2 = \frac{5}{2}X + 2$$

For Q,

$$y_1 = \frac{1}{2}(2X)t^2 = Xt^2 \Rightarrow v_2 = 2Xt$$

$$y_2 = (2Xt)t + \frac{1}{2}Xt^2 = \frac{5}{2}Xt^2$$

$$y_Q = y_1 + y_2 = \frac{7}{2}Xt^2 \Rightarrow y_Q > x_P$$

68. (b) Let a be the retardation produced by resistive force, ta and td be the time ascent and descent respectively.

If the particle rises up to a height h

Then $h = \frac{1}{2}(g+a)t_a^2$ and $h = \frac{1}{2}(g-a)t_d^2$

$$\therefore \quad \frac{t_a}{t_d} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}}$$

69. (d) Distance travelled from time $t - 1$ sec to t sec is

$$S = u + \frac{a}{2}(2t-1) \quad \text{...(i)}$$

from given condition $S = t$ ...(ii)

(i) and (ii) $\Rightarrow t = u + \frac{a}{2}(2t-1)$

$$\Rightarrow \quad u = \frac{a}{2} + t(1-a)$$

Since u and a are arbitrary constants, and they must be constant for every time.

$\Rightarrow$ Coefficient of t must be equal to zero.

$\Rightarrow \quad 1 - a = 0 \Rightarrow a = 1$ for $a = 1$, $u = \frac{1}{2}$ unit

Initial speed is $\frac{1}{2}$ unit.

70. (a) $u = +8$ m/s, $a = -4$ m/s^2

$v = 0$

$\Rightarrow \quad 0 = 8 - 4t \quad$ or $t = 2$ sec

displacement in first 2 sec.

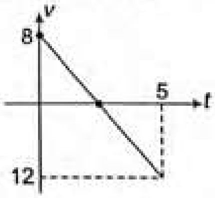
$$S_1 = 8\times 2+\frac{1}{2}\cdot(-4).\ 2^2 = 8 \text{ m}$$

displacement in next 3 sec.

$$S_2 = 0\times 3+\frac{1}{2}(-4)3^2 = -18 \text{ m}$$

distance travelled $= |S_1|+|S_2| = 26$ m

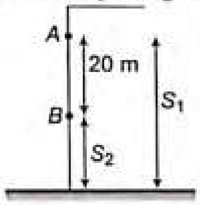
ALITER: We can draw velocity–time graph of the situation.



Total distance is equal to magnitude of area under velocity–time graph.

Hence, $d = \frac{1}{2}\times 2\times 8+\frac{1}{2}\times 3\ \times 12 = 8+18 = 26$ m.

71. (a) Velocity of 1[st] stone when passing at A



$V^2 = 0+2.10.5 \Rightarrow V = 10$ m/s

and $S_1 - S_2 = 20$ m

$$\Rightarrow \left(10\cdot t+\frac{1}{2}10\cdot t^2\right)-\left(\frac{1}{2}\cdot 10\cdot t^2\right) = 20$$

At $t = 2\ s$, $S_2 = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times 2^2 = 20$ m

Hence, height of the tower,

$$H = S_1+S_2 = 25+20 = 45 \text{ m}$$

72. (c) $v = 0+na \Rightarrow a = \frac{v}{n}$

displacement in last two seconds

$s(n)-s(n-2)$

$$= \left(0+\frac{1}{2}an^2\right)-\left(0\ +\ \frac{1}{2}a\ (n-2)^2\right)$$

$$= 2a(n-1) = 2v\frac{(n-1)}{n}$$

73. (d) A will be ahead of B when $x_A \geq x_B$

$$40(t-10) \geq (0)t+\frac{1}{2}(2)\ t^2$$

as A is 10 sec late than B.

$\Rightarrow\ t^2 - 40t+400 \leq 0$

$\Rightarrow\ (t-20)^2 \leq 0$

which is not possible, so A will never be ahead at B.

ARCHIVES

1. (a) For first part of penetration, by equation of motion,

$$\left(\frac{u}{2}\right)^2 = u^2 - 2a(3)$$

or $3u^2 = 24a \Rightarrow u^2 = 8a$ ...(i)

For latter part of penetration, $0 = \left(\frac{u}{2}\right)^2 - 2ax$

or $u^2 = 8ax$ ...(ii)

From (i) and (ii), $8ax = 8a \Rightarrow x = 1$ cm

2. (c) Let both the balls are thrown from height 'h', both will move under same acceleration i.e., acceleration due to the gravity.

Let the speed of ball 'A' when it reaches the ground be V_A.

For ball 'A', using $v^2 = u^2+2gh$;

$$(V_A)^2 = V^2 - 2gh \Rightarrow V_A = \sqrt{V^2-2gh}$$

Let the speed of ball 'B' when it reaches the ground be V_B.

For ball 'B', again using $v^2 = u^2+2gh$

$$(V_B)^2 = (-V)^2 - 2gh \Rightarrow V_B = \sqrt{V^2-2gh}$$

So for both the cases, velocity will be equal.

3. (d) Assuming constant retardation:

using $v^2 = u^2+2as \Rightarrow\ 0 = u^2 - 2as$

or $s = \frac{u^2}{2a}$

or $s \propto u^2 \Rightarrow \frac{s_1}{s_2} = \frac{u_1^2}{u_2^2}$

$$\Rightarrow\ \frac{s_1}{s_2} = \left(\frac{u}{4u}\right)^2 = \frac{1}{16}$$

4. (c) In first case:

Initial velocity: $u_1 = 50$ km/h $= 50\times\frac{5}{18} = \frac{125}{9}$ m/sec

using $v^2 = u^2+2as \Rightarrow\ 0 = u_1^2 - 2as_1$

$\therefore$ Acceleration, $a = -\frac{u_1^2}{2s_1} = -\frac{(125/9)^2}{2\times 6}$

$= -16$ m/sec^2

Now for second case: Initial velocity, $u_2 = 100$ km/h $= 100\times\frac{5}{18}$

$= \frac{250}{9}$ m/sec

Again using $v^2 = u^2+2as \Rightarrow\ 0 = u_2^2 - 2as_2$

Stopping distance, $s_2 = -\frac{u_2^2}{2a} = -\frac{(250/9)^2}{2\times 16} = 24$ m

5. (a) From polygon law of vector addition we know if the vectors are arranged along the sides of a polygon the resultant of vector becomes zero. Here in this case the forces are represented by the sides of a triangle taken in same order, so their resultant is zero. It means the acceleration of the particle should be zero, hence velocity remains constant.

6. (a) $h = ut+\frac{1}{2}gt^2\ \Rightarrow h = \frac{1}{2}gT^2$

After $\frac{T}{3}$ seconds, the position of ball from top,

$$h' = 0+\frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{1}{2}\times\frac{g}{9}\times T^2 = \frac{h}{9}$$

$\therefore$ Position of ball from ground $= h-\frac{h}{9} = \frac{8h}{9}$ m.

7. (d) Assuming constant retardation:

using $v^2 = u^2 + 2as \Rightarrow 0 = u^2 - 2as$ or $s = \dfrac{u^2}{2a}$ ...(i)

or $\dfrac{s_1}{s_2} = \dfrac{u_1^2}{u_2^2}$

$$\frac{20}{s_2} = \left(\frac{60}{120}\right)^2 \Rightarrow s_2 = 80 \text{ m}$$

8. (No option) The car starts from point A from rest and moves up to point B with constant acceleration f.

Using $v^2 = u^2 + 2as$

velocity of car at point B, $v = \sqrt{2fS}$

Car moves distance BC with this constant velocity in time t.

using $x = vt$

$$x = \sqrt{2fS}.t \quad ...(i)$$

Now the car moves from position C with initial velocity v and moves up to point D with constant retardation $f/2$.

Again using $v^2 = u^2 + 2as$

So the velocity of car at point C also will be $\sqrt{2fS}$ and finally car stops after covering distance y.

$$0^2 = (\sqrt{2fS})^2 - 2\frac{f}{2}y$$

Distance CD $\Rightarrow y = \dfrac{(\sqrt{2fS})^2}{2(f/2)} = \dfrac{2fS}{f} = 2S$ ...(ii)

So, the total distance $AD = AB + BC + CD = 15S$ (given)

$\Rightarrow \quad S + x + 2S = 15S \Rightarrow x = 12S$

Substituting the value of x in equation (i) we get

$$x = \sqrt{2fS}.t \Rightarrow 12S = \sqrt{2fS}.t \Rightarrow 144S^2 = 2fS.t^2$$

$\Rightarrow \quad S = \dfrac{1}{72}ft^2.$

9. (b) We are given the relation between time and distance as $t = \alpha x^2 + \beta x$ ...(i)

Differentiating equation (i) w.r.t 'x', we get

$\dfrac{dt}{dx} = 2\alpha x + \beta$ but $\dfrac{dt}{dx} = v \Rightarrow v = \dfrac{1}{2\alpha x + \beta}$ ...(ii)

Differentiating equation (ii) w.r.t 't', we get acceleration i.e.,

$$a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt}$$

$$\therefore \quad a = v\frac{dv}{dx} = \frac{-v.2\alpha}{(2\alpha x + \beta)^2} = -2\alpha.v.v^2 = -2\alpha v^3$$

The acceleration is negative here. It means that it is the case of retardation.

10. (b) After bailing out from position A, parachutist falls freely under gravity a distance 50 m. Let the velocity acquired by him at position B be 'v'.

$$v^2 = u^2 + 2as = 0 + 2 \times 9.8 \times 50 = 980$$

[As $u = 0$, $a = 9.8$ m/s^2, $s = 50$ m]

At position B, parachute opens and it moves with retardation of 2 m/s^2 and reach at ground (position C) with velocity of 3 m/s

For the journey of the part 'BC' again applying the equation $v^2 = u^2 + 2as$, where

$v = 3$ m/s, $u = \sqrt{980}$ m/s, $a = -2$ m/s^2, $s = h$

$\Rightarrow \quad (3)^2 = (\sqrt{980})^2 + 2 \times (-2) \times h \Rightarrow 9 = 980 - 4h$

$\Rightarrow \quad h = \dfrac{980 - 9}{4} = \dfrac{971}{4} = 242.7 \cong 243$ m.

So, the total height by which parachutist bailed out $= 50 + 243 = 293$ m.

11. (a) Let initial velocity of the bullet $= u$

After penetrating 3 cm its velocity becomes $\dfrac{u}{2}$.

From $v^2 = u^2 - 2as$, we get

$$\left(\frac{u}{2}\right)^2 = u^2 - 2a(3)$$

$\Rightarrow \quad 6a = \dfrac{3u^2}{4} \Rightarrow a = \dfrac{u^2}{8}$

Let further it will penetrate through distance x and stops at point C.

For distance BC, $v = 0, u = u/2, s = x, a = u^2/8$

From $v^2 = u^2 - 2as$,

$$0 = \left(\frac{u}{2}\right)^2 - 2\left(\frac{u^2}{8}\right).x$$

$\Rightarrow \quad x = 1$ cm.

12. (c) We are given $v = \alpha\sqrt{x}$

$$v = \frac{dx}{dt} = \alpha\sqrt{x} \text{ or } \frac{dx}{\sqrt{x}} = \alpha dt$$

Integrating both sides, we get

$$\int_0^x \frac{dx}{\sqrt{x}} = \alpha\int_0^t dt = \alpha \quad \text{or} \quad 2x^{1/2} = \alpha t \quad \text{or} \quad x = \left(\frac{\alpha}{2}\right)^2 t^2$$

or displacement is proportional to t^2.

13. (a) We are given velocity $v = v_0 + gt + ft^2$

$$\therefore \quad v = \frac{dx}{dt} \quad \text{or} \quad \int_0^x dx = \int_0^t v dt$$

$$x = \int_0^t (v_0 + gt + ft^2)dt \quad \text{or} \quad x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$$

At $t = 1$ sec,

$$x = v_0 + \frac{g}{2} + \frac{f}{3}$$

14. (b) For 1[st] particle:

It starts moving ($u_1 = 0$) with constant acceleration.

$$\therefore \quad x_1 = x_1(t) = u_1 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \quad ...(i)$$

For 2[nd] particle:

It is moving with constant velocity (v).

$$\therefore \quad x_2 = x_2(t) = vt \quad ...(ii)$$

Relative position of particle 1 w.r.t. 2,

$$x_1 - x_2 = x_{1,2} = \frac{1}{2}at^2 - vt \quad ...(iii)$$

Hence, graph should be parabola.

Differentiating equation (iii) w.r.t. time, we get the relative velocity of particle 1 w.r.t. 2

$$|v_{1,2}| = \frac{dx_{1,2}}{dt} = at - v$$

As particle 1 starts moving from rest, hence at $t = 0$, $v_{1,2}$ should be negative.

It means the slope of the parabola at $t = 0$ should be negative. The parabola should open upwards. Hence graph 'b' fulfils the requirements.

15. (b) $v = u + at$

$$\vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$$

$$\Rightarrow \quad \vec{v} = (3+4)\hat{i} + (4+3)\hat{j}$$

$$\Rightarrow \quad |\vec{v}| = \sqrt{49+49} = \sqrt{98} = 7\sqrt{2} \text{ unit}$$

(This value is about 9.9 units close to 10 units. If (a) is given that is also not wrong).

16. (a) Here, $\vec{v} = K(y\hat{i} + x\hat{j})$

$$\vec{v} = Ky\hat{i} + Kx\hat{j} \qquad \text{...(i)}$$

Also, $\vec{v} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j}$ ...(ii)

Equating equations (i) and (ii), we get

$$\frac{dx}{dt} = Ky; \frac{dy}{dt} = Kx$$

Now, $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} \quad \therefore \quad \dfrac{dy}{dx} = \dfrac{Kx}{Ky} = \dfrac{x}{y}$

Integrating both sides of the above equation, we get

$$\int y\,dy = \int x\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + \text{contsant.}$$

or $\quad y^2 = x^2 + \text{constant}$

17. (b) We are given $\dfrac{dv}{dt} = -0.25\sqrt{v}$ or $\dfrac{1}{\sqrt{v}}dv = 2.5\ dt$

On integrating, within limit (at $t = 0$, $v_1 = 6.25$ m s^{-1} and at any time, $v_2 = 0$), we get

$$\int_{6.25 \text{ms}^{-1}}^{0} v^{-1/2} dv = -2.5\int_0^t dt$$

$$2 \times [v^{1/2}]_{6.25}^{0} = -(2.5)t \Rightarrow t = \frac{-2 \times (6.25)^{1/2}}{-2.5} = 2\text{s}$$

18. (d) Let u be the velocity of projection of the stone.

The maximum height a boy can throw a stone,

$$H_{\max} = \frac{u^2}{2g} = 10 \text{ m} \qquad \text{...(i)}$$

The maximum horizontal distance the boy can throw the same stone is equal to maximum range

$$R_{\max} = \frac{u^2 \sin(2 \times 45^\circ)}{g} = \frac{u^2 \sin 90^\circ}{g} = \frac{u^2}{g}$$

$$H_{\max} = \frac{u^2}{g} = 20 \text{ m} \qquad \text{(Using (i))}$$

19. (c) For 1st stone: $y_1 = u_1 t - \dfrac{1}{2}gt^2$

For 2nd stone: $y_2 = u_2 t - \dfrac{1}{2}at^2$

Relative position of '2nd' w.r.t '1st'

$$\Delta y = y_2 - y_1 = (u_2 - u_1)t = (40 - 10)t = 30t \qquad \text{...(i)}$$

or $\quad \Delta y \propto t$

It means up to the time both the stones are in the air (From $t = 0$ to $t = 8$ sec) the graph should be a straight line.

When second stone hits the ground, first stone is still moving under gravitational acceleration, hence the graph should be parabolic. The speed of first stone will keep on increasing. Hence parabola should open downwards.

Hence, option (c) is correct.

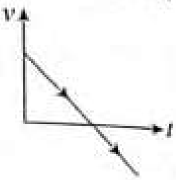
20. (a) Velocity at any time t is given by, $v = u + at$

If we take upward direction as positive and downward direction as negative, we can write

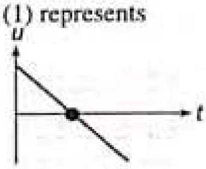
$$v = v_0 + (-g)t$$

$$v = v_0 - gt$$

$\Rightarrow$ Straight line with negative slope



21. (a) Graph in option (1) represents



- Negative acceleration
- Velocity is decreasing
- The situation is same as the object thrown upward and moving under gravity.

First moving up than moving down.

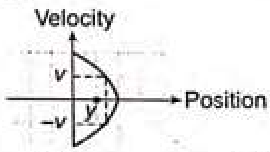
Mathematically the object is moving according to equation

$$v = u - at$$

For velocity position relation it should follow the equation

$$v = \sqrt{u^2 - 2ay} \qquad \text{(i)}$$

The graph in option (2)



The particle can have same magnitude of velocity at same position. Positive when moving away from initial position and negative when it is returning to initial position.

Hence this graph (2) represent same motion as (1)

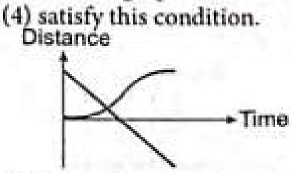
Now come to analyse graph of option (3).

We can represent the motion of the particle given in option (1) and (2) with the equation

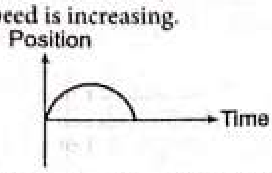
$$y = ut - \frac{1}{2}at^2$$

It means position time graph should be parabola open down.

The option (4) satisfy this condition.



But option (3) does not represent same motion as at $t = 0$, it slope is zero and slope is increasing with time, which represents speed is increasing.



While in first part of motion speed should decrease.

CHAPTER 4: MOTION IN TWO DIMENSIONS

Concept Application Exercise 4.1

1. For first particles angle of projection from the horizontal is α.

So $\quad T_1 = \dfrac{2u\sin\alpha}{g}$

For second particle, angle of projection from the vertical is α, it mean from the horizontal is $(\theta - \alpha)$

$\therefore \quad T_2 = \dfrac{2u\sin(90-\alpha)}{g} = \dfrac{2u\cos\alpha}{g}$

So, ratio of time of flight $\dfrac{T_1}{T_2} = \tan\alpha$.

2. $T = \dfrac{2u\sin\theta}{g} \quad \therefore \dfrac{T_1}{T_2} = \dfrac{g_2}{g_1} = \dfrac{g + \dfrac{g}{10}}{g} = \dfrac{11}{10}$

Fractional decrease in time of flight $= \dfrac{T_1 - T_2}{T_1} = \dfrac{1}{11}$

Percentage decrease = 9%

3. We have to simply calculate the range of projectile.

$R = \dfrac{u^2\sin 2\theta}{g}$

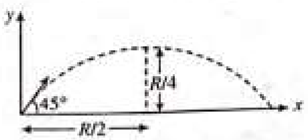
$= \dfrac{(10)^2\sin(2\times 30°)}{10}$

$= 5\sqrt{3} = 8.66\text{m}$

4. Range ∝ horizontal component of velocity. Graph 4 shows maximum range, so football possesses maximum horizontal velocity in this case. Hence ranking is 4, 3, 2, 1

5. For maximum horizontal range, $\theta = 45°$.

From $R = 4H\cot\theta = 4H \quad$ [As $\theta = 45°$, for maximum range.]



Speed of the particle will be minimum at the highest point of parabola.

So the co-ordinate of the highest point will be $(R/2, R/4)$.

6. The maximum area will be equal to the area of the circle with radius equal to the maximum range of projectile.

Maximum area, $\pi r^2 = \pi(R_{\max})^2 = \pi\left(\dfrac{u^2}{g}\right)^2 = \pi\dfrac{u^4}{g^2}$

[As $r = R_{\max} = u^2/g$ for $\theta = 45°$]

7. $R = \dfrac{u^2\sin 2\theta}{g} \therefore R \propto u^2\sin 2\theta$

$$\frac{R_2}{R_1} = \left(\frac{u_2}{u_1}\right)^2\left(\frac{\sin 2\theta_2}{\sin 2\theta_1}\right)$$

$$\Rightarrow \quad R_2 = R_1\left(\frac{2u}{u}\right)^2\left(\frac{\sin 90°}{\sin 30°}\right) = 8R_1$$

8. For equal ranges, body should be projected with angle θ or $(90° - \theta)$ from the horizontal.

And for these angles:

$$h_1 = \frac{u^2\sin^2\theta}{2g} \quad \text{and} \quad h_2 = \frac{u^2\cos^2\theta}{2g}$$

By multiplication of both height,

$$h_1h_2 = \frac{u^2\sin^2\theta\cos^2\theta}{4g^2} = \frac{1}{16}\left(\frac{u^2\sin 2\theta}{g}\right)^2$$

$$\Rightarrow \quad 16h_1h_2 = R^2 \quad \Rightarrow \quad R = 4\sqrt{h_1h_2}$$

Concept Application Exercise 4.2

1. For hitting the lowest plane, the ball will cover horizontal displacement equal to 20 cm and vertical displacement, 40 cm.

In vertical direction, $\dfrac{20}{100} = \dfrac{1}{2}\times 10\times t^2$ which gives, $t = 0.2$ sec

In horizontal direction, $\dfrac{40}{100} = u_x.t \Rightarrow u_x = 2$ m/s

2. Maximum range up the inclined plane,

$$(R_{\max})_{\text{up}} = \frac{u^2}{g(1+\sin\alpha)}$$

Maximum range down the inclined plane

$$(R_{\max})_{\text{down}} = \frac{u^2}{g(1-\sin\alpha)}$$

and according to: $\dfrac{u^2}{g(1-\sin\alpha)} = 3\times\dfrac{u^2}{g(1+\sin\alpha)}$

By solving $\alpha = 30°$

3. Here $u = 21\text{ ms}^{-1}$, $\alpha = 30°$, $\theta = \beta - \alpha = 60° - 30° = 30°$

Maximum range, $R = \dfrac{2u^2}{g}\dfrac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$

$$= \frac{2\times(21)^2\times\sin 30°\cos 60°}{9.8\times\cos^2 30°} = 30\text{ m}$$

4. Maximum range on horizontal plane, $R = \dfrac{u^2}{g} = 6$ km (given)

Maximum range on a inclined plane, $R_{\max} = \dfrac{u^2}{g(1+\sin\alpha)}$

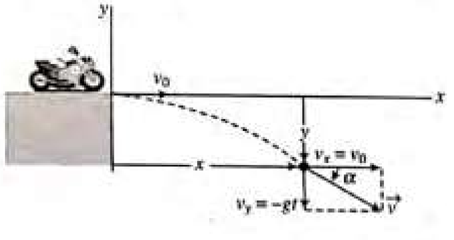
Putting $\alpha = 30°$

$$R_{\max} = \frac{u^2}{g(1+\sin 30°)} = \frac{2}{3}\left(\frac{u^2}{g}\right) = \frac{2}{3}\times 6 = 4 \text{ km}$$

5. At $t = 0.50$ s, the x- and y-coordinates are

$x = v_{0x}t = (9.0 \text{ m s}^{-1})(0.50 \text{ s}) = 4.5$ m

$$y = -\frac{1}{2}gt^2 = -\frac{1}{2}(10 \text{ m s}^{-2})(0.50 \text{ s})^2 = -\frac{5}{4} \text{ m}$$



The negative value of y shows that this time the motorcycle is below its starting point.

The motorcycle's distance from the origin at this time,

$$r = \sqrt{x^2+y^2} = \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{5}{4}\right)^2} = \frac{\sqrt{349}}{4} \text{ m}$$

The components of velocity at this time are

$v_x = v_{0x} = 9.0 \text{ m s}^{-1}$

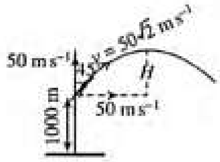
$v_y = -gt = (-10 \text{ m s}^{-2})(0.50 \text{ s}) = -5 \text{ m s}^{-1}$.

The speed (magnitude of the velocity) at this time is

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(9.0 \text{ m s}^{-1})^2 + (-5 \text{ m s}^{-1})^2} = \sqrt{106} \text{ m s}^{-1}$$

6. (a) $y = u_y t + \dfrac{1}{2}a_y t^2$



$$-1000 = 50t - \frac{1}{2}\times 10 \times t^2$$

$t^2 - 10t - 200 = 0$

$(t-20)(t+10) = 0$

$t = 20$ s

(b) $$H = \frac{u_y^2}{2g} = \frac{50^2}{2g} = \frac{50\times 50}{20} = 125 \text{ m}$$

Hence, maximum height above ground

$H = 1000 + 125 = 1125$ m

7. (a) Taking axis system as shown in the figure.

At highest point, $v_y = 0$. Therefore,

$v_y^2 = u_y^2 + 2a_y y$

$\Rightarrow \quad 0 = (30)^2 - 2g\cos 30° y$

$y = 30\sqrt{3}$ m (maximum height)

(b) Again for x coordinate,

$v_y = u_y + a_y t$

$0 = 30 - g\cos 30° \times t \quad \Rightarrow \quad t = 2\sqrt{3}$ s

Time of flight $T = 2 \times 2\sqrt{3}$ s

(c) Range, $x = u_x t + \dfrac{1}{2}a_x t^2$

$$= 40 \times 4\sqrt{3} - \frac{1}{2}g\sin 30° \times (4\sqrt{3})^2$$

$= 40(4\sqrt{3} - 3)$ m

(d) $$\theta = \frac{\pi}{4} - \frac{30°}{2} = 45° - 15° = 30°$$

(e) $$\frac{u^2}{g(1+\sin\beta)} = \frac{50\times 50}{10\left(1+\frac{1}{2}\right)} = \frac{2500}{15} = \frac{500}{3} \text{ m}$$

8. Equation of motion in x-direction, $100 = v \times t$

$$\Rightarrow \quad t = \frac{100}{v} \qquad \text{(i)}$$

In y-direction,

$0.1 = 1/2 \times 10 \times t^2$ (ii)

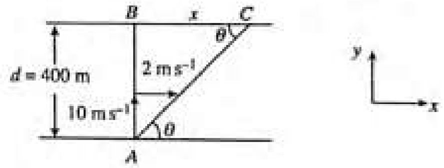
$0.1 = 1/2 \times 10 \times (100/v)^2$

From Eqs (i) and (ii), we get

$u = 700 \text{ m s}^{-1}$

Concept Application Exercise 4.3

1.



(a) Time taken to cross the river

$$t = \frac{d}{v_y} = \frac{400\text{m}}{10\text{ms}^{-1}} = 40 \text{ s}$$

(b) Drift $(x) = (vx)(t) = (2 \text{ m s}^{-1})(40\text{s}) = 80$ m

(c) Actual direction of boat,

$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = \tan^{-1} 5,$$

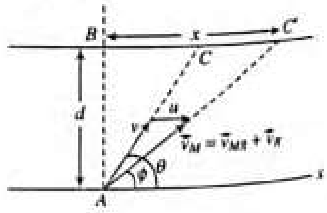
(downstream) with the river flow.

2. (a) $v_{MR} = v$, $v_R = u$

$\vec{v}_M = \vec{v}_{MR} + \vec{v}_R$

Therefore, velocity of man, $v_M = \sqrt{u^2 + v^2 + 2vu\cos\theta}$

(b) $$\tan\phi = \frac{v\sin\theta}{u + v\cos\theta}$$



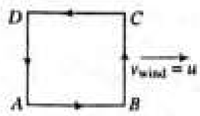
(c) $(v\sin\theta)t = d$

$$\Rightarrow \quad t = \frac{d}{v\sin\theta}$$

$x = (u + v\cos\theta)t$

$$= (u + v\cos\theta)\frac{d}{v\sin\theta}$$

3.



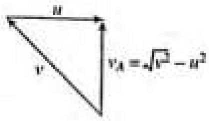
Velocity of aeroplane while flying through AB,

$$v_A = v + u$$

$$t_{AB} = \frac{a}{v+u}$$

Velocity of aeroplane while flying through BC,

$$v_A = \sqrt{v^2 - u^2}$$



$$t_{BC} = \frac{a}{\sqrt{v^2 - u^2}}$$

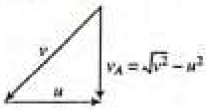
Velocity of aeroplane while flying through CD

$$v_A = v - u$$

$$t_{CD} = \frac{a}{v-u}$$

Velocity of aeroplane while flying through DA,

$$v_A = \sqrt{v^2 - u^2}$$



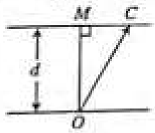
$$t_{DA} = \frac{a}{\sqrt{v^2 - u^2}}$$

Total time $= t_{AB} + t_{BC} + t_{CD} + t_{DA}$

$$= \frac{a}{v+u} + \frac{a}{\sqrt{v^2-u^2}} + \frac{a}{v-u} + \frac{a}{\sqrt{v^2-u^2}}$$

$$= \frac{2a}{v^2-u^2}\left(v + \sqrt{v^2-u^2}\right)$$

4. Let d m be the width of the river. The time to cross by quickest path (say) $t_1 = \frac{d}{v} = \frac{d}{17}$ s and the time to cross the shortest path,



$$(\text{say})\, t_2 = \frac{d}{\sqrt{v^2-u^2}} = \frac{d}{\sqrt{(17)^2 - (8)^2}}$$

From the problem, $t_2 - t_1 = 6$

$$d\left[\frac{1}{\sqrt{225}} - \frac{1}{17}\right] = 6$$

$$\Rightarrow \quad d\left[\frac{1}{15} - \frac{1}{17}\right] = 6 \quad \Rightarrow \quad d = \frac{6 \times 15 \times 17}{2} = 765 \text{ m}$$

5. We know that the time taken for a man to cross the river directly is given by $t_1 = \frac{d}{\sqrt{v^2-u^2}}$, where d is the width of the river.

Now, if the man swims downstream then the resultant velocity is $v + u$. The corresponding time, $t_2 = \frac{d}{u+v}$.

$$\text{Now,} \quad \frac{t_1}{t_2} = \frac{v+u}{\sqrt{v^2-u^2}} = \frac{\sqrt{v+u}\,\sqrt{v+u}}{\sqrt{v+u}\,\sqrt{v-u}}$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{\sqrt{v+u}}{\sqrt{v-u}}$$

Concept Application Exercise 4.4

1. Angular velocity of A with respect to O is

$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega$$

$$\therefore \quad \omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{v}{2r} = \frac{\omega}{2}$$

$$\text{and} \quad \omega_{AC} = \frac{(v_{AC})_\perp}{r_{AC}} = \frac{v}{3r} = \frac{\omega}{3}$$

2. Angular velocity of A with respect to O is

$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v}{r} = \omega$$

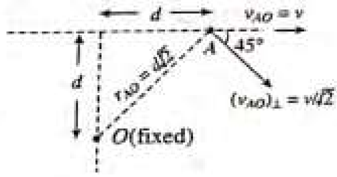
$$\text{Now,} \quad \omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} \quad \Rightarrow \quad v_{AB} = 2v$$

Since v_{AB} is perpendicular to r_{AB}

$$\Rightarrow \quad (v_{AB})_\perp = v_{AB} = 2v;\ r_{AB} = 2r$$

$$\Rightarrow \quad \omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{2v}{2r} = \omega$$

3. Angular velocity of A with respect to O is



$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}}$$

$$v_{AO} = v,\ (v_{AO})_\perp = \frac{v}{\sqrt{2}}$$

$$r_{AO} = d\sqrt{2}$$

$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v/\sqrt{2}}{d\sqrt{2}} = \frac{v}{2d}$$

4. Since speed increases uniformly, average tangential acceleration is equal to instantaneous tangential acceleration.
Therefore, the instantaneous tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\frac{6.0 - 5.0}{2.0}\,\text{m s}^{-2} = 0.5 \text{ m s}^{-2}$$

The angular acceleration is $\alpha = a_t / r$

$$= \frac{0.5 \text{ m s}^{-2}}{20 \text{ cm}} = 2.5 \text{ rad s}^{-2}$$

5. The distance covered in completing the circle is $2r = 2 \times 10$ cm.
The linear speed is

$$v = \frac{2r}{t} = \frac{2\pi \times 10 \text{ cm}}{4s} = 5 \text{ cm s}^{-1}$$

The acceleration is $a = \frac{v^2}{r} = \frac{(5\pi \text{ cm s}^{-1})^2}{10 \text{ cm}} = 2.5 \text{ cm s}^{-2}$

Exercises

Problems Based on Basic Concept of Projectile Motion

1. (a) Since $R = 2H$

or $\dfrac{v^2 \sin 2\theta}{g} = 2 \times \dfrac{v^2 \sin^2\theta}{2g}$

or $2 \sin\theta \cos\theta = \sin^2\theta$ or $\tan\theta = 2$

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$= \frac{v^2 2\sin\theta\cos\theta}{g} = \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$

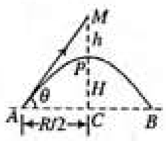
2. (d) We find that $H = R$ or $\dfrac{u^2 \sin^2\theta}{2g} = \dfrac{u^2 2\sin\theta\cos\theta}{g}$

or $\tan\theta = 4$ or $\theta = \tan^{-1}(4)$

3. (a) $AC = R/2$, $PC = H$

We have to find $h = MP$.

From the previous solution, we know that if $H = R$, then $\tan\theta = 4$



Now $\tan\theta = \dfrac{MC}{AC} = \dfrac{MP + PC}{AC}$

$$4 = \frac{h+H}{R/2} \Rightarrow 4 = \frac{(h+H)2}{H}$$

$\Rightarrow h = H$

4. (b) $H = 100$ m, $R = 2 \times 200 = 400$ m

$$\tan\theta = \frac{4H}{R} \Rightarrow \tan\theta = \frac{4 \times 100}{400} = 1$$

$$\Rightarrow \theta = 45^\circ \quad \left[\because \frac{H}{R} = \frac{\tan\theta}{4}\right]$$

5. (d) $\dfrac{R}{T^2} = \dfrac{u^2 \sin 2\theta / g}{4u^2 \sin^2\theta / g^2} = \dfrac{g}{2}\cot\theta$

i.e., $gT^2 = 2R\tan\theta$

If T is doubled, then R becomes 4 times.

6. (a) For the person to be able to catch the ball, the horizontal component of velocity of the ball should be same as the speed of the person, i.e.,

$$v_0 \cos\theta = \frac{v_0}{2} \text{ or } \cos\theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

7. (d) Range $= 150 = ut$ and $h = \dfrac{15}{100} = \dfrac{1}{2} \times gt^2$

or $t^2 = \dfrac{2 \times 15}{100 \times g} = \dfrac{30}{1000}$ or $t = \dfrac{\sqrt{3}}{10}$

$$u = \frac{150}{t} = \frac{150 \times 10}{\sqrt{3}} = 500\sqrt{3} \text{ ms}^{-1}$$

8. (b) The horizontal range is the same for the angles of projection θ and $(90^\circ - \theta)$.

$$t_1 = \frac{2u\sin\theta}{g}, \quad t_2 = \frac{2u\sin(90^\circ - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$t_1 t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g} = \frac{2}{g}\left[\frac{u^2 \sin 2\theta}{g}\right] = \frac{2}{g}R$$

where $R = \dfrac{u^2 \sin 2\theta}{g}$

Hence, $t_1 t_2 \propto R$ (as R is constant)

9. (c) $y_1 = \dfrac{u^2 \sin^2\theta}{2g}$, $y_2 = \dfrac{u^2 \sin^2(90^\circ - \theta)}{2g} = \dfrac{u^2 \cos^2\theta}{2g}$

$$\Rightarrow y_1 + y_2 = \frac{u^2}{2g}$$

10. (b) Given $y = 12x - \dfrac{3}{4}x^2$, $u_x = 3 \text{ m s}^{-1}$

$$v_y = \frac{dy}{dt} = 12\frac{dx}{dt} - \frac{3}{2}x\frac{dx}{dt}$$

At $x = 0$, $v_y = u_y = 12\dfrac{dx}{dt} = 12u_x = 12 \times 3 = 36 \text{ ms}^{-1}$

$$a_y = \frac{d}{dt}\left(\frac{dy}{dt}\right) = 12\frac{d^2x}{dt^2} - \frac{3}{2}\left[\left(\frac{dx}{dt}\right)^2 + x\frac{d^2x}{dt^2}\right]$$

But $\dfrac{d^2x}{dt^2} = a_x = 0$. Hence

$$a_y = -\frac{3}{2}\left(\frac{dx}{dt}\right)^2 = -\frac{3}{2}u_x^2 = -\frac{3}{2} \times (3)^2 = -\frac{27}{2} \text{ ms}^{-2}$$

Range, $R = \dfrac{2u_x u_y}{a_y} = \dfrac{2 \times 3 \times 36}{27/2} = 16$ m

Alternatively: We have $y = 12x - \dfrac{3}{4}x^2$. When projectile again comes to ground, $y = 0$ and $x = R$.

$$0 = 12R - \frac{3}{4}R^2 \Rightarrow R = 16 \text{ m}$$

11. (d) Range is same for angles of projection θ and $90^\circ - \theta$.

$$R = \frac{u^2 \sin^2\theta}{g}, \quad h_1 = \frac{u^2 \sin^2\theta}{2g}$$

and $h_2 = \dfrac{u^2 \sin^2(90 - \theta)}{2g} = \dfrac{u^2 \cos^2\theta}{2g}$.

Hence, $\sqrt{h_1 h_2} = \dfrac{u^2 \sin\theta\cos\theta}{2g} = \dfrac{1}{4}\left[\dfrac{u^2 \sin 2\theta}{g}\right] = \dfrac{R}{4}$

12. (b) $\dfrac{R}{T^2} = g\dfrac{\sin 2\theta}{4\sin^2\theta} = \dfrac{g}{2}\cot\theta = 5\cot\theta$

Given $\dfrac{R}{T^2} = 5$; Hence, $5 = 5\cot\theta$ or $\theta = 45^\circ$

13. (b) $t = \frac{2u\sin\theta}{g}$ or $2 = \frac{2u\sin\theta}{g}$ or $u\sin\theta = g$

$$h_m = \frac{u^2\sin^2\theta}{2g} = \frac{g^2}{2g} = \frac{g}{2} = 5\text{ m}$$

14. (c) $v^2 = u^2 - 2gh$ or $u^2 = v^2 + 2gh$

or $u_x^2 + u_y^2 = v_x^2 + v_y^2 + 2gh$, $u_x = v_x$

So, $u_y^2 = v_y^2 + 2gh$ or $u_y^2 = (2)^2 + 2\times10\times0.4 = 12$

$$u_y = \sqrt{12} = 2\sqrt{3}\text{ ms}^{-1},\ u_x = v_x = 6\text{ ms}^{-1}$$

$$\tan\theta = \frac{u_y}{u_x} = \frac{2\sqrt{3}}{6} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$$

15. (c) At the two points of the trajectory during projectile motion, the horizontal component of the velocity is same. Then

$$150\times\frac{1}{2} = v\times\frac{1}{\sqrt{2}} \text{ or } v = \frac{150}{\sqrt{2}}\text{ ms}^{-1}$$

Initially: $u_y = u\sin60° = \frac{150\sqrt{3}}{2}$ ms^{-1}

Finally: $v_y = v\sin45° = \frac{150}{\sqrt{2}}\times\frac{1}{\sqrt{2}} = \frac{150}{2}$ ms^{-1}

But $v_y = u_y + a_y t$ or $\frac{150}{2} = \frac{150\sqrt{3}}{2} - 10t$

$$10t = \frac{150}{2}(\sqrt{3}-1) \text{ or } t = 7.5(\sqrt{3}-1)$$

16. (a) A bullet fired at angle 45° will fall maximum away, and all other bullets will fall with this bullet fired at 45°.

$$R_{max} = \frac{u^2}{g}$$

Maximum area covered $= \pi(R_{max})^2 = \pi\left(\frac{u^2}{g}\right)^2$

17. (d) $\frac{h_2}{h_1} = \frac{u^2\sin^2\theta_2}{2g}\times\frac{2g}{u^2\sin^2\theta_1}$

$$= \frac{\sin^2\theta_2}{\sin^2\theta_1} = \frac{\sin^2\pi/6}{\sin^2\pi/3} = \frac{1}{3}$$

$$\Rightarrow h_2 = \frac{h_1}{3}$$

18. (c) Suppose the angle made by the instantaneous velocity with the horizontal be α. Then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$$

Given that $\alpha = 45°$, when $t = 1$ s; $\alpha = 0°$, when $t = 2$ s

This gives $u\cos\theta = u\sin\theta - g$ (i)

and $u\sin\theta - 2g = 0$ (ii)

Solving Eqs. (i) and (ii), we find $u\sin\theta = 2g$ and $u\cos\theta = g$.

Squaring and adding,

$$u = \sqrt{5}\,g = 10\sqrt{5}\text{ ms}^{-1}$$

19. (a) $u_x = 16\cos60° = 8$ ms^{-1}

Time taken to reach the wall = 8/8 = 1 s

Now $u_y = 16\sin60° = 8\sqrt{3}$ ms^{-1}

$$h = 8\sqrt{3}\times1 - \frac{1}{2}\times10\times1 = 13.86 - 5 = 8.9\text{ m}$$

20. (b) Let at any time t, the ball be at height of 15 m.

$$S_y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow 15 = u\sin\alpha\, t - \frac{1}{2}gt^2$$

$$\Rightarrow 15 = 52\times\frac{5}{13}t - \frac{1}{2}\times10t^2$$

$$\Rightarrow t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t-3) = 0$$

$\Rightarrow t = 1$ s, $t = 3$ s. Required time is $3 - 1 = 2$ s

21. (b) $h = (u\sin\theta)t - \frac{1}{2}gt^2$

$$d = (u\cos\theta)t \text{ or } t = \frac{d}{u\cos\theta}$$

$$h = u\sin\theta\frac{d}{u\cos\theta} - \frac{1}{2}g\frac{d^2}{u^2\cos^2\theta}$$

$$u = \frac{d}{\cos\theta}\sqrt{\frac{g}{2(d\tan\theta - h)}}$$

22. (c) Given $\frac{\sqrt{3}u}{2} = u\cos\theta$ = speed at maximum height

or $\cos\theta = \frac{\sqrt{3}}{2}$ or $\theta = 30°$ (i)

Given that $PH_{max} = R$ (ii)

We know $H_{max} = \frac{R\tan\theta}{4}$

$$P = \frac{R}{H_{max}} = \frac{4}{\tan\theta} = \frac{4}{\tan30°} = 4\sqrt{3}$$

23. (c) $y = ax - bx^2$; for height or y to be maximum:

$$\frac{dy}{dx} = 0 \text{ or } a - 2bx = 0 \text{ or } x = \frac{a}{2b}$$

i. $y_{max} = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2 = \frac{a^2}{4b}$

ii. $\left(\frac{dy}{dx}\right)_{x=0} = a = \tan\theta_0$, where θ_0 = angle of projection

$$\theta_0 = \tan^{-1}(a)$$

24. (a) $\tan\theta = \frac{u\sin\theta}{u\cos\theta} = \frac{2}{1}$

The desired equation is

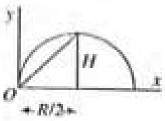
$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$= x\times2 - \frac{10x^2}{2(\sqrt{2^2+1^2})^2\left(\frac{1}{\sqrt{5}}\right)^2}$$

or $y = 2x - 5x^2$

25. (c) Average velocity $= \dfrac{\text{Displacement}}{\text{Time}}$

$$= \frac{\sqrt{H^2 + R^2/4}}{T/2}$$



Putting the required values, we get

$$v_{av} = \frac{v}{2}\sqrt{1 + 3\cos^2\theta}$$

26. (b) $\dfrac{u^2 \sin 2\theta}{g} = \dfrac{(u/2)^2 \sin 30°}{g} = \dfrac{u^2}{8g}$

$\therefore \quad \sin 2\theta = \dfrac{1}{8} \quad$ or $\quad \theta = \dfrac{1}{2}\sin^{-1}\left(\dfrac{1}{8}\right)$

27. (d) $H = \dfrac{u^2 \sin^2\theta}{2g}$, $R = \dfrac{u^2 \sin 2\theta}{g}$

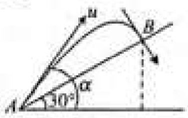
Maximum height will be same because acceleration $a = g/4$ is in horizontal direction

$$R' = u\cos\theta T + \frac{1}{2}aT^2$$

$$= R + \frac{1}{2}\frac{g}{4}\left(\frac{2u\sin\theta}{g}\right)^2 = R + H$$

28. (a) t_{AB} = time of flight of projectile $= \dfrac{2u\sin(\alpha - 30°)}{g\cos 30°}$

Now component of velocity along the plane becomes zero at point B.



$$0 = u\cos(\alpha - 30°) - g\sin 30° \times T$$

or $\quad u\cos(\alpha - 30°) = g\sin 30° \times \dfrac{2u\sin(\alpha - 30°)}{g\cos 30°}$

or $\quad \tan(\alpha - 30°) = \dfrac{\cot 30°}{2} = \dfrac{\sqrt{3}}{2}$

or $\quad \alpha = 30° + \tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

Projectile from Moving frame and from Inclined Plane

29. (c) The time taken to reach the ground depends on the height from which the bullets are fired when the bullets are fired horizontally. Here height is same for both the bullets, and hence the bullets will reach the ground simultaneously.

30. (c) If the ball hits the nth step, then horizontal distance traversed $= nb$. Here, velocity along horizontal direction $= u.n = ut$ (i)

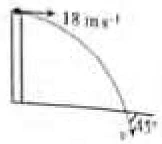
Initial velocity along vertical direction is 0.

$$nh = 0 + \frac{1}{2}gt^2 \quad \text{(ii)}$$

From $t = \dfrac{nb}{v}$, putting t in (ii), we get

$$nh = \frac{1}{2}g \times \left(\frac{nb}{u}\right)^2 \quad \text{or} \quad n = \frac{2hu^2}{gb^2}$$

31. (c) $v\cos 45° = u = 18\ \text{m s}^{-1}$



$\Rightarrow \quad v = 18\sqrt{2}\ \text{m s}^{-1}$

Vertical component

$$v\sin 45° = 18\sqrt{2} \times \frac{1}{\sqrt{2}} = 18\ \text{m s}^{-1}$$

32. (b) $v_x = u_x = 100\ \text{m s}^{-1}$, $v_y = u_y + a_y t = 0 + 10 \times 10$

$$\tan\theta = \frac{v_y}{v_x} = \frac{100}{100} = 1 \quad \Rightarrow \quad \theta = 45°$$

33. (c) $x = v\sqrt{\dfrac{2h}{g}}$, $2x = v'\sqrt{\dfrac{2(2h)}{g}}$ Solve to get $v' = \sqrt{2}\,v$

34. (b) $T = \dfrac{100}{25} = 4\ \text{s} \quad \Rightarrow \quad \dfrac{2u\sin\theta}{g} = 4 \Rightarrow u\sin\theta = 20\text{ms}^{-1}$

35. (a) Horizontal component of velocity, $u_H = u\cos 60° = \dfrac{u}{2}$

$$AC = u_H \times t = \frac{ut}{2}$$

and $AB = AC\sec 30° = \left(\dfrac{ut}{2}\right)\left(\dfrac{2}{\sqrt{3}}\right) = \dfrac{ut}{\sqrt{3}}$

36. (c) Vertical component of projectile relative to trolley.

$$v_{yT} = v\sin\theta$$

and vertical component of projectile relative to ground

$$v_{yG} = v\sin\theta$$

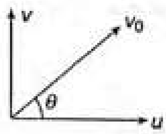
Vertical component is unchanged relative to trolley and ground.

Now time of flight $= \dfrac{2u\sin\theta}{g}$

Time of flight remains unchanged relative to trolley and ground. Hence, right choice is (c).

37. (b) $\quad v_0 \sin\theta = v \quad$ ...(i)

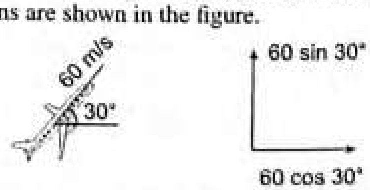
$v_0 \cos\theta = u \quad$ ...(ii)



Time of flight depends on vertical component of velocity.

$$T = \frac{2v_0\sin\theta}{g} = \frac{2v}{g}$$

38. (a) Component of velocity of aeroplane in vertical and horizontal directions are shown in the figure.



For the shell to fall along a straight line with respect to the observer on the ground horizontal component of velocity of shell with respect to ground to must be zero.

For this shell must be fired horizontal opposite direction with speed of aeroplane in horizontal

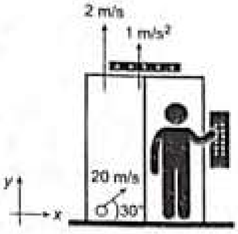
$$v_s = 30\sqrt{3}\ \text{m/s}$$

horizontally opposite to motion of plane.

39. (c) Initial velocity of stone w.r.t. lift

$= 20 \sin 30° \hat{j} + 20 \cos 30° \hat{i}$

$= (10\sqrt{3}\,\hat{i} + 10\hat{j})$ m/s



Initial velocity of stone w.r.t. ground

$= (10\sqrt{3}\,\hat{i} + 12\hat{j})$ m/s

The initial position of stone and lift are same and when they again meet their final positions will also be same. So both will have same displacement in vertical direction in same time

Displacement of lift $= 2(t) + \frac{1}{2} \times 1 \times t^2 = 2t + \frac{t^2}{2}$

Displacement of stone $= 12(t) - \frac{1}{2} \times 10 \times t^2 = 12t - 5t^2$

so $\quad 2t + \frac{t^2}{2} = 12t - 5t^2$

$\frac{11t^2}{2} = 10t$ or $t = \frac{20}{11}$ sec

So time taken by stone to return to the floor of lift is $\frac{20}{11}$ sec.

40. (c) $T = \frac{2u_y}{a_y} = \frac{2v_0 \sin 30°}{g \cos 30°} = \frac{2}{\sqrt{3}}$ s

41. (b) $t_{ascent} = \frac{u \sin\theta}{g} = \frac{10 \times \sin 60°}{g} = \frac{\sqrt{3}}{2}$ s

42. (d) $R = v_0 \cos 30° T - \frac{1}{2} g \sin 30° T^2$

$= 10\frac{\sqrt{3}}{2}\frac{2}{\sqrt{3}} - \frac{1}{2} \times 10 \times \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{20}{3}$

$R_{max} = \frac{v_0^2}{g(1+\sin\theta_0)}$

$= \frac{10^2}{10(1+\sin 30°)} = \frac{20}{3}$

So $\quad \frac{R}{R_{max}} = 1:1$

43. (c) Time of flight will be same in both cases, because uA is same in both cases.

44. (a) $R_1 = v_0 \cos 30° + \frac{1}{2} g \sin 30° T^2 = \frac{40}{3}$

$\frac{R}{R_1} = \frac{20/3}{40/3} = \frac{1}{2}$

45. (a) Perpendicular velocity will be same in both cases.

46. (d) For upward:

$v_{xf} = v_0 \cos 30° - g \cos 30° T$

$= 10\frac{\sqrt{3}}{2} - \frac{10}{2}\frac{2}{\sqrt{3}} = \frac{5}{\sqrt{3}}$

$v_{yf} = v_0 \sin 30° = \frac{10}{2} = 5$ m/s

$v_f = \sqrt{\left(\frac{5}{\sqrt{3}}\right)^2 + 5^2} = \frac{10}{\sqrt{3}}$ m/s

For downward:

$v_{xf} = v_0 \cos 30° + g \sin 30° T = \frac{25}{\sqrt{3}}$

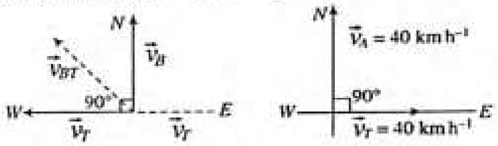
$v_{yf} = 5$ m/s

$v_f = \sqrt{\left(\frac{25}{\sqrt{3}}\right)^2 + 5^2} = 10\sqrt{\frac{7}{3}}$

Ratio $= 1 : \sqrt{7}$

Relative Velocity in Two Dimensions

47. (c) To find the relative velocity of bird w.r.t. train, superimpose velocity $-\vec{v}_T$ on both the objects. Now as a result of it, the train is at rest, while the bird possesses two velocities, $\vec{v}_B$ towards north and $-\vec{v}_T$ along west.

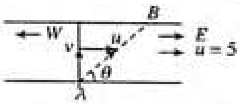


(a) (b)

$|\vec{v}_{BT}| = \sqrt{|\vec{v}_B|^2 + |-\vec{v}_T|^2}$ [By formula, $\theta = 90°$]

$= \sqrt{40^2 + 40^2} = 40\sqrt{2}$ km h^{-1} north–west

48. (b) Finally, he will swim along B. $\tan\theta = \frac{v}{u} = \frac{10}{5} = 2$

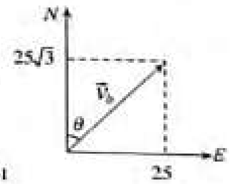


$\Rightarrow \quad \theta = \tan^{-1}(2)$ N of E

49. (c) Relative velocity of boat with respect to water is

$\vec{v}_b - \vec{v}_w = 3\hat{i} + 4\hat{j} - (-3\hat{i} - 4\hat{j}) = 6\hat{i} + 8\hat{j}$

50. (a) $\vec{v}_c = 25\hat{i}$, $\vec{v}_{b/c} = 25\sqrt{3}\hat{j}$



$\vec{v}_{b/c} = \vec{v}_b - \vec{v}_c \Rightarrow \vec{v}_b = \vec{v}_{b/c} + \vec{v}_c$

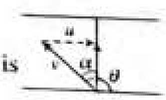
$\Rightarrow \quad \vec{v}_b = 25\hat{i} + 25\sqrt{3}\hat{j}$

$|\vec{v}_b| = \sqrt{25^2 + (25\sqrt{3})^2} = 50$ km h^{-1}

$\tan\theta = \frac{25}{25\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \quad \theta = 30°$

51. (a) Shortest time $= \frac{d}{v} = \frac{1/2}{3} = \frac{1}{6}$ h $= 10$ min

52. (d) Relative velocity of bird with respect to train is



$V_{BT} = V_B + V_T = 5 + 10 = 15$ ms^{-1}

[Because they are going in opposite directions]

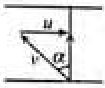
Time taken by the bird to cross the train is $\frac{150}{5} = 10$ s

53. (b) $\sin\alpha = \frac{u}{v} = \frac{\sqrt{3}}{2} \quad \Rightarrow \alpha = 60°$

$\Rightarrow \quad \theta = 90° + \alpha = 150°$

54. (d) We know that to cross the river by the shortest path,

$$\sin\alpha = \frac{u}{v}$$



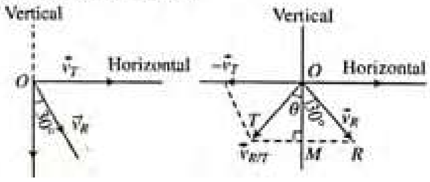
But $u > v \Rightarrow \sin\alpha > 1$, which is not possible.

55. (b) Speed of train $= 108 \times \frac{5}{18} = 30\ \text{ms}^{-1}$

Let $\vec{v}_R$ and $\vec{v}_T$ represent the respective velocities of rain and train.

Now, the relative velocity of rain w.r.t. person (train) is given by $v_{R,T} = \vec{v}_R - \vec{v}_T \Rightarrow \vec{v}_R + (-\vec{v}_T)$

Let $\overline{OR}$ and $\overline{RT}$ represent the vectors, respectively, in magnitude and direction.



$$OT^2 = OR^2 + RT^2 + 2\ OR \cdot RT \cos 120°$$

$$= 20^2 + 30^2 - 2 \times 20 \times 30 \times \frac{1}{2}$$

$$= 400 + 900 - 600 = 700 = \sqrt{700}\ \text{ms}^{-1}$$

$$= 10\sqrt{7}\ \text{ms}^{-1}$$

56. (b) $0 = (200)^2 - 2(a_{rel})(1000)$

or $\quad a_{rel} = 20\ \text{m/s}^2$

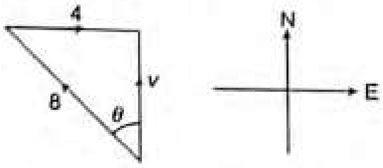
So to avoid the hit,

$\quad a_{rel} > 20\ \text{m/s}^2$

or $\quad a_p > 10\ \text{m/s}^2$

57. (c) In order to arrive at the opposite bank, the boat should start at an angle θ with north such that $\sin\theta = \frac{4}{8}$ or $\theta = 30°$. The real velocity of boat will be

$$v = \sqrt{8^2 - 4^2} = \sqrt{48}, \theta = 30°\ \text{W of N}$$



58. (a) Let v be the river velocity and u the velocity of swimmer in still water. Then

$$t_1 = 2\left(\frac{W}{\sqrt{u^2 - v^2}}\right)$$

$$t_2 = \frac{W}{u+v} + \frac{W}{u-v} = \frac{2uW}{u^2 - v^2} \quad \text{and} \quad t_3 = \frac{2W}{u}$$

Now we can see that $t_1^2 = t_2\, t_3$

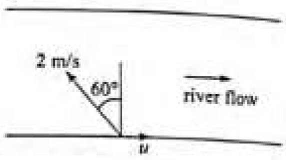
59. (a)

Let speed of current be u.

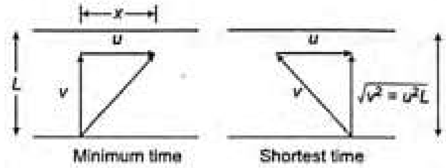
For net velocity of man to be normal to river flow.

$\quad 2\cos 60° = u$

or $\quad u = 1$ km/hr.



60. (a)



$$10 = \frac{L}{v} \qquad \text{...(i)}$$

$$12.5 = \frac{L}{\sqrt{v^2 - u^2}} = \frac{L}{v\sqrt{1 - u^2/v^2}} \qquad \text{...(ii)}$$

from (i) and (ii) $\frac{10}{12.5} = \frac{L}{v} \times \frac{v\sqrt{1 - u^2/v^2}}{L}$

$$\frac{4}{5} = \sqrt{1 - \frac{12^2}{v^2}}$$

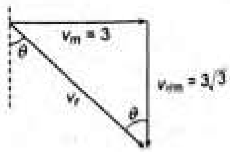
$$\frac{16}{25} = 1 - \frac{12^2}{v^2} \Rightarrow \frac{12^2}{v^2} = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\frac{12}{v} = \frac{3}{5} \Rightarrow v = \frac{12 \times 5}{3} = 20\ \text{m/min}$$

61. (b) Horizontal component of rain's velocity will be equal to velocity of wind which is 2 m/s in north direction. If cyclist goes towards north with velocity 2 m/s, then w.r.t. him rain's horizontal component of velocity will be zero, and he will see only vertical component.

62. (c) $\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$

$\vec{v}_r = \vec{v}_{r/m} + \vec{v}_m$



$$|\vec{v}_r| = \sqrt{v_{r/m}^2 + v_m^2}$$

$$= \sqrt{(3)^2 + (3\sqrt{3})^2} = 6\ \text{km/hr}$$

$$\tan\theta = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$$

63 (b) Time taken by man to cross the river

$$= \frac{\text{width of river}}{v_y}$$

$$12 = \frac{60}{v_y} \Rightarrow v_y = 5\ \text{m/s}$$

Let the x component of velocity of man w.r.t. river is v_x

Since velocity of man w.r.t. ground makes an angle of 45° with river flow x component of velocity of man w.r.t. ground

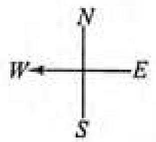
$= y$ component of velocity of man w.r.t. ground

$$\vec{v}_r + \vec{v}_x = \vec{v}_y$$

$$v_x = 0$$

So velocity of man w.r.t. water $= v_y = 5$ m/s

64. (a)



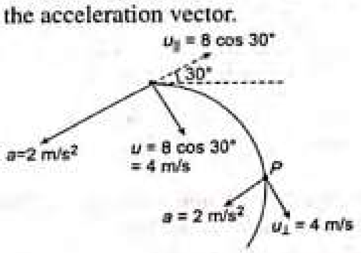
$$\vec{v}_A = -500\hat{i} \text{ and } \vec{v}_{GA} = 1500\hat{i}$$

$$\Rightarrow \quad \vec{v}_{GA} + \vec{v}_A = \vec{v}_{GAS}$$

$$\Rightarrow \quad \vec{v}_{GAS} = 1500\hat{i} - 500\hat{i} = 1000\hat{i}$$

Kinematics of Circular Motion

65. (c) The acceleration vector shall change the component of velocity $u_\parallel$ along the acceleration vector.



$$r = \frac{v^2}{a_n}$$

Radius of curvature r_{min} means v is minimum and a_n is maximum. This is at point P when component of velocity parallel to acceleration vector becomes zero, that is $u_\parallel = 0$.

$$\therefore \quad R = \frac{u_\perp^2}{a} = \frac{4^2}{2} = 8 \text{ m}$$

66. (c) $a_t = 2 \text{ ms}^{-2}$, $v = u + a_t t = 1 + 2 \times 2 = 5 \text{ ms}^{-1}$

$$a_c = \frac{v^2}{r} = \frac{5^2}{25} = 1 \text{ ms}^{-2}$$

Net acceleration $= \sqrt{a_c^2 + a_t^2} = \sqrt{1^2 + 2^2} = \sqrt{5} \text{ ms}^{-2}$

67. (c) $a_c = \frac{v^2}{r} \rightarrow$ Constant in magnitude if v is constant.

68. (a) Angular velocity is always directed perpendicular to the plane of the circular path. Hence, required change in angle is 0°.

69. (b) Angular acceleration and angular velocity are along the axis of circular path. So they cannot be perpendicular to each other.

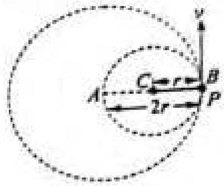
70. (d) $\alpha = (\omega_2 - \omega_1)/t = (400 - 100)/5 = 60 \text{ rev min}^{-2}$

$$= \frac{60 \times 2\pi}{(60)^2} = \frac{2\pi}{60} \text{ rad s}^{-2}$$

$$a_t = \alpha r = \frac{2\pi}{60} \times \frac{50}{100} = \frac{\pi}{60} \text{ m s}^{-2}$$

71. (c) Change in velocity $= 2v \sin(\theta/2) = 2v \sin 20°$

72. (b) Angular velocity of particle P about point A,



$$\omega_A = \frac{v}{r_{AB}} = \frac{v}{2r}$$

Angular velocity of particle P about point C,

$$\omega_C = \frac{v}{r_{BC}} = \frac{v}{r}$$

$$\text{Ratio } \frac{\omega_A}{\omega_C} = \frac{v/2r}{v/r} = \frac{1}{2}$$

73. (b) By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha(2\pi n) \Rightarrow \alpha = \frac{3}{4}\frac{\omega_0^2}{4\pi \times 36}, (n = 36) \quad \text{...(i)}$$

Now let fan completes total n' revolution from the starting to come to rest

$$0 = \omega_0^2 - 2\alpha(2\pi n') \Rightarrow n' = \frac{\omega_0^2}{4\alpha\pi}$$

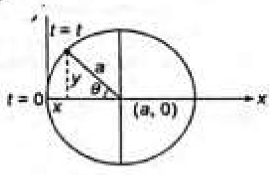
Substituting the value of α from equation (i)

$$n' = \frac{\omega_0^2}{4\pi}\frac{4 \times 4\pi \times 36}{3\omega_0^2} = 48 \text{ revolutions}$$

Number of rotation $= 48 - 36 = 12$

74. (b) $a_{\text{resultant}} = \sqrt{a_{\text{radial}}^2 + a_{\text{tangential}}^2} = \sqrt{\frac{v^4}{r^2} + a^2}$

75. (a) Particle is moving in a circle of radius a and center $(a, 0)$ with constant angular velocity ω. At time $t = 0$ particle is at origin and it starts rotating clockwise. At time t it has rotated an angle θ given by:



$$\theta = \omega t$$

$$y = a \sin \theta = a \sin \omega t$$

and $x = a - a \cos \theta = a(1 - \cos \omega t)$

$$\therefore \quad \vec{r} = a(1 - \cos\omega t)\hat{i} + a \sin \omega t\, \hat{j}$$

76. (a) $x = 2t \Rightarrow v_x = \frac{dx}{dt} = 2$

$$Y = 2t^2 \Rightarrow v_y = \frac{dy}{dt} = 4t$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{4t}{2} = 2t$$

$\therefore$

Differentiating with respect to time we get,

$$(\sec^2\theta)\frac{d\theta}{2} = 2$$

or $(1 + \tan^2\theta)\frac{d\theta}{2} = 2$; or $(1 + 4t^2)\frac{d\theta}{dt} = 2$

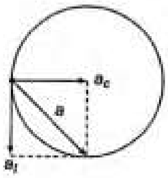
or $\frac{d\theta}{dt} = \frac{2}{1 + 4t^2}$; $\frac{d\theta}{dt}$ at $t = 2$ is

$$\frac{d\theta}{dt} = \frac{2}{1 + 4(2)^2} = \frac{2}{17} \text{ rad/s}$$

77. (c) The earth rotates west to east. So the velocity of train 2 increases. $ac = v^2/R$. Hence centripetal acceleration of train 2 is more.

78. (d) Magnitude of acceleration remains constant.

79. (a) Net acceleration is a as shown.



Problems Based on Mixed Concepts

80. (a) Horizontal component of both should be same.

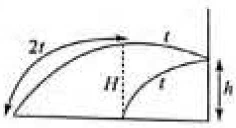
$$u\cos\theta = v_{rain}\cos 45°$$

$$\Rightarrow \quad u\cos 37° = 12\sqrt{2}\times\frac{1}{\sqrt{2}} \Rightarrow u = 15 \text{ m/s}$$

81. (c)

$$H = \frac{1}{2}g(2t)^2 = 2gt^2 \quad ...(1)$$

$$h = H - \frac{1}{2}gt^2 \quad ...(2)$$



Solving eqns (1) and (2), we get

$$h = H - \frac{H}{4} = \frac{3H}{4}$$

82. (d) As the relative acceleration is zero, the particles will meet for all values of u_1 and u_2 (Assuming they do not collide with the ground. It is given that the cliffs are very high)

83. (c) $R = \dfrac{u^2\sin 2\theta}{g}$ is same for angles θ & $90° - \theta$

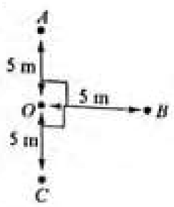
$$t_1 = \frac{2u\sin\theta_1}{g} = \frac{2u\sin\theta_1}{g}$$

$$t_2 = \frac{2u\sin\theta_2}{g} = 2u\sin\frac{(90° - \theta_1)}{g}$$

$$\frac{t_1}{t_2} = \frac{\sin\theta_1}{\sin(90° - \theta_1)}$$

$$= \frac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1$$

84. (c) Let the stones be projected at $t = 0$ sec with a speed u from point O. Then an observer, at rest at $t = 0$ and having constant acceleration equal to acceleration due to gravity, shall observe the three stones move with constant velocity as shown.



In the given time each ball shall travel a distance 5 metre as seen by this observer. Hence the required distance between A and B will be

$$= \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ metre}$$

85. (c) Let initial and final speeds of stone be u and v.

$\therefore \quad v^2 = u^2 - 2gh$ ...(i)

and $\quad v\cos 30° = u\cos 60°$ ...(ii)

solving 1 and 2 we get

$$u = \sqrt{3gh}$$

86. (c) Displacement of aircraft in time t = horizontal displacement of projectile

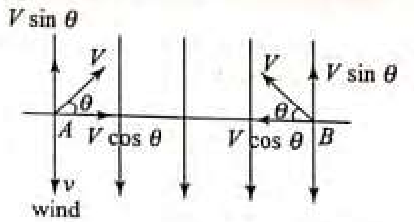
$$\Rightarrow \quad 972\times\frac{5}{18}t = V_0\cos\theta\cdot t$$

$$\Rightarrow \quad \cos\theta = \frac{54\times 5}{540} = \frac{1}{2} \quad \Rightarrow \quad \theta = 60°$$

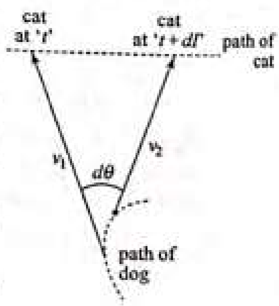
87. (a) For no drift: $V\sin\theta = v$

$$\sin\theta = \frac{v}{V} \quad \therefore \quad t = t_{AB} + t_{BA}$$

$$t = \frac{2\ell}{V\cos\theta} = \frac{2\ell}{V\sqrt{1 - \dfrac{v^2}{V^2}}} \Rightarrow t = \frac{2\ell}{\sqrt{V^2 - v^2}}$$



88. (a) The diagram shows the dog's velocity just before and just after the moment in question, with a very small angle between them. Extending those arrows to the path of cat forms an isosceles triangle with height x and width $v\,dt$. (velocity of dog is always directed towards cat.)



Magnitude of acceleration of dog at required moment

$$= \frac{|\vec{v}_2 - \vec{v}_1|}{dt} \quad ...(i)$$

Now $\quad |\vec{v}_2 - \vec{v}_1| = u\,d\theta$ ...(ii)

and $\quad d\theta = \dfrac{v\,dt}{x}$ ...(iii)

From equations (i), (ii) and (iii),

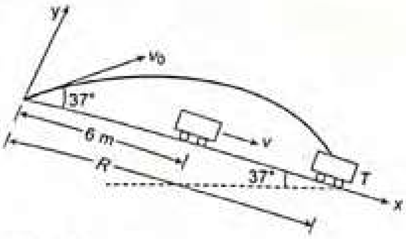
Required magnitude of acceleration = $\dfrac{uv}{x}$

89. (c) Horizontal component of velocity of A is $10\cos 60°$ or 5 m/s which is equal to the velocity of B in horizontal direction. They will collide at C if time of flight of the particles are equal or

$$t_A = t_B; \frac{2u\sin\theta}{g} = \sqrt{\frac{2h}{g}} \quad \left(h = \frac{1}{2}gt_B^2\right)$$

$$\text{or} \quad h = \frac{2u^2\sin^2\theta}{g}; = \frac{2(10)^2\left(\dfrac{\sqrt{3}}{2}\right)^2}{10} = 15 \text{ m}$$

90. (a)



Time of flight:

$$T=\frac{2v_0\sin 37^\circ}{g\cos 37^\circ}=\frac{2\times 10\times 3/5}{10\times 4/5}=1.5\text{ s}$$

$$R=(v_0\cos 37^\circ)T+\frac{1}{2}(g\sin 37^\circ)T^2$$

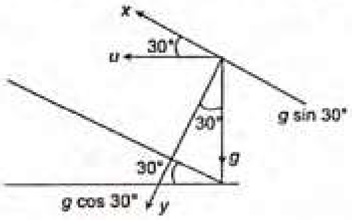
For cart:

$$R-6=vT+\frac{1}{2}(g\sin 37^\circ)T^2$$

Solve the equations to get $v = 4$ m/s

91. (c) In x direction: $v_f = u_f + a_f t$

$$\Rightarrow \quad 0=u\cos 30^\circ-g\sin 30^\circ t \Rightarrow u\sqrt{3}=gt \quad \ldots\text{(i)}$$



In y direction: $s_y = u_y t+\frac{1}{2}a_y t^2$

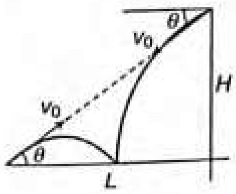
$$\Rightarrow \quad 20\cos 30^0=u\cos 60^0 t+\frac{1}{2}g\cos 30^0 t^2$$

$$\Rightarrow \quad 20\sqrt{3}=ut+\frac{1}{2}gt^2\sqrt{3}$$

$$\Rightarrow \quad 20\sqrt{3}=\frac{u^2\sqrt{3}}{g}+\frac{\sqrt{3}}{2}g.\frac{3u^2}{g^2}\Rightarrow 20=\frac{5u^2}{2g}$$

$$\Rightarrow \quad u=\sqrt{80}\text{ m/s}=4\sqrt{5}\text{ m/s}$$

92. (a)



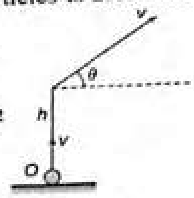
$$v_0\cos\theta\left(\frac{2v_0\sin\theta}{g}\right)+v_0\cos\theta\left(\frac{2v_0\sin\theta}{g}\right)=L$$

$$\sin\theta=\frac{H}{\sqrt{H^2+L^2}} \text{ and } \cos\theta=\frac{L}{\sqrt{H^2+L^2}}$$

$$v_0^2=\frac{gL}{4\sin\theta\cos\theta}$$

$$\Rightarrow \quad v_0=\sqrt{\frac{g(L^2+H^2)}{4H}}=\frac{1}{2}\sqrt{\frac{g(L^2+H^2)}{H}}$$

93. (d) Relative acceleration between the two particles is zero. The distance between then at time t is



$$s=\sqrt{\{h-(v-v\sin\theta t)\}^2+(v\cos\theta t)^2}$$

or $\quad s^2=\{h-(v-v\sin\theta)t\}^2+(v\cos\theta t)^2$

s is minimum when $\frac{d}{dt}(s^2)=0$

or $\quad 2\{h-(v-v\sin\theta)t\}\{v\sin\theta-v\}+2v^2\cos^2\theta t=0$

or $\quad t=\frac{h}{2v}$

94. (b) If we find relative velocity of B w.r.t. A, then it comes out to be directed towards A. Hence, both collide at some time and minimum distance between them will be zero.

Archives

1. (b) The kinetic energy of the ball at point of projection is E.

$$E=\frac{1}{2}mu^2$$

At highest point velocity has its horizontal component $u\cos\theta$. Therefore kinetic energy of the ball at highest point is:

$$E_H=\frac{1}{2}m(u\cos\theta)^2=\frac{1}{2}mu^2(\cos\theta)^2=E\cos^2\theta$$

$$\Rightarrow \quad E_H=E\cos^2 45^\circ=\frac{E}{2}$$

2. (b) Only horizontal component of velocity ($u\cos\theta$).

3. (c) $x=\alpha t^3$ and $y=\beta t^3$ (given)

x-component of the velocity: $v_x=\frac{dx}{dt}=3\alpha t^2$

and y-component of the velocity: $v_y=\frac{dy}{dt}=3\beta t^2$

Resultant velocity: $v=\sqrt{v_x^2+v_y^2}=3t^2\sqrt{\alpha^2+\beta^2}$

4. (a) When the ball again comes to the same horizontal level it covers a distance equal to range of the projectile motion.

Hence, range, $R=\frac{u^2\sin 2\theta}{g}=\frac{(10)^2\sin 60^\circ}{10}$

$$=5\sqrt{3}=8.66\text{ m}$$

5. (a) Range is same for angles of projection θ and $(90^\circ-\theta)$.

$$T_1=\frac{2u\sin\theta}{g} \text{ and } T_2=\frac{2u\sin\theta(90^\circ-\theta)}{g}$$

$$\Rightarrow \quad T_1T_2=\frac{4u^2\sin\theta\cos\theta}{g^2}=\frac{2}{g}\times\left(\frac{u^2\sin 2\theta}{g}\right)=\frac{2R}{g}$$

$\therefore \quad T_1T_2$ is proportional to R.

6. (b) In circular motion we know the velocity vector is tangent to the circle. If a particle moves in a circle with constant angular speed it perform uniform circular motion. In uniform circular motion the tangential acceleration of the particle is zero. The particle moves under radial acceleration only which points to the centre of the circle. Hence the velocity and acceleration vectors are perpendicular to each other.

7. (c) Person will catch the ball if its velocity will be equal to horizontal component of velocity of the ball.

$$\frac{v_0}{2}=v_0\cos\theta \Rightarrow \cos\theta=\frac{1}{2}\Rightarrow \theta=60^\circ$$

8. (b) For same range, angles of projection should be θ and $90°-\theta$.
So, time of flights,

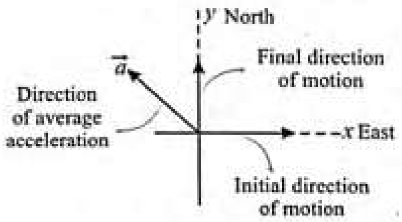
$$t_1 = \frac{2u\sin\theta}{g} \text{ and}$$

$$t_2 = \frac{2u\sin(90°-\theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore \quad t_1t_2 = \frac{4u^2\sin\theta\cos\theta}{g^2}$$

$$\Rightarrow \quad t_1t_2 = \frac{2}{g}\frac{(u^2\sin 2\theta)}{g} = \frac{2R}{g} \Rightarrow t_1t_2 \propto R$$

9. (a) Initial velocity of the particle, $\vec{v}_i = 5\hat{i}$ (m/s)



Final velocity of the particle, $\vec{v}_f = 5\hat{j}$ (m/s)

Change in velocity, $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 5\hat{j} - 5\hat{i}$ (m/s)

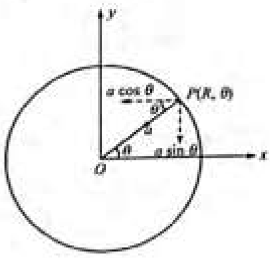
Average acceleration, $\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{(-5\hat{i}+5\hat{j})}{10}$

$$\Rightarrow \quad \vec{a} = -\frac{1}{2}\hat{i} + \frac{1}{2}\hat{j} \text{ (m/s}^2)$$

Hence, the direction of acceleration is along north-west.
Magnitude of acceleration,

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}} \text{m/s}^2$$

10. (c) For a particle in uniform circular motion, acceleration, $a = \frac{v^2}{R}$ is towards the centre.
From figure, we have



$$\bar{a} = -a\cos\theta\hat{i} - a\sin\theta\hat{j}$$

$$\Rightarrow \quad \vec{a} = -\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$$

11. (d) Position time relation of the particle, $s = t^3 + 3$

Speed of the particle, $v = \frac{ds}{dt} = \frac{d}{dt}(t^3+3) = 3t^2$

Tangential acceleration, $a_t = \frac{dv}{dt} = \frac{d}{dt}(3t^2) = 6t$

At time $t = 2$ s
Speed of the particle, $v = 3(2)^2 = 12$ m/s
Tangential acceleration, $a_t = 6(2) = 12$ m/s^2
Centripetal acceleration,

$$a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ m/s}^2$$

Net acceleration, $a = \sqrt{(a_c)^2 + (a_t)^2}$

$$= \sqrt{(7.2)^2 + (12)^2} \approx 14 \text{ m/s}^2$$

12. (b) Total area around the fountain that gets wet is equal to area of a circle whose radius is equal to maximum range of sprinkle water.

$$R_{max} = \frac{v^2\sin 90°}{g} = \frac{v^2}{g}$$

$$\text{Area} = \pi(R_{max})^2 = \frac{\pi v^4}{g^2}$$

13. (c) Centripetal acceleration, $a_c = \omega^2 r$

Angular velocity, $\omega = \frac{2\pi}{T}$

The cars make complete circle in same time.

As $T_1 = T_2 \Rightarrow \omega_1 = \omega_2$

$$\therefore \quad \frac{a_{c_1}}{a_{c_2}} = \frac{r_1}{r_2}$$

14. (a) Given initial velocity of the paeticle, $u = \hat{i} + 2\hat{j}$

$\therefore \quad u_x = 1$ m/s and $u_y = 2$ m/s

Also $\quad x = u_x t \;\therefore\; x = t$ ...(i)

and $\quad y = u_y t - \frac{1}{2}gt^2$

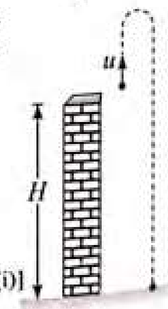
and $\quad y = 2t - \frac{1}{2}\times 10\times t^2 = 2t - 5t^2$ ...(ii)

From (i) and (ii)
Equation of trajectory is $y = 2x - 5x^2$.

15. (a) When a particle is thrown vertically upwards with a speed u, at highest point the velocity of the particle will be zero.
Using $v = u + at$, we get

$$0 = u - gt \quad \Rightarrow \quad t = \frac{u}{g} \qquad \text{...(i)}$$

The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. Let $t_1 = nt$.



Using, $s = ut_1 + \frac{1}{2}at_1^2$, we get

$$\Rightarrow \quad -H = u(nt) - \frac{1}{2}g(nt)^2$$

$$\Rightarrow \quad -H = u\times n\left(\frac{u}{g}\right) - \frac{1}{2}gn^2\left(\frac{u}{g}\right)^2 \text{ [using (i)]}$$

$$\Rightarrow \quad 2gH = 2nu^2 - n^2u^2 \Rightarrow 2gH = nu^2(n-2)$$

16. (a) Time to reach the maximum height, $t_1 = \frac{u}{g}$

If t_2 be the time taken to hit the ground, then

$$-H = ut_2 - \frac{1}{2}gt_2^2$$

But $t_2 = nt_1$ (given)

$$\Rightarrow \quad -H = u\frac{nu}{g} - \frac{1}{2}g\frac{n^2u^2}{g}$$

$$\Rightarrow \quad 2gH = nu^2(n-2)$$

CHAPTER 5: NEWTON'S LAWS OF MOTION (WITHOUT FRICTION)

Concept Application Exercise 5.1

1. Just before $t = 2$ s, the velocity of the particle is

$$u = \frac{2-0}{2-0} = 1 \text{ cm s}^{-1} = 0.01 \text{ m s}^{-1}$$

Just after $t = 2$ s, the velocity of the particle is

$$v = \frac{0-2}{4-2} = -1 \text{ cm s}^{-1} = -0.01 \text{ m s}^{-1}$$

The magnitude of impulse, $J = |m(v-u)|$

$= |0.04\,(-0.01-0.01)| = 8 \times 10^{-4}$ N s

The given x–t graph may represent the repeated rebounding of a particle between two elastic walls at $x = 0$ and $x = 2$ cm. The particle will get an impulse of 8×10^{-4} N s after every 2 s.

2. Velocity of the ball just before collision: $v^2 = 0 + 2gh$

$$v = v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m s}^{-1}$$

Therefore, momentum of the ball before collision,

$$\vec{P}_i = m\vec{v}_i$$

$$= 0.050 \times (-10)\hat{j} \text{ Ns} = -0.50\hat{j} \text{ Ns}$$

Velocity of the ball just after collision,

$$v_f = \sqrt{2gh} = \sqrt{2 \times 10 \times 1.25} = 5.0 \text{ m s}^{-1}$$

Therefore, momentum of the ball just after collision,

$$\vec{P}_f = m\vec{v}_f = 0.050 \times (5.0\hat{j}) = 0.25\hat{j} \text{ Ns}$$

Now impulse imparted by the ground on the ball

$$\vec{I} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i = 0.25\hat{J} - (-0.50\hat{j}) = 0.75\hat{j} \text{Ns}$$

Required force, $F = \dfrac{\Delta P}{\Delta t} = \dfrac{0.75}{0.1} = 7.5$ N

3. After 3 s of pouring, the bucket contains $(3 \text{ s})(0.25 \text{ L s}^{-1}) = 0.75$ L of water, with mass $0.75 \text{ L} \times (1 \text{ kg/l L}) = 0.75$ kg, and feeling gravitational force $0.75 \text{ kg } (10 \text{ m s}^{-2}) = 7.5$ N. The scale through the bucket must exert 7.35 N upward on this stationary water to support its weight. The scale must exert another 7.5 N to support the 0.75-kg bucket itself.

Water is entering the bucket with the speed given by

$$mgy_{top} = (1/2)mv_{impact}^2$$

$$v_{impact} = (2gy_{top})^{1/2} = [2(10 \text{ m s}^{-2})1.8 \text{ m}]^{1/2}$$
$$= 6.0 \text{ m s}^{-1} \text{ downward.}$$

The scale exerts an extra upward force to stop the downward motion of this additional water, as described by

$$mv_{impact} + F_{extra}\, t = mv_f$$

The rate of change of momentum is the force itself

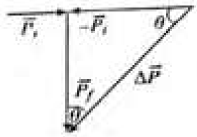
$$(dm/dt)v_{impact} + F_{extra} = 0$$

$$F_{extra} = -(dm/dt)\, v_{impact} = -(0.25 \text{ kg s}^{-1})(-6.0 \text{ m s}^{-1})$$
$$= +1.5 \text{ N}$$

Altogether the scale must exert,

$7.5 \text{ N} + 7.5 \text{ N} + 1.5 \text{ N} = 16.5$ N

4. Consider a mass Δm of liquid flowing across the corner in time Δt. We will apply Newton's second law of motion to the mass Δm.



Magnitude of initial momentum

$= P_i = (\Delta m)V$

Magnitude of final momentum

$= p_f = (\Delta m)V$

Change in momentum $= \Delta\vec{P} = \vec{P}_f - \vec{P}_i$

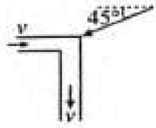
$\Delta\vec{P}$ can be calculated by the vector subtraction, geometrically.

As $P_i = P_f \Rightarrow \theta = 45°$

$$\Delta P = \sqrt{(P_i^2 + P_f^2)} = \sqrt{2(\Delta m)^2 V^2} = \sqrt{2}\Delta\, mV$$

Force exerted on the liquid $= \dfrac{\Delta P}{\Delta t} = \dfrac{\sqrt{2}\Delta mV}{\Delta t}$

$= \sqrt{2}V\rho AV$. Hence, the pipe must be pushed at the corners as shown in figure with force $F = \sqrt{2}\rho AV^2$ at an angle of 45°.



5. Impulse, $J = 2\, mu \cos 30° = 2 \times 3 \times 10 \times \dfrac{\sqrt{3}}{2} = 30\sqrt{3}$ N s

$$F_{av} = \frac{J}{\Delta t} = \frac{30\sqrt{3}}{0.2} = 150\sqrt{3} \text{ N}$$

6. (a) The impulse is to the right and equal to the area under the $F - t$ graph $J = \{[5+1]/2\} \times 4 = 12$ N s

(b) $m\vec{v}_i + \vec{F}t = m\vec{v}_f$

$\Rightarrow \quad 2.5 \times 0 + 12 = (2.5)v \quad \Rightarrow v = 4.8 \text{ ms}^{-1}$

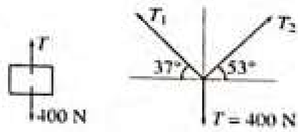
(c) From the same equation,

$\Rightarrow \quad 2.5 \times (-2) + 12 = (2.5)v \quad \Rightarrow v = 2.8 \text{ ms}^{-1}$

(d) $F_{avg}\,\Delta t = 12.0 \quad \Rightarrow F_{avg} = 2.40$ N

Concept Application Exercise 5.2

1. From the FBD of the block it is clear that tension in lower cord is 400 N. Figure shows the junction where the three cords meet.



As the system is in equilibrium, net sum of all the forces at the junction must be zero. In horizontal direction, we get

$$\cos 53°\, T_2 - \cos 37°\, T_1 = 0 \qquad \text{(i)}$$

In vertical direction, we get

$$\sin 37°\, T_1 + \sin 53°\, T_2 - 400 = 0 \qquad \text{(ii)}$$

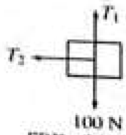
On solving the above equations, we get

$T_1 = 240$ N and $T_2 = 320$ N

2. A horizontal string cannot balance a vertical weight.

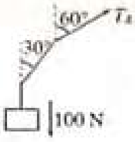
$$\sum F_y = 0 \Rightarrow T_1 = 100 \text{ N},$$

$$\sum F_x = 0 \Rightarrow T_2 = 0$$



FBD of the block

3. The free body diagram is drawn so that only the string T_4 is cut, as shown in the figure.



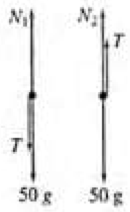
$$\sum F_y = 0$$

$\Rightarrow \quad T_4 \cos 60° = 100$

$\Rightarrow \quad T_4 = 200$ N

4. Pseudo force = ma in backward direction.

5. The FBD for the two cases is shown in the figure.



In first case, let the force exerted by the man on the floor is N_1. we have

$$N_1 = T + 50g$$

Block is to be raised without acceleration, so $T = 25g$

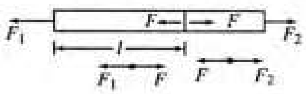
$\therefore \quad N_1 = 25g + 50g = 75g = 75 \times 10$
$= 750$ N

In second case, let the force exerted by the man on the floor is N_2. We have $N_2 = 50g - T$ and $T = 25g$

$\therefore \quad N_2 = 50g - 25g = 25g = 25 \times 10 = 250$ N

As the floor yields to a downward force of 700 N, so the man should adopt the second case.

6. Suppose $F_2 > F_1$. Then the motion of the rod will take place in the direction of the force F_2 with acceleration, say, a. Considering the motion of the entire rod, we have $F_2 - F_1$ = net force on the rod = $mL \times a$ where m is the mass of the rod per unit length of it.



Considering the motion of the length l from the first end $F - F_1 = mla$. Dividing, we get

$$\frac{F - F_1}{F_2 - F_1} = \frac{l}{L} \text{ or } F = \frac{(F_2 - F_1)l}{L} + F_1$$

7. From $t = 0$ to 2 s, the lift has a uniform acceleration

$$a_1 = \frac{\Delta v}{\Delta t} = \frac{3.6 - 0}{2 - 0} = 1.8 \text{ m s}^{-2}$$

From t = 2 to 10 s, the lift has a constant speed and hence acceleration a_2 during the period is 0 m/s^2.

From t = 10 s to 12 s, the lift has a uniform acceleration. If a_3 is the acceleration during the period, then

$$a_3 = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{0 - 3.6}{12 - 10} = -1.8 \text{ m s}^{-2}$$

(a) Here m = 1500 kg, $g = 9.8$ m s^{-2}

i. At t = 1 s, $a = a_1 = 1.8$ m s^{-2}

Tension $T_1 = m(g + a) = 1500(10 + 1.8) = 17700$ N

ii. At t = 6 s, $a = a_2 = 0$

Tension $T_2 = m(g + a_2) = mg = 1500 \times 10 = 15000$ N

iii. At t = 11 s $a = a_3 = -1.8$ m s^{-2}

Tension $T_3 = m(g + a_3) = 1500\ (10 - 1.8) = 12300$ N

(b) Height reached by the lift

= Area of enclosed by $v - t$ curve and t-axis

= Area of quadrilateral $OABC$ = 36 m

$$\text{Average velocity} = \frac{\text{Total diplacment}}{\text{Total time}} = \frac{26}{12} = 3 \text{ m s}^{-1}$$

$$\text{Average acceleration} = \frac{\text{Net change in velocity}}{\text{Total time}}$$

$$= \frac{0}{12} = 0 \text{ m s}^{-2}$$

8. (a) When the lift moves upward with an acceleration, then the apparent weight is

$$W' = W + ma \text{ or } W' = W + \frac{W}{g}a = W\left(1 + \frac{a}{g}\right)$$

Here

$W' = 50$ N, $a = 2.45$ m s^{-2}, $g = 9.8$ m s^{-2}

$$50 = \left(1 + \frac{2.45}{9.8}\right) = W \times \frac{5}{4} \Rightarrow W = \frac{4}{5} \times 50 = 40 \text{ N}$$

(b) Again if a is upward acceleration, then

$$W' = W\left(1 + \frac{a}{g}\right) \text{ gives}$$

$$30 = 40\left(1 + \frac{a}{g}\right)$$

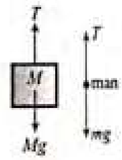
$$\Rightarrow \quad \frac{a}{g} = \frac{30}{40} - 1 = -\frac{1}{4} \quad \Rightarrow a = -\frac{g}{4}$$

i.e., lift has downward with acceleration $g/4$.

(c) When the elevator cable breaks, the lift falls freely with acceleration t.

Therefore, apparent weight $W' = W\left(1 - \frac{g}{g}\right) = 0$ N

9. Equation of motion for M:



Since M is stationary, $T - Mg = 0$

$\Rightarrow \quad T = Mg$ (i)

Since the boy moves up with an acceleration a,

$T - mg = ma$

$\Rightarrow \quad T = m(g + a)$ (ii)

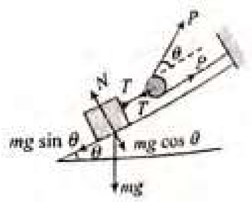
Equating (i) and (ii), we obtain

$Mg = m(g + a)$

$$\Rightarrow \quad a = \left(\frac{M}{m} - 1\right)g$$

That means, if $a > \left(\frac{M}{m} - 1\right)g$, block M can be lifted from floor.

10. From force diagram of pulley,



$P + P \cos \theta = T$

From force diagram of block

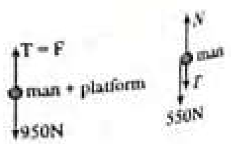
$T > mg \sin \theta$

or $\quad P + P \cos \theta > mg \sin \theta$

or $$P > \frac{mg\sin\theta}{1 + \cos\theta}$$

$$\therefore \quad P_{\min} = \frac{mg\sin\theta}{1 + \cos\theta}$$

11. Taking man and platform together on system with constant velocity.



$T = 950$ N = force applied by man

From FBD of man only,

$N = T + 550 = 950 + 550$
$= 1500$ N

12. Block C will not move and blocks A and B will move with common acceleration

$$a_A = a_B = \frac{50}{(1+2)} = \frac{50}{3} \text{ m s}^{-2}$$

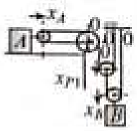
From free body diagram of B

$$N = m_B a_B = 2 \times \frac{50}{3} = \frac{100}{3} \text{ N}$$

Concept Application Exercise 5.3

1. Sum of change in length of segments of string (1) Should be zero $\Rightarrow (-2x_A) + (2x_{p1}) = 0$

or $\quad x_A = x_{p1} \Rightarrow a_A = a_{p1}$ (i)



Also sum of the change in segment length in string 2 should also be zero

$(-x_{p1} + x_B) + (x_B) = 0$

$2x_B - x_{p1} = 0 \Rightarrow 2a_B = a_{p1}$ (ii)

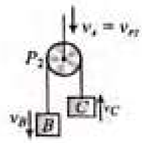
From (1) and (2) $a_A = 2a_B$

$\Rightarrow \quad a_B = \dfrac{a_A}{2} = 1\,\text{m s}^{-2}$

2. $v_A = v_{p2} = 10\ \text{m s}^{-1}$

Using shortcut method

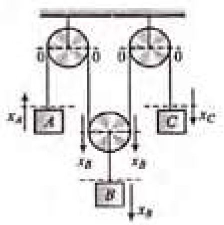
For pulley P_2



$v_{P2} = \dfrac{v_B + (-v_C)}{2}$

$10 = \dfrac{v_B - 2}{2} \Rightarrow v_B = 22\ \text{m s}^{-1}$

3. Sum of segment lengths should be zero.



$(-x_A) + (2x_B) + x_C = 0$

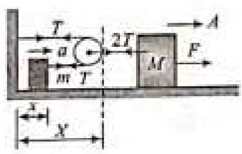
or $\quad x_A = 2x_B + x_C$

or $\quad a_A = 2a_B + a_C$

$5 = 2 \times 2 + a_C$

$\Rightarrow \quad a_C = 1\,\text{m s}^{-2}$

4. Let A be the acceleration of block M and pulley, and a be the acceleration of m.



Let x, X be the coordinates of m and pulley as shown.

Length of string passing over the pulley is

$L = X + (X - x) \Rightarrow L = 2X - x$

Differentiating twice with respect to time,

$0 = 2A - a \Rightarrow a = 2A$ (i)

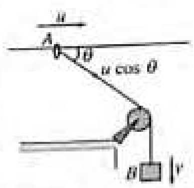
From force diagrams of m and M, we have

$T = ma$ (ii)

and $\quad F - 2T = MA$ (iii)

Comparing (i), (ii), and (iii), we get $A = \dfrac{F}{M + 4m}$ and $a = \dfrac{2F}{(M + 4m)}$

5. Let the speed of the ring is u, then $u\cos\theta = v$

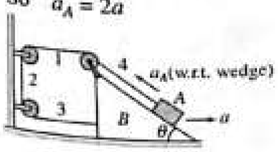


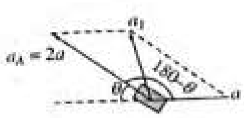
Because components of velocity along the string should be same.

$\Rightarrow \quad u = v/\cos\theta = v\sec\theta$

6. (a) Length of (1) and (3) will increase at the cost of part 4 (see figure). Hence decrease in length of (4) will be twice of increase in lengths of either 1 or 3.

So $\quad a_A = 2a$



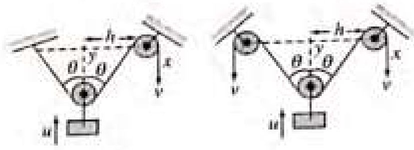


(b) Acceleration of block w.r.t. ground:

$$a_1 = \sqrt{a^2 + a_A^2 + 2a_A\cos(180 - \theta)} = a\sqrt{5 - 4\cos\theta}$$

7. (a) Length of the string: $l = x + 2\sqrt{h^2 + y^2}$

Differentiating, we get $\dfrac{dl}{dt} = \dfrac{dx}{dt} + 2\dfrac{2y}{2\sqrt{h^2 + y^2}}\dfrac{dy}{dt}$



$\Rightarrow \quad 0 = v + \dfrac{2y}{\sqrt{h^2 + y^2}}(-u)$

$\Rightarrow \quad 0 = v - 2u\cos\theta$

$\Rightarrow \quad u = v/(2\cos\theta)$

(b) Length of string, $l = 2x + 2\sqrt{h^2 + y^2}$

Again differentiate to get $u = v/\cos\theta$

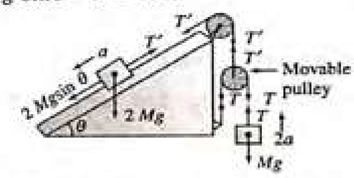
8. (a) For block $2M$, $2T = 2\,Ma$ (i)

For block M, $Mg - T = M2a$ (ii)

Solving equations (i) and (ii), we get $a = g/3$.

(b) For block $2M$,

$2Mg\sin\theta - T' = 2Ma$ (i)



For movable pulley,

$T' - 2T = 0 \times a \quad \Rightarrow T' = 2T$ (ii)

For block M, $T - Mg = M2a$ (iii)

Solving Eq. (i), (ii), and (iii), we get $a = \dfrac{g[\sin\theta - 1]}{3}$

This is negative, hence directions of accelerations will be opposite to those shown in the figure.

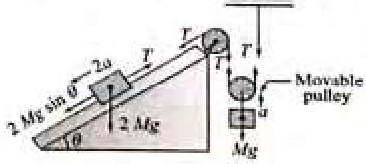
Note: *Here we assume that the block 2M is moving down the plane. You may assume that it is moving up the plane. The magnitude of the acceleration found will remain the same.*

(c) For block $2M$,

$2Mg\sin\theta - T = 2M(2a)$ (i)

For block M, $2T - Mg = Ma$ (ii)

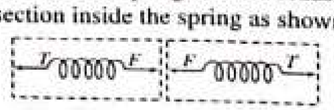
Solving Eqs (i) and (ii), we get $a = \dfrac{g[4\sin\theta - 1]}{9}$



Concept Application Exercise 5.4

1. In all the three cases the spring balance reads 10 kg.

Let us cut a section inside the spring as shown in the figure.



As each part of the spring is at rest, so $F = T$. As the block is stationary, so $T = F = 100$ N.

2. Let T be the reading of the spring balance. Then

For 20-kg block: $20g - T = 20a$ (i)

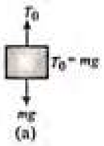
For 10-kg block: $T - 10g = 10a$ (ii)

Solving Eqs (i) and (ii), we get $a = \frac{g}{3}$, $T = \frac{40g}{3}$.

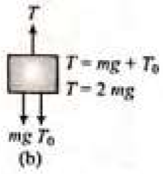
So the spring balance reading is $\frac{40}{3}$ kg.

3. When blocks A and B are in equilibrium position

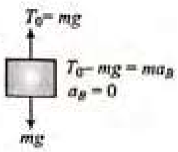
FBD of B



FBD of A



When string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw FBD of blocks A and B as shown in the figure.



FBD of B

$$mg + T_0 = ma_A$$

$$2mg = ma_A$$

$$a_A = 2g \text{ (downwards)}$$

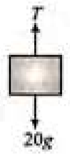
4. For calculating the reading, first we draw FBD of 20-kg block.

F.B.D of 20 kg.

$$mg - T = 0$$

$$T = 20\text{ g} = 200 \text{ N}$$

Since both balances are light so, both the scales will read 20 kg.



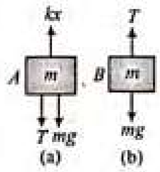
5. When string is not cut:

FBD of A block

$$kx = mg + T \quad \text{(i)}$$

FBD o B block

$$T = mg \quad \text{(ii)}$$



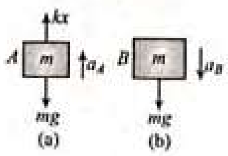
When string is cut:

FBD of A block

$$kx - mg = ma_A$$

$$2mg - mg = ma_A$$

$$= a_A = g \text{ (upwards)}$$



FBD of B block

$$ma_B = mg$$

$$a_B = g \text{ (downwards)}$$

6. From force diagram of 2-kg block:

$$10 - kx = 2a = 4$$

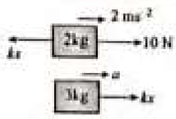
$\therefore$ $kx = 6$

From force diagram of 3kg block,

$$kx = 3a$$

or $6 = 3a$

$\therefore$ $a = \frac{6}{3} = 2\text{ms}^{-2}$



Exercises

Problems Based on Basic Concept of Projectile Motion

1. (a) Acceleration of the skaters will be in the ratio

$$a_1 : a_2 = \frac{F}{4} : \frac{F}{5} \quad \text{or} \quad 5:4$$

Here F is the tension in string.

Now according to the problem, $s = 0 + \frac{1}{2}at^2$, we get

$$\frac{s_1}{s_2} = \frac{a_1}{a_2} = \frac{5}{4}$$

2. (a) $a = \frac{\sqrt{R_1^2 + R_2^2}}{m} = \frac{R_3}{m}$ $\left[\because R_3 = \sqrt{R_1^2 + R_2^2}\right]$

3. (b) Change in momentum of one ball = 2 mu, time taken = 1 s

$$F_{av} = \frac{\text{Total change in momentum}}{\text{Time taken}} = \frac{n(2mu)}{1} = 2\,mnu$$

4. (c) Area under the force–time graph is impulse, and impulse is change in momentum.

Area of graph = change in momentum

$$\Rightarrow \frac{1}{2}TF_0 = 2\,mu \Rightarrow F_0 = \frac{4mu}{T}$$

5. (b) The force of 100 N acts on both the boats.

$$250\,a_1 = 100 \text{ and } 500a_2 = 100$$

or $a_1 = 0.4 \text{ ms}^{-2}$ and $a_2 = 0.2 \text{ ms}^{-2}$

The relative acceleration: $a = a_1 + a_2 = 0.6 \text{ ms}^{-2}$

Using $S = ut + \frac{1}{2}at^2$, we get

$$120 = (1/2) \times 0.6 \times t^2 \quad \text{or} \quad t = 20 \text{ s}$$

6. (d) $\vec{a} = \frac{\vec{F}}{m} = -10\hat{j}\,(\text{ms}^{-1})^2$

Displacement in y-direction

$$y = ut + \frac{1}{2}at^2$$

$$\Rightarrow 0 = 4 \times t \times -\frac{1}{2} \times 10 \times t^2$$

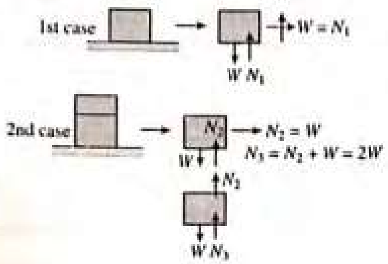
$$t = \frac{4}{5}\text{s} \Rightarrow x = 4t = 4 \times \frac{4}{5} = 3.2 \text{ m}$$

7. (d) Force on block $= v\frac{dm}{dt}$

$$\text{Acceleration of block} = \frac{F}{M} = \frac{v}{M}\frac{dm}{dt}$$

$$= \frac{5}{2} \times 1 = \frac{5}{2} \text{ m/s}^2$$

8. (c) Let the weight of each block be W (see figure)



So, $N_1 = N_2 = \dfrac{N_3}{2}$

9. (b) $\vec{u} = 4\hat{i} + 2\hat{j}, \quad \vec{a} = \dfrac{\vec{F}}{m} = \hat{i} - 4\hat{j}$

Let at any time, the coordinates be (x, y).

$$x - 2 = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \quad x - 2 = 4t + \frac{1}{2}t^2 \quad \text{and} \quad y - 3 = 2t - \frac{1}{2}4t^2$$

$$\Rightarrow \quad y - 3 = 2t - 2t^2$$

When $y = 3$ m, $t = 0, 1$ s; when $t = 0, x = 2$ m

When $t = 1$ s, $x = 6.5$ m

10. (c) From 0 to 2 s: at any time t, $F = 10\,t$

$$\Rightarrow \quad a = F/m = 10\,t/m$$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{10t}{m} dt \quad \Rightarrow \quad v = \frac{5t^2}{m}$$

Momentum: $p = mv = 5t^2$

At $t = 2$ s, $p = 5(2)^2 = 20$ kg m s^{-1}, $v = 20/m$

From 2 to 4 s: $F = 40 - 10\,t$

$$\int_{20/m}^{v} dv = \int_2^t \frac{40 - 10t}{m} dt \quad \Rightarrow \quad v = \frac{1}{m}[40t - 40 - 5t^2]$$

$$p = mv = 40t - 40 - 5t^2$$

11. (d) Speed, $v = \sqrt{{v_x}^2 + {v_y}^2}$

Rate of change of speed,

$$\frac{dv}{dt} = \frac{2v_x \dfrac{dv_x}{dt} + 2v_y \dfrac{dv_y}{dt}}{2\sqrt{{v_x}^2 + {v_y}^2}} = \frac{v_x a_x + v_y a_y}{\sqrt{{v_x}^2 + {v_y}^2}}$$

$$= \frac{(3 \times 2) + (4 \times 1)}{\sqrt{(3)^2 + (4)^2}} = 2 \text{m/s}^2$$

12. (b) u = velocity of bullet

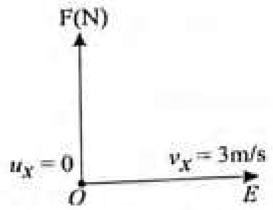
$\dfrac{dm}{dt}$ = Mass thrown per second by the machine gun

= Mass of bullet × Number of bullet fired per second

= 10 g × 10 bullet/sec = 100 g/sec = 0.1 kg/sec

$\therefore$ Thrust $= \dfrac{udm}{dt} = 500 \times 0.1 = 50$ N

13. (b)



Displacement of body in 4 sec along OE

$$s_x = v_x t = 3 \times 4 = 12 \text{ m}$$

Force along OF (perpendicular to OE) = 4 N

$$\therefore \quad a_y = \frac{F}{m} = \frac{4}{2} = 2 \text{ m/s}^2$$

Displacement of body in 4 sec along OF

$$\Rightarrow \quad s_y = u_y t + \frac{1}{2} a_y t^2 = \frac{1}{2} \times 2 \times (4)^2 = 16\,m \qquad [\text{As } u_y = 0]$$

$\therefore$ Net displacement $s = \sqrt{s_x^2 + s_y^2} = \sqrt{(12)^2 + (16)^2} = 20$ m

14. (a) Given that $\vec{p} = p_x\hat{i} + p_y\hat{j} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

$$\therefore \quad \vec{F} = \frac{d\vec{p}}{dt} = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$$

Now, $\vec{F} \cdot \vec{p} = 0$ i.e. angle between $\vec{F}$ and $\vec{p}$ is 90°.

15. (a) Force on $B = m_2 a_2 \quad \therefore$ Force on $A = m_1 a_1$

Acceleration of $A = \dfrac{m_2 a_2}{m_1}$

16. (b) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 = 2(4\hat{i})$ (i)

and $\vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 = 2(7\hat{j})$ (ii)

From (i) and (ii), $\vec{F}_1 = 8\hat{i} - 14\hat{j}$

$$\vec{a}_1 = \frac{\vec{F}_1}{m} = 4\hat{i} - 7\hat{j}$$

$$\Rightarrow \quad a_1 = \sqrt{16 + 49} = \sqrt{65} \text{ m/s}^2$$

17. (c) When all are pulling

$\vec{F}_{net} = 100 \times 3\,\hat{i}$ (1)

when A stops

$\vec{F}_{net} - \vec{F}_A = 100 \times 1(-\hat{i})$ (2)

when B stops

$\vec{F}_{net} - \vec{F}_B = 100 \times 24\,\hat{j}$ (3)

from these three get $\vec{F}_A + \vec{F}_B = (700\hat{i} - 2400\hat{j})$N

hence acceleration of the cart

$$\vec{a} = \frac{\vec{F}_A + \vec{F}_B}{m} = \frac{(700\hat{i} - 2400\hat{j})}{100} \text{m/s}^2$$

$$\vec{a} = (7\hat{i} - 24\hat{j}) \text{ m/s}^2 \Rightarrow |\vec{a}| = \sqrt{7^2 + 24^2} = 25 \text{ m/s}^2$$

18. (a) Change in linear momentum $\Delta P = \int F dt$

$$15(v_f + u) = \int_0^{15} 40\cos\left(\frac{\pi}{10}\right) t\, dt$$

$$v_f = -4 + \frac{40}{15}\left[\frac{\sin(\pi/10)\,t}{\pi/10}\right]_0^{15} = -4 + \frac{400}{15\,\pi}(-1) = -12.5 \text{m/s}$$

Application of Newton's Laws of Motion In Base Situations

19. (c) Let T be the tension in the rope and a the acceleration of rope.

The absolute acceleration of man is, therefore, $\left(\dfrac{5g}{4} - a\right)$. Equations of motion for mass and man gives:

$T - 100g = 100a$ (i)

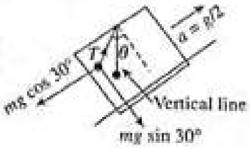
$T - 60g = 60\left(\dfrac{5g}{4} - a\right)$ (ii)

Solving Eqs (i) and (ii), we get $T = \dfrac{4875}{4}$ N

20. (b) $T \sin \theta - mg \sin 30° = ma$

$\Rightarrow \quad T \sin \theta = mg \sin 30° + mg/2$ (i)

$T \cos \theta = mg \cos 30°$ (ii)



Dividing Eqs. (i) by (ii), we get

$$\tan \theta = \frac{2}{\sqrt{3}}$$

21. (b) Suppose F = upthrust due to buoyancy

Then while descending, we find

$Mg - F = Ma$ (i)

When ascending, we have

$F - (M - m)g = (M - m)a$ (ii)

Solving Eqs. (i) and (ii), we get $m = \left[\dfrac{2a}{a+g}\right]M$

22. (a) Initial force = 2 g = 20 N

Initial acceleration $= \dfrac{\text{Force}}{\text{Mass}} = \dfrac{20}{5+1} = \dfrac{20}{6}\ \text{m s}^{-2}$

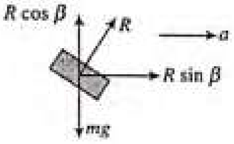
Final force = (Load + Mass of thread) × g

= (2 + 1) × 10 = 30 N

Therefore, final acceleration $= \dfrac{30}{6}\ \text{m s}^{-2}$

23. (a) Acceleration of the system:

$$a = \frac{P}{M+m} \quad \text{(i)}$$



The FBD of mass m is shown in the figure.

$R \sin \beta = ma$ (ii)

$R \cos \beta = mg$ (iii)

From Eqs. (ii) and (iii), we get

$a = g \tan \beta$

Putting the value of a in (i), we get

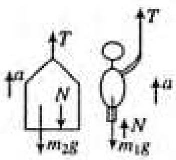
$P = (M + m)g \tan \beta$

24. (b) $m_1 = 100$ kg, $m_2 = 50$ kg, $a = 5\ \text{m s}^{-2}$

$T + N - m_1 g = m_1 a,$

$T - N - m_2 g = m_2 a$

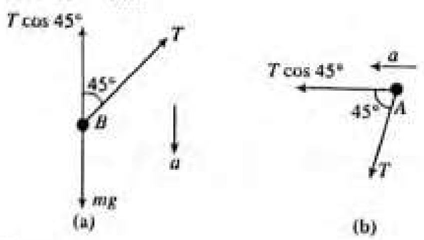
Solving these: $T = 1125$ N and $N = 375$ N



25. (d)

26. (d) As shown in Figs. (a) and (b) from FBD of A,

$T \cos 45° = ma$ (i)



From FBD of B:

$Mg - T \cos 45° = ma$ (2)

From (i) and (ii), we get $T = mg/\sqrt{2}$

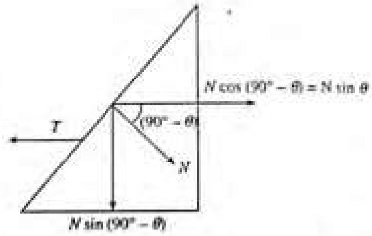
27. (a) As m_2 moves with constant velocity, there is no acceleration in the centre of mass. Net force should be zero. For this

$N = m_1 g + m_2 g = (m_1 + m_2)g$

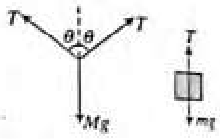
28. (b) $\vec{a}_{b,l} = \vec{a}_b - \vec{a}_l = (g - a)\downarrow \ \Rightarrow\ \vec{a}_b = g\downarrow$

29. (c) $T = N \sin \theta$ and $N = mg \cos \theta$

$$T = mg \cos\theta \sin\theta = \frac{mg}{2}\sin 2\theta$$



30. (c) At equation $2T \cos \theta = Mg$, $T = mg$



$$\cos\theta = \frac{Mg}{2mg} = \frac{M}{2m}$$

$\Rightarrow \quad \cos \theta < 1$

$$\frac{M}{2m} < 1 \quad \Rightarrow \quad M < 2m$$

31. (c) Acceleration is to be downward which is possible in option (c).

32. (c) If acceleration of M is 2 m s^{-2}, then acceleration of m w.r.t. M will be 2 m s^{-2}

$$\vec{a}_m = \vec{a}_{m/M} + \vec{a}_M$$

Acceleration of m w.r.t. ground

$$a_m = \sqrt{2^2 + 2^2 + 2 \times 2 \times 2\cos(180 - 37°)} = \sqrt{8/5}\ \text{m s}^{-2}$$

33. (b) $a = \dfrac{3T - mg \sin \theta}{m}$

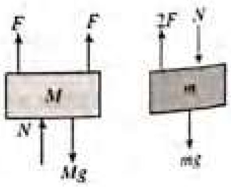
$$= \frac{3 \times 250 - 100 \times 10 \times \sin 30°}{100} = 2.5\ \text{m s}^{-2}$$

34. (c) From the figure

$2F + N - Mg = Ma$

$2F - mg - N = ma$

$4F - (M + m)g = (M + m)a$



$$a = \frac{4F - (M+m)g}{M+m}$$

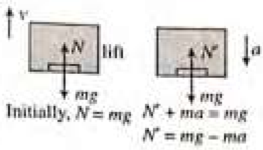
35. (d) The acceleration of block–rope system is $a = \dfrac{F}{(M+m)}$, where M is the mass of block and m is the mass of rope. So the tension in the middle of the rope will be

$$T = \{M + (m/2)\}\,a = \frac{M + (m/2)F}{M+m}$$

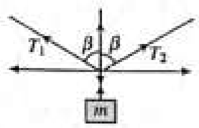
Given that $m = M/2$

$$\therefore \quad T = \left[\frac{M + (M/4)}{M + (M/2)}\right]F = \frac{5F}{6}$$

36. (d) The reading of the spring scale is the normal reaction between man and spring scale (see figure). As the reading is decreasing, it means normal reaction is decreasing. Firstly, the lift must be moving upwards with constant velocity and decelerated to rest.



37. (b) $\tan\beta = \frac{12}{5}$ $\quad\therefore\quad \cos\beta = \frac{5}{13}$



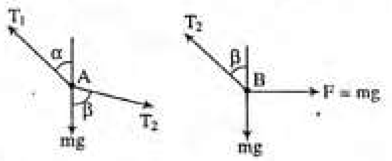
$$T_1\cos\beta + T_2\cos\beta = mg \qquad \text{(i)}$$

$$T_1\sin\beta = T_2\sin\beta \qquad \text{(ii)}$$

$\therefore\quad T_1 = T_2 = T$

$\therefore\quad 2T\cos\beta = mg \;\Rightarrow\; T = \frac{mg}{2\cos\beta} \;\Rightarrow\; T = \frac{13}{10}mg$

38. (c) From the figure $T_2\cos\beta = mg$, $\;T_2\sin\beta = mg$



$$T_1\sin\alpha = T_2\sin\beta = mg$$

$$T_1\cos\alpha = mg + T_2\cos\beta = 2\,mg$$

$$\Rightarrow\quad \frac{\cos\alpha}{\sin\alpha} = \frac{2mg}{mg}$$

$$\tan\alpha = \frac{1}{2} \;\Rightarrow\; \cos\alpha = \frac{2}{\sqrt{5}}$$

$$T_1\frac{2}{\sqrt{5}} = 2\,mg \;\Rightarrow\; T_1 = mg\sqrt{5}$$

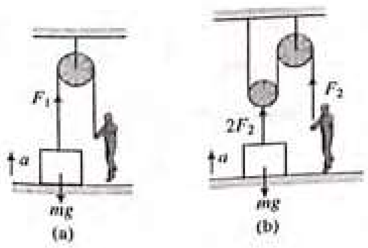
$$\frac{T_1}{T_2} = \frac{\sqrt{5}}{\sqrt{2}} \;\Rightarrow\; \sqrt{2}\,T_1 = \sqrt{5}\,T_2$$

39. (a) Since, $h = \frac{1}{2}at^2$, a should be same in both cases, because h and t are same in both cases as given.

In Fig. (a), $F_1 - mg = ma \;\Rightarrow\; F_1 = mg + ma$

In Fig. (b), $2F_2 - mg = ma \;\Rightarrow\; F_2 = \frac{mg + ma}{2}$

$\therefore\quad F_1 > F_2$



Problems Based on Constraint Relation

40. (c) Tx(Hanged part) $= 2Tx'$ (Sliding part)

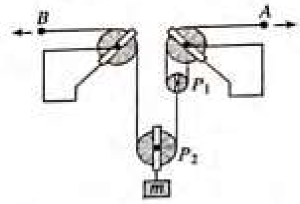
$\therefore\quad x = 3x' \;\Rightarrow\; x = 3\times 0.6 = 1.8\ \text{m s}^{-1}$

41. (a) $v_A = 2\ \text{m s}^{-1}$ (towards right)

$\therefore\quad v_{P_1} = \frac{v_A}{2} = 1\ \text{m s}^{-1}$ (upwards)

$v_B = 2\ \text{m s}^{-1}$ (towards left)

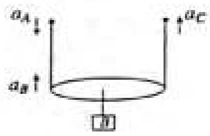
Now $2v_{P_2} = v_B + v_{P_1}$



$\therefore\quad v_{P_2} = \frac{v_B + v_{P_1}}{2} = \frac{2+1}{2} = 1.5\ \text{m s}^{-1}$

42. (d) From constraint relations we can see that the acceleration of block B in upward direction is

$$a_B = \left(\frac{a_C + a_A}{2}\right) \text{ with proper signs}$$



So $\;a_B = \left(\frac{3-12t}{2}\right) = 1.5 - 6t$

or $\;\frac{dv_B}{dt} = 1.5 - 6t\;$ or $\;\int_0^{v_B} dv_B = \int_0^{1}(1.5 - 6t)\,dt$

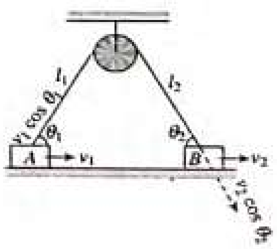
or $\;v_B = 1.5t - 3t^2\;$ or $\;v_B = 0$ at $t = 1/2$ s

43. (c) From figure $l_1 + l_2 = C$

or $\;\frac{dl_1}{dt} + \frac{dl_2}{dt} = 0$

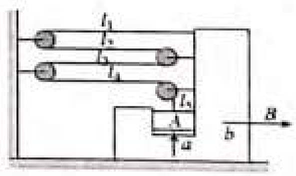
$-v_1\cos\theta_1 + v_2\cos\theta_2 = 0$

or $\;\frac{v_1}{v_2} = \frac{\cos\theta_2}{\cos\theta_1}$



44. (a) Acceleration of A in horizontal direction = the acceleration of $B = b$ rightwards

Acceleration of A in vertical direction = the acceleration of A with respect to b in upwards direction $= a = 4b$.



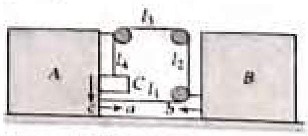
Hence, net acceleration of $A = b\hat{i} + 4b\hat{j}$

45. (a) From length constraint $l_1 + l_2 + l_3 + l_4 = C$

$l_1'' + l_2'' + l_3'' + l_4'' = 0$

$(-a-b) + 0 + (-a-b) + c = 0$

$c = 2a + 2b$



From wedge constraints, acceleration of C is right side is a.

acceleration of C w.r.t. ground $= a\hat{i} - 2(a+b)\hat{j}$

46. (d) For A: $5g - T = 5(2C)$

For C: $2T - 8g = 8C \quad \Rightarrow \quad C = \frac{8}{14} = \frac{5}{7}\ \text{m s}^{-2}$

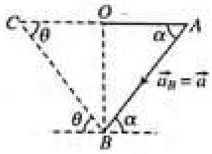
47. (c) By virtual work method:

Acceleration of B w.r.t. A will be $10\ \text{m s}^{-2}$ downward.

Apart from this, B also has an acceleration $5\ \text{m s}^{-2}$ in horizontal direction along with A, so net acceleration of B is

$$\sqrt{10^2 + 5^2} = \sqrt{100 + 25} = \sqrt{125} = 5\sqrt{5}\ \text{m s}^{-2}$$

48. (d) Direction of acceleration of B is along the fixed incline, and that of A is along horizontal towards left.

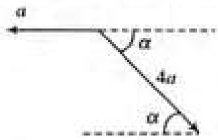


From diagram, acceleration of B is represented by $\overline{AB}$ while its horizontal and vertical components are shown by AO and OB, respectively. Acceleration of A is represented by $\overline{AC}$

$$\overline{OC} = a(\sin\alpha \cot\theta + \cos\alpha)$$

49. (b) If the wedge moves leftward by x, then the block moves down the wedge by $4x$, i.e., w.r.t. wedge the block comes down by $4x$.

So, acceleration of block w.r.t wedge $= 4a$ along the incline plane of wedge. Acceleration of wedge with respect to ground $= a$, along left. So acceleration of block w.r.t. ground is the vector sum of the two vectors shown in the figure. That is



$$|\vec{a}_{BG}| = \sqrt{a^2 + (4a)^2 + 2 \times a \times 4a \times \cos(\pi - \alpha)}$$

$$= (\sqrt{17 - 8\cos\alpha})\ a\ \text{m s}^{-2}$$

50. (d) Using constraint theory,

$l_1 + l_2 + l_3 =$ constant

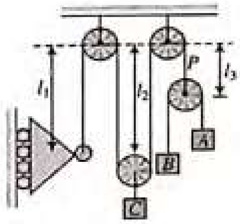
$\Rightarrow \quad v_1 + 2v_2 + v_3 = 0$

Take downward as positive and upward as –ve.

So $\quad 12 + 2(-4) + v_3 = 0$

$v_3 =$ velocity of pulley $P = -4\ \text{m s}^{-1}$

$= 4\ \text{m s}^{-1}$ in upward direction



$$\vec{v}_{AP} = -\vec{v}_{BP}$$

$$\Rightarrow \quad \vec{v}_{AP} = -(\vec{v}_B - \vec{v}_P)$$

$$\vec{v}_B = \vec{v}_P - \vec{v}_{AP} = -4 - (3) = -7\ \text{m s}^{-1}$$

i.e., block B will moves up with speed $7\ \text{m s}^{-1}$.

51. (d) Let at any time, their velocities are v_1 and v_2, respectively. Then

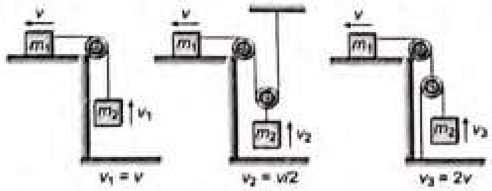
$v_1 = v_2 \cos\theta$

Differentiating: $a_1 = a_2 \cos\theta - v_2 \sin\theta \frac{d\theta}{dt}$

Hence, none of them is correct.

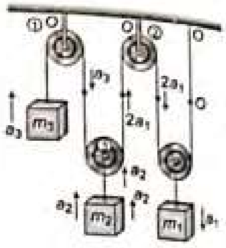
[Note: Option (a) is correct initially, because initially $v_2 = 0$.]

52. (a)



$v_1 : v_2 : v_3 = 1 : 0.5 : 2 = 2 : 1 : 4$

53. (d) From pulley (3)



$$a_2 = \frac{2a_1 - a_3}{2}$$

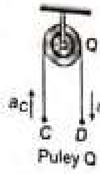
$$2a_2 = 2a_1 - a_3$$

$$2a_2 - 2a_1 + a_3 = 0$$

54. (c) Acceleration of pulley P

$$a_P = \frac{a_A + a_B}{2} = \frac{1+7}{2} = 4\ \text{m/s}^2 \quad \text{(upward)}$$

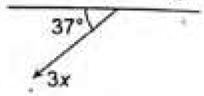
Acceleration pulley Q will be $4\ \text{m/s}^2$ downwards.



$$a_Q = \frac{a_D - a_C}{2}$$

$$4 = \frac{a_D - 2}{2} \Rightarrow a_D = 10\ \text{m/s}^2 (\downarrow)$$

55. (d) If system is released, the wedge will slide towards right. Let wedge slide towards right by distance x. The string of length $3x$ will be adjusted on the inclined face.



Let $\quad d_{\text{wedge}} = x$ (towards right)

Hence $\quad d_{\text{block, wedge}} = 3x$

$$\vec{d}_{\text{block}} = \vec{d}_{\text{block, wedge}} + \vec{d}_{\text{wedge}}$$

$$= \sqrt{(3x)^2 + x^2 + 2(3x)(x)\cos(180 - 37°)}$$

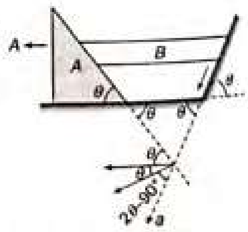
$$= \sqrt{10x^2 - 6x^2 \cos 37°} = x\sqrt{10 - 6\cos 37°}$$

$$= x\sqrt{10 - 6 \times \frac{4}{5}} = x\sqrt{\frac{26}{5}}$$

Hence, acceleration of block

$$|\vec{a}_m| = A\sqrt{\frac{26}{5}} = \sqrt{26}\ \text{m/s}^2$$

56. (d) Components of accelerations of A and B along normal $(\hat{n})$ to the incline of wedge should be same.



$\Rightarrow \quad A\sin\theta = a\cos(2\theta - 90°)$

$\Rightarrow \quad A\sin\theta = a\sin 2\theta$

$\Rightarrow \quad A = 2a\cos\theta$

Kinematics of Circular Motion

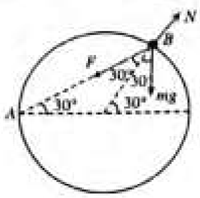
57. (d) Extension in the spring is

$$x = AB - R = 2R\cos 30° - R = (\sqrt{3} - 1)R$$

Spring force: $F = kx = \frac{(\sqrt{3}+1)mg}{R} \times (\sqrt{3} - 1)R = 2mg$

From the figure we have

$$N = (F + mg)\cos 30° = \frac{3\sqrt{3}mg}{2}$$



58. (d) Let spring does not get elongated, then net pulling force on the system is $Mg + mg - mg$ or simply Mg. Total mass being pulled is $M + 2m$. Hence, acceleration of the system is

$$a = \frac{Mg}{M+2m}$$

Now since $a < g$, there should be an upward force on M so that its acceleration becomes less than g. It means there is some tension developed in the string. Hence, for any value of M spring will be elongated.

59. (b) Before cutting the string, the tension in string joining m_4 and ground is $T = (m_1 + m_2 - m_3 - m_4)\,g$
And the spring force in the spring joining m_3 and m_4 is $T + m_4 g$.
As the string is cut, the spring forces do not change instantly, so just after cutting the string the equilibrium of m_1, m_2 and m_3 would be maintained but m_4 accelerates in upward direction with acceleration given by

$$a = \frac{T + m_4 g - m_4 g}{m_4}$$

60. (c) 5 N force will not produce any tension in spring without support of other 5 N force. So here the tension in the spring will be 5 N only.

61. (b) As the mass of 10 kg has acceleration 12 m/s^2 therefore it apply 120 N force on mass 20 kg in a backward direction.
$\therefore$ Net forward force on 20 kg mass = 200 – 120 = 80 N

Acceleration of 20 kg of mass $= \frac{80}{20} = 4$ m/s^2

62. (b) If reading of spring balance is T, then applying NLM on (man + ladder) system

$T - (25 + 5)g = 25\,a$

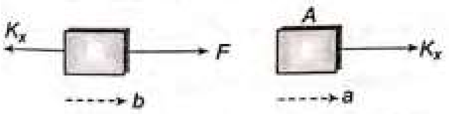
$T - 30\,g = 25\,a$

$\Rightarrow$ $T - 300 = 25(1)$

$\Rightarrow$ $T = 325$ N = 32.5 kg

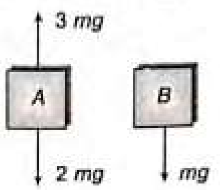
63. (a) As the spring balances are light, so tension in both the springs will be same and equal to weight of block suspended.

64. (c) $F - kx = mb$ and $kx = ma$



Hence $m\,(b - a) = F - 2kx$ or $b - a = \frac{F - 2kx}{m}$

65. (b) After string is cut, free body diagram of block A gives

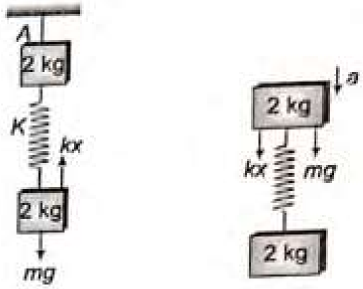


$2ma_A = 3mg - 2mg$ or $a_A = \frac{mg}{2m} = \frac{g}{2}$

Free body diagram of block B gives

$ma_B = mg$ or $a_B = g$

66. (d) Before the string A is cut, let x be elongation in the spring.



(a) (b)

As system is in equilibrium. (see Figure (a))
Then for lower block,

$kx = mg = 20$ N

Just after the string A is cut (see Figure (b))
For upper block,

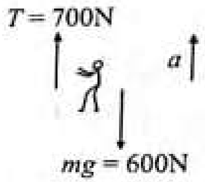
$ma = kx + mg$

$2a = 20 + 20$

$a = 20$ m/s^2

Problems Based on Mixed Concepts

67. (d)



For student A to just lift off the floor, tension T in string must be greater than or equal to 700 N.
The F.B.D. of student B is
Applying Newton's second law

$T - mg = ma \Rightarrow 700 - 600 = 6\,a$

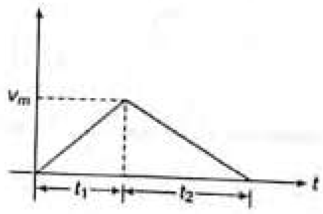
or $a = \frac{5}{3}$ m/s^2

68 (a) The magnitude of the force (from the string) is $T = 30$ N
The x-component = $T \sin\theta = 30 \times 3/5 = 18$ N
The y-component = $T \cos\theta = 30 \times 4/5 = 24$ N
The total force on the block is:
The x-component = 18 N
The y-component = $24 - mg = 24 - 20 = 4$ N
The x-component of the acceleration = 18 N/2 kg = 9 m/s^2
The y-component of the acceleration = 4 N/2 kg = 2 m/s^2

69. (d) Weight of lift = Mg
Maximum tension = nMg

$\therefore$ Maximum acceleration $= \frac{nMg - Mg}{M}$
$= (n - 1)g$

and maximum retardation = g
corresponding velocity–time graph for shortest time will be as follows:



Here $(n-1)g = \frac{v_m}{t_1}$ or $t_1 = \frac{v_m}{(n-1)g}$ ...(1)

and $g = \frac{v_m}{t_2}$ or $t_2 = \frac{v_m}{g}$ ...(2)

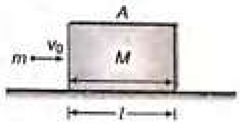
Area under v–t graph is total displacement h.

Hence $h = \frac{1}{2}(t_1 + t_2)v_m$ ...(3)

From (1), (2) and (3) we get,

$$v_m = \sqrt{2gh\left(\frac{n-1}{n}\right)}$$

70. (b) From Newton's third law a force F acts on the block in forward direction.



Acceleration of block $a_1 = \frac{F}{M}$

Retardation of bullet $a_2 = \frac{F}{m}$

Relative retardation of bullet

$$a_r = a_1 + a_2 = \frac{F(M+m)}{Mm}$$

Applying $v^2 = u^2 - 2a_r\ell$

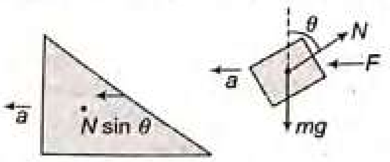
$$0 = v_0^2 - \frac{2F(M+m)}{Mm}\cdot \ell$$

Therefore, minimum value of v_0 is

or $v_0 = \sqrt{\frac{2F\ell(M+m)}{Mm}}$

71. (a) Since frictional force acts on the block, its deceleration is μg. So in the frame of wedge, a pseudo force μmg acts on the block and it is inclined at angle $\theta = \phi$ with vertical.

72. (c) For no relative motion between wedge and block, let the acceleration of both block and wedge be 'a' towards left.

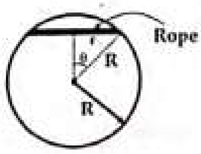


From FBD of block; $N\cos\theta = mg$ ...(i)

and $F - N\sin\theta = ma$ ...(ii)

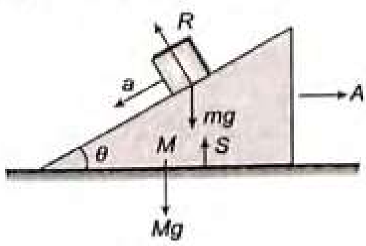
From FBD of wedge; $N\sin\theta = Ma$...(iii)

from equations (i), (ii) and (iii)



solving we get $F = \frac{m}{M}(M+m)g\tan\theta$

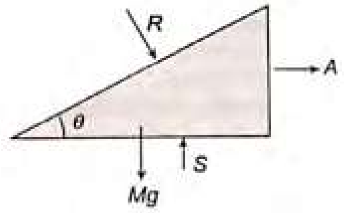
73. (a) As the particle moves down the inclined plane, let a be its acceleration. Now the inclined plane moves horizontally towards right with acceleration A.



Considering the vertical component of forces acting on the particle.

$mg - R\cos\theta = ma\sin\theta$ ...(i)

For horizontal components the motion of the particle is considered at the accelerated frame. So a horizontal fictious force acts opposite to A. This pseudo force mA acts towards the left.



$R\sin\theta + mA = ma\cos\theta$

$R\sin\theta = m(a\cos\theta - A)$ ...(ii)

Now consider the force on inclined plane. The forces are shown in figure.

$R\sin\theta = MA$ ...(iii)

$R\cos\theta + Mg = S$ ...(iv)

From equations (ii) and (iii), we get $A = \frac{ma\cos\theta}{(M+m)}$

74. (a) $bt\cos\alpha = m\frac{dv}{dt}$ ...(i)

$N + bt\sin\alpha = mg$ ...(ii)

For breaking off $N = 0$

Therefore from (ii), $t = \frac{mg}{b\sin\alpha} = t_0$(say)

From (i), $m\int_0^v dv = (b\cos\alpha)\int_0^{t_0} t\,dt$

$$mv = (b\cos\alpha)\frac{t_0^2}{2} = \frac{(b\cos\alpha)}{2}\frac{m^2g^2}{b^2\sin^2\alpha}$$

$$v = \frac{mg^2\cos\alpha}{2b\sin^2\alpha}$$

75. (d) Force acting on plate, $F = \frac{dp}{dt} = v\left(\frac{dm}{dt}\right)$

Mass of water reaching the plate per second,

$$\frac{dm}{dt} = Av\rho = A(v_1 + v_2)\rho = \frac{V}{v_2}(v_1 + v_2)\rho$$

($v = v_1 + v_2$ = velocity of water coming out of jet w.r.t. plate)

(A = Area of cross-section of jet $= \frac{V}{v_2}$)

$$\therefore\quad F = \frac{dm}{dt}v = \frac{V}{v_2}(v_1 + v_2)\rho \times (v_1 + v_2)$$

$$= \rho\left[\frac{V}{v_2}\right](v_1 + v_2)^2$$

76. (c) As no friction is involved, the tensions in the segments AC and AE of the string must be the same. Let its magnitude be T. For the ring A to be at rest on the smooth loop, the resultant force on it must be along AO, O being the center of the loop; otherwise there would be a component tangential to the loop.

Hence

$\angle OAE = \angle OAC = \angle AOE = 30°$.

The same argument applies to the segments BD and BE. Then by symmetry the point E at which the string carries the third weight must be on the radius HO, H being the highest point of the loop, and the tensions in the segments BD and BE are also T.

Now consider the point E. Each of the three forces acting on it, which are in equilibrium, are at an angle of 120° to the adjacent one. As two of the forces have magnitude T, the third force must also have magnitude T. Therefore, the three weights carried by the string are equal.

77. (d) FBD of C

$mg - 2T = mc$ ...(i)

If we assume acceleration of A to be 'a' upwards; by constraint equation we get:

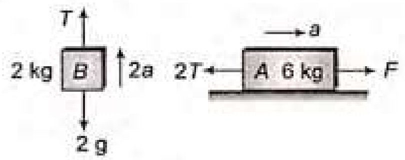
$-a - a + c + a + c = 0 \Rightarrow a = 2c$

and for block A:

$2T = m_A.a$ but as $m_A = 0 \Rightarrow T = 0$

so we get $c = g$ and $a = 2g\uparrow$.

78. (b) $a = \dfrac{v^2}{2s} = \dfrac{25}{10} = 2.5\ \text{m/s}^2$

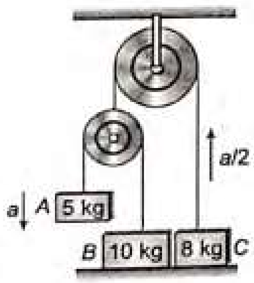


For 6 kg: $-F - 2T = 6a$ ...(1)

For 2 kg: $-T - 2g = 2\,(2a)$ ...(2)

From (1) and (2), $F = 75$ N

79. (d) Block B will not move.



$5g - T = 5a$ ...(1)

$2T - 8g = 8\dfrac{a}{2}$ ...(2)

Solving eqns. (1) and (2), we get

$2g = 14a$ or $a = \dfrac{g}{7}$

$\therefore\quad \dfrac{a}{2} = \dfrac{g}{14} = \dfrac{10}{14} = \dfrac{5}{7}\ \text{m/s}^2$

80. (a) Initially acceleration of B and A is same along the string which is given by

$$a = \frac{2\,mg\sin 30° - mg\sin 30°}{3m} = \frac{g}{6}$$

After 2 seconds, speed of both A and B is

$$v_B = v_A = \frac{g}{6} \times 2 = \frac{10}{3}\ \text{m/s}$$

Now, when B is caught for a moment and released again, speed of B becomes zero, while A is still having a speed $\dfrac{10}{3}$ m/s up the inclined due to which string will become slack. But A will decelerate and B will accelerate. Because of this the string will become tight again after A and B travel the same distance.

Let this time interval be t

$$\frac{10}{3} \times t - \frac{1}{2} g \sin 30° \times t^2 = \frac{1}{2} g \sin 30° \times t^2$$

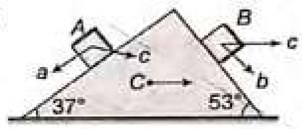
$$\Rightarrow\quad \frac{10}{3} = \frac{g}{2} t \text{ as } t \neq 0$$

$$t = \frac{2}{3}\text{ sec}$$

At this time speed of A is given by

$$v = \frac{10}{3} - \frac{g}{2} \times t = \frac{10}{3} - \frac{10}{2} \times \frac{2}{3} = 0$$

81. (a)



Let c be acceleration of wedge C.

a be acceleration of block A w.r.t. wedge c.

b be acceleration of block B w.r.t. wedge c.

Applying Newton's law in horizontal direction to system of $A + B + C$.

$mc + m\,(c - a\cos 37°) + m\,(c + b\cos 53°) = 0$ ...(1)

Applying Newton's law to block A and B along the incline gives.

In case of A: $mg\sin 37° = m\,(a - c\cos 37°)$ ...(2)

In case of B: $mg\sin 53° = m\,(b + c\cos 53°)$ ...(3)

Solving (1), (2), and (3), we get $c = 0$

ARCHIVES

1. (c) The monkey moves downwards with acceleration a.

From F.B.D of monkey; $mg - T = ma$

Tension in string $= m(g - a)$

This should not exceed breaking strength of the rope i.e., $360 \geq m(g - a)$

$\Rightarrow\quad 360 \geq 60(10 - a)$

$\therefore\quad a \geq 4\ \text{m/s}^2$

2. (a) For equilibrium of system,

$$F_1 = \sqrt{F_2^2 + F_3^2} \text{ as } \theta = 90°$$

In the absence of force F_1,

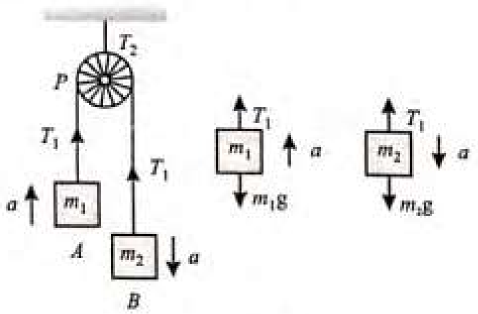
$$\text{Acceleration} = \frac{\text{Net force}}{\text{Mass}}$$

$$= \frac{\sqrt{F_2^2 + F_3^2}}{m} = \frac{F_1}{m}$$

3. (c) As the lift is descending with an acceleration a, so the man also has an acceleration a. Hence, the acceleration of the ball with respect to the man in the lift will be $(g - a)$ and with respect to the stationary man on the ground will be g.

4. (b) From F.B.D of m_1: $T - m_1g = m_1a$ ...(i)

From F.B.D of m_2: $m_2g - T = m_2a$ ...(ii)



From (i) and (ii), $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$

$$\Rightarrow \quad \frac{g}{8} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g \Rightarrow \frac{m_2}{m_1} = \frac{9}{7}$$

5. (b) $A + B = 18$ (i)

$$12 = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{(ii)}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

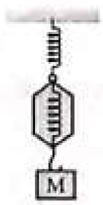
$$\Rightarrow \quad \tan 90° = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \quad \cos\theta = -\frac{A}{B} \quad \text{(iii)}$$

Solving (i), (ii), and (iii), we get

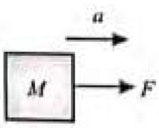
$A = 5$ N, $B = 13$ N

6. (b) The springs are connected in series and the spring balance are massless therefore both the scales read M kg each.



7. (a) If we take block and rope as system

Acceleration of the system $= \frac{P}{m + M}$

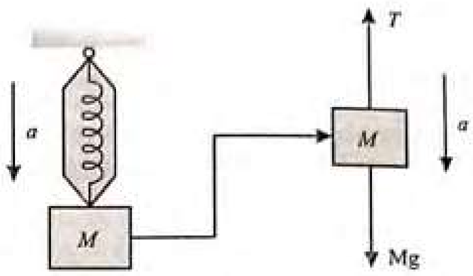


The force exerted by rope on the mass,

$$F = Ma = M\left(\frac{P}{m + M}\right)$$

8. (b) When the lift is stationary, $W = Mg$

$\Rightarrow \quad 49 = M \leftrightarrow 9.8 \Rightarrow M = 5$ kg



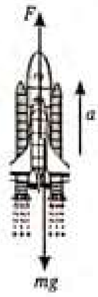
When the lift is moving downwards with an acceleration,

$Mg - T = Ma$

$\Rightarrow \quad T = M(g - a)$

$T = 5[9.8 - 5] = 24$ N

9. (c) From F.B.D of rocket,



$F - mg = ma$

$\Rightarrow \quad F = m(g + a)$

Hence, initial thrust must be

$F = m[g + a] = 3.5 \times 10^4(10 + 10)$

$= 7 \times 10^5$ N

10 (d) u = velocity of bullet

$\frac{dm}{dt}$ = Mass fired per second by the gun

$\frac{dm}{dt}$ = Mass of bullet (m_B) × Bullets fired per sec (N)

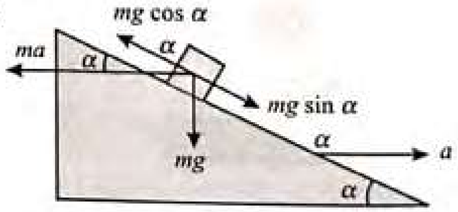
Maximum force that man can exert, $F = u\left(\frac{dm}{dt}\right)$

$\therefore \quad F = u \times m_B \times N$

$$\Rightarrow \quad N = \frac{F}{m_B \times u} = \frac{144}{40 \times 10^{-3} \times 1200} = 3$$

11. (c) $a = \frac{m_1 - m_2}{m_1 + m_2}g = \left(\frac{5 - 4.8}{5 + 4.8}\right)9.8 = 0.2 \text{ m/s}^2$

12. (a)



Let the mass of a block is m. It will remain stationary if forces acting on it are in equilibrium i.e.,

$ma\cos\alpha = mg\sin\alpha \Rightarrow a = g\tan\alpha$

Here ma = Pseudo force on block, mg = Weight.

13. (b) The force acting on particle, $F = -kx$

Force on particle at 20 cm away, $F = kx$

$F = 15 \times 0.2 = 3$ N [As $k = 15$ N/m]

$$\therefore \quad \text{Acceleration} = \frac{\text{Force}}{\text{Mass}} = \frac{3}{0.3} = 10 \text{ m/s}^2$$

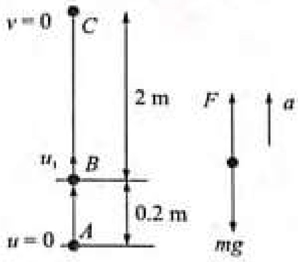
14. (a) We define average force as:

$$\left|\vec{F}_{\text{average}}\right| = \left|\frac{\Delta\vec{P}}{\Delta t}\right| = \frac{P_{\text{final}} - P_{\text{initial}}}{\text{duration}}$$

$$= \left|\frac{(0 - 0.150 \times 20)}{0.1}\right| = 30 \text{ N}$$

15. (b) From B to C, $v^2 = u_1^2 - 2 \times 10 \times 2$

$\Rightarrow \quad 0 = u_1^2 - 40 \quad \Rightarrow \quad u_1 = \sqrt{40}$ m/s



Let from A to B let the acceleration be a.

$u_1^2 = u^2 + 2a \times 0.2$

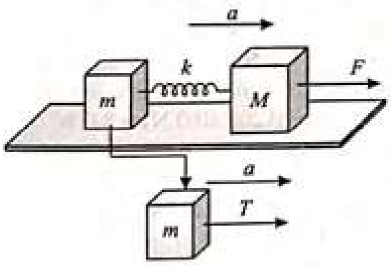
$\Rightarrow \quad 40 = 0^2 + 0.4a$

$\Rightarrow \quad a = 100 \text{ m/s}^2$

Now $F - mg = ma$

$\Rightarrow \quad F = m(g + a) = 0.2\,[10 + 100] = 22$ N

16. (b) Acceleration of the system, $a = \dfrac{F}{m+M}$



Force on block of mass m,

$$T = ma = m\left(\frac{F}{m+M}\right)$$

17. (d) Momentum is mv.

$m = 3.513$ kg; $v = 5.00$ m/s

$\therefore\ mv = 17.57 \text{ ms}^{-1}$

Because the values will be accurate up to second decimal place only, $17.565 = 17.57$.

18. (b) From the graph, it can be seen that it is a straight line. So the motion is uniform. Because of impulse, the direction of velocity changes as can be seen from the slope of the graph.

Initial velocity $= \dfrac{2}{2} = 1$ m/s

Final velocity $= -\dfrac{2}{2} = -1$ m/s

$\vec{P}_i = 0.4$ N-s

$\vec{P}_{f_i} = -0.4$ N-s

$\vec{J} = \vec{P}f - \vec{P}i = -0.4 = 0.8$ N-s $\quad (\vec{J} = \text{impulse})$

$|\vec{J}| = 0.8$ N-s

19. (d) $mg \sin\theta = ma$

$\therefore \quad a = g\sin\theta$,

where a is along the inclined plane

Therefore, the vertical component of acceleration is $g\sin^2\theta$.

Hence, the relative vertical acceleration of A with respect to B is

$$g\,[\sin^2 60 - \sin^2 30] = \frac{g}{2} = 4.9 \text{ m/s}^2 \text{ in vertical direction.}$$

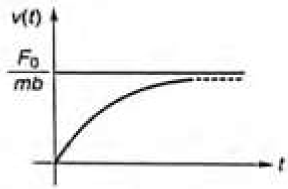
20. (c) $F = ma = F_0 e^{-bt}$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{F_0}{m} e^{-bt}$$

$$\Rightarrow \quad \int_0^v dv = \frac{F_0}{m}\int_0^t e^{-bt}\,dt$$

$$\Rightarrow \quad v = \frac{F_0}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t$$

$$\Rightarrow \quad v = \frac{F_0}{mb}(1 - e^{-bt})$$



CHAPTER 6: NEWTON'S LAWS OF MOTION (WITH FRICTION) AND DYNAMICS OF CIRCULAR MOTION

Concept Application Exercise 6.1

1. $f_l = \mu_s N = \mu_s mg = 0.3 \times 1 \times 10 = 3$ N

$f_K = \mu_K N = \mu_K mg = 0.25 \times 1 \times 10 = 2.5$ N

(a) $F = 1$ N, $F < f_l$, so the motion will not start and the friction will be static. So friction $f = F = 1$ N.

(b) $F = 2$ N, $F < f_l$, so the motion will not start and the friction will be static. So friction $f = F = 2$ N.

(c) $f = 3$ N, $F = f_l$. Here the motion is just about to start. Hence the friction force will be limiting. So friction $f = f_l = 3$ N.

(d) $F = 4$ N, $F > f_l$, so the motion will start and friction will be kinetic. Hence friction will be $f = f_K = 2.5$ N.

(e) $F = 20$ N, $F > f_l$, so motion will start and friction will be kinetic. Hence friction is $f = f_K = 2.5$ N.

2. Here $f_l = 10$ N $\Rightarrow \mu_s mg = 10$

$$\Rightarrow \quad \mu_S = \frac{10}{mg} = \frac{10}{5 \times 10} = 0.2$$

$$f_K = 8 \text{ N} \Rightarrow \mu_K mg = 8$$

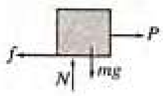
$$\Rightarrow \quad \mu_K = \frac{8}{mg} = \frac{8}{5 \times 10} = 0.16$$

3. Since the body is not moving.

So $f = P$ and $N = mg$

Net force applied by surface on the body:

$$F = \sqrt{f^2 + N^2} = \sqrt{P^2 + m^2 g^2}$$

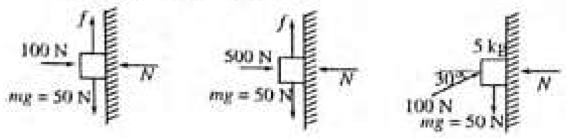


4. (a) $f_{max} = \mu N = (0.2)(100) = 20$ N

Since $mg > f_{max}$ therefore, friction force is equal to f_{max} i.e., $f = f_{max} = 20$ N

(b) $f_{max} = \mu N = 0.2 \times 500 = 100$ N

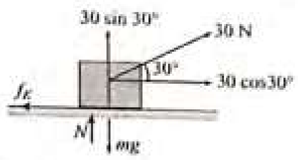
Since $mg < f_{max}$, therefore, friction force is equal to mg. This means that $f = mg = 50$ N



(c) $f_{max} = \mu N = (\mu 100 \cos 30°) = (0.2)100\frac{\sqrt{3}}{2} = 10\sqrt{3}$ N

Since the vertical component of the external force 100 sin 30° = 50 N can balance the weight of the block, therefore $f = 0$.

5.



$$N = mg - 30 \sin 30°$$

As the speed is constant, so the acceleration is zero.

(a) Hence friction $f_K = 30 \cos 30° = 15\sqrt{3}$ N

(b) $f_K = 30\cos 30° = \mu_K (mg - 30 \sin 30°)$

Solving, we get $\mu_K = 3\sqrt{3}/7$

6.

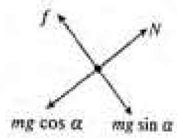
$$N = mg \cos\theta$$
$$f_K = \mu_K N = \mu_K mg \cos\theta$$

Hence, required force is $\sqrt{f_K^2 + N^2} = 6\sqrt{13}$ N

7. Apply $a = g \sin 60° - \mu_K g \cos 60°$ and get $\mu_K = \sqrt{3} - 1$

8. From FBD, friction $f = mg \sin\alpha$

$N = mg \cos\alpha$, $\frac{f}{N} = \tan\alpha$



For the maximum possible value of α, the friction becomes limiting friction, so

$\mu = \tan\alpha \Rightarrow \cot\alpha = 3$

9. (a) If no horizontal force is applied, no friction force is needed to keep the box in equilibrium.

(b) The maximum static friction force is,

$\mu_s N = \mu_s W = (0.40)(40.0) = 16.0$ N

so the box will not move and the friction force balances the applied force of 6.0 N. Hence friction = 6 N

(c) The maximum friction force found in part (b), 16.0 N.

(d) From $f_k = \mu_k N = (0.20)(40.0 \text{ N}) = 8.0$ N.

(e) The applied force is enough either to start the box moving or to keep it moving. The answer to part (d) is independent of speed (as long as the box is moving), so the friction force is 8.0 N. The acceleration is $(F - f_k)/m = 2.5$ m/s^2

10. $T = Mg$ (T = tension in the rope)

$N = 60\,g - T \sin 60°$ (N is normal reaction between the man and the ground) and $T \cos 60° = \mu N$

Solving these three equations with proper substitutions, we get

$$M = \frac{120}{\sqrt{3} + 2} \text{ kg}$$

Concept Application Exercise 6.2

1. $a_1 = -\frac{\mu m_1 g}{m_1} = -\mu g$ and $a_2 = \frac{\mu m_1 g}{m_2}$

$\therefore \quad a_{rel} = a_1 - a_2 = -\mu g - \mu\frac{m_1 g}{m_2} \Rightarrow a_{rel} = -5 - 0.5\frac{1}{2} \times 10$

$$a_{rel} = -5 - \frac{5}{2} = -7.5 \text{ ms}^{-2}$$

$$v_{rel} = 0,\ u_{rel} = v_0$$

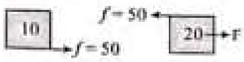
$\therefore \quad v_{rel} = u_{rel} + a_{rel} t$ or $0 = v_0 - 7.5\,t$

$\therefore \quad t = \frac{v_0}{7.5} = \frac{10}{7.5} = \frac{4}{3}$ s

2. At just sliding condition limiting friction is acting.

$F - 50 = 20a$ (i)

$f = 10a$ (ii)



$50 = 10a$

$\therefore \quad a = 5\ \text{m s}^{-2}$

Hence, $F = 50 + 20 \times 5 = 150$ N

$\therefore \quad F_{min} = 150$ N

3. $f_{max} = \mu N = 50$ N (available friction)

If move together,

$$a = \frac{50}{10+10} = 2.5\ \text{m s}^{-2}$$

Check friction for B:

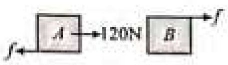
$f = 10 \times 2.5 = 25$

25 N is required which is less than available friction hence they will move together and $a_A = a_B = 2.5\ \text{m s}^{-2}$

4. $f_{max} = 50$ N

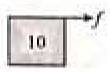
If they move together

$$a = \frac{120}{20} = 6\ \text{m s}^{-2}$$



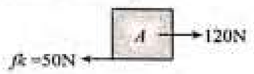
Check friction on B

$f_B = 10 \times 6 = 60$ N



60N > 50N ($\therefore$ required > available)

Hence, they will not move together.



Hence, they move separately so kinetic friction is involved.

$$\therefore \quad \text{for } a_A = \frac{120-50}{10} = 7.0\ \text{m s}^{-2}$$

$$\Rightarrow \quad a_B = \frac{50}{10} = 5\ \text{m s}^{-2}$$

5. $a = \dfrac{60}{30} = 2\ \text{m s}^{-2}$ No need to calculate.

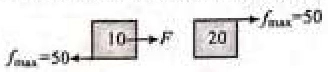
Check friction on 20 kg.

$f = 20 \times 2 = 40$ (which is required)

$40 < 50$ (therefore required < available)

Therefore, will move together.

Observing the critical situation where friction becomes limiting.



$\therefore \quad F - f_{max} = 10a$ (1)

$f_{max} = 20a$ (2)

$\therefore \quad F = 75$ N

6. For the block A, $f_{max} = ma_{max}$

a_{max} of A = maximum acceleration of plank in order to have no slipping between A and B.

$\therefore \quad f_{max} = \mu mg \qquad \therefore \ \mu mf = ma_{max}$

$$\therefore \quad \mu = \frac{a_{max}}{g} = \frac{2}{10} = 0.2$$

7. Let a_1 and a_2 be the retardation of m and M, respectively.

$\Rightarrow$

$a_{rel.} = |a_1 - a_2|$,

$$\text{Where} \quad a_1 = \frac{\mu/2mg}{m} = \frac{\mu}{2}g$$

$$\text{and} \quad a_2 = \frac{\mu(M+m)g - (\mu/2)\,mg}{M}$$

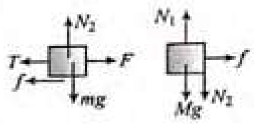
The relative acceleration between the blocks

$$= a_{rel} = |a_2 - a_1| = \frac{\mu(M+m)g}{2M}$$

Now, $l = \dfrac{1}{2}a_{rel}t^2$, where t = time of sliding of m over M.

$$\Rightarrow \quad t = \sqrt{\frac{2l}{a_{rel.}}} = \sqrt{\frac{4ml}{(M+m)\mu g}}$$

8. The force diagram and the FBD are shown in the figure respectively taking the ground as reference frame.



$N_2 = mg$ (i)

$F - T - f = 0$ (ii)

From figure,

$N_1 = N_2 + Mg$ (iii)

$T - f = 0$ (iv)

Again $f_{max} = \mu N_2$

i.e. $f_{max} = \mu mg$ (v)

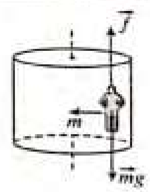
For m not to slide relatively on M,

$f \le f_{max}$ (vi)

Using Eq. (i) to (v) and the condition (vi), F_{max} can be found out as, $F_{max} = 2\mu mg$.

Concept Application Exercise 6.3

1. (a) $N = \dfrac{mv^2}{R}$, $f - mg = 0$, $f = \mu_s N$, $v = \dfrac{2\pi R}{T}$



Solving, we get $T = \sqrt{\dfrac{4\pi^2 R\mu_s}{g}}$

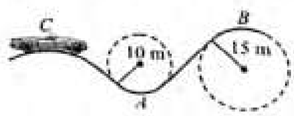
(b) $T = 0.8\pi$ s,

$$\frac{\text{rev}}{\text{min}} = \frac{60}{T} = \frac{60}{0.8\pi} = \frac{75}{\pi}\ \text{rev min}^{-1}$$

(c) The gravitational and the frictional forces remain constant. The normal force increases. The person remains in motion with the wall.

(d) The gravitational force remains constant. The normal and the frictional forces decrease. The person slides relative to the wall and downward into the pit.

2. (a) $v = 20.0\ \text{m s}^{-1}$, N = force of track on roller coaster, and $R = 10$ m.



$$N = Mg + \frac{Mv^2}{R}$$

$$= (500\ \text{kg})(10\ \text{m s}^{-2}) + \frac{(500\ \text{kg})(20\ \text{m/s})^2}{10.0\ \text{m}}$$

$N = 5000$ N + 20,000 N = 2.50×10^4 N

(b) $v_{max} = \sqrt{Rg} = \sqrt{15.0(10.0)} = 5\sqrt{6}\ \text{m s}^{-1}$

3. (a) Since the object of mass m_2 is in equilibrium,

$$\sum F_y = T - m_2 g = 0 \quad \text{or} \quad T = m_2 g$$

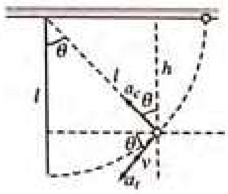
(b) The tension in the string provides the required centripetal acceleration of the puck.
Thus, $F_c = T = m_2 g$.

(c) From $F_c = \dfrac{m_1 v^2}{R}$, we have $v = \sqrt{\dfrac{RF_c}{m_1}} = \sqrt{\left(\dfrac{m_2}{m_1}\right) gR}$

(d) The puck will spiral inward, gaining speed as it does so. It gains speed because the extra-large string tension produces forward tangential acceleration as well as inward radial acceleration of the puck, pulling at an angle of less than 90° to the direction of the inward spiraling velocity.

(e) The puck will spiral outward, slowing down as it does so.

4. $v = \sqrt{2gh} = \sqrt{2gl\cos\theta}$, $a_t = g\sin\theta$



$$a_c = \frac{v^2}{l} = 2g\cos\theta$$

Net acceleration will be horizontal if the vertical components of a_c and a_t cancel each other.
For this, $a_c\cos\theta = a_t\sin\theta$

$\Rightarrow 2g\cos^2\theta = g\sin^2\theta$

$\Rightarrow \tan\theta = \sqrt{2}$

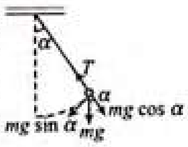
5. (a) Force on the particle when the fan is at full speed

$$F_n = m\omega^2 R = \left(\frac{1}{1000}\right)\left(\frac{2\pi\times 1500}{60}\right)^2\left(\frac{60}{100}\right) = \frac{3}{2}\pi^2 \text{ N}$$

(b) The friction force exerts this force.

(c) $F_{\text{particle}} = F_n = \dfrac{3}{2}\pi^2$

6. (a) The forces acting on the bob are:
i. the tension T
ii. the weight mg
As the bob moves in a circle of radius L with centre at O. A centripetal force of magnitude mv^2/L is required towards O. This force will be provided by the resultant of T and $mg\cos\alpha$.



Thus, $T - mg\cos\alpha = \dfrac{mv^2}{L}$

$$T = m\left(g\cos\alpha + \frac{v^2}{L}\right)$$

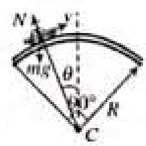
(b) $a_{\text{net}} = \sqrt{a_t^2 + a_r^2} = \sqrt{(g\sin\alpha)^2 + \left(\dfrac{v^2}{l}\right)^2}$

$$|\vec{F}_{\text{net}}| = ma_{\text{net}} = m\sqrt{g^2\sin^2\alpha + \frac{v^4}{L^2}}$$

7. Let the car losses the contact at angle θ with the vertical

$$mg\cos\theta - N = \frac{mv^2}{R}$$

$$N = mg\cos\theta - \frac{mv^2}{R} \qquad (1)$$



For losing the contact, $N = 0$

$\Rightarrow \quad v = \sqrt{Rg\cos\theta}$ [from Eq. (i)]

For minimum speed, cos θ should be minimum so that θ should be maximum.

$$\theta_{\max} = 45° \Rightarrow \cos 45° = \frac{1}{\sqrt{2}}$$

$$v_{\min} = \left(\frac{Rg}{\sqrt{2}}\right)^{1/2}$$

So that if car cannot lose the contact at initial or final point, car cannot be lose the contact anywhere.

EXERCISES

Basic Concept of Static and Kinetic Friction

1. (a) During downward motion:
$F = mg\sin\theta - mg\cos\theta$
During upward motion:
$2F = mg\sin\theta + \mu mg\cos\theta$
Solving above two equations, we get
$m = (\tan\theta)/3$

2. (c) The minimum force required to just move a body will be $f_1 = \mu_s mg$. After the motion is started, the friction will become kinetic. So the force which is responsible for the increase in velocity of the block is

$$F = (\mu_s - \mu_k)mg = (0.8 - 0.6)\times 4\times 10 = 8 \text{ N}$$

So $\quad a = \dfrac{F}{m} = \dfrac{8}{4} = 2 \text{ ms}^{-2}$

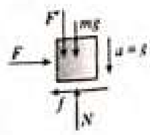
3. (b) Retardation of train = 20/4 = 5 ms^{-2}
It acts in the backward direction. Fictious force on suitcase = $5m$ Newton, where m is the mass of suitcase.
In acts in the forward direction. Due to this force, the suitcase has a tendency to slide forward. If suitcase is not to slide, then $5m$ = force f of friction

or $\quad 5m = m\,mg \quad$ or $\quad m = \dfrac{5}{10} = 0.5$

4. (d) Force of friction is independent of the area of contact.

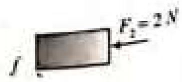
5. (c) When free fall of lift normal reaction on block by lift floor will be zero. Hence no friction.

6. (d) Here normal reaction $N = F' = 100$ N
$(F' + mg - N = mg)$
Hence $f_{\lim} = \mu_s N = 0.1\times 100 = 10$ N
Here F (driving force) = 5 N
Hence, block will not move friction will be static.
Hence, $f = 5$ N.



7. (c) The driving force on the block
$F_1 - F_2 = 10 - 2 = 8$ N

As the block is at rest the friction will be static and towards left.
$f = f_s = 8$ N
If F_1 is removed only $F_2 = 2$ N is acting on the block.
If the block is at rest $f = F_2 = 2$ N this value of friction is within static range as in previous case even $f = 8$ N is static case.



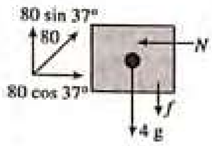
8. (a) Zero. No driving force. So friction is zero.

9. (c) Given horizontal force $F = 25$ N and coefficient of friction between block and wall $(\mu) = 0.4$.
We know that at equilibrium horizontal force provides the normal reaction to the block against the wall. Therefore, normal reaction to the block $(R) = F = 25$ N.
We also know that weight of the block (W) = Frictional force $= \mu R = 0.4 \times 25 = 10$ N.

10. (d) As there is no tendency of relative slipping between the block and cube, the friction force is zero.

11. (a) $\tan\theta = v^2/Rg \Rightarrow \dfrac{h}{b} = v^2/Rg \Rightarrow h = \dfrac{v^2 b}{Rg}$

12. (a) The FBD of the block is as shown in the figure.



$N = 80\cos 37° = 64$ N
So, $f_L = 0.2 \times 64 = 32$ N
As $4g < 80\sin 37°$, friction force will act downwards. Net applied force in upward direction (excluding friction force) is
$80\sin 37° - 40 = 48 - 40 = 8$ N
As F_{applied} in vertical direction is less than f_L, block will not move in vertical direction and value of static friction force is $f = 8$ N.

13. (b) Frictional force $= \mu R = \mu(mg + Q\cos\theta)$ and horizontal push $= P - Q\sin\theta$
For equilibrium, we have

$$\mu(mg + Q\cos\theta) = P - Q\sin\theta \Rightarrow \mu = \frac{P - Q\sin\theta}{mg + Q\cos\theta}$$

14. (c) From $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2, t = \sqrt{\frac{2s}{a}}$

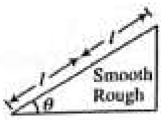
For smooth plane $a = g\sin\theta$
For rough plane, $a' = g(\sin\theta - \mu\cos\theta)$

$$\therefore \quad t' = nt \Rightarrow \sqrt{\frac{2s}{g(\sin\theta - \mu\cos\theta)}} = n\sqrt{\frac{2s}{g\sin\theta}}$$

$\therefore \quad n^2 g(\sin\theta - \mu\cos\theta) = g\sin\theta$
When $\theta = 45°$, $\sin\theta = \cos\theta = 1/\sqrt{2}$

Solving, we get $\mu = \left(1 - \dfrac{1}{n^2}\right)$

15. (a) For first half acceleration $= g\sin\phi$
Therefore, velocity after travelling half distance



$v^2 = 2(g\sin\phi)l$ (i)
For second half, acceleration
$= g(\sin\phi - \mu_k\cos\phi)$

So $0^2 = v^2 + 2g(\sin\phi - \mu_k\cos\phi)l$ (ii)

Solving (i) and (ii), we get $\mu_k = 2\tan\phi$

16. (a) If the plane makes an angle θ with horizontal, then $\tan\theta = 8/15$. If R is the normal reaction,

$$R = 170g\cos\theta = 170 \times 10 \times \left(\frac{15}{17}\right) = 1500 \text{ N}$$

Force of friction on $A = 1500 \times 0.2 = 300$ N
Force of friction on $B = 1500 \times 0.4 = 600$ N
Considering the two blocks as a system, the net force parallel to the plane is
$= 2 \times 170g\sin\theta - 300 - 600 = 1600 - 900 = 700$ N

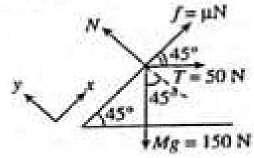
$\therefore$ Acceleration $= \dfrac{700}{340} = \dfrac{35}{17}\text{ m s}^{-2}$

Consider the motion of A alone.

$$170\,g\sin\theta - 300 - P = 170 \times \frac{35}{17}$$

(where P is pull on the bar)
$P = 500 - 350 = 150$ N

17. (a) The string is under tension. Hence, there is limiting friction between the block and the plane (see figure)



$$\sum F_x = 0$$

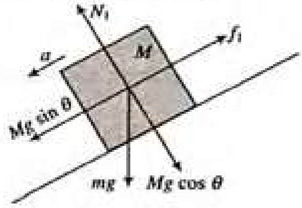
$\Rightarrow \quad \mu N + 50\cos 45° = 150\sin 45°$ (i)

$$\sum F_y = 0$$

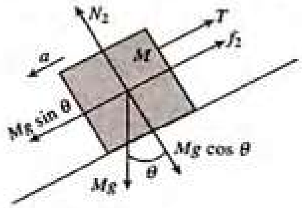
$\Rightarrow \quad N = 50\sin 45° + 150\cos 45°$ (ii)

Solving Eqs. (i) and (ii), we get $m = 1/2$

18. (c) $N_1 = mg\cos\theta$ and $f_1 = \mu\, mg\cos\theta$



$N_2 = Mg\cos\theta$ and $f_2 = \mu\, Mg\cos\theta$



Equations of motion are
$T - f_1 + mg\sin\theta = ma$ (i)
$Mg\sin\theta - T - f_2 = Ma$ (ii)
Solving Eqs. (i) and (ii), we get $T = 0$

19. (c) As sand particles are sliding down, the slope of the hill gets reduced. The sand particle stops coming down when component of gravity force along the hill is balanced by limiting friction force.
$mg\sin\theta = \mu_s\, mg\cos\theta$
$\Rightarrow \quad \theta = \tan^{-1}(\mu_s) = 37°$
where θ is the new slope angle of hill.

20. (b) For equilibrium,

$$\int \mu\, dmg\cos\theta \geq \int dmg\sin\theta$$

or $\displaystyle\int \mu\lambda\, dl\, g\cos\theta \geq \int \lambda\, dl\, g\sin\theta$

$$\mu\int dl\cos\theta \geq \int dl\sin\theta \left(\because \sin\theta = \frac{dy}{dl}, \cos\theta = \frac{dx}{dl}\right)$$

or $\mu\displaystyle\int dx \geq \int dy$ or $\mu l \geq h$ or $\mu_{\min} = \dfrac{h}{l}$

Application of Newton's Laws of Motion with Friction

21. **(d)** Here the force applied should be such that friction force acting on the upper block of m should not be more than the limiting friction ($= \mu_1 mg$). Let the system moves with acceleration a. Then for whole system,

$$F - \mu_2(M+m)g = (M+m)a \qquad \text{(i)}$$

For block of mass m,

$$f_1 = ma \quad \text{or} \quad \mu_1 mg = ma \quad \text{or} \quad a = \mu_1 g \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$F = (M+m)g(\mu_1 + \mu_2)$$

22. **(b)** For the equilibrium of block of mass M_1:

Frictional force, f = tension in the string, T

where $T = f = \mu(m + M_1)g$ (i)

For the equilibrium of block of mass M_2:

$T = M_2 g$ (ii)

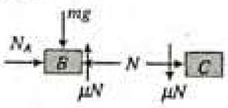
From Eqs. (i) and (ii), we get $\mu(m + M_1)g = M_2 g$

$$m = \frac{M_2}{\mu} - M_1$$

23. **(b)** Horizontal acceleration of the system is

$$a = \frac{F}{2m + m + 2m} = \frac{F}{5m}$$

Let N be the normal reaction between B and C. Free-body diagram of C gives



$$N = 2ma = \frac{2}{5}F$$

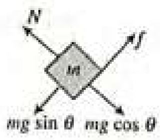
Now B will not slide downwards if $\mu N \geq m_B g$

$$\text{or} \quad \mu\left(\frac{2}{5}F\right) \geq mg \quad \text{or} \quad F \geq \left(\frac{5}{2\mu}\right)mg$$

$$\text{or} \quad F_{\min} = \left(\frac{5}{2\mu}\right)mg$$

24. **(a)** $N = mg\cos\theta$, $f = mg\sin\theta$

Net force applied by M on m (or m on M):



$$F = \sqrt{N^2 + f^2}$$

$$= \sqrt{(mg\cos\theta)^2 + (mg\sin\theta)^2}$$

$$= mg$$

25. **(b)** The friction force *on* block A is $\mu_k w_A = (0.25)(2\text{ N}) = 0.5\text{ N}$. This is the magnitude of the friction force that block A exerts on block B, as well as the tension in the string. The force F must then have magnitude

$$F = \mu_k(w_B + w_A) + \mu_k w_A + T$$

$$= \mu_k(w_B + 3w_A) = (0.25)(6\text{ N} + 3(2\text{ N})) = 3\text{ N}.$$

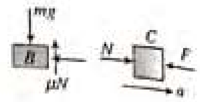
Note that the normal force exerted on block B by the table is the sum of the weights of the blocks.

26. **(b)** Horizontal acceleration of the system is

$$a = \frac{F}{2m + m + 2m} = \frac{F}{5m}$$

Let N be the normal reaction between B and C.

Free body diagram of C gives



$$F - N = 2ma$$

$$N = \frac{3F}{5}$$

Now B will not slide downward if

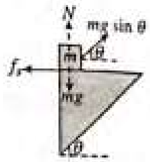
$$\mu N \geq m_B g$$

$$\text{or} \quad \mu\left(\frac{3}{5}F\right) \geq mg \quad \text{or} \quad F \geq \frac{5}{3\mu}mg$$

$$\text{So} \quad F_{\min} = \frac{5}{3\mu}mg$$

27. **(a)** Coefficient of friction is independent of the normal reaction. Hence, it will remain same.

28. **(d)** As the block does not slip on prism, the combined acceleration of the prism is $a = g\sin\theta$.



$mg\sin\theta$ is the pseudo force on m.

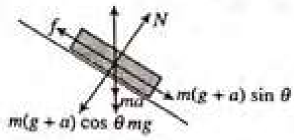
$N + mg\sin\theta\sin\theta = mg$

or $N = mg\cos^2\theta$

And for no slipping, $mg\sin\theta\cos\theta \leq \mu N$

$mg\sin\theta\cos\theta \leq \mu\, mg\cos^2\theta$ or $\mu \geq \tan\theta$

29. **(b)** In the figure, $F_c = \sqrt{f^2 + N^2} = m(g+a)$

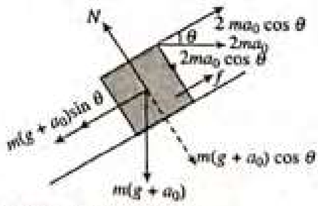


$$f = m(g+a)\sin\theta,\ N = m(g+a)\cos\theta$$

Net contact force: $F_C = \sqrt{f^2 + N^2} = m(g+a)$

30. **(d)** The FBD of block from lift frame is as shown in the figure.

From given data, as $m(g + a_0)\ \sin\theta > 2\,ma_0\cos\theta$



So friction force acts upwards.

$$f = m(g + a_0)\sin\theta - 2ma_0\cos\theta$$

$$= \frac{9g}{10} - \frac{4mg}{5} = \frac{mg}{10}$$

$$N = m(g + a_0)\cos\theta + 2ma_0\sin\theta = \frac{9mg}{5}$$

As $f_L = \mu_s N = \dfrac{18mg}{50} = \dfrac{9mg}{25} > f$ so static friction.

Reaction force,

$$R = \sqrt{f^2 + N^2} = \frac{mg}{5}\sqrt{\frac{1}{4} + 9^2} = \frac{mg\sqrt{13}}{2}$$

Alternative solution:

Net force, $\vec{F} - mg\hat{i} = m(a_0\hat{j} - 2a_0\hat{i})$

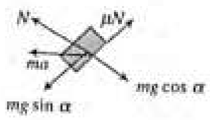
$$\Rightarrow \quad \vec{F} = m(-2a_0\hat{i} + (a_0 + g)\hat{j}) = m(-g\hat{i} + \frac{3g}{2}\hat{j})$$

$$\Rightarrow \quad F = m\sqrt{g^2 + \left(\frac{3g}{2}\right)^2} = \frac{\sqrt{13}mg}{2}$$

31. (b) FBD of m in frame of wedge,

$N = mg\cos\alpha - ma\sin\alpha$

Now $f = \mu N = ma\cos\alpha + mg\sin\alpha$



$$\mu = \frac{a\cos\alpha + g\sin\alpha}{g\cos\alpha - a\sin\alpha}$$

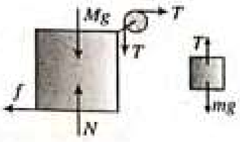
$$= \frac{a + g\tan\alpha}{g - a\tan\alpha} = \frac{5}{12}$$

32. (a) As sphere is at rest

$T = mg$ ...(i)

$f = T$ ...(ii)

and $f \le \mu N$



Here $N = Mg + T = (M + m)g$

$T \le \mu(M + m)g$ or $mg \le \mu(M + m)g$

$$\frac{m}{M} \le \frac{\mu}{(1-\mu)}$$

$$\frac{m}{M} \le \frac{0.25}{(1-0.25)} \Rightarrow \frac{m}{M} \le \frac{1}{3}$$

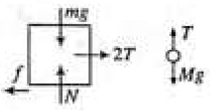
33. (a) $f = 2T$ ...(i)

and $T = Mg$ ...(ii)

If m be at rest friction will be static

$f \le \mu N$

$$2Mg \le \mu mg \Rightarrow M = \frac{\mu m}{2}$$

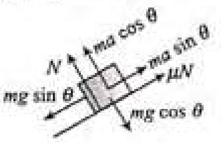


34. (a) If block is sliding with constant velocity

$mg \sin\theta = \mu N + ma \sin\theta$ ...(i)

and $N + ma\cos\theta = mg\cos\theta$

From (i), $N = m(g-a)\cos\theta$



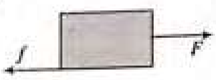
$mg\sin\theta = \mu m(g-a)\cos\theta + ma\sin\theta$

$$10 \times \frac{1}{2} = \mu(10-2)\frac{\sqrt{3}}{2} + 2 \times \frac{1}{2} \Rightarrow \mu = \frac{1}{\sqrt{3}}$$

35. (a) $f_l = \mu Mg$. If motion does not start, then

$f = F = F_0 t$

Motion will start when $f = f_l$.



$$\Rightarrow \quad F_0 T = \mu Mg \quad \Rightarrow \quad T = \frac{\mu Mg}{F_0}$$

36. (b)

(i) If both moves together $a = 2\ \text{m/s}^2$

Force required for $A = 4$ N

Max. friction force = 10 N

Hence there will be no slipping and friction force will be 4 N.

(ii) Max. Friction force = $\mu\, mg\cos\theta = 8$ N

Force along incline = $mg\sin\theta = 12$ N

Hence, block will move and friction force will be 8 N.

(iii) Max. friction force = $\mu N = 5$ N

Downward force = 20 N

Block will slip and friction force will be 5 N

(iv) Acceleration of the system = $\frac{10}{6}\ \text{m/s}^2$

Force required for $A = \frac{20}{6}$ N

Max. friction force = 10 N

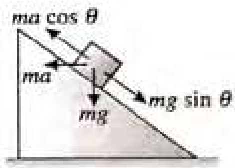
Hence A and B will move together and friction force will be $\frac{20}{6}$ N.

37. (c) Wedge will start slipping when

$F \ge \mu(2\,mg)$

38. (b) If wedge start slipping, common acceleration

$$a_C = \frac{F - 2\mu mg}{2m}$$



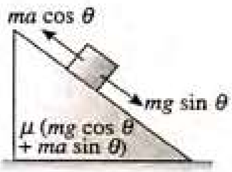
If $mg\sin\theta = ma\cos\theta$, then no force along the plane will be felt by the block and hence friction will be zero.

$$\Rightarrow \quad mg\sin\theta = m\left(\frac{F - 2\mu mg}{2m}\right)\cos\theta$$

$F = 2\,mg\tan\theta + 2\mu mg$

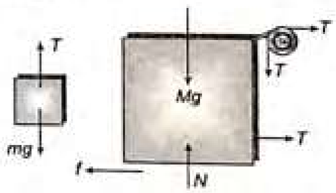
39. (b) Block will start sliding if

$ma\cos\theta \ge mg\sin\theta + \mu(mg\cos\theta + ma\sin\theta)$



get $$a > g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$$

40. (d) F.B.D. of m and M



From F.B.D.: $T = mg$ ...(i)

$2T = f \Rightarrow 2mg = f$ ...(ii)

and $N = Mg + T = Mg = mg$ ...(iii)

If M is not moving,

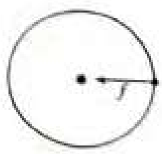
$f \le \mu N$ or $2mg \le \mu(M + m)g$

$(2m - \mu m) \le \mu M$

$$m \le \frac{\mu M}{(2-\mu)} \Rightarrow m \le \frac{0.5 \times 20}{(2-0.5)} \Rightarrow m \le \frac{20}{3}\text{kg}$$

Dynamics of Circular Motion

41. (b) $f_l = \mu\, mg$, friction will provide the necessary centripetal force $f = m\omega^2 r$

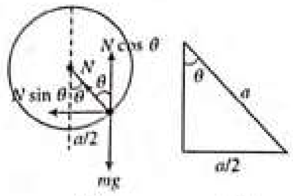


$f \le f_l \Rightarrow m\omega^2 r \le \mu mg$

$$\Rightarrow \quad \mu \ge \frac{\omega^2 r}{g} = \frac{2^2 \times 50/100}{10}$$

$\Rightarrow \quad \mu \ge 0.2$

42. (c)



(a) (b)

$$N\cos\theta = mg \quad \text{(i)}$$

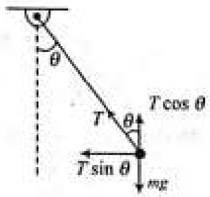
$$N\sin\theta = \frac{m\omega^2 a}{2} \quad \text{(ii)}$$

From Eqs. (ii)/(i), $\tan\theta = \frac{\omega^2 a}{2g} \Rightarrow \omega^2 = \frac{2g\tan\theta}{a}$

Now, $\sin\theta = \frac{a}{2}\times\frac{1}{a} = \frac{1}{2}$ or $\theta = 30°$

or $\omega^2 = \frac{2g}{\sqrt{3}a}$

43. (c) In conical pendulum,



$$T\cos\theta = mg = \text{constant}$$

$$\Rightarrow T_1\cos\theta_1 = T_2\cos\theta_2$$

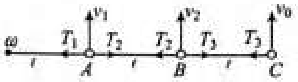
$$\frac{T_1}{T_2} = \frac{\cos\theta_1}{\cos\theta_2} = \frac{\cos 60°}{\cos 45°}$$

$$= \frac{1/2}{1/\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{1}{2}$$

44. (a) $v_0 = \omega 3l, v_2 = \omega 2l = \frac{2v_0}{3}$ and $v_1 = \omega l = \frac{v_0}{3}$

$$T_3 = m\omega^2 3l$$



$$T_2 - T_3 = m\omega^2 2l \Rightarrow T_2 = 5m\omega^2 l$$

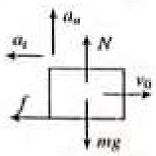
$$T_1 - T_2 = m\omega^2 l \Rightarrow T_1 = 6m\omega^2 l$$

$$T_3 : T_2 : T_1 = 3:5:6$$

$$T_1 : T_2 : T_3 = 6:5:3$$

45. (b) When the particle comes with the contact of hemisphere surface, it will slide and friction will be of kinetic nature. This kinetic friction will provide tangential acceleration to the particle.

Tangential acceleration of the particle,



$$a_t = \frac{\mu mg}{m} = \mu g = \frac{g}{2} \leftarrow$$

Normal acceleration of the particle,

$$a_n = \frac{v^2}{R} = \frac{gR}{R} = g \uparrow$$

Net acceleration of the particle,

$$a_{\text{net}} = \sqrt{a_t^2 + a_n^2} = \sqrt{\frac{g^2}{4} + g^2} = \frac{\sqrt{5}g}{2} \nwarrow$$

46. (d) As car travels with constant speed. The net force on the car should be towards the centre. The resultant of F_{air} and F_B may towards the centre.

47. (d) Change in linear momentum of block

$$|\Delta p| = 2mv$$

Time taken to move from A to B,

$$\Delta t = \frac{\pi R}{v}$$

Force exerted on the block $= \frac{2mv}{\Delta t} = \frac{2mv^2}{\pi R}$

48. (d) F is centrifugal force. Net force is along AB. So seat will be aligned along AB.

49. (d) $T = m\omega_1^2 l_1 \quad$ (i)

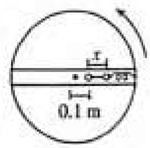
$$T\sin\theta = m\omega_2^2 l_2 \sin\theta \Rightarrow T = m\omega_2^2 l_2 \quad \text{(ii)}$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{\omega_2^2}{\omega_1^2}$$

50. (c) $v_{\max} = \sqrt{\frac{Rg(\tan\theta - \mu)}{1 - \mu\tan\theta}} = \sqrt{\frac{100\times 10(1-0.5)}{1-0.5\times 1}}$

$$= \sqrt{\frac{500}{0.5}} = 100 \text{ m/s}$$

51. (b) $k = 10^2$ N cm^{-1} = 10^4 N m^{-1}. Let the ball move distance x away from the center as shown in the figure.



$$kx = m\omega^2(0.1 + x)$$

$$\Rightarrow 10^4 x = \frac{90}{1000}\times(10^2)^2\times(0.1 + x)$$

Solve to get $x = 10^{-2}$

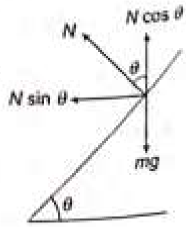
52. (c) At maximum velocity the mass will just loose contact with cone and will behave like free conical pendulum with time period

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}} \Rightarrow \omega = \frac{2\pi}{T}\sqrt{\frac{g}{L\cos\theta}}$$

Hence, $V_{\max} = (L\sin\theta)\omega = \sqrt{gL\sin\theta\tan\theta}$

53. (d) Car can make a turn without skidding at 40 km/hr.

The car will slip down the slope if it runs at a speed less than 40 km/hr.



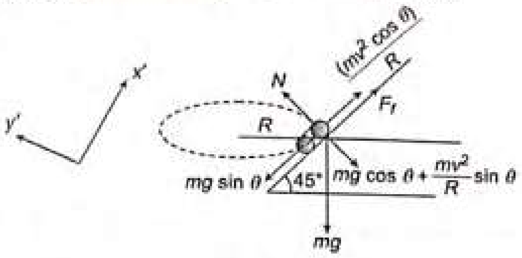
$$N\sin\theta = \frac{mv^2}{r} \Rightarrow N = \frac{1}{\sin\theta}\left(\frac{mv^2}{r}\right)$$

and $N\cos\theta = mg \Rightarrow N = \frac{mg}{\cos\theta}$

$\because$ $\sin\theta$ and $\cos\theta < 1$

So $N > mg$ and $N > \frac{mv^2}{r}$

54. (d) F.B.D. for minimum speed (w.r.t. automobile):



$$\Sigma f_{y'} = N - mg\cos\theta - \frac{mv^2}{R}\sin\theta = 0.$$

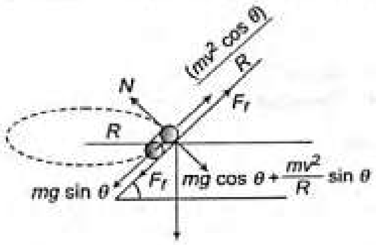
$$\Sigma f_{x'} = \frac{mv^2}{R}\cos\theta + \mu N - mg\sin\theta = 0$$

$$\Rightarrow \frac{mv^2}{R}\cos\theta + \mu\left(mg\cos\theta + \frac{mv^2}{R}\sin\theta\right) - mg\sin\theta = 0$$

$$\Rightarrow v^2 = \frac{(\mu Rg\cos\theta - Rg\sin\theta)}{(\cos\theta + \mu\sin\theta)}$$

for $\theta = 45°$ and $m = 1$; $v_{min} = \frac{Rg - Rg}{1+1} = 0$

F.B.D. for maximum speed (w.r.t. automobile)



$$\Sigma f_{x'} = \frac{mv^2}{R}\cos\theta - mg\sin\theta$$

$$-\mu\left(mg\cos\theta + \frac{mv^2}{R}\sin\theta\right) = 0$$

for $\theta = 45°$ and $\mu = 1$

$v_{max} = \infty$ (infinite)

Problems Based on Mixed Concepts

55. (b) As $\cos\theta = \frac{a}{2a}$

$\theta = 60°$

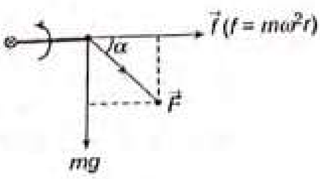
$\therefore$ $N\sin 60° = mg$

$N\cos 60° = m\frac{\omega^2 a}{2}$

$\therefore$ $\tan 60° = \frac{2g}{\omega^2 a} \Rightarrow \omega^2 = \frac{2g}{a\sqrt{3}}$

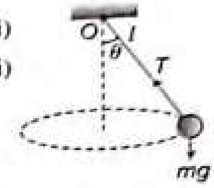
56. (c) $F = \sqrt{f^2 + (mg)^2}$

Now when the angular speed of the rod is increasing at constant rate, the resultant force will be more inclined towards $\vec{f}$.



Hence, the angle between $\vec{F}$ and horizontal plane decreases

57. (c) For conical pendulum of length l, mass m moving along horizontal circle as shown



$T\cos\theta = mg$ ....(i)

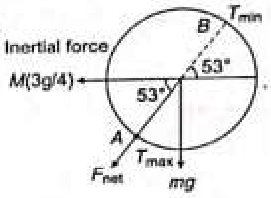
$T\sin\theta = m\omega^2 l\sin\theta$ ....(ii)

From equations (i) and (ii),

$$l\cos\theta = \frac{g}{\omega^2}$$

$l\cos\theta$ is the vertical distance of bob below O point of suspension. Hence, if ω of all three pendulums are same, they shall revolve in same horizontal plane.

58. (a)



F_{net} is shown in the figure. So, tension will be maximum at point A and will be minimum at point B.

59. (a) Retardation in upward motion $= g(\sin\theta + \mu\cos\theta)$

$\therefore$ Force required just to move up $F_{up} = mg(\sin\theta + \mu\cos\theta)$

Similarly for downward motion $a = g(\sin\theta + \mu\cos\theta)$

$\therefore$ Force required just to prevent the body sliding down

$$F_{dn} = mg(\sin\theta - \mu\cos\theta)$$

According to problem $F_{up} = 2F_{dn}$

$\Rightarrow$ $mg(\sin\theta + \mu\cos\theta) = 2mg(\sin\theta - \mu\cos\theta)$

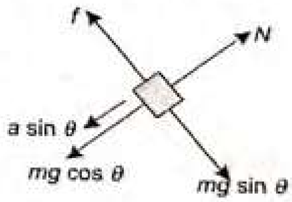
$\Rightarrow$ $\sin\theta + \mu\cos\theta = 2\sin\theta - 2\mu\cos\theta$

$\Rightarrow$ $3\mu\cos\theta = \sin\theta$ $\Rightarrow$ $\tan\theta = 3\mu$

$\Rightarrow$ $\theta = \tan^{-1}(3\mu) = \tan^{-1}(3\times 0.25) = \tan^{-1}(0.75) = 37°$

60. (c) The tension in the string initially is zero. If $\mu_1 > \tan\alpha$ and $\mu_2 > \tan\beta$, both the blocks will not move down the incline and the tension in the string shall continue to remain zero.

61. (c) The free body diagram of the block is as shown in the figure. N is the normal reaction exerted by wedge on the block. The wedge moves towards left with acceleration 'a', then the component of acceleration of block normal to the plane is 'a' $\sin\theta$.



Applying Newton's second law to the block normal to plane.

$mg\cos\theta - N = ma\sin\theta$

For N to be zero $a = g\cot\theta$

Hence, the friction shall be zero when $a = g\cot\theta$.

62. (b) For man + board system:

$f = 2T$ ...(i)

$T = N = (M+m)g$ ...(ii)

$N = (M+m)g - T$ ...(iii)

If board is not sliding $f \le \mu N$

$$2T \le \mu[(M+m)g - T]$$

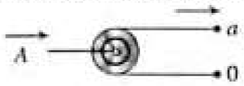
$$[2+\mu]T \le \mu(M+m)g;\ T \le \frac{\mu(M+m)g}{(2+\mu)}$$

Hence, $T_{max} = \frac{\mu(M+m)g}{(2+\mu)}$

After substituting the values, we get

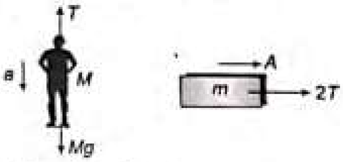
$$T_{max} = \frac{0.25(70+11)}{(2+0.25)} \times 100 = \frac{0.25 \times 81 \times 10}{2.25} = 90\,\text{N}$$

63. (a) Constraint relation: Considering movable pulley.



$$A = \frac{0+a}{2} \Rightarrow \frac{a}{A} = 2$$

64. (a) Free-body diagrams of board and man



Equation of motion of man:

$$Mg - T = Ma \quad \text{...(i)}$$

For board:

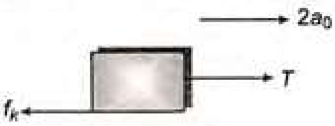
$$Ma = 2T = Ma = m\left(\frac{a}{2}\right) \quad \text{...(ii)}$$

From (i) and (ii),

$$a = \frac{4Mg}{(4M+m)}$$

$$a = \frac{4 \times 70 \times 10}{(4 \times 70 + 11)} = \frac{2800}{291}\,\text{m/s}^2$$

65. (a) Acceleration of crate will be $2a_0$.

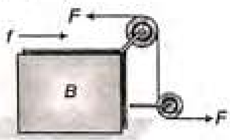


$$T - f_k = m(2a_0) \Rightarrow T - \mu mg = 2ma_0$$

$$T - 0.3 \times 50 \times 10 = 2 \times 50 \times 2 \Rightarrow T = 350\,\text{N}$$

66. (d) Maximum value of friction on A, $f_{max} = \mu mg$
$= 0.5 \times 2 \times 10 = 10$ N

Hence, the block B is subjected to this frictional force in the forward direction. Since the block A does not slip on B, both of them move with a common acceleration. The acceleration of B is the same as the acceleration of A.



To determine acceleration of a, we find that $m_B a_{max} = f_{max}$

$$a_{max} = \frac{10}{2} = 5\,\text{m/s}^2$$

$$F_{max} = (m_A + m_B)a_{max} = 4 \times 5 = 20\,\text{N}$$

67. (c) The retardation r of the block while moving up is $g(\sin\theta + \mu\cos\theta)$ while the acceleration a of the block while moving down is $g(\sin\theta - \mu\cos\theta)$.

$$t_1 \propto \frac{1}{\sqrt{r}} \text{ and } t_2 \propto \frac{1}{\sqrt{a}} \quad \text{since } r = a,\ t_2 > t_1$$

68. (a) Taking block as system, the friction is only force which work on the block. Using work energy theorem $W = \Delta K$.

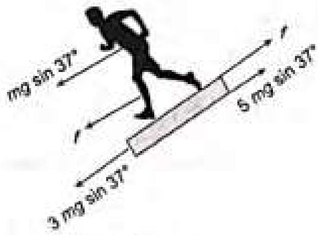
$$\mu mg \cdot s = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$s = \frac{1}{2}\frac{(v_2^2 - v_1^2)}{\mu g} = \frac{1}{2}\frac{(4^2 - 3^2)}{0.3 \times 10} = \frac{7}{6}\,\text{m}$$

69. (b) Since the blocks cannot accelerate in horizontal direction therefore the normal interaction force between the blocks as well as between 5 kg block and the wall is $F = 1000$ N. Again both the blocks accelerate downward with acceleration g m/s² and therefore the relative acceleration between the blocks is zero. Hence the friction force between the blocks is zero.

70. (a) For equilibrium of

$$3mg\sin 37° = f + 2mg\cos 37° \Rightarrow f = 2m$$



For man, $mg \sin 37° + f = ma$

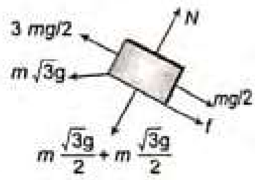
$$6m + 2m = ma \Rightarrow a = 8\,\text{m/s}^2$$

71. (a) $N = \sqrt{3}\,mg$

$$f_{max} = \sqrt{3} \times \frac{\sqrt{3}}{2}mg = \frac{3}{2}mg$$

$\therefore$ Block will not slide

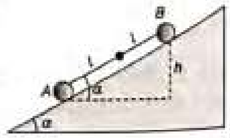
Since $f = \frac{3mg}{2} - \frac{mg}{2} = mg < f_{max}$



72. (d) $h = 2l\sin\alpha$

A is the lowest point and B the highest point. At B, in critical case tension is zero. Let velocity of particle at B at this instant be v_B.

Then



$$mg\sin\alpha = \frac{mv_B^2}{l} \text{ or } v_B^2 = gl\sin\alpha$$

Now $v_A^2 = v_B^2 + 2gh$

$= (gl\sin\alpha) + 2g(2l\sin\alpha)$

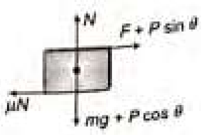
$\therefore \quad v_A = \sqrt{5gl\sin\alpha}$

73. (b) The minimum force required to pull a block of mass m place on rough horizontal surface is $\frac{\mu mg}{\sqrt{\mu^2+1}}$. Hence, the minimum force required to pull the system of mass $2m$ is $\frac{2\mu mg}{\sqrt{\mu^2+1}}$.

74. (c) The free-body diagram of block in the figure is as shown.

$\therefore \quad N = mg + P\cos\theta$

and acceleration of block



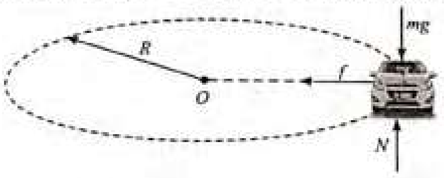
$$a = \frac{F + P\sin\theta - \mu(mg + P\cos\theta)}{m}$$

$$a = \frac{F + P\sin\theta - \frac{\sin\theta}{\cos\theta}(mg + P\cos\theta)}{m} = \frac{F - \mu mg}{m}$$

Hence, P does not change acceleration.

ARCHIVES

1. (b) The friction will provide required centripetal force



From F.B.D of car,

$$f = \frac{mv^2}{R} \quad \text{...(i)}$$

If the car does not skid, the friction should be static in nature.

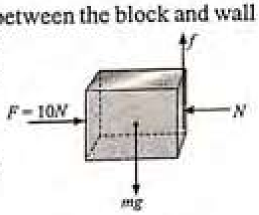
$\therefore \quad f \le \mu N$

$\Rightarrow \quad \frac{mv^2}{R} \le \mu mg \Rightarrow v \le \sqrt{\mu Rg}$

Maximum permitted velocity, $v = \sqrt{\mu Rg}$

$\therefore \quad v = \sqrt{0.6 \times 150 \times 10} = 30$ m/s

2. (a) If block does not slide, the friction between the block and wall should be static in nature. This friction force prevent the block from sliding on the wall.



From F.B.D of the block

$f = mg \quad \text{...(i)}$

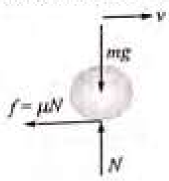
If block does not side, $f \le \mu N$

$\Rightarrow \quad mg \le \mu N \Rightarrow mg \le 0.2 \times 10$

or $\quad mg \le 2\text{N}$

Hence, force necessary to just hold the block should be 2 N.

3. (d) Retardation of the marble block because of friction,



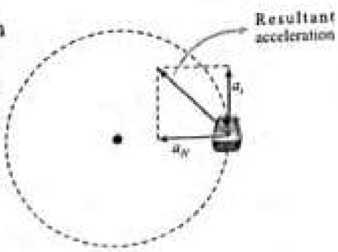
$$a = \frac{\mu N}{m} = \frac{\mu mg}{m} = \mu g$$

Now using $v = u + at$, we get

$0 = 6 - (m \times 10) \times 10$

Which gives $\mu = 0.06$

4. (c) It is the case of non-uniform circular motion. In this case the car will have radial as well as tangential acceleration.



Resultant acceleration is given by

$$|a_{net}| = \sqrt{a^2_{radial} + a^2_{tangential}}$$

Radial acceleration, $a_c = \frac{v^2}{r} = \frac{30 \times 30}{500} = \frac{9}{5}$ m/s^2, and

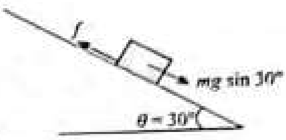
tangential acceleration, $a_t = 2$ m/s^2

$\therefore \quad |a_c| = \sqrt{\frac{81}{25} + 4} = 2.7$ m/s^2

5. (c) The angle of repose,

$$\alpha = \tan^{-1}(\mu) = \tan^{-1}(0.8) = 37^\circ$$

Here, the angle of inclined plane is $\theta = 30^\circ$. It means that block is at rest and therefore the friction should be static in nature. It will balance the component of weight parallel to inclined plane.

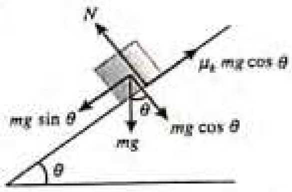


Static friction = component of weight in downward direction

$= mg \sin\theta = 10$ N

$\therefore \quad m = \frac{10}{9 \times \sin 30^\circ} = 2$ kg

6. (d) The acceleration of the block in case of smooth surface,



$$a_1 = \frac{mg\sin\theta}{m} = g\sin\theta$$

For smooth plane, using $s = ut + \frac{1}{2}at^2$, we get

$$d = \frac{1}{2}(g\sin\theta)t^2 \quad \text{...(i)}$$

For rough plane, the acceleration of the block,

$$a_2 = \frac{(mg\sin\theta - \mu_k mg\cos\theta)}{m} = g(\sin\theta - \mu_k\cos\theta)$$

Again using $s = ut + \frac{1}{2}at^2$, we get

$$d = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)(nt)^2$$

$\Rightarrow \quad \frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)n^2t^2$

$\Rightarrow \quad \sin\theta = n^2(\sin\theta - \mu_k\cos\theta)$

Putting $\theta = 45^\circ$, we get

$\sin 45^\circ = n^2(\sin 45^\circ - \mu_k \cos 45^\circ)$

$$\frac{1}{\sqrt{2}} = \frac{n^2}{\sqrt{2}}(1 - \mu_k) \text{ or } \mu_k = 1 - \frac{1}{n^2}$$

7. (c) Retardation due to friction $= \mu g$

we know $v^2 = u^2 + 2as$

$\therefore \quad 0 = (100)^2 - 2(\mu g)s$

or $\quad 2\,\mu gs = 100 \times 100$

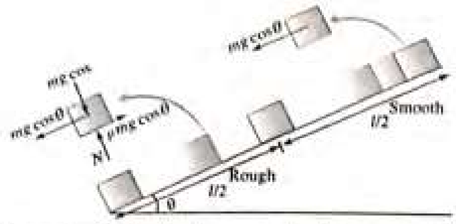
or $\quad s = \frac{100 \times 100}{2 \times 0.5 \times 10} = 1000$ m

8. (d) The acceleration of the body on upper half,

$$a_1 = \frac{mg\sin\theta}{m} = g\sin\theta$$

Let v be the velocity of the block at the end of smooth half. Using $v^2 = u^2 + 2as$, we get

$$v^2 = u^2 + 2a\frac{l}{2} = 2(g\sin\theta)\frac{l}{2} = gl\sin\theta$$



For lower half, the body comes back to rest at the bottom, it means the body retards on this section.
The retardation of the body on lower half,

$$a_2 = \frac{(\mu mg\cos\theta - mg\sin\theta)}{m} = g(\mu\cos\theta - \sin\theta)$$

Again using $v^2 = u^2 + 2as$, we get

$$0 = u^2 - 2g(\mu\cos\theta - \sin\theta)\frac{l}{2}$$

$$\Rightarrow \quad u^2 = 2g(\mu\cos\theta - \sin\theta)\frac{l}{2}$$

But velocity of the body at the starting of this section is equal to final velocity at the end of smooth section.

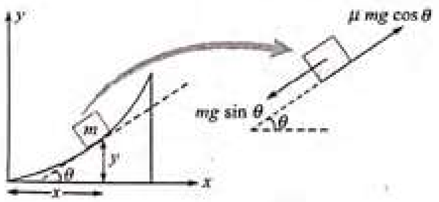
$$\therefore \quad gl\sin\theta = 2g(\mu\cos\theta - \sin\theta)\frac{l}{2}$$

$$\Rightarrow \quad \mu\cos\theta = 2\sin\theta \Rightarrow \mu = 2\tan\theta$$

9. (c) Block is under limiting friction, so

$$mg\sin\theta = \mu mg\cos\theta \Rightarrow \mu = \tan\theta \qquad \text{...(i)}$$

Equation of the surface, $y = \dfrac{x^3}{6}$



Slope of the curved surface,

$$\tan\theta = \frac{dy}{dx} = \frac{x^2}{2} \qquad \text{...(ii)}$$

From eqns (i) and (ii), we get

$$\Rightarrow \quad 0.5 = \frac{x^2}{2}$$

$$\Rightarrow \quad x^2 = 1 \text{ or } x = 1$$

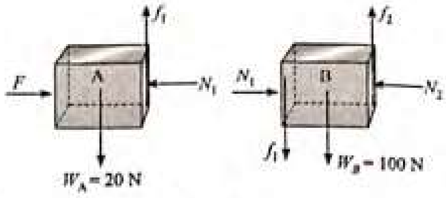
$$\text{So,} \quad y = \frac{x^3}{6} = \frac{(1)^3}{6} = \frac{1}{6}$$

10. (c) F.B.D of block A and block B

For block A:

If block A is at equilibrium, then

$f_l = W_A = 20$ N



If friction is static, $f_1 \le \mu_1 N_1$

$$\text{or} \quad 20 \le 0.1 \times F \Rightarrow F \ge \frac{20}{0.1} = 200 \text{ N}$$

Hence, limiting value of friction between the blocks,

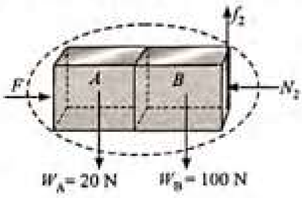
$f_1 = 0.1 \times 200 = 20$ N

From F.B.D of block B:

$f_2 = f_1 + W_B = 20 + 100 = 120$ N

Alternative method:

If we take both blocks together as system, them



For equilibrium of the system in vertical direction,

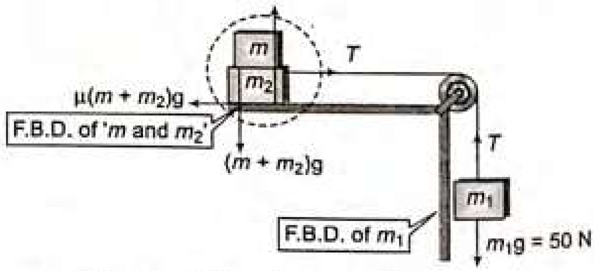
$f_2 = W_A + W_B = 20 + 100 = 120$ N

11. (d) From F.B.D of the block 'm_1'

$$T = m_1 g = 5 \times 10 = 50 \text{ N} \qquad \text{...(i)}$$

From F.B.D. of the block '$m + m_2$'

$$T = \mu(m + m_1)g$$
$$= 0.15(m + 10) \times 10 \qquad \text{...(ii)}$$



From (i) and (ii) $50 = 0.15\,(m + 10)10$

$$5 = \frac{3}{10}(m + 10) \Rightarrow m + 10 = \frac{100}{3} \Rightarrow m = 23.3 \text{ kg}$$

12. (d) Given: Force $\propto \dfrac{1}{R^n}$

Also Force $= m\omega^2 R$

$$\text{Hence } m\omega^2 R \propto \frac{1}{R^n} \Rightarrow \omega^2 \propto \frac{1}{R^{n+1}} \Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$$

$$\text{Time period } T = \frac{2\pi}{\omega} \text{ or } T \propto \frac{1}{\omega}$$

$$T \propto R^{\frac{n+1}{2}}$$

CHAPTER 7: WORK, ENERGY AND POWER

Concept Application Exercise 7.1

1. (a) The work done on the cycle by the road is the work done by the frictional force exerted by the road on the cycle.

Now,

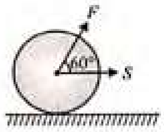
$$\vec{F}\cdot\vec{S} = F\cdot S\cos 180^\circ = -FS = -200\text{ N}\times 10\text{ m} = -2000\text{ J}$$

It is this negative work which brings the cycle to rest. This is clearly in accordance with work–energy theorem.

(b) The displacement of the road is zero. So, work done by the cycle on the road is zero.

(According to Newton's third law of motion, an equal and opposite force acts on the road due to the cycle. The magnitude of this force is 200 N).

2. Force $F = 50$ N; distance $S = 100$ m; angle $\theta = 60^\circ$



$$W = FS\cos\theta = 50\times 100\times\cos 60^\circ\text{ J}$$

$$= 50\times 100\times\frac{1}{2}\text{ J} = 2500\text{ J}$$

$$\text{Wages} = \frac{2500}{25}\times 10\text{ paise} = ₹10$$

3. Since the body is displaced 4 m along the z-axis only,

$$S = 0\hat{i} + 0\hat{j} + 4\hat{k}$$

Also, $F = -i + 2\hat{j} + 3\hat{k}$

Work done,

$$W = \vec{F}\cdot\vec{S} = (-\hat{i} + 2\hat{j} + 3\hat{k})\cdot(0\hat{i} + 0\hat{j} + 4\hat{k})$$

$$= 12(\hat{k}\cdot\hat{k})\text{J} = 12\text{ J}$$

4. $W = \int_{\vec{r}_i}^{\vec{r}_f}\vec{F}\cdot d\vec{r}$

$$= \int_{(2\text{m},3\text{m},4\text{m})}^{(1\text{m},2\text{m},3\text{m})}(2\hat{i} + 3\hat{j} + 4\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= [2x + 3y + 4z]_{2\text{m},3\text{m},4\text{m}}^{1\text{m},2\text{m},3\text{m}} = -9\text{ J}$$

5. Here force is variable so

$$W = \int_{\vec{r}_i}^{\vec{r}_f}\vec{F}\cdot\vec{dr} = \int_{\vec{r}_1}^{\vec{r}_2}(3x^2\hat{i} + 2y\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_{\vec{r}_1}^{\vec{r}_2}(3x^2dx + 2y\,dy) = \left[x^3 + y^2\right]_{(2,3)}^{(4,6)} = 83\text{ J}$$

6. Work done = Area under F–s graph

$$W_{AB} = W_{AP} + W_{PQ} + W_{QR} + W_{RB}$$

$$= 10\times 1\times\frac{1}{2}(10+15)\times 1 + \frac{1}{2}\times 1\times 15 - \frac{1}{2}\times 1\times 15$$

$$= 22.5\text{ J}$$

Concept Application Exercise 7.2

1. $E_{k_2} = \dfrac{121}{100}E_{k_1}$

or $\dfrac{1}{2}mv_2^2 = \dfrac{121}{100}\dfrac{1}{2}mv_1^2$

or $v_2 = \dfrac{11}{10}v_1$ or $mv_2 = \dfrac{11}{10}\dfrac{1}{2}mv_1$

or $p_2 = \dfrac{11}{10}p_1$

or $\dfrac{p_2}{p_1} - 1 = \dfrac{11}{10} - 1 = \dfrac{1}{10} = 0.1$

or $\dfrac{p_2 - p_1}{p_1}\times 100 = \dfrac{1}{10}\times 100 = 10$

So the percentage increase in the magnitude of linear momentum is 10%.

2. No, for a conservative force, the work done in a round trip should be zero.

3. Along each step of motion, the frictional force is opposite in direction to the incremental displacement. so in the work $\cos 180^\circ = -1$

(a) $W = (3\text{ N})(5\text{ m})(-1) + (3\text{ N})(5\text{ m})(-1) = -30.0\text{ J}$

(b) The distance CO is $(5^2 + 5^2)^{1/2}$ m = 7.07 m

$$W = (3\text{ N})(5\text{ m})(-1) + (3\text{ N})(5\text{ m})(-1) + (3\text{ N})(7.07\text{ m})(-1) = -51.2\text{ J}$$

(c) $W = (3\text{ N})(7.07\text{ m})(-1) + (3\text{ N})(7.07\text{ m})(-1) = -42.4\text{ J}$

(d) The force of friction is a non-conservative force.

4. $F_x = -\dfrac{\partial U}{\partial x} = -\dfrac{\partial(3x^3y - 7x)}{\partial x} = -(9x^2y - 7) = 7 - 9x^2y$

$F_y = -\dfrac{\partial U}{\partial y} = -\dfrac{\partial(3x^3y - 7x)}{\partial y} = -(3x^3 - 0) = -3x^3$

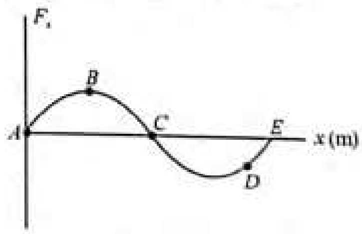
Thus, the force acting at point (x, y) is

$$\vec{F} = F_x\hat{i} + F_y\hat{j} = (7 - 9x^2y)\hat{i} - 3x^3\hat{j}$$

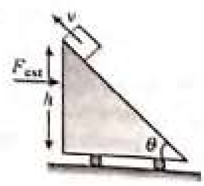
5. (a) F_x is zero at points A, C, and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c)



6. **W–E theorem:** Applying work–energy theorem for wedge bead system,



$W_{ext} + W_{gravity} = \Delta K$

We have $W_{ext} - mgy = \Delta K$

This gives $W_{ext} = mgy + \Delta K$ (i)

ΔK: The KE of the bead changes by

$$\Delta K = \frac{1}{2}mv^2 \quad \text{(ii)}$$

Using Eqs. (i) and (ii), we have $W_{ext} = \frac{1}{2}mv^2 + mgy$

Concept Application Exercise 7.3

1. Displacement of the point A of the string

$$\sqrt{(3\sqrt{3})^2 + (3)^2} - \sqrt{4^2 + 3^2} = 6 - 5 = 1 \text{ m}$$

ΔK = Work done by tension $= 50 \times 1 = 50$ J

2. (a) Only gravitational force acts on the ball, which is conservative; therefore, we can apply conservation of energy.

We assign reference level at the top of the building, i.e., $U_{gi} = 0$. At the topmost point, the ball is moving horizontally with velocity $u\cos\theta$.

Initial total mechanical energy:

$$E_i = 0 + \frac{1}{2}mu^2$$

Total mechanical energy at the topmost point:

$$E_f = \frac{1}{2}mu^2\cos^2\theta + mgh$$

From conservation of energy, we have $E_i = E_f$

$$\frac{1}{2}mu^2 = \frac{1}{2}mu^2\cos^2\theta + mgh$$

$$h = \frac{u^2 - u^2\cos^2\theta}{2g} = \frac{u^2\sin^2\theta}{2g}$$

(b) If v is the speed of the ball at the ground,

$$E_f = \frac{1}{2}mv^2 - mgH$$

From conservation of energy, we have $E_i = E_f$

$$\frac{1}{2}mv^2 = \frac{1}{2}mv^2 - mgH$$

$$v = \sqrt{u^2 + 2gH}$$

3. When the block is released, it moves horizontally with speed v till it leaves the spring.

By energy conservation, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$v^2 = \frac{kx^2}{m} \Rightarrow v = \sqrt{\frac{kx^2}{m}}$$

Time of flight, $t = \sqrt{\frac{2H}{g}}$

So horizontal distance travelled from the free end of the spring is

$$v \times t = \sqrt{\frac{kx^2}{m}} \times \sqrt{\frac{2H}{g}}a$$

$$= \sqrt{\frac{100 \times (0.05)^2}{0.1}} \times \sqrt{\frac{2 \times 2}{10}} = 1 \text{ m}$$

So at a horizontal distance of 1 m from the free end of the spring.

4. $v_1 = v_2 = v_3$. The first and third balls speed up after they are thrown, whereas the second ball initially slows down but then speeds up after reaching its peak. The paths of all the three balls are parabolas, and the balls take different time intervals to reach the ground because they have different initial velocities. All three balls, however, have the same speed at the moment they hit the ground because all start with the same kinetic energy and because the ball–earth system undergoes the same change in gravitational potential energy in all three cases.

5. (a) $(\Delta K)_{A \to B} = \Sigma W = W_g = mg\,\Delta h = mg(5.00 - 3.20)$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m(10)(1.80);$$

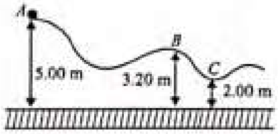
$$v_A = 0$$

$$\Rightarrow v_B = 6 \text{ ms}^{-1}$$

Similarly,

$$v_c = \sqrt{v_A^2 + 2g(5.00 - 2.00)} = 7.7 \text{ ms}^{-1}$$

(b) $W_g\big|_{A \to C} = mg(5.00 - 2.00) = 150$ J



6. Friction is only force which is changing mechanical energy of system.

$$W_{friction} = \Delta ME.$$

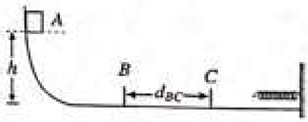
$$-\mu_k mgd\cos\theta = (mgy_{max} - mgh)$$

where $d = \frac{y_{max}}{\sin\theta}$

$$\therefore \quad mgy_{max} = mgh - \mu_k mgy_{max}\cot\theta$$

Solving, $y_{max} = \frac{h}{1 + \mu_k\cot\theta}$

7. $\Delta E_{mech} = -f\,dx \Rightarrow E_f - E_i = -f \cdot d_{BC}$

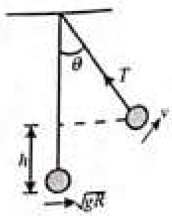


$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}}$$

Concept Application Exercise 7.4

1. $T = mg\cos\theta + \frac{mv^2}{l}$



$$\Rightarrow \quad mg = mg\cos\theta + \frac{mv^2}{l} \quad \text{(i)}$$

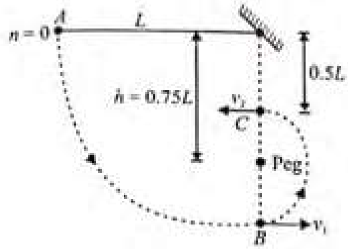
$$mgh = \frac{1}{2}m(\sqrt{gl})^2 - \frac{1}{2}mv^2$$

$$mg(l - l\cos\theta) = \frac{1}{2}mgl - \frac{1}{2}mv^2 \quad \text{(ii)}$$

From Eqs. (i) and (ii), $v = \sqrt{\frac{gl}{3}}$

2. Apply conservation of energy between A and B.

$$\frac{1}{2}mv_1^2 = mgL \Rightarrow v_1 = \sqrt{2gL}$$



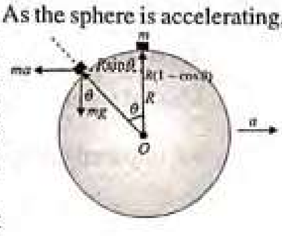
We see that $v_1 > \sqrt{5g(0.25L)}$

So the ball will complete the circular motion about peg.

Apply conservation of energy between A and C:

$$\frac{1}{2}mv_2^2 = mg(0.5L) \Rightarrow v_2 = \sqrt{gL}$$

3. The situation is shown in the figure. As the sphere is accelerating, it becomes a non-inertial frame for the particle and when particle is displaced, pseudo force acting on it will also do work in addition to gravity and will cause increment in its kinetic energy.



Using work–energy theorem at points A and B, we have

$$0 + mgR(1 - \cos\theta) + maR\sin\theta = \frac{1}{2}mv^2$$

or $$v = \sqrt{R[a\sin\theta + g(1 - \cos\theta)]}$$

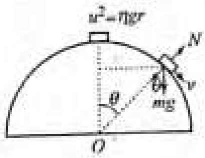
4. Let the velocity at B be v, then

$$\frac{1}{2}mv^2 = mg(h - 2R) \Rightarrow v = \sqrt{2g(h - 2R)}$$

$$N + mg = \frac{mv^2}{R}$$

Put $N = mg$, solve to get $h = 3R$

5. If v is the speed of the particle when the radius vector makes an angle θ with vertical, then



$$(\cos\theta)\, mg - N = \frac{mv^2}{r}$$

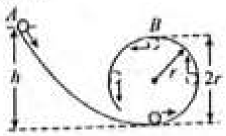
$$= \frac{m}{r}[u^2 + 2gr(1 - \cos\theta)]$$

When the particle loses contact, $N = 0$

$$mg\cos\theta = \frac{m}{r}[(\eta + 2)gr - 2gr\cos\theta]$$

$$\cos\theta = (\eta + 2) - 2\cos\theta \quad \Rightarrow \cos\theta = (\eta + 2)/3$$

6. To complete the circle of radius r, the velocity at the top must be $\geq \sqrt{rg}$.



From A to B;

loss in GPE = Gain in KE $mg(h - 2r)$

$$= \frac{1}{2}mv^2 = 0$$

$$mg(h - 2r)\frac{1}{2}m(\sqrt{rg})^2 \Rightarrow h = \frac{5}{2}r$$

Hence, h must be at least equal to $2.5\,r$.

Concept Application Exercise 7.5

1. Power of the first coolie $= \dfrac{\text{Work}}{\text{Time}}$

$$= \frac{M \times g \times S}{t} = \frac{M \times 9.8 \times 2}{60}\ \text{J s}^{-1}$$

Power of the second coolie is

$$\frac{M \times 9.8 \times 2}{30}\ \text{J s}^{-1} = 2\left(\frac{M \times 9.8 \times 2}{60}\right)\text{J s}^{-1}$$

$= 2 \times$ Power of the first coolie

So, the power of the second coolie is double that of the first one. Both the coolies spend the same amount of energy.

Alternative method: We know that $W = Pt$

For the same work, $W = P_1t_1 = P_2t_2$

or $$\frac{P_2}{P_1} = \frac{t_1}{t_2} = \frac{1\ \text{min}}{30\ \text{s}} = 2 \text{ or } P_2 = 2P_1$$

2. If A m^2 is the area, then power $= 200 \times A$ W

Useful electrical energy produced per second is

$$\frac{20}{100}(200\,A) = 40 \times A\ \text{W}$$

But $\quad 40\,A = 8000$ or $A = 200$ m^2

3. Weight of (elevator + passenger) $= mg = 1800 \times 10$ N $= 18000$ N

Frictional force $= 4000$ N

Total downward force on the elevator is

$(18000 + 4000)$ N $= 22000$ N

Clearly, the motor must have enough power to balance this force.

Now power, $P = Fv = 22{,}000\ \text{N} \times 2\ \text{m s}^{-1}$

$= 44{,}000$ W

$$= \frac{44000}{746}\ \text{hp} = 58.98\ \text{hp}$$

4. The kinetic energy is dependent on the square of the velocity, and work done is equal to the change in kinetic energy. Now, as the change in the kinetic energy of the second person is more than the first person, he does more work. As both of them increase their speeds in the same interval of time, the second person does more work in the same interval of time. Hence, the second person has more power.

5. Work done by the man in climbing the escalator,

$W = mgh = 90 \times 10 \times 20\sin 30° = 9000$ J

With the escalator stationary, man takes 10 s to climb up, therefore average power

$$P = \frac{W}{t} = \frac{9000}{10} = 900\ \text{W}$$

Speed of the man when escalator is stationary,

$$v = \frac{20}{10} = 2\ \text{m s}^{-1}$$

In the second case, the speed of the man relative to the moving escalator is also 2 m s^{-1}. His speed relative to the ground is $0.5 + 2 = 2.5$ m s^{-1}.

Accordingly, the time taken by the man in climbing is

$$\frac{20}{2.5} = 8\ \text{s}$$

The power developed by the man as seen from the ground reference is then, on the average:

$$\text{Power} = \frac{W}{t} = \frac{9000}{8} = 1125\ \text{W}$$

6. $P_{av} = \dfrac{W}{\Delta t} = \dfrac{K_f}{\Delta t} = \dfrac{mv^2}{2\Delta t} = \dfrac{0.875\text{ kg}(0.620\text{ ms}^{-1})^2}{2(21\times10^{-3}\text{ s})} = 8.0\text{ W}$

Exercises

Concept of Work and Work Energy Theorem

1. (c) The system consisting of the carts fixed, initial kinetic energy is the mechanical energy that can be transformed due to friction from the surface. Therefore, the loss of mechanical energy is $\Delta E_{mech} = -f_k d = -(6N)(0.06\ m) = 0.36$ J. This product must remain the same in all cases. For the cart rolling through gravel, $-(9\ N)(d) = 0.36$ J tells us $d = 4$ cm.

2. (b) Displacement w.r.t. ground $= \vec{s} + \vec{s}_0$

Since train is moving with constant velocity net force acting on block is $\vec{F}$. Therefore work done $= \vec{F}.(\vec{s} + \vec{s}_0)$

3. (c) $W = \dfrac{F^2}{2k}$

If both springs are stretched by same force then $W \propto \dfrac{1}{k}$

As $k_1 > k_2$ therefore $W_1 < W_2$

i.e. more work is done in case of second spring.

4. (d) $\int \vec{F}\cdot d\vec{x} = \int A(y^2\hat{i} + 2x^2\hat{j})\cdot(dx\hat{i} + dy\hat{j})$

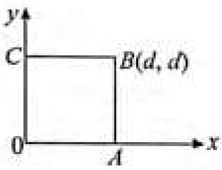
$= A\int(y^2dx + 2x^2dy)$

$W_{OA} = 0 + 0,\ W_{AB} = A[0 + 2d^2\, d]$

$W_{BC} = A[d^2(-d) + 0],\ W_{CD}$

$= A[0 + 0]$

$W = 0 + 2Ad^3 - Ad^3 + 0 = Ad^3$



5. (d) The work–energy theorem states that

$W_{net} = \Delta K = K_f - K_i$ Thus, if $W_{net} = 0$, then $K_f - K_i$

or $\dfrac{1}{2}mv_f^2 - \dfrac{1}{2}mv_i^2$,

which leads to the conclusion that the speed is unchanged ($v_f = v_i$). The velocity of the particle involves both magnitude (speed) and direction. The work–energy theorem shows that the magnitude or speed is unchanged when $W_{net} = 0$, but makes no statement about the direction of the velocity.

6. (c) When a force of constant magnitude which is perpendicular to the velocity of particle acts on a particle, work done is zero and hence change in kinetic energy is zero.

7. (a) The ball rebounds with the same speed. So change in its kinetic energy will be zero i.e. work done by the ball on the wall is zero.

8. (c) The net work needed to accelerate the object from $v = 0$ to v is

$$W_1 = KE_{1f} - KE_{1i} = \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}mv^2$$

The work required to accelerate the object from speed v to speed $2v$ is

$$W_2 = KE_{2f} - KE_{2i} = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$

$$= \frac{1}{2}m(4v^2 - v^2) = 3\left(\frac{1}{2}mv^2\right) = 3W_1$$

9. (c) $\Sigma W_{net} = \Delta K$

As particle is moving slowly, this means $\Delta K = 0$

$W_N + W_F + W_{mg} = \Delta K$

But $W_N = 0$ as $\vec{N} \perp d\vec{r}$

$0 + W_F - mgh = 0$

$W_F = mgh$

10. (b) $K = \dfrac{\text{Mass}}{\text{Length}} = \dfrac{dm}{dx}$, v = speed of water

$\text{KE} = \dfrac{1}{2}mv^2$

$\Rightarrow \quad \dfrac{d}{dt}(\text{KE}) = \dfrac{1}{2}\left(\dfrac{dm}{dt}\right)v^2$

$= \dfrac{1}{2}\left(\dfrac{dm}{dx}\cdot\dfrac{dx}{dt}\right)v^2 = \dfrac{1}{2}kvv^2 = \dfrac{kv^3}{2}$

Alternative method:

Let in time t, L length of water come out.

Then $t = \dfrac{L}{v}$

Mass of water that comes out in time t: $m = kL$

KE imparted per unit time $= \left(\dfrac{1}{2}\right)\dfrac{mv^2}{t}$

$= \left(\dfrac{1}{2}\right)\dfrac{kLv^2}{(L/v)} = \left(\dfrac{1}{2}\right)kv^3$

11. (a) First case: $\dfrac{1}{2}mv^2 = Fs$ (i)

$\dfrac{1}{2}\left(M + \dfrac{m}{2}\right)v^2 = Fs'$ (ii)

Dividing Eq. (ii) by Eq. (i), we get $\dfrac{s'}{s} = \dfrac{3}{2}$ or $s' = 1.5\,s$

12. (c) From work–energy theorem,

$\Delta\text{KE} = W_{net}$ or $K_f - K_i = \int P\,dt$

or $\dfrac{1}{2}mv^2 - 0 = \int_0^2\left(\dfrac{3}{2}t^2\right)dt$

or $v^2 = \left|\dfrac{t^3}{2}\right|_0^2$ or $v = 2\text{ ms}^{-1}$

13. (d) The initial potential energy of the object is mgh, which has been used up for work against the force of friction. In returning the body to its initial position, the force performs the same work against friction. In addition, it imparts to the object the initial potential energy. As a result, the total work will be $2mgh$.

14. (c) Minimum stopping distance = s

Work done against the friction = $W = \mu mgs$

Initial momentum gained by both toy carts will be same because same force acts for same time.

Initial kinetic energy of the toy cart $= \left(\dfrac{p^2}{2m}\right)$

Therefore, $\mu mgs = \dfrac{p^2}{2m}$ or $s = \left(\dfrac{p^2}{2\mu gm^2}\right)$

For the two toy carts, momentum is numerically the same. Further μ and g are the same for the toy carts.

So, $\dfrac{S_1}{S_2} = \left(\dfrac{m_2}{m_1}\right)^2$

15. (c) Work done by friction:

$$W = (\mu mg \cos\theta)\, S = (\mu mg \cos\theta)\,\frac{h}{\sin\theta} = \mu mgh \cot\theta$$

Now $\cot\theta_1 = \cot 30° = \sqrt{3}$

$$\cot\theta_2 = \cot 60° = \frac{1}{\sqrt{3}}$$

i.e., kinetic energy (KE = $mgh - W$) in first case will be less or $K_1 < K_2$.

16. (b) From work–energy theorem, net work done by all forces (internal and external) = change in kinetic energy.

$\Rightarrow$ Work done by gravity + work done by air drag

$$= \frac{1}{2} \times 70(20^2 - 0^2)$$

$\Rightarrow$ Work done by air drag = 14000 – 35000 = –21000 J

17. (c) Given $v = k\sqrt{x}$

or $\dfrac{dx}{dt} = k\sqrt{x}$ or $x^{-\frac{1}{2}}\, dx = k\, dt$

Integrating both sides, we get

$$\frac{x^{\frac{1}{2}}}{\frac{1}{2}} = kt + C\,; \text{ assuming } x(0) = 0$$

Therefore, $C = 0$

$$2\sqrt{x} = kt \Rightarrow x = \frac{k^2t^2}{4} \text{ or } v = \frac{k^2t}{2}$$

Therefore, work done,

ΔW = Increase in KE

$$= \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}m\left[\frac{k^2t}{2}\right]^2 = \frac{1}{8}mk^4t^2$$

18. (d) Statement I: Work done by gravity is same for motion from A to J and B to M for equal mass. So KE will be equal.

Statement II: Acceleration = $g\sin\theta$

$$\sin\theta_A > \sin\theta_B \Rightarrow \frac{h}{l} > \frac{h}{2l}$$

Statement III: $W_g + W_{ext} = 0$ (Because the block moves slowly)

$$W_{ext} = -W_g$$

From B to O: W_g is positive, so $W_{ext} < 0$.

19. (d) Velocity of a projectile at any instant of time (t) is

$$v^2 = v_x^2 + v_y^2 = (u\cos\theta)^2 + \left(u\sin\theta - g\frac{x}{u\cos\theta}\right)^2$$

$$\therefore \quad \text{KE} = \frac{1}{2}mu^2 - mgx\tan\theta + \frac{mg^2x^2}{u^2\cos^2\theta}$$

The given equation represents the equation of a parabola.

Concept of Potential Energy and Concept of Conservation of Mechanical Energy

20. (b) In case of non-conservative forces, the work done is dissipated as heat, sound, etc., i.e., it does not increase the potential energy. But in case of conservative forces, work done is responsible for increasing or decreasing the potential energy.

21. (b) We know that $dU = -dW$

where dU is the change in potential energy

and dW is the work done by conservative forces

Hence, work done by conservative forces on a system is equal to the negative of the change in potential energy.

22. (a) In the explosion of a bomb or inelastic collision between two bodies as force is internal, momentum is conserved while KE changes. Hence, the KE of a system can be changed without changing its momentum. Similarly, the reverse is also true, e.g., if a force acts perpendicular to motion, work done will be zero and so KE will remain constant. However, the force will change the direction of motion and so the momentum. Further, body may have energy (i.e., potential energy) without having momentum.

23. (c) Total gravitational energy gained is work done + energy released by the spring = $W + E$

24. (a) Decrease in height: $h = 14 - 7 = 7$ m

Also KE at C = Loss in PE = mgh = 140 J

25. (c) According to the law of conservation of energy, elastic potential energy stored in the spring = gravitational potential energy acquired by shot

$$\frac{1}{2}kx^2 = mgh \text{ or } h = \frac{kx^2}{2mg}$$

26. (b) $W_{gravity} = \Delta U_{spring}$

$$mg\sin\theta\,(d + 0.25) = \frac{1}{2}k\,(0.25)^2$$

which gives $d = 37.5$ cm

27. (d) $f(x) = -\dfrac{dU}{dx}(x)$ or $U(x) = -\int F(x)\, dx$

Here $F(x) = -kx$, where k is a positive constant.

28. (c) Let the particle be dropped from a height h and the spring be compressed by y. According to the conservation of mechanical energy, loss in PE of the particle = gain in elastic potential energy of the spring

$$mg\,(h + y) = \frac{1}{2}ky^2$$

Now, as the particle and spring are same for second case,

$$\frac{h_1 + y_1}{h_2 + y_2} = \left(\frac{y_1}{y_2}\right)^2 \text{ or } \left(\frac{0.24 + 0.01}{h_2 + 0.04}\right) = \left(\frac{0.01}{0.04}\right)^2$$

Solving, we get $h_2 = 3.96$ m

29. (b) Let x be the extension in the string when 2 kg block leaves the contact with ground. Then tension in the spring should be equal to weight of 2 kg block:

$$kx = 2g \text{ or } x = \frac{2g}{k} = \frac{2 \times 10}{40} = \frac{1}{2} \text{ m}$$

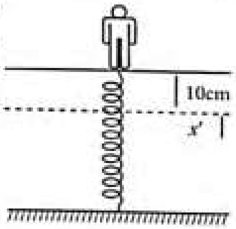
Now from conservation of mechanical energy,

$$mgx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{2gx - \frac{kx^2}{m}} = \sqrt{2\times10\times\frac{1}{2} - \frac{40}{4\times5}} = 2\sqrt{2}\ \text{ms}^{-1}$$

30. (d) Initially, 60 g = $kx = k$ (2.5) (i)

Let x' be the maximum compression when the person jumps on the balance, then $\frac{1}{2}kx'^2 = 60\,g\,(x' + 10)$



$$\Rightarrow \quad \frac{1}{2}\left[\frac{60\,g}{2.5}\right]x'^2 = 60\,g\,(x' + 10)$$

$$\Rightarrow \quad x'^2 = 5x' + 50$$

$$\Rightarrow \quad x'^2 - 5x' - 50 = 0$$

Solving for x', we get $x' = 10$ cm

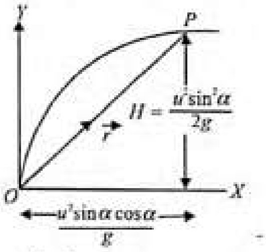
If m kg is the reading, then

$mg = k$ (10) (ii)

From Eqs. (i) and (ii), we get $m = 240$ kg

31. (d) $\vec{r} = \frac{u^2 \sin\alpha \cos\alpha}{g}\hat{i} + \frac{u^2 \sin^2\alpha}{2g}\hat{j}$

$$F = -mg\,\hat{j},\ W = \vec{F}\cdot\vec{r} = -\frac{mu^2 \sin^2\alpha}{2}$$



Alternative method:

Velocity at highest point = $u\cos\alpha$

$$W_{mg} = \Delta KE = \frac{1}{2}m(u\cos\alpha)^2 - \frac{1}{2}mu^2$$

$$= -\frac{mu^2 \sin^2\alpha}{2}$$

32. (b) The position of equilibrium corresponds to $F(x) = 0$

Since $F(x) = \frac{-dU(x)}{dx}$

so $F(x) = -\frac{d}{dx}\left(\frac{a}{x^4} - \frac{b}{x^2}\right)$ or $F(x) = \frac{4a}{x^5} - \frac{2b}{x^3}$

For equilibrium, $F(x) = 0$, therefore

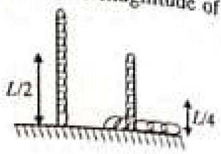
$$\frac{4a}{x^5} - \frac{2b}{x^3} = 0 \Rightarrow x = \pm\sqrt{\frac{2a}{b}}$$

$$\frac{d^2U(x)}{dx^2} = -\frac{20a}{x^6} + \frac{8b}{x^4}$$

Putting $x = \pm\sqrt{\frac{2a}{b}}$ gives $\frac{d^2U(x)}{dx^2}$ as negative

So U is maximum. Hence, it is position of unstable equilibrium

33. (c) The work done by the man is negative of the magnitude of decrease in potential energy of the chain.

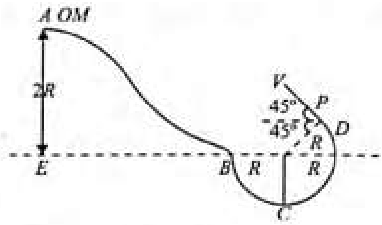


$$\Delta U = mg\,\frac{l}{2} - \frac{m}{2}g\,\frac{l}{4} = 3\,mg\,\frac{l}{g}$$

$$W = -\frac{3mgl}{8}$$

34. (a) Applying conservation of energy, we get

$$2mgR - mg\frac{R}{\sqrt{2}} = \frac{1}{2}mu^2 \Rightarrow u = \sqrt{4gR - \sqrt{2}\,gR}$$



$$H_{\max} = R + \frac{R}{\sqrt{2}} + \frac{u^2 \sin^2 45°}{2g}$$

$$= R + \frac{R}{\sqrt{2}} + \frac{4gR - \sqrt{2}gR}{2g\times2}$$

$$= 2R + \frac{R}{\sqrt{2}} - \frac{R}{2\sqrt{2}} = R\left[2 + \frac{1}{2\sqrt{2}}\right]$$

35. (a) KE of blocks at B = PE at A – PE at B

$$\frac{1}{2}mv^2 = mgh - mg2r = mg\,(h - 2r)$$

$$v^2 = 2g\,(h - 2r) \quad \text{(i)}$$

Also, $\frac{mv^2}{r} = x\,mg + mg$

or $v^2 = (x + 1)\,rg$ (ii)

Equating Eqs (i) and (ii), we get $2g\,(h - 2r) = (x + 1)\,gr$

or $2gh = (x + 1)gr + 4gr = (x + 5)gr$

$$h = \left(\frac{x+5}{2}\right)r$$

36. (d) Here PE = Work done

$$mgH = \mu_{\text{kinetic}}\,mgS \quad \text{or} \quad S = \frac{H}{\mu_{\text{kinetic}}}$$

The particle covers the length l or not or covers it repeatedly is determined by the above relation.

Mechanical Power

37. (b) Power $P = \frac{\text{Work}}{\text{Time}}$

Work done by both will be same.

Hence, $\frac{P_1}{P_2} = \frac{t_2}{t_1} = \frac{20}{15} = \frac{4}{3}$

38. (b) Power used to pump the water $= \dfrac{mgh}{t} = \dfrac{100 \times 10 \times 10}{5}$

$= 2000$ W

Power of engine $= 2000 \times \dfrac{100}{60} = 3.3$ kW

39. (b) If a liquid of density ρ is flowing through a pipe of cross section A at speed V, the mass coming out per second will be

$$\left(\frac{dm}{dt}\right) = \rho AV$$

In order to get n times water in the same time, we get

$$\left(\frac{dm}{dt}\right)' = n\left(\frac{dm}{dt}\right)$$

i.e., $A'V'\rho' = nAV\rho$

But as pipe and liquid are same, $\rho' = \rho$, $A' = A$

$V' = NV$,

So $\dfrac{F'}{F} = \dfrac{V'\left(\dfrac{dm}{dt}\right)}{V\left(\dfrac{dm}{dt}\right)} = \dfrac{nV\dfrac{ndm}{dt}}{V\left(\dfrac{dm}{dt}\right)} = n^2$

or $F' = n^2 F$

$\dfrac{P'}{P} = \dfrac{F'V'}{FV} = \dfrac{(n^2F)(nV)}{FV}$ or $P' = n^3 P$

40. (c) $P = Fv = m\dfrac{dv}{dt}v$

or $v\dfrac{dv}{dt} = \dfrac{P}{m}$

or $v\dfrac{dv}{dx}\dfrac{dx}{dt} = \dfrac{P}{m}$

or $v^2\dfrac{dv}{dx} = \dfrac{P}{m}$ or $v^2\,dv = \dfrac{P}{m}dx$

On integration, we get $\dfrac{v^3}{3} = \dfrac{Px}{m}$ or $v = \left(\dfrac{3xP}{m}\right)^{\frac{1}{3}}$

41. (c) Let us assume that the displacement of the body is directly proportional to t^n, i.e.,

$s = Kt^n$, $v = \dfrac{ds}{dt} = Knt^{n-1}$

and $a = \dfrac{dv}{dt} = Kn(n-1)t^{n-2}$

Force $F = ma = mKn(n-1)t^{n-2}$

Power, $P = Fv = [mKn(n-1)t^{n-2}][Knt^{n-1}]$

$= mKn^2(n-1)t^{2n-3}$

As power is constant, i.e, independent of time, hence

$2n - 3 = 0$ or $n = \dfrac{3}{2}$ or $s \propto t^{\frac{3}{2}}$

42. (c) $P = Fv = M\dfrac{dv}{dt}v$

Hence, $v\,dv = \dfrac{P}{m}dt$

On integration, we find $v \propto \sqrt{t}$

43. (c) $a_c = \dfrac{v^2}{r} = k^2rt^2$ or $v = krt$

$KE = \dfrac{1}{2}mv^2 = \dfrac{1}{2}mk^2r^2t^2$

By work–energy theorem,

$W = \Delta K = \dfrac{1}{2}mk^2r^2t^2 - 0$

$P = \dfrac{dW}{dt} = mk^2r^2t$

Alternative method: $a_t = \dfrac{dv}{dt} = kr$

Power is given by tangential force only. So, power $= F_t v = ma_t v = mk^2r^2t$

Power of centripetal force is zero.

44. (d) Average power $< P > = \dfrac{< W >}{t} = -\dfrac{mu^2\sin^2\alpha}{2\left(\dfrac{u\sin\alpha}{g}\right)}$

$= \dfrac{-mg\,u\sin\alpha}{2}$

[Since time taken to go from O to $P = u\sin\alpha/g$]

45. (b) Given $a_n = kt^2$

or $\dfrac{v^2}{r} = kt^2$ or $v^2 = krt^2$

Therefore, average power delivered $= \dfrac{\text{Total work done}}{\text{Total time elapsed}}$

$= \dfrac{\text{Increase in KE}}{\text{Total time elapsed}}$

or $<P> = \dfrac{\frac{1}{2}m(v^2 - 0^2)}{t_0} = \dfrac{m}{2}\dfrac{krt_0^2}{t_0} = \dfrac{mkrt_0^2}{2}$

Alternative: $v = \sqrt{kr}\,t \Rightarrow a_t = \sqrt{kr}$

$P = F_t v = ma_t v = mkrt$

$< P > = \dfrac{\int_0^{t_0} Pdt}{t_0} = \dfrac{1}{2}mkrt_0^2$

46. (d) $u = 10$ m s^{-1}, $v = 20$ m s^{-1}

Work done = Increase in kinetic energy

$= \dfrac{1}{2} \times 500\,[20^2 - 10^2] = \dfrac{500 \times 30 \times 10}{2}$

Power $= \dfrac{500 \times 30 \times 10}{2 \times 60} = 1250$ W

Circular Motion in Vertical Plane

47. (c) Conservation of energy gives

$\dfrac{1}{2}mv^2 = mgR\sin\theta \Rightarrow \dfrac{mv^2}{R} = 2mg\sin\theta = F_C$

Now, $N - mg\sin\theta = F_C \Rightarrow N = 3\,mg\sin\theta$

$\therefore \dfrac{F_C}{N} = \dfrac{2}{3}$

48. (c) Since the pendulum started with no kinetic energy, conservation of energy implies that the potential energy at Q_{max} must be equal to the original potential energy, i.e., the vertical position will be same. Therefore,

$$L\cos\alpha = l + (L - l)\cos\theta$$

$$\Rightarrow \cos\theta = \frac{L\cos\alpha - l}{L - l}$$

$$\Rightarrow \theta = \cos^{-1}\left[\frac{L\cos\alpha - l}{L - l}\right]$$

49. (c) Velocity at lowest position, $v = \sqrt{2gl(1-\cos 60^\circ} = \sqrt{gl}$

$$T = mg + \frac{mv^2}{l} = mg + \frac{m}{l}gl = 2mg$$

$$\therefore \quad T = \mu 4mg \Rightarrow \mu = \frac{1}{2}$$

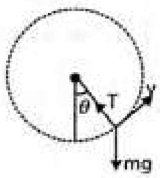
50. (a) Applying the work–energy theorem, we get

$$\frac{1}{2}\times mv^2 - 0 = F\times R + mg\times R$$

$$\frac{1}{2}\times\frac{1}{2}\times v^2 = 5\times 5 + \frac{1}{2}\times 10\times 5 = 50$$

$$v = \sqrt{200} = 14.14\ \text{ms}^{-1}$$

51. (a) $\frac{mv^2}{R} = \frac{1\times 4^2}{1} = 16\text{N}$



$$mg = 1\times 10 = 10\ \text{N}$$

$$\Rightarrow T = mg\cos\theta + \frac{mv^2}{R}$$

$$6 = 10\cos\theta + 16$$

$$\text{cosec}\ \theta = -1$$

$$\theta = 180^\circ$$

52. (a) $m = 1$ kg $\Rightarrow L = \frac{10}{3}$ m

$$\frac{T_{max}}{T_{min}} = 4 \qquad \text{(i)}$$

and $v_L = \sqrt{V_H^2 + 4gL}$

Tension at highest point, $T_{min} = \frac{mv_H^2}{K} - mg$

Tension at lowest point,

$$T_{max} = mv_L^2/L + mg = \frac{m(v_H^2 + 4gL)}{L} + mg$$

Now Eq. (i) can be written as

$$m\left[\frac{v_H^2 + 4gL}{L} + g\right] = 4\times m\left(\frac{v_H^2}{L} - g\right)$$

$$\Rightarrow v_H = \sqrt{3gL} = 10\ \text{ms}^{-1}$$

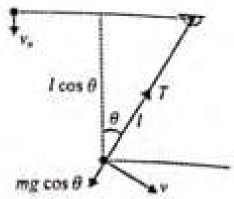
53. (a) $mgh = \frac{1}{2}mv^2 + 2mgR$

$$gh = \frac{v^2}{2} + 2gR$$

Required velocity at the top, $v = \sqrt{gR}$

$$gh = \frac{gR}{2} + 2gR \Rightarrow h = \frac{5}{2}R$$

54. (b) $v^2 = v_0^2 + 2gl\cos\theta$



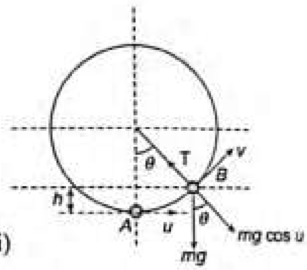
$$T = mg\cos\theta + \frac{mv^2}{l}$$

Given $T = 2mg$

$$2mg = mg\cos\theta + \frac{m}{l}(v_0^2 + 2gl\cos\theta)$$

$$\Rightarrow \cos\theta = \frac{1}{4}$$

55. (d) $T - mg\cos\theta = \frac{mv^2}{R}$



Given $T = mg$

$$mg - mg\cos\theta = \frac{mv^2}{R}$$

$$g(1-\cos\theta) = \frac{v^2}{R} \qquad \text{(i)}$$

C.O.M.E. at A and B; $\Delta K + \Delta U = 0$

$$\left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right) + mg(R - R\cos\theta) = 0$$

$$\Rightarrow v^2 - u^2 = -2gR(1-\cos\theta)$$

$$\Rightarrow v^2 - (\sqrt{gl})^2 = -2v^2$$

$$3v^2 = gl \Rightarrow v = \sqrt{\frac{gl}{3}}$$

56. (d) Velocity of ball to just reach the top of the tube should be given by

$$0 = v^2 - 2gh_0$$

Here, $h_0 = (R - h)$ and in critical case velocity will be zero at topmost point.

Thus $v = \sqrt{2g(2R - h)}$

57. (b) Consider energy-conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

$$\Delta K + \Delta U = 0$$

$$\left(0 - \frac{1}{2}mv_i^2\right) + \left[mg(2L) - 0\right] = 0$$

$$v_i = \sqrt{4gL} = \sqrt{4(10.0\ \text{m/s}^2)(0.40\ \text{m})}$$

$$v_i = 4.0\ \text{m/s}$$

58. (a) The tension vector $\vec{T}$ is always normal to velocity vector. Hence $\vec{T}\cdot\vec{v}$ is always zero.

$\Rightarrow$ (a) is correct choice.

59. (c) By law of conservation of mechanical energy

$$(K + U)_A = (K + U)_B$$

$$0 + mgR\cos\alpha = -mv \quad + mgR\sin\beta$$

$$v_B^2 = 2gR(\cos\alpha - \sin\beta)$$

At B since particle leaves contact with the surface, hence normal reaction equals zero.

$$mg\sin\beta = \frac{mv_B^2}{R}$$

$$mg\sin\beta = \frac{m}{R}[2gR(\cos\alpha - \sin\beta)]$$

$$3\sin\beta = 2\cos\alpha$$

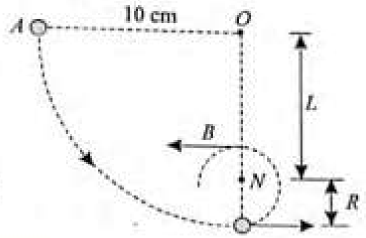
60. (c) Velocity of bob at C should be $\sqrt{5gR}$

$$\sqrt{5g(R-L)} = \sqrt{2gR}$$

$$5(R-L) = 2R$$

$$3R = 5L$$

$$L = \frac{3}{5}R = \frac{3}{5}\times 15 = 9\text{ cm}$$

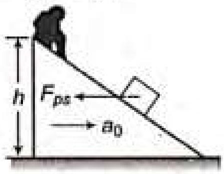


Problems Based on Mixed Concepts

61. (a) Since $\vec{F}_{ps} = -m\vec{a}_0$ is a constant force, $W_{ps} = \vec{F}_{ps}\cdot\vec{S}$

$$= -m\vec{a}_0\cdot\vec{S}$$

$$= -m(a_0\hat{i})\cdot(-l\hat{i}+b\hat{j}) = ma_0 l$$



62. (c) Work done in friction is

$$W = (\mu\, mg\cos\theta)\, s$$

$$= (\mu mg\cos\theta)\left(\frac{h}{\sin\theta}\right) = \mu mgh\cot\theta$$

Now $\cot\theta_1 = \cot 30° = \sqrt{3}$

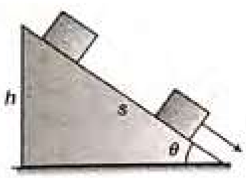
and $\cot\theta_2 = \cot 60° = \frac{1}{\sqrt{3}}$

$\therefore\ W_1 > W_2$

i.e., kinetic energy in first case will be less.

$(K = mgh - W)$

Or $K_1 < K_2$



63. (d) Inc in height of block $h = a\sin\theta$

Inc in length of spring $x = a\theta$

Work done by $p = mgh + \frac{1}{2}kx^2$

$$= Wa\sin\theta + \frac{1}{2}ka^2\theta^2$$

64. (a) Total energy at the time of throwing the ball

$$= mgh + \frac{1}{2}mv_0^2$$

Energy after collision with ground

$$= \frac{1}{2}(mgh + \frac{1}{2}mv_0^2)$$

The ball again rises to height h.

$$\therefore\ \frac{1}{2}(mgh + \frac{1}{2}mv_0^2) = mgh$$

or $\quad mgh + \frac{1}{2}mv_0^2 = 2mgh$

or $\quad \frac{1}{2}mv_0^2 = mgh$ or $v_0 = \sqrt{2gh}$

65. (c) Initially both masses are at rest, so their centre of mass is also at rest. Now we apply equal and opposite forces when means net external force is zero. So their centre of mass will always remain at rest. Elongation will be maximum when again their relative velocity becomes zero, or both simultaneously come to rest.

Let x_1 is displacement of m and x_2 that of M till maximum elongation, then work done by forces is equal to increase in PE of spring. There is no change in KE of blocks as they are at rest finally.

$$Fx_1 + Fx_2 = \frac{1}{2}k(x+x_2)^2$$

$$\Rightarrow\ x_1 + x_2 = \frac{2F}{k} \rightarrow \text{maximum elongation.}$$

66. (a) $KE \propto x \Rightarrow KE = C_x \Rightarrow \frac{1}{2}mv^2 = Cx$

$$\frac{1}{2}m2v\frac{dv}{dt} = C\frac{dx}{dt} \Rightarrow mv\frac{dv}{dt} = Cv$$

$$m\frac{dv}{dt} = C \Rightarrow ma = C$$

$$\Rightarrow\ F = C \rightarrow \text{constant}$$

67. (a) $F = k_A x_A = k_B x_B$

$$W_A = \frac{1}{2}k_A x_A^2;\ W_B = \frac{1}{2}k_B x_B^2$$

$$\frac{W_A}{W_B} = \frac{k_A}{k_B}\frac{x_A^2}{x_B^2} = \frac{k_A}{k_B}\frac{k_B^2}{k_A^2} = \frac{k_B}{k_A}$$

Hence $W_B > W_A$. So more work is done on B.

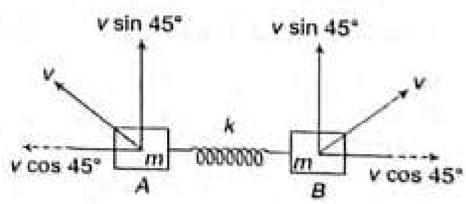
68. (c) Let at $x = 2$, $U = U_2$

$$\frac{10-U_2}{2-1} = \frac{10-0}{3.5-1} \Rightarrow U_2 = 6J$$

$$(KE+PE)_{x=2\text{m}} = (KE+PE)_{x=5\text{ m}}$$

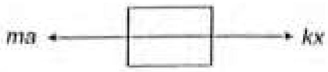
$$\Rightarrow\ 0+6 = \frac{1}{2}(2)v^2 + 2 \Rightarrow v = 2\text{ m/s}$$

69. (a) $\frac{1}{2}kx^2 = 2\left[\frac{1}{2}m\left(\frac{v}{\sqrt{2}}\right)^2\right] \Rightarrow \frac{1}{2}kx^2 = \frac{mv^2}{2}$



$\Rightarrow\ x = v\sqrt{\frac{m}{k}}$ [The velocities along y axis do not change]

70. (b)



$$mv\frac{dv}{dx} = (ma - kx)$$

$$\int_0^0 mv\,dv = \int_0^x (ma - kx)\,dx$$

$$0 = max - \frac{kx^2}{2} \Rightarrow x = \frac{2ma}{k}$$

71. (b) By energy conservation,

$$mgR(1+\cos 30^\circ) = \frac{1}{2}k\left(\frac{\pi R}{6}\right)^2$$

$$\Rightarrow \quad k = \frac{36mg(2+\sqrt{3})}{\pi^2 R}$$

72. (b) We first determine the energy output of the runner:

$$= (0.60 \text{ J/kg.step})(60 \text{ kg})\left(\frac{1\text{step}}{1.50 \text{ m}}\right) = 24.0 \text{ J/m}$$

From this we calculate the force exerted by the runner per step:

$F = (24 \text{ J/m})\ (1 \text{ N.m/J})\ 24\ N$

Then, from the definition of power, $P = Fv$, we obtain

$$v = \frac{P}{F} = \frac{72\ W}{24\ N} = 3.0 \text{ m/s}$$

73. (b) Work done by force F;

$$W = \int \vec{F}.dr = \int (y\hat{i} - x\hat{j})(dx\hat{i} + dy\hat{j}) = \int (ydx - xdy)$$

Equation of circular path

$$x^2 + y^2 = a^2$$

$$\therefore \quad xdx + y\,dy = 0$$

$$\Rightarrow \quad W = \int\left(y\left(\frac{-ydy}{x}\right) - xdy\right) = -\int \frac{(x^2+y^2)}{x}dy$$

$$= -\int_0^a \frac{a^2}{\sqrt{a^2-y^2}}dy = -\frac{\pi a^2}{2}\text{ J}$$

74. (d) For W to be maximum; $\frac{dW}{dx} = 0$;

i.e. $F(x) = 0 \Rightarrow x = l, x = 0$

Clearly for $d = l$, the work done is maximum.

Alternate Solution:

External force and displacement are in the same direction

$\therefore$ Work will be positive continuously so it will be maximum when displacement is maximum.

75. (a) Work done:

$= Mgh_1 + Mgh_2 + Mgh_3 + \mu_1 Mgl_1 + \mu_2 Mgl_2 + \mu 3Mgl_3$

$= Mg\,(h_1 + h_2 + h_3) + Mg\,(\mu_1 l_1 + \mu_2 l_2 + \mu_3 l_3)$

$= Mg\,(8 + 0.2 + 0.4 + 0.4)$

$= 90$ J

ARCHIVES

1. (b) From the conservation of energy, we know that the potential energy at height h is equal to the kinetic energy at ground. Therefore, at height h, the potential energy of ball A is $m_A gh$

Its kinetic energy at ground is, $K = \frac{1}{2}m_A v_A^2$

Applying C.O.M.E

$$\therefore \quad m_A gh = \frac{1}{2}m_A v_A^2$$

$$\Rightarrow \quad v_A = \sqrt{2gh}$$

Similarly, $v_B = \sqrt{2gh}$

Therefore, $v_A = v_B$.

[Note: In the question, it is not mentioned that the magnitudes of thrown velocity of both balls are same, which is assumed in the solution.]

2. (d) Using work–energy theorem, $W_{total} = \Delta K$

i.e., work done by stopping forces should be equal to change in the kinetic energy of the car.

$$-F_{stopping} \times \text{stopping distance} = (K_{final} - K_{initial})$$

$$\Rightarrow \quad -F_{stopping} \times s = \left(0 - \frac{1}{2}mv_{initial}^2\right)$$

$$\Rightarrow \quad F_{stopping} \times s = \frac{1}{2}mv_{initial}^2$$

As stopping force is same for both the cars, hence

$$s \propto v^2_{initial}$$

$$\Rightarrow \quad \frac{s_1}{s_2} = \frac{(v_{initial}^2)_1}{(v_{initial}^2)_2} = \frac{u^2}{(4u)^2} = \frac{1}{16}$$

3. (b) Here the work done in stretching a spring is stored as the potential energy of the spring, i.e., (1/2) kx^2. Thus

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}k(x_2^2 - x_1^2)$$

$$= \frac{1}{2} \times 800\left[\left(\frac{15}{100}\right)^2 - \left(\frac{5}{100}\right)^2\right] = 8 \text{ J}$$

4. (b) kinetic energy, $K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$

$$\Rightarrow \quad p = \sqrt{2mK}. \text{ or } p \propto \sqrt{m}$$

$$\therefore \quad \frac{p_1}{p_2} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

5. (d) Using work–energy theorem, $W_{total} = \Delta K$

i.e., work done by stopping forces should be equal to change in the kinetic energy of the car.

$$-F_{stopping} \times \text{stopping distance} = (k_{final} - k_{initial})$$

$$-F_{stopping} \times s = \left(0 - \frac{1}{2}mv_{initial}^2\right)$$

$$\Rightarrow \quad F_{stopping} \times s = \frac{1}{2}mv_{initial}^2$$

As stopping force is same for both the cars, hence

$s \propto v^2_{\text{initial}}$ Hence, $\frac{s_1}{s_2} = \frac{(v^2_{\text{initial}})_1}{(v^2_{\text{initial}})_2}$

or $\frac{6}{s_2} = \frac{(v^2_{\text{initial}})_1}{(v^2_{\text{initial}})_2} = \left(\frac{50}{100}\right)^2 \Rightarrow s_2 = 24$ m

6. (c) Here, power $P = F.v = \left(m\frac{dv}{dt}\right).v$

$\Rightarrow vdv = \frac{P}{m}dt$

Now integrating, $\int_0^v vdv = \frac{P}{m}\int_0^t dt$

$\Rightarrow \frac{v^2}{2} = \frac{Pt}{m}$ or $v = \left(\frac{2Pt}{m}\right)^{\frac{1}{2}}$

$\Rightarrow \frac{dx}{dt} = \left(\frac{2P}{m}\right)^{\frac{1}{2}} t^{\frac{1}{2}} \Rightarrow dx = \left(\frac{2P}{m}\right)^{\frac{1}{2}} t^{\frac{1}{2}} dt$

Integrating, we get

$$\int_0^x dx = \left(\frac{2P}{m}\right)^{\frac{1}{2}} \int_0^t t^{\frac{1}{2}} dt$$

or $x = \left(\frac{2P}{m}\right)^{\frac{1}{2}} \frac{t^{\frac{3}{2}}}{\frac{3}{2}}$ or $x \propto t^{\frac{3}{2}}$

7. (c) Here, $W = \frac{1}{2}k(x_2^2 - x_1^2)$

$$= \frac{1}{2} \times 5 \times 10^3 \left[\left(\frac{10}{100}\right)^2 - \left(\frac{5}{100}\right)^2\right]$$

$$= \frac{1}{2} \times \frac{5 \times 10^3}{10^4}[100 - 25] = \frac{75}{4} = 18.75 \text{ J}$$

8. (d) Because linear momentum is vector quantity whereas kinetic energy is a scalar quantity.

9. (c) According to problem,

Retardation $\propto$ displacement

or $\frac{dv}{dt} = kx$ or $v\frac{dv}{dx} = kx$ or $vdv = kx\,dx$

or $\int_{v_1}^{v_2} vdv = k\int_0^x x\,dx$ or $\frac{v_2^2}{2} - \frac{v_1^2}{2} = \frac{kx^2}{2}$

or $\frac{1}{m}\left(\frac{mv_2^2}{2} - \frac{mv_1^2}{2}\right) = \frac{kx^2}{2}$

or $(k_2 - k_1) = \frac{mk}{2}x^2$

Thus, loss of kinetic energy is proportional to x^2.

10. (b) Here, $W = \vec{F} \cdot \vec{S}$

$$= (5\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (2\hat{i} - \hat{j}) = 10 - 3 = 7 \text{ J}$$

11. (b) Here, applying $v = u + at$, we get

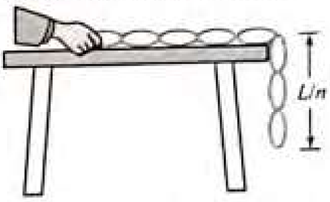
$v_1 = at_1$ or $a = \frac{v_1}{t_1}$

Also $v = at = \frac{v_1}{t_1} \times t$

Power $= |\vec{F} \cdot \vec{v}| = mav = m\left[\frac{v_1}{t_1} \times t\right]\left[\frac{v_1}{t_1}\right] = \frac{mv_1^2 t}{t_1^2}$

12. (a) When a force of constant magnitude which is perpendicular to the velocity of particle acts on a particle, work done is zero and hence change in kinetic energy is zero.

13. (b) A chain of length L and mass M is held on a frictionless table with y of its length hanging over the edge.



Let $m = \frac{M}{L}$ = mass per unit length of the chain and y is the length of the chain hanging over the edge. So the mass of the chain of length y will be ym and the force acting on it due to gravity will be mgy.

The work done in pulling the dy length of the chain on the table.

$dW = F(-dy)$ [As y is decreasing]

i.e., $dW = mgy\,(-dy)$

So the work done in pulling the hanging portion on the table.

$$W = -\int_y^0 mgy\,dy = -mg\left[\frac{y^2}{2}\right]_y^0 = \frac{Mg\,y^2}{2L} \quad [\text{As } m = M/L]$$

$\therefore$ $W = \frac{4 \times 10 \times (0.6)^2}{2 \times 2} = 3.6$ J

14. (d) When block of mass M collides with the spring, its kinetic energy gets converted into elastic potential energy of the spring. From the law of conservation of energy,

$$\frac{1}{2}Mv^2 = \frac{1}{2}kL^2 \;\therefore\; v = \sqrt{\frac{k}{M}}L$$

Where v is the velocity of block by which it collides with spring. So, its maximum momentum,

$$P = Mv = M\sqrt{\frac{k}{M}}L = \sqrt{Mk}\,L$$

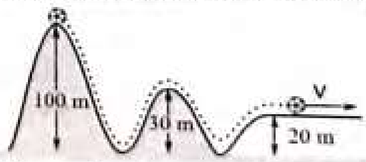
After collision, the block will rebound with same linear momentum.

15. (a) As per question, restoring force on the particle is

$F = 15 \times 0.2 = 3$ N

So the acceleration is $3/0.3 = 10$ m/s^2.

16. (d)



Ball starts from the top of a hill which is 100 m high and finally rolls down to a horizontal base which is 20 m above the ground, so from the conservation of energy

$$mg(h_1 - h_2) = \frac{1}{2}mv^2$$

$\Rightarrow \quad v = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 10 \times (100-20)}$

$= \sqrt{1600} = 40$ m/s.

17. (d)

18. (d) Using work–energy theorem, $W_{total} = \Delta K$

i.e., work done by gravity should be equal to change in the kinetic energy of the particle.

$$W_{gravity} = (K_{final} - K_{initial})$$

$$W_{gravity} = \left(0 - \frac{1}{2} \times \left(\frac{100}{1000}\right)(5)^2\right) = -1.25\text{ J}$$

Work done by force of gravity = –1.25 J

19. (c) Total mechanical energy of the particle,

$E_T = 2$ J.

When kinetic energy is maximum, the potential energy should be minimum.

The potential energy of the particle is given by

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2}$$

or $\quad \dfrac{dV}{dx} = \dfrac{4x^3}{4} - \dfrac{2x}{2} = x^3 - x = x(x^2-1)$

For V to be minimum, $\dfrac{dV}{dx} = 0$

$\therefore \quad x(x^2-1) = 0$, or $x = 0, \pm 1$

At $\quad x = 0, V(x) = 0$

At $\quad x = \pm 1, V(x) = -\dfrac{1}{4}$ J

$\therefore \quad (\text{Kinetic energy})_{max} = E_T - V_{min}$

or $\quad (\text{Kinetic energy})_{max} = 2 - \left(-\dfrac{1}{4}\right) = \dfrac{9}{4}$ J

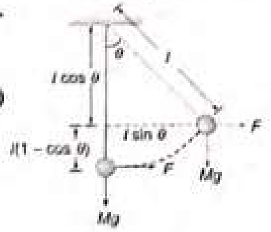
or $\quad \dfrac{1}{2} m v_m^2 = \dfrac{9}{4} \quad$ or $\quad v_m{}^2 = \dfrac{9 \times 2}{m \times 4}$

on solving, we get $v_m = \dfrac{3}{\sqrt{2}}$ m/s

20. (b) According to work energy theorem "work done by all forces is equal to change in kinetic energy".

$\therefore \quad W_{total} = \Delta K$

$W_F + W_{gravity} = (K_{final} - K_{initial})$
$= (0 - 0) = 0$



Work done by F,

$$W_F = F \times l \sin 45° = \frac{Fl}{\sqrt{2}}$$

Work done by gravity,

$$W_{gravity} = -Mg(l - l\cos 45°) = Mgl\left(1 - \frac{1}{\sqrt{2}}\right)$$

$\therefore \quad \dfrac{Fl}{\sqrt{2}} + \left(-\dfrac{Mgl(\sqrt{2}-1)}{\sqrt{2}}\right) = 0$

or $\quad F = Mg(\sqrt{2}-1)$

21. (a) Here the work done by hand should be equal to change in potential energy of the ball

i.e., $W_{hand} = \Delta U$

$\Rightarrow \quad F_{hand}\, s = mgh$

$\Rightarrow \quad F_{hand} = \dfrac{mgh}{s} = \dfrac{0.2 \times 10 \times 2}{0.2} = 20$ N

22. (d) Let the spring be compressed by x.

Loss in KE of block

= Gain in PE of spring + Work done against friction

$\Rightarrow \quad \dfrac{1}{2} \times 2 \times 4^2 = \dfrac{1}{2} 10000\, x^2 + 15x$

On solving, we get $x = 5.5$ cm.

23. (b) Let the kinetic energy of the particle at point of projection be K.

$$K = \frac{1}{2} m u^2$$

At highest point, velocity has its horizontal component $u \cos\theta$. Therefore kinetic energy of a particle at highest point is:

$$K_H = \frac{1}{2} m (u\cos\theta)^2 = \frac{1}{2} m u^2 (\cos\theta)^2 = K\cos^2\theta$$

$\Rightarrow \quad K_H = K\cos^2 60° = \dfrac{K}{4}$

24. (d) average velocity $v_{av} = \dfrac{s}{t} = \dfrac{100}{10} = 10$ m/s

Assuming an athelete has mass about 50 to 100 kg or average mass 75 kg,

His average kinetic energy,

$$k_{av} = \frac{1}{2} m v_{av}^2 = \frac{1}{2} \times 75 \times 100 = 3750\text{ J}$$

Minimum possible kinetic energy, (for 50 kg)

$$k_{min} = \frac{1}{2} m v_{av}^2 = \frac{1}{2} \times 50 \times 100 = 2500\text{ J}$$

Maximum possible kinetic energy (for 100 kg)

$$k_{max} = \frac{1}{2} m v_{av}^2 = \frac{1}{2} \times 100 \times 100 = 5000\text{ J}$$

Hence, the kinetic energy could be in the range 2000 to 5000 J.

25. (c) $U = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

Force, $\quad F = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(\dfrac{a}{x^{12}} - \dfrac{b}{x^6}\right)$

$$= -\left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right] = \left[\frac{12a}{x^{13}} - \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$.

$\therefore \quad \dfrac{12a}{x^{13}} - \dfrac{6b}{x^7} = 0 \quad$ or $\quad x^6 = \dfrac{2a}{b}$

$$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)}$$

$$= \frac{ab^2}{4a^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

$$U_{(x=\infty)} = 0$$

$$D = [U_{(x=\infty)} - U_{\text{at equilibrium}}$$

$$= \left[0 - \left(-\frac{b^2}{4a}\right)\right] = \frac{b^2}{4a}$$

26. (a) $k_1x_1 = k_2x_2 = F$

$$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1}$$

Similarly, $W_2 = \frac{F^2}{2k_1} \Rightarrow W \propto \frac{1}{K}$

$$W_1 > W_2 \Rightarrow k_1 < k_2$$

Statement II is true.

Now, $W_1 = \frac{1}{2}k_1X^2$

$$W_2 = \frac{1}{2}k_2X^2$$

So, $W_2 > W_1$. Hence, statement I is false.

27. (a) Restoring force on rubber-band, $F = ax + bx^2$

Work done in stretching the rubber-band by a small amount dx,

$dW = Fdx$

Net work done in stretching the rubber-band by L is

$$W = \int dW = \int_0^L Fdx = \int_0^L (ax + bx^2)dx$$

$$\Rightarrow \quad W = \left[a\frac{x^2}{2} + b\frac{x^3}{3}\right]_0^L = \frac{aL^2}{2} + \frac{bL^3}{3}$$

28. (c) From work–energy theorem, $W_g + W_{f_1} + W_{f_2} = \Delta K$

$$mgh + 2W = 0 \quad |\because W_{f_1} = W_{f_2} = W|$$

$$\Rightarrow W = -\frac{mgh}{2} \text{ hence } W_{f_1} = -\frac{mgh}{2}$$

But work done by friction,

$$W_{f_1} = -\mu N_1(PQ) = -\mu mg\cos 30°(4)$$

Hence, $-\mu mg\cos 30°(4) = -\frac{mgh}{2} \Rightarrow \mu = 0.29$

As $W_{f_1} = W_{f_2}$ hence $-\mu mgx = -\frac{mgh}{2}$

$$\Rightarrow x = \frac{h}{2\mu} = 3.5 \text{ m}$$

29. (d) The change in potential energy during exercise

$$\Delta PE = 1000 \times mgh$$

$$= 1000 \times 10 \times 9.8 \times 1 = 98000 \text{ J}$$

Mechanical energy per unit mass $= \frac{2}{100} \times 3.8 \times 10^7$

$$= 7.6 \times 10^5 \text{ J/kg}$$

$\therefore$ Fat burn $= \frac{9800}{7.6 \times 10^5} = 12.89 \times 10^{-3}$ kg

30. (a) $\frac{K_f}{K_i} = \frac{\frac{1}{8}mv_0^2}{\frac{1}{2}mv_0^2} = \frac{1}{4}$

which gives $v_f = \frac{v_0}{2}$

and $-kv^2 = \frac{mdv}{dt}$

$$\int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = \int_0^{t_0} \frac{-kdt}{m}$$

$$\left[-\frac{1}{v}\right]_{v_0}^{\frac{v_0}{2}} = \frac{-k}{m}t_0$$

On solving we get

$$k = \frac{m}{v_0t_0} = \frac{10^{-2}}{10 \times 10} = 10^{-4} \text{ kg m}^{-1}$$

31. (c) $F = 6t = ma$

$$\Rightarrow \quad a = 6t$$

$$\Rightarrow \quad \frac{dv}{dt} = 6t \qquad \int_0^v dv = \int_0^1 6t\, dt$$

$$v = (3t^2)_0^1 = 3 \text{ m/s}$$

From work energy theorem

$$W_F = \Delta KE = \frac{1}{2}m(v^2 - u^2)$$

$$= \frac{1}{2}(1)(9 - 0) = 4.5 \text{ J}$$

32. (d) $F = -\frac{dU}{dr} = -\frac{d\left(-K/2r^2\right)}{dr} = \frac{K}{2}\frac{dr^{-2}}{dr}$

$$= \frac{K}{2}\left(-2r^{-3}\right) = -\frac{K}{r^3}$$

Since it is performing circular motion $F = \frac{mv^2}{r} = \frac{K}{r^3}$

$$mv^2 = \frac{K}{r^2} \Rightarrow \text{K.E.} = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

$$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero}$$

CHAPTER 8: CENTRE OF MASS, CONSERVATION OF LINEAR MOMENTUM AND COLLISION

Concept Application Exercise 8.1

1. Block A slides with acceleration $g/2$ and block B slides with acceleration $\sqrt{3}g/2$. Now the acceleration of centre of mass of the system of blocks A and B can be given both in x and in y directions as

$$a_x = \frac{3\times\frac{\sqrt{3}g}{2}\cos 60° - 2\times\frac{g}{2}\cos 30°}{5} = \frac{\sqrt{3}g}{20}$$

and $$a_y = \frac{3\times\frac{\sqrt{3}g}{2}\sin 60° + 2\times\frac{g}{2}\sin 30°}{5} = \frac{11g}{20}$$

Thus acceleration of the centre of mass of the system is given as

$$a_{CM} = \sqrt{a_y^2 + a_x^2} = \sqrt{\left(\frac{\sqrt{3}g}{20}\right)^2 + \left(\frac{11g}{20}\right)^2} = \frac{\sqrt{31}}{10}g$$

2. The rectangular plate is shown in the figure of which one part is removed. We can find the x and y coordinates of the centre of mass of this system, taking origin at the centre of the plate. The coordinates of the three remaining rectangles are $(a/4, b/4)$, $(-a/4, +b/4)$ and $(-a/4, -b/4)$. By geometry, masses of these rectangles can be taken as $M/4$. Now x-coordinate of the centre of mass:

$$x_{CM} = \frac{\frac{M}{4}\frac{a}{4} - \frac{M}{4}\frac{a}{4} - \frac{M}{4}\frac{a}{4}}{3\frac{M}{4}} = \frac{a}{12}$$

and y-coordinate of the centre of mass:

$$y_{CM} = \frac{\frac{M}{4}\frac{b}{4} + \frac{M}{4}\frac{b}{4} - \frac{M}{4}\frac{b}{4}}{3\frac{M}{4}} = \frac{b}{12}$$

3. If m_1 and m_2 are placed at a distance l apart, their centre of mass will be located at a distance x from m_1, where

$$x = \frac{m_2 l}{m_1 + m_2}$$

If m_1 is displaced by l_1 towards C and m_2 is displaced by l_2 away from C, the new centre of mass C' now will be located at a distance x' from m_1, where

$$x' = \frac{m_2(l - l_1 + l_2)}{m_1 + m_2}$$

Displacement of the centre of mass is

$$\Delta x = x' + l_1 - x = \frac{m_1 l_1 + m_2 l_2}{m_1 + m_2}$$

4. First we write the three velocities in vectorial form, taking right direction as positive x-axis and upwards as positive y-axis.

$$\vec{v}_1 = -\frac{1}{2}v_1\hat{i} - \frac{\sqrt{3}}{2}v_1\hat{j}$$

$$\vec{v}_2 = v_2\hat{i},\quad \vec{v}_3 = -\frac{1}{2}v_3\hat{i} + \frac{\sqrt{3}}{2}v_3\hat{j}$$

Thus the velocity of centre of mass of the system is

$$\vec{v}_{CM} = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3}{3}$$

$$= \left(v_2 - \frac{1}{2}v_1 - \frac{1}{2}v_3\right)\hat{i} + \frac{\sqrt{3}}{2}(v_3 - v_1)\hat{j}$$

which can be written as $\vec{v}_{CM} = v_x\hat{i} + v_y\hat{j}$

Thus displacement of the centre of mass in time t is

$$\Delta\vec{r} = v_x t\,\hat{i} + v_y t\,\hat{j}$$

If $v_1 = v_2 = v_3 = v$, we have $\vec{v}_{CM} = 0$

Therefore, there is no displacement of centre of mass of the system.

5. Initial coordinates of CM = (0, 0, 0)

x coordinates of CM after 3 s,

$$x_{CM} = 120 \times 3 = 360 \text{ m}$$

y coordinates of CM after 3s,

$$y_{CM} = -\frac{1}{2}gt^2 = -45 \text{ m}$$

After 3 s,

$$(m_1 + m_2)x_{CM} = m_1x_1 + m_2x_2$$

$$x_2 = \frac{30\times 360 - 12\times 300}{18} = 400 \text{ m}$$

Similarly, $(m_1 + m_2)\,y_{CM} = m_1y_1 + m_2y_2$

$$-30\times 45 = 12\times 24 + 18\times y_2$$

$$y_2 = -91 \text{ m}$$

and in z coordinate, $m_1z_1 + m_2z_2 = 0 \Rightarrow z_2 = 32$ m

Concept Application Exercise 8.2

1. The velocity of shell at the highest point is

$$v = u\sin\theta = 100\times\sin 30° = 50 \text{ m/s}$$

Let m be the mass of the shell. Then the mass of the lighter fragment is $m/3$ and that of heavier fragment is $2m/3$.

Initial momentum of the shell before explosion is

$$mv = 50m$$

As no external force is acting on the shell, we can conserve momentum of the shell before and after its explosion.

In x-direction,

$$mv = \frac{2m}{3}v_2\cos\theta \Rightarrow v_2\cos\theta = \frac{3}{2}v \quad \text{(i)}$$

in y-direction,

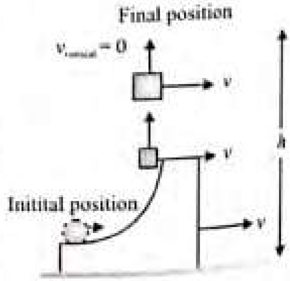
$$0 = \frac{m}{3}v_1 - \frac{2m}{3}v_2\sin\theta \Rightarrow v_2\sin\theta = \frac{v_1}{2} \quad \text{(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$v_2^2 = \frac{9}{4}v^2 + \frac{1}{4}v_1^2$$

$$\Rightarrow \quad v_2 = \frac{1}{2}\sqrt{9v^2 + v_1^2} = \frac{1}{2}\sqrt{9\times 2500 + 40000} = 125 \text{ m/s}$$

2. At the instant the block breaks contact with the wedge, they have common x-component of velocity. In addition, the block has a vertical component of velocity.



Due to this vertical component, the block rises upwards till the vertical component of velocity vanishes.

From momentum conservation along x-axis,

$$mv_0 = (m + M)v \quad \text{(i)}$$

$$v = \frac{mv_0}{(m+M)} \quad \text{(ii)}$$

From energy conservation between initial and final positions of the block,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(m+M)v^2 + mgh \quad \text{(iii)}$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}\left(\frac{m^2}{m+M}\right)v_0^2 + mgh \Rightarrow h = \frac{v_0^2}{2g}\left[\frac{M}{m+M}\right]$$

3. From conservation of momentum

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = 5 \text{ m/s}$$

Conservation of energy,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1+m_2)v^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad x^2 = \frac{m_1v_1^2 + m_2v_2^2 - (m_1+m_2)v^2}{k}$$

$$\Rightarrow \quad x = \sqrt{\frac{70}{1120}} = 0.25 \text{ m}$$

When the blocks have equal speeds, spring has maximum compression. After this instant, the spring again expands and after sometime m_1 loses contact with the spring.

Let v_1 and v_2 be the velocities of the blocks after they lose contact.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

and $\quad v_1 - v_2 = -1\,(u_1 - u_2)$

[$e = 1$, because there is no loss in KE]

Solving for v_1 and v_2, we get

$$v_1 = 0 \text{ m/s and } v_2 = 7 \text{ m/s}$$

4. (a) Velocity of the projectile after 5 s,

$$v = u - gt = 100 - 9.8 \times 5 = 51 \text{ m/s}$$

Therefore, momentum of projectile before explosion is

$$Mv = 50 \times 51 = 2550 \text{ kg m/s}$$

The momentum of the fragments after explosion is

$$m_1v_1 + m_2v_2 = 20 \times 150 + 30v = 3000 + 30\,v_2$$

Therefore, conservation of momentum principle yields

$$2550 = 3000 + 30v_2$$

or $\quad v_2 = -\dfrac{450}{30} = -15$ m/s

So velocity of the second fragment will be 15 m/s downward.

(b) Three seconds after explosion, velocity of first fragment upward

$$v_2' = u' - gt' = 150 - 9.8 \times 3 = 120.6 \text{ m/s}$$

Velocity of the second fragment downward

$$v_2'' = u'' + gt' = 15 + 9.8 \times 3 = 44.4 \text{ m/s}$$

$\therefore$ Sum of momenta,

$$m_1\vec{v}_1 + m_2\vec{v}_2 = 20 \times 120.6 - 30 \times 44.4 = 1080 \text{ kg m/s}$$

If the projectile had not exploded, its velocity after 8 s from firing would have been,

$$v' = u - gt' = 100 - 9.8 \times 8 = 21.6 \text{ m/s}$$

Therefore, momentum of the projectile would have been

$$Mv' = 50 \times 21.6 = 1080 \text{ kg m/s}$$

So net momentum at 8 s is independent of explosion.

5. Conservation of linear momentum along X and Y axes gives

$$mu = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

i.e., $\quad 2u = 1 \times v_1\cos\theta_1 + 1 \times v_2\cos\theta_2 \quad$ (i)

$\Rightarrow \quad v_1\cos\theta_1 + v_2\cos\theta_2 = 2u$

and $\quad 0 = m_1v_1\sin\theta_1 + m_2v_2\sin\theta_2$

As $\quad m_1 = m_2 = 1$ kg

$\Rightarrow \quad v_1\sin\theta_1 = v_2\sin\theta_2$

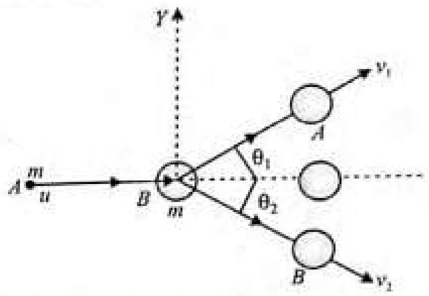
As $v_1 = v_2 = v$ (say)

$\Rightarrow \quad \theta_1 = \theta_2 = \theta$ (say)

From Eq. (i),

$\Rightarrow \quad v\cos\theta + v\cos\theta = 2u$

$\Rightarrow \quad v\cos\theta = u = 4 \quad$ (ii)



Also conservation of energy gives

$$\frac{1}{2}mu^2 + 48 = \frac{1}{2}(m_1)v^2 + \frac{1}{2}(m_2)v^2$$

$$\Rightarrow \quad \frac{1}{2} \times 2 \times (4)^2 + 48 = \frac{1}{2} \times 1v^2 + \frac{1}{2} \times 1v^2$$

$\Rightarrow \quad v = 8$ m/s

From Eq. (ii),

$$\cos\theta = \frac{u}{v} = \frac{4}{8} = \frac{1}{2}$$

$\Rightarrow \quad \theta = 60°$

6. Apply conservation of momentum:

$$\Rightarrow \quad m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$$

$$m(2\hat{i} + 5\hat{j} - 6\hat{k}) + 2m\vec{v}_2 + m(-4\hat{i} + 3\hat{j} + 2\hat{k}) = 0$$

$$\vec{v}_2 = \hat{i} - 4\hat{j} + 2\hat{k}$$

Concept Application Exercise 8.3

1. Let velocities of sphere 1 and 2 after collision be v_1 and v_2, respectively, then

$$\frac{v_2 - v_1}{u} = e \quad \text{and} \quad mv_2 + mv_1 = mu$$

From above equations: $v_2 = \dfrac{(1+e)u}{2}$

Now sphere 2 will collide with sphere 3 and after collision velocity of 3 can be formed as

$$v_3 = \left(\frac{1+e}{2}\right)^2 u$$

Hence, similarly velocity of nth sphere can be formed as

$$v_n = \left(\frac{1+e}{2}\right)^{n-1} u$$

2. Impact takes place between m_1 and m_3 horizontally. Since m_2 is kept on m_1 and all the surfaces in contact are smooth, friction does not act between m_2 and m_1 during the displacement of m_3 in the impact. Even though there is a friction between m_2 and m_3, which is very less than the impact force, the frictional force is assumed as non-impulsive force here.

Since the impact between m_1 and m_3 is inelastic, m_1 and m_3 will move together toward right and m_2 will not move due to the absence of friction.

The velocity of the combined mass $= v' = \dfrac{m_1 v}{m_1 + m_3}$

$$\frac{|\Delta KE|}{KE} = \frac{\frac{1}{2}m_1 v^2 - \frac{1}{2}(m_1 + m_3)v'^2}{\frac{1}{2}m_1 v^2}$$

$$= 1 - \frac{m_1}{m_1 + m_3} = \frac{m_3}{m_1 + m_3}$$

If m_2 and m_3 are rigidly attached, both together behave as a single mass $(m_2 + m_3)$ and the answer would have been $\dfrac{m_2 + m_3}{m_1 + m_2 + m_3}$.

3. Since m_2 is at rest $(u_2 = 0)$, velocity of m_1 after collision:

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1$$

Given, $v_1 < 0 \Rightarrow m_1 - em_2 < 0$

$\Rightarrow \quad m_1 < em_2 \Rightarrow m_1 < m_2$

4. $u_1 = 12$ m/s $\qquad m_1 = 4$ kg
$u_2 = 4$ m/s $\qquad m_2 = 8$ kg

Let v_1 and v_2 be the velocities after impact.

Conservation of momentum:

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

$\Rightarrow \quad 4v_1 + 8v_2 = 80 \qquad$ (i)

Newton's experimental law: $v_1 - v_2 = -e(u_1 - u_2)$

$v_1 - v_2 = -0.5\,(12 - 4) = -4 \qquad$ (ii)

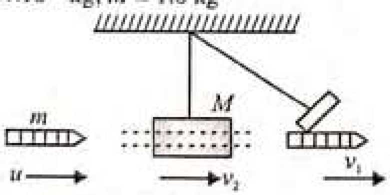
Solving Eqs. (i) and (ii), we get

$$v_1 = 4 \text{ m/s},\ v_2 = 8 \text{ m/s}$$

$$\text{Loss in KE} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$= \frac{1}{2}[4(144) + 8(16)] - \frac{1}{2}[4(16) + 8(64)] = 64 \text{ J}$$

5. Let $m = 2 \times 10^{-3}$ kg; $M = 1.0$ kg



$u = 500$ m/s $v_1 = 100$ m/s

v_2 = speed of the pendulum after the impact

$mu = mv_1 + Mv_2$ (conservation of momentum)

$$v_2 = \frac{m(u - v_1)}{M} = \frac{2}{1000}(500 - 100) = 0.8 \text{ m/s}$$

The block swings and its kinetic energy gets converted into potential energy.

$$\frac{1}{2}Mv_2^2 = Mgh$$

$$\Rightarrow \quad h = \frac{v_2^2}{2g} = \frac{0.8 \times 0.8}{2 \times 10} = 0.032 \text{ m}$$

6. Resolving the velocity before impact, velocity component along the plane $= 8 \sin 60° = 4\sqrt{3}$ m/s and that normal to the plane $= 8 \cos 60° = 4$ m/s.

If 'v' m/s is the velocity after the impact and θ the angle made by the velocity with the normal to the plane, then velocity component along the plane $= v \sin\theta$ and that normal to the plane $= v \cos\theta$.

As the velocity along the plane remains unchanged,

$\therefore \quad v \sin\theta = 4\sqrt{3} \qquad$ (i)

From Newton's experimental law,

$v \cos\theta = e(4) \qquad$ (in magnitude)

$= 2 \qquad$ (ii)

Squaring and adding Eqs. (i) and (ii),

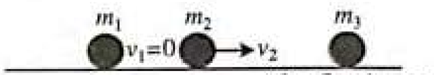
$$v^2 = 52 \Rightarrow v = 2\sqrt{13} \text{ m/s}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\tan\theta = 2\sqrt{3} \Rightarrow \theta = \tan^{-1}(2\sqrt{3})$$

7.

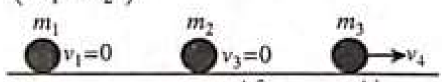
Before impact



After first impact

After first impact: $v_1 = 0$

$$\Rightarrow \quad \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 = 0 \Rightarrow m_1 = em_2 \qquad \text{(i)}$$



After second impact

After second impact: $v_3 = 0$

$$\Rightarrow \quad \left(\frac{m_1 - em_3}{m_1 + m_3}\right)v_2 = 0 \Rightarrow m_2 = em_3 \qquad \text{(ii)}$$

From (i) and (ii), $m_2^2 = m_1m_3$

Exercises

Centre of Mass and Its Motion

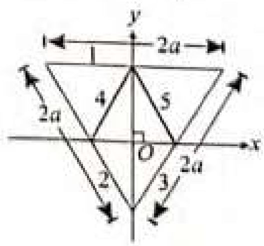
1. (c) $y_{cm} = \dfrac{m\cdot 0 + m\cdot 0 + m\cdot b + mb + m2b}{5m} = \dfrac{4b}{5}$

2. (a) $X_{cm} = \dfrac{m \times 0 + ml + m \times 2l + \frac{m}{2}\frac{3l}{2}}{3.5\text{ m}}$

$$X_{cm} = \frac{15ml}{4 \times \frac{7}{2}\text{m}} = \frac{15l}{14}$$

$$Y_{cm} = \frac{m \times 0 + ml + ml + \frac{m}{2}2l}{\frac{7}{2}\text{m}} \Rightarrow Y_{cm} = \frac{3ml}{\frac{7m}{2}} = \frac{6l}{7}$$

3. (b)



The y-coordinate of centre of mass is

$$\bar{y}=\frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4+m_5y_5}{m_1+m_2+m_3+m_4+m_5}$$

$$=\frac{2m\left(\frac{\sqrt{3}}{2}a\right)+2m(0)+2m(0)+m\left(\frac{\sqrt{3}}{4}a\right)+m\left(\frac{\sqrt{3}}{4}a\right)}{8m}$$

$$=\frac{9a}{16\sqrt{3}}$$

4. (c) Let the position of C.M. of remaining structure from O is $\vec{r}$

$$\vec{r}=\frac{8m\times 0-m\left(\frac{l}{2}\hat{i}+\frac{l}{2}\hat{j}+\frac{l}{2}\hat{k}\right)}{7\,m}=\frac{-l(\hat{i}+\hat{j}+\hat{k})}{14}$$

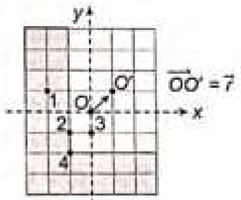
$$\Rightarrow\quad |\vec{r}|=\frac{\sqrt{3}l}{14}$$

5. (b) Let the mass of a small square $=m$

Hence the mass of the square plate $=48m$

And the mass of the L shaped plate $=24m$

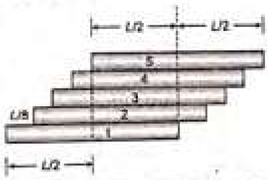
Let the position of centre of mass of remaining structure be $\vec{r}'$



$$\vec{r}'=\frac{48m\times 0-24m\times\vec{r}}{48m-24m}=-\vec{r}$$

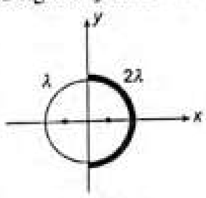
Hence the position of C.M. should be at '2'.

6. (c) As shown in the diagram, fifth brick is in limiting equilibrium. So, when sixth brick is placed over the fifth brick, both will fall down as fifth brick is in limiting equilibrium.



7. (b) Let the two half rings be placed in left and right of y-axis with centre as shown in figure.

Then the coordinate of centre of mass of left and right half rings are $\left(-\frac{2R}{\pi},0\right)$ and $\left(\frac{2R}{\pi},0\right)$.



$\therefore$ x-coordinates of centre of mass of complete ring is

$$\frac{m\left(-\frac{2R}{\pi}\right)+2m\left(\frac{2R}{\pi}\right)}{3m}=\frac{2R}{3\pi}$$

8. (d) Absence of external force, only confirms about the fact that $\vec{a}=0$. However, $\vec{v}$ may or may not be zero.

9. (a) $\vec{F}=m\frac{\Delta\vec{v}_{cm}}{\Delta t}=\frac{10(3\hat{j}-4\hat{i})}{2}=5(3\hat{j}-4\hat{i})$

$|\vec{F}|=5\sqrt{3^2+4^2}=25\text{N}$

10. (c) velocity of centre of mass will remain unaffected as no external forcer acts on system.

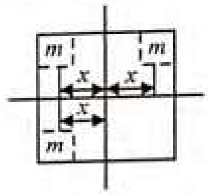
11. (c) As boy walks from left to right on the block, the block will recoil towards left. The friction acting on the block will act towards right. Hence, the centre of mass of the boy and block will shift towards right.

12. (a) Initially Y-coordinate of centre of mass is zero. After explosion Y-coordinate of centre of mass of the particles should be zero.

$$\frac{m}{4}\times(15)-\frac{3}{4}m(Y)=0\Rightarrow Y=-5\text{ cm}$$

13. (c) The mass move under the influence of gravitational pull which acts along the vertical. Thus CM changes along vertical while it remains unchanged in the horizontal direction.

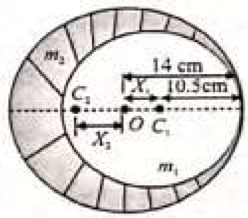
14. (b)



$x_1=\left(\frac{mx}{M-m}\right)\Big|\,x_2=\left(\frac{2mx}{M-2m}\right)\Big|\,x_3=\left(\frac{mx}{M-3m}\right)$

$x_1<x_3<x_2$

15. (c) C_1 is the centre of mass of cut portion and C_2 that of remaining portion. We have to find x_2.



$x_1=14-10.5=3.5$ cm

Mass will be proportional to area. So mass of the whole disc is

$M=k\pi\,(14)^2$

Mass of cut portion $m_1=k\pi\,(10.5)^2$

Mass of the remaining portion $m_2=M-m_1=k\pi\,(14^2-10.5^2)=k\pi\,(24.5)\times(3.5)$

Now, $m_1x_1=m_2x_2$

$$\Rightarrow\quad x_2=\frac{m_1x_1}{m_2}=\frac{k\pi\,(10.5)^2\times 3.5}{k\pi\,(24.5)\times 3.5}=4.5\text{ cm}$$

16. (d) As long as no external force acts on the system of 'man + boat', its centre of mass will remain stationary.

17. (c) **Method 1:** Acceleration of 3 kg:

$$a_2=\frac{6}{3}=2\text{ m/s}^2$$

Velocity of 3 kg at $t=5$ s: $v_2=u_2+a_2t\;=0+2\times 5=10$ m/s

$$v_{CM(t=5s)}=\frac{2\times 0+3\times 10}{2+3}=6\text{ m/s}$$

Method 2:

$$a_{CM}=\frac{F}{m_1+m_2}=\frac{6}{2+3}=1.2\text{ m/s}^2$$

$$v_{CM}=u_{CM}+a_{CM}\,t=0+1.2\times 5=6\text{ m/s}$$

18. (b) As no external force acts on the (ball + box) system, hence velocity of the system remains constant.

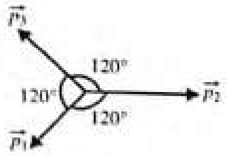
19. (a) Using $\vec{v}_{CM}=\hat{i}\,v_x+\hat{j}\,v_y$

where $v_x=\frac{mv_{x_1}+mv_{x_2}+mv_{x_3}}{3m}=0$

$$v_y=\frac{mv_{y_1}+mv_{y_2}+mv_{y_3}}{3m}=0$$

Net velocity is zero (using vector theory) i.e., CM is at rest. So, displacement of C.M. is zero. So choices (b), (c) and (d) are wrong.

20. (d) $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3}{m_1 + m_2 + m_3}$



$\Rightarrow \quad \vec{v}_{cm} = \dfrac{\text{Total momentum}}{\text{Total mass}}$

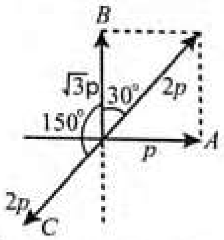
Here total momentum of system is zero, because momentum of each particle is same in magnitude and they are symmetrically oriented as shown.

So $\quad \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

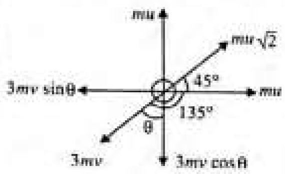
So, velocity of CM of the system will be zero

Conservation of Linear Momentum

21.(d) To conserve the momentum, momentum of C will be as shown, i.e. opposite to the resultant of A and B.



22. (a) Suppose m is the mass of each piece flying off perpendicular to one another with same speed u (= 30 m/s). Then $3m$ is the mass of the third piece. Let v be the velocity of the third piece. According to the figure,



$3\,mv\cos\theta = mu$, $3\,mv\sin\theta = mu$

$\tan\theta = 1$ or $\theta = 45°$

Hence, $3\,mv\cos 45° = mu$

or $\quad \dfrac{3v}{\sqrt{2}} = u = 30$ m/s $\quad$ or $\quad v = 10\sqrt{2}$ m/s

(inclined at 135° w.r.t. direction of each one)

23. (b) Since the body explodes into three equal parts, therefore

$m_1 = m_2 = m_3 = \dfrac{m}{3} = 1$ kg

Let the velocity of the third part be $\vec{v}$. According to the principle of conservation of linear momentum,

Momentum of system before explosion = Momentum of system after explosion

or $\quad mv = m_1v_1 + m_2v_2 + m_3v_3$

or $\quad 3 \times 0 = 1 \times 2\hat{i} + 3\hat{j} + 1 \times \vec{v}$

or $\quad v = -(2\hat{i} + 3\hat{j})$ m/s

Average force acting on the third particle is

$$\vec{F} = \frac{m\vec{v}}{t} = \frac{-1 \times (2\,\hat{i} + 3\,\hat{j})}{10^{-5}} = -(2\,\hat{i} + 3\,\hat{j}) \times 10^5 \text{ N}$$

24. (d) $u_x = 20\sqrt{2}\cos 45° = 20$ m/s

$u_y = 20\sqrt{2}\sin 45° = 20$ m/s

After 1 s, horizontal component remains unchanged while vertical component becomes

$v_y = u_y - gt = 20 - 10 = 10$ m/s

Due to explosion, one part comes to rest. Hence, from conservation of linear momentum, vertical component of second part will become $v_{y1} = 20$ m/s. Therefore, maximum height attained by the second part will be

$H = h_1 + h_2$

Here, h_1 = height attained in 1 s

$= (20)(1) - \dfrac{1}{2} \times 10 \times 1^2 = 15$ m

and h_2 = height attained after 1 s

$\dfrac{v_{y1}^2}{2g} = \dfrac{(20)^2}{2 \times 10} = 20$ m

$\therefore \quad H = 15 + 20 = 35$ m

25. (d) Let combined velocity of the system be v.

$(m + M)v = mu \Rightarrow v = \dfrac{mu}{m + M}$

$\text{KE} = \dfrac{1}{2}(m + M)v^2 = \dfrac{m^2u^2}{2(M + m)}$

26. (a) Let three of the fragments move along X, Y and Z axes. Therefore their velocities can be given as

$\vec{v}_1 = v\,\hat{i}$, $\vec{v}_2 = v\,\hat{j}$ and $\vec{v}_3 = v\,\hat{k}$

where v = speed of each of the three fragments. Since, in explosion no net external force is involved, the net of momentum of the system remains conserved just before and after the explosion.

$\Rightarrow \quad (p)_f = (p_i)$

$\Rightarrow \quad m\vec{v}_1 + m\vec{v}_2 + m\vec{v}_3 + m\vec{v}_4 = 0$

($p_i = 0$ because the body was stationary), putting the values of $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$, we obtain, $\vec{v}_4 = v(\hat{i} + \hat{j} + \hat{k})$

Therefore, $v_4 = \sqrt{3}v$.

The energy of explosion (ΔKE) system

$\Rightarrow \quad E = \text{KE}_f - \text{KE}_i$

$$= \left(\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 + \frac{1}{2}mv_4^2\right) - (0)$$

Putting $v_1 = v_2 = v_3 = \dfrac{v_4}{\sqrt{3}} = v$ and putting $\dfrac{1}{2}mv^2 = E_0$,

we obtain $E = 6E_0$.

27. (d) Conservation of momentum during explosion yields

$\Rightarrow \quad \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0 \Rightarrow \vec{p}_3 = |\vec{p}_1 + \vec{p}_2|$

$\Rightarrow \quad m_3v_3 = |m_1\vec{v}_1 + m_2\vec{v}_2|$

since $\quad \vec{v}_1 \perp \vec{v}_2 \Rightarrow v_3 = \sqrt{\dfrac{m_1^2v_1^2 + m_2^2v_2^2}{m_3}}$

$$\Rightarrow \quad v_3 = \sqrt{\left(\frac{m_1}{m_3}\right)^2 + v_1^2 + \left(\frac{m_2}{m_3}\right)^2 v_2^2}$$

$$\Rightarrow \quad \sqrt{\left(\frac{1}{3}\right)^2 (15)^2 + \left(\frac{1}{3}\right)^2 (15)^2} \qquad (\because m_1 : m_2 : m_3 = 1 : 1 : 3)$$

$\Rightarrow \quad v_3 = 5\sqrt{2}$ ms^{-1}

28. (b) Applying momentum conservation,

$(80)1 + 60(-2) = (80 + 60 + 100)\,v$

$v = \dfrac{-40}{240} = -\dfrac{1}{6}$ m/s

29. (d) Let the velocity of man after jumping be 'u' towards right. Then speed of cart is $v - u$ towards left. From conservation of momentum $mu = 2m(v - u)$

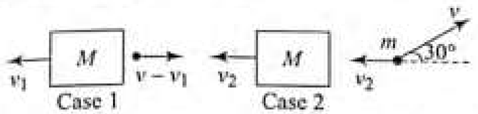
$\therefore \quad u = \dfrac{2v}{3}$

Hence, work done by man = change in KE of system

$$=\frac{1}{2}mu^2+\frac{1}{2}2m(v-u)^2$$

$$=\frac{1}{2}m\left(\frac{2v}{3}\right)^2+\frac{1}{2}2m\left(\frac{v}{3}\right)^2=\frac{mv^2}{3}$$

30. (b) Let mass of gun is M and that of shell is m. The two cases are shown in the figure as below.



Here v_1 and v_2 are the recoil speeds of the gun in two cases. Using conservation of linear momentum in horizontal direction in two cases:

$$m(v-v_1)=Mv_1$$

$$\therefore \quad v_1=\frac{mv}{M+m} \qquad (1)$$

$$m(v\cos 30^\circ - v_2)$$

$$\therefore \quad v_2=\frac{\sqrt{3}mv}{2(M+m)} \qquad (2)$$

From equations (1) and (2)

$$\frac{v_1}{v_2}=\frac{2}{\sqrt{3}}$$

31. (a) Let speed of block is v. Then from conservation of linear momentum in horizontal direction velocity of cylinder will by $2v$ in opposite direction $\left(\text{as } m=\frac{M}{2}\right)$.

Now from conservation of mechanical energy we have

$$mgh=\frac{1}{2}Mv^2+\frac{1}{2}m(2v)^2$$

Here $h=R-r=1.0$ m

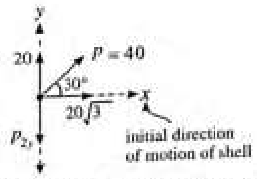
Substituting the values, we get,

$$(1)(10)(1)=\frac{1}{2}(2)(v^2)+\frac{1}{2}(1)(4v^2)$$

or $\quad 3v^2=10$

$$\therefore \quad v=\sqrt{\frac{10}{3}}\text{ m/s}$$

32. (c)



As shown in the figure, the components of momentum of one shell along initial direction and perpendicular to initial direction are $p_{1x}=20\sqrt{3}$ Ns and $p_{1y}=20$ Ns.

For momentum of the system to be zero in y-direction, p_{2y} must be 20 Ns. 2nd part of shell may or may not have momentum in x-direction

$\therefore \quad p_{2\min}=20$ Ns

33. (c) By conservation of linear momentum (consider man and plank as system)

$$mv+mv=m(-v)+mv'$$

(v' is the final speed of plank)

$v'=3v$

34. (b) Since $\Sigma\vec{F}_{ext}=\vec{0} \Rightarrow m_A\vec{a}_A+m_B\vec{a}_B=0$

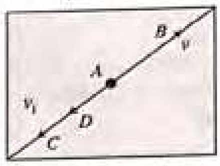
$\therefore$ Momentum of system will remain conserved, equal to zero.

$$m_A\vec{v}_A+m_B\vec{v}_B=0$$

But $\quad W_A+W_B\neq 0$

35. (c) Initial momentum of system of four particles is zero. So after collision net momentum will be zero. For this C and D will go opposite to B.

$$2mv_1=mv \Rightarrow v_1=v/2$$



36. (b) They can avoid the collision when separation between them starts.

First A throws ball towards B. Applying conservation of momentum on 'A + ball' system

$$80\times 1=70\,v_A+10\,(5+v_A)$$

where v_A is speed of A towards B after throwing the ball.

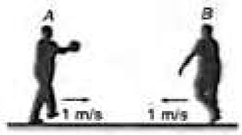
$$v_A=\frac{3}{8}\text{m/s}$$

B catches the ball and throws towards A. Let v_B is speed of B towards A after the throw. Therefore

$$70\times 1-10\times\frac{43}{8}=70\times v_B+10\,(5+v_B)$$

$$\frac{130}{8}-50=80\,v_B$$

$$-\frac{370}{640}=v_B \Rightarrow v_B=-\frac{1}{2}$$



i.e. B is going towards right with speed more than that of A (they are separating).

37. (b) The required work done by man = kinetic energy of man + kinetic energy of boat

$$=\frac{1}{2}\frac{p^2}{M}+\frac{1}{2}\frac{p^2}{m} \qquad \text{where } (p=Mv)$$

$$\therefore \quad W=\frac{1}{2}\left(\frac{M^2}{M}+\frac{M^2}{m}\right)v^2=\frac{1}{2}\left(M+\frac{M^2}{m}\right)v^2$$

38. (a) Compression is maximum when both blocks move with the same velocity v.

By conservation of momentum

$$v=\frac{m_1v_1+m_2v_2}{m_1+m_2}=5\text{ m/s}$$

The change in K.E. $=K_f-K_i=-35$ J

This is stored as spring PE.

Therefore $\frac{1}{2}kx^2=\Delta K \Rightarrow x=\sqrt{\frac{2\Delta K}{k}}$

on solving $x=0.25$ m = 25 cm

39. (c) Muzzle velocity $=v_{m/g}=v_0$

Along x-direction: $v_{m(x)}-v_{g(x)}=v_0\cos\theta$

By momentum conservation:

$$(M+m)(0)=m(v_0\cos\theta-v)-Mv$$

$$\Rightarrow \quad v=\frac{mv_0\cos\theta}{(M+m)}$$

40. (b) Given $\dfrac{v_v^2}{2g} = 20$ m

$v_y = 20$ m/sec = Vertical velocity of second part

For conservation of momentum

$m \times 0 + mv_y = (2m)v_{y0}$

v_{y0} = Vertical velocity of particle after one second before breaking of it.

$= \dfrac{v_v}{2} = \dfrac{20}{2} = 10$ m/s

But $v_{y0} = 10 = u_y - gt$

$u_y = 10 + (10)(1) = 20$ m/s

Now $u_y = u \sin 45°$

So, $u = \dfrac{u_y}{\sin 45^0} = 20\sqrt{2}$ m/s

Problems Based on Collision

41. (a) $m_1u_1 = m_2v_2$

$$\frac{1}{2}m_2v_2^2 = \frac{1}{2}\left[\frac{1}{2}m_1u_1^2\right]$$

$$\Rightarrow \quad (m_2v_2)v_2 = \frac{1}{2}(m_1u_1)u_1 \Rightarrow \quad v_2 = \frac{u_1}{2}$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - 0}{u_1 - 0} = \frac{v_2}{u_1} \Rightarrow \quad e = \frac{1}{2}$$

42. (c) $mv = mv_1 + nmv_2$

$v = v_1 + nv_2$

For elastic collision,

$$\Rightarrow \quad v_2 = \frac{2v}{(n+1)}, \; v_1 = \left(\frac{1-n}{1+n}\right)v$$

$$\frac{\text{KE}_1}{\text{KE}_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \left(\frac{n-1}{n+1}\right)^2$$

43. (d) As the collision is elastic, we can find

$$v_1 = \frac{1-n}{1+n}u, \; v_2 = \frac{2u}{1+n}$$

Hence, the required fraction is

$$\frac{\frac{1}{2}nmv_2^2}{\frac{1}{2}mu^2} = n\left(\frac{v_2}{u}\right)^2 = \frac{4n}{(1+n)^2}$$

44. (b) $\dfrac{\frac{1}{2}mv_1^2}{\frac{1}{2}nmv_2^2} = \dfrac{v_1^2}{nv_2^2} = \dfrac{(1-n)^2}{4n}$

45. (a) In this problem, the velocity of the earth before and after the collision may be assumed zero. Hence, coefficient of restitution will be

$$e^n = \frac{v_1}{v_0} \times \frac{v_2}{v_1} \times \frac{v_3}{v_2} \times \cdots \times \frac{v_n}{v_n - 1}$$

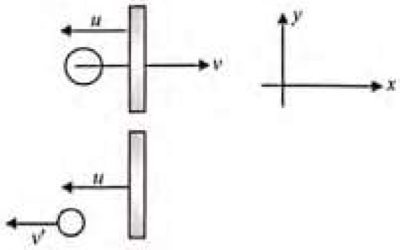
where v_n is the velocity after nth rebounding and v_0 is the velocity with which the ball strikes the earth for the first time. Hence,

$$e^n = \frac{v_n}{v_0} = \frac{\sqrt{2gh_n}}{\sqrt{2gh_0}}$$

where h_n is the height to which the ball rises after nth rebounding. Hence,

$$e^n = \frac{v_n}{v_0} = \frac{\sqrt{h_n}}{\sqrt{h_0}}$$

46. (d) $e = \dfrac{v_2 - v_1}{u_1 - u_2} \;\Rightarrow\; 1 = \dfrac{-u-(-v')}{v-(-u)}$



$\Rightarrow \quad v' = 2u + v$

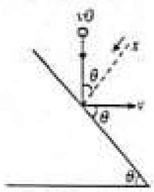
$\therefore \quad W = \Delta\text{KE} = \frac{1}{2}mv'^2 \; \frac{1}{2}mv^2$

$= \frac{1}{2}m[(2u+v)^2 \; v^2] = 2\,mu\,(u+v)$

47. (b) Writing coefficient of restitution equation

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{0 - v\sin\theta}{-v_0\cos\theta - 0} = \frac{v}{v_0}\tan\theta \qquad \text{(i)}$$

The velocity of ball will remain unchanged in the direction of sloping surface (common tangent)



$v_0 \sin\theta = v\cos\theta$

$\Rightarrow \quad \dfrac{v}{v_0} = \tan\theta \qquad$ (ii)

$\Rightarrow \quad e = \tan^2\theta.$

48. (b) Conservation of linear momentum just before and after yields

$m_1v\hat{i} = m_2v_2\hat{i} + m_1v_1(-\hat{i})$

$\Rightarrow \quad m_1v = m_2\,v_2 - m_1\,v_1 \qquad$ (i)

$e = -\dfrac{v_1 + v_2}{0 - v} = 1$

Therefore, $v = v_1 + v_2 \qquad$ (ii)

[for elastic collision, $e = 1$].

Eliminating v_2 from (i) and (ii), we obtain

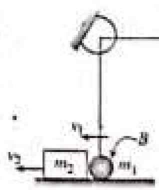
$m_1v = m_2\,(v - v_1) - m_1v_1.$

$\Rightarrow \quad v_1 = \dfrac{m_2 - m_1}{m_1 + m_2}\,v.$

Since $v_1 > 0$, $\dfrac{m_2 - m_1}{m_1 + m_2} > 1$

$\Rightarrow \quad m_2 - m_1 > 0 \Rightarrow m_1 < m_2.$

49. (b) Let the ball strike the block with a speed u_1. Since the initial speed (speed before collision) of the block $= u_2 = 0$ for the perfectly elastic collision.



$m_1u_1 = m_1v_1 + m_2v_2$

$e = 1 = \dfrac{v_1}{v_2 - v_1} \Rightarrow v_2 - v_1 = u_1$

$0.5\,u_1 = 0.5\,v_1 + 2.5\,v_2$

$u_1 = -v_1 + v_2 \Rightarrow \; v_2 = \dfrac{u_1}{3} = \dfrac{\sqrt{2gl}}{3} = \dfrac{20}{3}\,\text{m s}^{-1}$

50. (c) As $|\vec{p}_1| = |\vec{p}_2| = |\vec{p}| = p_0$ (say), $\vec{p}$ is final momentum
From conservation of linear momentum

$$\vec{p}_1 + \vec{p}_2 = \vec{p} \quad \therefore \quad p = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta}$$

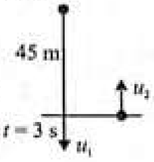
or $\quad p = \sqrt{p^2 + p^2 + 2p^2\cos\theta}$

$\therefore \quad \cos\theta = \dfrac{-1}{2} \Rightarrow \theta = 120°$

51. (c) Time taken to fall through 45 m $= \sqrt{\dfrac{2 \times 45}{10}} = 3$ s

$u_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 45} = 30$ m/s

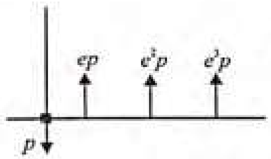
$u_2 = eu_1 = \dfrac{2}{3} \times 30 = 20$ m/s



Distance travelled in the fourth second
= distance travelled in 1 s after rebounding

$= 20 \times 1 - \dfrac{1}{2} \times 10\,(1)^2 = 15$ m

52. (d)



$$\Delta p = (p + ep) + (ep + e^2p) + (e^2p + e^3p) + \cdots$$

$$= p(1+e)\,[1 + e + e^2 + \cdots] = \frac{p(1+e)}{1-e}$$

53. (a) Initial momentum: $3 \times 4 + 4 \times (-3) = 0$
Final momentum will also be zero. But both stick together, hence both come at rest.

54. (d)

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

$= -u_1 + 2u_2$ (neglecting m_1 in comparison to m_2)

$= -v + 2(-v) = -3v$

Hence, (a) is correct.

Change in momentum of $m_1 = p_f - p_i = m_1v_1 - m_1v$
$= -m_13v - m_1v = -4\,m_1v$

Hence, (b) is correct.

Change in KE of $m_1 = K_f - K_i = \dfrac{1}{2}m_1v_1^2 - \dfrac{1}{2}m_1v^2$

$= \dfrac{1}{2}m_1\,(3v)^2 - \dfrac{1}{2}m_1v^2 = 4m_1v^2$

Hence, (c) is correct.

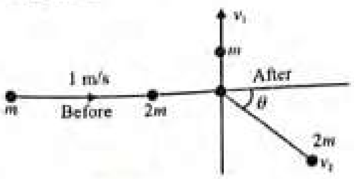
55. (d) From conservation of momentum along incident direction

$m \times 1 = 2\,mv_2\cos\theta$ (i)

From conservation of momentum along perpendicular direction,

$m \times 0 + 2\,m \times 0 = mv_1 - 2\,mv_2\sin\theta$ (ii)

$\Rightarrow \quad v_1 = 2v_2\sin\theta$



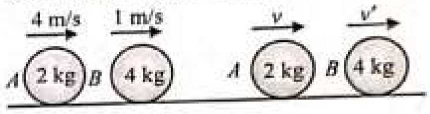
From energy conservation,

$\dfrac{1}{2}m \times (1)^2 = \dfrac{1}{2}mv_1^2 + \dfrac{1}{2}\,2\,mv_2^2$ (iii)

From Eqs. (i), (ii) and (iii), we get $\theta = 30°$.

56. (b) $\hat{j}$ component, i.e., component of velocity parallel to wall remains unchanged while $\hat{i}$ component will become $(-1/2)\,(2\hat{i})$ or $-\hat{i}$. Therefore, velocity vector of the sphere after it hits the wall is $-\hat{i} + 2\hat{j}$.

57. (d) First consider the collision of balls A and B. Let the velocities of these two balls after their collision be v and v'.
Momentum after impact = Momentum before impact



$\therefore \quad 2v + 4v' = 2 \times 4 + 4 \times 1$

$\therefore \quad 2v + 4v' = 12$

$\Rightarrow \quad v + 2v' = 6$ (i)

Relative velocity after impact = $-e$ × relative velocity before impact

$v - v' = -1\,(4 - 1)$

$v - v' = (-3)$ (ii)

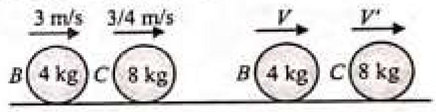
Subtracting Eq. (ii) from Eq. (i), we get

$3v' = 9$

$v' = 3$ m/s

Substituting in Eq. (ii), we get

$v - 3 = -3 \Rightarrow v = 0$



Hence, after the collision ball A is brought to rest, while ball B will move with a velocity of 3 m/s. Now consider the collision of balls B and C. Let the velocities of these balls after collision be V and V', respectively.

Total momentum after impact = Total momentum before impact

$4V + 8V' = 4 \times 3 + 8 \times \dfrac{3}{4} = 18$

$V + 2V' = \dfrac{9}{2}$ (iii)

Relative velocity after impact = $-e$ × relative velocity before impact

$V - V' = (-1)\,(3 - 3/4)$

$V - V' = -9/4$ (iv)

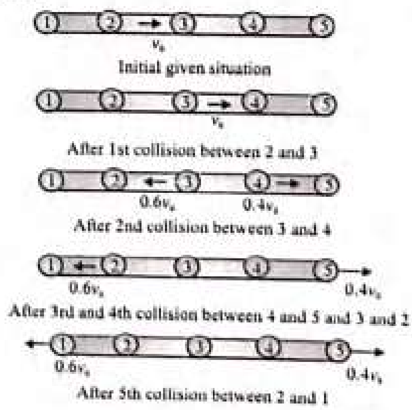
Subtracting Eq. (iv) from Eq. (iii), we get

$3V' = 27/4 \Rightarrow V' = 9/4$ m/s

$V = 0$

$V_A = 0,\ V_B = 0$

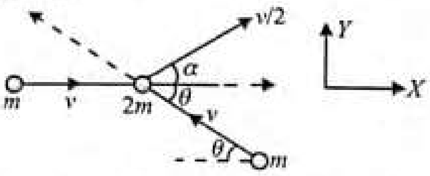
58. (d) The situation after various collisions is as shown in the figure.



After first collision, the momenta of 1 and 2 will be exchanged (property of elastic collision) and hence second ball starts to move towards 3 with velocity v_0.

59. (c) For elastic collision, $e = 1$. Let speed of the ball be v towards left after collision, then w.r.t wall, incident velocity = reflected velocity. We get $3 + 3 = v - 3 \Rightarrow v = 9$ m/s.

60. (a) Let angle between initial velocities be $\pi - \theta$ and the situation is as shown in the figure. Conserve the momentum along X- and Y-directions.



$$mv - mv\cos\theta = 2m\frac{v}{2}\cos\alpha, \text{ for } X\text{-axis}$$

$$mv\sin\theta = 2m\frac{v}{2}\sin\alpha, \text{ for } Y\text{-axis}$$

Solving above equations, $\theta = 60°$

Required angle $= \pi - 60° = 120°$

Problems Based on Mixed Concepts

61. (c) When the two balls collide with each other, as the mass of the two balls is equal, they exchange their velocities on colliding elastically. Let the speed of the ball B when it reaches back to the initial position be v. Then

$$4\,mgh = \frac{1}{2}mv^2 + mgh \Rightarrow v = \sqrt{6gh}$$

Height reached by particle B (from highest point on the incline) is

$$H_B = \frac{v^2\sin^2 60°}{2g} = \frac{9h}{4}; \text{ total height} = h + \frac{9h}{4} = \frac{13h}{4}$$

After collision the particle A reaches the maximum height $= h$

$$\text{Ratio} = \frac{H_A}{H_B} = \frac{4}{13}$$

62. (b) In both CM and ground frame, K_{max} is there, when x is zero in spring, which occurs simultaneously.

$$v_{CM} = \frac{m(v_0) + 0}{5m} = \frac{v_0}{5}$$

$$K_{max\,CM} = \frac{1}{2}m\left(\frac{4v_0}{5}\right)^2 + \frac{1}{2}(4m)\left(\frac{v_0}{5}\right)^2 = \frac{2}{5}mv_0^2$$

$$K_{max}\text{ ground} = \frac{1}{2}mv_0^2$$

$$K_{min\,CM} = 0$$

$$K_{min}\text{ ground} = \frac{1}{2}(m + 4m)\,v_{CM}^2 = \frac{mv_0^2}{10}$$

$$K_{max\,m} = \frac{1}{2}mv_0^2 \text{ (ground frame)}$$

$K_{min\,m} = 0$ (ground frame when energy is shared by spring and $4m$ only and m will reverse direction of motion)

63. (b) Applying the law of conservation of momentum,

$$m_1v_1 = m_1\frac{v_1}{3} + mv \text{ or } v = \frac{2m_1v_1}{3m}$$

To describe a vertical circle v should be $\sqrt{5gl}$. So,

$$\frac{2m_1v_1}{3m} = \sqrt{5gl} \text{ or } v_1 = \left(\frac{m}{m_1}\right)\frac{3}{2}\sqrt{5gl}$$

64. (a) Applying the law of conservation of momentum,

$$m_1v_1 = (m_1 + m_2)V \quad \text{(i)}$$

where $v_1 = \sqrt{2gd}$ is the velocity with which m_1 collides with m_2. Therefore,

$$V = \frac{m_1}{(m_1 + m_2)}\sqrt{2gd}$$

Now, let the centre of mass rise through a height h after collision. In this case, the kinetic energy of $m_1 + m_2$ system is converted into potential energy at maximum height h.

$$\Rightarrow \frac{1}{2}(m_1 + m_2)V^2 = (m_1 + m_2)gh$$

$$\Rightarrow \frac{1}{2}(m_1 + m_2)\left\{\frac{m_1}{m_1 + m_2}\right\}^2 2gd = (m_1 + m_2)gh$$

$$\Rightarrow h = d\left\{\frac{m_1}{m_1 + m_2}\right\}^2$$

65. (a) The bullet and block will meet after time

$$t = \frac{h}{u_{rel}} = \frac{100}{100} = 1$$

During this time, distance travelled by the block,

$$s_1 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 1^2 = 5 \text{ m}$$

Distance travelled by the bullet,

$$s_2 = 100 - s_1 = 95 \text{ m}$$

Velocity of the bullet before collision,

$$u_2 = u - gt = 100 - 10 \times 1 = 90 \text{ m/s}$$

Velocity of the block before collision,

$$u_1 = gt = 10 \text{ m/s}$$

Let V be the combined velocity after collision.

According to the law of conservation of momentum,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)V$$

or $\quad 0.01 \times (-10) + 0.01 \times 90 = 0.02V$

(Velocity in upward direction is considered positive.)

Solving, we get $V = 40$ m/s.

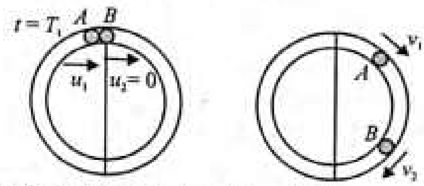
Maximum height risen by the block $= \frac{V^2}{2g} = 80$ m

Height reached above the top of the tower is

$$80 - s_1 = 80 - 5 = 75 \text{ m}$$

66. (d) $T_1 = \frac{\pi R}{u_1} \quad \text{(i)}$

$$\frac{v_2 - v_1}{u_1} = e \Rightarrow v_2 - v_1 = eu_1$$



Time taken to collide A and B again is

$$T_2 - T_1 = \frac{2\pi R}{v_2 - v_1} \Rightarrow T_2 - T_1 = \frac{2\pi R}{eu_1} \quad \text{(ii)}$$

Dividing (ii) by (i), we get $\frac{T_2}{T_1} = \frac{2 + e}{e}$

67. (b) Force on table due to collision of balls:

$$F_{dynamic} = \frac{dp}{dt} = 2 \times 20 \times 20 \times 10^{-3} \times 5 \times 0.5 = 2 \text{ N}$$

Net force on one leg $= \frac{1}{4}(2 + 0.2 \times 10) = 1$ N

68. (d) i. Since both have positive final velocities, hence, both moved in the same direction after collision.

ii. At $t = 2$ s, both had equal velocities.

iii. By conservation of linear momentum.

$$m_1(0.8) = m_1(0.2) + m_2(1) \Rightarrow \frac{m_1}{m_2} = \frac{5}{3} \Rightarrow m_1 > m_2$$

69. (a) Motion of centre of mass is exactly similar to the motion of a body had it not exploded.

$u_x = u\cos\theta = \frac{10}{\sqrt{2}}$ m/s, $u_y = u\sin\theta = \frac{10}{\sqrt{2}}$ m/s

$v_x = u_x = \frac{10}{\sqrt{2}}$ m/s

(since there is no change in the horizontal velocity)

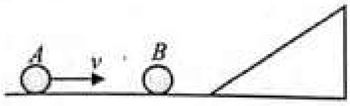
$v_y^2 - u_y^2 = 2(-g)(h)$

$\Rightarrow \quad v_y^2 = \frac{100}{2} - 2\times 10\times 1 = 30$

Therefore, net velocity of CM $= \sqrt{v_x^2 + v_y^2}$

$= \sqrt{\frac{100}{2} + 30} = \sqrt{80} = 4\sqrt{5}$ m/s

70. (b) $v = 16$ m/s



Let velocity of B after collision be v_2 and that of A be v_1. Then

$v_2 = \sqrt{2gs} = \sqrt{2\times 10\times 5} = 10$ m/s

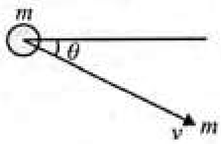
$mv_1 + mv_2 = mv$

$v_1 + 10 = 16 \Rightarrow v_1 = 6$ m/s

$e = \frac{v_2 - v_1}{v} = \frac{10-6}{16} = \frac{1}{4}$

71. (a) The force has to be applied at the centre of mass of the system for pure translation motion.

72. (d) The impact takes place along the line joining them. Since the particle at rest is constrained to move on the horizontal surface, we cannot conserve the momentum along the line joining them. Since no net force is acting on the system of particles parallel to the surface, conservation of linear momentum along that direction yields the following result.



$mv\cos\theta = (m+m)v' \Rightarrow v' = \frac{v}{2}\cos 60° = \frac{v}{4}$

73. (d) As only the gravity force is acting on the system, the centre of mass of the system follows a parabolic path.

At $t = 2$ s,

$x_{CM} = 30\cos 37° \times 2 = 48$ m

$y_{CM} = 30\sin 37° \times 2 - \frac{1}{2}\times 10\times 2^2 = 16$ m

Let coordinates of the second piece, i.e., 6 kg piece be (x, y). Then

$x_{CM} = 48 = \frac{6x + 4\times 105}{10} \Rightarrow x = 10$ m

$y_{CM} = 16 = \frac{6y + 4\times 43}{10} \Rightarrow y = -2$ m

Negative value of y shows that the second piece collides the ground before $t = 2$ s.

74. (b) $v_0 = \sqrt{2gh}$ before collision.

Apply conservation principle of momentum.

$mv_0 = (m+m)u$

$\therefore \quad u = \frac{v_0}{2}$

Applying conservation principle of momentum.

$\frac{1}{2}(2m)u^2 = 2mgh'$

$\therefore \quad h' = \frac{u^2}{2g} \Rightarrow h' = \frac{v_0^2}{4g}$

$h' = \frac{2gh}{4g} \Rightarrow h' = \frac{h}{2}$

75. (a) Since, A and B have same mass. So, after elastic collision, they interchange their velocity.

$\therefore \quad v_B = v$

After collision between B and C,

$v_B' = \left(\frac{m-4m}{m+4m}\right)v + \left(\frac{2\times 4m\times 0}{m+4m}\right) = -\frac{3}{5}v$

Again, collision takes place between A and B. So, velocity will be inter changed.

$\therefore \quad v_A' = (v_B') = \frac{3}{5}v$ toward left

76. (c) Let v_1, v_2 and v_3 be velocities of blocks 1, 2 and 3 after suffering collision each.

$mv = mv_1 + Mv_2$ and $v_1 - v_2 = -v$

Solving, we get $v_1 = \frac{m-M}{m+M} < 0$

$\therefore \quad m < M$

$\therefore \quad |v_1| = \frac{M-m}{m+M}v$ (i)

and $v_2 = \frac{2m}{m+M}\times v_2 = \frac{4Mmv}{(m+M)^2}$ (ii)

$\therefore \quad \frac{M-m}{M+m}v = \frac{4Mmv}{(M+m)^2}$

$M^2 - m^2 = 4Mm \Rightarrow \frac{M}{m} = 2+\sqrt{5}$

77. (a) $R = \frac{2u\cos\theta\cdot u\sin\theta}{g}$

After impact horizontal component remains the same $= u\cos\theta$

The vertical component becomes

$V = e(u\sin\theta)$

New range $R' = \frac{2u\cos\theta\cdot eu\sin\theta}{g}$

$R' = eR$

78. (d) As no external force acts, so centre of mass remains at rest.

79. (b) P_X (before collision) $= P_X$ (after collision)

$\Rightarrow \quad -4 + b = 3 + 4 \Rightarrow b = 11$

P_Y (before collision) $= P_Y$ (after collision)

$5 - 2 = a + 2 \Rightarrow a = 1$

80. (b) Conserving the momentum of two balls before and after collision in horizontal direction

$1\times 6 = (1+1)\times v$

$v = 3$ ms^{-1}

Now at the instant of maximum deflection ball will be moving horizontal with same speed as of trolley (i.e., velocity of ball w.r.t trolley will be 0)

Let it be v_0

$2\times 3 = (4+2)\times v_0$

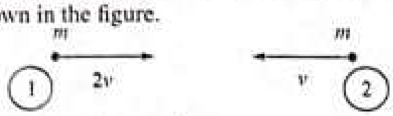
Now conserving energy

$$\frac{1}{2}\times 2\times 3^2 - 2\times 10\times 1.5(1-\cos\theta) = \frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 4\times v^2$$

$$\Rightarrow \quad \cos\theta = 0.8 \Rightarrow \theta = 37^\circ$$

Archives

1. (c) Let the mass of each body be m. Their motion is represented as shown in the figure.

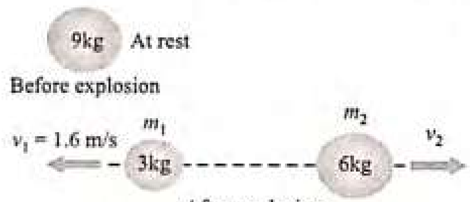


From $\vec{v}_{CM} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

$$v_{CM} = \frac{m\times 2v - mv}{m+m} = \frac{v}{2}$$

[The direction of motion of the first particle is taken as positive.] So the velocity of the centre of mass of the system is $v/2$ in the direction of motion of the particle having larger speed.

2. (c)



The bomb initially was at rest therefore

Initial momentum of bomb = 0

Final momentum of system = $m_1v_1 + m_2v_2$

As there is no external force

$\therefore \quad m_1v_1 + m_2v_2 = 0$

$\Rightarrow \quad 3\times 1.6 + 6\times v_2 = 0$

Velocity of 6 kg mass, $v_2 = 0.8$ m/s (numerically)

Its kinetic energy

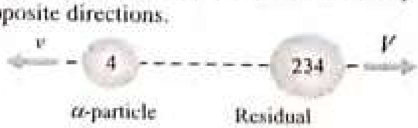
$$= \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}\times 6\times (0.8)^2 = 1.92\text{ J}$$

3. (d) If the kinetic energy of a system of particles is zero, it means all the particles are at rest. Hence, the momentum is zero. Thus II implies I.

But if the momentum of a system of particles is zero, it does not mean that all the particles are at rest. Hence, the particles may possess kinetic energy. Thus I does not imply II.

4. (a) Initially ^{238}U nucleus was at rest and after decay its part moves in opposite directions.

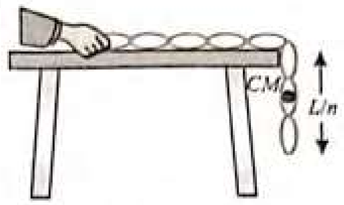


According to conservation of momentum,

$$4v + 234V = 238\times 0 \quad \Rightarrow \quad V = -\frac{4v}{234}$$

5. (b) Fraction of length of the chain hanging from the table

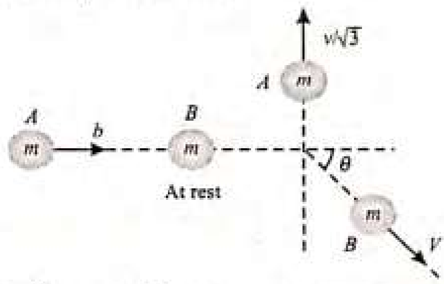
$$= \frac{1}{n} = \frac{60\text{cm}}{200\text{cm}} = \frac{3}{10} \quad \Rightarrow \quad n = \frac{10}{3}$$



The center of mass of the hanging part is at a distance $\frac{L}{2n}$ from table. For pulling the entire chain on the table we need to displace the center of the mass of the hanging part by a distance $\frac{L}{2n}$. Hence, work done in pulling the chain on the table

$$W = \frac{mgL}{2n^2} = \frac{4\times 10\times 2}{2\times (10/3)^2} = 3.6\text{ J}$$

6. (b) Let mass A moves with velocity v and collides inelastically with mass B, which is at rest.



According to problem, mass A moves in a perpendicular direction and let the mass B moves at angle θ with the horizontal with velocity v.

Initial horizontal momentum of system (before collision) = mv ...(i)

Final horizontal momentum of system (after collision) = $mV\cos\theta$ ...(ii)

From the conservation of horizontal linear momentum,

$$mv = mV\cos\theta$$

$$\Rightarrow \quad v = V\cos\theta \qquad ...(iii)$$

Initial vertical momentum of system (before collision) is zero.

Final vertical momentum of system = $\dfrac{mv}{\sqrt{3}} - mV\sin\theta$

From the conservation of vertical linear momentum

$$\frac{mv}{\sqrt{3}} - mV\sin\theta = 0$$

$$\Rightarrow \quad \frac{v}{\sqrt{3}} = V\sin\theta \qquad ...(iv)$$

By solving (iii) and (iv), we get

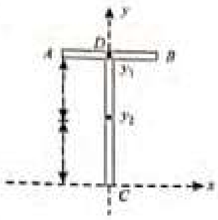
$$v^2 + \frac{v^2}{3} = V^2(\sin^2\theta + \cos^2\theta)$$

$$V = \frac{2}{\sqrt{3}}v$$

7. (c) If we take bodies B and C both as a system, no external force is acting on the system in horizontal direction. Hence, the center of mass of system (bodies B and C) does not shift.

8. (b) If the object has only the translational motion without rotation, the line of action of force should pass through center of mass of structure.

The masses are uniformly distributed hence the mass of DC should be twice of mass of AB.



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

mass of AB, $m_1 = m$, mass of CD, $m_2 = 2m$

Position of center of mass of AB from C, $\vec{r}_1 = 2l\,\hat{j}$, Position of center of mass of CD from C, $\vec{r}_2 = l\,\hat{j}$.

$$\Rightarrow \quad \vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{m2l\,\hat{j} + 2ml\,\hat{j}}{m + 2m} = \frac{4}{3}l\,\hat{j}$$

Hence, the location of P with respect to C is equal to $\frac{4}{3}l$.

9. (d) Linear momentum is conserved

$\therefore \quad 0 = m_1v_1 + m_2v_2 = (12 \times 4) + (4 \times v_2)$

or $\quad 4v_2 = -48 \Rightarrow v_2 = -12$ m/s

$\therefore$ Kinetic energy of mass m_2

$$= \frac{1}{2}m_2v_2^2 = \frac{1}{2} \times 4 \times (-12)^2 = 288 \text{ J}$$

10. (a) Let m_2 be moved by x, so as to keep the centre of mass at the same position.

$\therefore \quad m_1d + m_2(-x) = 0$

or $\quad m_1d = m_2x$ or $x = \frac{m_1}{m_2}d$.

11. (None) Let after cutting the disc of radius R, the mass of remaining disc be M' and mass of removed disc be m.

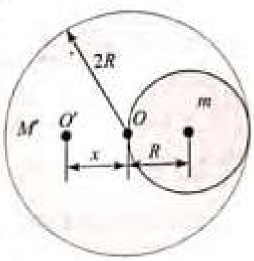
Let $(M' + m) = M = \pi(2R)^2.\sigma = 4\pi R^2.\sigma$

(where σ is mass per unit area)

$\sigma m = \pi R^2.\ \sigma \therefore M' = 4\pi R^2.\ \sigma - \pi R^2.\ \sigma = 3\pi R^2.\ \sigma$

For full disc, the centre of mass is at the centre O. After removing the disc of radius R let centre of mass shift by distance x.

$$\therefore \quad x = \frac{4\pi R^2\sigma.0 - \pi R^2\sigma.R}{4\pi R^2.\sigma - \pi R^2.\sigma} = -\frac{R}{3}$$



$\Rightarrow x = -\frac{R}{3} = \alpha R.$

$\therefore \quad \alpha = \frac{-1}{3}$

The centre of mass is at $R/3$ to the left on the diameter of the original disc.

12. (c) By the law of conservation of linear momentum

$mu = (M + m)v$

$$0.50 \times 2.00 = (1 + 0.50)v \Rightarrow v = \frac{1.00}{1.50}$$

Initial K.E. $= (1/2) \times 0.50 \times (2.00)^2$

$= 1.00$ J

$$\text{Final K.E.} = \frac{1}{2} \times 1.50 \times \frac{1.00^2}{(1.50)^2} = \frac{1.00}{3.00} = 0.33$$

$\therefore$ Loss of energy $= 1.00 - 0.33 = 0.67$ J

13. (a) Position of centre of mass,

$$x_{CM} = \frac{\int_0^L \left(\frac{k}{L^n}\cdot x^n.dx\right)x}{\int_0^L \frac{k}{L^n}\cdot x^n.dx} = \frac{\int_0^L x^{n+1}dx}{\int_0^L x^n dx} = \frac{L^{n+2}}{n+2}\cdot\frac{(n+1)}{L^{n+1}}$$

$$\Rightarrow \quad x_{CM} = \frac{L(n+1)}{(n+2)}$$

The variation of the centre of mass with x is given by

$$\frac{dx}{dn} = L\left\{\frac{(n+2)1 - (n+1)}{(n+2)^2}\right\} = \frac{L}{(n+2)^2}$$

If the rod has the same density as $x = 0$ i.e., $n = 0$, therefore uniform, the centre of mass would have been at $L/2$. As the density increases with length, the centre of mass shifts towards the right. Therefore it can only be option (a).

14. (c) As rubber ball is freely falling, the ball is dropped ($u = 0$). If we take downwards direction negative, the velocity at any time t can be given by the relation $v = -gt$. As when coming down, v increases, makes collision, the value of v becomes +ve, the speed decreases, momentarity comes to zero and then moves in downwards direction and speed increases. The change from $+v$ to $-v$ is almost instantaneous.

Further $h = -\frac{1}{2}gt^2$ is a parabola which opens downwards.

Therefore (c) is correct.

15. (a)

If it is a completely inelastic collision, then

$m_1v_1 + m_2v_2 = m_1v + m_2v$

$$\Rightarrow \quad v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$KE = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

As $\vec{p}_1$ and $\vec{p}_2$ both simultaneously cannot be zero, total KE cannot be lost.

16. (b) Here, mass of the body, $m = 0.4$ kg

Since position–time (x–t) graph is a straight line, so motion is uniform. Because of impulse direction of velocity change as it can be seen from the slopes of the graph.

From graph,

Initial velocity, $u = \frac{(2-0)}{(2-0)} = 1\text{ ms}^{-1}$

Final velocity, $v = \frac{(0-2)}{(4-2)} = -1\text{ ms}^{-1}$

Initial momentum,

$P_i = mu = 0.4 \times 1 = 0.4$ Ns

Final momentum,

$P_f = mv = 0.4 \times (-1) = -0.4$ Ns

Impulse = Change in momentum $= p_f = p_i$

$= (-0.4 - 90.4)$ Ns $= -0.8$ Ns

$|\text{Impulse}| = 0.8$ Ns

17. (c) Loss of energy is maximum when collision is inelastic as in an inelastic collision there will be maximum deformation.

KE in COM frame is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)v_{rel}^2$

$$KE_i = \frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2 \qquad KE_f = 0 \qquad (\because V_{rel} = 0)$$

Hence, loss in energy is $\frac{1}{2}\left(\frac{Mm}{M+m}\right)V^2$

$$\Rightarrow \quad f = \frac{M}{M+m}$$

18. **(c)** Applying conservation of linear momentum just before and just after collision.

Just before collision   Just after collision

$$m2v\hat{i} + 2mv\hat{j} = (m+2m)\vec{v}'$$

$$\vec{v}' = \frac{2}{3}v(i+j)$$

Hence $|\vec{v}'| = \frac{2\sqrt{2}}{3}v$

Initial kinetic energy of the masses just before collision,

$$K_{\text{initial}} = \frac{1}{2}m(2v)^2 + \frac{1}{2}2mv^2 = 3mv^2$$

Final kinetic energy of the system just after collision,

$$K_{\text{final}} = \frac{1}{2}(3m)\left(\frac{2\sqrt{2}}{3}v\right)^2 = \frac{4}{3}mv^2$$

Loss in kinetic energy,

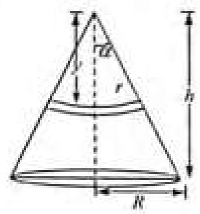
$$\Delta K = K_{\text{initial}} - K_{\text{final}}$$

$$= 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2$$

Hence, percentage loss in kinetic energy

$$\frac{\Delta K}{K}\% = \frac{\frac{5}{3}mv^2}{3mv^2} \times 100 = 56\%$$

19. **(b)** $dm = \pi r^2 \,.\, dy \,.\, r$



$$y_{CM} = \frac{\int y\,dm}{\int dm} = \frac{\int_0^h \pi r^2 dy \times \rho \times y}{\frac{1}{3}\pi R^2 h\rho} = \frac{3h}{4}$$

20. **(c)** Initial

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \qquad \text{...(i)}$$

From momentum conservation

$$mv_0 = m(v_1 + v_2) \qquad \text{...(ii)}$$

$$(v_1+v_2)^2 = v_0^2 \Rightarrow v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$$

$$2v_1v_2 = -\frac{v_0^2}{2}$$

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}v_0^2 + \frac{v_0^2}{2} = 2v_0^2$$

The relative velocity between the two particles, after collision,

$$v_1 - v_2 = \sqrt{2}v_0$$

21. **(b)** Let initial speed of neutron is v_0

By momentum conservation

$$mv_0 = mv_1 + 2mv_2 \Rightarrow v_1 + 2v_2 = v_0 \qquad \text{...(i)}$$

Coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{v_2 - v_1}{v_0 - 0}$$

$$v_2 - v_1 = v_0 \qquad \text{...(ii)}$$

From (i) and (ii) $v_2 = \frac{2v_0}{3}$ and $v_1 = -\frac{v_0}{3}$

Fractional loss $P_d = \dfrac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{3}\right)^2}{\frac{1}{2}mv_0^2}$

$$\Rightarrow \quad P_d = \frac{8}{9} \approx 0.89$$

Using conservation of linear momentum

$$mv_0 = mv_1 + 12mv_2 \Rightarrow v_1 + 12v_2 = v_0 \qquad \text{...(i)}$$

Coefficient of restitution $e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{v_2 - v_1}{v_0 - 0}$

$$v_2 - v_1 = v_0 \qquad \text{...(ii)}$$

From (i) and (ii) $v_2 = \frac{2v_0}{13}$ and $v_1 = -\frac{11v_0}{13}$

Fractional loss fraction loss of energy

$$P_c = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{11v_0}{13}\right)^2}{\frac{1}{2}mv_0^2} \Rightarrow P_c = \frac{48}{169} = 0.28$$

CHAPTER 9: ROTATIONAL DYNAMICS

Concept Application Exercise 9.1

1. Moment of inertia of the plate about axis 1 (by taking rods perpendicular to axis 1)

$$I_1 = \frac{Mb^2}{3}$$

Moment of inertia of the plate about axis 2 (by taking rods perpendicular to axis 2)

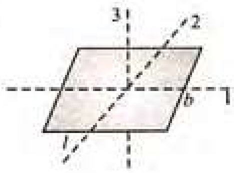
$$I_2 = \frac{Ml^2}{12}$$

Moment of inertia of the plate about axis 3 (by taking rods perpendicular to axis 3)

$$I_3 = \frac{Mb^2}{12}$$

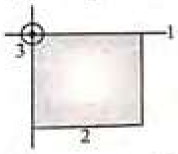
Moment of inertia of the plate about axis 4 (by taking rods perpendicular to axis 4)

$$I_4 = \frac{Mb^2}{3}$$



2. $I_3 = I_1 + I_2 = \frac{Mb^2}{12} + \frac{Ml^2}{12}$

3. $I_3 = I_1 + I_2 = \frac{Ml^2}{3} + \frac{Ml^2}{3} = \frac{2Ml^2}{3}$



4. (a) For calculation of the moment of inertia about centre, use parallel axis theorem

$$I = 2\left[\frac{Ml^2}{12} + m\left(\frac{b}{2}\right)^2\right] + 2\left[\frac{mb^2}{12} + m\left(\frac{l}{2}\right)^2\right]$$

$$I = \frac{2}{3}m(l^2 + b^2)$$

(b) Hence, we can also use parallel axis theorem.

$$I_{PQ} = 2\left(\frac{mb}{3}\right)^2 + mb^2 = \frac{5}{3}mb^2$$

5. (a) $I = m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{2}$

(b) $I = 4ml^2$

(c) $I = ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{5}{4}ml^2$

(d) $I = 2m(2l)^2 = 8\,ml^2$

6. $I = I_{\text{disc}} + I_{\text{ring}} + I_{\text{point masses}}$

$$= \frac{mR^2}{2} + mR^2 + 4\left[\frac{m}{4}\left(\frac{R}{2}\right)^2\right] = \frac{7mR^2}{4}$$

7. In the system shown in the figure, the total moment of inertia can be given as the sum of moment of inertia of the rod and that of the spherical ball.

Moment of inertia of the rod about the axis passing through an end is $I_{\text{rod}} = Ml^2/3$.

Moment of inertia of the spherical ball about the axis passing through its geometrical centre is $I = 2mr^2/5$.

Using parallel axis theorem, we get the moment of inertia about the axis passing through the end of rod as

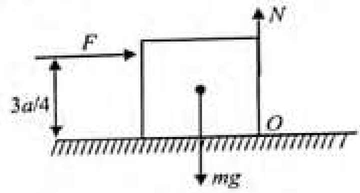
$$I_{AA'} = \frac{2}{5}mr^2 + ml^2$$

Thus total moment of inertia of the system can be given as

$$I_{\text{total}} = I_{\text{rod}} + I_{AA'} = \frac{Ml^2}{3} + \left(\frac{2}{5}mr^2 + ml^2\right)$$

Concept Application Exercise 9.2

1. In the limiting case normal reaction will pass through O. The cube will tip about O if torque of F exceeds the torque of mg.



Hence, $F\left(\frac{3a}{4}\right) > mg\left(\frac{a}{2}\right)$ or $F > \frac{2}{3}mg$

Therefore, minimum value of F is $2mg/3$.

2. $f_1 = \mu_1 N_1, f_2 = \mu_2 N_2$

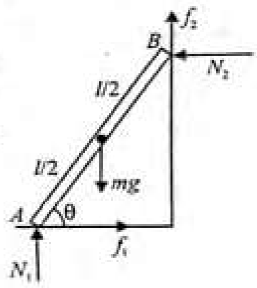
$N_2 = f_1 = \mu_1 N_1$ (i)

$N_1 + f_2 = mg$ (ii)

In limiting case $f_1 = \mu_1 N_1$ and $f_2 = \mu_2 N_2$

Substituting the values of f_1 and f_2 in Eqs. (i) and (ii), we get

$$N_2 = \frac{\mu_1 mg}{1 + \mu_1\mu_2}$$



Taking torque about A and putting it to be zero,

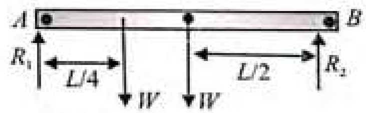
$$mg\,\frac{l}{2}\cos\theta = f_2\,l\cos\theta + N_2\,l\sin\theta$$

$$mg\cos\theta = 2\mu_2 N_2\cos\theta + 2N_2\sin\theta$$

Put the value of N_2 and simplify to get

$$\tan\theta = \frac{1 - \mu_1\mu_2}{2\mu_1}$$

3. $R_1 + R_2 = 2W$ (i)



Balancing torque about A:

$$R_2 L = W\frac{L}{4} + W\frac{L}{2}$$

$$R_2 = \frac{3W}{4} \text{ and } R_1 = \frac{5W}{4}$$

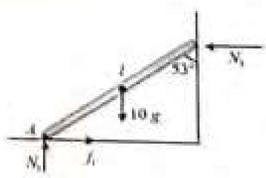
4. $N_1 = 10g = 100\,\text{N}$

Balancing torque about A:

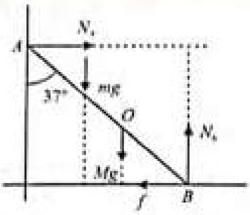
$$10g\,\frac{l}{2}\cos 37° = N_2\,l\cos 53°$$

$$N_2 = \frac{2\omega}{3}\,\text{N}$$

Now $f_1 = N_2 = \frac{200}{3}\,\text{N}$



5. If the electrician works safely, the ladder + man system should be in equilibrium



For equilibrium, $\sum F = 0$ and $\sum \tau = 0$

for $\quad \sum F_x = 0, N_A = f$

for $\quad \sum F_y = 0, N_B = mg + Mg = 60 \times 10 + 16 \times 10 = 760$ N

Taking torque about B,

$$N_A\, 10 \cos 37° = mg\, 8 \sin 37° + Mg\, 5 \sin 37°$$

$$N_A = \frac{\tan 37°}{10}(65 \times 10 \times 8 + 16 \times 10 \times 5)$$

From (i), $f = N_A = 420$ N

For maximum value of coefficient of friction,

$$f \le \mu N_B$$

$$\mu \ge \frac{f}{N_B} = \frac{420}{760} = 0.55$$

6. For equilibrium

$$\sum F_x = 0, \sum F_y = 0$$

and $\quad \sum \tau = 0$

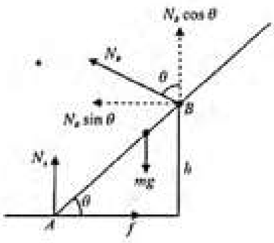
For $\quad \sum F_x = 0$

$$f = N_B \sin \theta \qquad \text{(i)}$$

For $\quad \sum F_y = 0$

$$N_A + N_B \cos \theta = mg \qquad \text{(ii)}$$

Taking torque of forces about A



$$N_B \frac{h}{\sin \theta} = mg\, \frac{L}{2} \cos \theta$$

That gives

$$N_B = \frac{mgL}{2h} \sin \theta \cos \theta \qquad \text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$N_A = mg\left[1 - \frac{mgL}{2h}\sin\theta.\cos^2\theta\right] \qquad \text{(iv)}$$

From Eqs. (i) and (iii), we get

$$f = \frac{mgL}{2h}\sin^2\theta.\cos\theta \qquad \text{(v)}$$

But $\quad f \le \mu N_A$

$$\mu \ge \frac{L \cos\theta \sin^2\theta}{2h - L\cos^2\theta \sin\theta}$$

Concept Application Exercise 9.3

1. (a) $\tau_H = I_H \alpha$

$$mg\frac{l}{2} = \frac{ml^2}{3}\alpha \Rightarrow \alpha = \frac{3g}{2l}$$

(b) $a_{tA} = \alpha l = \frac{3g}{2l} l$

$= a_{CA} = \omega^2 r = 0 \times l$

$= 0$ ($\omega = 0$ just after release)

2. Free-body diagram of the rod

In the previous problem

$$\alpha = \frac{3g}{2l}$$

Hence, $a_{CM} = a_t = \frac{\alpha l}{2} = \frac{3g}{4}$

From the free-body diagram in the vertical direction

$$mg - N_1 = ma_{CM} = m\left(\frac{3g}{4}\right)$$

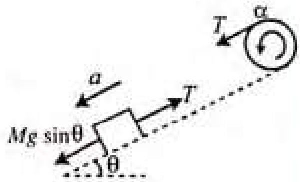
$\Rightarrow \quad N_1 = \frac{mg}{4}$

In horizontal direction

$$F_{ext} = ma_{CM} \Rightarrow N_2 = 0$$

($\because a_{CM}$ in horizontal $= 0$ as $\omega = 0$ just after release).

3. Suppose the deceleration of the block is a. The linear deceleration of the rim of the wheel is also a. The angular deceleration of the wheel is $\alpha = a/r$. If the tension in the string is T, the equations of motion are as follows:



$$Mg \sin\theta - T = Ma \text{ and } Tr = I\alpha = I\frac{a}{r}.$$

Eliminating T from these equations,

$$Mg\sin\theta - I.\frac{a}{r^2} = Ma$$

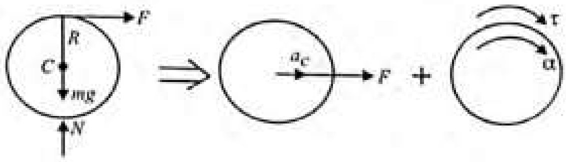
$$\Rightarrow \quad a = \frac{Mgr^2 \sin\theta}{(I + Mr^2)}$$

The initial velocity of the block up the incline is $v = \omega r$. Thus, the distance moved by the block before stopping is

$$x = \frac{v^2}{2a} = \frac{\omega^2 r^2 (I + Mr^2)}{2Mgr^2 \sin\theta} = \frac{(I + Mr^2)\,\omega^2}{2M\,g \sin\theta}$$

4. (i) The net force acting on the cotton reel is

$$F = ma_C$$



or $\quad a_C = \frac{F}{m}$

(ii) The net torque about the CM is $\tau_C (= FR) = I\alpha$

(as torques produced by mg and N about C are zero as they pass through the CM).

Then, $\alpha = \frac{FR}{I}$

5. The acceleration whole system, $a_1 = \frac{F}{m_1 + m_2}$

The acceleration of the point K w.r.t. the axis of the cylinder

$$a_2 = \alpha R$$

where α is given by

$$FR = I\alpha \quad \text{or} \quad \alpha = \frac{FR}{m_1 R^2/2} = \frac{2F}{m_1 R}$$

$$\Rightarrow \quad a_2 = \frac{2F}{m_1}$$

$\therefore$ The acceleration of the point K w.r.t. ground

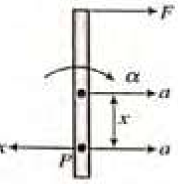
$$= a_1 + a_2 = \frac{F}{m_1 + m_2} + \frac{2F}{m_1} = F\frac{(3m_1 + 2m_2)}{m_1(m_1 + m_2)}$$

6. $a = \dfrac{F}{m}$

$$\alpha = \frac{\tau}{I} = \frac{Fl/2}{ml^2/12} = \frac{6F}{ml}$$

$$a_P = a - \alpha x = 0$$

$$\Rightarrow \quad \frac{F}{m} - \frac{6F}{ml}x = 0 \Rightarrow x = \frac{l}{6} = 1 \text{ m}$$



Concept Application Exercise 9.4

1. Using conservation of energy principle, we have fall in PE of the block = Gain in KE of the block + Rotational KE of the pulley + Rotational KE of the shell.

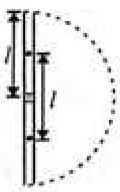
or $\quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}\left(\frac{2MR^2}{3}\right)\omega'^2$

where $\omega = \frac{v}{r}$ and $\omega' = \frac{v}{R}$

After substituting these values in the above equation and solving, we get

$$v^2 = \frac{mgh}{\left(\frac{m}{2} + \frac{I}{2r^2} + \frac{M}{3}\right)}$$

2. As the rod reaches its lowest position, the centre of mass is lowered by a distance l. Its gravitational potential energy is decreased by mg. As no energy is lost against friction, this should be equal to the increase in the kinetic energy. As the rotation occurs about the horizontal axis through the clamped end, the moment of inertia is $I = m^2/3$. Thus,



$$\frac{1}{2}I\omega^2 = mg\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 = mg$$

or $\quad \omega = \sqrt{\frac{6g}{l}}$

The linear speed of the free end is $v = l = \sqrt{6gl}$

3. Using principle of conservation of mechanical energy, we have

$$\frac{mgl}{2} + mgl = \frac{1}{2}I\omega^2$$

where ω is the angular speed of the rod when it becomes vertical,

and $I = (ml^2/3) + ml^2$, assuming mass of each rod is m.

Substituting value of I in the above equation, we get

$$\omega = \sqrt{\left(\frac{9g}{4l}\right)}$$

4. In the process, angular momentum remains constant

$$I_1\omega_1 = I_2\omega_2$$

where $I_1 = 2m\,(0.15)^2$, $\omega_1 = 4$ rad/s

$$I_2 = 2m\,(0.05)^2$$

Substituting these values in above equation, we get

$$\omega_2 = \frac{I_1\omega_1}{I_2} = 36 \text{ rad/s}$$

Work done on the collar

$$W = \frac{1}{2}I_2\,\omega_i^2 - \frac{1}{2}I_1\omega_i^2$$

or $\quad W = \frac{1}{2}(0.05)^2 \times 36^2 - \frac{1}{2}2m\,(0.015)^2 \times 4^2$

where $\quad m = \frac{500}{1000} = 0.05$ kg $= 1.44$ J

5. $\Delta K + \Delta U = 0$

$$\left[\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 - 0\right] + \left[-mg\frac{l}{2}\right] = 0 \Rightarrow \omega = \sqrt{\frac{3g}{l}}$$

Velocity of point P, $v_P = \omega l = \sqrt{3gl}$

Concept Application Exercise 9.5

1.(a) The frictional force between the two discs exerts a retarding torque on M_1 and an accelerating torque on M_2. Till both the discs start moving together with a common angular speed ω, the angular momentum of each disc changes but the angular momentum of the whole system $(M_1 + M_2)$ remains conserved as there is no external torque acting on the system.

A rotating disc is gently placed on a stationary disc.

Thus, applying conservation of angular momentum, we get

$$L_i = L_f$$

$$\left(\frac{M_1R^2}{2}\right)\omega_0 = \left(\frac{M_1R^2}{2} + \frac{M_2R^2}{2}\right)\omega$$

$$\omega = \frac{M_1\omega_0}{M_1 + M_2}$$

(b) The work done by friction is equal to the change in kinetic energy of the system.

$$W_f = K_f - K_i$$

$$K_i = \frac{1}{2}I_1\,\omega_o^2 = \frac{1}{2}\left(\frac{M_1R^2}{2}\right)\omega_o^2 = \frac{1}{4}M_1\,R^2\,\omega_o^2$$

$$K_f = \frac{1}{2}(I_1 + I_2)\,\omega^2 = \frac{1}{2}(M_1 + M_2)\,\frac{R^2}{2}\left[\frac{M_1\omega_0}{M_1 + M_2}\right]^2$$

$$= \frac{1}{4}\frac{M_1^2}{(M_1 + M_2)}R^2\,\omega_o^2$$

$$W_f = K_f - K_i = \frac{-1}{4}\left(\frac{M_1M_2}{M_1 + M_2}\right)R^2\,\omega_o^2$$

Note that the negative sign shows that work done by friction reduces the kinetic energy of the system.

(c) The fractional loss in kinetic energy is

$$\frac{\Delta K}{K_i} = \frac{M_2}{M_1 + M_2}$$

2. Initially the system was at rest, thus the initial angular momentum was zero. As child jumps off from the platform, it gains an angular momentum in the opposite direction. This implies that the platform must also gain the same amount of angular momentum in opposite direction.

When the child jumps off, the platform gains an angular velocity ω. Then the net velocity of child with respect to earth is $u - R\omega$. As no external torque is present, the net angular momentum of the system must be finally zero, thus

$$m\,(u - R\omega) = I\omega$$

$$\omega = \frac{muR}{mR^2 + I}$$

3. Again in this case no external torque is acting on the system so we can equate the angular momentum before catch and after catch. If after catching the ball, platform starts rotating with an angular speed ω, we have

Angular momentum of the ball about centre of platform before catch = Angular momentum of the platform plus child plus ball after catch

$$m_1uR = (I + mR^2 + m_1R^2)\omega$$

or $$\omega = \frac{m_1uR}{I + mR^2 + m_1R^2}$$

Here we can conserve angular momentum with respect to the centre pivot of the platform. This is because when the child catches the ball, it has a linear momentum and due to it the platform tends to gain a linear momentum, which develops a normal reaction on the pivot in opposite direction and it will not allow the platform to move translationally. If the platform was resting on a smooth plane without pivot at the centre, after catching the ball, the platform would also move translationally with its rotational motion and in that case linear momentum of system will also remain conserved.

4. As no external torque is present, we can conserve angular momentum before and after starting the walk by the child. As the child walks in opposite direction to the rotation of platform, its angular momentum will be in opposite direction which will tend to decrease the total angular momentum, hence the angular speed of the platform must increase to maintain the angular momentum conservation we have

$$(I + mR^2)\omega_1 = I\omega_2 - m(u - R\omega_2)R$$

The term on left side of the above expression is the angular momentum of platform plus child system when the child was at rest and the first term on right side of the expression is the angular momentum of the platform with increased angular speed (ω_2), when the child starts walking. The second term on right side is the final angular momentum of the child. This term is negative due to his walking in opposite direction and we have taken the velocity of the child as $(u - R\omega_2)$ because u is the relative speed of the child on platform which is rotating with angular speed ω_2 in opposite direction.

Again as we have used in the above problem, be careful, that angular momentum conservation should always be used with respect to earth or some inertial reference frame.

$$\omega_2 = \frac{(I + mR^2)\,\omega_1 + muR}{I + mR^2}$$

5. For circular motion of a body tied to a string on a horizontal plane

$$\left(\frac{mv^2}{r}\right) = T$$

Here as tension is provided by the hanging mass M, i.e., $T = Mg$, therefore

so, $$\left(\frac{mv^2}{r}\right) = Mg$$

According to the given problem,

$$\frac{(mv_1^2/r_1)}{(mv_2^2/r_2)} = \frac{M_1g}{M_2g} = \frac{8}{1}$$

or, $$\frac{v_1^2 r_2}{v_2^2 r_1} = \frac{8}{1} \qquad \text{(i)}$$

Now as force T is central, so angular momentum is also conserved, i.e.,

$$mv_1r_1 = mv_2r_2 \qquad \text{(ii)}$$

So substituting the value of v_1/v_2 from Eq. (ii) in Eq. (i),

$$\left[\frac{r_2}{r_1}\right]^2 \times \frac{r_2}{r_1} = \frac{8}{1}$$

i.e., $$\frac{r_2}{r_1} = 2 \qquad \text{(iii)}$$

so that $$\frac{\Delta r}{r_1} = \frac{r_2 - r_1}{r_1} = \frac{r_2}{r_1} - 1 = 2 - 1 = 1$$

Furthermore as in circular motion, $v = r\omega$

so, $$\frac{\omega_2}{\omega_1} = \frac{v_2}{v_1} \times \frac{r_2}{r_1} = \left[\frac{r_1}{r_2}\right]^2$$

$$\left[\text{as from Eq. (ii), } \frac{v_2}{v_1} = \frac{r_1}{r_2}\right]$$

$$\Rightarrow \quad \frac{\omega_2}{\omega_1} = \left[\frac{1}{2}\right]^2 = \frac{1}{4}$$

$$\left[\text{as from Eq. (iii), } \frac{r_2}{r_1} = 2\right]$$

so, $$\frac{\Delta\omega}{\omega} = \frac{\omega_2 - \omega_1}{\omega_1} = \frac{\omega_2}{\omega_1} - 1 = \frac{1}{4} - 1 = -\frac{3}{4}$$

6. When the girl lands on the plank, she exerts a force on it, similarly the plank exerts an equal and opposite force on the girl. For the system consisting of girl and plank these forces are internal; therefore angular momentum is conserved.

$$\vec{L}_i = \frac{1}{2}ml\sqrt{2gh}$$

$$\vec{L}_f = \frac{1}{6}ml^2\omega + \frac{1}{4}ml^2\omega$$

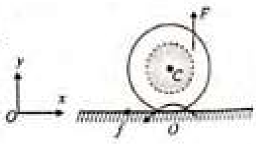
$$\frac{1}{2}ml\sqrt{2gh} = \frac{5}{12}ml^2\omega$$

$$\vec{L}_i = \vec{L}_f \Rightarrow \omega = \frac{6}{5}\sqrt{\frac{2gh}{l}}$$

Concept Application Exercise 9.6

1. (a) The CM moves along the positive x-axis because the net torque about the point of contact is clockwise.

(b) The friction force acts along the negative x-axis. In the absence of friction force, point O has a tendency to move along the positive x-axis.



(c) The CM moves along the negative x-axis because the net torque about the point of contact is anticlockwise.

(d) The friction force acts along the negative x-axis. No other force is acting along the x-axis that can accelerate the centre of mass along the negative x-axis.

2.(a) **True:** The friction force retards the motion of centre of mass and accelerates the rotational motion with respect to centre of mass.

(b) **False:** Energy is not conserved because friction force does negative work.

(c) **True:** The frictional force produces no torque about any point on the horizontal surface.

(d) **False:** Conserving angular momentum about a stationary point on the horizontal surface, we get

$$mv_0R = mvR + \left(\frac{2}{5}mR^2\right)\omega$$

Using the condition of rolling motion, we get

$$v = \omega R$$

$$mv_0R = mvR + \frac{2}{5}mvR = \frac{7}{5}mvR$$

$$v = \frac{5}{7}v_0$$

3. Zero. Since lowest point of the sphere is stationary with respect to the plank, therefore friction force acting on the plank is zero. Thus, no force is required to keep the plank stationary.

4. The velocity of the centre of mass of the tractor wheel is v. Therefore, velocity of the lower half of the belt at any instant

$= v - v = 0$

Velocity of the upper half $= v + v = 2v$

$\therefore T = \frac{1}{2}\left(\frac{m}{2}\right)(2v)^2 = mv^2$

5. Here the centre of mass of the marble will move in a circle of radius $(R - r)$ so for just looping the loop, at H

$$v = \sqrt{g(R-r)} \qquad \text{(i)}$$

Now as in rolling

$$K = K_T + K_R = \tfrac{1}{2} Mv^2 + \tfrac{1}{2} I\omega^2$$

and here

$$I = \left(\frac{2}{5}\right) Mr^2 \text{ with } v = r\omega$$

so, $$K = \frac{1}{2}Mv^2 + \frac{1}{2}\left[\frac{2}{5}Mr^2\right]\left[\frac{v}{r}\right]^2$$

i.e., $$= \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2 \qquad \text{(ii)}$$

So in the light of Eq. (i), Eq. (ii) becomes

$$K = \frac{7}{10}Mg(R-r) \qquad \text{(iii)}$$

As this kinetic energy is provided by loss in PE, applying conservation of mechanical energy between P and H.

$$0 + Mgh = \frac{7}{10}Mg(R-r) + Mg(2R-r)$$

$$h = \frac{1}{10}(27R - 17r)$$

6. The total kinetic energy of the sphere $= E = E_{tran} + E_{rot}$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{v^2}{R^2} = \frac{7}{10}mv^2 \Rightarrow v = \sqrt{\frac{10E}{7m}}$$

Since the spring force F passes through the centre O of the sphere, it causes no torque about O. Therefore, the angular momentum and hence angular velocity of the sphere remains constant. Since the surface AB is smooth no frictional loss take place. Therefore, conserving the energy of the system (sphere-spring) between the given (initial) position and the final position (maximum compression of the spring), we obtain

$$\Delta KE_{sphere} + \Delta PE_{spring} = 0$$

$$\Rightarrow \quad \Delta KE_{tran} + \Delta KE_{rot} + \Delta PE_{spring} = 0$$

Since the initial angular velocity of the sphere remains constant due to the absence of friction.

$$\Delta KE_{rotation} = 0 \quad \Rightarrow \quad \Delta KE_{tans} + \Delta PE_{spring} = 0$$

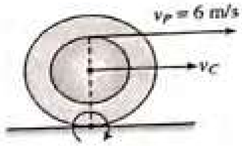
$$\Rightarrow \quad \left(0 - \frac{1}{2}mv^2\right) + \left(\frac{1}{2}kx^2 - 0\right) = 0$$

where $v = \sqrt{\frac{10E}{7m}}$ and x = maximum compression of the spring

$$\Rightarrow \quad \frac{1}{2}Kx^2 = \frac{1}{2}m\left(\sqrt{\frac{10E}{7m}}\right)^2 \Rightarrow x^2 = \frac{10E}{7k}$$

$$\Rightarrow \quad x = \sqrt{\frac{10E}{7k}}$$

7. As cotton reel rolls point of contact of reel with ground will acts as instantaneous axis of rotation.



$$v_p = \omega(R + r)$$

$$\Rightarrow \quad \omega = \frac{v_P}{R + r} = 20 \text{ rad/s}$$

Velocity of centre of reel,

$$v_C = \omega R = \frac{v_P R}{(R+r)} = \frac{6 \times 20}{(20+10)} = 4 \text{ m/s}$$

Exercises

Moment of Inertia

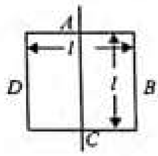
1. (c) Let M be the mass of each disc. Let R_A and R_B be the radii of discs A and B, respectively. Then $M = \pi R_A^2 t d_A = \pi R_B^2 t d_B$.

As $d_A = d_B$, so $R_A^2 < R_B^2$. Now,

$$I_A = \frac{1}{2}MR_A^2, \quad I_B = \frac{1}{2}MR_B^2$$

$$\frac{I_A}{I_B} = \frac{R_A^2}{R_B^2} < 1, \text{ i.e., } I_A < I_B$$

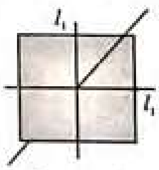
2. (d) $I_{\text{median line}} = I_A + I_B + I_C + I_D$



$$= 2 \times \frac{Ml^2}{12} + 2M\left(\frac{l}{2}\right)^2$$

$$= \frac{Ml^2}{6} + \frac{Ml^2}{2} = \frac{2}{3}Ml^2$$

3. (a)



$$I = I_1 + I_1 = 2 \times \frac{2}{3}Ml^2 = \frac{4}{3}Ml^2$$

4. (a) The moment of inertia about CM system $= \frac{4}{3}Ml^2$

From perpendicular axis theorem,

$$\frac{4}{3}Ml^2 = I_{d1} + I_{d2} \qquad (I_{d1} = I_{d2})$$

$$I_d = \frac{2}{3}Ml^2$$

or $$I = 4\frac{Ml^2}{3}(\sin 45°)^2 = \frac{2}{3}Ml^2$$

5. (d) Mass of disc $\propto$ area, $M_A = 4M_B$ (as $R_A = 2R_B$)

$$\frac{I_A}{I_B} = \frac{\frac{1}{2}M_A R_A^2}{\frac{1}{2}M_B R_B^2} = \frac{4M_B \times (2R_B)^2}{M_B \times R_B^2} = 16$$

6. (d) Moment of inertia of discs A and B about the axis through their centre of mass and perpendicular to the plane will be

$$I_{AA} = I_{BB} = \frac{1}{2}Mr^2$$

Now, moment of inertia of disc A about an axis through B, by theorem of parallel axis will be

$$I_{AA} = I_{BB} + M(2r)^2 = \frac{1}{2}Mr^2 + 4Mr^2 = \frac{9}{2}Mr^2$$

So $\quad I = I_{BB} + I_{AB} = \frac{1}{2}Mr^2 + \frac{9}{2}Mr^2 = 5Mr^2$

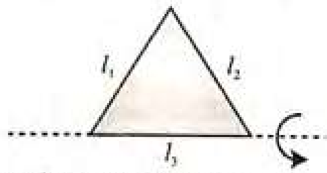
7. (a) Mass of each of the four parts $= \frac{M}{3}$

Mass of the plate including the cut piece $= \frac{4M}{3}$

Moment of inertia of the whole plate (including the cut piece) about the said axis $= \left(\frac{4M}{3}\right)\frac{l^2}{6}$

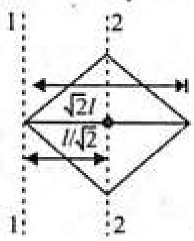
Now moment of inertia of the remaining portion should be 3/4 of the above $= Ml^2/6$.

8. (d) $I = I_1 + I_2 + I_3 = \frac{ml^2}{3} + \frac{ml^2}{3} + 0 = \frac{2ml^2}{3}$



9. (b) MOI is $\Sigma m_i r_i^2$. About BC masses are spread far away than about any other axis.

10. (d) Moment of inertia about 2:



$$I_2 = 4\left(\frac{ml^2}{3}\sin^2 45°\right) = \frac{2ml^2}{3}$$

Apply perpendicular axis theorem,

$$I_1 = I_2 + mh^2$$

$$= \frac{2ml^2}{3} + 4m\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{8}{3}ml^2$$

11. (a) $(\text{MI})_{CG} = M\left(\frac{a^2+b^2}{12}\right)$

According to the theorem of parallel axis,

$$(\text{MI})_{\text{required axis}} = (\text{MI})^{CG} + M(OA)^2$$

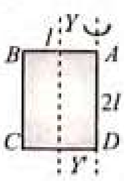
$$= M\left(\frac{a^2+b^2}{12}\right) + M\left(\frac{a^2+b^2}{4}\right)$$

$$= M\left(\frac{a^2+b^2}{3}\right)$$

12. (c) The distribution of mass is nearest to axis xx, hence moment of inertia is least about the xx-axis.

13. (b) Moment of inertia of ABC about $AC = \frac{1}{2} \times$ moment of inertia of square sheet $ABCD$ about $AC = \frac{1}{2} \times [2M] \times \frac{l^2}{12} = \frac{Ml^2}{12}$

14. (c) (About YY') $= \frac{ml^2}{12}$



Using parallel axis theorem,

$$I\text{ (about }AD) = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$

15. (b) Total MI $= I_1 + I_2 + I_3 + I_4$

$$= 2I_1 + 2I_2$$

$$= 2(I_1 + I_2) \qquad [I_2 = I_3;\ I_1 = I_4]$$

Now,

$$I_2 = I_3 = \frac{Ml^2}{3}$$

Using parallel axis theorem, we have

$$I = I_{CM} + Mx^2 \text{ and } x = \sqrt{l^2 + \left(\frac{l}{2}\right)^2}$$

$$I_1 = I_4 = \frac{Ml^2}{12} + M\left[\sqrt{l^2 + \left(\frac{l}{2}\right)^2}\,\right]^2$$

Putting all values , we get

$$I = \frac{10\,Ml^2}{3}$$

16. (b) Farther the mass from axis, greater will be the moment of inertia.

17. (c) The distribution of mass is nearest about xx.

Torque and Torque Equation

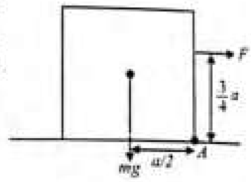
18. (c) Balancing the torque:

for the first case: $16l_1 = ml_2$

for the second case: $\quad ml_1 = 4l_2$

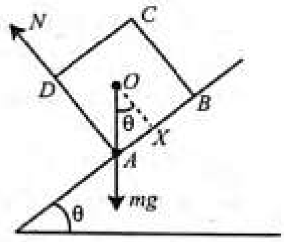
Divide them to get $\quad m = 8$ kg

19. (b) See the figure. For tilting about A the clockwise torque (due to F) should be greater than the anticlockwise torque about A.



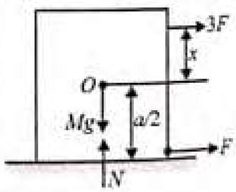
$$mg \times \frac{a}{2} = \frac{F3a}{4};\ F = \frac{2mg}{3}$$

20. (c) The cube will not topple until the line of action of CM passes through the cube.



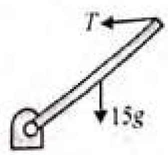
That is, $\tan\theta = \frac{AX}{OX} = \frac{a/2}{a/2}$ or $\theta = 45°$

21. (b) The free-body diagram of the block can be drawn as shown. As the body has to move in pure translational motion, the torque about the centre of gravity must be zero.



$$3F \times x = F \times \frac{a}{2} \quad \Rightarrow \quad x = \frac{a}{6}$$

22. (c) The free-body diagram of the rod is shown in the figure. (Force exerted by the hinge on the rod are not shown.) Take torque about the hinge,



$$T \times 3 - 15\,g \times 2 = 0$$

$\Rightarrow \quad T = 100$ N

23. (a) $T_1 - mg = ma \qquad$ (i)

$$r(T_2 - T_1) = I\alpha \qquad \text{(ii)}$$

$$Mg - T_3 = Ma \qquad \text{(iii)}$$

$$r(T_3 - T_2) = I\alpha \qquad \text{(iv)}$$

and $\quad a = R\alpha$

From Eqs. (ii) and (iv), we get

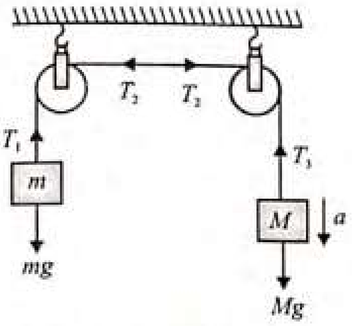
$$T_3 - T_1 = \frac{2Ia}{R^2}$$

From Eqs. (i) and (iii), we get

$$(M-m)g = (M+m)a + T_3 - T_1$$

$$(M-m)g = (M-m)a + \frac{2Ia}{r^2}$$

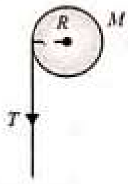
$$\Rightarrow \quad a = \frac{(M-m)g}{\left(M+m+\frac{2I}{r^2}\right)}$$



24. (d) Given $\alpha_A = 2a = 5$ m/s^2 $\Rightarrow$ $\alpha = \frac{5}{2}$ m/s^2

Hence, acceleration of B, $a_B = 1(\alpha) = \frac{5}{2}$ m/s^2

25. (c) Torque exerted on the disc



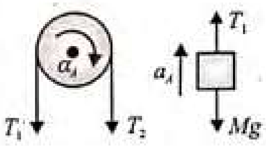
$$\tau = TR$$

Now $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$$

$$= \frac{2TR}{MR^2} = \frac{2T}{MR}$$

26. (b) In the figure, $(T_2 - T_1)R = \frac{MR^2}{2}\alpha_A$



$$T_1 - Mg = Ma_A$$

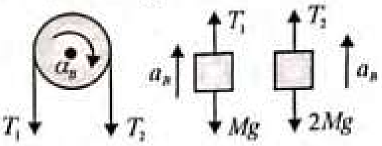
$$T_2 = 2Mg$$

$$a_A = R\alpha_A$$

$$\alpha_A = \frac{2g}{3R}$$

From the figure,

$$(T_2 - T_1) \times R = \frac{MR^2}{2}\alpha_B$$



$$T_1 - Mg = Ma_B$$

$$2Mg - T_2 = 2Ma_B$$

$$a_B = R\alpha_B$$

$$\alpha_B = \frac{2g}{7R}$$

So, $\alpha_A > \alpha_B$

27. (c) $\tau = Fd$ τ is same in magnitude for both cases.

Also, $\tau = I\alpha$

So, $I\alpha$ is constant.

For case (b), I is small.

So, α is large

28. (b) Net torque, $\tau = 30g \times 2 - 20g \times 2 = 20g$

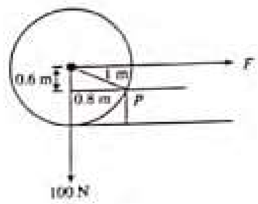
$$I = 30 \times 2^2 + 20 \times 2^2 = 120 + 80 = 200 \text{ kg m}^2$$

$$\alpha = \frac{\tau}{I} = \frac{20g}{200} = \frac{g}{10} = \frac{10}{10} = 1.0 \text{ rad s}^{-2}$$

29. (d) Taking moments of forces about point P, we get

$$F \times 0.6 = 100 \times 0.8$$

or $\quad F = \frac{100 \times 0.8}{0.6}\text{ N} = \frac{800}{6}\text{ N} = \frac{400}{3}\text{ N} = 133.3\text{ N}$



30. (b) Equation of motion is

$$Mg - T = Ma \quad \text{(i)}$$

Taking torque about the axis passing through the centre of the spool and perpendicular to it,

$$TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{2}\right)$$

$$T = \frac{1}{2}Ma \quad \text{(ii)}$$

From Eqs. (i) and (ii),

$$Ma = Mg - \frac{1}{2}Ma$$

$$a = \frac{2g}{3}$$

$$\therefore \quad T = \frac{Mg}{3}$$

31. (a) Force equation $F - 2F = ma \Rightarrow a = -\frac{F}{m}$

Hence, $\vec{a} = -\frac{F}{m}\hat{i}$

Torque equation about center of mass

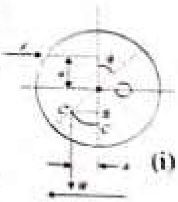
$$F\cdot\frac{l}{2} + \left(\frac{2}{3}l - \frac{l}{2}\right) = I_C\alpha$$

$$F\cdot\frac{l}{2} + F\frac{l}{6} = \frac{ml^2}{12}\cdot\alpha$$

$$\frac{3}{2}Fl = \frac{ml^2}{12}\cdot\alpha \Rightarrow \alpha = \frac{8F}{ml}$$

Hence, $\vec{\alpha} = -\frac{8F}{ml}\hat{k}$

32. (d) The sphere is in stable equilibrium. It will come back to original position after rotation. Rotating Moment = Fa. For stable equilibrium,



Restoring moment = Rotating moment

Since $F = W$, $W_a = Fa$ (i)

Let O be the centre and C and C' be the positions of CG before and after rotation.

$OC = OC'$ and $C'B = a$ [From eq. (i)]

In $\Delta OBC'$ and $C'B$ cosec $\theta = a$ cosec θ

Angular Momentum and Energy of Rigid Body

33. (d) As $\sum \tau = 0$, angular momentum and linear momentum remain conserved. As the two balls will move radially out, I changes. In order to keep the angular momentum ($L = I\omega$) conserved, angular speed (ω) should change.

34. (b) $E_1 = \frac{1}{2} I_1\omega_1^2 = \frac{1}{2}(I_1\omega_1)\omega_1 = \frac{1}{2}k\omega_1$

Now $I_1\omega_1 = I_2\omega_2 = k$ (say)

$$E_2 = \frac{1}{2} I_2\omega_2^2 = \frac{1}{2}(I_2\omega_2)\omega_2 = \frac{1}{2}k\omega_2$$

$I_1 > I_2$ and $\omega_2 > \omega_1$

so, $E_2 > E_1$

35. (b) $mg\left(\frac{l}{4}\right) = \frac{1}{2}\left[\frac{ml^2}{12} + m\left(\frac{l}{4}\right)^2\right]\omega^2$

$$\omega = \sqrt{\frac{24g}{7l}} = 2\sqrt{\frac{6g}{7l}}$$

36. (d) $\vec{L} = I_{CM}\vec{\omega} + m\vec{r} \times \vec{v}_{CM}$

$$= \frac{MR^2}{2}\omega(-\hat{k}) + M\frac{3}{2}R(-\hat{j}) \times v_0\hat{i} = MRv_0\hat{k}$$

37. (b) KE $= \frac{L^2}{2I}$. L remains constant. I doubles so KE becomes half.

38. (c) Using conversation of angular momentum about O, we get

$$mvR = (mR^2 + mR^2)\omega,\ \omega = \frac{v}{2R}$$

39. (d) Apply conservation of angular momentum about the hinge, we get

$$mvR = \frac{m}{2}R^2\omega + m(R\omega)R$$

$$R\omega = \frac{2v}{3} = \frac{2 \times 5}{3} = \frac{10}{3} \text{ m/s}$$

40. (d) $\omega_{rod} = \frac{(v_{rel})_\perp}{r}$

(V_{rel}) is the velocity of one point w.r.t. other perpendicular to rod.

$\omega_{rod} = \frac{3v - v}{r}$, and '$r$' being the distance between the points.

$\omega_{rod} = \frac{2v}{r}$

41. (c) The distance of $\vec{L}$ is perpendicular to the line joining the bob at point C. Since this line keeps changing its orientation in space, direction of $\vec{L}$ keeps changing, however, as ω is constant, magnitude of $\vec{L}$ remains constant.

Method-2: The torque about point is perpendicular to the angular momentum vector about point C. Hence it can only change the direction of L, and not its magnitude.

42. (d) $K_A = \frac{L_A^2}{2I_A} = \frac{25L_B^2}{2I_B}$

$$K_B = \frac{L_B^2}{2I_B}$$

Now, $\frac{K_A}{K_B} = 25 \times 4 = 100$

43. (c) Applying conservation of angular momentum,

$$(5 + 5)\omega^f = 5 \times 10 + 5 \times 20 = 150$$

or $\omega^f = 15 \text{ rad s}^{-1}$

Initial kinetic energy

$$= \frac{1}{2} \times 5 \times 10 \times 10 + \frac{1}{2} \times 5 \times 20 \times 20$$

$$= 250 + 1000 = 1250 \text{ J}$$

Final kinetic energy $= \frac{1}{2} \times 10 \times 15 \times 15 = 1125$ J

Loss of kinetic energy $= (1250 - 1125)$ J $= 125$ J

44. (b) I is doubled. ω is halved

Rotational K.E. $= \frac{1}{2}I\omega^2$

Clearly, rotational KE is decreases.

45. (d) As the inclined plane is smooth, the sphere can never roll, rather it will just slip down. Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passes through the centre of the ball.

46. (c) From the rigid object under a net torque model,

$$\sum\tau = I\alpha \to \alpha = \frac{\sum\tau}{I} = \frac{FR}{\frac{1}{2}MR^2} = \frac{2F}{MR}$$

From the definition of rotational kinetic energy and the rigid object under constant angular acceleration model,

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I(\omega_i + \alpha t)^2$$

$$= \frac{1}{2}I\alpha^2t^2 = \left(\frac{1}{2}MR^2\right)\left(\frac{2F}{MR}\right)^2 t^2 = \frac{F^2t^2}{M}$$

Substituting, $K = \frac{(50.0 \text{ N})^2(3.00\ s)^2}{(90.0 \text{ kg})} = 250$ J

47. (c) The conservation of energy yields

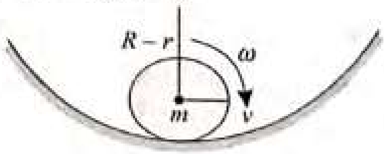
$\Delta K.E. = \Delta P.E.$

$$\frac{1}{2}I\omega^2 = mg(2r) + mgr$$

$$\Rightarrow \frac{1}{2}m(2r)^2\omega^2 + \frac{1}{2}\left\{\left(\frac{mr^2}{2}\right) + mr^2\right\}\omega^2 = 3mgr$$

$$\Rightarrow \frac{11}{4}mr^2\omega^2 = 3mgr \Rightarrow \omega = \sqrt{\frac{12g}{11r}}$$

48. (b) When the sphere reaches its lowest position it loses gravitational potential energy by ΔU and gains a kinetic energy by ΔK, where $\Delta U = -mg(R - r)$



and $\Delta K = \frac{1}{2}mv_c^2\left(1 + \frac{K^2}{R^2}\right) = \frac{1}{2}mv^2\left(1 + \frac{2}{5}\right) = \frac{7}{10}mv^2$

According to conservation of energy, $\Delta U + \Delta K = 0$

or $-mg(R - r) + \frac{7}{10}mv^2 = 0$

or $v = \sqrt{\frac{10}{7}g(R - r)}$

49. (b) From conservation of angular momentum for the isolated system of two disks:

$$(I_1 + I_2)\omega_f = I_1\omega_i \text{ or } \omega_f = \frac{I_1}{I_1 + I_2}\omega_i$$

50. (a) The initial angular momentum of the system is zero. The final angular momentum of the girl-plus-merry-go-round is $(I + MR^2)\,\omega$, which we will take to be positive. The final angular momentum we associate with the thrown rock is negative: $-mRv$, where v is the speed (positive, by definition) of the rock relative to the ground.

Angular momentum conservation leads to

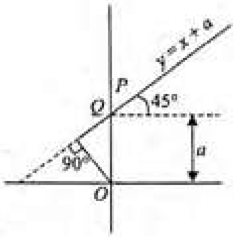
$$0 = (I + MR^2)\omega - mRv \Rightarrow \omega = \frac{mRv}{I + MR^2}$$

The girl's linear speed is $R\omega = \dfrac{mvR^2}{I + MR^2}$.

51. (c) $\angle POQ = 45°$

So, $OQ = a\cos 45° = a/\sqrt{2}$

$L = mv(OQ) = mva/\sqrt{2}$



52. (a) $I_1\omega_1 = I_2\omega_2$

Since, men move towards middle of turn table I_2 decreases hence ω_2 increases.

$$\therefore \quad \Delta K = \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}I_2\omega_2^2$$

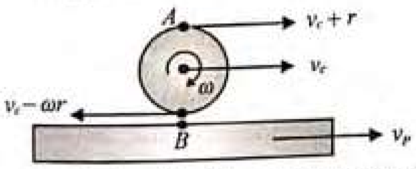
$$= \frac{1}{2}I_1\omega_1^2\left[1 - \frac{I_2}{I_1}\cdot\frac{\omega_2^2}{\omega_1^2}\right] \qquad \left\{\frac{\omega_2}{\omega_1} > 1\right\}$$

$$= \frac{1}{2}I_1\omega_1^2\left[1 - \frac{\omega_2}{\omega_1}\right] < 0$$

So kinetic energy increases.

Problems Based on Rolling

53. (c) For pure rolling velocity of the point of contact has to be equal to the velocity of the surface.



Let us say cylinder rolls with angular velocity ω.

At point B,

$$v_C - \omega r = v_P \Rightarrow \omega r = v_C - v_P$$

At point A,

$$v_A = v_C + \omega r = 2v_C - v_P$$

54. (c) As sphere rolls, the lowest point of the sphere should have the same acceleration as the plank.

Hence, $a_1 = \alpha R - a_2$

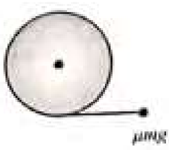
$$2 = 2\alpha - 4 \quad \Rightarrow \quad \alpha = 3 \text{ rad/s}^2$$

55. (d) Taking τ about CM

$$\mu mgR = MR^2\alpha$$

$$\mu g = R\alpha$$

$$\alpha = \frac{\mu g}{R}$$



$$\omega = \omega_0 - \frac{\mu g}{R}f = \frac{\omega_0}{2}$$

$$\frac{\omega_0}{2} = \frac{\mu g}{R}f \Rightarrow f = \frac{\omega_0 R}{2\mu g}$$

56. (b) $f_r = \dfrac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2}$, where $I = \frac{2}{5}mr^2$,

$$\omega = \frac{v}{r}$$

Solve to get $f_r = \frac{2}{7}$

57. (d) $F + f = ma$ (i)

$$FR - fR = \frac{mR^2}{2}\frac{a}{R} \quad \text{(ii)}$$

$$F - f = \frac{ma}{2} \quad \text{(iii)}$$

$$2F = \frac{2ma}{2} \Rightarrow a = \frac{4F}{3m}$$

58. (d) $\dfrac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2} = \dfrac{40}{60} = \dfrac{2}{3}$ or $\dfrac{\frac{Iv^2}{R^2}}{Mv^2} = \dfrac{2}{3}$ or $I = \dfrac{2}{3}MR^2$

Clearly, the body is a hollow sphere.

59. (a) As $v = \sqrt{\dfrac{2gh}{1 + \dfrac{I}{MR^2}}}$

Hence velocity is independent of the inclination of the plane and depends only on height h through which body descends.

But because $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left(1 + \dfrac{I}{MR^2}\right)}$ depends on the inclination also, hence greater the inclination lesser will be the time of descend. Hence, in present case, the speeds will be same (because h is same) but time is descend will be difference (because of different inclinations)

60. (d) For a sliding body of mass m,

$$v_{body} = \sqrt{2gh} = v \quad \text{(i)}$$

For a rolling ring of same mass m,

$$v_{ring} = \sqrt{\frac{2gh}{\beta}}$$

$$= \sqrt{\frac{2gh}{1 + \frac{I}{mR^2}}} = \sqrt{\frac{2gh}{1 + \frac{mR^2}{mR^2}}} = \sqrt{\frac{2gh}{2}} = \sqrt{gh} = \frac{v}{\sqrt{2}}$$

61. (a) For a body to roll without slipping on an incline of angle θ:

$$\mu \geq \frac{\tan\theta}{1 + \frac{mR^2}{I}}$$

I is minimum for the sphere, hence μ required is least for the sphere. Here it is given that μ is just sufficient to roll the sphere purely. It means the ring and the cylinder will not roll purely, so kinetic friction will act on them. Ultimately, friction acting on each of the three will be $\mu mg\cos\theta$ and each of them will have the same acceleration. Hence, all of them reach at the same time.

62. (d) In case of rolling in the inclined plane, friction is static and acts in the upward direction and is given by

$$f=\frac{mg\sin\theta}{1+\frac{R^2}{k^2}} \qquad \text{(i)}$$

For sphere, $k^2=\frac{2}{5}R^2$ (ii)

From Eqs. (i) and (ii), $f=\frac{2}{7}mg\sin\theta$ (upwards)

63. (b) $\tau=fR=I\alpha$ or $fR=\frac{1}{2}mR^2\times\frac{a}{R}$

or $f=\frac{1}{2}ma$ (i)

Now, $mg\sin\theta-f=ma$

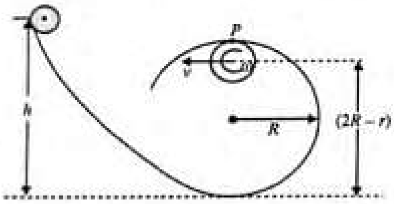
or $mg\sin\theta-\frac{1}{2}ma=ma$

or $\frac{3}{2}ma=mg\sin\theta$ or $\frac{1}{2}ma=\frac{mg\sin\theta}{3}$

From equation (i) $f=\frac{mg\sin\theta}{3}$

Substituting values, $f=\frac{3\times10\times0.5}{3}\text{ N}=5\text{ N}$

64. (d) The minimum velocity at P, top of the loop, should be $v=\sqrt{g(R-r)}$, if the sphere keeps on rolling at top $v=\omega R$



$$mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2+mg(2R-r)$$

$$=\frac{1}{2}mg(R-r)+\frac{1}{2}\left(\frac{2}{5}\right)mR^2 2\omega^2+mg(2R-r)$$

$$=\frac{7}{10}mg(R-r)+mg(2R-r)\left[\omega R=v=\sqrt{g(R-r)}\right]$$

$$=\frac{mg}{10}(27R-17r)\text{ or }h=\frac{1}{10}(27R-17r)$$

65. (c) $\frac{KE_{rot}}{KE_{tot}}=\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mv^2+\frac{1}{2}I\omega^2}=\frac{\frac{2}{5}mv^2}{\frac{7}{5}mv^2}=\frac{2}{7}$

66. (c) If the track is smooth (case A), only translational kinetic energy changes to gravitational potential energy.
But, if the track is rough (case B), both translational and rotational kinetic energies change to potential energy.
Therefore, potential energy (= mgh) will be more in case B than in case A.
Hence, $h_1>h_2$.

67. (d) $\frac{mv^2}{2}+\frac{I\omega^2}{2}=mg\times\frac{3v^2}{4g}\Rightarrow I=\frac{mR^2}{2}$

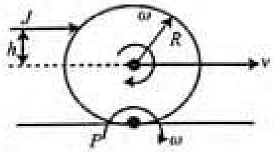
For pure rolling condition, $v=R\omega$.

68. (c) $a=\frac{g\sin\theta}{1+\frac{I}{MR^2}}$

a is minimum, if I is maximum.
$I_{max}=MR^2$ [for ring]

$$\Rightarrow\quad a_{min}=\frac{g\sin\theta}{1+\frac{MR^2}{MR^2}}=\frac{g\sin\theta}{2}$$

69. (b) Rolling is rotation about point of contact. Applying impulse momentum equation about P.



$J(R+h)=I_P\omega$ (i)

and $J=mv$ (ii)

As sphere rolls $v=\omega R$, and $I=\frac{2}{5}mR^2+mR^2=\frac{7}{5}mR^2$

After solving, we get $\frac{h}{R}=\frac{2}{5}$

70. (a) F will provide anticlockwise torque about the centre due to which the bottommost point will tend to move towards right, so friction will act towards left. So it will move towards left.

71. (d) Let J be the linear impulse imparted to the ball.
Applying impulse = change in momentum,
we have $J=mv_0$ (i)

$$J\cdot h=I\omega_0=\frac{2}{5}mr^2\omega_0 \qquad \text{(ii)}$$

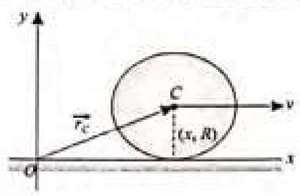
From equations (i) and (ii), we get $\omega_0=\frac{5}{2}\frac{v_0h}{r^2}$

Problems Based on Mixed concepts

72. (a) Angular momentum about point D is conserved.
$mva=I_D\omega$

But $I_D=\frac{5}{6}Ma^2$. So, $\omega=\frac{6mv}{5Ma}$

73. (b) The angular momentum of the disc about O is



$$\vec{L}_O=m\vec{r}_C\times\vec{v}_C+I_C\vec{\omega}$$

$$=m(x\hat{i}+R\hat{j})\times v\hat{i}+\frac{1}{2}mR^2\left(\frac{3v}{R}\hat{k}\right)$$

$$=m(x\hat{i}+R\hat{j})\times v\hat{i}+\frac{1}{2}mR^2\left(\frac{3v}{R}\hat{k}\right)mvR(\hat{j}\times\hat{i})+\frac{3}{2}mvR\hat{k}$$

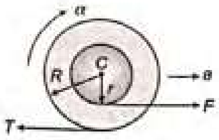
$$-mvR\hat{k}+\frac{3}{2}mvR\hat{k}=\frac{mvR}{2}\hat{k}$$

74. (a) $a = \alpha R$

$F - T = ma$ ..(i)

Torque about C:

$TR - Fr = I\alpha$



$\Rightarrow \quad TR - Fr = mk^2 \dfrac{a}{R}$

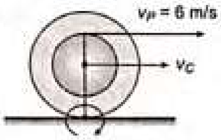
Put the value of T from equation (i)

$$(F - ma)R - Fr = \frac{mk^2 a}{r}$$

$$F(R-r) = a\left[\frac{mk^2}{R} + mR\right] \Rightarrow a = \frac{FR(R-r)}{(k^2+R^2)m}$$

$$= \frac{8\times 2\times(2-1)}{\left(\frac{4}{3}+4\right)\times 3} = \frac{8\times 2\times 1}{4\times\frac{4}{3}\times 3} = 1 \text{ m/s}^2$$

75. (c) As cotton reel rolls point of contact of reel with ground will act as instantaneous axis of rotation.



$v_p = \omega(R + r)$

$$\Rightarrow \quad \omega = \frac{v_p}{R+r} = 20 \text{ rad/s}$$

Velocity of centre of reel

$$v_C = \omega R = \frac{v_p R}{(R+r)} = \frac{6\times 20}{(20+10)} = 4 \text{ m/s}$$

76. (d) Let after collision velocity of rod be v and angular velocity be ω, then

$$mv_0 = mv + MV \Rightarrow mv_0 x = mvx + \frac{M\ell^2}{12}\omega$$

$$mv_0 = mv + \frac{M\omega\eta^2 x}{12}$$

From above equations,

$$\frac{M\omega\eta^2 x}{12} = MV \Rightarrow \omega\eta^2 x = 12\text{ V}$$

Velocity of farther end of the rod $= V - \dfrac{l\omega}{2}$

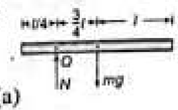
$$= \frac{\omega\eta^2 x}{12} - \frac{\omega\eta x}{2} = \frac{\omega\eta x}{2}\left(\frac{\eta}{6} - 1\right)$$

If this velocity is opposite to v_0, then

$$\frac{\eta}{6} - 1 < 0 \Rightarrow \eta < 6$$

77. (b) The torque applied by gravity about the edge

$O = \tau = mg\,(3l/4)$



$\Rightarrow \quad I_0\alpha = \dfrac{3}{4}mg$ ...(a)

where $I_0 = I_g + m\left(\dfrac{3\ell}{4}\right)^2$

$$= \frac{m\ell^2}{12} + \frac{9}{16}m\ell^2 = \frac{m\ell^2}{4}\left(\frac{1}{3} + \frac{9}{4}\right)$$

$I_0 = \dfrac{31 m\ell^2}{48}$ ...(b)

(a) and (b) yield, $\alpha = \dfrac{3mg\ell/4}{31m\ell^2/48}$

$\Rightarrow \quad \alpha = \dfrac{144g}{124\ell} = \dfrac{36g}{31\ell}$ ...(c)

Newton's second law of motion:

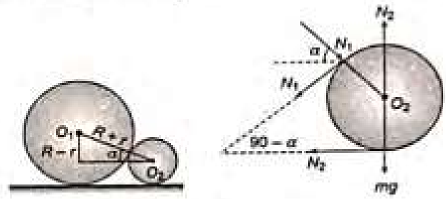
$mg - N = ma$ ...(d)

Kinematics: $a = \left(\dfrac{3\ell}{4}\right)\alpha = \dfrac{36g}{31}$

Using (d) and (c) we obtain

$$N = mg - m\left(\frac{27g}{31}\right) \Rightarrow N = \frac{4mg}{27}.$$

78. (d) Figure shows the forces acting on the smaller cylinder. N_1 is reaction between the two cylinders and N_2 the normal reaction between the smaller cylinder and the horizontal surface.



The conditions of equilibrium are

$\sum F_x = N_1\cos\alpha - \mu N_2 - \mu N_1\cos(90-\alpha) = 0$ ...(i)

Taking torque about O_2, centre of smaller sphere,

$\sum \tau = \mu N_1 r - \mu N_2 r = 0$

$N_1 = N_2 \ldots$(ii)

On substituting equations (ii) in (i), we obtain

$\mu N_1 + \mu N_1 \sin\alpha = N_1\cos\alpha$

$\mu(1+\sin\alpha) = \cos\alpha$ ...(iii)

From Figure (b), $\sin\alpha = \sqrt{\dfrac{R-r}{R+r}}$, $\cos\alpha = \dfrac{2\sqrt{Rr}}{R+r}$

Now equation (iii) becomes; $\mu\left(1 + \dfrac{R-r}{R+r}\right) = \dfrac{2\sqrt{Rr}}{R+r}\mu = \sqrt{\dfrac{r}{R}}$

Hence the required condition is $\mu \geq \sqrt{\dfrac{r}{R}}$.

79. (b) Let sphere rolls down with acceleration a'. Writing force and torque equations for sphere

$N = \dfrac{mg}{2}$ ...(i)

$mg - f = ma'$ ...(ii)

$fR = \left(\dfrac{2}{5}mR^2\right).\alpha$

$f = \dfrac{2}{5}mR\alpha$ ...(iii)

$\alpha R = a'$ ...(iv)

$$\Rightarrow \quad f = \frac{2}{5}ma' \text{ or } a' = \frac{5}{2}\frac{f}{m} \qquad \text{(v)}$$

from (ii) and (v)

$$mg - f = \frac{5}{2}f \Rightarrow mg = \frac{7}{2}f \text{ or } f = \frac{2}{7}mg$$

But, $f \le N$ hence $\frac{2}{7}mg \le \frac{\mu mg}{2} \Rightarrow \mu \ge \frac{4}{7}$

80. (a) Initially ball will gain both linear and angular velocity.

Linear impulse: $I_0 = mv_C$

Angular impulse: $I_0 x = I\omega_0 \Rightarrow mv_C x = I\omega_0$

I is moment of Inertia about C.

Apply conservation of angular momentum about lowest point.

$$I\omega_0 + mv_C r = I\omega + mvr$$

where $\omega = \frac{v}{r}$ at the time of pure rolling

$$\Rightarrow \quad mv_C x + mv_C r = \frac{2}{5}mr^2\frac{v}{r} + mvr$$

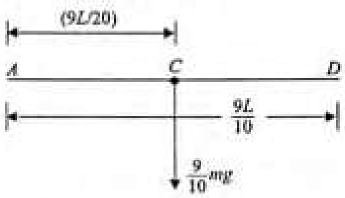
$$\Rightarrow \quad v_C(x+r) = \frac{7}{5}rv$$

$$\Rightarrow \quad v = \frac{5}{7}v_C\left[\frac{x+r}{r}\right]$$

81. (a) Torque $= \tau = \frac{9}{10}mg\left(\frac{9}{20}L\right) = I\alpha = \frac{m}{3}\left(\frac{9}{10}L\right)^2\alpha$

$$\alpha = \frac{3g}{2L}$$

Acceleration, $a_{CM} = \alpha\,(AC)$

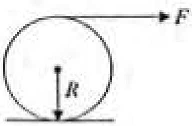


$$a_{CM} = \frac{3g}{2L}\left(\frac{9L}{20}\right) = \frac{27g}{40}$$

Now, $\quad \frac{9}{10}mg - N_A = ma_{CM} = m \cdot \frac{27g}{40}$

or $\quad N_A = \frac{9}{40}mg$

82. (d) $\tau = F \times R = I\alpha$



In one full rotation, $\theta = 2\pi = \frac{1}{2}\alpha t^2$

$$t^2 = \frac{4\pi}{\alpha} = \frac{4\pi I}{FR}$$

Linear acceleration, $a = F/M$

Distance travelled in one rotation $= 2\pi R$ (given)

$$= \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{M}\cdot\frac{4\pi I}{FR} = \frac{2\pi I}{MR}$$

It gives $I = MR^2$

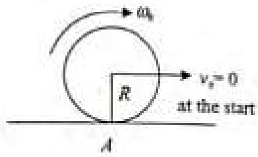
It is a hollow cylinder.

83. (d) Angular momentum about A is conserved.

$$L_1 = \frac{2}{5}mR^2\omega_0$$

$$L_2 = \frac{2}{5}mR^2\omega + mR^2\omega = \frac{7}{5}mR^2\omega$$

$$L_1 = L_2$$



$$\omega = \frac{2}{7}\omega_0$$

$$\omega_0 > \omega$$

and $v > v_0$

Sphere accelerates forward and rotation decelerates.

$$\omega = \omega_0 - \alpha t$$

$$\alpha = \frac{\omega_0 - \omega}{t} \qquad \text{(i)}$$

$$\omega^2 = \omega_0^2 - 2\alpha\theta$$

$$\theta = \frac{(\omega_0 - \theta)(\omega + \omega_0)}{2\alpha} = \frac{\alpha t \cdot \left(\omega + \frac{7}{2}\omega\right)}{2\alpha} = \frac{9}{4}\omega t$$

$$R\theta = \frac{9}{4}\omega R t = \frac{9}{4}\omega t \qquad \text{(ii)}$$

$$v = v_0 + at \qquad (v_0 = 0)$$

$$a = \omega R/t$$

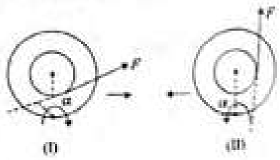
$$v^2 = v_0^2 + 2aS$$

$$S = \frac{v \cdot v}{2a} = \frac{(\omega R)(v \cdot t)}{2 \cdot \omega R} = \frac{vt}{2} \qquad \text{(iii)}$$

Ratio of distance $= 9/2$

84. (a) (Pure rotation about instantaneous point of contact)

Note: If line of action passes through point of contact, it only spins.



85. (d) Since linear acceleration is the same for all ($a = Mg \sin\theta - \mu Mg \cos\theta$) as they have the same mass 'M' and the same 'μ'.

Hence, all will reach the bottom simultaneously. Hence (d).

86. (b) For all the bodies, torque is the same.

Hence, angular momentum (L) is also the same.

Now, KE $= \frac{1}{2}Mv^2 + \frac{L^2}{2I}$

Linear velocity 'v' is the same for all as the same force acts on them. Therefore more value of moment of inertia implies lesser kinetic energy.

Among all, the hollow sphere has the maximum moment of inertia $I = \left(\frac{2}{3}MR^2\right)$.

ARCHIVES

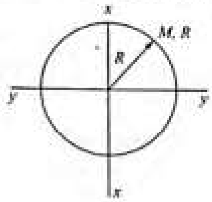
1. (c) Conservation of angular momentum gives

$$\frac{1}{2}MR^2\omega_1 = \left(\frac{1}{2}MR^2 + 2mR^2\right)\omega_2$$

$$\therefore \quad \omega_2 = \left(\frac{M}{M+4m}\right)\omega_1$$

2. (d) Since the inclined plane is frictionless, there will be no rolling and the mass will only slide down.
Hence, acceleration is the same for all the given bodies.

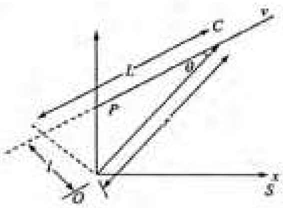
3. (a) The moment of inertia of the circular wire about its axis is MR^2. Consider two diameters XX and YY. Moment of inertia about any of these diameters is the same, say I.
From perpendicular axis theorem, we have



$$I + I = MR^2$$

$$I = \frac{MR^2}{2}$$

4. (b) Angular momentum of the particle about O,



$$\vec{L} = m(\vec{r}\times\vec{v})$$

$$= mv(r\sin\theta)$$

$$= mvl$$

5. (d) $\vec{\tau} = \vec{r}\times\vec{F}$

$\vec{\tau}$ is perpendicular to both

$\vec{r}$ and $\vec{F}$. So $\vec{r}\cdot\vec{\tau}$ as well as $\vec{F}\cdot\vec{\tau}$ has to be zero.

6. (d) Mass of disc X,

$$m_X = \pi R^2 t\,\rho.$$

where ρ is the density of the material of the disc.

$$\therefore \quad I_X = \frac{1}{2}m_X R^2 = \frac{1}{2}\pi R^2 t\,\rho R^2$$

$$= \frac{1}{2}\pi\rho t R^4$$

Mass of disc Y,

$$m_Y = \pi(4R)^2\frac{t}{4}\rho = 4\pi R^2 t\,\rho$$

$$\therefore \quad I_Y = \frac{1}{2}m_Y(4R)^2 = \frac{1}{2}4\pi R^2 t\,\rho\times 16R^2$$

$$= 32\pi t\,\rho R^4$$

$$\therefore \quad \frac{I_Y}{I_X} = \frac{32\pi t\,\rho R^4}{\frac{1}{2}\pi\rho t R^4} = 64$$

$$\Rightarrow \quad I_Y = 64 I_X$$

7. (a) Angular momentum

$$L = I\omega \qquad \text{(i)}$$

Kinetic energy

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega \qquad \text{[From (i)]}$$

$$\therefore \quad L = \frac{2K}{\omega}$$

Now, $L' = \dfrac{2\left(\dfrac{K}{2}\right)}{2\omega} = \dfrac{L}{4}$

8. (b) $L = I\omega =$ constant
I will increase, so ω will decrease. And rotational KE will decrease as $KE = L^2/2I$.

9. (a) $I_A = \dfrac{2}{5}MR^2,\ I_B = \dfrac{2}{3}MR^2$

Clearly $I_B > I_A$.

10. (b) The acceleration vector is along the radius of the circle.

11. (a) $F = m\omega^2 R \;\Rightarrow\; \dfrac{F_1}{F_2} = \dfrac{R_1}{R_2}$

12. (a)

13. (b) Angular momentum is conserved

$$\therefore \quad L_{\text{initial}} = L_{\text{final}}$$

$$\therefore \quad mR^2\omega = (mR^2 + 2MR^2)\omega'$$

$$\Rightarrow \quad m\omega = (m + 2M)\omega'$$

$$\text{or} \quad \omega' = \frac{m\omega}{m+2M}$$

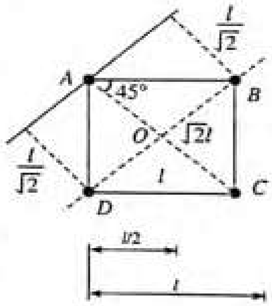
14. (d) Vector from A (1, – 1) to O

$$\vec{r} = (0-1)\hat{i} + [0-(-1)]\hat{j} = -\hat{i} + \hat{j}$$

$$\vec{\tau} = \vec{r}\times\vec{F} = (-\hat{i}+\hat{j})\times(-F\hat{k})$$

$$= -F\hat{j} - F\hat{i} = -F(\hat{i}+\hat{j})$$

15. (a) Moment of inertia of system



$$I = I_A + I_B + I_C + I_D$$

$$\Rightarrow \quad I = 0 + m(AO)^2 + m(AO)^2 + m(AC)^2$$

$$AO\cos 45° = \frac{l}{2} \Rightarrow AO = \frac{l}{\sqrt{2}}$$

$$\text{or} \quad I = \frac{2ml^2}{2} + m\left(\frac{2l}{\sqrt{2}}\right)^2$$

$$= \frac{2ml^2}{2} + \frac{4ml^2}{\cdot 2}$$

$$\text{or} \quad I = \frac{6ml^2}{2} = 3ml^2$$

16. (d) Acceleration of a uniform body of radius R and mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal is given by

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{MR^2}}$$

17. (b) Centre forces passes through axis of rotation so torque is zero. If no external torque is acting on a particle, the angular momentum of a particle is constant.

18. (b) Let the side of square be a.

$$I_{EF} = \frac{Ma^2}{12}$$

by perpendicular axes theorem, the moment of inertia of lamina about an axis passing through O and perpendicular to plane of lamina

$$I_z = \frac{Ma^2}{12} + \frac{Ma^2}{12} = \frac{Ma^2}{6}.$$

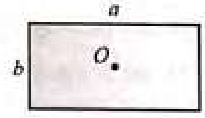
Again by perpendicular axes theorem,

$$I_{AC} + I_{BD} = I_z \Rightarrow I_{AC} = \frac{I_z}{2} = \frac{Ma^2}{12}$$

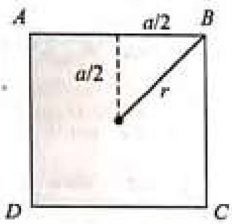
$\therefore \quad I_{AC} = I_{EF}$

19. (d) For a rectangular sheet moment of inertia passing through O, perpendicular to the plate is

$$I_O = M\left(\frac{a^2+b^2}{12}\right)$$



For square plate it is $\frac{Ma^2}{6}$ (as $a = b$)



$$r = \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{a}{\sqrt{2}}.$$

$\therefore \quad r^2 = \frac{a^2}{2}$

$\therefore$ Moment of inertia about B parallel to the axis through O is

$$I_B = I_O = Mr^2 = \frac{Ma^2}{6} + \frac{Ma^2}{2} = \frac{2Ma^2}{6}$$

$$I = \frac{2}{3}Ma^2$$

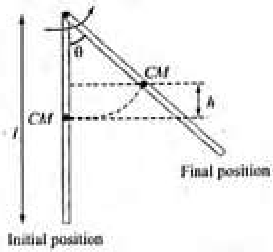
20. (d) The uniform rod of length l and mass m can swing about an axis passing through the end. When it is given angular velocity ω it starts rotating in vertical plane. When the centre of mass is raised through h, the increase in potential energy is mgh. This is equal to the decrease in kinetic energy.

Using conservation of mechanical energy $\Delta K + \Delta U = 0$

$$\left(0 - \frac{1}{2}I\omega^2\right) + (mgh) = 0$$

$$\Rightarrow \quad mgh = \frac{1}{2}\left(m\frac{l^2}{3}\right).\omega^2$$

$$\therefore \quad h = \frac{l^2.\omega^2}{6g}$$



21. (c) The position vector of the particle from the origin at any time t can be calculated by using $\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$$\Rightarrow \quad \vec{r} = v_0\cos\theta t\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j}$$

$\therefore$ Velocity vector at any time can be calculated by using $\vec{v} = \vec{u} + \vec{a}t$,

$$\Rightarrow \quad \vec{v} = v_0\cos\theta\hat{i} + (v_0\sin\theta - gt)\hat{j}$$

The angular momentum of the particle about the origin is

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$= m\left[v_0\cos\theta t\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\hat{j}\right.$$
$$\left.\times (v_0\cos\theta\hat{i} + (v_0\sin\theta - gt)\hat{j})\right]$$

$$= m\left[(v_0^2\cos\theta\sin\theta t - v_0gt^2\cos\theta)\hat{k}\right.$$
$$\left. + (v_0^2\sin\theta\cos\theta t - \frac{1}{2}gt^2v_0\cos\theta)(-\hat{k})\right]$$

$$= m\left[v_0^2\sin\theta\cos\theta t\hat{k} - v_0gt^2\cos\theta\hat{k}\right.$$
$$\left. - v_0^2\sin\theta\cos\theta t\hat{k} + \frac{1}{2}v_0gt^2\cos\theta\hat{k}\right]$$

$$= m\left[-\frac{1}{2}v_0gt^2\cos\theta\hat{k}\right] = -\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$$

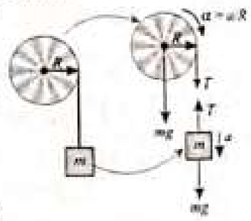
22. (d) From angular momentum conservation about vertical axis passing through centre: When insect is coming from circumference to center, moment of inertia decreases and when insect is going from centre to circumference, moment of inertia increases. So angular velocity first increases and then decreases.

23. (c) The free body diagram of pulley and mass

Equation of motion of block: $mg - T = ma$

$$\therefore \quad a = \frac{mg - T}{m} \quad \ldots\text{(i)}$$

$\therefore$ Angular acceleration of pulley



$$\alpha = \frac{\tau}{I} \quad \text{(ii)}$$

Here, $\tau = T \times R$

and $I = \frac{1}{2}mR^2$ (For circular disc)

$$\therefore \quad T = \frac{mR\alpha}{2} \quad \text{(Using (ii))}$$

Therefore, $a = \dfrac{mg - \frac{mR\alpha}{2}}{m}$ (Using (i))

$$ma = mg - \frac{ma}{2} \quad \left(\because \quad \alpha = \frac{a}{R}\right)$$

$$\therefore \quad a = \frac{2g}{3}$$

24. (b) $\tau = (20t - 5t^2)2 = 40t - 10t^2$

$$\alpha = \frac{\tau}{I} = \frac{40t - 10t^2}{10} = 4t - t^2$$

$$\omega = \int_0^t \alpha dt = 2t^2 - \frac{t^3}{3}$$

When direction is reversed, ω is zero. So

$$2t^2 - \frac{t^3}{3} = 0 \Rightarrow t^3 = 6t^2 \Rightarrow t = 6\text{ s}$$

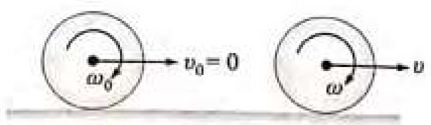
$$\theta = \int \omega dt$$

$$= \int_0^6 (2t^2 - \frac{t^3}{3})dt$$

$$= \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6 = 36 \text{ rad}$$

Number of revolution $= \frac{36}{2\pi}$ = Less than 6

25. (b)



From conservation of angular momentum about any fix point on the surface. Applying conservation at point of contact,

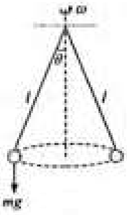
$$mr^2\omega_0 = mvr + mr^2\omega = mvr + mr^2\left(\frac{v}{r}\right)$$

$$\Rightarrow \quad mr^2\omega_0 = mvr + mvr = 2mvr$$

or $\quad v = \frac{r\omega_0}{2}$

26. (a) Torque of weight about point of suspension.

$$\tau_{\text{weight}} = mg \times l \sin\theta$$

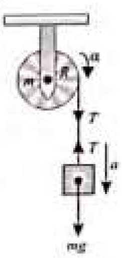


Direction of torque by weight is parallel to the plane of rotation of the particle.

As $\vec{\tau}$ is perpendicular to the angular momentum of the bob so the magnitude of angular momentum remains the same but direction changes.

27. (d) Applying Newton's second law on hanging block

$$mg - T = ma \quad \text{(i)}$$



Torque on cylinder due to tension in string about center of pulley

$$T \times R = I\alpha$$

$$T \times R = mR^2\alpha$$

($\because \ I = mR^2$ for hollow cylinder)

$$\Rightarrow \quad T = mR\alpha \quad \text{(ii)}$$

As string is not slipping over pulley.

$$a = R\alpha \quad \text{(iii)}$$

From eqns (ii) and (iii)

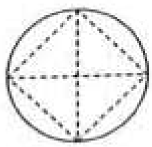
$$\Rightarrow \quad T = ma \quad \text{(iv)}$$

From eqns (i) and (iv)

$$mg = 2ma$$

$$\Rightarrow \quad a = \frac{g}{2}$$

28. (c) $d = 2R = a\sqrt{3}$



$$\Rightarrow \quad a = \frac{2}{\sqrt{3}}R$$

$$\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi$$

$$\Rightarrow \quad M' = \frac{2M}{\sqrt{3}\pi}$$

$$I = \frac{M'a^2}{6} = \frac{2M}{\sqrt{3}\pi} \times \frac{4}{3}R^2 \times \frac{1}{6}$$

$$I = \frac{4MR^2}{9\sqrt{3}\pi}$$

29. (b, d) When the particle moves along AB, $\vec{L} = mvr_\perp(-\hat{k})$

$$\vec{L}_{AB} = mg\frac{R}{\sqrt{2}}(-\hat{k})$$

When the particle moves from C to D, $\vec{L} = mvr_\perp(\hat{k})$

$$\vec{L}_{CD} = mv\left[\frac{R}{\sqrt{2}} + a\right](\hat{k})$$

When the particle moves from B to C,

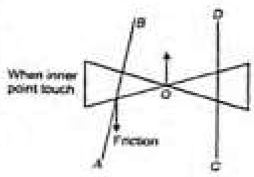
$$\vec{L}_{BC} = mvr_\perp(\hat{k}) = mv\left(\frac{R}{\sqrt{2}} + a\right)(\hat{k})$$

When the particle moves from D to A

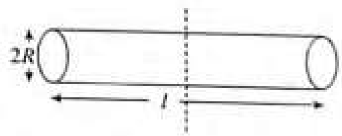
$$\vec{L}_{DA} = mvr_\perp(-\hat{k}) = mv\frac{R}{\sqrt{2}}(-\hat{k})$$

So (b) and (d) are false.

30. (a) When this cone proceeds then on AB rail the inner point will touch the rail whose velocity in forward direction slips forward and friction acts in backward direction. Due to this friction the system turns left.



31. (c)



$$I = \frac{ml^2}{12} + \frac{mR^2}{4}$$

$$\text{or} \quad I = \frac{m}{4}\left(\frac{l^2}{3} + R^2\right) \quad \ldots(1)$$

Also $m = \pi R^2 l\rho$

$$\Rightarrow \quad R^2 = \frac{m}{\pi l\rho} \text{ (put in equation)} \quad \ldots(2)$$

$$I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\pi l\rho}\right)$$

For maxima and minima

$$\frac{dI}{dl} = \frac{m}{4}\left(\frac{2l}{3} - \frac{m}{\pi l^2 \rho}\right) = 0$$

$$\frac{2l}{3} = \frac{m}{\pi l^2 \rho} \Rightarrow \frac{2l}{3} = \frac{\pi R^2 l \rho}{\pi l^2 \rho}$$

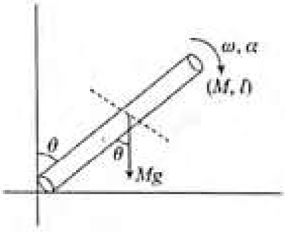
or $\frac{2l}{3} = \frac{R^2}{l}$

$\Rightarrow \frac{l^2}{R^2} = \frac{3}{2}$

or $\frac{l}{R} = \sqrt{\frac{3}{2}}$

32. (c) Torque at angle θ

$$\tau = Mg\sin\theta.\frac{l}{2}$$



Taking torque about pivot $\tau = I\alpha$

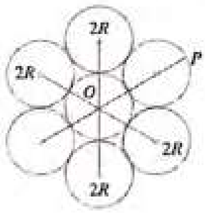
$$mg\sin\theta\frac{l}{2} = \frac{ml^2}{3}\alpha$$

$$\alpha = \frac{3g}{2l}\sin\theta$$

33. (a) The moment of inertia of the arrangement about the axis normal to the plane and passing through the point O is

$$I_O = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M\times(2R)^2\right)$$

$$= \frac{7MR^2}{2} + 24MR^2 = \frac{55\,MR^2}{2}$$



$$I_P = I_O + 7M(3R)^2$$

$$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$$

34. (b)

Surface mass density $= \frac{9M}{\pi R^2}$

Mass of removed disc,

$$m_2 = \sigma\times\pi\left(\frac{R}{3}\right)^2 = \frac{9M}{\pi R^2}\times\frac{\pi R^2}{9} = M$$

Moment of inertia about an axis passing through O of original disc of mass m;

$$I_1 = \frac{(9M)R^2}{2} = \frac{9MR^2}{2}$$

Moment of inertial of removed disc of mass m2 about an axis passing through O,

$$I_2 = \frac{M}{2}\left(\frac{R}{3}\right)^2 = \frac{MR^2}{18}$$

Now using parallel axis theorem to get moment of inertia of structure about O

$$I_0 = I_1 - \left[I_2 + m_2\left(\frac{2R}{3}\right)^2\right] = \frac{9MR^2}{2} - \left[\frac{MR^2}{18} + \frac{4}{9}MR^2\right]$$

$$= \frac{72\,MR^2}{18} = 4\,MR^2$$

CHAPTER 10: GRAVITATION

Concept Application Exercise 10.1

1. Let $m_1 = m$, then $m_2 = M - m$. Gravitational force of attraction between the two masses when placed distance r apart will be

$$F = \frac{Gm(M-m)}{r^2}$$

Differentiating it w.r.t. m, we get

$$\frac{dF}{dm} = \frac{G}{r^2}\left[m\frac{d}{dm}(M-m) + (M-m)\frac{dm}{dm}\right]$$

$$\frac{G}{r^2}[m(-1) + M - m] = \frac{G}{r^2}(M-2m)$$

If F is maximum, then $\frac{dF}{dm} = 0$

$\Rightarrow \quad \frac{G}{r^2}(M-2m) = 0$

or $\quad M = 2m$ or $m = \frac{M}{2}$

2. Here, $F = KR^{-5/2} = mR\left(\frac{2\pi}{T}\right)$

or $T^2 = \frac{4\pi^2 m R}{KR^{-5/2}} = \frac{4\pi^2 m}{K} R^{7/2}$

Hence, $T^2 \propto R^{7/2}$

3. Here, semimajor axis of the elliptical orbit of the planet around the sun is

$$r = \frac{(r_1 + r_2)}{2}$$

Therefore, $T^2 \propto \left(\frac{r_1+r_2}{2}\right)^3$

or $\quad T \propto (r_1 + r_2)^{3/2}$

4. $\frac{GMm}{R^n} = mR\left(\frac{2\pi}{T}\right)^2$

or $\quad T^2 = \frac{R \times 4\pi^2 \times R^n}{GM} = \frac{4\pi^2 R^{(n+1)}}{GM}$

or $\quad T \propto R^{(n+1)/2}$

5. Here, $r_j = 5.2\, r_e$

$T_e = 1$ year

$T_j = ?$

Now, $\frac{T_j^2}{T_e^2} = \frac{r_j^3}{r_e^3}$

or $\quad T_j = T_e\left(\frac{r_j}{r_e}\right)^{3/2} = 1 \times \left(\frac{5.2\, r_e}{r_e}\right)^{3/2} = 11.86$ years

6. Here, $r_1 = 10^{13}$ m and $r_2 = 10^{12}$ m

$\frac{T_1}{T_2} = ?$

Now, $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3} \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2}$

Now, $v = \frac{2\pi r}{T}$

Hence $\frac{v_1}{v_2} = \frac{r_1/r_2}{T_1/T_2} = \frac{10}{10^{3/2}} = \frac{1}{\sqrt{10}}$

Concept Application Exercise 10.2

1. Mass effect dominates over the distance effect. We know that

$$F = \frac{GMm}{R^2}, M \propto R^3$$

Hence, $F \propto R$

Hence, mass effect predominents.

2. Due to the rotation of the earth from west to east the stone will deviate slightly to the east.

3. $g' = g - R\omega^2\cos^2\lambda$

At equator $\lambda = 0$, $\cos\lambda = 1$

$g' = g - R\omega^2$

$\Delta g = g - g' = R\omega^2$

$= 6.67 \times 10^6 \times (7.27 \times 10^{-5})^2$

$= 3.36 \times 10^{-2}$ m/s^2 = 3.36 cm/s^2

4. $g' = \frac{g}{\left(\frac{1+h}{R}\right)^2}$

Given, $g' = \frac{10}{100}g = \frac{g}{10}$

$\frac{g}{10} = \frac{g}{\left(\frac{1+h}{R}\right)^2}$

$1 + \frac{h}{R} = \sqrt{10} = 3.16$

$\Rightarrow \quad \frac{h}{R} = 2.16$

$\Rightarrow \quad h = 2.16\,R = 2.16 \times 6.37 \times 10^6 = 1.37 \times 10^7$ m

5. (a) i. At height h, $g' = \frac{g}{\left(\frac{1+h}{R}\right)^2}$

$\Rightarrow \quad \frac{h}{R} = \frac{1600 \text{ km}}{6400 \text{ km}} = \frac{1}{4}$

$\therefore \quad g' = \frac{g}{\left(\frac{1+1}{4}\right)^2} = \frac{16}{25}g = 9.8 = 6.27$ m/s^2

ii. At depth x,

$g_x = g\left(\frac{1-x}{R}\right) = g\left(1 - \frac{1600\text{ km}}{6400\text{ km}}\right) = g \times \frac{3}{4} = 7.35$ m/s^2

(b) i. At distance r from centre of earth above the earth's surface,

$g = \frac{GM}{r^2} \quad \Rightarrow \quad \frac{dg}{dr} = -\frac{2}{r^3}GM$

As $r = R + h$, $dr = dh$, $\frac{dg}{dh} = -\frac{2GM}{(R+h)^3}$

ii. At distance x below earth's surface,

$g' = g\left(\frac{1-x}{R}\right)$

$\frac{dg}{dx} = -\frac{g}{R} = -\left(\frac{GM}{R^2}\right)\frac{1}{R} = -\frac{GM}{R^3}$

$$\frac{G\frac{4}{3}\pi R^2\rho}{R^3} = -\frac{4}{3}\pi G\rho$$

6. We know that $\omega = \frac{2\pi}{24h} = \frac{2\pi}{24\times60\times60}\, s^{-1}$

$$W' = W - mR\omega^2 \text{ (at equator)}$$

When bodies are weightless $W' = 0$ and $W = mg$. Therefore,
$$0 = mg - mR\omega^2$$
$$\Rightarrow \quad \omega = \sqrt{\frac{g}{R}}$$
$$\therefore \quad \frac{\omega'}{\omega} = \frac{(9.78\ \ 6.37\times10^6)^{1/2}}{(2\pi\ \ 24\times60\times60)} \approx 17$$

Concept Application Exercise 10.3

1. $v_e\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{M}\times\frac{4}{3}\pi R^3\rho} = R\sqrt{\frac{8\pi}{3}GK\rho}$

$$\frac{v_p}{v_e} = \frac{R_p}{R_e} = \frac{2R_e}{R_e} = 2 \quad \text{or} \quad v_p = 2v_e$$

2. We know that the value of escape velocity of a particle from the surface of earth is given by
$$v_e = \sqrt{2GM/R}$$
So, the minimum kinetic energy needed for the particle to escape is given by
$$\frac{1}{2}mv_e^2 = \frac{1}{2}m\times\frac{2GM}{R} = \frac{GMm}{R}$$
As per question, the kinetic energy given to the particle is $(GMm/2R)$.
If h is the height attained by the particle from the surface of the earth, then its potential energy is $-GMm/(R+h)$.
According to the law of conservation of total energy,
KE + PE on the surface of earth = PE at height h.
$$\frac{1}{2}\frac{GM\,m}{R} + \left(\frac{-GM\,m}{R}\right) = -\frac{GM\,m}{R+h} \Rightarrow h = R$$
or
$$\frac{GM\,m}{2R} = -\frac{GM\,m}{R+h} \Rightarrow h = R$$

3. Magnitude of the potential energy per unit mass of the object at the surface of earth is
$$\frac{GM}{R} = E \text{ (given)}$$
Escape velocity, $v = \sqrt{\frac{2GM}{R}} = \sqrt{2E}$

4. Conservation of energy implies
$$0 + 0 = \frac{1}{2}mv^2 - \frac{GMm}{R}$$
$$\Rightarrow \quad v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(R^2 g)}{R}}$$
$$= \sqrt{2Rg} \approx 11.2\times10^3 \text{ m/s}$$

5. $\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 - \frac{GMm}{R+h}$

If we put $GM = R^2 g$, we get
$$h = \frac{v^2R}{2gR - v^2} = 2.5\times10^4 \text{ km}$$

6. The potential energy of satellite is given by
$$U = -\frac{GMm}{r} \propto -\frac{1}{r}$$
This is negative. It means that larger the radius, greater is the potential energy.
$$\frac{U_1}{U_2} = \frac{-r_2}{-r_1} = \frac{-(R+h_2)}{-(R+h_1)}$$
$$= \frac{-(6400+19200)}{-(6400+6400)} = \frac{-2}{-1}$$
Kinetic energy of satellite, $k = \frac{GMm}{r^2}$
Hence $\frac{k_1}{k_2} = \frac{r_2}{r_1} = \frac{2}{1}$

Total energy $E = \text{KE} + \text{PE} = -\frac{GMm}{2r}$
$$\therefore \quad \frac{E_1}{E_2} = \frac{-r_2}{-r_1} = \frac{-2}{-1}$$
As $-1 > -2$, satellite B has greater value of total energy.

7. $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\left(\frac{4}{3}\pi R^3\rho\right)}{R}}$

$$R = v_e\sqrt{\frac{3}{8\pi G\rho}} = 30894 \text{ m} = 31 \text{ km}$$

Concept Application Exercise 10.4

1. The feeling of weight arises from the reaction of the ground on a man. In an orbiting satellite, the astronaut and the floor of the satellite both have the same acceleration (the centripetal acceleration towards the earth). Hence, the floor of the satellite offers no reaction to the astronaut and he feels weightless.
For circular motion of man,
$$\frac{GM\times m}{r^2} - N = \frac{mv^2}{r}$$
For circular motion of 'satellite + man' (mass m),
$$\frac{GMm}{r^2} = \frac{m\,v^2}{r} \Rightarrow \frac{GM}{r^2} = \frac{v^2}{r}$$
$$\therefore \quad \frac{mv^2}{r} - N = \frac{mv^2}{r} \Rightarrow N = 0$$

2. $T = 2p\sqrt{\frac{(R_e+h)^3}{GM_e}} = 2\pi\sqrt{\left\{\frac{\left(R_e+\frac{R_e}{2}\right)^3}{R_e^2 g}\right\}} = 2\pi\sqrt{\frac{27R_e}{8g}}$

$$= 2\times3.14\sqrt{\frac{27\times6.38\times10^6}{8\times9.8}} \approx 9308 \text{ s}$$

3. $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\left\{\frac{R^3}{\left(G\frac{4}{3}\pi R^3\rho\right)}\right\}} = \sqrt{\frac{3p}{Gr}}$

$$= \sqrt{\frac{3\times3.14}{6.67\times10^{-11}\times8000}} \approx 4202 \text{ s}$$

4. Initial total energy of each satellite $= -\frac{GMm}{2r}$

Hence, total energy of both satellites

$$2 \times \left(-\frac{GMm}{2r}\right) = \frac{GMm}{r}$$

When satellites travelling in opposite directions collide, the final velocity v is given by

$$(2mv) = mv - mv = 0$$

$$\Rightarrow \quad v = 0$$

Therefore, the wreckage comes to rest and it will fall freely under gravity.

After collision, total energy of satellite is

$$2\left(-\frac{GMm}{r}\right) = \left(-\frac{2GMm}{r}\right)$$

5. (a) $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$

$$\frac{\rho_1}{\rho_2} = \frac{M_1}{M_2}\left(\frac{R_2}{R_1}\right)^2 = 0.605$$

(b) $fg = -\dfrac{GM}{R^2}$

$$\frac{f_1}{f_2} = \frac{M_1}{M_2}\left(\frac{R_2}{R_1}\right)^2 = \frac{1}{11} \times \left(\frac{79}{42}\right)^2$$

(c) $v_e = \sqrt{\dfrac{2GM}{R}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{M_1}{M_2}\frac{R_2}{R_1}} = \sqrt{\frac{1}{11} \times \frac{79}{42}} = 0.4135$$

(d) Period near surface of planet is

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

$$\frac{T_1}{T_2} = \sqrt{\left(\frac{R_1}{R_2}\right)^3 \frac{M_2}{M_1}} = \sqrt{\left(\frac{42}{79}\right)^3 \times \frac{11}{1}} = 1.286$$

6. In elliptical orbit, total energy

$$E = -\frac{GMm}{2a} = \text{constant}$$

At position r, if v is orbital speed of satellite, then

$$\text{KE} = \frac{1}{2}mv^2 \text{ and PE} = \frac{GMm}{r}$$

$$\therefore \quad \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a} \quad \Rightarrow v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

Exercises

Kepler's Laws of Planetary Motion and Newton's Law of Gravitation

1. (b) Applying conservation of angular momentum at position A and B

$$mv_A \times OA = mv_B \times OB$$

Hence, $\dfrac{v_B}{v_A} = \dfrac{OA}{OB} = x$

2. (a) Conserving angular momentum.

$$mv_1 r_{\min} = mv_2 r_{\max} \text{ or } \frac{v_1}{v_2} = \frac{r_{\max}}{r_{\min}}$$

3. (d) $\dfrac{mv^2}{r} = \dfrac{GMm}{r^{5/2}} \left[\because F \propto \dfrac{1}{r^{5/2}}\right]$

or $r\omega^2 = \dfrac{GM}{r^{5/2}}$

or $r\dfrac{4\pi^2}{T^2} = \dfrac{GM}{r^{5/2}}$

or $T^2 \propto r^{5/2}$ or $T^2 \propto r^{3.5}$

4. (b) $F_G = \dfrac{GMm}{R}$ (as $F_G \propto \dfrac{1}{R}$ given)

So $\dfrac{mv^2}{R} = \dfrac{GMm}{R} \quad \Rightarrow v^2 \propto R^0$

5. (c) $F_G = \dfrac{Gm^2}{4R^2} \quad \Rightarrow \dfrac{Mv^2}{R} = \dfrac{Gm^2}{4R^2}$

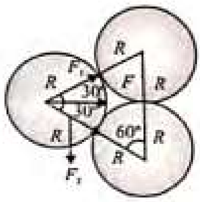
$\therefore \quad v = \dfrac{1}{2}\sqrt{\dfrac{Gm}{R}}$

6. (d) $\vec{F} = \vec{F}_1 + \vec{F}_2$

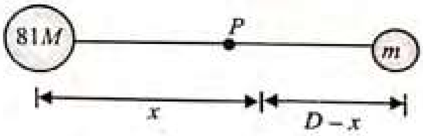
As $|\vec{F}_1| = |\vec{F}_2|$

$\therefore \quad |\vec{F}| = 2F_1 \cos 30°$

$$= 2\frac{GM^2}{(2r)^2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}\frac{GM^2}{r^2}$$



7. (d)



$$\frac{81GM}{x^2} = \frac{GM}{(D-x)^2}$$

$$\frac{9}{x} = \frac{1}{(D-x)}$$

$$D = \frac{9}{10}x$$

$$x = \frac{9}{10}D$$

8. (d) Let mass of the cavity = M'

Density of the sphere = $M/(4/3\,\pi R^3)$

Mass of the cavity cut out = $M' = \dfrac{4}{3}\pi\dfrac{R^3}{8} \times \dfrac{M}{\frac{4}{3}\pi R^3}$

$\therefore \quad M' = \dfrac{M}{8} \Rightarrow F_{\text{net}} = F_{Mm} - F_{M'm}$

$$= \frac{GMm}{4R^2} - \frac{GM'm}{\left(\frac{5}{2}R\right)^2} = \frac{GMm}{4R^2} - \frac{GMm}{50R^2}$$

$$F_{\text{net}} = \frac{23}{100}\frac{GMm}{R^2}$$

9. (c) As observed from the earth, the sun appears to move in an approximate circular orbit. The gravitational force of attraction between the earth and the sun always follows inverse square law.

Due to relative motion between the earth and mercury, the orbit of mercury, as observed from the earth, will not be approximately circular, since the major gravitational force on mercury is due to the sun.

10. (d) Asteroids are also being acted upon by central gravitational forces, hence they are moving in circular orbits like planets and obey Kepler's laws.

11. (c) Force on B due to $A = F_{BA} = \frac{G(2Mm)}{(AB)^2}$ towards BA

Force on B due to

$C = F_{BC} = \frac{GMm}{(BC)^2}$ towards BC

As $(BC) = 2AB$

$\Rightarrow \quad F_{BC} = \frac{GMm}{(2AB)^2} = \frac{GMm}{4(AB)^2} < F_{BA}$

Hence, m will move towards BA (i.e., $2M$)

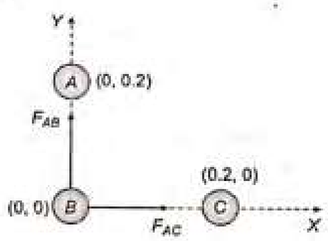
12. (c) $\frac{dA}{dt} = \frac{L}{2m} = \frac{dA}{dt} \propto vr \propto \omega r^2$

13. (a) By conservation of angular momentum mvr = constant

$v_{\min} \times r_{\max} = v_{\max} \times r_{\min}$

$\therefore \quad v_{\min} = \frac{60 \times 1.6 \times 10^{12}}{8 \times 10^{12}} = \frac{60}{5} = 12 \text{ m/s}$

14. (a) Let particle A lies at origin, particles B and C on y and X-axis respectively



$\vec{F}_{AC} = \frac{G m_A m_B}{r_{AB}^2}\hat{i} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(0.2)^2}\hat{i} = 1.67 \times 10^{-9}\,\hat{i}\,\text{N}$

Similarly $\vec{F}_{AB} = 1.67 \times 10^{-9}\,\hat{j}\,\text{N}$

$\therefore$ Net force on particle A

$\vec{F} = \vec{F}_{AC} + \vec{F}_{AB} = 1.67 \times 10^{-9}(\hat{i} + \hat{j})\,\text{N}$

15. (c) According to law of conservation of angular momentum

$mv_1 d_1 = mv_2 d_2$

$\therefore \quad v_2 = \frac{d_1 v_1}{d_2}$

Gravitational Field and Acceleration Due to Gravity

16. (c) At point P, we have $I_1 - I_2 = 0$ (because the gravitational field inside a shell is zero). Hence, $I_1 = I_2$.

17. (d) The gravitational field intensity at a point inside the spherical shell is zero.

NOTE: There is NO gravitational field inside a spherical shell.

18. (b) $g' = g\left[1 - \frac{2h}{R}\right] = g\left[1 - \frac{d}{R}\right]$

$\frac{2h}{R} = \frac{d}{R} \Rightarrow d = 2h$

19. (c) $g = \frac{Gm}{R^2}$

$M = \frac{4}{3}\omega R^2 \rho$

So $\rho = \frac{3g}{4\pi RG}$

20. (d) Inside the earth $g' = g\left[1 - \frac{h}{R}\right]$

At centre, $g' = g\left[1 - \frac{R}{R}\right] = 0$

21. (d) $g = \frac{GM}{R^2}$ (i)

$g' = \frac{Gm}{\left(\frac{90}{100}R\right)^2} = \frac{100}{81}\frac{GM}{R^2}$ (ii)

From Eqs. (i) and (ii),

$g' = \frac{100}{81}g \Rightarrow \frac{g'}{g} = \frac{100}{81}$

$\frac{g'}{g} - 1 = \frac{100}{81} - 1$

$\therefore \quad \Delta g = \frac{19}{81}g = 23\%$ of g

So increase is more than 19% of g.

22. (a) (i) The weight of the body at the centre of the earth is equal to zero because

$g_{\text{centre}} = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{R}{R}\right) = 0$

$\frac{g_1}{g} = \left(1 - \frac{d}{R}\right) = \frac{1}{4} \Rightarrow d = \frac{3R}{4}$

So from the centre, $d' = \frac{R}{4}$

23. (a) $3/5\, mg = mg - mR\omega^2$

$\omega^2 = g - \frac{3}{5}g \Rightarrow \omega = \sqrt{\frac{2}{5}\frac{g}{R}}$

24. (b) $mg = mR\omega^2$

R = radius of earth

$\omega = \sqrt{\frac{g}{R}}$

$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{64000}$

$= 2\pi \times 800 \text{ s} = \frac{2\pi \times 800}{3600} \text{ h} = 1.36 \text{ h} = 1.4 \text{ h}$

25. (a) $g_p = \frac{G\left[M - \frac{10}{100}M\right]}{\left[R + \frac{20}{100}R\right]^2} = \frac{G \times 9M}{10} \times \frac{25}{36R^2} = \frac{5}{8}g$

26. (c) $mg' = mg - mR\omega^2\cos^2\phi$

Now, $\frac{3}{5}mg = mg - mR\omega^2$

or $\quad mR\omega^2 = mg - \frac{3}{5}mg = \frac{2}{5}mg$ or $\omega = \sqrt{\frac{2g}{5R}}$

27. (a) $x = \frac{GM}{R^2}$

Again, $\frac{x}{4} = \frac{GM}{(R+h)^2}$ or $x = GM\left(\frac{2}{R+h}\right)^2$

$\therefore \quad \frac{1}{R^2} = \left(\frac{2}{R+h}\right)^2 = \frac{2}{R+h}$ or $R + h = 2R$ or $h = R$

28. (b) Gravitational pull depends upon the acceleration due to gravity on that planet.

$\therefore \quad M_m = \frac{1}{81}M_e, g_m = \frac{1}{6}g_e$

$$g = \frac{Gm}{R^2} \Rightarrow \frac{R_e}{R_m} = \left(\frac{M_e}{M_m} \times \frac{g_m}{g_e}\right)^{1/2} = \left(81 \times \frac{1}{6}\right)^{1/2}$$

$$R_e = \frac{9}{\sqrt{6}}R_m$$

29. (a) Acceleration due to gravity

$$g = \frac{4}{3}\pi\rho GR \quad \therefore \; g \propto \rho R$$

or $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e} \cdot \frac{R_m}{R_e}$ [As $\frac{g_m}{g_e} = \frac{1}{6}$ and $\frac{\rho_e}{\rho_m} = \frac{5}{3}$ (given)]

$$\therefore \quad \frac{R_m}{R_e} = \left(\frac{g_m}{g_e}\right)\left(\frac{\rho_e}{\rho_m}\right) = \frac{1}{6} \times \frac{5}{3} \quad \therefore \; R_m = \frac{5}{18}R_e$$

30. (d) Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$

$\therefore \quad g_1 : g_2 = R_1\rho_1 : R_2\rho_2$

Gravitational Potential, Energy and Escape Velocity

31. (b) As gravitational force is a conservative force, work done is independent of path.

$\therefore \quad W_1 = W_2 = W_3$

32. (a) At a distance x from the centre of earth, the gravitational force

$$F = \frac{GMm}{x^2}$$

$$dW = Fdx = \frac{GMm}{x^2}dx$$

We get $W = \int_R^{R+h} \frac{GMm}{x^2}dx = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$

$$= gR^2m\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

Therefore, the gain in potential energy is

$$gR^2m\left[\frac{1}{R} - \frac{1}{2R}\right] = \frac{mgR}{2}$$

33. (c) $v_e = 11.2$ km/s

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

$$v^2 = \frac{GM}{R^2}R = 8 \text{ km/s}$$

Therefore, the additional velocity = (11.2 – 8) = 3.2 km/s

34. (b) Energy of the skylab in the first orbit is

$$-\frac{GMm}{2(2R)} = -\frac{GMm}{4R}$$

Total energy required to place the skylab into the orbit of radius $2R$ from the surface of earth is

$$-\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = \frac{3GMm}{4R}$$

$$= \frac{3gR^2m}{4R} = \frac{3}{4}mgR$$

Energy of the skylab in the second orbit = $-(GMm)/6R$.

Energy needed to shift the skylab from the first orbit to the second orbit is

$$-\frac{GMm}{4R} - \frac{GMm}{6R} = \frac{GMm}{R} \times \frac{2}{24} = \frac{mgR}{12}$$

35. (b) According to the problem, as the potential at ∞ increases by $+10$ J kg^{-1}, hence potential will increase by the same amount everywhere (potential gradient will remain constant). Hence, potential at point $P = 10 - 5 = +5$ J kg^{-1}

36. (c)

M_1 ← (x) → $(R-x)$ M_2

$$\frac{GM_1}{x^2} = \frac{GM_2}{(R-x)^2} \quad \text{or} \quad \frac{M_2}{M_1}x^2 = R^2 + x^2 - 2Rx$$

Let $\frac{M_2}{M} = k$

$$x^2(k-1) + 2Rx - R^2 = 0$$

$$x = -\frac{2R + \sqrt{4R^2 + 4(k-1)R^2}}{2(k-1)} = \frac{R\sqrt{M_1}}{\sqrt{M_1} + \sqrt{M_2}}$$

$$R - x = \frac{R\sqrt{M_2}}{\sqrt{M_1} + \sqrt{M_2}}$$

Gravitational potential at point P is

$$-\left(\frac{GM_1}{x} + \frac{GM_2}{R-x}\right)$$

$$= -\left[\frac{GM_1(\sqrt{M_1} + \sqrt{M_2})}{R\sqrt{M_1}} + \frac{GM_2(\sqrt{M_1} + \sqrt{M_2})}{R\sqrt{M_2}}\right]$$

$$= -\left[\frac{G(\sqrt{M_2} + \sqrt{M_1})}{R}(\sqrt{M_1} + \sqrt{M_2})\right]$$

$$= -\frac{G(\sqrt{M_1} + \sqrt{M_2})^2}{R}$$

37. (a) Binding energy = GMm/R

M = mass of the Sun = 10^{30} kg

m = mass of the earth = 6×10^{24} kg

$R = 1.5 \times 10^{11}$ m

Binding energy of the system is

$$\frac{6.67 \times 10^{-11} \times 10^{30} \times 6 \times 10^{24}}{1.5 \times 10^{11}} = 2.7 \times 10^{33} \text{ J}$$

38. (c) Total energy, $E = \frac{1}{2}mv^2 - \frac{GmM}{r}$

$$= \frac{GmM}{2r} - \frac{GMm}{r} = -G\frac{mM}{2r}$$

$$r = 2R = R = 3R$$

$$E = \frac{GmM}{6R}$$

Potential energy = $-\frac{GMm}{R}$

Minimum energy required = $\frac{1}{6}\frac{GMm}{R} - \left(\frac{-GMm}{R}\right) = \frac{5}{6}\frac{GMm}{R}$

$$= \frac{5}{6}mgR$$

39. (c) $W = \left[-\frac{GMm}{5R}\right] - \left[\frac{GMm}{3R}\right]$

$$= \frac{GMm}{3R} - \frac{GMm}{5R}$$

$$= \frac{GMm}{R}\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{2}{15}\frac{GMm}{R}$$

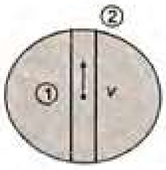
40. (c) Potential energy of the body at a distance 4 R_e from the surface of earth

$$U = -\frac{mgR_e}{1+h/R_e} = -\frac{mgR_e}{1+4} = -\frac{mgR_e}{5}$$

[As $h = 4R_e$ (given)]

So minimum energy required to escape the body will be $\frac{mgR_e}{5}$.

41. (a) $(KE+PE)_1 = (KE+PE)_2$



$$\frac{1}{2}mv^2 - \frac{3}{2}\frac{GMm}{R} = -\frac{GMm}{R}$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{gR^2}{R}} = \sqrt{gR}$$

42. (c) $(KE+PE)_1 = (KE+PE)_\infty$

$$\frac{1}{2}mv^2 - \frac{3}{2}\frac{GMm}{R} = 0$$

$$v = \sqrt{\frac{3GM}{R}} = \sqrt{3gR}$$

43. (b) $\frac{W_1}{W_2} = \frac{-\frac{GM}{R} - \left(-\frac{3}{2}\frac{GM}{R}\right)}{0 - \left(-\frac{GM}{R}\right)} = \frac{1}{2}$

44. (a) $\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 \Rightarrow v = \sqrt{\frac{2GM}{R}}$

45. (a) $v = -\frac{GMM}{R} = -\frac{GM^2}{R}$

Motion of Satellite

46. (d) $m_1 = 2m_2 \Rightarrow \frac{m_1}{m_2} = 2$

$$r_1 = 4r_2 \Rightarrow \frac{r_1}{r_2} = 4$$

$$T_A^2 \propto r_1^3 \text{ and } T_B^2 \propto r_2^3$$

$$\Rightarrow \frac{T_A}{T_B} = 8$$

47. (c) PE $= \frac{-GM}{2R}$ (at height R)

$$= -\frac{GMR_m}{2R^2} = -\frac{1}{2}mgR$$

48. (a) Total mechanical energy is given by

$$E = K + u = -\frac{GMm}{2a} - \frac{GMm}{a} = \frac{-GMm}{2a}$$

$$\frac{GM}{a} = v^2 \Rightarrow E = -\frac{1}{2}mv^2$$

49. (c) $\frac{m_1v_1^2}{r_1} = m_1\frac{GM}{r_1^2}$, $\frac{m_2v_2^2}{r_2} = m_2\frac{GM}{r_2^2}$

$$\frac{v_1^2}{v_2^2} = \frac{r_2}{r_1};\ r_2 < r_1 \Rightarrow v_1^2 < v_2^2.$$

Hence, $v_1 < v_2$.

50. (c) $V_A = \sqrt{\frac{GM}{r_A}}$ and $V_B = \sqrt{\frac{GM}{r_B}}$

Given $r_B = 3r_A$

Now $F_A = \frac{mv_A^2}{r_A} = \frac{M}{r_A}\frac{GM}{r_A} = \frac{GMm}{r_A^2}$

$$F_B = \frac{GMm}{r_B^2}$$

$$\frac{F_B}{F_A} = \frac{r_A^2}{r_B^2} = \frac{1}{9}$$

51. (c) Given $8 = \frac{2\pi}{\omega_1+\omega_2} = \frac{2\pi}{\frac{2\pi}{T_1}+\frac{2\pi}{T_2}}$, $T_1 = 24$ hours for earth.

$\Rightarrow$ $T_2 = 12$ hours (T_2 being the time period of satellite, it will remain same as the distance from the centre of the earth remains constant).

$$\Rightarrow T = \frac{2\pi}{\omega_2-\omega_1} = \frac{2\pi}{\frac{2\pi}{T_2}-\frac{2\pi}{T_1}} = 24 \text{ hours}$$

52. (b) In the circular motion around the Earth, the centripetal force on the satellite is a gravitational force. Therefore, $v^2 = GM/R$, where M is the mass of the Earth, R is the radius of the orbit of satellite and G is the universal gravitational constant. Therefore, the kinetic energy increases with the decrease in the radius of the orbit. The gravitational potential energy is negative and decreases with the decrease in radius.

53. (c) $v = \sqrt{\frac{GM}{R+h}}$

For first satellite $h = 0$, $v_1 = \sqrt{\frac{GM}{R}}$

For second satellite $h = \frac{R}{2}$, $v_2 = \sqrt{\frac{2GM}{3R}}$

$$v_2 = \sqrt{\frac{2}{3}}v_1 = \sqrt{\frac{2}{3}}v$$

54. (c)

(i) $T_{st} = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$ [As $h << R$ and $GM = gR^2$]

(ii) $T_{ma} = 2\pi\sqrt{\frac{R}{g}}$

(iii) $T_{sp} = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l}+\frac{1}{R}\right)}} = 2\pi\sqrt{\frac{R}{2g}}$ [As $l = R$]

(iv) $T_{is} = 2\pi\sqrt{\frac{R}{g}}$ [As $l = \infty$]

55. (d) Time period $T \propto r^{3/2} \Rightarrow r \propto T^{2/3}$ and

kinetic energy $\propto \frac{1}{r} \propto \frac{1}{T^{2/3}} \propto T^{-2/3}$

56. (d) Velocity of satellite will become $2v_{orbital}$ which is greater than $v_e(=\sqrt{2}v_{orbital})$ so satellite will escape with hyperbolic path.

57. (c) Angular speed of earth = angular speed of geostationary satellite

$$T \propto \frac{1}{\omega}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = \frac{1}{2} \Rightarrow T_2 = \frac{T_1}{2}$$

Also, $T \propto r^{3/2}$

$$\therefore \left(\frac{r_2}{r_1}\right)^{3/2} = \frac{T_2}{T_1} = \frac{1}{2} \Rightarrow r_2 = \frac{r_1}{4^{1/3}} = \frac{r_1}{4^{1/3}}$$

58. (c) For observer, $T' = \dfrac{2\pi}{\omega_S - \omega_E} = \dfrac{T_S T_E}{T_E - T_S}$

$= T_E$ (given) or $T_E^2 = 2T_S T_E \quad T_S = T_E/2$

59. (a) $v_a = \dfrac{dA}{dt} = \dfrac{1}{2} r v_0 = \dfrac{1}{2} r \sqrt{\dfrac{GM}{r}}$

$4v_a^2 = GMr = GM(R+h)$

$= gR^2(R+h) \quad h = \dfrac{4v_a^2}{gR^2} - R$

60. (b) Time period is minimum for the satellites with minimum radius of the orbit i.e. equal to the radius of the planet. Therefore,

$$\frac{GMm}{R^2} = \frac{mv^2}{R} \Rightarrow V = \sqrt{\frac{GM}{R}}$$

$$T_{\min} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R\sqrt{R}}{\sqrt{GM}}$$

using $M = \dfrac{4}{3} pR^3.\rho \quad T_{\min} = \sqrt{\dfrac{3\pi}{G\rho}}$

Using values $T_{\min} = 3000$ s

Problems Based on Mixed Concepts

61. (a) i. g' = acceleration due to gravity on the Moon

$= G\frac{4}{3}\pi R'\rho' = G\frac{4}{3}\pi \times \frac{R}{4} \times \frac{2}{3}\rho = \frac{g}{6}$

Now, $h_m = \dfrac{u^2}{2g}$ or $h_m \propto \dfrac{1}{g}$

Hence $h'_m g' = h_m g$

or $h'_m = 6h_m = 6 \times 0.5 = 3$ m

ii. $t = \dfrac{2u}{g}$ or $t \propto \dfrac{1}{g}$

$\therefore \quad t'g' = tg \quad$ or $\quad \dfrac{t'}{t} = \dfrac{g}{g'} = \dfrac{6}{1}$

62. (a) Energy of each satellite in the orbit $= \dfrac{-GMm}{2r}$.

Total energy of the system before collision,

$$E_i = E_1 = E_2 = 2E = -2 \times \frac{GMm}{2r} = -\frac{GMm}{r}$$

As the satellites of equal mass are moving in the opposite directions and collide inelastically, the velocity of the wreckage just after the collision is

$mv - mv = 2mV$, i.e., $V = 0$

The energy of the wreckage just after the collision will be totally potential and will be

$$E_f = \frac{GM(2m)}{r} = -\frac{2GMm}{r}$$

As after collision the wreckage comes to standstill in the orbit, it will move along the radius towards the earth under gravity.

63. (c) $\dfrac{1}{2} mv^2 = \dfrac{GMm}{R} - \dfrac{GMm}{R+h} = \dfrac{GMmh}{R(R+h)} = \dfrac{mghR}{R+h}$

Since $v = nv_e$ and $v_e = \sqrt{2gR}$, hence, $h = \left(\dfrac{Rn^2}{1+n^2}\right)$

64. (d) Let the mass of the particle be m

PE at a distance of $R' = (GMm)/R'$

PE at a distance of $R_e = -(GMm)/R_e$

Decrease in PE = Increase in KE

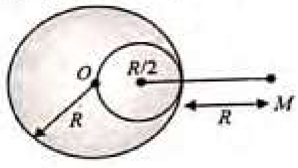
$$\Rightarrow -\frac{GMm}{R'} + \frac{GMm}{R_e} = \frac{1}{2}mv^2$$

$$v^2 = 2GM\left[\frac{1}{R_e} - \frac{1}{R'}\right] \Rightarrow v^2 = \frac{2GM}{R_e}\left[1 - \frac{R_e}{R'}\right]$$

$$\Rightarrow v^2 = \frac{2GM}{R_e}\left(1 - \frac{R_e}{R'}\right) \Rightarrow v = \sqrt{\frac{2GMR_e}{R_e^2}}\left(1 - \frac{R_e}{R'}\right)$$

$$\therefore \quad v = \sqrt{2gR_e\left(1 - \frac{R_e}{R'}\right)}$$

65. (d) Mass of the cavity $= M/8$ if mass of the sphere $= M$ as volume of the cavity is 1/8th of the sphere.



$$F_2 = \frac{GMm}{4R^2} - \frac{GMm}{8\left(\frac{3}{2}R\right)^2}$$

$$= \frac{GMm}{R^2}\left[\frac{1}{4} - \frac{1}{18}\right] = \frac{GMm}{R^2}\left[\frac{9-2}{36}\right]$$

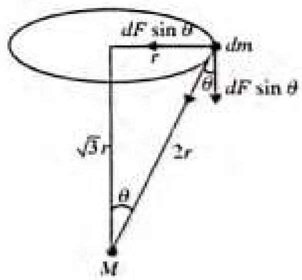
$$= \frac{GMm}{R^2}\frac{7}{36}$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{36}{4 \times 7} = \frac{9}{7}$$

66. (b) $F = (GMm)/R^2 = mg$

As the mass remains the same and radius, i.e., distance from the centre is also the same, therefore g will remain the same, i.e., 10 m s^{-2}.

67. (c) $dF = G\dfrac{Mdm}{4r^2}$



$F = \Sigma dF \cos\theta$

$= \Sigma \dfrac{GMdm}{4r^2} \cos\theta$

$= \dfrac{GM}{4r^2} \times \dfrac{\sqrt{3}r}{2r} \Sigma dm$

$= \dfrac{\sqrt{3}GMm}{8r^2}$

Alternative solution:

The gravitational field due to the ring at a distance $\sqrt{3}r$ is given by

$$E = \frac{Gm(\sqrt{3}r)}{[r^2 + (\sqrt{3}r)^2]^{3/2}} \text{ or } E = \frac{\sqrt{3}Gm}{8r^2}$$

The required force is EM, i.e., $(\sqrt{3}Gm)M/8r^2$.

68. (a) Let us first calculate the mass of the inner solid sphere of radius r.

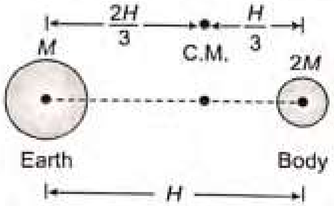
Mass of the inner solid sphere is

$$M' = \frac{M_e}{\frac{4}{3}\pi R_e^3} \times \frac{4}{3}\pi r^3 = \frac{M_e}{R_e^3} r^3$$

Now, $g = \frac{GM_e r^3}{R_e^3} \times \frac{1}{r^2}$ or $g = \frac{GM_e r}{R_e^3}$

Force on the particle of mass $m = mg = \frac{GM_e mr}{R_e^3}$

69. (c) As the masses of the body and the earth are comparable, they will move towards their centre of mass, which remains station

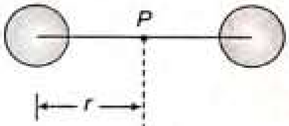


Hence the body of mass $2m$ move through distance $H/3$. and time to reach the earth surface

$$= \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2H/3}{g}} = \sqrt{\frac{2H}{3g}}$$

70. (b) Since the mass and radius of two stars are same, point P at which gravitational force will be equal, will be at equal distance (say r) from centre of each star. The particle should be projected to cross P, beyond which the particle will be attracted towards the other star.



KE = PE of body at P – PE at surface of the star

$$\frac{K_1 GMm}{a} = \left[-\frac{GMm}{r} - \frac{GMm}{r}\right] - \left[-K_2\frac{GMm}{a}\right]$$

On solving, $r = \frac{2a}{(K_2 - K_1)} \; 2r = \frac{4a}{(K_2 - K_1)}$

71. (d) We will be thrown into space, if weight mg is equal to gravitational force due to the planet. If y is the closest distance.

$$\frac{GMm}{R^2} = mg = \frac{G(KM)m}{(K'R + y)^2}$$

$$(K'R + y)^2 = \frac{KGM}{g}$$

$$y = \left(\frac{KGM}{g}\right)^{1/2} - K'R$$

72. (b) $m_1 r_1 = m_2 r_2$ or $r_1 = \frac{m_2 r}{m_1 + m_2}$ ...(i)

$$m_1 r_1 \omega^2 = \frac{Gm_1 m_2}{r^2} \Rightarrow \omega^2 = \frac{G(m_1 + m_2)}{r^3}$$

[From (i)]

or $r = \left[\frac{G(m_1 + m_2)}{\omega^2}\right]^{1/3}$

$(m_1 + m_2)^{1/3} = 2m_1 + m_2 = 8$

and $m_2 - m_1 = 6$ (given)

which gives $m_1 = 1$ and $m_2 = 7$ units

$$\frac{m_1}{m_2} = \frac{1}{7}$$

73. (a) $-\frac{Gm_1 m_2}{r} = -\frac{Gm_1 m_2}{(R_1 + R_2)} + \frac{1}{2}\mu v_r^2$

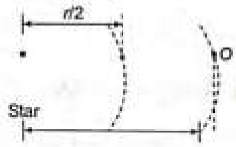
$$-\frac{G}{r} = -\frac{G}{(r/10)} + \frac{1}{2}\frac{1}{(m_1 + m_2)}v_r^2 \quad \left[\mu = \frac{m_1 m_2}{(m_1 + m_2)}\right]$$

But $m_1 + m_2 = 1 + 7 = 8$ units.

$$v_r = 12\sqrt{\frac{G}{r}}$$

74. (b) $T_O = \frac{2\pi r}{\omega} \quad T_I = \frac{2\pi r}{2\omega} = \frac{T_O}{2}$

Consider an imaginary comet moving along an ellipse. The extreme points of this ellipse are located on orbit of inner planet and the star. Semimajor axis of orbit of such comet will be half of the semi-major axis of the inner planet's orbit. According to Kepler's law, if T' is the time period of the comet.



$$\frac{T'^2}{(r/4)^3} = \frac{T_I^2}{(r/2)^3}$$

$$T'^2 = \frac{8}{64}T_I^2 = \frac{T_0^2}{32} \qquad (\because \; T_1 = T_0/2)$$

$$T' = \frac{T}{4\sqrt{2}}$$

$(T'/2)$ represents time in which inner planet will fall into star.

$$\left(\frac{T'}{2}\right) = \frac{T}{8\sqrt{2}} = \frac{T\sqrt{2}}{16}$$

75. (c) $T_{west} = \frac{2\pi R}{v_0 + R\omega}$ and $T_{east} = \frac{2\pi R}{v_0 - R\omega} \Rightarrow \Delta T = T_{east}$

$$\Rightarrow \quad T_{east} - T_{west} = 2\pi R\left[\frac{2\pi R}{v_0^2 - R^2\omega^2}\right] = \frac{4\pi\omega R^2}{v_0^2 - R^2\omega^2}$$

Archives

1. (d) Change in potential energy in displacing a body from r_1 to r_2 is given by

$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = GMm\left(\frac{1}{2R} - \frac{1}{3R}\right) = \frac{GMm}{6R}$$

2. (b) When gravitational force becomes zero, the centripetal force required cannot be provided. So the satellite will move with the velocity as it has at the instant when gravitational force becomes zero, i.e., it moves tangentially to the original orbit.

3. (a) Escape velocity = $\sqrt{2gR_e}$

So the escape velocity is independent of m. So it depends upon mass as m^0.

4. (c) The minimum kinetic energy required to project a body of mass m from the earth's surface to infinity is known as escape energy. Therefore,

$$KE = \frac{GM_e m}{R} = mgR \qquad \left(\because \quad gR = \frac{GM}{R}\right)$$

5. (c) $\frac{T_2}{T} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{4r}{r}\right)^{3/2} = 8$

$\therefore \quad T_2 = 8T_1 = 8 \times 5 = 40$ h

6. (c) If x_1 and x_2 are the distance covered by the two bodies, then

$x_1 + x_2 = 9R$

Also

$Mx_1 = 5Mx_2 \Rightarrow x_2 = \frac{x_1}{5}$

$\therefore \quad x_1 + \frac{x_1}{5} = 9R \Rightarrow x_1 = 7.5\,R$

7. (c) Work is done against gravitational force, which is a conservative force.

Therefore, the escape velocity is independent of the angle of projection.

8. (b) $v_0 = \sqrt{\frac{GM}{R+x}} = \sqrt{\frac{gR^2}{R+x}}$

9. (a) The time period of the satellite is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

where $R + h$ is the radius of the orbit of the satellite, M is the mass of the earth.

10. (b) $\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{1}{2}mgR \qquad (\because h = R)$

11. (a) $m\omega^2 R \propto \frac{1}{R^n} \quad \Rightarrow m\left(\frac{4\pi^2}{T^2}\right)R \propto \frac{1}{R^n}$

$\Rightarrow \quad T^2 \propto R^{n+1} \quad \therefore \quad T \propto R^{\left(\frac{n+1}{2}\right)}$

12. (c) $g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho$

$= \frac{4}{3}\pi R\rho G$

$\Rightarrow \quad \rho \propto g$

13. (c) At height h above the surface of the earth,

$$g' = g\left(1 - \frac{2h}{R}\right) \Rightarrow \Delta g_1 = \frac{2h}{R} \cdot g$$

At depth d below the surface of the earth,

$$g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \Delta g_2 = \frac{d}{R} \cdot g$$

Since $\Delta g_1 = \Delta g_2$,

$d = 2h$

14. (d) Work done = Change in GPE = $U_\infty - U_R$

$$W = 0 - \left(\frac{GMm}{R}\right) = \frac{GMm}{R}$$

$$= \frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{10^{-1}} = 6.67 \times 10^{-10}\text{ J}$$

15. (d) Electronic charge = 1.6×10^{-19} C

It is the fundamental property of an electron. Its value remains the same on the moon as well as on the earth.

$$\therefore \quad \frac{\text{Electronic charge on moon}}{\text{Electronic charge on earth}} = 1$$

16. (c) Mass of planet

$M_P = 10\,M_e$

where M_e is the mass of the earth.

Radius of planet $R_p = \frac{R_e}{10}$

where R_e is the radius of the earth.

$$v_P = \sqrt{\frac{2\,GM_P}{R_P}} = \sqrt{\frac{2G\,10\,M_e}{R_e/10}}$$

$= 10\,v_e = 110$ km/s

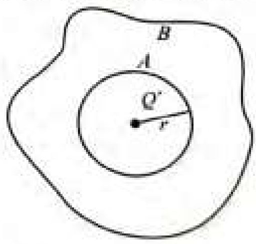
17. (a) $g' = \frac{GM}{(R+h)^2}$, acceleration due to gravity at height h

$$\Rightarrow \quad \frac{g}{9} = \frac{GM}{R^2}\frac{R^2}{(R+h)^2} = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{1}{9} = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{3}$$

$\Rightarrow \quad 3R = R + h \Rightarrow 2R = h$

18. (c) For solving this problem let us use the concept of electric flux. Let A be the Gaussian surface enclosing a spherical charge Q



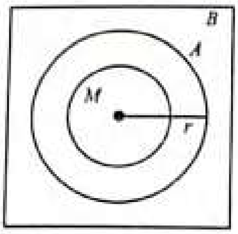
Flux enclosed by Gaussian surface

$$\phi = \oint \vec{E}.\vec{ds} = \frac{Q_{\text{enclose}}}{\varepsilon_0} \Rightarrow E.4\pi r^2 = \frac{Q}{\varepsilon_0}$$

Hence, electric field at a distance a from charge is $E = \frac{Q}{4\pi\varepsilon_0.r^2}$

Flux $\phi = \vec{E}.4\pi r^2 = \frac{Q}{\varepsilon_0}$

Every line passing through A, has to pass through B, whether B is a cube or any surface. It is only for Gaussian surface, the lines of field should be normal. Assuming the mass is a point mass.



Gravitational field $g = -\frac{GM}{r^2}$

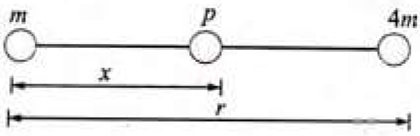
Flux $\phi_g = |\vec{g}.4\pi r^2| = \frac{4\pi r^2.GM}{r^2} = 4\pi GM.$

Here B is a cube. As explained earlier, whatever be the shape, all the lines passing through A are passing through B, although all the lines are not normal.

Statement 2 is correct because when the shape of the earth is spherical, area of the Gaussian surface is $4\pi r^2$.

This ensures inverse square law.

19. (d) Let x be the distance of the point P from the mass m where gravitational field is zero.



$$\therefore \quad \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2} \text{ or } \left(\frac{x}{(r-x)}\right)^2 = \frac{1}{4} \text{ or } x = \frac{r}{3} \qquad \text{(i)}$$

Gravitational potential at a point P is

$$V = -\frac{Gm}{x} - \frac{G(4m)}{(r-x)}$$

$$V = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r-\frac{r}{3}\right)} = -9\frac{Gm}{r} \qquad \text{(Using (i))}$$

20. (d) $W = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R} = gR^2 \times \frac{m}{R} = mgR$

$= 1000 \times 10 \times 6400 \times 10^3$

$= 64 \times 10^9 \text{J}$

$= 6.4 \times 10^{10} \text{J}$

21. (d) Energy of the satellite on the surface of the planet is

$$E_i = \text{KE} + \text{PE} = 0 + \left(-\frac{GMn}{R}\right) = -\frac{GMm}{R}$$

$$= \frac{1}{2}\frac{GMm}{3R} - \frac{GMm}{3R} = -\frac{GMm}{6R}$$

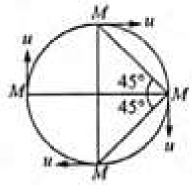
$\therefore$ Minimum energy required to launch the satellite is

$$\Delta E = E_f - E_i = -\frac{GMm}{6R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{6R} + \frac{GMm}{R} = \frac{5GMm}{6R}$$

22. (b) Net force on any one particle

$$= \frac{GM^2}{(2R)^2} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45° + \frac{GM^2}{(R\sqrt{2})^2}\cos 45°$$



$$= \frac{GM^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right]$$

This force will be equal to centripetal force so

$$\frac{Mu^2}{R} = \frac{GM^2}{R^2}\left[\frac{1+2\sqrt{2}}{4}\right]$$

$$u = \sqrt{\frac{GM}{R}\left[1+2\sqrt{2}\right]} = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$$

23. (b) $V = V_1 - V_2$

$$V_1 = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$V_2 = -\frac{3G\left(\frac{M}{8}\right)}{2\left(\frac{R}{2}\right)} \Rightarrow V = \frac{-GM}{R}$$

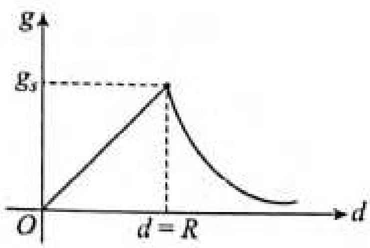
24. (d) Orbital velocity of a satellite revolving in a circular orbit at a height ($h << R$), $v_0 = \sqrt{Rg}$ and the velocity required to escape is $v_e = \sqrt{2Rg}$

So increase in velocity $\Delta v = v_e - v_0 = \left(\sqrt{2}-1\right)\sqrt{Rg}$

25.(b) $g = \frac{GMx}{R^3}$ inside the Earth (straight line)

$g = \frac{GM}{r^2}$ outside the Earth

where M is mass of Earth.



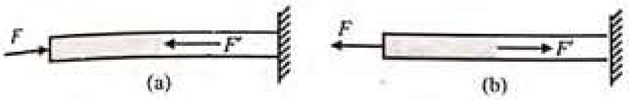
CHAPTER 11: ELASTICITY

Concept Application Exercise 11.1

1. Since the shaded segment is in translational equilibrium at rest;

$\sum F = |F - F'| = 0$ in both the figures.

This gives $F' = F$.



Since the rod is in rotational equilibrium,

$$\sum \tau = F'x - Fl = 0$$

This gives, $F' = \dfrac{Fl}{x}$

Compressive forces compress the rod; tensile forces elongate the rod and shearing forces twist (bend) the rod. The tensile and compressive forces are equal to the applied force when the rod is in equilibrium horizontally, whereas the shearing force need not be equal to the applied force.

2. The volumetric stain

$$\frac{\delta V}{V} = -\frac{P}{B}$$

where $P = \rho g h$

then $$-\frac{\delta V}{V} = \frac{\rho g h}{B} \qquad \text{(i)}$$

Since, the volume of the sphere is $V = 4/3\pi r^3$, we have

$$\frac{\delta V}{V} = \frac{3\delta r}{r} \qquad \text{(ii)}$$

Using Eqs. (i) and (ii), we have

$$\frac{\delta r}{r} = \frac{\rho g h}{3B}$$

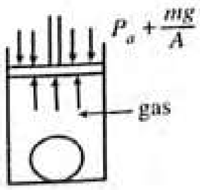
3. Longitudinal or normal stress

$$\sigma_l = \frac{100 \sin 30^\circ}{5 \times 2} = 5 \text{ N/m}^2$$

Tangential stress

$$\sigma_t = \frac{100 \cos 30^\circ}{5 \times 2} = 5\sqrt{3} \text{ N/m}^2$$

4. $$p_{gas} = \frac{mg}{A} + p_a = \frac{50 \times 10}{50 \times 10^{-4}} + 1 \times 10^5$$



$$= 2 \times 10^5 \text{ N/m}^2$$

Bulk stress $= p_{gas} = 2 \times 10^5$ N/m^2

5. $$\Delta L_B = \Delta L_{AB} = \frac{FL}{AY} = \frac{MgL}{AY}$$

$$= \frac{10 \times 10 \times 0.1}{10^{-7} \times 2.5 \times 10^{10}} = 4 \times 10^{-3} \text{m} = 4 \text{mm}$$

$$\Delta L_C = \Delta L_B + \Delta L_{BC}$$

$$= 4 \times 10^{-3} + \frac{100 \times 0.2}{10^{-7} \times 4 \times 10^{10}}$$

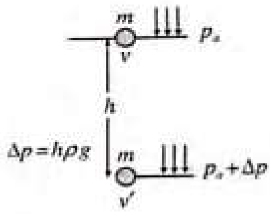
$$= 4 \times 10^{-3} + 5 \times 10^{-3} = 9 \text{ mm}$$

$$\Delta L_D = \Delta L_C + \Delta L_{CD}$$

$$= 9 \times 10^{-3} + \frac{100 \times 0.15}{10^{-7} \times 1 \times 10^{10}}$$

$$= 9 \times 10^{-3} + 15 \times 10^{-3} = 24 \text{ mm}$$

6. $$B = \frac{\Delta p}{-\frac{\Delta V}{V}} \Rightarrow \frac{\Delta V}{V} = -\frac{\Delta p}{B}$$



We know $p = p_{atm} + h\rho g$

or $m = \rho V = \text{const.}$

$d\rho v + dv\rho = 0$

$d\rho V + dV\rho = 0$

$$\Rightarrow \quad \frac{d\rho}{\rho} = -\frac{dV}{V}$$

i.e., $$\frac{\Delta\rho}{\rho} = \frac{\Delta p}{B}$$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{1}{100}$$

$$\frac{1}{100} = \frac{h\rho g}{B} \qquad \text{[assuming } \rho = \text{const.]}$$

$$h\rho g = \frac{B}{100} = \frac{1}{100\,k}$$

$$h\rho g = \frac{1 \times 1 \times 10^5}{100 \times 50 \times 10^{-6}}$$

$$h = \frac{10^5}{5000 \times 10^{-6} \times 1000 \times 10} = \frac{100 \times 10^3}{50} = 2 \text{km}$$

Exercises

Stress, Strain and Elasticity

1. **(d)** All of the statements are factual statements. Stress is defined as internal force developed in the system per unit cross-sectional area and is not defined as force applied per unit cross-section area, although in equilibrium, stress is numerically equal to applied force per unit area.

2. **(a)** Breaking strength = Breaking stress × $\dfrac{\pi D^2}{4}$

Breaking stress is unchanged.
D is halved. So, breaking strength becomes one-fourth, i.e.,
$\frac{1}{4}\times 1.5\times 10^5$ N or 0.375×10^5 N.

3. (c) Breaking stress $= \frac{\text{Maximum weight}}{\text{Area of cross section}}$

$$10^6 = \frac{al\rho g}{a} = l\rho g$$

or $l = \frac{10^6}{\rho g} = \frac{10^6}{4\times 10^3\times 10}$ m $= 25$ m

4. (a) $Y = \frac{F/a}{\Delta l/l} = \frac{Fl}{a\,\Delta l}$

or $Y = \frac{Fl\times 4}{\pi D^2\times \Delta l}$ or $\Delta l \propto \frac{1}{D^2}$ or $\frac{\Delta l_2}{\Delta l_1} = \frac{D_1^2}{D_2^2} = \frac{n^2}{1}$

5. (d) $Y = \frac{F/a}{\Delta l/l} = \frac{Fl}{a\,\Delta l}$ or $\Delta l \propto \frac{1}{D^2}$

(a) $\frac{100}{1^2} = 100$ (b) $\frac{200}{4} = 50$

(c) $\frac{300}{9} = 33.33$ (d) $\frac{50}{(1/2)^2} = 200$

6. (c) $Y = \frac{Fl}{a\Delta l}$ or $\Delta l \propto \frac{1}{a}$

Again, $m = al\rho$ or $m \propto a$

$\therefore$ $\Delta l \propto \frac{1}{m}$

$$\frac{\Delta l_1}{\Delta l_2} = \frac{m_2}{m_1} = \frac{2}{3}$$

7. (d) $Y = \frac{Fl}{a\,\Delta l}$

In the given problem, Y, l and Δl are constants.

$\therefore$ $F \propto a$

or $F \propto \pi r^2$ or $F \propto r^2$ or $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2} = \frac{1}{4}$

8. (b) $Y = \frac{Mg\times 4\times l}{\pi D^2\times \Delta l}$ or $\Delta l \propto \frac{1}{D^2}$

When D is doubled, Δl becomes one-fourth, i.e., $\frac{1}{4}\times 2.4$ cm, i.e., 0.6 cm. $\times$ 2.4 cm, i.e., 0.6 cm.

9. (b) $Y = \frac{F/a}{\Delta l/l}$ or $Y = \frac{Fl}{a\,\Delta l}$

or $\Delta l = \frac{Fl}{aY} = \frac{Fl}{\pi r^2 Y}$

In the given problem,

$$\Delta l \propto \frac{l}{r^2}$$

When both l and r are doubled, Δl is halved.

$\Delta l_{new} = 0.5$ mm

10. (d) 10 m column of water exerts nearly 1 atmosphere pressure. So, 100 m column of water exerts nearly 10 atmosphere pressure, i.e., 10×10^{-5} Pa or 10^6 Pa.

Now, $K = \frac{(\Delta P)V}{\Delta V} = \frac{10^6\times 100}{0.1}$ Pa $= 10^9$ Pa

11. (c) $\Delta l = \frac{Fl}{aY}$

$$\Delta l = \frac{2\times 10^5\times 10^{-5}\times 1}{2\times 10^{-4}\times 10^4} \text{ or } \Delta l = 1$$

So, new length is $2l$.

12. (c) $Y = \frac{Fl}{a\,\Delta l}$ or $\Delta l \propto \frac{1}{a}; \Delta l \propto \frac{1}{D^2}$

$$\frac{\Delta l_2}{\Delta l_1} = \frac{D_1^2}{D_2^2} = 4 \text{ or } \Delta l_2 = 4\Delta l_1 = 4 \text{ cm}$$

13. (c) $Y = \frac{Fl}{\pi r^2 \Delta l}$ or $\Delta l = \frac{Fl}{\pi r^2 Y}$

$$\Delta l \propto \frac{1}{r^2},\quad \Delta l' \propto \frac{2l}{(\sqrt{2}r)^2} \text{ or } \Delta l' \propto \frac{l}{r^2}$$

$\therefore$ $\frac{\Delta l}{\Delta l'} = 1:1$

14. (c) $Y \propto$ weight applied

$\therefore$ $\frac{Y_1}{Y_2} = \frac{W_1}{W_2} \Rightarrow \frac{W_1}{W_2} = \frac{3}{1}$

15. (c) Force = Weight suspended + Weight of $3L/4$ of wire

$$= W_1 + \frac{3W}{4}$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \left(W_1 + \frac{3W}{4}\right)/s$$

16. (d) $Y = \frac{Fl}{a\,\Delta l}$

Y, l and F are constants.

$\therefore$ $\Delta l \propto \frac{1}{D^2}$

$$\frac{\Delta l_2}{\Delta l_1} = \frac{D_1^2}{D_2^2} = \frac{1}{16}$$

$\therefore$ $\Delta l_2 = \frac{1}{16}$ mm

17. (b) $\frac{1}{K} = \frac{\Delta V/V}{\Delta P}$ or $\frac{\Delta V}{V} = \Delta P\left(\frac{1}{K}\right)$

or $\frac{\Delta V}{V}\times 100 = 10^5\times 8\times 10^{-12}\times 100 = 8\times 10^{-5}$

18. (b) $T = \frac{2m_1m_2}{m_1+m_2}g = \frac{2\times 1\times 2}{1+2}\times 10 \text{ N} = \frac{40}{3}$ N

If r is the minimum radius, then

$$\text{Breaking stress} = \frac{\frac{40}{3}}{\pi r^2} \text{ or } \frac{40}{3\pi}\times 10^6 = \frac{40}{3\pi r^2}$$

or $r^2 = \frac{1}{10^6}$ or $r = \frac{1}{10^3}$ m $= 1$ mm

19. (b) In ductile materials, yield point exists while in brittle material, failure would occur without yielding.

20. (b) $1.15\times10^{8}=\dfrac{900(10+a)}{\left(\dfrac{\pi d^{2}}{4}\right)}$

$$\Rightarrow\quad d=\frac{6}{\sqrt{10\,\pi}}\text{cm}=\frac{0.06}{\sqrt{10\,\pi}}\text{m}=\frac{6\times10^{-2}}{\sqrt{10\,\pi}}\text{m}$$

Energy Stored in a Deformed Body

21. (a) Energy density $=\dfrac{1}{2}\times\text{Stress}\times\text{Strain}$

$$=\frac{1}{2}\times\text{Stress}\times\frac{\text{Stress}}{Y}=\frac{(\text{Stress})^2}{2Y}\propto\frac{1}{D^4}$$

Now, $\dfrac{u_A}{u_B}=\dfrac{D_B{}^4}{D_A{}^4}=[2]^4=16$

22. (b) Energy per unit volume $=\dfrac{1}{2}\times\text{Stress}\times\text{Strain}$

$$=\frac{1}{2}\times\text{Stress}\times\frac{\text{Stress}}{Y}\quad\Big|\quad Y=\frac{\text{Stress}}{\text{Strain}}=\frac{S^2}{2Y}$$

23. (c) Work done $=\dfrac{1}{2}F\times\text{Extension}$

$$=\frac{1}{2}\times\frac{YA}{L}\times1\qquad\Big|\qquad Y=\frac{F\times L}{A\times1}$$

$$=\frac{YA}{2L}\qquad\Big|\qquad F=\frac{YA}{L}$$

24. (b) Work done $=\dfrac{1}{2}\times$ stretching force $\times$ extension.

Therefore, net work done in increasing the length from 0.164 mm to 1.02 mm is

$$W_2-W_1=\frac{1}{2}\times5\times9.8\times1.02\times10^{-3}-\frac{1}{2}$$

$$\times3\times9.8\times0.61\times10^{-3}=0.016\text{ J.}$$

25. (a) $W=\dfrac{1}{2}\tau\phi=\dfrac{1}{2}\times8\times\dfrac{\pi}{4}=\pi\text{ J}$

26. (b) The elastic potential is

Energy $=\dfrac{1}{2}$ Stress $\times$ Strain

$$=\frac{1}{2}Y(\text{strain})^2=\frac{1}{2}Y\left(\frac{\Delta l}{L}\right)^2$$

$$\therefore\quad\frac{U_2}{U_1}\propto\left(\frac{\Delta l_2}{\Delta l_1}\right)^2=\left(\frac{10}{2}\right)^2$$

$$\frac{U_2}{U_1}=25\Rightarrow U_2=25\,U_1=25\,U\qquad[U_1=U]$$

27. (d) $W=\dfrac{1}{2}F\,\Delta l$

$$W=\frac{1}{2}\times\frac{Y\pi r^2\Delta l}{l}\Delta l\quad\text{and}\quad Y=\frac{Fl}{\pi r^2\Delta l}$$

$$\text{or}\quad W=\frac{Y\pi r^2\Delta l^2}{2l}\quad\text{and}\quad F=\frac{Y\pi r^2\Delta l}{l}$$

$$\text{or}\quad W\propto\frac{r^2}{l},\ W'\propto\frac{(2r)^2 2}{l}$$

$$\frac{W'}{W}=8\quad\text{or}\quad W'=8\times2\text{ J}=16\text{ J}$$

28. (d) $U=\dfrac{1}{2}\left(\dfrac{YA}{L}\right)l^2\qquad\therefore\quad U\propto l^2$

$$\frac{U_2}{U_1}=\left(\frac{l_2}{l_1}\right)^2=\left(\frac{10}{2}\right)^2=25\ \Rightarrow\ U_2=25U_1$$

i.e., potential energy of the spring will be 25 V.

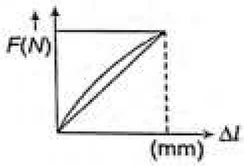
29. (a) $W=\dfrac{1}{2}Fl\ \therefore\ W\propto l$ $\qquad$ (F is constant)

$$\therefore\quad\frac{W_1}{W_2}=\frac{l_1}{l_2}=\frac{l}{2l}=\frac{1}{2}$$

30. (b) $K=\dfrac{F}{x}=\dfrac{40}{2\times10^{-2}}=0.2$ N/m

Work done $=\dfrac{1}{2}Kx^2=\dfrac{1}{2}\times(0.2)\times(0.05)^2=2.5$ J

31. (c) Area under the curve represents strain energy stored and it is greater than 40 mJ.



Area under curve (straight line)

$$=\frac{1}{2}.F.\Delta l$$

$$=\frac{1}{2}.(20).(4\times10^{-3})J=40\text{ mJ}$$

32. (a) Energy $=\dfrac{1}{2}Fl=\dfrac{1}{2}\times F\times\left(\dfrac{FL}{AY}\right)=\dfrac{1}{2}\times\dfrac{F^2L}{AY}$

$$=\frac{1}{2}\times\frac{(50)^2\times20\times10^{-2}}{2\times10^{-4}\times1.4\times10^{11}}=8.57\times10^{-6}\text{ J}$$

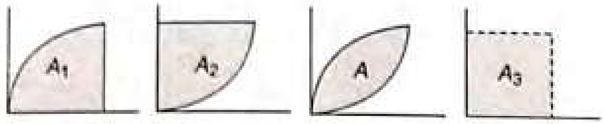
33. (c) $U=\dfrac{1}{2}\times\dfrac{YAl^2}{L}=\dfrac{1}{2}\times\dfrac{2\times10^{11}\times3\times10^{-6}\times(1\times10^{-3})^2}{4}$

$$=0.075\text{ J}$$

34. (b) $U=\dfrac{1}{2}\times Y\times(\text{Strain})^2=\dfrac{1}{2}\times9\times10^{11}\times\left(\dfrac{1}{100}\right)^2$

$$=4.5\times10^7\text{ J}$$

35. (d) Hysteresis loss corresponding to elasticity per unit volume of a substance is given by the area of hysteresis loop, i.e., stress-strain curve corresponding to one complete loading and deloading.



Area of an ellipse = π × semi-major axis × semi-minor axis

$$A_1 = \frac{1}{4}(\pi \times 8 \times 4 \times 10^2) \text{ and } A_2 = \frac{1}{4}(\pi \times 8 \times 4 \times 10^2)$$

Also, $A_3 = 8 \times 4 \times 10^2$

Area of hysteresis loop is $A = A_1 + A_2 - A_3$

$$A = 2\left[\frac{\pi}{4} \times 8 \times 4 \times 10^2\right] - [8 \times 4 \times 10^2]$$

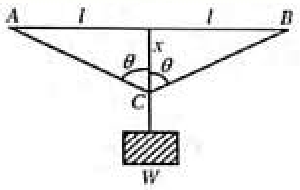
$$= \left[\frac{\pi}{2} - 1\right] \times 32 \times 10^2 \text{ J}$$

= work done per cycle

= energy lost per cycle per unit volume

Problems Based on Mixed Concepts

36. (c) Here change in length is



$$\Delta l = (AC + BC) - 2l$$

$$= 2(l^2 + x^2)^{1/2} - 2l$$

$$= 2l\left(1 + \frac{x^2}{l^2}\right)^{1/2}$$

$$= 2l\left(1 + \frac{1}{2}\frac{x^2}{l^2}\right) - 2l = \frac{x^2}{l}$$

$$\therefore \quad \text{Strain} = \frac{\Delta l}{2l} = \frac{x^2}{2l^2}$$

37. (d) We know $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{Wl}{2Ax} \times \dfrac{2l^2}{x^2}$

$$Y = \frac{Wl^3}{Ax^3} \text{ or } x = \left(\frac{W}{AY}\right)^{1/3} \times l$$

38. (c) Thermal stress = $Y\alpha t$

In the given problem,

$Y\alpha$ = constant

$$\frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

39. (c) $Y = \dfrac{Fl}{a\,\Delta l}$

Y, l and a are constants

$$\therefore \quad \frac{Fl}{\Delta l} = \text{constant or } \Delta l \propto F$$

Now, $l_1 - l = T_1$ and $l_2 - l = T_2$

Dividing, $\dfrac{l_1 - l}{l_2 - l} = \dfrac{T_1}{T_2}$

or $l_1T_2 - lT_2 = l_2T_1 - lT_1$ or $l(T_1 - T_2) = l_2T_1 - l_1T_2$

or $l = \dfrac{l_2T_1 - l_1T_2}{T_1 - T_2}$ or $l = \dfrac{l_1T_2 - l_2T_1}{T_2 - T_1}$

40. (a) $mr\omega^2$ = Breaking stress × Cross-sectional area

$$10 \times 0.3\omega^2 = 4.8 \times 10^7 \times 10^{-6} = 48$$

or $\omega^2 = \dfrac{48}{3} = 16$ or $\omega = 4$ rad/s

41. (b) $\Delta P = h\rho g = 200 \times 10^3 \times 10 \text{ N/m}^2 = 2 \times 10^6 \text{ N/m}^2$

$$K = \frac{\Delta P}{\dfrac{\Delta V}{V}} = \frac{2 \times 10^6}{\dfrac{0.1}{100}} = \frac{2 \times 10^8}{0.1} \text{N/m}^2 = 2 \times 10^9 \text{ N/m}^2$$

42. (c) $Y = \dfrac{Fl}{a\,\Delta l}$; Y, l and a are constants

$\therefore \quad F = \Delta l$

In the first case, $V\rho g = 16$

In the second case, $(V\rho g - V \times 1 \times g) = 14$

Dividing, $\dfrac{Vg(\rho - 1)}{Vg\rho} = \dfrac{14}{16} = \dfrac{7}{8}$

or $\dfrac{\rho - 1}{\rho} = \dfrac{7}{8}$ or $8\rho - 7\rho$ or $\rho = 8$ g/cm³

43. (a) We have, $Y = \dfrac{Fl}{a^2\Delta l}$

$$\Rightarrow \quad Y = \frac{F}{a} \times \frac{l}{\Delta l}$$

Now, $V = al$ or $l = \dfrac{V}{a}$ $\therefore Y = \dfrac{FV}{a^2\Delta l}$

$$\Delta l \propto \frac{1}{a^2} \text{ or } \Delta l \propto \frac{1}{D^4}$$

$$\frac{\Delta l_A}{\Delta l_B} = \frac{{D_B}^4}{{D_A}^4} = \frac{1^4}{\left(\frac{1}{2}\right)^4} = 16$$

44. (a) $Y = \dfrac{\text{stress}}{\Delta L / L}$ or $\dfrac{\Delta L}{L} = \dfrac{\text{stress}}{Y} = \dfrac{5 \times 10^7}{2 \times 10^{11}} = 2.5 \times 10^{-4}$

Now, $V = \pi r^2 L$

$$\frac{\Delta V}{V} = \frac{\pi \Delta(r^2 L)}{\pi r^2 L}$$

or $\dfrac{\Delta V}{V} = \dfrac{r^2\Delta L + L \times 2r\Delta r}{r^2 L}$ or $\dfrac{\Delta V}{V} = \dfrac{\Delta L}{L} + 2\dfrac{\Delta r}{r}$

or $2\dfrac{\Delta r}{r} = \dfrac{\Delta V}{V} - \dfrac{\Delta L}{L}$ or $2\dfrac{\Delta r}{r} = \dfrac{0.02}{100} - 2.5 \times 10^{-4}$

or $\dfrac{\Delta r}{r} = 1 \times 10^{-4} - \dfrac{2.5}{2} \times 100^{-4} = -0.25 \times 10^{-4}$

45. (c) $\dfrac{\rho'}{\rho} = \dfrac{\dfrac{M}{V-\Delta V}}{M/V} = \dfrac{V}{V-\Delta V}$

or $\dfrac{\rho'}{\rho} = V(V-\Delta V)^{-1}$ or $\dfrac{\rho'}{\rho} = 1 + \dfrac{\Delta V}{V}$

(using binomial theorem)

or $\dfrac{\Delta V}{V} = \dfrac{\rho'}{\rho} - 1$ or $\dfrac{\Delta V}{V} = \dfrac{\rho' - \rho}{\rho}$

Again, $k = k = \dfrac{\Delta P}{\dfrac{\Delta V}{V}} = \dfrac{\rho \Delta P}{\rho' - \rho}$ or $\rho' - \rho = \dfrac{\rho \Delta R}{K}$

But, $\Delta P = P$ (given)

$\therefore \quad \rho' - \rho = \dfrac{\rho P}{K}$

46. (b) If the rod is allowed to expand, then it will expand by $\Delta l = l\,\alpha\,\Delta T$ due to increase in temperature by ΔT. But as the length of the cylinder is kept constant by applying pressure, a stress is developed in the cylinder. The decrease in length of cylinder due to elasticity is $\Delta l = l\,\alpha\,\Delta T$ and a compressive stress will develop in it.

Stress = Excess pressure applied at ends

$= Y \times \dfrac{\Delta l}{l} Y \alpha \Delta T$

$= 2 \times 10^{11} \times 11 \times 10^{-6} \times 100 = 2.2 \times 10^{3}$ atm

47. (b) Consider a small element of the tube.

$2T \sin\theta = \Delta p \times A$

where $\Delta p = p_i - p_0$ and A is the area of element. As θ is very small, $\sin\theta = \theta$ so, $2T \times \theta = \Delta p \times l \times (2r\theta)$

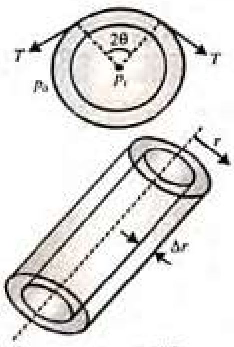


Fig. 2.157

$\Rightarrow \quad \Delta p = \dfrac{T}{lr} \quad \sigma$ (stress developed in tube) $= \dfrac{\Delta T}{\Delta r \times l}$

where $\Delta r \times i$ is the cross-sectional area.

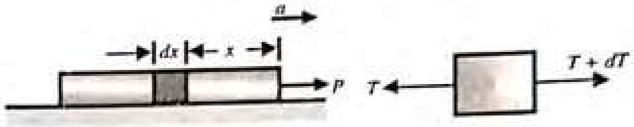
$\sigma = \dfrac{\Delta p \times lr}{\Delta r \times l} = \Delta p \times \dfrac{r}{\Delta r}$

For no rupturing, $\sigma \le \sigma_{max}$

So, $\Delta p \times \dfrac{r}{\Delta r} \le \sigma_{max}$

$\Delta p(\text{max, value}) = \sigma_{max} \times \dfrac{\Delta r}{r}$

48. (c) Let the force applied be p, then $P = ma$



$\Rightarrow \quad p = 5 \times 2 = 10$ N

For the element shown in the figure, $T = \dfrac{m}{l}(l - x) \times a$

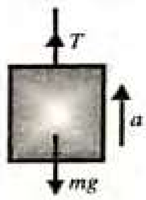
$\Rightarrow \quad \Rightarrow T = \dfrac{5}{1}(1 - x) \times 2 = 10(1 - x)$

Elongation in dx is $\dfrac{\Delta(dx)}{dx} = \dfrac{T/A}{A}$

$\Delta(dx) = \dfrac{10(1-x)}{(5 \times 10^{-2})^2} \times \dfrac{1}{5 \times 10^{9}} dx$

Total elongation, $\Delta l = \displaystyle\int_0^1 \dfrac{10(1-x)}{25 \times 10^{-4}} \times \dfrac{1}{5 \times 10^{9}} dx = 0.4 \times 10^{-6}$ m

49. (c) From the free-body diagram of the elevator,



$T - mg = ma$

$T = m(g + a)$

Stress in cable is $\sigma = \dfrac{T}{A} = \dfrac{m(g+a)}{A}$

From the given condition, $\sigma \le \dfrac{\sigma_{max}}{2}$

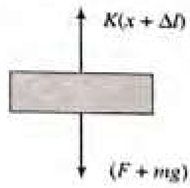
$\dfrac{m(g+a)}{A} \le \dfrac{\sigma_{max}}{2}$

$g + a \le \dfrac{2 \times 10^{9}}{2} \times \dfrac{10^{-4}}{2 \times 10^{3}} = 50$

$a \le 50 - g \le 40$ m/s^2

50. (b) The maximum load which a wire can sustain is a property of the material of wire and it depends on the cross-sectional area. $\sigma_{breaking}$ is a characteristic property of material of wire and is independent of length or breadth (dimensions) of the wire.

51. (a) Work done by applied force and gravity will be



$W = \displaystyle\int_0^{\Delta x} K(x + \Delta l)\,dx = K\left[\dfrac{x^2}{2} + x\,\Delta l\right]_0^{\Delta x}$

$= \dfrac{YA}{L}\left[\dfrac{(\Delta x)^2}{2} + (\Delta x)(\Delta l)\right]$

It will be work done by the force the wire exerts on the mass.

$= |-W| = W = \dfrac{YA}{L}\left[\dfrac{(\Delta x)^2}{2} + \Delta x\,\Delta l\right]$

But $\Delta l = \dfrac{mgl}{AY}$

So, $W = \dfrac{YA}{L}\Delta x\left[\dfrac{\Delta x}{2} + \dfrac{mgL}{AY}\right]$

$= \dfrac{YA}{L}\dfrac{(\Delta x)^2}{2} + mg\Delta x$

52. (b) Work done by the gravity $= mg\Delta x$.

53. (c) $\Delta l = \dfrac{FL}{AY}$

$$\frac{(\Delta l)_A}{(\Delta l)_B} = \left(\frac{F_A}{F_B}\right)\left(\frac{L_A}{L_B}\right)\left(\frac{A_B}{A_A}\right)\left(\frac{Y_B}{Y_A}\right)$$

$$= \left(\frac{KM}{3M}\right)(r)\left(\frac{1}{2r}\right)^2\left(\frac{1}{3r}\right) = \frac{K}{36r^2} = \frac{1}{6r^2} \quad \text{(given)}$$

$K = 6$

Mass of block $P = 6M$

ARCHIVES

1. (a) Energy stored $= \dfrac{1}{2}F\,\Delta l = \dfrac{1}{2} \times 200 \times 1 \times 10^{-3}$ J

$= 100 \times 10^{-3}$ J $= 0.1$ J

2. (c) $W = \dfrac{1}{2} \times 5 \times 10^3 \left[\left(\dfrac{10}{100}\right)^2 - \left(\dfrac{5}{100}\right)^2\right]$

$= \dfrac{5 \times 10^3}{5 \times 10^4}[100 - 25] = \dfrac{5 \times 75}{20}$ J

$= 18.75$ J $= 18.75$ N-m

3. (d) Work done $= \dfrac{1}{2}$ stress × strain × volume

$= \dfrac{1}{2}\dfrac{F}{A}\dfrac{l}{L}(AL) = \dfrac{1}{2}Fl$

4. (c) Energy stored/volume $= \dfrac{1}{2}$ × stress × strain

$= \dfrac{1}{2}$ × stress × $\dfrac{\text{stress}}{Y}$ $\qquad \left(Y = \dfrac{\text{stress}}{\text{strain}}\right)$

$= \dfrac{(\text{stress})^2}{2Y} = \dfrac{S^2}{2Y}$

5. (c) In both cases, the tension in the wire remains same. So the elongation will be same.

6. (d) $A_1 l_1 = A_2 l_2$

$\Rightarrow \quad l_2 = \dfrac{A_1 l_1}{A_2} = \dfrac{A \times l_1}{3A}$

$\Rightarrow \quad \dfrac{l_1}{l_2} = 3$

$\Delta x_1 = \dfrac{F_1}{A\gamma} \times l_1$

$\Rightarrow \quad x_2 = \dfrac{F_2}{3A\gamma} \times l_1$

Here $\Delta x_1 = \Delta x_2$

$\dfrac{F_2}{3A\gamma} l_2 = \dfrac{F_1}{A\gamma} l_1 \quad \Rightarrow \quad F_2 = 3F_1 \times \dfrac{l_1}{l_2} = 3F_1 \times 3 = 9F$

7. (c) $0.10 \times 1.1 \times 10^{-5} = \dfrac{\frac{F}{A}}{2 \times 10^{11}} \times 0.10$

$\therefore \dfrac{F}{A}$ = Pressure $= 1.1 \times 10^{-5} \times 100 \times 2 \times 10^{11}$

$= 2.2 \times 10^8$ Pa

8. (a) $T = 2\pi\sqrt{\dfrac{l}{g}}$ ...(a)

$T_M = 2\pi\sqrt{\dfrac{l + \Delta l}{g}}$ ...(b)

$Y = \dfrac{Fl}{A\Delta l} \Rightarrow \Delta l = \dfrac{Mgl}{AY}$ ...(c)

$\Rightarrow \quad \dfrac{1}{Y} = \dfrac{A}{Mg}\left[\left(\dfrac{T_M}{T}\right)^2 - 1\right]$

9. (c) $\dfrac{V_f}{V_i} = 9^3$

$\because$ Density remains same

So, mass ∝ volume

$\dfrac{m_f}{m_i} = 9^3$ and $\dfrac{(\text{Area})_f}{(\text{Area})_i} = 9^2$

Stress $= \dfrac{(\text{Mass}) \times g}{\text{Area}}$

$\dfrac{\sigma_2}{\sigma_1} = \left(\dfrac{m_f}{m_i}\right)\left(\dfrac{A_i}{A_f}\right) = \dfrac{9^3}{9^2} = 9$

10. (d) Bulk Modulus $= \dfrac{\text{volumetric stress}}{\text{volumetric strain}}$

$K = \dfrac{mg/a}{(dV/V)} \Rightarrow \dfrac{dV}{V} = \dfrac{mg}{Ka}$ ...(i)

volume of sphere $V = \dfrac{4}{3}\pi r^3$

Fractional change in volume $\dfrac{dV}{V} = \dfrac{3dr}{r}$ ...(ii)

Using eq. (i) and (ii) $\dfrac{3dr}{r} = \dfrac{mg}{Ka}$ hence $\dfrac{dr}{r} = \dfrac{mg}{3Ka}$.

CHAPTER 12: FLUID MECHANICS, SURFACE TENSION AND VISCOSITY

Concept Application Exercise 12.1

1. Pressure at any depth h from the free surface of the water is given by $P = \rho g h + P_a$

$$= (1000)\times 10\times 10 + 1.0\times 10^5 = 2.00\ \text{N/m}^2$$

2. The barometric height, $h = 75$ cm

If l is the length of the mercury column in the tube, then $h/l = \sin 60°$

or $l = \dfrac{h}{\sin 60°} = \dfrac{75}{(\sqrt{3}/2)} = 86.6$ cm

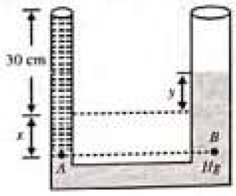
3. As the mercury column in the two arms of the U-tube is the same, so

$$P_A = P_B \Rightarrow \rho_w g h_w = \rho_s g h_s$$

$$\frac{\rho_s}{\rho_w} = \frac{h_w}{h_s} = \frac{10}{12.5} = 0.8$$

Thus, specific gravity of the spirit is 0.8.

4. Suppose area of cross section of narrow limb be a, then area of cross section of right limb will be $3(a)$



Let the level of mercury in left limb fall be x and the rise of level in right limb be y, then

$ax = (3a)y$

$x = 3y$

According to Pascal's law, $P_A = P_B$

$$(30\times x)\rho_w g = (x+y)\rho_{Hg} g$$

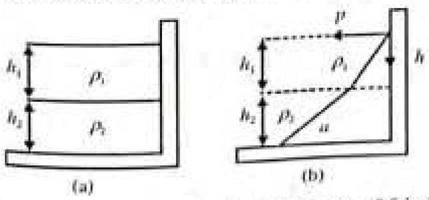
$$(30\times 3y)\times 1\times g = (3y+y)\times 13.6\times g$$

$y = 0.58$ cm

5. The inclination of the pressure diagram with the vertical wall is proportional with the density of the liquid. Since $\rho_2 > \rho_1$, therefore inclination of the lower diagram is always more than that of the upper one.

The diagram (a) is correct.

The diagrams (b) and (c) are not possible.



6. As plug secures the pipe opening the force of friction between plug and pipe wall,

$F = A(p_2 - p_1)$

But $p_1 = p_0$ and $p_2 = p_0 + h\rho g$

$F = Ah\rho g$

$F = \pi(2\times 10^{-2})^2\times 6\times 10^3\times 9.8 \Rightarrow F = 74$ N

Concept Application Exercise 12.2

1. At point A, $P_A = 0$

At point B, $P_B = P_A + \rho g h = \rho g h$

At point C, $P_C = P_B + \rho a L = \rho g h + \rho g L$

At point D, $P_D = P_C - \rho g h = \rho a L$

Or alternatively, $P_D = P_A + \rho a L = \rho a L$

2. Applying Pascal's law starting from point B, we get

$$P_B + \rho g H - \rho a L = P_A$$

Since $P_B = P_A = P_{atm}$, therefore $a = g\left(\dfrac{H}{L}\right)$

3. Let a be the acceleration of the elevator, then pressure inside the elevator is

$$P = \rho(g+a)h = \rho\times(g+a)\,0.76\ \text{N/m}^2$$

Atmospheric pressure,

$$P_a = \rho g\times 0.76\ \text{N/m}^2$$

Clearly, the air pressure inside the elevator will be greater than P_a, i.e., 76 cm of Hg.

4. Let a be the acceleration of the elevator and h be the barometer height, then

$$P_a = \rho(g+a)h \qquad \text{(i)}$$

For the static elevator with barometric height h_0

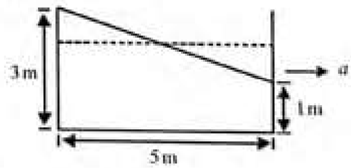
$$P_a = \rho g h_0 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$h = \frac{g h_0}{(g+a)}$$

Clearly, $h < h_0$ so barometric reading in an accelerating elevator will be less than 76 cm.

5. The level of liquid rises by 1 m on the left side and falls by 1 m on the right side.



$$\frac{a}{g} = \frac{3-1}{5} = \frac{2}{5}$$

or $a = 0.4g = 4\ \text{m/s}^2$

Concept Application Exercise 12.3

1. (a) If Mg of ice containing an air bubble is floating in water,

$Mg = V_D \sigma_w g$

i.e., $V_D = (M/\sigma_w)$

[as air bubble encloses air which is weightless]

Now when Mg ice melts, water formed will be Mg and it will occupy volume $V_F = (M/\sigma_W)$. As the volume of water formed by melting of ice is equal to the volume of water displaced by the floating ice, the level of water in the beaker will remain unchanged.

(b) Let the mass of ice be Mg and that of embedded piece m; then for floating

Weight = upthrust, i.e., $(M + m)g = V_D\sigma_w g$

So the volume of water displaced by the system when floating

$$V_D = \frac{M}{\sigma_W} + \frac{m}{\sigma_W} \qquad \text{(i)}$$

Now when ice melts, water formed will be Mg and so the volume of water formed by melting of ice

$$V_1 = (M/\sigma_W)$$

i. If the ice cube initially contains the lead piece, then as $\rho_{pb} > \sigma_W$, it will sink when ice melts and so the water displaced by it will be equal to its own volume.

$$V_2 = (m/\rho_{pb}) \text{ [as } \rho = m/V]$$

So finally total volume occupied by water formed by melting of ice and lead piece

$$V_1 + V_2 = \frac{M}{\sigma_W} + \frac{m}{\rho_{pb}} \qquad \text{(ii)}$$

Now as $\rho_{pb} > \sigma_W$, this volume will be lesser than initial water displaced by the floating system Eq. (i); so the level of water in beaker will go down as the ice melts if the ice contains a sinking impurity.

ii. If the ice initially contains a cork piece, then as $\rho_{rock} < \sigma_W$, the cork will float on water when ice melts. So the water displaced by the floating cork piece will be

$$mg = V_2'\sigma_W g$$

$$\Rightarrow \quad V_2' = m/\sigma_W$$

[and not its own volume (m/ρ_{rock})]

So in this case, water formed by melting of Mg of ice and water displaced by floating cork,

$$V_1 + V_2' = \frac{M}{\sigma_W} + \frac{m}{\sigma_W} \qquad \text{(iii)}$$

and as this is same as initial volume displaced by the floating system (Eq. i), the level of water in the beaker will remain unchanged.

2. If M is the mass of boat and m of pieces in it, then initially as the system is floating,

$$(M + m)g = V_D\sigma_W g$$

i.e., the system displaces water

$$V_D = \frac{M}{\sigma_W} + \frac{m}{\sigma_W}$$

When the pieces are dropped in the pond, the boat will still float, so it displaces water $M = V_1\sigma_W$, i.e., $V_1 = (M/\sigma_W)$

(a) Now if the unloaded pieces float in the pond, the water displaced by them

$$m = V_2\sigma_W$$

$$\Rightarrow \quad V_2 = (m/\sigma_W) \qquad \text{(i)}$$

So the total water displaced by the boat and the floating pieces,

$$V_1 + V_2 = \frac{M}{\sigma_W} + \frac{m}{\sigma_W}\text{ m} \qquad \text{(ii)}$$

which is same as the water displaced by the floating system initially (Eq. i); so the level of water in the pond will remain unchanged.

(b) Now if the unloaded pieces sink, the water displaced by them will be equal to their own volume, i.e.,

$$V_2' = \frac{m}{\rho} \quad \left[\text{as } \rho = \frac{m}{V}\right]$$

and so in this situation the total volume of water displaced by the boat and sinking pieces will be

$$V_1 + V_2' = \left(\frac{M}{\sigma_W} + \frac{m}{\rho}\right) \qquad \text{(iii)}$$

Now as the pieces are sinking $\rho > \sigma_W$, so this volume will be lesser than initial water displaced by the floating system (Eq. i); so the level of water in the pond will go down (or fall).

3. For a floating body in two liquids,

Weight of the body = Buoyant force of liquid I + buoyant force of liquid II

$$V\rho g = V_1\rho_1 g + V_2\rho_2 g$$

$$V\rho = V_1\rho_1 + V_2\rho_2 \qquad \text{(i)}$$

Also, $V = V_1 + V_2$ (ii)

After solving Eqs. (i) and (ii), we get

$$V_1 = V\left(\frac{\rho - \rho_2}{\rho_1 - \rho_2}\right) \text{ and } V_2 = V\left(\frac{\rho_1 - \rho}{\rho_1 - \rho_2}\right)$$

4. Volume of the cavities V_{cav} can be determined by taking difference between the volume V_{cast} of the casting as a whole and the volume of the iron in the casting.

$$V_{cav} = V_{cast} - V_{iron}$$

$$V_{iron} = \frac{W}{\rho_{iron} g}$$

W is the weight of casting; $W_{eff} = W - \rho_w g V_{cast}$

$$V_{cast} = \frac{W - W_{eff}}{\rho_w g}$$

$$\Rightarrow \quad V_{cav} = \frac{W - W_{eff}}{\rho_w g} - \frac{W}{g\rho_{iron}}$$

$$= \frac{6000 - 4000}{(10^3)(10)} - \frac{6000}{(10)(8 \times 10^3)} = 0.125 \text{ m}^3$$

5. Let m be the mass of copper in neckless.

Mass of gold = $(50 - m)$

Volume of copper = $V_1 = m/10$

Volume of gold = $V_2 = \dfrac{50 - m}{20}$

When immersed in water, $\rho_w = 1$ g/cm^3

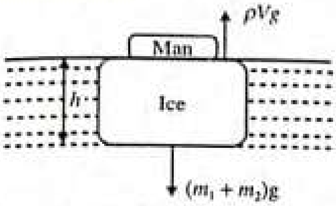
Decrease in weight = upthrust

$$(50 - 46)\, g = (V_1 + V_2)\, \rho_w\, g$$

$$4 = \frac{m}{10} + \frac{50 - m}{20}$$

$$m = 30 \text{ g}$$

6. For equilibrium, $(m_1 + m_2)\,g = \rho V g$



Here, m_1 = mass of man = 100 kg

m_2 = mass of ice = $0.917 \times 1000\ V = 917\ V$

$\rho = 1000$ kg/m^3, $h = 0.5$ m

$\therefore \quad 100\ g + 917\ Vg = \rho Vg = 1000\ Vg$

$$V = \frac{100g}{1000g - 917g} \Rightarrow Ah = \frac{100}{83}$$

$$\therefore \quad A = \frac{100}{83 \times 0.5} = 2.41\ \text{m}^2$$

7. When a body is immersed in a liquid, it experiences an upthrust u, which makes spring balance S_1 to read

Apparent weight = true weight – upthrust = $W_A - u = 2.5$ kg

S_2 reads: Weight of B + weight of L + reaction = upthrust of A

$W_B + W_L + u = 7.5$ kg weight

$W_B = 1$ kg weight, $W_L = 1.5$ kg weight

$1 + 1.5 + u = 7.5$ kg weight

$u = 5$ kg weight

$W_A = u + 2.5 = 7.5$ kg weight

Concept Application Exercise 12.4

1. Work done by per unit volume by force of gravity

= –change in potential energy per unit volume

$\therefore\ W = -\Delta U = -\rho g\,(h_2 - h_1) = -\rho g\,(6 - 3)$

$= -3 \times 500 \times 10 = -15 \times 10^4$ J/m^3

2. Since the flow is horizontal, so the reduced form of Bernoulli's equation can be applied as

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Given $P_1 = 50 \times 13.6 \times 1000 = 6.8 \times 10^5$ dyn/cm^2

and $P_2 = 20 \times 13.6 \times 1000 = 2.72 \times 10^5$ dyn/cm^2

and $\rho = 1$ g/cc and $v_1 = 500$ cm/s

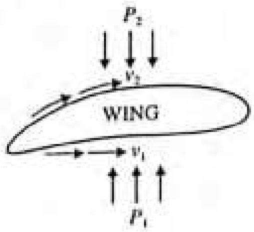
$\therefore \quad 6.8 \times 10^5 + 1/2 \times 1 \times (500)^2 = 2.72 \times 10^5 + 1/2 \times 1 \times v_2^2$

$\Rightarrow \quad \frac{1}{2}(v_2^2 - 250000) = 4.08 \times 10^5 \Rightarrow v_2 \approx 10.25\ \text{ms}^{-1}$

From the principle of continuity, $a_1 v_1 = a_2 v_2$

$$\frac{a_1}{a_2} = \frac{v_2}{v_1} = \frac{10.25}{5} = \frac{41}{20}$$

3. Applying Bernoulli's theorem to the lower and upper surface of a wing



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

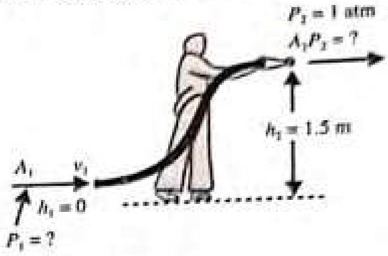
or, $$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$\Rightarrow \quad 1000 = \frac{1}{2} \times 1.30(v_2^2 - 50^2) \qquad [\because \text{lift} = P_1 - P_2]$

or $\quad v_2^2 - 2500 = 2000/1.3$

or, $\quad v_2 = 63.55$ m/s

4. (a) Figure illustrates the situation. From continuity equation, we can find the velocity of the fluid at the nozzle.



$$v_2 = \frac{A_1}{A_2}\,v_1 = \left(\frac{3.60}{0.250}\right)(50.0) = 720\ \text{cm/s} = 7.20\ \text{m/s}$$

(b) Thrust force experienced by gardener at hand due to flow of water

$$F_{\text{thrust}} = \rho A_2 v_2^2 = 1000 \times 0.25 \times 10^{-4} \times (7.2)^2 = 1.3\ \text{N}$$

(c) We will apply Bernoulli's equation to determine pressure P_1.

$h_1 = 0.00$ m, $h_2 = 1.50$ m, $P_2 = 1.01 \times 10^5$ Pa,

$v_1 = 0.50$ m/s, $v_2 = 7.20$ m/s, $\rho = 1.00 \times 10^3$ kg/m^3

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

$$= (1.01 \times 10^5) + \frac{1}{2}(1.00 \times 10^3)[(7.20)^2 - (0.50)^2] + (1.00 \times 10^3)(9.80)(1.50 - 0.00)$$

$$= 1.41 \times 10^5\ \text{Pa}$$

5. For horizontal pipe, gravitational head is same at two points, so using Bernoulli's theorem,

$$h_1 + \frac{P_1}{\rho g} + \frac{1}{2}v_1^2 = h_2 + \frac{P_2}{\rho g} + \frac{1}{2}\frac{v_2^2}{g}$$

(P_1 and P_2: pressures at two points, V_1 and V_2: velocities of water at two points)

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2$$

$\Rightarrow \quad P_1 = 10$ cm of Hg $= h\rho g$

$= 10 \times 13.6 \times 980$ CGS units

$V_1 = 40$ cm/s; $V_2 = 50$ cm/s and $\rho = 1$ g/cc

$$\Rightarrow \quad \frac{10 \times 13.6 \times 980}{1} + \frac{1}{2}(40)^2 = \frac{P_2}{1} + \frac{1}{2}(50)^2$$

$P_2 = 1.328 \times 10^5$ dyn/cm^2

$P_2 = 9.97$ cm of Hg

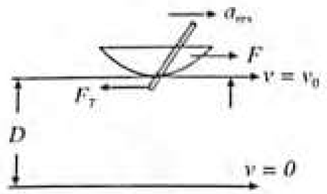
Concept Application Exercise 12.5

1. $F - F_T = ma_{\text{res}}$

As boat moves with constant velocity $a_{\text{res}} = 0$

$F = F_T$

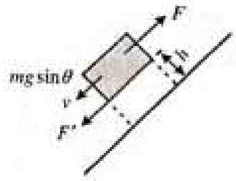
We know $F_T = \eta A \dfrac{dv}{dy}$, but $\dfrac{dv}{dy} = \dfrac{v_0 - 0}{D} = \dfrac{v_0}{D}$



then $\quad F = F_T = \dfrac{\eta A v_0}{D}$

2. $F = F' = \eta A \dfrac{dv}{dy} = mg \sin\theta \qquad \left[\dfrac{dv}{dy} = \dfrac{v}{h}\right]$

$$20 \times 10 \times \sin 30° = \eta \times 4 \times \frac{10}{h}$$



$$h = \frac{40 \times 10^{-2}}{100}\eta \quad [\eta = 10^{-1} \text{ poise} = 10^{-2} \text{ Ns/m}^2]$$

$= 4 \times 10^{-3}$ m = 4 mm

3. By Stoke's law, the terminal velocity of a water drop of radius r is given by

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where ρ is the density of water, σ is the density of air and h the coefficient of viscosity of air. Here σ is negligible and $r = 0.0015$ mm $= 1.5 \times 10^{-3}$ mm $= 1.5 \times 10^{-6}$ m. Substituting the values:

$$v = \frac{2}{9} \times \frac{(1.5 \times 10^{-6})^2 \times (1.0 \times 10^3) \times 9.8}{1.8 \times 10^{-5}} = 2.72 \times 10^{-4} \text{m/s}$$

4. The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \qquad \text{(i)}$$

This is the terminal velocity of the sphere in water. Hence by Stoke's law, we have

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere.

σ (= 1.0×10^3 kg/m^3) is the density of water and η is coefficient of viscosity of water.

$$\therefore \quad v = \frac{2 \times (1.0 \times 10^{-3})^2 (1.0 \times 10^4 - 1.0 \times 10^3) \times 10}{9 \times 1.0 \times 10^{-3}} = 20 \text{ m/s}$$

From Eq. (i), we have $\quad h = \dfrac{v^2}{2g} = \dfrac{20 \times 20}{2 \times 10} = 20$ m

5. $F = \eta A \dfrac{dy}{dt} = \eta A \dfrac{v}{h} = 15 \times 2 \times 10^4 \times \dfrac{10}{0.25} = 1.2 \times 10^7$ dyne

6. $F = 6\pi\eta r v$

$$\eta = \frac{F}{6\pi r v} = \frac{3.14}{(6)(3.14)(4)(5 \times 10^2)} = 8.3 \times 10^{-5} \text{poise}$$

Concept Application Exercise 12.6

1. If drop of radius R is sprayed into n droplets of equal radius r, then as a drop has only one surface, the initial surface area will be $4\pi R^2$ while final area is $n(4\pi r^2)$. So the increase in area

$$\Delta S = n(4\pi r^2) - 4\pi R^2$$

So energy expended in the process,

$$W = T\Delta S = 4\pi T\,[nr^2 - R^2] \qquad \text{(i)}$$

Now since the total volume of n droplets is same as that of initial drop, i.e.,

$$\frac{4}{3}\pi R^3 = n[(4/3)\pi r^3]$$

or $\quad r = R/n^{1/3} \qquad$ (ii)

Putting the value of r from Eq. (ii) in Eq. (i)

$$W = 4\pi R^2 T[(n)^{1/3} - 1].$$

2. Let n be the number of little droplets.

Since volume will remain constant, hence volume of n little droplets = volume of single drop

$$\therefore \quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \text{or} \quad nr^3 = R^3$$

Decrease in surface area $= n \times 4\pi r^2 - r\pi R^2$

or $\quad \Delta A = 4\pi\,[nr^2 - R^2] = 4\pi\left[\dfrac{nr^3}{r} - R^2\right]$

$$= 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Energy evolved $W = T \times$ decrease in surface area

$$= T \times 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Heat produced, $\quad Q = \dfrac{W}{J} = \dfrac{4\pi T R^3}{J}\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$

But $\quad Q = ms\,d\theta$

where m is the mass of big drop, s is the specific heat of water and $d\theta$ is the rise in temperature.

$\therefore \quad \dfrac{4\pi T R^3}{J}\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$ = volume of big drop × density of water × specific heat of water × $d\theta$

or, $\quad \dfrac{4}{3}\pi R^3 \times 1 \times 1 \times d\theta = \dfrac{4\pi T R^3}{J}\left(\dfrac{1}{r} - \dfrac{1}{R}\right)$

or, $\quad d\theta = \dfrac{3T}{J}\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$

3. The surface tension of the liquid is

$$T = \frac{rh\rho g}{2}$$

$$= \frac{(0.025\text{cm})\,(3.0\text{cm})\,(1.5\text{gm/cm}^3)\,(980\text{cm/sec}^2)}{2}$$

= 55 dyne/cm

Hence, excess pressure inside a spherical bubble

$$\Delta p = \frac{4T}{R} = \frac{4 \times 55\text{dyne/cm}}{(0.5\text{cm})} = 440 \text{ dyne/cm}^2$$

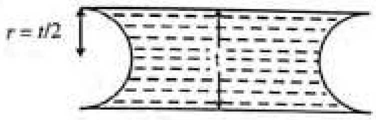
4. Pressure inside the film is less than outside by an amount

$$P = T\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$$

where r_1 and r_2 are the radii of curvature of the meniscus. Here $r_1 = t/2$ and $r_2 = \infty$, then the force required to separate the two glass plates, between which a liquid film is enclosed, is

$$F = P \times A = \frac{2AT}{t}$$

where t is the thickness of the film, A = area of film.



$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}}$$
$$= 45 \text{ N}$$

5. Let l be the length of the tube inside water. The forces acting on the tube are:

(a) Upthrust of water acting upward
$= \pi p^2 l \times 1 \times 980$
$= \frac{22}{7} \times (0.14)^2\, l \times 980 = 60.368\, l$ dyne

(b) Weight of the system acting downwards
$= mg = 0.2 \times 980 = 196$ dyne

(c) Force of surface tension acting downward
$= 2\pi p T = 64.24$ dyne

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,
$60.368\, l = 196 + 64.24 = 260.24$
$l = 4.31$ cm

6. Given that surface tension of water,
$T = 75$ dyne/cm
Radius $r = 0.1/2$ mm $= 0.05$ mm $= 0.005$ cm,
density $\rho = 1$ gm/cm³, angle of contact, $\theta = 0°$.
Let h be the height to which water rise in the capillary tube. Then

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times75\times\cos0°}{0.005\times1\times981} \text{ cm} = 30.58 \text{ cm}$$

But length of capillary tube, $h' = 5$ cm
(a) Because $h > h'/2$ therefore the first possibility does not exist.
(b) Because the tube is of insufficient length, therefore the water will rise up to the upper end of the tube.
(c) The water will not overflow out of the upper end of the capillary. It will rise only up to the upper end of the capillary.
The liquid meniscus will adjust its radius of curvature R' in such a way that

$$R'h' = Rh \quad \left[hR = \frac{2T}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length

$$R' = \frac{Rh}{h} = \frac{rh}{h'} \quad \left[R = \frac{r}{\cos\theta} = \frac{r}{\cos 0°} = r\right]$$
$$= \frac{0.005\times30.58}{5} = 0.0306 \text{ cm}$$

Exercises

Fluid Statics, Buoyancy and Floatation

1. (d) $P = h\rho g$

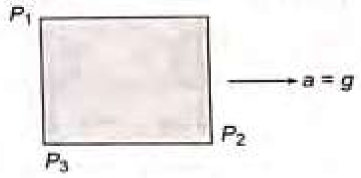
2. (c) Area of the plate = ½ × base × altitude
$= ½ \times 3 \times 3 = 4.5$ m²
Density of oil $= 0.8 \times 1000 = 800$ kg m⁻³
Depth of c.g. of the plate from the free surface
$= (1/3)\,3 = 1$ m.
Now, total thrust on the plate =
(plate area) × (pressure at the c.g. of the plate)
$= (4.5)\,(800 \times 10 \times 1) = 36$ kN

3. (b) Force acting on the base
$F = P \times A = hdg\,A = 0.4\times900\times10\times2\times10^{-3} = 7.2$ N

4. (a) The centre of liquid force passes through $y = \frac{h}{3}$.

5. (d) $P_0 + \rho_{glycerine}\, g \times 10 = P_0 + \rho_{oil} \times g \times h + \rho_{Hg} \times g \times (10 - h)$
$1.3\times10 = 0.8\times h + 13.6\times10 - 13.6\,h$
$12.8\,h = 12.3 \times 10 \Rightarrow h = 9.6$ cm

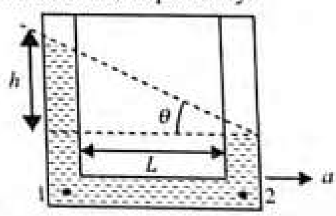
6. (a) Let pressure at the opening be P_o. Then,
$P_1 = P_o + \rho g b$ while $P_2 = P_o + \rho g h$.



Subtracting, $P_1 - P_2 = \rho g(b - h)$

7. (c) From Pascal's law, pressure is changing at every point by the same amount. Hence, buoyancy remains the same. So, the part of the block inside water remains the same.

8. (d) Let P_1 and P_2 be the pressures at the bottom of the left and right ends of the tube, respectively.

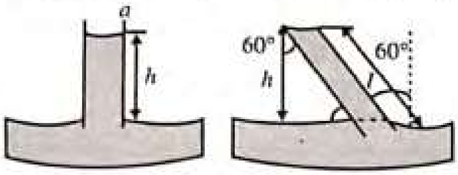


Then $F = (P_1 - P_2)A = \rho g h A$
where A is the cross section of the tube.
The mass of the liquid in the horizontal portion is
$m = \rho L A$
Now, $F = ma$
$\Rightarrow \rho g h A = \rho L A a$
$\therefore \quad h = \frac{aL}{g}$

9. (c) $P = 100 \text{ cm} \times 1 \text{ g cm}^{-3} \times 1000 \text{ cms}^{-2} = 10^5 \text{ dyn cm}^{-2}$
$F = 10^5 \times 100$ dyn $= 100$ N
Again, $V = 199$ cm³
Weight $= 199 \times 1 \times 1000$ dyn $= 1.99$ N

10. (d) The points A and C are in same horizontal level hence $P_C - P_A = \rho a l$
(refer to figure in question to identify A, B, C)
Now, $P_B - P_C = \rho g h$
$\Rightarrow \; P_B = (P_1 + \rho a l) = \rho g h \;\Rightarrow\; P_B - P_A = h\rho g + l\rho a$

11. (a) Since water rises to height of 2 cm in a capillary

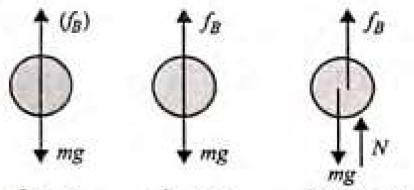


If tube is at 60°, in this case height must be equal to

$$h = 2\text{ cm} \;\Rightarrow \cos 60^\circ = \frac{h}{l}$$

$$\therefore \quad l = \frac{h}{\cos 60^\circ} = \frac{2}{1/2} = 4\text{ cm}$$

12. (a) W = weight of liquid.
f_B = buoyant force on the ball
mg = weight of the ball
N = normal reaction between the ball and the surface
The free-body diagrams of the balls in each vessel are as follows.



$f_B = mg$ (A) $\quad f_B = mg$ (B) $\quad f_B + N = mg$ (C)

At base, reaction force of buoyant force will act in downward direction.
The forces acting at the base of each tank are

$$F_A = W + f_B = W + mg$$
$$F_B = W + f_B = W + mg$$
$$F_C = W + f_B + N = W + mg$$

Thus, $F_A = F_B = F_C$

13. (a) Given that $m_{\text{real}} = 36$ g, $m_{\text{app}} = 34$ g.
Density of gold $\rho_{\text{Au}} = 19.3$ g/cc
Density of copper $\rho_{\text{Cu}} = 8.9$ g/cc
We know that loss of weight = weight of displaced water = 36 − 34 = 2 g = Buoyant force = B
Here, $m_{\text{real}} = m_{\text{Aw}} + m_{\text{Cu}} = 36\ g$ (i)
Let v be the volume of the ornament in centimetres. Then

$$B = v \times \rho_w \times g = 2 \times g$$

$$\Rightarrow \quad \left(\frac{m_{\text{Au}}}{\rho_{\text{Au}}} + \frac{m_{\text{Cu}}}{\rho_{\text{Cu}}}\right)\rho_w \times g = 2 \times g$$

$$m_{\text{Au}}\rho_{\text{Cu}} + m_{\text{Cu}}\rho_{\text{Au}} = 2\rho_{\text{Acc}}\rho_{\text{Cu}}$$

$$8.9m_{\text{Au}} + 19.3m_{\text{Cu}} = 2 \times 19.3 \times 8.9 = 343.54 \qquad \text{(ii)}$$

From Eqs. (i) and (ii), $8.9m_{\text{Au}} + 19.3m_{\text{Cu}} = 343.54$

$$\Rightarrow \quad 8.9(m_{\text{Au}} + m_{\text{Cu}}) + 10.4m_{\text{Cu}} = 343.54$$

$$\Rightarrow \quad 8.9 \times 36 + 10.4m_{\text{Cu}} = 343.54$$

$$m_{\text{Cu}} = 2.225\text{ g}$$

So the amount of copper in the ornament is 2.2 g.

14. (c) $\rho A = 0.75$, $\rho_P = 0.6$
$\rho B = 1.0$, $\rho_Q = 0.9$
As relative density of P is lesser than B, so it will float in liquid B and as relative density of Q is greater than liquid A so it will sink, because if density of the object is greater than that of the liquid in which it is immersed, then its weight is more than the upthrust and vice versa.

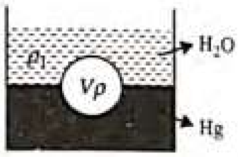
15. (b) Let V be the total volume of the ball and v be the volume of the ball in the upper liquid. Then $V - v$ is the volume of the lower liquid displaced.
Using the law of floatation, we have

$$V\rho g = v\rho_1 g + (V - v)\rho_2 g$$

$$V\rho = v\rho_1 + V\rho_2 - v\rho_2$$

or $\quad V(\rho - \rho_2) = v(\rho_1 - \rho_2)$

$$\frac{v}{V} = \frac{\rho - \rho_2}{\rho_1 - \rho_2} = \frac{\rho_2 - \rho}{\rho_2 - \rho_1}$$

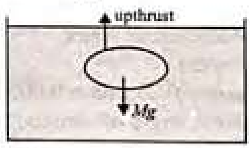


16. (a) As the water exerts upthrust on the stone, the stone also exerts the same force on water, according to Newton's third law. Upthrust force acting on the stone is

$$U = V_{\text{stone}}\,\rho_\omega g$$

$$= \left(\frac{m_{\text{stone}}}{\rho_{\text{stone}}}\right) = \frac{0.5}{10^4} \times 1000 \times 10 = 0.5\text{ N}$$

Also, the weight is exerted on the beaker. Therefore the reading will be 1.5 + 0.5 = 2 kg.



17. (a) Let ρ_S and ρ_L be the densities of silver and liquid, respectively, and m and V be the mass and volume, respectively, of the silver block. Therefore,
Tension in the string = mg − buoyant force

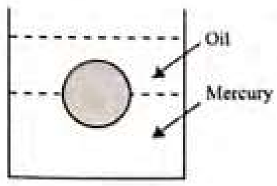
$$\Rightarrow \quad T = \rho_S Vg - \rho_L Vg = (\rho_S - \rho_L)Vg$$

Also, $V = \dfrac{m}{\rho_S}$

$$\therefore \quad T = \left(\frac{\rho_S - \rho_L}{\rho_S}\right)mg$$

$$= \frac{(10 - 0.72) \times 10^3}{10 \times 10^3} \times 4 \times 10 = 37.12\text{ N}$$

18. (c)



Weight = buoyant force $\frac{V}{2}\rho_{Hg}g + \frac{V}{2}\rho_{oil}g$

$$\therefore \quad \rho = \frac{\rho_{Hg} + \rho_{oil}}{2} = \frac{13.6 + 0.8}{2} = \frac{14.4}{2} = 7.2$$

19. (a) As the vessel is falling freely, the pressure at all the points in the liquids is same and equal to the atmospheric pressure and hence buoyancy becomes zero.

20. (b) $(V - \Delta V) \times 1.03 \times g = V \times 0.92 \times g$

or $\frac{V - \Delta V}{V} = \frac{0.92}{1.03}$

or $\frac{\Delta V}{V} = 1 - \frac{0.92}{1.03} = \frac{0.11}{1.03}$ or $\frac{\Delta V}{V} \times 100 = \frac{11}{1.03} \approx 11$

Fluid Dynamics

21. (b) Using equation of continuity, we have $v_2 = \frac{A_1}{A_2}v_1$

From Bernoulli's theorem, $p_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2$

$$p_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 \Rightarrow g(h_1 - h_2) = \frac{1}{2}(v_2^2 - v_1^2)$$

$$\Rightarrow \quad 60 = \left(\frac{A_1^2}{A_2^2} - 1\right)v_1^2 \Rightarrow \frac{A_1}{A_2} = \frac{4}{1}$$

22. (a) The air through the horizontal tube will decrease the pressure and more liquid will be pushed into the capillary tube.

23. (c) Let d_w and d_0 be the densities of water and oil, respectively. Then the pressure at the bottom of the tank is $h_w d_w g + h_0 d_0 g$. Let this pressure be equivalent to pressure due to water of height h. Then,

$$hd_w g = h_w d_w g + h_0 d_0 g$$

$$\therefore \quad h = h_w + \frac{h_0 d_0}{d_w} = 100 + \frac{400 \times 0.9}{1}$$

$$= 100 + 360 = 460 = 4.6 \text{ m}$$

According to Toricelli's theorem,

$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 4.6} \text{ m/s} = \sqrt{92} \text{ m/s}$$

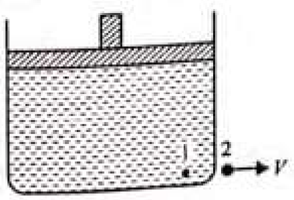
24. (a) Total pressure at the bottom = 3 atm

Pressure due to water in the tank = 3 atm − 1 atm = 2 atm = 20 m of water column

Height of water in the tank is h = 20 m

So, velocity of efflux $= \sqrt{2gh} = \sqrt{2} \times 10 \times 20 = \sqrt{400}$ m/s

25. (b) Applying Bernoulli's theorem at points 1 and 2, difference in pressure energy between 1 and 2 = difference in kinetic energy between 1 and 2.



Hence, $\rho hg + \frac{mg}{A} = \frac{1}{2}\rho v^2$

or $v = \sqrt{2gh + \frac{2mg}{\rho A}} = \sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$

26. (a) Let ρ be the density of the liquid. Then

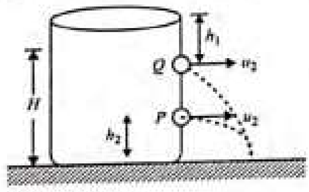
$$F_1 = (\Delta P)A = \rho g h A \quad \text{(i)}$$

In the second case, F_2 = rate of change of momentum

$$= \rho A u^2 = \rho A(\sqrt{2gh})^2 = 2\rho g h A \quad \text{(ii)}$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{1}{2}$$

27. (a) The two streams strike at the same point on the ground.



$$R_1 = R_2 = R$$

$$u_1 t_1 = u_2 t_2 \quad \text{(i)}$$

where u_1 = velocity of efflux at $Q = \sqrt{(2gh_1)}$ and u_2 = velocity of efflux at $P = \sqrt{[2g(H - h_2)]}$

t_1 = time of fall of water stream through Q is

$$= \sqrt{\frac{2(H - h_1)}{g}}$$

t_2 = time of fall of the water stream through $P = \sqrt{\frac{2h_2}{g}}$

Putting these values is Eq. (i), we get

$$(H - h_1)h_1 = (H - h_2)h_2$$

or $[H - (h_1 + h_2)][h_1 - h_2] = 0$

$H = h_1 + h_2$ is irrelevant because the holes are at two different heights. Therefore, $h_1 = h_2$ or $h_1/h_2 = 1$

28. (c) As $x = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2x}{r}}$

Velocity of efflux $v = \sqrt{2g(H - x)}$

Hence $R = vt = 2\sqrt{x(3H - x)}$

For range to be maximum $\frac{dR}{dx} = 0$

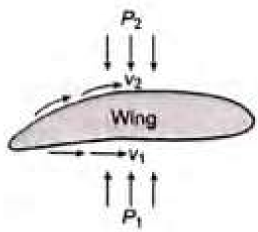
which gives $x = \frac{3}{2}H = 1.5\,H$

Alternatively:

Maximum range is also equal to average of two heights

$$x = \frac{H + 2H}{2} = \frac{3H}{2} = 1.5\,H$$

29. (b)



Applying Bernoulli's theorem to the lower and upper surface of a wing

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$

$\Rightarrow$ $1000 = \frac{1}{2} \times 1.30(v_2^2 - 50^2)$ $[\because \text{lift} = P_1 - P_2]$

or $v_2^2 - 2500 = 2000 / 1.3$

or $v_2 = 63.55$ m/s

30. (c) As the both points are at the surface of liquid and these points are in the open atmosphere. So both point possess similar pressure and equal to 1 atm. Hence, the pressure difference will be zero.

31. (b) Horizontal range will be maximum when $h = \frac{H}{2} = \frac{90}{2} = 45$ cm, i.e., hole 3.

32. (b) Bernoulli's equation between A and B gives

$$\frac{p_A}{\rho_a} = \frac{p_B}{\rho_a} + \frac{v^2}{2} \Rightarrow v^2 = 2\left[\frac{p_A - p_B}{\rho_a}\right]$$

Also equating pressures at horizontal level of E

$$p_A + \rho_a gy + \rho_a gh = p_B + \rho_a gy' + \rho_a gy + \rho_m gh.$$

$\Rightarrow$ $p_A + \rho_a gh = p_B + \rho_m gh$ $[\because y' = 0]$

$$p_A - p_B = (\rho_m - \rho_a)\, gh.$$

$$v^2 = \frac{2(\rho_m - \rho_a)gh}{\rho_a}$$

$$v = \sqrt{\frac{2(\rho_m - \rho_a)gh}{\rho_a}}$$

33. (a) Range is same for holes equal distance from top and bottom of tank.

34. (d) $x = \sqrt{2gh_1} \times \sqrt{\frac{2h_2}{g}}$ or $x = 2\sqrt{h_1 h_2}$

Now, imagine a hole at a depth h_2 below the free surface of the liquid. The height of this hole will be h_1. Clearly, x remains the same.

Surface Tension and Viscosity

35. (c) Radius of the larger drop = R

Suppose radius of the droplets = r

Since volume will be remain constant,

$$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^2 \quad \text{(as no. of droplets = 8)}$$

$$\therefore \quad r = \left(\frac{R^3}{8}\right)^{\frac{1}{3}} = \frac{R}{2}$$

Therefore, work done = Increase in surface area × Surface tension

$$= \left[8 \times 4\pi\left(\frac{R}{2}\right)^2 - 4\pi R^2\right] \times T$$

$$= (8\pi R^2 - 4\pi R^2) \times T = 4\pi R^2 T$$

36. (c) Work done = (increase in surface area) × surface tension

$$= 2 \times \left[4\pi\left(\frac{2D}{2}\right)^2 - 4\pi\left(\frac{D}{2}\right)^2\right] \times T$$

(Since for sap bubble, there are two free surfaces)

$$= 2 \times (4\pi D^2 - \pi D^2)\, T = 6\, \pi D^2 T$$

37. (b) Suppose, R = radius of water drop

and r = radius of droplets

$$\therefore \quad \frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$$

(Since volume remains constant)

$$\therefore \quad r = \frac{R}{2}$$

Since excess pressure inside drop $= \frac{2T}{R}$

(T—surface tension, R—radius)

Therefore, pressure difference between inner and outer surface of big drop will be half of that for smaller droplet.

38. (b) Since, ball gains the constant velocity.

Therefore, net force on the ball = zero.

$\Rightarrow$ Weight of the ball = Buoyancy force + Viscous force (F_v)

$$\Rightarrow \quad Mg = \frac{M}{d_1} \times d_2 \times g + F_v$$

$$\therefore \quad F_v = Mg\left(1 - \frac{d_2}{d_1}\right)$$

39. (b) $\left[P_0 + \frac{4\sigma}{R_2}\right] - \left[P_0 + \frac{4\sigma}{R_1}\right] = \frac{4\sigma}{R}$

or $\frac{1}{R} = \frac{1}{R_2} - \frac{1}{R_1}$

or $R = \frac{R_1 R_2}{R_1 - R_2} = \frac{50 \times 80}{30}\text{ mm} = \frac{400}{3}\text{ mm}$

$$= \frac{400}{3 \times 1000}\text{ m} = \frac{4}{30}\text{ m} = 0.133\text{ m}$$

40. (a) Total upward force due to surface tension = $\sigma(2r_1 + 2\pi_2)$. This supports the weight of the liquid column of height h. Weight of liquid column = $h\,[\pi r_2{}^2 - \pi r_1{}^2]\,\rho g$

Equating, we get $h\pi\,(r_2 + r_1)\,(r_2 - r_1)\,\rho g = 2\pi\sigma\,(r_1 + r_2)$

or $h\,(r_2 - r_1)\,\rho g = 2\sigma$ or $h = \frac{2\sigma}{(r_2 - r_1)\rho g}$

41. (d) Energy released = $[n \times 4\pi a^2 - 4\pi b^2]\,\sigma$

Now, $n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi b^3$ or $n = \frac{b^3}{a^3}$

Therefore, energy released is

$$= \left[\frac{b^3}{a^3} \times 4\pi a^2 - 4\pi b^2\right]\sigma = 4\pi b^2\left[\frac{b}{a} - 1\right]\sigma$$

Now, $\frac{1}{2}\left(\frac{4}{3}\pi b^3\right)\rho v^2 = 4\pi b^2\left[\frac{b}{a} - 1\right]\sigma$

or $v = \left[\frac{6\sigma}{\rho}\left(\frac{1}{a} - \frac{1}{b}\right)\right]^{1/2}$

42. (a) $F = (\sigma_1 - \sigma_2)\, l = (50 \times 6 - 40 \times 6)$ dyn

43. (a) $\left[2\pi \times \frac{8.7}{2} + 2\pi \times \frac{8.5}{2}\right]\sigma = 3.97 \times 980$

or $\sigma = \frac{3.97 \times 980 \times 7}{22 \times 17.2}$ dyn/cm $= 72$ dyn/cm

44. (c) $\frac{h_2}{h_1} = \frac{\sigma_2 \cos\theta_2}{\rho_2} \times \frac{\rho_1}{\sigma_1 \cos\theta_1}$

or $\frac{h_1}{h_2} = \frac{140 \times \frac{1}{2}}{2} \times \frac{1}{70 \times 1}$ or $h_2 = \frac{h_1}{2} = \frac{6}{2}$ cm $= 3$ cm

45. (d) $V \times \frac{4}{3}\pi R^3$; $2V = \frac{4}{3}\pi R'^3$; $2 \times \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R'^3$;

$R' = 2^{1/3} R$

$W'\ 2 \times 4\pi\,[2^{1/3} R]^2\, \sigma = 2^{2/3} \times 2 \times 4\pi R^2 \sigma = 4^{1/3}\, W$

46. (c) Final surface energy $= 1000 \times 4\pi r^2 \sigma$

Initial surface energy, $E = 4\pi R^2 \sigma$

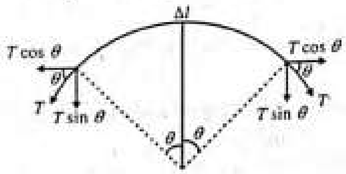
Again, $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^2$ or $R = 10r$

Now, final energy

$1000 \times 4\pi r^2 \sigma = 10 \times 4\pi R^2 \sigma = 10E$

47. (c) Work done $= 10E - E = 9E$.

48. (c) When the soap film enclosed by the thread is pricked, the thread would take up a circular shape. Consider a small element of the thread. While horizontal components of tension get balanced, the vertical components get added up.



or $2T \sin\theta = \sigma\, \Delta l$

or $2T\theta = \sigma\,(2\theta \times r)$

or $T = \sigma r = \sigma \times \frac{2\pi r}{2\pi} = 0.030 \times \frac{6.28 \times 10^{-2}}{2\pi}$ N

$= 3 \times 10^{-4}$ N

49. (d) $v_0 = \frac{2}{9}\frac{r^2(\rho - \rho')g}{\eta}$

Now, $x = \frac{4}{3}\pi r^3 \rho$ or $\rho \propto \frac{x}{r^3}$

Similarly, $\rho' \propto \frac{y}{r^3}$

$\therefore\ v_0 \propto \frac{x - y}{r}$

50. (b) $v_0 = \frac{2}{9}\frac{r^2(\rho - \rho')g}{\eta}$ or $\eta = \frac{2r^2(\rho - \rho')g}{9v_0}$

$= \frac{2 \times 0.2 \times 0.2 \times 9 \times 980}{9 \times 8}$ poise $= 9.8$ poise

51. (b) $V \propto r^4, V' \propto \left(r + \frac{1}{10}r\right)^4$

$\frac{V'}{V} = \frac{11 \times 11 \times 11 \times 11}{10 \times 10 \times 10 \times 10} = 1.4641$

or $\left(\frac{V'}{V} - 1\right) \times 100 = 46.4\%$

52. (b) For the same radius, terminal velocity is proportional to the density difference.

53. (a) $F = \frac{0.01 \times 100 \times 10}{0.1}$ dyn $= 100$ dyn

54. (c) Shearing stress $= \frac{|\text{viscous force}|}{\text{area}}$

$= \frac{\eta A \frac{dv}{dx}}{A} = \eta \frac{dv}{dx}$

$= 10^{-3} \times \frac{5}{10}\,\text{N/m}^2 = 0.5 \times 10^{-3}\,\text{N/m}^2$

55. (b) $m = \frac{4}{3}\pi r^3 \rho m$

Keeping m constant, if r is halved, ρ will increased by a factor of 8.

Now, $v_0 \propto r^2 \rho$

$v_0' \propto \frac{r^2}{4}(8\rho)$ or $v_0' \propto 2r^2\rho$

Dividing, $\frac{v_0'}{v_0} = 2$ or $v_0' = 2v_0$

or $v_0' = 2v$

56. (d) As in vacuum, there is no viscosity, no resistive force and hence no terminal velocity can be acquired.

57. (a) From the definition of bulk modulus $B = -\frac{dp}{dV/V}$

As we move from surface to placed where pressure changes to αp_0, let us assume volume changes by ΔV, then

$B = \frac{V\Delta p}{\Delta V} = \frac{V(\alpha - 1)p_0}{\Delta V}$

New volume, $V' = V - \Delta V = V\left[1 - \frac{(\alpha - 1)p_0}{B}\right]$

Density at the given depth, $\rho' = \rho V/V'$, where ρ is density at surface

$\rho' = \frac{\rho \times B}{B - (\alpha - 1)p_0}$

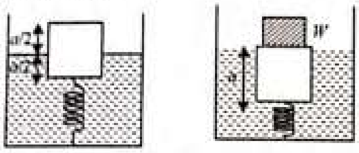
58. (b) $h = \frac{2S\cos\theta}{r\rho g}$

Cross-sectional area increases four times, which means radius gets doubled.

So, $h' = \frac{2S}{2(r\rho g)} = \frac{h}{2}$

Problems Based on Mixed Concepts

59. (d) Since density of block = 1/2(density of water), 50% of its volume is immersed in water.
In the second case, half of the volume of the block is further immersed in water.



Therefore, W = Extra upthrust + Spring force

$$= a \times a \times \frac{a}{2} \times 2\rho \times g + k\left(\frac{a}{2}\right)$$

$$= a\left(a^2 \rho g + \frac{k}{2}\right)$$

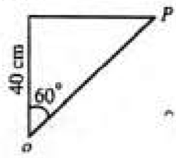
60. (b) Let v be the volume of the solid block of density ρ. Let ρ_1 be the density of water. Weight of body = $v\rho g$. When the body is immersed in water,
Tension in the string = Upward thrust − Weight of the body
$\Rightarrow \quad T = v\rho_1 g - v\rho g = vg(\rho_1 - \rho_2)$
When the lift is moving upwards with acceleration a, the tension in the string is $T = v(\rho_1 - \rho)(g + a)$

From Eqs. (i) and (ii), $T = T_0(1 + a/g)$.

61. (b) OP is the portion of the rod immersed in water.

$$\cos 60^\circ = \frac{40}{OP}$$

or $\quad OP = \dfrac{40}{\cos 60^\circ}\text{cm} = \dfrac{40}{1/2}\text{cm} = 80\text{cm}$



The centre of buoyancy is at the centre of the immersed part of the rod. So, the required distance is 40 cm.

62. (a) Let V be the volume of the ball.

Net upward force = $V\sigma g - V\rho g$

Net upward acceleration,

$$a = \frac{V\sigma g - V\rho g}{V\rho} = \frac{(\sigma - \rho)g}{\rho}$$

Velocity at the surface $= \sqrt{\dfrac{2(\sigma-\rho)gh}{\rho}}$

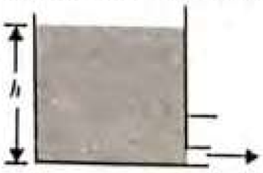
If h_a is the height in air to which the ball rises, then

$$0 - \frac{2(\sigma - \rho)gh}{\rho} = 2(-g)h_a$$

$$\therefore \quad h_a = \frac{(\sigma - r)gh}{g\rho} = \left(\frac{\sigma}{\rho} - 1\right)h$$

63. (c) The velocity with which the liquid comes out is $v_0 = \sqrt{2gh}$. Let cross-sectional area of the hole be A. Then the force exerted by the ejecting fluid due to change in momentum on the container is $F = \rho A v^2$.

$F = \rho A v^2 = ma$, where m is the mass of the liquid inside the container.



$$\rho A v^2 = \rho A h \times a$$

$\Rightarrow \quad a = 2g$

Differentiating the above equation, we can have $v = 2gt$ where t is the time in which the liquid comes out completely which is dependent on h.

64. (c) $\dfrac{40}{100} \times 1000 \times 9.8 = \dfrac{2 \times 7 \times 10^{-2}}{R}$

or $\quad R = \dfrac{14 \times 10^{-2} \times 100}{40 \times 1000 \times 9.8}\text{m} = \dfrac{14 \times 1000}{40 \times 1000 \times 9.8}\text{mm}$

$$= \frac{1}{28}\text{mm}$$

Diameter $= 2R = \dfrac{1}{14}$mm

65. (b) Assuming isothermal conditions,

$$\left(P + \frac{4\sigma}{a}\right)\left(\frac{4}{3}\pi a^3\right) + \left(P + \frac{4\sigma}{b}\right)\left(\frac{4}{3}\pi b^3\right)$$

$$= \left(P + \frac{4\sigma}{c}\right)\left(\frac{4}{3}\pi c^3\right)$$

or $\quad P[a^3 + b^3 - c^3] = 4\sigma\,[c^2 - a^2 - b^2]$

or $\quad \sigma = \dfrac{P(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

66. (b) Difference in apparent weights is due to differences in forces of surface tension. Due to 180°, the force surface tension in one case is opposite to the force of surface tension in the other case.

$\therefore \quad 2 \times \sigma_w \times \dfrac{10}{100} = 0.004 - 0.03$

or $\quad \sigma_w \times \dfrac{0.014}{1} \times 5$ N/m$= 0.07$N/m$= 7 \times 10^{-2}$ N/m

67. (a) In the condition of weightlessness, water rises to the whole of the available length.

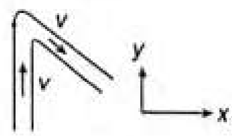
68. (a) The terminal velocity of the spherical raindrop of radius r is given by

$$v_t = \frac{2r^2 \rho g}{9\eta}$$

where ρ is the density of water and η the viscosity of air.
Substituting $r = 0.3$ mm $= 0.3 \times 10^{-3}$m, $\rho = 10^3$ kg/m^3
$g = 9.8$ ms^{-12} and $\eta = 1.8 \times 10^{-5}$ Ns/m^2

We get $\quad v_t = \dfrac{2 \times (0.3)^2 \times 10^{-3} \times 9.8}{9 \times 1.8 \times 10^{-5}} = 10.9$ m/s

69. (a) $|\overrightarrow{\Delta P}_x| = mv\sin 60^\circ = \dfrac{\sqrt{3}}{2}mv$



$$|\overrightarrow{\Delta P}_y| = \frac{mv}{2} + mv = \frac{3}{2}mv$$

$$\Rightarrow \quad |\overrightarrow{\Delta P}_{\text{net}}| = \sqrt{\Delta P_x^2 + \Delta P_y^2} = \sqrt{\left(\frac{9}{4} + \frac{3}{4}\right)}mv$$

$$|\overrightarrow{\Delta P}_{net}| = \sqrt{3}\, mv$$

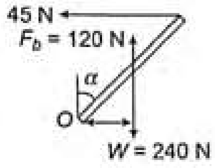
$$\Rightarrow \quad |\overrightarrow{\Delta F}_{net}| = \sqrt{3}\left(\frac{dm}{dt}\right).v = \sqrt{3}\,\rho A v^2$$

$$\left(\begin{array}{l}\text{Since, } dm = A(v\,dt)\rho \\ \Rightarrow \quad \dfrac{dm}{dt} = A\rho v\end{array}\right)$$

70. (b) F.B. of rod:

$$W = (0.012)(1)(2\times10^3)(10) = 240 \text{ N}$$

$$F_b = (0.012)(1)(10^3)(10) = 120 \text{ N}$$

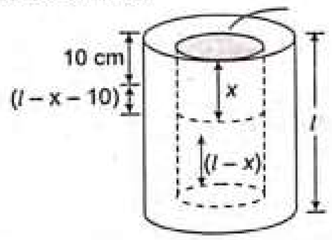


Torque about O:
(For equilibrium)

$$(240-120)\left(\frac{\sin\alpha}{2}\right) = 45\,(\cos\alpha)$$

$$\Rightarrow \quad \tan\alpha = \frac{90}{120} = \frac{3}{4} \Rightarrow \alpha = 37°$$

71. (d) After oil is filled up, pressure at the depth of lower end should equate if measured from inside and outside the tube. Suppose depth of oil is x cm then:



$$1000.g.[(l-10)\text{ cm}] = 800\,g.(x\text{ cm}) + 1000\,g\,[(l-x)\text{ cm}]$$

$$\Rightarrow \quad x = 50 \text{ cm}$$

Now, pressure difference due to x cm depth of oil is equal to pressure difference due to $(l-x-10)$ cm of water.

$$\Rightarrow \quad 800.g.x = 1000\,g\,(l-60)$$

$$\Rightarrow \quad l = 100 \text{ cm}$$

72. (b) Weight of sphere + chain = $(m + \lambda h)g$

$$\text{Buoyant force } = \left(3m + \frac{\lambda h}{7}\right)g$$

for equilibrium, weight = Buoyant force

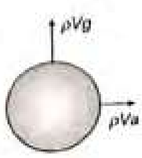
$$\text{or} \quad m + \lambda h = 3m + \frac{\lambda h}{7} \quad \text{or} \quad h = \frac{7m}{3\lambda}$$

73. (d) From the frame of reference of liquid, effective gravity (resultant of weight and pseudo force per unit mass)

$$= \sqrt{a^2+g^2}$$

$$\therefore \quad \text{Effective force due to liquid} = V\rho\sqrt{a^2+g^2}$$

Aliter:
The force exerted on spherical body by surrounding liquid is equal to sum of contact forces on displaced liquid by the surrounding liquid as shown in.
Hence, the force exerted on spherical body by surrounding liquid = $V\rho\sqrt{a^2+g^2}$.



ARCHIVES

1. (c) Let h be the height of the oil level above the water in the other tube.
Here, the pressure due to $(h+50)$ cm of oil is equal to the pressure due to 50 cm of water.

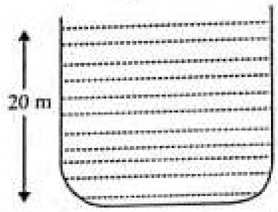
$$(h+50)\times 0.8 \times g = 50 \times 1 \times g$$

$$\Rightarrow \quad h = 12.5 \text{ cm of Hg}$$

2. (b) Apparent weight = True weight – Weight of liquid displaced

$$\therefore \quad W - \frac{W}{\rho}\rho_1 = W\left(1 - \frac{\rho_1}{\rho}\right)$$

3. (b) Apply Bernoulli's theorem inside and outside the hole. Take the reference line for gravitational potential energy at the bottom of the vessel.



Let P_0 be the atmospheric pressure, ρ be the density of the liquid, and v be the velocity at which water is coming out.

$$P_{\text{inside}} + \rho g h + 0 = P_{\text{outside}} + \frac{\rho v^2}{2}$$

$$\Rightarrow P_0 + \rho g h = P_0 + \frac{\rho v^2}{2}$$

$$\Rightarrow v = \sqrt{2gh} = \sqrt{2\times10\times20} = 20 \text{ m/s}$$

4. (c) $p = \dfrac{2\alpha}{R}$

R is less for the smaller bubble, p is more. For the larger bubble, R is more and p is less. Since air flows from higher pressure to lower pressure, it will flow from the smaller bubble to the larger bubble.

5. (b) The retarding viscous force is $6\pi\eta Rv$.

6. (a) In the condition of weightlessness, water rises to the whole of the available length.

7. (c) The terminal speed of the spherical body in a viscous liquid is given by

$$v_T = \frac{2r^2(\rho-\sigma)g}{9\eta},$$

where ρ is the density of the substance of the body and σ is the density of the liquid.
From the given data, we have

$$\frac{v_T(\text{Ag})}{v_T(\text{Gold})} = \frac{\rho_{\text{Ag}} - \sigma_l}{\rho_{\text{Gold}} - \sigma_l}$$

$$\Rightarrow \quad v_T(\text{Ag}) = \frac{10.5-1.5}{19.5-1.5}\times 0.2 = \frac{9}{18}\times 0.2 = 0.1 \text{ m/s}$$

8. (a) The forces acting on the ball are gravitational force, buoyancy force, and viscous force. When the ball acquires terminal speed, it is in dynamic equilibrium. Let the terminal speed of the ball be v_T. So

$$V\rho_2 g + kv_T^2 = V\rho_1 g$$

$$\Rightarrow \quad v_T = \sqrt{\frac{V(\rho_1-\rho_2)g}{k}}$$

9. (c) Soap solution has lower surface tension as compared to pure water, so h is less for soap solution.

10. (d) $\rho_1 < \rho_2$ because the denser liquid acquires the lowest position in the vessel. $\rho_3 > \rho_1$ because the ball sinks in liquid 1, and $\rho_3 < \rho_2$ because the ball doesn't sink in liquid 2. So

$$\rho_1 < \rho_3 < \rho_2$$

11. (c) $\rho_{oil} < \rho < \rho_{water}$
Oil is the least dense of them, so it will settle at the top with water at the base. Now the ball is denser than oil but less denser than water. So it will sink in oil but not in water. It will stay at the oil-water interface.

12. (b) $\Delta Q = MS\Delta T = 100 \times 10^{-3} \times 4184 \times 20 = 8.4 \times 10^3$

$\Delta Q = 84$ kJ and $\Delta W = 0$

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = 8.4$ kJ

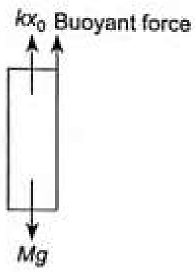
13. (d) $W = T\Delta A = 0.03[2 \times 4\pi(5^2 - 3^2)] \times 10^{-4}$

$= 0.384\ \pi \times 10^{-3}$ Joule $\cong 0.4\pi$ mJ

14. (d) $2TL = mg$

$$T = \frac{mg}{2L} = \frac{1.5 \times 10^{-2}}{2 \times 30 \times 10^{-2}} = \frac{1.5}{600} = 0.025 \text{ N/m}$$

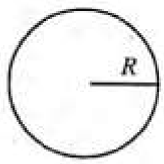
15. (b) At equilibrium, $\sum F = 0$



$$kx_0 + \left(\frac{AL}{2}\sigma g\right) - Mg = 0$$

$$x_0 = \frac{Mg}{k}\left[1 - \frac{LA\sigma}{2M}\right]$$

16. (c) $\rho 4\pi R^2 \Delta RL = T4\pi[R^2 - (R - \Delta R)^2]$



$\rho R^2 \Delta RL = T[R^2 - R^2 + 2R\Delta R - \Delta R^2]$

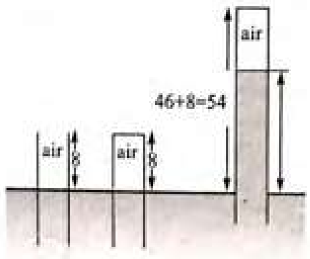
$\rho R^2 \Delta RL = T2R\Delta R$ (ΔR is very small)

$$R = \frac{2T}{\rho L}$$

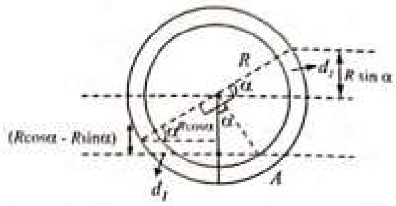
17. (c) (76) (8) = (54 – x) (76 – x)

$x = 38$ cm

Length of air column = 54 – 38 = 16 cm



18. (a) Equating pressure at A

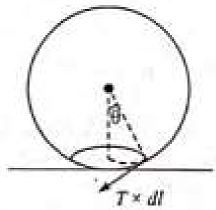


$(R \cos \alpha + R \sin \alpha)\ d_2\, g = (R \cos \alpha - R \sin \alpha)\ d_1 g$

$$\Rightarrow \frac{d_1}{d_2} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

19. (None)

$$(2\pi rT)\sin\theta = \frac{4}{3}\pi R^3 \rho_w g$$



$$T \times \frac{r}{R} \times 2\pi r = \frac{4}{3}\pi R^3 \rho_w g$$

$$r^2 = \frac{2}{3}\frac{R^4 \rho_w g}{T}$$

$$r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$

20. (d) $T = \dfrac{rhg}{2} \times 10^3$

$$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + 0$$

$$100 \times \frac{\Delta T}{T} = \left(\frac{10^{-2} \times .01}{1.25 \times 10^{-2}} + \frac{10^{-2} \times .01}{1.45 \times 10^{-2}}\right)100$$

$= (0.8 + 0.689) = (1.489)$

$100 \times \Delta T = 1.489\%$

$= 1.5\%$

CHAPTER 13: THERMAL EXPANSION, CALORIMETRY AND TRANSMISSION OF HEAT

Concept Application Exercise 13.1

1. The final temperature is obviously 0°C.

$600 \times 10^{-3} \times 0.5 \times 10 + (600-550) \times 10^{-3} \times 80$ kcal
$= m \times 0.1 \times 350$ kcal

or $m = 200 \times 10^{-3}$ kg or 200 g

2. The small ice crystal acts as the centre of condensation and so water begins to freeze instantaneously.

$mL = Mc\{0-(-t)\} = Mct$

or $m = \frac{Mct}{L} = \frac{1\times 4200\times 8}{336\times 10^3}$ kg $= 100$ g

Now $ML = Mct \Rightarrow t = \frac{L}{c} = \frac{336\times 10^3}{4200} = 80$°C

Hence temperature of overcooled water should be −80°C.

3. Energy supplied by heater $= Pt$.
$= 54 \times 3 \times 60 = 9720$ J

Energy absorbed by water $= (650 \times 10^{-3}) \times 4200 \times 3.4$
$= 9282$ J

Therefore, energy that passes out in the form of radiant heat
$= 9720 - 9282 = 438$ J

∴ Percentage loss $= 438/9720 \times 100 = 4.5\%$

4. Let m kg of water freeze on it.

Then heat lost $= m \times 80 \times 1000$ cal

Heat gained by ice $= 50 \times 10^{-3} \times 500 \times \{0-(-10)\}$ cal

Now,

Heat loss = Heat gain, $m = 3.125\times 10^{-3}$ kg $= 3.125$ g

5. Let t be the temperature of the mixture.

Net loss of heat by all

$= (V\rho_1)c_1(t_1-t) + (V\rho_2)c_2(t_2-t) + (V\rho_3)c_3(t_3-t)$

But net loss $= 0$

$V\rho_1c_1(t_1-t) + V\rho_2c_2(t_2-t) + V\rho_3c_3(t_3-t) = 0$

or $t = \frac{\rho_1c_1t_1 + \rho_2c_2t_2 + \rho_3c_3t_3}{\rho_1c_1 + \rho_2c_2 + \rho_3c_3}$

6. Consider mass m of water falling.

$mgy = mc\Delta t$

We express both sides in joules by noting

$c = 1$ kcal/kg, $K = 4184$ J/KgK

Then $9.8(122) = 4184\Delta t$

and $\Delta t = 0.29$ K

7. Let the specific heat capacities of A, B, C be s_1, s_2, s_3.

$\theta_{A+B} = 19 = \frac{10s_1 + 25s_2}{s_1+s_2}$ or $s_1 = \frac{2}{3}s_2$

$\theta_{B+C} = 35 = \frac{25s_2 + 40s_3}{s_2+s_3}$ or $s_3 = 2s_2$

$\theta_{A+B+C} = \frac{10s_1 + 25s_2 + 40s_3}{s_1+s_2+s_3}$

$= \frac{10\times\frac{2}{3}s_2 + 25s_2 + 40\times 2s_2}{\frac{2}{3}s_2 + s_2 + 2s_2} = 30.5$°C

Concept Application Exercise 13.2

1. $\alpha_{steel} = 12\times 10^{-6}\,K^{-1}$ and $\alpha_{brass} = 18\times 10^{-6}\,K^{-1}$

Let t be the required common temperature. Then $\Delta T = t - 25$. At the common temperature, both must have the same diameter.

∴ $D = 3.000(1 + 12 \times 10^{-6}\,\Delta T)$
$= 2.992\,(1 + 18 \times 10^{-6}\,\Delta T)$

⇒ $\Delta T = 448$°C ⇒ $t = 448 + 25 = 473$°C

2. We know; $\Delta T = \left(\frac{\alpha\Delta\theta}{2}\right)T$. Let correct reading occurs at θ°C, hence

$5 = \frac{\alpha(\theta-15)}{2} \times 86400$ ...(i)

and $10 = \alpha\left(\frac{30-\theta}{2}\right) \times 86400$ ...(ii)

From (i) and (ii); $\theta = 20$°C

From (1); $5 = \frac{\alpha}{2} \times 5 \times 86400$

or $\alpha = \frac{2}{86400}$/°C

or $\alpha = 23 \times 10^{-6}\,K^{-1}$

3. Since copper expands more than iron, the length of the iron rod must be great than that of the copper rod. Let their length be l_1 and l_2. Then $l_1 - l_2 = 10$ cm.

Since the difference is always same at all temperatures, their increase in length must be the same whatever the rise in temperature

∴ $\Delta l = l_1\,\alpha_{Fe}\,\Delta T = l_2\alpha_{Cu}\,\Delta T$

or $\frac{l_1}{l_2} = \frac{\alpha_{Cu}}{\alpha_{Fe}} = \frac{17}{11}$

Also $l_1 - l_2 = 10$ cm

Solving for l_1 and l_2

$l_1 = 28.3$ cm $l_2 = 18.3$ cm

4. $\Delta l = l\,\alpha\,\Delta T$ or $0.075 = 30 \times \alpha_A \times 100$

or $\alpha_A = 25 \times 10^{-6}$ °C^{-1}

and $0.045 = 45 \times \alpha_A \times 100$

or $\alpha_B = 10 \times 10^{-6}$ °C^{-1}

Let l_1 and l_2 be the lengths of the two sections of C.

Then $l_1 + l_2 = 45$ and

$0.04 = l_1 \times 25 \times 10^{-6} \times 50 + l_2 \times 10 \times 10^{-6} \times 50$

or $160 = 5l_1 + 2l_2$

solving $l_1 = 23.3$ cm

$l_2 = 21.7$ cm

5. Since the scale is graduated at 10°C,

1 cm of the scale at 10°C = exactly 1 cm

$\therefore$ 1 cm of the scale at 30°C

$= \text{exactly } (1 + 18 \times 10^{-6} \times 20)$ cm

$\therefore$ 60 cm of the scale at 30°C

$= \text{exactly } 60.00\,(1 + 36 \times 10^{-5})$

$= 60.02$ cm

6. $\left(\gamma_{\text{mercury}} = 18.2 \times 10^{-5}\,\text{K}^{-1} \quad \alpha_{\text{glass}} = 9 \times 10^{-6}\,\text{K}^{-1}\right)$

Let V_0, V_t = volume of mercury at 0°C and t°C, respectively. A_0, A_t = area of cross-section of capillary tube at 0°C and t°C, respectively.

Then $V_0 = l_0 A_0$, $V_t = l_t A_t$ and $V_t = V_0(1 + \gamma_r t)$

$\therefore \quad l_t A_t = l_0 A_0 (1 + \gamma_r t)$

or $\quad l_t A_0 (1 + 2\alpha_g t) = l_0 A_0 (1 + \gamma_r t)$

or $\quad l_t = l_0 \dfrac{1 + \gamma_r t}{1 + 2\alpha_g t}$

expanding and neglecting negligible terms

$l_t = l_0\,[1 + (\gamma_r - 2\alpha_g)\,t]$

$\therefore \quad l_{100} = 1\,[1 + (182 \times 10^{-6} - 2 \times 9 \times 10^{-6})\,100]$

$= 1.0164$ m

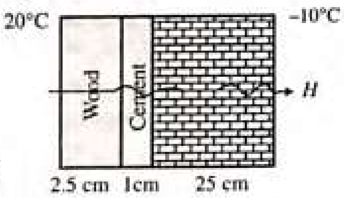
Let L = required reading of thread, on glass scale, at temperature t. Then this section of the glass (scale) has length L at 0°C and l_t at t°C.

or $\quad L = \dfrac{l_0\left[1 + (\gamma_r - 2\alpha_g)t\right]}{1 + \alpha_g t} = l_0\,[1 + (\gamma_r - 30_g)\,t]$

or $\quad L = 1[1 + (182 \times 10^{-6} - 3 \times 9 \times 10^{-6})\,100] = 1.0155$ m

Concept Application Exercise 13.3

1. Equivalent thermal conductivity of the wall



$$K = \frac{l_1 + l_2 + l_3}{\dfrac{l_1}{K_1} + \dfrac{l_2}{K_2} + \dfrac{l_3}{K_3}}$$

$$= \frac{0.025 + 0.01 + 0.25}{\left(\dfrac{0.025}{0.125} + \dfrac{0.01}{1.5} + \dfrac{0.25}{1.0}\right)}$$

$$= \frac{0.285}{0.457} = 0.624\ \text{W/m°C}$$

The rate of flow of heat is given by

$$H = KA\frac{T_1 - T_2}{L} = 0.624 \times 137 \times \frac{[20 - (-10)]}{0.285}$$

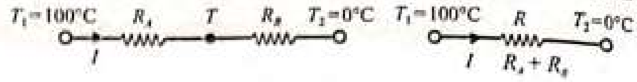
$$= \frac{0.624 \times 137 \times 30}{0.285} = 9000\ \text{W}$$

2. i. Thermal resistance is defined as $R = l/kA$

For rod A $\quad R_A = \dfrac{l}{k_A A} = \dfrac{l}{3kA}$

For rod B $\quad R_B = \dfrac{l}{k_B A} = \dfrac{l}{kA}$

ii. Using electrical analogy, the given problem may be modified as



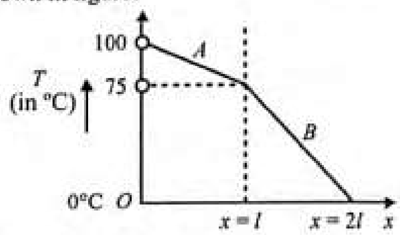
The equivalent resistance is $R = R_A + R_B = \dfrac{4}{3}\dfrac{l}{kA}$

The heat current is given by $I = \dfrac{T_1 - T_2}{R} = \dfrac{100 - 0}{\dfrac{4}{3}\dfrac{l}{kA}} = 75\left(\dfrac{kA}{l}\right)$

iii. The temperature of the junction is given by

$$T = T_1 - IR_A = 100 - \frac{75kA}{l}\left(\frac{l}{3kA}\right) = 75°\text{C}$$

iv. The variation of temperature along the length of the conductor is shown in figure.

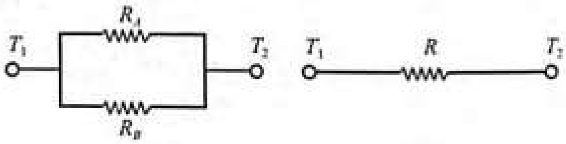


3. i. The thermal resistance of the two rods are

$$R_A = \frac{l}{3kA}; \quad R_B = \frac{l}{kA}$$

The equivalent resistance is

$$\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_B} = \frac{3kA}{l} + \frac{kA}{l} = \frac{4kA}{l}$$



or $\quad R = \dfrac{l}{4kA}$

ii. The heat current through each rod is

$$I_A = \frac{T_1 - T_2}{R_A} = \frac{100 - 0}{l/3kA} = 300\left(\frac{kA}{l}\right)$$

$$I_B = \frac{T_1 - T_2}{R_B} = \frac{100 - 0}{l/kA} = 100\left(\frac{kA}{l}\right)$$

Total heat current is

$$I = I_A + I_B = 400\frac{kA}{l}$$

4. Let the thermal conductivities of the rod AB, BC and BD be K, $2K$, and $3K$ respectively. Also, let the length be $2L$, L and L.

If T be the required temperature of the junction B and assuming $T_1 > T > T_2$ and $T > T_3$, we have

$$\left.\frac{\Delta Q}{\Delta t}\right]_{AB} = \left.\frac{\Delta Q}{\Delta t}\right]_{BC} + \left.\frac{\Delta Q}{\Delta t}\right]_{BD} \qquad \text{(Point rule)}$$

$$\frac{KA(T_1 - T)}{2L} = \frac{2KA(T - T_2)}{L} + \frac{3KA(T - T_3)}{L}$$

$$\frac{T_1 - T}{2} = 2(T - T_2) + 3(T - T_3)$$

$$T = \frac{1}{11}(T_1 + 4T_2 + 6T_3)$$

5. From equation, the rate of cooling of a body is given by

$$\frac{-dT}{dt} = \frac{Ae\sigma}{\rho Vs}(T^4 - T_0^4)$$

Since, substance is same for both bodies, so $e/\rho s$ = constant. Finally, they are allowed to cool under identical conditions, so $(T^4 - T_0^4)$ = constant.

$$\frac{-dT}{dt} \propto \frac{A}{V}$$

Let the edge of the cube or radius of the sphere be a, then, for cube; $A = 6a^2$ and $V = a^3$ so $A/V = 6a$

For the sphere; $A = 4\pi a^2$ and $V = (4/3)\pi a^3$; so $A/V = 3/a$

Evidently, the ratio A/V is more for cube, so, the cube cools at a faster rate. Note the special technique used in this problem.

6. It is given that the temperature of ball is constant. This means that the rate at which it loosing heat by radiation must be equal to the rate at which heat is supplied to this ball externally to keep its temperature constant.

The rate of heat loss by the ball is given as

$$\frac{dQ}{dt} = eA\sigma(T^4 - T_s^4)$$

$= 0.3 \times 4\pi(0.01)^2 \times 5.67 \times 10^{-8} \times [(1000)^4 - (300)^4] = 21.1$ W

Thus electrical energy must be supplied to the ball at a rate of 21.1 W.

Exercises

Thermal Expansion

1. (c) On heating the system; x, r and d all increase, since the expansion of isotropic solids is similar to true photographic enlargement.

2. (b) If the sheet is heated then both d_1 and d_2 will increase since the thermal expansion of isotropic solid is similar to true photographic enlargement.

3. (b) The effective value of α at a distance x from the left end is

$$\alpha_x = \alpha_1 + \left(\frac{\alpha_2 - \alpha_1}{L}\right)x$$

$$\Delta L = \int_0^L \alpha_x dx \Delta t$$

$$L = \left(\frac{\alpha_1 + \alpha_2}{2}\right)L\Delta T$$

$$\alpha_{\text{eff}} = \frac{\alpha_1 + \alpha_2}{2}$$

4. (c) Initial diameter of tyre = (1000 − 6) mm; = 994 mm, so initial radius of tyre

$$R = \frac{994}{2} = 497 \text{ mm}$$

and change in diameter $\Delta D = 6$ mm; so

$$\Delta R = \frac{6}{2} = 3 \text{ mm}$$

Given that after increasing temperature by ΔT tyre will fit onto wheel. Increment in the length (circumference) of the iron tyre

$$\Delta L = L \times \alpha \times \Delta T = L \times \frac{\gamma}{3} \times \Delta T \qquad \left(\text{As } \alpha = \frac{\gamma}{3}\right)$$

$$\Rightarrow \quad 2\pi\Delta R = 2\pi R\left(\frac{\gamma}{3}\right)\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{3}{\gamma}\frac{\Delta R}{R} = \frac{3 \times 3}{3.6 \times 10^{-5} \times 497}$$

[As $\Delta R = 3$ mm and $R = 497$ mm]

$$\Rightarrow \quad \Delta T = 500°\text{C}$$

5. (b) Apparent coefficient of volume expansion

$\gamma_{\text{app.}} = \gamma_L - \gamma_s = 7\gamma_s - \gamma_s = 6\gamma_s$ (given $\gamma_L = 7\gamma_s$)

Ratio of absolute and apparent expansion of liquid

$$\frac{\gamma_L}{\gamma_{\text{app.}}} = \frac{7\gamma_s}{6\gamma_s} = \frac{7}{6}$$

6. (a) As with the rise in temperature, the liquid undergoes volume expansion therefore the fraction of solid submerged in liquid increases.

Fraction of solid submerged at t_1°C $= f_1$ = Volume of displaced liquid

$= V_0(1 + \gamma t_1)$ (i)

and fraction of solid submerged at t_2°C $= f_2$ = Volume of displaced liquid $= V_0(1 + \gamma t_2)$ (ii)

From Eqs. (i) and (ii),

$$\frac{f_1}{f_2} = \frac{1 + \gamma t_1}{1 + \gamma t_2} \Rightarrow \gamma = \frac{f_1 - f_2}{f_2 t_1 - f_1 t_2}$$

7. (a) Loss of time due to heating a pendulum is given as

$$\Delta T = \frac{1}{2}\alpha\Delta\theta T$$

$$\Rightarrow \quad 12.5 = \frac{1}{2} \times \alpha \times (25 - 0)°\text{C} \times 86400$$

$$\Rightarrow \quad \alpha = \frac{1}{86400} /°\text{C}$$

8. (d) Due to heating the length of the wire increases.

∴ Longitudinal strain is produced

$$\Rightarrow \quad \frac{\Delta L}{L} = \alpha \times \Delta T$$

Elastic potential energy per unit volume

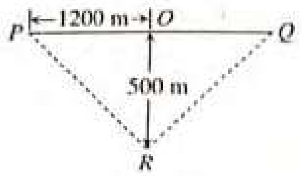
$$E = \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} \times Y \times (\text{Strain})^2$$

$$\Rightarrow \quad E = \frac{1}{2} \times Y \times \left(\frac{\Delta L}{L}\right)^2 = \frac{1}{2} \times Y \times \alpha^2 \times \Delta T^2$$

$$\text{or} \quad E = \frac{1}{2} \times Y \times \left(\frac{\gamma}{3}\right)^2 \times T^2 = \frac{1}{18}\gamma^2 Y T^2$$

[As $\gamma = 3\alpha$ and $\Delta T = T$ (given)]

9. (c) Span of bridge = 2400 m and bridge sags by 500 m at 30° (given)



From figure, $L_{PRQ} = 2\sqrt{1200^2 + 500^2} = 2600$ m

But $L = L_0(1+\alpha\Delta t)$ (Due to linear expansion)

$\Rightarrow$ $2600 = L_0(1+12\times10^{-6}\times30)$

$\therefore$ Length of the cable $L_0 = 2599$ m

Now change in length of cable due to change in temperature from 10°C to 42°C

$$\Delta L = 2599\times12\times10^{-6}\times(42-10) = 0.99 \text{ m}$$

10. (c) Thermal stress that is produced in an elastic wire is $Y\alpha\theta$ per unit area, where Y is Young's modulus, α coefficient of linear expansion and θ change of temperature. Thus, tension developed in the wire is

T = Thermal stress × area of cross section

$= Y\alpha\times(\Delta T)[\pi r^2]$ N

$= 0.91\times10^{11}\times2\times10^{-5}\times[27-(-33)][\pi\times1\times10^{-6}]$ N

$= 0.91\times2\times6\times3.14\times10$ N

$= 34.4\times10\ N = 0.34$ kN

11. (a) $U_0 = V_0\sigma_L^0 g = W_0$ and $U_t = V_t\sigma_L^t g = W$

$$\frac{W}{W_0} = \frac{V_t}{V_0}\times\frac{\sigma_L^t}{\sigma_L^0} = \frac{(1+\gamma_B\Delta\theta)}{(1+\gamma_L\Delta\theta)}$$

$= (1+\gamma_S\Delta\theta)(1+\gamma_L\Delta\theta)^{-1} = 1+\gamma_S\Delta\theta - \gamma_L\Delta\theta$

$= W = W_0[1+(\gamma_S-\gamma_L)\Delta\theta)]$

$= W_0[1+(\gamma_S-\gamma_L)t]$

12. (b) $\dfrac{T_{20}-T_0}{T_0} = \dfrac{1}{2}\times\alpha\times20 = 10\alpha$

Here T_{20} is the correct time period. The time period at 0°C is smaller so that the clock runs fast. The time gained in 24 h.

$= 24\text{ h}\times\alpha\times10$

$\Rightarrow$ $15\text{ s} = 24\text{ h}\times\alpha\times10$

$$\alpha = \frac{15s}{86400s\times10°C} = 1.7\times10^{-5}/°C$$

13. (c) Volume of mercury at 18°C

$V_0 = 50$ cc

Volume of mercury at 38°C

$(V_{38})_r = V_0(1+\gamma_m\Delta\theta)$

Volume of flask at 38°C

$(V_{38})_f = V_0(1+\gamma_f\Delta\theta)$

Volume of mercury at 38°C above the tank

$= V_0(1+\gamma_m\Delta\theta) - V_0(1+\gamma_f\Delta\theta)$

$= V_0(\gamma_m-\gamma_f)\Delta\theta$

$= 50[180\times10^{-6} - 3\times9\times10^{-6}](38-18)$

$= 0.153$ cc

14. (b) Increase in volume of flask

$= 40\times10^{-6}\times10^3\times10^2 = 4$ cc

Increase in volume of mercury

$= 180\times10^{-6}\times10^3\times10^2 = 18$ cc

Volume of mercury overflow

$= 18-4 = 14$ cc

15. (a) The 80 cm mark on the aluminium rod is really at a greater distance from the zero position than indicated because of the increase in temperature $\Delta\theta = 40$°C. The increased length is

$\Delta L = \alpha_{Al}L_{Al}\Delta\theta$

$= (2.50\times10^{-5})(80)(40) = 0.08$ cm

The correct length of the line is

$L = 80+0.08 = 80.08$ cm

16. (c) It is given that the volume of air in the flask remains the same. This means that the expansion in volume of the vessel is exactly equal to the volume expansion of mercury.

i.e., $\Delta V_G = \Delta V_L$

or $V_G\gamma_G\Delta\theta = V_L\gamma_L\Delta\theta$

$$\therefore\quad V_L = \frac{V_G\gamma_G}{\gamma_L} = \frac{1000\times(3\times9\times10^{-6})}{1.8\times10^{-4}} = 150 \text{ cc}$$

Calorimetry

17. (a) Work done in converting 1 g of ice at −10°C to steam at 100°C

= Heat supplied to raise temperature of 1 g of ice from − 10°C to 0°C ($m\times c_{ice}\times\Delta T$)

+ Heat supplied to convert 1 g ice into water at 0°C ($m\times L_{ice}$)

+ Heat supplied to raise temperature of 1 g of water from 0°C to 100°C ($m\times c_{water}\times\Delta T$)

+ Heat supplied to convert 1 g water into steam at 100°C [$m\times L_{vapour}$]

$= [m\times c_{ice}\times\Delta T] + [m\times L_{ice}] + [m\times c_{water}\times\Delta T] + [m\times L_{vapour}]$

$= [1\times0.5\times10] + [1\times80] + [1\times1\times100] + [1\times540]$

$= 725$ cal $= 725\times4.2 = 3045$ J

18. (c) Heat required to raise the temperature of m grams of substance by dT is given as

$$dQ = mc\,dT \quad \Rightarrow Q = \int mc\,dT$$

Therefore, to raise the temperature of 2 g of substance from 5°C to 15°C

$$Q = \int_5^{15} 2\times(0.2+0.14t+0.023t^2)dT$$

$$= 2\times\left[0.2t + \frac{0.14t^2}{2} + \frac{0.023t^3}{3}\right]_5^{15} = 82 \text{ cal}$$

19. (a) Same amount of heat is supplied to copper and water; so

$m_c c_c\Delta T_c = m_\omega c_\omega\Delta T_\omega$

$$\Rightarrow\quad (\Delta T)_\omega = \frac{m_c c_c\Delta T_c}{m_\omega c_\omega} = \frac{50\times10^{-3}\times420\times10}{10\times10^{-3}\times4200} = 5°C$$

20. (c) Heat lost by A = Heat gained by B

$\Rightarrow$ $m_A\times c_A\times(T_A-T) = m_B\times c_B\times(T-T_B)$

Since $m_A = m_B$ and temperature of the mixture (T) = 28°C

$\therefore$ $c_A\times(32-28) = c_B\times(28-24)$

$\Rightarrow$ $\dfrac{c_A}{c_B} = 1:1$

21. (b) Heat lost by hot water = Heat gained by cold water in beaker + Heat absorbed by beaker

$\Rightarrow$ $440\,(92 - T) = 200 \times (T - 20) + 20 \times (T - 20)$

$\Rightarrow$ $T = 68°C$

22. (a) $Q_4 = mL_w = 540$ cal

$Q_3 = ms_w\,(100 - 0) = 100$ cal

$Q_2 = mL_{ice} = 80$ cal

$Q_1 = ms_{ice}\,(20 - 0) = 20$ cal

$Q_4 > Q_3 > Q_2 > Q_1$

23. (c) Mass of water = 250 g

Mass of alcohol = 200 g

Water equivalent of calorimeter, W = 10 g

Fall of temperature = 60 − 55 = 5°C

Time taken by water to cool = 130 s

Time taken by alcohol to cool = 67 s

Heat lost by water and calorimeter

$= (250 + 10)\,5 = 260 \times 5 = 1300$ cal

Rate of loss of heat $= \dfrac{1300}{130} = 10$ cal/s

Heat lost by alcohol and calorimeter = $(200s + 10)5$

Rate of loss of heat $= \dfrac{(200s+10)5}{67}$ cal/s

Rates of loss of heat in the two cases are equal

$\therefore \quad \dfrac{(200s+10)s}{67} = 10$ or $s = 0.62\,\text{cal/g°C}$

24. (a) The kinetic energy of the bullet will be utilized to melt the bullet

$$\frac{1}{2}mv^2 = (ms\Delta\theta)\ J$$

$$\frac{1}{2} \times 2 \times 10^{-3} \times (200)^2 = 2 \times 0.03 \times \Delta\theta \times 4.2$$

$\Delta\theta = 158°C$

25. (c) $Q_1 = 10 \times 1 \times 10 = 100$ cal

$Q_2 = 10 \times 0.50\,[0 - (-20)] + 10 \times 80$

$= (100 + 800)$ cal $= 900$ cal

As $Q_1 < Q_2$, so ice will not completely melt and final temperature = 0°C.

As heat given by water in cooling up to 0°C is only just sufficient to increase the temperature of the ice from − 20°C to 0°C, hence mixture in equilibrium will consist of 10 g of ice and 10 g of water, both at 0°C.

26. (d) Let m grams of water whose temperature is θ_0 (> 30°C) and specific heat is 1 cal/g-°C be added to 20 g of water at 30°C and let θ be the final temperature of mixture.

$$m(1)(\theta_0 - \theta)(20)(1)(\theta - 30)$$

$$\therefore \quad \theta = \frac{600 + m\theta_0}{20 + m}$$

The right-hand side is maximum for option (d). Therefore, the correct answer is (d).

27. (a) By the law of conservation of energy, energy given by heater must be equal to the sum of energy gained by water and energy lost from the lid.

$Pt = ms\Delta\theta$ + energy lost

i.e., $1000t = 2 \times (4.2 \times 10^3) \times 50 + 160t$

or $840t = 8.4 \times 10^3 \times 50$

or $t = 500$ s = 8 min 20 s

28. (c) As water equivalent of pitcher is 0.5 kg, i.e., pitcher is equivalent to 0.5 kg of water, heat to be extracted from the system of water and pitcher for decreasing its temperature from 30 to 28°C is

$Q_1 = (m + M)\,c\,\Delta T$

$= (9.5 + 0.5)$ kg (1 kcal/kg°C)(30 − 28)°C = 20 kcal

And heat extracted from the pitcher through evaporation in t minutes

$$Q_2 = mL = \left[\frac{dm}{dt} \times t\right]L = \left[\frac{1\text{g}}{\text{min}} \times t\right]580\,\frac{\text{cal}}{\text{g}} = 580 \times t \text{ cal}$$

According to given problem $Q_2 = Q_1$, i.e., $580 \times t = 20 \times 10^3$

$t = 34.5$ min

29. (a) Here ice will absorb heat while hot water will release it. So if T is the final temperature of the mixture, heat given by water

$Q_1 = mc\,\Delta T = 5 \times 1 \times (30 - T)$

And heat absorbed by ice

$Q_2 = 5 \times (1/2)\,[0 - (-20)] + 5 \times 80 + 5 \times 1(T - 0)$

So, by principle of calorimetry $Q_1 = Q_2$, i.e.,

$150 - 5T = 450 + 5T$

$T = -30°C$

Which is impossible as a body cannot be cooled to a temperature below the temperature of cooling body. The physical reason for this discrepancy is the heat remaining after changing the temperature of ice from −20 to 0°C with some ice left unmelted and we are taking it for granted that heat is transferred from water at 0°C to ice at 0°C so that temperature of system drops below 0°C.

However, as heat cannot flow from one body (water) to the other (ice) at same temperature (0°C), the temperature of system will not fall below 0°C.

30. (b) Evidently the initial temperature of the water contained in the vessel (Mg) is 80°C, and the temperature of the water passed into it is 60°C, as the final temperature of the mixture tends to attain a value of 60°C.

$M \times 1\,(80 - 70) = m \times 10 \times 1\,(70 - 60)$

or, $M/m = 10$

Since the heat exchanged after a long time is 800 cal.

(Mg) (1 cal/m°C) (80 − 60°C) = 80 cal

or, $M = 40$ g

$\Rightarrow$ $m = 4$ g

Transmission of Heat

31. (c) $\dfrac{dQ/dt}{A} = K\left(\dfrac{\Delta\theta}{\Delta x}\right)$

$\Rightarrow$ Rate of flow of heat per unit area

= Thermal conductivity × Temperature gradient

Temperature gradient $(X) \propto \dfrac{1}{\text{Thermal conductivity (K)}}$

(As $\dfrac{dQ/dt}{A}$ = constant)

As $K_C > K_m > K_g$, therefore $X_c < X_m < X_g$

32. (b) $\left(\dfrac{dQ}{dt}\right) \times \dfrac{1}{A} = K_A\,\dfrac{(50-30)}{3}$ (for slab A)

$$= K_B \frac{(50-20)}{3} \qquad \text{(for slab } B\text{)}$$

$$2K_A = 3K_B$$

or, $\quad K_A/K_B = 3/2$

33. (b) Rate of flow of heat or power $P = \frac{KA\Delta\theta}{\Delta r} = \frac{K4\pi R^2 T}{\Delta r}$

$\therefore$ Thickness of shell $\Delta r = \frac{4\pi R^2 KT}{P}$

34. (d) Rate of flow of heat will be equal in both vest and shirt

$$\therefore \quad \frac{K_{vest} A.\Delta\theta_{vest} t}{l} = \frac{K_{shirt} A\Delta\theta_{shirt} t}{l}$$

$$\Rightarrow \quad \frac{K_{vest}}{K_{shirt}} = \frac{\Delta\theta_{shirt}}{\Delta\theta_{vest}} \quad \Rightarrow \frac{K_{vest}}{K_{shirt}} = \frac{25-22}{30-25} = \frac{3}{5}$$

35. (b) $Q = KA\frac{\Delta\theta}{l} t \quad \therefore t \propto \frac{l}{A}$ (As Q, K and $\Delta\theta$ are constant)

$$\frac{t_1}{t_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \left(\frac{l_1}{l_1/2}\right) \times \left(\frac{2A_1}{A_1}\right)$$

$$\frac{t_1}{t_2} = 4 \quad \Rightarrow t_2 = \frac{t_1}{4} = \frac{12}{4} = 3 \text{ s}$$

36. (c) Rate of flow of heat along PQ

$$\left(\frac{dQ}{dt}\right)_{PQ} = \frac{K_3 A\Delta\theta}{l} \qquad \text{(i)}$$

Rate of flow of heat along PRQ

$$\left(\frac{dQ}{dt}\right)_{PRQ} = \frac{K_s A\Delta\theta}{2l}$$

Effective conductivity for series combination of two rods of same length

$$K_s = \frac{2K_1K_2}{K_1+K_2}$$

So $\quad \left(\frac{dQ}{dt}\right)_{PRQ} = \frac{2K_1K_2}{K_1+K_2} \cdot \frac{A\Delta\theta}{2l} = \frac{K_1K_2}{K_1+K_2} \cdot \frac{A\Delta\theta}{l} \qquad \text{(ii)}$

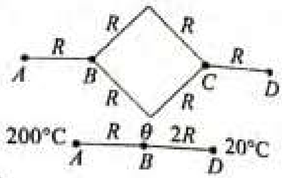
Equating Eqs. (i) and (ii) $K_3 = \frac{K_1K_2}{K_1+K_2}$

37. (a) $K_1 = 9K_2$, $l_1 = 18$ cm, $l_2 = 6$ cm, $\theta_1 = 100.C$, $\theta_2 = 0.C$

Temperature of the junction $\theta = \dfrac{\frac{K_1}{l_1}\theta_1 + \frac{K_2}{l_2}\theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$

$$\Rightarrow \quad \theta = \frac{\frac{9K_2}{18}100 + \frac{K_2}{6}\times 0}{\frac{9K_2}{18} + \frac{K_2}{6}} = \frac{50+0}{8/12} = 75°C$$

38. (c) Let the thermal resistance of each rod be R.



The two resistances connected along two paths from B to C are equivalent to $2R$ each and their parallel combination is R.

Effective thermal resistance between B and $D = 2R$

Temperature of interface $\theta = \frac{R_1\theta_2 + R_2\theta_1}{R_1 + R_2}$

$$\theta = \frac{R\times 20 + 2R\times 200}{R+2R} = \frac{420}{3} = 140°C$$

39. (c) Smooth and polished plates are poor radiators of heat. Hence, heat coming out from A is small, even though B being a black and rough plate is a good absorber. Effectively the heat coming to the left of pellet P is small.

Black and rough plates are good radiators of heat. Hence, plate B_2 radiates heat to a satisfactory level; however, plate A_2, being smooth and polished, is a bad absorber. Effectively, the heat coming to the right of P is also small.

40. (d) $\frac{dT}{dt} = \frac{\sigma A}{mc}(T^4 - T_0^4)$. If the liquids are put in exactly similar calorimeters and identical surrounding then we can consider T_0 and A constant. Then

$$\frac{dT}{dt} \propto \frac{(T^4 - T_0^4)}{mc} \qquad \text{(i)}$$

If we consider that equal masses of liquids (m) are taken at the same temperature then

$$\frac{dT}{dt} \propto \frac{1}{c}$$

So for same rate of cooling c should be equal, which is not possible because liquids are of different nature.

Again from Eq. (i)

$$\frac{dT}{dt} \propto \frac{(T^4 - T_0^4)}{mc} \quad \Rightarrow \quad \frac{dT}{dt} \propto \frac{(T^4 - T_0^4)}{V\rho c}$$

Now if we consider that equal volumes of liquids (V) are taken at the same temperature then

$$\frac{dT}{dt} \propto \frac{1}{\rho c}$$

So for same rate of cooling multiplication of $\rho \times c$ for two liquids of different nature can be possible. So option (d) may be correct.

41. (c) $\frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{\Delta x} \quad \Rightarrow \quad \Delta Q = KA\left(\frac{\Delta T}{\Delta x}\right)\Delta t$

Assuming the thickness of the spheres to be small, we have

For smaller sphere:

(rate of heat flow) (time) = (volume of ice melted) (ρL)

i.e., $\quad K_1(4\pi r^2)\frac{\Delta\theta}{d} \cdot 16 = \frac{4}{3}\pi r^3 \rho L \qquad \text{(i)}$

For larger sphere:

$$K_2[4\pi(2r)^2]\frac{\Delta\theta}{d/4}\cdot 25=\frac{4\pi}{3}(2r)^3\rho L \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i),

$$K_2/K_1 = 8/25$$

42. (b) From Wien's displacement law $\lambda_m T = b$

$$\therefore \quad T=\frac{b}{\lambda_m}=\frac{2.93\times10^{-3}}{2.93\times10^{-10}}=10^7\text{ K}$$

43. (d) Energy radiated by body per second $\frac{Q}{t}=A\sigma T^4$

or $\quad \frac{Q}{t}\propto l\times b\times T^4 \quad$ (Area $= l\times b$)

$$\therefore \quad \frac{E_2}{E_1}=\frac{l_2}{l_1}\times\frac{b_2}{b_1}\times\left(\frac{T_2}{T_1}\right)^4=\frac{(l_1/2)}{l_1}\times\frac{(b_1/2)}{b_1}\times\left(\frac{600}{400}\right)^4$$

$$=\frac{1}{2}\times\frac{1}{2}\times\left(\frac{3}{2}\right)^4 \quad \Rightarrow E_2=\frac{81}{64}E$$

44. (c) $\frac{dT}{dt}=\frac{eA\sigma}{mc}\left(T^4-T_0^4\right)=\frac{e(6a^2)\sigma}{(a^3\times\rho)c}(T^4-T_0^4)$

$\Rightarrow$ For the same fall in temperature, time $dt\propto a$

$$\frac{dt_2}{dt_1}=\frac{a_2}{a_1}=\frac{2\text{ cm}}{1\text{ cm}} \quad \Rightarrow dt_2=2\times dt_1=2\times100\text{ s}=200\text{ s}$$

(As $A=6a_2$ and $m=V\times\rho=a^3\times\rho$)

45. (a) $\frac{dT}{dt}=\frac{eA\sigma}{mc}(T^4-T_0^4)=\frac{eA\sigma}{V\rho c}(T^4-T_0^4)$

$\therefore$ Rate of cooling $R\propto A$

(As masses are equal, volume of each body must be equal because material is same)

i.e., rate of cooling depends on the area of cross section and we know that for a given volume the area of cross section will be minimum for sphere. It means the rate of cooling will be minimum in case of sphere.

So the temperature of sphere drops to room temperature at last.

46. (c) $Q=\sigma At\,(T^4-T_0^4)$

If T, T_0, σ and t are same for both bodies then

$$\frac{Q_{\text{sphere}}}{Q_{\text{cube}}}=\frac{A_{\text{sphere}}}{A_{\text{cube}}}=\frac{4\pi r^2}{6a^2} \quad \text{(i)}$$

But according to problem, volume of sphere = volume of cube

$$\Rightarrow \quad \frac{4}{3}\pi r^3=a^3 \quad \Rightarrow \quad a=\left(\frac{4}{3}\pi\right)^{1/3}r$$

Substituting the value of a in Eq. (i), we get

$$\frac{Q_{\text{sphere}}}{Q_{\text{cube}}}=\frac{4\pi r^2}{6a^2}=\frac{4\pi r^2}{6\left\{\left(\frac{4}{3}\pi\right)^{1/3}r\right\}^2}=\frac{4\pi r^2}{6\left(\frac{4}{3}\pi\right)^{2/3}r^2}$$

$$=\left(\frac{\pi}{6}\right)^{1/3}:1$$

47. (c) According to Newton's law of cooling the rate of cooling depends upon the difference of temperature between the body and the surrounding. It means that when the difference of temperature between the body and the surrounding is small, time required for same fall in temperature is more in comparison with the same fall at higher temperature difference between the body and surrounding. So according to problem $T_1<T_2<T_3$.

48. (d) According to Newton's law of cooling

$$\frac{\theta_1-\theta_2}{t}\propto\left[\frac{\theta_1+\theta_2}{2}-\theta\right]$$

For the first condition

$$\frac{80-60}{60}\propto\left[\frac{80+60}{2}-30\right] \quad \text{(i)}$$

and for the second condition

$$\frac{60-50}{t}\propto\left[\frac{60+50}{2}-30\right] \quad \text{(ii)}$$

By solving Eqs. (i) and (ii), we get $t=48$ s.

49. (b) According to Newton's law of cooling

$$\frac{\theta_1-\theta_2}{t}\propto\left[\frac{\theta_1+\theta_2}{2}-\theta\right]$$

For the first condition

$$\frac{62-61}{T}\propto\left[\frac{62+61}{2}-30\right] \quad \text{(i)}$$

and for the second condition

$$\frac{46-45.5}{t}\propto\left[\frac{46+45.5}{2}-30\right] \quad \text{(ii)}$$

By solving Eqs. (i) and (ii), we get $t=T$ minutes.

50. (b) $\frac{\theta_1-\theta_2}{t}\propto\left[\frac{\theta_1+\theta_2}{2}-\theta\right]$

For the first condition

$$\frac{60-50}{10}\propto\left[\frac{60+50}{2}-\theta\right] \quad \Rightarrow 1=K[55-\theta] \quad \text{(i)}$$

For the second condition

$$\frac{50-42}{10}\propto\left[\frac{50+42}{2}-\theta\right] \quad \Rightarrow 0.8=K(46-\theta) \quad \text{(ii)}$$

From Eqs. (i) and (ii), we get $\theta=10°\text{C}$

51. (a) According to Newton's law of cooling,

$$\left[\frac{\theta_1-\theta_2}{t}\right]=K\left[\left(\frac{\theta_1+\theta_2}{2}\right)-\theta_0\right]$$

So that $\left[\frac{60-40}{7}\right]=K\left[\left(\frac{60+40}{2}\right)-10\right]$

$$\Rightarrow \quad K=\frac{1}{14} \quad \text{(i)}$$

Now if after cooling from 40°C to 7 min the temperature of the body becomes θ, according to Newton's law of cooling,

$$\left[\frac{40-\theta}{7}\right]=K\left[\left(\frac{40+\theta}{2}\right)-10\right]$$

Which in the light of Eq. (i), i.e., $K=(1/14)$, gives

$$\left[\frac{40-\theta}{7}\right]=\frac{1}{14}\left[\left(\frac{20+\theta}{2}\right)\right]$$

$$160-4\theta=20+\theta;\ \theta=28°\text{C}$$

52. (b) According to Newton's law of cooling, the ratio of cooling is directly proportional to the temperature difference. When the average temperature difference is halved, the rate of cooling is also halved. So, the time taken is 10 s.

53. (b) According to Newton's law of cooling

$$\left(\frac{\theta_1-\theta_2}{t}\right)=K\left[\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)\right]$$

So that $\left(\frac{80-50}{5}\right)=K\left[\left(\frac{80+50}{2}-20\right)\right]$

And $\left(\frac{60-30}{t}\right)=K\left[\left(\frac{60+30}{2}\right)-20\right]$

Solving these for t, we get $t=9$ min.

Problems Based on Mixed Concepts

54. (d) Quantity of heat transferred through wall will be utilized in melting of ice.

$$Q=\frac{KA\Delta\theta t}{\Delta x}=mL$$

$\therefore$ Amount of ice melted $m=\dfrac{KA\Delta\theta t}{\Delta x L}$

$$\therefore\quad m=\frac{0.01\times1\times(30-0)\times86400}{5\times10^{-2}\times334\times10^{3}}=1.552\text{ kg or }1552\text{ gm}$$

55. (c) The change in length of wire $= l_{Al}\,\alpha_{Al}\,\Delta\theta+l_{st}\,\alpha_{st}\,\Delta\theta$ associated with temperature change of $\Delta\theta$, where α_{Al} and α_{St} are the coefficient of linear expansion of aluminium and steel, respectively

$$\alpha_{Al}=23\times10^{-6}/°\text{C}$$

$$\alpha_{st}=12\times10^{-6}/°\text{C}$$

The effective coefficient of linear expansion of the two segments of wire $=19\times10^{-6}/°$C

$$l_1\alpha_{Al}\Delta\theta+l_2\alpha_{st}\Delta\theta=(l_1+l_2)\alpha\Delta\theta$$

$$\frac{l_1}{(l_1+l_2)}=\frac{\alpha-\dfrac{l_2}{(l_1+l_2)}\alpha_{st}}{\alpha_{Al}}$$

$$\left[\frac{l_1}{l_1+l_2}=x,\ \ l_1+l_2=l,\ \left(\frac{l_2}{l_1+l_2}\right)=1-x\right]$$

$$x=\frac{\alpha-(1-x)\alpha_{st}}{\alpha_{Al}}$$

$$x=\frac{\alpha-\alpha_{st}}{\alpha_{Al}-\alpha_{st}}=\frac{19\times10^{-6}-12\times10^{-6}}{23\times10^{-6}-12\times10^{-6}}=\frac{7}{11}$$

56. (c) Fraction of wooden block immersed at 0°C,

$$\frac{V_1}{V_0}=\frac{(\rho_{wood})_{0°C}}{(\rho_{H_2O})_{0°C}}$$

$$f_1=\frac{V_0-V_1}{V_0}=\frac{(\rho_{H_2O})_{0°C}-(\rho_{wood})_{0°C}}{(\rho_{H_2O})_{0°C}}$$

V_1 – Volume of wood immersed in water at 0°C
V_0 – Volume of wood
$(\rho_{wood})_{0°C}$ – Density of wood at 0°C
When the temperature is raised to 10°C, the volume of wood immersed in water changes to V_2.

$$\frac{V_2}{V_0}=\frac{(\rho_{wood})_{10°C}}{(\rho_{H_2O})_{10°C}}$$

$$f_2=\frac{V_0-V_2}{V_0}=\frac{(\rho_{H_2O})_{10°C}-(\rho_{wood})_{10°C}}{(\rho_{H_2O})_{10°C}}$$

From 0°C to 4°C, the density of water increases, and from 4°C to 10°C the density of water decreases.
But for wood density decreases as temperature increases. The volume of block above water level will first increase and then decrease.

57. (b) Power sent to heat the water in the calorimeter

$$P'=\frac{ms\Delta\theta}{t}=\frac{V\rho s\Delta\theta}{t}=\frac{10^3\times10^{-6}\times10^3\times4200\times4}{420}=40\text{ W}$$

$$\text{Required ratio}=\frac{P-P'}{P}=\frac{54-40}{54}=\frac{14}{54}=26\%$$

58. (b) Since a ruler is used, the scale used does not expand with the tube. If the radius of the capillary be r, the increase due to thermal expansion is given by $dr=r\alpha dT$ for a temperature rise of dT. Since area of cross section is $A=\pi r^2$, we see that $dA/A=2dr/r$ or $dA=A(2\alpha)\,dT$. Thus if the temperature is increased from T to $T+dT$, the cross-sectional area changes from A to $A\,(1+2\alpha dT)$. The volume expansion of the liquid gives $V'=V+dV=V(1+\gamma dT)$, where γ is the coefficient of the volume expansion of the liquid. This causes change in length of thread and final length becomes $L'=L+dL$. The mass of liquid is constant; hence, $L'A'=V'=V\,(1+\gamma dT)=LA\,(1+\gamma dT)$.

But $\quad A'=A\,(1+2\alpha dT)$

Hence, $\quad L'=L\left(\dfrac{1+\gamma dT}{1+2\alpha dT}\right)$

$$=L[1+(\gamma-2\alpha)dT-2\alpha\gamma(dT)^2]$$

The last term is negligible.

Hence, $\quad L'=L[1+(\gamma-2\alpha)dT]$

$$\Delta L=L(\gamma-2\alpha)\Delta T$$

59. (d) Specific heat of lead = 0.03 kcal/kg°C. Gravitational potential energy is converted into thermal energy which is absorbed by the lead shot. If T_0 is the rise in temperature of the shot, then

$$(100)\,[mgh]=(m\times s\times T)\text{ J}$$

$$T=\frac{100\times g\times h}{Js}=\frac{100\times9.8\times1.5}{0.03\times4.18\times10^{3}}°\text{C}=11.3°\text{C}$$

60. (b) The entire kinetic energy of the fragment is changed to heat. Expressing mass m in *kilograms* everywhere, we have

$$\frac{1}{2}mv^2 = [m(30) + m(0.11)(1535° + 100°)]4184$$

$$v^2 = (8368)[30 + 180] = 1.76 \times 10^6$$

$$\Rightarrow v = (\sqrt{1.76}) \times 10^3 = 1.32 \text{ km/s}$$

61. (a) This is a problem on 'flow calorimeter' used to measure specific heat of a liquid.

Amount of heat supplied to the water per second by the heating coil $= Q_S = 250 \text{ J} = \frac{250}{4186}$ kcal

The volume of liquid flowing out per second $= 8.0 \text{ cm}^3$
$= 8 \times 10^{-6} \text{ m}^3$

Mass of this liquid $= (0.85) \times 1000 \times 8 \times 10^{-6}$ kg

Temperature rise of this mass of liquid = 15°C

Hence, $\frac{250}{4186} = mst = 0.85 \times 8 \times 10^{-3} \times s \times 15$

Hence, $s = \frac{250 \times 10^3}{4186 \times 0.85 \times 8 \times 15} = 0.6$ kcal/kg K

62. (b) According to principle of calorimetry,

$$ML_F + Ms\Delta T = (ms\Delta T)_{water} + (ms\Delta T)_{flask}$$

$$50L_F + 50 \times 1 \times (40 - 0)$$
$$= 200 \times 1 \times (70 - 40) + W(70 - 40)$$

$$50L_F + 2000 = (200 + W)\,30$$

$$5L_F = 400 + 3W \qquad \text{(i)}$$

Now the system contains (200 + 50) g of water at 40°C, so when further 80 g of ice is added

$$80L_F + 80 \times 1 \times (10 - 0)$$
$$= 250 \times 1 \times (40 - 10) + W(40 - 10)$$

$$80L_F = 670 + 3W \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii),

$$L_F = 90 \text{ cal/g} \quad \text{and} \quad W = \frac{50}{3} \text{ g}$$

63. (b) If the point is at a distance x from water at 100°C, heat conducted to ice in time t,

$$Q_{ice} = KA\frac{(200 - 0)}{(1.5 - x)} \times t$$

So ice melted by this heat

$$m_{ice} = \frac{Q_{ice}}{L_F} = \frac{KA}{80}\frac{(200 - 0)}{(1.5 - x)} \times t$$

Similarly heat conducted by the rod to the water at 100°C in time t,

$$Q_{water} = KA\frac{(200 - 100)}{x}t$$

Steam formed by this heat

$$m_{steam} = \frac{Q_{water}}{L_V} = KA\frac{(200 - 100)}{(540 \times x)}t$$

According to given problem $m_{ice} = m_{steam}$,

i.e., $\frac{200}{80(1.5 - x)} = \frac{100}{540 \times x} \Rightarrow x = \frac{6}{58} \text{ m} = 10.34$ cm

i.e., 200°C temperature must be maintained at a distance 10.34 cm from water at 100°C.

64. (b) Loss in energy $= mg(h - h') = 0.1 \times 10 \times (10 - 5.4) = 4.6$ J

Now, $4.6 \text{ J} = ms\,\Delta\theta = 0.1 \times 460 \times \Delta\theta$

$\therefore \Delta\theta = 0.1°\text{C}$

65. (a) According to Stefan's law, $E = \sigma T^4$

Total surface area of the sun $= 4\pi R_S^2$.

Therefore, the total energy radiated per second by the sun per unit solid angle

$$= \frac{\sigma T^4 \times 4\pi R_s^2}{4\pi} = \sigma T^4 R_s^2$$

Let R_{es} be the distance of earth from the sun. Hence, intensity of radiation of earth

$$I = \sigma T^4 R_s^2 / R_{es}^2$$

$$\therefore \quad 1400 = (5.6 \times 10^{-8})T^4\left[\frac{7.0 \times 10^8}{1.5 \times 10^{11}}\right]^2$$

$$T = 5801 \text{ K}$$

66. (b) Since the vessel is partly filled, volume of the vessel is greater than that of the liquid. When a body having volume V is heated through $\Delta\theta$, then increase in its volume is given by

$$\Delta V = V \cdot \gamma \cdot \Delta\theta$$

Since, $\gamma_V = \gamma_L$, therefore $\Delta V \propto V$. Hence, on heating expansion of vessel will be greater than that of liquid. It means unoccupied volume will necessarily increase. So, option (b) is correct.

67. (b) Energy supplied by the heater to the system in 10 min

$$Q_1 = P \times t = 90 \text{ J/s} \times 10 \times 60 \text{ s}$$

$$= 54000 \text{ J} = \frac{54000}{4.2} \text{cal} = 12857 \text{ cal}$$

Now if θ is the final temperature of the system, energy absorbed by it to change its temperature from 10°C to θ °C is

$$Q_2 = (ms\,\Delta T)_{water} + (ms\,\Delta T)_{coil + calorimeter}$$
$$= 360 \times 1 \times (\theta - 10) + 40(\theta - 10)$$
$$= 400\,(\theta - 10)$$

According to problem, $Q_1 = Q_2$

So $12857 = 400(\theta - 10)$ or $\theta = 42.14°\text{C}$

68. (b) If θ is the temperature of outside, heat passing per second through the glass window,

$$\frac{dQ}{dt} = KA\frac{(\theta_1 - \theta_2)}{L} = \frac{0.2 \times 1 \times (20 - \theta) \text{ cal}}{0.2 \times 10^{-2}} = 100(20 - \theta)$$

And heat produced per second by the heater in the room

$$P = \frac{V^2}{R}\frac{\text{J}}{\text{s}} = \frac{V^2}{RJ}\frac{\text{cal}}{\text{s}} = \frac{200 \times 200}{20 \times 4.2} = 476.2 \frac{\text{cal}}{\text{s}}$$

Now as the temperature of the room is constant, the heat produced per second by heater must be equal to the heat conducted through the glass window.

$$100\,(20 - \theta) = 476.2;\ \theta = 15.24°\text{C}$$

Archives

1. (a) $\frac{E_1}{E_2} = \frac{A_1 T_1^4}{A_2 T_2^4} = \frac{R_1^2 T_1^4}{R_2^2 T_2^4}$

$$= \frac{1^2 \times 4000^4}{4^2 \times 2000^4} = \frac{16}{16} = 1$$

2. (a) According to Newton's law of cooling, the rate of cooling of a body is directly proportional to the temperature difference between the body and the surroundings.

3. (b) When water is cooled at 0°C to form ice, then 80 calorie/gm (latent heat) energy is released. Because potential energy of the molecules decreases. Mass will remain constant in the process of freezing of water.

4. (b) $Q = m.c.\Delta\theta$; if $\Delta\theta = 1$ K, then $Q = mc$. Thermal capacity.

5. (b) Infrared radiations are detected by pyrometer

6. (c) When light incident on pin hole enters into the box and suffers successive reflection at the inner wall. At each reflection some energy is absorbed. Hence the ray once it enters the box can never come out and pin hole acts like a perfect black body.

7. (a) Factual information

8. (d) The surface area is increased by a factor of 4. The temperature is increased by a factor of 2. So the radiant energy is increased by a factor of 64. [Using Stefan's law]

9. (d)
$$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$$

$$\frac{K(\theta_1 - \theta_2)}{d} = \text{constant}$$

$$\frac{K(T_2 - T_0)}{x} = \frac{2K(T_0 - T_1)}{4x}$$

$$\Rightarrow \quad T_2 - T_0 = \frac{T_0 - T_1}{2}$$

$$\Rightarrow \quad 2T_2 - 2T_0 = T_0 - T_1$$

$$\Rightarrow \quad 3T_0 = 2T_2 + T_1$$

$$\Rightarrow \quad T_0 = \frac{2T + T_1}{3}$$

Rate of heat transfer $= \dfrac{Q}{T} = \dfrac{KA(T_2 - T_0)}{x}$

$$= \frac{KA}{x}\left[T_2 - \frac{2T_2 + T}{3}\right]$$

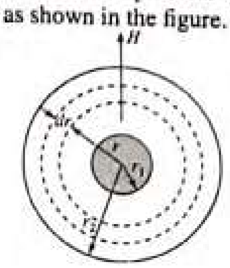
$$= \frac{KA}{x}\left[\frac{3T_2 - 2T_2 + T_1}{3}\right]$$

$$= \frac{KA}{x}\left[\frac{T_2 - T_1}{3}\right] = \frac{KA}{3x}(T_2 - T_1)$$

Now, calculating Q/t, we get

$$f = \frac{1}{3}$$

10. (a) Consider a concentric spherical shell of radius r and thickness dr as shown in the figure.



The radial rate of flow of heat through this shell in steady state will be

$$H = \frac{dQ}{dt} = -KA\frac{dT}{dr} = -K(4\pi r^2)\frac{dT}{dr}$$

$$\Rightarrow \quad \int_{r_1}^{r_2} \frac{dr}{r^2} = -\frac{4\pi K}{H}\int_{T_1}^{T_2} dT$$

Which on integration and simplification gives

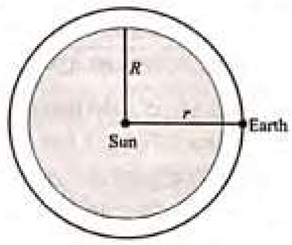
$$H = \frac{dQ}{dt} = \frac{4\pi K r_1 r_2 (T_1 - T_2)}{r_2 - r_1}$$

$$\Rightarrow \quad \frac{dQ}{dt} \propto \frac{r_1 r_2}{(r_2 - r_1)}$$

11. (d) Energy radiated by sun, according to Stefan's law,

$$E = \sigma T^4 \times (\text{area } 4\pi R^2)\ (\text{time})$$

This energy is spread around sun in space, in a sphere of radius r. earth (E) in space receives part of this energy.



$$\frac{\text{Energy}}{\text{Area of envelope}} = \frac{\sigma T^4 \times 4\pi R^2 \times \text{time}}{4\pi r^2}$$

Energy incident per unit area on earth

$$= \frac{\sigma T^4 R^2 \times \text{time}}{r^2}$$

$\therefore$ Power incident per unit area on earth $= \left(\dfrac{R^2 \sigma T^4}{r^2}\right)$

$\therefore$ Power incident on earth $-\pi r_0^2 \times \dfrac{R^2 \sigma T^4}{r^2}$

12. (d) From statement I:
Given $R = R_0(1 + \alpha\Delta t)$
Resistance at 27°C,

$$100 = R_0(1 + \alpha \times 27) \quad \text{...(i)}$$

Resistance at 227°C,

$$150 = R_0(1 + \alpha \times 227) \quad \text{...(ii)}$$

From (i) and (ii), $\dfrac{100}{150} = \dfrac{R_0(1 + 27\alpha)}{R_0(1 + 227a)}$

Which gives $\alpha = 268 \times 10^{-3}$/°C

Hence, statement II is correct as the value of α is constant if the change in temperature Δt is small.

13. (b) We know that

$$\frac{dQ}{dt} = KA\frac{d\theta}{dx}$$

In steady state flow of heat,

$$d\theta = \frac{d\theta}{dt} \times \frac{1}{KA} \times dx$$

$$\Rightarrow \quad \theta_H - \theta = K'x \Rightarrow \theta = \theta_H - K'x$$

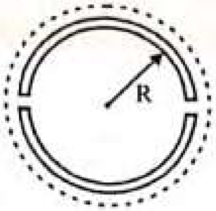
The equation $\theta = \theta_H - K'x$ represents a straight line.

14. (b) $\Delta Q = MS\Delta T = 100 \times 10^{-3} \times 4184 \times 20 = 8.4 \times 10^3$

$\Delta Q = 84$ kJ and $\Delta W = 0$

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = 8.4$ kJ

15. (d)



$L \Rightarrow S$

$\Delta L = L\,\alpha\,\Delta T$

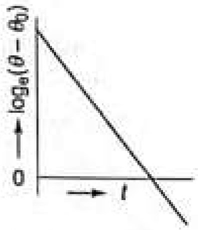
$\frac{F}{A} = \frac{\Delta L}{L} Y$

$F = \alpha \Delta T\, YS$

So $T = 2F$

$T = 2\alpha\Delta T\, YS$

16. (a) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$



$$\int_{\theta_0}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k \int_0^t dt$$

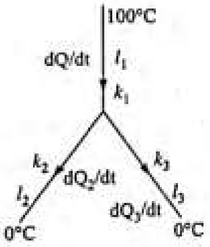
$(\theta - \theta_0) = -kt + \chi$

So graph is straight line.

17. (b) The temperature goes on decreasing with time (non-linearly). The rate of decrease will be more initially which is depicted in the second graph.

18. (a) $\frac{dQ_1}{dt} = \frac{dQ_2}{dt} + \frac{dQ_3}{dt}$

$$\Rightarrow \quad \frac{0.92(100 - T)}{46} = \frac{0.26(T - 0)}{13} + \frac{0.12(T - 0)}{12}$$

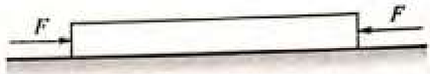


$\Rightarrow \quad T = 40°C$

$$\frac{dQ_1}{dt} = \frac{0.92 \times 4(100 - 40)}{40} = 4.8 \text{ cal/s}$$

19. (c) Thermal strain in the wire, $\frac{\Delta l}{l} = \alpha\,\Delta T$

Thermal stress in rod is the pressure due to the thermal strain



$$\text{Thermal stress} = Y \times \text{strain} = Y\left(\frac{\Delta l}{l}\right)$$

$$\text{Thermal stress} = \frac{F}{A} = \text{pressure} = Y\alpha\Delta T$$

Given, $\Delta T = 100°$ C, $Y = 2 \times 10^{11}$ N m^{-2}

$\alpha = 1.1 \times 10^{-5}$ K^{-1}

Pressure $= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100 = 2.2 \times 10^8$ Pa

20. (a) Let at temperature θ, clock gives correct time

$$\Delta T = \left(\frac{1}{2}\alpha\Delta\theta\right)T, \quad T = 1 \text{ day} = 86400\,\text{s}$$

$$12 = \frac{1}{2}\alpha(40 - \theta)T \qquad \text{...(i)}$$

$$4 = \frac{1}{2}\alpha(\theta - 20)T \qquad \text{...(ii)}$$

$$\frac{12}{4} = \frac{40 - \theta}{\theta - 20} \Rightarrow 3\theta - 60 = 40 - \theta$$

Hence, $\theta = 25°C$

from (ii) $4 = \frac{1}{2}\alpha(25 - 20) \times 86400$

$$\alpha = \frac{8}{5 \times 86400} = 1.85 \times 10^{-5}\,°C$$

21. (c) Due to thermal expansion

$$\frac{\Delta V}{V} = 3\alpha\Delta T \qquad \text{...(1)}$$

Volumetric strain due to external pressure

$$\frac{\Delta V}{V} = \frac{P}{K} \qquad \text{...(2)}$$

Equating both equations we get

$$3\alpha\Delta T = \frac{P}{K} \Rightarrow \Delta T = \frac{P}{3\alpha K}$$

22. (d) Heat given = Heat taken

$(100)(0.1)(T - 75) = (100)(0.1)(45) + (170)(1)(45)$

$10(T - 75) = 450 + 7650 = 8100$

$T - 75 = 810$

$T = 885°C$

CHAPTER 14: KINETIC THEORY OF GASES AND THERMODYNAMICS

Concept Application Exercise 14.1

1. $\frac{1}{2}Mv^2 = C_v\Delta T = \frac{R}{\gamma-1}\Delta T$

$$\left(C_v = \frac{R}{\gamma-1}\right) \Rightarrow \Delta T = \frac{(\gamma-1)Mv^2}{2R}$$

Now $pV = RT \Rightarrow p = \frac{RT}{V} \Rightarrow \Delta p = \frac{R}{V}\Delta T$

$$\therefore \quad \frac{\Delta p}{p} = \frac{\left(\frac{7}{5}-1\right)\times 28\times 10^{-3}\times 100^2}{2\times 8.3\times 300} = 0.0225 = 2.25\%$$

2. $C_v = \frac{R}{\gamma-1}$ and $C_v = Mc_v$

$$\therefore \quad M\times 650 = \frac{8.3}{\gamma-1} \qquad \text{(i)}$$

Further

$$\gamma = C_p/C_v = \frac{910}{650} \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii), $M = 32$ g/mol

$\gamma = 1.4$, so the gas is diatomic and the number of degrees of freedom is 5.

3. $p = \frac{1}{3}\rho v_{rms}^2 \Rightarrow v_{rms} = \sqrt{\frac{3p}{\rho}}$

$$\therefore \quad v_{rms} = \sqrt{\frac{3\times 1.00\times 10^{-2}\times 1.013\times 10^5}{1.24\times 10^{-2}}} = 495 \text{ m/s}$$

Again, $v_{rms} = \sqrt{\frac{3RT}{M}}$

or $M = \frac{3RT}{v_{rms}^2} = \frac{3\times 8.3\times 400}{(495)^2} = 40.6$ g/mol

This is the molecular weight of argon and so the gas is argon.

4. $C_v = \frac{R}{\gamma-1} = Mc_v$

$$\therefore \quad M\times(0.075\times 1000)\times 42 = \frac{8.3}{\frac{5}{3}-1}$$

$\Rightarrow \quad M = 39.65$ g

$\therefore$ Mass of one argon atom

$$m = \frac{M}{N_A} = \frac{39.65}{6\times 10^{23}} = 6.58\times 10^{-23} \text{ g}$$

5. U (total energy per mole) $= \frac{RT}{\gamma-1} = \frac{8.3\times 293}{\frac{7}{5}-1} = 6079.8$ J

$\therefore \quad E_t$ (translational energy) $= \frac{3}{5}\times 6079.8 = 3647.9$ J

Translation energy per molecule $= \frac{3647.9}{6.0\times 10^{23}} = 6.1\times 10^{-21}$ J

$$\Delta U = C_v\Delta T = \frac{R}{\gamma-1}\Delta T = \frac{8.3}{\frac{7}{5}-1} = 20.8 \text{ J}$$

6. Number of molecules in 1 g of water $= \frac{6.0\times 10^{23}}{18}$

$\therefore$ Number of molecules per unit area $= \frac{6.0\times 10^{23}}{18\times 4\pi\left(6.4\times 10^6\right)^2}$

$= 6.6\times 10^8$

Concept Application Exercise 14.2

1. W (work done) $= \int p\Delta V = p\frac{m}{M}\frac{R\Delta T}{p} = \frac{m}{M}R\Delta T$

$$\therefore \quad W = \frac{20}{28}8.3(250-17) = 1381 \text{ J}$$

$$\Delta U = \frac{m}{M}C_v\Delta T = \frac{m}{M}\frac{R}{\gamma-1}\Delta T$$

$$= \frac{20}{28}\times\frac{8.3}{\frac{7}{5}-1}\times(250-17) = 3453 \text{ J}$$

$\therefore \quad Q = W + \Delta U = 1381 + 3453 = 4834$ J

W is also equal to mgh (see the above problem).

$\therefore \quad 1381 = 75\times 9.8\times h$ or $h = 1.88$ m

2. $W = \int_V^{2V} pdV = p_0V = RT_0$

$$\Delta U = \int C_v dT = \int_V^{2V}\frac{R}{\gamma-1}dT$$

At constant pressure fixed at

$$p_0 dV = RdT \qquad (\because pV = RT)$$

$$\therefore \quad \Delta U = \frac{1}{\gamma-1}\int_V^{2V} p_0 dV = \frac{p_0V}{\gamma-1} = \frac{RT_0}{\gamma-1}$$

$$\therefore \quad Q = \Delta U + W = \frac{RT_0}{\gamma-1} + RT_0 = \frac{\gamma RT_0}{\gamma-1}$$

Since oxygen is diatomic, $\gamma = 1.4$

$$\therefore \quad Q = \frac{1.4\times 8.3\times 273}{1.4-1} = 7931 \text{ J}$$

3. We have $\Delta U = \int_{T_1}^{T_2} C_v dT = C_v(T_2 - T_1)$

Since the process is adiabatic, $Q = 0$

$Q = \Delta U + W$

$\therefore \quad W = -\Delta U = -C_v(T_2 - T_1) = C_v(T_1 - T_2)$

4. (a) $\Delta U = \int_T^{T+\Delta T}\frac{m}{M}C_v dT = \int_T^{T+\Delta T}\frac{m}{M}\frac{R}{\gamma-1}dT \quad \left(\because C_v = \frac{R}{\gamma-1}\right)$

$$\Rightarrow \quad \Delta U = \frac{m}{M}\frac{R}{\gamma-1}\Delta T$$

Similarly, $Q = \frac{m}{M}C\Delta T$

where C is the molar heat capacity in the process.

It is given that $\Delta Q = -\Delta U$

(b) $dQ = dU + dW \Rightarrow 2dQ = dW$ $\quad (\because dQ = -dU$ given)

$\Rightarrow \quad 2CdT = pdV$ $\quad (\because dQ = CdT)$

$$\Rightarrow \quad \frac{2R}{\gamma-1}dT + \frac{RT}{V}dV = 0$$

$$\Rightarrow \quad \frac{dT}{T} + \frac{\gamma-1}{2}\frac{dV}{V} = 0$$

Integrating, $TV^{(\gamma-1)/2}$ = constant

5. $\Delta U = \int_T^{T+\Delta T} C_v dT = \frac{R}{\gamma-1}\Delta T = \frac{8.3}{\frac{3}{5}-1}(-26) = -324$ J

and $W = \int_T^{T+\Delta T} pdV$

Here $pV^{1.5}$ = constant; $pV = RT$ (always)

$$V^{0.5} \propto \frac{1}{T} \quad \text{or} \quad V \propto \frac{1}{T^2}$$

or $V = \frac{a}{T^2}$, where a is a positive constant.

$$\therefore \quad p = \frac{RT}{V} = \frac{RT}{a/T^2} = \frac{R}{a}T^3$$

$$\therefore \quad W = \int_T^{T+\Delta T}\frac{R}{a}T^3\left(-\frac{2a}{T^3}dT\right) = -\int_T^{T+\Delta T}2RdT$$

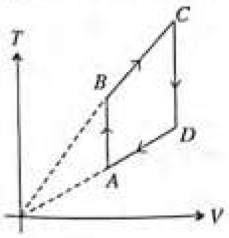
$\Rightarrow \quad W = -2R\,\Delta T = -2 \times 8.3 \times (-26) = +432$ J

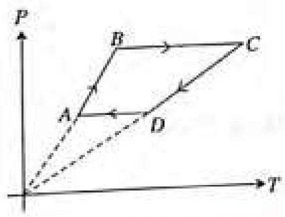
$Q = \Delta U + W = -324 + 432 = 108$ J

6. Since in P–V curves the area under the cycle is equal to work done, therefore work done by the gas is equal to P_0V_0.

Line AB and CD are isochoric line, line BC and DA are isobaric line.

$\therefore$ The T–V curve and P–T curve are drawn as shown in the figure.





Concept Application Exercise 14.3

1. Coefficient of performance of a Carnot refrigerator working between 30°C and 0°C is

$$\beta = \frac{T_2}{T_1 - T_2} = \frac{273°C}{303°C - 273°C} = \frac{273°C}{30°C} \approx 9$$

2. Efficiency of Carnot engine

$$\eta = \frac{T_1 - T_2}{T_1} \quad \frac{600-300}{600} = \frac{1}{2}$$

Again $\eta = \frac{\text{Work done}}{\text{Heat input}}$

$$\therefore \quad \text{Heat input} = \frac{\text{Work done}}{\eta} = \frac{800}{1/2} = 1600\text{ J}$$

3. Work done by the gas (as cyclic process is clockwise)

$\therefore \quad \Delta W$ = Area $ABCD$

So from the first law of thermodynamics ΔQ (net heat absorbed) $= \Delta W$ = Area $ABCD$

As change in internal energy in cycle $\Delta U = 0$.

4. Efficiency of Carnot engine $= \frac{T_1 - T_2}{T_1} = \frac{900-300}{900} = \frac{6}{9}$ or 66.6%.

Exercises

Kinetic Theory of Gases

1. (a) $P_1 = P$, $T_1 = T$, $P_2 = P + (0.4\%$ of $P)$

$$\Rightarrow \quad P_2 = P + \frac{0.4}{100}P = P + \frac{P}{250} \quad \text{and} \quad T_2 = T + 1$$

From Gay-Lussac's law

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \quad \frac{P}{P + \frac{P}{250}} = \frac{T}{T+1} \quad (\text{as } V = \text{constant for closed vessel})$$

By solving, we get $T = 250$ K

2. (a) By Dalton's law

$$P = P_1 + P_2 + P_3 = \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} + \frac{\mu_3 RT}{V}$$

$$= \frac{RT}{V}[\mu_1 + \mu_2 + \mu_3] = \frac{RT}{V}\left[\frac{m_1}{M_1} + \frac{m_2}{M_2} + \frac{m_3}{M_3}\right]$$

$$= 498 \times 10^3 = 500 \times 10^3 = 5 \times 10^5 \text{ N/m}^2$$

3. (b) Energy of 1 mol of gas $= \frac{f}{2}RT = \frac{f}{2}PV$

where f = degree of freedom

Monatomic or diatomic gases possess equal degree of freedom for translational motion and that is equal to 3, i.e., $f = 3$

$$\therefore \quad E = \frac{3}{2}PV$$

4. (c) Fraction of energy supplied for increment in internal energy $= 1/\gamma = 3/5$ (as $\gamma = 5/3$ for monatomic gas)

Therefore, percentage energy = 30/5 = 60%.

Fraction of energy supplied for external work done

$$= 1 - \frac{1}{\gamma} = \frac{\gamma-1}{\gamma} = \frac{\frac{5}{3}-1}{\frac{5}{3}} = \frac{2}{5}$$

$$\therefore \quad \text{Percentage energy} = \frac{2}{5} \times 100\% = 40\%$$

5. (b) If m is the total mass of the gas, then its kinetic energy $= 1/2\,mv^2$. When the vessel is suddenly stopped, total kinetic energy will increase the temperature of the gas (because process will be adiabatic), i.e.,

$$\frac{1}{2}mv^2 = \mu C_v \Delta T = \frac{m}{M} C_v \Delta T$$

$$\Rightarrow \quad \frac{m}{M}\frac{R}{\gamma-1}\Delta T = \frac{1}{2}mv^2 \qquad \left(\text{As } C_v = \frac{R}{\gamma-1}\right)$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2(\gamma-1)}{2R}$$

6. (b) $P = \dfrac{P_0}{1+(V/V_0)^3} = \dfrac{P_0}{2}$

$$T = \frac{P_0V_0}{2R}$$

Therefore translational kinetic energy is equal to

$$\frac{3}{2}RT = \frac{3R}{2}\frac{P_0V_0}{2R} = \frac{3P_0V_0}{4}$$

7. (c) For the gas in container A

$$\Delta P = (P_A)_{\text{final}} \quad (P_A)_{\text{initial}} = \frac{n_ART}{2V} - \frac{n_ART}{V}$$

$$\Delta P = -\frac{n_ART}{2V} \qquad \text{(i)}$$

For gas in container B

$$1.5\Delta P = (P_B)_{\text{final}} - (P_B)_{\text{initial}} = \frac{n_BRT}{2V} - \frac{n_BRT}{V}$$

$$1.5\Delta P = -\frac{n_BRT}{2V} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get $n_B = 1.5\,n_A$

$\Rightarrow \quad 2n_B = 3n_A \quad$ or $\quad 2m_B = 3m_A$

8. (d) Ideal gas equation $PV = \mu RT = \left(\dfrac{N}{N_A}\right)RT$, where N = Number of molecule, N_A = Avogadro number

$$\therefore \quad \frac{N_1}{N_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = \left(\frac{P}{2P}\right)\left(\frac{V}{V/4}\right)\left(\frac{2T}{T}\right) = \frac{4}{1}$$

9. (c) $v_{rms} = \sqrt{\dfrac{3RT}{M}}$. According to problem T will becomes $T/2$ and M will become $M/2$ so the value of v_{rms} will increase by $\sqrt{4} = 2$ times i.e., new root mean square velocity will be $2v$.

10. (c) Change in pressure will not affect the rms velocity of molecules. So we will calculate only the effect of temperature.

As $\quad v_{rms} \propto \sqrt{T}$

$$\therefore \quad \frac{v_{300^\circ}}{v_{400^\circ}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}} \Rightarrow \frac{200}{v_{400}} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow \quad v_{400} = \frac{200\times 2}{\sqrt{3}} = \frac{400}{\sqrt{3}}\text{ m/s}$$

11. (b) For n-molecules, we know that

$$v_{rms} = \sqrt{\frac{v_1^2+v_2^2+v_3^2+\cdots+v_n^2}{n}}$$

$[v_{rms}$ = root mean square velocity$]$

where $v_1, v_2, v_3 \ldots, v_n$ are individual velocities of n-molecules of the gas.

For two molecules,

$$v_{rms} = \sqrt{\frac{v_1^2+v_2^2}{2}} \qquad [v_1, v_2, v_3 \ldots, v_n \text{ are individual velocity}]$$

Given, $v_1 = 9\times 10^6$ m/s and $v_2 = 1\times 10^6$ m/s

$$\therefore \quad v_{rms} = \sqrt{\frac{(9\times10^6)^2+(1\times10^6)^2}{2}} = \sqrt{41}\times 10^6\text{ m/s}$$

12. (c) We know that for a given mass of a gas

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where R is gas constant, T is temperature in kelvin and M is molar mass of the gas.

Clearly for a given gas, $v_{rms} \propto \sqrt{T}$, as R, M are constants.

$$\text{Hence, } \frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}} \qquad \ldots\text{(i)}$$

Given, $(v_{rms})_1 = 100$ m/s

$T_1 = 27°\text{C} = 27+273 = 300$ K

$T_2 = 127°\text{C} = 127+273 = 400$ K

$\therefore$ From Eq. (i)

$$\frac{100}{(v_{rms})_2} = \sqrt{\frac{300}{400}} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad (v_{rms})_2 = \frac{2\times 100}{\sqrt{3}} = \frac{200}{\sqrt{3}}\text{ m/s}$$

13. (b) Given

Mass of the particle $m = 5\times 10^{-17}$ kg.

Absolute temperature = 273 K

The rms speed of the particle is given by

$$v_{rms} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3\times1.38\times10^{-23}\times273}{5\times10^{-17}}}$$

$$= \sqrt{226\times10^{-6}} = 15\times10^{-3}\text{ m/s}$$

14. (d) Mean free path of a molecule is given by

$$l = \frac{1}{\sqrt{2}d^2n}$$

where n = number of molecules/volume

d = diameter of the molecule

Now, we can write $l \propto \dfrac{1}{d^2}$

Given, $d_1 = 1\text{Å}, d_2 = 2A$

As $\quad l_1 \propto \dfrac{1}{d_1^2}$ and $l_2 \propto \dfrac{1}{d_2^2}$

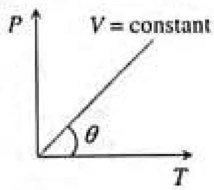
$$\Rightarrow \quad \text{So, } \frac{l_1}{l_2} = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$$

Hence, $l_1 : l_2 = 4 : 1$

15. (b) Total number of degrees of freedom in a thermodynamical system = number of degrees of freedom associated per molecule × number of molecules.
As given molecules are of hydrogen.
$\therefore$ Volume occupied by 1 mole
= 1 mole of the gas at NTP
= 22400 mL = 22400 cc
Any ideal gas has a molar volume of 22400 mL (cc) at NTP.
$\therefore$ Number of molecules in 1 cc of hydrogen
$$= \frac{6.023 \times 10^{23}}{22400} = 2.688 \times 10^{19}$$
H_2 is a diatomic gas, having a total of 5 degrees of freedom (3 translational + 2 rotational)
$\therefore$ Total degrees of freedom possessed by all the molecules
$= 5 \times 2.688 \times 10^{19} = 1.344 \times 10^{20}$

Ideal Gas Equation and Specific Heat Capacity of Gases

16. (a) From ideal gas equation,
$$PV = \mu RT$$
$$\therefore \quad P = \frac{\mu R}{V} T$$
Comparing, this equation with $y = mx$
Slope of line, $\tan\theta = m = \mu R/V$
i.e., $V \propto \dfrac{1}{\tan\theta}$



It means line of smaller slope represents greater volume of gas.
For the given problem figure
Points 1 and 2 are on the same line, so they will represent same volume, i.e., $V_1 = V_2$.
Similarly points 3 and 4 are on the same line, so they will represent same volume, i.e., $V_3 = V_4$.
But $V_1 > V_3 (= V_4)$ or $V_2 > V_3 (= V_4)$ as slope of line 1–2 is less than that of 3–4.

17. (b) As the quantity of gas remains constant, $\mu_A + \mu_B = \mu$
$$\frac{P_A V_A}{RT} + \frac{P_B V_B}{RT} = \frac{P(V_A + V_B)}{RT}$$
$$P = \frac{P_A V_A}{V_A + V_B} = \frac{1.4 \times 0.1 + 0.7 \times 0.15}{0.1 + 0.15}$$
$$\Rightarrow \quad P = 0.98 \text{ MPa}$$

18. (d) From Dalton's law, final pressure of the mixture of nitrogen and oxygen is
$$P = P_1 + P_2$$
$$= \frac{\mu_1 RT}{V} + \frac{\mu_2 RT}{V} = \frac{m_1}{M_1}\frac{RT}{V} + \frac{m_2}{M_2}\frac{RT}{V}$$
$$= \frac{8}{32}\frac{RT}{V} + \frac{7}{28}\frac{RT}{V} = \frac{RT}{2V} \Rightarrow 10 = \frac{RT}{2V} \qquad \text{(i)}$$
When oxygen is absorbed then for nitrogen let pressure is
$$P = \frac{7}{28}\frac{RT}{4V}$$
$$\Rightarrow \quad P = \frac{RT}{4T} \qquad \text{(ii)}$$
From Eqs. (i) and (ii), we get pressure of the nitrogen
$P = 5$ atm

19. (a) At constant pressure $Q_p = \mu C_p \Delta T$
$= 1 \times C_p \times (30 - 20) = 40$ cal
$\Rightarrow \quad C_p = 4$ cal/mol-K
$\therefore \quad C_v = C_p - R = 4 - 2 = 2$ cal/mol-K
Now $\quad Q_v = \mu C_v \Delta T = 1 \times 2 \times (30 - 20) = 20$ cal

20. (d) Given $c_p - c_v = 4150$ (i)
and $\quad c_p/c_v = 1.4 \quad \Rightarrow c_p = 1.4\, c_v$ (ii)
By substituting the value of c_p in Eq. (i), we get
$1.4\, c_v - c_v = 4150$
$\Rightarrow \quad 0.4\, c_v = 4150$
$$\therefore \quad c_v = \frac{4150}{0.4} = 10375 \text{ J/kg-K}$$

21. (a) Molar specific heat = molecular weight × gram specific heat
$C_v = M \times c_v$
2.98 cal/mol-K = $M \times 0.075$ kcal/kg-K
$$= M \times \frac{0.075 \times 10^3}{10^3} \text{ cal/g-K}$$
$\therefore$ Molecular weight of argon
$$M = \frac{2.98}{0.075} = 39.7 \text{ g}$$
i.e., mass of 6.023×10^{23} atom = 39.7 g
Therefore, mass of single atom
$$= \frac{39.7}{6.023 \times 10^{23}} = 6.60 \times 10^{-23} \text{ g}$$

22. (c) At constant volume the total energy will be utilized in increasing the temperature of gas
i.e., $\quad (\Delta Q)_v = \mu C_v \Delta T = \mu C_v (120 - 100) = 80$
$$\Rightarrow \quad \mu C_v = \frac{80}{20} = 4 \text{ J/K}$$
This is the heat capacity of 5 mol gas.

23. (b) Ideal gas equation for m grams of gas is
$PV = mrT$ (where r = specific gas constant)
or $\quad P = \dfrac{m}{V} rT = \rho rT$
$$\Rightarrow \quad r = \frac{P}{\rho T} = \frac{1.013 \times 10^5}{0.795 \times 273} = 466.7$$
Specific heat at constant volume
$$c_v = \frac{r}{\gamma - 1} = \frac{466.7}{\frac{4}{3} - 1} = 1400 \text{ J/kg-K}$$
($\gamma = 4/3$ for polyatomic gas)

24. (c) For state A, $C_p - C_v = R$, i.e., the gas behaves as an ideal gas.
For state B, $C_p - C_v = 1.06\, R (\neq R)$, i.e., the gas does not behave like an ideal gas.
We know that at high temperature and at low pressure, nature of gas may be ideal.
So we can say that $P_A < P_B$ and $T_A > T_B$.

25. (c) $\mu_1 = 1$, $\gamma_1 = 5/3$ (for monatomic gas) and $\mu_2 = 2$, $\gamma_2 = 7/5$ (for diatomic gas).

From formula

$$\gamma_{\text{mixture}} = \frac{\frac{\mu_1\gamma_1}{\gamma_1-1}+\frac{\mu_2\gamma_2}{\gamma_2-1}}{\frac{\mu_1}{\gamma_1-1}+\frac{\mu_2}{\gamma_2-1}} = \frac{\frac{1\times\frac{5}{3}}{\frac{5}{3}-1}+\frac{2\times\frac{7}{5}}{\frac{7}{5}-1}}{\frac{1}{\frac{5}{3}-1}+\frac{2}{\frac{7}{5}-1}} = \frac{\frac{5}{2}+7}{\frac{3}{2}+5} = \frac{19}{13}$$

26. (a) Let t be the temperature of mixture.

Heat gained by CO_2 = Heat lost by O_2

$$\Rightarrow \quad \mu_1 C_{v1}\Delta T_1 = \mu_2 C_{v2}\Delta T_2$$

$$\Rightarrow \quad \frac{22}{44}(3R)(t-27) = \frac{16}{32}\left(\frac{5}{2}R\right)(37-t)$$

$$\Rightarrow \quad 3(t-27) = \frac{5}{2}(37-t)$$

By solving we get $t = 31.5°C$

27. (d) Total internal energy of system

$$= U_{\text{oxygen}} + U_{\text{argon}} = \mu_1\frac{f_1}{2}RT + \mu_2\frac{f_2}{2}RT$$

$$2\frac{5}{2}RT + 4\frac{3}{4}RT = 5RT + 6RT = 11RT$$

[As $f_1 = 5$ (for oxygen) and $f_2 = 3$ (for argon)]

28. (a) AB is isobaric process, BC is isothermal process, CD is isochoric process and DA is isothermal process.

These processes are correctly represented by graph (a).

29. (b) For an isothermal process, PV = constant and for the given process PV^2 = constant.

Therefore the gas is cooled because volume expands by a greater exponent than in an isothermal process.

30. (c) From the given V–T diagram, we can see that in process AB, $V \propto T$. Therefore pressure is constant (as quantity of the gas remains same).

In process BC, V = constant and in process CA, T = constant. Therefore these processes are correctly represented on P–V diagram by graph (c).

31. (b) Process AB is an isothermal process, i.e., $P \propto 1/V$ and since $\rho \propto 1/V$, $\rho - V$ graph will be a rectangular hyperbola. Pressure is increasing; therefore, volume will increase. Process BC is an isochoric process. Therefore, V = constant and since $\rho = m/V$, density is also constant, i.e., $\rho - V$ graph is a dot. Process CD is inverse of process AB and process DA is inverse is BC.

32. (d) BC is isochoric. $V_B > V_A$, $V_B = V_C$, $V_D > V_C$

First Law of Thermodynamics, Internal Energy and Work Done

33. (b) We know fraction of given energy that goes to increase the internal energy $= 1/\gamma$.

So we can say the fraction of given energy that is supplied for external work $= 1 - (1/\gamma)$.

34. (b) As $f = 6$ (given), therefore

$$\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{6} = \frac{4}{3}$$

Fraction of energy given for external work

$$\frac{\Delta W}{\Delta Q} = \left(1 - \frac{1}{\gamma}\right) \Rightarrow \frac{25}{\Delta Q} = \left(1 - \frac{1}{4/3}\right) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \quad Q = 25 \times 4 = 100 \text{ J}$$

35. (b) Work done by the system = area of shaded portion on P–V diagram.

$$= (300-100)10^{-6} \times (200-100) \times 10^3 = 20\text{J}$$

And direction of process is anticlockwise so work done will be negative, i.e., $\Delta W = -20$ J.

36. (a) Work done = area enclosed by triangle ABC

$$= \frac{1}{2}AC \times BC = \frac{1}{2} \times (3V - V) \times (3P - P) = 2PV$$

37. (a) Area enclosed by curve 1 < Area enclosed by curve 2 < Area enclosed by curve 3

$\therefore \quad Q_1 < Q_2 < Q_3$ (As ΔU is same for all the curves)

38. (a) Change in internal energy

$$\Delta U = \mu C_v \Delta T \quad \Rightarrow U_2 - U_1 = \mu C_v (T_2 - T_1)$$

Let initially $T_1 = 0$ so $U_1 = 0$ and finally $T_2 = T$ and $U_2 = U$

$$U = \mu C_v T = \mu T \times C_v = \frac{PV}{R} \times \frac{R}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

(As $PV = \mu RT$, $\therefore \mu T = PV/R$ and $C_v = R/(\gamma - 1)$)

39. (a) By the graph, $W_{AB} = 0$ and

$$W_{BC} = 8 \times 10^4 [5 - 2] \times 10^{-3} = 240 \text{ J}$$

$$\therefore \quad W_{AC} = W_{AB} + W_{BC} = 0 + 240 = 240\text{J}$$

Now, $Q_{AC} = Q_{AB} + Q_{BC} = 600 + 200 = 800\text{J}$

From the first law of thermodynamics,

$$Q_{AC} = \Delta U_{AC} + W_{AC}$$

$$\Rightarrow \quad 800 = \Delta U_{AC} + 240 \text{ or } \Delta U_{AC} = 560\text{J}$$

40. (c) $Q = \Delta U$

$$= \mu C_v \Delta T = \mu\left(\frac{R}{\gamma - 1}\right)\Delta T = 2 \times \frac{R}{\frac{5}{3} - 1}[373 - 273] = 300R$$

(as for monatomic gas $\gamma = 5/3$)

41. (a) Given $P \propto T^3$. But for adiabatic process $P \propto T^{\gamma/\gamma-1}$. So,

$$\frac{\gamma}{\gamma - 1} = 3 \quad \Rightarrow \gamma = \frac{3}{2} \quad \Rightarrow \frac{C_p}{C_v} = \frac{3}{2}$$

42. (b) For an adiabatic process $TV^{\gamma - 1}$ = constant. Therefore,

$$\frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{\gamma - 1}$$

$$\Rightarrow \quad T_2 = T_1\left[\frac{V_1}{V_2}\right]^{\gamma-1} = 300\left[\frac{27}{8}\right]^{5/3-1} = 300\left[\frac{27}{8}\right]^{2/3} = 675\text{K}$$

$$\Rightarrow \quad \Delta T = 675 - 300 = 375\text{K}$$

43. (c) In second part there is a vacuum, i.e., $P = 0$. So work done in expansion $= P\Delta V = 0$. Also, $\Delta Q = 0$. From the first law of thermodynamics, $\Delta U = 0$ i.e., temperature of an ideal gas remains same due to free expansion.

44. (c) The work done = area of P–V diagram

$$a = \frac{V_2 - V_1}{2}, \quad b = \frac{P_2 - P_1}{2}$$

$$W = -\pi\left(\frac{V_2 - V_1}{2}\right)\left(\frac{P_2 - P_1}{2}\right)$$

But the cyclic process is anticlockwise. Hence, the work done is negative.

45. (a) $dU = 0$

Therefore by the first law of thermodynamics

$dQ_{\text{Cyclic}} = dW_{\text{Cyclic}}$

Since $B \to C$ is an isochoric process

$\Rightarrow \quad dW_{a \to C} = 0$

$\Rightarrow \quad 5 = dW_{A \to B} + dW_{a \to C} + dW_{C \to A}$

$\Rightarrow \quad 5 = 10(2-1) + 0 + dW_{c \to A}$

$\Rightarrow \quad dW_{C \to A} = -5\text{ J}$

46. (c) $W_{AB} = -P_0V_0$

$W_{BC} = 0$

and $W_{CD} = 4P_0V_0$

$\therefore \quad W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

47. (b) $W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} KV\,dV \qquad \left(\because \frac{P}{V} = K = \text{constant}\right)$

$\therefore \quad W = \frac{1}{2}k(V_2^1 - V_1^2)$

$PV = RT$

But $\quad p = KV$

$\therefore \quad KV^2 = RT \quad$ or $\quad K(V_2^2 - V_1^2) = R(T_2 - T_1)$

$\therefore \quad W = \frac{R}{2}(T_2 - T_1)$

48. (b) For 1 mol of gas, $Q = C_v\Delta T + P\Delta T$

At constant volume, $\Delta T = 0$

For 2 moles of gas, $\Delta = 2C_v\Delta T$

From $PV = nRT = 2R \times 300$

and $\quad \frac{P}{2}V = 2RT_f$

$\therefore \quad T_f = 150\text{ K}$

$\therefore \quad Q = 2C_v(T_f - T_i) = 2C_v(150 - 300) = -300C_v\text{ J}$

In the next process,

$Q = 2C_p\Delta T = 2C_p(300 - 150) = 300C_p\text{ J}$

$\therefore \quad$ Net heat absorbed $= -300C_v + 300C_p$

$= 300(C_p - C_v) = 300R\text{ J}$

49. (c) Work done = area of the ΔABC

$$= \frac{1}{2} \times AC \times AB = \frac{1}{2} \times (3V_1 - V_1) \times (4P_1 \quad P_1)$$

$$= \frac{1}{2} \times 2V_1 \times 3P_1 = 3P_1V_1$$

50. (c) As $\Delta U = 0$ in a cyclic process,

$Q = W =$ area of circle $= \pi r^2$

or $\quad W = 10^2\pi\text{ J}$

51. (d) Heat absorbed by gas in three processes is given by

$$Q_{ACB} = \Delta U + W_{ACB}$$

$$Q_{ADB} = \Delta U$$

$$Q_{AEB} = \Delta U + W_{AEB}$$

The change in internal energy in all the three cases is same and W_{ACB} is positive, W_{AEB} is negative.

Hence $\quad Q_{ACB} > Q_{ADB} > Q_{AEB}$

52. (a) $PV = RT$ for 1 mol

$$W = \int P\,dV = \int \frac{RT}{V}\,dV$$

$$V = CT^{2/3}$$

$$dV = \frac{2}{3}CT^{-1/3}dT \quad \text{or} \quad \frac{dV}{V} = \frac{2}{3}\frac{dT}{T}$$

$$W = \int_{T_1}^{T_2} RT\left(\frac{2}{3}\right)\frac{dT}{T} = \frac{2}{3}R(T_2 - T_1) = 166.2\text{ J}$$

Second Law of Thermodynamics

53. (b) In first case, $(\eta_1) = 1 - \frac{500}{800} = \frac{3}{8}$

and in second case, $(\eta_2) = 1 - \frac{600}{x}$

Since $\eta_1 = \eta_2$, therefore $\frac{3}{8} = 1 - \frac{600}{x}$

or $\quad \frac{600}{x} = 1 - \frac{3}{8} = \frac{5}{8}$ or $x = \frac{600 \times 8}{5} = 960\text{ K}$

54. (b) In first case $\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{(273+0)}{(273+200)} = \frac{200}{473}$

In second case $\eta_2 = 1 - \frac{(273-200)}{(273+0)} = \frac{200}{273}$

$$\Rightarrow \quad \frac{\eta_1}{\eta_2} = \frac{1}{\left(\frac{473}{273}\right)} = 1:1.73$$

55. (b) $\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{1}{2} = 1 - \frac{500}{T_1} \Rightarrow \frac{500}{T_1} = \frac{1}{2} \qquad$ ...(i)

$\frac{60}{100} = 1 - \frac{T_2'}{T_1} \Rightarrow \frac{T_2'}{T_1} = \frac{2}{5} \qquad$ ...(ii)

Dividing equation (i) by (ii),

$$\frac{500}{T_2'} = \frac{5}{4} \Rightarrow T_2' = 400\text{ K}$$

56. (b) In first case $\eta_1 = \frac{T_1 - T_2}{T_1}$

In second case $\eta_2 = \frac{2T_1 - 2T_2}{2T_1} = \frac{T_1 - T_2}{T_1} = \eta$

57. (c) Coefficient of performance

$$K = \frac{T_2}{T_1 - T_2} \Rightarrow 5 = \frac{(273-13)}{T_1 - (273-13)} = \frac{260}{T_1 - 260}$$

$\Rightarrow \quad 5T_1 - 1300 = 260 \Rightarrow 5T_1 = 1560$

$\Rightarrow \quad T_1 = 312K \to 39°\text{C}$

58. (a) Coefficient of performance $K = \frac{T_2}{T_1 - T_2}$

$$= \frac{(273-23)}{(273+27)-(273-23)} = \frac{250}{300-250} = \frac{250}{20} = 5$$

59. (a) $\eta = \frac{T_1 - T_2}{T_1} = \frac{(273+727)-(273+227)}{273+727} = \frac{1000-500}{1000} = \frac{1}{2}$

60. (c) $\eta = \frac{T_1 - T_2}{T_1} = \frac{W}{Q} \Rightarrow W = \frac{Q(T_1 - T_2)}{T_1}$

$$= \frac{6\times 10^4[(227+273)-(273+127)]}{(227+273)}$$

$$= \frac{6\times 10^4 \times 100}{500} = 1.2\times 10^4 \text{cal}$$

61. (c) $\eta_A = \frac{T_1 - T_2}{T_1} = \frac{W_A}{Q_1} \Rightarrow \eta_B = \frac{T_2 - T_3}{T_2} = \frac{W_B}{Q_2}$

$\therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \times \frac{T_2 - T_3}{T_1 - T_2} = \frac{T_1}{T_2}$ $\quad \therefore W_A = W_B$

$\therefore T_2 = \frac{T_1 + T_3}{2} = \frac{800+300}{2} = 550 \text{ K}$

62. (d) Initially $\eta = \left(1 - \frac{T_2}{T_1}\right) = \frac{W}{Q} = \frac{1}{6}$ ...(i)

Finally $\eta' = \left(1 - \frac{T_2'}{T_1}\right) = \left(1 - \frac{(T_2 - 62)}{T_1}\right) = 1 - \frac{T_2}{T_1} + \frac{62}{T_1}$

$$= \eta + \frac{62}{T_1} \quad \text{...(ii)}$$

It is given that $\eta' = 2\eta$. Hence solving equation (i) and (ii)
$T_1 = 372K = 99°\text{C}$ and $T_2 = 310 \text{ K} = 37°\text{C}$

63. (d) Initially $\eta = \frac{T_1 - T_2}{T_1} \Rightarrow 0.5 = \frac{T_1 - (273+7)}{T_1}$

$\Rightarrow \frac{1}{2} = \frac{T_1 - 280}{T_1} \Rightarrow T_1 = 560 \text{ K}$

Finally

$$\eta_1' = \frac{T_1' - T_2}{T_1'} \Rightarrow 0.7 = \frac{T_1' - (273+7)}{T_1'} \Rightarrow T_1' = 933 \text{ K}$$

$\therefore$ Increase in temperature $= 933 - 560 = 373 \text{ K} \approx 380 \text{ K}$

64. (c) Work done by the gas (as cyclic process is clockwise)
$\therefore W = \text{Area } ABCD$
So from the first law of thermodynamics ΔQ (net heat absorbed) $= W = \text{Area } ABCD$
As change in internal energy in cycle $\Delta U = 0$.

65. (b) Heat received (or produced by the burning of petrol) in one hour will be $= (2.4 \text{ kg/hour})(35 \text{ MJ/kg}) = 85.2 \times 10^6$ J/hour
$\therefore$ The rate at which heat is received

$$= \frac{85.2\times 10^6 \text{J}}{(3600 s)} = 2.37\times 10^4 \text{ J/s} = 23.7 \text{ kW}$$

Evidently, the rate of heat rejection
= rate at which heat is produced – rate at which work is obtained = 23.7 kW – 10 kW = 13.7 kW

66. (a) Given $Q_1 = 50$ kcal and $\eta = 20\%$

Using $\eta = \frac{W}{Q_1}$, we have $W = \eta \times Q_1$
i.e., work obtained per cycle $W = 20\% \times 50$ kcal
$= 10 \text{ kcal} = 42 \text{ kJ}$
Since, $Q_1 = Q_2 + W$ so, $Q_2 = Q - W$
i.e., heat rejected to the sink per cycle $Q_2 = 50 \text{ kcal} - 10 \text{ kcal}$
$= 40$ kcal

67. (d) Let T_1 be the initial temperature of the source, then, using $\eta = 1 - \frac{T_2}{T_1}$,

We have, $\frac{40}{100} = 1 - \frac{(273+27K)}{T_1}$

or $T_1 = 500$ K
For the efficiency to be 10% more, i.e., 50%, let T_1' be the new temperature of the sink,

then $\frac{50}{100} = 1 - \frac{(273+27\text{K})}{T_1'}$

or $T'_1 = 600$ K
The required increase in the temperature of the source
$T'_1 - T = 600 \text{ K} - 500 \text{ K} = 100 \text{ K}$

Problems Based on Mixed Concepts

68. (c) From ideal gas equation

$PV = RT$ (i)

$P\Delta V = R\Delta T$ (ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\Delta V}{V} = \frac{\Delta T}{T} \Rightarrow \frac{\Delta V}{V\Delta T} = \frac{1}{T} = \delta \quad \text{(given)}$$

$\therefore \delta = \frac{1}{T}$

So the graph between Δ and T will be a rectangular hyperbola.

69. (c) As we know that slopes of isothermal and adiabatic curves are always negative and slope of adiabatic curve is always greater than that of isothermal curve, in the given graph curves A and curve B represent adiabatic and isothermal changes, respectively.

70. (b) $\frac{C_v}{C_p} \times Q = nC_v dT$

$$dT = \frac{Q}{nC_p} = \frac{50}{2\times\frac{5}{2}\times R} = 5\text{K}$$

71. (d) $1 \to 2$: isothermal, $\Delta U_{12} = 0$
$2 \to 3$: isochoric, $W = 0$
$\Rightarrow Q_{23} = \Delta U_{23} \Rightarrow -40 = \Delta U_{23}$
For a cyclic process, $\Delta U = 0$
$\Delta U_{12} + \Delta U_{23} + \Delta U_{31} = 0$
$0 + (-40) + \Delta U_{31} = 0$
$0 + (-40) + \Delta U_{31} = 0$
$\Delta U_{31} = +40$ J

72. (b) For path acb:
$Q = \Delta U + W$

$\Rightarrow \quad 84 = \Delta U + 32 \Rightarrow \Delta U = 52$ kJ

Hence $\Delta U_{acb} = \Delta U_{ab} = \Delta U_{adb} = 52$ kJ

For path *adb*:

$$Q = \Delta U + W = 52 + 10.5 = 62.5 \text{ kJ}$$

So option (a) is correct.

For process ba, system will release the heat. So option (b) is wrong.

For path *ad*:

$$W_{adb} = W_{ad} + W_{db}$$

$$\Rightarrow \quad 10.5 = \Delta W_{ad} + 0$$

$$\Rightarrow \quad W_{ad} = 10.5 \text{ kJ}$$

$$Q_{ad} = \Delta U_{ad} + W_{ad} = (42 - 0) + 10.5 = 52.5 \text{ kJ}$$

So option (c) is correct.

$$Q_{adb} = 52 + 10.5 = 62.5 \text{ kJ}$$

$$Q_{db} = Q_{adb} - Q_{ad} = 62.5 - 52.5 = 10 \text{ kJ}$$

So option (d) is correct.

Hence answer of this question is (b).

73. (a) Change in temperature in process 1 will be greater and in process 3 will be least.

74. (d) $Q_{ab} = +7000 = \mu C_v(1000 - 300)$ (i)

For the process *ca*:

$$T_a = 300 \text{ K}, \quad T_c = T_b = 1000 \text{ K}$$

$$Q_{ca} = \mu C_p(300 - 1000) = -\mu C_p \times 700$$

$$= -\mu(C_v + R)700 \quad \text{(ii)}$$

For carbon monoxide:

$$T_a = 300 \text{ K}, \quad T_c = T_b = 1000 \text{ K}$$

$$(\Delta Q)_{ca} = \mu C_p(300 - 1000) = -\mu C_p \times 700$$

$$= -\mu(C_v + R)\, 700$$

For carbon monoxide: $\gamma = \dfrac{7}{5}$

$$C_v = \frac{R}{\gamma - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5R}{2} \quad \text{(iii)}$$

Hence, from Eq. (i)

$$7000 = \mu \frac{5R}{2} \times 700 \quad \text{or} \quad \mu R = \frac{20}{5} = 4$$

$$Q_{ca} = -(7000 + 4 \times 700) = -9800 \text{ J}$$

Negative sign shows that heat is ejected.

75. (c) $Q = Q_1 + Q_2 + Q_3 + Q_4$

$$= 5960 - 5585 - 2980 + 3645 = 1040 \text{ J}$$

$$W = W_1 + W_2 + W_3 + W_4$$

$$= 220 - 825 - 1100 + W_4 = 275 + W_4$$

For a cyclic process, $U_f = U_i$

$$\Delta U = U_f - U_i = 0$$

From the first law of thermodynamics,

$$Q = \Delta U + W$$

$$1040 = 0 = 275 + W_4 \quad \text{or} \quad W_4 = 765 \text{ J}$$

76. (c) For an adiabatic process,

$$PV^\gamma = \text{constant}$$

$$TV^{\gamma - 1} = \text{constant}$$

and $\quad T^\gamma P^{1-\gamma} = \text{constant}$

Putting, $\gamma = 5/3$ (argon being a monatomic gas), the equation becomes:

$$PV^{5/3} = \text{constant}$$

$$TV^{-2/3} = \text{constant}$$

$$T^{5/3} P^{-2/3} = \text{constant} \Rightarrow TP^{-2/5} = \text{constant}$$

77. (b) Let the initial pressure of the three samples be P_A, P_B and P_C, then

$$P_A(V)^{3/2} = (2V)^{3/2} P \quad (\because P_B = P)$$

or $\quad P_A = P(2)^{3/2}$

$$P_C(V) = P(2V)$$

or $\quad P_C = 2P$

$\therefore \quad P_A : P : P_C = (2)^{3/2} : 1 : 2 = 2\sqrt{2} : 1 : 2$

78. (b) Since P–V graph of the process is a straight line and two points $(V_0, 2P_0)$ and $(2V_0, P_0)$ are known, its equation will be

$$(P - P_0) = \frac{(2P_0 - P_0)}{(V_0 - 2V_0)}(V - 2V_0) = \frac{P_0}{V_0}(2V_0 - V)$$

$$\therefore \quad P = 3P_0 - \frac{P_0 V}{V_0}$$

According to equation for ideal gas,

$$T = \frac{pV}{nR} = \left(3P_0 - \frac{P_0 V}{V_0}\right)\frac{V}{nR}$$

$$= \frac{3P_0V_0V - P_0V^2}{nRV_0} \quad \text{(i)}$$

For T to be maximum, $\dfrac{dT}{dV} = 0$

$$3P_0V_0 - 2P_0V = 0$$

or $\quad V = \dfrac{3V_0}{2}$ (ii)

Putting this value in Eq. (i), we get

$$T_{\max} = \frac{3P_0V_0\left(\frac{3V_0}{2}\right) - P_0\left(\frac{9}{4}V_0^2\right)}{nRV_0} = \frac{9P_0V_0}{4nR}$$

79. (b) For an adiabatic process,

$$0 = dU + PdV$$

or $\quad d(a + bPV) + PdV = 0$

or $\quad (b+1)\dfrac{dV}{V} + b\dfrac{dP}{P} = 0$

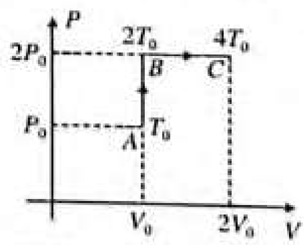
or $\quad (b+1)\log V + b\log P = \text{constant}$

$$V^{b+1}p^b = \text{constant}$$

or $\quad PV^{\frac{b+1}{b}} = \text{constant}$

$\therefore \quad \gamma = \dfrac{b+1}{b}$

80. (b) Let initial pressure, volume and temperature be P_0, V_0 and T_0, respectively, indicated by state A in P–V diagram. The gas is then isochorically taken to state B $(2P_0, V_0, 2T_0)$ and then taken from state B to state C $(2P_0, 2V_0, 4T_0)$ isobarically.



Total heat absorbed by 1 mol of gas

$$Q = C_v(2T_0 - T_0) + C_p(4T_0 - 2T_0)$$

$$= \frac{5}{2}RT_0 + \frac{7}{2}R \times 2T_0 = \frac{19}{2}RT_0$$

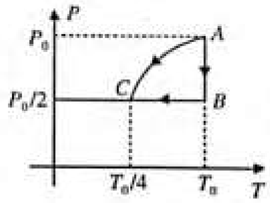
Total change in temperature from series A to C is $\Delta T = 3T_0$

Therefore,

$$\text{Molar heat capacity} = \frac{Q}{\Delta T} = \frac{\frac{19}{2}RT_0}{3T_0} = \frac{19}{6}R$$

81. (c) Process AB is isothermal expansion, BC is isobaric compression and in process CA

$$P \propto \frac{nRT}{P} \Rightarrow P^2 \propto T$$



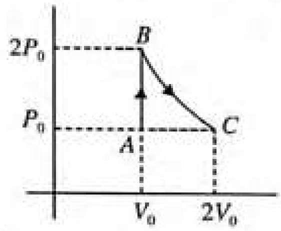
82. (c) $Q_{AB} = \Delta U_{AB} + W_{AB}$

$W_{AB} = 0$

$\Delta U_{AB} = \frac{f}{2}nR\Delta T$

$\frac{f}{2}(\Delta PV)\Delta U_{AB} = \frac{5}{2}(\Delta PV)$

$Q_{AB} = 2.5\,P_0V_0$



Process BC, $Q_{BC} = \Delta U_{BC} + W_{BC}$

$$= 0 + 2P_0V_0 \log 2 = 1.4\,P_0V_0$$

$$Q_{\text{net}} = Q_{AB} + Q_{BC} = 3.9\,P_0V_0$$

Archives

1. (c) The guiding principle in this problem is that the total number of moles of the system remains the same.

$$\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{P(V_1+V_2)}{RT}$$

$$\Rightarrow \quad T = \frac{P(V_1+V_2)T_1T_2}{P_1V_1T_2 + P_2V_2T_1}$$

By Boyle's law, we get

$$P(V_1+V_2) = P_1V_1 + P_2V_2$$

$$\Rightarrow \quad P = \frac{P_1V_1 + P_2V_2}{V_1+V_2}$$

$$\Rightarrow \quad T = \frac{(P_1V_1 + P_2V_2)T_1T_2}{P_1V_1T_2 + P_2V_2T_1}$$

2. (d) Change in internal energy does not depend upon both.

3. (b) $n_1 = 4, n_2 = \frac{1}{2}$

$$\gamma_1 = \frac{5}{3} \quad \text{(for monatomic gas)}$$

$$\gamma_2 = \frac{7}{5} \quad \text{(for diatomic gas)}$$

$$\frac{n_1+n_2}{\gamma-1} = \frac{n_1}{\gamma_1-1} + \frac{n_2}{\gamma_2-1}$$

$$\Rightarrow \quad \frac{4+0.5}{\gamma-1} = \frac{4}{\frac{5}{3}-1} + \frac{1/2}{\frac{7}{5}-1}$$

$$\Rightarrow \quad \frac{4.5}{\gamma-1} = \frac{12}{2} + \frac{5}{4} = \frac{29}{4}$$

$$\Rightarrow \quad \frac{\gamma-1}{4.5} = \frac{4}{29}$$

$$\Rightarrow \quad \gamma - 1 = \frac{4 \times 4.5}{29} = 0.62$$

$$\Rightarrow \quad \gamma = 1.62$$

4. (c) As the process is adiabatic,

$$\Delta U = -W = 1.46 \times 10^5\ \text{J} \qquad \text{...(i)}$$

Also $\quad \Delta U = nC_v\Delta T = \frac{nf}{2}R\Delta T$

$$= 10^3 \times \frac{f}{2} \times 8.3 \times 7\ \text{J} \qquad \text{...(ii)}$$

From (i) and (ii), we get $f = 5$ (diatomic).

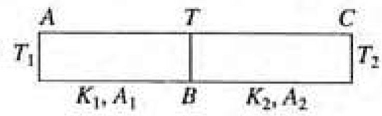
5. (c) $U_i = \frac{5}{2}RT_0 + \frac{3}{2}R\left(\frac{7}{3}T_0\right) = 6\,RT_0$

$$U_f = \frac{5}{2}RT_f + \frac{3}{2}RT_f = 4RT_f$$

As $\quad U_i = U_f$

$$\Rightarrow \quad T_f = \frac{3}{2}T_0$$

6. (c) Let the temperature at the interface be T.



For part AB, $\frac{Q_1}{t} \propto \frac{(T_1 - T)K_1}{l_1}$

For part BC, $\frac{Q_2}{t} \propto \frac{(T - T_2)K_1}{l_2}$

At equilibrium, $\frac{Q_1}{t} = \frac{Q_2}{t}$

$$\therefore \quad \frac{(T_1 - T)K_1}{l_1} = \frac{(T - T_2)K_2}{l_2}$$

$$\Rightarrow \quad T = \frac{T_1K_1l_2 + T_2K_2l_1}{K_1l_2 + K_2l_1}$$

7. (a) According to Mayer's relation,

$$C_P - C_v = \frac{R}{m} = \frac{R}{28}$$

8. (b) For Carnot engine used as a refrigerator,

$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

It is given

$$\eta = \frac{1}{10}$$

$$\Rightarrow \quad \eta = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{9}{10}$$

So, $Q_2 = 90$ J (as $W = 10$ J)

9. (a) From the first law of thermodynamics, we have

$$Q = \Delta U + W$$

For path *iaf*,

$$50 = \Delta U + 20$$

$$\therefore \quad \Delta U = U_f - U_i = 30 \text{ cal}$$

For path *ibf*,

$$Q = \Delta U + W$$

$$\Rightarrow \quad W = Q - \Delta U = 36 - 30 = 6 \text{ cal}$$

10. (a) As no work is done and the system is thermally insulated from the surrounding, the sum of the internal energy of the gas in the two partitions is constant, i.e.,

$$U = U_1 + U_2$$

If both gases have the same degrees of freedom, then

$$U = \frac{f(n_1 + n_2)RT}{2}$$

$$U_1 = \frac{fn_1RT_1}{2}, U_2 = \frac{fn_2RT_2}{2}$$

Solving, we get

$$T = \frac{(P_1V_1 + P_2V_2)T_1T_2}{P_1V_1T_2 + P_2V_2T_1}$$

11. (b) Thermal energy corresponds to internal energy.

Mass = 1 kg

Density = 8 kg/m^3

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{1}{8}\text{m}^3$$

Pressure = 8×10^4 N/m^2

$$\text{Internal energy} = \frac{5}{2}P \times V = 5 \times 10^4 \text{ J}$$

12. (c) $W_{ab} = \Delta Q - \Delta U = nC_P\,dT - nC_V\,dT$ (at constant pressure)

$$= n(C_P - C_V)dT$$

$$= nRdT$$

$$= 2 \times R \times (500 - 300) = 400R$$

13. (a) At constant temperature (isothermal process),

$$W_{DA} = nRT \log\left(\frac{P_1}{P_2}\right)$$

$$= 2.303 \times 2R \times 300 \log\left(\frac{10^5}{2 \times 10^5}\right)$$

$$= 2.303 \times 600\, R \log\left(\frac{1}{2}\right)$$

$$= 0.693 \times 600R$$

$$= -414R$$

14. (b) Net work done in a cycle is

$$W_{AB} + W_{BC} + W_{CB} + W_{BA}$$

$$= 400R + 2 \times 2.303 \times 500R \ln 2 - 400R - 414R$$

$$= 1000R \times \ln 2 - 600R \times \ln 2$$

$$= 400R \times \ln 2 = 276R$$

15. (b) The efficiency of cycle is

$$\eta = 1 - \frac{T_2}{T_1}$$

for adiabatic process

$$TV^{\gamma - 1} = \text{constant}$$

For a diatomic gas, $\gamma = 7/5$.

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow \quad T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\Rightarrow \quad T_1 = T_2(32)^{\frac{7}{5}-1} = T_2(2^5)^{2/5} = T_2 \times 4$$

$$\Rightarrow \quad T_1 = 4T_2$$

$$\therefore \quad \eta = \left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75$$

16. (b) Let $T_3 > T_2 > T_1$ and final temperature is T such that $T_3 > T > T_2 > T_1$. Now heat gained by first two gases is equal to heat lost by third gas:

$$n_1C(T - T_1) + n_2C(T - T_2) = n_3C(T_3 - T)$$

$$\Rightarrow \quad T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

17. (d)

$$\frac{1}{2}Mv^2 = 1 \times C_V\Delta T$$

$$\Rightarrow \quad \frac{1}{2}Mv^2 = \frac{R}{\gamma - 1}\cdot\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2(\gamma - 1)}{2R} = \frac{(\gamma - 1)Mv^2}{2R}\text{K}$$

18. (a)

$$\eta = 1 - \frac{T_2}{T_1} = \frac{1}{6} \quad \Rightarrow \quad \frac{T_2}{T_1} = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\frac{1}{3} = 1 - \frac{(T_2 - 62)}{T_1} \Rightarrow \frac{T_2 - 62}{T_1} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{5(T_2 - 62)}{6T_2} = \frac{2}{3} \Rightarrow T_2 = 310 \text{ K}$$

and

$$T_1 = \frac{6 \times 310}{5} = 372 \text{ K}$$

19. (c) For 1st case, Efficiency = $\eta = \left(1 - \dfrac{T_1}{T_2}\right) \times 100$

$$\left(1 - \frac{T_1}{500}\right) \times 100 = 40$$

$$T_1 = 300 \text{ K}$$

For 2nd case,

$$\eta = \left(1 - \frac{300}{T_2}\right) \times 100 = 60 \qquad T_2 = 750 \text{ K}$$

20. (a) Heat is extracted from the source in path DA and AB is

$$\Delta Q = \frac{3}{2}R\left(\frac{P_0V_0}{R}\right) + \frac{5}{2}R\left(\frac{2P_0V_0}{R}\right) = \frac{13}{2}P_0V_0$$

21. (b)

$$\Delta U_{AB} = nC_V(T_B - T_A) = 1 \times \frac{5R}{2}(800 - 400) = 1000\,R$$

$$\Delta U_{BC} = nC_V(T_C - T_B) = 1 \times \frac{5R}{2}(600 - 800) = -500\,R$$

$$\Delta U_{\text{total}} = 0$$

$$\Delta U_{CA} = nC_V(T_A - T_C) = 1 \times \frac{5R}{2}(400 - 600) = -500\,R$$

22. (c)

$$P = \frac{1}{3}\left(\frac{U}{V}\right) = \frac{1}{3}kT^4 \qquad \text{...(i)}$$

$$PV = \mu RT \qquad \text{...(ii)}$$

$$\frac{\mu RT}{V} = \frac{1}{3}kT^4 \Rightarrow V \propto T^3 \Rightarrow R \propto \frac{1}{T}$$

23. (b) We define the entropy change of the body $dS = \frac{dQ}{T} = ms\frac{dT}{T}$

Hence change in entropy for changing temperature from T_1 to T_2

$$\Delta S = \int_{T_2}^{T_1} ms\left(\frac{dT}{T}\right)$$

In case-I: $\Delta S_1 = \int_{100}^{200} 1\left(\frac{dT}{T}\right) = \ell n2$

In case-II: $\Delta S_2 = \int_{100}^{200} 1\left(\frac{dT}{T}\right) = \ell n2$

24. (c) $$\tau = \frac{\lambda}{v_{rms}} = \frac{1}{\sqrt{2}\pi d^2\left(\frac{N}{V}\right)\sqrt{\frac{3RT}{M}}} \quad \text{...(i)}$$

$$\tau \propto \frac{V}{\sqrt{T}} \quad \text{...(ii)}$$

$$TV^{\gamma-1} = k \quad \text{...(iii)}$$

$$\Rightarrow \quad \tau \propto V^{\frac{\gamma+1}{2}}$$

25. (b) We define molar heat capacity C,

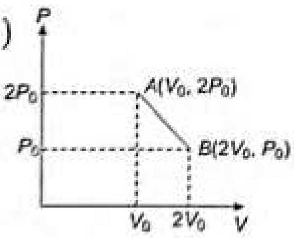
$$C = C_V + \frac{PdV}{ndT} = C_V + \frac{R}{1-n} = \frac{R}{\gamma-1} + \frac{R}{1-n}$$

$$C = \frac{R}{\frac{C_P}{C_V} - 1} + \frac{R}{1-n}, \text{ on solving we get, } n = \frac{C - C_P}{C - C_V}$$

26. (a)

$$P - 2P_0 = \frac{P_0 - 2P_0}{2V_0 - V_0}(V - V_0)$$

$$\Rightarrow P = -\frac{P_0}{V_0}V + 3P_0$$



Hence, $\frac{nRT}{V} = -\left(\frac{P_0}{V_0}\right)V + 3P_0$

$$T = \frac{1}{nR}\left[-\frac{P_0}{V_0}V^2 + 3P_0V\right] \quad \text{...(i)}$$

For temperature to be maximum, $\frac{dT}{dV} = 0$

$$\frac{1}{nR}\left[-\frac{2P_0}{V_0}V + 3P_0\right] = 0,$$

we get $V = \frac{3}{2}V_0$ ...(ii)

From (i) and (ii)

$$\therefore \; T_{max} = \frac{1}{nR}\left[-\frac{P_0}{V_0}\times\left(\frac{3}{2}V_0\right) + 3P_0 \times \frac{3}{2}V_0\right]$$

$$= \frac{1}{nR}\left[-\frac{9}{4}P_0V_0 + \frac{9}{2}P_0V_0\right] = \frac{9P_0V_0}{4nR}$$

27. (b) Using ideal gas equation, $PV = nRT$

Here n is number of moles.

$$P_0V_0 = n_iR \times 290 \quad \text{...(1)}$$

Initial temperature, $T_i = 273 + 17 = 290$ K

After heating

$$P_0V_0 = n_fR \times 300 \quad \text{...(2)}$$

Final temperature, $T_f = 273 + 27 = 300$ K

from equations (1) and (2)

$$n_f - n_i = \frac{P_0V_0}{R\times 300} - \frac{P_0V_0}{R\times 290}$$

difference in number of moles $-\frac{P_0V_0}{R}\left[\frac{10}{290\times 300}\right]$

Hence $n_f - n_i$ is

$$= \frac{P_0V_0}{R}\times\left[\frac{10}{290\times300}\right]\times 6.023\times10^{23}$$

Putting $P_0 = 10^5\ P_A$ and $V_0 = 30\ m^3$

Number of molecules $n_f - n_i = -2.5 \times 10^{25}$

28. (a) $C_p - C_v = R$

where C_p and C_v are molar specific heat capacities. As per the question

$$a = \frac{R}{2}b = \frac{R}{28} = 14b$$

29. (d) In an adiabatic process PV^γ = constant ...(i)

and $PV = nRT$ ...(ii)

From (i) and (ii) $\left(\frac{nRT}{V}\right)V^\gamma$ = constant

$$\Rightarrow \quad V^{\gamma-1} \propto \frac{1}{T}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right) \Rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

Monoatomic gas: $\gamma = \frac{5}{3} \Rightarrow \quad T_2 = (300\text{ K})\left(\frac{V}{2V}\right)^{\frac{5}{3}-1}$

= 189 K (final temperature)

Change in internal energy $\Delta U = n\frac{f}{2}R\Delta T$

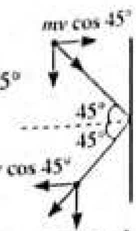
$$\Delta U = 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(189 - 300) = -2.7\text{ kJ}$$

30. (b)

$$|\Delta p| = 2mv\cos 45°$$

$$|F| = \frac{\Delta p}{\Delta t} = n(2mv\cos 45°)$$

$$|F| = \frac{\Delta p}{\Delta t} = n(2mv\cos 45°) = 2nmv\cos 45°$$



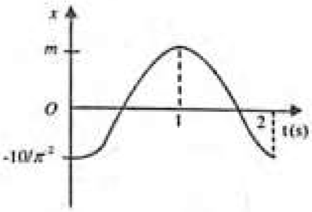
$$\text{Pressure} = \frac{F}{A} = \frac{2nmv\cos 45°}{\text{Area}}$$

$$= \frac{2\times10^{23}\times3.3\times10^{-27}\times\left(\frac{1}{\sqrt{2}}\right)}{2\times10^{-4}} = 2.35\times10^3\ \text{N/m}^2$$

CHAPTER 15: OSCILLATION AND SIMPLE HARMONIC MOTION

Concept Application Exercise 15.1

1. i.



ii. 0.5
iii. $10/\pi^2$ m
iv. zero
v. 0.5 s, 1.5 s
vi. 0; 1 s, and 2 s

2. (a) $x = 5\sin\left(20t + \frac{\pi}{3}\right)$

$$\frac{dx}{dt} = 100\cos\left(20t + \frac{\pi}{3}\right)$$

For particle to come into rest

$$\frac{dx}{dt} = 0$$

$$\cos\left(20t + \frac{\pi}{3}\right) = \cos\frac{\pi}{2}$$

$$20t = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6} \Rightarrow t = \frac{\pi}{120}$$

(b) The particle will have zero acceleration when it will be at equilibrium position.

As $\omega = 20$, $2\pi/T = 20$

$\Rightarrow$ $T = \frac{\pi}{10}$ s

Hence particle will come to equilibrium position.

$$t = \frac{\pi}{120} + \frac{\pi}{40} = \frac{\pi}{40}\left(\frac{1}{3} + 1\right) = \frac{\pi}{30} \text{ s}$$

3. Remember that whenever motion starts from origin or the mean position, then $x = a\sin(\omega t)$ and whenever it starts from extreme position then $x = a\cos(\omega t)$.

Because at $t = 0$, $x = 0$ for $x = a\sin(\omega t)$

and at $t = 0$, $x = a$ for $x = a\cos(\omega t)$

$\frac{a}{2} = a\sin(\omega t_1)$	$\frac{a}{2} = a\cos(\omega t_2)$	
$\sin(\omega t_1) = \frac{1}{2}$	$\cos(\omega t_2) = \frac{1}{2}$	$\Rightarrow \frac{t_1}{t_2} = \frac{1}{2}$
$\omega t_1 = \frac{\pi}{6}$	$\omega t_2 = \frac{\pi}{3}$	

4. Let the amplitudes of the individual motions be A each. The resultant amplitude is also A. If the phase difference between the two motions is δ,

$$A = \sqrt{A^2 + A^2 + 2AA\cos\delta}$$

or, $= A\sqrt{2(1+\cos\delta)} = 2A\cos\frac{\delta}{2}$

or, $\cos\frac{\delta}{2} = \frac{1}{2}$ or, $\delta = 2\pi/3$

5. In the case of SHM,

$x = A\sin(\omega t + \phi)$

or $5 = 10\sin(\omega \times 0 + \phi)$ (Here $A = 10$ mm $= 0.01$ m)

$\therefore$ $\sin\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}$

Here, $\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$\therefore$ $x = 0.01\sin\left(\pi t + \frac{\pi}{6}\right)$ where x is in metre.

6. We have $V = \omega\sqrt{a^2 - y^2}$

$\therefore$ $\frac{V_2}{V_1} = \frac{\sqrt{a^2 - y_2^2}}{\sqrt{a^2 - y_1^2}}$ or $\frac{49}{100} = \frac{a^2 - 16}{a^2 - 9}$

$\therefore$ $a = 4.76$ cm

Length of the path $= 2a = 9.5$ cm

Concept Application Exercise 15.2

1. When the bob is located to the right of point O, the obstacle acts as the pivot. The effective length of the pendulum is $l/4$ and when it is to the left of obstacle the effective length is l

$$\Rightarrow t = \frac{1}{2}\left(2\pi\sqrt{\frac{l}{4g}} + 2\pi\sqrt{\frac{l}{g}}\right) \Rightarrow t = \frac{3T}{4}$$

where $T = 2\pi\sqrt{\frac{l}{g}}$

2. $T_0 = 2\pi\sqrt{\frac{M}{k}}$ (i)

Let v_0 be the velocity of mass M as it passes the equilibrium position. If a_0 is the initial amplitude, then

$$\frac{1}{2}Mv^2 = \frac{1}{2}ka_0^2 \Rightarrow v = \left(\sqrt{\frac{k}{M}}\right)a_0$$

If V is the combined velocity of $(M + m)$ system in equilibrium position, then by law of conservation of linear momentum, we have

$$Mv = (M + m)V \Rightarrow V = \left(\frac{M}{M+m}\right)v$$

Let a be the new amplitude, then

$$\frac{1}{2}ka^2 = \frac{1}{2}(M+m)V^2 = \frac{1}{2}(M+m)\left(\frac{M}{M+m}\right)^2 v^2$$

$$= \frac{1}{2}(M+m)\frac{M^2}{(M+m)^2}\left(\frac{k}{M}\right)a_0^2$$

$\Rightarrow \quad a = a_0\sqrt{\frac{M}{M+m}}$

New time period is $T = 2\pi\sqrt{\frac{M+m}{k}}$

$= \sqrt{\frac{M+m}{M}}T_0 \qquad \left\{\because T_0 = 2\pi\sqrt{\frac{M}{k}}\right\}$

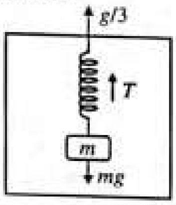
3. The angular frequency under all circumstances is

$\omega = \sqrt{(k/m)} = \sqrt{(200/1)} = 14$ rad/s

When elevator is moving up, the equation of motion

$T - mg = \frac{mg}{3}$

$\Rightarrow \quad T = \frac{4mg}{3}$



This tension elongates the spring by x

$T = kx$

$\Rightarrow \quad x = \frac{4mg}{3k} = 0.07\text{m}$

4. (a) Let us assume that the bob swings a maximum angle β in the left hand side of the mean position in the absence of the portion PC of the wall.

Conserving energy at A and D, we have

$U_A + K_A = U_D + K_D$

where $U_A = mgh_A, U_B = mgh_B$

and $\quad K_A = K_D = 0$

because the bob comes to rest at the extreme positions.

Then, we have $h_A = h_D$

Substituting $h_A = OA(1-\cos\theta)$

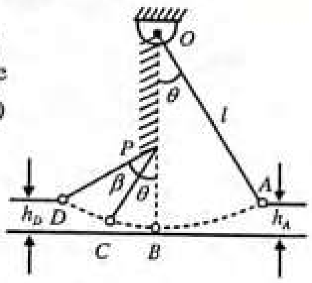
and $h_D = PA(1-\cos\beta)$, we have

$OA(1-\cos\theta) = PD(1-\cos\beta)$

Substituting $OA = l$,

$PD = (l-d)$, we have

$\beta = \left(\sqrt{\frac{l}{l-h}}\right)\theta$



(b) Since the bob swings from A to B, $t_{AB} = \frac{T}{4}$

where $T = 2\pi\sqrt{\frac{l}{g}}$ Then, $t_{AB} = \frac{\pi}{2}\sqrt{\frac{l}{g}}$

The time of motion from B to C can be found by substituting

$t = t_{BC}, \theta = 0$

and $\quad \theta = \beta\sqrt{(l/(l-h))\theta}$

in the equation $\omega = \beta\sqrt{(l/(l-h)\theta}$,

where $\omega = \sqrt{g/(l-h)}$

for the swinging position of the string from B to C. This gives

$\theta = \left(\sqrt{\frac{l}{l-h}}\right)\theta\sin\sqrt{\frac{g}{l-h}}t_{BC}$

Then, we have

$t_{AB} = \sqrt{\frac{l-h}{g}}\sin^{-1}\sqrt{\frac{l-h}{l}}$

Finally, substituting t_{AB} and t_{BC} in the expression of total time, $T = 2(t_{AB} + t_{BC})$, we have

$T = \pi\sqrt{\frac{l}{g}} + \sqrt{\frac{l-h}{l}}\sin^{-1}\sqrt{\frac{l-h}{l}}$

5. (a) For minimum time period, we have

$r = k \quad$ and $\quad k = \sqrt{I_C/m}$

Substituting $I_C = ml^2/12$ for the rod, we have

$r = \frac{l}{2\sqrt{3}}$

Then, $x = l/2 - r$, where $r = l/2\sqrt{3}$

This gives $x = \frac{l}{2}\left(1 - \frac{1}{\sqrt{3}}\right)$

(b) $T_{min} = 2\pi\sqrt{\frac{2k}{g}}$

where $k = \frac{l}{2\sqrt{3}}$

This gives $T_{min} = 2\pi\sqrt{\frac{l}{\sqrt{3}g}}$

6. $T \propto 1/\sqrt{k}$

Now, $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}$

or, $\quad \frac{1}{k} = \frac{4}{k'} \quad$ [Here $k_{eff} = k$ and $k_1 = k_2 = k_3 = k_4 = k'$]

$k' = 4k$

Therefore, time period will be twice the original value.

Exercises

Problems Based on Basic Theory

1. (c) The motion of particle will not be SHM but that of its projection on a diameter will be SHM. However, the motion of particle will be periodic because it has constant speed.

2. (d) Standard equation of S.H.M. $\frac{d^2y}{dt^2} = -\omega^2 y$ is not satisfied by $y = a\tan\omega t$ which will be periodic.

Clearly, this equation represents SHM.

3. (b) $x_1 = a\sin(\omega t + \phi_1)$

$x_2 = a\sin(\omega t + \phi_2)$

$\Rightarrow \quad |x_1 - x_2| = 2a\sin\left(\omega t + \frac{\phi_1+\phi_2}{2}\right)\cos\left(\frac{\phi_1-\phi_2}{2}\right)$

To maximize $|x_1 - x_2|$: $\sin\left(\omega t + \frac{\phi_1+\phi_2}{2}\right) = 1$

$\Rightarrow \quad a\sqrt{2} = 2a \times 1 \times \cos\left(\frac{\phi_1-\phi_2}{2}\right)$

$\Rightarrow \quad \frac{1}{\sqrt{2}} = \cos\left(\frac{\phi_1-\phi_2}{2}\right)$

$\Rightarrow \quad \frac{\pi}{4}=\frac{\phi_1-\phi_2}{2} \quad \Rightarrow \quad \phi_1-\phi_2=\frac{\pi}{2}$

4. (c) Equation of motion $y = a\cos\omega t$

$\Rightarrow \quad \frac{a}{2}=a\cos\omega t \Rightarrow \cos\omega t=\frac{1}{2}\Rightarrow \omega t=\frac{\pi}{3}$

$\Rightarrow \quad \frac{2\pi t}{T}=\frac{\pi}{3}\Rightarrow t=\frac{\frac{\pi}{3}\times T}{2\pi}=\frac{4}{3\times 2}=\frac{2}{3}\text{sec}$

5. (b) $a=-16\pi^2 x$

Standard equation of SHM is:

$a=-\omega^2 x$

Hence, comparing two equations, we get $\omega=4\pi$

and $\quad T=\frac{2\pi}{\omega}=\frac{2\pi}{4\pi}=\frac{1}{2}$ sec

6. (a) Distance travelled by the particle

$=A\sin\frac{5\pi}{12}-A\sin\frac{\pi}{12}=\frac{A}{\sqrt{2}}$

7. (c) Since $y=A\sin\phi$ and $a=-\omega^2 A\sin\phi$

and $-\sin\phi=\sin(\pi+\phi)$

So, acceleration is ahead of displacement by a phase π.

8. (d) Phase difference $\delta=\frac{2\pi}{3}=\delta_2-\delta_1=\delta_2=\delta_2-\frac{\pi}{6}$

$\Rightarrow \quad \delta_2=\frac{5\pi}{6}$; and $\sin\delta_1=\sin\delta_2=\frac{1}{2}$

Displacement is measured from starting point ($t=0$)

So, $\quad x=A\sin(\omega t+\delta_1)$; $A\sin\left(0+\frac{\pi}{6}\right)=\frac{A}{2}$

9. (b) At phase $3\pi/2$, $(\omega t+\phi)=3\pi/2$

$x_1=A\sin\frac{3\pi}{2}=-A$

$x_2=0$

So, at $t=0$, $x_1=-A\cos\omega t$ and $x_2=A\sin\omega t$

$x_1=x_2$

$-\cos\omega t=\sin\omega t \quad \Rightarrow \quad \tan\omega t=-1$

$\omega t=\frac{3\pi}{4} \quad \Rightarrow \quad \left(\frac{2\pi}{T}\right)t=\frac{3\pi}{4}$ or $\quad t=3T/8$

10. (c) Let T be the time period; then $\frac{T}{2}=\frac{5\pi}{64}-\frac{\pi}{64}=\frac{4\pi}{64}$

$T=\frac{\pi}{8}$ s $\quad \therefore \ \omega=\frac{2\pi}{T}=\frac{2\pi}{(\pi/8)}=16$ rad/s

Also, $A=10$ cm (from the graph)

The equation of the sinusoidal wave can be written as $y=10\sin(16t+\phi)$ cm, where ϕ is the initial phase.

From the graph, corresponding to the crest = 10 cm; when $t=3\pi/64$.

$10\text{ cm}=10\sin\left[16\left(\frac{3\pi}{64}\right)+\phi\right]\text{cm}$

$\sin\left[\frac{3\pi}{4}+\phi\right]=1 \quad$ or $\quad \frac{3\pi}{4}+\phi=\frac{\pi}{2} \Rightarrow \phi=-\frac{\pi}{4}$

$\therefore \quad y=10\sin\left(16t-\frac{\pi}{4}\right)$

11. (b) The slope of the length is $\frac{F}{x}=-\frac{0.5}{5}=-0.1$ N/cm $=-10$ N/m

But $\quad F=-m\omega^2 x \quad$ or $\quad F/x=-m\omega^2$

so, $\quad -m\omega^2=-10 \quad$ or $\quad m\omega^2=10$

or, $\quad \omega^2=10/m$

$\therefore \quad \omega^2=\frac{10}{4\times 10^{-1}} \quad \Rightarrow \omega=\frac{10}{2}=5$

$\therefore \quad f=\frac{\omega}{2\pi}=\frac{5}{2\pi}$/s

12. (d) $y_1=4\sin(10t+\phi)$, $\ y_2=5\cos 10t$

$v_1=\frac{dy_1}{dt}=40\cos(10t+\phi)$

$v_2=\frac{dy_2}{dt}=-50\sin 10t=50\cos\left(10t+\frac{\pi}{2}\right)$

Phase difference between v_1 and v_2 $=\left(\phi-\frac{\pi}{2}\right)$

13. (a) Let $x_1=a\sin\omega t$ and $x_2=a\sin(\omega t+\delta)$ be two SHMs.

$\frac{a}{3}=a\sin\omega t \quad$ and $\quad -\frac{a}{3}=a\sin(\omega t+\delta)$

$\sin\omega t=1/3 \quad$ and $\quad \sin(\omega t+\delta)=-1/3$

Eliminating t, $\frac{1}{3}\cos\delta+\sqrt{1-\frac{1}{9}}\sin\delta=-\frac{1}{3}$

$9\cos^2\delta+2\cos\delta-7=0$

$\cos\delta=-1 \quad$ or $\quad \frac{7}{9}$

i.e., $\quad \delta=180° \quad$ or $\quad \cos^{-1}\left(\frac{7}{9}\right)$

If we put 180°, we find that v_1 and v_2 are of opposite signs.

Hence $\delta=180°$ is not applicable.

$\therefore \quad \delta=\cos^{-1}\left(\frac{7}{9}\right)$

14. (d) $x=a\sin\omega t$

$x=a\sin\frac{2\pi}{T}\times\frac{T}{8} \quad$ or $\quad x=a\sin\frac{\pi}{4}=\frac{a}{\sqrt{2}}$

15. (d) Here $x=A\sin\omega t$

$x_1=A\sin\frac{2\pi}{8}\times 1=A\sin\frac{\pi}{4}=\frac{A}{\sqrt{2}}$

and $\quad x_2=A\sin\frac{2\pi}{8}\times 2=A$

Therefore, the distance traveled in 2nd second is

$x_2'=x_2-x_1=A-\frac{A}{\sqrt{2}}=\frac{(\sqrt{2}-1)A}{\sqrt{2}}$

$\therefore \quad \text{Ratio}=\frac{x_1}{x_2'}=\frac{A/\sqrt{2}}{\frac{(\sqrt{2}-1)A}{\sqrt{2}}}=\frac{1}{(\sqrt{2}-1)}$

$=\frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{(\sqrt{2}+1)}{2-1}=(\sqrt{2}+1)$

16. (c) Let the acceleration be f; $f = -\omega^2 x$

Therefore, distance of the particle from the centre at any time t is given by

$x = r\cos(\omega t)$, where r is the amplitude

when $t = 1$ s, $x = r - a$

$\therefore \quad (r-a) = r\cos\omega$

$$\cos\omega = \frac{r-a}{r} \quad \text{(i)}$$

When $t = 2$ s, $x = r - a - b$,

therefore $\quad r - a - b = \cos 2\omega$

$\therefore \quad r - a - b = r(2\cos^2\omega - 1) \quad$ (ii)

Substituting the value of $\cos\omega$ from Eq. (i) in Eq. (ii),

we get $r - a - b = r\left[2\dfrac{(r-a)^2}{r^2} - 1\right] = \dfrac{2(r-a)^2}{r} - r$

$\therefore \quad r(3a - b) = 2a^2 \Rightarrow r = \dfrac{2a^2}{3a-b}$

Velocity, Acceleration and Energy of SHM

17. (a) For A, time period $T_A = 16$ s (distance between two adjacent crests)

For B, time period $T_B = 2(20-8) = 24$ s (length between the crest and trough shown = 20 s − 8 s = 12 s)

Also, amplitudes $a_A = 10$ cm, $a_B = 5$ cm

$$\text{Now, } \frac{(V_{max})_A}{(V_{min})_B} = \frac{\omega_A a_A}{\omega_B a_B} = \frac{\left(\frac{2\pi}{T_A}\right)a_A}{\left(\frac{2\pi}{T_B}\right)a_B} = \frac{T_B a_A}{T_A a_B} = \frac{24\times 10}{16\times 5} = \frac{3}{1}$$

18. (b) From the graph $T = (5-1) = 4$ s

(distance between the two adjacent crests shown in the figure)

and $\quad v_{max} = 5$ m/s; $\quad \omega A = 5$ m/s

$$\left(\frac{2\pi}{T}\right)A = 5 \Rightarrow A = \frac{5T}{2\pi} = \frac{5\times 4}{2\pi} = \frac{10}{\pi}\text{ m}$$

Also, $\omega = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$ rad/s

The equation of velocity can be written as

$$v = 5\sin\left(\frac{\pi}{2}t\right)\text{ m/s}$$

At extreme position, $v = 0$; $\sin\left(\dfrac{\pi}{2}t\right) = 0 \quad$ or $\quad t = 2$ s

Phase of the particle velocity at that instant corresponding to the above equation = π.

Therefore, when a phase change of $\pi/6$ takes place, the resulting phase = $\pi + \pi/6$.

$$v = 5\sin\left(\pi + \frac{\pi}{6}\right) = -5\sin\frac{\pi}{6} = -5\left(\frac{1}{2}\right)$$

$= 2.5$ m/s (numerically)

19. (b) $\dfrac{dy}{dt} = v \quad \Rightarrow dy = v\,dt$

$$dy = \int 5\sin\left(\frac{\pi t}{2}\right)dt = \frac{10}{\pi}\left[-\cos\frac{\pi t}{2}\right] + C$$

Since at $t = 0$, the particle is at the extreme position, therefore

at $t = 0$; $y = -\dfrac{10}{\pi}$

$$-\frac{10}{\pi} = -\frac{10}{\pi}\cos\theta + C \quad \Rightarrow C = 0$$

$$y = -\frac{10}{\pi}\cos\frac{\pi t}{2}$$

Clearly a phase change of $\pi/6$ corresponds to a time difference of

$$\frac{T}{2\pi}\left(\frac{\pi}{6}\right) = \frac{T}{12} = \frac{4}{12} = \frac{1}{3}\text{ s}$$

$$y = -\frac{10}{\pi}\cos\frac{\pi}{6} = -\frac{10}{\pi}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{5\sqrt{3}}{\pi}\text{ m (numerically)}$$

20. (c) Acceleration, $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(5\sin\dfrac{\pi t}{2}\right) = \dfrac{5\pi}{2}\dfrac{\cos\pi t}{2}$

a at $\quad t = \dfrac{1}{3}$ s $= \dfrac{5\pi}{2}\cos\dfrac{\pi}{6} = \dfrac{5\pi\sqrt{3}}{4}$ m/s^2

21. (c) Maximum displacement, $x_{max} = A = \dfrac{10}{\pi}$ m

and maximum acceleration, $a_{max} = \omega^2 A$

$$= \left(\frac{\pi}{2}\right)^2 \times \frac{10}{\pi} = \frac{5\pi}{2}\text{ m/s}^2$$

22. (d) $x = A\sin\omega t$

$$v = \frac{dx}{dt} = A\omega\cos\omega t = \omega\sqrt{A^2 - x^2}$$

$$a = \frac{dv}{dt} = -A\omega^2\sin\omega t = \frac{dv}{dt} = -\omega^2 x$$

But $\quad x = -\dfrac{a}{\omega^2}$

$\therefore \quad v = \omega\sqrt{A^2 - \dfrac{a^2}{\omega^4}} \quad$ or $\quad v^2 = \omega^2\left(A^2 - \dfrac{a^2}{\omega^4}\right)$

23. (c) $x = a\cos\omega t$

$a/2 = a\cos\omega t_0$

or $\quad \cos\dfrac{\pi}{3} = \cos\omega t_0 \quad$ or $\quad \dfrac{\pi}{3} = \omega t_0 \quad \therefore t_0 = \dfrac{\pi}{3\omega}$

$$v = \frac{dx}{dt} = -a\omega\sin\omega t$$

$$\therefore \quad \bar{v} = \frac{\int_0^{\pi/3\omega} v\,dt}{\int_0^{\pi/3\omega} dt}$$

But $\quad \omega = \dfrac{2\pi}{T} \quad \therefore \quad \bar{v} = \dfrac{3a}{T}$

24. (c) $v_1 = \omega\sqrt{a^2 - x_1^2}$, $\quad v_2 = \omega\sqrt{a^2 - x_2^2}$

We get, $a = \sqrt{\dfrac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$

25. (c) $U = U_0 - U_0 \cos ax$

$$F = -\frac{dU}{dx} = -U_0 \sin ax = m\frac{d^2x}{dt^2}$$

This is the equation of SHM $\Rightarrow T = \frac{2\pi}{a}\sqrt{\frac{m}{U_0}}$

26. (a) Equation of SHM = $y \sin(\omega t + \phi)$

When $y = \frac{a}{2}, t = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega}$

$v = a\omega \cos(\omega t + \phi)$, velocity is negative

$$\frac{\pi}{2} < (\omega t + \phi) < \frac{3\pi}{2}$$

$$\frac{a}{2} = a\sin(\omega t + \phi) \Rightarrow \sin(\omega t + \phi) = \frac{1}{2}$$

$$\left(\frac{\pi}{2} + \phi\right) = \frac{5\pi}{6}$$

Substituting in the above equation, we get $\phi = \pi/3$.

27. (d) Average velocity $\bar{v} = \frac{\int_0^t \frac{dx}{dt} \cdot dt}{t} = \frac{\int_0^t dx}{t} = \frac{x(t) - x(0)}{t}$

$$= \frac{A(\cos \pi/6 - 1)}{\pi/6\omega} = \frac{3A\omega}{\pi}(\sqrt{3} - 2)$$

Since particle does not change its direction in the given interval, average speed becomes

$$|\bar{v}| = \frac{3A\omega}{\pi}(2 - \sqrt{3})$$

28. (b) $E_1 = \frac{1}{2}Kx^2 \Rightarrow x = \sqrt{\frac{2E_1}{K}}$, $E_2 = \frac{1}{2}Ky^2 \Rightarrow y = \sqrt{\frac{2E_2}{K}}$

and $E = \frac{1}{2}K(x+y)^2 \Rightarrow x + y = \sqrt{\frac{2E}{K}}$

$$\Rightarrow \sqrt{\frac{2E_1}{K}} + \sqrt{\frac{2E_2}{K}} = \sqrt{\frac{2E}{K}} \Rightarrow \sqrt{E_1} + \sqrt{E_2} = \sqrt{E}$$

29. (b) $E = \frac{1}{2}m\omega^2 A^2 \Rightarrow E = \frac{1}{2}m(2\pi f)^2 A^2 \Rightarrow A = \frac{1}{2\pi f}\sqrt{\frac{2E}{m}}$

Putting $E = K + U$ we obtain,

$$A = \frac{1}{2\pi\left(\frac{25}{\pi}\right)}\sqrt{\frac{2 \times (0.5 + 0.4)}{0.2}} \Rightarrow A = 0.06 \text{ m}$$

30. (b) The time period of potential energy and kinetic energy is half of time period of SHM.

31. (c) $y = x$

$$PE = \frac{1}{2}m\omega^2 y^2$$

$$KE = \frac{1}{2}m\omega^2(a^2 - y^2)$$

$$TE = PE + KE = \frac{1}{2}m\omega^2 a^2$$

Since PE is maximum at $x = a$ and KE is maximum at $x = 0$, therefore TE remains constant throughout the motion.

Spring Particle System and Simple Pendulum

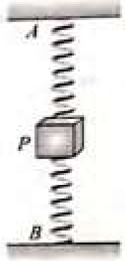
32. (b) $T_1 = 2\pi\sqrt{\frac{m}{k_1}}, T_2 = 2\pi\sqrt{\frac{m}{k_2}}, T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2} \text{ (for series)}, \ t_1^2 = \frac{2\pi m}{k_1}$$

$$t_2^2 = \frac{2\pi m}{k_2} \text{ and } T^2 = 2\pi m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\Rightarrow T^2 = t_1^2 + t_2^2$$

33. (b) $Mg = k \times 20 - k \times 10$



$$\Rightarrow k = \frac{mg}{10/100} \text{N/m}$$

$$T = 2\pi\sqrt{\frac{M}{k}}$$

$$= 2\pi\sqrt{\frac{1}{49 \times 4}} = \frac{\pi}{7} \text{ sec}$$

34. (d) Maximum velocity (ωA) are equal:

$$\omega_1 A_1 = \omega_2 A_2$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k_2}{k_1}} \quad (\because \ \omega = \sqrt{k/m})$$

35. (a) $k \cdot l = k' \times \frac{l}{n}$ = constant

New spring constant for each part is $k' = nk$

Now r part are taken in parallel.

so $k_{eq} = nrk$

Time period $= 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m}{nrk}}$

36. (b) When mass of 700 g is removed, the left out mass (500 + 400) g oscillates with a period of 3 sec.

$$\therefore \quad 3 = t = 2\pi\sqrt{\frac{(500 + 400)}{k}} \quad \text{...(i)}$$

When 500 g mass is also removed, the left out mass is 400 gm.

$$\therefore \quad t' = 2\pi\sqrt{\frac{400}{k}} \quad \text{...(ii)}$$

$$\Rightarrow \frac{3}{t'} = \sqrt{\frac{900}{400}} \Rightarrow t' = 2 \text{ sec}$$

37. (b) Let T_1 and T_2 are the time period of the two pendulums

$$T_1 = 2\pi\sqrt{\frac{100}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{121}{g}} \quad (T_1 < T_2 \text{ because } l_1 < l_2)$$

Let at $t = 0$, they start swinging together. Since their time periods are different, the swinging will not be in unision always. Only when number of completed oscillation differs by an integer, the two pendulum will again begin to swing together.

Let longer length pendulum complete n oscillation and shorter length pendulum complete $(n + 1)$ oscillation, for the unision swinging, then $(n + 1)T_1 = nT_2$

$$(n+1) \times 2\pi\sqrt{\frac{100}{g}} = n \times 2\pi\sqrt{\frac{121}{g}} \Rightarrow n = 10$$

38. (b) $n=\frac{1}{2\pi}\sqrt{\frac{g}{l}} \Rightarrow n\propto\frac{1}{\sqrt{l}} \Rightarrow \frac{n_1}{n_2}=\sqrt{\frac{l_2}{l_1}}=\sqrt{\frac{L_2}{2L_2}}$

$\Rightarrow \quad \frac{n_1}{n_2}=\frac{1}{\sqrt{2}} \Rightarrow n_2=\sqrt{2}\,n_1 \Rightarrow n_2>n_1$

Energy, $E=\frac{1}{2}m\omega^2a^2=2\pi^2mn^2a^2$

$\Rightarrow \quad \frac{a_1^2}{a_2^2}=\frac{m_2n_2^2}{m_1n_1^2}$ $\qquad (\because \quad E$ is same)

Given $n_2>n_1$ and $m_1=m_2 \Rightarrow a_1>a_2$

39. (c) $T=2\pi\sqrt{\frac{l}{|\vec{g}+\vec{a}|}}$, $\vec{a}$ is in horizontal direction in both cases.

Hence, $|\vec{g}+\vec{a}|>g$, so T decreases in both cases

40. (d) $T_1=2\pi\sqrt{\frac{l}{g}}, T_2=2\pi\sqrt{\frac{16}{g}}=4T_1$

at any time t, phases of pendulums are:

$$\phi_1=\omega_1t=\frac{2\pi}{T_1}t, \phi_2=\omega_2t=\frac{2\pi}{T_2}t$$

First pendulum is faster. Both will be in same phase again when faster pendulum has completed one oscillation more than slower pendulum:

$$\phi_1-\phi_2=2\pi$$

$$\Rightarrow \quad \frac{2\pi}{T_1}t-\frac{2\pi}{4T_1}t=2\pi \Rightarrow t=\frac{4T_1}{3}$$

Number of oscillations completed by shorter pendulum in time t:

$$n=\frac{t}{T_1}=\frac{4}{3}$$

41. (d) Let $T_1=T$ and $T_2=KT$; $\frac{2\pi}{T}t-\frac{2\pi}{KT}t=2\pi$

$$t\left(\frac{K-1}{KT}\right)=1; \frac{5T}{4}\left(\frac{K-1}{KT}\right)=1 \text{ which gives } K=5$$

If length of first pendulum $=l$, $T_1=2\pi\sqrt{l/g}$

$$T_2=5\times2\pi\sqrt{l/g}=2\pi\sqrt{25l/g}$$

So, ratio of lengths = 1 : 25.

42. (b) Time period for half part:

$$T=2\pi\sqrt{\frac{l}{g}}=2\pi\sqrt{\frac{1}{g}}=\frac{2\pi}{\pi}=2\text{ sec}$$

So 2° part will be covered in a time $t=\frac{T}{2}=1$ sec

For the left 1° part:

$$\theta=\theta_0\sin(\omega t)$$

$$1°=2°\sin\left(\frac{2\pi}{T}\times t\right) \Rightarrow \frac{1}{2}=\sin\left(\frac{2\pi}{T}\times t\right)$$

$$\Rightarrow \quad \frac{\pi}{6}=\pi\times t \Rightarrow t=1/6\text{ sec}$$

Total time $\Rightarrow \frac{T}{2}+2t$

$$\Rightarrow \quad 1+2\times\frac{1}{6}=1+\frac{1}{3}=\frac{4}{3}\text{ sec}$$

43. (b) The angular frequency is

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{200}{1}}=14\text{ rad/s}$$

44. (c) $T=\frac{25}{50}=\frac{1}{2}$ s $\Rightarrow \omega=\frac{2\pi}{T}=4\pi$ rad/s

Spring constant $k=m\omega^2=5\times(4\pi)^2=80\pi^2$ N/m

Force required to stretch the spring by 5 cm is

$F=kx=80\pi^2\times0.05$ N $=4\pi^2$

45. (d) Mean position of the particle is mg/k distance below the unstretched position of spring. Therefore, amplitude of oscillation is $A=\frac{mg}{k}$

$$\omega=\sqrt{\frac{k}{m}}=2\pi f=20\pi\ (f=10\text{ Hz})$$

$$\frac{m}{k}=\frac{1}{400\pi^2}$$

$$v_{max}=A\omega=\frac{g}{400\pi^2}\times20\pi=\frac{1}{2\pi}\text{ m/s}$$

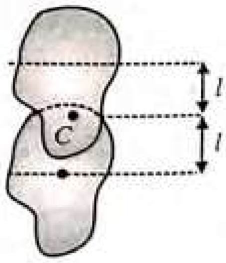
Compound Pendulum and Superposition of SHM

46. (b) $f_0=\frac{1}{2\pi}\sqrt{\frac{mgl}{I}}$

where l is the distance between point of suspension and centre of mass of the body. Thus, for the stick of length L and mass m.

$$f_0'=\frac{1}{2\pi}\sqrt{\frac{mg\frac{L}{4}}{\frac{m(L/2)^2}{2}\ \ \frac{}{12}}}=\frac{1}{2\pi}\sqrt{\frac{12g}{L}}=\sqrt{2}f_0$$

47. (a) $\frac{1}{2}I\omega^2=mg(2l) \Rightarrow \frac{I}{mgl}=\frac{4}{\omega^2}$



$$T=2\pi\sqrt{\frac{I}{mgl}}=2\pi\sqrt{\frac{4}{\omega^2}}=\frac{4\pi}{\omega}$$

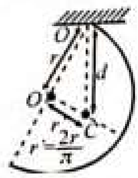
48. (d) $T=2\pi\sqrt{\frac{I_0}{mgd}}$;

$$I_0'=I_C+mr'^2 \Rightarrow mr^2=I_C+mr'^2$$

$$\Rightarrow \quad I_C=m(r^2-r'^2)$$

$$I_0=I_C+md^2m(r^2-r'^2)+m(r^2+r'^2)=2mr^2$$

$$d=\sqrt{r^2+\left(\frac{2r}{\pi}\right)^2}=r\sqrt{1+\frac{4}{\pi^2}}$$

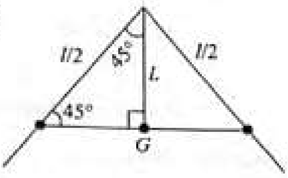


$$\Rightarrow \quad T=2\pi\sqrt{\frac{2r}{g\left(1+\frac{4}{\pi}\right)^{1/2}}}$$

49. (c) $T = 2\pi\sqrt{\dfrac{I}{mgL}}$

Here $I = \dfrac{ml^2}{3} + \dfrac{ml^2}{3} = \dfrac{2mgl^2}{3}$

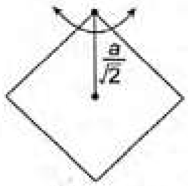
From figure, $\sin 45° = \dfrac{L}{l/2}$



$\therefore \quad L = \dfrac{l}{2\sqrt{2}}$

$\therefore \quad T = 2\pi\sqrt{\dfrac{2ml^2}{3 \times \dfrac{l}{2\sqrt{2}} mg}} = 2\pi\sqrt{\dfrac{2\sqrt{2}\,l}{3g}}$

50. (c)



$$T = 2\pi\sqrt{\frac{1}{mgd}} = 2\pi\sqrt{\frac{m\dfrac{(a^2+a^2)}{12} + m\left(\dfrac{a}{\sqrt{2}}\right)^2}{mg.\dfrac{a}{\sqrt{2}}}}$$

$$= 2\pi\sqrt{\frac{\left(\dfrac{a}{6}+\dfrac{a}{2}\right).\sqrt{2}}{g}} = 2\pi\sqrt{\frac{2\sqrt{2}a}{3g}}$$

51. (c) Given, $x = a\cos\omega t$ ...(i)

$y = a\sin\omega t$ ...(ii)

Squaring and adding Eqs. (i) and (ii),

$x^2 + y^2 = a^2 \qquad (\cos^2\omega t + \sin^2\omega t)$

$\Rightarrow \quad x^2 + y^2 = a^2 \qquad [\because \cos^2\omega t + \sin^2\omega t = 1]$

This is the equation, of a circle

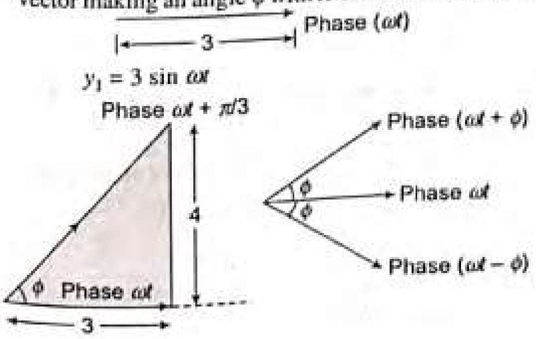
Clearly, the locus is a circle of constant radius a.

52. (b) $x = 3\sin(5\pi t + \pi/3) + \cos(5\pi t + \pi/3)$

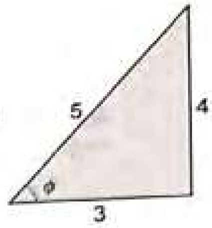
Amplitude $= x_{max} = \sqrt{3^2 + 1^2} = \sqrt{10}, \omega = 5\pi$

$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5\pi} = 0.4\,\text{s}$

53. (b) Representing a quantity with phase (ωt) along X-axis any quantity with a phase $(\omega t + \phi)$ can be represented by a vector making an angle ϕ with X-axis in the anticlockwise sense and any quantity with a phase $(\omega t - \phi)$ can be represented by a vector making an angle ϕ with X-axis in the clockwise sense.



Phase difference $\phi = \tan^{-1}\left(\dfrac{4}{3}\right)$



Alternatively, given $y_1 = 3\sin\omega t$ ...(i)

and $\quad y_2 = 4\sin\left(\omega t + \dfrac{\pi}{2}\right) + 3\sin\omega t$

$= 4\cos\omega t + 3\sin\omega t$

$= 5\left[\dfrac{4}{5}\cos\omega t + \dfrac{3}{5}\sin\omega t\right]$

$= 5[\sin\phi\cos\omega t + \cos\phi\sin\omega t]$

$= 5\sin(\omega t + \phi)$ ...(ii)

From (i) and (ii), the result follows.

$\cos\phi = \dfrac{3}{5}$; $\sin\phi = \dfrac{4}{5}$ and $\tan\phi = \dfrac{4}{3}$

54. (d) When two SHM's (which are perpendicular to each other) are given by $x = A_1\omega t$ and $y = A_2\sin(\omega t + \delta)$ and when they are combined the trajectory of the particle is given by

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1A_2}\cos\delta = \sin^2\delta$$

(This is equation of ellipse)

In the given equations, phase difference is $\delta = \dfrac{\pi}{6}$ and $A_1 = A_2 = A$

$\therefore \quad \dfrac{x^2}{A_1^2} + \dfrac{y^2}{A_2^2} - \dfrac{2xy}{A^2}.\dfrac{\sqrt{3}}{2} = \dfrac{1}{4}$

$x^2 + y^2 - \sqrt{3}xy = \dfrac{1}{4}A^2$ and this is an ellipse.

55. (b) Both the spring–mass system and torsional pendulum have no dependence on gravitational acceleration for their time periods.

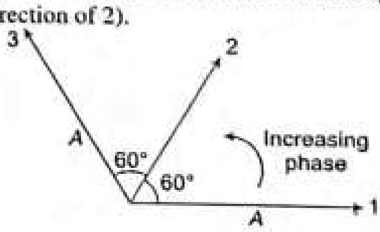
56. (a) Let the amplitudes of the individual motions be A each. The resultant amplitude is also A. if the phase difference between two motions is δ

$A = \sqrt{A^2 + A^2 + 2A.A\cos\delta} = A\sqrt{2(1+\cos\delta)}$

$= 2A\cos\dfrac{\delta}{2}$

$\cos\dfrac{\delta}{2} = \dfrac{1}{2} \Rightarrow \delta = \dfrac{2\pi}{3}$

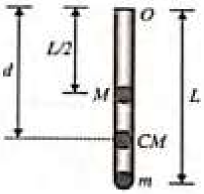
57. (d) The resultant of three vectors shown in the figure is $2A$ (along the direction of 2).



Problems Based on Mixed Concepts

58. (b) When the rod is replaced by the string the simple pendulm will act as compound pendulum for which time period is given by

$$T = 2\pi\sqrt{\frac{I_0}{Mgd}} \qquad \text{(i)}$$



I_0 = moment of inertia about point of suspension

$$= \frac{ML^2}{3} + mL^2 = \frac{(M+3m)L^2}{3}$$

M' = total mass = $(M + m)$

d = separation between point of suspension and centre of mass of the pendulum

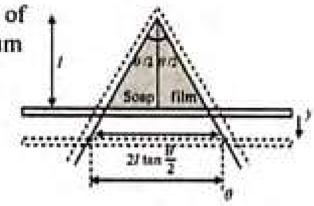
$$= \frac{M(L/2)+m(L)}{(M+m)} = \frac{(M+2m)L}{2(M+m)}$$

Substituting the values in Eq. (i), we get

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$$

59. (a) As the range of motion is 4 cm, the amplitude of motion is +2 cm. 10.5 s is equal to 10.5 time period of simple harmonic motion; so we have to find the height and position of cork after one-half of time period as $T/2 = 0.5$ s. As at $t = 10$ s, the particle is at its lowest position, after half a time period the cork would be at its maximum height and velocity of cork at extreme position is zero.

60. (a) Let S be the surface tension of the soap film. For equilibrium of rod



$$2(l+y)\tan\frac{\theta}{2}$$

$$mg = (F_{surface})_1$$

$$mg = \left(2l\tan\frac{\theta}{2}\right)S\times 2; \quad mg = 4Sl\tan\frac{\theta}{2}$$

If the rod is displaced from its mean position by small displacement y, then restoring force on the rod is

$$F_{rest} = -\left[(F_{surface})_2 - mg\right] - (F_{surface})_1$$

$$= -\left[4S(l+y)\tan\frac{\theta}{2} - mg\right] - \left[4S\tan\frac{\theta}{2}y\right]$$

$$a = -\frac{4S\tan\frac{\theta}{2}}{m}y = \frac{-4S\tan\frac{\theta}{2}}{\left(\frac{4Sl\tan\frac{\theta}{2}}{g}\right)}y$$

$$\frac{d^2y}{dt^2} = -\left(\frac{g}{l}\right)y \quad \therefore T = 2\pi\sqrt{\frac{l}{g}}$$

61. (a) Let the cylinder be depressed a distance x metres into water from the position of rest.

Increase in tension of spring = $100x$ metres into water from the position of rest

Increase in tension of spring = $100x$ kg-wt

($A = 100\text{ cm}^2 = 10^{-2}\text{ m}^2$)

$$\text{Increase in buoyancy} = \frac{100}{100^2}x\times 1000 \text{ kg-wt}$$

K = 1 kg-wt/cm = 100 kg-wt/m = $10x$ kg-wt

Total unbalanced vertical force on the cylinder = $100x + 10x = 110x$ kg-wt

$$\text{Acceleration of cylinder} = \frac{110x\times 10}{10}\text{ m/s}^2 = 110x \text{ m/s}^2$$

$$\text{Period of one oscillation} = \frac{2\pi}{\sqrt{110}} = 0.6\text{ s}$$

62. (d) When spring is compressed by $3x_0$. Amplitude, $A = 3x_0$. The time taken from extreme compressed position to mean position, $t_1 = T/4$.

If time taken (t_2) from mean position to $x = x_0$ is given by

$$x = A\sin\frac{2\pi t_2}{T} \Rightarrow x_0 = 3x_0\sin\frac{2\pi t_2}{T}$$

$$\sin\frac{2\pi t_2}{T} = \frac{1}{3} \Rightarrow \frac{2\pi t_2}{T} = \frac{\pi}{9} \Rightarrow t_2 = \frac{T}{18}$$

$$t_1 + t_2 = \frac{T}{4} + \frac{T}{18} = \frac{11}{18}T = \frac{11}{18}2\pi\sqrt{\frac{m}{k}} = \frac{11}{9}\pi\sqrt{\frac{m}{k}}$$

63. (d) The mean position of particle can be found by setting $F = -5x + 15 = 0$, so the mean position lies at $x = 3$. One extreme position is at $x = 6$. Hence the other extreme position for this particle undergoing SHM should be at $x = 0$. Time taken by particle to reach from $x = 6$ to $x = 0$ is 0.5 second, that is, $T/2 = 0.5$ sec, hence $T = 1$ second. Hence the equation of motion is $x = 3 + A(\sin\omega t + \phi_0)$ where $A = 3$, $\omega = 2\pi\times 1$ and $\phi_0 = \frac{\pi}{2}$. So $x = 3 + 3\cos(2\pi t)$.

64. (d) $ax^2 + bv^2 = k$

$bv^2 = k - ax^2$

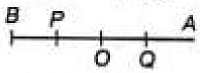
$$v^2 = \frac{k}{b} - \frac{a}{b}x^2$$

Compare with $v^2 = A^2\omega^2 - \omega^2x^2$

$\omega^2 = a/b$ and $A^2\omega^2 = k/b$

$$A = \sqrt{\frac{A^2\omega^2}{\omega^2}} = \sqrt{\frac{k/b}{a/b}} = \sqrt{\frac{k}{a}}$$

65. (a)



$$x = A\sin\left(\omega\times 0 + \frac{\pi}{6}\right) = A\sin\frac{\pi}{6} = \frac{A}{2} \quad (t = 0 \text{ at initial phase})$$

Let time $t = 0$ is considered from point Q where displacement $= A/2$

Time period $T = (t_{QA} + t_{AQ}) + (t_{QP} + t_{PQ}) + (t_{PB} + t_{BP})$

$= t + [(2t - t) + (2t - t)] + t = 4t = 2\pi/\omega$

$$\omega = \pi/2t;\ x = A\sin\left(\omega t + \frac{\pi}{6}\right)$$

$$\frac{dx}{dt} = A\omega\cos\left(\omega t + \frac{\pi}{6}\right) = A\omega\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$$

$$2 = A\times\frac{\pi}{2t}\times\frac{1}{2} \qquad \text{(sign not considered);}$$

$$A = \frac{8t}{\pi}$$

66. (b) $U = \beta x^4$ (Given)

$\therefore \quad U_{max} = \beta A^4$

$$K(x) = U_{max} - U(x) = \beta A^4 - \beta x^4 = \beta(A^4 - x^4)$$

$$U(x) = \frac{1}{3}K(x) \qquad \text{(Given)}$$

$$\therefore \quad \beta x^4 = \frac{1}{3}\beta(A^4 - x^4)$$

$$\Rightarrow \quad x = \pm\frac{A}{\sqrt{2}}$$

67. (c) By work energy theorem;

$$Fx_1 - \frac{1}{2}kx_1^2 = \frac{1}{2}mv^2 \qquad ...(1)$$

and $$Fx_2 - \frac{1}{2}k'x_2^2 = \frac{1}{2}mv'^2 \qquad ...(2)$$

where x_1, x_2 are initial and final extensions and v, v' are initial and final velocities.

In both cases, force applied is same, and velocity becomes maximum when $F = kx$. (after which the mass will decelerate)

$$\therefore \quad F = kx_1 = (4k)x_2$$

$$\Rightarrow \quad x_2 = \frac{x_1}{4}$$

Substituting in (2):

$$F\left(\frac{x_1}{4}\right) - \frac{1}{2}(4K)\left(\frac{x_1}{4}\right)^2 = \frac{1}{2}mv'^2$$

$$\Rightarrow \quad \frac{1}{4}[Fx_1 - \frac{1}{2}kx_1^2] = \frac{1}{2}mv'^2 \qquad ...(3)$$

Dividing (3)/(1), we get:

$$\frac{1}{4} = \frac{v'^2}{v^2} \Rightarrow v' = \frac{v}{2}$$

Hence, (c).

68. (c) Let the ball is pressed down by a small amount y, then the value of air decreases by Ay.

Then, excess pressure is $dp = -B\left(\frac{dV}{V}\right) = -B\left(\frac{yA}{V}\right)$

A restoring force $F(Adp)$ acts in the upward direction.

$$\therefore \quad F = A\left[-B\left(\frac{yA}{V}\right)\right] = -\frac{BA^2}{V}\cdot y$$

$$\therefore \quad a = -\frac{BA^2}{mV}y \text{ comparing this with } a = -\omega^2 y$$

$$\omega = \sqrt{\frac{BA^2}{mV}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{mV}{BA^2}}$$

69. (a) When a particle is executing SHM, at mean position $F = -\frac{dU}{dr} = 0$. Also, its potential energy is minimum at equilibrium position. So, $\frac{d^2U}{dr^2} > 0$.

$$\frac{dU}{dr} = -F \Rightarrow \frac{d^2U}{dr^2} = -\frac{dF}{dr}$$

At equilibrium, $dF = -\left|\frac{d^2U}{dr^2}\right|_{r=0} dr$

Comparing this standard SHM equation, i.e.

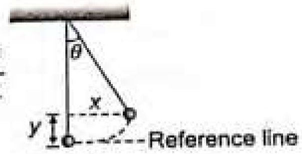
$F_{net} = -k_{eff}x$, we have $k_{eff} = \left|\frac{d^2U}{dr^2}\right|_{r=0}$

In general equilibrium happens at $r = r_0$ (say)

$$\therefore \quad k_{eff} = \left|\frac{d^2U}{dr^2}\right|_{r=r_0}$$

Also, at equilibrium $F = -\frac{dU}{dr} = 0$

Now, coming to the situation given, when the bob is given a small horizontal displacement as shown.



$$U = mgy$$

$$y = l(1 - \cos\theta) = 2l\sin^2\frac{\theta}{2}$$

$$\therefore \quad U = 2mgl\sin^2\frac{\theta}{2}$$

As θ is small $\sin\frac{\theta}{2} = \frac{\theta}{2}$

So $U = \frac{1}{2}mgl\left(\frac{\theta}{2}.\frac{\theta}{2}\right)$ or $U = \frac{1}{2}mgl\theta^2$

$$\theta = \frac{x}{l} \Rightarrow U = \frac{mg}{2l}x^2$$

$$\frac{dU}{dx} = \frac{mg}{2l}.2x = \frac{mg}{l}.x$$

$$\Rightarrow \quad \frac{d^2U}{dx^2} = \frac{mg}{l} \qquad \therefore \quad k_{eff} = \left|\frac{d^2U}{dx^2}\right|_{x=0} = \frac{mg}{l}$$

70. (d) $R_Q - R_P = \frac{mgx}{L}$

Frictional force at Q will be more than frictional force at P, and it will bring back the rod. Restoring force,

$$F = -(f_Q - f_P) = -\mu(R_Q - R_P) = -\frac{\mu mgx}{L}$$

So $K = \frac{\mu mg}{L}; T = 2\pi\left(\frac{m}{K}\right)^{1/2} = 2\pi\sqrt{\frac{L}{\mu g}}$

ARCHIVES

1. (b) As we know that the spring constant of a spring is inversely proportional to the length of the spring, so the new spring constant for each part is given by $k' = nk$, where k is the spring constant of the whole spring. From the theory of spring pendulum, we know that the time period of a spring pendulum is inversely proportional to the square root of spring constant, i.e.,

$$T \propto \frac{1}{\sqrt{k}}$$

$$T' \propto \frac{1}{\sqrt{nk}}$$

So $T = \frac{T}{\sqrt{n}}$

2. (c) The kinetic energy of a particle of mass m in SHM at any point is

$$\frac{1}{2}m\omega^2(a^2 - x^2)$$

and the potential energy is

$$\frac{1}{2}m\omega^2 x^2,$$

where a is the amplitude of the particle and x is the distance from the mean position.

So at the mean position, $x = 0$.

$\therefore$ $KE = \frac{1}{2}m\omega^2 a^2$ (maximum)

$PE = 0$ (minimum)

3. (b) As the child stands up, the effective lengths of the pendulum decreases due to the reason that the centre of gravity rises up. Hence, according to $T = 2\pi\sqrt{\frac{l}{g}}$, T will decrease.

4. (c) $T = 2\pi\sqrt{\frac{M}{k}}$

$$T' = 2\pi\sqrt{\frac{M+m}{k}}$$

$\therefore$ $T' = \frac{5}{3}T$,

$$2\pi\sqrt{\frac{M+m}{k}} = \frac{5}{3}2\pi\sqrt{\frac{M}{k}} \Rightarrow \frac{M+m}{k} = \frac{25}{9}\times\frac{M}{k}$$

$$\Rightarrow \frac{m}{M} = \frac{16}{9}$$

5. (d) $T = 2\pi\sqrt{\frac{1}{g}}$

and $T' = 2\pi\sqrt{\frac{1.21}{g}}$

$$\frac{T'}{T} = \sqrt{1.21} \Rightarrow T' = 1.1T \Rightarrow \Delta T = 0.1T$$

Percentage increase in time period is

$$\frac{\Delta T}{T}\times 100 = \frac{0.1T}{T}\times 100 = 10\%$$

6. (c) $X = 4\cos\pi t + 4\sin\pi t$

Since $\cos\pi t$ and $\sin\pi t$ are out of phase by 90°, amplitude

$$A = \sqrt{4^2+4^2} = 4\sqrt{2}$$

7. (a) We know that

$$KE = \frac{1}{2}m\omega^2(r^2 - X^2)$$

At mean position $X = 0$,

$$KE = \frac{1}{2}m\omega^2 r^2$$

8. (c) $v_{max} = a\omega = a\frac{2\pi}{T}$

$$= \frac{2\pi a}{2\pi\sqrt{\frac{m}{k}}} = a\sqrt{\frac{k}{m}}$$

Here

$$\frac{v_{max1}}{v_{max2}} = \frac{a_1}{a_2}\sqrt{\frac{k_1}{k_2}}$$

$\therefore$ $v_{max1} = v_{max2}$ (given).

$$\frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$$

9. (c) In air, the time period of the bob is

$$t_0 = 2\pi\sqrt{\frac{l}{g}}$$

In water apparent weight

$$mg' = \frac{4}{3}\pi r^3\rho_s g - \frac{4}{3}\pi r^3\rho_w g$$

$$mg' = \frac{4}{3}\pi r^3\rho g\left(1-\frac{\rho_w}{\rho_s}\right) = mg\left(1-\frac{3}{4}\right)\frac{mg}{4} \Rightarrow g' = g/4$$

In water, the time period of the bob

$$t = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{4l}{g}} = 2t_0$$

10. (d) $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$ and $t_2 = 2\pi\sqrt{\frac{m}{k}}$

$$\therefore \quad T = 2\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}\right)}$$

$$\Rightarrow \quad T^2 = 4\pi^2\frac{m}{k_1} + 4\pi^2\frac{m}{k_2} = t_1^2 + t_2^2$$

11. (c) Under simple harmonic motion, total energy $= \frac{1}{2}ma^2\omega^2$

Total energy is independent of x.

12. (b) For forced oscillations, the displacement is given by

$x = A\sin(\omega t + \phi)$ with

$$A = \frac{F_0/m}{\omega_0^2 - \omega^2}$$

13. (a) Both amplitude and energy get maximized when the frequency is equal to the natural frequency. This is the condition of resonance. Therefore,

$$\omega_1 = \omega_2$$

14. (b) $y = \sin^2\omega t = \frac{1-\cos 2\omega t}{2} \Rightarrow$ Period, $T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

The given function is not satisfying the standard differential equation of SHM $\frac{d^2y}{dx^2} = -\omega^2 y$. Hence, it represents periodic motion but not S.H.M.

15. (a) $v_1 = \frac{dy_1}{dt} = 0.1\times 100\,\pi\cos(100\,\pi t + \pi/3)$

$$v_2 = \frac{dy_2}{dt} = 0.1\times\pi\sin\pi t = 0.1\times\pi\cos(\pi t + \pi/2)$$

Phase difference

$$\Delta\phi_{12} = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$$

16. (b) $\frac{d^2x}{dt^2} + \alpha x = 0$

Comparing it with the standard equation of SHM, we get

$$\frac{d^2X}{dt^2}+\omega^2X=0$$

We have $\omega^2 = a$.

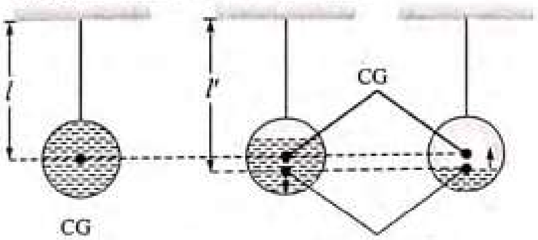
Time period

$$T = m\frac{2\pi}{2}=\frac{2\pi}{\sqrt{\alpha}}$$

17. (a) The given system is like a simple pendulum, whose effective length (l) is equal to the distance between point of suspension and CG (centre of gravity) of the hanging body.

When water slowly flows out the sphere, the CG of the system is lowered, and hence l increases, which in turn increases time period (as $T \propto \sqrt{l}$).

After some time weight of water left in sphere become less than the weight of sphere itself, so the resultant CG gets clear the CG of sphere itself, i.e., l decreases and hence T increases. Finally when the sphere becomes empty, the resulting CG is the CG of sphere, i.e., length becomes equal to the original length and hence the time period becomes equal to the same value as when it was full of water.



18. (d) For a particle to execute simple harmonic motion its displacement at any time t is given by

$$y(t) = A \sin \omega t$$

where, A= amplitude, ω = angular frequency,

Velocity of a particle $v=\frac{dy}{dt}=-A\,\omega\cos\omega t$

Acceleration of a particle $a=\frac{dv}{dt}=-A\,\omega^2\sin\omega t$

The force on particle $F = ma = -mA\omega^2 \sin \omega t$

Hence, force and time relation should be (d)

19. (a) $v_{max} = r\omega = r\times\frac{2\pi}{T}$

$$T=\frac{2\pi r}{v_{max}}=2\times\frac{2}{7}\times\frac{7}{1000}\times\frac{1}{4.4}=10^{-2}\text{ s}$$

20. (d) During simple harmonic motion,

kinetic energy $=\frac{1}{2}mv^2=\frac{1}{2}m(a\,\omega\cos\omega t)^2$

Total energy $E=\frac{1}{2}ma^2\omega^2$

$\therefore$ (Kinetic energy) $=\frac{75}{100}(E)$

or $\frac{1}{2}ma^2\omega^2\cos^2\omega t=\frac{75}{100}\times\frac{1}{2}ma^2\omega^2$

or $\cos^2\omega t=\frac{3}{4}\Rightarrow\cos\omega t=\frac{\sqrt{3}}{2}=\cos\frac{\pi}{6}$

$\therefore \quad \omega t=\frac{\pi}{6}$

or $t=\frac{\pi}{6\omega}=\frac{\pi}{6(2\pi/T)}=\frac{\pi}{6\times2\pi}=\frac{1}{12}$ sec.

21. (d) In vertical simple harmonic motion, maximum acceleration ($a\omega^2$) and so the maximum force ($ma\omega^2$) will be at extreme positions. At highest position, force will be towards mean position and so it will be downwards. At lowest position, force will be towards mean position and so it will be upwards. This is opposite to weight direction of the coin. The coin will leave contact will the platform for the first time when $m(a\omega^2) \geq mg$ at the lowest position of the platform.

22. (c) Here K_1 and K_2 are in parallel, so

$$K = K_1 + K_2$$

The frequency of oscillation is

$$f=\frac{1}{2\pi}\sqrt{\frac{K}{m}}=\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$$

When K_1 and K_2 are made four times their original values,

$$K' = 4K_1 + 4K_2 = 4(K_1 + K_2)$$

So the new frequency of oscillation is

$$f'=\frac{1}{2\pi}\sqrt{\frac{4(K_1+K_2)}{m}}$$

$$\therefore \quad \frac{f'}{f}=\sqrt{4}=2 \Rightarrow f'=2f$$

23. (d) For a particle to execute simple harmonic motion its displacement at any time t is given by

$$x(t) = a \sin \omega t$$

where a = amplitude, ω = angular frequency

Velocity of a particle $v=\frac{dx}{dt}=a\,\omega\cos\omega t$

Kinetic energy of a particle is $K=\frac{1}{2}mv^2$

$$\Rightarrow \quad K=\frac{1}{2}ma^2\omega^2\cos^2\omega t$$

Average kinetic energy, $\langle K\rangle=<\frac{1}{2}ma^2\omega^2\cos^2\omega t>$

$$=\frac{1}{2}m\omega^2a^2<\cos^2\omega t>$$

$$=\frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right) \qquad \left[\because <\cos^2\theta \geq \frac{1}{2}\right]$$

$$=\frac{1}{4}ma^2(2\pi v)^2 \qquad [\because \ \omega=2\pi v]$$

$$\Rightarrow \quad \langle K\rangle=\pi^2ma^2v^2.$$

24. (d) $x = 2 \times 10^{-2} \cos \pi t$

Comparing it with the standard form $x = \cos \omega t$, we have

$$\omega = \pi$$

$$\Rightarrow \quad \frac{2\pi}{T}=\pi$$

$$\Rightarrow \quad T = 2\text{ s}$$

Now in time $t = T/4$, the particle goes from the extreme position to the mean position where the KE becomes maximum. So

$$t=\frac{2}{4}=0.5\text{ s}$$

25. (c) $x = x\cos\left(\omega t - \frac{\pi}{4}\right)$

$\Rightarrow \quad v = \frac{dx}{dt} = -x_0\omega\sin\left(\omega t - \frac{\pi}{4}\right)$

Acceleration

$$a = \frac{dv}{dt} = -x_0\omega^2\cos\left(\omega t - \frac{\pi}{4}\right)$$

$$x_0\omega^2\cos\left(\pi + \omega t - \frac{\pi}{4}\right)$$

Now comparing it with $a = A\cos(\omega t + \delta)$, we have

$A = x_0\omega^2$ and $\delta = 3\pi/4$.

26. (b) For a simple harmonic motion, acceleration, $a = \omega^2 x$ where ω is a constant $\omega = \frac{2\pi}{T}$.

Hence, $a = -\frac{4\pi^2}{T^2}\cdot x \Rightarrow \frac{aT}{x} = -\frac{4\pi^2}{T}$.

The period of oscillation T is a constant

$\therefore \quad \frac{aT}{x}$ is a constant.

27. (d) $T_1 = 2\pi\sqrt{\frac{M}{k}}$ ...(i)

When a mass m is placed on mass M, the new system is of mass = $(M + m)$ attached to the spring. New time period of oscillation

$$T_2 = 2\pi\sqrt{\frac{(m+M)}{k}} \quad \text{...(ii)}$$

Consider v_1 is the velocity of mass M passing through mean position and v_2 velocity of mass $(m + M)$ passing through mean position.

Using, law of conservation of linear momentum

$Mv_1 = (m + M)v_2$

$M(A_1\omega_1) = (m + M)(A_2\omega_2)$

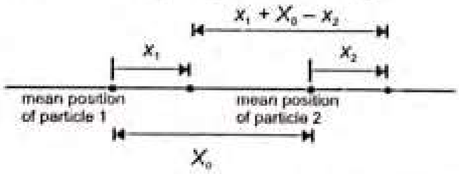
$(\because \; v_1 = A_1\omega_1$ and $v_2 = A_2\omega_2)$

or $\quad \frac{A_1}{A_2} = \frac{(m+M)}{M}\frac{\omega_2}{\omega_1}$

$$= \left(\frac{m+M}{M}\right)\times\frac{T_1}{T_2}\left(\because \omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}\right)$$

$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$ (Using (i) and (ii)).

28. (b) $x_1 = A\sin(\omega t + \phi_1)$, $x_2 = A\sin(\omega t + \phi_2)$



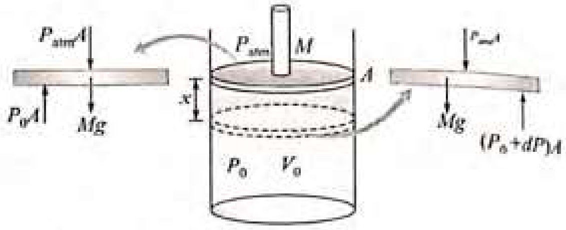
$$x_1 - x_2 = A\left[2\sin\left[\omega t + \frac{\phi_1 + \phi_2}{2}\right]\sin\left[\frac{\phi_1 - \phi_2}{2}\right]\right]$$

$(x_1 - x_2 + X_0)_{max} = X_0 + A$

$\Rightarrow \quad (x_1 - x_2)_{max} = A \Rightarrow 2A\sin\left(\frac{\phi_1 - \phi_2}{2}\right) = A$

$\Rightarrow \quad \frac{\phi_1 - \phi_2}{2} = \frac{\pi}{6} \Rightarrow \phi_1 - \phi_2 = \frac{\pi}{3}$

29. (b)



From FBD of piston at equilibrium

$P_{atm}A + Mg = P_0A$ ...(i)

From FBD of piston when piston is pushed down a distance x

$$(P_0 + dP)A - (P_{atm}A + Mg) = M\frac{d^2x}{dt^2} \quad \text{...(ii)}$$

The system is completely isolated from its surrounding hence the change is adiabatic.

For an adiabatic process, PV^γ = constant

$\therefore \quad V^\gamma dp + V^{\gamma-1}PdV = 0$

or $\quad dP = \frac{\gamma PdV}{V}$

But $\quad dV = Ax$

$\therefore \quad dP = -\frac{\gamma P_0(Ax)}{V_0}$ ...(iii)

Using (i) and (iii) in (ii), we get

$$M\frac{d^2x}{dt^2} = -\frac{\gamma P_0A^2}{V_0}x \text{ or } \frac{d^2x}{dt^2} = -\frac{\gamma P_0A^2}{MV_0}$$

Comparing it with standard equation of SHM,

$$\frac{d^2x}{dt^2} = -\omega^2x$$

We get $\omega^2 = \frac{\gamma P_0A^2}{MV_0}$

or $\quad \omega = \sqrt{\frac{\gamma P_0A^2}{MV_0}}$

Frequency, $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{MV_0}}$

30. (b) The amplitude of a damped oscillator at a given instant of time t is given by

$A = A_0e^{-bt/2m}$

Where A_0 is its amplitude in the absence of damping, b is the damping constant.

As per question

After 5 s (i.e. t = 15 s), its amplitude becomes

$0.9A_0 = A_0e^{-b(5)/2m} = A_0e^{-5b/2m}$

$0.9 = e^{-5b/2m}$ (i)

After 10 more second (*i.e.* t= 15 s), its amplitude becomes

$\alpha A_0 = A_0e^{-b(15)/2m} = A_0e^{-15b/2m}$

$\alpha = (e^{-5b/2m})^3 = (0.9)^3$ (Using (i))
$= 0.729$

31. (b) As the particle starts from rest so we choose

$x(t) = A \cos \omega t$

At $t = 0, x = A$

When $t = \tau, x = A - a$

When $t = 2\tau, x = A - 3a$

$\therefore \quad A - a = A \cos \omega\tau$ ...(i)

and $\quad A - 3a = A \cos 2\omega\tau = A(2\cos^2 \omega t - 1)$

$$\Rightarrow \quad (A-3a) = A\left[2\left(\frac{A-a}{A}\right)^2 - 1\right]$$

$$\Rightarrow \quad \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

On solving, $A = 2a$

Now $A - a = A \cos \omega\tau$,

$$\Rightarrow \quad \cos\omega\tau = \frac{1}{2} = \cos\frac{\pi}{3} \Rightarrow \quad \omega\tau = \frac{\pi}{3}$$

$$\Rightarrow \quad \frac{2\pi}{T}\tau = \frac{\pi}{3} \Rightarrow T = 6\tau.$$

32. (b) $KE = \frac{1}{2}m\omega^2(A^2 - d^2)$

$PE = \frac{1}{2}m\omega^2 d^2$

At $d = \pm A$,

PE = maximum while KE = 0.

33. (d) Velocity at any position is given by, $v = \omega\sqrt{A^2 - x^2}$

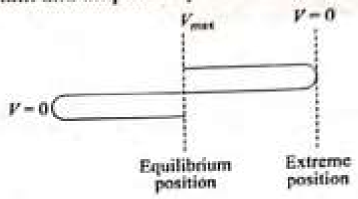
at $x = \frac{2A}{3}$, $v = \omega\sqrt{A^2 - \frac{4A^2}{9}} \Rightarrow v = \frac{\sqrt{5}}{3}A\omega$

If speed at $x = \frac{2A}{3}$ is tripled, i.e. $v' = 3v\sqrt{2}\, A\omega$

So at new amplitude A' is $v' = \omega\sqrt{A'^2 - \left(\frac{4A}{3}\right)^2}$

$$\sqrt{5}\, A\omega = \omega\sqrt{A'^2 - \frac{4A^2}{9}} \Rightarrow A' = \frac{7A}{3}$$

34. (b) Velocity of a particle at equilibrium position should be maximum and amplitude position should be zero



Time taken to reach the extreme position from equilibrium position is $\frac{T}{4}$. Velocity is maximum at equilibrium position and zero at extreme position.

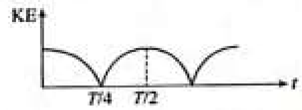
$v = A\omega \cos \omega t$

Hence kinetic energy, $KE = \frac{1}{2}mv^2$

(m is the mass of particle and v is the velocity of particle

$$\Rightarrow \quad KE = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$$

Hence graph of KE v/s time is square cos function



35. (c) Time period of SHM is given by $T = 2\pi\sqrt{\frac{m}{k}}$

Frequency $= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$

where m = mass of one atom $= \frac{108 \times 10^{-3}}{(6.02 \times 10^{23})}$kg

$$\frac{1}{2\pi}\sqrt{\frac{k}{108 \times 10^{-3}} \times 6.02 \times 10^{23}} = 10^{12}$$

On solving $k = 7.1$ N/m

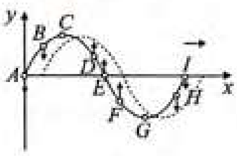
CHAPTER 16: WAVE AND ACOUSTICS

Concept Application Exercise 16.1

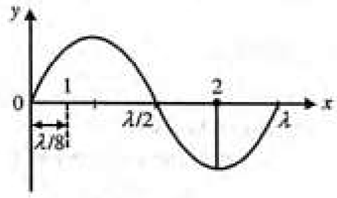
1. ABC represents compression (negative slope region) and CDE represents rarefaction (positive slope region).
2. Given, speed of wave, $v = 30$ cm/s
   And frequency of wave, $f = 20$ Hz
   $\therefore$ Wavelength of wave, $\lambda = \frac{v}{f} = \frac{30}{20} = 1.5$ cm
   Thus, the separation between consecutive compression = 1.5 cm.
3. The frequency of the note produced by the whistle is not equal to 1/20 or 0.05 Hz, it is only the frequency of pulse repetition.
4. i. On the same wavefront phase difference = 0.
   Linear distance between the points does not matter.
   ii. Between two successive crests, path difference = λ.
   $\therefore$ Phase difference = 2π radian
   iii. Again, path difference between two successive troughs = λ
   $\therefore$ Phase difference between A and B = 2π radian.

Concept Application Exercise 16.2

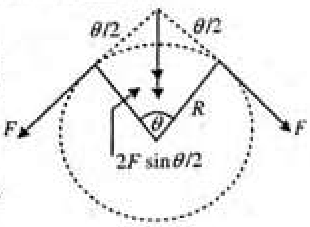
1. **Method 1:** The dotted curved in the figure represents the position of the wave at a later instant. It can be easily obtained from the figure that



   (a) Points D, E, and F move upward.
   (b) Points A, B, H, and I move downward.
   (c) Points C, and G have zero velocity.
   (d) Points A, E, and I have the maximum velocity.
2. (i) Given equation is $y = 5\sin(4.0t - 0.02x)$
   Comparing this with standard equation $y = A\sin(\omega t - kx)$, we get
   Amplitude $A = 5$ cm
   $\omega = 4.0\ \text{s}^{-1}$ and $k = 0.02\ \text{cm}^{-1}$
   $\therefore$ Frequency $f = \frac{\omega}{2\pi} = \frac{4.0}{2\pi} = \frac{2}{3.14} = 0.637\ \text{s}^{-1}$
   Velocity $v = \frac{\omega}{k} = \frac{4.0}{0.02} = 200$ cm/s
   Wavelength $\lambda = \frac{v}{f} = \frac{200}{0.637} = 314$ cm
   ii. The maximum velocity, $u_{max} = \omega A = 4 \times 5 = 20$ cm/s.
3. Tension in wire
   $T = 9$ kg wt $= 9$ g $= 9 \times 9.8$
   Mass per unit length $m = M/L$
   $$= \frac{12\times10^{-3}\ \text{kg}}{1.5\ \text{m}} = 8\times10^{-3}\ \text{kg/m}$$
   $\therefore$ Frequency of vibration of pth harmonic on string
   $$n = \frac{p}{2l}\sqrt{\frac{T}{m}} = \frac{2}{2\times1.5}\sqrt{\left\{\frac{9\times9.8}{8\times10^{-3}}\right\}} = 70\ \text{Hz}$$
4. Comparing the equation with the standard form of the wave equation
   $y = a\sin(\omega t - kx)$, where $\omega = 2\pi f$ and $k = 2\pi/\lambda$
   Given that
   i. $\omega = 500\pi$ or $f = 250$ Hz
   and $\frac{2\pi}{\lambda} = \frac{500\pi}{30}$ or $\lambda = \frac{3}{25}\ m = 0.12$ m
   ii. $\therefore C = f\lambda$ or $C = 250\times\frac{3}{25} = 30$ m/s
   iii. v (particle velocity) $= \frac{dy}{dt} = a\omega\cos(\omega t - kx)$
   $\therefore v_{max} = a\omega = 0.01 \times 500\pi = 15.7$ m/s
   iv. f (particle acceleration)
   $$= \frac{dv}{dt} = -a\omega^2\sin(\omega t - kx)$$
   $\therefore f_{max} = a\omega^2 = 0.01 \times (500\pi)^2 = 2.47 \times 10^4\ \text{m/s}^2$
5. The distance between the particles



   $$\Delta x = \left(\frac{\lambda}{2} - \frac{\lambda}{8}\right) + \frac{\lambda}{4} = \frac{5\lambda}{8}$$
   $$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\times\frac{5\lambda}{8} = \frac{5\pi}{4}$$
6. Suppose F is the tension in the string due to its rotation. Choose a small element of the string of length l. If μ is the mass per unit length of the string, then mass of the element,
   $m = \mu l = \mu(R\theta)$
   Using Newton's second law for the element, we have



   $2F\sin\theta/2 = mv^2/R$
   For small θ, $\sin\theta/2 \approx \theta/2$
   $$2F\left(\frac{\theta}{2}\right) = \frac{mv^2}{R}$$
   $$F\theta = (\mu R\theta)\frac{v^2}{R}$$
   $$F = \mu v^2$$
   The speed of the disturbance $= \sqrt{\frac{F}{\mu}} = \sqrt{\frac{\mu v^2}{\mu}} = v$

Concept Application Exercise 16.3

1. R_2 is approached by the source and the 'image source' at the same speed. So R_2 receives waves of the same frequency and hence, it does not register beats. R_1 is approached by the 'image source' while the source itself recedes from it. Hence, it receives waves of different wavelengths. So it registers beats.

N_1 (apparent frequency of the source wave)

$$= \frac{340-0}{340-(0.17)} \times 1000 = 1000.5 \text{ Hz}$$

N_2 (apparent frequency of the reflected wave)

$$= \frac{340-0}{340+0.17} \times 1000 = 999.5 \text{ Hz}$$

Therefore, beats registered $= 1000.5 - 999.5 = 1.0$ Hz

2. In general $n' = \frac{c - v_0}{c - v_s} \times n$

Here $v_0 = 0$ and $v_s = \omega r = 15 \times 2 = 30$ m/s

When the source moves away from the observer, the apparent frequency is minimum and when it moves towards the observer, the apparent frequency is maximum.

$$\therefore \quad n_{min} = \frac{330}{330-(-30)} \times 540 = 495 \text{ Hz}$$

$$n_{max} = \frac{330}{330-30} \times 540 = 594 \text{ Hz}$$

3. Here, the bat is the 'observer' and the 'image' is the source.

$$n' = \frac{c - v_0}{c - v_s} \times n = \left[\frac{340-(-6)}{340-6}\right] 450 = 466 \text{ kHz}$$

4. $n' = \frac{300-0}{300-u} \times 1000$

$$n'' = \frac{300-0}{300-(-u)} \times 1000$$

Dividing, $\frac{n'}{n''} = \frac{300+u}{300-u}$

It is given that $n'/n'' = 11/9$, therefore

$$\frac{11}{9} = \frac{300+u}{300-u}$$

or $\quad u = 30$ m/s

5. $v_s = r\omega = 1.5 \times 20 = 30$ m/s

$$n_{min} = \frac{v}{v+v_s} n = \frac{330}{330+30} \times 440 \text{ Hz} = 403 \text{ Hz}$$

$$\therefore \quad n_{max} = \frac{330}{330-30} \times 440 = 484 \text{ Hz}$$

6. (a) The frequency of sound heard directly,

$$f_1 = f_0\left(\frac{v}{v+v_s}\right)$$

$v_s = 8$ m/s

$$\therefore \quad f_1 = \left(\frac{330}{330+8}\right) \times 2000$$

$$f_1 = \frac{330}{338} \times 2000 = 1953 \text{ Hz}$$

(b) The frequency of the reflected sound is given by

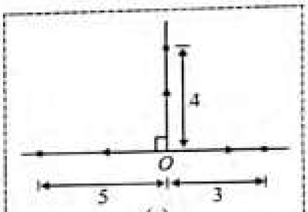
$$f_2 = f_0\left(\frac{v}{v-v_s}\right)$$

$$\therefore \quad f_2 = \left(\frac{330}{330-8}\right) \times 2000$$

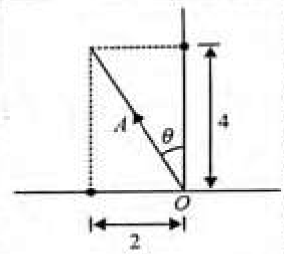
$$f_2 = \frac{330}{322} \times 2000 = 2050 \text{ Hz}$$

Concept Application Exercise 16.4

1. The amplitudes can be added vectorially, with angle between them as their phase difference. All these are shown in Figure (a). The resultant amplitude 'A' due to the interference of the three waves is evidently from Figure (b) given by



(a)



(b)

$$A = \sqrt{4^2 + 2^2} = 2\sqrt{5} \text{ cm}$$

Also, $\tan\theta = \frac{2}{4} = \frac{1}{2}$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Therefore, the resultant wave leads the first wave by an angle

$$\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right)$$

2. The path difference between the sounds going through the tubes

$$\Delta x = ABCD - AED$$

$$= (10 + 20 + 10) - (\sqrt{500} + 10) = 7.64 \text{ cm}$$

The wavelength of the sound

$$\lambda = \frac{v}{f} = \frac{340}{f} \text{ m}$$

For maximum intensity, $\Delta x = n\lambda$

$$7.64 \times 10^{-2} = n \times \frac{340}{f}$$

$$f = n\frac{340}{7.64 \times 10^{-2}} = n\, 4450 \text{ Hz}$$

Here $n \geq 1$. For $n = 1, f = 4450$ Hz and for $n = 2, f = 8900$ Hz. Thus, the frequency within the specified range is 4450 Hz.

3. $(SA + AD) - SD = n\lambda$ for reinforcement at D

Again $(SB + BD) - SD = n\lambda + \frac{\lambda}{2}$ for total annulment at D

Subtracting, $(SB + BD) - (SA + AD) = \frac{\lambda}{2}$

$$2(SB - SA) = \frac{\lambda}{2}$$

or $\quad \lambda = 2\sqrt{4(H+h)^2 + d^2} - 2\sqrt{4H^2 + d^2}$

4. Since intensity $\propto$ (amplitude)^2, the waves may be expressed as

$y_1 = a \sin(\theta)$ and $y_2 = 2a\sin(\theta - \pi/2)$

where $\theta = \omega t - kx$ and $I_0 = \text{constant} \times a^2$ (i)

The resultant wave is

$y = y_1 + y_2 = a \sin\theta - 2a\cos\theta$

Let $a = A\cos\varphi$, $2a = A\sin\varphi$ (ii)

Then $y = A\sin(\theta - \varphi) = A\sin(\omega t - kx - \varphi)$

This is a wave of amplitude A, and lags behind the y_1 wave by phase angle φ, where $\tan\varphi = 2$.

Resultant intensity:

$$I_R = I_0 + 4I_0 + 2\sqrt{I_0 4I_0}\cos\left(\frac{\pi}{2}\right) = 5I_0$$

5. In interference, amplitudes are added according to vector rules of addition.

∴ Resultant amplitude $= \sqrt{3^2 + 4^2 + 2\times3\times4\cos\pi/2} = 5$ cm

6. Length of the path of the reflected wave $= 2\sqrt{60^2 + 5^2}$

For constructive interference $2\sqrt{60^2+5^2} - 120 = n\lambda$

or $120.42 - 120 = n\lambda$ or $0.42 = n\lambda$

Let $n = 1, 2, 3, \ldots$

Then $\lambda = 0.42, 0.21, \ldots\ m$

Concept Application Exercise 16.5

1. Let f_1 and f_2 be the frequencies of the tuning forks A and B respectively.

Since, the beat frequency between A and B is 4

∴ $f_1 - f_2 = 4$ or $f_2 - f_1 = 4$

From, $n = \frac{1}{2l}\sqrt{T/\mu}$, we have, $n \propto 1/l$

∴ $f_1 \propto 1/32.4$ cm and $f_2 \propto 1/32$ cm

∴ Evidently, $f_2 > f_1$

∴ $f_2 - f_1 = 4\ s^{-1}$ (i)

Also, $\frac{f_2}{f_1} = \frac{324}{320}$ (ii)

Solving Eqs. (i) and (ii), we get

$f_1 = 320$ Hz and $f_2 = 324$ Hz

2. Wavelength of sound $= \frac{v}{f} = \frac{340\ \text{m/s}}{606\ s^{-1}} = 56.1$ cm

Since, closed pipe allows only odd harmonics, so

$$f = (2n+1)\frac{v}{4l} \quad \text{or,} \quad l = (2n+1)\frac{v}{4f}; \quad n \in I$$

or, $l = (2n+1) \times 14$ cm

∴ $l = 14$ cm, 42 cm, 70 cm, 98 cm, 126 cm, 154 cm, etc.

Since $l \le 150$ cm

∴ Number of resonances = 5

3. n_1, frequency of the first wave $= c/0.500$

n_2, frequency of the second wave $= c/0.505$

Obviously $n_1 > n_2$ ∴ $n_1 - n_2 = 6$

or $\frac{c}{0.500} - \frac{c}{0.505} = 6$ or $c = 303$ m/s

4. Using the relation $2\sin C\cos D = \sin(C+D) + \sin(C-D)$

$$y = 5\sin\frac{\pi x}{3}\cos 40\pi t = \frac{5}{2}\times 2\sin\frac{\pi x}{3}\cos 40\pi t$$

$$y = \frac{5}{2}\left[\sin\left(\frac{\pi x}{3} + 40\pi t\right) + \sin\left(\frac{\pi x}{3} - 40\pi t\right)\right]$$

$$= \frac{5}{2}\sin\left(40\pi t + \frac{\pi x}{3}\right) - \frac{5}{2}\sin\left(40\pi t - \frac{\pi x}{3}\right)$$

Thus, the given stationary wave is formed by the superposition of the progressive waves

$$y_1 = \frac{5}{2}\sin\left(40\pi t + \frac{\pi x}{3}\right)$$

and $$y_2 = \frac{5}{2}\sin\left(40\pi t - \frac{\pi x}{3} + \pi\right)$$

(a) Comparing each wave with the standard form of the progressive wave

$$y = a\sin\left(\omega t - \frac{2\pi}{\lambda}x + \alpha\right)$$

$a = 5/2 = 2.5$ cm

$\omega = 40\pi \Rightarrow 2\pi f = 40\pi \rightarrow f = 20\ s^{-1}$

and $\frac{2\pi}{\lambda} = \frac{\pi}{3}$ or $\lambda = 6$ cm $= 0.06$ m

∴ $c = f\lambda = 20 \times 0.06 = 1.2$ m/s

(b) Distance between the nodes $= \lambda/2 = 0.06/2 = 0.03$ m

(c) $y = 5\sin\frac{\pi x}{3}\cos 40\pi t$

$$v = \frac{dy}{dt} = -5\times 40\pi\sin\frac{\pi x}{3}\sin 40\pi t$$

$$\Rightarrow \quad v = -200\pi\sin\frac{\pi x}{3}\sin 40\pi t$$

∴ at $x = 1.5$ cm and $t = 9/8$ s,

$v = -200\pi\sin(\pi/2)\sin 45\pi = 0$

5. For open pipe:

$$f_1 = 300 = \frac{v}{2l_1} \Rightarrow l_1 = \frac{v}{600} = \frac{350}{600} = 0.58 \text{ m}$$

For closed pipe: $\frac{3v}{4l_2} = 2f_1$

$$\Rightarrow \quad \frac{3v}{4l_2} = 600 \Rightarrow l_2 = 0.44 \text{ m}$$

6. $n_A = \frac{c}{4\times0.32}$ and $n_B = \frac{c}{4\times0.33}$

Obviously $n_A > n_B$

∴ $n_A - n_B = \frac{40}{5}$

∴ $\frac{c}{4\times0.32} - \frac{c}{4\times0.33} = 8$ or $c = 337.92$ m/s

∴ $n_A = \frac{337.92}{4\times0.32} = 264$ Hz

and $n_B = 264 - 8 = 256$ Hz

EXERCISES

Progressive Waves and Sound Waves

1. (d) $y = 4\sin\left(4\pi t - \frac{\pi}{16}x\right)$

$\omega = 4\pi,\ k = \pi/16$

$v = \frac{\omega}{k} = \frac{4\pi}{\pi/16} = 64$ cm/s in positive x-direction.

2. (a) Standard equation $y = A\sin(\omega t - kx + \phi_0)$

In a given equation $\omega = 7\pi,\ k = 0.04\pi$

$v = \frac{\omega}{k} = \frac{7\pi}{0.04\pi} = 175$ m/s

3. (b) $y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$

$y_2 = a_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$

Phase difference

$$= \left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right) - \left(\omega t - \frac{2\pi x}{\lambda}\right) = \left(\phi + \frac{\pi}{2}\right)$$

Path difference $= \frac{\lambda}{2\pi} \times$ Phase difference $= \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

4. (a) $x = a\sin\left(\omega t + \frac{\pi}{6}\right)$

$x' = a\cos\omega t = a\sin\left(\omega t + \frac{\pi}{2}\right)$

Therefore, phase difference $= (\pi/2) - \pi/6 = (\pi/3)$

5. (b) Maximum particle velocity = 4 wave velocity

$A\omega = 4f\lambda$

$y_0 2\pi f = 4f\lambda$

$\lambda = \frac{\pi y_0}{2}$

6. (b) Maximum particle velocity $= a_0\omega = 2\pi a_0 v$

Wave velocity $= v\lambda$

Given that $2\pi a_0 v = 3v\lambda$ or $\lambda = (2\pi a_0/3)$

7. (b) $v = 1$ m/s, $f = 100$ Hz

$\lambda = \frac{v}{f} = \frac{1}{100}$ m $= 1$ cm

$\Delta\phi = \frac{2\pi}{\lambda} \times 2.75$ rad $= 5.5\,\pi$ rad $= \frac{11\pi}{2}$ rad

8. (d) For the wave, $y = A\sin(kx - \omega t)$, the wave speed is ω/k and the maximum transverse string speed is $A\omega$.

9. (c) At the moment shown in the figure, particle at 1 is moving in the downward direction.

We have, $T = 1/0.1\ s = 10$ s.

In one complete cycle, particle travels a distance, 4 times the amplitude. So, in time 10 min 15 s, i.e., 615 s which means 61 full + 1 half cycles, the distance travelled

$= (4 \times 3) \times 61 + (2 \times 3) \times 1 = 732 + 6 = 738$ cm

At time instant, the particle is moving in the upward direction.

10. (c) We start with a general form for a rightward moving wave,

$y(x, t) = A\sin(kx - \omega t + \phi)$

The amplitude given is $A = 2.0$ cm $= 0.02$ m.

The wavelength is given as, $\lambda = 1.0$ m

Wave number $= k = 2\pi/\lambda = 2\pi\ \text{m}^{-1}$

Angular frequency, $\omega = vk = 10\,\pi$ rad/s

$y(x, t) = (0.02)\sin[2\pi(x - 5.0\,t) + \phi]$

We are told that for $x = 0,\ t = 0$,

$y = 0$ and $\frac{\partial y}{\partial t} < 0$

i.e., $0.02\sin\phi = 0$ (as $y = 0$)

and $-0.2\,\pi\cos\phi < 0$

From these conditions, we may conclude that

$\phi = 2n\pi$ where $n = 0, 2, 4, 6, \ldots$

Therefore, $y(x,t) = (0.02\ \text{m})\sin[(2\pi\ \text{m}^{-1})x - (10\pi\ \text{s}^{-1})t]$ m

11. (b) The amplitude, $A = 0.06$ m

$\frac{5}{2}\lambda = 0.2$ m $\therefore\ \lambda = 0.08$ m

$f = \frac{v}{\lambda} = \frac{300}{0.08} = 3750$ Hz

$k = \frac{2\pi}{\lambda} = 78.5\ \text{m}^{-1}$ and $\omega = 2\pi f = 23562$ rad/s

At $t = 0, x = 0$, $\frac{dy}{dx} =$ positive

and the given curve is a sine curve.

Hence, equation of wave travelling in positive x-direction should have the form, $y(x, t) = A\sin(kx - \omega t)$

Substituting the values, we have

$y = (0.06\ \text{m})\sin[(78.5\ \text{m}^{-1})x - (23562\ \text{s}^{-1})t]$ m

12. (a) $v_p = -v\left(\frac{dy}{dx}\right)$

here v is negative and dy/dx is positive at point P. Hence v_p is positive. So P is moving upwards.

13. (a) Let equation of the wave is $y = A\sin(\omega t - kx + \phi_0)$

The particle at origin is moving downwards (from graph) at $t = 0$, so $\phi_0 = \pi$. Also $A = 2$ m from graph.

From graph, $\lambda = 8$ m, so $k = \frac{2\pi}{\lambda} = \frac{\pi}{4}$

$\omega = vk = \frac{4 \times \pi}{4} = \pi$

Putting all the values

$y = 2\sin[\pi t - \frac{\pi}{4}x + \pi] \Rightarrow y = 2\sin[\frac{\pi}{4}x - \pi t]$ m

14. (c) In a gas,

$v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow v \propto \sqrt{T}$

$\Rightarrow \frac{v}{2v} = \sqrt{\frac{300}{T}} \Rightarrow T = 300 \times 4 = 1200$ K $= 927°$C

15. (b) $P_0 = B.K.S_0 = B\left(\frac{2\pi}{\lambda}\right)S_0 \Rightarrow P_0 \propto \frac{1}{\lambda}$

Thus, pressure amplitude is highest for minimum wavelength, other parameters B and S_0 being same for all. From given graphs.

$\lambda_3 < \lambda_2 < \lambda_1$.

Hence $P_3 > P_2 > P_1$

16. (a) The equation of pressure variation due to sound is

$$p = -B\frac{ds}{dx} = -\frac{d}{dx}[s_0 \sin^2(\omega t - kx)] = Bks_0 \sin(2\omega t - 2kx)$$

17. (a) $v = \sqrt{\dfrac{B}{\rho}} \Rightarrow B = v^2\rho = (5.40 \times 10^3 \text{ m/s})^2 (2.7 \times 10^3)$

$$= 7.6 \times 10^{10} \text{ Pa}$$

18. (d) Let a be the amplitude due to S_1 and S_2 individually.

Loudness due to $S_1 = I_1 = Ka^2$

Loudness due to $S_1 + S_2 = I = K(2a)^2 = 4I_1$

$$\therefore \quad n = 10\log_{10}\left(\frac{4I_1}{I_1}\right) = 10 \log_{10}(4) = 6$$

19. (c) Bel is $\log_{10}\left(\dfrac{I}{I_0}\right)$. Here it is 6. In decibel, it is 60.

20. (c) $L = 10\log_{10}\dfrac{I}{I_0}$ decibel

$L' = 10\log_{10}\dfrac{2I}{I_0}$ decibel

$$L' - L = 10\left[\log_{10}\frac{2I}{I_0} - \log_{10}\frac{I}{I_0}\right]$$

$$= 10\log_{10} 2 = 10 \times 0.3010 = 3.01 \text{ dB}$$

Doppler Effect

21. (b) $f' = \dfrac{v}{v + v_s} f$

or $\dfrac{6}{7} f = \dfrac{330}{330 + v} f$ or $6 \times 330 + 6v = 7 \times 330$

or $6v = 330$ or $v = 55$ m/s

22. (a) As the source and the observer are approaching one another, so n' would be larger.

$$f = \left(\frac{v + v/15}{v - v/10}\right) 600 = 711 \text{ Hz}$$

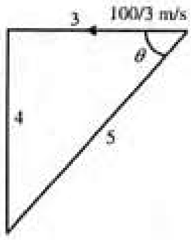
23. (b) $f' = \dfrac{v}{v - v_s} v$, $f'' = \dfrac{v}{v + v_s} f$

$\Rightarrow \dfrac{f'}{f''} = \dfrac{v + v_s}{v - v_s}$ or $\dfrac{6}{5} = \dfrac{330 + v_s}{330 - v_s}$

$11v_s = 330$ or $v_s = 30$ m/s

24. (b) Effective value of velocity of source is

$$v_s = \frac{100}{3}\cos\theta = \frac{100}{3} \times \frac{3}{5} = 20 \text{ m/s}$$



$$f' = \frac{v}{v - v_s} f$$

$$f' = \frac{340}{340 - 20} \times 640 \text{ Hz} = 680 \text{ Hz}$$

25. (c) $f = \left(\dfrac{v + v_m}{v + v_m - v_{source}}\right) 1000$

$$= \left(\frac{340 + 20\cos 60^\circ}{340 + 20\cos 60^\circ - 30}\right) 1000 = 1094 \text{ Hz}$$

26. (a) The frequency of direct and reflected sound is same. Therefore, no beats will be heard.

27. (d) Initially wall behaves as an approaching observer, so frequency of sound reaching the wall is

$$n_1 = \frac{c + v}{c} n$$

While reflecting, the wall behaves as an approaching source, so frequency received by stationary observer is

$$n_2 = \frac{c}{c - v} n_1 = \frac{c}{c - v} \times \frac{c + v}{c} n = \frac{c + v}{c - v} n$$

Direct frequency received by observer is n, the number of beats is

$$x = n_2 - n = \frac{c + v}{c - v} n - n = \frac{2nv}{c - v}$$

28. (b) The frequency of direct sound of whistle heard by observer is

$$n_1 = \frac{v}{v - v_s} n = \frac{340}{340 - 1} \times n = \frac{340}{339} n \qquad \text{(i)}$$

Frequency of sound of whistle reflected by wall is

$$n_2 = \frac{v}{v + v_s} n = \frac{340}{341} n \qquad \text{(ii)}$$

Given, $n_1 - n_2 = 4$

Therefore, $\dfrac{340}{339} n - \dfrac{340}{341} n = 4 \Rightarrow n = 680$ Hz

29. (b) $f = \left(\dfrac{v + v_0}{v}\right) f_1 = f_1 + f_1 \dfrac{v_0}{v}$

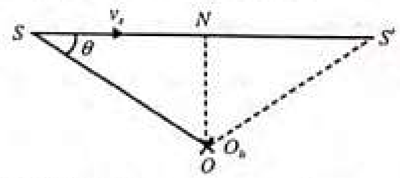
$v_0 = gt$

So, $f = f_1 + \left(\dfrac{f_1 g}{v}\right) t$

Slope of graph $= \dfrac{f_1 g}{v} \quad \dfrac{2 \times 10^3 - f_1}{30} = \dfrac{(f_1)(10)}{300}$

or $f_1 = 10^3$ Hz

30. (a) This frequency–time curve corresponds to a source moving at an angle to a stationary observer.



In the region SN, the source is moving towards the observer, i.e., the apparent frequency

$$n' = n_0\left(\frac{v}{v - v_s\cos\theta}\right)$$

$$n' = n_0\left(\frac{300}{300 - 30\cos\theta}\right)$$

When $\theta = \pi/2$, i.e., at N, $n' = n_0 = 1000$ Hz, i.e., natural frequency of source.

In the region NS' the source is moving away from the observer, i.e., apparent frequency

$$n' = n_0\left(\frac{300}{300 - 30\cos\theta}\right)$$

When $\theta = 0$, i.e., $\cos\theta = 1$,

$$n_{max} = n_0 \frac{v}{v - v_s} = \frac{(1000 \text{ Hz})(300 \text{ m/s})}{(300 \text{ m/s} - 30 \text{ m/s})}$$

$$= \frac{10}{9} \times 1000 \text{ Hz} = 1111 \text{ Hz}$$

$$n_{min} = n_0 \frac{v}{v + v_s} = \frac{1000 \times 300}{330} = 909 \text{ Hz}$$

31. (b)

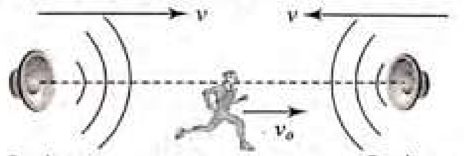
$$n' = n\left(\frac{v - v_o}{v + v_s}\right) = 750\left(\frac{330 - 180 \times \frac{5}{18}}{330 + 108 \times \frac{5}{18}}\right) = 625 \text{ Hz}$$

32. (c) For source $v_S = r\omega = 0.70 \times 2\pi \times 5 = 22$ m/sec

Minimum frequency is heard when the source is receding the man. It is given by

$$n_{min} = n \frac{v}{v + v_s} = 1000 \times \frac{352}{352 + 22} = 941 \text{ Hz}$$

33. (b) Observer is moving away from siren 1 and towards the siren 2.



Hearing frequency of sound emitted by siren 1

$$n_1 = n\left(\frac{v - v_0}{v}\right) = 330\left(\frac{330 - 2}{330}\right) = 328 \text{ Hz}$$

Hearing frequency of sound emitted by siren 2

$$n_2 = n\left(\frac{v + v_0}{v}\right) = 330\left(\frac{330 + 2}{330}\right) = 332 \text{ Hz}$$

Hence, beat frequency $= n_2 - n_1 = 332 - 328 = 4$

34. (b) At point A, source is moving away from observer so apparent frequency $n_1 < n$ (actual frequency). At point B source is coming towards observer so apparent frequency $n_2 > n$ and point C source is moving perpendicular to observer so $n_3 = n$.

Hence $n_2 > n_3 > n_1$

Superposition of Waves

35. (a) The resultant amplitude is given by

$$A_R = \sqrt{A^2 + A^2 + 2AA\cos\theta} = \sqrt{2A^2(1 + \cos\theta)}$$

$= 2A \cos \theta/2$ $\qquad (\because \ 1 + \cos\theta = 2\cos^2 \theta/2)$

36. (c) After two seconds each wave travel a distance of $2.5 \times 2 = 5$ cm, i.e., the two pulses will meet in mutually opposite phase and hence the amplitude of resultant will be zero.

37. (c) The wavelength of sound source $= \frac{330}{110} = 3$ metre.

The phase difference betwen interfering waves at P is

$$= \Delta\phi = \frac{2\pi}{\lambda}(S_2P - S_1P) = \frac{2\pi}{3}(5 - 4) = \frac{2\pi}{3}$$

$\therefore$ Resultant intensity at

$$P = I_0 + 4I_0 + 2\sqrt{I_0}\sqrt{4I_0}\cos\frac{2\pi}{3} = 3I_0$$

38. (c) $I = I_0 + I_0 + 2\sqrt{I_0 . I_0}\cos\phi = 2I_0$ (given)

So $\cos\phi = 0$

$\phi = \frac{n\pi}{2} = \frac{\pi}{2}$ $\qquad$ ($n = 1$ for S_2P to be minimum)

$\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \left(\frac{2\pi}{4}\right)\Delta x = \frac{\pi}{2}$

$\Delta x = 1$ m

$\Delta x = S_2P - S_1P = 1$

$S_2P = S_1P + 1 = (x + 1)$ metre.

39. (d) Beats are observed when intensity at a point varies with time and beat frequency is equal to the frequency of oscillations of intensity at the point.

Intensity at a point not only depends upon the frequency of medium particle also. Therefore, beats are observed when amplitude of oscillation of medium particles varies with time. If the beat frequency at a point is equal to n, it means, at that point amplitude of oscillation of medium particles varies with frequency n.

Amplitude of vibrations changes not only at the point of observation, but at all the points. Therefore, (a) and (b) are wrong. Since the frequency of variation of intensity is observed n times per second therefore, the maximum of intensity is observed n times per second and the intensity become zero n times per second. Hence, (c) is also wrong. Obviously, only option (d) is correct.

40. (b) As $y = A_b \sin(2\pi n_{av} t)$

where $A_b = 2A_b \cos(2\pi n_A t)$

where $n_A = \frac{n_1 - n_2}{2}$

Hence, the amplitude of vibration at any point changes simple harmonically with a frequency equal to the difference in the frequencies of the two waves.

Hence (b).

41. (a) $\lambda_1 = 1\, m, \lambda_2 = \frac{165}{164} m$

$f_1 = \frac{C}{1}; f_2 = \frac{164C}{165}$

$f_1 - f_2 = 2$

$\frac{C}{165} = 2; C = 330$ m/s

42. (b) $2A = 6$ or $A = 3$ cm

43. (a) $(2\pi/\lambda) = \pi/3$ or $\lambda = 6$ cm

44. (c) $(2\pi/T) = 40\pi$ or $T = (1/20)$ sec

45. (a) $n = (1/T) = 20$ Hz

46. (c) $V = n\lambda = 20 \times 6 = 120$ cm/sec.

47. (a) Separation between two consecutive antinodes
$$=\frac{\lambda}{2}=\frac{6}{2}=3\text{cm}$$

48. (a) Phase difference between two points in the same segment = 0.

49. (c) $y_2=5\,[\sin 3\pi t+\sqrt{3}\cos 3\pi t]$
$$=5\sqrt{1+3}\sin\left(3\pi t+\frac{\pi}{3}\right)=10\sin\left(3\pi t+\frac{\pi}{3}\right)$$
So, $A_1=10$ and $A_2=10$ Hence $A_1:A_2=1:1$.

50. (d) Probable frequency of A is 390 Hz and 378 Hz and after loading the beats are decreasing from 6 to 4 so the original frequency of A will 390 Hz.

51. (a) Standard equation
$$y=A\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi vt}{\lambda}$$
By comparing this equation with given equation,
$$\frac{2\pi x}{\lambda}=\frac{\pi x}{20}\Rightarrow\lambda=40\text{ cm}$$
Distance between two nodes = $\lambda/2$ = 20 cm.

52. (c) Node means a point at which medium particles do not displace from its mean position and antinode mean a point at which particles oscillate with maximum possible amplitude. Nodes and antinodes are obtained for both types of stationary waves, transverse and longitudinal. Hence, options (a) and (b) both are wrong. To obtain a stationary wave, two waves travelling in opposite directions, having same amplitude, same frequency are required. They must have same nature, means either both of the waves should be longitudinal or both of them should be transverse. Hence, option (c) is correct.

53. (b) Path difference = $(\pi r-2r)=(2n-1)\lambda/2$ for minima
Given λ = 0.40 m, for smallest radius n = 1
$$(3.14-2)r=\lambda/2$$
$$r=\frac{\lambda}{2\times 1.14}=\frac{0.40}{2\times 1.14}=0.175\text{ m}$$

54. (d) For maximum path difference $\Delta=n\lambda$
$$2\times 0.6\,l=\lambda$$
$$l=\frac{\lambda}{1.2}=\frac{6}{1.2}=5\text{m}$$

55. (b) $y=4\cos^2\left(\frac{t}{2}\right)\sin 1000\,t$
$$=2(1+\cos t)\sin 1000\,t$$
$$=2\sin 1000\,t+2\cos t\sin 1000t$$
$$=2\sin 1000\,t+\sin(1000t+t)+\sin(1000t-t)$$
$$=2\sin 1000\,t+\sin 1001\,t+\sin 999t$$
$$=y_1+y_2+y_3=\text{Three waves}$$

Vibration of String and Organ Pipes

56. (a) In both cases, the 'applied frequency' is same. So, the frequency of vibration has to be same. However, the mode of vibration of the string be different.

57. (b) For fundamental mode
$(\lambda/2)=100$ cm or $\lambda=200$ cm
As $n=330$ Hz, hence
$$v=n\lambda=330\times\frac{200}{100}=660\text{ m/s}$$

58. (c) $f=\frac{1}{2l}\sqrt{\frac{T}{\mu}}$
If radius is doubled, mass per unit length will become four times. Hence
$$f'=\frac{1}{2\times 2l}\sqrt{\frac{2T}{4\mu}}=\frac{f}{2\sqrt{2}}$$

59. (b) The string vibrates in two segments in the first overtone. Therefore the amplitude of vibration is maximum at (L/4) and (3L/4).

60. (c) Velocity of longitudinal waves $v_1=\sqrt{\frac{Y}{\rho}}$
and velocity of transverse waves $v_2=\sqrt{\frac{T}{m}}=\sqrt{\frac{T}{\rho s}}$
$$\therefore\quad\frac{v_1}{v_2}=\sqrt{\frac{Y}{T/s}}=\sqrt{\frac{Y}{Y\left(\frac{\Delta l}{l}\right)}}=\sqrt{n}\qquad\left[\because\Delta l=\frac{l}{n}\right]$$
Now $f\propto v\quad\therefore\quad\frac{f_1}{f_2}=\frac{v_1}{v_2}=\sqrt{n}$
In the above expression, ρ = density of string, s = area of cross-section of string, Y = Young's modulus.

61. (c) At 25 cm, there will be antinode. So wire will vibrate in two loops.
$$f=\frac{2}{2l}\sqrt{\frac{T\times l}{M}}=\sqrt{\frac{T}{Ml}}=\sqrt{\frac{20}{5\times 10^{-4}\times 1}}$$
$$=\sqrt{4\times 10^4}\text{ Hz}=200\text{ Hz}$$

62. (c) $3\times\frac{v}{4l_c}=2\times\frac{v}{2l_0}$ or $\frac{l_c}{l_0}=\frac{3}{4}$

63. (c) Beats $=\frac{V}{4l}-\frac{V}{4(l+\Delta l)}=\frac{V}{4}\left[\frac{\Delta l}{l(l+\Delta l)}\right]=\frac{V\Delta l}{4l^2}\quad(\because\Delta l<<l)$

64. (a) $n\propto 1/l$
On doubling the length, frequency is halved
The word 'nearly' in the statement has been used keeping is mind 'end correction.'

65. (d) $\frac{v}{4l_1}=3\left(\frac{v}{2l_2}\right)\Rightarrow\frac{l_1}{l_2}=\frac{1}{6}$

66. (b) Let l be the length of the pipes and v the speed of sound. Then frequency of open organ pipe of nth overtone is
$$f_1=(n+1)\frac{v}{2l}$$
and frequency of closed organ pipe of nth overtone
$$f_2=(2n+1)\frac{v}{4l}$$
Therefore, the desired ratio is
$$\frac{f_1}{f_2}=\frac{2(n+1)}{(2n+1)}$$

67. (b) $f_o-f_c=2$
or $\frac{v}{2l}-\frac{v}{4l}=2$ or $\frac{v}{4l}=2$ or $\frac{v}{l}=8$

When length of OOP is halved and that of COP is doubled, the beat frequency will be

$$f_o' - f_c' = \frac{v}{l} - \frac{v}{8l} = \frac{7}{8}\frac{v}{l} = \frac{7}{8} \times 8 = 7$$

68. (c) Beat frequency $= f_1 - f_2$

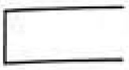
$$= \frac{v}{2l} - \frac{v}{2(l+x)} = \frac{v}{2l}\left[1 - \left(1 + \frac{x}{l}\right)^{-1}\right]$$

$$= \frac{v}{2l}\left[1 - 1 + \frac{x}{l}\right] = \frac{vx}{2l^2}$$

69. (b)

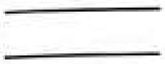
Fundamental frequency of wire $(f_{wire}) = v/2l$

(a)



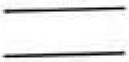
$f = \frac{v}{4l}, \frac{3v}{4l}, \frac{5v}{4l}$ cannot match with f_{wire}

(b)



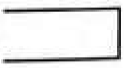
$f = \frac{v}{2(2l)}, \frac{2v}{2(2l)}, \frac{3v}{2(2l)}$ its second harmonic $\frac{2v}{2(2l)}$ matches with f_{wire}.

(c)



$f = \frac{v}{2(l/2)}, \frac{2v}{2(l/2)}$ cannot match with f_{wire}

(d)



$f = \frac{v}{4(l/2)}, \frac{3v}{4(l/2)}, \cdots$ cannot match with f_{wire}

70. (c) $f_1 : f_2 : f_3 = 50 : 150 : 250 = 1 : 3 : 5$

Hence this is close pipe of fundamental frequency 50 Hz

$$50 = \frac{v}{4L} \Rightarrow L = \frac{v}{200} = \frac{340}{200} = 1.7\,\text{m}$$

Problems Based on Mixed Concepts

71. (a) $y = 0.02 \sin(x + 30t)$

Comparing with standard equation

$y = A \sin(kx + \omega t)$, $\omega = 30$, $k = 1$

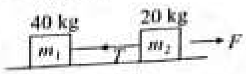
Velocity of wave, $v = \frac{\omega}{k} = \frac{30}{1} = 30$ m/s

Expression $v = \sqrt{\frac{T}{m}}$ gives

Tension $T = v^2 m = (30)^2 \times 10^{-4} = 0.09$ N

72. (b) Tension T in the wire $= v^2 \rho$

$= (400)^2 \times 10^{-3} = 160$ N

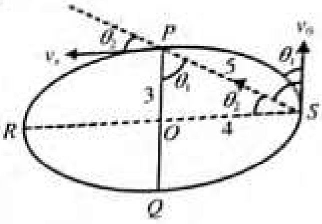


Force applied $F = \frac{T(m_1 + m_2)}{m_1} = 160 \times \frac{(40+20)}{40} = 240$ N

73. (a) $\lambda' = \left(\frac{v - v_s}{v}\right)\lambda = \left(\frac{320 - 20}{320}\right)60$

$= 56.25$ cm

74. (b) Frequency received by guard is



$$n_0 = n_0 \frac{(v + v_0 \cos\theta_1)}{(v + v_s \cos\theta_2)}$$

$v = 330$ m/s

Here, $v_0 = v_s = v/3$.

$\cos\theta_1 = 3/5$.

$\cos\theta_2 = 4/5$.

$$\therefore \quad n = n_0 \frac{\left(v + \frac{v}{3} \times \frac{3}{5}\right)}{\left(v + \frac{v}{3} \times \frac{4}{5}\right)} = \left(\frac{6}{5} \times \frac{15}{19}\right) n_0 = \frac{18 n_0}{19} = 1800 \text{ Hz}$$

75. (b) Time recorded in summer is more accurate. The velocity of sound is directly proportional to the square root of absolute temperature. Hence, the sound of the gun fired at the starting point will reach the finishing point quicker in summer than in winter. The lapse of time due to the time taken by the sound in reaching the finish point will be less in summer and hence the time recorded will be more accurate in summer than in winter.

76. (b) $v_1 = \sqrt{\frac{Y}{\rho}}$ = velocity of longitudinal waves

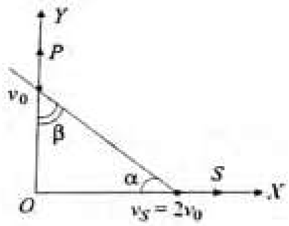
$v_2 = \sqrt{\frac{T}{\rho S}}$ = velocity of transverse waves

$$v_1 v_2 = \frac{1}{\rho}\sqrt{Y.Y\frac{\Delta l}{l}} = \frac{Y}{\rho}\sqrt{\frac{\Delta l}{l}} \qquad \left(\text{As } \frac{T}{S} = Y\frac{\Delta l}{l}\right)$$

But $\frac{\Delta l}{l} = \frac{l/n}{l} = \frac{1}{n}$, so, $v_1 v_2 = \frac{Y}{\rho n^{1/2}}$

or $v^2 = \frac{Y}{\rho n^{1/2}}$ or $v = \sqrt{\frac{V}{\rho n^{1/2}}}$

77. (c) OS = 2OP



$\cos\alpha = \frac{2}{\sqrt{5}}$, $\cos\beta = \frac{1}{\sqrt{5}}$

$$f' = f\left[\frac{v - v_0\cos\beta}{v + v_s\cos\alpha}\right]$$

$$= f\left[\frac{v - v_0/\sqrt{5}}{v + (2v_0)\times(2/\sqrt{5})}\right]$$

$$= f\left[\frac{v - v_0/\sqrt{5}}{v + 4v_0/\sqrt{5}}\right]$$

Let $\frac{v_0}{\sqrt{5}} = x$. Given $\frac{f'}{f} = \frac{3}{4} = \frac{v - x}{v + 4x}$

or $1 - \frac{3}{4} = \frac{5x}{v + 4x}$ or $v = 16x = \frac{16}{\sqrt{5}}v_0$

78. (a) $f \propto \sqrt{T}$

$$\frac{f+5}{f-5} = \sqrt{\frac{121}{100}}$$

$$10f + 50 = 11f - 55$$
$$f = 105 \text{ Hz}$$

79. (c) For pipe A, second resonant frequency is third harmonic thus

$$f = \frac{3v}{4L_A}$$

For pipe B, second resonant frequency is second harmonic thus

$$f = \frac{2v}{2L_B}$$

Equating, $\frac{3v}{4L_A} = \frac{2v}{2L_B} \Rightarrow L_B = \frac{4}{3}L_A = \frac{4}{3}\cdot(1.5) = 2$ m

80. (d) At $x_1 = \frac{\pi}{3k}, x_2 = \frac{3\pi}{2k}$, $\sin kx_1$ or $\sin kx_2$ is not zero.

Therefore, neither of x_1 or x_2 is a node

$$\Delta x = x_2 - x_1 = \left(\frac{3}{2} - \frac{1}{3}\right)\frac{\pi}{k} = \frac{7\pi}{6k}$$

Since $\frac{2\pi}{k} > \Delta x > \frac{\pi}{k}$ or $\lambda > \Delta x > \frac{\lambda}{2}$ $\left(k = \frac{2\pi}{\lambda}\right)$

therefore, $\phi_1 = \pi$

and $\phi_2 = k\cdot\Delta x = \frac{7\pi}{6}$ $\therefore \frac{\phi_1}{\phi_2} = \frac{6}{7}$

81. (c) Given $\frac{\Delta\lambda}{\lambda} = \frac{\lambda + \Delta\lambda}{51}$

$$\frac{\Delta\lambda}{\lambda(\lambda+\Delta\lambda)} = \frac{1}{51}; \frac{(\lambda+\Delta\lambda)-\lambda}{\lambda(\lambda+\Delta\lambda)} = \frac{1}{51}$$

$$\frac{1}{\lambda} - \frac{1}{(\lambda+\Delta\lambda)} = \frac{1}{51} \quad \text{... (i)}$$

But we know, $f_1 - f_2 = 6$

or $$\frac{v}{\lambda} - \frac{v}{(\lambda+\Delta\lambda)} = 6 \quad \text{...(ii)}$$

Comparing (i) and (ii),

$$\frac{6}{v} = \frac{1}{51}$$
$$v = 306 \text{ m/s}$$

82. (c) $\Delta x = x_2 - x_1 = \left(\frac{4}{3} - \frac{1}{4}\right)\frac{\pi}{k} = \frac{13\pi}{12k}$

$$\sin kx_1 = \sin k\left(\frac{\pi}{4k}\right) = \sin\frac{\pi}{4} \neq 0$$

$$\sin kx_2 = \sin k\left(\frac{4\pi}{3k}\right) = \sin\left(\pi + \frac{\pi}{3}\right) \neq 0$$

x_1 and x_2 are not the nodes

$$\frac{2\pi}{k} > \Delta x > \frac{\pi}{k} \Rightarrow \lambda > \Delta x > \frac{\lambda}{2}$$

For $\phi_1 = \pi, \phi_2 = k(\Delta x) = k\left(\frac{13\pi}{12k}\right) = \frac{13\pi}{12}$

$$\frac{\phi_1}{\phi_2} = \frac{\pi}{(13\pi/12)} = \frac{12}{13}$$

83. (c) $y(x, t) = A\sin(kx - 23562\,t)$

Compare with $y(x, t) = A\sin(kx - \omega t)$ where $\omega = 23562$

Hence $f = \frac{23562}{2\pi} = 3750$ Hz $\lambda = \frac{v}{f} = \frac{300}{3750} = 0.08$

For $y = A/2$ and $t = 0$

$$\frac{A}{2} = A\sin kx\left(\text{since } k = \frac{2\pi}{\lambda}\right) \text{ or } x = \lambda/12$$

Required distance $= 2\lambda + \frac{\lambda}{12} = \frac{25\lambda}{12}$

84. (c) $\frac{y}{A} = \sin 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right)$

$$y_1 = 1, A = 2, 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\text{rad}$$

$$y_1 = \sqrt{2}, A = 2, 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}\text{rad}$$

$$\frac{t}{T} - \frac{x_1}{\lambda} = \frac{1}{12} \Rightarrow \frac{t}{T} - \frac{x_2}{\lambda} = \frac{1}{8}$$

Subtract, $\frac{x_1}{\lambda} - \frac{x_2}{\lambda} = \frac{1}{24}$ $(x_1 - x_2) = \frac{\lambda}{24}$

85. (d) $y(x, t) = A\sin(kx - \omega t + \phi)$. For initial condition,

$$a_p = \frac{d^2y}{dt^2} = -\omega^2 A\sin\phi \quad (\text{for } x = 0 \text{ and } t = 0)$$
$$= 0 \text{ for } \phi = 2n\pi$$

Now, $a_p = -\omega^2 A\sin(kx - \omega t + (1/2)n\pi)$, $n = 0, 2, 4$

$$= 0 \text{ if } x = \lambda \text{ and } t = \frac{1}{f}$$
$$= 0 \text{ if } x = 2\lambda \text{ and } t = \frac{1}{f}$$

Because $a_p = -\omega^2 A\sin\left[\frac{2\pi}{\lambda}.\lambda - 2\pi f.\frac{1}{f} + \frac{1}{2}n\pi\right] = 0$

So, $x = \lambda$ and 2λ and $t = \frac{1}{f}$ are the required solutions.

86. (c) $I = \frac{p^2}{\rho v}$

$$\frac{I_2}{I_1} = \frac{p_2^2}{p_1^2} \times \frac{\rho_1}{\rho_2}\frac{v_1}{v_2} = \frac{9}{16} \times \frac{1.5}{3} \times \frac{400}{1200} = \frac{9}{16\times3\times2} = \frac{3}{32}$$

Archives

1. (c) The equation of a wave is

$$y = a\sin(\omega t - kx) \quad \text{...(i)}$$

Let the equations of another wave be

$$y = a\sin(\omega t + kx) \quad \text{...(ii)}$$

or $$y = -a\sin(\omega t + kx) \quad \text{...(iii)}$$

If (i) propagates with (ii), then we get

$$y = 2a\cos kx\sin\omega t \quad \text{...(iv)}$$

If (i) propagates with (iii), then we get

$$y = -2a\sin kx\cos\omega t \quad \text{...(v)}$$

After putting $x = 0$ in (iv) and (v), respectively, we get

$$y = 2a\sin\omega t \text{ and } y = 0$$

Hence, (iii) is an equation of the unknown wave.

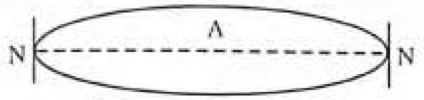
2. (c) The tuning fork of frequency 288 Hz is producing 4 beats per second with the unknown tuning fork, i.e., the frequency difference between them is 4. Therefore, the frequency of the unknown tuning fork is $288 \pm 4 = 292$ or 284.

On placing a little wax on the unknown tuning fork, its frequency decreases but now the number of beats produced per second is 2, i.e., the frequency difference now decreases. It is possible only when before placing the wax, the frequency of the unknown fork is greater than the frequency of the given tuning fork. Hence, the frequency of the unknown tuning fork is 292 Hz.

3. (b) $f_A = \frac{v}{2l}$ and $f_B = \frac{v}{4l}$

$\therefore \quad \frac{f_A}{f_B} = 2$

4. (b) When the string is plucked in the middle, it vibrates in one loop with nodes at fixed ends and an anti-node in the middle.



So $\quad \frac{\lambda_1}{\lambda_2} = l$

$\Rightarrow \quad \lambda_1 = 2 \times 40 = 80$ cm

5. (b) In condition of resonance, frequency of a.c. will be equal to natural frequency of wire

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2\times 1}\sqrt{\frac{10\times 9.8}{9.8\times 10^{-3}}} = \frac{100}{2} = 50 \text{ Hz}$$

6. (d) Suppose n_p = frequency of piano = ? ($n_p \propto \sqrt{T}$)

n_f = Frequency of tuning fork = 256 Hz

x = Beat frequency = 5 bps, which is decreasing ($5 \to 2$) after changing the tension of piano wire

Also, tension of piano wire is increasing so $n_p\downarrow$

Hence $n_p\uparrow - n_f = x\downarrow \longrightarrow$ Wrong

$n_f - n_p\uparrow = x\downarrow \longrightarrow$ Correct

$\Rightarrow \quad n_p = n_f - x = 256 - 5$ Hz.

7. (d) $y = 10^{-6}\sin[100t + 20x + (\pi/4)]$

Comparing it with the standard form

$y = r\sin(\omega t + kx + \phi)$,

we get

$\omega = 100\ \text{s}^{-1}$ and $k = 20$

$\therefore \quad v = \frac{\omega}{k} = 5$ m/s

8. (b) $v = \frac{\text{Coefficent of } t}{\text{Coefficent of } x} = \frac{\omega}{k} = \frac{100}{20} = 5$ m/s.

9. (a) When observer moves towards stationary source then apparent frequency

$$n' = \left[\frac{v + v_O}{v}\right]n = \left[\frac{v + v/5}{v}\right]n = \frac{6}{5}n = 1.2n$$

Increment in frequency = 0.2 n so percentage change in frequency = $\frac{0.2n}{n} \times 100 = 20\%$.

10. (b) $f_2 = f_1 \pm 4 = 200 \pm 4 = 204$ Hz or 196 Hz

Since on loading the unknown fork, beat frequency increases.

$f_2 = 196$ Hz

11. (d) $y_1 = 0.1\sin(100\pi t + \pi/3)$

$\therefore \quad v_1 = \frac{dy_1}{dt} = 0.1\cos(100\pi t + \pi/3)\,100\pi$

$$y_2 = 0.1\cos\pi t = 0.1\sin\left(\frac{\pi}{2} + \pi t\right)$$

$$v_2 = \frac{dy_2}{dt} = 0.1\cos\left(\frac{\pi}{2} + \pi t\right)\pi$$

Therefore, the initial phase difference between the velocities of first and second particles is

$$\phi_1 - \phi_2 = \left(\frac{\pi}{3} - \frac{\pi}{2}\right) = -\frac{\pi}{6}$$

12. (a) Let the successive loops formed be p and $(p + 1)$ for frequencies 315 Hz and 420 Hz

$\therefore \quad f = \frac{p}{2l}\sqrt{\frac{T}{\mu}} = \frac{pv}{2l}$

$\therefore \quad \frac{pv}{2l} = 315\text{Hz}$ and $\frac{(p+1)v}{2l} = 420\text{Hz}$

or $\quad \frac{(p+1)v}{2l} - \frac{pv}{2l} = 420 - 315$

or $\quad \frac{v}{2l} = 105 \Rightarrow \frac{1\times v}{2l} = 105\text{Hz}$

$p = 1$ for fundamental mode of vibration of string.

$\therefore$ Lowest resonant frequency = 105 Hz.

13. (c) When observer moves towards stationary observer then apparent frequency

$$f' = f\left[\frac{v}{v - v_s}\right] \Rightarrow \frac{f'}{f} = \left[\frac{v}{v - v_s}\right]$$

Where v is the velocity of sound in air.

$$\frac{10000}{9500} = \frac{300}{300 - v}$$

$$\Rightarrow (300 - v) = 285 \Rightarrow v = 15 \text{ m/s}.$$

14. (c) Let the initial loudness of sound wave intensity I_1 be 100 dB. Therefore,

$L = 10\log\frac{I_1}{I}$

$\Rightarrow \quad 100 = 10\log_{10}\frac{I_1}{I_0}$

$\Rightarrow \quad \frac{I_1}{I} = 10^{10}$

Since the sound absorber attenuates the sound level by 20 dB,

$(100 - 20)\text{ dB} = 10\log\frac{I_2}{I_1}$

$\Rightarrow \quad 80 = 10\log\frac{I_2}{I_0}$

$\Rightarrow \quad \frac{I_2}{I_0} = 10^8$

Dividing (ii) by (i), we get

$$\frac{I_2}{I_1} = \frac{10^8}{10^{10}} = \frac{1}{100}$$

$\Rightarrow \quad l_2 = \frac{l_1}{100}$

15. (b) If $f = 3v_2/4l_2$, where $l_2 = x$ according to the given situation and $v_1 < v_2$ as during summer, temperature would be higher, then

$$\frac{3v_2}{4l_2} = \frac{v_1}{4l_1}$$

$$\Rightarrow \quad l_2 = 3l_1 \times \frac{v_2}{v_1}$$

$\Rightarrow \quad x = 54 \times$ (a quantity greater than 1)

So $x > 54$.

16. (None)

The speed of sound is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v_{O_2} = \sqrt{\frac{\frac{7}{5}RT}{32}} \text{ and } v_{He} = \sqrt{\frac{\frac{5}{3}RT}{4}}$$

$$\frac{v_{O_2}}{v_{He}} = \sqrt{\frac{7 \times 3 \times 4}{5 \times 32 \times 5}}$$

$$\Rightarrow \quad v_{He} = 460 \times 10 \times \sqrt{\frac{2}{21}} = 1420 \text{ m/s}$$

No option matches with the answer.

17. (a) $y(x, t) = 0.005 \cos(\alpha x - \beta t)$

$$\frac{2\pi}{\lambda} = \alpha$$

and $\frac{2\pi}{T} = \beta$

So $\alpha = \frac{2\pi}{0.08} = 25\pi$

and $\beta = \frac{2\pi}{\pi} = \pi$

18. (b) For the motor cycle, $u = 0$ and $a = 2$ m/s^2

$$\therefore \quad n' = n\frac{v - v_0}{v + v_S}$$

$$\Rightarrow \quad \frac{94}{100}n = n\frac{330 - v_0}{330}$$

$$\Rightarrow \quad 330 - v_0 = \frac{330 \times 94}{100}$$

$$\Rightarrow \quad v_0 = 330 - \frac{94 \times 33}{10} = \frac{33 \times 6}{10} \text{ m/s}$$

$$\therefore \quad S = \frac{v^2 - u^2}{2a} = \frac{9 \times 33 \times 33}{100} = \frac{9 \times 1098}{100} = 98 \text{ m}$$

19. (c) Let the given sources of sound produce frequencies, $(f - 1)$, f and $(f + 1)$.

For two sources of frequencies f_1 and f_2,

$$y_1 = A \cos 2\pi f_1 t$$

$$y_2 = A \cos 2\pi f_2 t$$

If these two waves superpose then

$$y = 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\cos 2\pi\left(\frac{f_1 + f_2}{2}\right)t.$$

The resultant frequency obtained is $\frac{f_1 + f_2}{2}$ and this wave is modulated by a wave of frequency $\frac{f_1 - f_2}{2}$

Hence the intensity waxes and wanes. For a cosine curve (or sine curve), the number of beats = $f_1 \sim f_2$

Frequencies	Mean	Beats
$f + 1$ and f	$(f + 0.5)$ Hz	1
f and $f - 1$	$f - 0.5$	1
$(f + 1)$ and $f - 1$	f	2

Total number of beats = 2.

20. (d) The given equation of a wave is

$$y = 0.02\sin\left[2\pi\left(\frac{t}{0.04} - \frac{x}{0.50}\right)\right]$$

Compare it with the standard wave equation

$$y = A\sin(\omega t - kx)$$

we get

$$\omega = \frac{2\pi}{0.04}\text{ rad s}^{-1};\ k = \frac{2\pi}{0.5}\text{ rad m}^{-1}$$

$$\text{Wave velocity, } v = \frac{\omega}{k} = \frac{(2\pi/0.04)}{(2\pi/0.5)}\text{ms}^{-1} \quad \text{...(i)}$$

$$\text{Also} \quad v = \sqrt{\frac{T}{\mu}} \quad \text{...(ii)}$$

where T is the tension in the string and μ is the linear mass density Here, linear mass density $\mu = 0.04$ kg m^{-1}

Equating equations (i) and (ii), we get

$$\frac{\omega}{k} = \sqrt{\frac{T}{\mu}} \text{ or } T = \frac{\mu\omega^2}{k^2}$$

$$T = \frac{0.04 \times \left(\frac{2\pi}{0.04}\right)^2}{\left(\frac{2\pi}{0.05}\right)^2} = 6.25 \text{ N}$$

21. (b) $y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$

$$= e^{-(\sqrt{a}x + \sqrt{b}t)^2}$$

Comparing equation (i) with standard equation

$y(x, t) = f(ax + bt)$

As there is positive sign between x and t terms, hence wave travel in $-x$ direction.

$$\text{Wave speed} = \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \sqrt{\frac{b}{a}}$$

22. (a) $f = \frac{v}{2l}$

Now, it will act like one end open and other closed.

$$f=\frac{v}{4l'}=\frac{v}{4\frac{l}{2}}=\frac{v}{2l}=f$$

23. (a) Fundamental frequency of vibration of sonometer wire is

$$f_0=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where L is the length of the wire, T is the tension in the wire and μ is the mass per length of the wire

As $\mu=\rho A$

where ρ is the density of the material of the wire and A is the area of cross-section of the wire.

$$\therefore \quad f_0=\frac{1}{2L}\sqrt{\frac{T}{\rho A}}$$

But we have $Y=\frac{\text{Stress}}{\text{Strain}}=\frac{TL}{A\Delta L}$

The tension in the wire is due to elasticity of wire

$$\therefore \quad T=YA\left[\frac{\Delta L}{L}\right]$$

Hence, $f_0=\frac{1}{2L}\sqrt{\frac{Y\Delta L}{\rho L}}$

Here, $Y=2.2\times10^{11}\ \text{N/m}^2, \rho=7.7\times10^3\ \text{kg/m}^3$

$$\frac{\Delta L}{L}=0.01, \quad L=1.5\text{m}$$

Substituting the given values, we get

$$f_0=\frac{1}{2\times1.5}\sqrt{\frac{2.2\times10^{11}\times0.01}{7.7\times10^3}}$$

$$=\frac{10^3}{3}\sqrt{\frac{2}{7}}\ \text{Hz}=178.2\text{Hz}$$

24. (a) In fundamental mode

$$\frac{\lambda}{4}=0.85$$

$$\lambda=4\times0.85$$

$$f=\frac{v}{\lambda}=\frac{340}{4\times0.85}=100\text{Hz}$$

$\therefore$ Possible frequencies = 100 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz and 1100 Hz below 1250 Hz.

25. (b)

$$f_1=f\left[\frac{v}{v-v_s}\right]=f\left[\frac{320}{320-20}\right]=f\times\frac{320}{300}\text{Hz}$$

$$f_2=f\left[\frac{v}{v+v_s}\right]=f\times\frac{320}{340}\text{Hz}$$

$$100\times\left(\frac{f_2}{f_1}-1\right)=\left(\frac{f_2-f_1}{f_1}\right)\times100$$

$$=100\left[\frac{300}{340}-1\right]=12\%$$

26. (c) Let linear mass density (mass/length) is μ. Consider x length of the string, the tension at this position is T.

$$T=(x\mu)g$$

The velocity of wave pulse

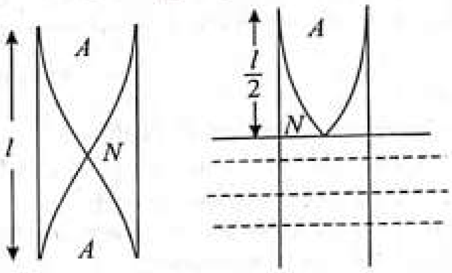
$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{(x\mu)g}{\mu}}=\sqrt{gx}$$

$$\frac{dx}{dt}=\sqrt{gx} \Rightarrow \frac{dx}{\sqrt{x}}=\sqrt{g}dt$$

$$\Rightarrow \int_0^{20}\frac{dx}{\sqrt{x}}=\sqrt{g}\int_0^t dt$$

On solving we get, $t=2\sqrt{2}$ s

27. (d)



For open organ pipe

$$\ell=\frac{\lambda}{4}+\frac{\lambda}{4}=\frac{\lambda}{2}\Rightarrow\lambda=2\ell$$

Hence frequency, $f=\frac{v}{\lambda}=\frac{v}{2\ell}$

The pipe is dipped vertically in water, it acts as closed pipe

$$\frac{\ell}{2}=\frac{\lambda}{4}\Rightarrow\lambda=2\ell$$

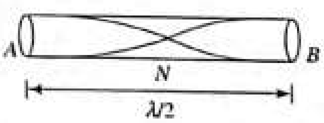
$$\therefore \quad f'=\frac{v}{\lambda}=\frac{v}{2\ell}=f$$

28. (b) Velocity of wave $=\sqrt{\frac{Y}{\rho}}$

$$=\sqrt{\frac{9.27\times10^{10}}{2.7\times10^3}}=\sqrt{3.433\times10^7}$$

$$\Rightarrow v=5.85\times10^3\ \text{m/sec}$$

The rod is clamped at middle fundamental wave shape is as follow



$$\frac{\lambda}{2}=L\Rightarrow\lambda=2L=2\times0.60=1.20\ \text{m}$$

Fundamental frequency

$$f=\frac{v}{\lambda}=\frac{5.85\times10^3}{1.2}=4.88\times10^3\text{Hz}=5\text{kHz}$$

CHAPTER 17: ELECTRIC CHARGE AND FIELD

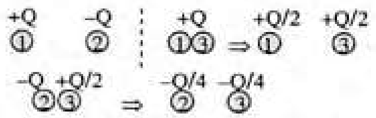
Concept Application Exercise 17.1

1. Number of molecules in 2.0 mol of H_2 gas is
$= 2(6.02 \times 10^{23}) = 12.04 \times 10^{23}$
As each H_2 molecule contains 2 electrons (and 2 protons), number of electrons (and protons) in 2.0 mol of H_2 gas is
$n = 2(12.02 \times 10^{23}) = 24.08 \times 10^{23}$
Thus, $q = ne = (24.08 \times 10^{23})(1.6 \times 10^{-19}\text{ C})$
$= 0.358 \times 10^6\text{ C} = 0.358\text{ MC}$

2. (a) Since the polythene piece acquires a negative charge (q) on rubbing with wool, electrons are transferred from wool to polythene. If n is the number of electrons transferred, then
$$q = ne \quad \text{or} \quad n = \frac{q}{e} = \frac{3 \times 10^{-7}\text{ C}}{1.6 \times 10^{-19}\text{ C}} = 1.875 \times 10^{12}$$
(b) Mass transferred from wool to polythene is
$(1.875 \times 10^{12})(9.1 \times 10^{-31}\text{ kg}) = 1.7 \times 10^{-18}\text{ kg}$
Of course, this mass is negatively small.

3. (a) This is due to the reason that the spheres are identical and they share the total charge equally.
(b) The charge on other sphere will be $+Q$. Because of induction the charge on other sphere will be $+Q$.

4. Due to conservation of charge, it is clear from figure that
$$Q_1 = \frac{Q}{2}, Q_2 = -\frac{Q}{4}, \text{ and } Q_3 = -\frac{Q}{4}$$



5. To calculate charge, we will apply formula $Q = ne$; for this, we must have number of electrons. Here, number of electrons is $n = 0.01\%$ of 5×10^{21}
i.e., $$n = \frac{5 \times 10^{21} \times .01}{100} = 5 \times 10^{21} \times 10^{-4} = 5 \times 10^{17}$$
So $\quad Q = 5 \times 10^{17} \times 1.6 \times 10^{-19} = 8 \times 10^{-2} = 0.08\text{ C}$
Since electrons have been removed, charge will be positive i.e., $Q = +0.08\text{ C}$

Concept Application Exercise 17.2

1. (a) Right, because third particle should be placed near the charge smaller in magnitude and not between the charges.
(b) The third particle should be negatively charged, only then net force on any charge due to other two charges can be zero.
(c) Equilibrium is unstable, because if we displace any of the charged particles from its equilibrium position, it may not return to its initial position for all directions of displacement.

2. When charges are brought in the medium, force between them will decrease by a factor of K, where K is known as dielectric constant of that medium. Hence,
$$\frac{F'}{F} = \frac{1}{\text{Dielectric constant}} = \frac{1}{81}$$

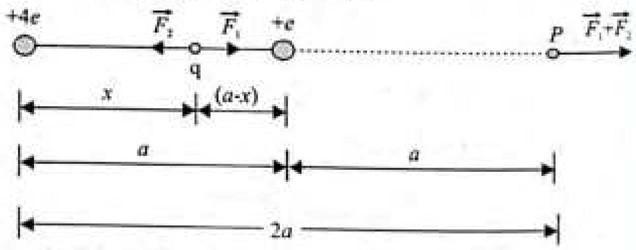
3. Because of the free fall, there will be weightlessness in the elevator. The balls will go maximum away from each other and finally angle between them will be 180°.

Repulsion force F between them will be balanced by tension T.
So $$T = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2L)^2} = \frac{q^2}{16\pi\varepsilon_0 L^2}$$

4. (a) Since the size of group is small, the group of n small charged particles must behave as single point charge, so that it can have a separation of 10 cm from the particle in question. Obviously, force on this particle due to the group of n particles is n times the force due to a single particle. Hence, force due to group of n particles is $F = n \times 3 \times 10^{-10}$ N.
(b) Here, $F = 6 \times 10^{-6}$ N
$$n = \frac{F}{3 \times 10^{-10}} = \frac{6 \times 10^{-6}}{3 \times 10^{-10}} = 2 \times 10^4$$

5. As $\quad F = k_e \dfrac{q_1 q_2}{r^2}$
$$3.7 \times 10^{-9} = (9 \times 10^9)\frac{q^2}{(5 \times 10^{-10})^2} \text{ whence } q = 3.2 \times 10^{-19}\text{ C}$$
As $q = ne$, $$n = \frac{q}{e} = \frac{3.2 \times 10^{-19}\text{ C}}{1.6 \times 10^{-19}\text{ C}} = 2$$

6. Let the third point charge (say q) be at a distance x from $+4e$ as shown in figure. Since the distance between $+4e$ and $+e$ is a, distance between q and $+e$ is $(a - x)$.



For q to be in equilibrium, force between q and $+4e$, i.e., F_1 = force between q and $+e$, i.e., F_2
$$k_e = \frac{q \times 4e}{x^2} = k_e \frac{q \times e}{(a-x)^2}$$
or $$\frac{4}{x^2} = \frac{1}{(a-x)^2} \quad \text{or} \quad \frac{2}{x} = \pm\frac{1}{(a-x)}$$
$$\frac{2}{x} = \frac{1}{(a-x)}$$
$$x = 2a - 2x \text{ or } x = \frac{2a}{3}$$
Thus, q should be at a distance of $2a/3$ from $+4e$ charge.

7. We are given that, charge on sphere A = charge on sphere $B = q = 6.5 \times 10^{-7}$ C

Force of repulsion between A and B is

$$F = k_e \frac{q \times q}{r^2} = k_e \frac{q^2}{r^2} = (9 \times 10^9)\frac{(6.5 \times 10^{-7})^2}{(0.5)^2} = 1.5 \times 10^{-2}\,\text{N}$$

Let the third sphere (say C), of the same size but uncharged, be brought in contact with A. Due to the flow of electrons, the two spheres share the charge equally. Therefore,

charge on A = charge on C $= \frac{q+0}{2} = \frac{q}{2}$

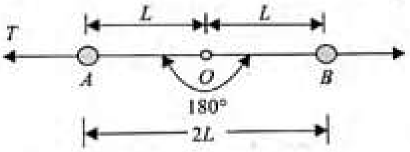
When the sphere C (having charge $q/2$) touches the sphere B (having charge q), total charge on B and C ($q + q/2 = 3q/2$) is equally distributed between them. Thus, charge on B = charge on $C = 3q/4$.

If F' is the new force of repulsion between A and B,

$$F' = k_e \frac{(q/2)(3q/4)}{r^2} = k_e \frac{3}{8}\frac{q^2}{r^2}$$

$$= \frac{3}{8}F = \frac{3}{8}(1.3 \times 10^{-2}\,\text{N}) = 5.7 \times 10^{-3}\,\text{N}$$

8. Let A and B be two charged balls suspended from hook, O as shown in figure.
   (i) In the absence of gravity, tension in the strings is due to only Coulomb's repulsive force. As such the two strings will be in the same straight line making an angle of 180° with each other, the separation between the balls being $2L$.



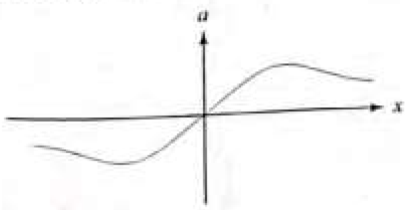
   (ii) Tension in each string T = Coulomb's force between the charged balls, i.e.,

$$T = k_e \frac{Q \times Q}{(2L)^2} = k_e \frac{Q^2}{4L^2}$$

Concept Application Exercise 17.3

1. Here we have assumed that the particle has started from the positive side of the x-axis.

Acceleration at infinity will be zero. When the particle comes toward the origin, first its acceleration will increase, becomes maximum, and again becomes zero at origin and similarly on the negative side of the x-axis.

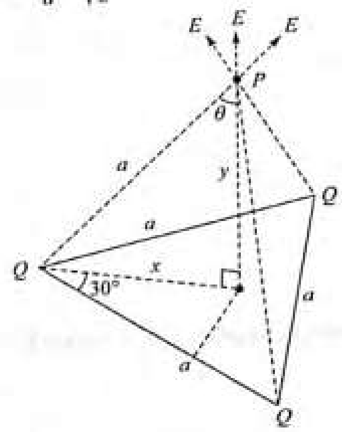


2. We have to find intensity at P.

$$E = \frac{kQ}{a^2}, \frac{a/2}{x} = \cos 30° \quad \text{or } x = \frac{a}{\sqrt{3}}$$

$$y = \sqrt{a^2 - x^2} = \sqrt{a^2 - \frac{a^2}{3}} = \left(\sqrt{\frac{2}{3}}\right)a$$

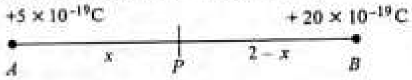
$$\cos\theta = \frac{y}{a} = \sqrt{\frac{2}{3}}$$



Net intensity at P is

$$E_0 = 3E\cos\theta = \frac{3kQ}{a^2}\sqrt{\frac{2}{3}} = \frac{3}{4\pi\varepsilon_0}\frac{Q}{a^2}\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{3}Q}{2\sqrt{2}\pi\varepsilon_0 a^2}$$

3. Let electric field at point P is zero, then



$$\frac{1}{4\pi\varepsilon_0}\frac{5 \times 10^{-19}}{X^2} = \frac{1}{4\pi\varepsilon_0}\frac{20 \times 10^{-19}}{(2-X)^2}$$

or $\quad \frac{2-X}{X} = 2 \quad$ or $\quad X = \frac{2}{3}$ m

4. The acceleration experienced by a charged particle under the action of an electric field E is

$$a = \frac{eE}{m}$$

If it falls through a distance h, starting from rest,

$$h = \frac{1}{2}at^2 \qquad (\text{as } u = 0)$$

or $\quad t^2 = \frac{2h}{a} = \frac{2h}{eE/m} = \frac{2mh}{eE}$

or $\quad t \propto \sqrt{m} \qquad$ (i)

From Eq. (i) the time of fall will be

$$\frac{t_{\text{electron}}}{t_{\text{proton}}} = \sqrt{\frac{m_e}{m_p}}$$

Now since $m_p > m_e$, so $t_p > t_e$, i.e., time of fall through the same distance is greater for a heavier particle (i.e., proton), which is in contrast with the situation of free fall under gravity where the time of fall is independent of the mass of the body.

5. The electron will move with constant velocity in the x-direction. Time taken by the electron to come out of the plate is $t = \ell/v_0$.

Hence $h = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\ell}{v_0}\right)^2 = \left(\frac{eE}{2mv_0^2}\right)\ell^2$

Velocity of electron when it comes out of the plate is

$$\vec{v}=\vec{u}+\vec{a}t=v_0\hat{i}+\left(\frac{eE}{m}\right)\hat{j}\cdot t$$

$$=v_0\hat{i}+\left(\frac{eE}{m}\hat{j}\right)\left(\frac{\ell}{v_0}\right)=v_0\hat{i}+\left(\frac{eE\ell}{mv_0}\right)\hat{j}$$

After coming out of field, the electron moves in straight line with constant velocity $\vec{v}_0$. Further time taken by electron to hit the screen is $t'=\ell/v_0$. Hence, further displacement of electron outside the electric field is

$$y'=(v_y)\cdot t'=\left(\frac{eE\ell}{mv_0}\right)\frac{\ell}{v_0}=\frac{eE\ell^2}{mv_0^2}$$

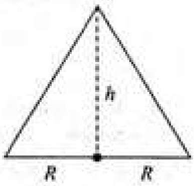
Hence $H=h+y'=\dfrac{3eE\ell^2}{2mv_0^2}$

Concept Application Exercise 17.4

1. (a) and (b): Since the point charge Q is located just above the centre of the flate face, then almost half the lines emitting from Q will pass through the flate surface. Any line passing through the flat surface will also pass through the curved surface below it. Hence the required flux is $\phi=Q/2\varepsilon_0$. Here flux through curved surface will be positive and through flat surface will be negative having same magnitude.
   (c) If the charge is placed exactly at the centre, then any line emitting from the charge will be parallel to the flat surface. Hence, there is no flux through the flat surface. But the flux through the curved surface will remain the same as $\phi=Q/2\varepsilon_0$.

2. Effective area of the cone through which flux passes is

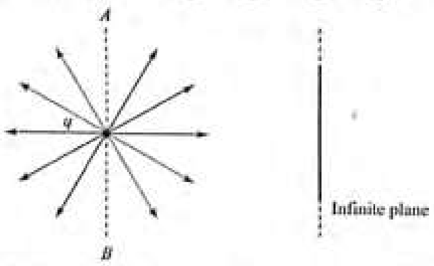
$$A=2\left[\frac{1}{2}Rh\right]=Rh$$



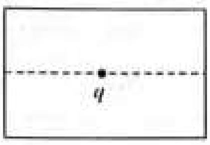
Flux $\phi=EA=ERh$

3. $\vec{E}=a\hat{i}+b\hat{j}$
   (a) $\vec{A}=A\hat{i}\quad\Rightarrow\phi=\vec{E}\cdot\vec{A}=Aa$
   (b) $\vec{A}=A\hat{j}\quad\Rightarrow\phi=\vec{E}\cdot\vec{A}=Ab$
   (c) $\vec{A}=A\hat{k}\quad\Rightarrow\phi=\vec{E}\cdot\vec{A}=0$

4. (a) Any line emitting from charge q to the right of line AB will pass through the infinite plane. It means half flux will pass through the plane. So the flux passing through the plane is $q/2\varepsilon_0$.



   (b) q is above the centre at a small distance. So this becomes similar to the case (a).

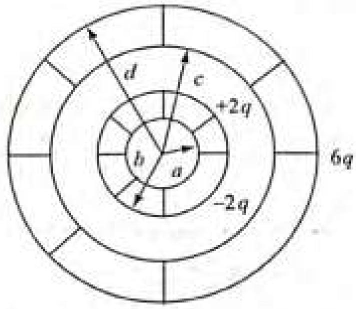


5. Effective area $=A=\omega r^2$
   $\therefore\quad \phi=E_0A=E_0\omega r^2$

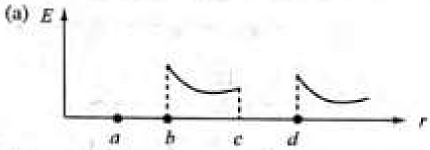
6. (a) Since Q_1 is inside the surface, so flux due to Q_1 is Q_1/ε_0.
   (b) Since Q_2 is outside, so there is no contribution to the flux due to Q_2.

Concept Application Exercise 17.5

1. (a)



   (b) **i, ii.** For $r<b$, electric field is zero, because there is no charge inside.
   iii. For $c>r>b$, electric field will be similar to that as if a point charge $2q$ is placed at the centre.
   iv. For $d>r>c$, electric field will be zero, as there is no charge inside.
   v. For $r>d$, electric field will be similar to that as if a point charge $6q$ is placed at the centre.
   The resulting plot is as shown in figure



   (c) **i.** 0
   ii. $+2q$
   iii. $-2q$
   iv. $+6q$

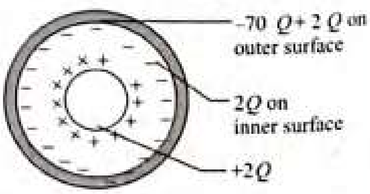
2. (a) $E=0$, electric field will be cancelled due to both.
   (b) σ/ε_0 to the right, field of both will be added.
   (c) $E=0$, electric field will be cancelled due to both.

3. According to Gauss' law,

$$\phi_1=\frac{Q}{\varepsilon_0},\ \phi_2=\frac{(Q+2Q)}{\varepsilon_0}\quad\therefore\quad\frac{\phi_1}{\phi_2}=\frac{Q/\varepsilon_0}{3Q/\varepsilon_0}=\frac{1}{3}$$

4. The flux passing through curved surface
   = the flux passing through the plane $ABCD$
   $=E\,2R\,l$

5. (a) The charge on the inner sphere induces equal magnitude of charge, but opposite in sign, on the inner surface of the outer sphere. Sum of all the induced charges is always zero. Therefore, an equal amount of charge must appear on the outer surface. Thus outer and



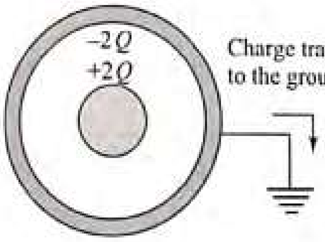
inner surface of the outer sphere have charges $-5Q$ and $-2Q$, respectively.

(b) When outer and inner spheres are connected by a wire, the entire charge is transferred to the outer surface from the inner sphere. In electrostatic equilibrium, charge does not reside inside a conductor. Total charge on the outer surface of the outer sphere is $-5Q$.
Total charge on the inner surface is 0.
The electric field at the surface of the inside sphere goes to zero after connection. Consider a Gaussian surface just on the surface of the inner sphere.

$$\phi_E = E(4\pi r^2) = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0, \text{ as } Q_{\text{enclosed}} = 0$$

Thus, we have $E = 0$

(c) When the outer sphere is grounded the charge on the outer surface is transferred to ground, thus charge is reduced to zero on the outer surface. The final charge distribution is shown in figure.

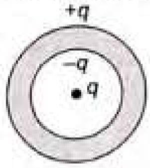


EXERCISES

Electric Charge and Coulomb's Law

1. (b) When charged metallic ball touches the bottom of can, all of its charge goes to the outer surface of the can. (Net charge on a conductor lies only on its surface).
When ball is withdrawn from the can, it is uncharged. When ball is placed on the disc of electroscope. Final charge on the leaves is reduced so leaves of electroscope converges.

2. (b) Let the charge on the ball at centre is q, then $-q$ charge will be induced on the inner surface and $+q$ on the outer surface. Field at any point outside the sphere due to inner charges q and $-q$ will combinely be zero.
Outer charge q will produce the electric field at outside points as it would be produced by a charge at the centre. Hence, presence of sphere does not affect the electric field outside the sphere.



3. (a) To give positive charge to the copper sphere, a certain number of free electrons must be removed.
The total number of electrons is $29(2 \times 10^{22}) = 5.8 \times 10^{23}$
Since a positive charge of 2 μC is to be given the number of

$$\text{electrons to be removed} = \frac{2 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{13}$$

The mass of the electrons removed

$$= 1.25 \times 10^{13} \times 0.91 \times 10^{-30} \text{ kg}$$

$$= 1.13 \times 10^{-17} \text{ kg} = 1.13 \times 10^{-14} \text{ g}$$

This mass will be lost by the copper sphere.

4. (a) Suppose the balls have charges Q_1 and Q_2 respectively.
Initially,

Finally,

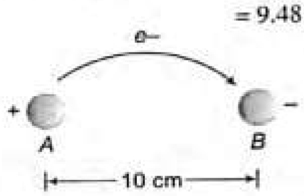
$$F' = \frac{k\left(\frac{Q_1+Q_2}{2}\right)^2}{\left(\frac{r}{2}\right)^2} = \frac{k(Q_1+Q_2)^2}{r^2}$$

It is given that $F = 4.5$ F so $\frac{k(Q_1+Q_2)^2}{r^2} = 4.5k \cdot \frac{Q_1Q_2}{r^2}$

$\Rightarrow \quad (Q_1+Q_2)^2 = 4.5Q_1Q_2$. On solving it gives $\frac{Q_1}{Q_2} = \frac{2}{1}$.

5. (c) Coulomb forces between unequal charges are equal.

6. (c) Number of atoms in given mass $= \frac{10}{63.5} \times 6.02 \times 10^{23}$
$= 9.48 \times 10^{22}$



Transfer of electron between balls $= \frac{9.48 \times 10^{22}}{10^6}$
$= 9.48 \times 10^{16}$

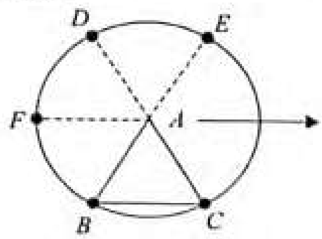
Hence, magnitude of charge gained by each ball.

$$Q = 9.48 \times 10^{16} \times 1.6 \times 10^{-19} = 0.015 \text{ C}$$

Force of attraction between the balls

$$F = 9 \times 10^9 \times \frac{(0.015)^2}{(0.1)^2} = 2 \times 10^8 \text{ N}.$$

7. (c) Two similar charges should be placed at D and E so that resultant force at A is zero. A similar charge should not be placed at F, so that resultant force at A will be $\frac{q^2}{4\pi\varepsilon_0 a^2}$ towards x-axis.



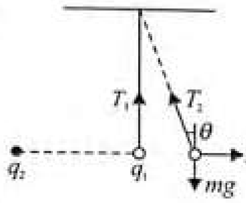
8. (c) $F_1 \propto q_1q_2 \Rightarrow F_2 \propto \frac{(q_1+q_2)^2}{4}$
Thus, $F_2 > F_1$

9. (c) (i) is incorrect because '1' and '2' are repelled by '3'. So '1' and '2' have charge of same nature. Hence they should repel each other. For this reason, (ii) is correct.
(iii) is correct if charge on '2' and '3' is of same nature and on '1' opposite of them.
(iv) All attraction is not possible.

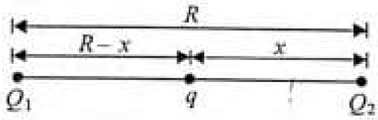
10. (b) Initial tension: $T_1 = mg$
Final tension: $T_2 \cos\theta = mg$

or $T_2 = \dfrac{mg}{\cos\theta}$

Obviously, $T_2 > mg$.
Here we have assumed θ to be small so that F is almost horizontal.



11. (c) Force on Q_2 is zero (q should be negative)



$$\frac{kQ_1Q_2}{R^2} = \frac{kqQ_2}{x^2} \text{ or } \frac{x}{R} = \sqrt{\frac{q}{Q_1}}$$

Force on q is zero:

$$\frac{kQ_1q}{(R-x)^2} = \frac{kqQ_2}{x^2}$$

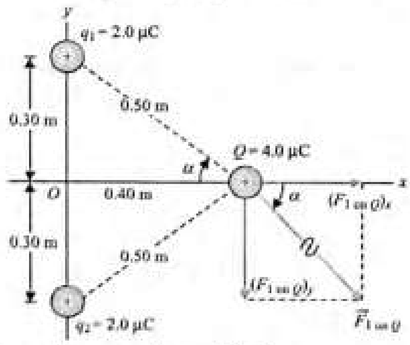
or $$\frac{R-x}{x} = \frac{\sqrt{Q_1}}{\sqrt{Q_2}}$$

or $$\frac{R}{x} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}} \text{ or } \frac{\sqrt{Q_1}}{\sqrt{q}} = \frac{\sqrt{Q_1}+\sqrt{Q_2}}{\sqrt{Q_2}}$$

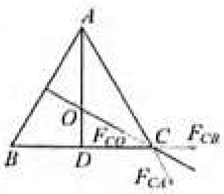
or $$q = \frac{Q_1Q_2}{(\sqrt{Q_1}+\sqrt{Q_2})^2}$$

12. (b) $F_{net} = 2|F_{31}|\cos\alpha$

$$= 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{2\times4\times10^{-12}}{(0.5)^2}\times\frac{4}{5} = 0.46 \text{ N}$$



13. (a) To keep the system in equilibrium, net force experienced by charges at 'A', 'B', and 'C' should be zero. For this, another charge of opposite sign should be placed at the centroid of the triangle. Let this charge be '$-Q$'.



$$AD = l\cos 30^\circ = \frac{l\sqrt{3}}{2},\ AO = \frac{2}{3}AD = \frac{1}{\sqrt{3}}$$

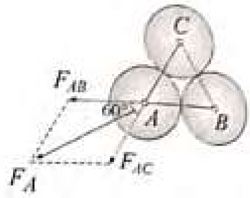
$$2|\vec{F}_{CA}|\cos 30^\circ = |\vec{F}_{CO}|$$

$$2\times\frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{l^2}\times\frac{\sqrt{3}}{2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(l/\sqrt{3})^2} \text{ or } Q = -\frac{q}{\sqrt{3}}$$

14. (c) For external points, a charged sphere behaves as if the whole of its charge is concentrated at its center.
Force on A due to B,

$$F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{along } \overline{BA}$$



And force on A due to C,

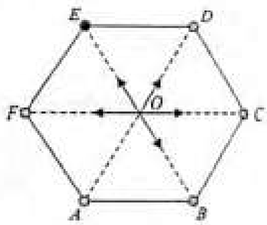
$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{4R^2} \text{along } \overline{CA}$$

Now as angle between BA and CA is 60° and
$|F_{AB}| = |F_{AC}| = F$

$$\therefore \quad F_A = \sqrt{F^2+F^2+2FF\cos 60} = \sqrt{3}F$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{3}}{4}\left(\frac{q}{R}\right)^2$$

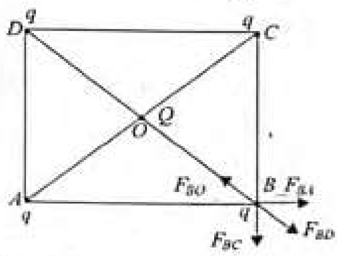
15. (d) If there had been a sixth charge $+q$ at the remaining vertex of hexagon, force due to all the six charges on $-q$ at O will be zero.
Now if f is the force due to the sixth charge and F due to the remaining five charges, then



$$\vec{F}+\vec{f} = 0, \quad \text{i.e., } \vec{F} = -\vec{f}$$

$$|\vec{F}| = |\vec{f}| = \frac{1}{4\pi\varepsilon_0}\frac{q\times q}{L^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{L}\right)^2$$

16. (d) $AC = \sqrt{2}l = BD$ or $BO = \dfrac{l}{\sqrt{2}}$

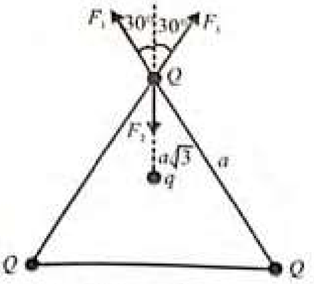


$$F_{BO} = F_{BD} + (F_{BA}+F_{BC})\cos 45^\circ$$

Solving, we get

$$Q = \frac{q}{4}(1+2\sqrt{2}) \text{ and } Q \text{ should be negative of } q.$$

17. (b) Net force on q will be zero for any values of q and Q. Net force on each Q should also be zero. For this q should be negative of Q.



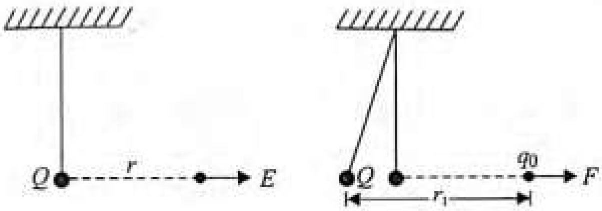
$F_2 = 2F_1 \cos 30°$

or $\dfrac{kqQ}{(a/\sqrt{3})^2} = 2\dfrac{kQ^2}{a^2}\dfrac{\sqrt{3}}{2}$ or $\dfrac{q}{Q} = \dfrac{1}{\sqrt{3}}$

18. (d) Force between two charges does not depend upon the presence or absence of third charge.

Electric Field

19. (a) $E = \dfrac{kQ}{r^2}$ (i)



$F = \dfrac{kQ\,q_0}{r_1^2}$ or $\dfrac{F}{q_0} = \dfrac{kQ}{r_1^2}$ (ii)

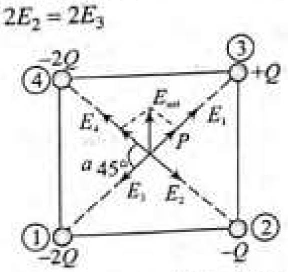
As $r_1 > r$, so from Eqs. (i) and (ii), we get $E > F/q_0$

20. (b) $\vec{r} = (9-3)\hat{i} + (12-4)\hat{j} = 6\hat{i} + 8\hat{j}$

or $r = \sqrt{6^2 + 8^2} = 10$ m

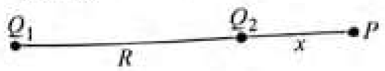
$$E = \frac{9\times10^9 \times 100\times10^{-6}}{10^2} = 9000 \text{ V m}^{-1}$$

21. (b) $E_1 = E_4 = 2E_2 = 2E_3$



Horizontal components will be canceled, net field will be upward.

22. (d) $\dfrac{kQ_2}{x^2} = \dfrac{kQ_1}{(x+R)^2}$ or $x = \dfrac{R}{2}$



23. (c) $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\hat{r}$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(1.2-0)\hat{i} + (-1.6-0)\hat{j}}{\sqrt{(1.2)^2 + (1.6)^2}}$$

$$\hat{r} = \frac{1}{2}(1.2\hat{i} - 1.6\hat{j})$$

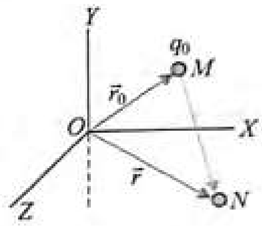
$$\vec{E} = 9\times10^9 \times \frac{(-8\times10^{-9})}{2^2} \times \left[\frac{1}{2}(1.2\hat{i} - 1.6\hat{j})\right]$$

$$\vec{E} = -10.8\hat{i} + 14.4\hat{j} \text{ NC}^{-1}$$

24. (c) As $\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{|\vec{r} - \vec{r}_0|^3}(\vec{r} - \vec{r}_0)$

$$\overrightarrow{MN} = \vec{r} - \vec{r}_0 = (8\hat{i} - 5\hat{j}) - (2\hat{i} + 3\hat{j}) = (6\hat{i} - 8\hat{j})$$

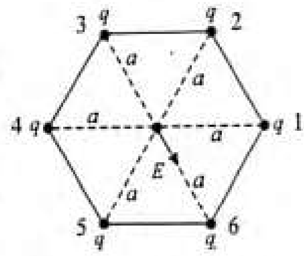
$$|\vec{r} - \vec{r}_0| = \sqrt{6^2 + 8^2} = 10 \text{ m}$$



$$\vec{E} = 9\times10^9 \times \frac{50\times10^{-6}}{(10)^3}(6\hat{i} - 8\hat{j})$$

$$\vec{E} = (2.7\hat{i} - 3.6\hat{j}) \text{ kNC}^{-1}$$

25. (a) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a^2}$



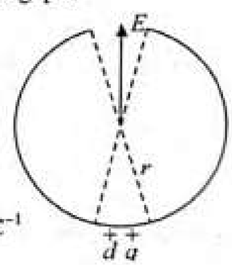
Suppose the charge is present at the sixth vertex also, then electric field at center would be zero. Now, if charge is not present at this vertex, the electric field at center would be because of other five charges, which should be equal and opposite to the field produced due to single charge at the sixth vertex.

26. (b) Charge on the element opposite to the gap is

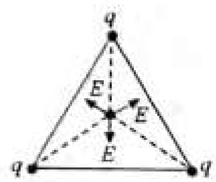
$$dq = \frac{Q}{2\pi r}(0.002\pi)$$

$$= \frac{1}{2\pi(0.5)} \times \frac{2\pi}{1000} = 2\times10^{-3} \text{ C}$$

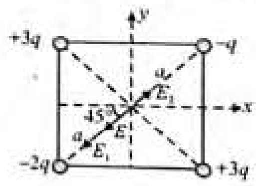
$$E = \frac{9\times10^9 \times 2\times10^{-3}}{(0.5)^2} = 7.2\times10^7 \text{ NC}^{-1}$$



27. (d) Each charge will produce the same magnitude of intensity, say E, at the centroid. These are directed at angles of 120° with each other. So, their vector sum will be zero.



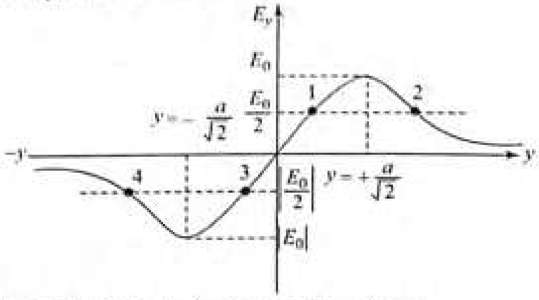
28. (b) At center electric due to both $3q$ will be canceled. Now net field at center is



$$E = E_1 - E_2 = \frac{k2q}{(a)^2} - \frac{kq}{(a)^2} = \frac{kq}{(a)^2}$$

This is 45° below negative x-axis.

29. (d) Four points as shown.



30. (c) Electric field due to dipole at axial position is

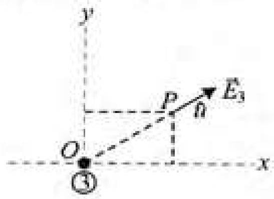
$$E_{\text{axial}}=\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \text{ (along the dipole moment)}$$

Electric field due to dipole at equatorial position

$$E_{\text{eq}}=\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \text{ (opposite to dipole moment)}$$

Hence $\vec{E}_A=-2\vec{E}_B$

31. (b) Let us consider the electric field due to wire (3) only.



$$\vec{E}_3=E\hat{u}$$

$$\vec{E}_3=\frac{\lambda}{2\pi\varepsilon_0(a^2+a^2)^{1/2}}(\hat{i}\cos 45^\circ+\hat{j}\cos 45^\circ)$$

$$=\frac{\lambda}{2\sqrt{2}\pi\varepsilon_0 a}\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

$$\vec{E}_3=\frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i}+\hat{j})$$

Similarly, electric field due to wires (1) and (2)

$$\vec{E}_1=\frac{\lambda}{4\pi\varepsilon_0 a}(\hat{j}+\hat{k}) \text{ and } \vec{E}_2=\frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i}+\hat{k})$$

$$\vec{E}_{\text{net}}=\vec{E}_1+\vec{E}_2+\vec{E}_3 \quad \vec{E}_{net}=\frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i}+\hat{j}+\hat{k})$$

32. (c) From the directions of fields from graph, it is clear that q_1 is negative and q_2 is positive. Now since electric field is zero to the right of q_2, q_2 should be smaller in magnitude.

33. (c) Electric field is zero inside the shell due to the outside changes.

34. (c) q is +ve because lines of force emerge from it and $|Q|<|q|$ because more lines emerge from q and less lines terminate at Q.

35. (c) Electric lines of force never intersect the conductor. They are perpendicular and slightly curved near the surface of conductor.

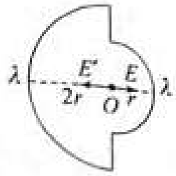
36. (d) Electric field at a point on z-axis distant r from origin is

$$E=\frac{1}{4\pi E_o}\left(\frac{Qr}{(r^2+R^2)^{3/2}}-\frac{\sqrt{8}\,Qr}{(r^2+4R^2)^{3/2}}\right)=0$$

Solving we get $r=\sqrt{2}\,R$

37. (d) The electric field due to both straight wires shall cancel at common centre O. The electric field due to larger and smaller semi-circular rings at O be E and E' respectively.

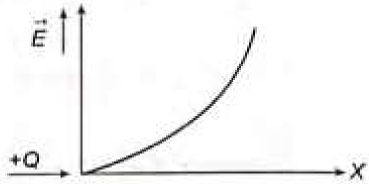
$$E=\frac{1}{4\pi\varepsilon_o}\frac{2\lambda}{2r}\; E'\,\frac{1}{4\pi\varepsilon_o}\frac{2\lambda}{r}$$



$\therefore$ Magnitude of electric field at O is

$$=E-\frac{1}{4\pi\varepsilon_o}\left(\frac{2\lambda}{r}-\frac{\lambda}{r}\right)=\frac{1}{4\pi\varepsilon_o}\frac{\lambda}{r}$$

38. (a) Vertical velocity changes due to electric field, but no change in horizontal velocity.



39. (b) For electron $s=\frac{eE}{m_e}\times t_1^2$, for proton $s=\frac{eE}{m_p}\times t_2^2$

$$\therefore \quad \frac{t_2^2}{t_1^2}=\frac{m_p}{m_e}\Rightarrow\frac{t_2}{t_1}=\sqrt{\frac{m_p}{m_e}}=\left(\frac{m_p}{m_e}\right)^{1/2}$$

40. (c) The time required to fall through distance d is

$$d=\frac{1}{2}\left(\frac{qE}{m}\right)t^2 \text{ or } t=\sqrt{\frac{2dm}{qE}}$$

Since $t^2\propto m$, a proton takes more time.

Electric Flux and Gauss Law

41. (b) We should know the total charge inside. There is no contribution in the flux due to outside charges. Also, we should know both magnitude and direction of electric field at any point on the surface.

42. (b) Net charge on the conductor will be zero. So, net charge inside S_1 will be the charge on the rod. Hence, flux through S_1 is q/ε_0.

43. (a) Charge inside $S_1=q_1+q_2=3\times10^{-6}$ C

Charge inside $S_2=q_2+q_3=-1\times10^{-6}$ C

Charge inside $S_3=0$

Charge inside S_1 is greatest. So, flux through S_1 is maximum.

44. (b) Net flux is due to charges inside S only.

45. (b) $\phi=\frac{q_{\text{in}}}{\varepsilon_0}$ or $0=\frac{q_{\text{in}}}{\varepsilon_0}\Rightarrow q_{\text{in}}=0$

46. (b) $\vec{E}=8\hat{i}+4\hat{j}+3\hat{k}, \vec{A}=100\,\hat{k}$

So $\quad \phi=\vec{E}\cdot\vec{A}=300$ units

47. (c) $\phi=\frac{q}{\varepsilon_0}$ (Independent of dimensions)

New flux, $\phi'=\frac{\phi}{2}$

48. (c) $\vec{E}=2000\,\hat{k}, \vec{A}=10\times20\times10^{-4}\hat{k}$

$\phi=\vec{E}\cdot\vec{A}=40$ Vm

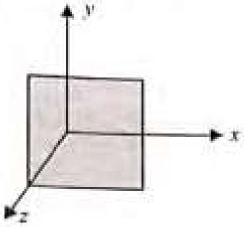
49. (c) Flux through both will be same as the net charge enclosed by both is same.

50. (c) $1-7-4+10+2-5-3+6=0$

Sum of all the charges is zero, so net flux is zero.

51. (c) Imagine a charge q at the center of a cube of edge length $2L$ (figure). Then $\phi = q/\varepsilon_0$.

Here, the square is 1/24 of the surface area of the imaginary cube, so it intercepts 1/24 of the flux. That is,



$$\Phi = \frac{q}{24\varepsilon_0}$$

52. (a) To find the charge enclosed, we need the flux through the parallelepiped:

$\Phi_1 = AE_1 \cos 60°$

$= (0.0500 \text{ m})(0.0600 \text{ m})(2.50 \times 10^4 \text{ NC}^{-1}) \cos 60°$

$= 37.5 \text{ Nm}^2\text{C}^{-1}$

$\Phi_2 = AE_2 \cos 120°$

$= (0.0500 \text{ m})(0.0600 \text{ m})(7.00 \times 10^4 \text{ NC}^{-1}) \cos 120°$

$= -105 \text{ Nm}^2\text{C}^{-1}$

So, the total flux is

$\Phi = \Phi_1 + \Phi_2 = (37.5 - 105) \text{ Nm}^2\text{C}^{-1} = -67.5 \text{ Nm}^2\text{C}^{-1}$

$q = \Phi\varepsilon_0 = (-67.5 \text{ Nm}^2\text{C}^{-1})\,\varepsilon_0 = -5.97 \times 10^{-10}\text{ C}$ or $-67.5\varepsilon_0\text{C}$

There must be a net charge (negative) in the parallelepiped since there is a net flux flowing into the surface. Also, there must be an external field, otherwise all lines would point toward the slab.

53. (a) If portion outside the cylinder is removed, electric field at points on curved surface will decrease.

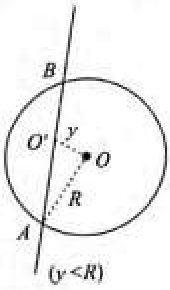
54. (c) Electric flux $\oint_S \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$

q_{in} is the charge enclosed by the Gaussian surface, which, in the present case, is the surface of the given sphere. As shown, length AB of line lies inside the sphere. In $\Delta OO'A$,

$$R^2 = y^2 + (O'A)^2$$

$\therefore \quad O'A = \sqrt{R^2 - y^2}$

and $\quad AB = 2\sqrt{R^2 - y^2}$



Charge on length AB is $2\sqrt{R^2 - y^2} \times \lambda$

Therefore, electric flux is

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{2\lambda\sqrt{R^2 - y^2}}{\varepsilon_0}$$

55. (c) $\alpha = 60°$. Solid angle subtended by BCD is

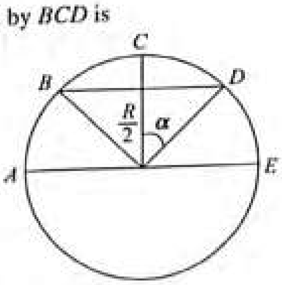
$\omega = 2\pi(1 - \cos\alpha) = \pi$

Solid angle subtended by $ABDE$ is

$\omega_{(ABCDE)} - \omega_{(BCD)}$

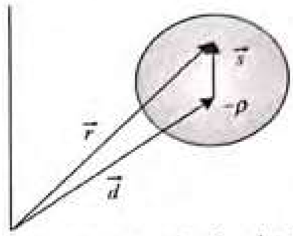
$= 2\pi - \pi = \pi$

Hence, flux through $ABDE$ is



$$\phi = \frac{q}{\varepsilon_0}\frac{\pi}{4\pi} = \frac{q}{4\varepsilon_0}$$

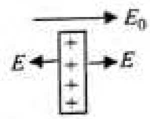
56. (c) $\vec{E}_P = \frac{-\rho}{3\varepsilon_0}\vec{s} = -\frac{\rho}{3\varepsilon_0}(\vec{r} - \vec{d}) = \frac{\rho}{3\varepsilon_0}(\vec{d} - \vec{r})$



57. (b) From the figure, it is clear that the plate is placed in an external electric field. Let the electric field due to plate be E, and E_0 is the external electric field. Therefore,

$\therefore \quad E_0 + E = 12 \text{ Vm}^{-1} \quad$ (i)

$E_0 - E = 8 \text{ Vm}^{-1} \quad$ (ii)



Solving Eqs. (i) and (ii), $E = 2 \text{ Vm}^{-1}$

and $E_0 = 10 \text{ Vm}^{-1}$

Now, electric field due to plate is $\sigma/2\varepsilon_0 = 2$. Therefore, $\sigma = 4\varepsilon_0$

58. (d) In each case: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0}$

$$E\oint dA = \frac{Q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

As charged enclosed within the Gaussian surface is the same, the radius of Gaussian surface is equal. Hence, the magnitude of electric field will be equal.

59. (a) We know that if a point charge subtends a half-angle α on the circular cross-section of a disc, then flux passing through the disc is:

$$\phi = \frac{q}{2\varepsilon_o}(1 - \cos\alpha)$$

Also, if a point charge q lies on the axis of a charged disc of charge density σ and subtends a half-angle α, then it experiences a force

$$F = \frac{q\sigma}{2\varepsilon_o}(1 - \cos\alpha) = \sigma\phi$$

60. (c) Flux going in pyramid $= \frac{Q}{8\varepsilon_0}$

Which is divided equally among all 4 faces

$\therefore \quad$ Flux through one face $= \frac{Q}{8\varepsilon_0}$

Problems Based on Mixed Concepts

61. (a) As charge on both particles is same, so electric force acting on them will be same. Since the particles are allowed to move for the same time, their final momentum will be same. Because

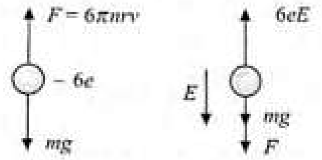
change in momentum = impulse = force × time, so

$$\text{KE} = \frac{p^2}{2m} \text{ gives KE} \propto \frac{1}{m}$$

$$\frac{\text{KE}_1}{\text{KH}_2} = \frac{m_2}{m_1} = 2:1$$

62. (c) For the first case, $F = mg$.
For the second case
$F + mg = 6eE$ or $2mg = 6eE$

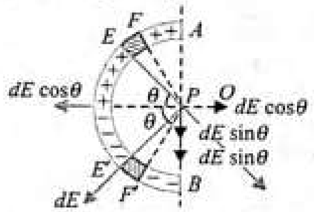
or $$E = \frac{mg}{3e} = \frac{1.6\times10^{-15}\times10}{3\times1.6\times10^{-19}} = 3.3\times10^4\ \text{NC}^{-1}$$



63. (b) Force on the block: $F = qE$ toward left.
Let spring be compressed maximum by x. Then

$$Fx = \frac{1}{2}kx^2 \text{ or } qEx = \frac{1}{2}kx^2 \text{ or } x = \frac{2qE}{k}$$

64. (a) Take PO as the x-axis and PA as the y-axis. Consider two elements EF and $E'F'$ of width $d\theta$ at angular distance θ above and below PO, respectively.



The magnitude of the field at P due to either element is

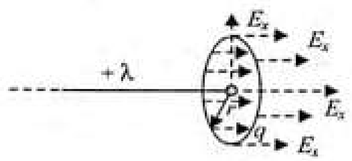
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{rd\theta\times Q/(\pi r/2)}{r^2} = \frac{Q}{2\pi^2\varepsilon_0 r^2}d\theta$$

Resolving the fields, we find that the components along PO sum up to zero, and hence the resultant field is along PB. Therefore, field at P due to pair of elements is $2dE\sin\theta$

$$E = \int_0^{\pi/2} 2dE\sin\theta = 2\int_0^{\pi/2}\frac{Q}{2\pi\varepsilon_0 r^2}\sin\theta d\theta = \frac{Q}{\pi^2\varepsilon_0 r^2}$$

65. (a) Net force $F_{net} = qE_x$

$$F = q\frac{\lambda}{4\pi\varepsilon_0} = \frac{\lambda q}{4\pi\varepsilon_0 r}$$



66. (c) $F = \dfrac{kQrq}{R^3} = -ma$

$$a = -\left(\frac{Qq}{4\pi\varepsilon_0 mR^3}\right)r = -\omega^2 r \quad\text{or}\quad \omega = \sqrt{\frac{Qq}{4\pi\varepsilon_0 mR^3}}$$

67. (d) Both angles will remain same at any time. It is because, force on the balls will be equal and opposite, although they have different charges. $\phi > \theta$, because force has increased. Angles would have been different if masses were different.

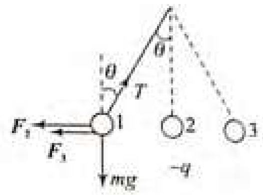
68. (a) Time period is independent of a constant force acting on the block of spring-block system. Its time period will remain same as

$$T = 2\pi\sqrt{\frac{m}{k}}$$

69. (a) $F_e = F_2 + F_3$

$$= \frac{kq^2}{(L\sin\theta)^2} + \frac{kq^2}{(2L\sin\theta)^2}$$

$$F_e = \frac{5}{4}\frac{kq^2}{L^2\sin^2\theta} \qquad \text{(i)}$$



$T\sin\theta = \text{Fe}$ (ii)
$T\cos\theta = mg$ (iii)
From (i), (ii), and (iii)

$$q = \sqrt{\frac{16}{5}\pi\varepsilon_0 mgL^2\sin^2\theta\tan\theta}$$

70. (d) $E = \dfrac{\sigma}{2\varepsilon_0}$

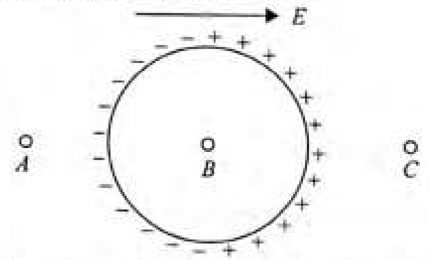
$$F = E\,dr = \frac{\sigma dA\sigma}{2\varepsilon_0}$$

$$P = \frac{F}{dA} = \frac{\sigma^2}{2\varepsilon_0}$$

$$\text{Force} = P\pi R^2 = \frac{\sigma^2}{2\varepsilon_0}\pi R^2 = \frac{Q^2}{16\pi^2R^4}\frac{\pi R^2}{2\varepsilon_0} = \frac{Q^2}{32\pi\varepsilon_0 R^2}$$

$$= \frac{(20\times10^{-6})^2\times9\times10^9}{8(1.5\times10^{-2})^2} = 2000\ \text{N}$$

71. (c) The dielectric gets polarized as shown. Induced charges will also contribute in electric field.



So, intensity at points A and C will increase and at B intensity will decrease.

72. (b) $\rho = Cr^2$

$$q = \int_0^r 4\pi r^2 dr\rho = \int_0^r 4\pi Cr^4 dr = \frac{4}{5}\pi Cr^5$$

$$E_{r=2R} = \frac{kq_{(r=R)}}{(2R)^2} = \frac{k(4/5)\pi CR^5}{4R^2}$$

$$= \frac{kq_{(r=R/2)}}{(R/2)^2} = \frac{k(4/5)\pi C(R/2)^5}{(R/2)^2}$$

Now solve to get $\dfrac{E_{r=2R}}{E_{r=R/2}} = 2$

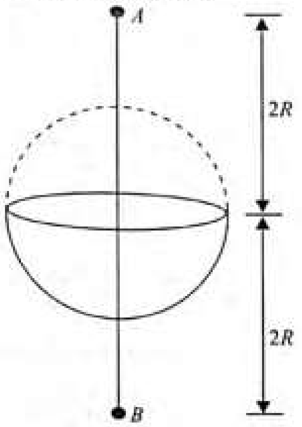
73. (b) Net electric field at point P should be zero. For this electric field due to induced charges is equal to the applied electric field. So

$$\frac{\sigma}{\varepsilon_0}=E \text{ or } \sigma=\varepsilon_0 E$$

74. (c) Electric field is zero inside a metallic cavity if no charge is present in the cavity, whatever be the field outside.

75. (b) Let us complete the sphere. Electric field due to lower part at A is equal to electric field due to upper part at $B=E$ (given)

Electric field due to lower part at B = electric field due to full sphere – electric field due to upper part



$$=\frac{kQ}{(2R)^2}-E$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{\rho(4/3)\pi R^3}{4R^2}-E$$

$$=\frac{\rho R}{12\varepsilon_0}-E$$

76. (a) Electric field on surface of a uniformly charged sphere is given by

$$\frac{Q}{4\pi\varepsilon_0 R^2}=\frac{\rho R}{3\varepsilon_0}$$

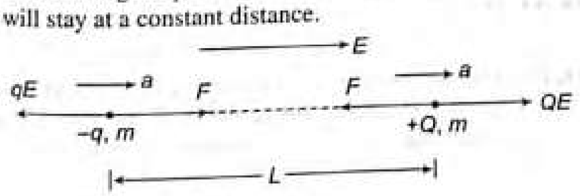
Electric field at outside point is given by

$$E=\frac{Q}{4\pi\varepsilon_0 r^2}=\frac{\rho R^3}{3\varepsilon_0 r^2}$$

$$E_B=E_{\text{whole sphere}}-E_{\text{cavity}}$$

$$=\frac{\rho r_0}{3\varepsilon_0}-\frac{\rho\left(\frac{r_0}{2}\right)^3}{3\varepsilon_0\left(\frac{3r_0}{2}\right)^2}=\frac{17\rho r_0}{54\varepsilon_0}$$

77. (c) On releasing the particles, if their acceleration is same, they will stay at a constant distance.



$$QE-F=Ma,\ F-qE=ma$$

where, $F=\dfrac{kqQ}{L^2}$

From above: $\dfrac{QE-F}{F-qE}=\dfrac{M}{m}$

$$\Rightarrow\quad \frac{KqQ}{L^2}=\frac{(mQ+qM)E}{m+M}\ \Rightarrow\ L=\sqrt{\frac{(M+m)kqQ}{E(qM+mQ)}}$$

78. (b) The net electrostatic force on 10 kg charge should be $F_e=100$ N upwards

$\therefore$ Net electrostatic force on remaining four charges = 100 N downwards

$\therefore$ Net force on remaining four charges = Net electrostatic force + net weight = 100 N + 100 N = 200 N downwards

$\therefore$ Acceleration of centre of mass of the four charges immediately after the release

$$=\frac{\text{Net Force}}{\text{Net mass}}=\frac{200}{10}=20 \text{ m/s}^2 \text{ downwards}$$

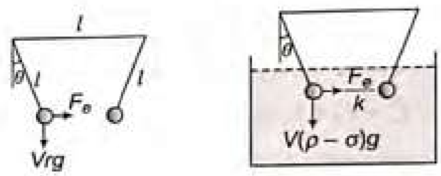
79. (b) The pressure due to surface tension $=\dfrac{4T}{R}$

The pressure due to electrostatic forces $=\dfrac{\sigma^2}{2\varepsilon_0}$

Just before the bubble bursts,

$$\frac{4T}{R}=\frac{\sigma^2}{2\varepsilon_0}\quad\text{or}\quad R=\frac{8T\varepsilon_0}{\sigma^2}$$

80. (c)



$$\tan\theta=\frac{F_e}{V\rho g}=\frac{F_e/k}{V(\rho-\sigma)g}$$

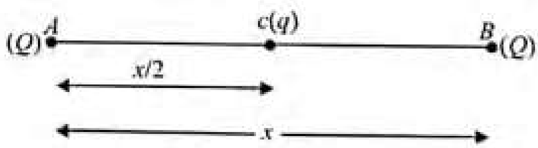
$$\Rightarrow\quad k=\frac{\rho}{\rho-\sigma}=\frac{2.4}{2.4-0.8}=1.5$$

Archives

1. (None) We define flux through any surface by $\phi=\int \vec{E}.d\vec{A}$.

The electric field at each and every point on face $ABCD$ is the resultant of electric field due to both charges. The angle between $\vec{E}$ and $d\vec{A}$ at every point on the surface $ABCD$ is less than 90°, this gives non zero electric flux through the surface $ABCD$. Hence option (b) cannot be the answer. The options (a), (c) and (d) are dimensionally incorrect. Hence, no option is correct.

2. (d)



The system is in equilibrium. So two oppositely directed forces must be equal, i.e.,

$$\frac{1}{4\pi\varepsilon_0}\times\frac{Q\times q}{\left\{\frac{x}{2}\right\}^2}+\frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{x^2}=0$$

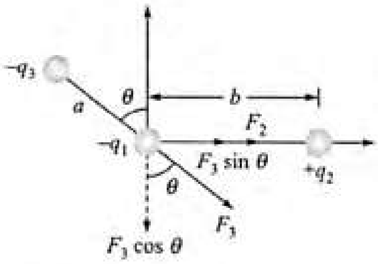
$$\Rightarrow\quad q=-\frac{Q}{4}$$

3. (b) When a negatively charged pendulum oscillates over a positively charged plate then effective value of g increases, so according to $T = 2\pi\sqrt{\frac{l}{g}}$, T decreases.

4. (c) $$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1Q_2}{r};$$

Net potential energy,
$$U_{\text{net}} = 3\times\frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{l}$$

5. (b)



F_2 = Force applied by q_2 on $-q_1$
F_3 = Force applied by $-q_3$ on $-q_1$
x-component of net force on $-q_1$ is:
$$F_x = F_2 + F_3\sin\theta = k\frac{q_1q_2}{b^2} + k\frac{q_1q_3}{a^2}\sin\theta$$
$$\Rightarrow \quad F_x = k\left[\frac{q_1q_2}{b^2} + \frac{q_1q_3}{a^2}\sin\theta\right]$$
$$\Rightarrow \quad F_x = k\cdot q_1\left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$
$$\Rightarrow \quad F_x \propto \left(\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right)$$

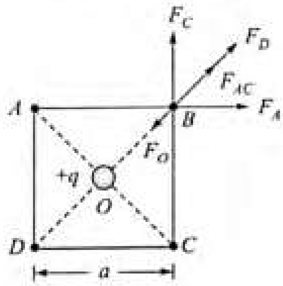
6. (a) From Gauss's law, we have
$$\frac{\text{Charge enclosed}}{\varepsilon_0} = \text{Flux leaving the surface}$$
$$\Rightarrow \quad \frac{q}{\varepsilon_0} = \phi_2 - \phi_1$$
$$\Rightarrow \quad q = (\phi_2 - \phi_1)\varepsilon_0$$

7. (d) We have
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$$

When A (which is of the same size as that of B) is brought in contact with B, the charge on both becomes $q/2$. When A, now containing a charge $q/2$, is brought in contact with C, the charge on each becomes
$$\frac{(q/2)+q}{2} = \frac{3q}{4}$$
The new force of repulsion between B and C is
$$F' = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{2}\right)\left(\frac{3q}{4}\right)}{d^2} = \frac{3F}{8}$$

8. (b) If all charges are in equilibrium, system is also in equilibrium. Charge at centre: charge q is in equilibrium because no net force is acting on its corner charge.

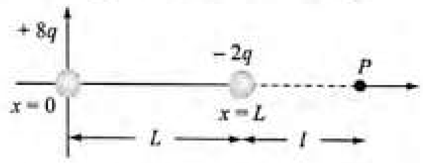
If we consider the charge at corner B. This charge will experience following forces:
$$F_A = k\frac{Q^2}{a^2},$$
$$F_C = \frac{kQ^2}{a^2},$$
$$F_D = \frac{kQ^2}{(a\sqrt{2})^2} \text{ and } F_O = \frac{kQq}{(a\sqrt{2})^2}$$



Force at B away from the centre $= F_{AC} + F_D$
$$= \sqrt{F_A^2 + F_C^2} + F_D = \sqrt{2}\frac{kQ^2}{a^2} + \frac{kQ^2}{2a^2} = \frac{kQ^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$
Force at B towards the centre $= F_O = \frac{2kQq}{a^2}$

For equilibrium of charge at B, $F_{AC} + F_D = F_O$
$$\Rightarrow \quad \frac{kQ^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right) = \frac{2kQq}{a^2} \Rightarrow q = \frac{Q}{4}\left(1+2\sqrt{2}\right)$$

9. (a) The net field will be zero at a point outside the charges and near the charge which is smaller in magnitude.



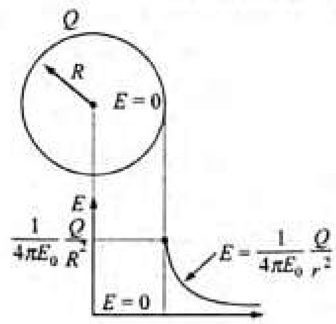
Suppose E.F. is zero at P as shown.

Hence, at P; $k\cdot\frac{8q}{(L+l)^2} = \frac{k\cdot(2q)}{l^2} \Rightarrow l = L.$

So distance of P from origin $= L + L = 2L$.

10. (c) In a non-uniform field, a dipole experiences a force $F = q\vec{E}_1 - q\vec{E}_2$ as well as torque.

11. (c) The electric field for a uniformly charged spherical shell is given in the figure. Inside the shell, the field is zero and it is maximum at the surface and then decreases $\propto 1/r^2$.

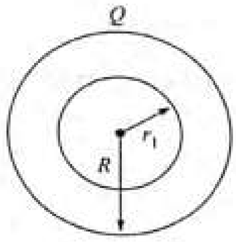


$E=\frac{Q}{4\pi\varepsilon_0\cdot r^2}$ outside shell and zero inside.

12. (c) If the charge density, $\rho=\frac{Q}{\pi R^4}r$,

The electric field at the point P distant r_1 from the centre, according to Gauss's theorem,

$$\phi=\int\vec{E}.d\vec{A}=\frac{q_{\text{enclosed}}}{\varepsilon_0}\Rightarrow E.4\pi r_1^2=\frac{q_{\text{enclosed}}}{\varepsilon_0}$$



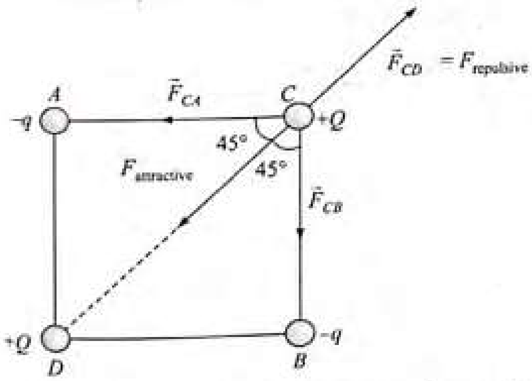
$$E.4\pi r_1^2=\frac{1}{\varepsilon_0}\int\rho dV$$

$$\Rightarrow\quad E.4\pi r_1^2=\frac{1}{\varepsilon_0}\int_0^{r_1}\frac{Qr}{\pi R^4}.4\pi r^2dr$$

$$\Rightarrow\quad E=\frac{Qr_1^2}{4\pi\varepsilon_0R^4}$$

13. (a) For electric force to be zero at charge Q, the sign of charge q should be negative. The force of repulsion by Q is cancelled by the resultant attracting force due to q^- and q^- at A and B. Force on charge at C due to charge at D,

$$F_{\text{repulsive}}=\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{\left(\sqrt{a^2+a^2}\right)^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}$$



Net force on charge placed at C due to charges placed at A and B.

$$F_{\text{attractive}}=2\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}\cos 45^\circ=\frac{1}{4\pi\varepsilon_0}\left\{\frac{Qq\sqrt{2}}{a^2}\right\}$$

For resultant force zero at C,

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{2a^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq\sqrt{2}}{a^2}$$

$$\frac{Q^2}{2a^2}=\frac{Qq\sqrt{2}}{a^2}\Rightarrow\frac{Q}{q}=2\sqrt{2}$$

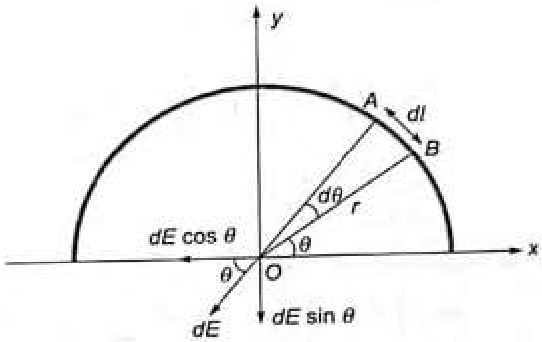
As q is negative charge, hence $\frac{Q}{q}=-2\sqrt{2}$.

14. (c) Linear charge density, $\lambda=\frac{q}{\pi r}$

Consider a small element AB of length dl subtending an angle $d\theta$ at the centre O as shown in the figure.

$\therefore$ Charge on the element,

$$dq=\lambda dl=\lambda rd\theta\qquad\left(\because d\theta=\frac{dl}{r}\right)$$



The electric field at the centre O due to the charge element,

$$dE=\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}=\frac{\lambda rd\theta}{4\pi\varepsilon_0r^2}$$

Resolve dE into two rectangular components.

By symmetry, $\int dE\cos\theta=0$

The net electric field at O,

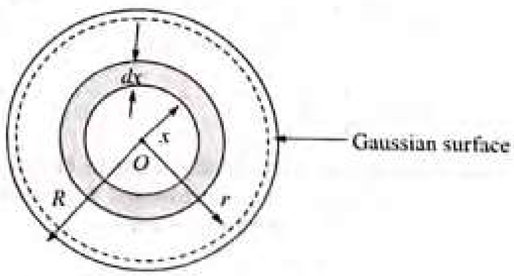
$$\vec{E}=\int_0^{\pi}dE\sin\theta(-\hat{j})=\int_0^{\pi}\frac{\lambda rd\theta}{4\pi\varepsilon_0r^2}\sin\theta(-\hat{j})$$

$$=-\int_0^{\pi}\frac{qr\sin\theta d\theta}{4\pi^2\varepsilon_0r^3}\hat{j}\qquad\left(\because\lambda=\frac{q}{\pi r}\right)$$

$$=-\int_0^{\pi}\frac{q\sin\theta d\theta}{4\pi^2\varepsilon_0r^2}\hat{j}=-\frac{q}{4\pi^2\varepsilon_0r^2}[-\cos\theta]_0^{\pi}\hat{j}$$

$$=-\frac{q}{2\pi^2\varepsilon_0r^2}\hat{j}$$

15. (b) Let us take a thin spherical shell of radius x and thickness dx as shown in the figure.



Volume of the shell, $dV=7\pi x^2dx$

Let us draw a Gaussian surface of radius $r(r<R)$ as shown in the figure above.

Total charge enclosed inside the Gaussian surface,

$$Q_{in} = \int_0^r \rho dV = \int_0^r \rho_0\left(\frac{5}{4} - \frac{x}{R}\right)4\pi x^2 dx$$

$$= 4\pi\rho_0 \int_0^r \left(\frac{5}{4}x^2 - \frac{x^3}{R}\right)dx$$

$$= 4\pi\rho_0\left[\frac{5}{12}x^3 - \frac{x^4}{4R}\right]_0^r$$

$$= 4\pi\rho_0\left[\frac{5}{12}r^3 - \frac{r^4}{4R}\right]$$

$$= \pi\rho_0\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right]$$

According to Gauss's law,

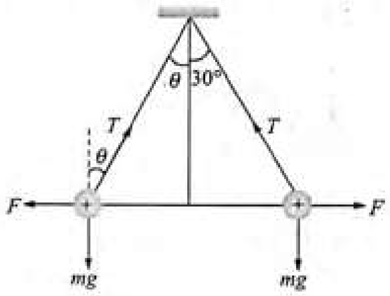
$$\phi = \int \vec{E}.d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \Rightarrow E.4\pi r^2 = \frac{Q_{in}}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{\pi\rho_0}{\varepsilon_0}\left[\frac{5}{3}r^3 - \frac{r^4}{R}\right]$$

$$E = \frac{\pi\rho_0 r^3}{4\pi r^2\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

16. (c) Initially, the forces acting on each ball are shown in figure.



For its equilibrium along vertical,

$$T\cos\theta = mg \quad \text{...(i)}$$

and along horizontal,

$$T\sin\theta = F \quad \text{...(ii)}$$

Dividing equation(ii) by (i), we get

$$\tan\theta = \frac{F}{mg} \quad \text{...(iii)}$$

When the balls are suspended in a liquid of density σ and dielectric constant K, the electrostatic force will become $(1/K)$ times, i.e. $F' = (F/K)$ while weight

$$mg' = mg - \text{Upthrust}$$

$$= mg - V\sigma g \qquad [\text{As Upthrust} = V\sigma g]$$

$$= mg\left[1 - \frac{\sigma}{\rho}\right] \qquad \left[\text{As } V = \frac{m}{\rho}\right]$$

For equilibrium of balls,

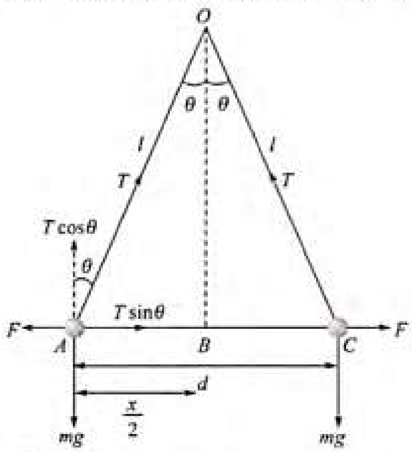
$$\tan\theta' \frac{F'}{mg'} = \frac{F}{Kmg[1-(\sigma/\rho)]} \quad \text{...(iv)}$$

According to given problem, $\theta' = \theta$.
From equations (iv) and (iii), we get

$$K = \frac{1}{\left(1 - \frac{\sigma}{\rho}\right)}$$

$$K = \frac{\rho}{(\rho - \sigma)} = \frac{1.6}{(1.6 - 0.8)} = 2$$

17. (a) The forces acting on each spheres are shown in FBD



$\therefore \quad T\cos\theta = mg$

and $\quad T\sin\theta = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}$

$\therefore \quad \tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2 mg}$

When charge begins to leak from both the spheres at a constant rate, then

$$\tan\theta = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg}$$

or $\quad \frac{x}{2l} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2 mg} \qquad \left(\because \tan\theta = \frac{x}{2l}\right)$

or $\quad \frac{x}{2l} \propto \frac{q^2}{x^2}$

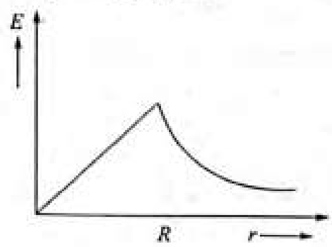
or $\quad q^2 \propto x^3$

$\Rightarrow \quad \theta \propto x^{3/2}$

or $\quad \frac{dq}{dt} \propto \frac{3}{2}x^{1/2}\frac{dx}{dt}$

or $\quad v \propto x^{-1/2} \qquad \left(\because \frac{dq}{dt} = \text{constant}\right)$

18. (c) For uniformly charged sphere



$$E=\frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\quad(\text{For } r<R)$$

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\quad(\text{For } r=R)$$

$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\quad(\text{For } r>R)$$

The variation of E with distance r from the centre is shown in given figure.

19. (c) The electric field inside the charged sphere,

$$E=\frac{1}{4\pi\varepsilon_0}\frac{qr}{R^2}=\frac{\rho r}{3\varepsilon_0}$$

Hence, statement 2 is true.

Potential inside the sphere, $V=\frac{V_{\text{surface}}}{2}\left[3-\frac{r^2}{R^2}\right]$

Potential at the centre of the sphere, $V_C=\frac{3V_{\text{surface}}}{2}$

Potential at the surface of the sphere,

$$V_S=\frac{1}{4\pi\varepsilon_0}\frac{q}{R}=\frac{\rho R^2}{3\varepsilon_0}$$

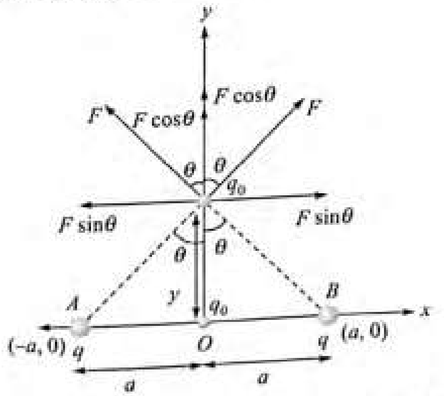
When a charge q is taken from the centre to the surface, the charge in potential energy,

$$\Delta U=q(V_C-V_S)=q\left(\frac{3V_S}{2}-V_S\right)$$

$$\Rightarrow\quad \Delta U=q\frac{V_S}{2}=q\frac{\left(\frac{\rho R^2}{3\varepsilon_0}\right)}{2}=\frac{\rho q R^2}{6\varepsilon_0}$$

Hence, statement 1 is false.

20. (d) When a particle of mass m and charge $q_0=\left(\frac{q}{2}\right)$ placed is at the origin and it is given a small displacement along the y-axis, then the situation is as shown in the figure.



By symmetry, the components of forces on the particle of charge q_0 due to charges at A and B along x-axis will cancel each other where along y-axis will add up.

$\therefore$ The net force acting on the particle,

$$F_{\text{net}}=2F\cos\theta=2\frac{1}{4\pi\epsilon_0}\frac{q_0}{\left(\sqrt{y^2+a^2}\right)}\frac{y}{\sqrt{(y^2+a^2)}}$$

$$=\frac{2}{4\pi\epsilon_0}\frac{q\left(\frac{q}{2}\right)}{(y^2+a^2)}\frac{y}{\sqrt{(y^2+a^2)}}\quad\left(\because q_0=\frac{q}{2}(\text{Given})\right)$$

$$=\frac{1}{4\pi\epsilon_0}\frac{q^2y}{(y^2+a^2)^{3/2}}$$

As $y<<a$

$$\therefore\quad F_{\text{net}}=\frac{1}{4\pi\epsilon_0}\frac{q^2y}{a^3}\text{ or } F_{\text{net}}\propto y$$

21. (a)

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{R^2}$$

$$\Rightarrow\quad \varepsilon_0=\frac{q_1q_2}{4\pi FR^2}$$

$$=\frac{C^2}{N.m^2}=\frac{[AT]^2}{MLT^{-2}\cdot L^2}$$

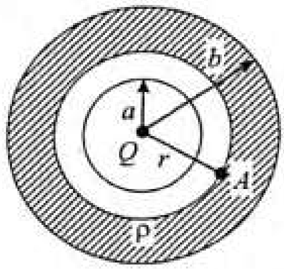
$$=[M^{-1}L^{-3}T^4A^2]$$

22. (a)

$$\oint \bar{E}\cdot d\bar{s}=\frac{q_{in}}{\epsilon_0}$$

$$E\cdot 4\pi r^2=\frac{Q+\int_{r=a}^{r}\rho dV}{\epsilon_0}$$

$$E\cdot 4\pi r^2=\frac{Q+\int_a^r\frac{A}{r}4\pi r^2dr}{\epsilon_0}$$



$$E=\frac{1}{4\pi\epsilon_0 r^2}\left[Q+4\pi A\left(\frac{r^2-a^2}{2}\right)\right]$$

$$E=\frac{1}{4\pi\epsilon_0 r^2}\left[Q+2\pi A(r^2-a^2)\right]$$

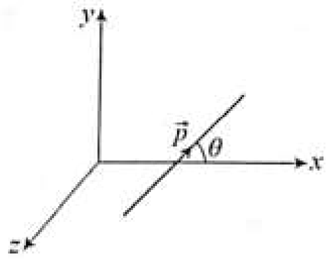
$$E=\frac{Q}{4\pi\epsilon_0 r^2}+\frac{A(r^2-a^2)}{2\epsilon_0 r^2}=\frac{Q}{4\pi\epsilon_0 r^2}+\frac{A}{2\epsilon_0}-\frac{Aa^2}{2\epsilon_0 r^2}$$

If E is constant it should be independent of r

$$\therefore\quad \frac{Q}{4\pi\epsilon_0 r^2}=\frac{Aa^2}{2\epsilon_0 r^2}$$

which gives, $A=\frac{Q}{2\pi a^2}$

23. (a) From $\vec{\tau} = \vec{p} \times \vec{E}$



$$\tau\hat{k} - \tau\hat{k} = \left(p_x\hat{i} + p_y\hat{j}\right) \times \left(E\hat{i} + \sqrt{3}E\hat{j}\right)$$

$$= p_x \times \sqrt{3}E\,\hat{k} + p_y E\left(-\hat{k}\right)$$

$$0 = E\hat{k}\left(\sqrt{3}p_x - p_y\right)$$

$$\frac{p_y}{p_x} = \sqrt{3}$$

$$\therefore \quad \tan\theta = \sqrt{3}$$

$$\theta = 60°$$

CHAPTER 18: ELECTRIC POTENTIAL AND CAPACITANCE

Concept Application Exercises 18.1

1. $(W_{ext})_{\infty P} = -(W_{el})_{\infty P} = (W_{el})_{P\infty} = 10\ \mu J$

Because $\Delta KE = 0$,

$$V_P = \frac{(W_{ext})_{\infty P}}{q} = \frac{10\ \mu J}{10\ \mu C} = 1\ V$$

So if now the charge is doubled and taken from infinity, then

$$1 = \frac{(W_{ext})_{\infty P}}{20\ \mu C} \quad \text{or} \quad (W_{ext})_{\infty P} = 20\ \mu J$$

$$(W_{el})_{\infty P} = -20\ \mu J$$

2. $W_{el} = \Delta K = K_f - 0$

$$(W_{el})_{P\to\infty} = qV_P = 60\ \mu J$$

So $\quad K_f = 60\ \mu J$

3. (a) The potential difference between two points in electric field is

$$|\Delta V_{AB}| = -\vec{E}\cdot\overrightarrow{AB} = Ed\cos\theta = E\cdot d$$
$$= Ed = 20\times 2\times 10^{-2} = 0.4$$

So $\quad V_A - V_B = 0.4\ V$

Negative because in the direction of electric field, potential always decreases.

(b) $|\Delta V_{BC}| = Ed = 20\times 2\times 10^{-2} = 0.4$

d is the distance between the equipotential passing through B and C, so $V_B - V_C = 0.4\ V$

(c) $|\Delta V_{CA}| = Ed = 20\times 4\times 10^{-2} = 0.8$

So $\quad V_C - V_A = -0.8\ V$

Negative because in the direction of electric field, potential always decreases.

(d) $|\Delta V_{DC}| = Ed = 20\times 0 = 0$

So $\quad V_D - V_C = 0$

Because the effective distance between D and C is zero. Also potential difference between C and D is zero as both the points lie on same equipotential.

(e) $|\Delta V_{AD}| = Ed = 20\times 4\times 10^{-2} = 0.8$

Here d is the distance between equipotential passing through A and D.

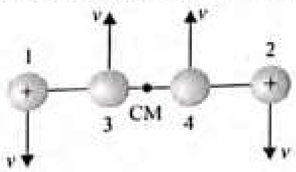
So $\quad V_A - V_D = 0.8\ V$

Because in the direction of electric field, potential always decreases.

(f) The order of potential is

$$V_A > V_B > V_C = V_D$$

4. Take the illustration presented with the problem as an initial picture. No external horizontal forces act on the set of four balls, so its center of mass stays fixed at the location of the center of the square. As the charged balls 1 and 2 swing out and away from each other, balls 3 and 4 move up with equal y-components of velocity. The maximum-kinetic energy point is illustrated. System energy is conserved, so

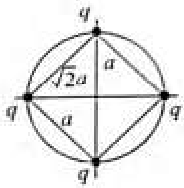


$$\frac{k_e q^2}{a} = \frac{k_e q^2}{3a} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

or $\quad \dfrac{2k_e q^2}{3a} = 2mv^2 \quad \text{or} \quad v = \sqrt{\dfrac{k_e q^2}{3am}}$

5. U_i is the electrical potential energy in square arrangement. So

$$U_i = \frac{4q^2}{4\pi\varepsilon_0 a} + \frac{2q^2}{4\pi\varepsilon_0\sqrt{2}a}$$



U_f is the electric potential energy of circular arrangement of charge.

$$U_f = \frac{4q^2}{4\pi\varepsilon_0\sqrt{2}a} + \frac{2q^2}{4\pi\varepsilon_0(2a)}$$

$$w_{ext} = \Delta U = U_f - U_i = -\frac{q^2}{4\pi\varepsilon_0 a}(3-\sqrt{2})$$

6. $\vec{E} = -\left[\dfrac{\partial V}{\partial x}\hat{i} + \dfrac{\partial V}{\partial y}\hat{j} + \dfrac{\partial V}{\partial z}\hat{k}\right]$

Here, $\dfrac{\partial V}{\partial x} = \dfrac{\partial}{\partial x}(2x+3y-z) = 2$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x+3y-z) = 3$$

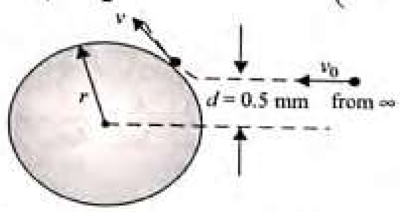
$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x+3y-z) = -1$$

$\therefore \quad \vec{E} = -2\hat{i} - 3\hat{j} + \hat{k}$

Concept Application Exercises 18.2

1. Applying conservation principle of angular momentum, we get $m v_0 d = mv r$

or $\quad v = \dfrac{v_0 d}{r} = \dfrac{v_0}{2} \qquad \left(\because d = \dfrac{r}{2} = 0.5\ \text{mm}\right)$



Applying mechanical energy conservation principle, we get

$$U_i + K_i = U_f + K_f$$

or $\quad 0 + \dfrac{1}{2}mv_0^2 = \dfrac{q^2}{4\pi\varepsilon_0 r} + \dfrac{1}{2}mv^2$

or $\quad \dfrac{1}{2}mv_0^2 = \dfrac{q^2}{4\pi\varepsilon_0 r} + \dfrac{1}{2}mv^2$

or $\quad \dfrac{1}{2}mv_0^2 = \dfrac{q^2}{4\pi\varepsilon_0 r} + \dfrac{1}{2}m\left(\dfrac{v_0}{2}\right)^2$

On solving

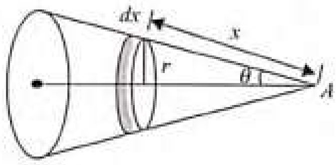
or $\quad v_0=\sqrt{\frac{8}{3}}=2\sqrt{\frac{2}{3}}\ \text{ms}^{-1}$

2. $W_{ext}=\Delta U=U_f-U_i=q(V_A-V_\infty)$

$W_{ext}=q(V_A-0)=qV_A \qquad$ (i)

Here V_A is electric potential at point A. The electric charge on the considered ring is

$$dq=\frac{Q2\pi r dx}{\pi RL}$$



Therefore, electric potential due to considered ring is

$$dV=\frac{dq}{4\pi\varepsilon_0 x}$$

$$dV=\frac{Q2\pi r dx}{\pi RL4\pi\varepsilon_0 x}$$

$\because \quad \sin\theta=\frac{r}{x}=\frac{R}{L} \qquad \therefore \quad r=\frac{Rx}{L}$

$$\therefore \quad dV=\frac{2\pi Q\left(\frac{R}{L}x\right)dx}{\pi RL4\pi\varepsilon_0 x}$$

$$V_A=\int dV=\frac{Q}{2\pi\varepsilon_0 L^2}\int_0^L dx=\frac{Q}{2\pi\varepsilon_0}L$$

$$\therefore \quad W_{ext}=q\,V_A=\frac{qQ}{2\pi\varepsilon_0 L} \qquad \text{from Eq. (i)}$$

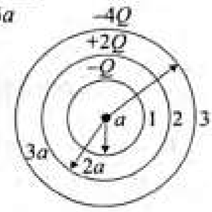
3. (i) $r<a$

$$V=V_{1,in}+V_{2,in}+V_{3,in}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{a}+\frac{1}{4\pi\varepsilon_0}\frac{(2Q)}{2a}+\frac{1}{4\pi\varepsilon_0}\frac{(-4Q)}{3a}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\left[-1+1-\frac{4}{3}\right]$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\left[-\frac{4}{3}\right]=-\frac{Q}{3\pi\varepsilon_0 a}$$



(ii) $a,\ r<2a$

$$V=V_{1,out}+V_{2,out}+V_{3,out}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{r}+\frac{1}{4\pi\varepsilon_0}\frac{(2Q)}{2a}+\frac{1}{4\pi\varepsilon_0}\frac{(-4Q)}{3a}$$

$$=\frac{1}{4\pi\varepsilon_0}Q\left[-\frac{1}{r}+\frac{1}{a}-\frac{4}{3a}\right]=\frac{1}{4\pi\varepsilon_0}Q\left[-\frac{1}{r}-\frac{1}{3a}\right]$$

$$=-\frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}+\frac{1}{3a}\right]$$

(iii) $2a<r,\ 3a$

$$V=V_{1,out}+V_{2,out}+V_{3,in}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{r}+\frac{1}{4\pi\varepsilon_0}\frac{(2Q)}{r}+\frac{1}{4\pi\varepsilon_0}\frac{(-4Q)}{3a}$$

$$=\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}+\frac{2}{r}-\frac{4}{3a}\right]=\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{4}{3a}\right]$$

(iv) $r>3a$

$$V=V_{1,out}+V_{2,out}+V_{3,out}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{r}+\frac{1}{4\pi\varepsilon_0}\frac{(2Q)}{r}+\frac{1}{4\pi\varepsilon_0}\frac{(-4Q)}{r}$$

$$=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}[-1+2-4]=-\frac{3}{4\pi\varepsilon_0}\frac{Q}{r}$$

4. (i) If we connect the conductors 1 and 2, the potential of spheres 1 and 2 will be equal.

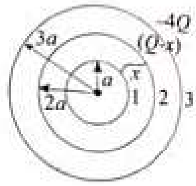
Here $V_1=V_2$

or $\quad K\frac{x}{a}+K\frac{Q-x}{2a}+K\frac{(-Q)}{3a}=K\frac{x}{a}+K\frac{Q-x}{2a}+K\frac{(-4Q)}{3a}$

or $\quad x=0$

The charge of sphere 1 will come to sphere 2.

Hence charge flown in this case is equal to $-Q$.



(ii) If we connect the spheres 2 and 3, their potential will be same.

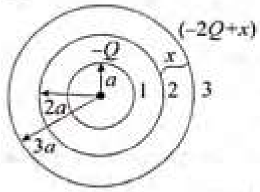
$V_2=V_3$

or $\quad K\frac{(-Q)}{2a}+K\frac{x}{2a}+K\frac{(-2Q-x)}{3a}$

$\quad =K\frac{(-Q)}{3a}+K\frac{x}{3a}+K\frac{(2Q-x)}{3a}$

or $\quad x=Q$

Hence, charge of amount Q will flow from 2 to 3.



(iii) If we connect 1 and 3, their potential should be equal.

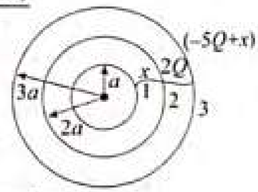
$V_1=V_3$

or $\quad K\frac{x}{a}+K\frac{2Q}{2a}+K\frac{(-5Q-x)}{3a}$

$\quad =K\frac{x}{3a}+K\frac{2Q}{3a}+K\frac{(-5Q-x)}{3a}$

or $\quad \frac{x}{a}-\frac{x}{3a}=\frac{2Q}{3a}-\frac{2Q}{2a}$

or $\quad \frac{2x}{a}=-\frac{Q}{3}\quad$ or $\quad x=\frac{-Q}{2}$



Hence, $-Q/2$ charge will flow from 1 to 3.

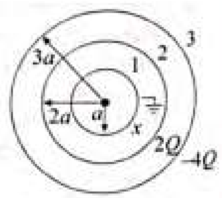
(iv) Now sphere 1 is connected with ground. The potential of sphere 1 will be zero.

$$V_1=0=K\frac{x}{a}+K\frac{2Q}{2a}+K\frac{(-4Q)}{3a}$$

or $\quad x=-Q+\frac{4Q}{3}=\frac{Q}{3}$

Hence, $Q+\frac{Q}{3}=\frac{4Q}{3}$

should flow from ground to sphere.

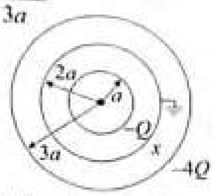


(v) Now sphere 2 is connected with earth. The potential of sphere 2 should be zero.

$$V_2 = 0 = K\frac{(-Q)}{2a} + K\frac{x}{2a} + K\frac{(-4Q)}{3a}$$

or $\quad \frac{x}{2} = \frac{Q}{2} + \frac{4Q}{3}$

or $\quad x = Q + \frac{8Q}{3} = \frac{11Q}{3}$



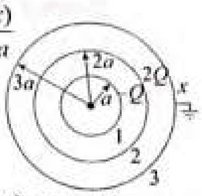
Hence, $\frac{11Q}{3} - 2Q = \frac{5Q}{3}$ charge will flow from earth to sphere.

(vi) Now sphere 3 is connected with ground.

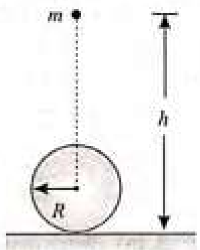
Here, $V_3 = 0 = K\frac{(-Q)}{3a} + K\frac{(2Q)}{3a} + K\frac{(x)}{3a}$

or $\quad x = Q - 2Q = -Q$

Hence, $-3Q$ charge will flow from sphere to earth or $3Q$ charge will flow from earth to sphere.



5. Each charged drop that falls into the conducting vessel increases the charge of the vessel by Q. The accumulated charge on the conducting vessel is uniformly distributed on the surface of the vessel. The charged spherical vessel crates its own electric field that can be calculated by assuming the entire charge of the sphere to be concentrated at its center. Let n drops have accumulated in the vessel and $(n + 1)$th be in the state of equilibrium at a height h. The equilibrium exists under the influence of repulsive Coulomb force (directed upward) and force of gravity (downward). Thus, we have



$$\frac{(nQ)Q}{4\pi\varepsilon_0(h-R)^2} = mg$$

Hence $\quad n = \frac{4\pi\varepsilon_0 mg(h-R)^2}{Q^2}$

Concept Application Exercises 18.3

1. Here, $q = \pm 8 \times 10^{-9}$ C; $2a = 4$ cm $= 0.04$ m; $\tau = 4\sqrt{3}$ Nm, and $\theta = 60°$.
   (a) Now, $p = q(2a) = 8 \times 10^{-9} \times 0.04 = 3.2 \times 10^{-10}$ Cm
   Torque on the electric dipole is $\tau = pE\sin\theta$

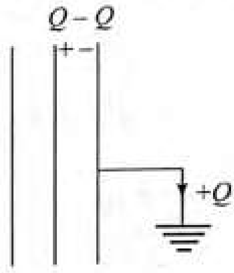
   or $\quad E = \frac{\tau}{p\sin\theta} = \frac{4\sqrt{3}}{3.2\times10^{-10}\times\sin 60°}$

   $$= \frac{4\sqrt{3}\times 2}{3.2\times10^{-10}\times\sqrt{3}} = 2.5\times10^{10}\ \text{NC}^{-1}$$

   (b) Potential energy of the electric dipole is
   $U = -pE\cos\theta$
   $= -3.2 \times 10^{-10} \times 2.5 \times 10^{10} \times \cos 60°$
   $= -3.2 \times 2.5 \times 0.5 = -4$ J
2. For 2 and 4, $\tau = pE\sin\theta$, $U = -pE\cos\theta$
   For 1 and 3, $\tau = pE\sin(180 - \theta) = pE\sin\theta$
   $U = -pE\cos(180 - \theta) = pE\cos\theta$
   Hence, torque is same for all, and U is greater for 1 and 3 than 2 and 4.
3. (a) Electric field will do positive work, if dipole rotates such that its angle decreases with E.
   (b) 2 and 4 orientations are identical. Hence, work done will be same.
4. (a) $U = -pE\cos\theta = pE\cos(180 - \theta)$
   The more is U, the more is θ. So $\angle 4 > \angle 3 > \angle 1 > \angle 2$.
   (b) If angle between p and E is more closer to 90°, then torque is more. If angle is closer to 0° or 180°, then torque is less. It means the lesser is the magnitude of potential energy, the more is torque and vice versa. So $3 > 1 = 4 > 2$.
5. Potential energy of dipole will be zero because electric field due to point charges at dipole will cancel out. Also electric field due to any point charge is perpendicular to the dipole moment.

Concept Application Exercises 18.4

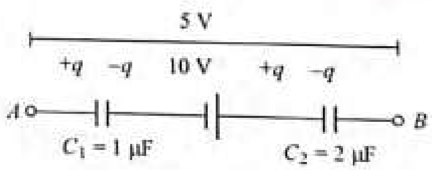
1. (a) The charge from one plate gets transferred to another plate through the battery. The battery pumps the charge from one plate to another.
   (b) Yes, size of plates does not matter.
2. On connecting with earth, the charge on central plate will shift to its right face. $-Q$ charge will be induced on the left face of the earthed plate. Thus, $+Q$ charge will flow to the earth.



3. As $q = CV$, C decreases in both the cases.
   i. In the first case, V remains the same; hence, q decreases, due to which force of attraction will decrease.
   ii. But in the second case, q will remain the same; hence, force of attraction remains the same. So more work will be done in the second case.
4. No, because electric field is conservative, so net work done in a closed path should be zero.
5. At point 1, distance between the plates is less, so attraction between the charges here can be more due to which concentration of charge at point 1 will be more.
6. (a) True
   (b) True
   (c) True

Concept Application Exercises 18.5

1.



$$V_A - \frac{q}{1} + 10 - \frac{q}{2} = V_B \quad \text{or } V_A - V_B = q + \frac{q}{2} - 10$$

or $\quad 5 = \frac{3}{2}q - 10 \quad$ or $q = 10\ \mu$C

2. Capacitors 4 and 5 are short-circuited. So, they are useless. '1',

'2', and '3' will be in parallel and then 6 will be in series with them, $C_{eq} = 3C/4$.

3. Let q_1, q_2, q_3 be charges on capacitors C_1, C_2, C_3, respectively. Then we have

$$V_1 - V_0 = \frac{q_1}{C_1},\ V_0 - V_2 = \frac{q_2}{C_2},\ V_0 - V_3 = \frac{q_3}{C_3}$$

By junction rule, we have $q_1 = q_2 + q_3$

$$(V_1 - V_0)C_1 = (V_0 - V_2)C_2 + (V_0 - V_3)C_3$$

or $$V_0 = \frac{V_1C_1 + V_2C_2 + V_3C_3}{C_1 + C_2 + C_3}$$

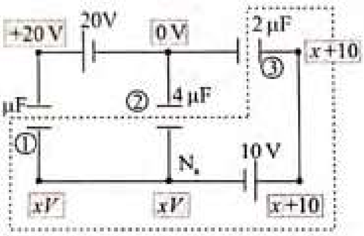
4. $V = 10\text{ V}$

$C_{eq} = C_1 + C_2$ [C_1 and C_2 are in parallel]

$= 6 + 5 = 11\ \mu\text{F}$;

$Q = CV = 11 \times 10 = 110\ \mu\text{C}$

5. Figure shows an isolated system.



For isolated system, net charge is zero.

$2(x - 20) + 4x + (x + 10)2 = 0$

or $8x = 20$ or $x = 2.5\text{ V}$

Thus, charge on capacitor (1) is 35 μC

Thus, charge on capacitor (2) is 10 μC

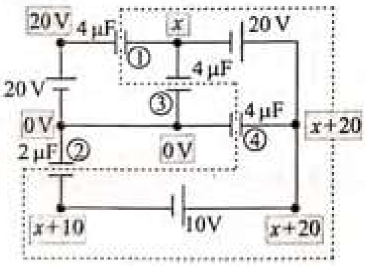
Thus, charge on capacitor (3) is 25 μC

6. The figure shows an isolated system that includes all the capacitor plates isolated from other parts of circuit. We can conclude net sum of charge should be zero.

$2(x + 10) + 4(x + 20) = 6(0 - x) + 4(x - 20) = 0$

or $4x + 20 = 0$ or $x = -5V$

Hence $q_1 = 4[20 - (5)] = 100\ \mu\text{C}$

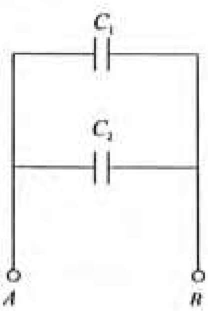


$q_2 = 2[((-5) + 10) - 0)] = 10\ \mu\text{C}$

$q_3 = 6[0 - (-5)] = 30\ \mu\text{C}$

$q_4 = 4[(-5 + 20) - 0] = 60\ \mu\text{C}$

7. The given system can be made equivalent to two capacitors C_1 and C_2 in parallel.



For upper part

$$C_1 = \frac{\varepsilon_0\varepsilon_r\left(\frac{A}{2}\right)}{d} = \frac{\varepsilon_0 A}{2d}$$

For lower part

$$C_2 = \frac{\varepsilon_0\left(\frac{A}{2}\right)}{\frac{d/2}{\varepsilon_{r_1}} + \frac{d/2}{\varepsilon_{r_2}}} = \frac{\varepsilon_0 A/2}{\frac{d}{2}\left(\frac{1}{2} + \frac{1}{3}\right)} = \frac{6\varepsilon_0 A}{5d}$$

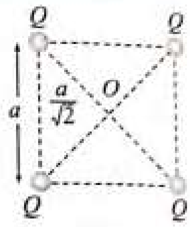
Then $C_{AB} = C_1 + C_2$ (because C_1 and C_2 are in parallel)

$$= \frac{\varepsilon_0 A}{2d} + \frac{6\varepsilon_0 A}{5d} = \frac{17\varepsilon_0 A}{10d}$$

Exercises

Electric Potential and Potential Energy

1. (c) Potential at centre O of the square



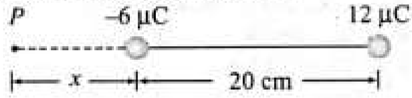
$$V_O = 4\left(\frac{Q}{4\pi\varepsilon_0(a/\sqrt{2})}\right)$$

Work done in shifting $(-Q)$ charge from centre to infinity

$$W = -Q(V_\infty - V_O) = QV_0$$

$$= \frac{4\sqrt{2}Q^2}{4\pi\varepsilon_0 a} = \frac{\sqrt{2}Q^2}{\pi\varepsilon_0 a}$$

2. (c) Point P will lie near the charge which is smaller in magnitude, i.e., $-6\ \mu\text{C}$. Hence potential at P,



$$V = \frac{1}{4\pi\varepsilon_0}\frac{(-6 \times 10^{-6})}{x} + \frac{1}{4\pi\varepsilon_0}\frac{(12 \times 10^{-6})}{(0.2 + x)} = 0 \Rightarrow x = 0.2\text{ m}$$

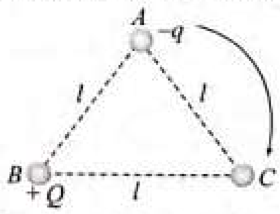
3. (a) Work done $W = 3 \times 10^{-6}(V_A - V_B)$, where

$$V_A = 10^{10}\left[\frac{(-5 \times 10^{-6})}{15 \times 10^{-2}} + \frac{2 \times 10^{-6}}{5 \times 10^{-2}}\right] = \frac{1}{15} \times 10^6\text{ volt}$$

and $$V_B = 10^{10}\left[\frac{(2 \times 10^{-6})}{15 \times 10^{-2}} - \frac{5 \times 10^{-6}}{5 \times 10^{-2}}\right] = -\frac{13}{15} \times 10^6\text{ volt}$$

$$\therefore\quad W = 3 \times 10^{-6}\left[\frac{1}{15} \times 10^6 - \left(-\frac{13}{15} \times 10^6\right)\right] = 2.8\text{ J}$$

4. (d) According to figure, potential at A and C is equal. Hence work done in moving $-q$ charge from A to C is zero.



5. (d) Length of the diagonal of a cube having each side b is $\sqrt{3}b$.

So distance of the centre of cube from each vertex is $\frac{\sqrt{3}b}{2}$.

Hence, potential energy of the given system of charge is

$$U = 8 \times \left\{\frac{1}{4\pi\varepsilon_0} \cdot \frac{(-q)(q)}{\sqrt{3}b/2}\right\} = \frac{-4q^2}{\sqrt{3}\pi\varepsilon_0 b}$$

6. (c) $U=\dfrac{kq^2}{a}\left(-1+2-\dfrac{2}{\sqrt{2}}\right)$

$=\dfrac{kq^2}{a}(1-\sqrt{2}) \Rightarrow U=-\dfrac{kq^2}{a}(\sqrt{2}-1)$

7. (d) $U_i=\dfrac{3KQ^2}{a}$ where a = side of equilateral triangle

$U_f=\dfrac{3kQ^2}{a}+\dfrac{3kQ.q}{a/\sqrt{3}}$ where q is the charge brought at the center

$U_f=0 \Rightarrow q=\dfrac{-Q}{\sqrt{3}}$

8. (a) Work done for moving a charge q_0 from B to A.

$$W_{B\to A}=q_0(V_A-V_B)$$

Also, V_A = potential at point A

$$=\frac{kq_1}{r_1}+\frac{kq_2}{r_2}=k\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}\right)$$

$$=10^{10}\left\{\frac{-5\times10^{-6}}{15\times10^{-2}}+\frac{2\times10^{-6}}{5\times10^{-2}}\right\}=\frac{10^6}{15}\text{ V}$$

Similarly,

$$V_B=10^{10}\left\{\frac{2\times10^{-6}}{15\times10^{-2}}+\frac{5\times10^{-6}}{5\times10^{-2}}\right\}=\frac{-13}{15}\times10^6\text{ V}$$

$$\Rightarrow \quad W_{B\to A}=3\times10^{-6}\left\{\frac{1}{15}\times10^6-\left(\frac{-13}{15}\times10^6\right)\right\}$$

$$=3\times10^{-6}\times\frac{10^6}{15}[1-(-13)]\;=\frac{1}{5}\times14=2.8\text{ J}$$

9. (c) Work done by external force = ΔU [It is state function]

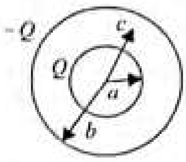
10. (b) $W_{eq}=q(V_A-V_B)$

or $\quad 50\times10^{-6}=2\times10^{-6}(V_A-V_B)$

or $\quad V_A-V_B=25$ V

11. (c) As potential at A and B is the same, $V_A=V_B=kQ/d$. So work done in both the cases will be the same. i.e., $W_A=W_B$

12. (d) $V=\dfrac{kQ}{c}-\dfrac{kQ}{b}=k\left(\dfrac{Q}{c}-\dfrac{Q}{b}\right)$



13. (a) $V_C=V_1=V_2$

$$\frac{kq_1}{r_1}=\frac{kq_2}{r_2}$$

$$\frac{q_1}{q_2}=\frac{r_1}{r_2} \qquad \text{(i)}$$

$$q_1+q_2=Q \qquad \text{(ii)}$$

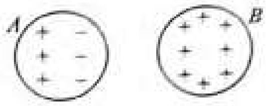
From Eqs. (i) and (ii), we get

$$q_1=\frac{Qr_1}{r_1+r_2}$$

Putting V_1, we get

$$V_C=\frac{kQ}{r_1+r_2}$$

14. (b) Charge will be induce on A, but total charge on A will remain zero. Negative charge of A will be more closer to B than positive charge on A. So potential of B will decrease.



15. (c) Apply conservation of mechanical energy between points a and b

$$(\text{KE}+\text{PE})_a=(\text{KE}+\text{PE})_b$$

$$0+\frac{k(3\times10^{-9})q_0}{0.01}-\frac{k(3\times10^{-9})q_0}{0.02}$$

$$=\frac{1}{2}mv^2+\frac{k(3\times10^{-9})q_0}{0.02}-\frac{k(3\times10^{-9})q_0}{0.01}$$

Put the values and get $v=12\sqrt{15}=46\text{ ms}^{-1}$

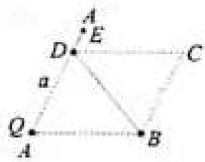
16. (b) $U=-\dfrac{kqQ}{r}-\dfrac{kqQ}{r}+\dfrac{kq^2}{2r}=0$ or $Q/q=1/4$

17. (a) Charge can be considered as located at a distance R from the center. Total charge is $(Q-2Q+3Q)=2Q$

Hence $V=\dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{R}=\dfrac{Q}{2\pi\varepsilon_0 R}$

18. (b) When only $+Q$ charge is placed at point A, electric field at D is

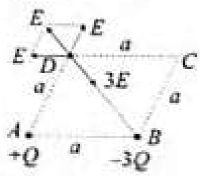
$$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{a^2} \qquad \text{(i)}$$



Electric potential at D is

$$V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{a} \qquad \text{(ii)}$$

When charges $-3Q$ and $+Q$ are placed at B and C, respectively, from figure it is clear that electric field at D will be $2E$ toward B.



Electric potential at D will be

$$V_D=\frac{1}{4\pi\varepsilon_0}\frac{Q}{a}-\frac{1}{4\pi\varepsilon_0}\frac{3Q}{a}+\frac{1}{4\pi\varepsilon_0}\frac{Q}{a}=\frac{-1}{4\pi\varepsilon_0}\frac{Q}{a}=-V$$

19. (d) $F=qE$

$W=q_E\times2\cos60^\circ$

or $\quad 4=0.2E\times2\times\dfrac{1}{2}$ or $E=20\text{ NC}^{-1}$

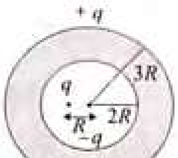
20. (b) $V_1=\dfrac{k\times2\times10^{-6}}{0.1}+\dfrac{k\times4\times10^{-6}}{\sqrt{(0.1)^2+(0.5)^2}}$

$V_2=\dfrac{k\times4\times10^{-6}}{0.1}+\dfrac{k\times2\times10^{-6}}{\sqrt{(0.1)^2+(0.5)^2}}$

Work done is $q(V_2 - V_1) = 0.72$ J

21. (c) $V = k\frac{q}{R} - \frac{kq}{2R} + \frac{kq}{3R} = \frac{kq}{R}\left[1 - \frac{1}{2} + \frac{1}{3}\right]$

$= \frac{k}{6}\frac{5q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{5q}{6R}$



22. (b) $W_{AB} = W_{AC} + W_{CB}$

W_{CB} should be zero, because in moving from C to B, we always move perpendicular to field. Hence, force applied by field and displacement will be at 90°.

$$W_{AC} = -e(V_C - V_A)$$

$$\therefore \quad V_C - V_A = -E \times AC = -10 \times 4 = -40$$

$$W_{AC} = -e \times (-40) = 40e$$

So $\quad W_{AB} = 40e$ J $= 40$ eV

Relation Between Electric Field and Potential

23. (a) $dV = -E\,dr\cos\theta$

Along the line of force, θ is 0°, hence dV is maximum. So the variation of potential is maximum along the line of force.

24. (d) V is a scalar quantity, and E is a vector quantity.

25. (d) Equipotential surfaces are perpendicular to the electric lines of forces. So, only option (d) is correct.

26. (d) We know, $E = -\frac{dV}{dx}$.

At B, $dV/dx = 0$, hence $E_x = 0$.

At A, dV/dx is positive, hence E_x is negative.

At C, dV/dx is negative, hence E_x is positive.

27. (d) Potential decreases in the direction of electric field. So it depends on whether the lines of forces are from A to B or from B to A.

28. (b) $W_{el.} = q(V_i - V_f)$

or $\quad 6.4 \times 10^{-19} = -1.6 \times 10^{-19}(V_A - V_B)$

or $\quad V_A - V_B = -4$V

or $\quad V_A - V_C = -4$V $\quad (\because V_B = V_C)$

or $\quad V_C - V_A = 4$V

29. (a) $E = -\frac{dV}{dr} =$ negative of the slope of V–r graph.

$= -\left(\frac{5-0}{4-6}\right)$

$= +2.5$ Vm^{-1}

30. (a) Potential decreases in the direction of electric field. So potential is minimum at A.

31. (b) $E_x = -\frac{-2V}{-2 \times 10^{-2}\text{ m}} = -100$ Vm^{-1}

$E_y = -\frac{-2V}{1 \times 10^{-2}\text{ m}} = 200$ Vm^{-1}

Time-saving solution

Clearly, x-component is negative and y-component is positive.

32. (d) $E_x = -\frac{dV}{dx} = -4x = -4 \times 2 = -8$

$E_y = 0, E_z = 0$

Hence, $\vec{E} = -8\hat{i}$ NC^{-1}

33. (c) $E_x = -\frac{dV}{dx} = -\frac{1}{2}[-4] = 2$

$E_y = -\frac{dV}{dy} = -\frac{1}{2}[2y] = -y = -1$

$\therefore \quad \vec{E} = E_x\hat{i} + E_y\hat{j}$

$\vec{E} = (2\hat{i} - \hat{j})$Vm^{-1}

34. (a) $V_B - V_A = -\int_1^2 2dx + \int_2^1 3dy$

$= -[2(2-1) + 3(1-2)]$

$= -[2-3] = 1$V

Hence, $V_A - V_B = -1$V

35. (b) $\vec{E} = a(y\hat{i} + x\hat{j})$

$V_2 - V_1 = -\int(ay\,dx + ax\,dy)$, take $V_1 = C$ and $V_2 = V$

Take $V_1 = C$ and $V_2 = V$. Then

$V = -a\int(ydx + xdy) + C = -a\int d(yx) + C = -axy + C$

36. (b) $\vec{E} = -\nabla V = +\frac{1}{x^2}\hat{i} + \frac{1}{y^2}\hat{j} + \frac{1}{z^2}\hat{k} = \hat{i} + \hat{j} + \hat{k}$

37. (d) $E_{PQ} = -\frac{dV_{PQ}}{X_{PQ}} = -\frac{(V_Q - V_P)}{X_{PQ}} = \frac{-(0-5)}{1.0} = 5$ N/C

and $\quad E_{QR} = -\frac{(-10-0)}{0.5} = 20$ N/C

38. (a) $dV = -\vec{E} \cdot d\vec{r}$

$= -(y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j})$

$= -(ydx + xdy) = -d(xy)$

Integrating, we get $V = -xy +$ constant.

Capacitance and Combination of Capacitors

39. (b) Extra charge will flow through battery, so work is done by battery. External agent will do negative work.

40. (d) $V = Ed$. As d increases, V also increases. Note that E remains the same.

41. (d) Check each option separately.

42. (d) $W = U_2 - U_1 = \frac{q^2}{2}\left[\frac{1}{C_2} - \frac{1}{C_1}\right]$

$C_1 = \frac{\varepsilon_0 A}{d}, C_2 = \frac{C_1}{2} = \frac{\varepsilon_0 A}{2d}$

$q = C_1 V = \frac{\varepsilon_0 AV}{d}$

Solve to get $\quad W = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}$

Alternatively:

$W = Fd = \frac{Q^2}{2A\varepsilon_0}d = \frac{C_1^2V^2}{2\varepsilon_0 A}d = \frac{1}{2}\frac{\varepsilon_0 AV^2}{d}$

43. (a) $V_C = \frac{C_1V_1}{C_1 + C_2} = 100$ V

Energy lost $= U_i - U_f$

$$= \frac{1}{2}C_1V_1^2 - \frac{1}{2}(C_1 + C_2)V_c^2 = 6 \times 10^{-6}\ \text{J}$$

44. (d) $V_c = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{CV + 2CV}{KC + 2C} = \dfrac{3V}{K+2}$

45. (c) Initial charge on C_1 is $Q_1 = C_1V = 110\ \mu\text{C}$

Let x charge flow through wires.

$$\frac{Q_1 - x}{C_1} = \frac{x}{C_{eq}} \text{ where } C_{eq} = \frac{C_2 \times C_3}{C_2 + C_3}.$$

Solve to get $x = 60\ \mu\text{C}$.

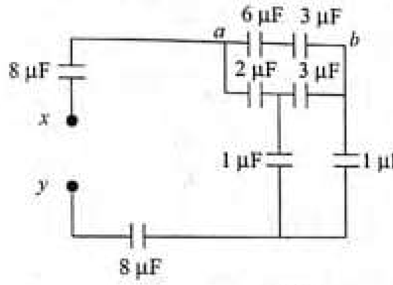
46. (b) In series, all the potentials will be added.

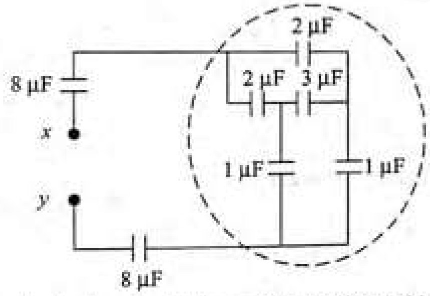
Potential of this combination $= 10 \times V = 10V$

47. (a) $V_1 = \dfrac{C \times 100}{3C + C} = 25\ \text{V},\ V_2 = 100 - 25 = 75\ \text{V}$

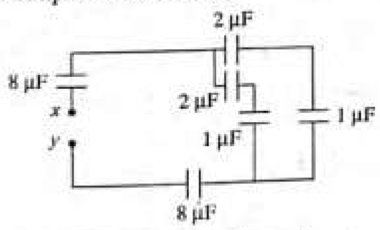
48. (d) Start with C_3 and C_4 in parallel, then C_2 in series, then C_5 in parallel, then C_1 in series, and finally C_6 in parallel.

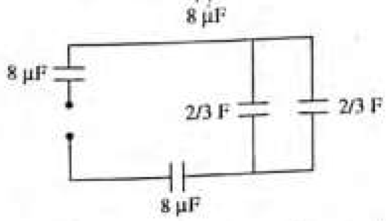
49. (b)

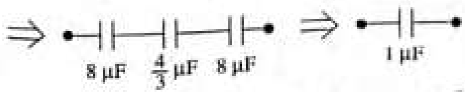




Dotted circuit makes balanced Wheatstone bridge. The circuit can be simplified as follows:







50. (d) Figure (a) (see Exercises) can be simplified as follows:

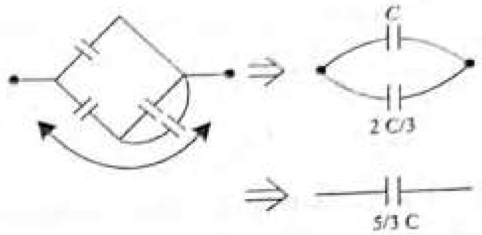


Figure (b) (see Exercises) can be simplified as follows:

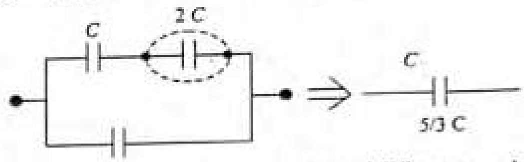
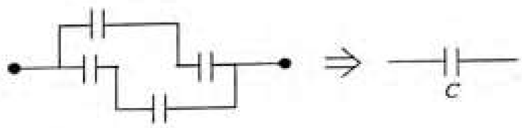


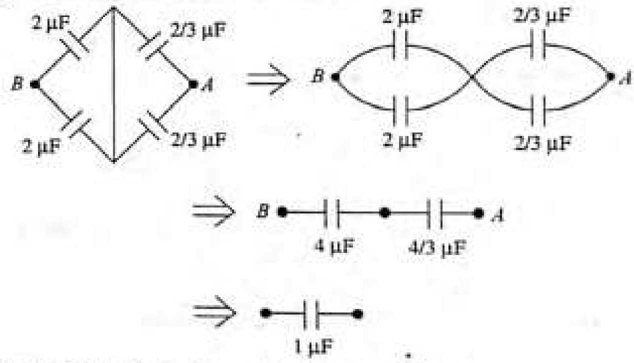
Figure (c) (see Exercises) is balanced Wheatstone bridge, which can be simplified as follows



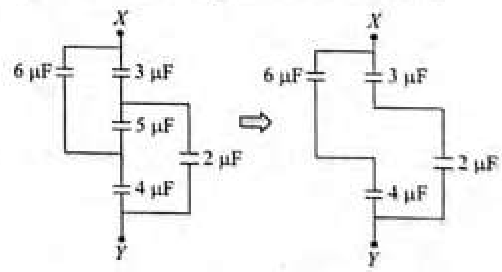
Hence ratio is

$$\frac{5}{3}C : \frac{5}{3}C : C = 5:5:3$$

51. (c) The redrawn circuit is as follows:



52. (c) Equivalent circuit may be drawn as follows:



As the structure is balanced Wheatstone bridge, so 5 μF capacitor will draw no charge from battery and may be removed. Now the circuit is a simple circuit.

$$C_{eq} = \frac{4 \times 6}{(6+4)} + \frac{3 \times 2}{(3+2)} = \frac{18}{5}\ \mu\text{F}$$

53. (c) $V_A - \dfrac{q}{C_1} - E - \dfrac{q}{C_2} = V_B$ or $V_A - V_B - E = q\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)$

or $q = -\dfrac{40}{3}\ \mu\text{C}$

54. (b) Capacitors B and C are in parallel, then A is in series.

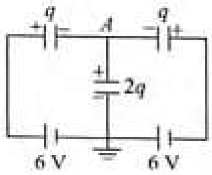
$$C_{eq}=\frac{2\times(3+4)}{2+(3+4)}=\frac{14}{9}\,\mu F$$

$$Q=C_{eq}V=\frac{14}{9}(7-6)=\frac{14}{9}\,\mu C$$

Q will be divided between B and C. So charge on B is

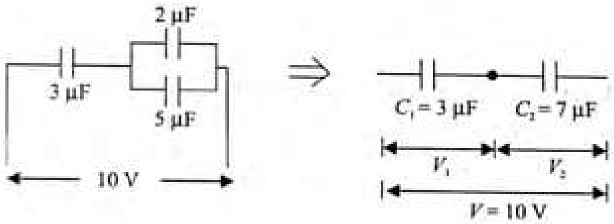
$$q=\frac{3\times(14/9)}{3+4}=\frac{2}{3}\,\mu C$$

55. (a) $6=\frac{q}{2}+\frac{2q}{4}$ or $q=6\,\mu C$



56. (a) $V_A-0=\frac{2q}{4}=3\text{ V}$

57. (c) Potential difference across upper branch is 6 V.

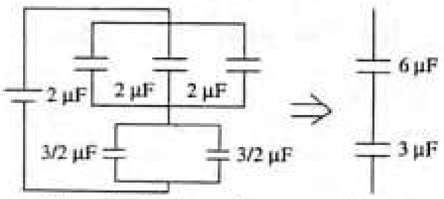


$$V_2=V\left(\frac{C_1}{C_1+C_2}\right)=10\left[\frac{3}{3+7}\right]=3\text{ V}$$

Hence, potential difference across 5 μF capacitor is 3 V.
Thus, charge in 5 μF capacitor is

$$Q_{5\mu F}=5\times 3=15\,\mu C$$

58. (a) The circuit can be simplified as follows:



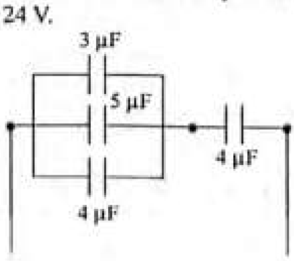
Hence $C_{eq}=2\,\mu F$. Thus, charge supplied by battery is $Q=2\times 20=40\,\mu C$.
Charge on required capacitor is 20 μC

59. (a) Potential difference across 5F capacitor is 120/5 = 24V. Hence, potential difference across all three capacitors connected in parallel is $V_1=24$ V.

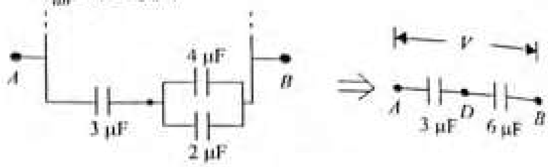
$$V_1=V\left(\frac{C_2}{C_1+C_2}\right)$$

or $$V=V_1\left(\frac{C_1+C_2}{C_2}\right)$$

$$=24\left[\frac{12+4}{4}\right]=96\text{ V}$$



$\therefore\quad V_{ab}=V=96$ V



$$V_{AD}=V\left(\frac{6}{30+6}\right)=96\left[\frac{6}{30+6}\right]=16\text{ V}$$

Problems Based on Mixed Concepts

60. (a) Their potential will be same, i.e., $V_1=V_2$

or $\frac{kq_1}{R_1}=\frac{kq_2}{R_2}$ or $\frac{q_1}{q_2}=\frac{R_1}{R_2}$

or $\frac{E_1}{E_2}=\frac{kq_1/R_1^2}{kq_2/R_2^2}=\frac{q_1}{q_2}\left(\frac{R_2}{R_1}\right)^2=\frac{R_1}{R_2}\left(\frac{R_2}{R_1}\right)^2=\frac{R_2}{R_1}$

61. (d) $Q=nq,\ R=n^{1/3}r$

$$E_{small}=\frac{kq}{r^2},\ E_{big}=\frac{kQ}{R^2}$$

$$E_{big}=\frac{k\,n\,q}{n^{2/3}r^2}=(n^{1/3})\frac{kq}{r^2}=n^{1/3}\,E_{small}$$

$$V_{small}=\frac{kq}{r},\ V_{big}=\frac{kQ}{R}$$

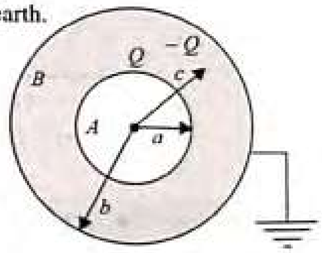
$$V_{big}=\frac{k\,n\,q}{n^{1/3}r}=n^{2/3}\frac{kq}{r}=n^{2/3}\,V_{small}$$

62. (b) Charge $+Q$ will flow into earth.

$$V_C=\frac{kQ}{r}-\frac{kQ}{b}$$

$$V_A=\frac{kQ}{a}-\frac{kQ}{b}$$

Potential at B is zero.



63. (a) $E=\frac{V}{d},\ F=eE=eV/d$

$$a=\frac{F}{m}=\frac{eV}{md}$$

$d=\frac{1}{2}at^2$ or $t=\sqrt{\frac{2d}{a}}$ or $t=\sqrt{\frac{2d\,md}{eV}}=\sqrt{\frac{2\,md^2}{eV}}$

64. (c)

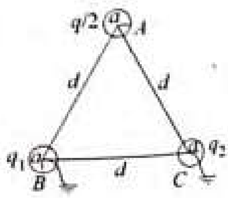
$$U=2kq^2\left[-\frac{1}{a}+\frac{1}{2a}-\frac{1}{3a}+\frac{1}{4a}+\cdots\right]$$

$$=-\frac{2q^2}{4\pi\varepsilon_0 a}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots\right]=-\frac{2q^2\log_e 2}{4\pi\varepsilon_0 a}$$

65. (c) $\frac{k\,q_1}{a}+\frac{k\,q}{2d}=0$ or $q_1=\frac{-q\,a}{2d}$

$$\frac{kq_2}{a}+\frac{kq}{2d}+\frac{kq_1}{d}=0$$

or $q_2=\frac{-qa}{2d}\left(\frac{d-a}{d}\right)$



66. (b) On connecting, the entire amount of charge will shift to the outer sphere. Heat generated is

$$U_i - U_f = \frac{q^2}{8\pi\varepsilon_0 R_1} - \frac{q^2}{8\pi\varepsilon_0 R_2}$$

$$= \frac{(20\times10^{-6})\times9\times10^9}{2}\left[\frac{1}{0.10}-\frac{1}{0.20}\right] = 9\text{ J}$$

67. (d) Let us proceed process-wise. After each process the charge of spheres A, B and C is given below.

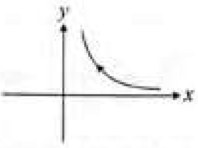
(i) $Q/2, Q/2, 0$
(ii) $Q/2, Q/4, Q/4$
(iii) $3Q/8, Q/4, 3Q/8$
(iv) $5Q/16, 5Q/16, 3Q/8$
(v) $5Q/16, 11Q/32, 11Q/32$

So, option (d) is correct answer.

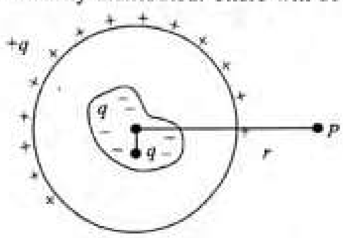
68. (a) $V = x^2 - y^2$

$$E_x = -\frac{dV}{dx} = -2x$$

$E_y = 2y$



69. (a) A negatively charge $-q$ will be induced on the inner surface of cavity and positive charge $+q$ will be induced on the outer surface, which will be uniformly distributed. There will be no field or potential at point P due to inner charges $+q$ and $-q$. Potential at P will only be due to outer $+q$ charge. So

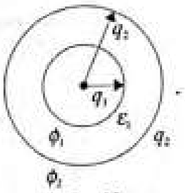


$$V_P = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

70. (a) $\phi_1 = \frac{kq_1}{a_1} + \frac{kq_2}{a_2}$ or $\phi_2 = \frac{kq_1}{a_2} + \frac{kq_2}{a_2}$

Solve to get

$$q_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{\phi_1-\phi_2}{a_2-a_1}\right]a_1a_2$$

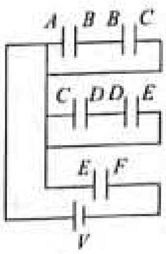


71. (c) At junction A, Q_1 will be divided into Q_2 and Q_3. Hence, $Q_1 = Q_2 + Q_3$. C_2 and C_3 are in parallel, so potential on them will be the same, i.e., $V_2 = V_3$.
V will be divided into V_1 and V_2 (or V_3).
Hence, $V = V_1 + V_2$ or $V = V_1 + V_3$

72. (c) After closing S_1, charge on C_1 is $q = 6\times20 = 120\ \mu$C. Now, S_1 is opened. On closing S_2, charge q will be distributed between C_1 and C_2 according to their capacitances. So charge on C_2 is

$$q_2 = \frac{C_2 q}{C_1 + C_2} = \frac{3\times120}{3+6} = 40\ \mu\text{C}$$

73. (c) $V_A = V_C$ and $V_C = V_E$
Hence the capacitors connected across points A and C also between C and E can be removed. Now only one capacitor is connected across battery, i.e., EF.

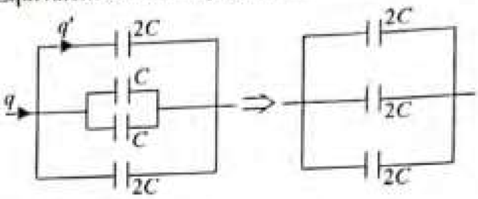


Net energy stored is $U = \frac{1}{2}CV^2$

where, $C = \frac{\varepsilon_0 A}{d}$ (For EF)

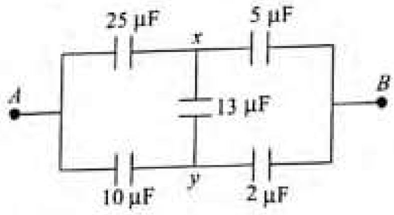
$\therefore \quad U = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$

74. (b) Equivalent circuit may be drawn as follows:



Here $C_{eq} = 6C = 6\ \mu$F.
Charge supplied by battery is $q = 6\times6 = 36\ \mu$C. Hence, charge on each branch will be $12\ \mu$C, and so charge on required capacitor is $6\ \mu$C.

75. (a) Circuit can be redrawn as follows:

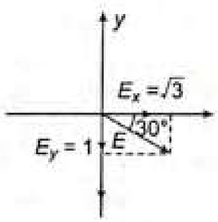


Now, $V_x = V_y$.

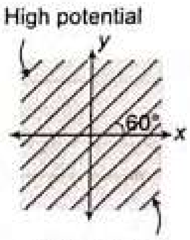
Hence $\frac{25\ \mu\text{F}}{5\ \mu\text{F}} = \frac{7\ \mu\text{F}}{2\ \mu\text{F}}$

$$C_{eq} = \left(\frac{5\times25}{5+25}\right)+\left(\frac{10\times2}{10+2}\right) = \frac{35}{6} = \frac{35}{6}\ \mu\text{F}$$

76. (c) The direction of uniform electric field E in xy-plane is as shown in Figure 1.



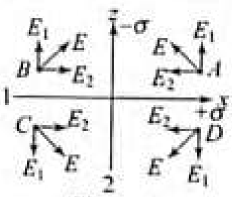
(Figure 1)



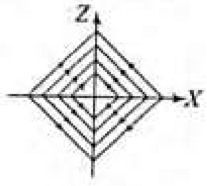
(Figure 2)

The equipotential lines will be perpendicular to electric field. Also electric field points from high potential region towards low potential region. Therefore nature of equipotential lines in x-y plane is given by Figure 2.

77. (c) The electric field intensity due to each uniformly charged infinite plane is uniform. The electric field intensity at points A, B, C and D due to plane 1 and plane 2 and both planes are given by E_1, E_2 and E as shown in Figure 1. Hence the electric lines of forces are as given in Figure 2



(Figure 1)



(Figure 2)

Aliter: Electric lines of forces originate from positively charged plane and terminate at negatively charged plane. Hence, the correct representation of electric lines of forces is as shown Figure 2.

78. (d) $C_1 = \dfrac{K_1\varepsilon_0 \frac{A}{2}}{\left(\frac{d}{2}\right)} = \dfrac{K_1\varepsilon_0 A}{d}$

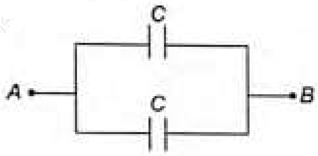
$$C_2 = \frac{K_2\varepsilon_0\left(\frac{A}{2}\right)}{\left(\frac{d}{2}\right)} = \frac{K_2\varepsilon_0 A}{d} \text{ and } C_3 = \frac{K_3\varepsilon_0 A}{2d}$$

Now, $C_{eq} = C_3 + \dfrac{C_1C_2}{C_1+C_2} = \left(\dfrac{K_3}{2} + \dfrac{K_1K_2}{K_1+K_2}\right)\cdot\dfrac{\varepsilon_0 A}{d}$

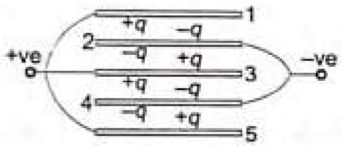
79. (a) The given circuit is equivalent to a parallel combination of two identical capacitors

Hence equivalent capacitance between A and B is

$$C = \frac{\varepsilon_0 A}{d} + \frac{\varepsilon_0 A}{d} = \frac{2\varepsilon_0 A}{d}$$



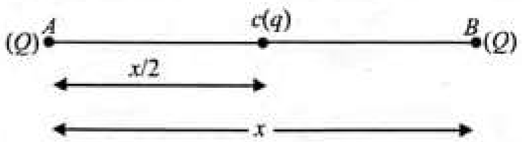
80. (b)



Therefore $q_2 = -2q$, $q_3 = +2q$, $q_4 = -2q$ and $q_5 = +q$

ARCHIVES

1. (d)



The system is in equilibrium. So two oppositely directed forces must be equal, i.e.,

$$\frac{1}{4\pi\varepsilon_0}\times\frac{Q\times q}{\left\{\frac{x}{2}\right\}^2} + \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{x^2} = 0$$

$$\Rightarrow \quad q = -\frac{Q}{4}.$$

2. (b) The energy stored by any system of capacitor is

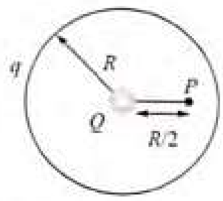
$$E = \sum \frac{1}{2}CV^2 = \frac{1}{2}nCV^2$$

3. (a) The capacity of a spherical conductor is

$$C = 4\pi\varepsilon_0 R = 1.1 \times 10^{-10}\text{ F}$$

4. (c) Electric potential at P

$$V = \frac{k.Q}{R/2} + \frac{k.q}{R} = \frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$$



5. (d) Capacitance $C = 100\ \mu\text{F} = 100 \times 10^{-6}\text{ F} = 10^{-4}\text{ F}$

Charge $Q = 8 \times 10^{-18}$ C

Hence, the required work done is

$$W = \frac{1}{2}\frac{Q^2}{C^2}$$

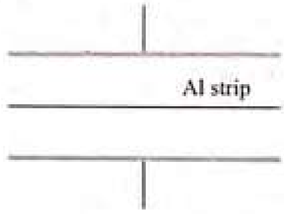
$$= \frac{1}{2}\times\frac{(8\times10^{-8})^2}{(10^{-4})}$$

$$= 32 \times 10^{-32}\text{ J}$$

6. (b) The capacitance of the capacitor with air as dielectric is given by

$$C = \frac{\varepsilon_0 A}{d},$$

where A is the area of each plate and d is the distance between them.



The capacitance of the capacitor with a conducting shell of thickness t is given by

$$C = \frac{\varepsilon_0 A}{d-t}$$

Now, if $t \to 0$, then $C = \varepsilon_0 A/d$

Therefore, when a sheet of aluminium foil of negligible thickness is introduced between the plates of the capacitor, its capacitance remains unchanged.

7. (a) In equilibrium,

$$qE = mg$$

$$\Rightarrow \quad q = 3.3 \times 10^{-18}\text{ C}$$

8. (d) Charge q will momentarily come to rest at a distance r from charge Q when all it's kinetic energy is converted to potential energy.

Energy is conserved in the phenomenon.

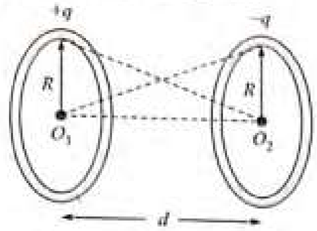
Initially, $\dfrac{1}{2}mv^2 = \dfrac{kqQ}{r}$ ...(i)

Finally, $\dfrac{1}{2}m(2v)^2 = \dfrac{kqQ}{r_1}$ ...(ii)

From equations (i) and (ii), we get

$$\frac{1}{4} = \frac{r_1}{r} \Rightarrow r_1 = \frac{r}{4}$$

9. (b) Potential at the centre of rings are:



$$V_{O_1}=\frac{k.q}{R}+\frac{k(-q)}{\sqrt{R^2+d^2}},\ V_{O_2}=\frac{k(-q)}{R}+\frac{kq}{\sqrt{R^2+d^2}}$$

$$\Rightarrow\quad V_{O_1}-V_{O_2}=2kq\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$$

$$=\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\sqrt{R^2+d^2}}\right]$$

10. (a) For n plates joined alternately, there will be $(n-1)$ capacitors in parallel.

$$C_{eq}=(n-1)C$$

11. (b)

$$\frac{1}{2}CV^2=m.s.\Delta T$$

$$\Rightarrow\quad V=\sqrt{\frac{2ms\Delta T}{C}}$$

12. (c) Here $V_1=V_2$. So

$$\frac{1}{4\pi\varepsilon_0}\times\frac{q_1}{R_2}=\frac{1}{4\pi\varepsilon_0}\times\frac{q_2}{R_1}$$

$$\Rightarrow\quad \frac{q_1}{q_2}=\frac{R_1}{R_2}$$

Also,

$$\frac{E_1}{E_2}=\frac{q_1}{q_2}\times\left(\frac{R_2}{R_1}\right)^2=\frac{R_1}{R_2}\times\frac{R_2^2}{R_2^1}=\frac{R_2}{R_1}=2:1$$

13. (a) An electron on plate 1 has electrostatic potential energy. When it moves, potential energy is converted into kinetic energy.

$\therefore$ Kinetic energy = Electrostatic potential energy

or $\quad \frac{1}{2}mv^2=e\Delta V$

or $\quad v=\sqrt{\frac{2e\times\Delta V}{m}}=\sqrt{\frac{2\times1.6\times10^{-19}\times20}{9.11\times10^{-31}}}$

or $\quad v=2.65\times10^6$ m/s

14. (c) Given: potential $V(x)=\frac{20}{x^2-4}$

Electric field, $E=\frac{-dV}{dx}=\frac{-d}{dx}\left(\frac{20}{x^2-4}\right)=\frac{40x}{(x^2-4)^2}$

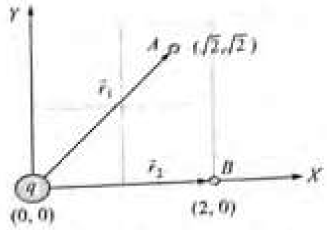
At $\quad x=4$ mm,

$$E=\frac{40\times4}{[16-4]^2}=\frac{160}{144}=\frac{10}{9}\ \text{V/}\mu\text{m}$$

Positive sign indicate E is in +ve x direction.

15. (a) The position vector of A, $\vec{r}_1=\sqrt{2}\hat{i}+\sqrt{2}\hat{j}$

$$|\vec{r}_1|=r_1=\sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2}=2$$



The position vector of B, $\vec{r}_2=2\hat{i}+0\hat{j}$

or $\quad |\vec{r}_2|=r_2=2$

Potential at point A is:

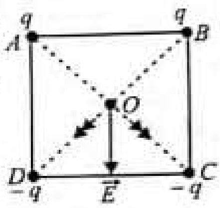
$$V_A=\frac{1}{4\pi\varepsilon_0}\frac{q}{r_1}=\frac{1}{4\pi\varepsilon_0}\frac{10^{-3}\times10^{-6}}{2}$$

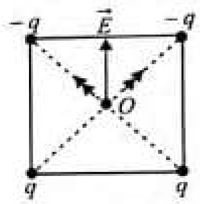
Potential at point B is:

$$V_B=\frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}=\frac{1}{4\pi\varepsilon_0}\frac{10^{-3}\times10^{-6}}{2}$$

$\therefore\quad V_A-V_B=0$

16. (c) V remains unchanged. Because with the change in position of charges, distances AO, BO, CO, and DO remain same. The direction of $\vec{E}$ changes.





17. (c) The potential energy of a charged capacitor,

$$U_i=\frac{q^2}{2C}$$

where U_i is the initial potential energy.

If a dielectric slab is slowly introduced, the energy is $\frac{q^2}{2KC}$.

Once it is taken out, again the energy increases to the old value.

Therefor after it is taken out, the potential energy comes back to the old value. Total work done is zero.

18. (c) Let E be emf of the battery.

Work done by the battery, $W=CE^2$

Energy stored in the capacitor,

$$U=\frac{1}{2}CE^2$$

$$\therefore\quad \frac{U}{W}=\frac{\frac{1}{2}CE^2}{CE^2}=\frac{1}{2}.$$

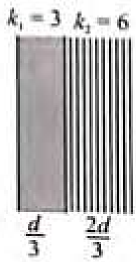
19. (d)

$$C=\frac{\varepsilon_0A}{d}=9\times10^{-12}\text{F}$$

With dielectric,

$$C=\frac{\varepsilon_0kA}{d}$$

$$C_1=\frac{\varepsilon_0A\times3}{d/3}=9C$$



$$C_2 = \frac{\varepsilon_0 A \times 6}{2d/3} = 9C$$

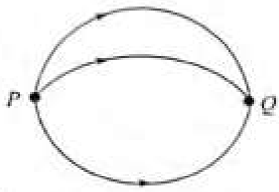
$$C_{total} = \frac{C_1 C_2}{C_1 + C_2}, \text{ as they are in series}$$

$$= \frac{9C \times 9C}{18C} = \frac{9}{2}C$$

$$= \frac{9}{2} \times 9 \times 10^{-12}\,\text{F} = 40.5\text{ pF}$$

20. (b) Work done, $W = (V_Q - V_P) \times q$

The potential difference between the points P and Q depends on the initial and final positions and not on the path. As electrostatic force is a conservative force.



If the loop is completed, $V_Q - V_P = 0$. No net work is done as the initial and final potentials are the same. Both the statements are true but statement 2 is not the reason for statement 1.

21. (d) $W = QdV = Q(V_q - V_P) = -100 \times (1.6 \times 10^{-19}) \times (-4 - 10)$
$= +100 \times 1.6 \times 10^{-19} \times 14$
$= +2.24 \times 10^{-16}$ J

22. (d) The electrostatic potential, $\phi = ar^2 + b$

Electric field, $E = \frac{-d\phi}{dr} = -2ar$

According to Gauss's theorem,

$$\phi = \oint \vec{E}.d\vec{S} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow \quad \oint E.ds\cos 0 = \frac{q_{enclosed}}{\varepsilon_0} \Rightarrow E\oint ds = \frac{q_{enclosed}}{\varepsilon_0}$$

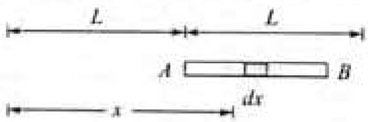
$$\text{or} \quad -2ar(4\pi r^2) = \frac{q_{enclosed}}{\varepsilon_0} \Rightarrow q_{enclosed} = -8\varepsilon_0 a\pi r^3$$

Charge density inside the ball,

$$\rho_{inside} = -\frac{q_{enclosed}}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \quad \rho_{inside} = -\frac{-8\varepsilon_0 a\pi r^3}{\frac{4}{3}\pi r^3} = -6a\varepsilon_0$$

23. (c) Let us take a small element dx at a distance x from O.



Potential at O due to the element, $dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x}$

Charge on the element, $dQ = \frac{Q}{L}dx$

$$\Rightarrow \quad dV = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{x} = \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\right)\frac{dx}{x}$$

$$\text{Hence, } V = \int dV = \frac{Q}{4\pi\varepsilon_0 L}\int_L^{2L}\frac{dx}{x}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}[\ln x]_L^{2L} = \frac{Q\ln 2}{4\pi\varepsilon_0 L}$$

24. (a) $120C_1 = 200C_2$
$6C_1 = 10C_2$
$3C_1 = 5C_2$

25. (None) $\vec{E} = 30x^2\hat{i}$

$$dV = -\int E.dx$$

$$\int_{V_0}^{V_A} dV = -\int_0^2 30x^2 dx$$

$V_A - V_0 = -80$ volt.

(Units given in question are incorrect)

26. (c) By formula of electric field between the plates of a capacitor

$$E = \frac{\sigma}{k\varepsilon_0}$$

$$\Rightarrow \quad \sigma = Ek\varepsilon_0 = 3 \times 10^4 \times 2.2 \times 8.85 \times 10^{-12}$$
$$= 6.6 \times 8.85 \times 10^{-8} = 5.841 \times 10^{-7}$$
$$\cong 6 \times 10^{-7}\text{ C/m}^2$$

27. (a) The options (b) and (c) are not possible since field lines should originate from positive and terminate to negative charge. Option (d) is not possible since field lines must be smooth. Also the field lines should resemble that of dipole hence option (a) satisfies all required condition

28. (c, d) Electric potential at the surface,

$$V_0 = k\frac{Q}{R} \qquad \text{(i)}$$

Electric potential inside,

$$V_I = \frac{kQ}{2R^3}(3R^2 - r^2)$$

At $R = 0$, $V = \frac{3}{2}V_0$ Hence $R_1 = 0$

Case II; $\frac{5}{4}\frac{kQ}{R} = kQ\frac{(3R^2 - R_2^2)}{2R^3}$

$$\Rightarrow \quad R_2 = \frac{R}{\sqrt{2}}$$

Case III; $\frac{3}{4}\frac{kQ}{R} = \frac{kQ}{R_3} \quad \Rightarrow \quad R_3 = \frac{4R}{3}$

Case IV; $\frac{1}{4}\frac{kQ}{R} = \frac{kQ}{R_4}$

$\Rightarrow \quad R_4 = 4R \qquad \Rightarrow \quad R_4 > 2R$ and $R_2 < (R_4 - R_3)$

29. (b) $$C_{eq} = \frac{3C}{3+C} \qquad \text{...(i)}$$

$$\text{Total charges } q = \left(\frac{3C}{3+C}\right)E \qquad \text{...(ii)}$$

Charge upon capacitor 2 μF,

$$Q_2 = \frac{2}{3} \times \frac{3CE}{(3+C)} = \frac{2CE}{3+C} = \frac{2E}{1+\frac{3}{C}}$$

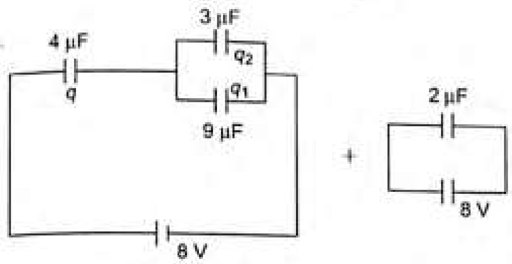
If $C = 1\ \mu F$; $q' = \frac{E}{4}$

and if $C = 3\ \mu F$; $q' = E$

Hence Q_2 is maximum at $C = 3\ \mu R$

Hence graph (b) is correct.

30. (c) We can break this circuit into two parts



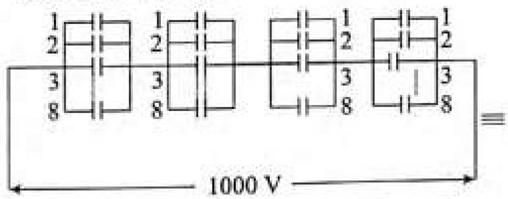
$$q = C_{eq}V = \left(\frac{4\times 12}{4+12}\right)\times 8 = 24\mu C$$

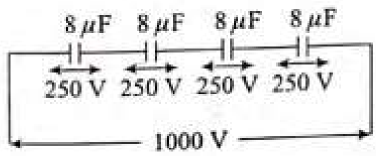
$$q_1 = \frac{9}{9+3}\times 24\mu C = 18\mu C$$

$$Q = q + q_1 = 24 + 18 = 42\ \mu C$$

$$E = \frac{kQ}{r^2} = \frac{9\times 10^9\times 42\times 10^{-6}}{(30)^2} = 420\,\text{N/C}$$

31. (b) To hold 1 kV potential difference minimum four capacitors are required in series





$\Rightarrow \quad C_1 = \frac{1}{4}$ for one series.

So for C_{eq} to be 2 μF, 8 capacitors of 1 μF in parallel with four such branches in series. Following arrangement will do the needful

$\Rightarrow \quad$ Minimum number of capacitors = 8 × 4 = 32

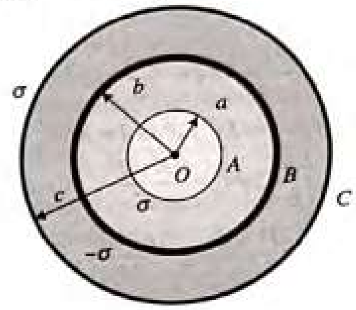
32. (c) $V_B = V_{A\,out} + V_{B\,surface} + V_{C\,in}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q_C}{c}$$

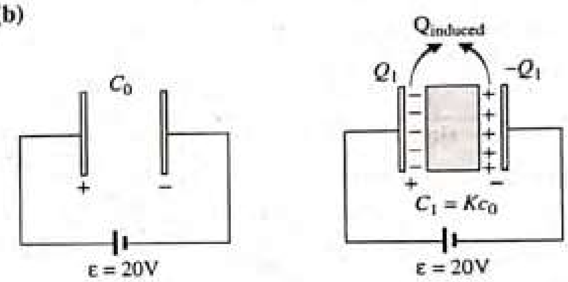
$$= \frac{1}{4\pi\varepsilon_0}\cdot\left[\frac{\sigma.4\pi a^2}{b} + \frac{(-\sigma.4\pi b^2)}{b} + \frac{\sigma.4\pi c^2}{c}\right]$$

$$\Rightarrow \quad V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b} - b + c\right)$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$



33. (b)



The charge in capacitor after placing the dielectric

$$Q_1 = C_1V$$

or $\quad Q_1 = (KC_0)\varepsilon = \frac{5}{3}\times 90\times 20 = 3000\ \text{pC}$

or $\quad Q_1 = 3\,\text{nC}$

The induced charge on dielectric surface

$$Q_{induced} = Q_1\left(1 - \frac{1}{K}\right)$$

$$Q_{induced} = 3\left(1 - \frac{1}{5/3}\right) = 3\times\frac{2}{5} = \frac{6}{5} = 1.2\ \text{nC}$$

CHAPTER 19: ELECTRIC RESISTANCE AND SIMPLE CIRCUITS

Concept Application Exercise 19.1

1. We need $R_1(1+\alpha_1\Delta t)+R_2(1+\alpha_2\Delta t)=20$. Because $R_1+R_2=20$ when $\Delta t=0$, we must have $R_1\alpha_1=-R_2\alpha_2$ with $\alpha_1=-0.5\times10^{-3}$ and $\alpha_2=5\times10^{-3}$. Solving the two equations $R_1+R_2=20$ and $R_1=10R_2$ simultaneously leads to $R_1=18.18\ \Omega$ and $R_2=1.82\ \Omega$.

2. (a) $E_1=\dfrac{V}{L},\ E_2=\dfrac{2}{2L}=\dfrac{V}{L},\ E_3=\dfrac{2V}{3L}$

Hence, $E_1=E_2>E_3$

(c) $V_d\propto E$. Hence, $V_{d_1}=V_{d_2}>V_{d_3}$

(b) $I=neAv_d=\dfrac{ne\pi d^2}{4}v_d=\dfrac{ne\pi}{4}v_d d^2$

or $\quad I\propto v_d d^2$ or $I\propto Ed^2$ or $I=KEd^2$

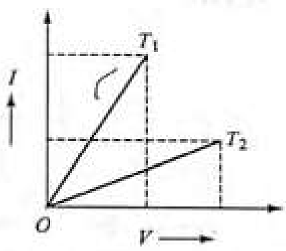
$$I_1=\frac{KV}{L}(3d)^2=\frac{9KVd^2}{L},\quad I_2=K\frac{V}{L}d^2A$$

$$I_3=K\frac{2V}{3L}(2d)^2=\frac{8KVd^2}{3L}.$$

Hence, $I_1>I_3>I_2$.

3. We know $R=V/I$ and slope of IV curve is I/R.

or $\quad R=\dfrac{1}{\text{slope}}$

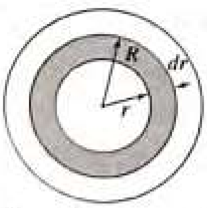


Since the slope of OT_2 is smaller than OT_1, the resistance of wire at T_2 is greater than that at T_1 as resistance increases with temperature, so temperature T_2 is greater than T_1.

4. $R=\dfrac{\rho\ell}{A}$, so $\rho\propto R$. But $R_Y>R_X$

Because for same V, current in Y is less. So, $\rho_Y>\rho_X$.

5. $dI=J2\pi r\,dr=J_0\left(1-\dfrac{r}{R}\right)2\pi r\,dr=2\pi J_0\left(1-\dfrac{r}{R}\right)r\,dr$



$$I=\int dI=2\pi J_0\int_0^R\left(1-\frac{r}{R}\right)r\,dr=2\pi J_0\left[\int_0^R r\,dr-\frac{1}{R}\int_0^R r^2dr\right]$$

$$=2\pi J_0\left[\frac{R^2}{2}-\frac{R^2}{3}\right]=2\pi J_0\frac{R^2}{6}=\frac{J_0A}{3}$$

6. Let L be the length and A the cross-sectional area of the wire and d be the density of the material of wire.

Mass M = Volume × density = ALd

or $\quad AL=\dfrac{M}{d}=\dfrac{2.23\times10^{-3}}{8.92\times10^3}=\dfrac{1}{4}\times10^{-6}\,\text{m}^3 \quad$ (i)

Resistance of wire, $R=\dfrac{V}{\ell}=\dfrac{1.7}{1}=1.7\ \Omega$

$$R=\frac{\rho L}{A}\quad\text{or}\quad\frac{L}{A}=\frac{R}{\rho}=\frac{1.7}{1.7\times10^{-8}}=10^8\,\text{m}^{-1}\qquad\text{(ii)}$$

Multiplying (i) and (ii), we get

$$L^2=\frac{1}{4}\times10^{-6}\times10^8=\frac{10^2}{4}\quad\text{or}\quad L=5\,\text{m}$$

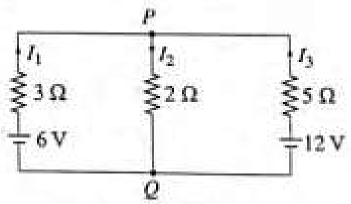
From (i), $A=\dfrac{\frac{1}{4}\times10^{-6}}{L}=\dfrac{\frac{1}{4}\times10^{-6}}{5}=\dfrac{10^{-6}}{20}=5\times10^{-8}\,\text{m}^2$

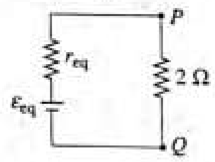
When wire is stretched to length L, its cross-sectional area decreases. So $R\propto L^2$. If R' is new resistance of wire, then

$$\frac{R'}{R}=\left(\frac{L'}{L}\right)^2\quad\text{or}\quad R'=\left(\frac{L'}{L}\right)^2R=(2)^2\times1.7=6.8\ \Omega$$

Concept Application Exercise 19.2

1. $\varepsilon_{eq}=\dfrac{\frac{6}{3}+\frac{12}{5}}{\frac{1}{3}+\frac{1}{5}}=\dfrac{33}{4}\text{V},\ r_{eq}=\dfrac{1}{\frac{1}{3}+\frac{1}{5}}=\dfrac{15}{8}\Omega$





Hence, $I_2=\dfrac{\frac{33}{4}}{2+\frac{15}{8}}=\dfrac{66}{31}\text{A}$

Potential difference across PQ is

$$V_{PQ}=I_2R=\frac{66}{31}\times2=\frac{132}{31}\text{ V}$$

Current through 6 V battery:

$$\frac{132}{31}=6-3I_1\text{ or }I_1=\frac{18}{31}\text{A}$$

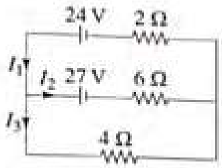
Hence, $I_3=I_2-I_1=\dfrac{66}{31}-\dfrac{18}{31}=\dfrac{48}{31}\text{A}$

2. Traversing the upper and lower loops anticlockwise, we get

$2I_1+6I_2=24-27\quad$ or $\quad 2I_1+6I_2=-3 \quad$ (i)

and $\quad 4I_3-6I_2=27\quad$ or $\quad 4(I_1-I_2)-6I_2=27$

or $\quad 4I_1-10I_2=27 \quad$ (ii)

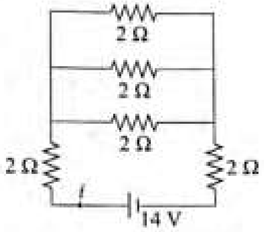


Solving Eqs. (i) and (ii), we get

$I_1 = 3$ A, $I_2 = -1.5$ A,

$I_3 = I_1 - I_2 = 3 + 1.5 = 4.5$ A

3. The given circuit can be redrawn as follows:

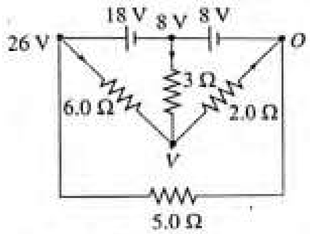


$$\frac{1}{R'} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \text{ or } R' = \frac{2}{3}\Omega$$

$$\therefore \quad R_{Total} = 2 + \frac{2}{3} + 2 = \frac{14}{3}\Omega.$$

Hence, $i = \frac{V}{R} = \frac{14 \times 3}{14} = 3$ A

4. Applying Kirchhoff's current law, we get



At node V, $\frac{26-V}{6} + \frac{8-V}{3} = \frac{V-0}{2}$ or $V = 7$ V

Current through the 6 Ω resistor is $I_6 = \frac{26-7}{6} = \frac{19}{6}$ A

Current through the 3 Ω resistor is $I_3 = \frac{8-7}{3} = \frac{1}{3}$ A

Current through the 2 Ω resistor is $I_2 = \frac{7}{2}$ A

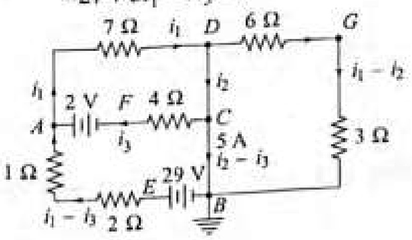
Current through the 5 Ω resistor is $I_6 = \frac{26}{5}$ A

5. Given $i_2 - i_3 = 5$

In $ADC, -7i_1 - 4i_3 + 2 = 0$ (i)

In $FAEBC, 2 + 3(i_1 - i_3) - 29 - 4i_3 = 0$

or $-27 + 3i_1 - 7i_3 = 0$ (ii)



(i) × 3 + (ii) × 7 gives $i_3 = -3$ A

Putting in Eq. (i), we get $i_1 = 2$ A

Putting in Eq. (A), we get $i_2 = 2$A

In ACB, $V_A - V_B = 2 - (-3 \times 4)$ or $V_A - 0 = 14$ V

(a) $|V_A| = 14$ V, $V_A - V_D = 7 \times 2 = 14$

(b) $V_D = 14 - 14 = 0$

$V_D - V_G = 6 \times (i_1 - i_2) = 6(2-2)$ or $V_G = 0 + V_D$

(c) $V_G = 0$ V

6. (a) $I_R = 6\text{ A} - 4\text{ A} = 2$ A

(b) Using a Kirchhoff's loop around the outside of the circuit, we get

$$28\text{ V} - (6\text{ A})(3\ \Omega) - (2\text{ A})R = 0$$

or $R = 5\ \Omega$

(c) Using a counterclockwise loop in the bottom half of the circuit, we get

$$\varepsilon - (6\text{ A})(3\ \Omega) - (4\text{ A})(6\ \Omega) = 0 \quad \text{or } \varepsilon = 42\text{ V}$$

(d) If the circuit is broken at point x, then the current in the 28 V battery is

$$I = \frac{\Sigma\varepsilon}{\Sigma R} = \frac{28\text{ V}}{3\ \Omega + 5\Omega} = 3.5\text{ A}$$

Concept Application Exercise 19.3

1. (a) At $t = 0$, C will behave as a short circuit, so no current passes through R_2. And $I = I_2 = \varepsilon/R_1$.

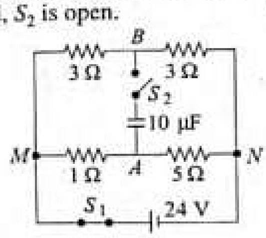
(b) At $t = \infty$, C will block the current. So $I_2 = 0$. And

$$I = I_1 = \frac{\varepsilon}{R_1 + R_2}$$

2. (a) Immediately after the switch is closed, C_1 will act as a simple wire due to which R_2 and R_3 will be short-circuited. So, $I = E/R_1$

(b) After a long time, C_1 and C_2 will block the current. Current will pass through only R_1 and R_3. $I = \frac{E}{R_1 + R_3}$

3. (a) S_1 is closed, S_2 is open.



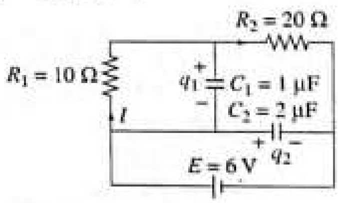
$$V_M - V_A = \frac{1 \times 24}{1+5} = 4\text{ V}, \; V_M - V_B = \frac{3 \times 24}{3+3} = 12\text{ V}$$

$V_A - V_B = 8$ V

(b) i. Just after closing S_2, A and B will come to the same potential, so $V_A - V_B = 0$.

ii. After a long time, no current will flow through AB, so $V_A - V_B = 8$ V.

4. $I = \frac{E}{R_1 + R_2} = \frac{6}{10+20} = \frac{1}{5}$A



$$q_1 = C_1V_1 = C_1 I\, R_1 = 1 \times \frac{1}{5} \times 10 = 2\ \mu\text{C}$$

$$q_2 = C_2V_2 = 2 \times 6 = 12\ \mu\text{C}$$

5. (a) At $t = 0$, capacitor acts as short-circuit. There will not be any current in the 40 Ω resistance. Therefore,

$$I_0 = \frac{10}{20} = 0.5\text{ A}$$

(b) Capacitor acts as open circuit, so

$$I=\frac{10}{60}=\frac{1}{6}\text{A}$$

(c) Voltage across capacitor is

$$V_{40}=IR=\frac{1}{6}\times 40, Q=CV=0.50\times\frac{1}{6}\times 40=\frac{10}{3}\,\mu\text{C}$$

(d) $q=q_0(e^{-t/RC})$, capacitor will discharge through the 40 Ω resistor when S is open, i.e.,

$$\frac{20}{100}q_0=q_0\left(e^{-\frac{t}{40\times 0.5\times 10^{-6}}}\right)\text{ or }\frac{1}{5}=\frac{1}{e^{\frac{t}{20\times 10^{-6}}}}$$

or $\quad t=20\times 10^{-6}\ln(5)$

EXERCISES

Resistivity and Drift Velocity

1. (c) $\frac{R_1}{R_2}\left(\frac{l_1}{l_2}\right)$ if $l_1=100$, then $l_2=110$

$$\Rightarrow\quad \frac{R_1}{R_2}=\left(\frac{100}{110}\right)^2\Rightarrow R_2=1.21R_1$$

$$\%\text{ change}=\frac{R_2-R_1}{R_1}\times 100=21\%$$

2. (c) Two wires carry the same current

$$I=n_1eA\,V_{d_1}=n_2eA\,V_{d_2}$$

So $\quad n_1V_{d_1}=n_2V_{d_2}\quad$ or $\quad\frac{V_{d_1}}{V_{d_2}}=\frac{n_1}{n_2}=4\Rightarrow V_{d_1}:V_{d_2}=4:1$

3. (b) Current density $\Rightarrow J=\frac{I}{ds}$

$\therefore\quad (ds)_A<(ds)_B$

$\therefore\quad J_A>J_B$

4. (d) $j=\frac{I}{A}=nev_d\;\Rightarrow\;\frac{4I}{\pi d^2}=nev\qquad$ ...(i)

or $\quad\frac{16I}{\pi d^2}=nev'\qquad$ ...(ii)

From equation (i) and (ii)

$$\frac{4I}{16I}=\frac{v}{v'}\Rightarrow v'=4v$$

5. (c) Let ρ be the resistivity of the material

Resistance for contact A-A

$$R_{AA}=\rho\frac{x}{2x\times 4x}=\frac{\rho}{8x}$$

Similar for contacts B-B and C-C are respectively

$$R_{BB}=\rho\cdot\frac{2x}{x\times 4x}=\frac{\rho}{2x}=\frac{4\rho}{8x}$$

and $\quad R_{CC}=\rho\frac{4x}{x\times 2x}=\frac{2\rho}{x}=\frac{16\rho}{8x}$

It is clear that maximum resistance will be for contact C-C.

6. (d) $R_1+R_2=R_1(1+\alpha t)+R_2(1-\beta t)$

$$\Rightarrow\quad R_1+R_2=R_1+R_2+R_1\alpha t-R_2\beta t\;\Rightarrow\;\frac{R_1}{R_2}=\frac{\beta}{\alpha}$$

7. (b) Let initial resistance of the wires are R_1 and R_2 respectively. Then $R_1'+R_2'=R_1+R_2$

$\Rightarrow\quad R_1(1+\alpha_1\Delta T)+R_2(1+\alpha_2\Delta T)=R_1+R_2$

$\Rightarrow\quad R_1\alpha_1+R_2\alpha_2=0$

$\Rightarrow\quad \frac{\rho_1L_1}{A}\alpha_1+\frac{\rho_2L_2}{A}\alpha_2=0$

$\Rightarrow\quad \rho_1L_1\alpha_1+r_2L_2\alpha_2=0.$

8. (c) The resistance of the square X is given by

$$R_X=\rho\frac{L}{Lt}=\frac{\rho}{t}$$

where ρ is the resistivity on the metal. Similarly,

$$R_Y=\rho\frac{2L}{(2L)t}=\frac{\rho}{t}$$

same as before. Hence, $R_X/R_Y=1$

9. (b) In series combination, I is constant, therefore $V\propto R$.

$$\frac{V_{AB}}{V_{CA}}=\frac{R_{AB}}{R_{CA}}=\frac{\rho\frac{l}{\pi a^2}}{\rho\frac{l}{\pi(2a)^2}}=\frac{4}{1}$$

10. (c) Charge = area under the current–time graph

$$q_1=2\times 1=2,\ q_2=1\times 2=2,\text{ and }q_3=\frac{1}{2}\times 2\times 2=2$$

$$q_1:q_2:q_3=2:2:2=1:1:1$$

11. (c) $\alpha=\frac{1}{R_t}\frac{dR_t}{dt}=\frac{1}{R_t}R_0[\alpha+2\beta t]$

$$\alpha=\frac{R_0[\alpha+2\beta t]}{R_0\left[1+\alpha t+\beta t^2\right]}=\frac{\alpha+2\beta t}{1+\alpha t+\beta t^2}$$

12. (d) $R=\frac{\rho\ell}{A}=\frac{\rho\ell^2}{A\ell}=\frac{\rho\ell^2}{V}=\frac{\rho\ell^2}{m/d}=\frac{\rho d\ell^2}{m}\quad$ or $\quad R\propto\frac{\ell^2}{m}$

$$R_1:R_2:R_3=\frac{\ell_1^2}{m_1}:\frac{\ell_2^2}{m_2}:\frac{\ell_3^2}{m_3}=\frac{25}{1}:\frac{9}{3}:\frac{1}{5}=125:15:1$$

13. (c) $\alpha(T)=\frac{1}{R_0}\frac{dR}{dT}\quad$ or $\quad(3T^2+2T)=\frac{1}{R_0}\frac{dR}{dT}$

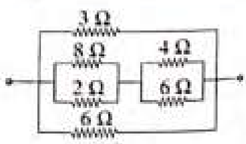
or $\quad dR=R_0(3T^2+2T)\,dT$

or $\quad\int_{R_0}^{R}dR=R_0\left[3\int_0^T T^2dT+2\int_0^T T\,dT\right]$

or $\quad R=R_0[1+T^2+T^3]$

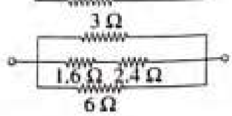
Combination of Resistance and Cells

14. (a) The equivalent of the network is given in figure.



The equivalent of the above network is a parallel combination of 3 Ω, 4 Ω, and 6 Ω, i.e.,

$$\frac{1}{R}=\frac{1}{3}+\frac{1}{4}+\frac{1}{6}\text{ or }R=\frac{4}{3}\Omega$$



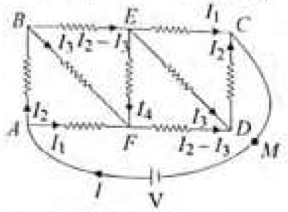
15. (c) A careful observation will reveal that one end of each resistor is connected to A and the other end of each resistor is connected to B. Hence, the resistors are in parallel. So $R_{eq} = R/5$.

16. (d) At junction E or F:

$$I_1 + I_3 + I_4 = I_2 - I_3 \quad \text{or} \quad I_1 - I_2 + 2I_3 + I_4 = 0$$

Loop $AFBA$ or loop $ECDE$:

$$rI_1 = rI_2 + rI_3 \quad \text{or} \quad I_1 = I_2 + I_3$$



Loop $BEFB$ or loop $EDFE$:

$$r(I_2 - I_3) + rI_4 = rI_3 \text{ or } I_2 - 2I_3 + I_4 = 0$$

Loop $AFDCMA$ or $ABECMA$:

$$V = rI_1 + r(I_2 - I_3) + rI_2 = rI_1 + 2rI_2 - rI_3$$

Solve to get $I_1 = 2V/5r$, $I_2 = V/3r$

$$R_{eq} = \frac{V}{I} = \frac{V}{I_1 + I_2} = \frac{V}{2V/5r + V/3r} = \frac{15r}{11}$$

17. (d) Let current in AB be I_1, in BF be I_2, in BC be $2I_2$, in FC be I_3, in FE be I_3, and in CD be $(I_3 + 2I_2)$.

At point B, $I_1 = 3I_2$

At point F, $I_2 = 2I_3$

Along $ABFCD$,

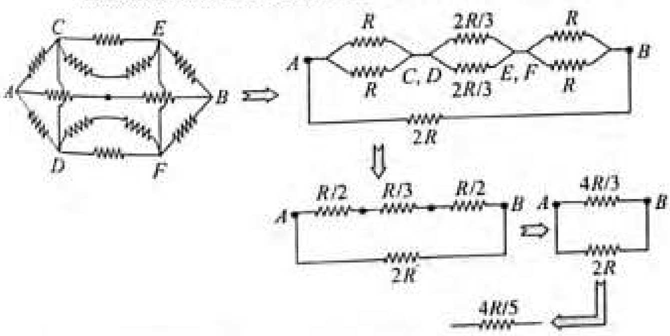
$$V_A - V_D = V = I_1\left(\frac{2R}{5}\right) + I_2\left(\frac{R}{5}\right) + I_3\left(\frac{R}{5}\right) + (I_3 + 2I_2)\frac{R}{5} = 2I_2R$$

Along $ABCD$,

$$V_A - V_D = V = 2I_2R = I_1\left(\frac{2R}{5}\right) + 2I_2R' + (I_3 + 2I_2)\frac{R}{5}$$

or $\quad 2I_2R' = \dfrac{3}{10}I_2R \quad$ or $\quad R' = \dfrac{3}{20}R$

18. (d) The current in CO, OE, DO, and OF will be same. So these branches will not touch each other at O.



19. (b) Current in the circuit is $i = \dfrac{\varepsilon}{R + r}$

Potential difference across cell = potential difference across R

$$V = iR = \frac{\varepsilon R}{R + r}$$

Set up two equations with the given data and solve for ε, r.

$$1.6 = \frac{\varepsilon \times 4}{4 + r} \qquad \text{...(i)}$$

$$1.8 = \frac{\varepsilon \times 9}{9 + r} \qquad \text{...(ii)}$$

On solving we get $\varepsilon = 2\text{V}$ and $r = 1\Omega$

20. (b) For series connection, $I_{min} = \dfrac{N\varepsilon}{Nr} = \dfrac{\varepsilon}{r}$

For parallel connection, $I_{max} = \dfrac{\varepsilon}{(r/N)} = \dfrac{N\varepsilon}{r}$

21. (a) Current in circuit, is $i = \dfrac{n\varepsilon}{nr} = \dfrac{\varepsilon}{r}$

The equivalent circuit of one cell is shown in figure. The potential difference across the cell is

$$V_A - V_B = -\varepsilon + ir = -\varepsilon + \frac{\varepsilon}{r}r = 0$$

22. (a) $i = \dfrac{(n-2)\varepsilon}{nr}$

$$V_B - V_A = -ir + \varepsilon = \varepsilon - \frac{(n-2)\varepsilon}{nr}r = \varepsilon\left[1 - \frac{n-2}{n}\right] = \frac{2\varepsilon}{n}$$

23. (b) $V = E - ir$. When $i = 0$, the potential reading is 2 V. Hence emf is 2 V. When $V = 0$, $i = 5$ A. This gives $r = 0.4\ \Omega$.

24. (b) Current through R before short-circuiting second cell

$$I = \frac{E_1 + E_2}{R + (r_1 + r_2)}$$

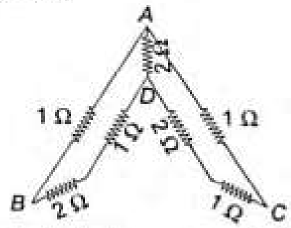
Current through R after short-circuiting second cell

$$I' = \frac{E_1}{R + r_1}$$

Here, $I' > I$ or $E_1r_2 > E_2(R + r_1)$

25. (b) By symmetric path method Points E, F and B, C are Equipotential

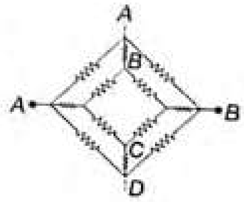
$\Rightarrow \quad R_{AD} = 1\ \Omega$



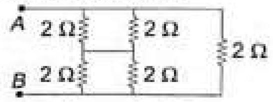
26. (c) By perpendicular Axis symmetry all points 1, 2, 3 are at same potential therefore junction on this line can be redrawn as

$$R_{AB} = \frac{22}{35}R$$

27. (a) By applying perpendicular axis symmetry. Points lying on the line 'AD' have same potential therefore Resistance between AB and CD can be removed $R_{AB} = 9\ \Omega$



28. (a) The figure can be redrawn as:

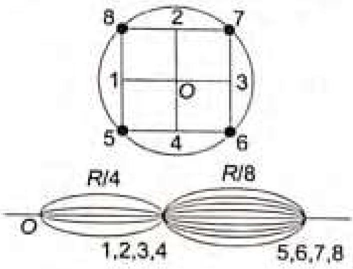


So equivalent resistance is 1 Ω.

29. (a)

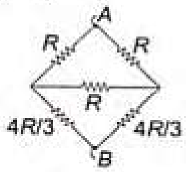
$$R_{AB}=\frac{5R}{9}$$

30. (c) 1, 2, 3, 4 are at same potential; 5, 6, 7, 8 are at same potential.



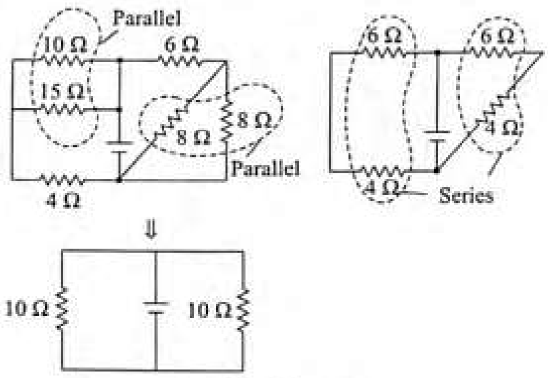
$$R_{eq}=\frac{R}{4}+\frac{R}{8}=\frac{3R}{8}$$

31. (b)



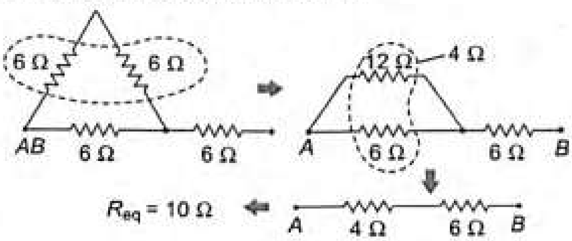
Given circuit can be reduce to $R_{AB}=\frac{7}{6}R$.

32. (c) Given circuit can be reduced to a simple circuit as shown in the following figures



So equivalent resistance = $\frac{10\times 10}{10+10}\Omega=5\Omega$

33. (b) Given resistance of each part will be

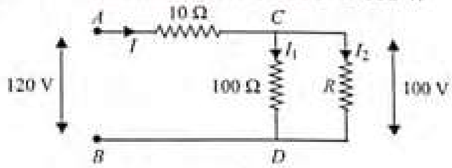


Kirchhoff's Law and Simple Circuits

34. (c) Current through 1 Ω resistance will be 2 A in the upward direction.

$V_G-2\times 4+3-2\times 2+2\times 1=V_H$ or $V_G-V_H=7$ V

35. (a) Potential difference across C and D is 100 V.



Hence, $I_1=\frac{100}{100}=1\text{A},\ V_{AC}=120-100=20$ V

And $I=\frac{20}{10}=2$A. Hence, $I_2=2-1=1$ A

$R=100/I_2=100\ \Omega$

36. (a) $\frac{(20R)/(20+R)}{5}=\frac{20}{10}$ or $R=20\,\Omega$

37. (c) Notice the polarities of the batteries. The batteries will cancel each other and finally there will be no current anywhere in the circuit.

38. (b) By symmetry, we see that the current in the left and right arms should be the same. It means no current should flow from A to B.

39. (d) Emf = 6 V, total resistance = 6 Ω, I = 6/6 = 1 A

For the direction of current, look at the direction of emf of the cell of 10 V.

40. (c) $2=\frac{12}{\frac{R}{3}+0.6}$ or $\frac{R}{3}+0.6=6$ or $R=16.2\,\Omega$

41. (a) The equivalent resistance of resistors is

$$R=2+\frac{4}{2}+\frac{15}{3}=9\,\Omega,\quad I=\frac{E}{r+R}=\frac{10}{1+9}=1\text{ A}$$

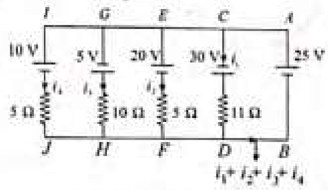
42. (c) Applying KVL in loops $CDBAC$, $EFBAE$, $GHBAG$, and $IJBAI$, we get

$30-i_1\times 11=-25$ or $i_1=5$ A

$20+i_2\times 5=25$ or $i_2=1$ A

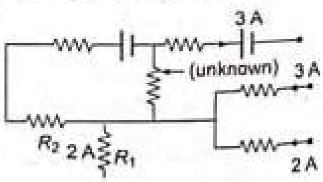
$5-i_3\times 10=-25$ or $i_3=3$ A

$10+i_4\times 5=25$ or $i_4=3$ A



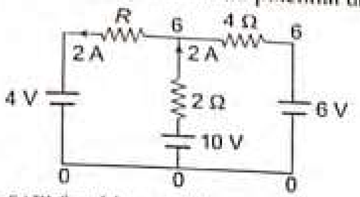
Current through 25 V cell is $i_1+i_2+i_3+i_4=12$ A.

43. (d) Since 3 A is in upper part of circuit. Therefore out of 5 A coming in lower part, 3 A has to go to the upper part. Out of which some part will flow through R_2 and rest through the unknown resistance.



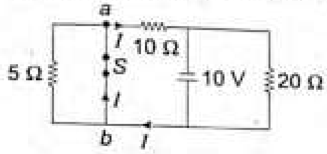
∴ 2 A will go through R_1.

44. (b) By taking 'O' as a reference potential as current through '4 Ω' is zero there should be no potential drop across it



Value of 'R' for this condition = 1 Ω

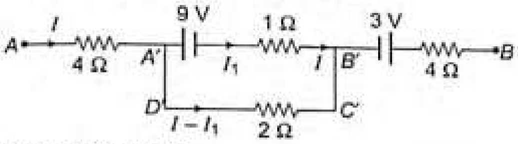
45. (d) After closing the switch, 5 Ω will be shorted.



$I = \frac{10}{10} = 1$ A → this is from b to a.

So current from a to $b = -I = -1$ A.

46. (b)



By applying K.V.L

$$V_A - 4I - 9 - I_1 - 3 - 4I = V_B$$
$$16 = +8I + I_1 + 12$$
$$8I + I_1 = 4V \quad \text{...(i)}$$

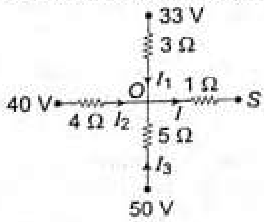
By applying K.V.L. in loop $A'B'C'D'A'$

$$-9 - I_1 + 2(I - I_1) = 0$$
$$-3I_1 + 2I = 9 \quad \text{...(ii)}$$

By solving (i) and (ii), current in 2 Ω resistance is 3.5A.

47. (b) Right end of 2 R will be at higher potential than left end. So current in 2 R will flow from right to left end.

48. (b)



$$I = \frac{V_0 - V_S}{1} = 18 \text{ A}$$

$$I_1 + I_2 + I_3 = I$$

$$\frac{33 - V_0}{3} + \frac{40 - V_0}{4} + \frac{50 - V_0}{5} = 18$$

$$\Rightarrow \quad V_0 = 780/47 \text{ V}$$

Resistance- Capacitor Circuits

49. (b) Charge on capacitor at any time is

$$q = Qe^{-t/\tau} \quad \text{(i)}$$

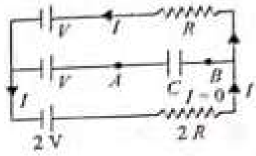
Differentiating Eq. (i) with respect to time, we get the current

$$I = -\frac{dq}{dt} = \frac{Q}{\tau}e^{-t/\tau} \quad \text{(ii)}$$

Required ratio is $q/I = \tau$ [by dividing Eq. (i) by Eq. (ii)]

50. (c) Initial rate of discharging means initial current, which is equal to V/R, where V is initial potential. Here V and R are the same for both.

51. (c) In the steady-state conduction, no current will flow through the capacitor C.



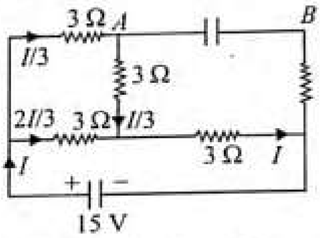
Current in the outer circuit

$$I = \frac{2V - V}{2R + R} = \frac{V}{3R}$$

Potential difference between A and B is

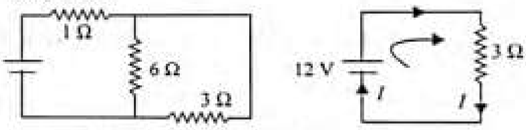
$$V_A - V + V + IR = V_B \quad \text{or} \quad V_B - V_A = IR = \left(\frac{V}{3R}\right)R = \frac{V}{3}$$

52. (c) In the steady state, no current flow through C.



$$15 = 3 \times 2I/3 + 3I \text{ or } I = 3 \text{ A}$$
$$V_A - 3I/3 - 3I = V_B$$
$$\text{or} \quad V_A - V_B = 4I = 4 \times 3 = 12 \text{ V}$$

53. (b) At $t = 0$, the capacitor behaves as a short circuit. The corresponding circuit is shown in figure. According to the loop rule,



$$12 - 3I = 0 \quad \text{or} \quad I = 4 \text{ A}$$

54. (d) At $t = 0$, the potential difference across the top 2 μF capacitor is 0. Therefore, $I = 10/2 = 5$ A.

55. (a) In the steady state, no current is flowing through the capacitor branch, so the capacitor branches may be removed from the circuit, the equivalent circuit is shown in figure.

$$\therefore \quad R_{AB} = R_1 + R_2 = r + r = 2r$$

56. (a) The current in the circuit is $I = (12/12) = 1$ A.

Potential across d and e is 12 V − 3 × 1 V = 9 V

Capacitance across d and e is $C = \frac{1 \times 2}{1 + 2} = \frac{2}{3}$ μF

Therefore, charge on either capacitor is $\frac{2}{3} \times 9 = 6$ μC.

57. (c) In the case of discharging, $I = I_0 e^{-t/RC}$

$$\text{or} \quad 2.5 \times 10^{-6} = \frac{q_0}{RC}e^{-t/RC} = 5 \times 10^{-6}\, e^{-t/10}$$

$$\text{or} \quad e^{t/10} = 2$$

Taking log on both the sides, we get

$$\frac{t}{10} = \ln 2 \quad \text{or} \quad t = 10\ln 2 = 6.9 \cong 7 \text{ s}$$

58. (a) $\dfrac{q^2}{2C} = \dfrac{q_0^2 e^{-2t/RC}}{2C}$ or $U_0 e^{-2} = \dfrac{U_0}{e^2}$

59. (c) During charging, $\tau_1 = RC$; during discharging, $\tau_2 = 2RC$. Therefore, ratio is

$$\frac{\tau_1}{\tau_2} = \frac{1}{2} = 1:2$$

60. (a) The resistance in the middle plays no part in the charging process of C, as it does not alter either the potential difference across the RC combination or the current through it.

61. (b) C discharges through $R + R$ in series, so time constant is $2RC$.

62. (c) $I = I_0 e^{-t/RC}$ or $\dfrac{I_0}{2} = I_0 e^{-t/RC}$ or $t = RC \ln 2$

or $\quad 10^{-6} \times \ln 4 = (2 + r) \times 0.5 \times 10^{-6} \ln 2$

Solve to get $r = 2\ \Omega$

63. (c) Here, $I_1 = \dfrac{V}{R} e^{-t/RC}$, $I_2 = \dfrac{V}{R} e^{-t/2RC}$

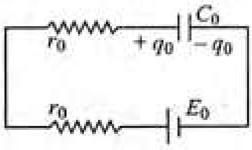
$$\therefore \quad \frac{I_1}{I_2} = e^{-t/RC + t/2RC} = e^{-t/2RC} = \frac{1}{e^{t/2RC}}$$

From this expression, it is clear that when t increases, ratio decreases.

64. (d) Uncharged capacitor behaves as a short circuit just after closing the switch. But charged capacitor behaves as the battery of emf V just after closing the switch. Therefore,

$$I = \frac{q_0}{C_1(2R)} = \frac{q_0}{2RC_1} = \frac{V}{2R}$$

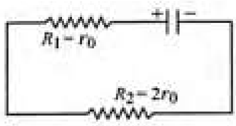
65. (c) **Step I: Switch at position 1:** Since circuit is in the steady state, the current through circuit is zero.



According to the loop rule,

$E_0 - \dfrac{q_0}{C_0} = 0$ or $q_0 = C_0 E_0$

Step II: Switch at position 2: In this case, the total energy stored in the capacitor appears as heat energy in the resistor.



$$\Delta H = I^2 RT \quad \therefore \Delta H \propto R$$

$$\therefore \quad \frac{\Delta H_1}{\Delta H_2} = \frac{R_1}{R_2} = \frac{r_0}{2r_0} = \frac{1}{2}$$

or $\quad \Delta H_2 = 2\Delta H_1$

But $\quad \Delta H = \Delta H_1 + \Delta H_2 = \dfrac{\Delta H_2}{2} + \Delta H_2 = \dfrac{3}{2}\Delta H_2$

$$\therefore \quad \Delta H_2 = \frac{2}{3}\Delta H = \frac{2}{3} \times \frac{1}{2} C_0 E_0^2 = \frac{1}{3} C_0 E_0^2$$

66. (b) As the capacitors are identical, each of them finally have charge $Q/2$.

Initial energy of the system is $E_i = \dfrac{Q^2}{2C}$

Final energy of the system is $E_f = 2\left[\dfrac{(Q/2)^2}{2C}\right] = \dfrac{Q^2}{4C}$

Heat produced = loss in energy $= E_i - E_f = \dfrac{Q^2}{4C}$

67. (b) From the given situation, we have

$$i_1 = \frac{V}{R}\left(e^{-t/RC}\right) \text{ and } i_2 = \frac{V}{R}\left(e^{-t/3RC}\right)$$

$$\frac{i_1}{i_2} = (e)^{\frac{2t}{3RC}} \text{ which increases with time}$$

68. (d) No current will flow in the branch containing capacitor. Hence, no energy is stored in the capacitor.

Problems Based on Mixed Concepts

69. (b) Let R be the resistance of the wire and let R' be the resistance of the wire. Energy released in t seconds is $(3V^2)/R \times t$.

$R' = 2R$ (as length is twice)

Therefore, energy released in t seconds is $\dfrac{(NV^2)}{2R} \times t$

But $\quad Q = mc\Delta T$

$$\therefore \quad Q' = \frac{(N^2V^2)}{2R} \times t$$

$$\therefore \quad mc\Delta T = \frac{(9V^2)}{R} \times t \quad \text{(i)}$$

Applying $Q' = m'c\Delta T$,

$$2mc\Delta T = \frac{(N^2V^2)}{2R} \times t \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{mc\Delta T}{2mc\Delta T} = \frac{9V^2 \times t/R}{N^2V^2\, t/2R}$$

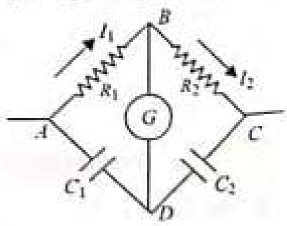
$$\therefore \quad \frac{1}{2} = \frac{9 \times 2}{N^2} \text{ or } N = 6$$

70. (c) The bridge is balanced and the current in ADC is larger than in ABC. Also $I_3 = 0$.

71. (a) All the four resistances are in parallel to E. So current in them flows independently. Hence, no change in current flowing in A after closing the switch.

72. (b) On connecting the switch, the current drawn by the resistances through the battery will increase. This will decrease the terminal potential difference ($V = E - Ir$) across the cell and hence the potential difference across A will also decrease. So the current through A will decrease.

73. (b) In the steady state, no current is passing through the capacitor. Let the charge on each capacitor be q. Since the current through galvanometer is zero,



$$I_1 = I_2$$

The potential difference between the ends of the galvanometer will be zero. Therefore

$$V_A - V_B = V_A - V_D$$

$$I_1 R_1 = \frac{q}{C_1} \quad \text{(i)}$$

Similarly, $V_B - V_C = V_D - V_C$

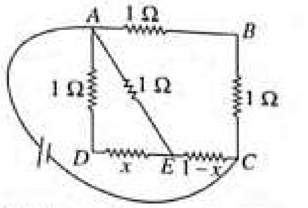
$$I_2R_2 = \frac{q}{C_2} \quad \text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{I_1R_1}{I_2R_2} = \frac{q/C_1}{q/C_2} = \frac{C_2}{C_1} \text{ or } \frac{C_1}{C_2} = \frac{R_2}{R_1}$$

74. (d) Equivalent resistance between A and E is

$$y = \frac{x+1}{x+2}$$



For B and E to be equipotent, we get

$$\frac{R_{AE}}{R_{AB}} = \frac{R_{EC}}{R_{BC}} \text{ or } \frac{x+1}{(x+2)\times 1} = \frac{1-x}{1}$$

Solve to get $x = \sqrt{2} - 1\,\Omega$

Now $\dfrac{CE}{ED} = \dfrac{1-x}{x} = \sqrt{2}\,\Omega$

75. (d) In the steady state condition, no current will flow through the capacitor C.

Current in the outer circuit is $I = \dfrac{2V - V}{2R + R} = \dfrac{V}{3R}$

Potential difference between A and B is

$$V_A - V + V + IR = V_B$$

$$\therefore \quad V_B - V_A = IR = \left(\frac{V}{3R}\right)R = \frac{V}{3}$$

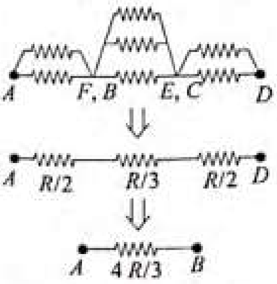
Here, $V = 6\text{V}$

$V_B - V_A = 2\text{V}$

as $V_B = 0\text{V}$

So, $V_A = 2\text{V}$

76. (b)



$$V = I R_{eq} = I\frac{4R}{3} \quad \text{or} \quad I = \frac{3V}{4R}$$

$$V_1 = V_A - V_F = I\frac{R}{2}$$

$$V_2 = V_A - V_C = I\left(\frac{R}{2} + \frac{R}{3}\right) = \frac{5}{6}IR$$

$$V_2 - V_1 = V_F - V_C = IR\left(\frac{5}{6} - \frac{1}{2}\right) = \frac{IR}{3}$$

$$V_F - V_C = \frac{IR}{3} = \left(\frac{4IR}{3}\right)\frac{1}{4} = \frac{V}{4}$$

77. (c) Let $V_A - V_F = V$

if $\quad V_A - V_C = V_A - V_B \quad$ (i)

Equivalent resistance along $ADCEF$

$$= R\left(\frac{2}{7} + \frac{3}{7} + \frac{1}{7} + \frac{1}{7}\right) = R \quad \text{(Remove } CB\text{)}$$

Along $ADCEF$, $V_A - V_F = V = I_1R$

$$= \frac{V}{R};\ I_1 = \text{current in path } ADCEF$$

$$V_A - V_C = I_1\frac{5}{7}R = \frac{5V}{7} = V_A - V_B \quad \text{(ii)}$$

[From Eq. (i)]

Along path ABF, equivalent resistance

$$R_{AF} = R_{AB} + \frac{2}{3}R$$

$$V_A - V_F = V = I_2\left(R_{AB} + \frac{2}{3}R\right)$$

I_2 = current in path ABF

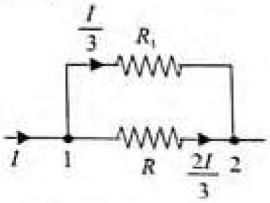
or $\quad I_2 = \dfrac{V}{R_{AB} + \frac{2}{3}R} \quad$ or $\quad V_A - V_B = I_2 R_{AB} = V_A - V_C$

$$\left[\frac{V}{R_{AB} + \frac{2}{3}R}R_{AB}\right] = \frac{5}{7}R \text{ [from Eq. (ii)]}$$

or $\quad R_{AB} = \dfrac{5}{3}R$

78. (b) Here we will apply superposition principle.

i. Let us feed a current I to point 1, but do not draw the current from 2 and allow the current to spread to infinity. Then current in branch 12 will be $I/3$.



ii. Secondly, feed current I to the current at take it out from 2, now again current through branch 12 will be $I/3$.

iii. Now carry out both the above steps (i) and (ii) simultaneously. Then current in branch 12 will be sum of above two currents,

i.e., $\dfrac{I}{3} + \dfrac{I}{3} = \dfrac{2I}{3}$

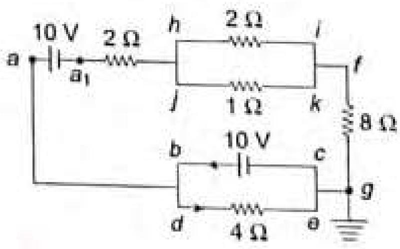
Resistance between 1 and 2 is R. Let remaining resistance is R_1. Then

$$R_1\frac{I}{3} = R\frac{2I}{3} \quad \text{or} \quad R_1 = 2R$$

$$R_{12} = R_{eq} = \frac{RR_1}{R + R_1} = \frac{R2R}{R + 2R} = \frac{2R}{3}$$

79. (d) Potentials of B and D will be same.

80. (a)



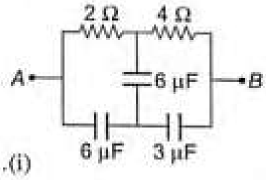
Since battery is in parallel with 4 Ω resistance, it carries current

$$I = \frac{10}{4} = 2.5\,\text{A}$$

In open loop $a_1 a\, d\, e\, g$,

$$V_{a_1} + 10 - 10 = 0 \Rightarrow V_{a_1} = 0\,\text{V}$$

81. (d) Consider the shown circuit.
Current through resistors



$$I = \frac{\varepsilon}{6}$$

$$\therefore \quad V_A - V_C = 2 \times \frac{\varepsilon}{6} = \frac{\varepsilon}{3} \quad \ldots\text{(i)}$$

$$V_A - V_B = \frac{\varepsilon}{3} \quad \ldots\text{(ii)}$$

From equations (i) and (ii)

$\Rightarrow \quad V_C = V_B$

$\therefore$ If 6 μF is connected through B and C it will be short circuited.

So net capacitance $= \dfrac{6 \times 3}{6+3} = 2\,\mu\text{F}$

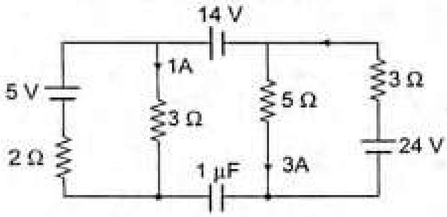
82. (d) Word done by cell $= 2q.V$

$$= 4\left(\frac{1}{2}q.V\right)$$

$= 4$ (energy stored in capacitor)

83. (d) For any values of capacitors and resistors, finally there will be no current through capacitors.

84. (a)



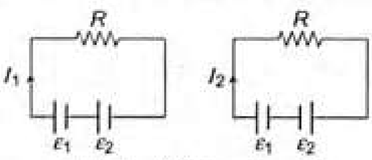
$$V_A + 3 \times 1 - 14 - 5 \times 3 = V_B$$

$\Rightarrow \quad V_A - V_B = 26\,\text{V}$

Energy stored in capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 10^{-6} \times (26)^2 = 338\,\mu\text{J}$$

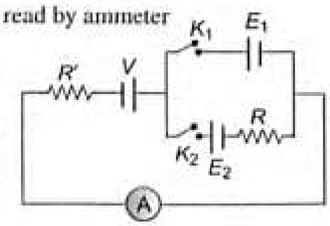
85. (a)



$$\varepsilon_1 + \varepsilon_2 = I_1 R,\ \varepsilon_1 - \varepsilon_2 = I_2 R$$

Dividing $\varepsilon_1 = \left(\dfrac{I_1 + I_2}{I_1 - I_2}\right)\varepsilon_2$

86. (c) If K_2 is closed, the current read by ammeter



$$I = \frac{E_2 - V}{R' + R} \quad \ldots\text{(i)}$$

If K_1 is closed, K_2 is open

$$I = \frac{E_1 - V}{R'} \quad \ldots\text{(ii)}$$

The reading of ammeter is same in both cases

$$\Rightarrow \quad I = \frac{E_2 - V}{R' + R} = \frac{E_1 - V}{R'} \Rightarrow I = \frac{(E_2 - V) - (E_1 - V)}{(R' + R) - R'}$$

Hence, reading of ammeter $\dfrac{E_1 - E_2}{R}$

Archives

1. (c) Metallic conductors have positive value of temperature coefficient of resistance. On the other hand, semiconductors have negative value of temperature coefficient of resistance.

2. (c) Resistance

$$R = \frac{6 \times 3}{6+3}\,\Omega = 2\,\Omega$$

$$I = \frac{3}{2}\,\text{A} = 1.5\,\text{A}$$

3. (a) The new length is $2l$ if the original length is l. Clearly, the new cross-sectional area is $a/2$ if a is the initial cross-sectional area. This is because the volume of the wire has to remain constant.

Now, $R' = \rho \dfrac{2l}{9/2} = 4R$

The increase in resistance is $4R - R = 3R$.

The percentage increase in resistance is $\dfrac{3R}{R} \times 100 = 300\%$

4. (c) 2 Ω and 6 Ω are in parallel.

This gives $\dfrac{2 \times 6}{2+6}\,\Omega = \dfrac{12}{8}\,\Omega = \dfrac{3}{2}\,\Omega = 1.5\,\Omega$

This is in series with 1.5 Ω. This is in parallel with 3 Ω. This gives 1.5 Ω.

Now

$$I = \frac{6\,\text{V}}{1.5\,\Omega} = 4\,\text{A}$$

5. (a) $S = R_1 + R_2$

$$P = \frac{R_1 R_2}{R_1 + R_2}$$

$\because \quad S = nP,$

$$R_1 + R_2 = \frac{nR_1R_2}{R_1 + R_2} \quad \Rightarrow \quad (R_1 + R_2)^2 = nR_1R_2$$

For the minimum value of n, $R_1 = R_2$. Therefore,

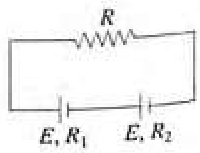
$$(2R_1)^2 = nR_1^2 \Rightarrow n = 4$$

6. (b) $\dfrac{l_1}{l_2} = \dfrac{R_2}{R_1}$

$$= \frac{\rho L_2}{A_2} \times \frac{A_1}{\rho L_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{A_1}{A_2}\right) = \left(\frac{L_2}{L_1}\right)\left(\frac{\pi r_1^2}{\pi r_2^2}\right)$$

$$= \frac{L_2 r_1^2}{L_1 r_2^2} = \frac{3}{4}\left(\frac{2}{3}\right)^2 = \frac{3}{4} \times \frac{4}{9} = \frac{1}{3}$$

7. (c) $V = E - IR_2$



$$V = 0$$

$$E = IR_2$$

$$E = \frac{2\,ER_2}{R + R_1 + R_2}$$

$$\Rightarrow \quad R + R_1 + R_2 = 2R_2$$

$$\Rightarrow \quad R = R_2 - R_1$$

8. (b) According to Kirchhoff's first law, a junction can act neither as a source of charge nor as a sink of charge. This supports the law of conservation of charges. According to Kirchhoff's second law, the energy per unit charge transferred to the moving charges is equal to the energy per unit charge transferred from them. This supports the law of conservation of energy.

9. (d) Using $R_t = R_0(1 + \alpha t)$ twice, we get

$100 = R_0(1 + \alpha \times 100)$ and $200 = R_0(1 + \alpha \times t)$

Dividing, we get

$$2 = \frac{1 + \alpha \times t}{1 + \alpha \times 100}$$

$$\Rightarrow \quad 2 + 200\alpha = 1 + \alpha t$$

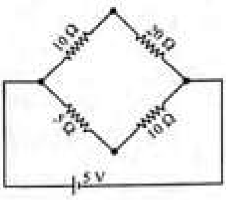
$$1 = \alpha(t - 200)$$

$$\Rightarrow \quad t - 200 = \frac{1}{\alpha} = \frac{1}{0.005} = 200$$

$$t = 400°\text{C}$$

However, if we read the mind of the examiner, considering the given options, we find that we need not use this formula.

10. (d) The given network of resistances constitutes a balanced Wheatstone's bridge. So the given circuit may be redrawn as under:



The resistance of a parallel combination of 30 Ω and 15 Ω is

$$\frac{30 \times 15}{30 + 15}\,\Omega = \frac{450}{45}\,\Omega = 10\,\Omega.$$

Now $I = \dfrac{5\text{ V}}{10\,\Omega} = 0.5\text{ A}$

11. (b) $R = \rho = \dfrac{el}{a} = \dfrac{\rho l \times 4}{\pi D^2}$

For the given data,

$$l \propto \frac{D^2}{\rho}$$

$$\therefore \quad \frac{l_B}{l_A} = \left(\frac{D_B}{D_A}\right)^2 \left(\frac{\rho_A}{\rho_B}\right) = \left(\frac{2D_A}{D_A}\right)^2 \left(\frac{\rho_A}{2\rho_A}\right) = \frac{4}{2} = \frac{2}{1}$$

12. (c) From $R_t = R_0(1 + \alpha t)$, we get

$$5 = R_0(1 + 50\alpha) \qquad \text{(i)}$$

and $$6 = R_0(1 + 100\alpha) \qquad \text{(ii)}$$

$$\therefore \quad \frac{5}{6} = \frac{1 + 50\alpha}{1 + 100\alpha}$$

$$\Rightarrow \quad \alpha = \frac{1}{200}$$

Putting the value of α in (i), we get

$$5 = (R_0 + 50 \times 1/200)$$

$$\Rightarrow \quad R_0 = 4\,\Omega$$

13. (c) $U = \dfrac{1}{2}\dfrac{q^2}{C} = \dfrac{1}{2C}(q_0 e^{-t/T})^2 = \dfrac{q_0^2}{2C} e^{-2t/T}$ (where $\tau = CR$)

$$= U_i e^{-2t/\tau}$$

$$\Rightarrow \quad \frac{1}{2}U_i = U_i e^{-2t_1/\tau} \Rightarrow \frac{1}{2} = e^{-2t_1/\tau} \Rightarrow t_1 = \frac{T}{2}\log_e 2$$

Now $q = q_0 e^{-t/T} \Rightarrow \dfrac{1}{4}q_0 = q_0 e^{-t/2T}$

$$\Rightarrow \quad t_2 = T\log_e 4 = 2T\log_e 2$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{1}{4}$$

14. (d) Let R_0 be the initial resistance of both conductors.

At temperature q, their resistance will be

$R_1 = R_0(1 + \alpha_1\theta)$ and $R_2 = R_0(1 + \alpha_2\theta)$

For series combination,

$$R_s = R_1 + R_2$$

$$R_{s0}(1 + \alpha_s\theta) = R_0(1 + \alpha_1\theta) + R_0(1 + \alpha_2\theta),$$

where $R_{s0} = R_0 + R_0 = 2R_0$

$$2R_0(1 + \alpha_s\theta) = 2R_0 + R_0\theta\,(\alpha_1 + \alpha_2)$$

$$\Rightarrow \quad \alpha_s = \frac{\alpha_1 + \alpha_2}{2}$$

For parallel combination,

$$R_p = \frac{R_1R_2}{R_1 + R_2}$$

$$R_{p0}(1 + \alpha_p\theta) = \frac{R_0(1 + \alpha_1\theta)\,R_0(1 + \alpha_2\theta)}{R_0(1 + \alpha_1\theta) + R_0(1 + \alpha_2\theta)} = \frac{R_0R_0}{R_0 + R_0} = \frac{R_0}{2}$$

where

$$R_{p0} = \frac{R_0R_0}{R_0 + R_0} = \frac{R_0}{2}$$

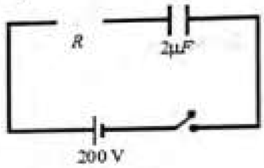
$$\frac{R_0}{2}(1 + \alpha_p\theta) = \frac{R_0^2(1 + \alpha_1\theta + \alpha_2\theta + \alpha_1\alpha_2\theta)}{R_0(2 + \alpha_1\theta + \alpha_2\theta)}$$

α_1 and α_2 are small quantities. So $\alpha_1\alpha_2$ is negligible. Hence,

$$\alpha_p = \frac{\alpha_1 + \alpha_2}{2 + (\alpha_1 + \alpha_2)\theta} = \frac{\alpha_1 + \alpha_2}{2}[1 - (\alpha_1 + \alpha_2)\theta]$$

$(\alpha_1 + \alpha_2)^2$ is negligible. So $\alpha_p = \dfrac{\alpha_1 + \alpha_2}{2}$

15. (c) $V = 200(1 - e^{-t/\tau})$



$$120 = 200(1 - e^{-t/\tau})$$

$$\Rightarrow \quad e^{-t/\tau} = \frac{200 - 120}{200} = \frac{80}{200}$$

$$\frac{t}{\tau} = \log_e 2.5$$

$$= 2.302 \times \log_{10} 2.5 = 2.302 \times 0.4 = 0.9210$$

$$5 = (0.9210) \times R \times 2 \times 10^{-6}$$

$$\Rightarrow \quad R = 2.7 \times 10^6\,\Omega$$

16. (b) $R = \dfrac{\rho\ell}{A}$ $\qquad (\because V = A\ell$ constant)

By differtiation

$$0 = \ell dA + Ad\ell \qquad \text{(i)}$$

By differentiation

$$dR = \frac{\rho(Ad\ell - \ell dA)}{A^2} \qquad \text{(ii)}$$

$$dR = \rho\frac{2Ad\ell}{A^2}$$

$$dR = \frac{2\rho d\ell}{A}$$

or $$\frac{dR}{R} = 2\cdot\frac{d\ell}{\ell}$$

So, $$\frac{dR}{R}\% = 2\cdot\frac{d\ell}{\ell}\% = 2 \times 0.1\%$$

$$\frac{dR}{R}\% = 0.2\,\%$$

17. (d) $Q = c\varepsilon_0 e^{-t/cR}$

$$4\varepsilon = 4\varepsilon_0 e^{-t/t}$$

When $t = 0 \Rightarrow \varepsilon_0 = 25$

$$\varepsilon = \varepsilon_0 = 25$$

When $t = 200 \Rightarrow \varepsilon = 5$

$$5 = 25e^{-\frac{200}{\tau}}$$

$$\ln 5 = \frac{200}{\tau}$$

$$\tau = \frac{200}{\ln 5} = \frac{200}{\ln 10 - \ln 2} = \frac{200}{\ln 10 - 0.693}$$

Alternative

Time constant is the time in which 63% discharging is completed.

So remaining charge = 0.37 × 25 = 9.25 V

Which time in $100 < t < 150$ sec.

18. (b) Charge on the capacitor at any time 't' is

$$q = CV(1 - e^{t/\tau})$$

At $t = 2\tau$

$$q = CV(1 - e^2)$$

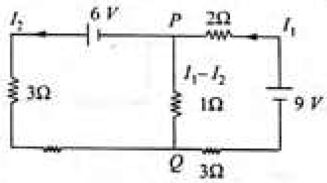
19. (d) $V = IR = I\rho\dfrac{l}{A}$

$$\Rightarrow \quad \rho = \frac{VA}{Il} = \frac{VA}{lneAv_d} = \frac{V}{l \times n \times e \times v_d}$$

$$\Rightarrow \quad \rho = \frac{5}{0.1 \times 2.5 \times 10^{-19} \times 1.6 \times 10^{-19} \times 8 \times 10^{28}}$$

$$= 1.6 \times 10^{-5}\,\Omega\,\text{m}$$

20. (c)



From KVL,

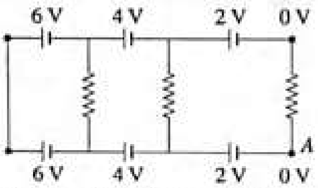
$$9 = 6I_1 - I_2 \qquad \text{(i)}$$

$$6 = 4I_2 - I_1 \qquad \text{(ii)}$$

Solving, $I_1 - I_2 = -0.13$ A

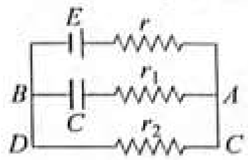
Therefore, 0.13A current will flow from Q to P.

21. (b)



Taking voltage of point A as = 0. Then voltage at other points can be written as shown in the figure. Hence voltage across all resistance is zero. Hence current = 0.

22. (a) In steady state, current through $AB = 0$

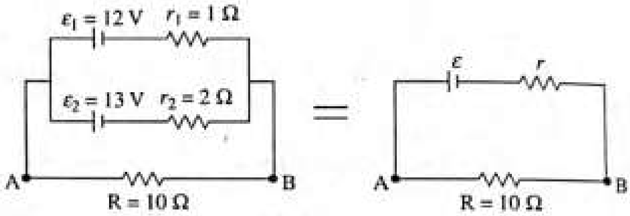


$$\Rightarrow \quad V_{AB} = V_{CD}$$

$$\Rightarrow \quad V_{AB} = \left(\frac{\varepsilon}{r + r_2} \times r_2\right) - V_{CD}$$

$$\Rightarrow \quad Q_C = CV_{AB} = CE\left(\frac{r_2}{r + r_2}\right)$$

23. (c)



Electric emf of the batteries connected in parallel

$$\varepsilon = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3}\text{ V}$$

and effective internal resistance of the combination

$$r = \frac{1}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{2}{3}\Omega$$

Potential difference across load resistance

$$V_{AB} = IR = \frac{\varepsilon}{(r + R)} \times R$$

or $$V_{AB} = \left(\frac{37/3}{2/3 + 10}\right) \times 10 = \frac{37}{32} \times 10 = 11.56\text{ V}$$

Hence, option (c) should be correct.

CHAPTER 20: HEATING EFFECT OF CURRENT AND ELECTRICAL MEASURING INSTRUMENTS

Concept Application Exercise 20.1

1. For balanced Wheatstone bridge, $\dfrac{X}{40}=\dfrac{3}{60}$ or $X = 2\,\Omega$

2. Given, $G = 12.0\,\Omega$, $I_g = 2.5$ mA $= 2.5 \times 10^{-3}$ A
   (a) $I = 7.5$ A, $S = ?$

$$S = G\times\frac{I_g}{I-I_g}=12\times\frac{2.5\times10^{-3}}{7.5-2.5\times10^{-3}}\,\Omega=4.0\times10^{-3}\,\Omega$$

   (b) $V = 10.0$ V, $R = ?$

$$\therefore \quad R=\frac{V}{I_g}-G=\frac{10.0}{2.5\times10^{-3}}-12.0=3988\,\Omega$$

3. We have $I_2 = 1.00$ mA $= 1.00 \times 10^{-3}$ A and $R_2 = 20.0\,\Omega$, and the ammeter should be able to handle maximum current, $I = 50.0 \times 10^{-3}$ A.
   Solving equation $I_g R_g = (I - I_g)R_s$ for R_s, we get

$$R_s=\frac{I_gR_g}{I-I_g}=\frac{(1.00\times10^{-3}\,\text{A})(20.0\,\Omega)}{50.0\times10^{-3}\,\text{A}-1.00\times10^{-3}\,\text{A}}=0.408\,\Omega$$

4. Solving equation for R_S, we have

$$R_s=\frac{V}{I_g}-R_G=\frac{10.0\text{ V}}{0.00100\text{ A}}-20.0\,\Omega=9980\,\Omega$$

   At full-scale deflection, $V_{ab} = 10.0$ V. The voltage across the meter is 0.0200 V, the voltage across R_S is 9.98 V, and the current through the voltmeter is 0.00100 A. In this case, most of the voltage appears across the series resistor. The equivalent meter resistance is $R_{eq} = 20.0 + 9980 = 10{,}000\,\Omega$. Such a meter is described as a "1000 ohms per volt", referring to the ratio of resistance to full-scale deflection voltage. The voltmeter draws off only a small fraction of the current and disturbs only slightly the circuit being measured.

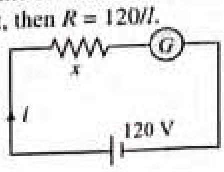
5. Internal resistance of the cell is given by

$$r=R\left(\frac{\ell_1-\ell_2}{\ell_2}\right)=1\times\frac{60-30}{30}\,\Omega=1\,\Omega$$

6. $k=\dfrac{E_B}{L}=\dfrac{1.1}{440}=0.0025\text{ Vcm}^{-1}$

   Potential difference across R is $0.0025 \times 220 = 0.550$ V
   Error in the reading of voltmeter is
   Reading of voltmeter − Reading of potentiometer
   $= 0.5 - 0.55 = -0.05$ V

7. We should connect this galvanometer in series with the given resistor. In general, resistance of galvanometer is small, so it can be neglected. If I is the current in circuit, then $R = 120/I$.



$$I_{max} = 40 \times 10^{-6}\text{ A}$$

$$R_{min}=\frac{120}{40\times10^{-6}}=3\text{ M}\Omega$$

Concept Application Exercise 20.2

1. Resistance of a bulb is given by $R = V^2/P$. Now, 100 W bulb will have less resistance. So we are connecting less resistance in series with the heater. It means potential difference across the heater will increase. So the heater will give more heat.

2. $P_{max}=(i_{max})^2R$ or $18=i_{max}^2\times2$ or $i_{max}=3$ A
   Here $R_{eq} = 3\,\Omega$.
   So the maximum power of circuit is $i_{max}^2\ R_{eq} = 27$ W

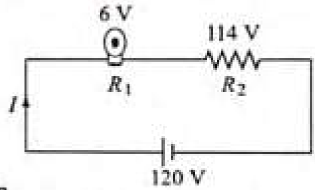
3. We know $R = V^2/P$. Therefore, resistance of the first bulb is $R_1 = V^2/P_1$. And resistance of the second bulb is $R_2 = V^2/P_2$.
   In series, same current will pass through each bulb. Therefore, power developed across the first bulb is $P_1' = I^2 = (V^2/P_1)$ and that across the second bulb is $P_2' = I^2 = (V^2/P_2)$.

$$\frac{\text{Power in bulb 1}}{\text{Power in bulb 2}}=\frac{P_2}{P_1}<1\,(\text{as } P_2<P_1)$$

   Hence, the bulb rated 220 V and 40 W will glow more.

4. Here, $P = 30$ W, $V = 6$ V. Therefore, resistance of the bulb is
   $R_1 = V^2/P = (6)^2/30 = 1.2\,\Omega$
   Current capacity of the bulb

$$I=\frac{P}{V}=\frac{30}{6}=5\text{ A}$$



   Supply voltage, $V' = 120$ V
   Let R_2 be the resistance used in series with the bulb to have a current of 5 A in the circuit. Total resistance is

$$R_2 + R_1 = R_2 + 1.2$$

   Therefore, current $I = V'/(R_2 + 1.2)$

   or $\quad 5=\dfrac{120}{R_2+1.2}$ or $R_2=\dfrac{120}{5}-1.2=22.8\,\Omega$ in series

   Alternatively
   Potential difference across the bulb should not exceed 6 V.
   For this $\dfrac{6}{114}=\dfrac{R_1}{R_2}$ or $R_2=\dfrac{114}{6}R_1=\dfrac{114}{6}\times1.2=22.8\,\Omega$

5. According to Joule heating, the rate at which heat is produced in a resistance is given by $P = I^2R = V^2/R$. And in parallel, V is same, i.e., $V_1 = V_2$. So,

$$\frac{P_2}{P_1}=\frac{R_1}{R_2}=\frac{L_1}{L_2}\left[\frac{r_2}{r_1}\right]^2 \qquad \left[\text{as } R=\rho\frac{L}{\pi r^2}\right]$$

$$=\frac{L_1}{2L_1}\left[\frac{2r_1}{r_1}\right]^2 \qquad [\text{as } L_2=2L_1 \text{ and } r_2=2r_1]$$

   or $\quad P_2 = 2P_1$

   That is heat produced in the thicker coil is more (i.e., double) than that produced in the other coil.

6. As the three bulbs are in series and identical, initially when switch S is open,

$$V_A = V_B = V_C = \frac{1}{3}\text{V}$$

Also $P_A = P_B = P_C = \dfrac{(V/3)^2}{R} = \dfrac{V^2}{9R} = P$

Now, the switch S is closed.

(a) The bulb C is short-circuited and hence potential difference across it is $V'_C = 0$ and so $V'_A = V'_B = V/2$ with $V'_C = 0$. And hence,

$$P'_A = P'_B = \frac{(V/2)^2}{R} = \frac{V^2}{4R}$$

i.e., $P'_A = P'_B = \dfrac{9}{4}P.$

i.e., intensities of bulbs A and B will increase and become 2.25 of their initial values.

(b) As $V'_C = 0$, no current will pass through bulb C, so it will give no light, i.e., $P'_C = 0$.

(c) As R_T changes from $3R$ to $2R$, so the current in the circuit will change from

$$I = \frac{V}{3R} \text{ to } I' = \frac{V}{2R}, \quad \text{i.e.,} \quad I' = \frac{3}{2}I$$

Thus, current in the circuit will increase and will become 1.5 times its initial value.

(d) As explained earlier, initially the voltage V divides equally across A, B, and C, i.e., $V_A = V_B = V_C = V/3$. Now when the switch S is closed, bulb C is short-circuited, i.e., $V'_C = 0$, so voltage V now will divide equally across A and B, i.e., $V'_A = V'_B = V/2$. So voltage across bulbs A and B will change from $V/3$ to $V/2$, while across C from $V/3$ to zero, i.e., voltage drop across A and B will increase while across C it will decrease and become zero. Further as R_T changes from $3R$ to $2R$, so power consumed in the circuit changes from

$$P_T = \frac{V^2}{3R} \text{ to } P'_T = \frac{V^2}{2R}, \quad \text{i.e.,} \quad P'_T = \frac{3}{2}P_T$$

Thus, power dissipated in the circuit increases and becomes 1.5 times its initial value.

Exercises

Heating Effect of Current

1. (a) Power $P = I^2R = \left(\dfrac{V}{R+3r}\right)^2 R$ or $P \propto V^2$

Power ratio $= \left(\dfrac{V_1}{V_2}\right)^2 = \left(\dfrac{4.5}{1.5}\right)^2 = 3^2 = 9$

2. (a) Maximum current flows through bulb 1.

3. (b) When switch S_2 is closed, the whole of current shall flow through the connecting wire only, which is supposed to have zero resistance.

4. (d) Suppose V is the voltage of the supply and R is the resistance of each bulb. Now, $R_p = R/3$ and current in ammeter is $I = V/R_p = 3V/R$, provided all three bulbs are working properly. If one bulb has broken down, then

$$R_p = \frac{R}{2} \quad \text{and} \quad I = 2\frac{V}{R}$$

Therefore, current decreases and since current through each bulb is V/R, the same as before, brightness of bulbs is not affected.

5. (a) Consider two extreme cases: (i) When the resistance of the rheostat is zero, the current through Q is zero since Q is short-circuited. The circuit is then essentially a battery in series with lamp P. (ii) When the resistance of the rheostat is very large, almost no current flows through it. So the currents through P and Q are almost equal. The circuit is essentially a battery in series with lamps P and Q.

6. (c) Let the resistance of the two heaters be denoted by R_1 and R_2, respectively. Then

$$R_1 = \left(\frac{V^2}{P_1}\right) \text{ and } R_2 = \left(\frac{V^2}{P_2}\right)$$

If the resistance of the series combination is denoted by R_S and the corresponding power by R_S, then $R_S = R_1 + R_2$. So

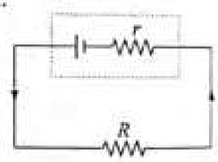
$$\frac{V^2}{P_S} = \frac{V^2}{P_1} + \frac{V^2}{P_2}$$

or $\quad P_s = \dfrac{P_1 P_2}{P_1 + P_2} = \dfrac{1000 \times 1000}{2000} = 500\text{ W}$

7. (c) Given $P_A = 60$ W, $P_B = 100$ W. We know that current flowing through the bulb is $I = P/V$. We also know that as both the bulbs are connected in parallel, potential difference (V) across both the bulbs is the same. Thus, $I \propto P$. Since the power of bulb B is greater than that of bulb A, bulb B draws more current than bulb A.

8. (a) After disconnecting B_2 and B_3, the whole of potential difference will be across B_1. Hence, potential difference across B_1 increases, so its brightness increases.

9. (d) Let current I flow through the circuit. Energy dissipated in load is I^2R. Energy dissipated in the complete circuit is $I^2(r + R)$. Therefore, the efficiency is



$$\frac{I^2R}{I^2(R+r)} = \frac{R}{R+r}$$

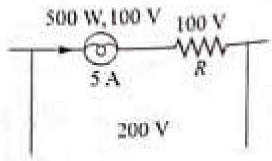
10. (b) Let both the batteries be connected in series. Power output is maximum when external resistance is 2 Ω. Current in the circuit is 4 V/4 Ω = 1 A and power in the external circuit is $(1)^2 \times 2 = 2$ W.

11. (d) Power consumed by each lamp is 24 W. Hence, using $R = (V^2/P)$, we find $R = (36/24) = 1.5$ Ω.

12. (b) $P = \dfrac{V^2}{R}, \; \dfrac{P_1}{P_2} = \dfrac{R_2}{R_1}$ or $r_2 = 2r_1$

13. (a) $P = i^2R, \; \dfrac{dP}{P} = 2\dfrac{dI}{I} = 2 \times 0.5\% = 1\%$

14. (b) $P = VI$ or $I = \dfrac{P}{V}$



or $\quad I = \dfrac{500\text{ W}}{100\text{ V}} = 5\text{ A}$

Now, $5R = 100$ or $R = 20$ Ω

15. (b) Power $= \dfrac{V^2}{R}$

or $V = \sqrt{PR} = \sqrt{2 \times 10} = \sqrt{20} = 2\sqrt{5}$ V

Clearly, voltage across single resistor of 10 Ω cannot exceed $2\sqrt{5}$ V. Note that the resistance of parallel combination is half of 10 Ω. Thus, the maximum possible voltage between A and B is $3\sqrt{5}$ V.

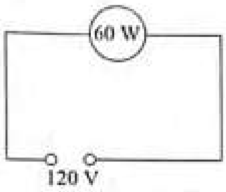
16. (d) Resistance of bulb is $\dfrac{1.5 \times 1.5}{4.5}\ \Omega = 0.5\ \Omega$

Resistance of parallel combination,

$$R = \frac{1 \times \frac{1}{2}}{1 + \frac{1}{2}}\ \Omega = \frac{1}{3}\ \Omega$$

Now, $r = \dfrac{E - V}{V} R$ or $\dfrac{8}{3} = \dfrac{E - 1.5}{1.5} \times \dfrac{1}{3}$ or $E = 13.5$ V

17. (d) $R_{60} = \dfrac{120 \times 120}{60}\ \Omega = 240\ \Omega$

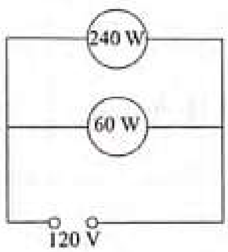


Current $= \dfrac{120}{240 + 6}$ A $= \dfrac{120}{246}$ A

Voltage across bulb $= \dfrac{120}{246} \times 240$ V

$= 117.1$ V

$R_{240} = \dfrac{120 \times 120}{240}\ \Omega = 60\ \Omega$



Resistance of parallel combination is

$$\frac{60 \times 240}{60 + 240}\ \Omega = 48\ \Omega$$

Total resistance is

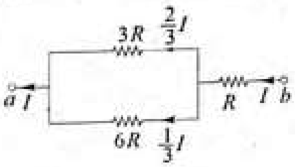
$(48 + 6)\ \Omega = 54\ \Omega$.

Current I is 120/54A. Voltage across parallel combination is

$$\frac{120}{54} \times 48 \text{ V} = 106.7 \text{ V}$$

Change in voltage is (117.1 – 106.7) V = 10.4 V.

18. (c) Let current flow from b to a as shown (figure).



Ratio is $\left(\dfrac{2}{3}I\right)^2 3R : \left(\dfrac{1}{3}I\right)^2 6R : I^2R$

or $\dfrac{4}{3} : \dfrac{2}{3} : 1$ or $4 : 2 : 3$

19. (a) Let resistors P, Q, and R have resistance r. The effective resistance across the source is

$$R_{\text{eff}} = r + r \,||\, r = r + \frac{(r)(r)}{r + r} = r + \frac{r}{2} = \frac{3r}{2}$$

Current drawn from source is

$$I_s^2 R_{\text{eff}} = 12 \quad \text{or} \quad I_s = \sqrt{\frac{12}{R_{\text{eff}}}} = \sqrt{\frac{8}{r}} \text{ A}$$

Since Q and R have equal resistance r, each draws a current of I, which is given by

$$I = \frac{1}{2} I_s = \sqrt{\frac{2}{r}} \text{ A}$$

Heat dissipation in R can now be determined and is given by

$$I^2 r = \left(\frac{2}{r}\right) r = 2 \text{ W}$$

20. (a) Since all the three resistors are in series, the same current will flow through them. Their resistances are given by $R = V^2/P$.

$$R_{40} = \frac{240 \times 240}{40} = 1440\ \Omega$$

$R_{60} = 960\ \Omega$ and $R_{100} = 576\ \Omega$

Total resistance is $R = 2976\ \Omega$

$$I = \frac{240}{2976} = 0.0806 \text{ A}$$

Potential difference across the 40 W bulb is

$1440 \times 0.0806 = 116$ V

21. (b) Let current in 5 Ω resistor be I_1 and in 4 Ω resistors be I_2, then $I_2/I_1 = 1/2$. The heat generated in the 5 Ω resistor is 10 cal s^{-1} Given $I^2_1 \times 5 = 10$ or $(2I_2)^2 \times 5 = 10$ or $I_2^2 = 1/2$. Heat in the 4 Ω resistor will be $(I_2)^2 \times 4 = 2$ cal s^{-1}.

Ammeter and Voltmeter

22. (b) We know that

$$R = \frac{V}{I_g} - G$$

The voltmeter gives the full-scale deflection for potential difference V. Its resistance is G. Hence, $I_g = (V/G)$. Given that $V = nV$. Therefore,

$$R = \frac{nV}{(V/G)} - G = (n - 1)\, G$$

23. (b) $\dfrac{I_g}{I} = \dfrac{S}{S + G}$ or $\dfrac{1}{34} = \dfrac{S}{S + G}$

∴ $S = (G/33) = (3663/33) = 111\ \Omega$

24. (c) $R_s = \dfrac{SG}{S + G} = \dfrac{111 \times 3663}{111 + 3663} = 107.7\ \Omega$

25. (d) Compensation external resistance

$$G - \frac{SG}{S + G} = 3663 - 107.7 = 3555.3\ \Omega$$

26. (a) Total range is doubled, i.e., $4I_g$

Now shunt required is

$$S = \frac{I_g}{4I_g - I_g} \times G = 10\ \Omega$$

$$\frac{1}{30} + \frac{1}{x} = \frac{1}{10} \quad \text{or} \quad x = 15\ \Omega$$

27. (d) $\dfrac{I_g}{I} = \dfrac{S}{S + G} = \dfrac{(G/10)}{(G/10) + G} = \dfrac{1}{11}$

Initially, $\alpha_i = \theta/I_g$ (i)

Finally, after the shunt is used,

$\alpha_f = \theta/I$ (ii)

$$\therefore \quad \frac{\alpha_i}{\alpha_i} = \frac{\theta/I}{\theta/I_g} = \frac{I_g}{I} = \frac{1}{11}$$

So current sensitivity becomes 1/11-fold.

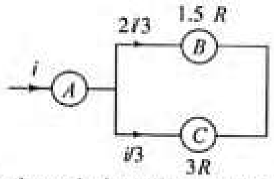
28. (c) $i_g = \frac{V}{G+R}$ or $10^{-3} = \frac{2.5}{10+R}$ or $R = 2490\ \Omega$

29. (c) $S = \frac{i_g G}{(I - i_g)} = \frac{0.01 \times 1}{1 - 0.01} = \frac{1}{99}\ \Omega$

30. (a) $V_A = iR,$

$V_B = \frac{2i}{3} \times 1.5R = iR,$

$V_C = (i/3)(3R) = iR$



31. (b) Let a current of x ampere pass through the voltmeter; then $(4 - x)$ ampere passes through the resistance R. Therefore, voltmeter reading is

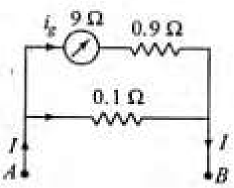
$$20 = (4 - x)R, \text{ i.e., } R = \frac{20}{4 - x}, \text{ i.e., } R > 5\ \Omega$$

32. (c) $i_g = 10\text{ mA} = 0.01\text{ A}$

$V_A - V_B = (I - i_g)\,0.1$

$= i_g(9 + 0.9)$

or $\quad I = \frac{10 \times 0.01}{0.1} = 1\text{ A}$



33. (a) $3 = IR_1$ or $3 = 1 \times R_1$

or $\quad R_1 = 3\ \Omega$

When points A and B are connected by a conducting wire, R_2 is short-circuited. Therefore,

$10.5 = I'R_1$ or $10.5 = I' \times 3$

or $\quad I' = \frac{10.5}{3} = 3.5\text{ A}$

But $\quad 10.5 = E - I'r$ or $10.5 = 12 - 3.5r$

$\therefore \quad r = \frac{1.5}{3.5} = \frac{3}{7}\ \Omega$

34. (c) Current through R is $12/(500 + R)$

Voltage across R is $\frac{12R}{500 + R}$

Since galvanometer shows zero deflection, so

$$\frac{12R}{500 + R} = 2 \quad \text{or} \quad R = 100\ \Omega$$

35. (b) A careful analysis would show that the voltage along R is 1.03 V. So

$1.03 = 1 \times R$ or $R = 1.03\ \Omega$

36. (c) If V is the potential difference applied across P and Q, the current through M is determined as follows:

Circuit	Current
(a)	$V/4$
(b)	$3\ V/8$
(c)	$V/2$
(d)	$V/3$

Hence, circuit arrangement (c) gives the largest reading in ammeter M.

37. (a) $I = \frac{k}{NBA}\theta$. Given that $I_1 = I_2$

$$\therefore \quad \frac{K\theta_1}{N_1 BA_1} = \frac{K\theta_2}{N_2 BA_2} \quad \text{So} \quad \frac{\theta_1}{\theta_2} = \frac{A_1 N_1}{A_2 N_2}$$

Meter Bridge and Potentiometer

38. (b) When Wheatstone bridge is balanced,

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad \frac{P}{R} = \frac{Q}{S}$$

If the galvanometer is replaced with a cell in balanced Wheatstone bridge, the condition for balanced bridge will be $P/R = Q/S$, which is there. Hence, balance point will remain unchanged, where galvanometer shows no current.

39. (a) At null point, $R_1/R_2 = R_3/R_4 = x/(100 - x)$. If radius of the wire is doubled, then the resistance of AC will change and the resistance of CB will also change. But since R_1/R_2 does not change, so R_3/R_4 should also not change at null point. Therefore, point C does not change.

40. (a) $\frac{X}{Y} = \frac{20}{80} = \frac{1}{4}$ or $Y = 4X$

$\frac{4X}{Y} = \frac{l}{100 - l}$ or $\frac{4X}{4X} = \frac{l}{100 - l}$ or $l = 50$ cm

41. (b) $P/Q = R/S$. If P is increased, then either R or Q should be increased or S should be decreased.

42. (a) Using the principle of potentiometer, $V \propto l$. So

$$\frac{V}{E} = \frac{l}{L} \quad \text{or} \quad V = \frac{l}{L}E = \frac{30}{100}E = \frac{30E}{100}$$

43. (a) $r = \frac{l_1 - l_2}{l_2}R = \frac{240 - 120}{120} \times 2\ \Omega = 2\ \Omega$

44. (b) Sensitivity of potentiometer means the smallest potential difference it can measure. It can be increased by reducing the potential gradient. The same is possible by increasing the length of the potentiometer.

45. (b) In case of internal resistance measurement by potentiometer,

$$\frac{V_1}{V_2} = \frac{\ell_1}{\ell_2} = \frac{[ER_1/(R_1 + r)]}{[ER_2/(R_2 + r)]} = \frac{R_1(R_2 + r)}{R_2(R_1 + r)}$$

Here $\ell_1 = 2$ m, $\ell_2 = 3$ m, $R_1 = 5\ \Omega$, and $R_2 = 10\ \Omega$. So

$$\frac{2}{3} = \frac{5(10 + r)}{10(5 + r)} \quad \text{or} \quad r = 10\ \Omega$$

46. (c) $k = 0.05\text{ mVcm}^{-1} = 5\text{ mVm}^{-1}$

$$I = \frac{kl}{r} = \frac{5 \times 10^{-3} \times 1}{5} = 10^{-3}\text{ A}$$

Now, $2 = I(R + r) = 10^{-3}(R + 5)$ or $R = 1995\ \Omega$

47. (a) The value of E = Potential gradient × Length

$= (5\text{ mVm}^{-1})(60\text{ cm}) = 3\text{ mV}$

48. (a) Current in $AB = I = \frac{E}{R + r}$

Potential difference across $AB = IR = \frac{ER}{R + r}$

To obtain balanced point, $\frac{ER}{R+r} > \frac{E}{3}$ or $R > r/2$

49. (b) When X is connected to Y, the balance length l is proportional to the potential difference across the 100 Ω resistor. When X is connected to Z, the balance length is proportional to the potential difference across the 100 Ω resistor and resistor R.

$$\frac{100+R}{100} = \frac{588}{400} \text{ or } R = 47\ \Omega$$

50. (b) The voltage per unit length on the meter wire PQ is

$$\frac{6.00\ \text{mV}}{0.60\ \text{m}} \text{ or } 10\ \text{mVm}^{-1}$$

Hence, potential across the meter wire PQ is 10 mVm^{-1} (1 m) = 10 mV. Current drawn from the driver cell is

$$I = \frac{10\,\text{mV}}{5\,\Omega} = 2\ \text{mA}$$

Resistance of the resistor R is

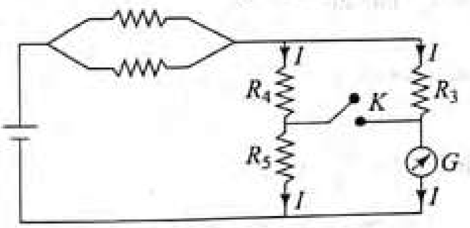
$$R = \frac{2\,\text{V} - 10\,\text{mV}}{2\,\text{mA}} = \frac{1990\,\text{mV}}{2\,\text{mV}} = 995\ \Omega$$

51. (b) Let current through R is I_0 and voltmeter reading is V_0, then $R' = V_0/I_0$, also $V_0 = I_0(R + R_A)$

$$\frac{V_0}{I_0} = R + R_A$$

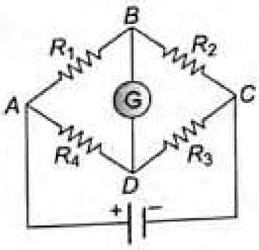
$$R' = R + R_A \quad \text{or} \quad R = R' - R_A$$

52. (c) If the reading of galvanometer does not change, then Wheatstone bridge with R_3, R_4, R_5, G and key K is balanced.



$$I(R_4) = I(R_5) \quad \text{and} \quad I(R_3) = I_G$$

53. (b)



So, $R_1R_3 = R_2R_4$

54. (b) In the part $c\ b\ d$,

$$V_c - V_b = V_b - V_d \Rightarrow V_b = \frac{V_c + V_d}{2}$$

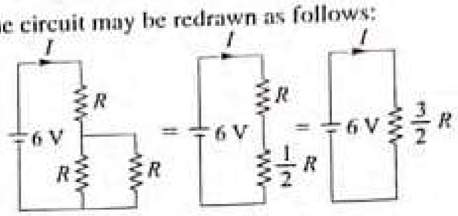
In the part $c\ a\ d$,

$$V_c - V_a > V_a - V_d \Rightarrow \frac{V_c + V_d}{2} > V_a \Rightarrow V_b > V_a$$

55. (c) In balance condition, no current will flow through the branch containing S.

Problems Based on Mixed Concepts

56. (b) The circuit may be redrawn as follows:



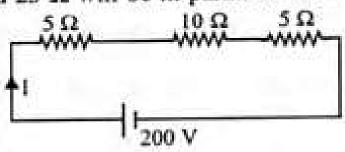
Current is given by $I = \frac{6}{\frac{3}{2}R} = \frac{4}{R}$ A

Therefore, current through the voltmeter is $I/2$ or $2/R$ A. Hence, the reading of the voltmeter is $(2/R)(R)$ or 2 V.

57. (c) Initially, when switch is open, V_2 and V_3 will be out of circuit or they are short-circuited. So no potential difference is across V_2 and V_3. On closing the switch, all three voltmeters will be in parallel. Now, there is some potential difference across V_2 and V_3. So potential difference across V_2 and V_3 increases and potential difference across V_1 remains the same.

58. (b) $P = \frac{V^2}{R}$ or $P \propto \frac{1}{R}$

59. (b) Resistors 20 Ω, 100 Ω, and 25 Ω will be in parallel. Their equivalent is 10 Ω.



$$I = \frac{200}{5+10+5} = 10\ \text{A}$$

Potential difference across 10 Ω is $10\,I = 10 \times 10 = 100$ V

This will be the voltmeter reading. Also, this will be the potential difference across each of 20 Ω, 100 Ω, and 25 Ω resistors.

Ammeter reading = current through 25 Ω = $\frac{100}{25} = 4$ A

60. (c) It is a case of weak current.

61. (b) For the maximum power, external resistance is equal to internal resistance. Therefore, $2R = 4$ or $R = 2\ \Omega$.

62. (a) Net resistance of circuit, $R = \frac{10\times5}{10+5} + \frac{10\times5}{10+5} = \frac{20}{3}\ \Omega$

Heat generated in circuit per minute is

$$Q = I^2Rt = (10)^2 \times \frac{20}{3} \times (10\times60)$$

$$= 4\times10^5\ \text{J} = \frac{4\times10^5}{4.2\times10^3}\ \text{kcal}$$

$$m = \frac{Q}{L} = \frac{4\times10^5}{4.2\times10^3\times80} = 1.19\ \text{kg}$$

63. (a) $P_I = I^2(3R)$, $P_{II} = I^2\left(\frac{2R}{3}\right)$, $P_{III} = I^2\left(\frac{R}{3}\right)$

$$P_{IV} = I^2\left(\frac{3R}{2}\right)$$

$\therefore$ III < II < IV < I

64. (a) As both bulbs are in series, current through them will be the same. But resistance of 25 W bulb is more. Hence, from the relation $P = I^2R$, the 25 W bulb will glow more brightly.

65. (b) Let R be the resistance of each lamp. If E is the applied emf, then the current in the circuit I_1 is given by

$$I_1 = \frac{E}{R+(R/2)} = 2E/3R$$

Current flowing through L_2 or L_3 is

$$\frac{1}{2}\left[\frac{2E}{3R}\right] = \frac{E}{3R}$$

When L_3 is fused, the whole current flows through L_1 and L_2. Thus $I_2 = E/2R$. So current through L_1 decreases and the current through L_2 increases.

66. (a) emf should be 125 V.

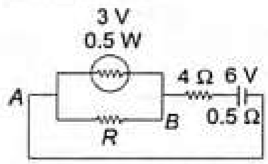
for second case: $\frac{100}{2500} = \frac{25}{R} \Rightarrow R = 625\ \Omega$

67. (c) The percentage error in R can be minimised by adjusting the balance point near the middle of the bridge, i.e., when l_1 is close to 50 cm. This requires a suitable choice of S.

Since $\frac{R}{S} = \frac{Rl_1}{R(100-l_1)} = \frac{l_1}{100-l_1}$

Since here, $R : S :: 2.9 : 97.1$ imply that the S is nearly 33 times to that of R. In order to make this ratio 1:1, it is necessary to reduce the value of S nearly $\frac{1}{33}$ times i.e., nearly 3 Ω.

68. (b) The electric bulb consumes the rated power if the voltage across electric bulb is 3 V.



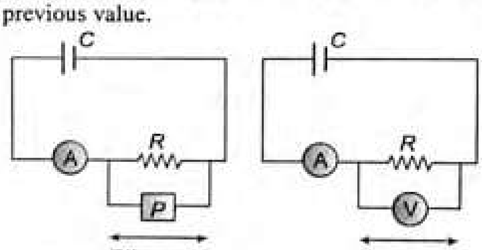
i.e., resistance across AB is 4.5 Ω.

Resistance of bulb, $R_b = \frac{V^2}{P_b} = \frac{3^2}{0.5} = 18\Omega$

$$\frac{RR_b}{R_b+R} = 4.5\Omega$$

$$18R = \frac{9}{2}(18+R) \Rightarrow R = 6\Omega$$

69. (a) Effective resistance of R and V is less than effective resistance of R and P, hence p.d. across voltmeter (or R) is less than previous value.



Eff res = R Eff res < R

Also overall resistance decreases, so overall current increases, hence reading of ammeter increases.

70. (b) When S_1 is closed and S_2 is open, W_1 is in circuit.

Current through W_1.

$$I_1 = \frac{\varepsilon_P}{2+1} = \frac{\varepsilon_P}{3}$$

Potential difference across $\frac{l}{2}$ length of W_1.

$$= \frac{1}{2}[2I_1] = \frac{\varepsilon_P}{3}$$

This should be equal to ε.

So $\frac{\varepsilon_P}{3} = \varepsilon \Rightarrow \varepsilon_P = 3\varepsilon$

Similarly for second case:

$$I_2 = \frac{\varepsilon_P}{R+1} = \frac{3\varepsilon}{R+1}$$

Potential difference $2l/3$ length of W_2.

$$= \frac{2}{3}I_2R = \frac{2}{3}\left(\frac{3\varepsilon}{R+1}\right)R = \frac{2\varepsilon R}{R+1}$$

This should be equal to ε, so $\frac{2\varepsilon R}{R+1} = \varepsilon$

$\Rightarrow \quad R = 1\ \Omega$

71. (b) In series, potential difference $\propto R$

When only S_1 is closed, $V_1 = \frac{3}{4}E = 0.75\ E$

When only S_2 is closed, $V_2 = \frac{6}{7}E = 0.86\ EPl$

and when both S_1 and S_2 are closed combined resistance of $6R$ and $3R$ is $2R$.

$$V_3 = \left(\frac{2}{3}\right)E = 0.67\ E$$

Hence, $V_2 > V_1 > V_3$.

72. (b) $P = \frac{V^2}{R}$ Initially, $I = \frac{V}{2R}$

Power across $= P_X = P_Y\left(\frac{\varepsilon^2}{4R}\right)R$

Finally, $I = \frac{2V}{3R}$, Power $P_X = \frac{4V^2}{9R}$, $P_Y = P_Z = \frac{2V^2}{9R}$

Hence, P_X increases, P_Y decreases.

73. (a) In position (1)

$\varepsilon - ir = 0 \quad i = 2$

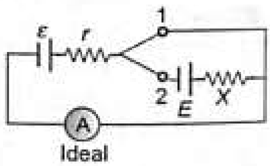
$\Rightarrow \quad \varepsilon = 2r.$

Now, in position (2)

$\varepsilon - ir + E - ix = 0$

$\Rightarrow \quad 2r - 2r + 10 - 2x = 0$

$\Rightarrow \quad x = 5\ \Omega$



74. (a) $P = I^2R = \left(\frac{V}{R}\right)^2 R = \frac{\varepsilon^2}{(R+r)^2}R$

ε is constant and $(R+r)$ increases rapidly then $P\downarrow$

Archives

1. (b) $150 = \frac{15\times15}{R} \Rightarrow R = \frac{15\times15}{150}\Omega$

$\Rightarrow \quad R = \frac{15}{10}\Omega = \frac{3}{2}\Omega$

Now $\frac{2R}{2+R} = \frac{3}{2}$

$\Rightarrow \quad 4R - 3R = 6 \Rightarrow R = 6\ \Omega$

2. (b) $P = \dfrac{V^2}{R}$

R is reduced by a factor of 4. So P is increased by a factor of 4.

3. (a) Using the principle of potentiometer, we get

$V \propto l$

$\therefore \quad \dfrac{V}{E} = \dfrac{l}{L} \Rightarrow V = \dfrac{l}{L}E = \dfrac{30}{100}E$

4. (d) $P = \dfrac{V^2}{R}$

If V is halved, P is reduced by a factor of 4.

So the new power is $\dfrac{1000}{4}$ W = 250 W

5. (a) $\dfrac{X}{Y} = \dfrac{20}{80} = \dfrac{1}{4}$

$\Rightarrow \quad Y = 4X \quad \Rightarrow \quad \dfrac{4X}{Y} = \dfrac{l}{100} - 1$

$\Rightarrow \quad \dfrac{4X}{4X} = \dfrac{l}{100 - l} \quad \Rightarrow \quad l = 100 - l$

$\Rightarrow \quad 2l = 100\text{ cm} \quad \Rightarrow \quad l = 50\text{ cm}$

6. (c) $Pt = mS\theta$

$\Rightarrow \quad t = \dfrac{mS\theta}{P} \times \dfrac{1 \times 4180 \times 30}{836}\text{ s} = 150\text{ s}$

7. (c) Current through R is $\dfrac{12}{500 + R}$

Voltage across R is $\dfrac{12R}{500 + R}$

Since the galvanometer shows zero deflection,

$\dfrac{12R}{500 + R} = 2$

$\Rightarrow \quad 12R = 1000 + 2R$

$\Rightarrow \quad 10R = 1000$

$\Rightarrow \quad R = 100\ \Omega$

8. (b) Factual information

9. (a) $r = \dfrac{l_1 - l_2}{l_2}R = \dfrac{240 - 120}{120} \times 2\ \Omega = 2\ \Omega$

10. (d) $Q = \dfrac{V^2}{R}t$

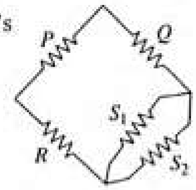
When R is halved, Q is doubled.

11. (d) $R_{\text{hot}} = 10\, R_{\text{cold}}$

$R_{\text{hot}} = \dfrac{V^2}{P} = \dfrac{20 \times 200}{100} = 400\ \Omega$

$\therefore R_{\text{cold}} = \dfrac{400}{10}\ \Omega = 40\ \Omega$

12. (d) Using the condition for Wheatstone's bridge to be balanced, we get

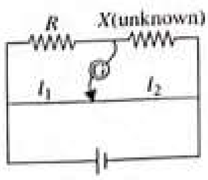


$\dfrac{P}{Q} = \dfrac{R}{\dfrac{S_1 S_2}{S_1 + S_2}} = \dfrac{R(S_1 + S_2)}{S_1 S_2}$

13. (a) $R = \dfrac{V^2}{P} = \dfrac{220 \times 220}{100}\ \Omega = 484\ \Omega$

$P' = \dfrac{V'^2}{R} = \dfrac{100 \times 100}{484}\text{ W} = 25\text{ W}$

14. (d) $\dfrac{R}{l_1} = \dfrac{X}{l_2}$



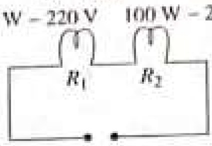
On increasing the temperature, X increases.

Therefore, R should be increased to keep the null point same.

15. (d) For a balanced meter bridge (null deflection),

$\dfrac{55}{R} = \dfrac{20}{80} \quad \Rightarrow \quad R = 220\ \Omega$

16. (c) 25 W – 220 V   100 W – 220 V



As $\quad R_1 = \dfrac{220}{25} \times 220$ and $R_2 = \dfrac{220}{100} \times 220$

$R = R_1 + R_2$

$= 220 \times 220\left(\dfrac{1}{25} + \dfrac{1}{100}\right)$

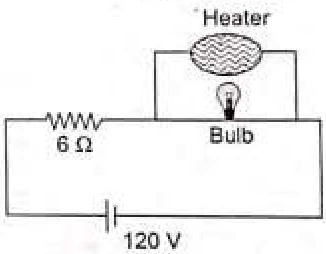
$= 220 \times 220 \dfrac{1}{20}$

$\therefore \quad I_{\text{live}} = \dfrac{440}{\dfrac{220 \times 220}{20}} = \dfrac{40}{220}$

$\therefore \quad 1^{\text{st}}$ bulb (25 W) will fuse only.

17. (c) Resistance of bulb $= \dfrac{120 \times 120}{60} = 240\ \Omega$

Resistance of heater $= \dfrac{120 \times 120}{240} = 60\ \Omega$



Voltage across bulb before heater is switched on,

$V_1 = \dfrac{120}{246} \times 240$

Voltage across bulb after heater is switched on,

$V_2 = \dfrac{120}{54} \times 48$

Decrease in the voltage is $V_1 - V_2 = 10.04$ (approximately)

NOTE: Here supply voltage is taken as rated voltage.

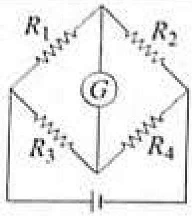
18. (a)

Item	Number	Power
40 W bulb	15	600 watt
100 W bulb	5	500 watt
80 W fan	5	400 watt
1000 W heater	1	1000 watt

Total Wattage = 2500 Watt

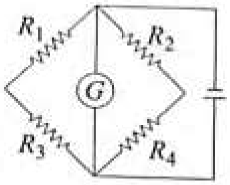
So current capacity $i = \dfrac{P}{V} = \dfrac{2500}{220} = \dfrac{125}{11} = 11.36 \cong 12$ A

19. (d) Options (a), (b) and (c) are theoretically correct statements. Now let us check, if the cell and the galvanometer are exchanged.



On balancing condition

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$



On balancing condition

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

We observe here that in both cases we are getting same equations, hence the null point does not disturb. So (d) statement is false.

20. (c) $i_g = 5 \times 10^{-3}$ A

$G = 15\ \Omega$

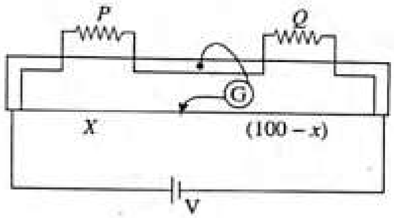
Let series resistance be R.

$V = i_g(R + G)$

$10 = 5 \times 10^{-3}(R + 15)$

$R = 2000 - 15 = 1985 = 1.985 \times 10^3\ \Omega$

21. (d)



Initially $\dfrac{P}{Q} = \dfrac{x}{100 - x}$ ....(i)

When resistances are interchanged

$\dfrac{Q}{P} = \dfrac{x - 10}{(110 - x)}$ ....(ii)

Also given $P + Q = 1000$ ....(iii)

From (i) $\dfrac{P}{P+Q} = \dfrac{x}{x + 100 - x} = \dfrac{x}{100}$ ....(iv)

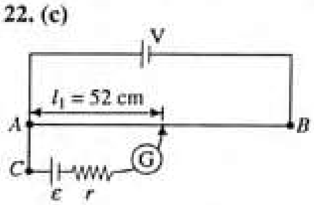
or $\dfrac{P}{1000} = \dfrac{x}{100}$ or $P = 10x$ ....(v)

From (ii) $\dfrac{Q+P}{P} = \dfrac{(x-10)+(110-x)}{(110-x)}$

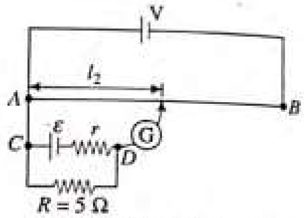
or $\dfrac{1000}{P} = \dfrac{100}{(110-x)}$ ...(vi)

From (v) and (vi) $P = 550\Omega$

22. (c)



Case I: Initially



Case II: where R is shunted

In case I: The potential difference across the length l1 will be balanced by the emf of the cell

or $\varepsilon = K\ell_1$ ....(i)

In case II: The potential difference across the length l2 will be balanced by the potential difference across the shunt resistance 'R'

$R\left(\dfrac{\varepsilon}{R+r}\right) = K\ell_2$ ....(ii)

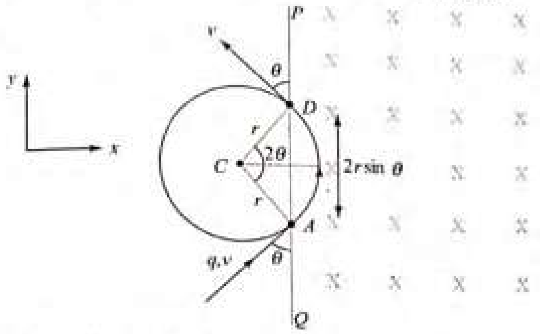
From (ii) / (i) we get $\dfrac{R}{R+r} = \dfrac{\ell_2}{\ell_1}$

or $r = R\left(\dfrac{\ell_1}{\ell_2} - 1\right) = 5\left(\dfrac{52}{40} - 1\right) = 5\left(\dfrac{52-40}{40}\right) = 1.5\Omega$

CHAPTER 21: SOURCE AND EFFECTS OF MAGNETIC FIELD

Concept Application Exercise 21.1

1. The particle will move in the field as shown in figure.



Angle subtended by the arc at the centre = 2θ

(a) Time spent by the charge in magnetic field

$$t = \frac{2\theta}{\omega} \Rightarrow t = \frac{m2\theta}{qB}$$

(b) Distance travelled by the charge in magnetic field

$$= r(2\theta) = \frac{mv}{qB} \times 2\theta$$

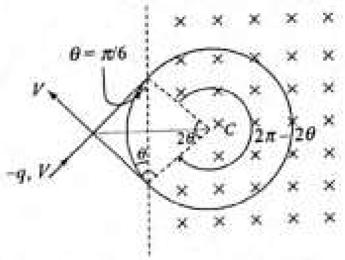
(c) Impulse = change in momentum of the charge

$$(-mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}) - (mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j})$$
$$= -2mv\sin\theta\,\hat{i}$$

2. (a) $2\pi - 2\theta = 2\pi - 2\times\frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} = \omega t = \frac{qB}{m}t$

$$\Rightarrow t = \frac{5\pi m}{3qB}$$

(b) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$



(c) Impulse = charge in linear momentum

$$= m(-v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j}) - m(v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j})$$
$$= -2mv\sin\theta\,\hat{i} = -2mv\sin\frac{\pi}{6}\,\hat{i} = -mv\,\hat{j}$$

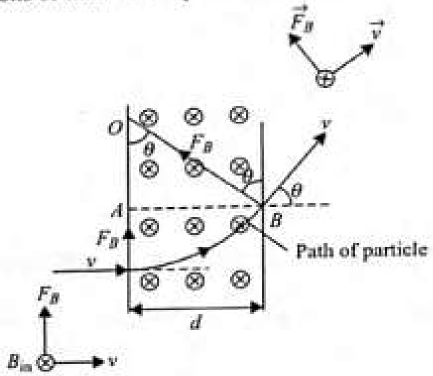
3. $qV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2qV}{m}}$

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2mV}{qB^2}} \Rightarrow m \propto r^2$$

$$\frac{m_{\text{heavy}}}{m_{\text{light}}} = \left(\frac{l+d}{l}\right)^2$$

4. The radius of circulation is $R = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2mK}}{Bq}$.

As given, angle of deviation is θ; it is the angle between directions of final velocity and initial velocity.



In ΔOAB, $\sin\theta = \frac{d}{R} = \frac{dBq}{mv} = \frac{dBq}{p} = \frac{dBq}{\sqrt{2mK}}$

(a) $p_\alpha = p_p$, $\frac{\sin\theta_p}{\sin\theta_\alpha} = \frac{q_p}{q_\alpha} = \frac{1}{2}$

(b) $K_\alpha = K_p$; hence $\frac{\sin\theta_p}{\sin\theta_\alpha} = \frac{q_p}{q_\alpha}\times\sqrt{\frac{m_\alpha}{m_p}} = \frac{1}{2}\times\sqrt{\frac{4}{1}} = 1$

(c) When a charged particle is accelerated through a potential difference V, gain of kinetic energy is qV.

$$K_p = q_\alpha V,\ K_\alpha = q_\alpha V$$

Now, $\sin\theta = \frac{dBq}{\sqrt{2mqV}} = \frac{dB\sqrt{q}}{\sqrt{2mV}}$

Hence, $\frac{\sin\theta_p}{\sin\theta_\alpha} = \sqrt{\frac{q_p}{q_\alpha}}\times\sqrt{\frac{m_\alpha}{m_p}} = \sqrt{\frac{1}{2}}\times\sqrt{\frac{4}{1}} = \sqrt{2}$

5. Let R_1 be the radius of trajectory of the isotope $^{12}C_6$ and R_2 that of the unknown isotope. The trajectory of the unknown isotope has a greater radius, and so the mass of the unknown isotope is greater than that of the $^{12}C_6$ isotope. From the figure provided in the question,

$$d = 2r = \frac{2mv}{qB} \Rightarrow m \propto d$$

$$\frac{mA_2}{mA_1} = \frac{d_2}{d_1} = \frac{A_2}{12} = \frac{35}{30} \Rightarrow A_2 = 14$$

Thus, the unknown isotope is $^{14}C_6$

6. In first case, force is along $-\hat{k}$, so $\vec{B}$ should be in xy plane or z component of B should be zero. In second case, force is along x-axis, so B can not be along x-axis. Finally we find that B is along negative y direction.

Let $\vec{B} = -B_0\hat{j}$ and applying $\vec{F} = q\vec{v} \times \vec{B}$

$$-(1.28\times10^{-13}\hat{k}) = (1.6\times10^{-19})\times[(2\hat{i}+3\hat{j})\times(-B_0\hat{j})]\times10^6$$

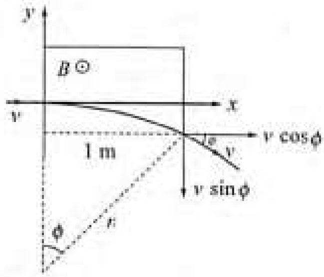
$1.28 = 1.6 \times 2 \times B_0$ or $B_0 = 0.4$ T

$\vec{B} = -(0.4\,\hat{j})$ T

7. (a) $E = 5$ MeV $= 5 \times 1.6 \times 10^{-13}$ J

$$\frac{1}{2}mv^2 = 5\times1.6\times10^{-13}$$

$$v = \sqrt{\frac{16\times10^{-13}}{m}}$$



$$r = \frac{mv}{eB}$$

$1 = r\sin\phi$

$$\Rightarrow \sin\phi = \frac{1}{r} = \frac{eB}{mv}$$

y-component of the velocity is $v\sin\phi$.

Hence y-component of momentum is $mv\sin\phi = eB$

Concept Application Exercise 21.2

1. Suppose the field and the current have directions as shown in the figure provided.

The force on PQ is

$$\vec{F} = i\vec{l}\times\vec{B}$$

or $F_1 = 5.0\text{ A} \times 10\text{ cm} \times 2.0\text{ T} = 1.0\text{ N}$

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and directed inside the triangle.

The three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

2. The net force from A to B is $d\vec{F} = I(d\vec{L}\times\vec{B})$

$$\int_A^B d\vec{F} = \int_A^P I[d\vec{L}_1\times\vec{B}] + \int_P^Q I[d\vec{L}_2\times\vec{B}] + \int_Q^R I[d\vec{L}_3\times\vec{B}]$$

$$+\int_R^T I[d\vec{L}_4\times\vec{B}] + \int_T^B I[d\vec{L}_5\times\vec{B}]$$

The entire path can be broken down into elemental vectors joined to each other in sequence. We know, from polygon law of addition of vectors, that vector joining the tail of the first vector to the head of the last vector is the resultant.

$$\vec{F} = I(\vec{L}\times\vec{B}),$$

where $|\vec{L}| = a + \sqrt{c^2-b^2} + 2r + d$

$F_{\text{net}} = IB(a+\sqrt{c^2-b^2}+2r+d)$ and its direction is upward on the plane of paper.

3. $\vec{F} = I\vec{l}\times\vec{B}$

$F = IlB\sin\theta$

(a) When the current is flowing from east to west, $\theta = 90°$

Hence $F = IlB = (1\text{ A})(1\text{ m})(3\times10^{-5}\text{ T}) = 3\times10^{-5}$ N

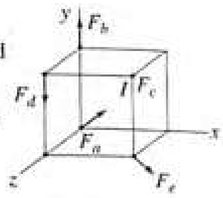
The direction of the force is downwards. This direction may be obtained by either Fleming's left hand rule or the directional property of cross product of vectors.

(b) When the current is flowing from south to north,

$\theta = 0° \Rightarrow F = 0$

Hence, there is no force per unit length on the conductor.

4. In figure, let $F_0 = qvB$, then:



$F_a = F_0$ in the $-\hat{k}$ direction

$F_b = F_0$ in the $+\hat{j}$ direction

$F_c = 0$, since magnetic field and velocity are parallel

$F_d = F_0\sin45°$ in the $-\hat{j}$ direction

$F_e = F_0$ in the $-(\hat{j}+\hat{k})$ direction

5. (a) $\vec{F} = I\vec{l}_{ab}\times\vec{B} = I(l_{ab}B)\hat{j}\times\hat{i}$

$= -(6.58\text{ A})(0.750\text{ m})(0.860\text{ T})\hat{k}$

$= (-4.24\text{ N})\hat{k}$

(b) $\vec{F} = I\vec{l}_{bc}\times\vec{B} = I(l_{bc}B)\left[\frac{(\hat{i}-\hat{k})}{\sqrt{2}}\times\hat{i}\right]$

$= -(6.58\text{ A})(0.750\text{ m})(0.860\text{ T})\hat{j}$

$= (-4.24\text{ N})\hat{j}$

(c) $\vec{F} = I\vec{l}_{cd}\times\vec{B} = I(l_{cd}B)\left[\frac{(\hat{k}-\hat{j})}{\sqrt{2}}\times\hat{i}\right]$

$= (6.58\text{ A})(0.750\text{ m})(0.860\text{ T})\left[\hat{j}+\hat{k}\right]$

$\Rightarrow \vec{F} = (4.24\text{ N})\left[\hat{j}+\hat{k}\right]$

(d) $\vec{F} = I\vec{l}_{de}\times\vec{B} = Il_{de}B\left[-\hat{k}\times\hat{i}\right]$

$= -(6.58\text{ A})(0.750\text{ m})(0.860\text{ T})\hat{j}$

$= (-4.24\text{ N})\hat{j}$

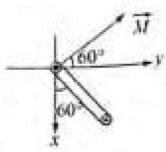
(e) $\vec{F} = I\vec{l}_{ef}\times\vec{B} = I(l_{ef}B)(-\hat{i})\times\hat{i} = 0$

(f) Summing all the forces in parts (a)–(e), we have

$\vec{F}_{\text{total}} = (-4.24\text{ N})\hat{j}$

Concept Application Exercise 21.3

1. (a) $\phi = 90°$: $\tau = NIAB\sin(90°) = NIAB$



Direction: $\hat{k}\times\hat{j} = -\hat{i}$, $U = -MB\cos\phi = 0$

(b) $\phi = 0°$: $\tau = NIAB\sin(0) = 0$, no direction,

$M = NIA$, $U = -MB\cos\phi = -NIAB$

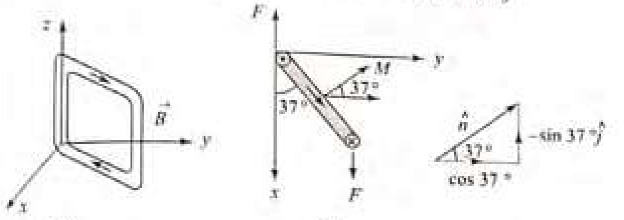
(c) $\phi = 90°$: $\tau = NIAB\sin(90°) = NIAB$

Direction: $-\hat{k}\times\hat{j} = -\hat{i}$, $U = -MB\cos\phi = 0$

(d) $\phi = 180°$: $\tau = NIAB \sin(180°) = 0$, no direction,
$U = -MB\cos\phi = -NIAB$

2. (a) From Figure (b), we see that the unit vector normal to loop

$$\hat{n} = -\sin 37°\hat{i} + \cos 37°\hat{j} = -0.6\hat{i} + 0.8\hat{j}$$



(a) (b) (c)

The magnetic moment is

$$\vec{M} = NIA\hat{n} = (5)(2)(0.2)^2(-0.6\hat{i} + 0.8\hat{j})$$
$$= -0.24\hat{i} + 0.32\hat{j}\ \text{A m}^2$$

(b) The torque,

$$\vec{\tau} = \vec{M} \times \vec{B} = (-0.24\hat{i} + 0.32\hat{j}) \times (0.5\hat{j}) = -0.12\hat{k}\ \text{N m}$$

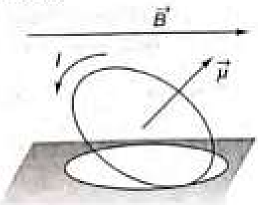
(c) The potential energy of the loop is $U = -MB\cos\theta$ where $M = NIA = 0.4\ \text{Am}^2$ and the position of minimum energy is $\theta = 0$.

Thus, the external work, $W_{ext} = +\Delta U$, needed to rotate it to the given orientation, is given by

$$W_{ext} = U_f - U_i$$
$$= (-MB\cos 37°) - (-MB\cos 0°)$$
$$= (0.4)(0.5)(1 - 0.8) = 0.04\ \text{J}$$

The external work is positive since the dipole moment is rotated away from alignment with the field.

3. The loop will start to lift off when the magnetic torque equals the gravitational torque as shown in figure



The magnetic torque acting on the loop, $\tau_m = MB = I\pi R^2 B$.

The gravitational torque exerted on the loop, $\tau_g = mgR$.

So $I\pi R^2 B = mgR \Rightarrow I = \dfrac{mg}{\pi RB}$

4. The magnetic moment of the loop is in the positive z-direction (right hand thumb rule).

(a) The magnetic moment of the loop is given by

$$\vec{M} = NIA\hat{k} = (12)(3)(0.40)^2\hat{k} = 5.76\ \text{Am}^2\hat{k}$$

(b) The torque on the current loop is given by

$$\vec{\tau} = \vec{M} \times \vec{B} = (5.76\hat{k}) \times (0.3\hat{i} + 0.4\hat{k}) = 1.73\ \text{Nm}\ \hat{j}$$

(c) The potential energy is the negative dot product of $\vec{\mu}$ and $\vec{B}$:

$$U = -\vec{M} \cdot \vec{B} = -(5.76\hat{k}) \cdot (0.3\hat{i} + 0.4\hat{k}) = -2.30\ \text{J}$$

5. The normal to the loop, OP, makes an angle $\theta = 60°$ with the $+x$ direction, the field direction. Hence,

$$\tau = NIAB\sin\theta$$
$$= (1)(14\ \text{A})(\pi \times 25 \times 10^{-4}\ \text{m}^2)(0.03\ \text{T})\sin 60°$$
$$= 2.9 \times 10^{-3}\ \text{Nm}$$

The right hand rule shows that the loop will rotate about the y-axis, so as to decrease the angle labelled 60°.

6. $\theta = \dfrac{NBAI}{k}$, $\theta' = \dfrac{10NB(A/16)I}{k} \Rightarrow \theta' = \dfrac{5}{8}\theta$

Concept Application Exercise 21.4

1. From the figure in question, $\vec{B}_0 = \vec{B}_{AB} + \vec{B}_{BCD} + \vec{B}_{DE}$

$$\Rightarrow \quad \vec{B}_0 = \left(\frac{\mu_0 I}{4\pi R}\right)(-\hat{j}) + \left[\frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}\right](-\hat{i})$$

2. The contributions from the straight segments is zero since

$$d\vec{\ell} \times \vec{r} = 0$$

The magnetic field from the curved wire is just one quarter of a full loop:

$$B = \frac{1}{4}\left(\frac{\mu_0 I}{2R}\right)$$

and is out of the page.

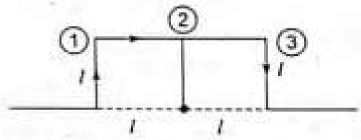
3. At the centre of the circular loop the current I_2 generates a magnetic field that is into the page. So the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude

$$\frac{\mu_0 I_1}{2\pi D} = \frac{\mu_0 I_2}{2R}$$

Thus $I_1 = \dfrac{\pi D}{R} I_2$

4. **Case I:** $\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$$= \left[\frac{\mu_0 I}{4\pi\ell}[0 + \sin 45°] + \frac{\mu_0 I}{4\pi\ell}[\sin 45° + \sin 45°] + \frac{\mu_0 I}{4\pi\ell}[0 + \sin 45°]\right] \otimes$$



$$\therefore \quad |\vec{B}_{net}| = \frac{\mu_0 I}{\sqrt{2}\pi\ell} = B_1$$

Case II: $2\pi r = 4\ell \Rightarrow r = \dfrac{2\ell}{\pi}$

Magnetic field at the centre of semicircle,

$$B_{semicircle} = \frac{\mu_0 I}{4r} = \frac{\mu_0 I\pi}{8\ell} = B_2$$

$$\frac{B_1}{B_2} = \frac{\mu_0 I}{\sqrt{2}\pi\ell} \times \frac{8\ell}{\mu_0 I\pi} = \frac{4\sqrt{2}}{\pi^2}$$

5. The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

(a) At (0, 0, 1 m):

$$\vec{B} = \vec{B}_0 - \frac{\mu_0 I}{2\pi r}\hat{i} = (1.50 \times 10^{-6}\ \text{T})\hat{i} - \frac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{i}$$
$$= -(1.0 \times 10^{-7}\ \text{T})\hat{i}$$

(b) At (1 m, 0, 0):

$$\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{k}$$

$$= (1.50 \times 10^{-6}\ \text{T})\hat{i} + \frac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{k}$$

$$\vec{B} = (1.50 \times 10^{-6}\ \text{T})\hat{i} + (1.6 \times 10^{-6}\ \text{T})\hat{k}$$

$$= 2.19 \times 10^{-6}\ \text{T}$$

at $\theta = 46.8°$ from x to z.

(c) At (0, 0, – 0.25 m):

$$\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{i} = (1.50 \times 10^{-6}\ \text{T})\hat{i} + \frac{\mu_0(8.00\ \text{A})}{2\pi(0.25\ \text{m})}\hat{i}$$

$$= (7.9 \times 10^{-6}\ \text{T})\hat{i}$$

6. There is no contribution from the straight wires, and now we have two oppositely oriented contributions from the two semicircles

$$B = (B_1 - B_2) = \frac{1}{2}\left(\frac{\mu_0}{2R}\right)|I_1 - I_2|,$$

into the page. Note that if the two currents are equal, the magnetic field goes to zero at the centre of the loop.

Concept Application Exercise 21.5

1. $F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a} a(-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i}) = \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$

2. Magnetic field due to ring will be parallel to wire, hence force on the wire will be zero.

3. (a) $F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$

$$= \frac{\mu_0(5.00\ \text{A})(2.00\ \text{A})(1.20\ \text{m})}{2\pi(0.400\ \text{m})} = 6.00 \times 10^{-6}\ \text{N}$$

The force is repulsive since the currents are in opposite directions.

(b) Doubling the current makes the force increase by a factor of 4 to

$$F = 2.40 \times 10^{-5}\ \text{N}$$

4. Net force on circular parts will be zero because magnetic field produced by I_1 on these parts will be tangent at any point.

On straight parts, forces will be in same direction.

$$\text{Net force} = 2\left[\frac{\mu_0}{4\pi}\frac{2I_1I_2}{R}L\right] = \frac{\mu_0 I_1 I_2 L}{\pi R}$$

5. For L_1, $\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I_1 - I_2)$. Here I_1 is taken positive because magnetic lines of force produced by I_1 is anti-lockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $d\vec{\ell}$ is anticlockwise as seen from top.

For L_2: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I_1 - I_2 + I_4)$

For L_3: $\oint \vec{B} \cdot d\vec{\ell} = 0$

Exercises

Motion of a Charged Particles In Magnetic Field

1. (b) The charged particle has circular path in the case when only magnetic field is present.

2. (a) It is clear from Right Palm Rule that the charges are positively charged.

As $r = \frac{mv}{qB}$

The radius of path is not the only function of either m or q. For (b), (c) and (d) we cannot make clear statement, but statement (a) is certainly true.

3. (c) $T = \frac{2\pi m}{Bq}$ or $T \propto \frac{m}{q}$

$$\frac{T_\alpha}{T_p} = \frac{4m}{2q} \times \frac{q}{m} = 2$$

or $T_\alpha = 2\left[\frac{25}{5}\right]\mu\text{s} = 10\ \mu\text{s}$

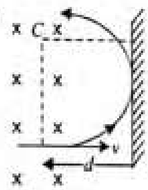
4. (a) Time interval in which $\vec{v}$ returns to its initial value is same as time period of the particle, hence the required time $= \frac{2\pi m}{eB}$

5. (d) Depending on the direction of magnetic field, tension may increase or decrease.

6. (a) $\vec{F} \propto (\vec{v} \times \vec{B}) = \hat{k}[aD - dA]$

If $F = 0$, $aD = dA$

7. (a) The particle moves in a circular path with radius d if it is to just miss the wall.



$\Rightarrow \quad mv = Bqr,\ r = d$

or $\quad B = \frac{v}{(q/m)d} = \frac{v}{sd}$

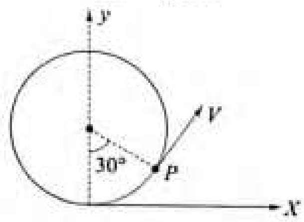
8. (b) For a particle moving in any combination of electric and magnetic fields, work is done only by the electric field.

Energy of the particle = work done by the electric field = electric field × displacement in the direction of the electric field.

9. (c) $p = \frac{2\pi m}{Bq}(v \cos 45°) = \frac{2\pi m}{Bq}(v \sin 45°)$

$\therefore \quad \frac{mv\sin 45°}{Bq} = \frac{p}{2\pi}$ = radius of helix

10. (b) Time period, $T = \frac{2\pi m}{qB} = \frac{2\pi}{\pi \times 2} = 1\ \text{s}$



Thus, particle will be at point P after $t = \frac{1}{12}$ s

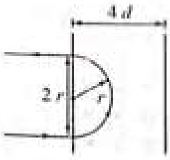
$$\vec{v} = 10[\cos 30\hat{i} + \sin 30\hat{j}]$$

$$\vec{v} = 10\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right] = 5\left[\sqrt{3}\hat{i} + \hat{j}\right]\ \text{ms}^{-1}$$

11. (b) $r = \frac{mv}{qB} = \frac{\sqrt{2mE}}{qB} = \frac{\sqrt{2mqV}}{qB} = \sqrt{\frac{m}{q}} \times \frac{\sqrt{2V}}{B}$

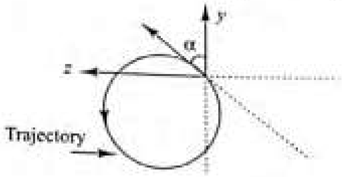
$$\Rightarrow \quad \frac{r_2}{r_1} = \sqrt{\frac{4}{1}} \times \sqrt{\frac{1}{2}} = \sqrt{2} \quad \Rightarrow \quad r_2 = \sqrt{2} r_1 = 5\sqrt{2} \text{ cm}$$

12. (b) $r = \dfrac{mv}{qB} = \dfrac{v}{B\alpha} = \dfrac{(2\alpha d)(B)}{(B\alpha)} = 2d$



i.e., the electron will move out after travelling on a semicircular path of radius $r = 2d$.

13. (c) We can see from trajectory that both y- and z-coordinates can become negative. This trajectory lies in y–z plane. So, x-coordinate is always zero.



14. (c) The particles will not collide if

$$d > 2(r_1 + r_2)$$

or $\quad d > 2\left(\dfrac{mv_1}{Bq} + \dfrac{mv_2}{Bq}\right) \quad$ or $\quad d > \dfrac{2m}{Bq}(v_1 + v_2)$

15. (a) Direction of field can be find using Fleming left-hand rule.

16. (d) Using and we obtain

$$\vec{F} = q(v_x B_y - v_y B_x)\hat{k} = q(v_x(3B_x) - v_y B_x)\hat{k}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\hat{k}$ where $F_z = 6.4 \times 10^{-19}$ N, then we are led to the condition

$$q(3v_x - v_y)B_x = F_z \Rightarrow B_x = \frac{F_z}{q(3v_x - v_y)}.$$

Substituting $v_x = 2.0$ m/s, $v_y = 4.0$ m/s, and $q = -1.6 \times 10^{-19}$ C, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)}$$

$$= \frac{6.4 \times 10^{-19}\text{ N}}{(-1.6 \times 10^{-19}\text{C})[3(2.0\text{ m/s}) - 4.0\text{ m}]} = -2.0\text{ T}$$

17. (c) $T = \dfrac{2\pi m}{qB}$ and $r = \dfrac{mv}{qB} \Rightarrow \dfrac{r}{T} \propto v$

18. (d) Path of particle is helix with increasing pitch

$$v = (v_x^2 + v_y^2 + v_z^2)^{1/2}$$

Here $\quad v_x^2 = \left(\dfrac{qE}{m}t\right)^2$ and $v_y^2 + v_z^2 = v_0^2$

Also $\quad v = 2v_0$

$$(2v_0)^2 = \frac{q^2E^2t^2}{m^2} + v_0^2$$

$$t = \frac{\sqrt{3}\, mv_0}{qE} = \frac{\sqrt{3}\, mv_0}{\sqrt{2}\, qE_0} \text{ given}$$

$$\vec{E} = \sqrt{2} E_0 \hat{i}$$

19. (d) $v_\perp = -\dfrac{v_0}{2}$ will contribute to circular motion

$v_\| = \dfrac{v_0\sqrt{3}}{2}$ will contribute to helical path

$$T = 2\pi\frac{m}{B_0 q}$$

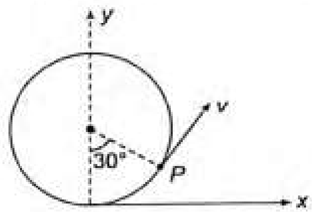
Pitch, $P = v_\| T = \dfrac{v_0\sqrt{3}}{2}T$

Number of turns $= \dfrac{R}{P} = \dfrac{v_0 T\sqrt{3}(2)}{v_0\sqrt{3}\,T} = 2$

20. (b) Time period, $T = \dfrac{2\pi m}{qB}$

$$T = \frac{2\pi}{\pi \times 2} = 1\text{ s}$$

Thus, particle will beat point P after $t = \dfrac{1}{12}$ s

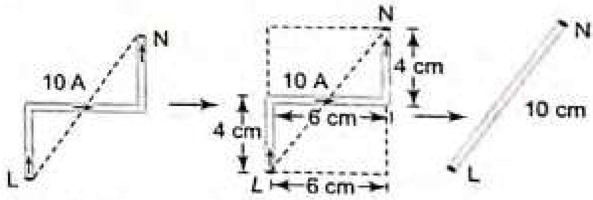


$$\vec{v} = 10[\cos 30\hat{i} + \sin 30\hat{j}]$$

$$\vec{v} = 10\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right] = 5\left[\sqrt{3}\,\hat{i} + \hat{j}\right]\text{ m/s}$$

Force on a Conductor And Torque on A Current Carrying Loop In Magnetic Field

21. (b) The given wire can be replaced by a straight wire as shown below



Hence force experienced by the wire

$$F = BIl = 5 \times 10 \times 0.1 = 5\text{ N}$$

22. (c) For radial component of magnetic field, the total force will be either in the upward or in the downward direction depending on the direction of current.

$\therefore \quad$ Force $= ILB_\perp = I(2\pi a)B\sin\theta$

23. (b) Initially $F_1 + mg + IaB$ (downwards)

When the direction of current is reversed

$F_2 = mg = -IaB$ (downwards) $\Rightarrow \Delta F = 2IaB$

24. (a) $\vec{F}_{CAD} = \vec{F}_{CD} = \vec{F}_{CED}$

$\therefore \quad$ Net force on frame $= 3\vec{F}_{CD}$

$= (3)(2)(1)(4)\ (F = ilB) = 24$ N

25. (a) $\vec{\tau} = \vec{m} \times \vec{B}$

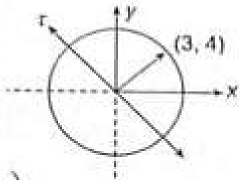
$$= I_0 L^2\hat{k} \times \left(\frac{B}{\sqrt{2}}\hat{i} + \frac{B}{\sqrt{2}}\hat{j}\right) = \frac{I_0 L^2 B}{\sqrt{2}}(\hat{j} - \hat{i})$$

26. (a) Megnetic moment $\vec{M} = \pi r^2 i\,\hat{i}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$

$\therefore \quad \vec{\tau} = \vec{M} \times \vec{B} = \pi r^2 (3\,\hat{i} - 4\hat{i})$

$\vec{\tau}$ will be along the direction shown.

Hence, the point about which the loop will be lift up will be (3, 4)



27. (c) Angle between $\vec{M}$ and $\vec{B}$ is $\left(\frac{\pi}{2} + \theta\right)$

28. (a) $F_{CAD} = F_{CD} = F_{CAD}$

$\therefore$ Net force on the frame $= 3F_{CD}$

$= (3)(2)(1)(4)$

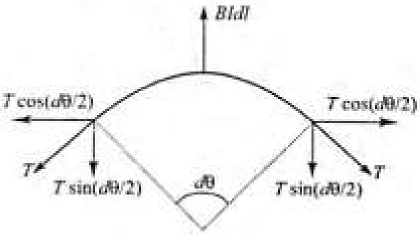
$= 24$ N $\quad (F = i\ell B)$

29. (c) $BI(dl) = 2T\sin(d\theta/2)$

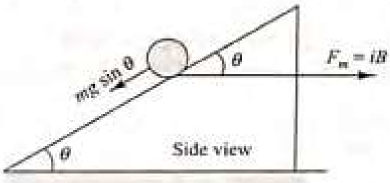
$\Rightarrow \quad BI(rd\theta) = 2T(d\theta/2)$

(θ is small, $\sin\theta = \theta$)

$\Rightarrow \quad T = BIr = BIl/2\pi$



30. (b) Magnetic force acts in the direction shown in figure



Rod will move downward with constant velocity if net force on it is zero.

or $\quad F_m \cos\theta = mg\sin\theta$

or $\quad ilB\cos\theta = mg\sin\theta$

$\therefore \quad B = \left(\frac{mg}{il}\right)\tan\theta$

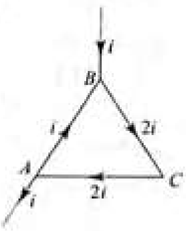
31. (a) Effective length from A to C is $2\sqrt{2}a$

Magnetic force on the structure $= Bil_{eff} = 2\sqrt{2}\,Bia$

32. (a) The equivalent figure can be redrawn as shown in figure.

Force on $AB = ilB$, Force on $BCA = 2ilB$ in opposite direction to that on BA.

Hence, net force $= 2ilB - ilB = ilB$



33. (a) Wire abc can be replaced by a straight wire ac for the computation of force.

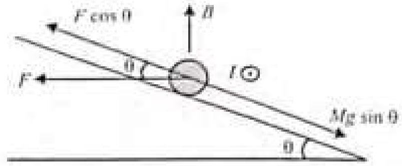
Length of $\overline{ac}$ can be written as,

$\vec{l} = \vec{r}_C - \vec{r}_A = \left[(\hat{i} + \hat{j}) - (\hat{i} + \hat{k})\right] 50 \times 10^{-2}$

$= 0.50(\hat{j} - \hat{k})$

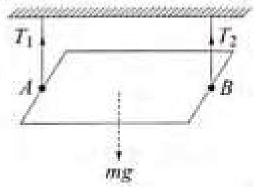
Required force, $\vec{F} = I\left(\vec{l} \times \vec{B}\right) = 0.6\hat{i}$

34. (c) $F\cos\theta = Mg\sin\theta$



$$IBL\cos\theta = Mg\sin\theta \quad \Rightarrow \quad I = \frac{Mg\tan\theta}{LB}$$

35. (c) Taking moments about point B to be zero.



$$T_1.\ell + ib\ell B = mg\frac{\ell}{2}; \quad T_1 = \frac{mg - 2ibB}{2}$$

36. (a) Due to torque of magnetic field, ring will rotate about vertical diameter. $\tau = I\alpha \quad \Rightarrow \quad MB = I\alpha$

$\Rightarrow \quad i\pi r^2 B = \frac{1}{2}mr^2\alpha$

$\Rightarrow \quad \alpha = \frac{2iB\pi}{m} = \frac{2 \times 4 \times 10\pi}{2} = 40\pi$ rad s^{-2}

37. (b) Torque acts on the coil due to magnetic field when current flows in it. Hence, it is based on magnetic effect of current.

38. (c) Knowledge based.

39. (b) Knowledge based.

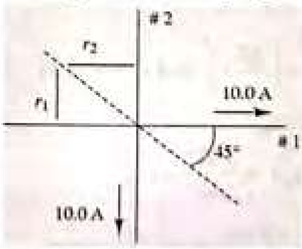
40. (b) Due to linear scale, $i \propto \theta$.

Magnetic Field Due to Current Carrying Wire and Wire Loop

41. (c) $\frac{\mu_0 I_1}{4\pi(a+x)} = \frac{\mu_0 I_2}{4\pi(a-x)}$

$$\frac{a-x}{a+x} = \frac{I_2}{I_1} \quad \Rightarrow \quad x = \left(\frac{I_1 - I_2}{I_1 + I_2}\right)a$$

42. (c) Along the dashed line, $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions. If the line has slope -1.00, then $r_1 = r_2$ and $B_1 = B_2$. So, $B_{tot} = 0$



43. (a) Using $B = \frac{\mu_0 I}{4\pi d}[\sin\theta_1 + \sin\theta_2]$

But $\quad \theta_1 + \phi_1 = 90° \quad$ or $\quad \theta_1 = 90° - \phi$,

$\sin\theta_1 = \sin(90° - \phi_1) = \cos\phi_1$

Similarly, $\quad \sin\theta_2 = \cos\phi_2$

$$B_{net} = \frac{\mu_0 I}{2\pi d}(\cos\phi_1 + \cos\phi_2)$$

44. (d) To get magnetic field in resultant direction, current in X should be in and that in Z should be out; current in W should be in and that in Y should be out.

45. (b) $B \propto I$, because distance of centre for all wires is zero.

$B_1 = kI_1 = 10k \otimes$, $B_2 = kI_2 = 8k \otimes$

$B_3 = kI_3 = 20k \odot$, $B_4 = kI_4$

Now $B_1 + B_2 + B_3 + B_4 = 0$

$\Rightarrow \quad (10k + 8k - 20k) \otimes + kI_4 = 0$

$\Rightarrow \quad kI_4 = 2k \otimes$

$\Rightarrow \quad I_4 = 2$ A towards the bottom.

46. (c) $x = r\sin\dfrac{\alpha}{2}$

$$\therefore B_P = 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{i}{x}\right)\left[\sin\left(90° - \frac{\alpha}{2}\right) + \sin 90°\right]$$

$$= \frac{\mu_0}{2\pi}\frac{i}{r}\frac{\left(1 + \cos\dfrac{\alpha}{2}\right)}{\sin\dfrac{\alpha}{2}}$$

47. (d) No current will flow through section BC as the potentials of points B and C are same. Therefore magnetic field at centre O will be equal and opposite due to section AB and AC. Hence, they cancel out.

48. (b) $B = \dfrac{3}{4}\left[\dfrac{\mu_0 I}{2a}\right] + \dfrac{1}{4}\left[\dfrac{\mu_0 I}{2b}\right]$

$$B = \frac{3\mu_0 I}{8a} + \frac{\mu_0 I}{8b} \otimes$$

49. (a) $\vec{B}_1 = \dfrac{\mu_0 I}{4r} \odot; \vec{B}_2 = \dfrac{\mu_0 I}{4r} \otimes$

$$\vec{B}_3 = \frac{\mu_0 I}{2r}\left(\frac{3}{4}\right)\otimes + \frac{\mu_0 I}{4\pi r}\odot = \frac{\mu_0 I}{4r}\left[\frac{3}{2}\otimes + \frac{1}{\pi}\odot\right]$$

$$= \frac{\mu_0 I}{4r}\left[\frac{3\pi - 2}{2\pi}\right]\otimes = \frac{\mu_0 I}{4r}\left(\frac{2}{\pi}\right)\left(\frac{3\pi}{4} - \frac{1}{2}\right)\otimes$$

$$B_1 : B_2 : B_3 = -\left(\frac{\pi}{2}\right) : \left(\frac{\pi}{2}\right) : \left(\frac{3\pi}{4} - \frac{1}{2}\right)$$

50. (c) $B_{AB} = \dfrac{\mu_0 I}{4\pi(OC)}[2\sin\theta]$

But $\quad OC = r\cos\theta$ or $B_{AB} = \dfrac{\mu_0 I}{2\pi r}\tan\theta$

Magnetic field due to circular portion,

$$B_{AB} = \frac{\mu_0 I}{2r}\left(\frac{2\pi - 2\theta}{2\pi}\right) = \frac{\mu_0 I}{2\pi r}(\pi - \theta)$$

Total magnetic field

$$= \frac{\mu_0 I}{2\pi r}\tan\theta + \frac{\mu_0 I}{2\pi r}(\pi - \theta) = \frac{\mu_0 I}{2\pi r}[\tan\theta + \pi - \theta]$$

51. (c) Magnetic field at the centre

$$B = \frac{\mu_0 I}{2c}\otimes + \frac{\mu_0 I}{2b}\left(\frac{3}{4}\right)\odot + \frac{\mu_0 I}{2a}\left(\frac{3}{4}\right)\otimes$$

As per problem:

$$0 = \frac{\mu_0 I}{2}\left[\frac{1}{c} - \frac{3}{4b} + \frac{3}{4a}\right] \Rightarrow \frac{3}{4}\left[\frac{1}{b} - \frac{1}{a}\right] = \frac{1}{c}$$

If $\quad c = 2a, \dfrac{3}{4}\left[\dfrac{1}{b} - \dfrac{1}{a}\right] = \dfrac{1}{2a} \quad \Rightarrow \quad a = \dfrac{5}{3}b$

52. (c) Magnetic field at the centre due to either arm

$$B_1 = \frac{\mu_0}{4\pi} \times \frac{I}{(L/2)}[\sin 45° + \sin 45°]$$

$$= \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}\,I}{L}$$

Field at centre due to the four arms of the square

$$B = 4B_1 = 4 \times \frac{\mu_0}{4\pi} \times \frac{2\sqrt{2}\,I}{L}$$

i.e., $\quad B \propto \dfrac{1}{L}$

53. (b) $B_{axis} = \dfrac{\mu_0}{4\pi} \times \dfrac{2\pi IR^2}{\left(R^2 + x^2\right)^{3/2}}$

At centre, $B_{centre} = \dfrac{\mu_0}{4\pi} \times \dfrac{2\pi I}{R}$

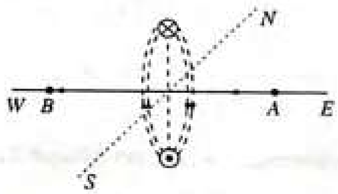
In the given problem,

$$\frac{\mu_0}{4\pi} \times \frac{2\pi IR^2}{\left(R^2 + x^2\right)^{3/2}} = \frac{1}{8}\left[\frac{\mu_0}{4\pi} \times \frac{2\pi I}{R}\right]$$

or $\quad (R^2 + x^2)^{3/2} = 8R^3$

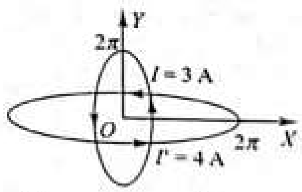
Solving, we get $x = R\sqrt{3}$

54. (d) Using right hand rule, the direction of magnetic field is clearly toward west at A and B



55. (c) Due to current shown in figure, the magnetic field within loop is outwards. So field at O should be inward because magnetic field lines form closed loop. But if $\theta > \pi$, then point O will lie within loop and for a point within loop magnetic field is outward.

56. (a)



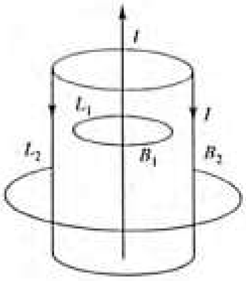
$$B_x = \frac{\mu_0}{2}\frac{I}{2\pi \times 10^{-2}} = \frac{\mu_0}{4\pi}3 \times 10^2 = 3 \times 10^{-5}\ \text{T}$$

$$B_y = \frac{\mu_0}{2}\frac{I'}{2\pi \times 10^{-2}} = 4 \times 10^{-5}\ \text{T}$$

$$B_{net} = \sqrt{B_x^2 + B_y^2} = \sqrt{25 \times 10^{-10}} = 5 \times 10^{-5}\ \text{T}$$

57. (c) Apply Ampere's circuital law to the coaxial circular loops L_1 and L_2. The magnetic field is B_1 at all points on L_1 and B_2 at all points on L_2. $\Sigma I \neq 0$ for L_1 and 0 for L_2. Hence, $B_1 \neq 0$ but $B_2 = 0$

$$\left[\text{As} \oint \vec{B} \cdot d\vec{l} = \mu_0 \sum I\right]$$



58. (c) Loop B: $\mu_0 (2i - i) = \oint B \cdot d\ell$

loop C: $\mu_0(i - 2i) = \oint B \cdot d\ell$

Loop A: $\mu_0(3i - 3i) = \oint B \cdot d\ell$

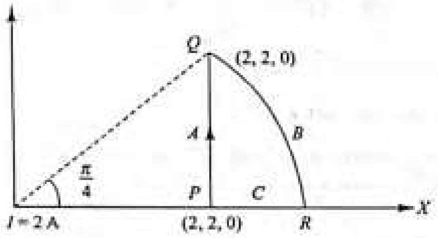
Loop D: $\mu_0 (0 - i) = \oint B \cdot d\ell$

(c) $B > A > C = D$

59. (a) $\int_{PAQ} \vec{B} \cdot d\vec{\ell} = \int_{RBQ} \vec{B} \cdot d\vec{\ell}$ $\quad \left(\because \int_{PCR} \vec{B} \cdot d\vec{\ell} = 0\right)$

$$= \left(\frac{\mu_0 I}{2\pi R}\right)\left(\frac{\pi}{4} R\right)$$

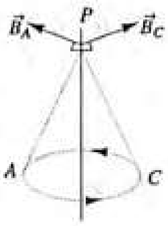
$$= \frac{(4\pi \times 10^{-7})(2)}{16} = \frac{\pi \times 10^{-7}}{2} \text{ S.I. units}$$



60. (a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I = 4\pi \times 10^{-7} \times \dfrac{1}{4\pi} = 10^{-7}\,\text{Wb m}^{-1}$

Problems Based on Mixed Concepts

61. (a) The point charge moves in circle as shown in figure. The magnetic field vectors at a point P on the axis of circle are $\vec{B}_A$ and $\vec{B}_C$ at the instant the point charge is at A and C, respectively, as shown in figure. Hence, as the particle rotates in circle, only magnitude of magnetic field remains constant at P but its direction changes.



62. (b) $F = qvB = \dfrac{qv\mu_0 i}{2\pi r}$

$$\frac{mv^2}{R} = \frac{qv\mu_0 i}{2\pi r} \Rightarrow R = \frac{2\pi rmv}{q\mu_0 i}$$

63. (c) Net magnetic field at P will be zero, because the magnetic field at centre P is cancelled by opposite pairs like AD–EF, etc.

64. (d) $B = \dfrac{\mu_0 i}{2r} - \dfrac{\mu_0 i}{2(2r)} + \dfrac{\mu_0 i}{2(2^2 r)} + \cdots \infty$

$$B = \frac{\mu_0 i}{2r}\left[1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots \infty\right]$$

$$B = \frac{\mu_0 i}{2r}\left[\frac{1}{1 - \left(-\frac{1}{2}\right)}\right] = \frac{\mu_0 i}{3r}$$

65. (d) Current density: $\rho = \dfrac{I}{\pi R^2 - \pi (R/2)^2} \Rightarrow \rho = \dfrac{4I}{3\pi R^2}$

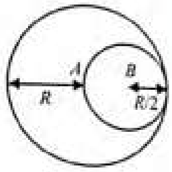
Current in smaller cylinder (if there were): $I_1 = \rho\pi\left(\dfrac{R}{2}\right)^2 = \dfrac{I}{3}$

For A: $B_A = B_{\text{whole-cylinder}} - B_{\text{small-cylinder}}$

$$\Rightarrow \quad B_A = 0 - \frac{\mu_0 (I/3)}{2\pi (R/2)} = \frac{-\mu_0 I}{3\pi R}$$

For B: $B_B = B_{\text{whole-cylinder}} - B_{\text{small-cylinder}}$

$$= \frac{\mu_0 (I + I/3)(R/2)}{2\pi R^2} - 0 = \frac{\mu_0 I}{3\pi R}$$



66. (a) To find the magnetic field outside a thick conductor, the current may be assumed to flow along the axis. As points 1, 2 and 3 are equidistant from the axis.

$B_1 = B_2 = B_3$

67. (b) Consider an element of length dx on AB, at a distance x from XY. Force on the element,

$$dF = (\mu_0 I/2\pi x) I' dx$$

Total force on AB,

$$F = \int_{L/2}^{3L/2} \frac{\mu_0 II'}{2\pi x} dx \quad \text{or} \quad F = \frac{\mu_0 II'}{2\pi} \log 3$$

68. (a) $F = \dfrac{4\pi \times 10^{-7} \times 20 \times 10 \times 10 \times 10^{-2}}{2\pi \times 2 \times 10^{-2}}$

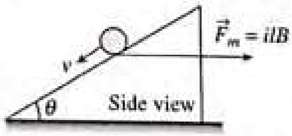
$$- \frac{4\pi \times 10^{-7} \times 10 \times 30 \times 10 \times 10^{-2}}{2\pi \times 10 \times 10^{-2}}$$

$= 1.4 \times 10^{-4}$ N toward right.

69. (b) $R = \dfrac{mv}{qB}$

The radius may be decreased if v decreases or B increases.

70. (b) Magnetic force acts in the direction shown in figure below:



Road will move downwards with constant velocity if net force on it is zero.

or $\quad F_m \cos \theta = mg \sin \theta$

or $\quad ilB \cos \theta = mg \sin \theta$

$$\Rightarrow \quad B = \left(\frac{mg}{il}\right)\tan\theta$$

71. (b) Torque of magnetic force about PQ

$$\tau_m = (ILB)\, L \cos \theta = IL^2 B \cos \theta$$

Torque of gravitational force about PQ

$$\tau_g = [(\lambda L)gL \sin\theta + 2(\lambda L)g(1/2)L \sin\theta]\ \lambda L^2 g \sin\theta$$

$$\tau_m = \tau_g \Rightarrow \tan\theta = \frac{IB}{2\lambda g} = \frac{10\sqrt{3}\times 2}{2\times\sqrt{3}\times 10} = 1$$

$\Rightarrow \quad \theta = 45°$

72. (b) $\vec{\tau} = \vec{M}\times\vec{B} = \frac{IA}{2}(-\hat{k})\times B(-\hat{j}) = \frac{IAB}{2}(-\hat{i})$ (leftward)

73. (a) In a cylindrical cavity like this, magnetic field is uniform, i.e., same both in magnitude and direction.

74. (a) $di = \frac{dq}{\left(\frac{2\pi}{\omega}\right)} = \frac{\omega}{2\pi}(\sigma . 2\pi r dr) = \omega(A - Br)\, r\, dr$

$$dB = \frac{\mu_0\omega(A - Br)r\,dr}{2r};\ B = \frac{\mu_0\omega}{2}\int_0^2 (A - Br)\,dr$$

$$0 = \left[Ar - B\frac{r^2}{2}\right]_0^2$$

$$2A - 2B = 0, \frac{A}{B} = 1$$

75. (c) Radius R of particle in spiral path at any time when velocity is v.

$$r = \frac{mv}{qB}\ \omega = \frac{v}{r} = \frac{qB}{m} \rightarrow \text{constant}$$

Angular momentum, $L = mvr = qBr^2 \rightarrow$ decreases as r decreases

If particle is moving in a plane then its velocity and acceleration will be in that plane. The force on the particle will also be in that plane. It means magnetic field should be perpendicular to that plane.

Magnetic force on the particle = qvB

Resistive on the particle = kv, where k is a constant

As velocity decreases, so net force acting on the particle will decrease.

76. (c) Mercury is a conductor. So current can flow through mercury.

$F = ilB$

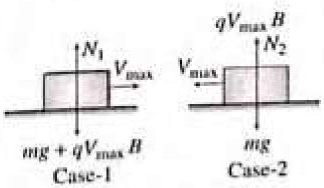
$\int F dt = mv$ or $\int ilB\,dt = m\sqrt{2gh}$

$$q = \int i\,dt = \frac{m\sqrt{2gh}}{lB} = \frac{10\times10^{-3}\times\sqrt{2\times10\times3}}{20\times10^{-2}\times0.1} = \sqrt{15}\ \text{C}$$

77. (c) $v = (g \sin\theta)t$

$$N = mg\cos\theta - qvB = 0 \Rightarrow t = \frac{m\cot\theta}{qB}$$

78. (d) The velocity is maximum at mean position. Hence, the magnetic force on block is maximum at its mean position. The magnetic force on the block while it crosses the mean position towards right and left is as shown.



Hence, normal reaction is maximum in case 1 and minimum in case 2.

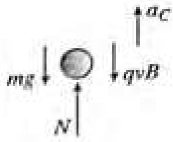
Hence, correct option is (d).

79. (b) The electrostatic force on proton is along positive x-axis and the magnetic force is along negative x-axis. Initially net force on charge Q is zero. Since the velocity of proton is increased in repeat experiment, the magnetic force on proton shall increase and the proton would then be deflected in negative x-direction.

80. (b) Maximum force will be at lowest point.

Magnetic force will not do any work.

Velocity at lowest point: $v = \sqrt{2gR}$



$$N - mg - qvB = ma_C$$

$$N = mg + \frac{mv^2}{R} + qBv$$

$\Rightarrow$

$\Rightarrow \quad N = 3mg + qB\sqrt{2gR}$

Archives

1. (b) Like currents attract each other.

2. (a) $Bvq = \frac{mv^2}{r}$

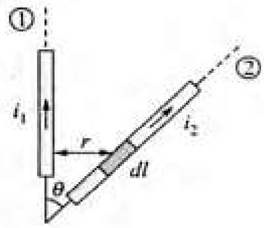
$\Rightarrow \quad r = \frac{mv}{Bq} \Rightarrow R = \frac{p}{Bq}$

All quantities on the right hand side remain unchanged. So R remains unchanged.

3. (a) $B = \frac{\mu_0 I}{2R}$

In the given problem, $1/R$ is unchanged.

4. (c) Length of the component dl which is parallel to wire (1) is $dl \cos\theta$, so force on it.



$$F = \frac{\mu_0}{4\pi}\cdot\frac{2i_1i_2}{r}(dl\cos\theta)$$

$$= \frac{\mu_0 i_1 i_2 dl\cos\theta}{2\pi r}$$

5. (a) $T = \frac{2\pi m}{Bq}$

Clearly, T is independent of speed.

6. (c) The work done by the magnetic field is zero.

7. (b) $v = \frac{E}{B}$

$\Rightarrow \quad B = \frac{E}{v} = \frac{10^4}{10}\ \text{Wb/m}^2 = 10^3\ \text{Wb/m}^2$

8. (b) No current is enclosed. Using Ampere's circuital law, we get $B = 0$.

9. (b) $B = \frac{\mu_0 I}{2r} = \frac{\mu_0 I}{2(l/2\pi)}$

But $\quad l = 2\pi r \times n$

$\Rightarrow \quad r = \frac{l}{2\pi n}$

$\therefore \quad B = \frac{\mu_0 n^2 I\times 2\pi}{2l} = n^2\frac{\mu_0 I}{2(l/2\pi)} = n^2 B$

This leads us to the right choice.

10. (a) $B=\dfrac{\mu_0 Ir^2}{2(r^2+x^2)^{3/2}}$

$=\dfrac{\mu_0 I}{2r}$

$\therefore \quad \dfrac{B'}{B}=\dfrac{(r^2+x^2)^{3/2}}{r^3}$

$=\left(1+\dfrac{x^2}{r^2}\right)^{3/2}=\left(1+\dfrac{16}{9}\right)^{3/2}$

$\left(\dfrac{25}{9}\right)^{3/2}=\left(\dfrac{5}{3}\right)^{3/2}=\dfrac{125}{27}$

$\Rightarrow \quad B'=\dfrac{125}{27}B=\dfrac{125}{27}\times 54\,\mu\text{T}=250\,\mu\text{T}$

11. (c) $F\propto\dfrac{I_1I_2}{d}$

$F'\propto\dfrac{2I_1I_2}{3d}$

$\therefore \quad \dfrac{F'}{F}=-\dfrac{2}{3}\Rightarrow F'=-\dfrac{2F}{3}$

12. (b) $B_1=\dfrac{\mu_0 I_1}{2r}, B_2=\dfrac{\mu_0 I_2}{2r}$

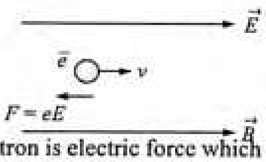
$B=\sqrt{B_1^2+B_2^2}$

$=\dfrac{\mu_0}{2r}\sqrt{I_1^2+I_2^2}$

$=\dfrac{4\pi\times10^{-7}}{2\times2\pi\times10^{-2}}\sqrt{9+16}$

$=5\times10^{-5}\ \text{Wb/m}^2$

13. (d) Since electron is moving parallel to the magnetic field, hence magnetic force on it $F_m=0$.



The only force acting on the electron is electric force which reduces its speed.

14. (b) Time period of a particle is given by

$$T=\frac{2\pi}{\omega}=\frac{2\pi r}{v} \qquad \ldots\text{(i)}$$

The centripetal force for rotation of the charged particle will be provided by magnetic force.

$$\therefore \quad \frac{mv^2}{r}=qvB\Rightarrow v=\frac{qBr}{m} \qquad \ldots\text{(ii)}$$

From (i) and (ii), $T=\dfrac{2\pi r\times m}{qBr}=\dfrac{2\pi m}{qB}$

15. (d) $\dfrac{F}{l}=\dfrac{\mu_0}{4\pi}\dfrac{2I_1I_2}{d}=\dfrac{\mu_0 i^2}{2\pi d}$

Since like currents attract, the force is attractive.

16. (c) In first case, $B_1=\mu_0 n_1 I_1$

In second case, $B_2=\mu_0 n_2 I_2$

$\therefore \quad \dfrac{B_2}{B_1}=\dfrac{n_2}{n_1}\times\dfrac{I_2}{I_1}=\dfrac{100}{200}\times\dfrac{i/3}{i}=\dfrac{1}{6}$

$\therefore \quad B_2=\dfrac{B_1}{6}=\dfrac{6.28\times10^{-2}}{6}=1.05\times10^{-2}\ \text{Wb/m}^2.$

17. (d) $\dfrac{1}{2}mV^2=\text{KE}$

Since speed does not change, the KE remains same. Due to change in the direction of motion, momentum changes.

18. (a) Magnetic field due to AOB is

$$B_1=\frac{\mu_0 I_1}{2\pi d} \qquad \text{(i)}$$

Magnetic field due to COD is

$B_2=\dfrac{\mu_0 I_2}{2\pi d}$

$\therefore \quad B=\sqrt{B_1^2+B_2^2}=\dfrac{\mu_0}{2\pi d}\sqrt{I_1^2+I_2^2}$

19. (a) $B=\dfrac{\mu_0 i}{2\pi r}$

$B\propto\dfrac{1}{r}$

$\therefore \quad \dfrac{B_1}{B_2}=\dfrac{2a}{a/2}=4$

20. (b) We have Ampere law, $\oint\vec{B}.d\vec{l}=\mu_0 i_{\text{enclosed}}$ (i)

If we take an ampere loop inside the straight thin walled tube, the current enclosed in the loop will be zero.

Hence from (i), $\oint\vec{B}.d\vec{l}=0$

$\Rightarrow \quad B=0$

It means the magnetic field at any point inside the pipe is zero.

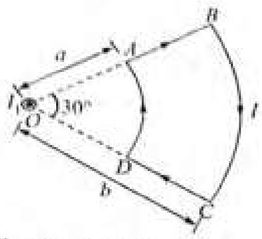
21. (b) When $\overline{E}$ and $\overline{B}$ are perpendicular and velocity has no changes then $qE=qvB$ i.e., $v=\dfrac{E}{B}$. The two forces oppose each other if v is along $\overline{E}\times\overline{B}$ i.e., $\bar{v}=\dfrac{\overline{E}\times\overline{B}}{B^2}$. As $\overline{E}$ and $\overline{B}$ are perpendicular to each other,

$$\frac{\overline{E}\times\overline{B}}{B^2}=\frac{EB\sin90^\circ}{B^2}=\frac{E}{B}$$

For historic and standard experiments like Thomson's e/m value, if v is given only as E/B, it would have been better from the pedagogic view, although the answer is numerically correct.

22. (a) $B=\dfrac{\mu_0}{4\pi}\dfrac{2i}{r}=5\times10^{-6}\,\text{T}$

23. (b) The direction of current in the wires AB and CD pass through O, hence these wires do not contribute to the magnetic induction at O. The field due to wire DA is out of the paper and that due to wire BC is into the paper.



Let us take the direction out of the paper as positive and the direction into the paper as negative.
The total magnetic field due to loop $ABCD$ at O, $B = B_{AB} + B_{BC} + B_{CD} + B_{DA}$

$$\Rightarrow \quad B = 0 - \frac{\mu_0 I}{4\pi b}\times\frac{\pi}{6} + 0 + \frac{\mu_0 I}{4\pi a}\times\frac{\pi}{6}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{24ab}(b-a)\text{, out of the paper or positive.}$$

24. (b) The forces on AD and BC are zero because the magnetic field due to a straight wire on AD and BC is parallel to the elementary length of the loop.

25. (a) The magnetic field in between because of each will be in the opposite direction.

$$B_{\text{in between}} = \frac{\mu_0 i}{2\pi x}\hat{j} - \frac{\mu_0 i}{2\pi(2d-x)} - (-\hat{j})$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{2d-x}\right](\hat{j})$$

At $x = d$, $B_{\text{in between}} = 0$

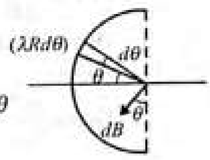
For $x < d$, $B_{\text{in between}} = \hat{j}$

For $x > d$, $B_{\text{in between}} = -\hat{j}$

Towards x, the net magnetic field will add up and direction will be $-\hat{j}$

Towards x', the net magnetic field will add up and direction will be $\hat{j}$

26. (a) Current per unit length: $\lambda = \frac{I}{\pi R}$

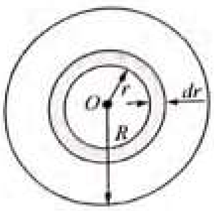


$$dB = \left(\frac{\mu_0}{2\pi}\right)\frac{dl}{R}, \qquad dI = \lambda R\, d\theta$$

$$\Rightarrow \quad B = \int_{-\pi/2}^{\pi/2} dB\cos\theta = \frac{\mu_0\lambda}{2\pi}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

$$= \frac{\mu_0\lambda}{\pi} = \frac{\mu_0 I}{\pi^2 R}$$

27. (a) Let us consider an elementary ring of radius R and a disc of thickness dr.



Charge on the ring, $dq = \frac{Q}{\pi R^2}(2\pi r dr) = \frac{2Qr}{R^2}dr$

Current due to rotation of charge on ring,

$$I = \frac{dq\omega}{2\pi} = \frac{Qr\omega dr}{\pi R^2}$$

Magnetic field at the centre due to elementary ring,

$$dB = \frac{\mu_0 I}{2r} = \frac{\mu_0 Qr\omega dr}{2\pi R^2 r}\frac{\mu_0 Q\omega dr}{2\pi R^2}$$

Magnetic field at the centre due to the complete disc,

$$B = \int dB = \frac{\mu_0 Q\omega}{2\pi R^2}\int_0^R dr = \frac{\mu_0 Q\omega R}{2\pi R^2} = \frac{\mu_0 Q\omega}{2\pi R}$$

Since, Q and ω are constants, $B \propto \frac{1}{R}$

Hence, variation of B with R should be a rectangular hyperbola as represented in option (a).

28. (b) Kinetic energy of a charged particle, $K = \frac{1}{2}mv^2$

Hence velocity of the charge particle, $v = \sqrt{\frac{2K}{m}}$

The radius of the circular path of a charged particle in the magnetic field, $r = \frac{mv}{Bq}$

$$\therefore \quad r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

As K and B are constants,

Hence $r \propto \frac{\sqrt{m}}{q}$

$$\Rightarrow \quad r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$= \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} = 1 : \sqrt{2} : 1$$

$$\Rightarrow \quad r_\alpha = r_p < r_d$$

29. (d)

$$P = \frac{\text{Work done}}{\text{Time}} = \frac{\int F dx}{t} = \frac{\int I\ell dB dx}{t}$$

$$= \frac{\int_0^2 (10)(3)(3\times 10^{-4}e^{-0.2x})dx}{5\times 10^{-3}}$$

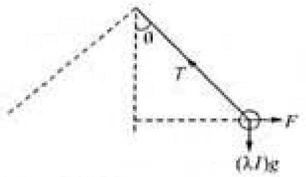
$$= \frac{9\times 10^{-3}}{5\times 10^{-3}}\left[\frac{e^{-0.2x}}{-0.2}\right]_0^2 = 9\left[1-e^{-0.4}\right] = 2.97\text{ W}$$

30. (b)

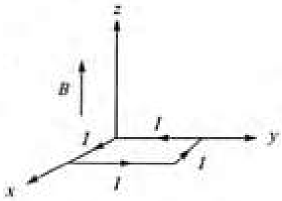
$$T\cos\theta = \lambda g l \qquad \text{(i)}$$

$$T\sin\theta = \frac{\mu_0}{2\pi}\cdot\frac{I\times Il}{(2L\sin\theta)} \qquad \text{(ii)}$$

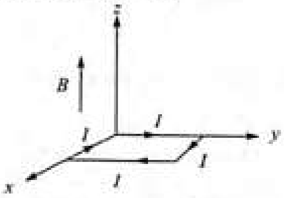
$$\Rightarrow \quad I = 2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0\cos\theta}}$$



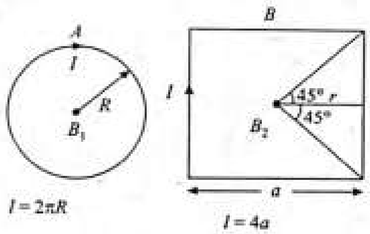
31. (c) Stable equilibrium $\vec{M} \parallel \vec{B}$



Unstable equilibrium $\vec{M} \parallel (-\vec{B})$



32. (d)



$$R = \frac{\ell}{2\pi} \qquad a = \frac{\ell}{4}$$

$$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{2 \times \frac{\ell}{2\pi}} = \frac{\mu_0 \pi I}{\ell}$$

$$B_B = \frac{\mu_0 I}{4\pi r}[\sin\alpha + \sin\beta] \times 4$$

$$= \frac{\mu_0 I}{4\pi \frac{a}{2}}[\sin 45° + \sin 45°] \times 4$$

$$\Rightarrow B_B = 2\sqrt{2}\frac{\mu_0 I}{\pi a} = 2\sqrt{2}\frac{\mu_0 I}{\frac{\pi \ell}{4}} = 8\sqrt{2}\frac{\mu_0 I}{\pi \ell}$$

$$\frac{B_A}{B_B} = \frac{\frac{\mu_0 \pi I}{\ell}}{8\sqrt{2}\frac{\mu_0 I}{\pi \ell}} = \frac{\pi^2}{8\sqrt{2}}$$

33. (c) Radius of circular path in magnetic field

$$R = \frac{mv}{qB} = \frac{\sqrt{2Km}}{qB}$$

where K = kinetic energy of particle
m = mass of particle
q = charge on particle
B = magnetic field intensity
R = radius of path

For electron $r_e = \dfrac{\sqrt{2Km_e}}{eB}$ ...(i)

For proton $r_p = \dfrac{\sqrt{2Km_p}}{eB}$ ...(ii)

For a particle

$$r_\alpha = \frac{\sqrt{2Km_\alpha}}{q_\alpha B} = \frac{\sqrt{2K\,4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB} \quad ...(iii)$$

as $m_e < m_p$ so $r_e < r_p = r_\alpha$

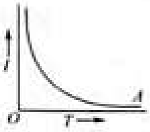
34. (d) We know magnetic dipole moment of a coils is; $m = IA$.
In our case current is kept constant and dipole moment is doubled, it means $m \propto A$
i.e., the area will be doubled as coil is circular it means the radius will become $\sqrt{2}$ times.
We know magnetic field at the centre of a circular coil is given by $B = \dfrac{\mu_0 I}{2R}$ of for constant current $B \propto \dfrac{I}{R}$
Hence magnetic field of the centre of loop will become $\dfrac{1}{\sqrt{2}}$ times.
Hence $\dfrac{B_1}{B_2} = \dfrac{R_2}{R_1} = \dfrac{\sqrt{2}R}{R}$ or $\dfrac{B_1}{B_2} = \sqrt{2}$

CHAPTER 22: MAGNETISM AND MATTER

Concept Application Exercises 22.1

1. (a) Paramagnetic materials have a small positive susceptibility (I/H), while ferromagnetic materials have a much larger positive susceptibility. Therefore, material A is paramagnetic and material B is susceptibility.

   (b) The intensity of magnetisation of a paramagnetic material is inversely proportional to its absolute temperature. Therefore, variation of intensity of magnetisation with temperature for the material A will be as shown in figure.



2. The magnetic field at a point on the equatorial line of the magnetic dipole (earth as magnetic dipole) is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$

Here, $M = 8 \times 10^{22}$ JT^{-1}

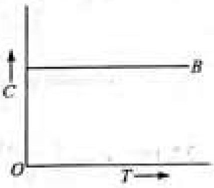
and $r = 6.4 \times 10^6\ m$ (radius of the earth)

$$\therefore \quad B = 10^{-7} \times \frac{8 \times 10^{22}}{(6.4 \times 10^6)^3} = 0.3 \times 10^{-4}\ \text{T} = 0.3\text{G}$$

It is really of the order of the observed field of the earth.

3. (a) Paramagnetic materials have a small positive susceptibility, while diamagnetic materials have a small negative susceptibility. Therefore, material A is paramagnetic and material B is diamagnetic.

   (b) The susceptibility of a diamagnetic substance does not change with temperature for practical purposes. Therefore, variation of susceptibility with temperature for the material B will be shown in figure.



4. Here, $B_H = 0.22$ T; $B_V = 0.38$ T

Now, $B = \sqrt{B_H^2 + B_V^2}$

$$= \sqrt{(0.22)^2 + (0.38)^2} = \sqrt{0.1928} = 0.44\ \text{T}$$

5. Here, $B_H = \sqrt{3}\ B_V$

Now, $\tan\delta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3}B_V} = \frac{1}{\sqrt{3}}$

Or $\delta = 30°$

6. Here, $\delta = 60°$; $B_H = 0.3 \times 10^{-4}$ T

Now, $B_H = B\cos\delta$

or $B = \frac{B_H}{\cos\delta} = \frac{0.3 \times 10^{-4}}{\cos 60°} = \frac{0.3 \times 10^{-4}}{0.5} = 0.6 \times 10^{-4}$ T

7. Here, $B_H = 0.2$ G; $B = 0.4$ G

Now, $B_H = B\cos\delta$

or $\cos\delta = \frac{B_H}{B} = \frac{0.2}{0.4} = 0.5$

or $\delta = 60°$

Exercises

Magnetic Dipole, Magnet and Its Properties

1. (a) The electron configuration is $1s^2\ 2s^2\ 2p^6$ i.e. the octet (8 electron) is complete. Hence, the dipole moment is zero.

2. (b) $M = IA$

$$M = \frac{e}{T}\pi r^2$$

$$M = \frac{ev}{2\pi r} \times \pi r^2$$

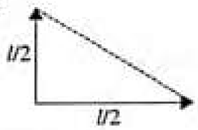
$$M = \frac{evr}{2}$$

But $l = mvr$

or $r = \frac{l}{mv}$ $\therefore$ $M = \frac{ev}{2} \times \frac{l}{mv}$ or $M = \frac{e}{2m}l$

3. (b) Magnetic moment of each part is $= M/2$

So, the net magnetic moment is



$$= \sqrt{\left(\frac{M}{2}\right)^2 + \left(\frac{M}{2}\right)^2} = \frac{M}{\sqrt{2}}$$

4. (c) Here, $W = MB(\cos\theta_1 - \cos\theta_2)$

$= MB(\cos 0° - \cos 180°)$

or $W = 2MB = 2NiAB$

$= 2 \times 100 \times 0.1 \times 3.14 \times (0.05)^2 \times 1.5$

$= 0.2355$ J

5. (a) $W = \Delta U = U_2 - U_1 = \frac{\mu_0 m_1 m_2}{4\pi r_0} - 0 = \frac{\mu_0 m_1 m_2}{4\pi r_0}$

6. (c) Pole strength becomes half due to change in breadth. Change in length does not affect pole strength.

7. (d) New magnetic moment,

$$M' = \frac{2M\sin(\theta/2)}{\theta} = \frac{2M\sin(\pi/6)}{\theta/3} = \frac{2M \times \frac{1}{2}}{\pi/3} = \frac{3M}{\pi}$$

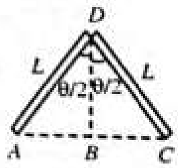
8. (d) When the magnet is bent, the length of the magnet is:

$AC = AB + BC$

$= L\sin\frac{\theta}{2} + L\sin\frac{\theta}{2}$

$= 2L\sin(\theta/2) = 2L\sin(60°/2)$

$= 2L\sin 30° = L$



9. (a) All the magnitude moment vectors can be represented both in magnitude and direction by the three sides of a triangle taken in the same order.

10. (c) The hysteresis loss for soft iron is low. On the other hand, hysteresis loss for steel is high.

11. (b) $\tau = MB\sin\theta$

τ and B are constants.

Now, $\sqrt{3}\sin\theta = 1 \times \sin(90° - \theta)$

or $\tan\theta = \frac{1}{\sqrt{3}}$ or $\theta = 30°$.

12. (b) $\tau = MB\sin\theta$

$\tau = \tau_0 \sin\theta$

$\frac{1}{2}\tau_0 = \tau_0 \sin\theta$ or $\sin\theta = \frac{1}{2}$ or $\theta = 30°$

13. (c) $\tau = (m \times 2l)\, B \sin\theta$

$= 48 \times 25 \times 10^{-2} \times 0.15 \times \sin 30°$ N m

$= 0.9$ N m.

14. (c) $B_1 = \frac{\mu_0}{4\pi}\frac{2M \times 10}{(10^2 - l^2)^2}$

$B_2 = \frac{\mu_0}{4\pi}\frac{2M \times 20}{(20^2 - l^2)^2}$

$\frac{B_1}{B_2} = \frac{10}{(10^2 - l^2)^2} \times \frac{(20^2 - l^2)^2}{20}$

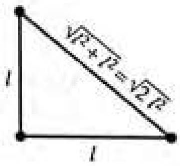
or $\frac{125}{10} = \frac{(20^2 - l^2)^2}{2(10^2 - l^2)^2}$, on solving we get

or $l = 5$ cm

Length $= 2l = 10$ cm.

15. (c) Here, distance between two poles $= \sqrt{2}l$

Hence, $M = m \times \sqrt{2}l = \sqrt{2}\, ml$



16. (d) $W = mB\,[\cos 0° - \cos 60°] = \frac{mB}{2}$

$\tau = mB \sin 60°$, $\tau = mB \times \frac{\sqrt{3}}{2} = W \times \sqrt{3}$

17. (b) $\tau = MH \sin\theta$

$\therefore \quad \sin\theta = \frac{\tau}{MH} = \frac{1}{2} \quad \therefore \quad \theta = 30°$

18. (a) $\tau = MB \sin 90° = 1.42$ Nm

19. (d) $F = \frac{\mu_0}{4\pi} \cdot \frac{6M_1M_2}{r^4}$

i.e., $\propto \frac{1}{r^4} \quad \therefore F' = \frac{F}{16} = \frac{8}{16} = 0.5$ N

20. (d) $W = MB\,(\cos\theta_1 - \cos\theta_2)$

When the magnet is rotated from 0° to 60°, then work done is 0.8 J.

$0.8 = MB\,(\cos 0° - \cos 60°) = \frac{MB}{2}$

$MB = 1.6$ N-m

In order to rotate the magnet through an angle of 30°, i.e., from 60° to 90°, the work done is

$W' = MB\,(\cos 60° - \cos 90°)$

$= MB\left(\frac{1}{2} - 0\right) = \frac{MB}{2} = \frac{1.6}{2} = 0.8$ J

$= 0.8 \times 10^7$ ergs

Earth Magnetism

21. (c) $\tan 30° = \frac{\tan\delta}{\cos\theta}$ or $\cos\theta = \frac{\tan\delta}{\tan 30°} = \sqrt{3}\tan\delta$

Again, $\tan 45° = \frac{\tan\delta}{\sin\theta}$ or $\sin\theta = \tan\delta$

Now, $\sin^2\theta + \cos^2\theta = 1 \quad \therefore \quad \tan^2\delta + 3\tan^2\delta = 1$

or $\tan^2\delta = \frac{1}{4}$ or $\tan\delta = \frac{1}{2}$

or $\frac{1}{\cot\delta} = \frac{1}{2}$ or $\delta = \cot^{-1}(2)$

22. (c) Here, $\delta = 30°$ and $H = 0.5$ oersted.

Now, $B\cos\delta = H$

or $B = \frac{H}{\cos\delta}$, $B = \frac{0.5}{\cos 30°} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$ oersted.

23. (c) $B = \mu_0 H = 4\pi \times 10^{-7} \times 28 \times 10^4$ gauss

$B = 0.352$ gauss

24. (c) If $X = 45°$ is the angle of rotation away from magnetic meridian, then

$\frac{\tan\theta'}{\tan\theta} = \frac{1}{\cos X} = \frac{1}{\cos 45°} = \sqrt{2}$

25. (c) $\tan\theta_1 = \frac{\tan\delta}{\cos\alpha}$ or $\cos\alpha = \frac{\tan\delta}{\tan\theta_1}$ (i)

Again, $\tan\theta_2 = \frac{\tan\delta}{\tan\theta_2}$

or $\sin\theta = \frac{\tan\delta}{\tan\theta_2}$ (ii)

$\frac{\text{(i)}}{\text{(ii)}}$ gives $\tan\alpha = \frac{\tan\theta_1}{\tan\theta_2}$

26. (b) $2 = 2\pi\sqrt{\frac{I}{MH}}$

$1 = 2\pi\sqrt{\frac{I}{M(F-H)}}$

Dividing, $\frac{2}{1} = \sqrt{\frac{F-H}{H}} \Rightarrow \frac{H}{F} = \frac{1}{3}$

27. (a) $v = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$

$v' = \frac{1}{2\pi}\sqrt{\frac{M(B_H + B)}{I}}$

$v'' = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$

$\left(\frac{v'}{v}\right) = \frac{B_H + B}{B_H} = \frac{14 \times 14}{10 \times 10} = \frac{B_H + B}{B_H}$

or $1.96\, B_H = B_H + B$ or $0.96\, B_H = B$

Now $\left(\frac{v''}{v}\right)^2 = \frac{B_H - B}{B_H}$ or $\frac{(v'')}{100} = \frac{B_H - 0.96\, B_H}{B_H}$

or $(v'')^2 = 4$ or $v'' = 2$

28. (a) $\tau = MB\sin\theta$

or $\sin\theta = \frac{\tau}{MB} = \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = 0.5$

or $\theta = 30°$

29. (b) $T = 2\pi\sqrt{\frac{I_1 + I_2}{(2M + M)H}} = 3$ (i)

$$T' = 2\pi\sqrt{\frac{I_1 + I_2}{(2M - M)H}} \quad \text{(ii)}$$

$$\frac{T'}{T} = \sqrt{3} \quad \text{or} \quad T' = 3\sqrt{3}\ \text{s}$$

30. (c) In the first case, it is vibrating in the total field of earth $n \propto \sqrt{B}$. In the second case, it is vibrating in vertical component only as the horizontal component is zero.

$\therefore \quad n' \propto \sqrt{V}$ but $V < B \quad \therefore\ n' < n$.

31. (a) $n \propto \sqrt{H}$ or $\dfrac{9}{16} = \dfrac{H}{35 \times 10^{-5}}$

or $H = \dfrac{31.5 \times 10^{-5} \times 9}{16} = 1.98 \times 10^{-5}\ \text{T}$

32. (b) Here, $\dfrac{M_1}{M_2} = \dfrac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \dfrac{n_1^2 + n_2^2}{n_1^2 - n_2^2} = \dfrac{400 + 225}{400 - 225} = \dfrac{625}{175} = \dfrac{25}{7}$

33. (b) Since $T \propto \sqrt{I}$ and $I \propto m$, when mass is made 4 times, the time period becomes $2T$.

34. (a) $T = 2\pi\sqrt{\dfrac{I}{M\,B}}$

$$T_1 = 2\pi\sqrt{\frac{I_1}{M_1\,B}} \qquad \left(\because\ I_1 = \frac{I}{4} \text{ and } M_1 = \frac{M}{2}\right)$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad T_1 = \frac{T_0}{\sqrt{2}}$$

35. (b) $T = 2\pi\sqrt{\dfrac{I}{MB\cos\delta}}$

$$v = \frac{1}{2\pi}\sqrt{\frac{MB\cos\delta}{I}}$$

$$v = \sqrt{B\cos\delta}$$

or $B \propto \dfrac{v^2}{\cos\delta}$

$$\frac{B_1}{B_2} = \frac{400}{\cos 30^\circ} \times \frac{\cos 60^\circ}{225} = \frac{16 \times 2}{9 \times \sqrt{3}} \times \frac{1}{2} = 16 : 9\sqrt{3}$$

36. (a) Theory based

37. (d) $B_0 = V_0$ also total intensity

$$B = \sqrt{B_0^2 + V_0^2} \Rightarrow B = \sqrt{2}B_0$$

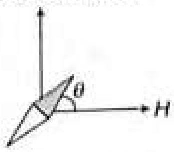
38. (a) Here, $\tan\delta = \dfrac{V}{H}$ and $\tan\delta' = \dfrac{V}{H\cos\theta}$

$$\therefore \quad \frac{\tan\delta'}{\tan\delta} = \frac{V}{H\cos\theta} \times \frac{H}{V} = \frac{1}{\cos\theta}$$

39. (a) In given case, H and H_0 are perpendicular to each other.

From figure $\tan\theta = \dfrac{H_0}{H}$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{H_0}{H}\right)$$



40. (a) Let the real dip be ϕ, then $\tan\varphi = \dfrac{B_V}{B_H}$

For apparent dip,

$$\tan\varphi' = \frac{B_V}{B_H\cos\beta} = \frac{B_V}{B_H\cos 30^\circ} = \frac{2B_V}{\sqrt{3}B_H}$$

or $\quad \tan 45^\circ = \dfrac{2}{\sqrt{3}} \cdot \tan\varphi \quad$ or $\quad \varphi = \tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

Magnetic Materials

41. (c) Susceptibility of diamagnetic substance is negative and it does not change with temperature.

42. (d) When a ferromagnetic material in heated above its curie temperature then it behaves like paramagnetic material.

43. (a) Soft iron is highly ferromagnetic.

44. (b) On heating, different domains have net magnetisation in them which are randomly distributed. Thus the net magnetisastion of the substance due to various domains decreases to minimum.

45. (d) From the characteristic of B-H curve.

46. (b) For a diamagnetic substance χ is small, negative and independent of temperature.

47. (a) Susceptibility of a paramagnetic substance is independent of magnetising field.

48. (a) Susceptibility of a ferromagnetic substance falls with rise of temperature $\left(\chi = \dfrac{C}{T - T_c}\right)$ and the substance becomes paramagnetic above Curie temperature, so magnetic susceptibility becomes very small above Curie temperature.

49. (a) $X = C \times \dfrac{1}{T} = \dfrac{0.4}{7 \times 10^{-3}} = 57\ \text{K}$

50. (b) With rise in temperature their magnetic susceptibility decreases i.e.

$$\chi_m \propto \frac{1}{T}$$

51. (c) Diamagnetic substances are repelled by magnetic field.

52. (c) If the temperature is decreased, the thermal vibrations will be reduced. So, there would not be negative effect on magnetisation.

53. (b) By Curie's law, $\chi_m \propto \dfrac{1}{T}$

$0.5\ T = K\,(273 + 27)$ or $T = 600\ \text{K}$

$\therefore \quad t = (600 - 273)^\circ\text{C} = 327^\circ\text{C}$

54. (b) Steel of an alloy Alnico (Al + Ni + Co) is used for making permanent magnets. Hysteresis loss is high. Moreover, coercivity is high and retentivity is low. Soft iron is used for electromagnets. Retentivity is high. Coercivity is low.

Problems Based on Mixed Concepts

55. (d) $F_m = B_H qv = 0.15 \times 1.6 \times 10^{-19} \times 3 \times 10^5\ \text{N}$

$= 7.2 \times 10^{-15}\ \text{N}$

Note that B_V would not exert any force on the protons.

56. (b) $B = \mu_0 nI$

Also, $B = \mu_0 H$

$\therefore \quad \mu_0 H = \mu_0 nI$

$\therefore \quad H = nI$

57. (d) $M_1 B \sin 30^\circ \propto (180^\circ - 30^\circ)$

and $M_2 B \sin 30^\circ \propto (270^\circ - 30^\circ)$

$\therefore \quad \dfrac{M_1}{M_2} = \dfrac{150^\circ}{240^\circ} = \dfrac{5}{8}$.

58. (c) In equilibrium, $\tau_1 = \tau_2$

$M B \sin\theta = \sqrt{3}\, M B \sin(90^\circ - \theta)$

$\therefore \quad \tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

59. (c) In the given figure, two magnets are perpendicular to each other.

$$B = \frac{\mu_0}{4\pi r^3}\sqrt{4+1} = \frac{\mu_0 M}{4\pi d^3}\sqrt{5}$$

60. (b) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{4\pi\times10^{-7}\times18}{2\pi\times0.2}\,\text{T} = 18\,\mu\text{T}$

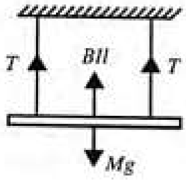
Now, $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$ and $T' = 2\pi\sqrt{\dfrac{I}{M(B_H - B)}}d$

Dividing $\dfrac{T'}{T} = \sqrt{\dfrac{B_H}{B_H - B}}$ or $\dfrac{T'}{T} = \sqrt{\dfrac{24}{24-18}} = 2$

$T' = 2\times0.1\text{ s} = 0.2\text{ s}$

61. (d) $2T + BIl = Mg$

or $\quad T = \dfrac{Mg - BIl}{2}$



62. (a) Here, $\tau = iAB\sin\theta$; $i = 0.1$ A

$A = \dfrac{1}{2}\times\text{base}\times\text{height}$

or $\quad A = \dfrac{1}{2}\times a\times\dfrac{a\sqrt{3}}{2} = \dfrac{\sqrt{3}a^2}{4} = \dfrac{\sqrt{3}\times(0.02)^2}{4}$

$= \sqrt{3}\times10^{-4}\,\text{m}^2$; $\theta = 90^\circ$

$\tau = 0.1\times\sqrt{3}\times10^{-4}\times5\times10^{-2}\sin 90^\circ$

$= 5\sqrt{3}\times10^{-7}\,\text{N-m}$

63. (b) As magnets are perpendicular to each other the resultant magnetic moment

$= \sqrt{M^2 + M^2} = \sqrt{2}\,M$

$\therefore \quad T_1 = 2\pi\sqrt{\dfrac{2I}{\sqrt{2}\,MH}} \qquad$ (i)

In the second case $T_2 = 2\pi\sqrt{\dfrac{I}{MH}}$

$\dfrac{T_2}{T_1} = \dfrac{1}{(2)^{1/4}}$

$\therefore \quad T_2 = \dfrac{4}{(2)^{1/4}} = 3.36$ s

64. (b) Since B and H are $\perp$ to each other and the resultant field is inclined at an angle 45° with

So, $B = H$

$\dfrac{\mu_0}{4\pi}\dfrac{2M}{r^3} = H$

$\therefore \quad r^3 = \dfrac{\mu_0}{4\pi}\dfrac{2M}{H} = 0.5$ m

65. (b) $B_1 = 1.5\times10^{-2}$ T

$B_2 = ?$

In equilibrium position,

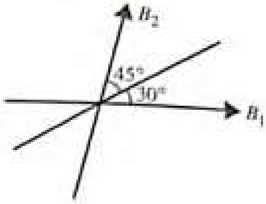
$\tau_1 = \tau_2$

$MB_1\sin\theta_1 = MB_2\sin\theta_2$

$1.5\times10^{-21}\times\sin 30^\circ$

$= B_2\times\sin 45^\circ$

Then $B = \dfrac{1.5}{\sqrt{2}}\times10^{-2}$ T



66. (b) $\tan\delta = \dfrac{B_V}{B_H}$ and $\tan\delta_1 = \dfrac{B_V}{B_H\cos\theta}$

or $\quad \tan\delta_1 = \dfrac{\tan\delta}{\cos\theta} = \tan\delta\sec\theta$

67. (a) $I = \dfrac{M}{V} = \dfrac{\mu N}{V} = \dfrac{1.5\times10^{-23}\times2\times10^{26}}{1} = 3\times10^3$ A/m

68. (c) The number of atoms per unit volume in a specimen,

$n = \dfrac{\rho N_A}{A}$

For iron, $\rho = 7.8\times10^{-3}$ kg m^{-3},

$N_A = 6.02\times10^{26}$/ kg mol, $A = 56$

$\Rightarrow \quad n = \dfrac{7.8\times10^3\times6.02\times10^{26}}{56} = 8.38\times10^{28}\,m^{-3}$

Total number of atoms in the bar is

$N_0 = nV = 8.38\times10^{28}\times(5\times10^{-2}\times1\times10^{-2}\times1\times10^{-2})$

$N_0 = 4.19\times10^{23}$

The saturated magnetic moment of bar

$= 4.19\times10^{23}\times1.8\times10^{-23} = 7.54$ Am^2

69. (d) We have, $B = \mu_0 H + \mu_0 I$

or $\quad I = \dfrac{B - \mu_0 H}{\mu_0}$ or $I = \dfrac{\mu H - \mu_0 H}{\mu_0} = \left(\dfrac{\mu}{\mu_0} - 1\right)H$

$I = (\mu_r - 1)H$

For a solenoid of n-turns per unit length and current i

$H = ni$

$\therefore \quad I = (\mu_r - 1)ni = (1000 - 1)\times500\times0.5$

$I = 2.5\times10^5$ Am^{-1}

$\therefore \quad$ Magnetic moment $M = IV$

$M = 2.5\times10^5\times10^{-4} = 25$ Am^2

70. (d) The bar magnet coercively 4×10^3 Am^{-1} i.e., it requires a magnetic intensity $H = 4\times10^3$ Am^{-1} to get demagnetized. Let i be the current carried by solenoid having n number of turns per metre length, then by definition $H = ni$. Here $H = 4\times10^3$ Amp turn metre^{-1}

$n = \dfrac{N}{l} = \dfrac{60}{0.12} = 500$ turn metre^{-1}

$\Rightarrow \quad i = \dfrac{H}{n} = \dfrac{4\times10^3}{500} = 8.0$ A

71. (b) $I = \dfrac{M}{V} = \dfrac{M}{\text{mass/density}}$, given mass = 1 gm = 10^{-3} kg,

and density $= 5\text{ gm/cm}^3 = \dfrac{5\times10^{-3}\text{kg}}{(10^{-2})^3\text{m}^3} = 5\times10^3\text{ kg/m}^3$

Hence $I = \dfrac{6\times10^{-7}\times5\times10^3}{10^{-3}} = 3\text{ A/m}$

72. (a) $E = nAVt = nA\dfrac{m}{d}t = \dfrac{50\times250\times10\times3600}{7.5\times10^3} = 6\times10^4\text{ J}$

73. (b) $T = 2\pi\sqrt{\dfrac{I}{MB_H}} \Rightarrow T \propto \dfrac{1}{\sqrt{M}} \Rightarrow \dfrac{T_1}{T_2} = \sqrt{\dfrac{M_2}{M_1}}$

If $M_1 = 100$ then $M_2 = (100-36) = 64$

So $\dfrac{T_1}{T_2} = \sqrt{\dfrac{64}{100}} = \dfrac{8}{10} \Rightarrow T_2 = \dfrac{10}{8}T_1 = 1.25\,T_1$

So % increase in time period = 25%

Archives

1. (c) Moment of inertia $= \dfrac{\text{mass}\times(\text{length})^2}{12}$

When the magnet is divided into two equal halves, its mass is reduced by a factor of 2 and length is also reduced by a factor of 2. So the new moment of inertia is 1/8 of the initial moment of inertia.

Also, magnetic moment = pole strength × length

Pole strength is unchanged and the length is halved. So the new magnetic moment is one-half of the initial magnetic moment.

Now,

$$T' = 2\pi\sqrt{\frac{I'}{M'B}}$$

$$= 2\pi\sqrt{\frac{I/8}{\frac{M}{2}B}} = \frac{T}{\sqrt{4}} = \frac{T}{2}$$

$$\therefore \quad \frac{T'}{T} = \frac{1}{2}$$

2. (b) Curie temperature is that temperature above which a ferromagnetic material becomes paramagnetic.

3. (b) $W = MB\,(I - \cos 60°) = \dfrac{MB}{2}$

$\Rightarrow \quad MB = 2\text{ W}$

Again,

$$\tau = MB\sin 60°$$

$$= MB\times\frac{\sqrt{3}}{2} = 2\text{ W}\times\frac{\sqrt{3}}{2} = \sqrt{3}\text{ W}$$

4. (a) The magnetic lines of force inside a bar magnet are south pole to north pole. but outside the magnet the force lines are from north to south pole

5. (b) Initially, the time period of the magnet

$$T = 2 = 2\pi\sqrt{\frac{I}{MH}} \qquad \text{...(i)}$$

Moment of inertia $I = \dfrac{ml^2}{12}$

Now the magnet is cut into three equal parts, the moment of inertia of each part

$$I' = \frac{(m/3)(l/3)^2}{12} = \frac{1}{27}\frac{ml^2}{12} = \frac{I}{27}$$

The magnetic moment M = pole strength × length

After cutting the magnet the magnetic moment of each part

$$M' = \frac{M}{3}$$

Now each parts of magnets are placed together with like poles together

$\therefore$ Moment of inertia of system

Magnetic moment of system

Time period of system

$$T_s = 2\pi\sqrt{\frac{I_s}{M_sB}} = \frac{1}{3}\times2\pi\sqrt{\frac{I}{MB}} = \frac{T}{3} = \frac{2}{3}\text{sec}$$

6. (c) The electromagnets are made of soft iron. Soft iron has high retentivity and low coercivity.

7. (a) The current required for full-scale deflection is

$$I_s = \frac{150}{10}\text{ mA} = 15\text{ mA}$$

The voltage corresponding to full-scale deflection is

$$\frac{150}{10}\text{ mA} = 75\text{ mA}$$

Galvanometer resistance

$$G = \frac{75\text{ mV}}{15\text{ mA}} = 5\,\Omega$$

The voltage to be measured is 150 V. Therefore,

$$V = I_g(R+G)$$

$$\Rightarrow \quad R = \frac{V}{I_g} - G$$

$$= \frac{150}{15\times10^{-3}}$$

$$= 1000 - 5 = 9995\,\Omega$$

8. (b) A magnetic needle is a magnetic dipole. A magnetic dipole experiences both force and torque when placed in non-uniform magnetic field, hence magnetic needle also experiences both force and torque in non-uniform magnetic field.

9. (d) Force exerted by magnetic field

$= Bev\sin\theta = Bev\sin 0° = 0$

Electric field exerts force along a straight line. The path of charged particle will be a straight line.

10. (c) Ferromagnetic substances are strongly attracted. Paramagnetic substances are weakly attracted. Diamagnetic substances are repelled.

11. (a) Use $F = iBl$

$$[\text{MLT}^{-2}] = \left[\frac{C}{T}\right][B][L]$$

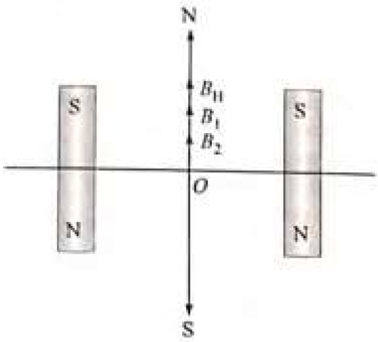
$$\therefore \quad [B] = [\text{MT}^{-1}\,\text{C}^{-1}]$$

12. (d) For diamagnetic materials, $0 < \mu_r < 1$, and for any material, $\varepsilon_r > 1$.

13. (c) For ammeter, $S = \dfrac{I_gG}{I - I_g}$

So for I to increase, S should decrease, so additional S can be connected across it.

14. (a) The situation is as shown in the figure. As the point O lies on broad-side position with respect to both the magnets. Therefore,



The net magnetic field at point O is

$B_{net} = B_1 + B_2 + B_H$

$$B_{net} = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} + \frac{\mu_0}{4\pi}\frac{M_2}{r^3} + B_H = \frac{\mu_0}{4\pi r^3}(M_1 + M_2) + B_H$$

Substituting the given values, we get

$$B_{net} = \frac{4\pi \times 10^{-7}}{4\pi \times (10 \times 10^{-2})^3}[1.2 + 1] + 3.6 \times 10^{-5}$$

$$= \frac{10^{-7}}{10^{-3}} \times 2.2 + 3.6 \times 10^{-5}$$

$$= 2.2 \times 10^{-4} + 0.36 \times 10^{-4}$$

$$= 2.56 \times 10^{-4}\ \text{Wb/m}^2$$

15. (a) Here,

$L = 10$ cm $= 0.1$ m, $N = 100$, $I = ?$

As, $B = \mu_0 nI = \mu_0 \dfrac{N}{L} I$

$$\Rightarrow \quad I = \frac{B}{\mu_0} \times \frac{L}{N} = 3 \times 10^3 \times \frac{0.1}{100} = 3\ \text{A}$$

16. (d) For electromagnet and transformers, we require the core that can be magnetised and demagnetised quickly when subjected to alternating current. Transformer core and electromagnets are made by soft magnetic materials, which are easy to magnetise and demagnetise whereas for electric generators we use hard magnetic materials.

Fig. A → Hard magnetic material: Fig. B → soft magnetic material. Hence graph B is suitable.

17. (c) $T = 2\pi\sqrt{\dfrac{I}{MB}}$

$I = 7.5 \times 10^{-6}$ kg-m^2

$M = 6.7 \times 10^{-2}$ Am^2

By substituting value in the formula

$T = 0.665$ sec

For 10 oscillations, time taken will be

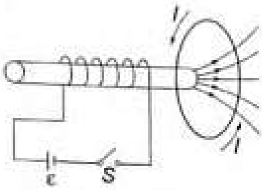
Time $= 10T = 6.65$ sec

Option (c) is correct.

CHAPTER 23: ELECTROMAGNETIC INDUCTION

Concept Application Exercises 23.1

1. (a) As the switch is closed, the situation changes from one in which no magnetic flux passes through the ring to the one in which flux passes through the ring in the direction shown. The induced current flows in the anticlockwise sense as seen from the side on which the solenoid lies.



(b) When the switch has been closed for a long time, no change in the magnetic flux through the loop occurs; hence, no current is induced in the loop.

(c) When the switch is opened, flux decreases in the direction shown above. Hence current will be opposite to the direction of situation (a), i.e., in clockwise direction.

2. (a) Induced current should be in anticlockwise direction, so that upper face becomes north pole.

(b) Induced current should be clockwise so that upper face becomes south pole.

(c) Flux is increasing in the inward direction, so induced current should be in anticlockwise direction. It will oppose the increase of flux.

(d) Flux is decreasing in upward direction, so induced current will be in anticlockwise direction. It will oppose the decrease in flux.

3. (a) When the switch is opened, the magnetic field to the right decreases. Therefore, the second coil's induced current produces its own field to the right. That means the current must pass through the resistor from point a to point b.

(b) If coil B is moved closer to coil A, more flux passes through it towards the right. Therefore, the induced current must produce its own magnetic field to the left to oppose the increased flux. That means the current must pass through the resistor from point b to point a.

(c) If the variable resistor R is decreased, then more current flows through coil A, and so a stronger magnetic field is produced, leading to more flux to the right through coil B. Therefore, the induced current must produce its own magnetic field to the left to oppose the increased flux. That means the current must pass through the resistor from point b to point a.

4. (a) When current is passing from a to b and is increasing, the magnetic field becomes stronger to the left, so the induced field points towards right, and the induced current must flow from right to left through the resistor.

(b) If the current passes from b to a, and is decreasing, then there is less magnetic field pointing right, so the induced field points towards right, and the induced current must flow from right to left through the resistor.

(c) If current passes from b to a and is increasing, then there is more magnetic field pointing right, so the induced field points left, and the induced current must flow from left to right through the resistor.

5. (a) Φ_B is $\odot$ and increasing so the flux Φ_{ind} of the induced current is clockwise.

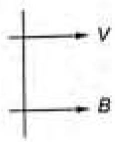
(b) The current reaches a constant value so Φ_B is constant $d\Phi_B/dt = 0$ and there is no induced current.

(c) Φ_B is $\odot$ and decreasing, so Φ_{ind} is $\odot$ and current is counterclockwise.

Concept Application Exercises 23.2

1. In loop (i), no emf will be induced because there is no flux change. In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field in inward direction. Therefore, to oppose the decrease in flux in inward direction, current will be induced such that its magnetic field will be inwards. For this, direction of current should be clockwise.

2. (a)

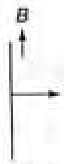


emf = 0 as $\vec{B} \parallel \vec{v}$

(b)

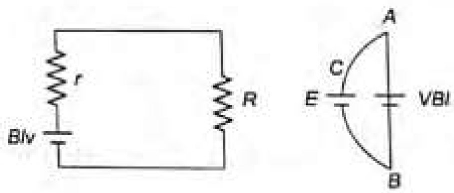
emf = 0 as $\vec{\ell} \parallel \vec{v}$

(c)



emf = 0 as $\vec{\ell} \parallel \vec{B}$

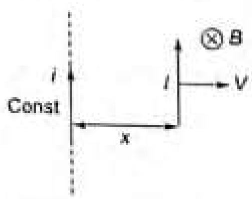
3.



4. $E = Blv = \dfrac{\mu_0 i l v}{2\pi x}$

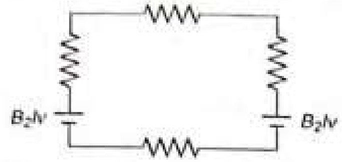
Alternatively:

emf is equal to the rate with which magnetic field lines are cut. In dt time the area swept by the rod is $lv\,dt$. The magnetic field lines cut in dt time $= Blv\,dt = \dfrac{\mu_0 i l v\,dt}{2\pi x}$.



$\therefore$ The rate at which magnetic field lines are cut $= \dfrac{\mu_0 i l v}{2\pi x}$ and this will be induced emf.

5. $E = B_1 lv - B_2 lv = \frac{\mu_0 i}{2\pi x} lv - \frac{\mu_0 i}{2\pi(x+b)} lv = \frac{\mu_0 i l b v}{2\pi x(x+b)}$



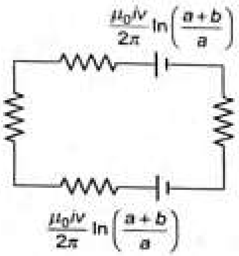
6. Consider a segment of rod of length dx, at a distance x from the wire, emf induced in the segment,

$$d\varepsilon = \frac{\mu_0 i}{2\pi x} dx\, v$$

$$\therefore \quad \varepsilon = \int_a^{a+l} \frac{\mu_0 i v dx}{2\pi x} = \frac{\mu_0 i v}{2\pi} \ln\left(\frac{\ell + a}{a}\right)$$

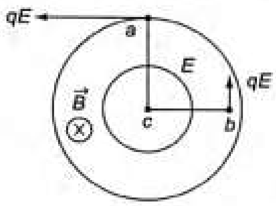
7. Net emf = 0

The equivalent circuit is



Concept Application Exercises 23.3

1. Since the field is increasing in inward direction, so electric field will be in anticlockwise direction. So, force on a and b are as shown in figure.



$E 2\pi r = \pi r^2 \frac{dB}{dt} \Rightarrow E = \frac{r}{2}\frac{dB}{dt}$

So force on a and b: $F = qE = \frac{qr}{2}\frac{dB}{dt}$

At centre, electric field will be zero, so no force on charge at c.

2. $\frac{dB}{dt} = 0.060\, t$

$$E_{\text{at } P_1} = \frac{r}{2}\frac{dB}{dt} = \frac{0.020}{2} \times 0.060 = 6 \times 10^{-4} \text{ V/m}$$

Here magnetic field is increasing with time hence electric field will be in anticlockwise direction.

3. At $t = 0$, C will act as a simple wire, so R and L both will be short-circuited. So that there is no current in R and L.

$$I_1 = 0, I_2 = 0, I_3 = \frac{\varepsilon}{r}$$

At $t = \infty$, L will act as a simple wire so, R and C will be short-circuited.

$$I_1 = 0, I_2 = \frac{\varepsilon}{r}, I_3 = 0$$

4. (a) $U = Pt = (200 \text{ W})(24 \text{ h/day} \times 3600 \text{ s/h}) = 1.728 \times 10^7$ J

(b) $U = \frac{1}{2}LI^2 \Rightarrow L = \frac{2U}{I^2} = \frac{2(1.728 \times 10^7 \text{ J})}{(80.0 \text{ A})^2} = 5400$ H

5. (a) $i_{max} = \frac{30 \text{ V}}{1000\ \Omega} = 0.030 \text{ A} = 30$ mA, long time after closing the switch.

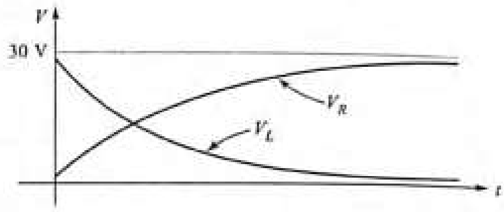
(b) $i = i_{max}(1 - e^{-t/(L/R)}) = 0.030\left(1 - e^{-\frac{20\ \mu s}{10\ \mu s}}\right) = 0.0259$ A

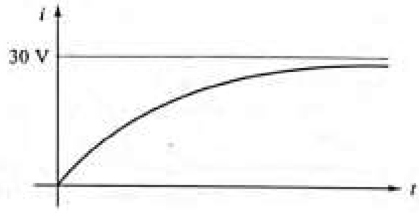
$V_R = Ri = (1000\ \Omega)(0.0259 \text{ A}) = 26$ V

$V_L = \varepsilon_{\text{Battery}} - V_R = 30 \text{ V} - 26 \text{ V} = 4.0$ V

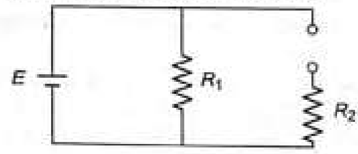
(or could use $V_L = L\frac{di}{dt}$ at $t = 20\ \mu$s)

(c)





6. (i) At $t = 0$, inductor acts as open circuit:



(a) $i_1 = \frac{E}{R_1} = 2$ A

(b) $i_2 = 0$ A

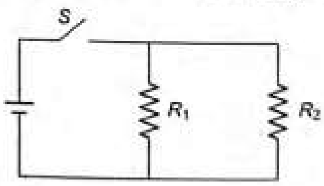
(c) $i = i_1 + i_2 = 2 + 0 = 2$ A

(d) V across resistor $R_2 = 0$

(e) V across $L = 10$ V

(f) $L\frac{di_2}{dt} = 10 \Rightarrow \frac{di_2}{dt} = \frac{10}{5} = 2$ A/s

(ii) At $t = \infty$, inductor acts as a conducting wire:



(a) $i_1 = \frac{E}{R_1} = 2$ A

(b) $i_2 = \frac{E}{R_2} = \frac{10}{10} = 1$ A

(c) $i_{\text{switch}} = i_1 + i_2 = 3$ A

(d) $V_{R2} = 10$ V

(e) $V_L = 0$ V

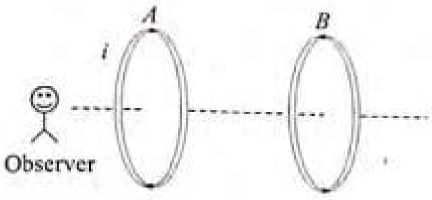
(f) Current in R_2 is not changing with time, so

$$\frac{di_2}{dt} = 0$$

Exercises

Faraday's Law and Lenz's Law

1. (d)



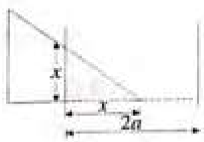
If current through A increases, crosses (X) linked with coil B increases, hence anticlockwise current induces in coil B. As shown in the figure both the current produces repulsive effect.

2. (d) At B, flux is maximum, so from $|\varepsilon| = \frac{d\phi}{dt}$ at B $|\varepsilon| = 0$

3. (a) Increasing the resistance causes a decrease in the current on the left. This reduces the field strength for the solenoid $B = \mu_0 nI$. By the right-hand rule, the field through the solenoid is directed to the right. Hence, in the right-hand circuit. Since there will be fewer field lines directed to the right, there is an induced electric field that will produce a current in the wires to "replace" the lost field lines. This means that current will be directed with the same orientation as the current in the left circuit. ∴ the galvanometer will deflect to the left.

4. (c) For $0 \le x \le a$ The flux enclosed $\phi = \frac{1}{2}Bx^2$

Hence induced emf, $e = -\frac{d\phi}{dt} = -Bxv$

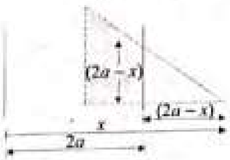


At $x = 0$, $e = 0$

At $x = \frac{a}{2}$, $e = -\frac{Bav}{2}$

At $x = a$, $e = -Bav$

Now considering for $a < x \le 2a$, The flux enclosed with triangle will be constant hence no induced emf



Finally for $2a \le x \le 3a$,

$$\phi = B\left[\frac{a^2}{2} - \frac{(x-2a)^2}{2}\right]$$

$$e = -\frac{d\phi}{dt} = \frac{B}{2}\left[2(x-2a)v\right] = Bvx - 2Bav$$

Hence graph for this part is straight line with positive slope but negative intercept with y-axis.

5. (b) Apply Lenz's law.

6. (c) Because A and C are at equal distance from B, and their flux across B is in opposite direction, so at any time flux in B will be zero. Hence no emf is induced.

7. (b) In the r–t graph, it is clear that from a to b there is no change in radius and hence no change in area and magnetic flux. Same is the situation from c to d.

Now, $|\varepsilon| = \frac{d}{dt}(\phi)$

$$|\varepsilon| = B\frac{d}{dt}(\pi r^2) = B\pi 2r\frac{dr}{dt}$$

Since $r \propto t$, ∴ $\frac{dr}{dt}$ = constant

∴ $|\varepsilon| \propto r$

8. (b) $Q = \frac{\Delta\phi}{R} = \frac{\phi_2 - \phi_1}{R} = \frac{BA - 0}{R} = \frac{BA}{R}$

9. (c) By Lenz's law, clockwise current is induced in both loops. Greater the area, large will be the induced emf Outer loop has greater area.

10. (b) ϕ(flux linked)

$$= a^2B\cos 0° - b^2B\cos 180°$$

$$= (a^2 - b^2)B$$

$$E = -\frac{d\phi}{dt} = -(a^2 - b^2)\frac{dB}{dt}$$

$$= (a^2 - b^2)\, B_0\, \omega \cos \omega t$$

where $B = B_0 \sin \omega t$, $B_0 = 10^{-3}$ T, $\omega = 100$

∴ $I_{max} = (a^2 - b^2)\frac{B_0\omega}{R}$

and $R = (4a + 4b)\, r = 4\,(a + b)\, r$

∴ $I_{max} = \frac{(a-b)B_0\omega}{4r} = \frac{(1-0.4)\times 10^{-3}\times 100}{4\times 5\times 10^{-3}} = 3$ A

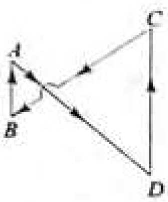
11. (d) Magnetic flux in ⊗ direction through the coil is increasing. Therefore, induced current will produce magnetic field in ⊙ direction. Thus, the current in the loop is anticlockwise. Magnitude of induced current at any instant of time is

$$i = \frac{e}{R} = \frac{Bv(FG)}{\rho(FG + GD + DF)}$$

When the wire AH moves downwards FG, GD and DF all increase in the same ratio. Therefore, i is constant.

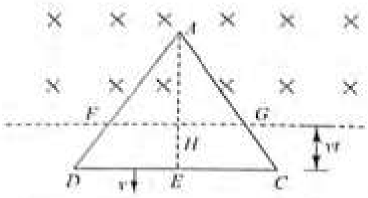
12. (d) When the coil is within the field, there is no change in the magnetic flux passing through it. Thus, no current will be induced and the acceleration will be g. But according to Lenz's law, the induced current will oppose its motion when it enters or leaves the field. Therefore, acceleration will be less than g.

13. (a) Magnetic field in ⊗ direction is increasing. Therefore, induced current will produce magnetic field in ⊙ direction. Thus, current in both the loops should be anticlockwise. But as the area of the loop on the right side is more, induced emf in this side will be more compared to the left side loop.



Therefore, net current in the complete loop will be in a direction shown below:

14. (b) Let $2a$ be the side of the triangle and b be the length AE.



$$\frac{AH}{AE} = \frac{GH}{EC} \Rightarrow GH = \left(\frac{AH}{AE}\right) EC$$

or $\quad GH = \dfrac{(b - vt)}{b} . a = a - \left(\dfrac{a}{b} vt\right)$

$\therefore \quad FG = 2GH = 2\left[a - \dfrac{a}{b} vt\right]$

$\therefore \quad$ Induced emf, $e = Bv\,(FG) = 2Bv\left(a - \dfrac{a}{b} vt\right)$

$\therefore \quad$ Induced current, $i = \dfrac{e}{R} = \dfrac{2Bv}{R}\left[a - \dfrac{a}{b} vt\right]$

or $\quad i = k_1 - k_2\, t$

Thus i–t graph is a straight line with negative slope and positive intercept.

15. (a) Speed of the loop should be

$$v = \frac{\ell}{t} = \frac{0.5}{2} = 0.25 \text{ m s}^{-1}$$

Induced emf, $\varepsilon = Bv\ell = (1.0)\,(0.25)\,(0.5) = 0.125$ V

$\therefore \quad$ Current in the loop, $i = \dfrac{\varepsilon}{R} = \dfrac{0.125}{10}$

$$= 1.25 \times 10^{-2} \text{ A}$$

The magnetic force on the left arm due to the magnetic field is

$$F_m = i\ell B = (1.25 \times 10^{-2})\,(0.5)\,(1.0)$$

$= 6.25 \times 10^{-3}$ N

To pull the loop uniformly an external force of 6.25×10^{-3} N towards right must be applied.

$\therefore \quad W = (6.25 \times 10^{-3}\text{N})\,(0.5\text{m}) = 3.125 \times 10^{-3}$ J

16. (d) Induced emf depends upon vertical edge.

17. (b) $B = \mu_0 nI = 4\pi \times 10^{-7} \times 200 \times 10^2 \times 1.5 = 3.8 \times 10^{-2}$ T

$$\phi = BA = 3.8 \times 10^{-2} \times 3.14 \times 10^{-4} = 1.2 \times 10^{-5} \text{ Wb}$$

When the current in the solenoid is reversed, the change in magnetic flux,

$$d\phi = 2 \times (1.2 \times 10^{-5}) = 2.4 \times 10^{-5} \text{ Wb}$$

$\therefore \quad$ Induced e.m.f.,

$$\varepsilon = N\left(\frac{d\phi}{dt}\right) = 100 \times \left(\frac{2.4 \times 10^{-5}}{0.05}\right) = 0.048 \text{ V}$$

18. (b) $|\varepsilon| = B\dfrac{dA}{dt}$

$$|\varepsilon| = B\frac{(A_2 - A_1)}{(t_2 - t_1)}$$

$$|\varepsilon| = \frac{1\left(\frac{\pi}{4} - 0\right)}{(1 - 0)}$$

$$\varepsilon = \frac{\pi}{4} \text{ V}$$

Motional EMF

19. (a) Consider the force on an electron in PQ. This electron experiences a force towards Q. Free electrons in PQ tend to move towards N. So M will be positively charged.

20. (b) Emf induced across the rod AB is

$$e = \vec{B} \cdot (\vec{\ell} \times \vec{v}) = B\ell v \sin\theta = 2 \times 2 \times 2 \times \sin 30$$

$$e = 4 \text{ V}$$

Free electrons of the rod shift towards right due to force $q(\vec{v} \times \vec{B})$

Thus, end P is at higher potential

or $\quad V_P - V_Q = 4$ V. Thus, choice (b) is correct.

21. (c) $\vec{F}_m \perp \vec{v} \;\therefore\; \theta = 90° + 30° = 120°$

22. (c) Induced emf $= Blv$. R is internal resistance of seat of emf, i.e., of rod

Total resistance of circuit $= R + \dfrac{R_1 R_2}{R_1 + R_2}$

$$\therefore \quad I = \frac{Blv}{R + \dfrac{R_1 R_2}{(R_1 + R_2)}} = \frac{Blv(R_1 + R_2)}{R_1 R_2 + R(R_1 + R_2)}$$

23. (a) Let v is the velocity of conductor at any time, then induced emf: $e = Blv$ ...(i)

Charge on capacitor: $q = Ce = CBlv$

Current in circuit: $I = \dfrac{dq}{dt} = CBl\dfrac{dv}{dt}$

for conductor: $mg - IBl = \dfrac{mdv}{dt}$

$$\Rightarrow \quad mg - CB^2l^2\frac{dv}{dt} = \frac{mdv}{dt} \quad \Rightarrow \quad \frac{dv}{dt} = \frac{mg}{m + CB^2l^2}$$

This is the acceleration of conductor which is constant.

24. (a) Component of weight along the inclined plane $= mg \sin\theta$

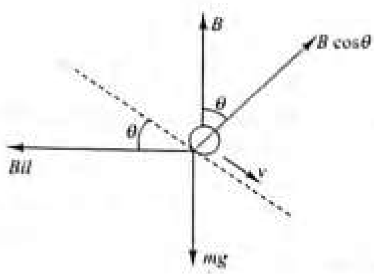
Again, $F = BIl = B\dfrac{Blv}{R}\,l = \dfrac{B^2l^2v}{R}$

Now, $\dfrac{B^2l^2v}{R} = mg\sin\theta \quad$ or $\quad v = \dfrac{mgR\sin\theta}{B^2l^2}$

25. (c) $Bil\cos\theta = mg\sin\theta$ (i)

Here induced emf across slider is $(B\cos\theta)lv$

$\therefore \quad$ Induced current $I = \dfrac{Blv\cos\theta}{R}$



From Eq. (i)

$$Bl\cos\theta\,\frac{Blv\cos\theta}{R}=mg\sin\theta$$

$$\therefore\quad v=\frac{mgR\sin\theta}{B^2l^2\cos^2\theta}$$

26. (a) $|E|$ = Magnitude of induced emf

$$=\frac{B\ell^2}{2}\omega,\ \ell=\sqrt{2}\,r$$

$$E=\frac{B(2r^2)\omega}{2}=B\omega r^2$$

27. (b) Let there be an element dx of rod at a distance x from the wire.

emf developed in the element, $dE = B\,dx\,v$

$$\therefore\quad dE=\left(\frac{\mu_0}{4\pi}\frac{2I}{x}\right)dxv$$

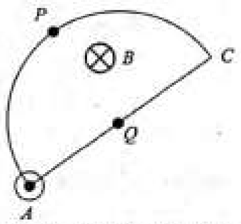
$$\therefore\quad E=\frac{\mu_0 Iv}{2\pi}\int_a^b\frac{dx}{x}=\frac{\mu_0 Iv}{2\pi}\log_e\frac{b}{a}$$

$$\therefore\quad E=\frac{4\pi\times10^{-7}\times100\times5}{2\pi}\log_e\frac{100}{1}$$

$$=4.6\times10^{-4}\ \text{V}=0.46\ \text{mV}$$

28. (b) We connect a conducting wire from A to C and complete the semicircular loop.

The emf in the semicircular loop is zero because its magnetic flux does not change.



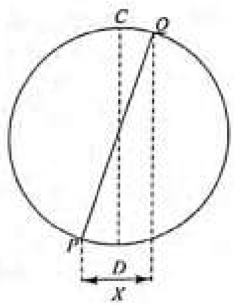
$\therefore$ emf of section APC + emf of section $CQA = 0$

$\therefore$ emf of section APC = emf of section $AQC = 2BR^2\omega$

29. (d) Since all the wires are connected between rim and axle, they will generate induced emf in parallel, hence it is same for any number of spokes.

30. (b) When the ring falls vertically, there will be an induced emf across A and B ($e = Bv(2r)$).

Note that there will be a potential difference across any two points on the ring, and the line joining these has a projected length in the horizontal plane. For example, between points P and Q there is a projected length x in the horizontal plane.



$\therefore$ P.D. across P and Q is

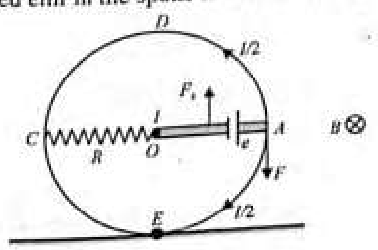
$V = B\,vx$

But for points C and D, $x = 0$

Therefore, P.D. = 0

Hence (b)

31. (d) Induced emf in the spoke is shown in figure below.



$$e=\frac{1}{2}B\omega r^2,\ I=\frac{e}{R}=\frac{B\omega r^2}{2R}$$

There will be no induced emf separately in parts ADC or AEC.

$$F_b=IrB=\frac{B^2\omega r^3}{2R}$$

Balancing torque about E: $Fr = F_b r/2$

$$\Rightarrow\quad F=\frac{F_b}{2}=\frac{B^2\omega r^3}{4R}$$

Note: Force due to currents $I/2$ will act on circular parts also, but their torque about E will be zero.

32. (d) Induced emf between O and Q is

$$\frac{1}{2}B\omega(2r)^2=B\omega 2r^2=2Bvr.$$

But net induced emf in the ring will be zero. Hence no current flows in the ring.

33. (d) Potential difference across capacitor

$V = Bv\ell$ = constant

Therefore, charge stored in the capacitor is also constant. Thus, current through the capacitor is zero.

Induced Electric Field, Self and Mutual Inductance

34. (d) Perpendicular distance between BC and centre O is 10 cm.

Component of induced electric field along the rod $=\dfrac{d}{2}\dfrac{dB}{dt}$,

where d = perpendicular distance from centre to the rod. Hence, potential difference between the ends of rod.

$$V=El=l\cdot\frac{d}{2}\frac{dB}{dt}$$

$$=\frac{10}{2}\times10^{-2}\times20\times10^{-2}2=20\ \text{mV}$$

35. (c) Perpendicular distance between CD and O is 20 cm. Therefore, induced emf in CD.

$$=\frac{d}{2}l\frac{dB}{dt}=\frac{20}{2}\times10^{-2}\times20\times10^{-2}\times2=40\ \text{mV}$$

36. (c) $\displaystyle\int E.dl=\frac{-d\phi}{dt}=-\pi r^2\frac{dB}{dt};\ N=R\int\lambda E dl$

$$=-R\lambda\pi r^2\frac{dB}{dt}$$

$$\int Ndt=-R\lambda\pi r^2\int dB=R\lambda\pi r^2B=I\omega$$

$$\Rightarrow\quad \pi r^2B\frac{q}{2\pi r}=mr^2\omega\quad\Rightarrow\quad\omega=\frac{qB}{2m}$$

37. (b) $\displaystyle\tau=\frac{\Delta L}{\Delta t}=\frac{I\omega}{\Delta t}=\frac{mR^2\omega}{\Delta t}$

but $\displaystyle\tau=\lambda(2\pi R)ER=\lambda\left(\pi a^2\frac{\Delta B}{\Delta t}\right)R=\frac{\lambda\pi a^2BR}{\Delta t}$

$$\Rightarrow\quad\frac{mR^2\omega}{\Delta t}=\frac{\lambda\pi a^2BR}{\Delta t}\Rightarrow\omega=\frac{\lambda\pi a^2B}{mR}$$

38. (b) The changing magnetic flux through the ring (due to changing field B) produced an electrical field along the ring (tangential to the ring at every point), the magnitude of the electrical

field being $\vec{E} = -\frac{R}{2}\frac{dB}{dt}$, the minus sign indicating that the electrical field opposes the change of the magnetic field. This electric field produces a tangential force F on the charge q carried by the ring, $|F| = |E|.q = \frac{Rq}{2}\frac{dB}{dt}$.

Angular acceleration $= \frac{d\omega}{dt} = \frac{|F|R}{I} = \frac{|F|R}{mR^2}$, where I is the moment of inertia of the ring (since the ring is non-conducting, the force on the charge tending to produce a current act on the ring itself)

Hence, $\frac{d\omega}{dt} = \frac{1}{mR}\frac{Rq}{2}\frac{dB}{dt} = \frac{q}{2m}\frac{dB}{dt}$

Integrating, $|\omega| = \frac{q}{2m}|\vec{B}(t)|$

39. (d) We use equation, $|\varepsilon_2| = \left|-M\frac{di_1}{dt}\right|$, from which we obtain the mutual inductance:

$$M = \frac{|\varepsilon_2|}{|di_1/dt|} = \frac{0.0960\text{ V}}{1.20\text{ A/s}} = 0.080\text{ H} = 80\text{ mH}$$

40. (c) A current i in the large loop of radius R produces a magnetic field of magnitude $B = \frac{\mu_0 i}{2R}$ at its center. Since the radius of the small loop $r << R$, we may treat the flux through the small loop as being approximately $\Phi_B = BA\cos 0°$ $= \frac{\mu_0 i}{2R}A = \frac{\mu_0 \pi r^2 i}{2R}$. The mutual inductance of the loops is then

$$M = \frac{\Phi_B}{i} = \frac{\mu_0 \pi r^2}{2R}$$

41. (b) Total magnetic flux linked with coil

$$= \frac{5\times 10}{10} \times 500 = 2.5\times 10 \quad \text{Wb}$$

$$L = \frac{\phi}{i} = \frac{2.5\times 10^{-2}}{5} = 5\times 10^{-3}\text{ H}$$

42. (a) $L = \frac{\mu_0 N^2 \pi r^2}{l}$

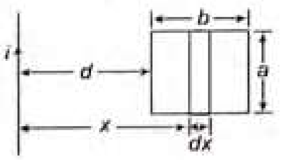
Length of wire $= N\,2\pi r =$ Constant ($= C$, suppose)

$\therefore \quad L = \mu_0\left(\frac{C}{2\pi r}\right)^2 \frac{\pi r^2}{l}$

$\therefore \quad L \propto \frac{1}{l}$

$\therefore$ Self-inductance will become $2L$.

43. (a) The flux in rectangular loop due to current i in wire is



Mutual inductance is $M = \frac{\phi}{i} = \frac{\mu_0 a}{2\pi}\ln\frac{b+d}{d}$

$\therefore$ Mutual inductance is proportional to a.

44. (b) As anticlockwise direction is positive, so area vector out wards is positive. So net flux through the given loop is

$$\phi = -BA - BA + BA = -BA$$

$$\oint \vec{E}\cdot d\vec{r} = -\frac{d\phi}{dt} = -\frac{d(-BA)}{dt} = A\frac{dB}{dt} = A(-\alpha)$$

$$\Rightarrow \quad \oint \vec{E}\cdot d\vec{r} = -\alpha A$$

45. (c) The mutual inductance M remains the same whether X or Y is used as the primary.

46. (b) Magnetic field at the centre of a large coil $E \propto \frac{\mu_0 I}{2R}$.

Magnetic flux linked with smaller coil $= \frac{\mu_0 I}{2R}\times \pi r^2$

$$M = \frac{\phi}{I} = \frac{\mu_0 \pi r^2}{2R} \quad \therefore M \propto \frac{r^2}{R}$$

47. (d) When inductances are connected like this in series, then $L = L_1 + L_2 + 2M$

48. (d) Induced current in $B = 0.006\text{ A} = 6\times 10^{-3}\text{ A}$

Induced e.m.f. in $B = 6\times 10^{-3}\times 4\text{ V} = 24\times 10^{-3}\text{ V}$

Now, $M\frac{dI}{dt} = 24\times 10^{-3}$

or $\quad dI = \frac{24\times 10^{-3}\times 0.02}{3\times 10^{-3}}\text{ A} = 0.16\text{ A}$

49. (b) $\phi_2 = N_2 B_1 A$ or $\phi_2 = N_2\frac{\mu_0 N_1 I_1}{L}A$

or $\quad \phi_2 = \frac{\mu_0 N_1 N_2 A}{L}I_1$

Comparing with $\phi_2 = MI_1$, we get $M_2 = \frac{\mu_0 N_1 N_2 A}{L}$

50. (c) Induced e.m.f. $e = L\frac{dI}{dt} = A\frac{dB}{dt}$

$$\Rightarrow \quad \int_0^I dI = \int_0^B \frac{A}{L}dB \Rightarrow I = \frac{A}{L}B$$

$$\Rightarrow \quad I_{max} = \frac{A}{L}B_{max} = \frac{10^{-2}}{10\times 10^{-3}}\times 0.1 = 0.1\text{ A} = 100\text{ mA}$$

L.R and L.C Circuits

51. (a) $U = \frac{1}{2}Li^2$

Rate $= \frac{dU}{dt} = (Li)\left(\frac{di}{dt}\right)$

At $t = 0$, $i = 0$, $\therefore$ rate $= 0$

at $t = \infty$, $i = i_0$ but $\frac{di}{dt} = 0$.

Therefore, rate $= 0$.

52. (b) $e = E - iR$

Clearly, the graph is a straight line with negative slope.

53. (a) For $t < t_0$, $E = L\dfrac{di}{dt} \Rightarrow i = \dfrac{E}{L}t$

For $t > t_0$, $L\dfrac{di}{dt} = 0 \Rightarrow i = \text{constant}$

54. (c) $U_{max} = \dfrac{1}{2}LI_0^2,\quad U = \dfrac{U_{max}}{2}$

$$\Rightarrow \frac{1}{2}LI^2 = \frac{1}{2}\left[\frac{1}{2}LI_0^2\right] \Rightarrow I_0^2\left[1-e^{-t/\tau}\right]^2 = \frac{I_0^2}{2}$$

$$\Rightarrow e^{-t/\tau} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\Rightarrow -\frac{t}{\tau} = \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) \Rightarrow t = \tau\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

Since $\tau = \dfrac{L}{R}$

$$t = \frac{L}{R}\ln\left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)$$

55. (a) At $t = 0$, i.e., when the key is just pressed, no current exists inside the inductor. So 10 Ω and 20 Ω resistors are in series and a net resistance of (10 + 20) = 30 Ω exists across the circuit.

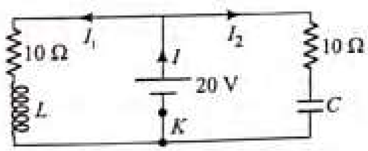
Hence, $I_1 = \dfrac{2}{30} = \dfrac{1}{15}$ A

As $t \to \infty$, the current in the inductor grows to attain a maximum value, i.e., the entire current passes through the inductor and no current passes through 10 Ω resistor.

Hence, $I_2 = \dfrac{2}{20} = \dfrac{1}{10}$ A

56. (c) Current in branches containing L and R will flow independently

$$I_1 = \frac{20}{10}\left(1 - e^{-\frac{t}{5\times10^{-4}}}\right) = \frac{3}{2} = 1.5\text{ A}$$



$$I_2 = \frac{20}{10}e^{-\frac{t}{10^{-3}}} = 1.0\text{ A}$$

$$I = I_1 + I_2 = 2.5\text{ A}$$

57. (b) The power dissipated in the resistor,

$$P = \frac{dW}{dt} = I^2R$$

Since the current through resistor varies with time, we must integrate.

The total energy produced as heat in the resistor

$$W = \int_0^\infty I^2R\,dt$$

The current in an RL circuit is $I = I_0e^{-(R/L)t}$

$$W = \int_0^\infty I_0^2e^{-(2R/L)t}R\,dt = \frac{I_0^2R}{-2R/L}\left[e^{-\frac{2R}{L}t}\right]_0^\infty = \frac{1}{2}LI_0^2$$

We can integrate by substituting

Note that the total heat produced equals the energy $(1/2)LI_0^2$ originally stored in the conductor.

58. (b) Inductors 5 mH and 10 mH are connected in parallel, hence equivalent inductance $L_{eq} = \dfrac{5\times10}{5+10} = \dfrac{50}{15} = \dfrac{10}{3}$ mH

Current at steady state, $I = \dfrac{20}{5} = 4$ A

As L_1 and L_2 are in parallel

$$I_1 = I\left(\frac{L_2}{L_1+L_2}\right) = 4\left[\frac{10}{10+5}\right] = \frac{8}{3}\text{ A}$$

59. (d) $E = L\dfrac{dI}{dt}$ or $dI = \dfrac{E}{L}dt$

or $\quad I = \dfrac{2}{4}t = 0.5\,t \Rightarrow t = 2I = 2\times5 = 10$ s

60. (b) In steady state: $P = I^2R$

Energy in inductor $= \dfrac{1}{2}LI^2$

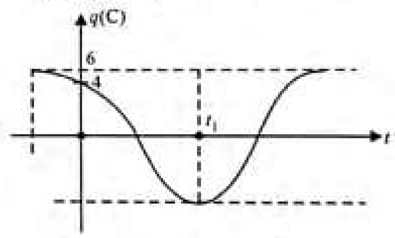
After connecting x and z, the whole energy of inductor will go into heat so heat produced: $H = \dfrac{1}{2}LI^2 = \dfrac{1}{2}L\dfrac{P}{R} = \dfrac{1}{2}P\tau$

61. (c) Constancy of flux implies that $\dfrac{E}{R}L_1 = i(L_1 + L_2)$

i.e., $\quad i = \dfrac{EL_1}{R(L_1+L_2)}$

62. (a) $i = \sqrt{5}$ A

$$\frac{q_m^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}Li^2 \Rightarrow q_{max} = 6\text{ C}$$



$q = Q\cos(\omega t + \delta)$, at $t = 0$, $q = 4$

$$\Rightarrow 4 = 6\cos\delta \Rightarrow \cos\delta = \frac{2}{3} \Rightarrow \delta = \cos^{-1}\left(\frac{2}{3}\right)$$

Charge on the capacitor is maximum at $t = t_1$

for this $\omega t_1 + \delta = \pi$

$$\Rightarrow t_1 = \frac{1}{\omega}(\pi - \delta) = \sqrt{LC}(\pi-\delta) = 2\left[\pi - \cos^{-1}\left(\frac{2}{3}\right)\right]$$

63. (b) Equivalent inductance

$$L_{eq} = L + 2L = 3L,\ C_{eq} = C + 2C = 3C$$

∴ Frequency of oscillation

$$f = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{6\pi\sqrt{LC}}$$

64. (a) When energy on both is same, means energy on capacitor is half of its maximum energy.

$$\frac{q^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \Rightarrow q = \frac{Q}{\sqrt{2}}$$

$$\Rightarrow Q\cos\omega t = \frac{Q}{\sqrt{2}} \Rightarrow \cos\omega t = \frac{1}{\sqrt{2}}$$

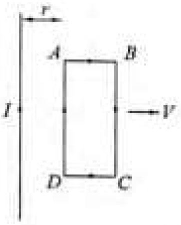
$$\Rightarrow \quad \omega t = \frac{\pi}{4} \quad \Rightarrow \quad t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}$$

65. (b) At $t = 0$, charge on C is zero, so p.d. across C is zero, p.d. across R is also zero. Hence there is no current in R.
At $t = \infty$, current through L is maximum and constant, so p.d. across L is zero, therefore p.d. across R is zero. Hence no current in R.

66. (a) $\omega = \dfrac{1}{\sqrt{L_{eq}C_{eq}}} = \dfrac{1}{\sqrt{2LC/2}} = \dfrac{1}{\sqrt{LC}}$

Problems Based on Mixed Concepts

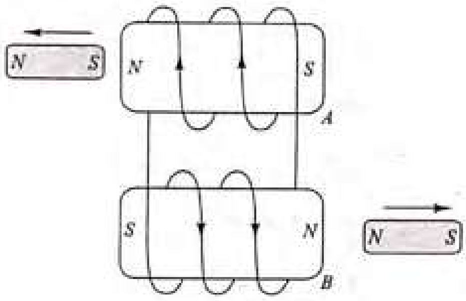
67. (d)



As the flux decreases, to maintain flux, current in the loop is clockwise. Force on DA due to the long wire is towards left while on BC is towards right.

68. (a) For first case: $2\text{ mV} = M \times 5$
For second case: $\varepsilon = M \times 2$
Solve to get $\varepsilon = 0.8\text{ mV} = 8 \times 10^{-4}\text{ V}$

69. (a)

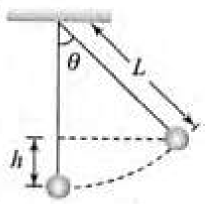


70. (b) $f = \dfrac{1}{2\pi\sqrt{LC}}, f + 50 = \dfrac{1}{2\pi\sqrt{LC/K}}$

$$\Rightarrow \quad K = \left(\frac{f+50}{f}\right)^2 = \left(1 + \frac{50}{f}\right)^2$$

$$\Rightarrow \quad K = 1 + \frac{100}{f} = 1 + \frac{100}{10{,}000} = 1.01$$

71. (a) From figure,
$\Rightarrow \quad h = L(1 - \cos\theta)$
Maximum velocity at equilibrium is given by
$\therefore \quad v^2 = 2gh = 2g\,L(1 - \cos\theta) = 2g\,L\left(2\sin^2\dfrac{\theta}{2}\right)$



$\Rightarrow \quad v = 2\sqrt{gL}\,\sin\dfrac{\theta}{2}$

Thus, maximum potential difference

$$V_{max} = BvL = B \times 2\sqrt{gL}\,\sin\frac{\theta}{2}L$$

$$= 2BL\sin\frac{\theta}{2}(gL)^{1/2}$$

72. (a) As the loop $PQRS$ moves in positive x-direction, the flux through loop does not change. Hence induced emf and induced current through the loop is zero.

73. (a) Induced emf $\displaystyle\int_a^b Bv\,dx = \int_a^b \frac{\mu_0 I}{2\pi x} v\,dx$

$\Rightarrow \quad$ Induced emf, $E = \dfrac{\mu_0 Iv}{2\pi}\ln\left(\dfrac{b}{a}\right)$

$\Rightarrow \quad$ Power dissipated $= \dfrac{E^2}{R}$

Also, power $= F.v \Rightarrow F = \dfrac{E^2}{vR}$

$F = \left[\dfrac{\mu_0 Iv}{2\pi}\ln\left(\dfrac{b}{a}\right)\right]^2 \dfrac{1}{vR}$

74. (b) $\oint \vec{E}.d\vec{l} = 10\pi R^2 k$

$E(6\pi R) = 10\pi R^2 k$

$E = \dfrac{10}{6}Rx = \dfrac{5}{3}Rk$

Induced emf across $AB = \dfrac{5}{3}Rk\,3R\theta = 5R^2k\theta$

75. (b) $\phi = MI = MI_0 t$

Hence, induced emf $\varepsilon = \dfrac{d\phi}{dt} = MI_0$

Current through ring $= \dfrac{\varepsilon}{R} = \dfrac{MI_0}{R}$

76. (a) $\varepsilon = -L\dfrac{di}{dt}, -\dfrac{di}{dt}$ is more for 1

$\varepsilon = \left[\dfrac{L(E/R)^2}{4(L/R)}\right] = \dfrac{E^2}{4R}$

77. (a) Magnetic field energy stored $= U = \dfrac{1}{2}Li^2$

$32 = \dfrac{1}{2}L(4)^2$ or $L = 4$ Henry

Power dissipated as heat is $P = i^2R$
$320 = 4^2R$ or $R = 20$ ohm

Time constant of circuit $= \tau = \dfrac{L}{R} = \dfrac{4}{20} = 0.2$ sec

78. (b) Initially inductor will block the current and finally capacitor.

79. (b) Since resistance of the circuit is increasing and hence current in the circuit is decreasing so $\dfrac{di}{dt}$ is negative.

Current in the circuit is given by $i = \dfrac{6 - L\dfrac{di}{dt}}{12}$

Since di/dt is negative so the value of numerator will be more than 6V and hence current in the circuit at that instant will be more than 0.5 A.

80. (a) When switch K_1 is opened and K_2 is closed it becomes L–C circuit so applying energy conservation:

$$\frac{\quad}{2C} = \frac{1}{2}Li^2 ; Q = C_{eq}V = \frac{C_1C_2}{C_1 + C_2}.V = (20 \times 10^{-6})$$

$$\frac{(20\times10^{-6})^2}{2\times2\times10^{-6}} = \frac{1}{2}(0.2\times10^{-3})i^2 \Rightarrow i = 1\text{ A}$$

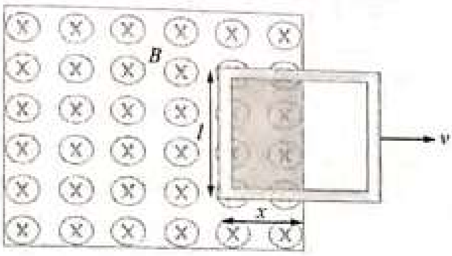
ARCHIVES

1. (d) The given arrangement is a parallel combination of inductances. Therefore,

$$\frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

$\Rightarrow$ $L = 1$ H

2. (d) The emf will be induced due to change in flux.



$$\phi = BA = Bl.\,x$$

$$|e| = \frac{d\phi}{dt} = \frac{d(Blx)}{dt} = Bl\cdot\frac{dx}{dt} = Blv$$

3. (d) Mutual inductance between two coils depends on the materials of the wires of the coils.

4. (a)
$$8 = L\frac{4}{0.05}$$

$\Rightarrow$ $L = 2\times0.05\text{ H} = 0.1\text{ H}$

5. (d)
$$\frac{1}{2}\frac{Q^2}{2C} = \frac{q^2}{2C}$$

$$\Rightarrow \quad q = \frac{Q}{\sqrt{2}}$$

6. (b) $I = -\dfrac{nd\phi}{R'dt} = -\dfrac{n(W_2 - W_1)}{5Rt}$

7. (b)
$$\phi = BA\cos\theta$$

$$= B\left(\frac{\pi r^2}{2}\right)\cos\omega t$$

$$E = -\frac{d}{dt}\phi = \frac{B\pi r^2}{2}\omega\sin\omega t$$

$$P = \frac{E^2}{R} = \frac{B^2\pi^2r^4\omega^2}{4R}\sin^2\omega t$$

But $<\sin^2\omega t> = \dfrac{1}{2}$

$$\therefore \quad <P> = \frac{(B\pi r^2\omega)^2}{8R}$$

8. (b) $E = \dfrac{1}{2}B\omega R^2 = \dfrac{1}{2}\times0.2\times10^{-4}\times5\times1\times1$

$= 0.5\times10^{-4}\text{ V} = 54\times10^{-1}\text{ V}$

$= 50\times10^{-6}\text{ V} = 50\ \mu\text{V}$

9. (a) Two emf's, each of value Blv, are produced. Both the emf's tend to send currents in the same direction. So they are added up. Net emf = $2Blv$

10. (b)
$$v = \frac{1}{2\pi\sqrt{LC}}$$

$$50 = \frac{1}{2\pi\sqrt{10C}}$$

$$\Rightarrow \quad 2500 = \frac{1}{40\pi^2C}$$

$$\Rightarrow \quad C = \frac{1}{2500\times40\times10}\text{ F} = 10^{-6}\text{ F} = 1\ \mu\text{F}$$

11. (c)
$L = 300\times10^{-3}$ H

$R = 1\ \Omega$

$I = \dfrac{I_0}{2}$

Using $I = I_0(1 - e^{-Rt/L})$, we get

$$\frac{I_0}{2} = I_0(1 - e^{-Rt/L})$$

$$\Rightarrow \quad \frac{1}{2} = 1 - e^{-Rt/L} \Rightarrow e^{-Rt/L} = \frac{1}{2}$$

$$\Rightarrow \quad e^{Rt/L} = 2 \Rightarrow \frac{R}{L}t = \log_e 2 = 0.693$$

$$\Rightarrow \quad t = \frac{0.693\times300\times10^{-3}}{2}\text{ s} = 0.1\text{ s}$$

12. (c)
$$I = I_0e^{-Rt/L}$$

$$= \frac{E}{R}e^{-Rt/L}$$

$$= \frac{100}{100}e^{-\frac{100}{100\times10^{-3}}\times10^{-3}}$$

$$= \frac{1}{e}\text{ A}$$

13. (d)
$$E = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}(10t^2 - 50t + 250)$$

$$= -(20t + 50)$$

$$E_{t=3s} = -(20\times3 + 50)\text{ V} = -10\text{ V}$$

14. (c) During the growth of current in LR circuit is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

or
$$I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{5}{5}\left(1 - e^{-\frac{5}{10}\times2}\right)$$

$$I = (1 - e^{-1})$$

15. (b) $M = \mu_0 n_1 n_2 \pi r_1^2 l.$

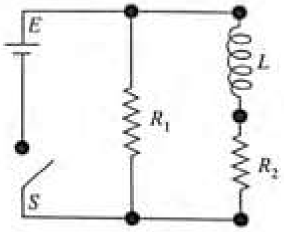
From $\phi_2 = \pi r_1^2 (\mu_0 n i) n_{2l}$

$A = \pi r_1^2 = 10\text{ cm}^2, l = 20\text{ cm}, N_1 = 300, N_2 = 400.$

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi \times 10^{-7} \times 300 \times 400 \times 10 \times 10^{-4}}{0.20}$$

$$= 2.4\pi \times 10^{-4}\text{ H}$$

16. (d)



$$I_1 = \frac{12}{2} = 6\text{ A}$$

$$E = L\frac{dI_2}{dt} + R_2 \times I_2$$

$$I_2 = I_0(1 - e^{-t/t_C})$$

$$\Rightarrow \quad I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\text{ A}$$

$$t_C = \frac{L}{R} = \frac{400 \times 10^{-3}}{2} = 0.2$$

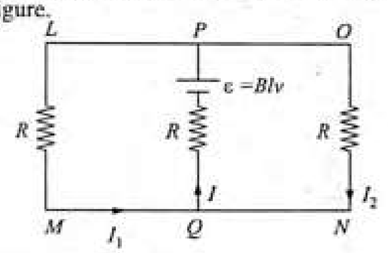
$$I_2 = 6(1 - e^{-t/0.2})$$

Potential drop across L is

$$E - R_2 I_2 = 12 - 2 \times 6(1 - e^{-5t}) = 12e^{-5t}$$

17. (b) Emf induced across sliding connector, $\varepsilon = Blv$.

The equivalent circuit diagram can be drawn as shown in the figure.



At junction Q we can write

$$I = I_1 + I_2 \qquad \text{(i)}$$

Applying Kirchhoff's loop law for the closed loop $PLMQP$, we get

$$-I_1 R - IR + \varepsilon = 0$$

$$I_1 R + IR = Blv \qquad \text{(ii)}$$

Again, applying Kirchhoff's loop law for the closed loop $PONQP$, we get

$$-I_2 R - IR + \varepsilon = 0$$

$$I_2 R + IR = 2Blv \qquad \text{(iii)}$$

Adding equations (ii) and (iii), we get

$$2IR + I_1 R + I_1 R = 2\,Blv$$

$$2IR + R(I_1 + I_2) = 2Blv$$

$$2IR + IR = 2Blv \qquad \text{(Using (i))}$$

$$3IR = 2Blv$$

$$I = \frac{2Blv}{3R} \qquad \text{(iv)}$$

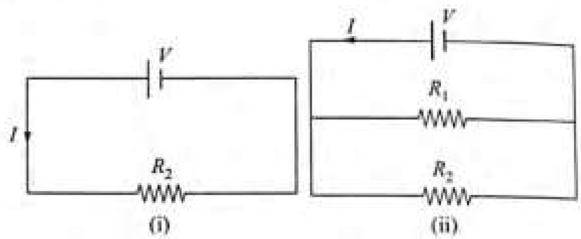
Substituting this value of I in equation (ii), we get $I_1 = \frac{Blv}{3R}$

Substituting the value of I in equation (iii), we get

$$2IR + I_1 R + I_2 R = I_2 = \frac{Blv}{3R}$$

Hence, $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

18. (b) Just after closing the key (say at time $t = 0$) the inductor coil offers infinite resistance, hence it acts as an open circuit. The corresponding equivalent circuit diagram is as shown in the figure (i).



The current through battery is $I = \frac{V}{R_2}$

At time $t = \infty$, the inductor acts as a conducting wire. The corresponding equivalent circuit diagram is as shown in the figure (ii).

$\therefore$ The current through the battery is

$$I = \frac{V}{R_{eq}} = \frac{V}{\frac{R_1 R_2}{R_1 + R_2}} \quad (R_1 \text{ and } R_2 \text{ are in parallel})$$

$$= \frac{V(R_1 + R_2)}{R_1 R_2}$$

19. (b) Charge on the capacitor at any time 't',

$$q = q_0 \cos \omega t \qquad \text{(i)}$$

At time when energy stored equally in electric and magnetic field, at this time

Energy of a capacitor $= \frac{1}{2}$ Total energy

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right) \Rightarrow q = \frac{q_0}{\sqrt{2}}$$

From equation (i)

$$\frac{q_0}{\sqrt{2}} = q_0 \cos \omega t$$

$$\cos \omega t = \frac{1}{\sqrt{2}} \Rightarrow \omega t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} \quad \left(\because \ \omega = \frac{1}{\sqrt{LC}}\right)$$

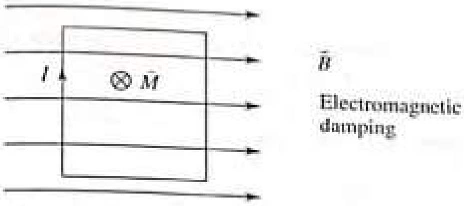
20. (d) Here earth magnetic field $B_H = 5.0 \times 10^{-5}\text{ N A}^{-1}\text{ m}^{-1}$

Length of vertical aerial $l = 2$ m

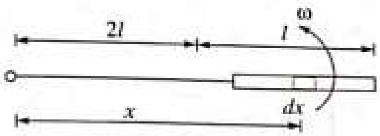
Velocity of this aerial, $v = 1.5\ ms^{-1}$
This aerial moves in horizontal magnetic field hence motional e.m.f will be induced across the aerial
Induced e.m.f, $\varepsilon = B_H vl = 5 \times 10^{-5} \times 1.50 \times 2$

$= 15 \times 10^{-5}\ V = 0.15\ mV$

21. (d)



22. (c) As rod is rotating in magnetic field thus cutting magnetic field lines, motional emf will develop across the rod. As each and every element of the rod is moving with different velocity. For calculating emf across rod we first consider an element of length dx at a distance x from the fixed end of the string.



e.m.f. induced in the element is

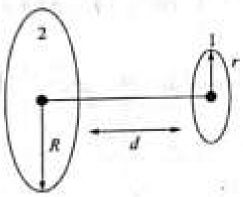
$d\varepsilon = B(\omega x)dx$

Hence, the e.m.f. induced across the ends of the rod is

$$\varepsilon = \int_{2l}^{3l} B\omega x dx = B\omega\left[\frac{x^2}{2}\right]_{2l}^{3l} = \frac{B\omega}{2}\left[(3l)^2 - (2l)^2\right]$$

$$= \frac{5B\omega l^2}{2}$$

23. (d) Let M_{12} be the coefficient of mutual induction between loops



$\phi_1 = M_{12} i_2$

$\Rightarrow \quad \dfrac{\mu_0 i_2 R^2}{2(d^2 + R^2)^{3/2}} \pi r^2 = M_{12} i_2$

$\Rightarrow \quad M_{12} = \dfrac{\mu_0 R^2 \pi r^2}{2(d^2 + R^2)^{3/2}}$

$\phi_2 = M_{12} i_1 \Rightarrow \phi_2 = 9.1 \times 10^{-11}$ weber

24. (d) Applying Kirchhoff's law in closed loop,

$-V_R - V_C = 0$

$\Rightarrow \quad V_R/V_C = -1$

[Note: The sense of voltage drop has not been defined.]
The answer could have been 1.

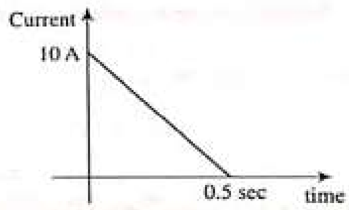
25. (d) $\quad I = I_0 e^{-\frac{t}{\tau}}, \tau = \dfrac{L}{R} = \dfrac{15}{150} e^{-\frac{1 \times 10^{-3}}{1/5 \times 10^{3}}} = 0.67$ mA

26. (a) For a damped pendulum, $A = A_0 e^{-bt/2m}$

$\Rightarrow \quad A = A_0 e^{-\left(\frac{R}{2L}\right)t}$

(Since L plays the same role as m)

27. (a)



Charge flowing in the coil, $q = \dfrac{\Delta\phi}{R}$

$\Delta\phi$ = change in flux and R = resistance of the coil

Also, $q = \int I\, dt$ = Area of current–time graph

$\Rightarrow \quad q = \dfrac{1}{2} \times 10 \times 0.5 = 2.5$ coulomb

Change in flux $\Delta\phi = qR$

$\Rightarrow \quad \Delta\phi = 2.5 \times 100 = 250$ Wb

CHAPTER 24: ALTERNATING CURRENT

Concept Application Exercises 24.1

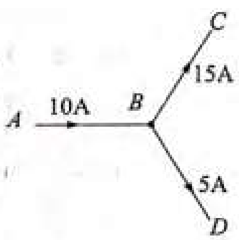
1. Yes, because here also (as when 50 Hz), fluctuation in current will be so rapid [30 times/s] that the bulb will appear glowing continuously due to persistence (lesser than 30/s) of vision.
2. The brightness will decrease, because L increases on inserting the iron rod and impedence $Z = \sqrt{R^2 + (\omega L)^2}$ increases. This decreases the current, hence brightness decreases.
3. The brightness will decrease, because Z increases on increasing w.
4. First decreases, then becomes same. Because as the rod is inserted, L increases. From $f = LI$, I decreases. But soon the current will become same as original.
5. $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$ decreases on increasing ω. This increases current, hence the brightness will increase.
6. $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$ increases on decreasing C. This decreases current, hence the brightness will decrease.

Exercises

Properties of Alternating Current

1. (c) $i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \frac{T^2}{\sqrt{5}}$

2. (a) Yes, in AC if branch AB has R, BC has a capacitor C, and BD has a pure inductance L



3. (a) $V = \frac{V_0}{T/4} t$ or $V = \frac{4V_0}{T} t$

The average value of voltage in one complete cycle will be zero in the given case.

4. (d) $$<V> = \frac{\int_0^T v\,dt}{\int_0^T dt} = 0$$

5. (c) Phase difference $\Delta\varphi = \varphi_2 - \varphi_1 = \frac{\pi}{6} - \left(\frac{-\pi}{6}\right) = \frac{\pi}{3}$

6. (c) $I = I_1 \cos \omega t + I_2 \sin \omega t$

$$(I^2)_{mean} = I_1^2 \overline{\cos^2 \omega t} + I_2^2 \overline{\sin^2 \omega t} + 2I_1 I_2 \overline{\cos \omega t \cdot \sin \omega t}$$

$$= I_1^2 \cdot \frac{1}{2} + I_2^2 \cdot \frac{1}{2} + 2I_1 I_2 \times 0$$

$$I_{r.m.s.} = \frac{(I_1^2 + I_2^2)^{1/2}}{\sqrt{2}}$$

7. (b) At $t = 0$, phase of the voltage is zero, while phase of the current is $-\frac{\pi}{2}$ i.e., voltage leads by $\frac{\pi}{2}$

8. (b) Given $I = 5 + 10 \sin \omega t$

$$I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (5 + 10 \sin \omega t)^2 dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T (25 + 100 \sin \omega t + 100 \sin^2 \omega t)\right]^{1/2}$$

But as $\frac{1}{T}\int_0^T \sin \omega t\, dt = 0$

and $\frac{1}{T}\int_0^T \sin^2 \omega t\, dt = \frac{1}{2}$

so $I_{eff} = \left[25 + \frac{1}{2} \times 100\right]^{1/2} = 5\sqrt{3}$ A

9. (d) As current at any instant in the circuit will be

$I = I_{dc} + I_{ac} = a + b \sin \omega t$

so, $$I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (a + b \sin \omega t)^2 dt\right]^{1/2}$$

i.e., $$I_{eff} = \left[\frac{1}{T}\int_0^T (a^2 + 2ab \sin \omega t + b^2 \sin^2 \omega t)\, dt\right]^{1/2}$$

but as $\frac{1}{T}\int_0^T \sin \omega t\, dt = 0$ and $\frac{1}{T}\int_0^T \sin^2 \omega t\, dt = \frac{1}{2}$

So, $I_{eff} = \left[a^2 + \frac{1}{2} b^2\right]^{1/2}$

10. (c) $E_1 = E_0 \sin \omega t$; $E_2 = E_0 \sin[\omega t + (\pi/3)]$

$E = E_2 + E_1 = E_0 \sin[\omega t + (\pi/3)] + E_0 \sin \omega t$

$= E_0 [2 \sin\{\omega t + (\pi/6)\} \cos(\pi/6)]$

$= \sqrt{3}\, E_0 \sin[\omega t + (\pi/6)]$

11. (b) Root mean square current of the sinusoidal waveform.

$I = \frac{I_0}{\sqrt{2}}$.

Power output of the heater,

$$P = I^2 R = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \frac{I_0^2 R}{2}$$

12. (a) $E = E_0 \cos \omega t$

$\therefore \quad \omega = 50\pi$

$2\pi f = 50\pi \Rightarrow f = 25$ Hz

In one cycle ac current becomes zero twice.

Therefore, 50 times the current becomes zero in 1 s.

13. (d) Time for reaching maximum or peak value from 0

$$= \frac{T}{4} = \frac{1}{4} \times \frac{1}{50}\,\text{s} = \frac{1}{200}\,\text{s} = 5 \times 10^{-3}\,\text{s}$$

$$I_0 = 10\sqrt{2}\ \text{A} = 14.14\ \text{A}$$

14. (b) Given that $E_0 = 10$ V, $t = \frac{1}{600}$ s

$$\therefore \quad E = E_0 \cos 2\pi f t = 10 \cos\left[2\pi \times 50 \times \frac{1}{600}\right]$$

$$= 10 \cos(\pi/6) = 10(\sqrt{3}/2) = 5\sqrt{3}\ \text{V}$$

15. (b) $V = V_0 \sin \omega t$

Voltage read is rms value.

$\therefore \quad V_0 = \sqrt{2} \times 234\ \text{V} = 331\ \text{V}$

and $\quad \omega t = 2\pi f t = 2\pi \times 50 \times t = 100\pi t$

Thus, the equation of the line voltage is given by

$$V = 331 \sin(100\pi t)$$

16. (c) $i = 3 \sin \omega t + 4 \cos \omega t$

$$= 5\left[\frac{3}{5}\sin \omega t + \frac{4}{5}\cos \omega t\right] = 5[\sin(\omega t + \delta)]$$

(i) rms value $= \frac{5}{\sqrt{2}}$ A

If voltage applied is $V = V_m \sin \omega t$ then I, given by Eq. (i), indicates that it is ahead of V by δ where $0 < \delta < 90$ which indicates that the circuit contains R and C.

If we find average value for the first half period (0 to $T/2$), then it will be $6/\pi$ A. But for different time interval, it will be different.

Different A-C Circuits

17. (d) Both V and I are in the same phase. So, let us calculate the time taken by the voltage to change from peak value to rms value. Now $220 = 220 \sin 100\pi t_1$

or $\quad 100\pi t_1 = \frac{\pi}{2}$ or $t_1 = \frac{1}{200}$ s

Again, $\frac{220}{\sqrt{2}} = 220 \sin 100\pi t_2$

or $\quad \frac{1}{\sqrt{2}} = \sin 100\pi t_2$ or $100\pi t_2 = \frac{3\pi}{4}$

or $\quad t_2 = \frac{3}{400}$ s

Required time $= t_2 - t_1 = \frac{1}{400}$ s $= 2.5 \times 10^{-3}$ s

18. (c) Clearly, $X_L = R$

or $\quad L \times 2 \times 3.14 \times 1000 = 100$

or $\quad L = \frac{100}{2 \times 3.14 \times 1000}$ H

$= 15.9 \times 10^{-3}$ H $= 15.9$ mH ≈ 16 mH

19. (b) $V_{rms} = \sqrt{16^2 + 20^2} = 25.6$ V

20. (b) As resistance of the lamp

$$R = \frac{V_S^2}{P_0} = \frac{100^2}{50} = 200\ \Omega$$

The rms current $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}$ A.

So when the lamp is put in series with a capacitance and run at 200 V ac, from $V = IZ$, we have

$$Z = \frac{V}{I} = \frac{200}{(1/2)} = 400\ \Omega$$

Now as in case of C–R circuit,

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

i.e., $\quad R^2 + \left(\frac{1}{\omega C}\right)^2 = 160000$

or $\quad \left(\frac{1}{\omega C}\right)^2 = 16 \times 10^4 - (200)^2 = 12 \times 10^4$

$$\frac{1}{\omega C} = \sqrt{12} \times 10^2$$

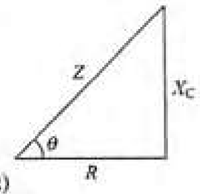
$$C = \frac{1}{100\pi \times \sqrt{12} \times 10^2}\ \text{F} = \frac{100}{\pi\sqrt{12}}\ \mu\text{F} = \frac{50}{\pi\sqrt{3}} = 9.2\ \mu\text{F}$$

21. (b) Here, $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 1 = 100\ \pi\ \Omega$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = \frac{10^3}{\pi}\ \Omega$$

So, $\quad X = |X_L - X_C| = \left|100\pi - \frac{10^3}{\pi}\right| = \left|10^2\left[\frac{\pi^2 - 10}{\pi}\right]\right|\ \Omega$

22. (b) $V_C^2 + V_R^2 = V^2$



$50^2 + V_R^2 = 110^2$

$\Rightarrow \quad V_R = \sqrt{160 \times 60} = 98$ V

Then, $I_V = \frac{\sqrt{160 \times 60}}{50} \quad (\because \quad R = 50\ \Omega)$

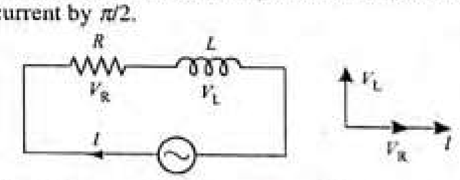
Also, $I_V = \frac{110}{\sqrt{R^2 + X_C^2}} \quad \therefore \quad \frac{98}{50} = \frac{110}{\sqrt{50^2 + X_C^2}}$

Flux X_C and now using $X_C = \frac{1}{\omega C}$, we get $C = 104\ \mu$F

23. (c) $V_R = I_r R = 1 \times 50 = 50\ \Omega$

$$V_L = \sqrt{E_v^2 - V_R^2} = \sqrt{220^2 - 50^2} = 214\ \text{V}$$

24. (b) Here inductance and resistance are connected in series. We know that in case of resistance, both current and potential difference are in the same phase. In inductor, voltage leads current by $\pi/2$.



25. (a) Since the capacitor is connected in series to the resistor, current I_C from the supply and I_R through the resistor is in phase as represented by choice (a).

26. (a) $R = \frac{E}{I} = \frac{100}{1} = 100\ \Omega$

for ac: $Z = [R^2 + (2\pi fL)^2]^{1/2} = \frac{E_v}{I_v} = \frac{100}{0.5} = 200\ \Omega$

or $200 = [(100)^2 + (100\pi L)^2]^{1/2}$

Solving, we get $L = 0.55$ H

27. (a) Current remains unchanged in R. However, it becomes half in L, because reactance is doubled on doubling the frequency.

28. (a) $X_L = L\omega = 5 \times 10^{-3} \times 2000 = 10\ \Omega$

$$X_C = \frac{1}{C\omega} = \frac{1}{50 \times 10^{-6} \times 2000} = \frac{100}{10}\ \Omega = 10\ \Omega$$

Since

$X_L = X_C$, therefore,

$Z = R = 5.9 + 0.10 + 4 = 10\ \Omega$

$$I_v = \frac{E_v}{Z} = \frac{20}{10}\ \text{A} = 2\ \text{A}$$

29. (d) If an ac source $E = E_0 \sin\omega t$ is applied across an inductance and capacitance in parallel, the current in inductance will lag the applied voltage while that across the capacitor will lead, and so,

$$I_L = \frac{E_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) = -0.8\sqrt{2}\cos\omega t$$

$$I_C = \frac{V}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) = 0.6\sqrt{2}\cos\omega t$$

So the current drawn from the source

$$I = I_L + I_C = -0.2\sqrt{2}\ \cos\ \omega t$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{0.2\sqrt{2}}{\sqrt{2}} = 0.2\ \text{A}$$

30. (b) Here $R = X_L = X_C$ ($\therefore$ voltage across them is same)

When capacitor is short circuited,

$$I = \frac{10}{(R^2 + X_L^2)^{1/2}} = \frac{10}{\sqrt{2}R}$$

$\therefore$ Potential drop across inductance $= IX_L = IR = 10/\sqrt{2}$ V

31. (c) According to the given question,

$$\tan 60^\circ = \frac{\omega L}{R} \text{ and } \tan 60^\circ = \frac{1/\omega C}{R}$$

$\therefore$ $\omega L = (1/\omega C)$(case of resonance)

Now $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 100\ \Omega$

$$\therefore \quad I_{rms} = \frac{E_{rms}}{Z} = \frac{200\ \text{V}}{100\ \Omega} = 2\ \text{A}$$

32. (a) Here $V_L = V_C$. They are in opposite phase. Hence, they will cancel each other. Now the resultant potential difference is equal to the applied potential difference = 100 V

$Z = R$ ($\therefore X_L = X_C$)

$$\therefore \quad I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{100}{50} = 2\ \text{A}$$

33. (c) The current of 1.6 A lags emf in phase by $\pi/2$. The current of 0.4 A leads emf in phase by $\pi/2$. So, these two currents are 180° out of phase with each other.

$\therefore$ Net current, $I_1 = (1.6 - 0.4)$ A = 1.2 A

$$1.6 = \frac{E_v}{X_L} \text{ and } 0.4 = \frac{E_v}{X_C}$$

$$\Rightarrow \frac{X_C}{X_L} = 4 \Rightarrow \frac{1}{\omega C \omega L} = 4$$

$$\Rightarrow \omega = \frac{1}{2\sqrt{LC}}$$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{4\pi\sqrt{LC}}$$

34. (c) $X_L = X_C$ at resonance

$\frac{X_L}{X_C} = 1$ for both circuits.

Impedance may be different if applied voltage is different.

35. (d) Since $\cos\phi = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$

(Also $\cos\phi$ can never be greater than 1)

Hence, (c) is wrong.

Also $IX_C > IX_L \Rightarrow X_C > X_L$

$\therefore$ Current will be leading.

In an LCR circuit,

$$V = \sqrt{(V_L - V_C)^2 + V_R^2} = \sqrt{(6-12)^2 + 8^2}$$

$V = 10$; which is less than voltage drop across capacitor.

36. (b) At resonance, the series combination of L and C gives zero impedance.

At resonance, the voltages across L and C are equal but opposite in phase.

Power in A-C Circuits

37. (d) Brightness $\propto P_{consumed} \propto \frac{1}{R}$ for Bulb, $R_{ac} = R_{dc}$, so brightness will be equal in both the cases.

38. (b) Power $= I^2R = \left(\frac{I_p}{\sqrt{2}}\right)^2 R = \frac{I_p^2 R}{2}$

39. (a) If the current is wattless then power is zero. Hence phase difference $\varphi = 90^\circ$

40. (c) $P = V_{r.m.s.} \times i_{r.m.s.} \times \cos\varphi = \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} \times \cos\frac{\pi}{3}$

$$= \frac{10^4 \times 10^{-3}}{2} \times \frac{1}{2} = \frac{10}{4} = 2.5 \text{ watt}$$

41. (a) As heat produced in an L-C-R circuit, i.e. Heat $= (i_{rms})^2 Rt$.

the rms value of current $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{25}{Z\sqrt{2}}$

where $Z = \sqrt{R^2 + (X_L - X_C)^2} = 5\ \Omega$

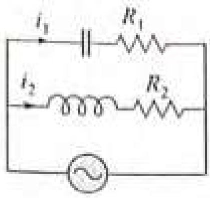
$$\text{Heat} = \left(\frac{25}{5\sqrt{2}}\right)^2 4 \times 80 = 4000\ \text{J}$$

Amplitude of wattless current is $i_0 \sin\phi$

where $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \Rightarrow \phi = 37^\circ$

Amplitude of wattless current $= i_0 \sin 37° = \frac{25}{5} \times \frac{3}{5} = 3$ A

42. (c) $i_{1_{rms}} = \frac{E_{rms}}{\sqrt{X_C^2 + R_1^2}} = \frac{130}{13} = 10$ A



$i_{2_{rms}} = \frac{E_{rms}}{\sqrt{X_L^2 + R_2^2}} = 13$ A

Power dissipated

$= i_{1rms}^2 R_1 + i_{2rms}^2 R_2 = 10^2 \times 5 + 13^2 \times 6$

= power delivered by battery = 500 + 169 × 6

= 1514 watt

43. (a) Concept based

44. (a) $P = E_v I_v \cos\phi$; $P = E_v \frac{E_v}{Z} \frac{R}{Z}$

or $P = \frac{E_v^2 R}{Z^2} = \frac{110 \times 110 \times 11}{22 \times 22} = 275$ W

45. (a) In an ac circuit, a pure inductor does not consume any power. Therefore, power is consumed by the resistor only.

$\therefore$ $P = I_v^2 R$

or $108 = (3)^2 R$ or $R = 12\ \Omega$

46. (b) In an ac circuit, capacitor does not consume any power. Therefore, power is consumed by the resistor only.

$\therefore$ $P = I_v^2 R$ or $100 = (2)^2 R$ or $R = 25\ \Omega$

47. (c) Impedance $Z = \frac{E_v}{I_v}$ or $Z = \frac{200}{2} = 100\ \Omega$

But $Z^2 = R^2 + \left(\frac{1}{\omega C}\right)^2$

or $\left(\frac{1}{\omega C}\right)^2 = Z^2 - R^2 = (100)^2 - (25)^2 = 125 \times 75$

or $X_C = \frac{1}{\omega C} = \sqrt{125 \times 75}\ \Omega$

48. (d) $\frac{1}{\omega C} = \sqrt{125 \times 75}$

Here $\omega = 2\pi f = 2\pi \times 50 = 100\pi$ rad s^{-1}

$\therefore$ $C = \frac{1}{100\pi\sqrt{125 \times 75}}$ F

49. (b) Wattless component of ac

$= I_v \sin\phi = \frac{E_v}{Z} \frac{X_L}{Z} = \frac{E_v X_L}{Z^2} = \frac{220 \times \omega L}{(R^2 + \omega^2 L^2)}$

As $\omega L = 0.7 \times 2\pi \times 50 = 220\ \Omega$

Hence, wattless component of ac $= \frac{200 \times (220)}{(220^2 + 220^2)} = 0.5$ A

50. (c) $\cos\phi = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$

Putting the values, $C = 500\ \mu$F

Problems Based on Mixed Concepts

51. (c) Let $E = E_0 \sin \omega t$, then $I_1 = \frac{E_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$

$I_2 = \frac{E_0}{\sqrt{R^2 + X^2}} \sin(\omega t + \phi)$

where $\tan\phi = -\frac{X_L}{R} \Rightarrow \phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$

Phase difference between I_1 and I_2

$= \left(\omega t + \frac{\pi}{2}\right) - (\omega t + \phi) = \frac{\pi}{2} - \phi = \frac{\pi}{2} + \tan^{-1}\left(\frac{X_L}{R}\right)$

52. (a) The equivalent primary load is

$R_1 = \left(\frac{N_1}{N_2}\right)^2 R_2 = \left(\frac{20}{1}\right)^2 (6.0) = 2400\ \Omega$

Current in the primary coil $= \frac{240}{R_1} = \frac{240}{2400} = 0.1$ A

53. (b) $X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi 50 \times 500} = \frac{20}{11}\ \Omega$

$X_L = 2\pi f L = 2\pi \times 50 \times 10 \times 10^{-3} = \pi\Omega$

Since $X_L < X_C$, so inductive branch has less impedance, and so more current. Hence B_2 will be brighter.

54. (a) $R = \frac{125}{12.5} = 10\ \Omega$

$X_L = \omega L = 2\pi f L = \frac{V}{I} = \frac{125}{10} = 12.5$

$\therefore$ $X_L' = 2\pi L \times f' = 0.25 \times 40 = 10\ \Omega$

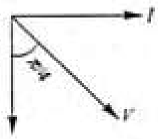
Impedance of the circuit

$Z = \sqrt{R^2 + X_L^2} = 10\sqrt{2}\ \Omega$

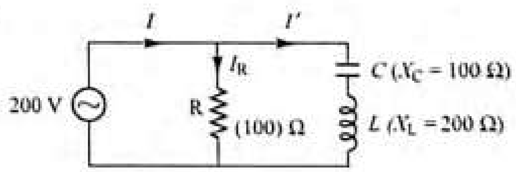
$\therefore$ Current $= \frac{100}{10\sqrt{2}} = 10/\sqrt{2}$ A

55. (a) $V = V_0 \sin[\omega t + (\pi/4)] = V_0 \cos[\omega t - (\pi/4)]$

Since V lags current, an inductor can bring it in phase with current.



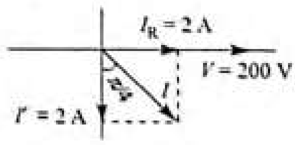
56. (b)



$I_R = \frac{V}{R} = \frac{200}{100} = 2$ A

$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2$ A

$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2}$ A



57. (c) The circuit will have inductive nature if

$$\omega > \frac{1}{\sqrt{LC}}\left(\omega L > \frac{1}{\sqrt{LC}}\right)$$

Hence (a) is false. Also, if circuit has inductive nature, the current will lag behind voltage. Hence, (d) is also false.

If $\omega = \frac{1}{\sqrt{LC}}\left(\omega L = \frac{1}{\omega C}\right)$, the circuit will have resistance nature. Hence, (b) is false.

Power factor, $\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = 1$

If $\omega L = \frac{1}{\omega C}$. Hence, (c) is true.

58. (c) Since current is leading emf, capacitor must be present in the circuit. Along with capacitor, there can be either (i) only resistor or (ii) resistor and inductor if $X_C > X_L$.
In both of the above conditions, we can obtain a phase difference of $\pi/4$ with current leading emf.

59. (a) Given:
Voltage in primary coil, $V_P = 200$ V
Current in primary coil, $i_p = 2$ A
Voltage in secondary coil, $V_s = 2000$ V
The relation for the current in the secondary coil is

$$\frac{V_s}{V_P} = \frac{i_p}{i_s} \Rightarrow \frac{2000}{200} = \frac{2}{i_s} = \frac{2\times 200}{2000} = 0.2\text{ A}$$

60. (b) For power to be consumed at the rate of

$$\frac{1100}{5} = 220\text{ W}, \text{ We have } P = E_v I_v \cos\theta$$

$$220 = \frac{220\times 220}{\sqrt{R^2 + L^2\omega^2}} \times \frac{R}{\sqrt{R^2 + L^2\omega^2}}$$

$$\text{where } R = \frac{V^2}{P} = \frac{220^2}{1100} = 44\ \Omega$$

$$220 = \frac{(220)^2 \times 44}{44^2 + (L\omega)^2};\ 44^2 + (L\omega)^2 = 220$$

$$(L\omega)^2 = \sqrt{220\times 44 - 44^2}$$

$$= \sqrt{44(220-44)} = \sqrt{44\times 176} = 88\ \Omega$$

$$L = \frac{88}{2\pi\times f} = \frac{88}{2\pi\times 50} = \frac{88}{2\times 22}\times\frac{7}{50} = 0.28\text{ H}$$

61. (a) Capacitance of wire

$$C = 0.014 \times 10^{-6} \times 200 = 2.8 \times 10^{-6}\ F = 2.8\ \mu\text{F}$$

For impedance of the circuit to be minimum $X_L = X_C$

$$\Rightarrow\quad 2\pi\nu L = \frac{1}{2\pi\nu C}$$

$$\Rightarrow\quad L = \frac{1}{4\pi^2\nu^2 C} = \frac{1}{4(3.14)^2\times(5\times 10^3)^2\times 2.8\times 10^{-6}}$$

$$= 0.35 \times 10^{-3}\text{ H} = 0.35\text{ mH}$$

62. (c) Given $X_L = X_C = 5\Omega$, this is the condition of resonance. So $V_L = V_C$, so net voltage across L and C combination will be zero.

63. (d) At resonance net voltage across L and C is zero.

64. (b) The peak value of the current is

$$I_0 \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{V_0}{\sqrt{2}R}$$

when the angular frequency is changed to $\frac{\omega}{\sqrt{3}}$

The new peak value is

$$I_0' = \frac{V_0}{\sqrt{R^2 + \frac{3}{\omega^2 C^2}}} = \frac{V_0}{\sqrt{4R^2}} = \frac{V_0}{2R}$$

$$\therefore\quad I_0' = \frac{I_0}{\sqrt{2}}$$

65. (b) $i_{rms} = \frac{E_0}{\sqrt{2}Z} \Rightarrow \frac{i_{rms,1}}{i_{rms,2}} = \frac{Z_2}{Z_1} = \frac{\sqrt{R^2 + \left(\frac{1}{\omega kC}\right)^2}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$

Solving: $\Rightarrow \frac{i_{rms,1}}{i_{rms,2}} = 0.3$

66. (b) Voltage across the capacitor and inductor are same.
i.e., $V_L = V_C$
$\Rightarrow$ $IX_L = IX_C$
$\Rightarrow$ $X_L = X_C$ (as they are in series)
The circuit is in resonance. $\therefore z = R$.
Hence, the voltmeter reading across resistor will be 200 V.

67. (d) Since current leads voltage by 45°, there must be a resistor and a capacitor. We can say nothing about inductor surely. It may or may not be present.

68. (c) For given parallel L-C-R circuit, we get

$$V_S = V_S \sin\omega t$$

$$I_1 = I_{01}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I_2 = I_{02}\sin(\omega t + \theta)$$

Then, $\tan\theta = \frac{X_C}{R}$

So, phase difference $= \theta + \frac{\pi}{2} = \tan^{-1}\left(\frac{X_C}{R}\right) + \frac{\pi}{2}$

Archives

1. (b) Power factor

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

2. (b)

$$\frac{n_s}{n_p} = \frac{280}{140} = 2$$

$$\frac{I_s}{I_p} = \frac{n_p}{n} = \frac{1}{2}$$

$$\Rightarrow\quad I_s = \frac{I_p}{2}\text{ A} = \frac{4}{2} = 2\text{ A}$$

3. (b) $P = Vi\cos\varphi = V\left(\frac{V}{Z}\right)\left(\frac{R}{Z}\right) = \frac{V^2R}{Z^2} = \frac{V^2R}{(R^2+\omega^2L^2)}$

4. (d) The energy loss due to eddy currents is reduced by using laminated core in a transformer.

5. (c) $\omega_0 = \frac{1}{\sqrt{LC}}$

Now

$L'(2C) = LC \Rightarrow L' = \frac{L}{2}$

6. (d) In DC ammeter, a coil is free to rotate in the magnetic field of a fixed magnet.

If an alternating current is passed through such a coil, the torque will reverse its direction each time the current changes direction and the average value of the torque will be zero.

AC instruments are based on "heating effect of current".

7. (d) The voltage across L and C are in opposite phases.

So they cancel out.

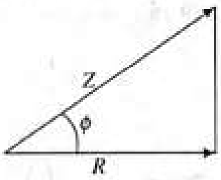
8. (c) The resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

If C changes to $2C$ then for keeping v_0 constant L must change to $L/2$.

9. (c) The presence of R forbids a phase difference of $\pi/2$.

10. (d) $\cos\phi = \frac{R}{Z}$

$= \frac{12}{15} = 0.8$



11. (a) Current at resonance

$I = \frac{100\text{ V}}{100\ \Omega} = 0.1\text{ A}$

$V_C = IX_C = I\frac{I}{\omega C}$

$= \frac{0.1}{200\times2\times10^{-6}}\text{ V}$

At resonance, $V_C = V_L$.

$\therefore V_L = 250\text{ V}$

12. (b) In an a.c. generator, maximum emf $= NAB\omega$.

13. (a) $P_{av} = E_vI_v\cos\phi$

Here ϕ = Phase difference $= \frac{\pi}{2}$

$\therefore \quad P_{av} = 0$

14. (d) When only the capacitance is removed, the phase difference between the current and voltage is

$\tan\phi = \frac{X_L}{R}$

$\tan 30° = \frac{X_L}{R}$ or $X_L = \frac{1}{\sqrt{3}}R$

When only the inductance is remove, the phase difference between current and voltage is

$\tan\phi' = \frac{X_C}{R} \Rightarrow \tan 30° = \frac{X_C}{R}$ or $X_C = \frac{1}{\sqrt{3}}R$

As $X_L = X_C$, therefore the given series LCR is in resonance.

$\therefore$ Impedance of the circuit is $Z = R = 200\ \Omega$

The power dissipated in the circuit is

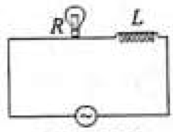
$P = V_{rms}I_{rms}\cos\phi$

$= \frac{V_{rms}^2}{Z}\cos\phi \qquad \left(\because I_{rms} = \frac{V_{rms}}{Z}\right)$

At resonance, power factor $\cos\phi = 1$

$\therefore \quad P = \frac{V_{rms}^2}{Z} = \frac{(220V)^2}{(200\Omega)} = 242\text{ W}$

15. (d) Resistance of lamp $R = \frac{V}{i} = \frac{80}{10} = 8\Omega$



220 V–50 Hz

$V_R = iR = 80$

$\left(\frac{220}{\sqrt{R^2+X_L^2}}\right)R = 80 \Rightarrow \sqrt{R^2+X_L^2} = \frac{11}{4}R$

$X_L^2 = \frac{105}{16}R^2 = \frac{105}{16}\times8^2$

$X_L^2 = 420 \Rightarrow X_L = 20.5$

$2\pi fL = 20.5 \Rightarrow 2\times3.14\times50L = 20.5$

On solving we get, $L = 0.065$ H

16. (b) Quality factor $Q = \frac{\omega_0}{\Delta\omega}$

Band width $\Delta\omega = \frac{R}{L}$

Hence $Q = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R}$

17. (c) $P_{avg} = V_{rms}I_{rms}\cos\phi$

$= \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\phi = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45°$

$= \frac{1000}{\sqrt{2}}$ watt

Wattless current $= I_{rms}\sin\phi$

$= \frac{I_0}{\sqrt{2}}\sin\phi = \frac{20}{\sqrt{2}}\sin 45° = 10$ A.

CHAPTER 25: ELECTROMAGNETIC WAVES

EXERCISES

Problems Based on Basic Theory

1. (b) Intensity is given by, $\frac{P}{A}=\frac{3.9\times10^{26}}{4\pi r^2}=5.62\times10^7\ \text{W/m}^2$

2. (a) Electric or magnetic field has zero average value in e.m.f. waves

3. (c) $Z=\sqrt{\frac{\mu_r}{\varepsilon_r}}\times\sqrt{\frac{\mu_0}{\varepsilon_0}}=\sqrt{\frac{50}{2}}\times376.6\Omega=1883\ \Omega$

4. (a) $\nu=1057$ MHz

$\lambda=\frac{c}{\nu}=\frac{3\times10^8}{1057\times10^6}\text{m}=0.28\ \text{m}=28\ \text{cm}$ (radio waves)

5. (b) $\lambda_m T=$ constant

$\Rightarrow\quad \lambda_m\times(2.7\ \text{K})=0.2896$ cm K

$\therefore\quad \lambda_m=\frac{0.2896}{2.7}\text{cm}=0.10\ \text{cm}=1\ \text{mm}$

6. (c) $E=h\nu\ \therefore\ E\propto\nu$

7. (b) $E=\frac{hc}{\lambda},\ E\propto\frac{1}{\lambda}$ or $eV\propto\frac{1}{\lambda}$

8. (a) X-rays are not reflected from the target, so cannot be used for radar.

9. (c) $E=h\nu$

$V=E/h=5\times10^{17}$ Hz

10. (d) $eV=h\nu=\frac{hc}{\lambda}\quad\therefore\quad\lambda=\frac{hc}{eV}$

11. (b) Area surrounding the TV tower is

$A=\pi r^2\quad(r=d)$

$=\pi d^2=\pi\left(\sqrt{2hR}\right)^2=\pi.2hR$

$a\propto h\quad\therefore\quad a'=2a,\ h'=2h$

12. (a) Here, $\lambda=\frac{c}{\nu}=\frac{3\times10^8}{8.2\times10^6}=36.6$ m

13. (b) Comparing the given equation with equation of plane electromagnetic wave, $E_z=E_0\cos(\omega t+kx)$, we have $\omega=6\times10^{-8}$ and $k=4$.

Velocity of light in medium

$v=\frac{\omega}{k}=\frac{6\times10^8}{4}=\frac{3}{2}\times10^8$ m/s

Refractive index, $\mu=\frac{c}{v}=\frac{3\times10^8}{(3/2)\times10^8}=2$

Maxwell Equations and Properties of Electromagnetic Waves

14. (c) $\mu=\frac{c}{\nu}$

Now $c=\frac{1}{\sqrt{\mu_0K_0}}$ and $v=\frac{1}{\sqrt{\mu_0K_0\mu_rK_r}}$

$\therefore\quad\mu=\sqrt{\mu_rK_r}$

But $\mu_r=\mu_0$ and $K_r=K_0$

$\therefore\quad\mu=\sqrt{\mu_0K_0}$

15. (b) $\mu=\frac{c}{v}=\frac{1}{\sqrt{\mu_0\varepsilon_0}}\times\frac{\sqrt{\mu\varepsilon}}{1}=\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$

16. (c) $c=\frac{E_0}{B_0}$

$\therefore\quad B_0=\frac{E_0}{c}=1.6\times10^{-6}\ \text{Wb/m}^2$

17. (a) As $B\propto r$, since the point is on the axis, where $r=0$, so $B=0$.

18. (c) $I_{av}=\frac{1}{2}\varepsilon_0E_0^2\times c=1.719\ \text{W/m}^2$

19. (a) Average energy density of electromagnetic wave

$U_{av}=\frac{1}{2}\varepsilon_0E_0^2=\frac{1}{2}\times(8.85\times10^{-12})\times(50)^2$

$\approx10^{-8}\ \text{J/m}^3$

20. (c) $C=\frac{\varepsilon_0KA}{d}=\frac{(8.85\times10^{-12})\times10\times1}{10^{-3}}=8.85\times10^{-8}$ F

$i=\frac{d}{dt}(CV)=C\frac{dV}{dt}=8.85\times10^{-8}\times25=2.2\times10^{-6}$ A

$=2.2\ \mu$A

21. (b) Charge on capacitor plates at time t is, $q=it$

Electric field between the plates at this instant.

$E=\frac{q}{A\varepsilon_0}=\frac{it}{A\varepsilon_0}$

Electric flux through the given area

$\phi_E=\left(\frac{A}{2}\right)E=\frac{it}{2\varepsilon_0}$

Therefore, displacement current

$i_d=\varepsilon_0\frac{d\phi_E}{dt}=\varepsilon_0\frac{d}{dt}\left(\frac{it}{2\varepsilon_0}\right)=\frac{i}{2}$

22. (c) Total power = Solar constant × Area

$=10^4\times(10\times10)\times10^6$ W

23. (d) Electric energy density

$u_e=\frac{1}{2}\varepsilon_0E_{rms}^2$

$E_{rms}=\frac{E_0}{\sqrt{2}}$

$\therefore\quad u_e=\frac{1}{4}\varepsilon_0E_0^2$

24. (d) Power = Area × Solar constant = 1.62×10^5 W

25. (d) Force of radiation $= \frac{\text{Power}}{c} = \frac{1.6 \times 10^5}{3 \times 10^8} = 5.32 \times 10^{-4}$ N

26. (a) Transferred momentum/second to mirror is

$$P = \frac{2AS}{C} = 1.21 \times 10^{-10} \text{ kg m s}^{-1}$$

Problems Based on Mixed Concepts

27. (b) Number of oscillation in coherence length

$$\frac{\ell}{\lambda} = \frac{0.024}{5900 \times 10^{-10}} = 4.068 \times 10^4$$

28. (d) $B = \frac{E}{c} = \frac{10^{-4}}{3 \times 10^8} = 3.3 \times 10^{-13}$ T

29. (b) $B_0 = \frac{E_0}{c} = \frac{10^{-3}}{3 \times 10^8} = 3.3 \times 10^{-12}$ T

30. (c) $\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$

or $\quad E\, 2\pi r = \frac{d}{dt}(Kt \times \pi r^2)$

or $\quad E = \frac{Kr}{2}$

31. (c) Radiation force = Momentum transferred per second by electromagnetic wave to the mirror

$$= \frac{2I_{av}A}{c} = \frac{2 \times (10) \times (20 \times 10^{-4})}{(3 \times 10^8)} = 1.33 \times 10^{-10} \text{ N}$$

32. (d) $d = \sqrt{2hR}$

Population covered = πd^2 × Population density

= π × 2 hR × Population density

$$= \frac{3.14 \times 2 \times 100 \times 6.37 \times 10^6 \times 1000}{1000 \times 1000}$$

= 4 × 10^6 = 40 lakh

33. (d) The pointing vector $\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$

$$= \frac{1}{4\pi \times 10^{-7}} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 80 & 32 & -64 \\ 0.2 & 0.08 & 0.29 \end{vmatrix} = 11.52\hat{i} - 28.8\hat{j}$$

34. (b) $\omega = 2\pi v \frac{2\pi c}{\lambda} = \frac{2\pi \times 3 \times 10^8}{6 \times 10^{-3}}$

$= \pi \times 10^{11}$ rad/s

The equation for the electric field, along y-axis in the electromagnetic wave is

$$E_y = E_0 \sin \omega\left(t - \frac{x}{c}\right) = 33 \sin \pi \times 10^{11}(t - x/c)$$

35. (d) $B_0 = \frac{E_0}{c} = \frac{48}{3 \times 10^8} = 16 \times 10^{-8}$ T

$$B_{rms} = \frac{B_0}{\sqrt{2}} = \frac{16 \times 10^{-8}}{\sqrt{2}} = 8\sqrt{2} \times 10^{-8} \text{T} = 11.3 \times 10^{-8} \text{T}$$

36. (d) Wave is propagating along positive X-axis; magnetic field is directed along Z-axis so electric field must be directed along $(\hat{k} \times \hat{i}) = \hat{j}$, along Y-axis.

Amplitude of electric field

$$E_0 = B_0 c = 2 \times 10^{-7} \times 3 \times 10^8 = 60 \text{ V/m}$$

So choice (d) is correct.

Archives

1. (a) The polarisation phenomenon of light proves its transverse nature.

2. (c) β-rays are only streams of electrons, not rays.

3. (c) $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \Rightarrow \frac{1}{\mu_0 \varepsilon_0} = c^2$

$$\left[\frac{1}{\mu_0 \varepsilon_0}\right] = [L^2T^{-2}]$$

4. (a) γ-rays have maximum energy

As $\quad E = \frac{hc}{\lambda}$,

$E \propto \frac{1}{\lambda}$

So, γ-rays have least wavelength.

5. (d) $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}, c' = \frac{1}{\sqrt{\mu_0 4\varepsilon_0}} = \frac{1}{2}c$

$\because \quad c = v\lambda$,

$\Rightarrow \quad c \propto \lambda$

So when

$$c' = \frac{c}{2}, \lambda' = \frac{\lambda}{2}$$

6. (b) In electromagnetic waves, electric energy density is equal to magnetic energy density.

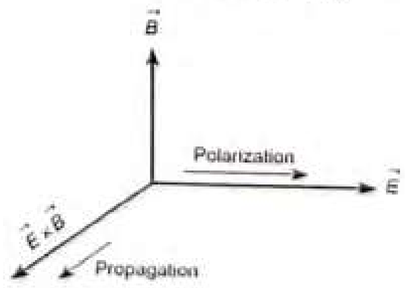
Total energy density = 2 × electric energy

$$= 2\left(\frac{1}{2}\varepsilon_0 E^2\right)$$

$= 2 \times 12 \times 8.854 \times 10^{-12} \times (720)^2$

$= 4.58 \times 10^{-6}$ J/m^3

7. (b)



8. (a) $E_0 = CB_0$

$= 3 \times 10^8 \times 20 \times 10^{-9}$

$= 6$ V/m

9. (b) Infrared waves → To treat muscular strain
Radio waves → for broadcasting
X-rays → To detect fracture of bones
Ultraviolet rays → Absorbed by the ozone layer of the atmosphere

10. (a) Energy is equally divided between electric and magnetic field.

11. (b) $I = \dfrac{P}{4\pi r^2} = U_{av} \times c$ (i)

$U_{av} = \dfrac{1}{2}\varepsilon_0 E_0^2$ (ii)

$\Rightarrow \quad \dfrac{P}{4\pi r^2} = \dfrac{1}{2}\varepsilon_0 E_0^2 \times c$

$\Rightarrow \quad E_0 = \sqrt{\dfrac{2P}{4\pi r^2 \varepsilon_0 c}} = 2.45$ V/m

12. (a) Doppler effect in light (speed of observer is not very small as compared to speed of light)

$$f' = \left[\sqrt{\frac{1+v/c}{1-v/c}}\right] f_{source} = \left[\sqrt{\frac{1+1/2}{1-1/2}}\right](10\text{ GHz})$$

$= 17.3$ GHz

13. (d) In air $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\dfrac{z}{c} - t\right)\right]$

In medium $\vec{E}_2 = E_{02}\,\hat{x}\cos[k(2z - ct)]$

During refraction, frequency remains unchanged, whereas wavelength gets changed.

Velocity of wave in medium $v = \dfrac{\omega}{k} = \dfrac{kc}{2k} = \dfrac{c}{2}$

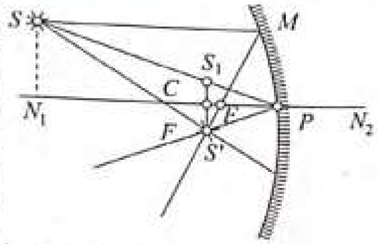
$\therefore \quad \dfrac{1}{\sqrt{\mu_0 \varepsilon_{r_2}}} = \dfrac{1}{2}\dfrac{1}{\sqrt{\mu_0 \varepsilon_{r_1}}} \Rightarrow \sqrt{\dfrac{\varepsilon_{r_1}}{\varepsilon_{r_2}}} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \dfrac{1}{4}$

CHAPTER 26: GEOMETRICAL OPTICS

Concept Application Exercises 26.1

1. Since the ray incident on the mirror at its pole is reflected symmetrically w.r.t. the major optical axis, let us plot point S_1 symmetrical to S' and draw ray SS_1 until it intersects the axis at point P (see figure).

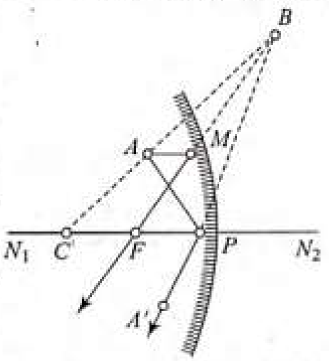


This point will be the pole of the mirror.

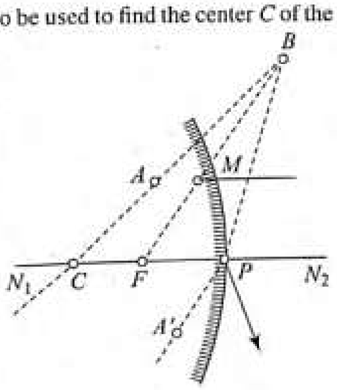
The optical center C of the mirror can obviously be found as the point of intersection of ray SS' with axis N_1N_2.

The focus can be found by the usual construction of ray SM parallel to the axis. The reflected ray must pass through focus F (lying on the optical axis of the mirror) and through S'.

2. (a) Let us construct, as in the previous example, the ray BAC and find point C (optical center of the mirror) (see figure). Pole P can be found by constructing the path of the ray APA' reflected in the pole with the aid of symmetrical point A'. The position of the mirror focus F is determined by means of the usual construction of ray AMF parallel to the axis.



(b) This construction can also be used to find the center C of the mirror and pole P (figure). The reflected ray BM will pass parallel to the optical axis of the mirror. For this reason, to find the focus, let us first determine point M at which straight line AM, parallel to the optical axis, intersects the mirror, and then extend BM to the point of intersection with the axis at the focus F.



3. Mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Here, $v=+4$ cm; $f=+\frac{R}{2}=+\frac{24}{2}=+12$ cm;

$\therefore \quad u=\frac{vf}{v-f}=\frac{(4)(12)}{4-12}=-6$ cm

The negative sign shows that the object is real and it is placed in front of the mirror.

The magnification, $m=-\frac{u}{v}=\frac{-(+4)}{-6}=\frac{2}{3}$

4. We know that $m=-\frac{v}{u}=\frac{f}{f-u}$

Here, $m_1=\frac{f}{f-(-25)}=\frac{f}{f+25}$ and

$$m_2=\frac{f}{f-(-25-15)}=\frac{f}{f+40}$$

Since $\frac{m_1}{m_2}=4$, therefore $\frac{f+40}{f+25}=4$. Thus

$f+40=4f+100$

or $\quad f=-20$ cm

The negative sign shows that the mirror is concave.

5. (a) Mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

On differentiating both sides, we get

$$-\frac{dv}{v^2}-\frac{du}{u^2}=0 \quad \text{or} \quad dv=-du\left(\frac{v}{u}\right)^2$$

Since $\frac{v}{u}=\frac{f}{u-f}$, therefore

$$dv=-du\left(\frac{f}{u-f}\right)^2$$

Lateral magnification, $m_2=\frac{dv}{du}=-\left(\frac{f}{u-f}\right)^2$

The negative sign shows that the image is longitudinally inverted.

(b) Since velocity of object, $U=\frac{du}{dt}$

and velocity of image,

$$V=\frac{dv}{dt}$$

$$\therefore \quad V=-U\left(\frac{f}{u-f}\right)^2 \quad \text{or} \quad \frac{V}{U}=-\left(\frac{f}{u-f}\right)^2$$

The negative sign shows that the object and image always move in opposite directions.

$$v=\frac{-xf_0}{f_0-x}$$

v becomes negative (real image) only when $x<f_0$.

6. For point B, $u=-25$ cm, $f=+10$ cm

$\therefore \quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

or $\quad \frac{1}{v}-\frac{1}{25}=\frac{1}{10}$

or $\quad \frac{1}{v}=\frac{1}{10}+\frac{1}{25}$

or $\dfrac{1}{v} = \dfrac{5+2}{50}$

$\therefore \quad v = \dfrac{50}{7}$

For point A, $u' = -30$ cm and $f' = 10$ cm

$$\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f'} \quad \text{or} \quad \frac{1}{v'} - \frac{1}{30} = \frac{1}{10}$$

or $\dfrac{1}{v'} = \dfrac{1}{30} + \dfrac{1}{10} = \dfrac{1+3}{30} = \dfrac{4}{30}$

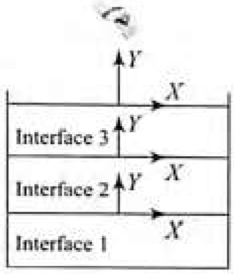
$\therefore \quad v' = \dfrac{30}{4} = \dfrac{15}{2}$ cm

$\therefore \quad A'B' = \dfrac{15}{2} - \dfrac{50}{7} = \dfrac{5}{14}$ cm

Concept Application Exercises 26.2

1. Method of interfaces:

A ray of light from the object undergoes refraction at three interfaces: (1) Water–oil, (2) Oil–glycerine, and (3) Glycerine–air. The coordinate system for each of the interfaces is shown in the figure below.



Water–oil interface:

$d_1 = -8$ cm, $\mu_1 = \dfrac{4}{3}$, $\mu_2 = 1.5$

$\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$, we get

$d_2 = -9$ cm

Oil–glycerine inteface:

$d_1 = -(9+9) = -18$ cm, $\mu_1 = 1.5$, $\mu_2 = 2$

As $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$

$d_2 = -24$ cm

Glycerine–air interface

$d_1 = -(4+24) = -28$ cm, $\mu_1 = 2$, $\mu_2 = 1$

As $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$, we get $d_2 = -14$ cm

Thus, the final image is 14 cm below the glycerine-air interface.

Method 2: Method of shifting

Net shifting

$$s = d_1\left(1 - \frac{1}{\mu_1}\right) + d_2\left(1 - \frac{1}{\mu_2}\right) + d_3\left(1 - \frac{1}{\mu_3}\right)$$

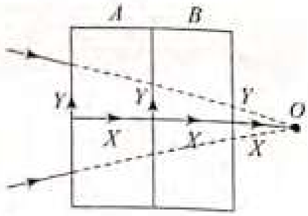
$$= 8\left(1 - \frac{1}{4/3}\right) + 9\left(1 - \frac{1}{3/2}\right) + 4\left(1 - \frac{1}{2}\right)$$

$= 7$ cm

The shifting will be in the direction of ray travelling, i.e., upwards.

Hence, apparent depth $h' = (21-7) = 14$ cm.

2. A ray of light from the object undergoes refraction at three interfaces: (1) Air–medium A, (2) Medium A–medium B, (3) Medium B–air. The coordinate system for each of the interface is shown in the following figure.



Air–medium A interface:

$d_1 = +14$ cm, $\mu_1 = 1$, $\mu_2 = 1.5$

As $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$

we get

$d_2 = +21$ cm

Medium A–medium B interface:

$d_1 = (21-6) = 15$ cm, $\mu_1 = 1.5$, $\mu_2 = 2$

As $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$, we get

$d_2 = +20$ cm

Medium B–air interface:

$d_1 = +(20-4) = +16$ cm, $\mu_1 = 2$, $\mu_2 = 1$

As $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$ or $d_2 = \dfrac{\mu_2}{\mu_1} d_1$, we get

$d_2 = +8$ cm

Thus, the final image is 8 cm in front of the medium B-air interface.

Method 2: Method of shifting

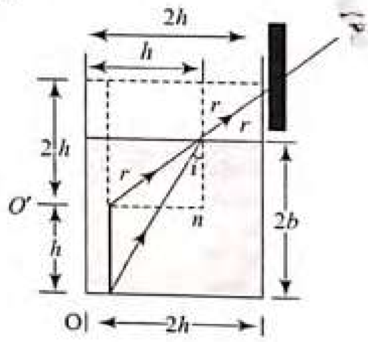
$$\text{Net shifting } S = 6\left(1 - \frac{1}{3/2}\right) + 4\left(1 - \frac{1}{2}\right)$$

$= 4$ cm

The shifting will be in the direction of ray travelling, i.e., towards right.

Hence, rays will converge finally = 14 + 4 = 18 cm from left surface or 8 cm from right surface.

3. $\dfrac{\sin i}{\sin r} = \dfrac{1}{n}$



Since $\tan r = \dfrac{2h}{2h} = 1 \Rightarrow r = 45°$

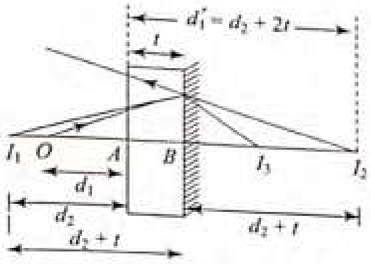
$\Rightarrow \quad \sin i = \dfrac{h}{h\sqrt{5}} \Rightarrow \sin i = \dfrac{1}{\sqrt{5}}$

$\therefore \quad \dfrac{1}{n} = \dfrac{1/\sqrt{5}}{1/\sqrt{2}} \Rightarrow n = \sqrt{\dfrac{5}{2}}$

4. At first surface,

$$\frac{\mu_1}{d_1} - \frac{\mu_2}{d_2} \Rightarrow \frac{1}{(-8)} = \frac{\mu_2}{d_2} \Rightarrow d_2 = -8\mu$$

Image I_1 will serve as an object for the mirror and form an image I_2 behind it at a distance of $(8\mu + 6)$ cm.



I_2 will serve as an object for the first surface. The rays will reflect from the mirror.

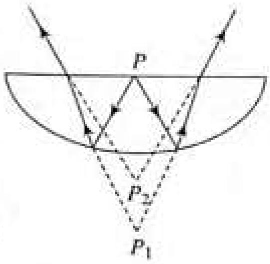
Again using $\dfrac{d_1'}{\mu} = \dfrac{d_2'}{1}$

$$d_1' = d_2 + 2t = -(8\mu + 12)$$

$$d_2' = -(10 + 6) = -16 \text{ cm}$$

After substituting the values, $\mu = 1.5$.

5. The ray diagram is shown in the following figure. Let us first locate the image formed by the concave mirror. Let us take vertically upward as the negative axis. Then, $R = -40$ cm. The object distance is $u = -5$ cm. Using the mirror equation:



$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} \Rightarrow \frac{1}{v} = \frac{2}{R} - \frac{1}{u} = \frac{2}{-40 \text{ cm}} - \frac{1}{-5 \text{ cm}} = \frac{6}{40} \text{ cm}$$

or $\quad v = 6.67$ cm

The positive sign shows that the image P_1 is formed below the mirror and hence, it is virtual. These reflected rays are refracted at the water surface and go to the observer. The depth of the point P_1 from the surface is 6.67 cm + 5.00 cm = 11.67 cm. Due to refraction at the water surface, the image P_1 will be shifted above by a distance (11.67 cm)

$$\left(1 - \frac{1}{1.33}\right) = 2.92 \text{ cm}$$

Thus, the final image is formed at a point (11.67 − 2.92) cm = 8.75 cm below the water surface.

6. Let the object be placed at a height 'x' above the surface of water. The apparent position of the object with respect to the mirror should be at the center of curvature so that the image is formed at the same position.

Since $\dfrac{\text{Real depth}}{\text{Apparent depth}} = \dfrac{1}{\mu}$

(with respect to the mirror), therefore

$$\frac{x}{R - h} = \frac{1}{\mu}$$

or $\quad x = \dfrac{R - h}{\mu}$

Concept Application Exercises 26.3

1. In the case of minimum deviation,

$$i_1 = i_2, r_1 = r_2$$

$$\delta = 2i - A$$

$$r = A/2$$

According to problem, $i = 2r = A$

$$\delta_{\min} = 2A - A = A$$

$$n = \frac{\sin\left(\dfrac{A + \delta_{\min}}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \frac{\sin A}{\sin \dfrac{A}{2}} = \frac{2 \sin \dfrac{A}{2} \cos \dfrac{A}{2}}{\sin \dfrac{A}{2}}$$

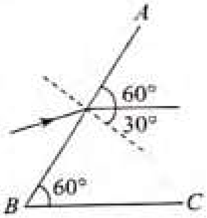
$$\cos \frac{A}{2} = \frac{n}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$\therefore \quad A = 90°$

2. The given parameters are $\delta = 30°$ and $A = 60°$. Let us test whether the prism is in the position of minimum deviation.

$$n = \frac{\sin\left(\dfrac{30° + 60°}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

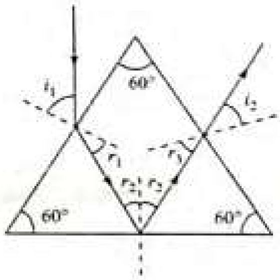


As $n = \sqrt{2}$, the ray suffers minimum deviation through the prism. Thus,

$$r_1 = r_2 = r = \frac{A}{2} = 30°$$

Inside the prism, the ray makes an angle of 60° with the face AB, so it is parallel to the base.

3. In the figure shown,



$r_2 = C$

$r_1 = 60 - r_2 = 60 - C$

$r_3 = 60 - r_2 = 60 - C$

$r_1 = r_3 = r$ (say)

$i_1 = i_2 = i$ (say)

Net deviation, $\delta = (1 - r) + (180 - 2r_2) + (i - r) = 108°$

$$r_2 = r - i = 36°$$

$$C + 60 - C - i = 36°$$

$$i = 24°, \sin 24° = 0.40$$

From Snell's law, we have

$$\sin 24° = \mu \sin r$$

$$0.4 = \mu \sin(60° - C)$$

$$0.4 = \mu\left[\frac{\sqrt{3}}{2}\cos C - \frac{1}{2\mu}\right]$$

$\Rightarrow \quad \mu = 1.447$

4. As the ray of light grazes the second surface, r_2 is the critical angle.

$$\sin r_2 = \frac{1}{\mu}$$

$$r_2 = (45° - r_1)$$

$$\sin r_1 = \frac{\sin 45°}{\mu} = \frac{1}{\sqrt{2}\,\mu}$$

$$\sin r_2 = \sin(45° - r_1)$$

$$= \frac{1}{\sqrt{2}}[\cos r_1 - \sin r_1]$$

$$\frac{1}{\mu}=\frac{1}{\sqrt{2}}\left[\sqrt{1-\frac{1}{2\mu^2}}-\frac{1}{\sqrt{2}\mu}\right]$$

$\Rightarrow \quad \mu^2=5 \quad \text{or} \quad \mu=\sqrt{5}$

At minimum deviation,

$$r_1=r_2\frac{45°}{2}=22.5°$$

$$\mu=\frac{\sin i_1}{\sin r_1} \Rightarrow \sin i_1=(\sqrt{5})\sin(22.5°)$$

$$i_1=58.8°$$

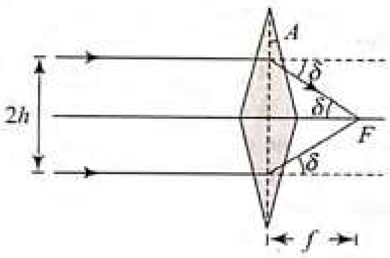
5. For minimum deviation:

$$\mu=\frac{\sin[(A+\delta_m)/2]}{\sin(A/2)}$$

$$\frac{3}{2}=\frac{\sin[(60°+\delta_m)/2]}{\sin(60°/2)}=\sin\left(\frac{60+\delta_m}{2}\right)=\frac{3}{4}$$

$$\Rightarrow \quad i=\frac{A+\delta_m}{2}=\sin^{-1}\left(\frac{3}{4}\right)=48.5°$$

6. Since $\tan\delta=\frac{h}{f} \quad \Rightarrow \quad f=\frac{h}{\tan\delta}$



Further, $\delta=(\mu-1)A \quad \Rightarrow \quad f=\frac{h}{(\mu-1)A}$

Concept Application Exercises 26.4

1. (a) $\mu_1=1,\ \mu_2=1.5,\ R=+20$ mm, $u=+80$ mm

$$\frac{1.5}{v}-\frac{1}{-80}=\frac{1.5-1}{-20}$$

$$v'=+120 \text{ mm}$$

The image is therefore formed at the right of the vertex. (v is positive and at a distance of 120 mm from it.)

(b) $\frac{1.5}{v}+\frac{-1.33}{-80}=\frac{1.5-1.33}{+20}$, $v=-180$ mm

The fact that v is negative means that the rays after refraction by the surface are not converging but appear to diverge from a point 180 mm to the left of the vertex. In this illustration, then, the surface forms a virtual image 180 mm to the left of the vertex.

2. Let the object be placed at a distance x from the pole P_1 of the sphere. If a real image is to be formed at equidistant from the sphere, then the ray must pass symmetrical through the sphere, as shown in the figure in question.

Applying the equation at the first surface, we get

$$\frac{\mu_2}{+\infty}-\frac{\mu_1}{-x}=\frac{\mu_2-\mu_1}{+R}$$

or $\quad x=\left(\frac{\mu_1}{\mu_2-\mu_1}\right)R$

3. According to cartesian sign convention,

$u=-30$ cm, $R=+10$ cm; $\mu_1=1$; $\mu_2=1.5$

Applying the equation, we get $\frac{1.5}{v}-\frac{1}{-30}=\frac{1.5-1}{+10}$

or $\quad v=90$ cm (real image)

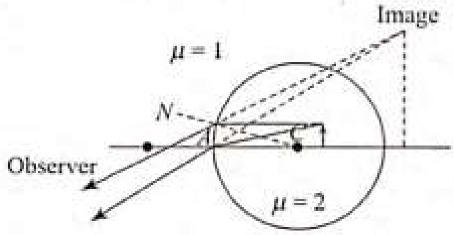
Let h_i be the height of the image, then

$$\frac{h_i}{h_0}=\frac{\mu_1 v}{\mu_2 u}=\frac{(1)(90)}{(1.5)(-30)}=-2$$

$\Rightarrow \quad h_i=-2h_0(0.5)=-2(0.5)=-1$ cm

The negative sign shows that the image is inverted.

4. For refraction near point A, $u=-30$; $R=-20$; $n_1=2$; $n_2=1$.



Applying refraction formula,

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R} \quad \Rightarrow \quad \frac{1}{v}-\frac{2}{-30}=\frac{1-2}{-20}$$

$v=-60$ cm

$$m=\frac{h_2}{h_1}=\frac{n_1 v}{n_2 u}=\frac{2(-60)}{1(-30)}=4$$

$\therefore \quad h_2=4$ mm

5. Deviation by a sphere is $2(i-r)$

Here, deviation $\delta=60°=2(i-r)$

or $\quad i-r=30°$

$\Rightarrow \quad r=i-30°=60°-30°=30°$

$\therefore \quad \mu=\frac{\sin i}{\sin r}=\frac{\sin 60°}{\sin 30°}=\sqrt{3}$

6. The observer sees the image formed due to refraction at the spherical surface when the light from the bubble goes from the glass to the air.

Here, $u=-4.0$ cm, $R=-10$ cm, $\mu=1.5$ and $\mu_2=1$.

$$\frac{1}{v}=\frac{0.5}{-4.0\text{cm}}=\frac{1-1.5}{-10\text{cm}}$$

$$\frac{1}{v}=\frac{0.5}{10\text{cm}}-\frac{1.5}{4.0\text{cm}}$$

$v=-3.0$ cm

Thus, the bubble will appear 3.0 cm below the surface.

Concept Application Exercises 26.5

1. Two images will be formed, one from each type of material.
2. (a) $\mu_1<\mu_2$, because it is behaving like a divergence lens.
(b) $\mu_1=\mu_2$, because no refraction is taking place.

3. We have, $\mu = 1.5$, $R_1 = 20$ cm, and $R_2 = -40$ cm.
Therefore, the focal length of the lens is

$$\frac{1}{f} = (1.5-1)\left[\frac{1}{20} - \frac{1}{-40}\right]$$

or $\quad f = \frac{80}{3}$ cm

Now, $f = \frac{80}{3}$, $u = -40$ cm

Therefore, from equation, we get

$$\frac{1}{v} = \frac{3}{80} - \frac{1}{40} \text{ or } \quad v = +80 \text{ cm}$$

Therefore, the image is formed 80 cm on the other side of the lens.

4. Here, $\mu = 1.5$, $R_1 = -20$ cm, and $R_2 = +20$ cm.
Substituting in equation, we get $f = -20$ cm
Now, $u = +10$ cm, $f = -20$ cm
Substituting in equation, we get

$$\frac{1}{v} = \frac{1}{-20} + \frac{1}{10} \quad \text{or} \quad v = +20 \text{ cm}$$

Thus, the rays now converge to a point 20 cm in front of the lens.

5. Here, $u = -15$ cm, $m = -2$ (Note the negative sign. As the image is on a screen, it is real which implies m must be negative)
$\therefore \quad v = mu = +30$ cm
Applying equation, we get

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{-15} \text{ or } f = 10 \text{ cm}$$

Therefore, the lens is convex, with focal length of 10 cm.

6. Here, $f = +10$ cm and $m = +2$. (Magnification is now positive as the image is erect.) Let the object distance from the lens be x.
Then, $u = -x$ and $v = mu = 2x$.
Substituting in equation, we get

$$\frac{1}{10} = \frac{1}{-2x} - \frac{1}{-x} \text{ or } x = 5 \text{ cm}$$

Therefore, the lens is to be held 5 cm from the object.

Concept Application Exercises 26.6

1. For convex lens:

$$v = \frac{uf}{u+f} = \frac{-15 \times 10}{-15+10} = 30 \text{ cm}$$

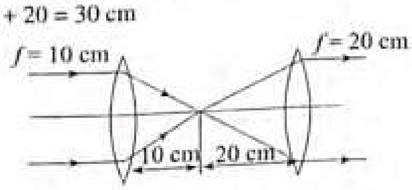
For concave lens:

$$u = (30-25) = 5 \text{ cm}$$

$$v = \frac{uf}{u+f} = \frac{5(-10)}{5-10} = 10 \text{ cm}$$

So, final image is formed 10 cm to the right of the concave lens.

2. $d = 10 + 20 = 30$ cm



3. Here, $R_1 = +12$ cm, $R_2 = \infty$, $\mu_p = 1.5$, $\mu_a = 1$, therefore

Focal length of the lens, $\frac{1}{f_1} = (1.5-1)\left[\frac{1}{12} - \frac{1}{\infty}\right]$

or $\quad f_1 = 24$ cm

The plane side of the lens is silvered; focal length of the mirror

$$f_m = \frac{R_2}{2} = \frac{\infty}{2} = \infty$$

Therefore, effective focal length of the silvered lens is

$$\frac{1}{f_e} = \frac{1}{f_m} - \frac{2}{f_1} \quad \text{or} \quad f_e = -12 \text{ cm}$$

The object is placed 24 cm in front of the lens.

$$u = -24 \text{ cm}, f = -12 \text{ cm}$$

And from the mirror Eq. (i), we have

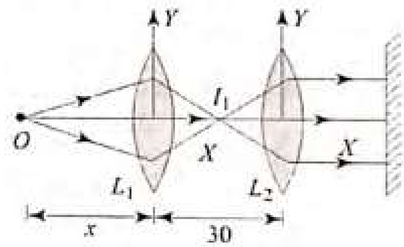
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \text{(i)}$$

or $\quad v = -24$ cm

The final image is formed on the object itself. The behavior is like that of a concave mirror.

4. The convex lenses and the plane mirror are shown in figure.
The combination behaves like a concave mirror.
Let the distance of the object from the first lens be x.
For the ray to retrace its path, it should be incident normally on the plane mirror.
From the diagram, we see that for lens L_2

$$v = \infty, f = +10 \text{ cm}, u = ?$$



From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad u = -10 \text{ cm}$$

From the diagram, we see that for lens L_1

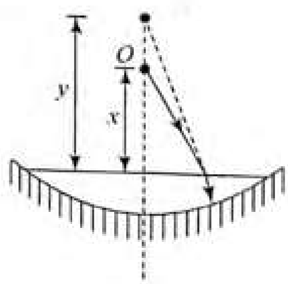
$$v = 30 - 10 = 20 \text{ cm}, f = +10 \text{ cm}, u = -x$$

From the lens equation, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{20} - \frac{1}{-x} = \frac{1}{10} \quad \text{or} \quad x = 20 \text{ cm}$$

5. $y = \mu x = \frac{4}{3}x$

For the mirror, $u = -y = \frac{4}{3}x$



$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{3}{4x} = \frac{1}{-30}$$

$$v = -\left(\frac{60x}{2x-45}\right)$$

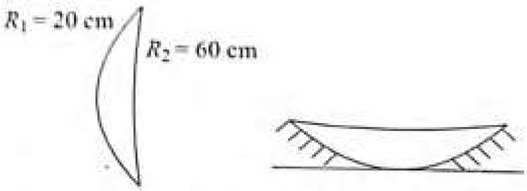
Let the final image is formed at a distance z from the mirror, then

$$\left(\frac{60x}{2x-45}\right)\frac{1}{z}=\mu=\frac{4}{3} \text{ and } \frac{z}{x}=2 \text{ (magnification)}$$

Solving, we get $x = 33.75$ cm

6. $\frac{1}{f_l}=(1.5-1)\left[\frac{1}{20}-\frac{1}{60}\right]=\frac{1}{60}$

$f_l = 60$ cm



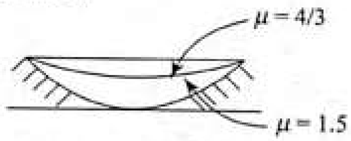
When convex side is silvered and water is not filled:

$$\frac{1}{F}=\frac{2}{f_1}-\frac{1}{f_m},\ f_m=\frac{-R_1}{2}=-10 \text{ cm}$$

$$\frac{1}{F}=\frac{1}{-10}-2\left[\frac{1}{60}\right]$$

$$F=\frac{-30}{4}=-7.5 \text{ cm}$$

After water is filled:



Let f_{l_1} is the focal length of water lens:

$$\frac{1}{f_{l_1}}=\left(\frac{4}{3}-1\right)\left(\frac{1}{60}\right)=\frac{1}{180}$$

$$\frac{1}{F}=\frac{2}{f_{l_1}}-\frac{2}{f_{l_2}}-\frac{1}{f_m}$$

Solving, we get $F=\frac{-90}{13}$ cm

Exercises

Reflection through Plain and Spherical Mirror

1. (a) $m=+\frac{1}{n}=-\frac{v}{u}\Rightarrow v=\frac{u}{n}$

By using mirror formula $\frac{1}{f}=\frac{1}{-\frac{u}{n}}+\frac{1}{u}\Rightarrow u=-(n-1)f$

2. (a) Mirror formula

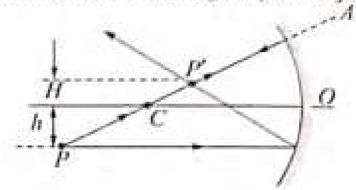
$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}\Rightarrow\frac{1}{f}=\frac{1}{-20}+\frac{1}{(-10)}\Rightarrow f=\frac{20}{3} \text{ cm}$$

If object moves towards the mirror by 0.1 cm then.

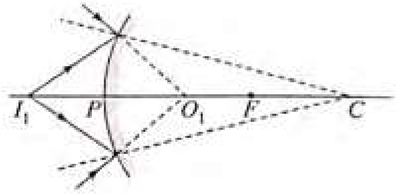
$u = (10 - 0.1) = 9.9$ cm. Hence again from mirror formula

$\frac{1}{-20/3}=\frac{1}{v'}+\frac{1}{-9.9}\Rightarrow v' = 20.4$ cm i.e. image shifts away from the mirror by 0.4 cm.

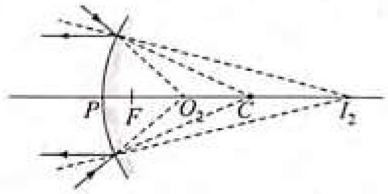
3. (b) The object, center of curvature and image always lie on a straight line. Hence the image of point object P is on line PA.



4. (b)

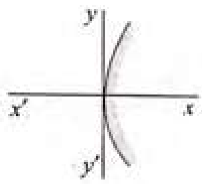


For I_1, virtual object will be between P and F



For I_2, virtual object will be between F and C.

5. (b) For point B, $u = -25$ cm, $f = +10$ cm



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

or, $\frac{1}{v}-\frac{1}{25}=\frac{1}{10}$

or, $\frac{1}{v}=\frac{1}{10}+\frac{1}{25}$

or, $\frac{1}{v}=\frac{5+2}{50}$ $\quad v=\frac{50}{7}$

For point A, $u' = -30$ cm, $f' = 10$ cm

$\frac{1}{v'}+\frac{1}{u'}=\frac{1}{f'}$ $\quad$ or, $\frac{1}{v'}-\frac{1}{30}=\frac{1}{10}$

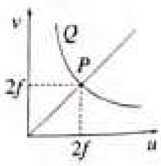
or, $\frac{1}{v'}=\frac{1}{30}+\frac{1}{10}=\frac{1+3}{30}=\frac{4}{30}$

$$v'=\frac{30}{4}=\frac{15}{2} \text{ cm}$$

$$A'B'=\frac{15}{2}-\frac{50}{7}=\frac{5}{14} \text{ cm}$$

6. (c) At P, $u = v$ which happened only when $u = 2f$

At another point Q on the graph (above P) $v > 2f$



7. (a) Since $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}\Rightarrow\frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$

Putting the sign convention properly

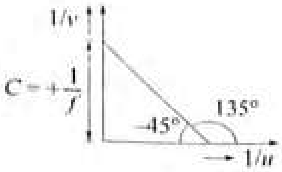
$$\frac{1}{(-v)}=\frac{-1}{(-u)}+\frac{1}{(-f)}\Rightarrow\frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$$

Comparing this equation with $y = mx + c$

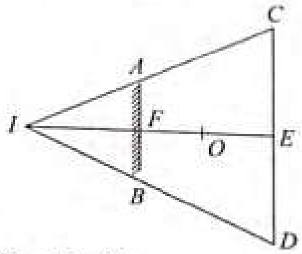
Slope $= m = \tan\theta = -1$

$\Rightarrow \theta = 135^\circ$ or -45° and

intercept $C = +\frac{1}{f}$



8. (d) As IAB and ICD are similar.



$$\frac{CD}{AB} = \frac{IE}{IF} = \frac{3L}{L} = 3$$

$$CD = 3AB = 3d$$

9. (d) $f = \frac{1.6}{2}\text{m} = 0.8\text{m},\ u = -1\text{m}$

$$\frac{1}{v} = \frac{1}{0.8} - \frac{1}{-1} = \frac{10}{8} + 1 = \frac{18}{8} = \frac{9}{4}$$

or $\quad v = \frac{4}{9}\text{m}$

10. (a) $m = -\frac{v}{u} = -\frac{\frac{4}{9}}{-1} = \frac{4}{9}$

11. (d) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$\frac{1}{f - x_1} + \frac{1}{f - x_2} = \frac{1}{f} \text{ or } \frac{f - x_2 + f - x_1}{(f - x_1)(f - x_2)} = \frac{1}{f}$$

or $\quad f^2 - fx_2 - fx_1 + x_1x_2 = 2f^2 - f(x_1 + x_2)$

or $\quad f^2 = x_1x_2$ or $f = \sqrt{x_1x_2}$

This is Newton's mirror formula.

12. (d) When there is no water in the mirror, the rays of light are incident normally on the mirror and retrace their path. So, we get an image coincident with the object as shown in the figure. When the mirror is filled with water, then the equivalent focal length F is given by

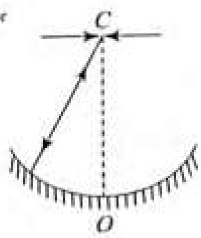
$$\frac{1}{F} = \frac{1}{f_{\text{water lens}}} + \frac{1}{f_{\text{concave mirror}}} + \frac{1}{f_{\text{water lens}}}$$

or $\quad \frac{1}{F} = 2 \times \frac{1}{f_{\text{water lens}}} + \frac{1}{f_{\text{concave mirror}}}$

or $\quad \frac{1}{F} = 2(\mu - 1)\left(\frac{1}{R}\right) + \frac{1}{R}$

or $\quad \frac{1}{F} = \frac{2(\mu - 1)}{R} + \frac{2}{R}$

or $\quad \frac{1}{F} = \frac{2\mu}{R}$ or $F = \frac{R}{2\mu}$



Clearly, focal length of the new optical system in less than the original. So, the object is effectively at a distance greater than twice the focal length. So, the real image will be formed between F and C.

13. (d) A thick glass mirror produces a number of images. There is an apparent shift of actual silvered surface toward the unsilvered face.

Effective distance of the reflecting surface from unsilvered face

$$= \frac{d}{\mu} = \frac{3}{3/2}\text{cm} = 2\text{ cm}$$

Distance of the point object from effective reflecting surface $= 9\text{ cm} + 2\text{ cm} = 11\text{ cm}$.

Distance of image from the point object $= 11\text{ cm} + 11\text{ cm} = 22\text{ cm}$

Distance of image from unsilvered face $= (22 - 9)\text{ cm} = 13\text{ cm}$

14. (a) Clearly, the distance of I from P is 15 cm.

Now, $\quad u = -25\text{ cm},\ v = 15\text{ cm},\ f = ?$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \text{or} \quad \frac{1}{f} = \frac{1}{-25} + \frac{1}{15}$$

or $\quad \frac{1}{f} = \frac{-3+5}{75} \quad$ or $\quad f = \frac{75}{2}\text{cm} = 37.5\text{ cm}$

15. (b) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$-\frac{du}{u^2} - \frac{dv}{v^2} = 0 \quad \text{or} \quad \frac{dv}{v^2} = -\frac{du}{u^2}$$

or $\quad dv = -\frac{v^2}{u^2}du$

$$= -\frac{10 \times 10}{30 \times 30} \times 9\text{ m s}^{-1} = -1\text{ m s}^{-1}$$

16. (a) For a concave mirror,

$$u = -\frac{15}{2}\text{cm},\ v = ?$$

$$f = -\frac{10}{2}\text{cm} = -5\text{ cm}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-5} - \frac{1}{-15/2}$$

$$= -\frac{1}{5} + \frac{2}{15} = \frac{-1}{15}$$

or $\quad v = -15\text{ cm}$

Clearly, the position of the final image is on the pole of the convex mirror.

17. (b) For upright portion,

$$m = \frac{f}{f-u} = \frac{\frac{-10}{2}}{\frac{-10}{2} - (-20)} = \frac{-5}{-5+20} = \frac{-5}{15} = -\frac{1}{3}$$

For horizontal portion, magnification is $\left(-\frac{1}{3}\right)^2$ i.e., $\frac{1}{9}$

Required ratio is $\frac{-1/3}{1/9} = -3:1$

18. (d) Let $u = -x$

$$\frac{1}{-x} + \frac{1}{-x-10} = -\frac{1}{12}$$

or $\quad x^2 - 14x - 120 = 0$

$$x^2 - 20x + 6x - 120 = 0$$

or $\quad x(x - 20) + 6(x - 20) = 0$

Here $\quad u = -20$ cm

$$v = -(20 + 10) \text{ cm} = -30 \text{ cm}$$

Then magnification

$$m = -\frac{u}{v} = -\left(\frac{-30}{-20}\right)$$

$$= -1.5$$

19. (b). $m = \dfrac{f}{f - u}$

$$3 = \frac{-24}{-24 - u}$$

or $\quad -24 - u = -8$ or $u + 24 = 8$

or $\quad u = (8 - 24)$ cm $= -16$ cm

If $\quad m = -3$, then

$$-3 = \frac{-24}{-24 - u}$$

$$u + 24 = -8$$

or $\quad u = -32$ cm.

Note that the magnification is greater than 1, so mirror cannot be convex.

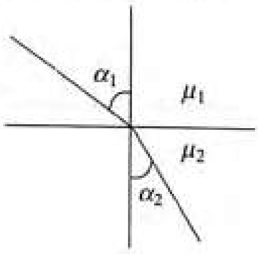
20. (d) For convex mirror, $u + v = 12 \times 2 = 24$ cm (because for plane mirror, distance of object = distance of image). Also, here for the convex mirror $u = 20$ cm, therefore $v = 4$ cm. Hence, using

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

We find $f = 5$ cm

Refraction of Light at Plane and Curved Surface

21. (b) $\mu_1 \sin \alpha = \mu_2 \sin \alpha_2$



$$\frac{c}{v_1} \sin \alpha_1 = \frac{c}{v_2} \sin \alpha_2$$

$$\frac{\sin \alpha_1}{f\lambda_1} = \frac{\sin \alpha_2}{f\lambda_2}$$

$$\lambda_2 = \frac{\sin \alpha_2}{\sin \alpha_1} \lambda_1$$

22. (b) $\mu_1 \sin \theta = \mu_2 \times \sin(90° - \theta)$

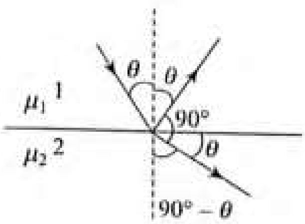
$$\Rightarrow \quad \frac{\mu_1}{\mu_2} = \tan \theta$$

For θ_c

$$\Rightarrow \mu_1 \times \sin \theta_C = \mu_2 \times \sin(90°)$$

$$\sin \theta_C = \frac{\mu_1}{\mu_2} = \tan \theta$$

$$\Rightarrow \quad \theta_C = \sin^{-1}(\tan \theta)$$



23. (a) The refractive index, ${}_1n_2$ (from medium 1 to medium 2) for two given media 1 and 2 is given by

$${}_1n_2 = \frac{\text{speed of light in medium 1 } (c_1)}{\text{speed of light in medium 2 } (c_2)}$$

$$= \frac{\text{wavelength of light in medium 1 } (\lambda_1)}{\text{wavelength of light in medium 2 } (\lambda_2)}$$

Now, ${}_1n_2 = \dfrac{n_2}{n_1} = \dfrac{5}{4}$

$$c_1 = 2.0 \times 10^{-8} \text{ m s}^{-1}$$

$$\lambda_1 = 500 \text{ nm}$$

$$\therefore \quad \frac{5}{4} = \frac{2.0 \times 10^8}{c_2} = \frac{500}{\lambda_2}$$

Hence, $\lambda_2 = 400$ nm

$$c_2 = 1.6 \times 10^8 \text{ m s}^{-1}$$

24. (d) The two slabs will shift the image a distance

$$d = 2\left(1 - \frac{1}{\mu}\right)t = 2\left(1 - \frac{1}{1.5}\right)(1.5) = 1.0 \text{ cm}$$

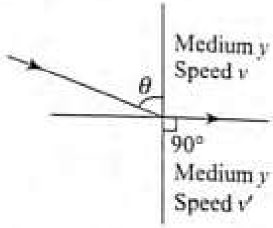
Therefore, final image will be 1 cm above point P.

25. (d) Clearly, x is denser medium.

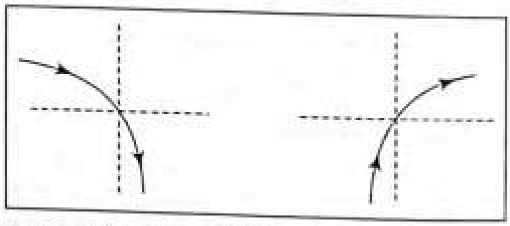
Now, $\dfrac{\sin \theta}{\sin 90°} = \dfrac{1}{{}_y\mu_x}$

$$= {}_x\mu_y = \frac{v}{v'}$$

or $\quad v' = \dfrac{v}{\sin \theta}$



26. (a) Since the refractive index is changing, the light cannot travel in a straight line in the liquid as shown in options (c) and (d). Initially, it will bend towards normal and after reflecting from the bottom it will bend away from the normal as shown below.



27. (b) Distance of first image (I_1) formed after refraction from the plane surface of water is

$$\frac{10}{4/3} = 7.5 \text{ cm}$$

from water surface $\left(d_{\text{app}} = \dfrac{d_{\text{actual}}}{\mu}\right)$.

Now, distance of this image is 5 + 7.5 = 12.5 cm from the plane mirror. Therefore, distance of second image (I_2) will also be equal to 12.5 cm from the mirror.

28. (b) $v_m = \dfrac{1}{2}c$

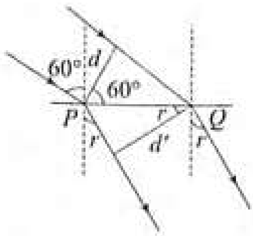
$$\mu = \frac{c}{V_m} = \frac{c}{\frac{1}{2}c} = 2 \text{ or } \frac{1}{\sin i_c} = 2$$

or $\sin i_c = \frac{1}{2}$ or $i_c = 30°$

29. (c) Let d' be the diameter of refracted beam. Then,

$d = PQ \cos 60°$

and $d' = PQ \cos r$



i.e. $\frac{d'}{d} = \frac{\cos r}{\cos 60°} = 2 \cos r$

or $d' = 2d \cos r$

$\sin r = \frac{\sin i}{\mu} = \frac{\sqrt{3}/2}{3/2} = \frac{1}{\sqrt{3}}$

$\therefore \cos r = \sqrt{1 - \sin^2 r} = \sqrt{\frac{2}{3}}$

$\therefore d' = (2)(2)\sqrt{\frac{2}{3}}$

$= 4\sqrt{\frac{2}{3}}$ cm $\approx$ 3.26 cm

30. (c) $\sin \theta_1 = \frac{1}{\mu_g}$ and $\sin \theta_2 = \frac{1}{\mu_w}$

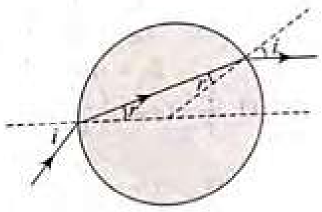
Since $\mu_g > \mu_w$, $\theta_1 < \theta_2$

Critical angle θ between glass and water will be given by

$\sin \theta = \frac{\mu_w}{\mu_g}$

or $\theta > \theta_2$

31. (b) Deviation by a sphere is $2(i-r)$



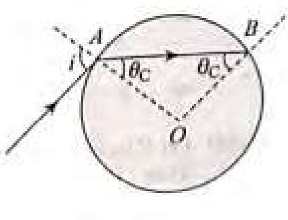
Here, deviation $\delta = 60°$
$= 2(i - r)$

or $i - r = 30°$

$\therefore r = i - 30° = 60°$
$- 30° = 30°$

$\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$

32. (c) $\angle ABO = \angle OAB = \theta_C$



$\sin \theta_C = \frac{1}{\mu} = \frac{2}{3}$

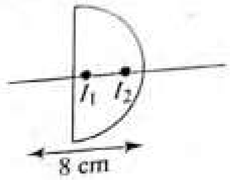
Applying Snell's law at A

$\frac{\sin i}{\sin \theta_C} = \frac{3}{2}$

$\sin i = \left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$

or $i = 90°$

33. (d) Distance of image from the plane surface is as follows:



$x_1 = \frac{4}{1.6}$

$= 2.5$ cm $\left(d_{app} = \frac{d_{actual}}{\mu}\right)$

For the curved side

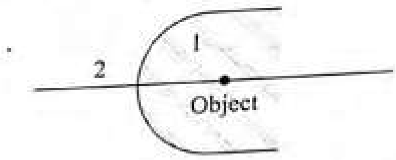
$\frac{1.6}{4} + \frac{1}{x_2} = \frac{1-1.6}{8}$

$\therefore x_2 = -3.0$ cm

The minus sign means the image is on the object side.

$\therefore I_1 I_2 = (8 - 2.5 - 3.0)$ cm $= 2.5$ cm

34. (a). Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



$\frac{1}{v} - \frac{1.5}{(-u)} = \frac{1-(1.5)}{-R}$ or $\frac{1}{v} + \frac{3}{2u} = \frac{1}{2R}$

For v to be positive, $\frac{1}{2R} > \frac{3}{2u}$

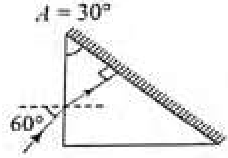
or $u > 3R$

35. (b) Critical angle $\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$

$= \sin^{-1}\left(\frac{1}{2}\right) = 30°$

If $A > 2\theta_c$, the ray does not emerge from the prism. So, maximum angle can be 60°.

36. (b) $r_2 = 0°$



$r_1 = A = 30°$

and $i_1 = 60°$

$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$

37. (a) $A = \delta_m = 60°$

At minimum deviation $i = \left(\frac{A + \delta_m}{2}\right) = 60°$

38. (c) $\sqrt{3} = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$

$\sqrt{3} = \sin\left(\frac{60° + \delta_m}{2}\right)$

$\sin 60° = \sin\left(\frac{60° + \delta_m}{2}\right)$

or $\frac{60° + \delta_m}{2} = 60°$

or $\delta_m = 60° \Rightarrow i = \frac{A + \delta_m}{2} = \frac{60° + 60°}{2} = 60°$

39. (b) $_g\mu_a = \dfrac{\sin i}{\sin r}$

$$\frac{1}{_a\mu_g} = \frac{\sin i}{\sin 2i}$$

$$\frac{1}{16} = \frac{\sin i}{2\sin i \cos i} \text{ or } \frac{1}{\cos i}\frac{1}{10} = \frac{5}{8}$$

or $\dfrac{1}{\cos i} = \dfrac{5}{4}$ or $\cos i = \dfrac{4}{5}$

Now, $\sin i = \sqrt{1-\cos^2 i}$

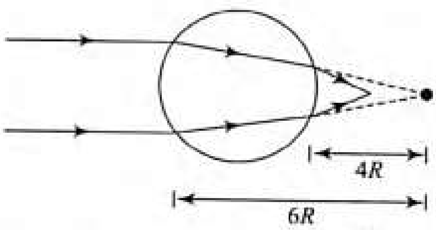
or $\sin i = \sqrt{1-\dfrac{16}{25}} = \sqrt{\dfrac{9}{25}} = \dfrac{3}{5}$

or $i = \sin^{-1}\left(\dfrac{3}{5}\right)$

40. (b) $\dfrac{\mu_1}{-u} + \dfrac{\mu_2}{v} = \dfrac{\mu_2-\mu_1}{R}$

$$\frac{1.25}{-(-\infty)} + \frac{1.5}{v} = \frac{1.5-1.25}{R} \quad \text{or} \quad \frac{1.5}{v} = \frac{0.25}{R}$$

or $v = \dfrac{1.5R}{0.25} = 6R$



Again, $\dfrac{\mu_2}{-u} + \dfrac{\mu_1}{v} = \dfrac{\mu_1-\mu_2}{R}$

$$\frac{1.5}{-4R} + \frac{12.5}{v} = \frac{1.25-1.5}{-R}$$

or $\dfrac{1.25}{v} = \dfrac{1}{4R} + \dfrac{1.5}{4R} = \dfrac{2.5}{4R} = \dfrac{1.25\times 4R}{2.5} = \dfrac{5R}{2.5}$

or $v = 2R$.

Distance from the center = $3R$

Convex and Concave Lenses and Prism

41. (a) A diverging lens is ruled out because both x and y are positive values. Both x and y equal 20 cm at their smallest sum, which occurs when

$x + y = 40$ cm $= 4f$

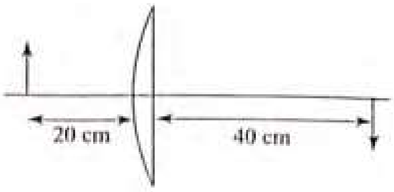
$\therefore$ $f = 10$ cm

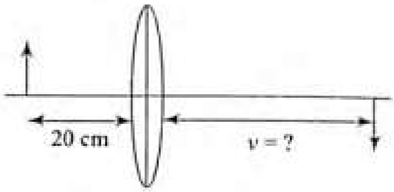
This indicates a converging lens of focal length = 10 cm

42. (a) $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$

$\Rightarrow$ $\dfrac{1}{f} = \dfrac{1}{40} - \dfrac{1}{(-20)}$ $\Rightarrow$ $f = \dfrac{40}{3}$

Focal length for the combination,





$f_{eq} = \dfrac{20}{3}$

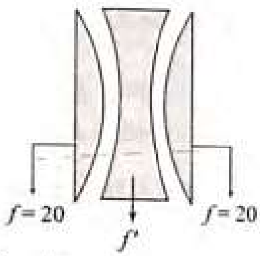
$\dfrac{1}{f_{eq}} = \dfrac{1}{v} - \dfrac{1}{u}$

$\Rightarrow$ $\dfrac{30}{20} = \dfrac{1}{v} + \dfrac{1}{20}$

$\Rightarrow$ $\dfrac{1}{v} = \dfrac{3}{20} - \dfrac{1}{20}$

$v = 10$ cm (to the right)

43. (c) $\dfrac{1}{f} = (\mu-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$



$R = 10$ cm

$f' = (3-1)\left(\dfrac{1}{-10} - \dfrac{1}{10}\right)$

$f' = \dfrac{-10}{4}$

$\dfrac{1}{f_{eq}} = \dfrac{1}{20} - \dfrac{4}{10} + \dfrac{1}{20} = \dfrac{2}{20} - \dfrac{4}{10}$

$f_{eq} = -10/3$ cm

44. (c) $\dfrac{1}{f} = (\mu-1)\left(\dfrac{2}{R}\right)$

or $f = \dfrac{R}{2(\mu-1)}$

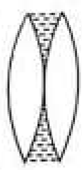
Now, $f > R$

$\therefore$ $\dfrac{R}{2(\mu-1)} > R$

or $\dfrac{1}{2(\mu-1)} > 1$ or $2(\mu-1) < 1$

or $\mu - 1 < \dfrac{1}{2}$ or $\mu < \left(1+\dfrac{1}{2}\right)$ or $\mu < 1.5$

45. (d) Let R be the radius of curvature of each surface. Then



$\dfrac{1}{f} = (1.5-1)\left(\dfrac{1}{R} + \dfrac{1}{R}\right)$.

For the water lens,

$\dfrac{1}{f'} = \left(\dfrac{4}{3}-1\right)\left(-\dfrac{1}{R} + \dfrac{1}{R}\right) = \dfrac{1}{3}\left(-\dfrac{2}{f}\right); \dfrac{1}{f'} = -\dfrac{2}{3f}$

Now, using $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$, we have

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}+\frac{1}{f'}=\frac{2}{f}-\frac{2}{3f}=\frac{4}{3f}$$

$\Rightarrow \quad F=\frac{3f}{4}$

46. (d) Only convex lens can form a real well as virtual image. So, the given lens is a convex lens.

Let f is the focal length of the lens and n the magnitude of magnification.

In the first case, when the image is real:

$\mu v=-16$

$v=+16n$

So, applying $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{16n}+\frac{1}{16}=\frac{1}{f}$$

or $\quad 1+\frac{1}{n}=\frac{16}{f}$ (i)

In the second case, when image is virtual:

$u=-6,\ v=-6n$ and $f=+f$

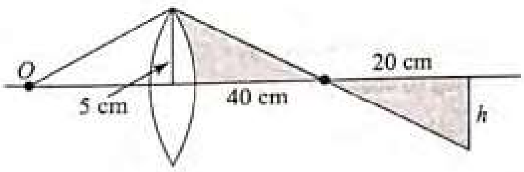
$\therefore \quad \frac{1}{-6n}+\frac{1}{6}=\frac{1}{f}$

$\therefore \quad 1-\frac{1}{n}=\frac{6}{f}$ (ii)

Adding (i) and (ii), we get

$2=\frac{22}{f}$ or $f=11$ cm

47. (c)



$$\frac{\ }{20}=\frac{5}{40}=\frac{1}{8}$$

or $\quad h=\frac{20}{8}\text{ cm}=\frac{5}{2}\text{ cm}=2.5$ cm

48. (d) Clearly, power of system is zero.

$\therefore \quad 0=\frac{1}{20}+\frac{1}{f}-\frac{5}{20f}$

or $\quad -\frac{1}{20}=\frac{15}{20f}$ or $f=-15$ cm

49. (c) $P=(1.5-1)\left(\frac{200}{5}\right)=20$ D

$P'=(1.5-1)\left(\frac{200}{5}\right)=16.67$ D

Decrease in power = 20 D − 16.67 D = 3.33 D

50. (d) $\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

$\frac{1}{v}-\frac{1}{-15}=\frac{1}{f}$

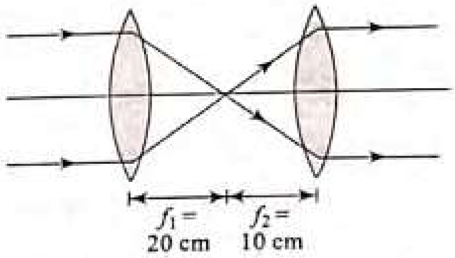
or $\quad \frac{1}{v}-\frac{1}{15}+\frac{1}{10}$ or $\frac{1}{v}=\frac{-2+3}{30}$

or $\quad v=30$ cm

51. (a) $\frac{1}{F}=(\mu-1)\left(\frac{1}{\infty}+\frac{1}{R}\right)+(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)$

$=\frac{\mu_1-\mu_2}{R}$ or $F=\frac{R}{\mu_1-\mu_2}$

52. (b)



As shown in figure the distance between the lenses should be 30 cm.

53. (d) Image will be formed at infinity if object is placed at focus of the lens, i.e., at 20 cm from the lens. Hence,

shift $=25-20=\left(1-\frac{1}{\mu}\right)\mu$

or $\quad 5=\left(1-\frac{1}{1.5}\right)t$ or $t=\frac{5\times1.5}{0.5}=15$ cm

54. (c) $m=\frac{f}{f+u},\ \frac{1}{2}=\frac{f}{f-2}$; or $2f=f-2$ or $f=-2$ m

$|f|=2$ metre. Since the image is virtual as well as diminished, therefore the lens is concave.

55. (b) Power of the system decreases due to separation between the lenses. So, the focal length increases.

56. (d) $\frac{I}{O}=\frac{v}{u}$

$\frac{I}{15}=\frac{-25}{-10}\quad I=15\times2.5\text{ cm}=37.5$ cm

57. (b) Power of combination = (6 − 4)D = 2D

Power $=\frac{100}{f\text{ (in cm)}}$

$2=\frac{100}{f}$ (in cm) or f (in cm) = 50

Since the net power is positive, therefore the combination shall behave like a convex lens.

58. (a) Power of system

$$\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}=\frac{1}{1}+\frac{1}{-0.25}-\frac{0.75}{(1)(-0.25)}$$

$=1-4+3=-3+3=0$

Since power of the system is zero, therefore the incident parallel beam of light will remain parallel after emerging from the system.

59. (b) $P = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

$$0 = \frac{1}{20} - \frac{1}{5} - \frac{d}{20(-5)}$$

$$\frac{d}{100} = \frac{1}{5} - \frac{1}{20} = \frac{4-1}{20} = \frac{3}{20} \quad \text{or} \quad d = 15 \text{ cm}$$

60. (d) Clearly, $2f = 20$ cm

or $f = 10$ cm

Now, $u = -15$ cm, $v = ?$

$f = 10$ cm

$$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$$

or $\frac{1}{v} + \frac{1}{15} = \frac{1}{10}$ or $v = 30$ cm

The change in image distance is (30 – 20) cm, i.e., 10 cm.

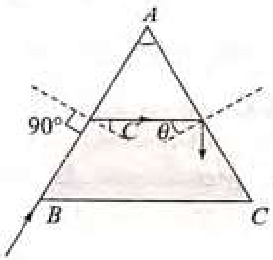
61. (d) $\dfrac{\sin\frac{A+\delta m}{2}}{\sin\frac{A}{2}} = \mu$, But $\frac{A+\delta_m}{2} = i = 45°$

So $\frac{\sin 45°}{\sin(A/2)} = \sqrt{2} \Rightarrow \frac{1}{2} = \sin\frac{A}{2} \Rightarrow A = 60°$

62. (c) From ray diagram

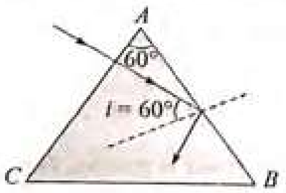
$A = C + \theta$ for TIR at AC

$\theta > C$ so $A > 2C$



63. (d) Critical angle for the material of prism $C = \sin^{-1}\left(\frac{1}{\mu}\right) = 42°$

since angle of incidence at surface AB(60°) is greater than the critical angle (42°) so total internal reflection takes place.



64. (b) $A = 60°$, $\delta_m = 30°$ so $\mu = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\mu = \frac{\sin\left(\frac{60°+30°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°} = \sqrt{2}$$

Also $\mu = \frac{1}{\sin C} \Rightarrow C = \sin^{-1}\left(\frac{1}{\mu}\right) \Rightarrow C = 45°$

65. (c) Given $\delta_m = A$, as $\mu = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\Rightarrow \quad \mu = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = 2\cos\frac{A}{2} \Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

Optical Instruments

66. (a) When final image is formed at infinity.

Length of the tube $= v_o + f_e$

$\Rightarrow 15 = v_o + 3 \Rightarrow v_o = 12$ cm

For objective lens $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

$$\frac{1}{(+2)} = \frac{1}{(+12)} - \frac{1}{u_o} \quad \Rightarrow \quad u_o = -2.4 \text{ cm}$$

67. (c) $m = m_o \times m_e \Rightarrow m = m_o \times \left(1 + \frac{D}{f_e}\right)$

$$\Rightarrow \quad 100 = 10 \times \left(1 + \frac{25}{f_e}\right) \Rightarrow f_e = \frac{25}{9} \text{ cm}$$

68. (d) In this case $|m| = -\frac{f_o}{f_e} = 5$ ... (i)

and length of telescope $= f_o + f_e = 36$ ... (ii)

Solving (i) and (ii), we get $f_e = 6$ cm, $f_o = 30$ cm

69. (c) $f_o = \frac{1}{1.25} = 0.8$ m and $f_e = \frac{1}{-20} = -0.05$ m

$\therefore \quad |L_\infty| = |f_o| - |f_e| = 0.8 - 0.05 = 0.75 \text{ m} = 75 \text{ cm}$

and $|m_\infty| = \frac{f_o}{f_e} = \frac{0.8}{0.05} = 16$

70. (a) $f_o + f_e = 54$ and $\frac{f_o}{f_e} = m = 8 \Rightarrow f_o = 8f_e$

$$\Rightarrow \quad 8f_e = f_e = 54 \Rightarrow f_e = \frac{54}{9} = 6$$

$$\Rightarrow \quad f_o = 8f_e = 8 \times 6 = 48$$

71. (c) Minimum angular separation $\Delta\theta = \frac{1}{R.P.} = \frac{1.22\lambda}{d}$

$$= \frac{1.22 \times 5000 \times 10^{-10}}{2} = 0.3 \times 10^{-6} \text{ rad}$$

72. (b) $m = \dfrac{v_o}{u_o}\left(1+\dfrac{D}{f_e}\right) = m_o\left(1+\dfrac{D}{f_e}\right)$

$\Rightarrow \quad 30 = m_o\left(1+\dfrac{25}{5}\right) = m_o \times 6 \quad \Rightarrow \quad m_o = 5$

73. (a) $m = \dfrac{f_o}{f_e} \Rightarrow \dfrac{100}{f_e} = 50 \Rightarrow f_e = 2$ cm

Normal distance $f_o - f_e = 100 - 2 = 98$ cm

74. (a) For objective lens $\dfrac{1}{f_o} = \dfrac{1}{v_o} - \dfrac{1}{u_o}$

$\Rightarrow \quad \dfrac{1}{v_o} = \dfrac{1}{f_o} + \dfrac{1}{u_o} = \dfrac{1}{4} + \dfrac{1}{-5} = \dfrac{1}{20} \quad \Rightarrow \quad v_o = 20$ cm

Now $\quad M = \dfrac{v_o}{u_o}\left(1+\dfrac{D}{f_e}\right) = \dfrac{20}{5}\left(1+\dfrac{20}{10}\right) = 12$

Problems Based on Mixed Concepts

75. (c) Critical angle between glass and liquid face is

$$\sin\theta_c = \frac{3/2}{\mu} = \frac{3}{2\mu} \qquad \text{(i)}$$

Angle of incidence at face AC is 60°

i.e., $\quad i = 60°$

For total internal reflection to take place,

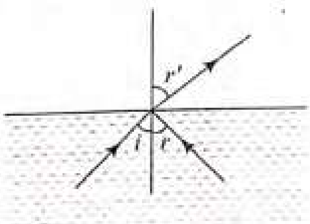
$i > \theta_c$

or $\quad \sin i > \sin\theta_c$

or $\quad \sin 60° > \dfrac{3}{2\mu}$

or $\quad \mu > \sqrt{3}$

76. (c) $\mu = \dfrac{\sin r'}{\sin i}$



But $\quad r' + r = 90°$

or $\quad r' = 90° - r$

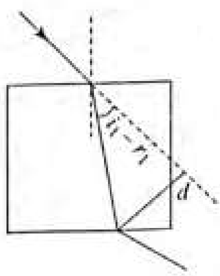
$r = 90° - r$

$\therefore \quad \mu = \dfrac{\cos i}{\sin i}$

or $\quad \dfrac{1}{\sin i_c} = \dfrac{1}{\tan i}$

or $\quad i_c = \sin^{-1}(\tan i)$

77. (c) $\mu_g \sin\theta_c = \mu_l \sin 90°$



or $\quad \mu_g \sin\theta_c = 1$

When water is poured,

$\mu_w \sin r = \mu_g \sin\theta_c$

or $\quad \mu_w \sin r = 1$

Again, $\mu_a \sin\theta = \mu_w \sin r$

or $\quad \mu_a \sin\theta = 1$

or $\quad \sin\theta = 1 \quad$ or $\quad \theta = 90°$

78. (b) $\dfrac{\sin 60°}{\sin r_1} = \sqrt{3}$

or $\quad \sin r_1 = \dfrac{\sin 60°}{\sqrt{3}}$

or $\quad \sin r_1 = \dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{3}} = \dfrac{1}{2}$

or $\quad r_1 = 30°$

Now, $\sin(i_1 - r_1) = \dfrac{d}{5}$

or $\quad d = 5\sin(i_1 - r_1)$

or $\quad d = 5\sin(60° - 30°)$

$= 5\sin 30° = \dfrac{5}{2}$ cm

79. (c) $\sin i_c = \dfrac{1}{\sqrt{2}} \quad i_c = 45°$

Now, $75° = r + 45°$

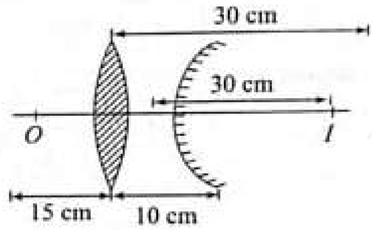
or $\quad r = 30°$

Now, $\dfrac{\sin i}{\sin 30°} = \sqrt{2}$

or $\quad \sin i = \sqrt{2} \times \dfrac{1}{2} = \dfrac{1}{\sqrt{2}}$

$\therefore \quad i = 45°$

80. (b) $\dfrac{1}{v} - \dfrac{1}{-15} = \dfrac{1}{10}$



or $\quad \dfrac{1}{v} = \dfrac{1}{10} - \dfrac{1}{15}$

or $\quad \dfrac{1}{v} = \dfrac{3-2}{30}$

or $\quad v = 30$ cm

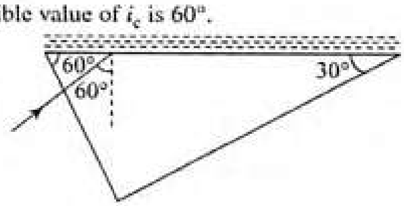
Clearly, the rays coming from the convex lens should fall normally on the convex mirror. In other words, the rays should be directed toward the center of curvature of the convex mirror.

$\therefore \quad 2f = 20$ cm $\quad$ or $\quad f = 10$ cm

81. (b) Clearly, $\quad i_c \le 60°$

So, maximum possible value of i_c is 60°.

Now, $_a\mu_g = \dfrac{1}{\sin i_c}$



$\dfrac{\mu_g}{\mu_l} = \dfrac{1}{\sin i_c}$

or $\quad \mu_l = \mu_g \sin i_c$

$= 1.5\sin 60° = 1.5 \times \dfrac{\sqrt{3}}{2}$

$= 1.5 \times 0.866 = 1.299 = 1.3$

82. (d) If the refractive index of the material of the lens is greater than the refractive index of the surrounding medium, then a concave lens would behave as a concave lens.

83. (a) Only one image will be formed by this lens system because optic axis of both the parts coincide. Two images would have formed if their optic axis were different.

84. (a) In the first case, neither the radii of curvature nor the material of the lens is affected. In the second case,

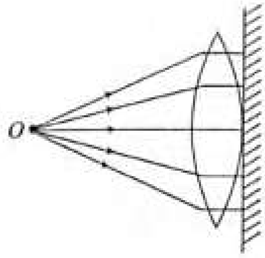
$$\frac{1}{f''} = (\mu - 1)\left(\frac{1}{R}\right) \text{ or}$$

$$\frac{1}{f''} = \frac{1}{2}(\mu - 1)\left(\frac{2}{R}\right) = \frac{1}{2f}$$

or $f'' = 2f$.

85. (c) "O" act as the focal point.

Also, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

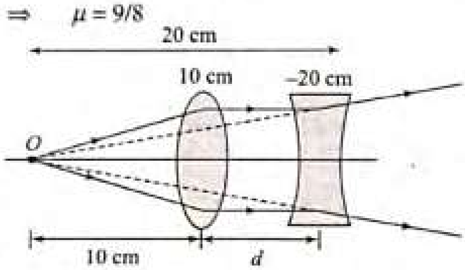


$$\Rightarrow \quad \frac{1}{40} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{10}\right)$$

$$\frac{1}{40} = (\mu - 1)\left(\frac{2}{10}\right)$$

$$\Rightarrow \quad \mu = 9/8$$

86. (a)



From figure, $d = 10$ cm.

87. (d) For A:

$u = -3$ m, $v_1 = ?$, $f = -1$ m

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-3} = \frac{1}{3} - 1 = -\frac{2}{3}$$

or $\quad v_1 = -\frac{3}{2}$

For B:

$$\frac{1}{v_2} = \frac{1}{-1} - \frac{1}{-5}$$

or $\quad \frac{1}{v_2} = \frac{1}{5} - 1 = -\frac{4}{5} \quad$ or $\quad v_2 = -\frac{5}{4}$ m

Now, $v_1 - v_2 = \frac{3}{2} - \left(-\frac{5}{4}\right)$

$$= -\frac{3}{2} + \frac{5}{4} = -\frac{1}{4}\text{m} = -0.25 \text{ m}$$

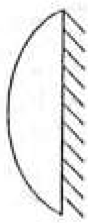
Again, $\frac{I_1}{O} = -\frac{v_1}{u}$

or $\quad I_1 = -\frac{v_1}{u}O = -\left(\frac{-3}{2}\right)\left(\frac{-1}{3}\right) = -1$ m

Again, $\frac{I_2}{O} = -\frac{v_2}{u}$

or $\quad I_2 = -\left(-\frac{5}{4}\right)\left(\frac{1}{-5}\right)2 = -0.5$ m

88. (b) $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_1} + \frac{1}{f_m}$



$$\frac{1}{F} = \frac{2}{f_1} + \frac{2}{f_m}$$

or $\quad \frac{1}{F} = 2(\mu - 1)\left(\frac{1}{R}\right) + \frac{1}{\infty}$

or $\quad F = \frac{R}{2(\mu - 1)}$

Now, $-60 = \frac{R}{2(\mu - 1)}$

or $\quad 120(\mu - 1) = R \qquad$ (i)

Again, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_1} + \frac{1}{f_m}$

$$= \frac{2}{f_1} + \frac{1}{R/2}$$

$$= 2(\mu - 1)\left(\frac{1}{R}\right) + \frac{2}{R} = \frac{2}{R}(\mu - 1 + 1)$$

or $\quad F = \frac{R}{2\mu}$

Now, $-20 = \frac{-R}{2\mu}$

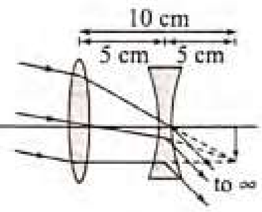
or $\quad 40\mu = R \qquad$ (ii)

Dividing (i) by (ii), we get

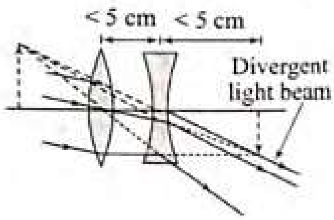
$$\frac{120(\mu - 1)}{40\mu} = \frac{R}{R} = 1$$

or $\quad \mu = \frac{120}{80} = \frac{3}{2} = 1.5$

89. (d) At 5 cm from the lens, the second lens has a virtual object (image of the first lens) at its focal length. The emergent rays are therefore parallel.



When the second lens is closer than 5 cm to the first lens, its object is outside the focal length of the diverging second lens. This produces a virtual image outside $2f$ of the second lens. The emergent rays are therefore *divergent*.



Archives

1. (b) The larger the aperture of the telescope, the more the resolving power.

2. (c) Number of images $= \left(\frac{360}{\theta} - 1\right) = \left(\frac{360}{60} - 1\right) = 5$

3. (d) In optical fibre the concept of total internal reflection is used.

4. (a) Intensity of scattered light $I \propto \frac{1}{\lambda^4}$, since λ_{blue} is least hence sky looks blue.

5. (d) Resolving power is inversely proportional to wavelength of the light

$$R.P. \propto \frac{1}{\lambda} \Rightarrow \frac{(R.P.)_1}{(R.P.)_2} = \frac{\lambda_2}{\lambda_1} = \frac{5}{4}$$

6. (d) Note that the objective of a compound microscope is a convex lens.

7. (b) Large aperture leads to high resolving power of telescope.

8. (c) The options (a), (b) and (d) are correct regarding telecommunication through optical fibres. In optical fibre the concept of total internal reflection is used, it is not interference hence option (c) is incorrect.

9. (c) $$n=\left(\frac{360}{\theta}-1\right)\Rightarrow 3=\left(\frac{360}{\theta}-1\right)\Rightarrow\theta=90^\circ$$

10. (d) For total internal reflection $\theta > C$

$$\Rightarrow \quad \sin\theta > \sin C \Rightarrow \sin\theta > \frac{1}{n}$$

or $$n > \frac{1}{\sin\theta} \Rightarrow n > \frac{1}{\sin 45^\circ} \Rightarrow n > \sqrt{2}$$

$$\Rightarrow \quad n > 1.41$$

11. (a) The effective focal length is given by

$$\frac{1}{F}=\frac{2}{f_1}+\frac{1}{f_m}$$

But

$$\frac{1}{f_1}=(1.5-1)\left(\frac{1}{\infty}-\frac{1}{30}\right)=\frac{1}{60}$$

$$\Rightarrow \quad \frac{2}{f_1}=\frac{1}{30}$$

Again, $R = 30$ cm

$$f_m=\frac{R}{2}=15\text{ cm}$$

Now

$$\frac{1}{F}=\frac{1}{30}+\frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{F}=\frac{1+2}{30}=\frac{3}{30}=\frac{1}{10}$$

$$\Rightarrow \quad F = 10\text{ cm}$$

To have a real image of the size of the object, the object must be placed at the centre of curvature of the equivalent mirror.

So the required distance is 2×10 cm, i.e., 20 cm.

12. (d) According to Brewster's law,

$$n = \tan i_p$$

$$\Rightarrow \quad i_p = \tan^{-1}(n)$$

13. (None)

$$P=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad -5=\left(\frac{1.5}{1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{-5}{0.5}=\frac{1}{R_1}-\frac{1}{R_2}$$

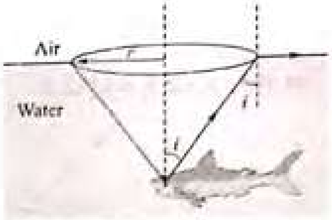
$$\Rightarrow \quad \frac{1}{R_1}-\frac{1}{R_2}=-10$$

Again,

$$P=\left(\frac{1.5}{1.6}-1\right)(-10)$$

$$=\frac{(-0.1)(-10)}{1.6}=\frac{5}{8}\text{D}$$

14. (d) $\tan i_c = \dfrac{r}{h}$



$$\Rightarrow \quad r = h\tan i_c$$

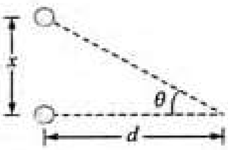
But $\sin i_c = \dfrac{1}{\mu}$

$$\tan i_c=\frac{1}{\sqrt{\mu^2-1}}$$

$$\therefore \quad \frac{r}{h}=\frac{1}{\sqrt{\mu^2-1}}$$

$$\Rightarrow \quad r=\frac{h}{\sqrt{\mu^2-1}}=\frac{12}{\sqrt{\frac{16}{9}-1}}=\frac{36}{\sqrt{7}}\text{ cm}$$

15. (c) Resolving limit $R.L.=\dfrac{1.22\lambda}{a}$



Again resolving limit from figure.

$$R.L.=\frac{x}{d}\Rightarrow d=\frac{x}{RL}$$

$$\Rightarrow \quad d=\frac{x\times a}{1.22\lambda}=\frac{1\times10^{-3}\times3\times10^{-3}}{1.22\times500\times10^{-9}}=5\text{ m}$$

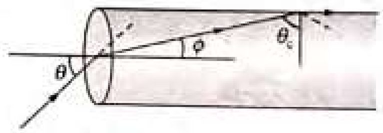
16. (d) $D = (\mu - 1)A$

$$D_1 = (\mu_r - 1)A$$
$$D_2 = (\mu_v - 1)A$$
$$\mu_r < \mu_v$$
$$\therefore \quad D_1 < D_2$$

17. (a) $P = D_1 + D_2 = -15 + 5 = -10$

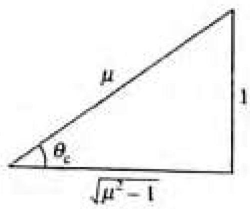
$$\therefore \quad f=\frac{1}{P}=\frac{1}{-10}\text{ m}=-10\text{ cm}$$

18. (d)



If θ_c has to be the critical angle,

$$\theta_c=\sin^{-1}\frac{1}{\mu}$$



But $\theta_c = 90^\circ - \phi,\ \theta_i = \theta$

$$\frac{\sin\theta_i}{\sin\phi}=\mu=\frac{2}{\sqrt{3}}\Rightarrow\frac{\sin\theta}{\cos\theta_c}=\mu.$$

But

$$\cos\theta_c=\frac{\sqrt{\mu^2-1}}{\mu}\ \therefore \sin\theta=\mu\frac{\sqrt{\mu^2-1}}{\mu}=\sqrt{\mu^2-1}$$

$$\therefore \quad \theta=\sin^{-1}\sqrt{\frac{4}{3}-1}=\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

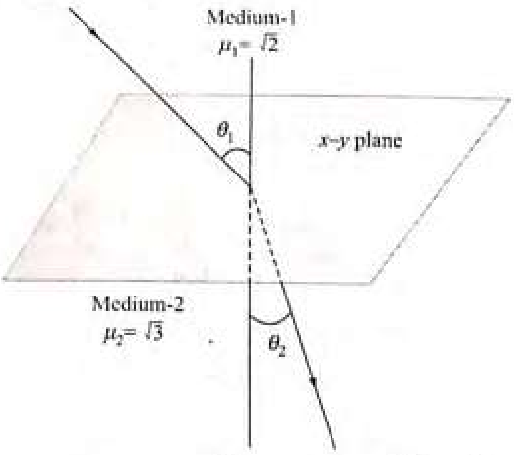
So that θ_c is making total internal reflection.

19. (b) Mirror formula: $\dfrac{1}{v}+\dfrac{1}{-280}=\dfrac{1}{20}\Rightarrow v=\dfrac{280}{15}$ cm

$$v_{image} = -\left(\frac{v}{u}\right)^2 v_{object} \Rightarrow v_{image} = -\left(\frac{280}{15 \times 280}\right)^2 15$$

$$\Rightarrow \quad v_{image} = -\frac{1}{15} \text{ m/s}$$

20. (b) $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$



$$\cos\theta_1 = \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + 100}} = \frac{10}{\sqrt{400}} = \frac{10}{20}$$

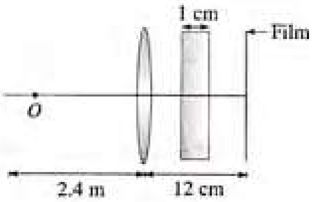
$$\cos\theta_1 = \frac{1}{2} \quad \Rightarrow \theta_1 = 60°$$

$$\sqrt{2}\sin 60° = \sqrt{3}\sin\theta_2 \Rightarrow \sqrt{2} \times \frac{\sqrt{3}}{2} = \sqrt{3}\sin\theta_2$$

$$\Rightarrow \quad \sin\theta_2 = \frac{1}{\sqrt{2}} \quad \Rightarrow \theta_2 = 45°$$

21. (d) $\frac{1}{f} = \frac{1}{12} + \frac{1}{240} = \frac{20+1}{240}$

$$f = \frac{240}{21} \text{m}$$



$$\text{shift} = 1\left(1 - \frac{2}{3}\right) = \frac{1}{3}$$

Now $v' = 12 - \frac{1}{3} = \frac{35}{3}$ cm

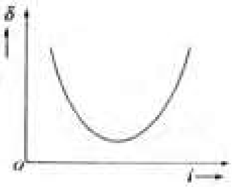
$$\therefore \quad \frac{21}{240} = \frac{3}{35} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left(\frac{3}{7} - \frac{21}{48}\right)$$

$$\frac{5}{u} = \left|\frac{144 - 147}{48 \times 7}\right|$$

$$u = 560 \text{ cm} = 5.6 \text{ m}$$

22. (b) The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is as shown in the following figure.

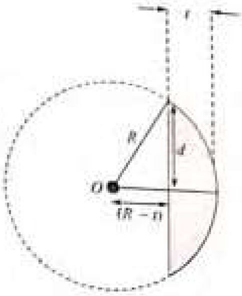


23. (b) $R^2 = d^2 + (R - t)^2$

$$R^2 - d^2 = R^2\left\{1 - \frac{t}{R}\right\}^2$$

$$1 - \frac{d^2}{R^2} = 1 - \frac{2t}{R}$$

$$R = \frac{(3)^2}{2 \times (0.3)} = \frac{90}{6} = 15 \text{cm}$$



$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$$F = 30 \text{ cm}$$

24. (d) Given $\mu = \frac{3}{2}$ (crown glass) and focal length $= f$

Focal length f_1 when lens is placed in liquid of refractive index $\mu_1 = \frac{4}{3}$

Focal length $= f_2$ when lens is placed in liquid of refractive index $\mu_2 = \frac{5}{3}$

Using Lens maker's formula

$$\frac{1}{f_1} = \left(\frac{\mu}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Similarly, $\frac{1}{f_2} = \left(\frac{\mu}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

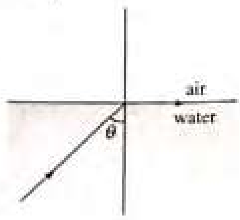
$$\Rightarrow \quad \frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{-1}{10}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Hence, $f_1 = 4f$ and $f_2 = -5f$.

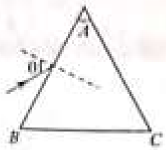
25. (d) For critical angle we can write, $\sin\theta = \frac{1}{\mu}$

The refractive index (μ) of the medium inversely proportional to the wavelength of the light. μ is less for the greater wavelength (i.e., lesser frequency).



So, θ will be more for lesser frequency of light. Hence the spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

26. (a) $\sin\theta = \mu \sin r_1$



$$\Rightarrow \quad \sin r_1 = \frac{\sin\theta}{\mu}$$

$$\Rightarrow \quad r_1 = \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

$$r_2 = A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

$$\Rightarrow \quad r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$A - \sin^{-1}\left(\frac{\sin\theta}{\mu}\right) < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow \quad A - \sin^{-1}\left(\frac{1}{\mu}\right) < \sin^{-1}\left(\frac{\sin\theta}{\mu}\right)$$

$$\Rightarrow \quad \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right) < \frac{\sin\theta}{\mu}$$

$$\Rightarrow \quad \mu\left(\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right) < \sin\theta$$

$$\Rightarrow \quad \sin^{-1}\left(\mu\sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right) < \theta$$

27. (c) Magnifying power $MP = \left(\frac{\beta}{\alpha}\right) = 20$

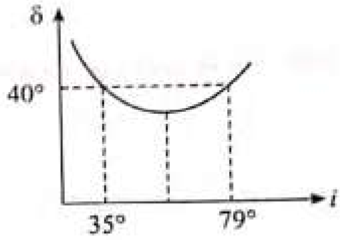
The object is of height 10 m which is very far away and when image also very far away, then height of image h.

$$\beta = \frac{h}{x} \text{ and } \alpha = \frac{10\text{ m}}{x}$$

Hence $\frac{\beta}{\alpha} = \frac{h}{10}$ or $\frac{h}{10} = 20$

Hence $h = 20 \times 10$ m or 20 times taller.

28. (a) Given $i = 35°$, $\delta = 40°$ and $e = 79°$.



From the given data, $\delta = i + e - A$

$$40° = 35° + 79° - A \Rightarrow A = 74°$$

$$\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{40° + 74°}{2}\right)}{\sin 37°} = \frac{\sin 57°}{\sin 37°} = 1.44$$

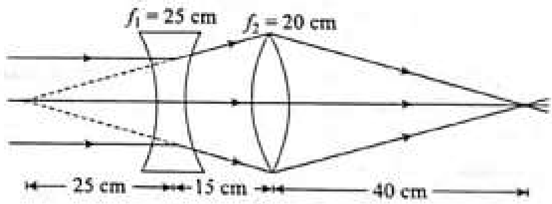
Nearest value is 1.5.

29. (c) As parallel beam incident on diverging lens if forms virtual image at $v_1 = -25$ cm from the diverging lense which works as an object for the converging lense ($f = 20$ cm)

So for converging lens $u = -40$ cm, $f = 20$ cm

$\therefore$ Final image $\frac{1}{v} - \frac{1}{-40} = \frac{1}{20}$

$v = 40$ cm from converging lens.



CHAPTER 27: WAVE OPTICS

Concept Application Exercise 27.1

1. If a_1, a_2 are amplitudes of the superposing waves and I_1, I_2 are intensities, then

$$\beta = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \quad \text{or} \quad \frac{a_1}{a_2} = \sqrt{\beta}$$

$\therefore \quad I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$

and $\quad I_{min} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 - a_2)^2$

$$\Rightarrow \quad \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a_1+a_2)^2 - (a_1-a_2)^2}{(a_1+a_2)^2 + (a_1-a_2)^2}$$

$$= \frac{4a_1a_2}{2(a_1^2 + a_2^2)} = \frac{2\left(\frac{a_1}{a_2}\right)}{\left(\frac{a_1^2}{a_2^2} + 1\right)} = \frac{2\sqrt{\beta}}{(1+\beta)}$$

2. The resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

(a) The sources are said to be coherent if they have constant phase difference between them. The intensity will be maximum when $f = 2n p$; the sources are in same phase.

Thus, I_{max} $I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

Similarly, for n identical waves,

$$I_{max} = (\sqrt{I_0} + \sqrt{I_0} + \cdots)^2 = n^2 I_0$$

(b) The incoherent sources have phase difference that varies randomly with time.

Thus, $[\cos\phi]_{av} = 0$

Hence, $I = I_1 + I_2$

Hence, for n identical waves, $I = I_0 + I_0 + \ldots = nI_0$

3. The light from the flashlights consists of many different wavelengths (that is why it is white) with random time difference between the light waves; therefore there is no coherence between the two sources; and no possibility of an interference pattern.

4. Condition for maximum interference in the reflected light, in case of thin film interference, can be expressed as

$2\mu t \cos r = (2n-1)\,\lambda/2,\ n = 1, 2, \ldots$

[for plane parallel films]

where μ is the refractive index of film relative to the surrounding:

t is the thickness of film; and

r is the angle of refraction.

For normal incidence, $r = 0$

$\therefore \quad 2\mu t = (2n-1)\lambda/2,\ n = 1, 2, 3, \ldots$

For minimum thickness, $n = 1$

$$\therefore \quad 2\mu t_{min} = \lambda/2 \quad \text{or} \quad t_{min} = \frac{\lambda}{4\mu}$$

Given: $\lambda = 5360$ Å and $\mu = 1.34$

$$\therefore \quad t_{min} = \frac{5360}{4 \times 1.34} \quad \text{or} \quad t_{min} = 1000$$

5. Conditions for maximum and minimum intensity in reflected light, in case of thin film interference, are

Maxima: $2\mu t = (2n-1)\lambda/2,\ n = 1, 2, \ldots$

Minima: $2\mu t = n\lambda,\ n = 0, 1, 2, 3, \ldots$

Given: $\mu = 1.5$, $t = 3000$ Å $= 3000 \times 10^{-10}$ m

(normal incidence)

Maxima: $2\mu t = (2n-1)\lambda/2$

$\therefore \quad 2 \times 1.5 \times (3000 \times 10^{-10}) = (2n-1)\lambda/2$

$$\text{or} \quad \lambda = \frac{4 \times 1.5 \times 3000 \times 10^{-10}}{2n-1} \quad \therefore \quad \lambda = \frac{18000}{2n-1} \times 10^{-10}\ \text{m}$$

For $n = 1$, $\lambda = 18000$ Å

$n = 2$, $\lambda = 6000$ Å

$n = 3$, $\lambda = 3600$ Å

and so on

Only $\lambda = 6000$ Å for $n = 2$ lies in the visible range. Intensity will be maximum for $\lambda = 6000$ Å which belongs to the oranges–red part of the visible spectrum.

Minima: $2\mu t \cos r = n\lambda$

$2 \times 1.5 \times (3000 \times 10^{-10}) = n\lambda$

$$\therefore \quad \lambda = \frac{9000 \times 10^{-10}}{n} = \frac{9000}{n}\ \text{Å}$$

In this case, only $\lambda = 4500$ Å for $n = 2$ lies in the visible range. Intensity will be minimum for $\lambda = 4500$ Å which falls in the violet–blue part of the visible spectrum.

When observed in white light, the red–orange part will be strongly reflected and the violet–blue part will be quite reduced in intensity. Hence, color of the pot will appear to be orange–red.

6. We must find which colours in the visible region, having vacuum wavelengths from 400 nm (violet) to 700 nm (red), will interfere constructively and which destructively. From the problem, with $n_1 = 1$ (air) maximum constructive interference occurs for

$$\lambda_1 = \frac{2(n_2/n_1)t}{m + \frac{1}{2}} = \frac{(900\ \text{nm})}{m + \frac{1}{2}} \quad (m = 0, 2, \ldots)$$

Only the value corresponding to $m = 1$, that is, $\lambda_1 = 450$ nm (violet) is in the visible range.

We infer that the red–orange–yellow end of the spectrum will be strongly reflected, while the violet–blue end will be greatly diminished in intensity as compared with the illuminating white light.

Concept Application Exercise 27.2

1. The fringe width is given by $\beta = \dfrac{\lambda D}{d}$

If the extent of field of vision is ℓ.

$\ell = N_1\beta_1$

where N_1 is the number of fringes formed with wavelength ℓ.

As the field of vision is fixed, $\ell = N_1\beta_1 = N_2\beta_2$

$$N_1\left(\frac{\lambda_1 D}{d}\right)=N_2\left(\frac{\lambda_2 D}{d}\right)$$

$$N_1\lambda_1=N_2\lambda_2$$

Thus, $N_2=\frac{N_1\lambda_1}{\lambda_2}=\frac{60\times 5890}{5460}=64.72=65$

2. Angular fringe width is given by

$$\beta\theta=\frac{\lambda}{d}$$

$$\beta_\theta^{\text{water}}=\frac{\lambda_{\text{air}}}{d}, \beta_\theta^{\text{water}}=\frac{\lambda_{\text{water}}}{d}$$

$$\frac{\beta_\theta^{\text{air}}}{\beta_\theta^{\text{water}}}=\frac{\lambda_{\text{air}}}{\lambda_{\text{water}}}=\frac{n_{\text{air}}}{n_{\text{water}}}=\frac{3}{4}$$

$\therefore \quad \beta_\theta^{\text{water}}=\frac{3}{4}\beta_\theta^{\text{air}}=0.40°\times\frac{3}{4}=0.30°$

3. The angular position of maxima is given by

$$d\sin\theta=n\lambda$$

$$d\theta=n\lambda$$

[For small θ, $\sin\theta=\theta$]

The angular separation of two adjacent maxima is

$$\Delta\theta=\frac{\lambda}{d}$$

Let angular separation be 10% greater for wavelength λ'.

$\therefore \quad \frac{1.10\lambda}{d}=\frac{\lambda'}{d}$

Then, $\lambda'=1.10\lambda=(1.10\times 589)=648$ nm

4. $\beta_1=\frac{\lambda_1 D}{d}=\frac{6000\times10^{-10}\times1}{10^{-3}}=0.6$ mm

$\beta_2=\frac{\lambda 2D}{d}=0.45$ mm

Let n_1^{th} maxima of λ_1 and n_2^{th} maxima of λ_2 coincide at a position y.

Then, $y=n_1P_1=n_2P_2=$ LCM of β_1 and β_2

$\Rightarrow \quad y=$ LCM of 0.6 mm and 0.45 mm

or $\quad y=1.8$ mm

At this point, 3rd maxima for 6000 Å and 4th maxima for 4500 Å coincide.

5. $\Delta x=\frac{yd}{D}=9\times10^{-4}\times1\times10^{-3}$ m $=900$ nm

For minima, $\Delta x=(2n-1)\lambda/2$

$\Rightarrow \quad \lambda=\frac{2\Delta x}{(2n-1)}=\frac{1800}{(2n-1)}=\frac{1800}{1},\frac{1800}{3},\frac{1800}{5},\frac{1800}{7},\ldots$

Of these, 600 nm and 360 nm lie in the visible range. Hence, these will be the missing lines in the visible spectrum.

6. As derived earlier, the total fringe shift $=\frac{w}{\lambda}(\mu-1)t$

As each fringe width $=w$,

The number of fringes that will shift is

$$\frac{\text{Total fringe shift}}{\text{Fringe width}}=\frac{\frac{w}{\lambda}(\mu-1)t}{w}=\frac{(\mu-1)t}{\lambda}$$

$$=\frac{(1.6-1)\times1.8\times10^{-5}\,m}{600\times10^{-9}\,m}=18$$

Concept Application Exercise 27.3

1. (i) Here, $\lambda=5.4\times10^{-5}$ cm $=5.4\times10^{-7}$ m

$a=0.12$ m $=0.12\times10^{-2}$ m, $D=2.7$ m

Now, $\beta_0=\frac{2D\lambda}{a}$

$$=\frac{2\times2.7\times5.4\times10^{-7}}{0.12\times10^{-2}}=2.43\times10^{-9}\text{ m}=2.43\text{mm}$$

(ii) When apparatus is immersed in liquid:

$$\beta_0'=\frac{2D\lambda'}{\alpha}=\frac{2D\lambda}{\alpha\mu}=\frac{\beta_0}{\mu}=\frac{2.43}{1.35}=1.8\text{mm}$$

2. Here, $\lambda=5890$ Å $=5890\times10^{-10}$ m

$a=0.25$ mm $=0.25\times10^{-3}$ m

Angular separation between central and first order maximum.

$$\theta_1'=\frac{3\lambda}{2a}=\frac{3\times5890\times10^{-10}}{2\times0.25\times10^{-9}}=3.534\times10^{-3}\text{ rad}$$

3. Here, $\lambda=600$ nm $=600\times10^{-9}$ m; $D=0.8$ m

and $\quad y_2'=15$ mm $=15\times10^{-3}$ m

For second order maximum,

$$\frac{5\lambda}{2a}=\frac{y_2'}{D}$$

or $\quad a=\frac{5\lambda D}{2y_2'}=\frac{5\times600\times10^{-9}\times0.8}{2\times15\times10^{-9}}=8\times10^{-5}$ m

4. The intensity of unpolarised light transmitted through the polarising sheets arranged with their planes of transmission making an angle of 30° is given by

$$I'=\frac{I_0}{2}\cos^2 30°=\frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^2=\frac{I_0}{2}\times\frac{3}{4}$$

Therefore, intensity of transmitted light through the third polarising sheet,

$$I''=I'\cos^2 30°=\frac{I_0}{2}\times\frac{3}{4}\times\left(\frac{\sqrt{3}}{2}\right)^2=\frac{I_0}{2}\times\left(\frac{3}{4}\right)^2$$

Finally, intensity of light transmitted through the fourth polarising sheet,

$$I=I''\cos^2 30°=\frac{I_0}{2}\times\left(\frac{3}{4}\right)^2\times\left(\frac{\sqrt{3}}{4}\right)^2$$

$$=\frac{I_0}{2}\times\left(\frac{3}{4}\right)^3=\frac{27I_0}{128}\quad\text{or}\quad\frac{I}{I_0}=\frac{27}{128}$$

5. Now, $\tan p=\mu=1.33$ or $p=53.06°$

Exercises

Huygens's Wave Principle and Interference of Light

1. (c) A slit would give divergent; a biprism would give double; a glass slab would give a parallel wavefront. Edge is downward.
2. (b) P to Q: convergence increasing; Q to R: direction changing.
3. (b) Velocity of light is perpendicular to the wavefront.

4. (c) Velocity of wave in medium (μ) is less than that in air. Hence, wavefront reaches earlier at P through air.

5. (c) Here, direction of light is given by normal vector $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Angle made by the $\vec{n}$ with y-axis is given by

$$\cos\beta = \frac{2}{\sqrt{1^2+2^2+3^2}} = \frac{2}{\sqrt{14}}$$

6. (b) Different wavelengths would correspond to different frequencies. Lights of different intensities can give coherence even if contrast is poor.

7. (b) Effective path difference is $\mu_1 L_1 - \mu_2 L_2$.

8. (a) Detector receives both the direct as well as the reflected waves. Distance between two consecutive maxima = $\lambda/2$.

For 14 maxima, distance $= 14 \times \dfrac{\lambda}{2} = 0.14$ m

$\therefore \quad \lambda = 0.02$ m

$$v = \frac{c}{\lambda} = \frac{3\times 10^8}{0.02} = 1.5\times 10^{10}\text{ Hz}$$

9. (b) $I(\phi) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

Here, $I_1 = I$ and $I_2 = 4I$

At point A,

$$\phi = \frac{\pi}{2}$$

$\therefore \quad I_A = I + 4I = 5I$

At point B,

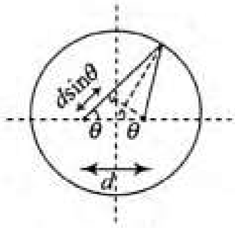
$$\phi = \pi$$

$\therefore \quad I_B = I + 4I - 4I = I$

$\therefore \quad I_A - I_B = 4I$

10. (a) Condition for maxima is

$d\sin\theta = n\lambda$



$$\sin\theta = \frac{n\lambda}{d} = n\left(\frac{0.50}{2.0}\right) = 0.25n$$

As $\sin\theta$ lies between -1 and 1, so we wish to find all values of n for which

$$|0.25n| \le 1$$

These values are $-4, -3, -2, -1, 0, +1, +2, +3, +4$. For each of these, there are two different values of θ except for -4 and $+4$. A single value of θ, $-90°$ and $+90°$, is associated with $n = -4$ and $n = +4$, respectively. Thus, there are 16 different angles in all and therefore 16 maxima.

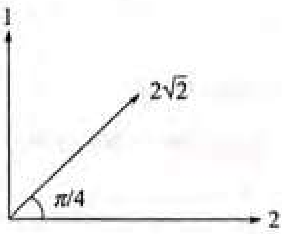
11. (d) Resultant intensity $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$

For maximum I_R, $\varphi = 0°$

$$\Rightarrow \quad I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

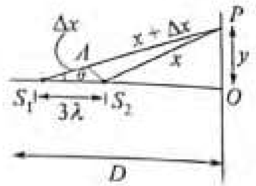
12. (c) Here,

$$a_x = 2 + 2\sqrt{2}\cos\frac{\pi}{4}$$
$$= 4 \text{ unit}$$
$$a_y = 2\sqrt{2}\sin 45° + 1$$
$$= 3 \text{ unit}$$
$$A = \sqrt{A_x^2 + a_y^2}$$
$$= \sqrt{4^2+3^2} = 5 \text{ unit}$$



13. (b) $\Delta x = n\lambda$

in this case path difference is $d\cos\theta$.



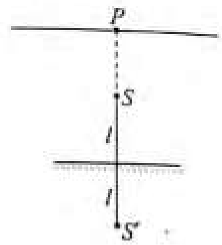
So $\quad n\lambda = \lambda = 3\lambda\cos\theta$

$$\Rightarrow \quad \cos\theta = \frac{1}{3}$$

$$\Rightarrow \quad \frac{D}{\sqrt{D^2+y^2}} = \frac{1}{3}$$

$$\Rightarrow \quad 3D = \sqrt{D^2+y^2} \quad \Rightarrow \quad y = \sqrt{8}\,D = 2\sqrt{2}\,D$$

14. (c) The path difference at point P is



$$\Delta x = S'P + \frac{\lambda}{2} - SP$$
$$= 2l + \frac{\lambda}{2} = m\lambda$$

or $\quad 2l = m\lambda - \dfrac{\lambda}{2}$

$$\therefore \quad l = \frac{m\lambda}{2} - \frac{\lambda}{4} \qquad \text{...(i)}$$

When the mirror is shifted downward a distance, y_0

$$\Delta x' = 2(l + l_0) + \frac{\lambda}{2} = (m+1)\lambda$$

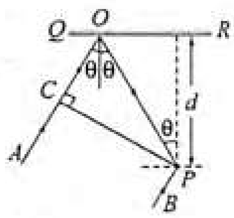
or $\quad l + l_0 = (m+1)\dfrac{\lambda}{2} - \dfrac{\lambda}{4}$

or $\quad m\dfrac{\lambda}{2} - \dfrac{\lambda}{4} + l_0 = m\dfrac{\lambda}{2} + \dfrac{\lambda}{2} - \dfrac{\lambda}{4}$

or $\quad l_0 = \dfrac{\lambda}{2} = \dfrac{600}{2} = 300\,\text{nm}$

15. (b) $\because PR = d \Rightarrow PO = d\sec\theta$ and $CO = PO\cos 2\theta = d\sec\theta\cos 2\theta$ is

Path difference between the two rays



$$\Delta = CO + PO$$
$$= (d\sec\theta + d\sec\theta\cos 2\theta)$$

Phase difference between the two rays is

$\phi = \pi$ (One is reflected, while another is direct)

Therefore condition for constructive interference should be

$$\Delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots$$

or $\quad d\sec\theta(1+\cos 2\theta) = \dfrac{\lambda}{2}$

or $\quad \dfrac{d}{\cos\theta}(2\cos^2\theta) = \dfrac{\lambda}{2} \quad \Rightarrow \quad \cos\theta = \dfrac{\lambda}{4d}$

Young's Double Slit Experiment

16. (d) Let intensity of one slit be I.

For maxima,

$$\Delta\phi = 0$$

$$\Rightarrow \quad I_0 = I + I + 2\sqrt{II}\cos(0) \quad \Rightarrow \quad I = \frac{I_0}{4}$$

17. (a) Path difference, $\Delta x = \dfrac{yd}{D}$

Here, $y = \frac{5\lambda}{2}$

and $D = 10d = 50\lambda$ (as $d = 5\lambda$)

So,

$$\Delta x = \left(\frac{5\lambda}{2}\right)\left(\frac{5\lambda}{50\lambda}\right) = \frac{\lambda}{4}$$

Corresponding phase difference will be

$$\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x) = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) = \frac{\pi}{2}$$

or $\frac{\phi}{2} = \frac{\pi}{4}$

$\therefore$ $I = I_0 \cos^2\left(\frac{\phi}{2}\right) = I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$

18. (d) The contrast between bright and dark fringes is determined by intensity ratio.

19. (a) $I_{\min} \propto (A_1 - A_2)^2$

$I_{\min} \propto (2a - a)^2$

Clearly, the intensity of minima increases. Again,

$I_{\max} \propto (A_1 + A_2)^2$

$I_{\max} \propto (2a + a)^2$

Clearly, the intensity of maxima increases.

20. (d) $\beta_w = \frac{\lambda D}{\mu d}$

We need to increase $\beta \Rightarrow D$ increases; d decreases.

21. (a) $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right) \Rightarrow I_0 = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

$\Rightarrow$ $\cos\left(\frac{\phi}{2}\right) = \frac{1}{2}$ or $\frac{\phi}{2} = \frac{\pi}{3}$

or $\phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right)\Delta x$

or $\frac{1}{3} = \left(\frac{1}{\lambda}\right) y \frac{d}{D}$ $\left(\Delta x = \frac{yd}{D}\right)$

$\therefore$ $y = \frac{\lambda}{3 \times \frac{d}{D}} = \frac{6 \times 10^{-7}}{3 \times 10^{-4}} = 2 \times 10^{-3}\text{ m} = 2\text{ mm}$

22. (c) In the first case,

$I = I_0 + I_0 + 2I_0 \cos 0°$

or $I = 4I_0$

In the second case,

$I' = I_0 + I_0 = 2I_0$ $\therefore$ $\frac{I}{I'} = \frac{4I_0}{2I_0} = \frac{2}{1}$

23. (c) Fringe width, $\beta \propto \lambda$. Therefore, λ and hence β will decrease 1.5 times when immersed in the liquid. The distance between central maxima and 10th maxima is 3 cm in vacuum. When immersed in the liquid, it will reduce to 2 cm. Position of central maxima will not change while 10th maxima will be obtained at $y = 4$ cm.

24. (c) Intensity at the center will be zero if path difference is $\lambda/2$. That is,

$(\mu - 1)t = \frac{\lambda}{2}$ or $t = \frac{\lambda}{2(\mu - 1)}$

25. (b) For maximum intensity on the screen,

$d \sin\theta = n\lambda$

or $\sin\theta = \frac{n\lambda}{d} = \frac{(n)(2000)}{(7000)} = \frac{n}{3.5}$

$\sin\theta \geq 1 \Rightarrow n = 0, 1, 2, 3$ only

Thus, only seven maxima can be obtained on both sides of the screen.

26. (a) Fringe width, $\beta = \lambda D/d$. When the apparatus is immersed in a liquid, λ and hence β is reduced μ (refractive index) times.

$10\beta' = (5.5)\beta$

or $10\lambda'\left(\frac{D}{d}\right) = (5.5)\frac{\lambda D}{d}$

or $\frac{\lambda}{\lambda'} = \frac{10}{5.5} = \mu$ or $\mu = 1.8$

27. (c) $30\beta = n\beta'$

$\Rightarrow$ $30\frac{D \times 4000}{d} = n\frac{D \times 6000}{d} \Rightarrow n = 20$

28. (a) $y = \frac{d}{2} = \frac{(2n-1)D\lambda}{2d}$

$\Rightarrow$ $\lambda = \frac{d^2}{D(2n-1)}$, $n = 1, 2, 3, \ldots$

$$\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D}, \ldots$$

29. (a) White fringe is formed at the center of screen. Position of central fringe will remain unchanged on moving the screen.

30. (a) $y_n(\max) = n\frac{D\lambda}{d}$

Here, $y_n(\max) = d/2$

So, $n\frac{D\lambda}{d} = \frac{d}{2}$ or $n = \frac{d^2}{2\lambda D}$

31. (b) Fringe width, $\beta = \frac{\lambda D}{d}$, i.e., $\beta \propto \lambda$

When the wavelength is decreased from 600 nm to 400 nm, fringe width will also decrease by a factor of 4/6 or 2/3 or the number of fringes in the same segment will increase by a factor of 3/2. Therefore, number of fringes observed in the same segment is $12 \times \frac{3}{2} = 18$.

32. (a) Path difference due to slab should be integral multiple of λ. Hence,

$\Delta x = n\lambda$

or $(\mu - 1)t = n\lambda$, $n = 1, 2, \ldots$

or $t = \frac{n\lambda}{\mu - 1}$

For minimum value of t, $n = 1$.

$\therefore$ $t = \frac{n\lambda}{\mu - 1} = \frac{\lambda}{1.5 - 1} = 2\lambda$

33. (b) Angular separation is λ/d.

For angular separation to be 10% greater, λ should be 10% greater.

$\therefore$ New wavelength $= \left(589 + \frac{589}{10}\right)$ nm

$= (589 + 58.9)$ nm

$= 647.9$ nm (≈ 648 nm)

34. (a) $I \propto 4a^2 \cos^2 \frac{\phi}{2}$

In the first case, $\phi = 2\pi$

$\therefore \quad I' \propto 4a^2$

In the second case, $\phi = \frac{2\pi}{3}$

$\therefore \quad I' \propto 4a^2 \cos^2 \frac{2\pi}{3} \quad$ or $\quad I' \propto a^2$

$\frac{I'}{I} = \frac{1}{4} \quad$ or $\quad I' = \frac{I}{4}$

35. (a) The experimental set-up is in a liquid, therefore the wavelength of light will change

$$\lambda_{\text{liquid}} = \frac{\lambda_{\text{air}}}{\mu} = \frac{6300}{1.33} = \frac{6300 \times 10^{-10}}{1.33} \text{ m}$$

Fringe width,

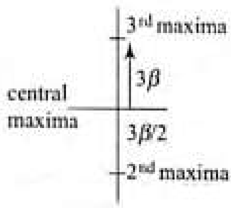
$$\beta = \frac{\lambda_{\text{liquid}} D}{d} = \frac{\lambda_{\text{air}} D}{\mu d} = \frac{6300 \times 10^{-10}}{1.33} \times \frac{1.33}{10^{-3}}$$

$= 6.3 \times 10^{-4}$ m

36. (d) $d = 0.5$ mm and $D = 0.5$ m

Separation $= 3\beta + 1.5\beta = 4.5\beta$

$$= 4.5 \times \frac{\lambda D}{d} = 2.25 \text{ mm}$$



37. (c) In Young's double slit experiment,

$\sin\theta = \theta = y/D$

So, $\Delta y/D$ and hence angular fringe width

$\theta_0 = \Delta\theta$ (with $\Delta y = \beta$) will be

$$\theta_0 = \frac{\beta}{D} = \frac{D\lambda}{d} \times \frac{1}{D} = \frac{\lambda}{d}$$

$\Rightarrow \quad \theta_0 = 1^\circ = \left(\frac{\pi}{180}\right)$ rad, and $\lambda = 6 \times 10^{-7}$ m

or $\quad d = \frac{\lambda}{\theta_0} = \frac{180}{\pi} \times (6 \times 10^{-7}) = 3.44 \times 10^{-5}$ m

or $\quad d = 0.0344$ mm

38. (a) As a bright fringe is formed in front of slit, therefore

$d/2$ = integral multiple of fringe width

$$\frac{d}{2} = \frac{n\lambda D}{d} \quad \Rightarrow \quad n = \frac{d^2}{2\lambda D}$$

39. (a) According to the question,

shift = 5 fringe widths

$$\Rightarrow \quad \frac{(\mu - 1)tD}{d} = \frac{5\lambda D}{d}$$

$$\therefore \quad t = \frac{5\lambda}{\mu - 1} = \frac{25000}{1.5 - 1} = 50{,}000 \text{ Å} = 5 \times 10^{-6} \text{ m}$$

Diffraction and Polarization

40. (c) For first minima $\theta = \frac{\lambda}{a}$ or $a = \frac{\lambda}{\theta}$

$$\therefore \quad a = \frac{6500 \times 10^{-8} \times 6}{\pi} \qquad \left(\text{As } 30^\circ = \frac{\pi}{6} \text{ radian}\right)$$

$= 1.24 \times 10^{-4}$ cm $= 1.24$ microns

41. (a) Using $d \sin\theta = n\lambda$, for $n = 1$

$$\sin\theta = \frac{\lambda}{d} = \frac{550 \times 10^{-9}}{0.55 \times 10^{-3}} = 10^{-3} = 0.001 \text{ rad}$$

42. (a) For single slit diffraction pattern $d \sin\theta = \lambda$ (d = slit width)

Angular width $= 2\theta = 2\sin^{-1}\left(\frac{\lambda}{d}\right)$

It is independent of D i.e. distance between screen and slit.

43. (a) For secondary maxima $d \sin\theta = \frac{5\lambda}{2}$

$$\Rightarrow \quad d\theta = d.\frac{x}{D(\approx f)} = \frac{5\lambda}{2}$$

$$\Rightarrow \quad 2x = \frac{5\lambda f}{d} = \frac{5 \times 0.8 \times 10^{-7}}{4 \times 10^{-4}} = 6 \times 10^{-3} \text{ m} = 6 \text{ mm}$$

44. (c) Angular width $= \frac{2\lambda}{d} = \frac{2 \times 6000 \times 10^{-10}}{12 \times 10^{-5} \times 10^{-2}} = 1$ rad

45. (d) For n^{th} secondary maxima path difference

$$d \sin\theta = (2n+1)\frac{\lambda}{2} \Rightarrow a \sin\theta = \frac{3\lambda}{2}$$

46. (b) Angular width $\beta = \frac{2\lambda}{d}$

$$\Rightarrow \quad \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{\beta}{\frac{70}{100}\beta} = \frac{6000}{\lambda_2} \Rightarrow \lambda_2 = 4200 \text{ Å}$$

47. (c) It is given that $r_4 = \sqrt{4b\lambda_1}$ and $r_5 = \sqrt{5b\lambda_2}$ are equal. Therefore $\sqrt{4b\lambda_1} = \sqrt{5b\lambda_2}$

or $\quad 4b\lambda_1 = 5b\lambda_2 \quad$ or $\quad \frac{\lambda_1}{\lambda_2} = \frac{5}{4}$

48. (a) $\frac{3\lambda}{2\alpha} = \frac{\lambda_R}{\alpha} \quad \Rightarrow \quad \lambda = \frac{2}{3}\lambda_R = \frac{2}{3} \times 600 = 400$ nm

49. (a) It magnitude of light vector varies periodically during its rotation, the tip of vector traces an ellipse and light is said to be elliptically polarised. This is not in nicol prism.

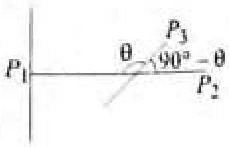
50. (c) If an unpolarised light is converted into plane polarised light by passing through a polaroid, its intensity becomes half.

51. (c) Intensity of polarized light from first polarizer $= \frac{100}{2} = 50\%$

$$I = 50\cos^2 60^\circ = \frac{50}{2} = 12.5\%$$

52. (b) $I' = \frac{I}{2}\cos^2\theta = \frac{I}{6}$ or $\cos\theta = \frac{1}{\sqrt{3}}$ $\therefore \theta = 55^\circ$

53. (a) No light is emitted from the second polaroid, so P_1 and P_2 are perpendicular to each other



Let the initial intensity of light is I_0. So intensity of light after transmission from first polaroid $= \frac{I_0}{2}$.

Intensity of light emitted from P_3 $I_1 = \frac{I_0}{2}\cos^2\theta$

Intensity of light transmitted from last polaroid i.e. from

$$P_2 = I_1 \cos^2(90^\circ - \theta) = \frac{I_0}{2}\cos^2\theta.\sin^2\theta$$

$$= \frac{I_0}{8}(2\sin\theta\cos\theta)^2 = \frac{I_0}{8}\sin^2 2\theta$$

54. (a) Specific rotation (α)

$$= \frac{\theta}{lc} \Rightarrow c = \frac{\theta}{\alpha l} = \frac{0.4}{0.01 \times 0.25} = 160\,\text{kg}/\text{m}^3$$

Now percentage purity of sugar solution

$$= \frac{160}{200} \times 100 = 80\%$$

55. (d) If I is the final intensity and I_0 is the initial intensity then

$$I = \frac{I_0}{2}(\cos^2 30^\circ)^5 \text{ or } \frac{I}{I_0} = \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^{10} = 0.12$$

$$= 12\%$$

Problems Based on Mixed Concepts

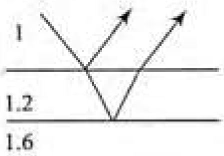
56. (d) Path difference $= d \sin\phi + d\sin\theta$

For maxima, $\Delta x = m\lambda$

$$\Rightarrow \sin\phi + \sin\theta = \frac{m\lambda}{d}$$

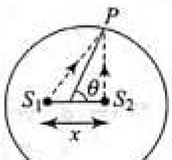
57. (b) $2\mu t = n\lambda \Rightarrow t = \dfrac{n\lambda}{2\mu}$

$$\Rightarrow \quad t = \frac{\lambda}{2\mu} = 200\text{ nm}$$



58. (a) Path difference at P is

$$\Delta x = 2\left(\frac{x}{2}\cos\theta\right) = x\cos\theta$$



For intensity to be maximum,

$$\Delta x = n\lambda \qquad (n = 0, 1, 2, 3, \ldots)$$

or $\quad x\cos\theta = n\lambda$

or $\quad \cos\theta = \dfrac{n\lambda}{x} \geq 1$

$\therefore \quad n \geq \dfrac{x}{\lambda}$

Substituting $x = 5\lambda$, we get

$n \geq 5$ or $n = 1, 2, 3, 4, 5, \ldots$

Therefore, in all four quadrants there can be 20 maxima. There are more maxima at $\theta = 0^\circ$ and $\theta = 180^\circ$. But $n = 5$ corresponds to $\theta = 90^\circ$ and $\theta = 270^\circ$ which are coming only twice while we have multiplied it four times. Therefore, total number of maxima are still 20, i.e., $n = 1$ to 4 in four quadrants (total 16) plus more at $\theta = 0^\circ, 90^\circ, 180^\circ$ and 270°.

59. (a) Consider light to be incident at near normal incidence. We wish to cause destructive interference between rays r_1 and r_2 so that maximum energy passes into the glass. A phase change of $\lambda/2$ occurs in each ray because at both the upper and lower surfaces of the MgF_2 film the light is reflected by a medium of greater index of refraction. When striking a medium of lower index of refraction, the light is reflected with no phase change.

Since in this problem both rays 1 and 2 experience the same phase shift, no net change of phase is introduced by these two reflections. Hence, the only way a phase change can occur is if the two rays travel through different optical path lengths. The optical path length is the product of the geometric path difference a ray travels through different media and the refractive index of the medium in which it is travelling. For destructive interference, the two rays must be out of phase by an odd number of half wavelengths. Hence, the optical path difference needed for destructive interference is

$$2\mu d = (2n+1)\,\frac{\lambda}{2},\ n = 0, 1, 2, \ldots$$

Note that $2\mu d$ is the total optical path length that the rays traverse when $n = 0$.

$$\therefore \quad d = \frac{\lambda/2}{2\mu} = \frac{\lambda}{4\mu} = \frac{350 \times 10^{-9}}{4 \times 1.38} = 100\text{ nm} = 1 \times 10^{-7}\text{m}$$

60. (a) Condition for observing bright fringe is

$$2nd = \left(m + \frac{1}{2}\right)\lambda$$

$$\therefore \quad \lambda = \frac{2nd}{\left(m+\frac{1}{2}\right)} = \frac{2 \times 1.5 \times 4 \times 10^{-5}}{m + \frac{1}{2}} = \frac{12 \times 10^{-5}}{m + \frac{1}{2}}$$

The integer m that gives the wavelength in the visible region (4000 Å to 7000 Å) is $m = 2$. In that case,

$$\lambda = \frac{12 \times 10^{-5}}{2 + \frac{1}{2}} = 4.8 \times 10^{-5} = 4800\text{ Å}$$

61. (a) In this case, both the rays suffer a phase change of 180° and the conditions for destructive interference is

$$2nd = \left(m + \frac{1}{2}\right)\lambda_1$$

$$2nd = \left(m + \frac{3}{2}\right)\lambda_2$$

$$\therefore \quad \frac{m + \frac{1}{2}}{m + \frac{3}{2}} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{7000} = \frac{5}{7}$$

and $\quad d = \dfrac{\left(m + \frac{1}{2}\right)\lambda_1}{2n} = \dfrac{2.5 \times 7000}{2 \times 1.3}$

$$= 6738\text{ Å} = 6.738 \times 10^{-5}\text{ cm}$$

62. (a) $\mu = \dfrac{c}{v} = \dfrac{v\lambda}{v\lambda'}$

$$\frac{3}{2} = \frac{\lambda}{\lambda'} \quad \text{or} \quad \lambda' = \frac{2\lambda}{3}$$

Note that the frequency remains unchanged.

63. (a) $\beta = \dfrac{D\lambda}{d} = \dfrac{f\lambda}{d} = \dfrac{1 \times 4890 \times 10^{-10}}{0.2 \times 10^{-3}}$

$$= 0.29 \times 10^{-2}\text{ m} = 2.9\text{ mm} = 3\text{ mm}$$

64. (d) If $d\sin\theta = (\mu - 1)t$, central fringe is obtained at O.
If $d\sin\theta > (\mu - 1)t$, central fringe is obtained above O.
If $d\sin\theta < (\mu - 1)t$, central fringe is obtained below O.

65. (b) $S_1 = \dfrac{\Delta_1 D}{d} = 11 \times 10^{-3}$

$S_2 = \dfrac{\Delta_2 D}{d} = 12 \times 10^{-3}$

$\Rightarrow \quad \Delta_1/\Delta_2 = 11/12 \quad \Rightarrow \quad 12\Delta_1 = 11\Delta_2$

66. (c) $I_p = \dfrac{I_{max}}{2}[1 + \cos\phi] = \dfrac{I_{max}}{2}\left(1 + \cos\dfrac{2\pi y}{\beta}\right)$

where $\beta = D\lambda/d$.

First maxima is observed at P, i.e., $\cos\dfrac{2\pi y}{\beta} = 1$. As D increases β will increase and the value of $\cos\dfrac{2\pi y}{\beta}$ should be negative. Hence, the ratio I_p/I_{max} starts decreasing but starts increasing again as $\cos\dfrac{2\pi y}{\beta}$ again starts becoming positive.

67. (b) The wavelength missing from the reflected spectrum must satisfy the condition, $2\mu t = n\lambda$, where t is thickness of air film.

$2\mu t = n\lambda_1 = (n + 1)\lambda_2$

or $\quad n \times (7200) = (n + 1)\, 5400$

$\therefore \quad n = 3$

The next wavelengths must satisfy the condition,

$n\lambda_1 = (n + 2)\lambda_2$

or $\quad 7200 \times 3 = (3 + 2)\lambda_2 = 5\lambda_2$

$\Rightarrow \quad \lambda_2 = 4320$ Å

68. (d) The path difference introduced due to introduction of transparent sheet is given by $\Delta x = (m - 1)t$.

If the central maxima occupies position of nth fringe, then

$(\mu - 1)t = n\lambda = d \sin\theta$

$\Rightarrow \quad \sin\theta = \dfrac{(\mu - 1)t}{d} = \dfrac{(1.17 - 1) \times 1.5 \times 10^{-7}}{3 \times 10^{-7}}$

$= 0.085$

Hence, the angular position of central maxima is

$\theta = \sin^{-1}(0.085) = 4.88°$

For small angles,

$\sin\theta = \theta = \tan\theta$

$\Rightarrow \quad \tan\theta = \dfrac{y}{D} \quad \therefore \quad \dfrac{y}{D} = \dfrac{(\mu - 1)t}{d}$

Shift of central maxima is

$y = \dfrac{D(\mu - 1)t}{d}$

This formula can be used if D is given.

69. (d) Fringe width, $\beta = \dfrac{\lambda D}{d}$

β becomes (1/4th) when λ is halfed and d is doubled. The separation between successive dark fringes reduces. It does not remain unchanged.

70. (a) $\beta = 0.03$ cm, $D = 1$ m $= 100$ cm

Distance between images of the source = 0.8 cm

Distance of image from lens, $v = 80$ cm

Distance of slit from lens = u

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} \quad \Rightarrow \quad \dfrac{1}{60} + \dfrac{1}{u} = \dfrac{1}{16} \quad \Rightarrow \quad u = 20$ cm

Magnification $= \dfrac{v}{u} = \dfrac{80}{20} = 4$

Magnification $= \dfrac{\text{Distances between images of slits}}{\text{Distance between slits}}$

$= \dfrac{0.8}{d} = \dfrac{0.8}{d} = 4$

$\Rightarrow \quad d = 0.2$ cm $\quad \Rightarrow \quad \beta = \dfrac{D\lambda}{d} = \dfrac{100\lambda}{2} = 0.03$

$\Rightarrow \quad \lambda = 6000$ Å

71. (a) When mica sheet of thickness t and refractive index μ is introduced in the path of one of the interferring beams, optical path increases by $(\mu - 1)t$. Therefore, the shift on the screen is given by

$$y_0 = \frac{D}{d}(\mu - 1)t \qquad \text{(i)}$$

When the distance between the plane of slits and screen is changed from D to $2D$, then

$$\beta = \frac{2D}{d}\lambda \qquad \text{(ii)}$$

$\therefore \quad \dfrac{D}{d}(\mu - 1)t = \dfrac{2D(\lambda)}{d} \quad \Rightarrow \quad \lambda = \dfrac{1}{2}(\mu - 1)t$

72. (a) y_9 = Position of 9th bright fringe $= 9\left(\dfrac{\lambda D}{d}\right)$

y_2 = Position of 2nd dark fringe $= \left(2 - \dfrac{1}{2}\right)\dfrac{\lambda D}{d} = \dfrac{3}{2}\dfrac{\lambda D}{d}$

$y_9 - y_2 = 7.5$ mm $\quad \Rightarrow \quad \dfrac{\lambda D}{d}\left(9 - \dfrac{3}{2}\right) = 7.5 \times 10^{-3}$

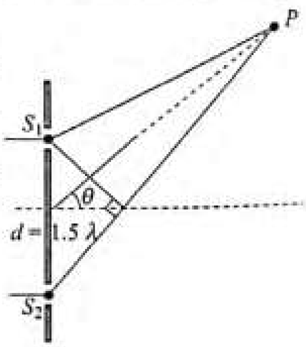
$\therefore \quad \lambda = (7.5 \times 10^{-3})\left(\dfrac{2}{15}\right)\left(\dfrac{0.5 \times 10^{-3}}{100 \times 10^{-2}}\right)$

$= (75)\left(\dfrac{2}{15}\right)(5)\,(10^{-4-4}) = 50 \times 10^{-8}$ m

$= 5000$ Å

73. (a) δ = phase difference between the waves from S_1 and S_2 at

$P = \dfrac{\pi}{2} - \dfrac{2\pi}{2}(d \sin\theta)$



For maximum intensity at P, $\delta = n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$

$\therefore \quad \dfrac{2\pi}{2}(1.5\lambda \sin\theta)\dfrac{\pi}{2} = n\pi$

$\Rightarrow \quad n - \dfrac{1}{2} = 3\sin\theta$

$\Rightarrow \quad \sin\theta = \left(\dfrac{n - \frac{1}{2}}{3}\right)$

For $n = 0$, $\quad \sin\theta = -\dfrac{1}{6}$

For $n = \pm 1$, $\quad \sin\theta = \dfrac{1}{6}, -\dfrac{1}{2}$

For $n = \pm 2$, $\quad \sin\theta = \dfrac{1}{2}, -\dfrac{5}{6}$

74. (c) The maxima in this case is obtained whenever

$x_{cm} = \dfrac{n\lambda D}{d}$

We can write for first wavelength,

$$(x_4)\lambda_1 = \frac{4\lambda_1 D}{d}$$

This must be equal to $(x_3)\lambda_2 = \frac{3\lambda_2 D}{d}$, since $(x_4)\lambda_1 = (x_3)\lambda_2$.

$$\therefore \quad \frac{4\lambda_1 D}{d} = 3\lambda_2 \frac{D}{d} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{3}{4}$$

Hence, when both wavelengths are incident simultaneously, the maxima due to two will coincide at a point where the fourth maxima due to λ_1 occured. This point will have maximum intensity and intensity will not be zero at any point.

ARCHIVES

1. (d) Resolving power is proportional to $1/\lambda$.

Required ratio $= \frac{5000}{4000} = 5:4$

2. (c) It is polarization of light through which we can observe transverse nature of light

3. (a) Coherent sources are those sources which have the same frequency and which have a constant phase relationship.

4. (b) The condition for interference maximum is

$$d \sin\theta = n\lambda$$

But $d = 2\lambda$ (given)

$$\therefore \quad 2\lambda \sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n}{2}$$

Clearly, n can have only integral values $-2, -1, 0, +1, +2$.

[Note that the value of $\sin\theta$ can vary between -1 and $+1$ only].

So the maximum number of possible interference maximum is 5.

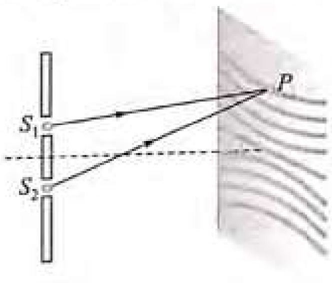
5. (d) According to Brewster's law of polarization

$$\mu = \tan\theta_P \Rightarrow \mu_P = \tan^{-1} n$$

θ_p is the angle of incidence

6. (d) $S_2P - S_1P =$ constant

This is the equation of a hyperbola with focii S_1 and S_2.



7. (d) $\frac{1.22\lambda}{a} = \frac{x}{d}$

$$\Rightarrow \quad d = \frac{xa}{1.22\lambda}$$

$$= \frac{1 \times 10^{-3}}{1.22 \times 550 \times 10^{-9}} m \approx 5 \text{ m}$$

8. (c) When the slit width is doubled, the half angular width of the central maximum, which is λ/a, reduces to half. The area of the central diffraction band becomes 1/4. So the intensity of the central maximum becomes four times.

9. (d) $I = I_0 \cos^2\theta$

Intensity of polarised light is $I_0/2$.

Therefore, the intensity of untransmitted light is

$$I_0 - \frac{I_0}{2} = \frac{I_0}{2}$$

10. (c) For maximum intensity $\phi = 0$.

$$I_0 = a^2 + a^2 + 2a \times a \cos\theta = 4a^2$$

When $x = \lambda/6$,

$$\phi = \frac{2\pi}{\lambda} x = \frac{2\pi}{\lambda}\frac{\lambda}{6} = \frac{\pi}{3} = 60°$$

Then

$$I = a^2 + a^2 + 2a \times a \cos 60° = 3a^2$$

$$\therefore \quad \frac{I}{I_0} = \frac{3a^2}{4a^2} = \frac{3}{4}$$

11. (c) In Young's double slits experiment,

Path difference for bright fringes $\Delta x = \frac{xd}{D} = n\lambda$

Hence position of any point on screen,

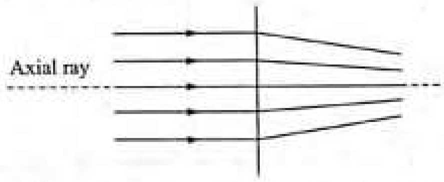
$$x = \frac{\Delta x D}{d} = \frac{n\lambda D}{d}$$

The central fringes when $\Delta x = 0$ coincide for all wavelengths.

The third bright fringe of $\lambda_1 = 590$ nm coincides with the fourth bright fringe of unknown wavelength λ.

$$\therefore \quad \Rightarrow x = \frac{3 \times 590 D}{4d} = \frac{4\lambda D}{d} \Rightarrow \lambda = 442.5 \text{ nm}$$

12. (b) As the beam enters the medium, axial ray will travel slowest. So, it will lag behind. To compensate for the path, the rays will bend towards axis.



13. (d) As the beam is initially parallel, the shape of wave front is planar.

14. (a) The speed of light in a medium of refractive index μ is given by

$$v = \frac{\text{velocity of light in vaccum}}{\text{Refractive index of medium}} = \frac{c}{\mu(I)}$$

The refractive index is given as $\mu(I) = \mu_0 + \mu_2 I$

It is given that intensity decreases with increasing radius, hence the intensity is maximum at the axis of beam. It means the refractive index will be maximum on the axis of the beam. Hence the speed of the light will be minimum at the axis of the light beam.

15. (b) As light enters from air to glass it suffers a phase change on π and therefore at centre there will be destructive interference.

Statement 1 is true as light enters from air to glass it suffers a phase change on π ($\therefore$ rarer to denser propagation)

Statement 2 is true, the centre of interference pattern is dark, showing that the phase difference between two interfering waves is π.

16. (d) It is given, $A_2 = 2A_1$

We know, Intensity $\propto$ (Amplitude)^2

Hence $\frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2A_1}{A_1}\right)^2 = 4$

$\Rightarrow \quad I_2 = 4I_1$

Maximum intensity, $I_m = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$$= \left(\sqrt{I_1} + \sqrt{4I_1}\right)^2 = \left(3\sqrt{I_1}\right)^2 = 9I_1$$

Hence $I_1 = \frac{I_m}{9}$

Resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$= I_1 + 4I_1 + 2\sqrt{I_1(4I_1)}\cos\phi$$

$$= 5I_1 + 4I_1\cos\phi = I_1 + 4I_1 + 4I_1\cos\phi$$

$$= I_1 + 4I_1(1 + \cos\phi)$$

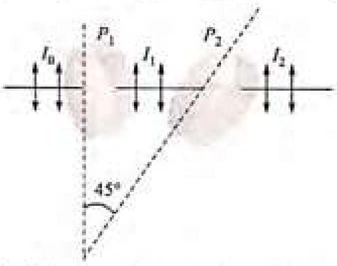
$$= I_1 + 8I_1\cos^2\phi \qquad \left(\because \ 1 + \cos\phi = 2\cos^2\frac{\phi}{2}\right)$$

$$I = \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$$

Putting the value of I_1 from eqn. (i), we get

$$I = \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$$

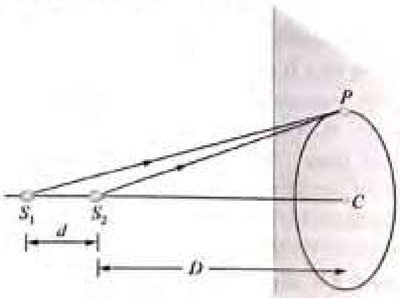
17. (b) Intensity of light after passing polaroid P_1 is $I_1 = \frac{I_0}{2}$



Now this light pass through the second polaroid P_2 whose axis is inclined at an angle of 45° to the axis of polaroid P_1. So in accordance with Malus law, the intensity of light emerging from polaroid B is

$$I_2 = I_1\cos^2 45° = \left(\frac{I_0}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$$

18. (c) For bright fringe $S_1P - S_2P = n\lambda$



So fringes are concentric circles (centre of origin)

19. (b) By law of Malus, $I = I_0\cos^2\theta$

Now, $I_A = I_A\cos^2 30°$

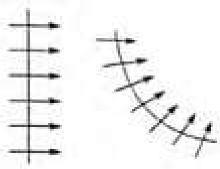
and $\quad I_B = I_B\cos^2 60°$

As $I_{A'} = I_{B'}$

$$\Rightarrow \quad I_A \times \frac{3}{4} = I_B \times \frac{1}{4}$$

$$\frac{I_A}{I_B} = \frac{1}{3}$$

20. (d) Consider a plane wavefront travelling horizontally. As it moves, its different parts move with different speeds. So, its shape will change as shown.

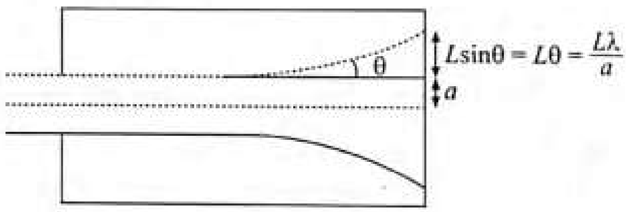


$\Rightarrow \quad$ Light bends upward

21. (b) $$RP = \frac{1.22\lambda}{2\mu\sin\theta} = \frac{1.22 \times (500 \times 10^{-9}\text{ m})}{2 \times 1 \times \left(\frac{1}{100}\right)}$$

or $\quad RP = 3.05 \times 10^{-5}\text{ m} = 30\ \mu\text{m}$

22. (c) Let b is the radius of spot = geometrical spread + spread due to diffraction



$$b = a = \frac{\lambda L}{a}$$

$$\frac{db}{da} = 0 \Rightarrow \quad 1 - \frac{\lambda}{a^2 L} = 0$$

$$\Rightarrow \quad a^2 = L\lambda \quad \text{or} \quad a = \sqrt{L\lambda}$$

Hence $\quad b_{\min} = \sqrt{L\lambda} + \frac{\lambda L}{\sqrt{L\lambda}}$

$$b_{\min} = 2\sqrt{L\lambda} \quad \text{or} \quad b_{\min} = \sqrt{4L\lambda}$$

23. (d) For λ_1, $y = \frac{m\lambda_1 D}{d}$

For λ_2, $y_2 = \frac{n\lambda_2 D}{d}$

The point where the bright fringes due to both the wavelengths coincide, $y_1 = y_2$

$$\Rightarrow \quad \frac{m}{n} = \frac{\lambda_2}{\lambda_1} = \frac{4}{5}$$

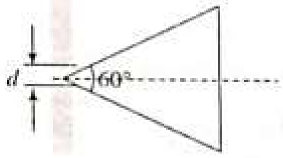
The least distance from the common central maximum for λ_1

$$y = \frac{4\times650\times10^{-9}\times0.15}{0.5\times10^{-3}} = 7.8\times10^{-3}\text{ m}$$

$= 7.8$ mm

24. (b) In case of single slit diffraction $d\sin30^\circ = \lambda$

$$d\times\frac{1}{2} = \lambda \text{ or } \lambda = \frac{d}{2} \Rightarrow \lambda = \frac{10^{-6}}{2}m$$



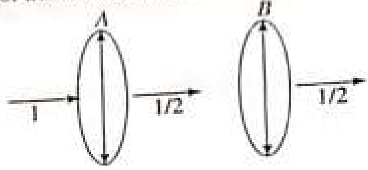
We know Young's fringe width $\beta = \frac{D\lambda}{d'}$

where d' = separation between two slits.
Substituting the corresponding values

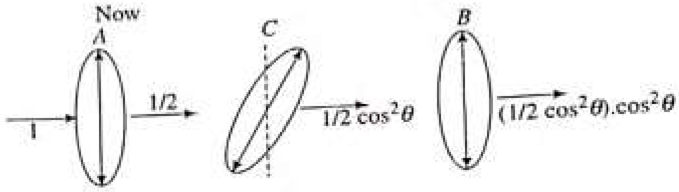
$$10^{-2} = \frac{50\times10^{-2}\times10^{-6/2}}{d'} = \frac{25\times10^{-8}}{d'}$$

We get $d' = 25\mu$m

25. (d) Axis of transmission of A & B are parallel.



Now



$$\frac{I}{2}\cos^4\theta = \frac{I}{8} \Rightarrow \cos^4\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

CHAPTER 28: DUAL NATURE OF RADIATION AND MATTER

Concept Application Exercise 28.1

1. Energy of proton, $E = 2$ MeV $= 2 \times 1.6 \times 10^{-13}$ J

$\therefore$ de Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$

$\Rightarrow \lambda = \dfrac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.64 \times 10^{-27} \times 2 \times 1.6 \times 10^{-13}}}$

$\Rightarrow \lambda = 2.044 \times 10^{-14}$ m

2. Given: $k = 1.38 \times 10^{-23}$ J molecule^{-1} K^{-1}
and $T = 273 + 127 = 400$ K
KE of the neutron

$$E = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 400$$

$\Rightarrow E = 8.25 \times 10^{-21}$ J

$\therefore$ de Broglie wavelength, $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2mE}}$

or $\lambda = \dfrac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.66 \times 10^{-27} \times 8.28 \times 10^{-21}}}$

or $\lambda = 1.264 \times 10^{-10}$ m

3. For completely absorbing surface,

$$P_{rad} = \frac{I}{c} = \frac{1.4 \times 10^3}{3.0 \times 10^8} = 4.66 \times 10^{-6} \text{ m}^{-2}$$

4. For an electron, de Broglie wavelength is given by

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{25}} = \sqrt{6} \text{ Å}$$

$= 2.5$ Å

5. The value of de Broglie wavelength associated with a moving particle is given by

$$\lambda = \frac{h}{mv}$$

Consider a ball of 0.5 kg moving with a velocity of 10 ms^{-1}. It is an object from our daily observations. The de Broglie wavelength of the ball will be

$$\lambda = \frac{6.625 \times 10^{-34}}{0.5 \times 10} = 13.25 \times 10^{-34} \text{ m}$$

The value of λ is very small as compared to the size a ball of 0.5 kg would possess. Hence, wave nature of matter is not apparent in our daily observations.

6. If λ is the de Broglie wavelength of the particle of kinetic energy K, then

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Suppose that the de Broglie wavelength of the particle becomes λ', when its kinetic energy is $K/4$, then

$$\lambda' = \frac{h}{\sqrt{2mk/4}} = 2\left(\frac{h}{\sqrt{2mK}}\right) = 2\lambda$$

7. In terms of accelerating potential V, the de Broglie wavelength of a charged particle is given by

$$\lambda = \frac{h}{\sqrt{2meV}} \quad \text{(i)}$$

where e is charge and m is mass of the particle.

Equation (i) represents a straight line, whose slope is $h/\sqrt{2me}$. The slope of the line is inversely proportional to $\sqrt{m}$. Since the slope of line A is lesser, it represents the particle of heavier mass.

Concept Application Exercise 28.2

1. The energy of each photon of incident light is

$$E = h\nu = \frac{hc}{\lambda} = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{(4560 \times 10^{-10})} = 4.35 \times 10^{-19} \text{ J}$$

Number of photons in one milliwatt source

$$= \frac{10^{-3}}{4.35 \times 10^{-19}} = 2.29 \times 10^{15} \text{ s}^{-1}$$

As quantum efficiency = 0.5%. Hence, number of electrons liberated per second $= 2.29 \times 10^{15} \times \dfrac{0.5}{100}$

$= 1.14 \times 10^{13}$ per second

$\therefore$ Photoelectric current $= (1.14 \times 10^{13})(1.6 \times 10^{-19})$

$= 1.824 \times 10^{-6}$ A $= 1.824\ \mu$A

2. According to Einstein's photoelectric equation,

$$\frac{1}{2}mv_{max}^2 = h\nu - W = 4.9 - 4.5 = 0.4 \text{ eV}$$

If E be the energy, then

$$E = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2E}{m}}$$

Momentum $= mv = \sqrt{2Em}$

We know that change of momentum is impulse

Hence, impulse $= mv - 0$

$\therefore$ Impulse $= mv = \sqrt{2Em}$

Substituting the values, we get

Maximum impulse $= \sqrt{\{2 \times (0.4 \times 1.6 \times 10^{-19}) \times 9.1 \times 10^{-31}\}}$

$= 3.45 \times 10^{-25}$ kg m s^{-1}

3. According to Einstein's photoelectric equation,

$$\frac{hc}{\lambda} = W + K_{max}$$

Here, $\dfrac{hc}{\lambda} = W + 30$ eV and $\dfrac{hc}{2\lambda} = W + 10$ eV

$\therefore \dfrac{hc}{\lambda} - \dfrac{hc}{2\lambda} = \dfrac{hc}{2\lambda} = 20$ eV

$$\lambda = \frac{hc}{40 \text{ eV}} = \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{40 \times (1.6 \times 10^{-19})} = 310.3 \text{ Å}$$

Now, $W = \frac{hc}{\lambda} - K_{max} = (40 - 30)\text{ eV} = 10\text{ eV}$

$\therefore \quad \lambda_0 = \frac{(6.62\times10^{-34})(3\times10^8)}{10\times(1.6\times10^{-19})} = 1241\text{ Å}$

4. Minimum energy of the photon falling on photoelectric plate

$= h\nu = \frac{hc}{\lambda} = \frac{(6.625\times10^{-34})(3\times10^8)}{250\times10^{-19}}$

According to Einstein's photoelectric equation,

$$\frac{1}{2}mv_{min}^2 = \frac{hc}{\lambda} - W$$

$$[v_{min}]^2 = \frac{2}{m}\left[\frac{hc}{\lambda} - W\right]$$

$$= \frac{2}{9.1\times10^{-31}}\left[\frac{(6.625\times10^{-34})(3\times10^8)}{250\times10^{-9}} - 1.9\times(1.6\times10^{-19})\right]$$

$= 1.08\times10^{12}$

$\therefore \quad v_{min} = 1.04\times10^6\text{ m s}^{-1}$

5. Let the number of photons hitting the photocell per second be n, then

$nh\nu = 1.5$ mW

or $\quad n = \frac{(1.5\times10^{-3})\times400\times10^{-9}}{(6.63\times10^{-34})\times(3\times10^8)}\left[\because \nu = \frac{c}{\lambda} = \frac{3\times10^8}{400\times10^{-9}}\right]$

$\therefore \quad n = 3\times10^{15}$

Number of photoelectrons produced per second is

$3\times10^{15}\times\frac{0.1}{100} = (3\times10^{12})$

So, current in the photocell is

$(3\times10^{12})(1.6\times10^{-19}) = 4.8\times10^{-7}$ A

6. The intensity of radiation, I, is defined as the energy passing per unit time per unit area normal to the direction of the beam. If N be the number of photons crossing unit area per unit time, then

$I = N\times$ energy carried by one photon

$I = N\frac{hc}{\lambda}$ or $N = \frac{I\lambda}{hc}$

If N_i be the number of incident photons per unit area per unit time, then

$$N_i = \frac{I_i\lambda}{hc} = \frac{10^{-8}\times365\times10^{-19}}{6.62\times10^{-34}\times3\times10^8} = 18.35\times10^9$$

The number of photons absorbed, N_{ab}, by the surface per unit area per unit time is given by

N_{ab} = absorption coefficient of surface $\times N_i$

$= 0.8\times18.35\times10^9 = 1.47\times10^{10}\text{ m}^{-2}\text{s}^{-1}$

Now, assuming that each photon ejects only one electron, the rate of electrons emitted per unit area is given by

$N = N_{ab} = 1.47\times10^{10}\text{ m}^{-2}\text{s}^{-1}$

Power absorbed per m^2

$= \text{Absorption coefficient}\times\frac{\text{incident power}}{m^2}$

$= 0.8\times10^{-8} = 8\times10^{-9}\text{ W m}^{-2}$

From Einstein's equation, maximum kinetic energy is given by

$$KE_{max} = h\nu - W_0 = \frac{hc}{\lambda} - W_0$$

$$= \frac{(6.62\times10^{-34})(3\times10^8)}{365\times10^{-9}} - 1.6\times1.6\times10^{19}$$

$= 2.89\times10^{-19}\text{ J} = 1.80\text{ eV}$

Exercises

Photon Momentum and Energy, De-Broglie Hypothesis

1. (c) Energy is given by

$$E = \frac{m_0c^2}{\sqrt{1-(v^2/c^2)}} \quad\text{or}\quad E^2 = \frac{m_0^2c^2}{c^2-v^2}$$

Momentum p is given by

$$p = \frac{m_0v}{\sqrt{1(v^2/c^2)}} \quad\text{or}\quad p^2c^2 = \frac{m_0^2c^4v^2}{c^2-v^2}$$

$\therefore \quad E^2 - p^2c^2 = m_0^2c^4$

or $\quad E^2 = p^2c^2 + m_0^2c^4$

For photon, rest mass

$m_0 = 0$, so $E = pc$

For electron, $m_0 \neq 0$, so $E \neq pc$

2. (a) $E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{632.8\times10^{-9}}$ J

Number of photons per second, $n = \frac{P}{E}$

$\Rightarrow \quad n = \frac{5\times10^{-3}}{3.14\times10^{-19}} = 1.6\times10^{16}$

3. (a) Energy radiated as visible light is $\frac{5}{100}\times100 = 5\text{ J s}^{-1}$

Let n be the number of photons emitted per second. Then,

$\frac{nhv}{\lambda} = 5$

$\therefore \quad n = \frac{5\lambda}{hc} = \frac{5\times5.6\times10^{-7}}{(6.62\times10^{-34})(3\times10^8)} = 1.4\times10^{19}$

4. (c) Effective power $= \frac{25}{100}\times200\text{ W} = 50\text{ W}$

Now, $50 = nh\nu = \frac{nhc}{\lambda}$

$n = \frac{50\lambda}{hc} = \frac{50\times0.6\times10^{-6}}{6.6\times10^{-34}\times3\times10^8} = 1.5\times10^{20}$

5. (b) The momentum of photon $= h/\lambda$

If n is the number of photons falling per second on the plate, then total momentum per second of the incident photons is

$p = n\times\frac{h}{\lambda}$

Since the plate is blackened, all photons are absorbed by it.

$\frac{\Delta p}{\Delta t} = n\frac{h}{\lambda}$

Since $F = \frac{\Delta p}{\Delta t} = n\frac{h}{\lambda}$

$\therefore \quad n = \frac{F\lambda}{h} = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}} = 5 \times 10^{22}$

6. (d) If q is the charge on the particle and V the potential difference through which it is accelerated, then

$$qV = \frac{1}{2}mv^2 \quad \text{or} \quad mv = \sqrt{2mqV}$$

de Broglie's wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}} \quad \therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

7. (b) We know that mass m in motion and the rest mass m_0 is related through the equation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As $\quad v = c,\ m = \frac{m_0}{\sqrt{1-1}} = \frac{m_0}{0} = \infty$

Therefore, de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{h}{(\infty)(c)} = 0$$

8. (d) Order of magnitude calculation is enough.

$(2m_e eV)^{1/2} = (2 \times 9 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19})^{1/2}$

$\approx 5 \times 10^{-24}$ kg m s^{-1}

and $\quad h = 6 \times 10^{-34}$ J s

So, $\quad \lambda \approx 10^{-10}$ m

Mid-wavelength in visible region is

$\lambda_0 \approx 5000 \times 10^{-10}$ m

Thus, $\quad \lambda = \lambda_0/5000$

9. (c) $\lambda = \frac{h}{\sqrt{2mqV}} \quad mV = \text{constant}$

$$1837\ V' = 1 \times V \quad \text{or} \quad V' = \frac{V}{1837} \text{ volt}$$

10. (b) $\lambda = \frac{h}{\sqrt{2meV}}$

$$\frac{hc}{\lambda_{min}} = eV$$

$$\lambda \times \frac{hc}{\lambda_{min}} = \frac{h}{\sqrt{2meV}}eV \quad \text{or} \quad \frac{\lambda}{\lambda_{min}} \propto \sqrt{V}$$

11. (b) The gain of kinetic energy by an electron is eV.

$$\frac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{eV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19})(50)}{(9.11 \times 10^{-31})}}$$

$= 4.19 \times 10^6$ m s^{-1}

Thus, the electron's de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{34}}{(9.11 \times 10^{31})(4.19 \times 19^6)}$$

$= 1.74 \times 10^{-10}$ m

12. (a) Kinetic energy gained by a charge q after being accelerated through a potential difference V volt.

$$qV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2qV}{m}}$$

$$mv = \sqrt{2mqV}$$

de Broglie wavelength $= \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}$$

Putting $V_\alpha = V_p$, $\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4 \times 2}{1 \times 1}} = 2\sqrt{2}$

13. (d) Speed of electron which enters into electric field may increase or decrease while for second electron, it remains constant.

So, from $\lambda = \frac{h}{mv'}$

$\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

14. (c) $\lambda = \frac{h}{mv}$. Since v is increasing in case (i), but it is not changing in case (ii). Hence, in the first case de-Broglie wavelength will change, but it second case, it remains the same

15. (d) Longest wavelength is specified by $\lambda = 2L$, next by $\lambda = L$, then $\lambda = L \times \frac{2}{3}$, etc.

In general $\lambda_n = \frac{2L}{n}$, $n = 1, 2, 3, \ldots$

For (a) $\lambda = \frac{2L}{3}$

For (b) $\lambda = L$

For (c) $\lambda = 2L$. Hence, all are possible.

16. (b) The radiation pressure

$$P = \frac{F}{A} = \left(\frac{2h}{\lambda}\right)\frac{N}{A} = 2\frac{I}{C}$$

is reflected completely. It is independent of wavelength. It will depend on the nature of the surface and the intensity of light.

17. (c) $\lambda = \frac{h}{p} \Rightarrow \lambda - \frac{0.5}{100}\lambda = \frac{h}{p + \Delta p} \Rightarrow \frac{199\lambda}{200} = \frac{h}{p + \Delta p}$

$$\Rightarrow \frac{199}{200}\frac{h}{p} = \frac{h}{p + \Delta p} \Rightarrow p = 199\ \Delta p$$

18. (a) $p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{4400 \times 10^{-10}} = 1.5 \times 10^{-27}$ kg.m/s

and mass $m = \frac{p}{c} = \frac{1.5 \times 10^{-27}}{3 \times 10^8} = 5 \times 10^{-36}$ kg

19. (b) $F = \frac{dp}{dt} \Rightarrow F = \frac{(2p\sin\theta)}{\Delta t}$

$\Rightarrow \quad F = 2np\sin\theta$ [n = number of photon]

$\Rightarrow \quad 1N = 2(n)\ \frac{h}{\lambda}\ \sin 30° \Rightarrow n = 10^{27}$

20. (c) $E = nh\nu \Rightarrow \nu \propto \frac{1}{n} \Rightarrow \frac{n_1}{n_2} = \frac{\gamma_2}{\gamma_1}$

21. (a) $hv = eV$

$\Rightarrow \quad v = \dfrac{eV}{h} = \dfrac{1.6\times10^{-19}\times10\times10^{3}}{6.6\times10^{-34}} = 2.4\times10^{18}\,\text{Hz}$

Photoelectric Effect

22. (c) $KE_{max} = hv - \phi$

$\Rightarrow \quad \dfrac{1}{2}mv^2_{max} = hv - \phi \quad \Rightarrow \quad v_{max} = \sqrt{\dfrac{2(hv-\phi)}{m}}$

Hence, (a) is incorrect.

Since $n = (IA/hv)$, therefore rate of emission of electrons is proportional to the intensity (I).

$KE_{max} = hv - \phi$

Hence, (c) is true.

23. (a) Option (a) correctly explains the photoelectric effect on the basis of electromagnetic theory. Its correct explanation is given by Quantum theory of light.

24. (d) Since the number of photoelectrons emitted is directly proportional to the intensity of incident radiation, the number of photoelectrons emitted becomes four times.

The energy of photoelectrons does not change with the intensity light.

25. (a) $\dfrac{hc}{\lambda} < W$ (for no emission) $\Rightarrow \lambda > \dfrac{hc}{W}$

26. (c) $\dfrac{hc}{\lambda_{max}} = 3\times1.6\times10^{-19}\,\text{J}$

$\Rightarrow \quad \lambda_{max} = \dfrac{6.6\times10^{-34}\times3\times10^{8}}{3\times1.6\times10^{-19}} = 4.125\times10^{-7}\,\text{m}$

27. (a) As work function $W = h\upsilon_0$, where υ_0 is the threshold frequency.

Greater the work function, greater is the threshold frequency. Therefore, the threshold frequency of sodium will be lesser than that for aluminium.

28. (b) Maximum KE depends on the frequency of incident radiation, not on intensity.

29. (c) Let $h\nu_0 - W_0 = K$

If frequency is doubled, let kinetic energy of photoelectrons be K_1.

$2h\nu_0 - W_0 = K_1$

$\Rightarrow \quad 2(h\nu - W_0) + W_0 = K_1$

$\Rightarrow \quad 2K + W_0 = K_1$

i.e., kinetic energy is more than doubled.

30. (c) If E is the energy of incident photon and W the work function, then $E - W_0$ = available energy.

$E - W_0 = \dfrac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{\dfrac{2(E-W_0)}{m}}$

$\therefore \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{1-0.5}{2.5-0.5}} = \sqrt{\dfrac{0.5}{2}} = \dfrac{1}{2}$

31. (d) $\lambda_p = \lambda_\alpha$

or $\quad \dfrac{h}{\sqrt{2m_pQ_pV}} = \dfrac{h}{\sqrt{2m_\alpha Q_\alpha V_\alpha}}$

$\therefore \quad m_pQ_pV_p = m_\alpha Q_\alpha V_\alpha$

$\therefore \quad V_\alpha = \left(\dfrac{m_p}{m_\alpha}\right)\left(\dfrac{Q_p}{Q_\alpha}\right)V = \left(\dfrac{1}{4}\right)\left(\dfrac{1}{2}\right)V = \dfrac{V}{8}$

32. (b) Since $E \propto \dfrac{1}{\lambda}$, so energy corresponding to 5000 Å is

$E = 2.46\ \text{eV}$

Now, $h\upsilon - W = eV_s$

or $\quad 2.46\,\text{eV} - W = 1.36\,\text{eV}$

$W = (2.46 - 1.36)\,\text{eV} = 1.1\,\text{eV}$

33. (a) Change in intensity from I_0 to $4I_0$ does not affect the stopping potential.

34. (c) When intensity is increased from I_0 to $4I_0$, i.e., four times, then the saturation current increases by a factor of 4, i.e., the saturation current becomes

$= 4\times(0.40\times10^{-6})\,\text{A}$

35. (c) $V_s = 1.36\,\text{V}$

$\therefore \quad eV_s = 1.36\ \text{eV}$

or $\quad \dfrac{1}{2}m(v_{max})^2 = 1.36\ \text{eV}$

i.e., various electrons have KE between zero and 1.36 eV.

36. (d) Let energy corresponding to wavelength of 4000 Å be E. Then,

$\dfrac{E}{E'} = \dfrac{\lambda'}{\lambda} \quad \text{or} \quad \dfrac{E}{1.23} = \dfrac{10{,}000}{4000}$

$\therefore \quad E = 1.23\times2.5 = 3.075\,\text{eV}$

But $\quad h\upsilon - h\upsilon_0 = eV_s$

or $\quad 3.075\,\text{eV} - 1.1\,\text{eV} = eV_s$

$\therefore \quad V_s = 1.975\,\text{V}$

37. (a) $eV = hv - \phi_0 = \left(\dfrac{12375}{2000} - 5.01\right)\text{eV}$

$V = (6.1875 - 5.01)\,\text{V} = 1.18\,\text{V} = 1.2\,\text{V}$

38. (b) $W = hv - eV_s$

$h\upsilon$ = energy of incident photon

Here $\quad h\upsilon = \dfrac{12400}{1240}\,\text{eV} = 10\,\text{eV}$

$\therefore \quad W = 10 - 8 = 2\,\text{eV}$

So, $\quad \lambda_0$ = Threshold wavelength

$= \dfrac{12400}{2eV}\,\text{Å} = 6200\,\text{Å}$

39. (a) $\lambda_1 = \dfrac{12375}{E_1(eV)}\,\text{Å} = 1000\,\text{Å}$

$\therefore \quad E_1 = 12.375\,\text{eV}$

Similarly, $\dfrac{12375}{\lambda_2(\text{Å})}\,\text{eV} = \dfrac{12375}{2000} = 6.1875\,\text{eV}$

Now, $\quad E_1 - W_0 = eV_s$

and $\quad E_2 - W_0 = eV_S$

Hence, $12.375 - W_0 = 7.7\text{eV}$

and $\quad 6.1875 - W_0 = eV'_s$

Solving, we get $V'_s = 1.5\,\text{V}$

40. (b) $eV = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ (i)

$$\frac{eV}{3} = hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right) \quad \text{(ii)}$$

Dividing Eq. (i) and (ii), we get $\lambda_0 = 4\lambda$.

41. (c) $\frac{hc}{\lambda} = E + \phi_0$

$$\frac{hc}{\lambda'} = 2E + \phi_0$$

Dividing, we get $\frac{\lambda'}{\lambda} = \left(\frac{E+\phi_0}{2E+\phi_0}\right)$ or $\frac{\lambda'}{\lambda} < 1$

$\therefore \quad \lambda' < \lambda$ or $\lambda > \lambda'$ (i)

Also, $\frac{\lambda'}{\lambda} = \frac{1}{2}\left[\frac{E+\phi_0}{E+\frac{\phi_0}{2}}\right]$

or $\frac{\lambda'}{\lambda} > \frac{1}{2}$ or $\lambda' > \frac{\lambda}{2}$ (ii)

It follows from Eqs. (i) and (ii) that

$$\lambda > \lambda' > \frac{\lambda}{2}$$

42. (a) $E = \frac{hc}{\lambda/2} - \phi_0$

$$2E = \frac{hc}{\lambda/3} - \phi_0$$

or $2\left(\frac{2hc}{\lambda} - \phi_0\right) = \frac{3hc}{\lambda} - \phi_0$

or $\frac{4hc}{\lambda} - 2\phi_0 = \frac{3hc}{\lambda} - \phi_0$ or $\phi_0 = \frac{hc}{\lambda}$

43. (c) In the first case,

$$\frac{1}{2}mv_{\max}^2 = 2h\upsilon_0 - h\upsilon_0 = h\upsilon_0$$

In the second case,

$$\frac{1}{2}mv_{\max}^2 = 5h\upsilon_0 - h\upsilon_0 = 4h\upsilon_0$$

Clearly, $v_{\max}$ is doubled, i.e., $v_{\max} = 8 \times 10^6$ ms^{-1}

44. (b) Energy corresponding to 2000 Å = 12375/2000 eV = 6.2 eV

Maximum kinetic energy is

$$(6.2 - 5.01) \text{ eV} = 1.19 \text{ eV}$$

Now, $\frac{1}{2} \times 9.1 \times 10^{-31} \times v_{\max}^2 = 1.19 \times 1.6 \times 10^{-19}$

or $v_{\max}^2 = \frac{1.19 \times 1.6 \times 10^{-19} \times 2}{9.1 \times 10^{-31}}$

$$= 0.418 \times 10^{12} = 41.8 \times 10^{10}$$

or $v_{\max} = 6.46 \times 10^5$ m s^{-1}

45. (c) $E_k = \frac{12375}{4000} - \phi = 3.1 - \phi_0$

$$2E_k = \frac{12375}{3100} - \phi = 3.99 - \phi_0$$

$$6.2 - 2\phi_0 = 3.99 - \phi_0$$

or $\phi_0 = 6.2 - 3.99 = 2.21$ eV

46. (d) $eV_s = h\nu - \phi_0$

$$V_s = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

Now, $\frac{h}{e}$ = slope = $\frac{1.656}{4 \times 10^{14}} = 0.414 \times 10^{-14}$ V s

$$= 4.14 \times 10^{-15} \text{ V s}$$

47. (c) $\phi_0 = h\nu_0$

or $\phi_0 = e \times 4.14 \times 10^{-15} \times 1 \times 10^{14} = 0.414$ eV

48. (b) $\frac{1}{2}mv^2 = h\nu - \phi_0$

$$\frac{1}{2}mv'^2 = 4h\nu - \phi_0$$

$$\frac{v'^2}{v^2} = \frac{4h\nu - \phi_0}{h\nu - \phi_0}$$

or $\frac{v'^2}{v^2} = \frac{4[h\nu - \phi] + 3\phi_0}{h\nu - \phi_0}$

Clearly, $v' > 2v$

49. (d) $\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W_0$ (i)

Let the speed of the fastest electron be V_1 when excitation wavelength is changed to $3\lambda/4$.

$\therefore \quad \frac{1}{2}mv_1^2 = \frac{4hc}{3\lambda} - W_0$

$\Rightarrow \quad \frac{1}{2}mv_1^2 = \frac{4}{3}\left(\frac{hc}{\lambda} - W_0\right) + \frac{W_0}{3}$

$\Rightarrow \quad \frac{1}{2}mv_1^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{W_0}{3}$ [using Eq. (i)]

$\Rightarrow \quad v_1^2 = \frac{4v^2}{3} + \frac{2W_0}{3m}$

$\therefore \quad v_1 > \sqrt{\frac{4}{3}}\,v$

50. (a) $K_{\max} = h\nu - W$

ω is the intercept on y-axis and h is the slope.

$\therefore \quad h = \frac{2.4 \times 10^{-15}}{4 \times 10^{18}} = 6 \times 10^{-34}$ Js

$$W = 2 \times 10^{-15} \text{ J}$$

$\Rightarrow \quad h\nu_0 = 2 \times 10^{-15}$ or $\nu_0 = 3.33 \times 10^{18}$ s

Problems Based on Mixed Concepts

51. (c) Using photoelectric equation,

$$h\nu - h\nu_0 = \frac{1}{2}m\upsilon^2 = eV_s$$

or $\left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0}\right) = eV_s$

For the first case, $\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = e(3V_0)$ (i)

For the second case, $\frac{hc}{2\lambda} - \frac{hc}{\lambda_0} = e(V_0)$ (ii)

Solving $\lambda_0 = 4\lambda$

52. (c) Einstein's equation for photoelectric effect is

$$h\upsilon - h\upsilon_0 = \frac{1}{2}m\upsilon^2_{max}$$

When $\upsilon = 2\upsilon_0$, $\upsilon_{max} = 4 \times 10^8$ cm s^{-1}

$$2h\upsilon_0 - h\upsilon_0 = (1/2)\, m\, (4 \times 10^8)^2$$

$$h\upsilon_o = \frac{1}{2}m\,(4 \times 10^8)^2 \quad \text{(i)}$$

When $v = 5v_o$, $v_{max} = v'$

$$h(5v_o) - hv_o = \frac{1}{2}mv'^2 \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$v' = 8 \times 10^8 \text{ cm s}^{-1}$$

53. (b) $E - W_0 = \frac{1}{2}mv^2 = eV_s$

or $\frac{hc}{\lambda} - W_0 = eV_s$

Hence, $\frac{hc}{0.6 \times 10^{-6}} - W_0 = e(0.5)$ (i)

and $\frac{hc}{0.4 \times 10^{-6}} - W_0 = e(1.5)$ (ii)

Solving, we get $W_0 = 1.5$ eV

54. (c) $n = \frac{\text{power}}{hc/\lambda} = \frac{300 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^8}$

$= 1.5 \times 10^{18}$ m^{-2} s^{-1} $= 1.5 \times 10^{14}$ cm^{-2} s^{-1}

As only 1 percent of photons cause emission of photo-electrons, number of photo electrons is

$$n_e = 1.5 \times 10^{12} \text{ s}^{-1}$$

55. (c) According to Einstein's equation,

$E = W_0 + \text{KE}$

$W_{0max} = 4.2$ eV

$\text{KE} = 2.6$ eV

$\therefore$ $E_{min} = W_{0max} + \text{KE} = (4.2 + 2.6)$ eV $= 6.8$ eV

56. (c) $I \propto 1/d^2$

When source is placed 2 m away, then $I' = (I/4)$. The number of electrons emitted $\propto$ intensity. Hence, the number of emitted electrons is reduced to one-fourth.

57. (b) In both the cases, the intensity is same.

58. (b) Momentum imparted per unit time $= np$

$\Rightarrow$ $F = \frac{nh}{\lambda}$

$\therefore$ Acceleration $= \frac{nh}{m\lambda}$

59. (a) Energy received by the eye,

$$E = \frac{nhc}{\lambda} = \frac{5 \times 10^4 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}}$$

$$= 0.2 \times 10^{-13} \text{ W m}^{-2}$$

So, eye is more sensitive by a factor of $\frac{1}{0.200} = 5.00$

60. (c) $E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 4 \times 1.67 \times 10^{-27} \times (0.1 \times 10^{-10})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{43.82 \times 10^{-68}}{21.376 \times 10^{-68}} = 2.05 \text{ eV}$$

61. (c) Kinetic energy is same, that settles for (c). Intensity 4-fold, so n 4-fold.

62. (d) Magnetic force experienced by a charged particle in a magnetic field is given by

$$F_B = q\vec{\upsilon} \times \vec{B} = qvB \sin\theta$$

In our case, $F_B = qvB$ [as $\theta = 90°$]

Hence, $Bqv = \frac{m\upsilon^2}{r}$ $\Rightarrow$ $mv = qBr$

The de Broglie wavelength,

$$\lambda = \frac{h}{m\upsilon} = \frac{h}{qBr}$$

$$\frac{\lambda_{\alpha\text{-particle}}}{\lambda_{\text{proton}}} = \frac{q_p r_p}{q_\alpha r_\alpha}$$

Since $\frac{r_\alpha}{r_p} = 1$ and $\frac{q_\alpha}{q_p} = 2$

$\Rightarrow$ $\frac{\lambda_\alpha}{\lambda_p} = \frac{1}{2}$

63. (d) The electrons ejected with maximum speed V_{max} are stopped by electric field $E = 4N/C$ after traveling a distance $d = 1$ m.

$$\frac{1}{2}mv^2_{max} = eE\,d = 4 \text{ eV}$$

The energy of incident photon $= \frac{1240}{200} = 6.2$ eV

From equation of photo electric effect

$$\frac{1}{2}mv^2_{max} = h\nu - \phi_0$$

$\therefore$ $\phi_0 = 6.2 - 4 = 2.2$ eV

64. (a) We have $\text{KE} = \frac{P^2}{2m_e} = \frac{hc}{\lambda_{min}}$

$$P = \sqrt{\frac{2hcm_e}{\lambda_{min}}}$$

Also, $\lambda_{\text{de Broglie}} = \frac{h}{p} = \sqrt{\frac{h\lambda_{min}}{2m_e c}}$

For $\lambda_{min} = 10$ Å,

$\lambda_{\text{de Broglie}} \cong 0.3$ Å

65. (b). The intensity of light at the location of your eye is

$$I = \frac{P}{4\pi r^2} = \frac{60}{4\pi \times 4^2} \text{ W m}^{-2}$$

The energy entering into your eye per second is

$$P_1 = I \times \frac{\pi d^2}{4}$$

where d is the diameter of pupil.

$$P_1 = \frac{60}{4\pi \times 4^2} \times \frac{\pi \times (2\times 10^{-3})^2}{4}$$

$$= 9.375 \times 10^{-7}\ \text{J s}^{-1}$$

Let n be the number of photons entering into the eye per second, then

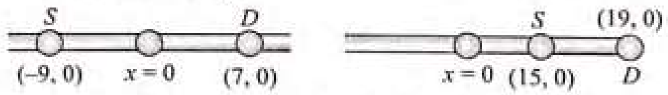
$$P_1 = n \times \frac{hc}{\lambda}$$

$$9.375 \times 10^{-7} = n \times \frac{1240 \times 1.6 \times 10^{-19}}{600}$$

$$n = 2.84 \times 10^{12}\ \text{photon s}^{-1}$$

So, the number of photons entering the eye in 0.1 s = 0.1 n = 2.84×10^{11}

66. (b) The separation between source and photosensitive material at $t = 0$ is 16 m. Therefore, intensity received by photosensitive material at $t = 0$ is $I_0 = P/(4\pi \times 16^2)$, where P is the power of source of light.



At $t = 3$ s, the source is at (15, 0) and detector is at (19, 0), so the separation between them is 4 m.

$$I_2 = \frac{P}{4\pi \times 4^2}$$

So, $\frac{I_1}{I_2} = \frac{1}{16}$

67. (d) $K_{max} = h\nu - \phi$

$$= \left(\frac{6.63\times 10^{-34} \times 3\times 10^8}{0.3\times 10^{-6} \times 1.6\times 10^{-19}} - 2.46\right)\text{eV} = 1.68\ \text{eV}$$

Cut-off wavelength, $\lambda_0 = \frac{hc}{\phi} = 505$ nm

The minimum energy required to eject the photo-electrons is equal to work function.

68. (a) Let m be the mass of particle.

$\frac{mv^2}{2} = \frac{hc}{\lambda_{photon}}$, where symbols have their usual meanings.

$$\frac{p^2}{2m} = \frac{hc}{\lambda_{photon}}$$

and $p = \frac{h}{\lambda_{particle}} \Rightarrow \frac{h^2}{2m\lambda^2_{particle}} = \frac{hc}{\lambda_{photon}}$

$$\Rightarrow \frac{\lambda_{photon}}{\lambda_{particle}} = \frac{2mc}{h} \times \lambda_{particle} = \frac{2mc}{h} \times \frac{h}{mv}$$

$$= \frac{2c}{0.05c} = 40$$

69. (d) Radiation force

$$\frac{2I}{c} \times \text{Area} = \frac{I}{c}\cdot\frac{1}{2}\cdot 2R.\text{H} = \frac{IRH}{c}$$

70. (c) $n = \frac{E\lambda}{hc} = \frac{1\times 10^{-7} \times 200 \times 10^{-9}}{6.6\times 10^{-34} \times 3\times 10^8} = 1\times 10^{11}$

Number of electrons ejected $\frac{10^{11}}{10^3} = 10^8$

$\therefore \quad V = \frac{q}{4\pi\varepsilon_0 r} = \frac{(10^8 \times 1.6\times 10^{-19})\times 9\times 10^9}{4.8\times 10^{-2}} = 3$ V

71. (c) K.E. $= 2E_0 - E_0 = E_0$ (for $0 \le x \le 1$)

$$\Rightarrow \lambda_1 = \frac{h}{\sqrt{2mE_0}}$$

K.E. $= 2E_0$ (for $x > 1$) $\Rightarrow \lambda_2 = \frac{h}{\sqrt{4mE_0}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{2}$

72. (d) $K = \frac{hc}{\lambda} - \phi_0$ (i)

and $K' = \frac{4hc}{3\lambda} - \phi_0$ (ii)

From Eqs. (i) and (ii), we get

$$\Rightarrow K' - K = \frac{4hc}{3\lambda} - \frac{hc}{\lambda}$$

$$K' - K = \frac{hc}{3\lambda}$$

But from Eq. (i) $\frac{hc}{\lambda} = K + \phi_0$

$$\therefore K' - K = \frac{K}{3} + \frac{\phi_0}{3}$$

$$\Rightarrow K' = \frac{4K}{3} + \frac{\phi_0}{3}$$

Or $K' > \frac{4K}{3}$

73. (c) As, $hf = E$

$$\Rightarrow \frac{hc}{\lambda} = E \quad \Rightarrow \quad \frac{1}{\lambda} = \frac{E}{hc}$$

Now, $hc \approx 1240$ eV-nm

$\Rightarrow$ $hc \approx 1240 \times 10^{-7}$ eV-cm

So, $\frac{1}{\lambda} = \frac{1\ \text{eV}}{1240\times 10^{-7}\ \text{eV-cm}}$ (when $E = 1$ eV)

$= 8065.8\ \text{cm}^{-1}$

So, a wavelength reciprocal $\frac{1}{\lambda}$ of $8065.8\ \text{cm}^{-1}$ corresponds to an energy of 1 eV.

74. (d) Only 4% of energy is absorbed and only 3% of this can cause photoemission, part of intensity available for photoemission is

$$I' = 0.03 \times 0.04 I_0 = 1.2\ \text{nW cm}^{-2}$$

$\therefore$ Number of electrons/second

$$= \frac{1.2\times 10^{-9}\ \text{W}}{hf} = 1.5\times 10^9\ \text{electrons per second}$$

75. (a) Energy $= \frac{1}{2} = mv^2 = 5000$ eV

$= 5000 \times 1.6 \times 10^{-19}$ joule

$$mv = \sqrt{2\times 5000 \times (1.6\times 10^{-19})\,m} = 4\times 10^{-8}\sqrt{m}$$

Number of electron striking per second

$$= n = \frac{q}{e}\frac{It}{e} = \frac{50\times 10^{-6} \times 1}{1.6\times 10^{-19}} = 31.25 \times 10^{13}$$

Force = change of momentum per second

$= n(mv) = 31.25\times10^{13}\times4\times10^{-8}\sqrt{m}$

$= 125\times10^{5}\sqrt{9.1\times10^{-31}}$

$= 1.1924\times10^{-8}$ newton

ARCHIVES

1. (c) $\phi_0 = \dfrac{hc}{\lambda}$

$\therefore \quad \dfrac{\lambda_1}{\lambda_2} = \dfrac{4.5}{2.3} = \dfrac{2}{1}$

2. (c) The formation of a molecule by sharing of electrons between combining atoms is called covalency. Covalent bonding is formed by sharing of electrons of opposite spins between two atoms and can be explained by wave nature of electrons.

3. (b) $\dfrac{1}{2}mv_1^2 = hf_1 - \phi_0$

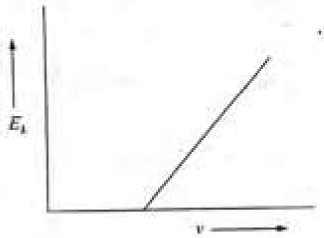
and $\dfrac{1}{2}mv_2^2 = hf_2 - \phi_0$

Subtracting, we get

$\dfrac{1}{2}m(v_1^2 - v_2^2) = h(f_1 - f_2)$

$\Rightarrow \quad v_1^2 - v_2^2 = \dfrac{2h}{m}(f_1 - f_2)$

4. (d)



According to Einstein's photoelectric equation, $(KE)_{max} = h\nu - \phi_0$. Comparing with the straight line equation $y = mx + c$, we find that the slope of the graph is h. Clearly, the slope is the same for all metals and is independent of the intensity of the radiation.

5. (c) $\lambda_{max} = \dfrac{hc}{\phi_0}$

$= \dfrac{6.6\times10^{-34}\times3\times10^{8}}{4\times1.6\times10^{-19}} = 310$ nm

6. (a) $qE = mg$

$q = \dfrac{mg}{E} = \dfrac{9.9\times10^{-15}\times10}{3\times10^{4}}$ C

$= 3.3\times10^{-18}$ C

7. (c) Intensity $\propto \dfrac{1}{(\text{distance})^2}$

Since the distance is halved, intensity is increased by a factor of 4. Moreover, photoelectric current and, hence the number of photoelectrons, is proportional to intensity.

8. (b) $\lambda = \dfrac{h}{mv} = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE_k}}$

$\lambda \propto \dfrac{1}{\sqrt{E_k}},$

$\lambda' \propto \dfrac{1}{\sqrt{2E_k}}$

$\dfrac{\lambda'}{\lambda} = \dfrac{1}{\sqrt{2}}$

9. (b) $I' = Ie^{-\mu x} \Rightarrow x = \dfrac{1}{\mu}\log_e\dfrac{I}{I'}$ (where I = original intensity, I' = changed intensity)

$$36 = \frac{1}{\mu}\log_e\frac{I}{I/8} = \frac{3}{\mu}\log_e 2 \qquad \text{...(i)}$$

$$x = \frac{1}{\mu}\log_e\frac{I}{I/2} = \frac{1}{\mu}\log_e 2 \qquad \text{...(ii)}$$

From equations (i) and (ii), $x = 12$ mm.

10. (b) The photoelectron emission,

(Incident energy E) = $(\text{K.E.})_{max}$ + (Work function ϕ)

or $\quad E = K_m + \phi$

or $\quad E = 5 + 6.2 = 11.2$ eV

$= 11.2\times(1.6\times10^{-19})$ J

$\therefore \quad \dfrac{hc}{\lambda} = 11.2\times1.6\times10^{-19}$

or $\quad \lambda = \dfrac{(6.63\times10^{-34})\times(3\times10^{8})}{11.2\times1.6\times10^{-19}}$ m

or $\quad \lambda = 1110\times10^{-10}$ m $= 1110$ Å.

The incident radiation lies in ultraviolet region.

11. (c) Emission of photo-electron starts from the surface after incidence of photons in about 10^{-10} s.

12. (c) $p = \dfrac{E}{c} \Rightarrow p = \dfrac{h\nu}{c}$

13. (d) Bragg's equation for X-rays, which is also used in electron diffraction given $n\lambda = 2d\cos i$

For electron diffraction, $d = 1$, $i = 30°$

$\therefore \quad \lambda = \dfrac{2\times1(\text{Å})\times\cos30°}{1}$ (assuming first order $n = 1$)

$\Rightarrow \quad \lambda = \sqrt{3}$Å.

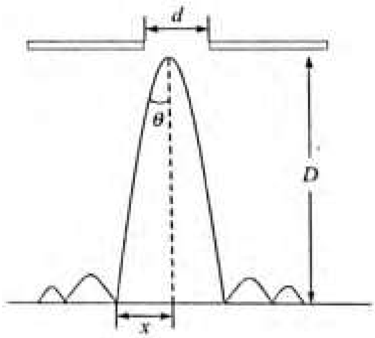
de-Broglie wavelength associated with electron

$$\lambda = \frac{12.27}{\sqrt{V}}\text{Å} \Rightarrow \sqrt{V} = \frac{12.27}{\sqrt{3}}$$

or $\quad V = 50.18$ volt.

14. (d) For strong peak, path difference is $n\lambda_{dB}$.

15. (b) The electron diffraction pattern from a single slit will be as shown below.



$d\sin\theta = \dfrac{\lambda}{2\pi}$,

the line of maximum intensity for the zeroth order will exceed d very much.

16. (b) $\frac{1}{2}mv^2 = 1.68$ eV

$$hv = \frac{hc}{\lambda} = \frac{1240 \text{ eVnm}}{400 \text{ nm}} = 3.1 \text{ eV}$$

$\Rightarrow$ $3.1 \text{ eV} = W_0 + 1.68 \text{ eV}$

$\therefore$ $W_0 = 1.42$ eV

17. (d) Since the frequency of ultraviolet light is less than the frequency of X-rays, the energy of each incident photon will be more for X-rays.

KE of photoelectron $= hv - \varphi$

Stopping potential stops the fastest photoelectron.

$$V_0 = \frac{hv}{e} - \frac{\varphi}{e}$$

So KE_{max} and V_0 both increase.

But KE ranges from zero to KE_{max} because of loss of energy due to subsequent collisions before getting ejected and not due to the range of frequencies in the incident light.

18. (a) Here,

Power of the source, $P = 4$ kW $= 4 \times 10^3$ W

Number of photons emitted per second, $N = 10^{20}$

Energy of photon, $E = hf = \frac{hc}{\lambda}$ ...(i)

$\therefore$ Also energy of photon, $E = \frac{P}{N}$ ...(ii)

Equating (i) and (ii), $\frac{hc}{\lambda} = \frac{P}{N}$

or $$\lambda = \frac{Nhc}{P} = \frac{10^{20} \times 6.63 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^3}$$

$$= 4.972 \times 10^{-9}\text{m} = 49.72 \text{ Å}$$

It lies in the X-ray region.

19. (c) $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

$U(x = \infty) = 0$

$$F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$.

$$x^6 = \frac{2a}{b}$$

$$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$D = [U(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

20. (d) The maximum kinetic energy of the electron

$$K_{max} = h\upsilon - h\upsilon_0$$

Here, υ_0 is threshold frequency.

The stopping potential is $eV_0 = K_{max} = h\upsilon - h\upsilon_0$

Therefore, if v is doubled K_{max} and V_0 is not doubled.

21. (c) Davisson–Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystals. This shows the wave nature of electrons as waves can exhibit interference and diffraction.

22. (c) As λ is increased, there will be a value of λ above which photoelectrons will be ceased to come out. So photocurrent will become zero. Hence, (c) is the correct answer.

23. (c) Franck– Hertz experiment—Discrete energy level.

Photoelectric effect—Particle nature of light

Davison–Germer experiment—Diffraction of electron beam.

24. (a) $(KE)_{max}$ for fastest emitted electron

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi \quad ...(i)$$

$$\frac{1}{2}m(v')^2 = \frac{hc}{\frac{3\lambda}{4}} - \phi$$

$$\frac{1}{2}m(v')^2 = \frac{4}{3}\frac{hc}{\lambda} - \phi \quad ...(ii)$$

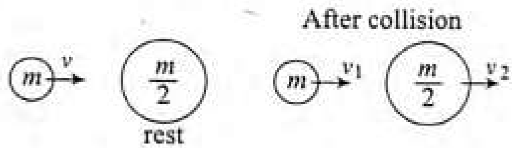
From (i) and (ii)

$$\frac{1}{2}m(v')^2 = \frac{4}{3}\left(\frac{1}{2}mv^2 + \phi\right) - \phi$$

$$\frac{1}{2}m(v')^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$$

or $$\frac{1}{2}m(v')^2 > \frac{4}{3}\left(\frac{1}{2}mv^2\right) \Rightarrow v' > \sqrt{\frac{4}{3}}v$$

25. (d)



By conservation of linear momentum

$$mv = mv_1 + \frac{m}{2}v_2$$

$$2v = 2v_1 + v_2 \quad ...(1)$$

By law of collision

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow u = v_2 - v_1 \quad ...(2)$$

By equations (1) and (2) $v_1 = \frac{v}{3}$; $v_2 = \frac{4v}{3}$

$$\lambda_A = \frac{h}{p_1}; \; \lambda_B = \frac{h}{p_2} \Rightarrow \frac{\lambda_A}{\lambda_B} = \frac{2}{1}$$

Option (d) is correct.

CHAPTER 29: ATOMIC STRUCTURE

Concept Application Exercise 29.1

1. $$\frac{1}{\lambda_1} = R_\infty Z^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right] \quad \text{(i)}$$

$$\frac{1}{\lambda_2} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right] \quad \text{(ii)}$$

$$\frac{1}{\lambda_3} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right] \quad \text{(iii)}$$

On adding (i) and (ii), we get

$$\frac{1}{\lambda_1}+\frac{1}{\lambda_2} = R_\infty Z^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right]$$

Thus, $\frac{1}{\lambda_1}+\frac{1}{\lambda_2}=\frac{1}{\lambda_3} \Rightarrow \lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$

2. (a) Let n_1 be the initial state of electron. Then,

$$E_1 = -\frac{13.6}{n_1^2}\text{eV}$$

Here, $E_1 = -0.85$ eV

$$\therefore \quad -0.85 = -\frac{13.6}{n_1^2}$$

or $n_1 = 4$

(b) Let n_2 be the final excitation state of the electron. Since excitation energy is always measured with respect to the ground state, therefore

$$\Delta E = 13.6\left[1-\frac{1}{n_2^2}\right]$$

Here, $\Delta E = 10.2$ eV

$$\therefore \quad 10.2 = 13.6\left[1-\frac{1}{n_2^2}\right] \quad \text{or} \quad n_2 = 2$$

Thus, the electron jumps from $n_1 = 4$ to $n_2 = 2$.

(c) The wavelength of the photon emitted for a transition between $n_1 = 4$ to $n_2 = 2$, is given by

$$\frac{1}{\lambda} = R_\infty\left[\frac{1}{n_2^2}-\frac{1}{n_1^2}\right]$$

$$\frac{1}{\lambda} = 1.09\times 10^7\left[\frac{1}{2^2}-\frac{1}{4^2}\right] \Rightarrow \lambda = 4860 \text{ Å}$$

3. (a) H_β line of Balmer series corresponds to the transition from $n = 4$ to $n = 2$ level. The corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097\times 10^7)\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = 0.2056\times 10^7 \text{ m}$$

$$\lambda = 4.9\times 10^{-7} \text{ m}$$

(b) $$f = \frac{c}{\lambda} = \frac{3.0\times 10^8}{4.9\times 10^{-7}}$$

$$= 6.12\times 10^{14} \text{ Hz}$$

4. The transition equation for Lyman series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2}-\frac{1}{n^2}\right), n = 2, 3, \ldots$$

The largest wavelength is corresponding to $n = 2$

$$\therefore \quad \frac{1}{\lambda_{max}} = 1.097\times 10^7\left(\frac{1}{1}-\frac{1}{4}\right)$$

$$= 0.823\times 10^7$$

$$\Rightarrow \quad \lambda_{max} = 1.2154\times 10^{-7} \text{ m} = 1215 \text{ Å}$$

The shortest wavelength corresponds to $n = \infty$

$$\therefore \quad \frac{1}{\lambda_{min}} = 1.097\times 10^7\left(\frac{1}{1}-\frac{1}{\infty}\right)$$

or $\lambda_{min} = 0.911\times 10^{-7} \text{ m} = 911$ Å

Both of these wavelengths lie in the ultraviolet (UV) region of electromagnetic spectrum.

5. (a) $z = 3$ for Li^{2+}.

Further, we know that

$$r_n = \frac{n^2}{z}a_0$$

Substituting $n = 3, z = 3$, and $a_0 = 0.529$ Å, we have

$$r_3 \text{ for } Li^{+2} = \frac{(3)^2}{(3)}(0.529) \text{ Å} = 1.587 \text{ Å}$$

(b) $z = 2$ for He^+. Also, we know that

$$v_n = \frac{z}{n}v_1$$

Substituting $n = 4$, $z = 2$, and $V_1 = 2.19\times 10^6 \text{ m s}^{-1}$, we get

$$V_4 \text{ for } He^+ = \left(\frac{2}{4}\right)(2.19\times 10^6)\text{ms}^{-1}$$

$$= 1.095\times 10^6 \text{ ms}^{-1}$$

6. $E_1 = -13.60$ eV $\Rightarrow K_1 = -E_1 = 13.60$ eV

$V_1 = -2K_1 = -27.2$ eV

$$E_2 = \frac{E_1}{(2)^2} = -3.40 \text{ eV}$$

$\Rightarrow K_2 = 3.40$ eV and $U_2 = -6.80$ eV

Now, $U_1 = 0$, i.e., potential energy of each shell has been increased by 27.20 eV while kinetic energy will remain unchanged. Changed values in tabular form are as under.

Orbit	K (eV)	U (eV)	E (eV)
First	13.6	0	13.60
Second	3.40	20.40	23.80

Concept Application Exercise 29.2

1. When an electron of charge e is accelerated through a potential difference V, it acquires energy eV. If m be the mass of the electron and v_{max} the maximum speed of the electron, then

$$\frac{1}{2}mv_{max}^2 = eV \quad \text{or} \quad v_{max} = \sqrt{\left(\frac{2eV}{m}\right)}$$

Substituting the given values, we get

$$v_{max} = \sqrt{\left(\frac{2\times(1.6\times10^{-19})\times 20{,}000}{9\times10^{-31}}\right)}$$

$$= 8.4\times10^7 \text{ ms}^{-1}$$

2. (a) $\lambda_{min} = 0.45$ Å

$$E_{max} = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$= \frac{12431}{0.45} = 27624.44 \text{ eV} = 27.624 \text{ keV}$$

(b) The minimum accelerating voltage for electrons is

$$\frac{27.6 \text{ keV}}{e} = 27.6 \text{ kV, i.e., of the order of 30 kV.}$$

3. According to Moseley's law, the frequency for K series is proportional to $(Z-1)^2$

or $\quad \frac{c}{\lambda} \propto (Z-1)^2$

or $\quad \frac{1}{\lambda} = k(Z-1)^2 \qquad$ (i)

where k is a constant. Let λ' be the wavelength of K_α line emitted from molybdenum, then

$$\frac{1}{\lambda'} = k(Z-1)^2 \qquad \text{(ii)}$$

Dividing (i) and (ii), we get

$$\lambda' = \left(\frac{Z-1}{Z-1}\right)^2\lambda = \left(\frac{30-1}{42-1}\right)^2 \times 1.415 \text{ Å} = 0.708 \text{ Å}$$

4. The short series limit of the Balmer series is corresponding to transition $n = \infty$ to $n = 2$ which is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

or $\quad R = \frac{4}{\lambda} = \frac{4}{3644}$ (Å)$^{-1}$

The shortest wavelength corresponds to transition from $n = \infty$ to $n = 1$ which is given as

$$\frac{1}{\lambda_c} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$$

or $\quad (Z-1)^2 = \frac{1}{\lambda_c R} = \frac{1}{1\text{ Å}\times\frac{4}{3644}(\text{Å})^{-1}} = \frac{3644}{4} = 911$

or $\quad Z - 1 = 30.2$

or $\quad Z = 31.2 \simeq 31.$

Thus, the atomic number of the element is 31 which is gallium.

5. The binding energy for K shell in eV is

$$E_k = \frac{hc}{\lambda_k} = \frac{12431}{0.2} \text{ eV} = 62.155 \text{ keV}$$

The energy of the incident photon in eV is

$$E = \frac{hc}{\lambda} = \frac{12431}{0.15} = 82.873 \text{ keV}$$

Therefore, the maximum energy of the photoelectrons emitted from the K shell is

$$E_{max} = E - E_k = (82.873 - 62.155) \text{ keV}$$
$$= 20.718 \text{ keV}$$

6. Tungsten is a multielectron atom. Due to the shielding of the nuclear charge by the negative charge of the inner core electrons, each electron is subject to an effective nuclear charge Z_{eff}, which is different for different shells.

For an electron in the K shell, $\sigma = 1$.

Thus, $Z_{eff} = Z - \sigma = Z - 1$

Here, as electron drops from M shell ($n = 3$) to K shell ($n = 1$), we call the radiated emission K_β X-ray and from Moseley's law the wavelength emitted of K_β X-ray is given as

$$\frac{1}{\lambda_{K_\beta}} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

or $\quad \frac{1}{\lambda_{K_\beta}} = 10967800 \times (74-1)^2\left[\frac{8}{9}\right]$

or $\quad \lambda_{K_\beta} = 0.192$ Å

Exercises

Bohr's Hydrogen Atom Model

1. (c) The minimum energy to ionize an atom is the energy required to remove an outermost electron in the atom.

2. (d) a. No, since Balmer formula was known.

b. No, since Rutherford scattering experiment was known.

c. No, since Einstein's photon theory was known.

d. Bohr chose 'allowed' energy levels $\propto 1/n^2$ and these led to angular momentum quantized as a derivation.

3. (d) $F = \frac{mv^2}{r}$

But $\quad v \propto \frac{1}{n}$ and $r \propto n^2 \Rightarrow F \propto \frac{1}{n^4}$

4. (a) $\frac{\text{Magnetic moment}}{\text{Angular momentum}} = \frac{e}{2m}$

$\therefore$ Magnetic moment $\propto$ angular momentum

$$\propto n \left(\because L = n\frac{h}{2\pi}\right)$$

5. (d) $r = \frac{\varepsilon_0 n^2 h^2}{e^2 \pi m}$

$$= \frac{\varepsilon_0 (2\pi L)^2}{e^2 \pi m} \qquad \left(L = n\frac{h}{2\pi} \text{ or } nh = 2\pi L\right)$$

$\therefore \quad Lr^{-\frac{1}{2}} = \text{constant}$

6. (a) $mvr = \frac{nh}{2\pi} \quad \therefore \quad \frac{h}{mv} = \frac{(2\pi r)}{n}$

$\frac{h}{mv}$ = de Broglie wavelength

7. (b). Maximum angular speed will be in its ground state. Hence,

$$\omega_{max} = \frac{v_1}{r_1} = \frac{2.2\times10^6}{0.529\times10^{-10}} = 4.1\times10^{16} \text{ rad s}^{-1}$$

8. (c) This is Bohr's postulate.

9. (c) Potential energy = $-C/r^2$ and total energy = $-Rhc/n^2$. With higher orbit, both r and n increase. So, both become less negative; hence both increase.

10. (b) $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}; \lambda = \frac{36}{5R}$

11. (a) In this case, there is the widest energy gap.

12. (b) $v = \frac{1}{137}\frac{c}{n}$ or $v \propto \frac{1}{n}$

Since v is reduced to one-third, therefore $n = 3$

Now, $r \propto n^2$

So, radius of new orbit will be $9r$.

13. (a) $E_n = -3.4$ eV, $E_n \propto \frac{1}{n^2}$

$E_1 = -13.6$ eV

Clearly, $n = 2$

Angular momentum $= \frac{nh}{2\pi} = \frac{2h}{2\pi} = \frac{h}{\pi} = 2.11 \times 10^{-34}$ J s

14. (a) $13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$ eV = 1.9 eV

15. (c) $\frac{13.6}{4}$ eV = 3.4 eV

16. (d) $E = -\frac{13.6}{5^2}$ eV $= -0.544$ eV

$E_p = -2 \times 0.544$ eV $= -1.088$ eV

17. (d) $v = 2\pi rf$

$\Rightarrow \quad f = \frac{v}{2\pi r}$

18. (a) Total energy for nth level $= -\frac{13.6}{n^2}$ eV

$$E_2 - E_1 = -13.6\left(\frac{1}{4} - \frac{1}{1}\right)$$
$$= \frac{13.6 \times 3}{4} = 0.75 \times 13.6 \text{ eV}$$
$$E_3 - E_2 = -13.6\left(\frac{1}{9} - \frac{1}{4}\right)$$
$$= \frac{13.6 \times 5}{36} = 0.14 \times 13.6 \text{ eV}$$
$$E_4 - E_3 = -13.6\left(\frac{1}{16} - \frac{1}{9}\right)$$
$$= \frac{13.6 \times 7}{144} \text{ eV} = 0.05 \times 13.6 \text{ eV}$$

Obviously, the difference of energy between consecutive energy levels decreases.

19. (d) $v_n = k\frac{2\pi e^2}{nh}$

We know that in cgs system $k = 1$

$\therefore \quad v_n = \frac{2\pi e^2}{nh} \Rightarrow v_1 = \frac{2\pi e^2}{h}$

So $\quad \frac{v_1}{c} = \frac{2\pi e^2}{ch}$

20. (b) $N = \Sigma 2n^2$

$N = 2(1^2 + 2^2 + 3^2 + 4^2) = 60$

21. (c) $E_n = -\frac{13.6}{n^2} \Rightarrow n^2 = -\frac{13.6}{-0.54}$

or $\quad n^2 = 25.2$ or $n = 5$(nearly)

As $v \propto 1/n$, so $v_n = \frac{v}{5}$

22. (a) $\frac{1}{\lambda} = 1.09 \times 10^7\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$\Rightarrow \quad \lambda = 6.606 \times 10^{-7}$ m $\Rightarrow 6606$ Å

23. (c) Making potential energy zero increases the value of total energy by $13.6 - (-13.6) = 27.2$ eV

Now, actual energy in second orbit $= -3.4$ eV

Hence, new value is $(-3.4 + 27.2)$ eV $= 23.8$ eV

24. (a) Number of possible emission lines are $n(n-1)/2$ when an electron jumps from n^{th} state to ground state. In this question, this value should be $(n-1)(n-2)/2$.

Hence, $10 = \frac{(n-1)(n-2)}{2}$

Solving this, we get $n = 6$.

Hydrogen Like Atom and Atomic Spectrum

25. (a) Radius of first orbit, $r \propto 1/Z$. For doubly ionized lithium, Z will be maximum. Hence, for doubly ionized lithium r will be minimum.

26. (b) For third excited state, $n = 4$

$$r_n = r_0\frac{n^2}{2}$$

or $\quad r_1 = 0.5 \times \frac{4 \times 4}{2}$ Å = 4 Å

27. (d) $L = \frac{nh}{2\pi}$

Clearly, L is constant and independent of Z.

28. (c) $E_{max} = 13.6$ eV; $E_{min} = 31.6\left(1 - \frac{1}{2^2}\right) = \frac{3}{4} \times 13.4$ eV

$\Rightarrow \quad \frac{E_{max}}{E_{min}} = \frac{4}{3}$

29. (b) $f = cZ^2R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\Rightarrow \quad 2.7 \times 10^{15} = cZ^2R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

$f' = cZ^2R\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$

Divide and solve to get: $f = 3.2 \times 10^{15}$ Hz

30. (d) $E = \frac{Z^2}{n^2}E_0 = \frac{(11)^2}{1} \times 13.6$ eV

31. (d) $B_n = \frac{\mu_0 I_n}{2r_n}$ or $B_n \propto \frac{I_n}{r_n}$

or $\quad B \propto \frac{(f_n)}{r_n} \quad \therefore \quad B_n \propto \frac{(v_n/r_n)}{r_n}$

Hence, $B \propto \frac{v_n}{(r_n)^2} \propto \frac{(z/n)}{(n^2/z)^2} \propto \frac{z^3}{n^5}$

32. (b) $a = \frac{v^2}{r}$

$\therefore \quad a \propto \frac{(z)^2}{(1/z)}$ (for $n = 1$)

or $\quad a \propto z^3$

$\therefore \quad \frac{a_1}{a_2} = \left(\frac{2}{1}\right)^3 = 8$

33. (b) Possible transitions are: $4 \to 3$, $4 \to 2$, $4 \to 1$, $3 \to 2$, $3 \to 1$, and $2 \to 1$

So, six transitions are possible.

34. (d) $\lambda \propto \frac{1}{Z^2}$

Now, $\lambda_{Na} = \frac{1216}{11 \times 11} \approx 10$ Å

35. (a) Series limit means the shortest possible wavelength (maximum photon energy) and first line means the largest possible wavelength (minimum photon energy) in the series.

$$v = C\left[\frac{1}{n^2} - \frac{1}{m^2}\right] \quad \text{(where } C \text{ is a constant)}$$

For series limit of Lyman series:

$n = 1, m = \infty \Rightarrow v_1 = C$

For first line of Lyman series:

$n = 1, m = 2 \Rightarrow v_2 = 3C/4$

For series limit of Balmer series:

$n = 2, m = \infty \Rightarrow v_3 = C/4$

Only option (a) is correct.

36. (d) $13.6 - 0.85 = 12.75$ eV

So, 12.75 V is the required potential difference.

37. (d) $\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$

or $\quad \frac{1}{\lambda} = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4}$

Hence, wavenumber

$$\frac{1}{\lambda} = \frac{3R}{4}$$

38. (a) The wavelengths of the hydrogen spectrum could be arranged in a formula or series named after its discoverer. For ultraviolet spectrum the series is called Lyman series, for visible spectrum the Blamer series, and for infrared region we have the Paschen series.

The ultraviolet series is obtained when the energy of the atom falls from higher states to the energy level corresponding to $n = 1$. Thus, ultraviolet radiation can only be possible with transition from E_2 to E_1 out of the given transitions.

39. (d) By quantum theory of radiation, the energy change ΔE between energy levels is proportional to the frequency of electromagnetic radiation f and is given by

$$\Delta E = hf = \frac{hc}{\lambda}$$

Hence, $\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_1 - E_2}$

40. (d) For Lyman series, the series limit wavelength is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = R \quad \text{or} \quad \lambda = \frac{1}{R}$$

For Balmer series, the series limit wavelength is given by

$$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{R}{4} \quad \text{or} \quad \lambda' = \frac{4}{R}$$

Clearly,

$$\lambda' = 4\left[\frac{1}{R}\right] \quad \text{or} \quad \lambda' = 4\lambda = 4 \times 912 \text{ Å}$$

41. (a) From conservation of momentum:

$$MV = \frac{h}{\lambda} = hR\left(1 - \frac{1}{4}\right) \Rightarrow V = \frac{3}{4}\frac{hR}{M}$$

42. (d) $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$

or $\quad \frac{f}{c} = R\left[\frac{1}{4} - \frac{1}{16}\right]$

or $\quad f = cR\left[\frac{1}{4} - \frac{1}{16}\right] = 3 \times 10^8 \times 10^7 \times \frac{3}{16}$

$$= \frac{9}{16} \times 10^{15} \text{ Hz}$$

43. (a) Shortest wavelength of Brackett series corresponds to the transition of electron between $n_1 = 4$ and $n_2 = \infty$ and the shortest wavelength of Balmer series corresponds to the transition of electron between $n_1 = 2$ and $n_2 = \infty$. So,

$$(z^2)\left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right)$$

$\therefore \quad z^2 = 4$ or $z = 2$

44. (d) For Lyman series, $n_1 = 1$ and $n_2 = 2$ for first line

$$\frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[\frac{1}{1} - \frac{1}{4}\right] = \frac{3R}{4}$$

For Paschen series, $n_1 = 3$ and $n_2 = 4$ for first line

$$\therefore \quad \frac{1}{\lambda_2} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = R\left[\frac{1}{9} - \frac{1}{16}\right] = \frac{7R}{144}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{4/3R}{144/7R} = \frac{7}{108}$$

45. (d) $\frac{E_{4n} - E_{2n}}{E_{2n} - E_n} = \frac{\frac{E_1}{16n^2} - \frac{E_1}{4n^2}}{\frac{E_1}{4n^2} - \frac{E_1}{n^2}} = \frac{1}{4} = \text{constant}$

X-rays

46. (c) When applied voltage is greater than energy of K-electron, continuous and all characteristic X-rays are emitted.

47. (d) According to Mosley's law $v \propto (Z - b)^2$

For k_α line, $b = 1$, and it has maximum frequency so $v_{max} \propto (Z - 1)^2$

48. (c) According to the energy diagram of X-ray spectra

$$\because \quad \Delta E = \frac{hc}{\lambda} \Rightarrow \lambda \propto \frac{1}{\Delta E}$$

So, $E_\beta = E_\alpha + E'_\alpha$

or $\dfrac{1}{\lambda_\beta} = \dfrac{1}{\lambda_\alpha} + \dfrac{1}{\lambda'_\alpha}$

49. (c) The energy of K_α x-ray photons is directly proportional to $(Z-1)^2$. The energy ratio of two K_α photons obtained in x-ray from two metal targets of atomic numbers Z_1 and Z_2 is $\left(\dfrac{Z_1-1}{Z_2-1}\right)^2$.

50. (d) $\lambda_{min} \propto \dfrac{12375}{V}$ Å $\Rightarrow V = \dfrac{12375}{2.5} = 4950\text{ V} \approx 5\text{ kV}$

51. (b) $\dfrac{1}{\lambda} = Z^2 R_\infty \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

For K_α line, $n_1 = 1$ and $n_2 = 2$

$$\frac{1}{\lambda} = Z^2 R_\infty \left(\frac{3}{4}\right)$$

$$Z = \sqrt{\frac{4}{3\lambda R_\infty}} = 39.9 \approx 40$$

52. (c) $\lambda_{min} = \dfrac{hc}{eV_{max}}$

As, $\lambda_{min} \propto \dfrac{1}{V_{max}}$

So, the shortest wavelength produced in an X-ray tube operating at 0.5 MV is double of the shortest wavelength at 1 MV.

53. (b) Use Moseley's law

54. (d) $\lambda_{min} = \dfrac{hc}{eV_{max}}$

55. (d) $\Delta\lambda = \lambda_{K_\alpha} - \lambda_{min}$

When V is halved, λ_{min} becomes two times but λ_{K_α} remains the same.

$\therefore \quad \Delta\lambda' = \lambda_{K_\alpha} - 2\lambda_{min}$

$\qquad = 2(\Delta\lambda) - \lambda_{K_\alpha}$

$\therefore \quad \Delta\lambda' < 2(\Delta\lambda)$

56. (c) Using Moseley's law, $v^{1/2} = a(z-b)$

$$\left(\frac{c}{\lambda_1}\right)^{1/2} = a(z_1 - b) \text{ and } \left(\frac{c}{\lambda_2}\right)^{1/2} = a(z_2 - b)$$

$$\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}} = \frac{a(z_1-b)}{a(z_2-b)} \Rightarrow \left(\frac{7.12}{15.42}\right)^{\frac{1}{2}} = \frac{(29-b)}{(42-b)}$$

$(42-b) = 1.47\,(29-b) \Rightarrow b = 1.44$

$$\left(\frac{\lambda_1}{\lambda}\right)^{\frac{1}{2}} = \frac{(z-1.44)}{(z_1-1.44)}$$

$$\left(\frac{15.42}{22.85}\right)^{\frac{1}{2}}(27.56) = z - 1.44 \Rightarrow z = 24$$

Problems Based on Mixed Concepts

57. (c) (a) Z was taken from X-ray scattering experiments.

(b) Validity not known earlier; established by Rutherford's experiments.

(c) Yes, the experiments said $r < \dfrac{Ze^2}{2\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$.

This sets upper limit for r.

(d) Lower limit of r not set.

58. (c) Photon energy $= hf = 13.6\left[1 - \dfrac{1}{25}\right]$ eV $= 13$ eV

Photon momentum = momentum of hydrogen atom:

$\Rightarrow \quad p = \dfrac{hf}{c}$ or $mv = \dfrac{hf}{c}$

$$v = \frac{hf}{mc} = \frac{13 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27} \times 3 \times 10^8} = 4\text{ m s}^{-1}$$

59. (d) Rydberg's constant determines the frequencies. We have $R \propto m$. So, modified R for positronium atom is half of H atom. Hence, frequencies are reduced to half.

60. (a) $\dfrac{1}{\lambda} = R\left[\dfrac{1}{1^2} - \dfrac{1}{3^2}\right] \Rightarrow \lambda = \dfrac{9}{8R}$

Again, $\dfrac{1}{\lambda'} = R\left[\dfrac{1}{2^2} - \dfrac{1}{4^2}\right] \Rightarrow \lambda' = \dfrac{16}{3R}$

Now, $\dfrac{\lambda'}{\lambda} = \dfrac{16}{3R} \times \dfrac{8R}{9}$ or $\lambda' = \dfrac{128}{27}\lambda$

61. (c) Volume occupied by one mole of gold

$$= \frac{197\text{ g}}{19.7\text{ g cm}^{-3}} = 10\text{ cm}^3$$

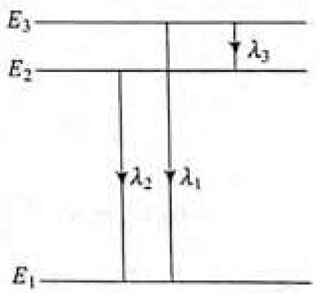
Volume of one atom

$$= \frac{10}{6 \times 10^{-23}} = \frac{5}{3} \times 10^{23}\text{ cm}^3$$

Let r be the radius of the atom. Therefore,

$$\frac{4}{3}\pi r^3 = \frac{5}{3} \times 10^{23} \quad \text{or} \quad r \cong 1.5 \times 10^{-10}\text{ m}$$

62. (c) Let the three energy levels be E_1, E_2, and E_3. The wavelengths λ_1, λ_2, and λ_3 of the spectral lines corresponding to the three energy transitions are depicted as shown in figure.



$E = hf = \dfrac{h}{\lambda}$ or $E \propto \dfrac{1}{\lambda}$

(given $\lambda_1 < \lambda_2 < \lambda_3$)

Thus, for the three wavelengths, we have

$$\begin{cases} E_3 - E_2 = \dfrac{h}{\lambda_3} & \text{(i)} \\ E_2 - E_1 = \dfrac{h}{\lambda_2} & \text{(ii)} \\ E_3 - E_1 = \dfrac{h}{\lambda_1} & \text{(iii)} \end{cases}$$

Now, $E_3 - E_1 = (E_3 - E_2) + (E_2 - E_1)$

$\Rightarrow \quad \dfrac{h}{\lambda_1} = \dfrac{h}{\lambda_3} + \dfrac{h}{\lambda_2} \Rightarrow \dfrac{1}{\lambda_1} = \dfrac{1}{\lambda_2} + \dfrac{1}{\lambda_3}$

63. (a) Barrier height

$$= \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_c}\,\text{J} = \frac{1}{4\pi\varepsilon_0}\frac{e}{r_c}\,\text{eV}$$

$$= \frac{9\times10^9\times1.6\times10^{-19}}{10^{-14}}\,\text{eV} = 1.44\times10^5\,\text{eV}$$

$$= 0.14\ \text{MeV}$$

64. (a) The recoil momentum of atom is same as that of photon but in opposite direction.

Hence, recoil momentum:

$$P = \frac{E}{c} = \frac{12.09\times1.6\times10^{-19}}{3\times10^8}\,\text{Ns} = 6.45\times10^{-27}\ \text{Ns}$$

Note that almost whole of the energy will be carried away by the photon because it is very light in comparison to H atom.

65. (a) $T^2 \propto r^3$ and $r \propto n^2 \Rightarrow T^2 \propto n^6 \Rightarrow T \propto n^3$

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3 \Rightarrow 8 = \left(\frac{n_1}{n_2}\right)^3 \text{ or } \frac{n_1}{n_2} = 2$$

Only (a) satisfies the above, hence this is right choice.

66. (a) $M = IA = ef\pi r^2$

$= 1.6\times10^{-19}\times10^{16}\times3.14\times(0.5\times10^{-10})^2$ A m^2

$= 1.256\times10^{-23}$ Am^2

67. (d) $E_p = -\dfrac{ke^2}{r}$, $E = -\dfrac{ke^2}{2r}$

So, $E_p = 2E = 2(-13.6)$ eV $= -27.2$ eV.

Potential energy of electron in the ground state of Li^{2+} ion is $-3^2\times27.2$ eV or -244.8 eV.

68. (c) Required energy $= \left[\left(\dfrac{-13.6}{9}\right)-\left(\dfrac{-13.6}{1}\right)\right]\times9$

$$= \left[13.6-\frac{13.6}{9}\right]9 = 8\times13.6\ \text{eV}$$

Wavelength $= \dfrac{12375}{8\times13.6} = 113.7$ Å

69. (b) $U = -\dfrac{ke^2}{2R^3}$, $F = -\dfrac{dU}{dR} = -\dfrac{3ke^2}{2R^4}$

But, $F = \dfrac{mv^2}{R} \Rightarrow \dfrac{mv^2}{R} = \dfrac{3ke^2}{2R^4}$

Also, $mvR = \dfrac{nh}{2\pi}$

Solve to get: $R = \dfrac{6\pi^2ke^2m}{n^2h^2}$

70. (c) Use $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(Ze)(e)}{r_0}$

71. (b) For an elastic collision to take place, there must be no loss in the energy of electron. The hydrogen atom will absorb energy from the colliding electron only if it can go from ground state to first excited state, i.e., from $n = 1$ to $n = 2$ state. For this, hydrogen atom must absorb energy $E_2 - E_1 = -3.4-(-13.6) = 10.2$ eV

So, if the electron possesses energy less than 10.2 eV, it would never lose it and hence collision would be elastic.

72. (b) $\text{KE}_{\lambda_1} = \dfrac{hc}{\lambda_1} - \psi = e\Delta V$

$$\text{KE}_{\lambda_2} = \frac{hc}{\lambda_2} - \psi = 2e\Delta V$$

$$\Rightarrow\quad 3\left(\frac{hc}{\lambda_1}-\psi\right) = \frac{hc}{\lambda_2}-\psi$$

$$\Rightarrow\quad \psi = hc\left(\frac{3}{2\lambda_1}-\frac{1}{2\lambda_2}\right)$$

$$\Rightarrow\quad \text{KE}_{\lambda_3} = \frac{hc}{\lambda_3} - hc\left[\frac{3}{2\lambda_1}-\frac{1}{2\lambda_2}\right]$$

$$= hc\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$$

$$e\Delta V = hc\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$$

$$\Delta V = \frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$$

73. (c) Assuming that ionization occurs as a result of a completely inelastic collision, we can write

$$mv - 0 = (m + m_H)\,u$$

where m is the mass of incident particle, m_H the mass of hydrogen atom, v_0 the initial velocity of incident particle, and u the final common velocity of the particle after collision. Prior to collision, the KE of the incident particle was

$$E_0 = \frac{mv_0^2}{2}$$

The total kinetic energy after collision

$$E = \frac{(m+m_H)u^2}{2} = \frac{m^2v_0^2}{2(m+m_H)}$$

The decrease in kinetic energy must be equal to ionization energy. Therefore,

$$E_1 = E_0 - E = \left(\frac{m_H}{m+m_H}\right)E_0$$

i.e., $\dfrac{E_1}{E_0} = \dfrac{1}{1+\dfrac{m}{m_H}}$

i.e., the greater the mass m, the smaller the fraction of initial kinetic energy that be used for ionization.

74. (c) $\dfrac{hc}{\lambda} = Rhc(1-1/n^2)$

or $n = \sqrt{\dfrac{\lambda R}{\lambda R - 1}}$

75. (b) Let v = speed of neutron before collision,

v_1 = speed of neutron after collision,

v_2 = speed of proton or hydrogen atom after collision,

and ΔE = energy of excitation

From conservation of linear momentum,

$$mv = mv_1 + mv_2 \tag{i}$$

From conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E \tag{ii}$$

From Eq. (i),

$$v^2 = v_1^2 + v_2^2 + 2v_1v_2$$

From Eq. (ii),

$$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$$

$\therefore \quad 2v_1v_2 = \frac{2\Delta E}{m}$

$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$

$\Rightarrow \quad (v_1 - v_2)^2 = v^2 - 4\frac{\Delta E}{m}$

As $v_1 - v_2$ must be real, therefore

$$v^2 - 4\frac{\Delta E}{m} \geq 0$$

or $\quad \frac{1}{2}mv^2 \geq 2\Delta E$

The minimum energy that can be absorbed by hydrogen atom in ground state to go into excited state is 10.2 eV. Therefore,

$$\frac{1}{2}mv_{\min}^2 = 2 \times 10.2 \text{ eV} = 20.4 \text{ eV}$$

76. (d) $E_n \propto \frac{1}{n^2}$ and $r_n \propto n^2$

Therefore, E_nr_n is independent of n.

Hence, $E_1r_1 = (13.6 \text{ eV})\ (0.53 \text{ Å})$

$= 7.2$ eV-Å = constant

77. (b) $\lambda = \frac{h}{p}$ (for electron)

or $\quad p = h/\lambda$

and $\quad E = \frac{hc}{\lambda}$ (for photon)

$\therefore \quad \frac{p}{E} = \frac{1}{c} = \frac{1}{3 \times 10^8} = 3.33 \times 10^{-9} \text{ s m}^{-1}$

78. (b) Potential energy = $-2 \times$ Kinetic energy = $-2E$

Total energy = $-2E + E = -3.4$ eV = $-E$

or $\quad E = 3.4$ eV

p = momentum, m = mass of electron

$$E = \frac{p^2}{2m}$$

or $\quad p = \sqrt{2mE}$

$$= \sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}} = 10^{-24}$$

de Brogile wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = 6.6 \times 10^{-10}$ m

79. (b) de Broglie wavelength of electron in hydrogen atom

$$= \frac{h}{mv} = \frac{2\pi r_n}{n}$$

For second Bohr orbit, $\lambda = \frac{600 \times 10^{-9}}{2} = 3000 \times 10^{-9}$ m

$$\lambda = \sqrt{\frac{150}{V}} \text{ Å} = 300 \text{ Å}$$

$\therefore \quad V = \frac{150}{(3000)^2} = \frac{5}{3} \times 10^{-5} \text{ V}$

80. (d) Angular momentum, $L = 4.2176 \times 10^{-34} = \frac{n_2h}{2\pi}$

$\Rightarrow \quad n_2 = 4$

For the transition from $n_2 = 4$ to $n_1 = 3$, the wavelength of spectral line = λ

$$\frac{1}{\lambda} = \frac{13.6}{hc}\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{13.6 \text{ eV}}{1240 \text{ eV nm}}\left(\frac{7}{9 \times 16}\right)$$

$$\lambda = \frac{1240 \times 144}{13.6 \times 7} = 1876 \text{ nm} = 18760 \text{ Å}$$

$$= 1.876 \times 10^4 \text{ Å}$$

Archives

1. (c) $E_2 = \frac{13.6}{4}$ eV = 3.4 eV

2. (d) $E_{n_1 \to n_2} = -13.6\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]; n_1 = 2$ and $n_2 = 1$

$$= -13.6 \times \frac{3}{4} = -10.2 \text{ eV}$$

3. (d) The masses of the two nuclei are different hence the wavelengths involved in the spectrum of deuterium ($^2_1$D) are slightly different from that of hydrogen ($^1_1$H) spectrum

4. (b) $E_n = \frac{13.6}{n^2} \times Z^2$. For first excited state $n = 2$ and for Li^{++}, $Z = 3$

$\Rightarrow \quad E = \frac{13.6}{4} \times 9 = 30.6$ eV

5. (d) $E_n = \frac{13.6Z^2}{n^2}$ eV

$\Rightarrow \quad E_n = -\frac{k}{n^2}$ eV

$\therefore \quad E_1 = -\frac{k}{1}, E_2 = -\frac{k}{4}, E_3 = -\frac{k}{9}, E_4 = -\frac{k}{16}$

Now, $E_2 - E_1 = -\frac{k}{4} - \left(-\frac{k}{1}\right)$

$$= k - \frac{k}{4} = \frac{3k}{4}$$

$$E_4 - E_2 = -\frac{k}{16} - \left(-\frac{k}{4}\right)$$

$$= \frac{k}{4} - \frac{k}{16} = \frac{3k}{16}$$

$$E_4 - E_3 = -\frac{k}{16} - \left(-\frac{k}{9}\right)$$

Clearly, $E_2 - E_1 > E_4 - E_2 > E_4 - E_3$

So, transition III is most energetic.

6. (b) Transition from $n = 2 \to n = 1$

$$(hf)_{2\to1} = -13.6\left(\frac{1}{2^2} - \frac{1}{1^2}\right) \text{eV} = 10.2 \text{ eV}$$

Transition from $n = 6 \rightarrow n = 2$

$$(hf)_{6\to 2} = -13.6\left(\frac{1}{6^2} - \frac{1}{2^2}\right) = +13.6 \times \frac{2}{9}$$

Hence $E_{n=6} \rightarrow E_{n=2} < E_{n=2} \rightarrow E_{n=1}$.

It means the frequency for transition $n = 2$ to $n = 1$ is highest.

Therefore, f is highest.

7. (d) $L = \frac{nh}{2\pi}$

$\Rightarrow \quad mvr_n = \frac{nh}{2\pi}$

Also

$\frac{mv^2}{r_n} = \frac{k}{r_n}$

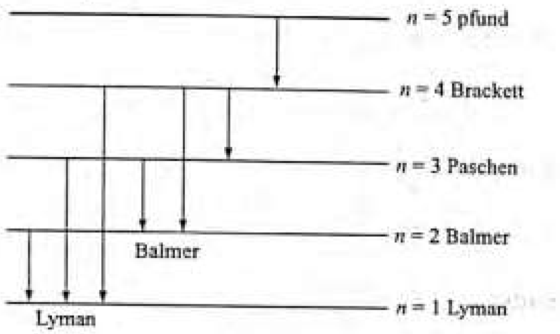
$\Rightarrow \quad mv^2 = k$

$\Rightarrow \quad T_n = \frac{1}{2}mv^2 = \frac{1}{2}k$, which is independent of n

$r_n = \frac{nh}{2\pi mv} = \frac{nh}{2\pi\sqrt{km}}$

$\therefore \quad r_n \propto n$

8. (d)



The transition $4 \rightarrow 3$ belongs to Paschen series. This is not in the ultraviolet region but this is in the infrared region.

The transition $5 \rightarrow 4$ also belongs to Brackett series and it is in the infrared region.

9. (c) $E_1 = -\frac{13.6(3)^2}{(1)^2}$, $E_2 = -\frac{13.6(3)^2}{(3)^2}$

$\therefore \quad \Delta E = E_2 - E_1$

$$= 13.6(3)^2\left[1 - \frac{1}{9}\right] = \frac{13.6 \times 9 \times 8}{9} = 108.8 \text{ eV}$$

10. (d) If $n = 4$

lines $= \frac{n(n-1)}{2} = 6$

11. (c) Let the masses of two atoms of the molecule be m_1 and m_2 at a distance r apart. The distances of the atoms from the centre of mass of the molecule be r_1 and r_2. The moment of inertia of this molecule about an axis passing through its centre of mass and perpendicular to a line joining the atoms, $I = m_1r_1^2 + m_2r_2^2$

Also we can write, $m_1r_1 = m_2r_2$ or $r_1 = \frac{m_2}{m_1}r_2$

But $\quad r_1 + r_2 = r$

Hence $r_1 = \frac{m_2}{m_1}(r - r_1) \Rightarrow r_1 = \frac{m_2 r}{m_1 + m_2}$

Similarly, $r_2 = \frac{m_1 r}{m_1 + m_2}$

Therefore, the moment of inertia can be written as

$$I = m_1\left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1 + m_2}\right)^2$$

$$= \frac{m_1 m_2}{m_1 + m_2}r^2 \qquad \text{(i)}$$

According to Bohr's quantization condition

$$L = \frac{nh}{2\pi} \text{ or } L^2 = \frac{n^2h^2}{4\pi^2} \qquad \text{(ii)}$$

Rotational energy, $E = \frac{L^2}{2I}$

$$E = \frac{n^2h^2}{8\pi^2 I} \qquad \text{(Using (ii))}$$

$$= \frac{n^2h^2(m_1 + m_2)}{8\pi^2(m_1m_2)r^2} \qquad \text{(Using (i))}$$

$$= \frac{n^2\hbar^2(m_1 + m_2)}{2m_1m_2r^2} \qquad \left(\because \hbar = \frac{h}{2\pi}\right)$$

12. (c) When an electron makes a transition from an energy level with n to $n - 1$, for a hydrogen like atom, the frequency of emitted radiation is

$$f = RcZ^2\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= RcZ^2\left[\frac{n^2 - (n-1)^2}{(n^2)(n-1)^2}\right] = \frac{RxZ^2(2n-1)}{n^2(n-1)}$$

As $\quad n >> 1$

$\therefore \quad f = \frac{RcZ^2 2n}{n^4} = \frac{2RcZ^2}{n^3}$ or $f \propto \frac{1}{n^3}$

13. (d) Radius of a charged particle rotating in a constant magnetic field is given by $R = \frac{mv}{qB}$

or $\quad R^2 = \frac{m^2v^2}{q^2B^2} = \frac{2m\left(\frac{1}{2}mv^2\right)}{q^2B^2} = \frac{2m(K)}{q^2B^2}$

Hence kinetic energy of the charged particle,

$\Rightarrow \quad K = \frac{q^2B^2R^2}{2m}$

$\Rightarrow \quad K_{max} = \frac{q^2B^2R^2_{max}}{2m} = 0.80 \text{ eV}$

Energy of photon corresponding transition from orbit $3 \to 2$ in hydrogen atom,

$$E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89 \text{ eV}$$

Using Einstein photoelectric equation,

$$E = K_{max} + \phi$$

$$\Rightarrow \quad 1.89 = 0.8 + \phi$$

$$\Rightarrow \quad \phi = 1.09 \approx 1.1 \text{ eV}$$

14. (a)

$$\text{PE} = -27.2\frac{z^2}{n^2} \text{ eV}$$

$$\text{TE} = -\frac{13.6 z^2}{n^2} \text{ eV}$$

$$\text{KE} = \frac{13.6 z^2}{n^2} \text{ eV}$$

$\text{KE} = \frac{13.6}{n^2}$ eV, As n decreases, KE $\uparrow$

$\text{PE} = -\frac{27.2}{n^2}$ eV, as n decreases, PE $\downarrow$

$\text{TE} = -\frac{13.6}{n^2}$ eV, as n decreases, TE $\downarrow$

15. (c) In X-ray tube, $\frac{hc}{\lambda_{min}} = eV \quad \Rightarrow \frac{1}{\lambda_{min}} = \frac{eV}{hc}$ ...(1)

Taking log on both sides

$$\ln\left(\frac{1}{\lambda_{min}}\right) = \ln V + \ln\frac{e}{hc}$$

$$-\ln(\lambda_{min}) = \ln V + \ln\frac{e}{hc}$$

$$\ln(\lambda_{min}) = -\ln V - \ln\left(\frac{e}{hc}\right)$$

It is a straight line with –ve slope.

16. (b) Using $\Delta E = \frac{hC}{\lambda}$

For λ_1, $E - (-2E) = \frac{hC}{\lambda_1}$

$$\Rightarrow \quad \lambda_1 = \frac{hC}{E} \qquad ...(1)$$

For λ_2, $E - \left(-\frac{4E}{3}\right) = \frac{hC}{\lambda_2}$

$$\Rightarrow \quad \lambda_2 = \frac{3hC}{E} \qquad ...(2)$$

From (1) and (2), $\frac{\lambda_1}{\lambda_2} = r = \frac{1}{3}$

17. (b) de Broglie wavelength of the electron in the n^{th} state

$$\lambda_n = \frac{h}{mu} = \frac{h}{\sqrt{2mk_n}} \Rightarrow k_n = \frac{h^2}{2m\lambda_n^2} = -E_n$$

Similarly for ground state $K_g = \frac{h^2}{2m\lambda_g^2} = -E_g$

$$\Rightarrow K_g - K_n = \frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]$$

for emitted photon

$$\frac{hc}{\Lambda_n} = E_n - E_g = K_g - K_n \Rightarrow \Lambda_n = \frac{hc}{K_g - K_n}$$

$$\Lambda_n = \frac{hc}{\frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]} = \frac{2mc}{h\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2\lambda_n^2}\right)} = \frac{2mc\lambda_g^2\lambda_n^2}{h(\lambda_n^2 - \lambda_g^2)}$$

$$\Lambda_n = \frac{2mc\lambda_g^2\lambda_n^2}{h\left(\lambda_n^2 - \lambda_g^2\right)} = \frac{2mc\lambda_g^2\lambda_n^2}{h\lambda_n^2\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)} = \frac{2mc\lambda_g^2}{h}\left[1 - \left(\frac{\lambda_g}{\lambda_n}\right)^2\right]^{-1}$$

$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\left[1 + \left(\frac{\lambda_g}{\lambda_n}\right)^2 + \text{higher power of } \frac{\lambda_g}{\lambda_n}\right]$$

But $\lambda_n >> \lambda_g$

$$\Lambda_n \approx \frac{2mc\lambda_g^2}{h}\left(1 + \left(\frac{\lambda_g}{\lambda_n}\right)^2\right) \approx \left(\frac{2mc\lambda_g^2}{h} + \frac{2mc\lambda_g^4}{h}\left(\frac{1}{\lambda_n^2}\right)\right)$$

$$\Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

where $A = \frac{2mc\lambda_g^2}{h}$ & $B = \frac{2mc\lambda_g^4}{h}$

18. (a) The energy of a photon in transition of an electron in atom

$$h\nu = E_0\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

for Lyman series for series limits $n_2 = \infty$, $n_1 = 1$

$$h\nu_L = E_0[1] \qquad ...(1)$$

for P fund series limits $n_2 = \infty$, $n_1 = 5$

$$h\nu_P = E_0\left[\frac{1}{25}\right] \qquad ...(2)$$

By dividing equation (1) and (2)

$$\frac{\nu_L}{\nu_P} = \frac{25}{1} \Rightarrow \nu_P = \frac{\nu_L}{25}$$

CHAPTER 30: NUCLEAR PHYSICS

Concept Application Exercise 30.1

1. Number of protons in nucleus = atomic number = 11
Number of electrons in an atom = Number of protons = 11
Number of neutrons = mass number (A) – atomic number (Z)
$= 24 - 11 = 13$

2. In order to compute binding energy, let us first find the total mass of all protons and neutrons in Nb and subtract mass of the Nb:
Given: $m_p = 1.007276$ u and $m_n = 1.008665$ u
Number of protons: $N_p = 41$
Number of neutrons: $N_n = 93 - 41 = 52$
Mass difference: $\Delta m = 41m_p + 52m_n - m_{Nb}$
$= 41(1.007825\text{ u}) + 52(1.008665\text{ u}) - (92.9063768\text{ u})$
$= 0.865028$ u
Thus, binding energy per nuclear is
$$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.865028\,u)(931.5\text{MeV/u})}{93}$$
$= 8.66$ MeV nucleon^{-1}

3. Nuclei are stable because of the presence of another, short-range (about 2 fm) force, the *nuclear force*. This is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force. The attractive nuclear force also acts between pairs of neutrons and between neutrons and protons.

The nuclear attractive force is stronger than the Coulomb repulsive force within the nucleus (at short ranges). If this were not the case, stable nuclei would not exist.

4. $\left(\frac{BE}{A}\right)_P = \frac{100}{10} = 10,$
$\left(\frac{BE}{A}\right)_Q = \frac{60}{5} = 12$, and $\left(\frac{BE}{A}\right)_R = \frac{66}{6} = 11$
Therefore, stability order is $Q > R > P$.

5. After releasing 10 MeV, it will become more stable and its binding energy will increase.
New binding energy = 100 + 10 = 110 MeV

6. $(B_3 + B_4) - (B_1 + B_2)$

Concept Application Exercise 30.2

1. (a) The initial number of nuclei is $N_0 = \frac{mN_A}{M}$

(b) The number of undecayed nuclei after a time t is
$$N = N_0e^{-\lambda t} = \left(\frac{mN_A}{M}\right)e^{-\lambda t}$$
Therefore, the number of decayed nuclei is
$$N' = N_0 - N = \frac{mN_A}{M}(1 - e^{-\lambda t})$$

(c) The activity of the sample is $A = A_0e^{-\lambda t} = \lambda N_0e^{-\lambda t}$
$$A = \left(\frac{mN_A\lambda}{M}\right)e^{-\lambda t}$$

2. We know that $N = N_0e^{-\lambda t}$, where $\lambda = \frac{\ln 2}{T} = \frac{1}{T_{av}}$
Here, $N = (1-\eta)N_0$
$$(1-\eta)N_0 = N_0e^{-\lambda t}$$
$$t = \frac{1}{\lambda}\ln\left|\frac{1}{(1-\eta)}\right|$$
(a) Thus, $t = \frac{T}{\ln 2}\left[\ln\left|\frac{1}{1-\eta}\right|\right] = \frac{T}{0.693}\ln\left|\frac{1}{(1-\eta)}\right|$

(b) $t = T_{av}\ln\left|\frac{1}{(1-\eta)}\right|$

3. $A_1 = A_0e^{-\lambda t_1}$ and $A_2 = A_0e^{-\lambda t_2}$
$$\therefore \quad \frac{A_1}{A_2} = e^{\lambda(t_2 - t_1)}$$
$$\Rightarrow \quad \lambda(t_2 - t_1) = \ln\left|\frac{A_1}{A_2}\right| \quad \Rightarrow \quad T = \frac{1}{\lambda} = \frac{t_2 - t_1}{\ln\left|\frac{A_1}{A_2}\right|}$$

4. As the net electric charge is conserved, hence number of protons before the reaction is equal to the total number after the reaction. The total number of nucleons is conserved as well, so we can equate the total number before and after the reaction.
The results are listed in the table as follows:

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	2 + 13	$= Z + 0$
Total number of nucleons	4 + 27	$= A + 1$

Thus, we get $Z = 15$ and $A = 30$. Since $Z = 15$ identifies the element as phosphorus, the nucleus produced is $^{30}_{15}P$.

5. A deuteron is the nucleus of deuterium, the isotope of hydrogen containing one proton and one neutron, ^2_1H, we have the reaction
$$^1_0n + ^{16}_8O \to ^A_ZX + ^2_1H$$
Proceeding similar to previous problem,

Conserved quantity	Before reaction	After reaction
Total electric charge (number of protons)	1 + 16	$= Z$
Total number of nucleons	0 + 8	$= A$

Hence, the product nucleus must have $Z = 7$ and $A = 15$. From the periodic table we find that it is nitrogen that has $Z = 7$, so the nucleus is $^{15}_7N$. The reaction can be written $^{16}_8O$ (n, d) $^{15}_7N$, where d represents deuterium ^2_1H

6. The equation for decay is ${}^{238}_{92}U \rightarrow {}^{A}_{Z}X + {}^{4}_{2}He$
Proceed similar to previous problem, to get $Z = 90$ and $A = 234$. Thus, the nucleus is ${}^{234}_{90}Th$.

Exercises

Nuclear Reaction, Mass Energy and Binding Energy

1. (b) $A = 238 - 4 = 234$ and $Z = 92 - 2 = 90$.
2. (b) Mass of nucleus is less than the sum of the masses of the constituent particles.
3. (d) Packing fraction $= \frac{M - A}{A}$
4. (c) Energy released while forming a nucleus is known as binding energy (by definition).
5. (a) B.E. per nucleon is maximum for Fe^{56}. For further detail refer theory.
6. (d) $E = \Delta mc^2 = 1 \times (3 \times 10^8)^2 = 9 \times 10^{10}$ J

$$\Rightarrow \quad E = \frac{9 \times 10^{16}}{1.6 \times 10^{-19}} = 5.625 \times 10^{35} \text{ eV} = 5.625 \times 10^{29} \text{MeV}$$

7. (c) Total binding energy of helium atom ($_2He^4$)
$= 4 \times 7 = 28$ MeV
Total binding energy of deuteron $_1H^2(1p + 1n)$
$= 2 \times 1.1 = 2.2$ MeV
$\therefore$ Binding energy of 2 deuterons $= 2 \times 2.2 = 4.4$ MeV
So, the energy released in forming helium nucleus from two deuterons $= (28 - 4.4)$ MeV $= 23.6$ MeV
8. (a) When a free neutron decays to a proton along with an electron and an antineutrino, the Q value of the reaction is positive which means the reaction is possible all by itself, while a free proton cannot convert itself into a neutron due to negative Q value.
In beta minus decay, the electron originates from nucleus only, by the transformation of neutron into a proton, with simultaneous emission of an antineutrino.
9. (b) Nuclear forces are charge independent.
10. (c) The minimum energy needed to carry out an endothermic reaction is greater than the Q value of the reaction. This is because to conserve the momentum some extra energy has to be provided.
$KE_{min} = \left(1 + \frac{m}{M}\right) \times |Q|$, where m is the mass of the incident particle and M is the mass of target.
11. (b) During fusion, binding energy of daughter nucleus is always greater than the total binding energy of the parent nuclei. The difference of binding energies is released. Hence,
$Q = E_2 - 2E_1$
12. (d) Energy released would be
ΔE = total binding energy of $_2He^4$
$- 2 \times$ (total binding energy of $_1He^4$)
$= 4 \times 7.0 - 2\,(1.1)\,(2)$
$= 23.6$ MeV
13. (a) $_{92}U^{238} + {}_0n^1 \rightarrow {}_{92}U^{239} \rightarrow {}_{-1}e^0 + {}_9Np^{239} \rightarrow {}_{-1}e^0 + {}_{94}Pu^{239}$
14. (d) The emission of antineutrino is a must for the validity of different laws.
15. (a) ${}^{A}_{Z}X \rightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y$
${}^{A-4}_{Z-2}Y \rightarrow e^+ + {}^{A-4}_{Z-3}Y'$
During β^+ emission,
$_1p^1 \rightarrow {}_0n^1 + \beta^+$
The proton changes into neutron. So, charge number decreases by 1 but mass number remains unchanged.
16. (d) Atomic mass M(H) of hydrogen and nuclear mass (M_n) are
$M(H) = 1.007825$ u and $M_n = 1.008665$ u
Mass defect,
$\Delta m = [M(H) + M_n - M(D)]$
$M(D)$ = mass of deuteron $= 2.016490$ u $- 2.014102$ u
$= 0.002388$ u
As 1 u corresponds to 931.494 MeV energy, therefore, mass defect corresponds to energy,
$E_b = 0.002388 \times 931.5 = 2.224$ MeV
17. (d) Disintegration of deuteron to a proton and a neutron can be represented by
${}^{2}_{1}H + Q \rightarrow {}^{1}_{1}H + {}^{1}_{0}n$
The energy captured is the γ-ray photon E_γ is given by
$E_\gamma + 1876 = 939 + 940$
$\Rightarrow \quad E_\gamma = (939 + 940) - 1876 = 3$ MeV.
18. (d) Refer to the definition of mass defect.

Radioactivity

19. (b) For α-decay: ${}_xA^y \rightarrow {}_{x-2}B^{y-4} + \alpha$
For β^- decay: ${}_xA^y \rightarrow {}_{x+1}B^y + {}_{-1}\beta^0$
For β^+ decay: ${}_xA^y \rightarrow {}_{x-1}B^y + {}_{+1}\beta^0$
For k-capture, there will be no change in the number of protons. Hence, only case in which number of protons increases is β^- decay.
20. (c) $N = N_0 e^{-\lambda t}$, $N_Y = N_0(1 - e^{-\lambda t})$
Rate of formation of Y is
$$\frac{dN}{dt} = +\lambda N_0 e^{-\lambda t}$$
which decreases exponentially with time.
21. (d) α-decay decreases mass number by 4 and reduces charge number by 2. β-decay keeps mass number unchanged and increases charge by 1. Clearly, option (d) is the right choice.
22. (b) Decrease in mass number $= 232 - 208 = 24$
Number of α-particles emitted $= \frac{24}{4} = 6$
Due to emission of 6 particles, decrease in charge number is 12. But actual decrease in charge number is 8. Clearly, 4 β-particles are emitted.
23. (d) $\frac{9}{16} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$
$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{2T_{1/2}}} \Rightarrow \left(\frac{N}{N_0}\right)^2 = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

or $\left(\dfrac{N}{N_0}\right)^2 = \dfrac{9}{16}$ or $\dfrac{N}{N_0} = \dfrac{3}{4}$

Note the special technique used in the problem.

24. (a) Two α-particles reduce mass number by 8.
Therefore, new mass number = 180 − 8 = 172
Emission of two α-particles reduces charge number by 4. Emission of β-particles increases charge number by 1.
Therefore, the new charge number = 72 − 4 + 1 = 69.

25. (b) $\dfrac{N_0}{4} = \dfrac{N_0}{2n} \Rightarrow n = 2$

Thus, 10 days = 2 half-lives

$\therefore$ Half-life = 8 days

26. (b) $T_{1/2} = \dfrac{0.693}{\lambda}$ or $T_{1/2} = 0.693\left[\dfrac{1}{\lambda}\right]$

or $T_{1/2} = 0.693\ \tau$

Clearly, $x = 0.693$.

27. (c) $\dfrac{A_0}{3} = A_0\left(\dfrac{1}{2}\right)^{\frac{9}{T_{1/2}}}$

$A' = \dfrac{A_0}{3}\left(\dfrac{1}{2}\right)^{\frac{9}{T_{1/2}}}$

Dividing, we get $\dfrac{A' \times 3}{A_0} = \dfrac{1}{3}$ or $A' = \dfrac{A_0}{9}$

28. (d) After n half-lives, the radioactive nuclei remaining is $\dfrac{N_0}{2^n}$.

So, number of nuclei disintegrated in n half-lives is $\left(N_0 - \dfrac{N_0}{2^n}\right)$

For $n = \dfrac{1}{2}$, the fraction distintegrated is $\left(1 - \dfrac{1}{\sqrt{2}}\right)$.

29. (c) Since four half-lives have elapsed.

$$A = \frac{A_0}{2^4} = \frac{A_0}{16} = \frac{1.6}{16}\text{ curie} = 0.1\text{ curie}$$

30. (c) Decay constant, $\lambda = 10^{-6}\ s^{-1}$. The half-life $T_{1/2}$ is thus given by

$$T_{1/2} = \frac{0.639}{\lambda} = \frac{0.639}{10^{-6}} = 0.639 \times 10^6\text{ s}$$
$$= 192.5\text{ h} \approx 8\text{ days} = 1.14\text{ week} \approx 1\text{ week}$$

31. (b) The radioactive decay constant λ is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{32}\text{h}^{-1}$$

From the equation $N = N_0e^{-\lambda t}$, the fraction of a sample remaining after 16 h is given by

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-\left(\frac{0.693}{32}\right)} = e^{-0.3465} = 0.71$$

32. (a) From given information, $\dfrac{dN}{dt} = \dfrac{-0.04N}{3600}$

Comparing above equation with standard decay equation,

$$\frac{dN}{dt} = -\lambda N$$

$$\lambda = 1.1 \times 10^{-5}\ s^{-1}$$

$$\therefore \quad \tau = \frac{1}{\lambda} = \frac{3600}{0.04}\text{s} = 25\text{ h}$$

33. (a) Probability of survival for any nucleus at time t is

$$P = \frac{N}{N_0} = \frac{N_0e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

So, in one mean life, required probability is $e^{-\lambda \times \frac{1}{\lambda}} = \dfrac{1}{e}$

34. (d) $N_{x_1} = N_0e^{-10\lambda t}$

$N_{x_2} = N_0e^{-\lambda t}$

$$\frac{N_{x_1}}{N_{x_2}} = \frac{1}{e} = \frac{e^{-10\lambda t}}{e^{-\lambda t}} = e^{-9\lambda t}$$

$$9\lambda t = 1 \Rightarrow t = \frac{1}{9\lambda}$$

35. (c) Let N be the number of nuclei at any time t. Then,

$$\frac{dN}{dt} = 200 - \lambda N$$

$$\therefore \quad \int_0^N \frac{dN}{200 - \lambda N} = \int_0^t dt$$

or $N = \dfrac{200}{\lambda}(1 - e^{-\lambda t})$

Given: $N = 100$ and $\lambda = 1\ s^{-1}$

$\therefore \quad 100 = 200\,(1 - e^{-t})$

or $e^{-t} = \left(\dfrac{1}{2}\right) \quad \therefore \quad t = \ln(2)$ s

36. (c) Activity, $R = \lambda N$. Number of nuclei (N) per mole are equal for both the substances.

$$\therefore \quad R \propto \lambda \quad \text{or} \quad \frac{R_1}{R_2} = \frac{\lambda_1}{\lambda_2} = \frac{4}{3}$$

37. (b) Activity of a radioactive substance,

$$R = \lambda N \quad \therefore \quad \lambda = \frac{R}{N}$$

Here, $R = N_2$ particles per second and $N = N_1$.

$$\therefore \quad \lambda = \frac{N_2}{N_1}$$

38. (a) Let $t = 0$, $M_0 = 10$ g

$$t = 2\tau = 2\left(\frac{1}{\lambda}\right)$$

Then,

$$M = M_0e^{-\lambda t} = 10e^{-\lambda\left(\frac{2}{\lambda}\right)} = 10\left(\frac{1}{e}\right)^2 = 1.35\text{ g}$$

39. (a) $N = \dfrac{N_0}{2^{t/T_{1/2}}}$

$$\frac{N_0}{16} = \frac{}{t\ T_{1/2}}$$

$$2^{t/T_{1/2}} = 16 = 2^4$$

or $\dfrac{t}{T_{1/2}} = 4$ or $T_{1/2} = \dfrac{t}{4} = \dfrac{24}{4}\text{h} = 6\text{ h}$

40. (a) $\frac{1}{16}=\frac{1}{2^{\frac{t}{100}}}$

or $\frac{1}{2^4}=\frac{1}{2^{t/100}}$ or $4=\frac{t}{100}$ or $t=400\ \mu s$

41. (b) Number of α-particles emitted $=\frac{232-208}{4}=6$

Decrease in charge number due to α-emission = 12.
But actual decrease in charge number = 90 – 82 = 8.
Clearly, four β-particles are emitted.

42. (a) $\frac{A_2}{A_1}=\frac{N_2}{N_1}$

$\frac{A_2}{10^3}=\frac{1}{2}$ or $A_2=\frac{1000}{2}=500\ s^{-1}$

43. (b) $\frac{N}{N_0}=\frac{1}{2^{5T_{1/2}/T_{1/2}}}$

$\frac{N}{N_0}=\frac{1}{2^5}$ $\therefore$ $\frac{N}{N_0}\times 100=\frac{100}{32}=3.125$

44. (d) $N=N_0e^{-\lambda t}$

$\Rightarrow$ $\frac{N_0}{N}=e^{\lambda t}$

$\lambda t=\log_e\frac{N_0}{N}$

$t=\frac{1}{\lambda}\log_e\frac{N_0}{N}$ $\Rightarrow$ $t\propto\log_e\frac{N_0}{N}$

$5\propto\log_e\frac{100}{90}$ and $20\propto\log_e\frac{N_0}{N}$

Dividing, $\frac{5}{20}=\frac{\log_{10}\frac{100}{90}}{\log_{10}\frac{N_0}{N}}$

or $\log_{10}\frac{N_0}{N}=4\log_{10}\frac{10}{9}$

or $\frac{N_0}{N}=\left(\frac{10}{9}\right)^4$ $\Rightarrow$ $\frac{N}{N_0}=0.6561$

Percentage of substance decayed is
$(1-0.6561)\times 100=34.39$

45. (d) Here, $T=26.8$ min $=26.8\times 60$ s

$\therefore$ Decay constant,

$\lambda=\frac{0.693}{T}=\frac{0.693}{26.8\times 60}=4.32\times 10^{-4}\ s^{-1}$

Now, 1 curie is equal to 3.71×10^{10} disintegrations per second $=3.71\times 10^{10}$.
If N be the number of atoms in one curie, then

$-\frac{dN}{dt}=\lambda N$

or $3.71\times 10^{10}=431\times 10^{-4}N$

$\therefore$ $N=\frac{3.71\times 10^{10}}{4.31\times 10^{-4}}=8.607\times 10^{13}$

Further, atomic weight of RaB = 214 and Avogadro's number $=6.025\times 10^{23}$.

Mass of one atom $=\frac{214}{6.025\times 10^{23}}$ g

Mass of N atoms $=\left(\frac{214}{6.025\times 10^{23}}\right)\times(8.607\times 10^{13})$
$=3.064\times 10^{-8}$ g

Nuclear Fission, Fusion and Nuclear Energy

46. (b) Power P of fission reactor,
$P=10^6$ watt $=10^6$ joule/second
Time $=t=1$ day $=24\times 36\times 10^2$ s

Energy produced, $U=Pt$

or $U=10^6\times 24\times 36\times 10^2$
$=24\times 36\times 10^8$ joule

Energy released per fission of $U^{235}=200$ MeV $=32\times 10^{-12}$ J

Number of U^{235} atoms used $=\frac{24\times 36\times 10^8}{32\times 10^{-12}}=27\times 10^{20}$

Mass of 6×10^{23} atoms of $U^{235}=235$ g

Mass of 27×10^{20} atoms of U^{235}
$=\left(\frac{235}{6\times 10^{23}}\right)(27\times 10^{20})=1.058\text{ g}=1\text{ g}$

47. (b) The nuclear fission differs from other nuclear reactions in three respects.

(a) The nucleus is deeply divided into two large fission fragments or nuclei of roughly equal mass. The nuclei or fission fragments fly apart at great speed and thus possess large kinetic energies that carry off the greater part of the energy released.

(b) The mass decrease is appreciable and hence large energy is released.

(c) Other neutrons, called fission neutrons, are emitted in the process. Small amount of energy is released in the form of radiation.

48. (c) The number of nuclei in 1 kg ^{235}U is

$N=\frac{N_A}{235}\times(1\times 10^3)$

$N=\frac{6.023\times 10^{23}}{235}\times 10^3=2.56\times 10^{24}$ nuclei

Total energy released is
$E=N\times 200\text{ MeV}=5.12\times 10^{26}$ MeV

49. (a) Since scheme A releases more energy than scheme B, scheme A is more likely to occur. This is because the more the energy released, the more stable the daughter nucleus is. A heavy nucleus undergoes fission such that its products will be more stable than the parent nucleus.

50. (c) The net reaction is

$3(^2_1H)\rightarrow {^4_2He}+n+p$

$Q=[3\times m(^2H)-m(^4He)-m(n)-m(p)]\times 931$ MeV
$=3.87\times 10^{-12}$ J

This is the energy produced by the consumption of 3 deuteron atoms. So, the total energy released by 10^{40} deuterons is

$$\frac{3.87\times10^{-12}}{3}\times10^{40}=1.29\times10^{28}\text{J}$$

Let total supply of deuterons in star be exhausted in t seconds. Then,

$$10^{16}\times t=1.29\times10^{28}$$

$$\Rightarrow \quad t=1.29\times10^{12}\text{ s}$$

51. (b) Number of atoms in 2 kg fuel

$$\frac{2}{235}\times6.02\times10^{26}=5.12\times10^{24}$$

Fission rate = Number of atoms fissioned in one second

$$=\frac{5.12\times10^{24}}{30\times24\times60\times60}=1.975\times10^{18}\text{ s}^{-1}$$

Each fission gives 185 MeV. Hence, energy obtained in one second,

$$E=185\times1.975\times10^{18}\text{ MeV s}^{-1}$$

$$=185\times1.975\times10^{18}\times1.6\times10^{-19}\times10^{6}\text{J s}^{-1}=58.46\text{ MW}$$

52. (c) Let the kinetic energy of the α-particle be E_α and that of the thorium Th be E_{th}. The ratio of kinetic energies is

$$\frac{E_\alpha}{E_{th}}=\frac{\frac{1}{2}m_\alpha v_\alpha^2}{\frac{1}{2}m_{th}v_{th}^2}=\left(\frac{m_\alpha}{m_{th}}\right)\left(\frac{v_\alpha}{v_{th}}\right)^2 \quad \text{(i)}$$

By conservation of momentum, the momentum of α-particle and that of the recoiling thorium must be equal. Thus,

$$m_\alpha v_\alpha=m_{th}v_{th}$$

or

$$\frac{v_\alpha}{v_{th}}=\frac{m_{th}}{m_\alpha} \quad \text{(ii)}$$

Substituting Eq. (ii) in Eq. (i), we have

$$\frac{E_\alpha}{E_{th}}=\left(\frac{m_\alpha}{m_{th}}\right)\left(\frac{m_{th}}{m_\alpha}\right)^2=\frac{m_{th}}{m_\alpha}=\frac{234}{4}=58.5$$

Thus, the kinetic energy of the α-particle expressed as the fraction of the total kinetic energy T is given by

$$E_\alpha=\frac{58.5}{1+58.5}T=\frac{58.5}{59.5}T=0.98\,T$$

which is slightly less than T.

53. (b) Let, ${}^A_ZX\rightarrow{}^{A-4}_{A-2}Y+{}^4_2\text{He}$

$$K_\alpha=\frac{m_y}{m_y+m_\alpha}Q=\frac{A-4}{A}Q$$

or $\quad 48=\frac{A-4}{A}\times50 \quad\Rightarrow\quad A=100$

54. (c) Energy released is

$$(80\times7+120\times8-200\times6.5)=220\text{ MeV}$$

55. (d) Energy released in the fission of one nucleus = 200 MeV

$$=200\times10^6\times1.6\times10^{-19}\text{ J}=3.2\times10^{-11}\text{ J}$$

$$P=16\text{ kW}=16\times10^3\text{ watt}$$

Now, number of nuclei required per second

$$n=\frac{P}{E}=\frac{16\times10^{-13}}{3.2\times10^{-11}}=5\times10^{14}$$

56. (a) Number of fissions per second

$$=\frac{\text{Power output}}{\text{Energy released per fission}}$$

$$=\frac{3.2\quad10^6}{200\quad10^6\times1.6\times10^{-19}}=1\times10^{17}$$

$\Rightarrow$ Number of fission per minute $=60\times10^{17}=6\times10^{18}$

57. (d) Energy radiated = 1.4 kW/m^2

$$=1.4\text{ kJ/s m}^2=\frac{1.4\text{ kJ}}{\frac{1}{86400}\text{day m}^2}=\frac{1.4\times86400\text{ kJ}}{\text{day m}^2}$$

Total energy radiated/day

$$=\frac{4\pi\times(1.5\times10^{11})^2\times1.4\times8640}{1}\frac{\text{kJ}}{\text{day}}=E$$

$$\therefore \quad E=mc^2\Rightarrow m=\frac{E}{c^2}$$

$$\therefore \quad m=\frac{4\pi(1.5\times10^{11})^2\times1.4\times864000}{(3\times10^8)^2}=3.8\times10^{14}\text{kg}.$$

58. (b) $P=\frac{nE}{t}\Rightarrow300\times10^6=\frac{n\times170\times1.6\times10^{-19}}{t}$

$\therefore$ Number of atoms per second $\frac{n}{t}=1.102\times10^{19}$

Number of atoms per hour $=1.02\times10^{19}\times3600$
$=3.97\times10^{22}$

59. (b) $P=10^6$ watt

Time = 1 day = $24\times36\times10^2$ s

Energy produced,

$$U=Pt=10^6\times24\times36\times10^2=24\times36\times10^8\text{ joule}$$

Energy released per fusion reaction

$$=20\text{ MeV}=32\times10^{-13}\text{ joule}$$

Energy released per atom of ${}_1\text{H}^2=32\times10^{-13}$ joule

Number of ${}_1\text{H}^2$ atoms used $=\frac{24\times36\times10^8}{32\times10^{-13}}=22\times10^{21}$

Mass of 6×10^{23} atoms = 2g

$\therefore$ Mass of 27×10^{21} atoms $=\frac{2}{6\times10^{23}}\times27\times10^{21}=0.1\text{ g}$

Problems Based on Mixed Concepts

60. (b) Mass of one atom of U^{235} = 235.121420 amu

Mass of one neutron = 1.008665 amu

Sum of the masses of U^{235} and neutron = 236.130085
= 236.130 amu

Mass of one atom of U^{236} = 236.123050 amu = 236.123 amu

Increase in mass when one neutron is removed from
U^{236} = 236.136 − 236.123 = 0.007 amu

$\therefore$ Energy required to remove one neutron
= 0.007 × 931 MeV
= 6.517 MeV = 6.5 MeV

61. (a) $\frac{dN_A}{dt}=(-\lambda N_A)+(-2\lambda N_A)+(-2\lambda N_A)=-6\lambda N_A$

62. (a) After first half hours,

$$N=N_0\frac{1}{2}$$

For $\quad t = \frac{1}{2}$ h to $t = 1\frac{1}{2}$ h, 1 h = four half-lives

Hence, $N = \left(N_0 \frac{1}{2}\right)\left[\frac{1}{2}\right]^4 = N_0\left(\frac{1}{2}\right)^5$

For $\quad t = \frac{1}{2}$ to $t = 2$ h

$\left[\text{for both } A \text{ and } B, \frac{1}{T_{1/2}} = \frac{1}{T_{1/2}} + \frac{1}{T_{1/4}} = 2 + 4 = 6 \Rightarrow T_{1/2} = \frac{1}{6}\right]$

$\frac{1}{2}$ h = three half-lives

$\therefore \quad N = \left[\left(N_0 \frac{1}{2}\right)^5\right]\left(\frac{1}{2}\right)^3 = N_0\left(\frac{1}{2}\right)^8$

63. (b) ${}^{A}_{Z}X \xrightarrow{\text{Proton}} {}^{A}_{Z-1}Y \xrightarrow{2\alpha} {}^{A-8}_{A-5}Y$

Given: $A - 8 = 224$ and $Z - 5 = 89$

$\Rightarrow \quad A = 237, Z = 94$

64. (b) $\frac{dN_2}{dt} = \lambda N_1 - 2\lambda N_2$

For n_2 to be maximum, $\frac{dN_2}{dt} = 0$

$\Rightarrow \quad \lambda N_1 = 2\lambda N_2 \quad$ or $\quad \frac{N_1}{N_2} = 2$

65. (c) Let n collisions are required for the given condition. Then,

$$\left(\frac{1}{2}\right)^n \times 2 \text{ MeV} = 0.04 \times 10^{-6} \text{ MeV}$$

$$2^n = \frac{2}{0.04} \times 10^6 = 50 \times 10^6$$

After solving above equation, $n = 26$.

66. (c) Expected atomic mass of Cu must be less than that of zinc, but it is not so. So, it means Cu is radioactive and unstable and decays to Zn through β-decay.

67. (a) As the alpha particle decays, the daughter nucleus recoils. In such a process, the momentum conservation holds good. So,

$$p_\alpha = p_D = p$$

$$K_\alpha = \frac{p^2}{2M_\alpha} \quad \text{and} \quad K_D = \frac{p^2}{2M_D}$$

As $\quad M_D > M_\alpha$, so, $K_\alpha > K_D$.

68. (a) At present,

$$\frac{\text{Number of K atoms}}{\text{Number of Ar atoms}} = \frac{1}{7}$$

Let age of rock be n half-lives of K-nuclide. Then,

$$\left(\frac{1}{2}\right)^n = \frac{\text{Number of K-atoms present now}}{\text{Number of K-atom present initially}} = \frac{1}{1+7}$$

where number of K atoms present initially = number of K atoms + number of Ar atoms present now.

$\therefore \quad n = 3$

So, age of rock is 3 half-lives of K nuclides, i.e., 4.2×10^9 years.

69. (b) The complete fission reaction is

$${}^{235}_{92}\text{U} + n \to {}^{94}_{40}Zr + {}^{140}_{58}\text{Ce} + 2n + 6e^{-1}$$

$$Q = [m({}^{235}\text{U}) - m({}^{94}Zr) - m({}^{140}\text{Ce}) - m(n)]c^2$$

$$= 208 \text{ MeV}$$

70. (d) $\frac{3}{5}N_0 = N_0 e^{-\lambda t} \quad \Rightarrow \quad e^{\lambda t} = \frac{5}{3}$

$\log_e e^{\lambda t} = \log_e \frac{5}{3}$ or $\lambda t = \log_e \frac{5}{3}$

or $\quad t = \frac{1}{\lambda}\log_e \frac{5}{3}$

$= \frac{T}{0.693} \times 0.5 \qquad \left[\because \; T = \frac{0.693}{\lambda}\right]$

$= \frac{5570 \times 0.5}{0.693}$ years = 4018.7 years

= 4000 years

71. (c) $A = A_0 e^{-\lambda t}$; $2100 = 16000\, e^{-12\lambda} \Rightarrow e^{12\lambda} = 7.6$

$\Rightarrow \quad 12\lambda = \log_e 7.6 = 2 \Rightarrow \lambda = \frac{2}{12} = \frac{1}{6}$

$\therefore \quad T = \frac{0.6931 \times 6}{1} = 4$

72. (b) Total mass of the products = 2.0165 a.m.u., which is greater than the mass of the deuteron by 0.0024 a.m.u. The extra mass must be provided by the energy of the photon so that minimum possible frequency must be given by

$$\nu = \frac{0.0024 \text{ a.m.u. } c^2}{h} \qquad (1 \text{ a.m.u.} = 1.66 \times 10^{-27} \text{ kg})$$

$$= 5.4 \times 10^{20} \text{ Hz}$$

73. (d) Given:

X has activity A_0 at $t = 0$ and its half-life is 24 years.

Y has activity A_0 at $t = 0$ and its half-life is 16 years.

At $t = 48$ years, activity of $X = \frac{1}{4}A_0$

(2 half-lives have elapsed)

At $t = 48$ years, activity of $Y = \frac{1}{8}A_0$

(3 half-lives have elapsed)

Thus, total activity of the mixture of X and Y at $t = 48$ years is

$$\frac{1}{4}A_0 + \frac{1}{8}A_0 = \frac{3}{8}A_0$$

74. (b) After a time t, a sample of ^{238}U originally consisting of N atoms will have decayed to $Ne^{-\lambda t}$. The number of ^{206}Pb atoms,

$$N_{Pb} = N(1 - e^{-\lambda t})$$

$$\therefore \quad \frac{N_{Pb}}{N_U} = N\frac{(1 - e^{-\lambda t})}{Ne^{-\lambda t}} = 0.0058$$

$$e^{\lambda t} - 1 = 0.0058 \quad \Rightarrow \quad e^{\lambda t} = 1.0058$$

$$\therefore \quad t = \frac{1}{\lambda}\ln(1.0058) = \frac{(4.5 \times 10^9 \text{ years})}{\ln 2}\ln(1.0058)$$

$$= 0.0376 \times 10^9 \text{ years} = 38 \times 10^6 \text{ years}$$

75. (d) Nuclear reactions conserve total charge, and also conserve the total approximate mass. The other particles in the reaction will have mass = 236 – 140 – 94 = 2

The other particles are two neutrons. Hence, (a) is not correct. For nuclei, number of protons tells the charge. So, the other particles must have charge Z such that

$$92 = 54 + 38 + Z$$

$$\therefore \quad Z = 0$$

Therefore, the other particles have a total atomic mass 2 and total charge 0. Hence, only (d) is correct.

76. (c) All neutrons attract each other with the same strong nuclear force. So, the strong nuclear force holds together three protons and one neutron ($^4_3$Li) just as vigourously as it holds together two protons and two neutrons ($^4_2$He). Specifically, protons electrostatically repel other protons. This repulsion tries to make a nucleus fly apart. Since $^4_2$He contains only two protons, the attractive strong nuclear forces overcome the repulsion of the protons. Hence, the nucleus holds together. But in $^4_3$Li, the mutual repulsion of the three protons overcomes the strong nuclear attractions and the nucleus falls apart (or undergoes radioactive decay into a more stable nucleus). Therefore, the answer will be (c).

77. (d) As, $m(^2_1\text{H}) = 2.014082\ u$

and $m(^3_2\text{He}) = 3.016029$

and $m(^1_0n) = 1.008665$

Δm = mass defect

$= 2m\,(^2_1\text{H}) - m3\,(^3_2\text{He}) - m(^1_0 n)$

$= 2 \times 2.014082 - 3.016029 - 1.008665$

$\Rightarrow$ Q-value = Energy liberated in reaction

= [mass (reactants) – mass (products)] × 931.5

= $\Delta m \times 931.5$ MeV = 3.232 MeV

As Q-value is positive, so no threshold energy is required.

78. (c) $C > B > A$

α-particles have large ionisation power, they have same energy as γ-rays. So, α-rays are more dangerous than γ-rays.

Archives

1. (a) $$N = \frac{N_0}{2^{15/2}} \Rightarrow N = \frac{N_0}{2^3} = \frac{N_0}{8}$$

2. (a) Charged particles deflect in magnetic field.

3. (c) β-rays are emitted from nucleus and they carry negative charge.

4. (b) Factual information

5. (b) $$N = N_0 e^{-\lambda t}$$

$$\frac{dN}{dt} = -\lambda N e^{-\lambda t}$$

$$-5000 = -\lambda N_0\, \lambda e^{-\lambda \times 0}$$

$$-1250 = \lambda N_0\, e^{-\lambda \times 5}$$

$$\Rightarrow \quad e^{-5\lambda} = \frac{1250}{5000}$$

$$\Rightarrow \quad \lambda = \frac{2}{5}\log_e 2 = 0.4 \log_e 2$$

6. (c) The decrease in Z due to the emission of 8 α-particles is 16.

The increase in Z due to the emission of 4 β-particles is 4.

The decrease in Z due to the emission of $2\beta^+$-particles is 2.

Therefore, Z of the resultant nucleus is

$$92 - 16 + 4 - 2 = 78.$$

7. (c) As $^{133}_{55}$Cs has a larger size among the four given atoms, the electrons present in the outermost orbit will be away from the nucleus and the electrostatic force experienced by electrons due to the nucleus will be minimum. Therefore, the energy required to liberate electrons from the outer orbit will be minimum in case of $^{133}_{55}$Cs.

8. (a) To initiate the reaction,

$$\frac{3}{2}kT = 7.7 \times 10^{-14}$$

$$\Rightarrow \quad \frac{3}{2} \times 1.38 \times 10^{-23}\, T = 7.7 \times 10^{-14}$$

$$\Rightarrow \quad T = \frac{7.7 \times 10^{-14} \times 2}{3 \times 1.38 \times 10^{-23}} \text{K} = 3.7 \times 10^9 \text{ K}$$

9. (b) The energy of the first excited state of helium

$$= -\frac{3^2 \times 13.6}{2^2}\text{ eV} = -30.6 \text{ eV}$$

The energy required for the conservation of momentum

$$= -(-30.6)\text{ eV} = 30.6 \text{ eV}$$

10. (d) Using the principle of conservation of momentum, we get

$$m_1 v_1 = m_2 v_2$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{v_2}{v_1} \Rightarrow \frac{m_1}{m_2} = \frac{1}{2}$$

$$\Rightarrow \quad m_1 = \frac{m_2}{2}$$

We know that $R = R_0 A^{1/3}$,

where R is the radius of the nucleus and A is the mass number.

$$\frac{R_1}{R_2} = \frac{A_1^{1/3}}{A_2^{1/3}} = \frac{m_1^{1/3}}{m_2^{1/3}} = \left(\frac{m_1}{m_2}\right)^{1/3} = \left(\frac{m_2}{2m_2}\right)^{1/3}$$

$$= \left(\frac{1}{2}\right)^{1/3} = 1 : 2^{1/3}$$

11. (c) The binding energy per nucleon for $^2_1$H is 1.1.

Therefore, the binding energy is 2.2 MeV.

The binding energy per nucleon for $^4_2$He is 7.

Energy released = total binding energy of product

– total binding energy of reactions

= 28 × (2 × 2.2) = 28 × 4.4

= 23.6 MeV

12. (c) At closest distance of approach

Kinetic energy = Potential energy

$$\Rightarrow \quad 5 \times 10^6 \times 1.6 \times 10^{-19} = \frac{1}{4\pi\varepsilon_0} \times \frac{(ze)(2e)}{r}$$

For uranium, $z = 92$, so $r = 5.3 \times 10^{-12}$ cm

13. (b) $$N = \frac{N_0}{2^{\frac{t}{T_{1/2}}}} \text{ or } \frac{N}{N_0} = \frac{1}{2^{\frac{15}{T_{1/2}}}}$$

$$\Rightarrow \quad 1 - \frac{7}{8} = \frac{1}{2^{\frac{15}{T_{1/2}}}} \Rightarrow \frac{1}{2^3} = \frac{1}{2^{\frac{15}{T_{1/2}}}}$$

$$\therefore \quad \frac{T_1}{2} = \frac{15}{3} = 5 \text{ min}$$

14. (d) $R = R_0 A^{1/3}$

$$\frac{R_2}{R_1} = \left[\frac{125}{27}\right]^{1/3} = \frac{5}{3}$$

$\Rightarrow \quad R_2 = \frac{5}{3}R_1 = \frac{5}{3} \times 3.6$ fm

$= 6$ fm

15. (c) $X + {}_0n^1 \Rightarrow {}_3Li^7 + {}_2He^4$

So X is ${}_5B^{10}$.

16. (a) $I' = Ie^{-\mu x}$

$\Rightarrow \quad e^{\mu x} = \frac{I}{I'}$

$\Rightarrow \quad \mu x = \log_e \frac{I}{I'}$

Now, $36\mu = \log_e \frac{I \times 8}{I} = 3\log_e 2$

$x\mu = \log_e \frac{I \times 2}{I}$

$\Rightarrow \quad \frac{x}{36} = \frac{\log_e 2}{3\log_e 2} = \frac{1}{3}$

$\Rightarrow \quad x = \frac{36}{3}$ mm $= 12$ mm

17. (a) $X(n,\alpha)^7_3$ Li $\Rightarrow {}_ZX^A + {}_0n^1 \rightarrow 3Li^7 + {}_2He^4$

$Z = 3 + 2 = 5$ and $A = 7 + 4 - 1 = 10$

$\therefore \quad {}_5X^{10} = {}_5B^{10}$

18. (a) $\frac{7}{8}$ part decays, i.e. remaining part is r_n.

$$N = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} \Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{15}{T_{1/2}}} \Rightarrow T_{1/2} = 5\text{min}$$

19. (d) Energy of proton = 2 × binding energy of

${}^4_2$He – binding energy of ${}^7_3$Li

$= (2 \times 4 \times 7.06 - 7 \times 5.60)$ MeV

$= (56.48 - 39.2)$ MeV

$= 17.28$ MeV

20. (a) $Li^7 + {}_1H^1 \rightarrow {}_4B^8 + zX^A$

Z for the unknown X nucleus = (3 + 1) – 4 = 0

A for the unknown X nucleus = (7 + 1) – 8 = 0

Hence particle emitted has zero Z and zero A

It is a gamma photon.

21. (d) $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{m}$

$\Rightarrow \quad r \propto \frac{1}{m}$

22. (a) Factual information

23. (d) $T_{1/2}(X) = \tau(Y)$

$\Rightarrow \quad \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$

$\Rightarrow \quad \lambda_Y = \frac{\lambda_X}{0.693}$

$\Rightarrow \quad \lambda_Y > \lambda_X$

So Y will decay faster than X.

24. (a) Binding energy $= [ZM_P + (A - Z)M_N - M]c^2$

$= [8M_p + (17 - 8)M_N - M_O]c^2$

$= [8M_p + 9M_N - M_O]c^2$

[But the option given is negative of this].

25. (a) γ-ray emission takes place due to de-excitation of the nucleus. Therefore during γ-ray emission, there is no change in the proton and neutron number.

26. (b) Since electronic charge (1.6×10^{-19} C) universal constant. It does not depend on g.

$\therefore$ Electronic charge on the moon = electronic charge on the earth

or $\quad \frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1.$

27. (a) Statement I states that energy is released when heavy nuclei undergo fission and light nuclei undergo fusion is correct. Statement 2 is wrong.

The binding energy per nucleon, B/A, starts at a small value, reaches to a maximum at ^{62}Ni, then decreases to 7.5 MeV for the heavy nuclei. The answer is (a).

28. (a) When two nucleons combine to form a third one, and energy is released, one has fusion reaction. If a single nucleus splits into two, one has fission. The possibility of fusion is more for light elements and fission takes place for heavy elements.

Out of the choices given for fusion, only A and B are light elements and D and E are heavy elements. Therefore $A + B \rightarrow C + \varepsilon$ is correct. The possibility of fission is only for F and not C. Therefore

$F \rightarrow D + E + \varepsilon$ is the correct choice.

29. (c) After decay, the daughter nuclei will be more stable. Hence, the binding energy per nucleon will be more than that of their parent nucleus.

30. (b) Let the speed of each daughter nuclei be v_1 and v_2 respectively

Applying law of conservation of linear momentum,

$$(M + \Delta m) \times 0 = \frac{M}{2} \times v_1 - \frac{M}{2} \times v_2$$

Which gives $v_1 = v_2$

Mass defect, $\Delta M = (M + \Delta m) - \left(\frac{m}{2} + \frac{m}{2}\right)$

$= [M + \Delta m - M] = \Delta m$

Energy released, $Q = \Delta mc^2$ (i)

Also,

$$Q = \frac{1}{2}\left(\frac{M}{2}\right)v_1^2 + \frac{1}{2}\left(\frac{M}{2}\right)v_2^2 - \frac{1}{2}(M + \Delta m) \times (0)^2$$

$$\Rightarrow \quad Q = \frac{M}{2}v_1^2 \quad (\because v_1 = v_2) \qquad \text{(ii)}$$

Equating equations (i) and (ii), we get

$$\left(\frac{M}{2}\right)v_1^2 = \Delta mc^2 \Rightarrow v_1^2 = \frac{2\Delta mc^2}{M}$$

Which gives, $v_1 = c\sqrt{\frac{2\Delta m}{M}}$

31. (b) In positive beta decay, a proton is transformed into a neutron and a positron is emitted.

$p^+ \rightarrow n^0 + e^+$

The number of neutrons initially was $A - Z$.

The number of neutrons after decay is

$(A - Z) - 3 \times 2$ (due to alpha particles) $+ 2 \times 1$ (due to positive beta decay) $= A - Z - 4$

The number of protons

$= Z - 3 \times 2$ (due to alpha particles)-2 (due to positive beta particles)

$= Z - 8$

Required ratio $= \dfrac{A - Z - 4}{Z - 8}$

32. (c) Number of undecayed atoms after time t_2,

$$\frac{N_0}{3} = N_0 e^{-\lambda t_2} \qquad \text{(i)}$$

Number of undecayed atoms after time t_2,

$$\frac{2}{3} N_0 = N_0 e^{-\lambda t_1} \qquad \text{(ii)}$$

Dividing (ii) by (i), we get

$$2 = e\lambda(t_2 - t_1) \text{ or } \ln 2 = \lambda(t_2 - t_1)$$

or $\quad (t_2 - t_1) = \dfrac{\ln 2}{\lambda}$

As per question, $t_{1/2}$ = half-life time = 20 min

$\therefore \quad t_2 - t_1 = 20$ min $\quad \left[\because t_{1/2} = \dfrac{\ln 2}{\lambda}\right]$

33. (None)

Mass defect, $\Delta m = m_p + m_e - m_n$

$= (1.6725 \times 10^{-27} + 9 \times 10^{-31} - 1.6725 \times 10^{-27})$ kg

$= 9 \times 10^{-31}$ kg

Energy released $= \Delta mc^2$

$= 9 \times 10^{-31} \times (3 \times 10^8)^2$ J

$$= \frac{9 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-13}} \text{MeV} = 0.51 \text{ MeV}$$

$\therefore$ None of the given option is correct.

34. (a) $\dfrac{1}{\lambda} = Rz^2\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)$

$\therefore \quad \lambda = \dfrac{4}{3Rz^2}$

$$\lambda_1 = \frac{4}{3R}$$

$$\lambda_2 = \frac{4}{3R}$$

$$\lambda_3 = \frac{4}{12R}$$

$$\lambda_4 = \frac{4}{27R}$$

$\Rightarrow \quad \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

35. (d) Given $t = 80$ min $= 4\,T_A = 2T_B$

$\therefore$ No. of nuclei of A decayed $= N_0 - \dfrac{N_0}{2^4} = \dfrac{15N_0}{16}$

$\therefore$ No. of nuclei of B decayed $= N_0 - \dfrac{N_0}{2^2} = \dfrac{3N_0}{4}$

Hence required ratio (ratio of decayed Nuclei)

$$= \frac{15N_0}{16} / \frac{3N_0}{4} = \frac{5}{4}$$

36. (d) At time t

$$\frac{N_B}{N_A} = 0.3 \quad \Rightarrow N_B = 0.3\,N_A$$

Also let initially there are total N_0 number of nuclei

$$N_A = \frac{N_0}{1.3}$$

Also as we know $N_A = N_0 e^{-\lambda t}$

$$\frac{N_0}{1.3} = N_0 e^{-\lambda t}$$

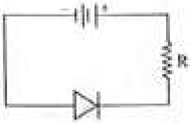
$$\frac{1}{1.3} = e^{-\lambda t} \Rightarrow \ln(1.3) = \lambda \text{ or } t = \frac{\ln(1.3)}{\lambda}$$

$$t = \frac{\ln(1.3)}{\frac{\ln(2)}{T}} = \frac{\ln(1.3)}{\ln(2)} T$$

CHAPTER 31: ELECTRONIC DEVICES

Concept Application Exercise 31.1

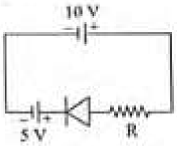
1. The circuit shown in Figure (a) of question can be redrawn as follows:



As the p-section is connected to negative terminal of the battery, the diode has been reverse biased.

By redrawing the circuit diagrams, it can be shown that diode in circuit shown in Figure (b) is reverse biased and in the circuit shown in Figure (c), the diode is forward biased.

2. The circuit shown in Figure (a) can be redrawn as shown in figure.



The battery of 10 V makes p-section at +10 V, while the battery of 5 V makes n-section at +5 V. Therefore, p-section is at net = 5 V w.r.t. n-section.

Hence, the diode in the circuit shown in Figure (a) is forward biased.

Similarly, by redrawing the circuit shown in Figure (b), it can be shown that the p-section is at net +7 V w.r.t n-section.

Hence the diode is forward biased.

For diagram showing reverse bias, refer Figure (a).

3. In the circuit containing two junction diodes D_1 and D_2, the diode D_2 is forward biased and D_1 is reverse biased. Therefore, the path of appreciable current will be from positive pole of the battery to its negative pole through resistance R and junction diode D_2.

4. During forward bias: $\Delta V = 2.4 - 2 = 0.4$ V

And $\Delta I = 80 - 60 = 20$ mA $= 0.02$ A

$$\therefore \quad r = \frac{\Delta V}{\Delta I} = \frac{0.4}{0.02} = 20\Omega$$

During reverse bias: $\Delta V = 0 - (-2) = 2$ V

and $\Delta I = 0 - (-0.25) = 0.25\ \mu A = 0.25 \times 10^{-6}$ A

$$\therefore \quad r = \frac{\Delta V}{\Delta I} = \frac{2}{0.25 \times 10^{-6}} = 18 \times 10^6 \Omega$$

5. A junction diode conducts during alternate half cycles of a.c. input supply. During a half cycle of conduction, the capacitor will charge itself to peak value of the supply voltage. Therefore, voltage across the capacitor,

$$V = E_0 = E_{r.m.s} \times \sqrt{2} = 220 \times \sqrt{2} = 311.1 \text{ V}$$

6. In the circuit shown in figure, the lamp will light. It is because the *n-p-n* transistor in the circuit can conduct only when its base is positive.

Concept Application Exercise 31.2

1. The truth table of the combination of gates shown in figure is produced as below:

A	B	y′	y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

≡

A	B	y
0	0	0
1	0	0
0	1	0
1	1	1

Since it is the truth table of AND gate, the given combination of gates represents AND gate.

2. The truth table of the combination of gates shown in figure is produced as below:

A	B	y_1	y_2	y
0	0	1	1	0
1	0	0	1	0
0	1	1	0	0
1	1	0	0	1

≡

A	B	y
0	0	0
1	0	0
0	1	0
1	1	1

Since it is the truth table of AND gate, the given combination of gates represents AND gate.

3. The truth table of the combination of gates shown in figure is produced as below:

A	B	y_1	y_2	y
0	0	1	1	0
1	0	0	1	0
0	1	1	0	0
1	1	0	0	1

≡

A	B	y
0	0	0
1	0	0
0	1	0
1	1	1

4. The truth table of combination of gates shown in figure is produced as below:

A	B	y′	y_1	y_2	y
0	0	1	1	1	0
1	0	1	0	1	1
0	1	1	1	0	1
1	1	0	1	1	0

≡

A	B	y
0	0	0
1	0	1
0	1	1
1	1	0

Since it is the truth table of XOR gate, the given combination of gates represents XOR gate.

5. In dotted line box *a*: When the two inputs of NAND gate are joined, it acts as NOT gate.

In dotted line box *b*: When outputs of two NOT gates are fed to a NAND gate, the arrangement acts as OR gate.

In dotted line box *c*: When the output of NAND gate is fed to a NOT gate, the arrangement acts as AND gate.

Let y′ be output of logic gate in dotted line box *a* and y″ be output of logic gate in box *b*.

The truth table of the whole arrangement will be as given below:

A	B	y′	y″	y
0	0	1	1	0
1	0	0	0	0
0	1	1	1	1
1	1	0	1	1

≡

A	B	y
0	0	0
1	0	0
0	1	1
1	1	1

EXERCISES

Semiconductors and Diodes

1. **(a)** The energy gap in the case of germanium is 0.74 eV. In the case of silicon, the energy gap is nearly 1 eV.

2. (b) Resistivity decreases with rise in temperature; conductivity increases.

3. (d) $T = \dfrac{30 \times 10^{-3}}{8.62 \times 10^{-5}}\,\text{K} = 348\,\text{K}$

4. (c) $n_e h_h = n^2$ or $h_e = \dfrac{n^2}{n_h} = \dfrac{10^{16} \times 10^{16}}{4.5 \times 10^{22}} = \dfrac{10^{32}}{4.5 \times 10^{22}}\,\text{m}^{-3}$

5. (c) When p-n junction is reverse biased, there is practically no flow of current.

6. (c) Net voltage = 1 V

$$R = \frac{1}{1 \times 10^{-3}} = 10^3\ \text{V}$$

7. (a) The barrier potential depends upon temperature, forward-biased and doping density.

8. (b) Reverse bias opposes the movement of majority charge carriers.

9. (a) There is a considerable change in the electrical properties due to the addition of impurity.

10. (b) Not that if p-n junction diode is made from germanium or silicon, the energy released due to recombination of free electrons and holes is in the infrared region. However, if the p-n junction diode is made from gallium arsenide or indium phosphide, then the energy released due to recombination of free electrons and holes is in the visible region.

11. (c) A junction diode conducts during alternate half-cycles of ac input supply. During the half-cycle of conduction, the capacitor will charge itself to peak value of supply voltage.

$\therefore$ Voltage across capacitor

$$= E_{\text{rms}}\sqrt{2} = 200 \times \sqrt{2}\ \text{V} = 282.8\ \text{V} \approx 283\ \text{V}$$

12. (b) Diode D_2 is reverse biased. Current through 100 W resistance

$$= \frac{6}{(\text{Resistance of } D_1) + 100 + 150}$$

$$= \frac{6}{50 + 100 + 150} = 0.02\ \text{A}$$

Given $X = 0$, $Y = 1$ the output state of each of the three gates is thus 0 for the NOR gate, 0 for the AND gate and 1 for the NAND gate, respectively.

13. (c) In intrinsic semiconductors, the creation or liberation of one free electron by the thermal energy has created one hole. Thus, in intrinsic semiconductors, $n_e = n_h$.

14. (d) In semiconductors, the forbidden energy gap between the valence band and conduction band is very small, almost equal to kT. Moreover, valence band is completely filled whereas conduction band is empty.

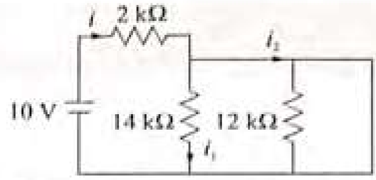
15. (b) Because in case (1), N is connected with N. This is not a series combination of transistor.

16. (b) In forward biasing, P-side is connected with positive terminal and N-side with negative terminal of the battery.

17. (b) In Figures 2, 4 and 5, P-crystals are more positive as compared to N-crystals.

18. (a) The diode is in reverse biasing. So current through it is zero.

19. (d) The equivalent circuit can be redrawn as follows:



From figure, it is clear that current drawn from the battery

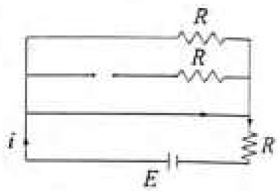
$i = i_2 = \dfrac{10}{2} = 5\text{mA}$ and $i_1 = 0$

20. (c) According to the given figure, A is at lower potential w.r.t. B. Hence, both diodes are in reverse biasing. So equivalent circuit can be redrawn as follows:

$\Rightarrow$ Equivalent resistance between A and B

$R = 8 + 2 + 6 = 16\ \Omega$

21. (a) Diodes D_1 and D_3 are forward biased and D_2 is reverse biased. So the circuit can be redrawn as follows:



$\Rightarrow \quad i = \dfrac{E}{R}$

Junction Transistor

22. (b) Number of holes in base region increases. Hence, recombination of electron and hole also increases in this region. As a result, base current increases which in turn decreases the collector current.

23. (d) Given $i_c = \dfrac{80}{100} \times i_e \Rightarrow 24 = \dfrac{80}{100} \times i_e \Rightarrow i_e = 30$ mA

By using $i_e = i_b + i_c \Rightarrow i_b = 30 - 24 = 6$ mA

24. (a) Use $V_0 = AV_s$

Now $A = \dfrac{24 \times 10k}{10k + 10k} = \dfrac{24 \times 10}{20} = 12$

Therefore, $V_0 = 12 \times 0.4$ volt(r.m.s) = 4.8 V

25. (c) $\alpha = \dfrac{i_c}{i_e} = 0.96$ and $i_e = 7.2$ mA

$\Rightarrow \quad i_c = 0.96 \times i_e = 0.96 \times 7.2 = 6.91$ mA

$\therefore \quad i_e = i_c + i_b \Rightarrow 7.2 = 6.91 + i_b \Rightarrow i_b = 0.29$ mA

26. (c) $\alpha = 0.8$

$\Rightarrow \quad \beta = \dfrac{0.8}{(1 - 0.8)} = 4$

Also $\quad \beta = \dfrac{\Delta i_c}{\Delta i_b}$

$\Rightarrow \quad \Delta i_c = \beta \times \Delta i_b = 4 \times 6 = 24$ mA

27. (a) $\Delta i_c = \alpha \Delta i_e = 0.98 \times 2 = 1.96$ mA

$\therefore \quad \Delta i_b = \Delta i_e - \Delta i_c = 2 - 1.96 = 0.04$ mA

28. (b) $i_e = i_b + i_c \Rightarrow i_c = i_e - i_b$

29. (b) $V_b = i_b R_b \Rightarrow R_b = \frac{9}{35\times10^{-6}} = 257\ \text{k}\Omega$

30. (a) Theory based.

31. (b) $i_e = i_b + i_c \Rightarrow \frac{i_e}{i_c} = \frac{i_b}{i_c} + 1 \Rightarrow \frac{1}{\alpha} = \frac{1}{\beta} + 1 \Rightarrow \alpha = \frac{\beta}{(1+\beta)}$.

32. (c) The figure is for *p-n-p*, not *n-p-n*. So, that identifies the one incorrect statement. Rest is not needed.

33. (d) $\frac{\mu R_L}{R_L + R_i} = \frac{98\times1\times10^6}{1\times10^6\times600} = 98$

34. (c) $\beta \frac{\alpha}{1-\alpha} = \frac{0.995}{1-0.995} = \frac{0.995}{0.005} = 199 \approx 200$

35. (b) $\beta = \frac{\Delta I_c}{\Delta I_b} = \frac{2\text{ mA}}{20\ \mu\text{A}} = \frac{2\times10^{-3}}{20\times10^{-6}} = 100.$

36. (a) Voltage gain $= \frac{2\text{ mA}\times5\text{ k}\Omega}{20\text{ mV}} = \frac{1\times5\times10^3}{10} = 500$

Resistance gain $= \frac{\text{Voltage gain}}{\text{Current gain}} = \frac{500}{100} = 5$

or Input resistance $= \frac{\text{Output resistance}}{5} = \frac{5\text{ k}\Omega}{5} = 1\text{ k}\Omega$

37. (a) Transconductance, gm

$$= \frac{\Delta I_C}{\Delta V_{BE}} = \frac{\Delta I_C}{\Delta V_{BE}} \times \frac{\Delta I_B}{\Delta I_B} = \frac{\Delta I_C}{\Delta V_B} \times \frac{\Delta I_B}{\Delta I_B}$$

$$= \beta \times \frac{1}{R_{in}} = \frac{100}{1\times10^3} = \frac{1}{10}\text{mho} = 0.1\text{ mho}$$

38. (b) Input resistance $= \frac{0.02}{20\times10^{-6}} = \frac{2\times10^4}{20}\ \Omega = 10^3\ \Omega$

39. (a) Again, current gain $= \frac{2\times10^{-3}}{20\times10^{-3}} = \frac{2\times10^{-3}}{2\times10^{-5}} = 100$

40. (c) Now, transconductance $= \frac{\beta_{aC}}{R_{in}} = \frac{100}{1\times10^3} = 0.1\ \Omega^{-1}$

41. (b) Voltage gain $= g_m\, g_L = 0.1\times5\times10^3 = 500$

42. (c) Change in output across load $= 500\times0.22$ V $= 10$ V.

$$V = \frac{P}{I} = \frac{P}{ne} = \frac{448\times10^{-3}}{14\times10^{15}\times1.6\times10^{-19}}\text{V} = 200\text{ V}$$

43. (c) $R_1 = \frac{9}{30\times10^{-6}} = 3\times10^5 = 300\text{ k}\Omega$

44. (c) $V_{BE} = V_{CC} - I_B R_B = 5.5 - 10\times10^{-6}\times5\times10^5 = 0.5$ V

Digital Electronics

45. (b) It is the symbol of 'NOR' gate.

46. (a) The given symbol is of NAND gate.

47. (d) "Single input" not acceptable.

48. (d) This is a truth table for OR logic gate because for OR gate, $X = A + B$.

49. (c) This a truth table for NAND gate because $X = \overline{A.B}$.

50. (b) To form AND gate, 2 NAND gates are use

51. (a) $(100010)_2 = 2^5\times1 + 2^4\times0 + 2^3\times0 + 2^2\times0 + 2^1\times1 + 2^0\times0$
$= 32 + 0 + 0 + 0 + 2 + 0 = (34)_{10}$

and $(11011)_2 = 2^4\times1 + 2^3\times1 + 2^2\times0 + 2^1\times1 + 2^0\times1$
$= 16 + 8 + 0 + 2 + 1 = (27)_{10}$

$\therefore$ Sum $(100010)_2 + (11011)_2 = (34)_{10} + (27)_{10} = (61)_{10}$

Now

2	61	Remainder
2	30	1 LSD
2	15	0
2	7	1
2	3	1
2	1	1
	0	1 MSD

$\therefore$ Required sum (in binary system)
$(100010)_2 + (11011)_2 = (111101)_2$

52. (d) (a) $\overline{\bar{A}\,.\,\bar{B}} = \bar{\bar{A}} + \bar{\bar{B}} = A + B$

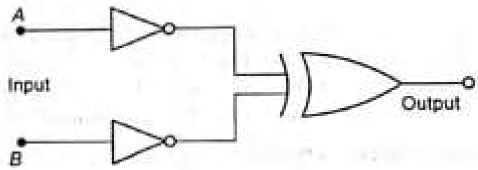
(b) $\overline{\bar{A} + \bar{B}} = \bar{\bar{A}}\,.\,\bar{\bar{B}} = A\,.\,B$

(c) $\overline{(\overline{A+B})\cdot(\overline{A\,.\,B})} = A\,.\,B + A\,.\,B = A\,.\,B + A\,.\,B$

(d) $0 + 0 = 0$

Clearly, d. is incorrect.

53. (c)



A	B	C	D	X
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	0	0	1	1
1	1	0	0	0

The combination of logic gates can be replaced by a single EX-OR gate.

54. (d) The truth table of the resulting logic circuit by connecting X to Y is as follows:

P	Q	R	R
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

Hence, from the truth table, the combination is equivalent to a single NAND gate.

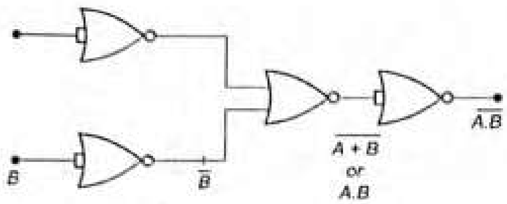
(OR X is an AND gate and Y is a NOT gate, thus the combination is NOT AND gate, i.e., a NAND gate.)

55. (c) $A + B = 1$

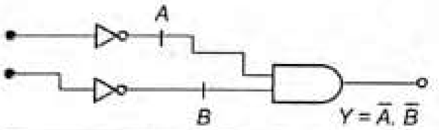
$C = 1$

So, AND gate responds.

56. (d)



57. (d)

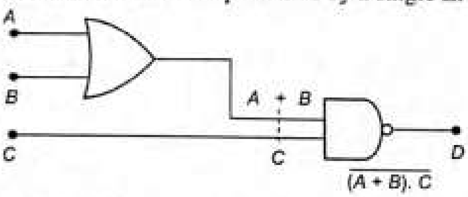


58. (c)

A	B	Output
0	0	0
1	0	0
0	1	1
1	1	0

The network can be represented by a single EX-OR gate.

59. (b)



60. (d) To get a half-adder, we require one AND gate and one XOR (also called EOR) gate. For 1 AND gate, 2 NAND gates are required. For 1 XOR gate, 5 NAND gates are required. So, a total of 7 NAND gates are required for making a half-adder.

61. (a) A half-adder has the following true table.

Inputs		Outputs	
P	Q	Sum	Carry
0	0	0	1
1	0	1	1
0	1	1	0
0	0	0	1

From the table, sum output can be implanted using an EXOR gate while carry output can be realized using an AND gate.

62. (d) The output of both AND gates will be zero. So, OR gate does not respond.

63. (c) The following arguments shall help us to arrive at the right choice.

(i) AND gate shall not respond to zero input.

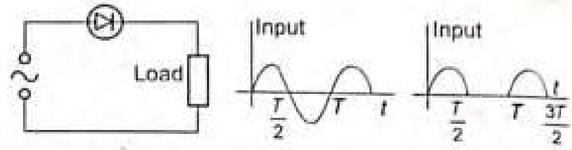
(ii) As R is zero and S is 1, NOR gate gives zero; NOT gives 1 corresponding to 0.

(iii) NOR gate would not respond to 0 and 1.

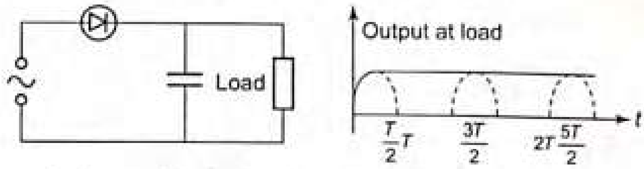
Problems Based on Mixed Concepts

64. (d) The purpose of negative feedback resistor in an operating amplifier (OP-AMP) is to increase the frequency bandwidth of the OP-AMP but at the expense of its gain, i.e., at a lower constant gain.

65. (d) Only circuit d gives the correct connection of the capacitor to the half-wave rectifier to provide smoothening of the half-wave rectified waveform. Without the capacitor, the half-wave rectifier is connected as shown. The output waveform is attached net to it.

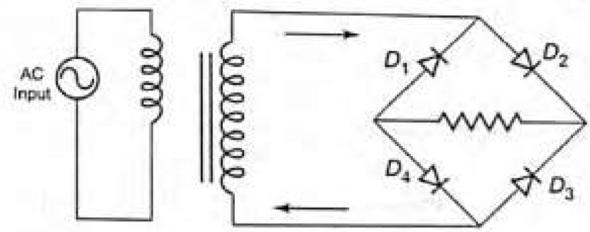


To smooth out the waveform, the capacitor has to be connected parallel to the load as shown below.



During positive cycle, the capacitor is charged to the peak value of the applied ac voltage and then is discharged through R when the applied voltage begins to decrease. The discharge takes place slowly because of the large time-constant. RC and then C become charged to the peak value of the voltage again. The charge–discharge process takes place continuously and so the voltage across the C–R combination is a ripple voltage as shown diagrammatically above. With the C–R arrangement, the average dc voltage is nearly equal to the peak voltage value, which is better, and so the voltage variation.

66. (c) The given question is based on Bridge Rectifier. This is the most widely used full-wave rectifier. It makes use of four diodes D_1, D_2, D_3, D_4 connected in the four arms of a bridge.



Bridge rectifier does not require a centre-tapped transformer. It has several advantages over full-wave rectifier.

67. (c) In common-emitter mode, the transistor is current amplifier

$$\Delta I_B = \frac{10\times10^{-3}}{2000} = 5\times10^{-6}\text{ A}$$

Asign, $\beta = \dfrac{\Delta I_C}{\Delta I_B}$

or $\quad \Delta I_C = \beta\times\Delta I_B$

$$= 50\times5\times10^{-6}\text{ A} = 250\times10^{-6}\text{ A}$$

$$= 2.5\times10^{-4}\text{ A}$$

Peak value of output voltage

$$= 2.5\times10^{-4}\times5000\text{ V} = 1.25\text{ V}$$

68. (a)

$$\text{Voltage gain} = \frac{\text{Output voltage}}{\text{Input voltage}} = \frac{1.25}{10\times10^{-3}} = 125$$

Power gain = voltage gain × current gain = 125 × 50

69. (d) The truth table of the NAND gate is as shown below:

S_1	S_2	Output
0	0	1
0	1	1
1	0	1
1	1	0

where a logic level 0 at the input, S_1 or S_2, corresponds to a closed switch (at 0 V) and a logic level 0 at the output means the lamp will be lit. From the truth table, both switches S_1 and S_2 have to be at open position in order to light up the lamp at the output of the NAND gate.

70. (c) The current must only pass through the meter in the direction +1 to −1. The diodes in a. and b. will not stop the reverse current, so can be eliminated. Remember the reverse diode conducts in the direction in which the triangle of the symbol is pointing.

71. (b) Note that drift velocity in semiconductors is affected by an increase in temperature in the same way as in metals, i.e., reduced due to an increase in the number of collisions with the lattice ions, which vibrate with greater amplitude at a higher temperature.

72. (b) When a diode is reverse-biased, it does not conduct. So, if the resistor and diode are in series, then the current should be zero in one of the two given cases. But this is not the case. So, clearly, the two are connected in parallel. Clearly, $I = -25$ mA corresponds to reverse-biasing.

Now, $R = \dfrac{1\text{ V}}{25\times10^{-3}\text{ A}} = \dfrac{1000}{25}\,\Omega = 40\,\Omega$

Again, $I = 50$ mA

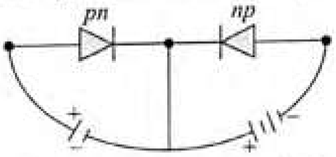
Now, current shall flow through the diode also because diode is forward-biased.

If R_p is the combined resistance of diode and resistor, then

$$R_p = \frac{1}{50\times10^{-3}}\,\Omega = \frac{1000}{50}\,\Omega = 20\,\Omega$$

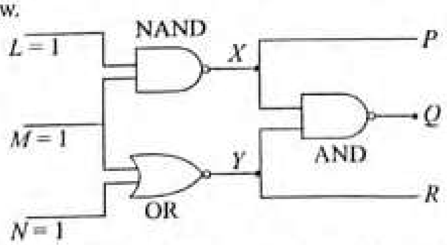
Clearly, it is a parallel combination of 40 Ω and 40 Ω.

73. (a) We put biasing batteries to clear the figure.



So, (i) is a p-n-p type and (ii) is an n-p-n type configuration.

74. (b) The outputs X, Y, P, Q and R are summarized in the table below.



75. (b) Since the operational amplifier is assumed to be ideal, it (Q) does not sink nor source current.

By Kirchhoff's current law, $I_f = I$.

76. (b) The circuit configuration is a non-inverting amplifier. The gain of the amplifier is given by the equation

$$\text{Gain} = \frac{V_0}{V_i} = 1 + \frac{R_2}{R_1} = 1 + \frac{10}{2} = 6.$$

Thus, the output voltage V_0 is

$$V_0 = 6V_i = 6(0.20) = 1.2\text{ V}$$

Archives

1. (b) Resistance of conductor (Cu) decreases with decrease in temperature while that of semiconductor (Ge) increases with decrease in temperature.

2. (b) $E = \dfrac{12375}{\lambda(\text{in Å})}\text{ eV} = \dfrac{12375}{24800}\text{ eV} = 0.5\text{ eV}$

3. (b) In full wave rectifier, the fundamental frequency in ripple is twice that of input frequency.

4. (b) In CB amplifier input and output voltage signal are in the same phase.

5. (d) It is covalent binding.

6. (d) $v_d = \dfrac{I}{neA}$

$$\frac{v_e}{v_h} = \frac{I_e}{I_h}\times\frac{n_h}{n_e} = \frac{7}{5}\times\frac{5}{7} = \frac{5}{4}$$

7. (b) $\alpha = \dfrac{I_c}{I_e} = \dfrac{5.488}{5.60} = 0.98$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49$$

8. (d) The value of E_c and E_v will decrease but the value of E_g will increase.

9. (a) In the first figure, p-type is at a lower potential than that of n-type. So the diode is reverse biased.

10. (b) C, Si, and Ge have equal number of free electrons as Z for them are 6, 14, and 32, respectively.

11. (a) As the diode will conduct only at + 5 V .

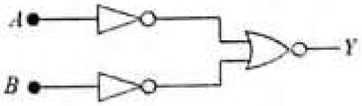
12. (d) It is pnp transistor with R as collector.

13. (b) It is OR gate. When either of them conducts, the gate conducts.

14. (c) The given figure is a half-wave rectifier.

15. (a) Truth Table

A	B	Y
1	1	1
1	0	0
0	1	0
0	0	0



16. (a) $\psi = \overline{\left(\overline{A.\overline{A.B}}\right).\left(\overline{B.\overline{A.B}}\right)}$

$$= \left(\overline{\overline{A.\overline{A.B}}}\right)\left(\overline{\overline{B.\overline{A.B}}}\right)$$

$$= A.\left(\overline{A} + \overline{B}\right) + B.\left(\overline{A} + \overline{B}\right)$$

$= A.\bar{A} + A.\bar{B} + B.\bar{A} + B.\bar{B}$

$Y = 0 + A.\bar{B} + B.\bar{A} + 0$

Alternative method

The truth table of the given circuit is as shown in the table

A	B	y'	y_1	y_2	y
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0

17. (c) $f_C = \dfrac{1}{2\pi RC} = \dfrac{2}{2\times 3.14\times 100\times 10^3\times 250\times 10^{-12}}$

$= 6.37$ kHz

f_C = cut off frequency

As we know that $f_m = f_C$

$\therefore$ (c) is correct.

NOTE: The maximum frequency of modulation must be less than f_m, where

$$f_m = f_C\frac{\sqrt{1-\mu^2}}{\mu}$$

$\mu \Rightarrow$ modulation index

18. (d) For LED, in forward bias, intensity increases with voltage.

19. (c) $5 = e^{1000\frac{V}{T}} - 1$

$\Rightarrow \quad e^{1000\frac{V}{T}} = 6$ ...(i)

Again, $I = e^{1000\frac{V}{T}} - 1$

$$\frac{dI}{dV} = e^{\frac{1000}{T}V}\frac{1000}{T}$$

$$dI = \frac{1000}{T}e^{\frac{1000}{T}V}dV$$

Using (i) $\Delta I = \dfrac{1000}{T}\times 6\times 0.01 = \dfrac{60}{T} = \dfrac{60}{300} = 0.2\,\text{mA}$

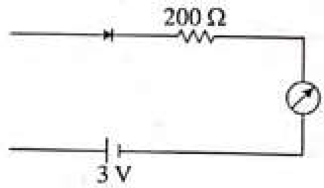
20. (c) 

For forward bias, p-side must be at higher potential than n-side.

21. (b) In common emitter amplifier circuit input and output voltage are out of phase. When input voltage is increased then i_b is increased, i_c also increases so voltage drop across R_c is increased. However, increase in voltage across R_c is in opposite sense.

22. (d) Silicon diode is in forward bias.

Hence across diode potential barrier



$\Delta V = 0.7$ volts

$$I = \frac{V - \Delta V}{R} = \frac{3 - 0.7}{200} = \frac{2.3}{200} = 11.5\text{ mA}$$

CHAPTER 32: COMMUNICATION SYSTEMS

EXERCISES

1. (a) By using $f_c = 9(N_{max})^{1/2} \Rightarrow f_c = 2$ MHz
2. (a) A maximum frequency deviation of 75 kHz is permitted for commercial FM broadcast stations in the 88 to 108 MHz VHF band.
3. (b) Velocity factor $= \frac{1}{\sqrt{k}} = \frac{1}{\sqrt{2.6}} = 0.62$
4. (d) Here $\frac{n_1 - n_2}{n_1} = \frac{0.88}{100} \Rightarrow \frac{n_2}{n_1} = 0.9912$

   $\therefore$ Critical angle,

   $$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}(0.9912) = 84°24'$$
5. (b) The energy flux, $\phi = \frac{\text{Pulse power}}{\text{Area}} = \frac{10^{12}}{10^{-4}} = 10^{16}\,\frac{\text{W}}{\text{cm}^2}$
6. (c) $v = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\sqrt{10\times10^{-6}\times1\times10^{-9}}}$ = 1592 kHz
7. (a) VHF (very high frequency) band having frequency range 30 MHz to 300 MHz is typically used for TV and radar transmission.
8. (c) $\text{MUF} = \frac{f_c}{\cos\theta} = \frac{60}{\cos 70°} = 175\,\text{MHz}$
9. (d) Laser beams are perfectly parallel. So that they are very narrow and can travel a long distance without spreading. This is the feature of laser while they are monochromatic and coherent these are characteristics only.
10. (b) The formula for modulating index is given by

    $$m_f = \frac{\delta}{v_m} = \frac{\text{Frequency variation}}{\text{Modulating frequency}} = \frac{10\times10^3}{2\times10^3} = 5$$
11. (d) Here, $V_{max} = \frac{24}{2} = 12\,\text{mV}$ and $V_{min} = \frac{8}{2} = 2\,\text{mV}$

    Now, $m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{12-4}{12+4} = \frac{8}{16} = \frac{1}{2} = 0.5 = 50\%$
12. (a) Here, $f_c = 1.5$ MHz = 1500 kHz, $f_m = 10$ kHz

    $\therefore$ Low side band frequency

    $= f_c - f_m$ = 1500 kHz – 10 kHz = 1490 kHz

    Upper side band frequency

    $= f_c + f_m$ = 1500 kHz + 10 kHz = 1510 kHz
13. (a) Frequency modulation requires much wider channel (7 to 15 times) as compared to AM.
14. (b) $P_t = P_c\left(1 + \frac{m_a^2}{2}\right)$; Here $m_a = 1$

    $\Rightarrow \quad 1800 = P_c\left(1 + \frac{(1)^2}{2}\right) \Rightarrow P_c = 1200$ W
15. (c) An antenna is a metallic structure used to radiate or receive EM waves.
16. (a) Carrier swing $= \frac{\text{Frequency deviation}}{\text{Modulating frequency}} = \frac{50}{7} = 7.143$
17. (c) In optical fibre, light travels inside it, due to total internal reflection.
18. (d) Few advantages of optical fibres are that the number of signals carried by optical fibres is much more than that carried by the Cu wire or radio waves. Optical fibres are practically free from electromagnetic interference and problem of cross talks whereas ordinary cables and microwave links suffer a lot from it.
19. (d) The process of changing the frequency of a carrier wave (modulated wave) in accordance with the audio frequency signal (modulating wave) is known as frequency modulation (FM).
20. (d) Following are the problems which are faced while transmitting audio signals directly.
    (i) These signals are relatively of short range.
    (ii) If everybody started transmitting these low frequency signals directly, mutual interference will render all of them ineffective.
    (iii) Size of antenna required for their efficient radiation would be larger i.e. about 75 km.
21. (d) Remote sensing is the technique to collect information about an object in respect of its size, colour, nature, location, temperature etc. without physically touching it. There are some areas or location which are inaccessible. So to explore these areas or locations, a technique known as remote sensing is used. Remote sensing is done through a satellite.
22. (a) The critical frequency of a sky wave for reflection from a layer of atmosphere is given by $f_c = 9(N_{max})^{1/2}$

    $\Rightarrow \quad 10\times10^6 = 9(N_{max})^{1/2}$

    $\Rightarrow \quad N_{max} = \left(\frac{10\times10^6}{9}\right)^2 = 1.2\times10^{12}\,\text{m}^{-3}$
23. (b) Core of acceptance angle, $\theta = \sin^{-1}\sqrt{n_1^2 - n_2^2}$
24. (c) $m_a = \frac{E_m}{E_c} = \frac{15}{60}\times100 = 25\%$
25. (a) Using $Z = \sqrt{\frac{L}{C}}$, we get $Z = \sqrt{\frac{0.40\times10^{-6}}{10^{-11}}} = 2\times10^2\,\Omega$
26. (b) Optical source frequency, $f = \frac{c}{\lambda} = \frac{3\times10^8}{1.3\times10^{-6}} = 2.3\times10^{14}$ Hz

    $\therefore$ Number of channels or subscribers $= \frac{2.3\times10^{14}}{20\times10^3}$

    $= 1.15\times10^{10}$

27. (a) Carrier swing $= \dfrac{\text{Frequency deviation}}{\text{Modulating frequency}} = \dfrac{50}{7} = 7.143$

28. (c) $f_c \propto (N)^{1/2}$

$\Rightarrow \quad (f_c)_E : (f_c)_{F1} : (f_c)_{F2}$

$= (2 \times 10^{11})^{1/2} : (5 \times 10^{11})^{1/2} : (8 \times 10^{11})^{1/2}$

$= 2 : 3 : 4$

29. (a) Attenuation $= 10 \log \dfrac{120}{4}$

$= 10 \log 30 = 10 \times 1.4771 = 14.77$ dB.

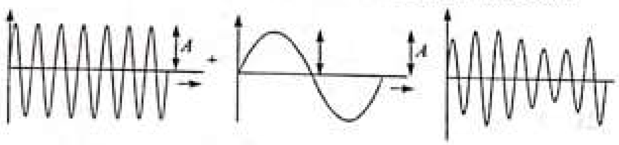
30. (a) $I_{\text{Carrier}} = \dfrac{I_{\text{rms}}}{\sqrt{1 + \dfrac{m_a^2}{2}}} = \dfrac{11}{\sqrt{1 + \dfrac{(0.5)^2}{2}}} = 10.35\,\text{A}$

31. (b) $P_c = \dfrac{P}{\left(1 + \dfrac{m_a^2}{2}\right)} = \dfrac{10000}{\left(1 + \dfrac{(0.5)^2}{2}\right)} = \dfrac{10000}{1.125} = 8.89\,\text{kW}$

32. (a) 1% of 10 GHz $= 10 \times 10^9 \times \dfrac{1}{100} = 10^8\,\text{Hz}$

Number of channels $= \dfrac{10^8}{5 \times 10^3} = 2 \times 10^4$

33. (a) When signal amplitude is equal to the carrier amplitude, the amplitude of carrier wave varies between 2A and zero.



$m_a = \dfrac{\text{Amplitude charge of carrier}}{\text{Amplitude of normal carrier}} = \dfrac{2A - A}{A} \times 100 = 100\%$

Archives

1. (c) Optical fibres are not subjected to electromagnetic interference from outside.

2. (d) Theory based.

3. (a) $r = R + h \equiv R$

$x = \sqrt{(R+h)^2 - R^2}$

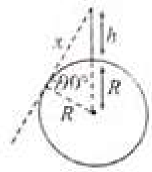
$= \sqrt{h^2 + 2hR}$

$x^2 = 250000 + 2 \times 500 \times 6.4 \times 100000$

$= 250000 + (64 \times 100000000)$

$= 10^4\,(640025)$

$\Rightarrow \quad x = 80$ km



4. (c) Modulated wave has frequency range.

$\omega_c \pm \omega_m$

$\therefore \quad \omega_c >> \omega_m$

$\therefore \quad \omega_m$ is excluded.

5. (d) As the carrier frequency is distributed as band width frequency, so available bandwidth

$= 10\%$ of 10 GHz

$= \dfrac{10}{100} \times 10 \times 10^9 = 10^9\,\text{Hz}$

Bandwidth for each telephonic channel = 5 kHz

Number of channels $n = \dfrac{10^9}{5 \times 10^3}$

$n = 2 \times 10^5$ telephonic channels

CHAPTER 33: EXPERIMENTAL SKILLS

ARCHIVES

1. (b) The length of the arms of the tuning fork will increase.

2. (a) We have from the meter bridge experiment,

$$\frac{R_1}{R_2}=\frac{l_1}{l_2}\text{ where } l_2=(100-l_1)\text{ cm}$$

In the first case,

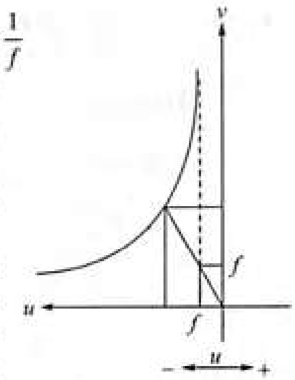
$$\frac{X}{Y}=\frac{20}{80}$$

In the second case,

$$\frac{4X}{Y}=\frac{l}{100-l}$$

$$l=50\text{ cm}$$

3. (a) We have lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$



According to the new cartesian system, for a convex lens, u has to be negative.

If $v=\infty$, $u=f$ and if $u=\infty$, $v=f$

A parallel beam ($u=\infty$) is focused at f and if the object is at f, the rays are parallel. The point which meets the curve at $u=v$ gives $2f$. Therefore v is +ve, u is negative, both are symmetrical and this curve satisfies all the conditions for a convex lens.

4. (c) A travelling microscope moves horizontally on a main scale provided with a vernier scale, provided with the microscope.

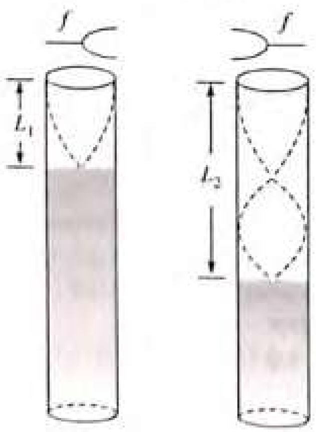
5. (d) For a balanced meter bridge (null deflection),

$$\frac{55}{R}=\frac{20}{80}$$

$$\Rightarrow\quad R=220\ \Omega$$

6. (d) The speed of the sound in air can be written as $v_1=\sqrt{\frac{\gamma RT}{M}}$

Here we are assuming M is the average molar mass of the air and g is the adiabatic constant of air.



1st Resonance 2nd Resonance

Hence we can write, $v_1=\sqrt{\frac{\gamma RT_1}{M}}$; $v_2=\sqrt{\frac{\gamma RT_2}{M}}$ where T_1 and T_2 stand for winter and summer temperatures.

For 1st resonance: $L_1=\frac{\lambda}{4}=18$ cm.

$$f=\frac{v_1}{\lambda}=\frac{v_1}{4L_1}$$

At temperature T_1

$$L_1=\frac{v_1}{4f}=18\text{cm}\Rightarrow v_1=72f\text{ cm/s}$$

For 2nd resonance: $L_2=\frac{3\lambda}{4}$

$$f=\frac{v_2}{\lambda}=\frac{3v_2}{4L_2}$$

At temperature T_2

$$v_2=\frac{4fL_2}{3}$$

At T_2, in summers, $v_2>v_1$.

i.e., $72f<\frac{4fx}{3}\Rightarrow x>54$ cm $\quad [L_2=x]$

7. (a) The least count of the screw gauge is

$$\frac{0.5\text{ mm}}{50}=0.01\text{ mm;}$$

The main scale reading is 3 mm.

The vernier scale reading is 35.

Observed reading = 3 + 0.35 = 3.35

Zero error = – 0.03

Therefore, the actual diameter of the wire is 3.35 – (–0.03) = 3.38 mm

8. (a) Least count of the instrument

$$\text{L.C.}=\frac{\text{value of main scale division}}{\text{no. of divisions on vernier scale}}$$

Here n vernier scale divisions = $(n-1)$ M.S.D.

$$\therefore\quad 1\text{ V.S.D.}=\frac{n-1}{n}\text{M.S.D}$$

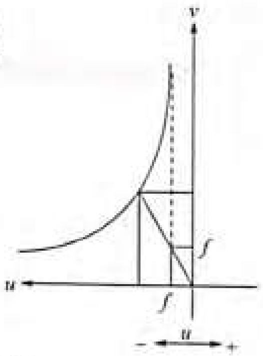
$$\text{L.C.}=1\text{ M.S.D.}-1\text{ V.S.D}$$

$$=1\text{M.S.D}-\frac{(n-1)}{n}\text{M.S.D.}$$

$$\Rightarrow\quad \text{L.C.}=0.5-\frac{29}{30}\times0.5^\circ$$

$$\Rightarrow\quad \text{L.C.}=\frac{0.5}{30}=\frac{1}{30}\times\frac{1}{2}=\left(\frac{1}{60}\right)^\circ=1\text{min.}$$

9. (a) We have lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$



According to the new cartesian system, for a convex lens, u has to be negative.

One has to take that u is negative again for calculation,

it effectively comes to

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

If u = radius of curvature, $2f$, $v = 2f$

i.e., $\frac{1}{2f}+\frac{1}{2f}=\frac{1}{f}$.

v and u have the same value when the object is at the centre of curvature. The solution is (a).

According the real and virtual system, u is +ve and v is also +ve as both are real. If $u = v$, $u = 2f$ = radius of curvature.

$\therefore \quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{2f}+\frac{1}{2f}=\frac{1}{f}$.

The answer is the same (a).

(The figure given is according to New Cartesian system).

10. (b) Least count of screw gauge

$$=\frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

$$=\frac{1}{100}\text{mm} = 0.01 \text{ mm}$$

Diameter of wire = main scale reading

+ circular scale reading × Least count

= 0 + 52 × 0.01 = 0.52 mm 0.052 cm

11. (a) $R=\frac{V}{I} \Rightarrow \pm\frac{\Delta R}{R}=\pm\frac{\Delta V}{V}\pm\frac{\Delta I}{I}=3+3=6\%$

12. (c) 30 V.S.D. → 29 M.S.D.

1 V.S.D. → $\frac{29}{30}$ M.S.D.

$=\frac{29}{30}\times 0.5$

Least count of vernier = 1 M.S.D. – 1 V.S.D.

$$=0.5^\circ-\frac{29}{30}\times 0.5^\circ$$

$$=\frac{0.5^\circ}{30}$$

Reading of vernier = M.S. reading + V.S reading × Least count

$$=58.5^\circ+9\times\frac{0.5^\circ}{30}=58.65^\circ$$

13. (d) As measured value is 3.50 cm, the least count must be 0.01 cm = 0.1 mm

For vernier scale with 1 MSD = 1 mm and 9 MSD = 10 VSD,

Least count = 1 MSD – 1 VSD = 0.1 mm

14. (b) $g=4\pi^2.\frac{l}{T^2}$

$$\Rightarrow \quad \frac{\Delta g}{g}\times 100=\frac{\Delta l}{l}\times 100+2\frac{\Delta T}{T}\times 100$$

$$=\frac{\Delta l}{l}\times 100+2.\frac{\Delta t}{t}\times 100$$

$$=\frac{0.1}{20.0}\times 100+2\times\frac{1}{90}\times 100$$

$$=\frac{100}{200}+\frac{200}{90}=\frac{1}{2}+\frac{20}{9}\cong 3\%$$

15. (a) Measured time period of 100 oscillations are 90 sec, 91 sec, 95 sec and 92 sec.

Mean value of time = $t_m=\frac{90+91+95+92}{4}=92$ sec

Absolute error in measurement

$$|\Delta t_1|=|t_m-t_1|=2\text{ sec}$$

$$|\Delta t_2|=|t_m-t_2|=1\text{ sec}$$

$$|\Delta t_3|=|t_m-t_3|=3\text{ sec}$$

$$|\Delta t_4|=|t_m-t_4|=0\text{ sec}$$

Mean absolute error $\Delta t_{\text{mean}}=\frac{2+1+3+0}{4}=1.5$ sec

But the least count of the measuring clock is 1 sec, so it cannot measure up to 0.5 second, so we have to round it off. So mean error will be 2 second.

Hence mean time $(92\pm 2\text{ s})$.

16. (b) Least count LC = $\frac{\text{pitch}}{\text{no. of div. on circular scale}}$

$$\Rightarrow \quad LC=\frac{0.5\text{mm}}{50}=0.01\text{mm}$$

When jaws are closed, the zero error will be

= main scale reading + (circular scale reading) (least count)

= –0.5 mm + (45)(0.01)

Hence zero error e = –0.05 mm.

When the sheet is placed between the jaws:

measured thickness = 0.5 mm + (25)(0.01) = 0.75 mm

Hence actual thickness or true reading = observed reading + zero error

= 0.75 mm – (–0.05) = 0.80 mm

Thanks to Raj Chauhan ✨

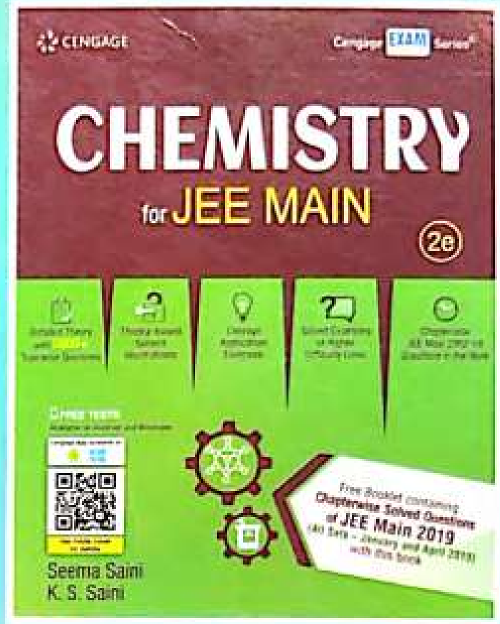
PHYSICS for JEE Main 3e

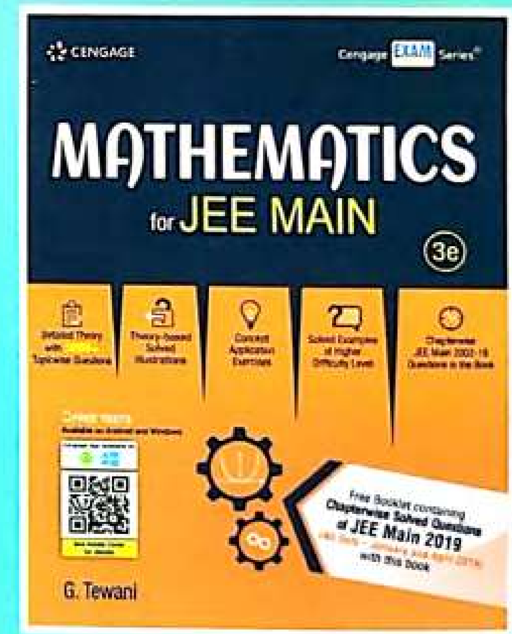
Physics for JEE Main, 3e, a Cengage Exam Crack Series® title, has been designed in keeping with the needs and expectations of students appearing for JEE Main. Its coherent presentation and compatibility with the latest prescribed syllabus and pattern of JEE will prove extremely useful to JEE aspirants.

KEY FEATURES

- Conforms to the pattern and syllabus prescribed for JEE Main
- Comprises revised content with more refinement in theory and question bank
- Enhances understanding of the concepts with a large number of in-text illustrations and concept application exercises
- Helps in honing problem-solving skills to solve trickier problems with a vast bank of multiple concepts based solved problems
- Includes topic-wise, end-of-chapter exercises that contain exam level multiple choice questions requiring special attention
- Includes past years' questions updated up to solved JEE Main 2018 paper that help students understand the trend(s) behind JEE
- Free booklet containing chapterwise solved questions of JEE Main 2019 (All sets - January and April 2019) with this book
- Includes access to FREE TESTS on Cengage App (Android/Windows)

OTHER TITLES IN THIS SERIES





CENGAGE
For product information, visit www.cengage.co.in

₹985.00
ISBN 978-93-5350-132-7
ISBN 93-5350-132-6